The Cross-section of Subjective Expectations: Understanding Prices and Anomalies

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ABSTRACT

We decompose cross-sectional differences in the level of price-earnings ratios using professional forecasts. High price-earnings ratios are accounted for by both low expected returns and overly high expected earnings growth. The magnitudes and timing of the comovements between prices, earnings growth, and returns are consistent with gradual learning rather than expectations being highly sensitive to recent realizations. Earnings growth surprises do not translate 1-1 into one-period returns, but instead are gradually reflected in returns over time. A structural risk-premia model incorporating constant-gain learning about mean earnings growth replicates our findings and generates realistic dispersion and persistence in price-earnings ratios.

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It has been known since Basu (1975) and Stattman (1980) that high price ratio stocks (e.g., price-earnings ratios, price-book ratios) earn lower returns than their peers. While one-month differences between Growth and Value stocks have declined over time (Schwert, 2003; Fama and French, 2020), return differences at longer horizons have remained substantial (Delao, Han, and Myers, 2024)[1] and play a large role in accounting for the level of prices (van Binsbergen et al., 2023; Cho and Polk, 2024). Given that a stock's price is the risk-adjusted value of expected future cash flows, these realized return differences imply that high price ratio stocks have low risk exposure, overly high expected cash flows, or a mix of both.

There is a long-standing debate over whether cross-sectional return differences are driven by risk exposure or incorrect cash flow expectations.[2] Our innovation is twofold.

First, using professional forecasts of both returns and cash flows, we decompose cross-sectional differences in the *level* of price ratios. While one-month or one-year returns may be relevant for traders, the level of price ratios is arguably of equal or greater relevance for understanding the allocation of resources across firms (e.g., why are some firms valued at five times earnings while others are valued at fifty times earnings?). To the best of our knowledge, we are the first to quantify the fraction of cross-sectional dispersion in price ratios that is explained by high price ratio stocks having lower subjective return expectations and the fraction that is explained by high price ratio stocks having overly high subjective cash flow expectations. We find that both components play a non-trivial role, however, incorrect cash flow expectations are the quantitatively larger component.

Second, we document that errors in subjective cash flow expectations are "stubborn" in the sense that they are insensitive to recent earnings surprises. On average, high price ratio stocks have high subjective cash flow expectations which are not met by future realized earnings. However, these negative earnings surprises do not lead to a large immediate revision in subjective expectations of future cash flows nor to a large immediate price change.

[1] Table I also confirms that long-term return differences are large even for 1999-2020.

[2] See Fama and French (1995) and Daniel and Titman (1997) for early evidence and Hou, Karolyi, and Kho (2011) and Koijen, Lustig, and Van Nieuwerburgh (2017) for more recent explanations.

Instead, earnings surprises predict a sequence of gradual adjustments in subjective cash flow expectations and prices over the next several years. This aligns with the literature on post earnings announcement drift but emphasizes that these stubborn expectations play an important role in the level of prices, not just one-month to one-year alphas around quarterly earnings announcements.[3] In particular, the presence of stubborn expectations helps to explain how errors in cash flow expectations can have a large impact on the level of prices, as it becomes riskier for informed traders to bet against these expectations. The intuition follows the famous adage, often attributed to Keynes, that "markets can remain irrational longer than you can remain solvent."

For our empirical analysis, we utilize a cross-sectional version of the Campbell-Shiller decomposition.[4] Using professional forecasts, we find that 43.3% of dispersion in price-earnings ratios is accounted for by high price ratio firms having higher expected four-year earnings growth and 12.7% of dispersion is accounted for by high price ratio firms having lower expected four-year returns.[5] Thus, both higher expected earnings growth and lower expected returns help to explain high valuation stocks. Interestingly, while Greenwood and Shleifer (2014) show that expected returns are positively correlated with price ratios in the aggregate time series, in the cross-section investors correctly expect lower returns for high price-ratio firms.[6] The remaining dispersion is explained by expectations of future price-earnings ratios, which reflect expectations of earnings growth and returns beyond four years.

For comparison, realized four-year earnings growth and negative returns account for 9.9% and 32.0% of price-earnings ratio dispersion, respectively. This means that, empirically, high price ratio firms are primarily characterized by lower future returns than their peers, rather

[3]For example, Chan, Jegadeesh, and Lakonishok (1996) and Skinner and Sloan (2002) show significant 3- to 12-month return differences across portfolios sorted by earnings surprises.

[4]Because this decomposition is derived from an identity, it holds even if expectations differ from the objective distribution.

[5]For concision, we shorten "subjective expected earnings growth" and "subjective expected returns" to simply "expected earnings growth" and "expected returns." Any time we refer to FIRE (full-information rational expectations) beliefs, we clearly specify that we are using the FIRE-implied distribution.

[6]Dahlquist and Ibert (2024), Bastianello (2024), Büsing and Mohrschladt (2023), and Couts et al. (2024) also find evidence that expected returns are negatively related to price ratios.

than by higher future earnings growth. Thus, investors overestimate the earnings growth of high price ratio firms, which leads to consistent disappointment in earnings growth for these firms. While investors do expect lower returns for high price ratio firms, they understate the magnitude of this relationship. Consistent with the fact that investors are disappointed by realized earnings growth, the realized returns on high price ratio firms are even lower than investors expected.

Given these errors in expectations, a natural question is how prices and expectations adjust over time as future earnings are realized. Under FIRE (full-information rational expectations), realized earnings being higher than expected should lead to an almost 1-for-1 immediate revision in expected future earnings and should not predict any subsequent revisions in earnings expectations. Empirically, we find that earnings surprises predict an immediate revision in expected earnings of only 0.11 to 0.14 and predict a sequence of additional revisions of 0.09 to 0.12 in each of the subsequent years. To confirm that this gradual adjustment is not merely an artifact of the subjective expectations data, we show that returns follow a similar path both qualitatively and quantitatively. Earnings surprises do not translate 1-for-1 into immediate returns. Instead, earnings surprises predict a sequence of returns of 0.12 to 0.15 in the current year and each of the subsequent years.

While we mainly focus on cross-sectional dispersion in price ratios, we can extend these tests to understand anomaly returns. Focusing on portfolios sorted by 20 different anomalies from Hou, Xue, and Zhang (2015), we consistently find that earnings surprises result in only minor revisions to future earnings expectations, in line with our results for high and low price ratio stocks. Further, we find that realized one-year returns on anomalies are much higher than investors expected and that errors in earnings expectations are large enough to account for the entirety of these unexpected anomaly returns. In fact, we find that one-year earnings surprises are larger than the one-year unexpected returns, again indicating that earnings surprises do not immediately translate 1-1 into one-year returns.

How do these findings fit with FIRE and non-FIRE models? While evidence of errors in expectations obviously points against FIRE, the more surprising result is that standard FIRE models struggle to match the magnitude of the *empirical* relationship between price-earnings ratios and future returns. While risk premia related to growth options or adjustment costs (Berk, Green, and Naik, 1999; Zhang, 2005) can generate return differences between high and low price-earnings ratio stocks, we find that these models predict a relationship that is an order of magnitude smaller than what we observe in the data. In contrast, we find that several common behavioral models struggle to match our evidence of stubborn expectations. For example, if agents extrapolate from current earnings growth or have diagnostic expectations of earnings growth, then prices should be highly sensitive to recent realized earnings, and earnings surprises should translate into large immediate returns.

To explain our empirical findings, we show that constant-gain learning generates volatile prices and reversals (i.e., high price ratios predicting lower future returns) while still matching our evidence of stubborn expectations. In the spirit of Nagel and Xu (2022), we propose a model in which agents learn about mean earnings growth, and we make a small but qualitatively important change by including temporary shocks to the level of earnings. Relative to a FIRE benchmark, constant-gain learning with temporary level shocks increases the volatility of prices and returns substantially due to waves of optimism/pessimism about future earnings. Because of the temporary level shocks, these errors in expectations are stubborn and prices only adjust gradually as future earnings are realized, as earnings surprises are largely attributed to the temporary level shocks.[7] Expanding the model to include risk premia based on cash flow timing à la Lettau and Wachter (2007) allows the model to also match our results on subjective returns expectations and improves the model's ability to match the persistence and cross-sectional dispersion of price ratios.

We then estimate and quantitatively test our constant-gain learning model. We set the constant-gain parameter to match previous studies on constant-gain learning (Milani,

[7] In models of learning about mean earning growth without these temporary level shocks, such as Lewellen and Shanken (2002) and Nagel and Xu (2022), the agent believes log earnings follow a random walk. Because of this, prices adjust at least 1-1 to earnings surprises.

2007; Malmendier and Nagel, 2016; Nagel and Xu, 2022) and estimate the remaining 5 parameters solely using realized earnings growth and average aggregate returns. Despite not using any cross-sectional information, the model successfully replicates our decomposition results. Both in terms of magnitudes and timing, the model outperforms standard FIRE models in matching the realized dynamics of price-earnings ratios, earnings growth, and returns and outperforms common behavioral models in matching the dynamics of subjective expectations. Further, the model matches several untargeted aggregate and cross-sectional asset pricing moments.

The quantified structural model allows us to extend our empirical results in two ways. First, we can go beyond the four-year horizon to estimate that expected earnings growth and expected returns for all horizons account for two-thirds (65.7%) and one-third (34.3%) of price-earnings ratio dispersion, respectively. This is largely due to errors in earnings growth expectations, which account for half (50.8%) of all price-earnings ratio dispersion.

Second, we examine how constant-gain learning interacts with risk premia related to cash flow timing to drive expected earnings growth and expected returns. Their combined effect is crucial for generating realistic price volatility, dispersion in price-earnings ratios and persistence in price-earnings ratios. For example, compared to an economy with no learning and no risk premia, introducing only risk premia has little impact on the dispersion in price-earnings ratios and introducing only learning increases the dispersion by a factor of 2.1. However, introducing both increases the dispersion by a factor of 4.5. This highlights the benefit of unifying non-FIRE earnings growth expectations and risk premia related to cash flow timing, as the interaction magnifies the sensitivity of prices to changes in beliefs.

