A Generalized Control Function Approach to Production Function Estimation*

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Abstract

We develop a generalized control function approach to production function estimation. Our approach accommodates settings in which productivity evolves jointly with other unobservable factors such as latent demand shocks and the invertibility assumption underpinning the traditional proxy variable approach fails. We provide conditions under which the output elasticity of the variable input—and hence the markup—is nonparametrically point-identified. A Neyman orthogonal moment condition ensures oracle efficiency of our GMM estimator. A Monte Carlo exercise shows a large bias for the traditional approach that decreases rapidly and nearly vanishes for our generalized control function approach.

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As the difference of revenue and cost, a firm's profit depends on the demand that it faces and on its production technology. Demand and cost are linked through product innovations, foreign market access, and the firm's advertising decisions. For example, a firm may move downmarket with products that are cheaper to produce but appeal to a broader set of consumers or it may move upmarket with products that are more costly to produce but appeal to more quality-conscious foreign consumers. Cost also depends directly on the firm's R&D decisions that, in turn, may be subject to financial constraints.

In this paper, we estimate the production function that describes the firm's production technology. To capture the interrelation of demand and cost, we allow the productivity ω_{it} of firm i in period t to evolve jointly with an unobservable factor δ_{it}. Besides latent demand shocks, δ_{it} may capture unobserved variation in investment opportunities or financial constraints across firms or time.

Our setting poses two difficulties for the traditional proxy variable approach to production function estimation developed by G. Steven Olley & Ariel Pakes (1996), James Levinsohn & Amil Petrin (2003), and Daniel A. Ackerberg, Kevin Caves & Garth Frazer (2015) (henceforth, OP, LP, and ACF). First, the OP/LP/ACF framework is underpinned by an invertibility assumption that in particular requires that there is no other unobservable, such as our δ_{it}, besides productivity ω_{it} (the scalar unobservable assumption in ACF). Second, this framework assumes that productivity is governed by a Markov process with law of motion $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$. In contrast, we generalize the law of motion for productivity to $\omega_{it} = g(\omega_{it-1}, \delta_{it-1}) + \xi_{it}$ to allow ω_{it} and δ_{it} to evolve jointly.[1]

To tackle these difficulties, we develop a generalized control function approach to production function estimation. We show that our approach generalizes the control function already present in the OP/LP/ACF framework. We focus on estimating the production function and make no attempt to identify or estimate the law of motion for productivity. We provide conditions under which the output elasticity of the variable input v_{it}—and hence the markup following Jan De Loecker &

[1]Replacing the autonomous Markov process in the OP/LP/ACF framework with a controlled Markov process is straightforward if the control is observed (see Jan De Loecker (2013) for learning-by-exporting and Ulrich Doraszelski & Jordi Jaumandreu (2013) for R&D) but not if the control is unobserved.

Frederic Warzynski (2012)—is nonparametrically point-identified and can be estimated by solving a straightforward GMM problem. A Neyman orthogonal moment condition ensures oracle efficiency of our GMM estimator.

1 Setup

Our setup follows Ulrich Doraszelski & Lixiong Li (2025) (henceforth, DL), except that we generalize the law of motion for productivity from $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$ to $\omega_{it} = g(\omega_{it-1}, \delta_{it-1}) + \xi_{it}$, where $g(\omega_{it-1}, \delta_{it-1}) = \mathrm{E}\left[\omega_{it}|\omega_{it-1}, \delta_{it-1}\right]$ and ξ_{it} is the productivity innovation. For concreteness we think of δ_{it} as latent demand shocks. Firm i in period t uses inputs k_{it} and v_{it} to produce output q_{it} according to the production function

$$q_{it} = f(k_{it}, v_{it}) + \omega_{it} + \varepsilon_{it},$$

where lower case letters denote logs. Capital k_{it} is a predetermined input that is chosen in period $t-1$ whereas v_{it} is freely variable and decided on in period t after the firm observes ω_{it} and δ_{it}. The disturbance ε_{it} sits between the firm's output q_{it} as recorded in the data and the output $q^*_{it} = q_{it} - \varepsilon_{it} = f(k_{it}, v_{it}) + \omega_{it}$ that the firm planned on when it decided on the variable input v_{it}. It can be interpreted alternatively as measurement error or as the untransmitted component of productivity.