Broadly, this paper contributes to the growing literature using subjective expectations to understand asset prices.[8] In the cross-section, errors in firm-level professional earnings forecasts have been strongly linked to future returns (La Porta, 1996; Frankel and Lee, 1998;

[8] Amromin and Sharpe (2014); Greenwood and Shleifer (2014); Piazzesi, Salomao, and Schneider (2015); Cassella and Gulen (2018); Delao and Myers (2021); Nagel and Xu (2022); Bordalo et al. (2024a); and Gandhi et al. (2024) utilize survey expectations for aggregate outcomes such as returns, cash flows and yields.

Da and Warachka, 2011; So, 2013; Weber, 2018, Bouchaud et al., 2019, van Binsbergen, Han, and Lopez-Lira, 2022).[9] We differ from these studies in two important ways. First and foremost, we focus on explaining cross-sectional differences in the *level* of price ratios, rather than short-term returns. While understanding short-term returns or fluctuations in price ratios is important, understanding the level of a firm's price ratio is of first-order importance for a firm owner or a policy maker interested in allocations. We document stubborn errors in subjective cash flow expectations, which do not translate immediately into large one-period returns, but do play a substantial role in generating large differences between firms in the level of price ratios. Second, by utilizing expectations of both earnings growth and returns, we quantify the relative importance of these two expectations in accounting for cross-sectional dispersion in price-earnings ratios and returns.[10] This decomposition sheds light on the relative importance of risk (discount rates) and mispricing in stock prices.

Our structural model builds on the literature on learning about mean consumption or cash flow growth (Lewellen and Shanken, 2002; Collin-Dufresne, Johannes, and Lochstoer, 2016; Nagel and Xu, 2022) and incorporates risk premia related to cash flow timing, similar in spirit to Lettau and Wachter (2007). We provide new evidence supporting these types of learning models using the cross-sectional dynamics of stocks and show that incorporating learning about temporary shocks to the level of earnings generates distinct qualitative predictions for the timing of earnings growth surprises and returns. We also highlight that learning about cash flows naturally complements risk premia related to cash flow timing. Even if the objective timing of cash flows is relatively similar across all firms (Chen, 2017), these risk premia can still play an important role in stock prices so long as investors *believe* there is a

[9]Further, Kozak, Nagel, and Santosh (2018) and Engelberg, Mclean, and Pontiff (2018) find that short legs of multiple long-short anomaly strategies comprise stocks with more optimistic earnings forecasts and Engelberg, McLean, and Pontiff (2020) find that anomaly short legs comprise stocks with more optimistic return forecasts. Similarly, Décaire and Graham (2024) use subjective expectations to study fluctuations in price ratios, and Bordalo et al. (2024b) use errors in earnings expectations to explain the level and fluctuations in anomaly returns such as HML and RMW.

[10]This differs from the implied cost of capital approach (Chen, Da, and Zhao, 2013; Hommel, Landier, and Thesmar, 2023) in which discount rates are inferred using earnings expectations for observable horizons and assumptions about long-term industry growth or GDP growth.

large difference in the timing of cash flows. In other words, as argued in Jensen (2024), once we depart from FIRE, the compensation for risk that investors require should be disciplined by data on investors' believed risks, not the objective risks.

The rest of the paper is organized as follows. Sections I and II discuss the decomposition and data utilized in our empirical exercises. Section III shows the results of the decomposition of price-earnings ratio dispersion and compares these findings to the predictions of several FIRE models. Section IV shows the evidence of stubborn expectations, discusses how this relates to several non-FIRE models, and extends our results to anomaly returns. Section V discusses robustness tests. Section VI proposes a structural model of stubborn expectations and volatile prices based on constant-gain learning. Section VII quantifies and tests the structural model, estimates the infinite horizon decomposition of price-earnings ratio dispersion, and analyzes the interaction of learning and risk premia in generating realistic cross-sectional asset price moments.

I. Decomposing the cross-section of price ratios

While a large amount of the asset pricing literature has focused on the cross-section of short-term returns, relatively less attention has been paid to the cross-section of prices or price ratios.[11] In particular, we want to understand what can account for the large empirical dispersion in price ratios across stocks, e.g., why do some stocks trade at 50 times earnings while others only trade at 10 times earnings?

To understand dispersion in stock price ratios and how this dispersion relates to subjective cash flow growth expectations and subjective discount rates, we focus on a cross-sectional version of the Campbell-Shiller decomposition. In terms of notation, $E_t^*[\cdot]$ denotes subjective expectations. All other operators use the objective probability distribution. For example, $Var(\cdot)$ and $Cov(\cdot,\cdot)$ denote the observable variance or covariance of variables.

[11] See Cochrane (2011) for a discussion, "When did our field stop being 'asset pricing' and become 'asset expected returning'?"

For any stock or portfolio of stocks i, the one-year ahead return $r_{i,t+1}$ can be approximated in terms of the price-earnings ratio $px_{i,t}$, future earnings growth $\Delta x_{i,t+1}$, and the future price-earnings ratio, all in logs:

$$r_{i,t+1} \approx \kappa + \Delta x_{i,t+1} + \rho p x_{i,t+1} - p x_{i,t}, \tag{1}$$

where κ and $\rho < 1$ are constants.[12] To understand cross-sectional dispersion in price-earnings ratios, let $\tilde{px}_{i,t}$ be the cross-sectionally demeaned price-earnings ratio of portfolio i and let $\Delta\tilde{x}_{i,t+1}$ and $\tilde{r}_{i,t+1}$ be the cross-sectionally demeaned earnings growth and returns. Rearranging equation (1) and applying subjective expectations $E_t^*[\cdot]$, we see that a higher than average price-earnings ratio must be explained by higher than average expected earnings growth, lower than average expected returns, or a higher than average expected future price-earnings ratio,

$$\tilde{px}_{i,t} \approx \sum_{j=1}^{h} \rho^{j-1} E_t^*\left[\Delta\tilde{x}_{i,t+j}\right] - \sum_{j=1}^{h} \rho^{j-1} E_t^*\left[\tilde{r}_{i,t+j}\right] + \rho^h E_t^*\left[\tilde{px}_{i,t+h}\right]. \tag{2}$$

Importantly, equation (2) does not require that expectations are rational. Because this equation is derived from an identity, it holds under any subjective probability distribution.

To measure the relative contribution of subjective cash flow growth expectations and subjective discount rates to the dispersion in price-earnings ratios, we decompose the variance of $\tilde{px}_{i,t}$ into three components:

$$1 \approx \underbrace{\frac{Cov\left(\sum_{j=1}^{h}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{CF_h} + \underbrace{\frac{Cov\left(-\sum_{j=1}^{h}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{DR_h} + \underbrace{\rho^h\frac{Cov\left(E_t^*\left[\tilde{px}_{i,t+h}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{FPX_h}. \tag{3}$$

Note that $Var\left(\tilde{px}_{i,t}\right)$ is the average squared cross-sectionally demeaned price-earnings ratio, which means it measures the average cross-sectional dispersion in price-earnings ratios. The

[12]Note that this approximation still holds even for non-dividend paying firms. Appendix B discusses the log-linearization in more detail including the role of the payout ratio.

coefficients CF_h and DR_h give a quantitative measure of how much dispersion in price-earnings ratios is accounted for by dispersion in earnings growth expectations and how much is accounted for by dispersion in discount rates. Applying the decomposition to multiple horizons h provides information about the timing of expected earnings growth and discount rates. Additionally, the terms in equation (3) can be interpreted as the coefficients from univariate regressions with time fixed effects, e.g., a one unit increase in $px_{i,t}$ is associated with a CF_1 unit increase in expected one-year earnings growth.

When we estimate equation (3) using professional forecasts, we will use expectations of price growth $E_t^*[\Delta p_{i,t+j}]$ as a proxy for expectations of returns $E_t^*[r_{t+j}]$. Empirically, realized price growth and returns are closely related with a correlation of 0.997 to 0.999 for the $j = 1, ..., 4$ horizons that we study in our analysis. However, to ensure that the use of this proxy does not impact the results, we also estimate an exact decomposition based on price growth in Appendix C.1. Because this alternative decomposition is an exact identity, it also addresses any concerns that cross-sectional differences in payout ratios between high and low price-earnings ratio firms may impact the approximation error in equation (3). As shown in Tables I and AI, the results of this exact decomposition closely match the results from equation (3). Further, Delao, Han, and Myers (2024) show that payout ratios do not account for cross-sectional differences in price-earnings ratios, i.e., high price-earnings ratios are not associated with higher or lower dividend-earnings ratios.

II. Data

The firm-level realized earnings and prices are collected from Compustat and CRSP. The firm-level expected earnings and prices are collected from I/B/E/S (Institutional Brokers' Estimate System) and Value Line. To perform the decomposition from Section I, we sort these firms into the classic Value and Growth portfolios.[13] Specifically, for each month t,

[13]We focus on these portfolios to connect to a long literature studying Value and Growth portfolios. Additionally, by aggregating firms into portfolios, we can study cash flow growth and discount rates without

we construct five value-weighted portfolios sorted by book-to-market.[14] For these portfolios, we measure the expectations at time t for earnings growth, price growth, and the future price-earnings ratio over the next four years. We also track the realized buy-and-hold future earnings growth, returns, and price-earnings ratios over the next four years. The *main sample*, which contains expectations of both earnings growth and price growth, ranges from 1999 to 2020. For robustness tests, we also use a *long sample* which ranges from 1982 to 2020 and contains earnings growth expectations. The subsections below provide more detail on the firm-level variable measurements.