In what follows, we let $x_{it} = (k_{it}, v_{it}, \ldots)$ denote a vector of observables and $z_{it} = (k_{it}, k_{it-1}, v_{it-1}, \ldots)$ a vector of instruments. As in DL, these vectors can contain additional variables besides the ones listed.

In the OP/LP/ACF framework, estimation proceeds in two steps. The first step assumes $\mathrm{E}[\varepsilon_{it}|x_{it}] = 0$ and flexibly or nonparametrically estimates the conditional expectation $\mathrm{E}[q_{it}|x_{it}]$. The second step assumes $\mathrm{E}[\xi_{it} + \varepsilon_{it}|z_{it}] = 0$ and estimates the production function and the law of motion for productivity using GMM.

Notation. For any two vectors a and b, we write $a \setminus b$ for the elements of a that are not contained in b. To avoid clutter, equalities involving random variables and conditional expectations are understood to hold almost surely. Proofs are deferred to the Supplemental Appendix.

2 Results

We choose the *special instrument* $z_{it}^s \subseteq z_{it}$ and partition z_{it} as $z_{it} = (z_{it}^s, z_{it}^c)$. In the following assumption, z_{it}^c serve as control variables whereas the special instrument z_{it}^s is excluded from the control function.

Assumption 1. $\mathrm{E}\left[\omega_{it} + \varepsilon_{it} - h\left(z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right)\middle| z_{it}, \mathrm{E}[q_{it-1}|x_{it-1}]\right] = 0$ for the control function $h\left(z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right)$.

The special instrument z_{it}^s can consist of one or several components of z_{it}. As our notation for the control function $h\left(z_{it}^c, \mathrm{E}\left[q_{it-1}|x_{it-1}\right]\right)$ emphasizes, Assumption 1 requires that conditional on $(z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}])$, the special instrument z_{it}^s is uncorrelated with the sum of the transmitted and untransmitted components of productivity $\omega_{it} + \varepsilon_{it}$. In contrast, Assumption 1 allows the control variables z_{it}^c to be correlated with productivity.

Because it controls for a part of the variation in productivity, Assumption 1 is less demanding than the condition $\mathrm{E}[\omega_{it} + \varepsilon_{it}|z_{it}, \mathrm{E}[q_{it-1}|x_{it-1}]] = 0$ used in a conventional instrumental variables approach to production function estimation. It is widely understood that finding instruments in the latter approach is difficult, or perhaps even impossible, in practice (Zvi Griliches & Jacques Mairesse 1998).

We further illustrate Assumption 1 with two examples. Example 1 shows that Assumption 1 holds in the OP/LP/ACF framework.

Example 1 (OP/LP/ACF framework)**.** In the OP/LP/ACF framework, the law of motion for productivity is $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$. The GMM estimation in the second step uses the moment condition[2]

$$\mathrm{E}\left[q_{it} - f(k_{it}, v_{it}) - g\left(\mathrm{E}\left[q_{it-1}|x_{it-1}\right] - f(k_{it-1}, v_{it-1})\right)\right| z_{it}, \mathrm{E}[q_{it-1}|x_{it-1}]] = 0. \tag{1}$$

Because $q_{it} - f(k_{it}, v_{it}) = \omega_{it} + \varepsilon_{it}$, moment condition (1) implies Assumption 1 if we choose $z^c_{it} = (k_{it-1}, v_{it-1})$, $z^s_{it} = z_{it} \setminus z^c_{it}$, and $h\left(z^c_{it}, \mathrm{E}\left[q_{it-1}|x_{it-1}\right]\right) = g\left(\mathrm{E}\left[q_{it-1}|x_{it-1}\right] - f(k_{it-1}, v_{it-1})\right)$.

Note that in the OP/LP/ACF framework the invertibility assumption ensures that lagged productivity $\omega_{it-1} = \mathrm{E}\left[q_{it-1}|x_{it-1}\right] - f(k_{it-1}, v_{it-1})$ can be recovered from observables. Moment condition (1) and Assumption 1 are therefore closely related. The difference is that moment condition (1) restricts the control function to take on the specific form implied by the Markov process and invertibility assumptions while Assumption 1 leaves it unrestricted. ■

Example 1 clarifies that the OP/LP/ACF framework can be seen as a control function approach. A key insight of OP is that all that remains of current productivity ω_{it} after controlling for lagged productivity ω_{it-1} via the control function $h\left(z^c_{it}, \mathrm{E}\left[q_{it-1}|x_{it-1}\right]\right) = g\left(\mathrm{E}\left[q_{it-1}|x_{it-1}\right] - f(k_{it-1}, v_{it-1})\right)$ is the productivity innovation ξ_{it}. Because ξ_{it} is an independent shock, this facilitates finding instruments.

Following DL, Assumption 1 can also be implied without the invertibility assumption if we choose $x_{it-1} = z_{it}$ in addition to $z^c_{it} = (k_{it-1}, v_{it-1})$ and $z^s_{it} = z_{it} \setminus z^c_{it}$. Without invertibility, however, the control function $h\left(z^c_{it}, \mathrm{E}\left[q_{it-1}|x_{it-1}\right]\right)$ generally no longer equals the law of motion $g\left(\mathrm{E}\left[q_{it-1}|x_{it-1}\right] - f(k_{it-1}, v_{it-1})\right)$. DL maintain that the law of motion for productivity is $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$.

Relative to the OP/LP/ACF framework and DL, Assumption 1 generalizes the control function approach to production function estimation by allowing us to add variables to the control function. As the conditioning set $(z^c_{it}, \mathrm{E}[q_{it-1}|x_{it-1}])$ contains more variables and thus controls for a larger part of the variation in productivity, the special instrument z^s_{it} contains fewer variables. Adding variables to the control function therefore makes Assumption 1 less demanding. This increases

[2]In the OP/LP/ACF framework, $\mathrm{E}[q_{it-1}|x_{it-1}]$ can be used as an additional instrument. Because the invertibility assumption ensures $\omega_{it-1} = \mathrm{E}\left[q_{it-1}|x_{it-1}\right] - f(k_{it-1}, v_{it-1})$ and the timing and Markov process assumptions further ensure $\mathrm{E}[\xi_{it} + \varepsilon_{it}|z_{it}, \omega_{it-1}] = 0$, we have $\mathrm{E}\left[\xi_{it} + \varepsilon_{it}|z_{it}, \mathrm{E}[q_{it-1}|x_{it-1}]\right] = 0$.

robustness.

Perhaps even more importantly, Assumption 1 allows us to go beyond the OP/LP/ACF framework by not relying on invertibility and accommodating the joint evolution of productivity ω_{it} and latent demand shocks δ_{it}. It shifts the focus from the law of motion for productivity to specifying a model for the special instrument z^s_{it}. As the researcher chooses the special instrument, one can leverage institutional features or auxiliary data to justify Assumption 1. Example 2 illustrates this point.

Example 2 (Independent input price shocks)**.** Assume that the price of the variable input p^V_{it} evolves according to $p^V_{it} = \kappa(p^V_{it-1}, \eta_{it}, \tau_i)$ for some function κ, where η_{it} is a firm- and time-specific input price shock and τ_i is a firm-specific shifter such as the type of inputs the firm uses. The shifter τ_i may be correlated with δ_{it-1} in the law of motion for productivity. This accommodates settings in which the type of inputs links productivity and demand.

Assume that η_{it} is an independent shock and therefore in particular independent of $(\xi_{it}, \varepsilon_{it}, \varepsilon_{it-1})$ and of any variables known or chosen by firm in period $t-1$. Because $\omega_{it} = g(\omega_{it-1}, \delta_{it-1}) + \xi_{it}$, it follows that

$$(\omega_{it} + \varepsilon_{it}, \omega_{it-1} + \varepsilon_{it-1}) \perp\!\!\!\perp \eta_{it} \middle| k_{it}, k_{it-1}, v_{it-1}, \left(p^V_{it'}\right)_{t'<t}.$$

Assume further that the shifter τ_i can be identified from $\left(p^V_{it'}\right)_{t'<t}$.[3] It follows that

$$(\omega_{it} + \varepsilon_{it}, \omega_{it-1} + \varepsilon_{it-1}) \perp\!\!\!\perp p^V_{it} \middle| k_{it}, k_{it-1}, v_{it-1}, \left(p^V_{it'}\right)_{t'<t}.$$