A. *Realized data*

The sample of stocks consists of all common stocks (share code 10 and 11) listed on NYSE, AMEX, and NASDAQ. We obtain monthly prices, returns, and shares outstanding from the Center for Research in Security Price (CRSP). The firm-level accounting variables are constructed from the quarterly Compustat database. Following Davis, Fama, and French (2000) and Cohen, Polk, and Vuolteenaho (2003), we define book value as stockholders' book equity, plus deferred taxes and investment tax credit if available, minus the book value of preferred stock. If stockholders' book equity is not available at Compustat, we define it as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities in that order. Depending on availability, we use redemption or par value for the book value of preferred stock. To be consistent with the I/B/E/S's definition of earnings, we define earnings as Compustat net income (item NIq) excluding non-I/B/E/S items, which comprise extraordinary items and discontinued operations (item XIDOq), special items (item SPIq), and non-recurring income taxes (item NRTXTq). This aligns with the measure of earnings proposed in Hillenbrand and McCarthy

needing to drop firms with negative future earnings or negative forecasted earnings, whereas studies of firm-level implied cost of capital are generally forced to exclude these firms.

[14] The book-to-market ratio is measured using the market-cap in the portfolio formation month and the total book value from the most recent four quarters. To account for potential data errors, we exclude firms with book-to-market ratios over 100 or below 0.01.

(2022). At every month, annual earnings at the firm level are defined as the sum of quarterly earnings from the most recent four quarters.[15] The main sample includes all firms which have observable returns $r_{i,t+j}$, earnings growth $\Delta x_{i,t+j}$, and price-earnings ratios $px_{i,t+j}$ in future years $j = 1, 2, 3, 4$. We require a future observation so that we can calculate forecast errors for the subjective expectations. However, for robustness, in Appendix C.4, we drop this requirement and estimate a decomposition using delisting returns to reinvest any delisting firms and find similar results.

B. *Subjective expectations*

The subjective earnings and short-term price expectations are extracted from the I/B/E/S Database. The Summary Statistics of the I/B/E/S Database contains the median forecasts for EPS (earnings per share) since 1976 for shorter horizons and 1982 for longer horizons for U.S. publicly traded firms and the median forecasts for prices at the 12-month horizon since 1999. I/B/E/S gathers their forecasts from hundreds of brokerage and independent analysts who track firms as part of their investment research work. Because the forecasts are not anonymous, analysts have a strong incentive to accurately report their expectations.[16] Furthermore, research on I/B/E/S suggests that financial companies' trades are consistent with their own analysts' forecasts and recommendations, which adds to the evidence that reported forecasts genuinely reflect the beliefs of the companies.[17] More importantly, market participants take seriously these analyst forecasts and trade in line with them, with stock prices increasing (decreasing) shortly after upward (downward) revisions in analyst earnings forecasts (Kothari, So, and Verdi 2016).

The long-term price expectations are obtained from the three-to-five-year price targets from the Value Line Investment Survey. Value Line is an independent investment research

[15]To account for possible data errors or extreme outliers, we winsorize annual earnings cross-sectionally at the 1% level.

[16]See Mikhail, Walther, and Willis (1999) and Cooper, Day, and Lewis (2001).

[17]Bradshaw (2004) shows that individual earnings forecasts are correlated to Buy/Sell recommendations, while Chan, Chang, and Wang (2009) show that financial companies' own stock holding changes are significantly positively related to recommendation changes.

and financial publishing firm. The price targets cover approximately 1,700 actively traded U.S. companies every period, approximately 90% of the US publicly listed firms market value.[18] Value Line does not have any investment banking relation with the analyzed firms, nor any other obvious reason for providing biased forecasts. To the best of our knowledge, this is the only widely available survey containing firm-level price forecasts at long horizons.

We construct monthly earnings expectations for every firm in I/B/E/S at different horizons by using the EPS forecasts for up to three Annual Fiscal Periods (FY1-FY3) and the Long-Term Growth measure (LTG) meant to forecast earnings growth over the next "three-to-five years." For each month, we first interpolate across the different horizons in the annual fiscal periods to estimate an expectation over the next twelve months. We repeat this procedure to calculate two-year expectations. To estimate the three-year expectations, we use the two-year expectations and compound them with the long-term growth forecasts. We repeat this procedure to get four-year earnings expectations. We exclude from the main sample the following firms: a) firms without a LTG forecast, b) firms that do not have sufficient forecasts to calculate a 12-month interpolated forecast $E_t^* [\Delta x_{i,t+1}]$, and c) firms that do not have sufficient forecasts in the next year to calculate a 12-month interpolated forecast, $E_{t+1}^* [\Delta x_{i,t+2}]$.[19]

To estimate the price expectations, we obtain the one-year price expectations from the price target in I/B/E/S. We then calculate the four-year price expectation as the three-to-five year price targets from Value Line. We exclude from the main sample those firms missing either a one-year or a three-to-five year price forecast. Since analysts update earnings and price forecasts every month, our expectation data are also in monthly frequency. The main sample covers on average 79.7% of the total market size of firms listed for at least four years in CRSP.

[18]Value Line is an industry standard to the extent that it has been documented that a large portion of investment newsletters herds towards Value Line recommendations (Graham, 1999).

[19]This last point ensures that for every firm in the main sample we can calculate revisions $E_{t+1}^* [\Delta x_{i,t+2}] - E_t^* [\Delta x_{i,t+2}]$. This allows us to study how expectations of future earnings growth are revised after earnings growth surprises.

III. Empirical decomposition

Table I and Figure 1 show the results of decomposition (3) applied both in a FIRE (Full Information Rational Expectations) benchmark and using the subjective expectations. The results show the fraction of price-earnings ratio dispersion that is explained by cumulative earnings growth expectations $\sum_{j=1}^{h}\rho^{j-1}E_t^*[\Delta\tilde{x}_{i,t+j}]$ at $h = 1, 2, 3$, and 4 years, as well as the fraction that is explained by cumulative return expectations $\sum_{j=1}^{h}\rho^{j-1}E_t^*[\tilde{r}_{i,t+j}]$.

We first apply the decomposition under the FIRE benchmark using realized values and compare the results with several FIRE models of risk premia. Overall, we find that these FIRE models struggle to match the decomposition results for realized earnings growth and returns. We then compare this FIRE benchmark with a decomposition using subjective expectations. In Section IV, we examine in more detail the dynamics of these subjective expectations, compare them with several behavioral and learning models, and extend our analysis to anomaly returns.

A. FIRE Benchmark

Let $E_t^{FIRE}[\cdot]$ denote expectations under FIRE. Because forecast errors $\Delta\tilde{x}_{i,t+j} - E_t^{FIRE}[\Delta\tilde{x}_{i,t+j}]$ are uncorrelated with time t variables under FIRE, we know that $Cov\left(E_t^{FIRE}[\Delta\tilde{x}_{i,t+j}], \tilde{px}_{i,t}\right) = Cov\left(\Delta\tilde{x}_{i,t+j}, \tilde{px}_{i,t}\right)$. The same logic also applies to FIRE expectations of future returns and future price-earnings ratios. Thus, the FIRE columns of Table I show the estimates of CF_h, DR_h, FPX_h for each year $h = 1, 2, 3, 4$ using the covariance of $\tilde{px}_{i,t}$ with realized future earnings growth, returns, and price-earnings ratios. For every coefficient, we report the Driscoll-Kraay standard errors, which account for very general forms of spatial and serial correlation, as well as the block-bootstrap standard errors, following the Martin and Wagner (2019) procedure.

Empirically, high price-earnings ratios are associated with lower future returns and slightly higher future earnings growth. The first column of Table I shows that 10.3% of dispersion

Table I

Decomposition of dispersion in price-earnings ratios

This table decomposes the variance of price-earnings ratios using equation (3) at multiple horizons. The *FIRE* columns report the elements CF_h, DR_h and FPX_h of the decomposition using future earnings growth, future negative returns and future price-earnings ratios. The *Expected* columns report the elements of the decomposition using expected earnings growth, expected returns and expected price-earnings ratios. The *Error* columns report the contribution of the forecast errors. For instance, for year $h = 1$, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the *FIRE* column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decomposition estimated over the longer sample period of 1982-2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	$h=1$			$h=2$			$h=3$			$h=4$		
	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error
Main Sample: 1999-2020												
CF_h	0.103***	0.331***	−0.228***	0.075	0.344***	−0.268***	0.071	0.389***	−0.317***	0.099*	0.433***	−0.335***
	[0.037]	[0.024]	[0.032]	[0.052]	[0.020]	[0.046]	[0.052]	[0.020]	[0.048]	[0.054]	[0.020]	[0.053]
	[0.052]	[0.027]	[0.044]	[0.068]	[0.022]	[0.063]	[0.074]	[0.023]	[0.062]	[0.074]	[0.022]	[0.074]
DR_h	0.143***	0.033***	0.110**	0.266***			0.305***			0.320***	0.127***	0.192**
	[0.050]	[0.013]	[0.053]	[0.057]			[0.058]			[0.080]	[0.043]	[0.082]
	[0.051]	[0.014]	[0.054]	[0.070]			[0.075]			[0.102]	[0.045]	[0.096]
FPX_h	0.746***	0.620***	0.126**	0.642***			0.599***			0.550***	0.385***	0.165***
	[0.050]	[0.019]	[0.056]	[0.034]			[0.039]			[0.057]	[0.027]	[0.062]
	[0.044]	[0.024]	[0.051]	[0.038]			[0.048]			[0.064]	[0.028]	[0.076]
Long Sample: 1982-2020												
CF_h	0.137***	0.312***	−0.175***	0.102**	0.335***	−0.232***	0.107**	0.399***	−0.291***	0.147***	0.462***	−0.316***
	[0.026]	[0.021]	[0.027]	[0.048]	[0.023]	[0.041]	[0.043]	[0.025]	[0.035]	[0.040]	[0.027]	[0.034]
	[0.026]	[0.021]	[0.027]	[0.049]	[0.022]	[0.042]	[0.042]	[0.026]	[0.036]	[0.040]	[0.027]	[0.033]