Choosing $z_{it} = \left(k_{it}, k_{it-1}, v_{it-1}, p^V_{it}, \left(p^V_{it'}\right)_{t'<t}\right)$, $x_{it-1} = z_{it}$, $z^s_{it} = p^V_{it}$, and $z^c_{it} = z_{it} \setminus z^s_{it}$, we have

$$(\omega_{it} + \varepsilon_{it}, \omega_{it-1} + \varepsilon_{it-1}) \perp\!\!\!\perp z^s_{it} \middle| z^c_{it}. \tag{2}$$

Because $\omega_{it-1} + \varepsilon_{it-1} \perp\!\!\!\perp z^s_{it} | z^c_{it}$, we know that $q_{it-1} \perp\!\!\!\perp z^s_{it} | z^c_{it}$ and thus that $\mathrm{E}[q_{it-1}|x_{it-1}]$ depends only on z^c_{it}. Equation (2) therefore implies

$$\omega_{it} + \varepsilon_{it} \perp\!\!\!\perp z^s_{it} \middle| z^c_{it}, \mathrm{E}[q_{it-1}|x_{it-1}]$$

[3] Alternatively, assume that we have other information that pins down τ_i. For example, τ_i may be the location of the firm if there are regional differences in input markets or it may be the countries from which the firm imports inputs.

and Assumption 1 holds for the control function $h\left(z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}]\right) = \mathrm{E}\left[\omega_{it} + \varepsilon_{it}|z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}]\right]$. Furthermore, because $\mathrm{E}[q_{it-1}|x_{it-1}]$ depends only on z_{it}^{c}, the control function simplifies from $h(z_{it}^{c}$, $\mathrm{E}[q_{it-1}|x_{it-1}])$ to $h(z_{it}^{c})$. ■

Assumption 1 ensures that the moment condition

$$\mathrm{E}\left[q_{it} - f(k_{it}, v_{it}) - h\left(z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}]\right)\middle| z_{it}, \mathrm{E}[q_{it-1}|x_{it-1}]\right] = 0 \tag{3}$$

holds at the true production function.

We are interested in the output elasticity $\frac{\partial f(k_{it},v_{it})}{\partial v_{it}}$ of the variable input v_{it} as the key to estimating the markup. The choice of the special instrument z_{it}^{s} entails a tradeoff. As noted above, as the conditioning set $(z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}])$ contains more variables and thus controls for a larger part of the variation in productivity, the special instrument z_{it}^{s} contains fewer variables. This leaves less exogenous variation in z_{it}^{s} for identifying $\frac{\partial f(k_{it},v_{it})}{\partial v_{it}}$. To ensure that z_{it}^{s} has sufficient identification power, we make the following assumption:

Assumption 2. Let $k_{it} \in z_{it}^{c}$. Conditional on $(z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}])$, z_{it}^{s} is a complete instrument for v_{it}.

Assumption 2 places restrictions on the underlying economic model. In particular, it rules out the case where the law of motion for productivity is $\omega_{it} = g(\omega_{it-1}) + \xi_{it}$ and the variable input demand $v_{it} = \kappa(k_{it}, \omega_{it})$ is some function κ that depends only on capital and productivity and can be inverted for productivity.[4] In this case, for any choice of the special instrument $z_{it}^{s} \subseteq z_{it} \setminus (k_{it}, k_{it-1}, v_{it-1})$, z_{it}^{s} is independent of ω_{it} conditional on $(z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}])$ because ω_{it-1} is pinned down by $(z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}])$ and the productivity innovation ξ_{it} is an independent shock. Consequently, z_{it}^{s} cannot generate any variation in $v_{it} = \kappa(k_{it}, \omega_{it})$ after controlling for $(z_{it}^{c}, \mathrm{E}[q_{it-1}|x_{it-1}])$. Note that this lack-of-variation argument breaks down if latent demand shocks δ_{it} enter the law of motion for productivity or the variable input demand. Hence, latent demand shocks δ_{it} make room for Assumption 2 to hold.

[4] Assumption 2 therefore also rules out the nonidentification result in Amit Gandhi, Salvador Navarro & David A. Rivers (2020).

Our main identification result is the following:

Theorem 1. *Under Assumptions 1 and 2, moment condition (3) nonparametrically point-identifies* $\frac{\partial f(k_{it}, v_{it})}{\partial v_{it}}$.

Turning from identification to estimation, for any weighting function $\varphi(z_{it})$ of the instruments z_{it}, moment condition (3) implies that the moment condition

$$\mathrm{E}\left[\varphi(z_{it})\left(q_{it} - f(k_{it}, v_{it}) - h\left(z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right)\right)\right] = 0 \tag{4}$$

holds at the true production function. GMM estimation based on moment condition (4) can be conducted by viewing the control function $h\left(z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right)$ as a nuisance parameter and estimating it alongside the production function, as in the OP/LP/ACF framework.[5] However, depending on the number of control variables z_{it}^c, the dimension of the control function may be high. This increases the asymptotic variance of the estimates and can create numerical challenges for minimizing the GMM objective function.

Our main estimation result shows that these drawbacks can be avoided:

Theorem 2. *For any weighting function $\varphi(z_{it})$ of the instruments z_{it}, moment condition (3) implies that the moment condition*

$$\mathrm{E}\left[\left(\varphi(z_{it}) - \tilde{\varphi}_{it}\right)\left(q_{it} - f(k_{it}, v_{it}) - \left(\tilde{q}_{it} - \tilde{f}_{it}\right)\right)\right] = 0, \tag{5}$$

where

$$\tilde{\varphi}_{it} = \mathrm{E}\left[\varphi(z_{it})|z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right], \quad \tilde{q}_{it} - \tilde{f}_{it} = \mathrm{E}\left[q_{it} - f(k_{it}, v_{it})|z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right],$$

holds at the true production function. Moreover, if a production function satisfies moment condition (5), then it satisfies moment condition (4) for the control function $h\left(z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right) = \tilde{q}_{it} - \tilde{f}_{it}$. Finally, if $\varphi(z_{it})$ and $q_{it} - f(k_{it}, v_{it})$ have finite L^2 norms, then moment condition (5) is Neyman orthogonal with respect to L^2-integrable perturbations of $\left(\tilde{\varphi}_{it}, \tilde{q}_{it} - \tilde{f}_{it}\right)$.

[5] The law of motion for productivity cannot generally be recovered from estimates of the production and control functions.

Applying results from the double-debiased machine learning literature (Victor Chernozhukov, Denis Chetverikov, Mert Demirer, Esther Duflo, Christian Hansen, Whitney Newey & James Robins 2018), Neyman orthogonality ensures that the GMM estimator based on moment condition (5) is oracle efficient as long as the estimators for $\tilde{\varphi}_{it}$, $\tilde{q}_{it}$, and $\tilde{f}_{it}$ converge sufficiently fast. This means that the asymptotic distribution of the GMM estimator is *as if* the true values of $\tilde{\varphi}_{it}$ and $\tilde{q}_{it} - \tilde{f}_{it}$ are known.

A complication arises because estimating $\tilde{\varphi}_{it}$ and $\tilde{q}_{it} - \tilde{f}_{it}$ requires itself a plugin estimator for $\mathrm{E}[q_{it-1}|x_{it-1}]$. To the best of our knowledge, the literature has not yet developed a treatment for such "double plugin" estimators. In what follows, we therefore focus on the case where the control function simplifies from $h\left(z_{it}^c, \mathrm{E}[q_{it-1}|x_{it-1}]\right)$ to $h\left(z_{it}^c\right)$ as in Example 2. Thus, the plugin estimator for $\mathrm{E}[q_{it-1}|x_{it-1}]$ is no longer required, and $\tilde{\varphi}_{it} = \mathrm{E}\left[\varphi(z_{it})|\, z_{it}^c\right]$ and $\tilde{q}_{it} - \tilde{f}_{it} = \mathrm{E}\left[q_{it} - f(k_{it}, v_{it})|\, z_{it}^c\right]$.

While our generalized control function approach accommodates the joint evolution of productivity ω_{it} and latent demand shocks δ_{it}, it is not costless. If moment condition (1) holds, then for any choice of the special instrument $z_{it}^s \subsetneq z_{it} \setminus (k_{it-1}, v_{it-1})$ it is less efficient than the OP/LP/ACF procedure. This reflects a general tradeoff between robustness and efficiency in nonparametric estimation. Our generalized control function approach makes this tradeoff explicit by allowing a researcher to target greater robustness (smaller z_{it}^s) or greater efficiency (larger z_{it}^s).