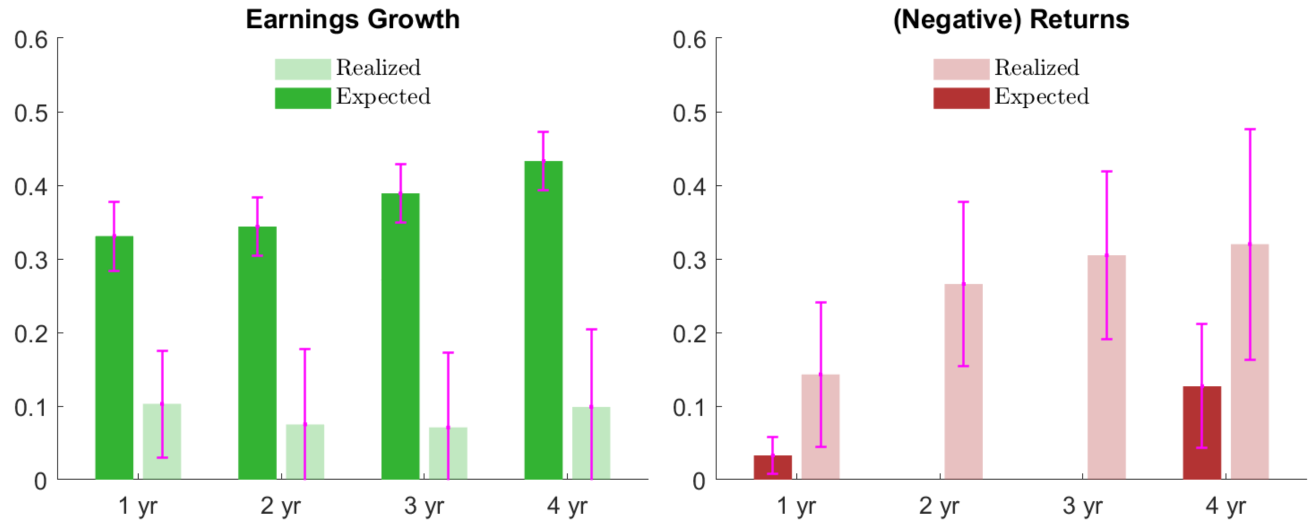


Figure 1. Expected and realized decomposition of price-earnings ratios. This figure illustrates the earnings growth and returns components of the cross-sectional decomposition of $\tilde{px}_{i,t}$ in equation (3). The light bars show the contribution of realized earnings growth and realized returns to the dispersion of price-earnings ratios obtained in the *FIRE* columns of Table I. The dark bars show the contribution of expected earnings growth and expected returns to the dispersion of price-earnings ratios obtained in the *Expected* columns of Table I. Each bar shows Driscoll-Kraay 95% confidence intervals.

in price-earnings ratios is accounted for by differences in one-year future earnings growth and 14.3% is accounted for by differences in one-year future returns. The remaining 74.6% is accounted for by the future price-earnings ratio.[20] At first glance, the statistically significant DR_1 may be surprising given that the value premium has generally been found to be insignificant over recent years (Gonçalves and Leonard, 2023). This is because the value premium is simply measured as the difference in average returns between the highest and lowest price ratio portfolios, which is equivalent to regressing returns onto a dummy variable. Regressing returns onto price-earnings ratios utilizes more information and shows that there are predictable differences in returns between portfolios even in the recent 1999-2020 sample.[21] Gonçalves and Leonard (2023) find a similar result, showing that regressing returns

[20]Note that the three coefficients CF_h, DR_h and FPX_h are not mechanically set to equal one. However, the sum of these coefficients is very close to unity, summing to 0.992, 0.983, 0.975 and 0.969 for each of the four yearly decompositions, which shows that equation (3) holds very tightly, and any potential deviations from the approximation (2) are not correlated with firms' price-earnings ratios.

[21]For example, the average low-minus-high return spread between our lowest price ratio portfolio and highest price ratio portfolio is insignificant. However, regressing this return spread onto the difference in price-earnings ratios $(\tilde{px}_{L,t} - \tilde{px}_{H,t})$ gives a significant relationship even at the 1% confidence level.

onto price-to-fundamentals ratios gives significant results despite the insignificant average value premium.

Continuing with the FIRE results, we see that DR_h accounts for a larger and larger share of price-earnings ratio dispersion as we extend to longer horizons. As shown in the $h = 4$ column of Table I, differences in future four-year returns account for nearly a third of all price-earnings ratio dispersion (32.0%).[22] In comparison, the contribution of future cash flows CF_h remains fairly flat at around 0.1 at all horizons.

The large role of returns in explaining price ratio dispersion poses a quantitative challenge for traditional FIRE asset pricing models. Even models designed to generate a short-term value premium (i.e., low expected returns for high price ratio stocks) struggle to generate enough dispersion in expected returns to match our findings. In Table II, we simulate three FIRE models for the value premium (Berk et al., 1999; Zhang, 2005; Lettau and Wachter, 2007) using their benchmark specifications and calculate the model-implied CF_h and DR_h. As shown by the value of DR_h, in these models, differences in expected returns only account for a small fraction of the dispersion in price-earnings ratios.

Specifically, the three models imply that future returns over the next four years should account for less than 6% of dispersion in price-earnings ratios while, empirically, we find that they account for 32%. In the data and (to some extent) in the models, DR_h increases as we include more horizons. Thus, we also calculate in the model the maximum amount of dispersion that can be explained by returns DR_∞ and find that it is still an order of magnitude smaller than what we observe in the data using just the first four years of realized returns. These results highlight the importance of a quantitative framework. While there are certainly FIRE models in which high price ratio stocks have lower exposure to systematic risk, it is difficult to generate a risk premium that is quantitatively large enough to match the observed relationship between price-earnings ratios and future returns.

[22] These results are consistent with Delao, Han, and Myers (2024), who use a longer sample (1963-2020) to show that at least 43.6% of dispersion in price-earnings ratios are reflected in differences in returns after ten years.

Table II

Decompositions in FIRE Asset Pricing Models

This table calculates the variance decomposition for the price-earnings ratio in different asset pricing models and reports the implied cash flow and discount rate components for one year (CF_1, DR_1) and four years (CF_4, DR_4), as well as the infinite-horizon DR_∞. The first, second, and third rows show the results for models of risk premia. These three models are the model of growth options in Berk et al. (1999), the model of costly reversibility of capital in Zhang (2005), and the model of duration risk in Lettau and Wachter (2007). The last row shows the values measured in the data. All models are solved and estimated using the original author calibrations and simulated over a 20-year sample.

Models	CF_1	CF_4	DR_1	DR_4	DR_∞
Berk, Green, & Naik 1999 (Growth Options)	0.61	0.85	0.01	0.03	0.04
Zhang 2005 (Costly Reversibility of Capital)	−0.31	0.69	−0.01	−0.03	−0.03
Lettau & Wachter 2007 (Duration Premium)	0.03	0.24	0.02	0.06	−0.04
Observed Data (Main Sample)	0.10	0.10	0.14	0.32	n.a.

Why do these models struggle to generate large DR_h? Broadly, these models cover three distinct mechanisms for generating a value premium. However, all three can be thought of as settings in which agents exhibit preferences for the timing of cash flows. In Berk et al. (1999) and Zhang (2005), existing projects (or capital) cannot be adjusted easily in response to aggregate shocks. Instead, firms primarily adjust their choices about initiating new projects or installing new capital when aggregate shocks occur. Thus, firms whose value primarily comes from future potential projects (capital) rather than existing projects (capital) carry a lower risk premium, as they can more easily respond to aggregate shocks. In Lettau and Wachter (2007), aggregate shocks are partly reversed over time, reducing the exposure of longer-horizon cash flows to aggregate risk. Firms whose value mostly comes from future cash flows rather than current cash flows therefore carry a lower risk premium.

Because agents have rational expectations and know the objective parameters in these models, each firm's risk premium is tied to the objective timing of its cash flows. High price ratio stocks can only carry a low risk premium if they objectively have much more backloaded cash flows (i.e., much higher cash flow growth) than their peers. Thus, these models inherently struggle to match the empirical results, in which firms only differ slightly in their objective future cash flow growth but differ substantially in their objective future returns.[23] Appendix E discusses the three models and the simulations in more detail. In Section VI, we propose a model that incorporates this type of preference for the timing of cash flows but, importantly, we do not impose FIRE. In the quantified model, we find that these preferences play an important role in generating cross-sectional dispersion in price ratios because agents believe that firms differ substantially in their future cash flow growth, even though the objective differences in future cash flow growth are minimal.

B. *Subjective Expectations*

The subjective columns of Table I and Figure 1 show the results of the decomposition when we use subjective expectations of earnings growth, returns, and future price-earnings ratios rather than assuming FIRE. Given that price forecasts are only available at the one-year and four-year horizon, we show DR_h and FPX_h for $h = 1, 4$ and CF_h for all horizons $h = 1, 2, 3, 4$. Comparing the subjective results to the FIRE results, there are two important findings.

First, investors substantially overestimate the extent to which high price-earnings ratio stocks will have high future earnings growth. Differences in expected one-year earnings growth account for nearly a third (33.1%) of all dispersion in price-earnings ratios and differences in expected four-year earnings growth account for 43.3% of all price-earnings ratio dispersion. Given that realized one-year and four-year earnings growth only account for 10.3% and 9.9% of the dispersion, respectively, this means that high price-earnings ratios are consistently associated with disappointment in future earnings growth. Rephrased, more than a third of all dispersion in price-earnings ratios is accounted for by the fact that current price-earnings ratios significantly negatively predict future forecast errors (as shown in the "Error" columns). The final row of Table I shows that our earnings growth results are qualitatively and quantitatively similar over the longer 1982-2020 sample.

[23] In general, these models would require extremely large risk aversion in order for the primary effect of differences in future cash flow growth (i.e., CF_h) to be dominated by the secondary effect that differences in cash flow growth generate differences in discount rates (i.e., DR_h). These levels of risk aversion would imply that these models would no longer match the aggregate equity premium.

Second, investors understand that expensive stocks will have lower returns (i.e., a high price-earnings ratio is associated with lower expected returns), but they underestimate the magnitude of the relationship. As shown in the second row of Table I, differences in expected one-year returns account for 3.3% of dispersion in price-earnings ratios and differences in expected four-year returns account for 12.7%. This contrasts sharply with previous findings for aggregate return expectations, which positively comove with aggregate price ratios (Amromin and Sharpe, 2014; Greenwood and Shleifer, 2014; Delao and Myers, 2021). Consistent with the fact that investors overestimate future earnings growth for high $\tilde{px}_{i,t}$, we find that they consistently overestimate the returns for high $\tilde{px}_{i,t}$. In other words, while investors expect lower returns for high $\tilde{px}_{i,t}$ stocks, the realized returns are even worse than expected.

Combined, these two findings emphasize that the mistakes in investors' expectations are about magnitudes, not directions. Investors understand that high price-earnings ratios are associated with higher future earnings growth and lower future returns, but they overestimate the magnitude of the earnings growth relationship and underestimate the magnitude of the return relationship. This highlights the benefit of using a quantitative decomposition which captures magnitudes as well as correlations to study these expectations.

IV. Stubbornness in expectations

The results of Section III indicate that errors in earnings growth expectations account for a notable amount of dispersion in price-earnings ratios. Given that we observe earnings expectations at multiple horizons, in Section IV.A we examine how earnings expectations are revised as future earnings are realized. The key finding is that earnings expectations are "stubborn," meaning that positive earnings growth surprises actually lower expected next period earnings growth, as earnings are forecasted to return to their previous levels. This can be more easily understood when we frame the result in levels rather than growth. We show that earnings surprises only lead to small revisions in expected $\tilde{x}_{i,t+2}$, $\tilde{x}_{i,t+3}$, and

$\tilde{x}_{i,t+4}$. In Section IV.B, we discuss how these stubborn expectations differ from expectation-formation models such as Hirshleifer, Li, and Yu (2015), Bordalo et al. (2024a), Lewellen and Shanken (2002) and Nagel and Xu (2022), in which expectations for future earnings are sensitive to recent realizations. In Section IV.C, we extend these results to 20 different anomaly returns.

A. *Gradual adjustment to surprises*

To understand the response to earnings surprises, we regress revisions in expectations on the earnings surprise $\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]$. To give a benchmark, we first consider the regression

$$\tilde{x}_{i,t+h} - E_t^*[\tilde{x}_{i,t+h}] = \alpha_h + \gamma_h(\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]) + \varepsilon_{t+h} \quad (4)$$

for $h = 2, 3, 4$ years. We estimate $\gamma_2 = 0.91^{***}$, $\gamma_3 = 0.89^{***}$, and $\gamma_4 = 0.89^{***}$, and the black dashed line of Figure 2 shows these results for realized future earnings.

We then estimate the revision in expectations after an earnings surprise,

$$E_{t+j}^*[\tilde{x}_{i,t+h}] - E_{t+j-1}^*[\tilde{x}_{i,t+h}] = \alpha_{h,j}^* + \gamma_{h,j}^*(\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]) + \eta_{h,t+j} \quad (5)$$

for $1 \leq j < h$. The coefficient $\gamma_{h,j}^*$ captures how much earnings surprises at $t+1$ predict forecast revisions at time $t+j$.[24] Under FIRE, these revisions should happen immediately and there should be no further predictable revisions nor predictable forecast errors, i.e., $\gamma_{h,1}^* = \gamma_h$ and $\gamma_{h,j}^* = 0$ for $j > 1$.

Columns 1-3 of Table III Panel A show the results for $\gamma_{h,j}^*$. Figure 2 shows the cumulative sum of the $\gamma_{h,j}^*$ coefficients, i.e., how much $E_{t+j}^*[\tilde{x}_{i,t+h}] - E_t^*[\tilde{x}_{i,t+h}]$ responds to $\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]$. Rather than an immediate large revision, we find a sequence of gradual revisions over the next three years. After a one unit positive earnings surprise, earnings are expected to largely revert back to their previous levels, as shown by the first green line of Figure

[24] This is the complement of Coibion and Gorodnichenko (2015) regressions. Rather than studying how a single revision predicts future forecast errors over time, this regression tests how a single forecast error predicts a sequence of future revisions. To measure these revisions, in Table III and Figures 2 and 5, we restrict our portfolios to firms for which revisions are observed for $h = 2, 3, 4$ which only reduces the total market value in our portfolios by 1.8%. This restriction is not imposed for any other tables.

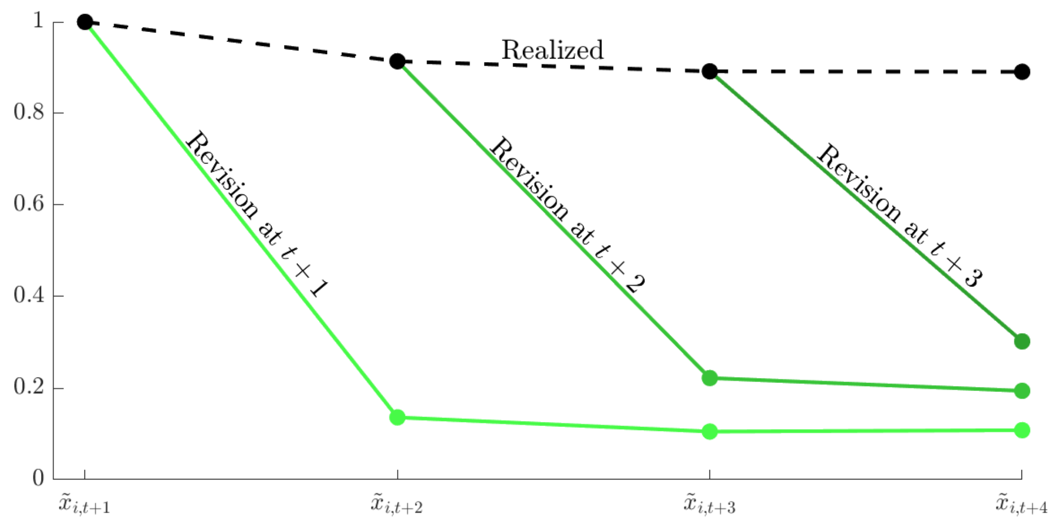


Figure 2. Revisions after an earnings surprise. This figure illustrates the gradual revision in expectations of future earnings following an earnings surprise at $t+1$. Each solid line represents the revised expected path of earnings, $E^*_{t+j}[\tilde{x}_{i,t+h}] - E^*_t[\tilde{x}_{i,t+h}]$, after a one unit earnings surprise, $\tilde{x}_{i,t+1} - E^*_t[\tilde{x}_{i,t+1}]$. The line *Revision at t+1* represents the coefficients $\gamma^*_{h,1}$, *Revision at t+2* shows the cumulative sum $\sum_{j=1}^{2}\gamma^*_{h,j}$, and *Revision at t+3* shows the cumulative sum $\sum_{j=1}^{3}\gamma^*_{h,j}$. The dashed line shows the benchmark γ_h from equation (4), representing the impact of earnings surprises on future realized earnings.

2. Expectations of $\tilde{x}_{i,t+2}$, $\tilde{x}_{i,t+3}$, $\tilde{x}_{i,t+4}$ are only revised upwards by 0.14, 0.11, and 0.11 respectively. Because of this small revision, in the following year $(t+2)$, investors are on average positively surprised by the $\tilde{x}_{i,t+2}$ earnings and revise their expectations for $\tilde{x}_{i,t+3}$ and $\tilde{x}_{i,t+4}$ by an additional 0.12 and 0.09. This small revision means that investors are again positively surprised in period $t+3$ by $\tilde{x}_{i,t+3}$ and revise their expectations for $\tilde{x}_{i,t+4}$ by another 0.11.