3 Monte Carlo Exercise

Data generating process. Similar to DL, we specify the CES production function

$$f(k_{it}, v_{it}) = \frac{\nu}{\rho} \ln\left(\alpha \exp(\rho k_{it}) + (1-\alpha)\exp(\rho v_{it})\right)$$

with $\alpha = 0.3$, $\rho = -1$, and $\nu = 0.95$, the disturbance $\varepsilon_{it} \sim N\left(0, 0.5^2\right)$, and the CES demand $q_{it}^* = \delta_{1it} - (1 + \exp(-\delta_{2it}))p_{it}$, where p_{it} is the output price and $\delta_{it} = (\delta_{1it}, \delta_{2it})$ captures shocks to the demand the firm faces and unobserved rivals.

Different from DL, the price of capital p_{it}^K, the price of the variable input p_{it}^V, and the latent demand shocks δ_{1it} and δ_{2it} follow Gaussian $AR(1)$ processes. We parameterize these processes so that $\mathrm{E}[p_{it}^K] = \mathrm{E}[p_{it}^V] = 0$, $\mathrm{E}[\delta_{1it}] = 10$, $\mathrm{E}[\delta_{2it}] = -1.3543$, $\mathrm{Var}(p_{it}^K) = \mathrm{Var}(p_{it}^V) = \mathrm{Var}(\delta_{2it}) = 0.5^2$, $\mathrm{Var}(\delta_{1it}) = 5^2$, and the autocorrelation is 0.7. Short-run profit maximization implies the markup $\mu_{it} = \frac{P_{it}}{MC_{it}} = 1 + \exp(\delta_{2i})$, where MC_{it} is marginal cost, and thus $\mathrm{E}\left[\ln \mu_{it}\right] = 0.25$ and $\mathrm{Var}\left(\ln \mu_{it}\right) = 0.0126$.

We specify the law of motion for productivity

$$g(\omega_{it-1}, \delta_{it-1}) = \mu_\omega + \rho_\omega \omega_{it-1} + \rho_{\delta_1} \delta_{1it-1} + \rho_{\delta_2} \delta_{2it-1}$$

and the productivity innovation $\xi_{it} \sim N\left(0, \sigma_\omega^2\right)$. We parameterize μ_ω, ρ_ω, ρ_{δ_1}, ρ_{δ_2}, and σ_ω^2 so that $\mathrm{E}[\omega_{it}] = 0$, $\mathrm{Var}(\omega_{it}) = 0.5^2$, $\mathrm{corr}(\omega_{it}, \omega_{it-1}) = 0.7$, $\mathrm{corr}(\omega_{it}, \delta_{1it}) = 0.3$, and $\mathrm{corr}(\omega_{it}, \delta_{2it}) = -0.3$. This aligns with the notion that more productive firms participate in larger and more competitive markets.

We simulate $S = 1,000$ datasets with $N = 5,000$ firms and $T = 20$ periods. We refer the reader to DL for further details on the data generating process.

Estimation. We use GMM estimation based on moment condition (5) to estimate the production function parameters $\theta = (\alpha, \rho, \nu)$. Our instruments are $z_{it} = \left(k_{it}, k_{it-1}, v_{it-1}, p_{it-1}, p_{it}^V, p_{it-1}^V\right)$ and our special instrument is $z_{it}^s = p_{it}^V$ as in Example 2. Our weighting function $\varphi(z_{it})$ is the complete set of Hermite polynomials of total degree 4 in the variables in z_{it}. We estimate $\tilde{\varphi}_{it} = \mathrm{E}\left[\varphi(z_{it}) | z_{it}^c\right]$ by OLS using the complete set of Hermite polynomials of total degree d in the variables in z_{it}^c. We proceed similarly to estimate $\tilde{q}_{it} - \tilde{f}_{it} = \mathrm{E}\left[q_{it} - f(k_{it}, v_{it}) | z_{it}^c\right]$. The latter must be re-estimated at each iteration of the GMM problem. Even though the control function $h\left(z_{it}^c\right) = \mathrm{E}\left[\omega_{it} + \varepsilon_{it} | z_{it}^c\right]$ is absent from moment condition (5), the total degree d implicitly determines how well we can approximate it. We accordingly explore $d \in \{2, 3, 4, 5\}$. We provide further details on the GMM estimator in the Supplemental Appendix.