A natural question is whether this stubborn adjustment to new information is unique to analyst forecasts or if it extends to market prices more broadly. To investigate this, we examine how stock returns respond to earnings surprises using the following specification:

$$\tilde{r}_{i,t+j} \quad = \quad \lambda_j + \theta_j \left(\tilde{x}_{i,t+1} - E^*_t\left[\tilde{x}_{i,t+1}\right]\right) + \upsilon_{t+j} \qquad (6)$$

where θ_j captures the extent to which contemporaneous and future returns respond to earnings surprises. Table III Panel B shows the results. Returns do, indeed, respond to earnings surprises, but the reaction is gradual. In line with the gradual adjustment of earnings fore-

Table III

Gradual adjustment of expectations

Panel A shows the gradual adjustment of expectations about future earnings $\tilde{x}_{i,t+h}$ after an earnings surprise at $t+1$, i.e., the coefficients $\gamma^*_{h,j}$ estimated using equation (5). For example, the first row shows $\gamma^*_{2,1}$, the effect of an earnings surprise $\tilde{x}_{i,t+1} - E^*_t\left[\tilde{x}_{i,t+1}\right]$ on the revisions to two-year earnings $E^*_{t+1}\left[\tilde{x}_{i,t+2}\right] - E^*_t\left[\tilde{x}_{i,t+2}\right]$. The second row shows $\gamma^*_{3,1}$ and $\gamma^*_{3,2}$, the effect of an earnings surprise $\tilde{x}_{i,t+1} - E^*_t\left[\tilde{x}_{i,t+1}\right]$ on revisions about $\tilde{x}_{i,t+3}$ occurring in years $t+1$ and $t+2$. Panel B shows the estimated coefficient θ_j from equation (6) which estimates the reaction of returns $\tilde{r}_{i,t+1}, \tilde{r}_{i,t+2}$, and $\tilde{r}_{i,t+3}$ after an earnings surprise at $t+1$. The sample period is 1999 to 2020. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

Panel A: Earnings revisions			
	$j=1$	$j=2$	$j=3$
$\left(E^*_{t+j} - E^*_{t+j-1}\right)\left[\tilde{x}_{i,t+2}\right]$	0.14*		
	(0.07)		
$\left(E^*_{t+j} - E^*_{t+j-1}\right)\left[\tilde{x}_{i,t+3}\right]$	0.11***	0.12***	
	(0.03)	(0.04)	
$\left(E^*_{t+j} - E^*_{t+j-1}\right)\left[\tilde{x}_{i,t+4}\right]$	0.11***	0.09**	0.11***
	(0.03)	(0.03)	(0.03)
Panel B: Returns			
$\tilde{r}_{i,t+j}$	0.13**	0.15***	0.12***
	(0.05)	(0.05)	(0.03)

casts by 9 to 14pp each year, we see a gradual return of 12 to 15pp each year after the earnings surprise. This pattern of gradual returns provides strong evidence that the stubborn adjustment to earnings information extends beyond analyst forecasts to market prices themselves, suggesting a broader phenomenon of gradual information incorporation in financial markets.

Another natural question is whether these stubborn expectations could be due to certain analysts not updating their forecasts. The fact that returns follow a similar pattern already points against the idea that this is a mechanical feature of the analyst data. However, to show this point more directly, in Section V, we repeat the exercise excluding any analyst forecasts that are unchanged from their prior value, and we find nearly identical results. This is because our tests deal with one-year revisions in expectations, $E^*_{t+j}\left[\tilde{x}_{i,t+h}\right] - E^*_{t+j-1}\left[\tilde{x}_{i,t+h}\right]$. While analysts may not update their forecasts every month, it is rare for an analyst to leave a forecast unchanged for an entire year.

Overall, our results align with a long literature on post earnings announcement drift

(PEAD). However, our results emphasize that this gradual adjustment over multiple years is relevant for understanding the level of prices and price ratios, not just one-month to one-year returns around quarterly earnings announcements. For example, we can already see evidence of stubborn expectations in the decomposition of price-earnings ratios in Table I. As shown in Column 3 of Table I, one-year earnings growth surprises substantially exceed one-year unexpected returns. Specifically, while 22.8% of the $\tilde{px}_{i,t}$ dispersion is reflected in one-year earnings growth forecast errors, only 11.0% appears in one-year unexpected returns. In other words, disappointment in one-year earnings growth does not translate into an equally large disappointment in one-year returns. This statistically significant 11.8% difference suggests that investors respond stubbornly to earnings disappointment, i.e., stock prices adjust less than 1 for 1 to negative earnings surprises.

B. Connection to expectation-formation models

How do these findings compare to common behavioral or learning models of cash flow expectation formation? Agents may overstate the persistence of growth (e.g., Hirshleifer, Li, and Yu 2015), or have diagnostic expectations of growth (e.g., Bordalo et al. 2024a). We also consider models in which agents are learning about the mean of an i.i.d. growth process (e.g., Lewellen and Shanken 2002 and Nagel and Xu 2022). These mechanisms can all potentially explain the result from Section III that investors' cash flow growth expectations overstate the objective relationship between current price-earnings ratios and future earnings growth.[25]

However, as detailed in Appendix F, these models all imply large immediate returns and revisions in expected earnings after an earnings surprise. In the analyst forecast data, a positive earnings surprise lowers expected next period growth, as earnings are expected to largely revert to their previous level. In contrast, these models imply that a positive earnings surprise should raise expected next period earnings growth, meaning that the expected level

[25]In Appendix F, we also discuss the diagnostic expectations model of Bordalo et al. (2019). Because this model features diagnostic expectations about earnings levels, rather than earnings growth, it predicts that high price-earnings ratio stocks have low one-year earning growth expectations, which is not consistent with our Section III findings.

of future earnings increases more than 1 for 1 with a positive earnings surprise, i.e., $\gamma^*_{h,1} > 1$. Similarly, the data shows that a one unit earnings surprise is only associated with a moderate contemporaneous return, which aligns with the fact that expected earnings are only moderately revised. In these models, a positive surprise $\tilde{x}_{i,t+1} - E^*_t[\tilde{x}_{i,t+1}]$ raises current returns more than 1 for 1 , i.e., $\theta_1 > 1$, as the earnings surprise raises the expected level of all future cash flows more than 1 for 1.

In Section VI, we modify the constant-gain learning model of Nagel and Xu (2022) by including temporary shocks to the level of earnings and show that this generates stubborn earnings expectations and gradual returns.[26] This is only a small modification, but it is important for explaining why rational arbitrageurs would not offset these errors in expectations. While the model of Section VI focuses on a single representative agent, Appendix I.1 generalizes the model to allow for rational arbitrageurs. In this extended model, non-stubborn errors in expectations have limited impact on asset prices, as they are largely traded away by the rational arbitrageurs. If non-stubborn earnings expectations are too high, arbitrageurs know these expectations will on average be disappointed in the next period, leading to a large revision in expectations and a large one-period return from shorting the asset. In comparison, biases due to stubborn expectations have a large impact on prices. If stubborn earnings expectations are too high, arbitrageurs know that the next period return from shorting the asset will be small, as prices gradually adjust over many periods. If arbitrageurs have shorting costs or liquidity costs, then they prefer to bet against stocks whose returns are realized quickly.

Importantly, our results do not mean that the biases studied in these previous papers are not relevant for explaining return predictability or other patterns in asset prices. They simply indicate that if one is interested in explaining the level of prices or price ratios, then biases due to stubborn expectations are particularly relevant. In line with the old adage that

[26] In this modification, earnings surprises are largely attributed to temporary shocks to the level of earnings. This aligns with evidence of overconfidence (see Daniel and Hirshleifer (2015) for a summary), as overconfident investors will place less weight on new public information and will stick tightly to their prior beliefs.

"markets can remain irrational longer than you can remain solvent," errors in expectations that are stubborn may be more difficult for informed traders to bet against, as they may have to wait years for prices to correct.

C. Extending to anomaly returns

This section shows that the evidence on stubbornness in earnings expectations (i.e., that earnings surprises lead to small revisions) is not limited to cross-sectional differences in valuation ratios. Instead, this pattern is reflected across a wide range of one-year cross-sectional anomalies.

To demonstrate this, we use our high and low price-earnings ratio portfolios from the previous section, as well as 20 other annual anomalies documented in Hou, Xue, and Zhang (2015).[27] We also calculate a representative anomaly that sorts stocks based on the 20 different variables and uses the average ranking across these variables in the sorting and in the regressions. For each anomaly, we sort stocks into five equal-value portfolios based on the anomaly variable, and then measure earnings expectations and revisions for each of the portfolios.[28] Using the regression specified in equation (5), we estimate $\gamma^*_{2,1}$, which is the revision in two-year earnings expectations after a $t+1$ earnings surprise.

As illustrated in Figure 3, we find that for most anomalies, positive earnings surprises $\tilde{x}_{i,t+1} - E^*_t\left[\tilde{x}_{i,t+1}\right]$ result in only minor revisions to future earnings expectations $E^*_{t+1}\left[\tilde{x}_{i,t+2}\right] - E^*_t\left[\tilde{x}_{i,t+2}\right]$. Across all anomalies, this coefficient is small and significantly less than one. This means that positive earnings surprises actually *decrease* expected next period growth $E^*_{t+1}\left[\Delta\tilde{x}_{i,t+2}\right]$ as earnings are forecasted to largely return to their previous levels. Since we do not need return expectations to perform this test, we repeat the test over the longer 1982-2020 sample for robustness and find similar results in Table AIX.

[27] Beyond the price-earnings ratio, we find 21 anomalies documented in Hou, Xue, and Zhang (2015) which are applicable to annual returns. We then drop the size anomaly because the analyst forecasts are provided primarily for large firms and are thus not suited to cover portfolios over this anomaly.

[28] To perform these tests, stocks are required to have one-year expected and realized earnings growth, returns, and price-earnings ratios. We also require that stocks have a current two-year earnings expectation $E^*_t\left[\tilde{x}_{i,t+2}\right]$ and a future one-year earnings expectation $E^*_{t+1}\left[\tilde{x}_{i,t+2}\right]$ for our test of revisions.

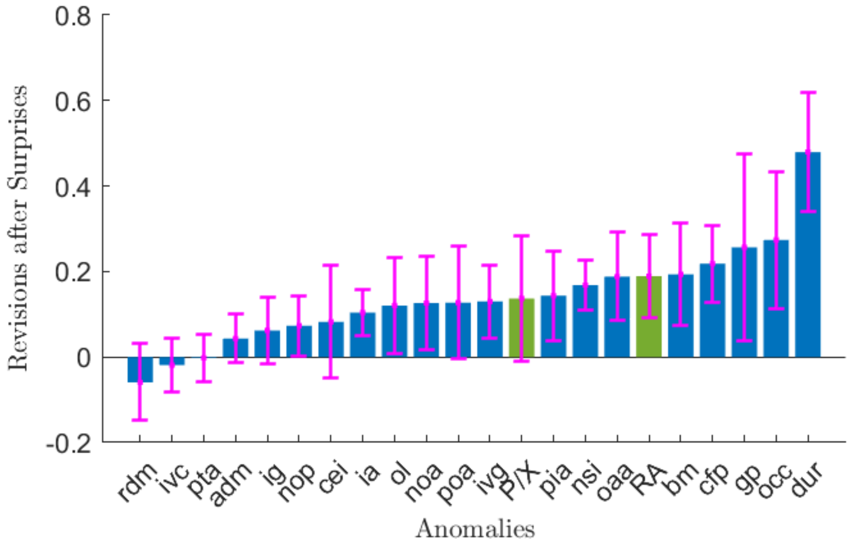


Figure 3. Revisions in anomaly expected future earnings. This figure shows the effect of earnings surprises on revisions for each set of anomaly portfolios. Each bar shows the coefficient from regressing the revision in expected earnings $E^*_{t+1}[\tilde{x}_{i,t+2}] - E^*_t[\tilde{x}_{i,t+2}]$ on the earnings surprise $\tilde{x}_{i,t+1} - E^*_t[\tilde{x}_{i,t+1}]$. The coefficients are shown in ascending order. Individual anomalies are shown in blue. In green, we show the Representative Anomaly (RA) that sorts stocks based on their average ranking across all of the individual anomalies, as well as the results for the portfolios used to study cross-sectional variation in price-earnings ratios (P/X).

Section IV.A illustrated how this stubbornness in expectations affects market prices through the slow adjustment of returns. Here, we quantify how this sluggish adjustment contributes to the returns of specific anomaly portfolios. Consider an anomaly variable $\tilde{a}_{i,t}$, such as profitability or investment, which predicts next-year returns. To facilitate comparisons, we normalize $\tilde{a}_{i,t}$ to have unit variance and positive covariance with future returns. From equation (1), we have that

$$\underbrace{Cov\left(\tilde{r}_{i,t+1} - E^*_t\left[\tilde{r}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,r}} \approx \underbrace{Cov\left(\tilde{x}_{i,t+1} - E^*_t\left[\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,x}} + \underbrace{\rho Cov\left(\tilde{px}_{i,t+1} - E^*_t\left[\tilde{px}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,px}}. \tag{7}$$

Note that unexpected earnings growth $\Delta\tilde{x}_{i,t+1} - E^*_t[\Delta\tilde{x}_{i,t+1}]$ is identical to unexpected earnings $\tilde{x}_{i,t+1} - E^*_t[\tilde{x}_{i,t+1}]$. For robustness, Appendix C.1 shows an exact decomposition based on price growth, which gives very similar results.

Table IV

Unexpected anomaly returns

This table measures and decomposes unexpected anomaly returns. The Representative Anomaly is the average ranking of each stock across 20 different anomalies, and $\tilde{px}_{i,t}$ is the demeaned price-earnings ratio. For each anomaly variable, we sort stocks into five equal-value portfolios based on the anomaly variable. Each column shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. Both anomaly variables are scaled to have unit variance and to positively comove with future returns. The three dependent variables are the unexpected return $\tilde{r}_{i,t+1} - E_t^* [\tilde{r}_{i,t+1}]$, the earnings forecast errors $\tilde{x}_{i,t+1} - E_t^* [\tilde{x}_{i,t+1}]$, and the price-earnings ratio forecast errors $\rho \left(\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]\right)$. The sample period is 1999 to 2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	Representative Anomaly	$\tilde{px}_{i,t}$
$\tilde{r}_{i,t+1} - E_t^* [\tilde{r}_{i,t+1}]$	0.034***	0.033**
	[0.013]	[0.016]
	[0.013]	[0.016]
$\tilde{x}_{i,t+1} - E_t^* [\tilde{x}_{i,t+1}]$	0.064***	0.069***
	[0.020]	[0.010]
	[0.020]	[0.014]
$\rho \left(\tilde{px}_{i,t+1} - E_t^* \left[\tilde{px}_{i,t+1}\right]\right)$	−0.032***	−0.038**
	[0.009]	[0.017]
	[0.009]	[0.015]

Under FIRE, we would have $\sigma_{a,r}, \sigma_{a,x}, \sigma_{a,px} = 0$, i.e., any predictable anomaly returns would be fully anticipated and $\tilde{a}_{i,t}$ would not predict forecast errors. For example, a higher $\tilde{a}_{i,t}$ might be related to higher risk exposure and investors would require higher returns on these stocks as compensation. If expectations deviate from FIRE, then positive values of $\sigma_{a,r}$ indicate that investors understate the relationship between $\tilde{a}_{i,t}$ and future returns. In other words, the high returns on high $\tilde{a}_{i,t}$ stocks are not fully anticipated. In comparison, $\sigma_{a,x}$ and $\sigma_{a,px}$ reflect the extent to which unexpected returns are explained by predictable errors in one-year earnings growth expectations and expectations of the future price-earnings ratio, respectively. A relatively large $\sigma_{a,x}$ indicates that unexpected returns of high $\tilde{a}_{i,t}$ stocks can be largely explained by unexpectedly high earnings.

For each of the 21 anomaly variables, we measure forecast errors for one-year returns, earnings, and price-earnings ratios, and regress each on the anomaly variable to estimate the decomposition in equation (7). Table IV shows that the results for the representative anomaly are qualitatively and quantitatively similar to those for the $\tilde{px}_{i,t}$ portfolios in Section III.B. The first row shows that a one standard deviation increase in either of the anomaly variables is associated with a roughly 3pp increase in unexpected returns (0.034 and 0.033, respectively). This increase in unexpected returns is more than accounted for by the roughly 6pp increase in unexpected earnings (0.064 and 0.069, respectively). Since expectations of future earnings are only modestly revised in response to an earnings surprise, as shown in Figure 3, these surprises do not immediately translate 1-1 into unexpected returns. Rephrased, prices do not move 1-1 with the earnings surprise, meaning that the 6pp earnings surprise also leads to the price-earnings ratio being roughly 3pp lower than expected.

Figure 4 shows the results for each of the 22 anomalies (the 20 individual anomalies, our price-earnings ratio portfolios and the representative anomaly). For almost every anomaly, we find positive $\sigma_{a,r}$, meaning that investors do not fully anticipate the high returns on high $\tilde{a}_{i,t}$ stocks. Further, most anomalies (17 out of 22) are associated with large positive one-year earnings forecast errors, as shown by the magnitude of $\sigma_{a,x}$. Appendix Table AX provides the full decomposition for each anomaly.

Comparing $\sigma_{a,r}$ and $\sigma_{a,x}$ across anomalies, we see that anomalies with higher $\sigma_{a,r}$ generally have higher $\sigma_{a,x}$, i.e., larger unanticipated returns are associated with larger one-year earnings surprises, and $\sigma_{a,x}$ is generally larger than $\sigma_{a,r}$, i.e., earnings surprises translate less than 1-1 into unexpected returns. This means that our findings on the dynamics of earnings surprises and returns from Section IV.A also extends to most anomaly portfolios. Rephrased, many anomaly returns can be accounted for by prices gradually responding to errors in earnings expectations.

To summarize, consistent with the results from Section IV.A, we find only moderate revisions to earnings surprises in anomaly portfolios. Our results highlight that unexpected returns are positively associated with earnings surprises but with a less-than-proportional relationship. This evidence, once again, points against models in which unexpected realized

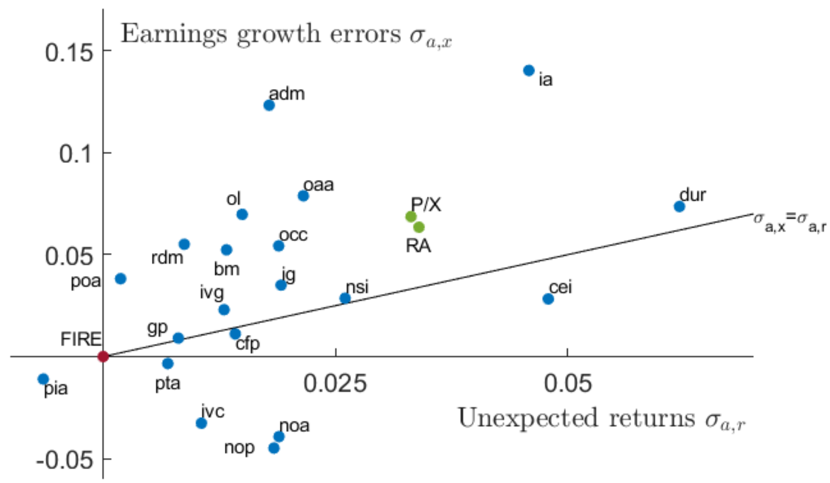


Figure 4. Unexpected anomaly returns and earnings surprises. This figure shows the decomposition results $(\sigma_{a,r}, \sigma_{a,x})$ for each anomaly $\tilde{a}_{i,t}$. The x-axis shows $\sigma_{a,r} = Cov\left(\tilde{r}_{i,t+1} - E_t^*\left[\tilde{r}_{i,t+1}\right], \tilde{a}_{i,t}\right)$, which measures how much the anomaly variable predicts unexpected returns. The y-axis shows $\sigma_{a,x} = Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)$, which measures how much the anomaly variable predicts one-year earnings growth forecast errors. The anomalies are shown in blue. In red, we show the FIRE benchmark, which is that $\sigma_{a,r}$ and $\sigma_{a,x}$ should equal 0 for all anomalies. In green, we show a Representative Anomaly (RA) that sorts stocked based on their average ranking across all of the individual anomalies, as well as the results for the portfolios used to study cross-sectional variation in price-earnings ratios (P/X). Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to positively comove with future returns.

returns are highly sensitive to recent earnings surprises and highlights the benefit of quantitative decompositions which allow for these types of comparisons. In Section VI, we propose a model that can replicate these dynamics of earnings surprises and unexpected returns.