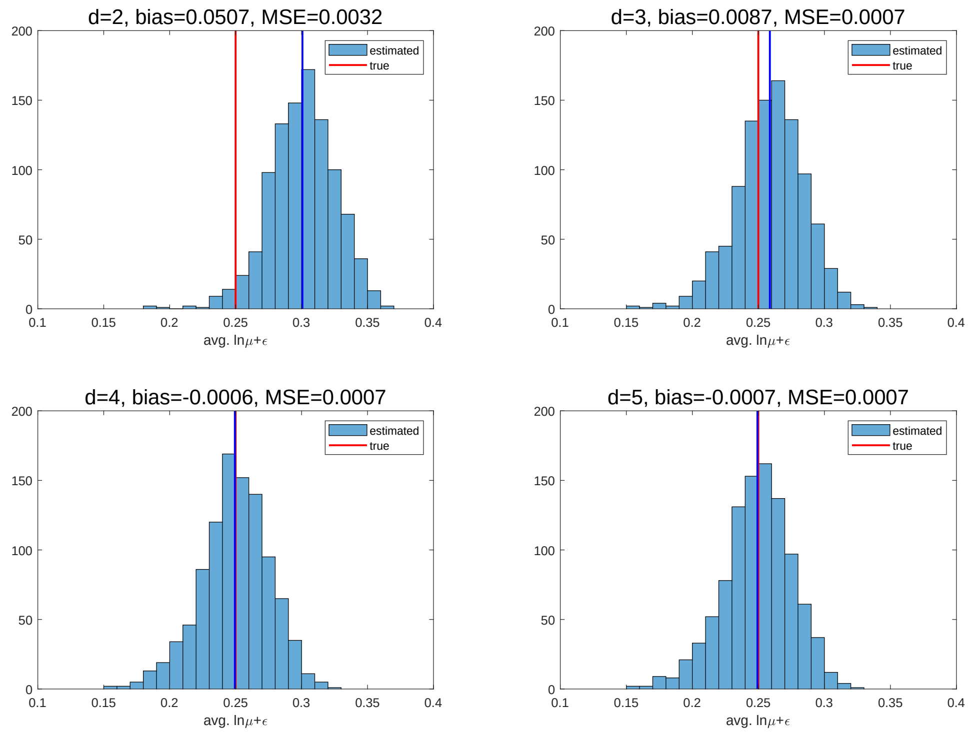

Figure 1: Distribution of average log markup for Hermite polynomials of total degree $d \in \{2, 3, 4, 5\}$.

With an estimate of θ in hand, we estimate the markup μ_{it} of firm i in period t as

$$\ln \mu_{it} + \varepsilon_{it} = p_{it} + q_{it} - p_{it}^V - v_{it} + \ln \frac{\partial f(k_{it}, v_{it})}{\partial v_{it}}.$$

The right-hand side is the log of the output elasticity minus the log of the expenditure share of the variable input. Noting that the disturbance ε_{it} averages out as $\mathrm{E}[\varepsilon_{it}] = 0$, we refer to the average of $\ln \mu_{it} + \varepsilon_{it}$ across firms and time simply as the average log markup.

Results. As a baseline, we implement the modified OP/LP/ACF procedure described in DL by choosing $x_{it-1} = z_{it}$ and explicitly including the first-order bias correction. We use a univariate Hermite polynomial of order 4 to approximate the law of motion for productivity. The bias in the average log markup is 0.1277 (compared to its true value of 0.25) and the mean squared error is 0.0164. The large bias is not surprising given that the joint evolution of productivity ω_{it} and latent demand shocks δ_{it} is outside the scope of the OP/LP/ACF procedure.

Turning to the generalized control function approach in this paper, Figure 1 shows the distribution of the average log markup. As can be seen, the results rapidly improve with the total degree d of the complete set of Hermite polynomials in the variables in z_{it}^c. The bias decreases from 0.0507 for $d = 2$ to -0.0006 for $d = 4$ and the mean squared error from to 0.0032 to 0.0007. There are no further improvements going from $d = 4$ to $d = 5$.

4 Concluding Remarks

We provide conditions for consistently estimating the production function in empirically relevant settings that are outside the scope of the OP/LP/ACF framework. As such, our approach complements the OP/LP/ACF framework. It generalizes the control function that is already present in the OP/LP/ACF framework and requires solving a straightforward GMM problem. We hope that it proves valuable for applied researchers seeking to estimate the production function and the markup from it.

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