V. Robustness checks

We perform a number of robustness checks for the main results from Sections III and IV. First, in addition to the Driscoll-Kraay and block-bootstrap standard errors reported in Table I, we also calculate the significance of our results under a worst-case scenario for overlapping observations. Specifically, in Appendix C.2, we perform Bauer and Hamilton (2018) simulations, which account for trends and potential small-sample bias, and assume a worst-case scenario for overlapping observations in which residuals are MA(47). Note that this substantially overstates the measured persistence of our residuals. We find that all of the earnings growth and return coefficients which are significant at the 5% level in Table I remain significant at the 5% level in Table AIII, even under this worst-case scenario.

Second, we estimate an exact decomposition to remove the approximation in equation (3). Table AI shows that this exact decomposition gives nearly identical results to Table I.

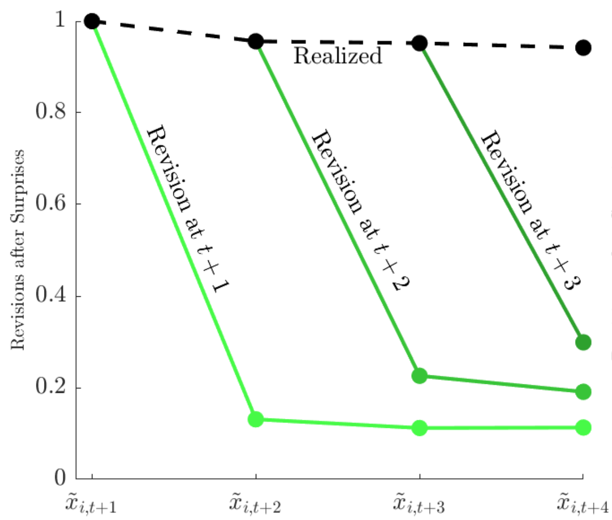
Third, in Table AIV, we address the concern that dispersion in price-earnings ratios may potentially be driven by fluctuations in one-year earnings rather than cross-sectional differences in prices. We find that the dispersion in price-earnings ratios is nearly identical to the dispersion in price-to-smoothed-three-year-earnings ratios.[29] Further, we show that the decomposition results are not changed in any noticeable way when we repeat the decomposition for price-to-smoothed-three-year-earnings ratios.

Fourth, we address potential survivorship bias. For our main estimation, we require that stocks have observed future prices and future earnings, as this allows us to study forecast errors in subjective expectations. However, in Table AV, we remove this requirement and calculate future portfolio outcomes by reinvesting delisted stocks based on the delisting return. We find almost no change in our results.

Finally, we assess whether the evidence on the gradual adjustment of earnings expectations can be attributed to infrequent updating by analysts or stale forecasts. Table III Panel B shows that we find a similar gradual adjustment in returns, which already points against the idea that gradual adjustment is purely a mechanical feature of the analyst data. However, to address this more thoroughly, we follow Bouchaud et al. (2019) and construct alternative consensus forecasts using detailed analyst-by-analyst data from the I/B/E/S Detail History File. Specifically, we include only those price forecasts and earnings forecasts that were explicitly updated within the current quarter.[30] This avoids any mechanical staleness

[29]The standard deviation of cross-sectionally demeaned price-earnings ratios is 30.1% and the standard deviation of cross-sectionally demeaned price-to-three-year-earnings ratios is 32.6%. This demonstrates that smoothing the denominator does not reduce the cross-sectional dispersion in price ratios.

[30]The reason why we do not make this our benchmark specification is that an analyst may choose not to



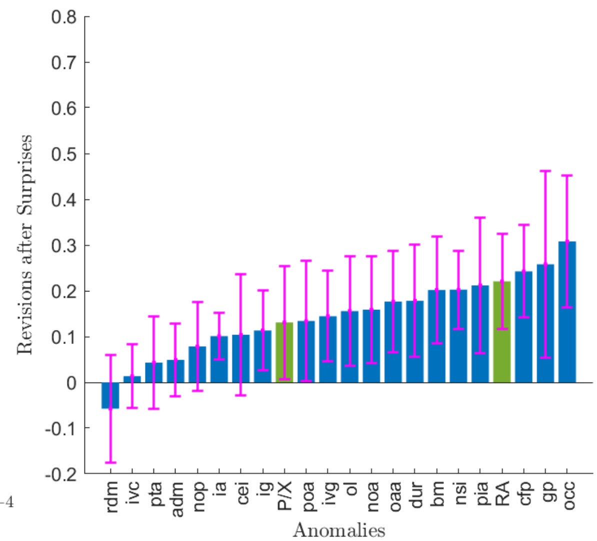


Figure 5. Evidence of stubborn expectations using only actively updated forecasts. These figures replicate Figures 2 and 3 shown in Section IV. To eliminate the possibility that revisions to expected earnings are driven by stale forecasts, the consensus expectations are constructed exclusively from those individual earnings forecasts that were actively updated each quarter.

in the data and ensures that all included forecasts reflect recent revisions.

Using this alternative consensus forecast, we re-estimate the impact of earnings surprises on the revision of expectations in our main $p\tilde{x}_{i,t}$ portfolios and the other 20 anomalies. Figure 5 reveals that the magnitude of the gradual adjustment in forecast revisions is consistent with the results in Figures 2 and 3. Tables AVI and AVII show the detailed estimates. This indicates that the documented stubbornness cannot be attributed to stale forecasts. We also calculate the subjective and realized decompositions using the alternative consensus forecasts and find no material differences in the main results, as demonstrated by Table AVIII.

The key reason why this change has such a limited impact on our results is because we are studying fairly long horizons. Our shortest horizon test is the one-year forecast revision. While staleness can occur at the monthly level, annual forecast revisions—which underpin our analysis—are infrequently stale, with most analysts providing updates within this timeframe. Combined, these results confirm that the gradual adjustment of expectations and our decomposition results are robust to concerns about staleness in forecasts.

update her forecasts specifically because her beliefs have not changed. In that scenario, dropping the analyst would be removing useful information about expectations.

VI. Model of stubborn expectations and volatile prices

In this section, we introduce a structural model of cash flow expectations and discount rates. The main component of the model is gradual learning about cash flow growth. To match our results on subjective return expectations, we also include risk premia related to cash flow timing à la Lettau and Wachter (2007).

The quantified model fulfills three key purposes. First, quoting Brunnermeier et al. (2021), *"Research focus should be on motivating, building, calibrating, and estimating models with non-RE beliefs rather than on merely rejecting RE models. To make further progress, we need structural models of belief dynamics that can compete with RE models in explaining asset prices and empirically observed beliefs."*[31] This model intends to be a step in this direction. It provides a quantitative model that generates realistic asset pricing moments and outperforms the FIRE models of Table II in matching the empirical decomposition results. Second, the model demonstrates that stubborn errors in expectations increase price volatility, even though these two features seem opposed at first glance. Third, the quantified model allows us to extend the decomposition in equation (3) beyond the four-year horizon to estimate the full role of subjective expected earnings growth and subjective discount rates in accounting for the dispersion in price-earnings ratios.

A. Cash flows and the stochastic discount factor

Throughout this section, we use lowercase letters to denote log values, $z \equiv \log(Z)$. For each firm i, the log cash flow $x_{i,t}$ has an aggregate and a firm-level component,

[31]Note that they use RE as a shorthand for full information rational expectations and specifically highlight learning about parameters as a promising form on non-RE models to explore: *"For example, models of Bayesian learning relax the RE assumption that agents know the model of the world and its parameter values".*

$$x_{i,t} = x_t^{agg} + \tilde{x}_{i,t} \tag{8}$$

$$x_t^{agg} = \phi x_{t-1}^{agg} + u_t \tag{9}$$

$$\tilde{x}_{i,t} = g_i t + v_{i,t}. \tag{10}$$

The aggregate component is an AR(1) process, which can be thought of as business-cycle fluctuations. The firm-level component is a firm-specific trend $g_i t$ plus noise to capture potential cross-sectional differences in growth rates. The shocks $u_t, v_{i,t}$ are uncorrelated and have variances σ_u^2, σ_v^2.

The agent has a log stochastic discount factor

$$m_{t+1} = -r^f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} \tag{11}$$

which depends on the aggregate shock u_{t+1}.

B. Subjective cash flow expectations

Objectively, the value of g_i is identical across firms, $g_i = \bar{g}$.[32] However, the agent does not know each firm's g_i and forms her subjective expectation $E_t^*[g_i]$ using constant-gain learning,

$$E_t^*[g_i] = E_{t-1}^*[g_i] + \beta\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right) \tag{12}$$

$$E_t^*[v_{i,t}] = (1-\beta)\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right) \tag{13}$$

where β is the constant-gain parameter. Specifically, after observing the surprise $\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]$, she attributes portion β to firm-specific growth and portion $(1-\beta)$ to the noisy shock $v_{i,t}$. Her expectation for the future growth of the firm-level component is then

$$E_t^*[\Delta\tilde{x}_{i,t+1}] = E_t^*[g_i] - E_t^*[v_{i,t}]. \tag{14}$$

Her expectation for the future level of the firm-level component is

[32] Given that our empirical analysis focuses on price-earnings ratios, we normalize $\bar{g}$ to 0 without loss of generality.

$$E_t^*\left[\tilde{x}_{i,t+n}\right] = \tilde{x}_{i,t} + nE_t^*\left[g_i\right] - E_t^*\left[v_{i,t}\right]. \tag{15}$$

C. Prices and subjective risk premia

Sections VI.A and VI.B lay out all of the elements and assumptions of the model. In this subsection, we simply combine the agent's beliefs and the stochastic discount factor to calculate the price for various claims. Appendix A gives the details for all of the equations.

To start, let $P_t^{(n)}$ be the price of an n-period aggregate strip, i.e., a claim that pays X_{t+n}^{agg} in n periods. The aggregate strip price is

$$\begin{aligned} P_t^{(n)} &= E_t^*\left[\left(\prod_{j=1}^{n} M_{t+j}\right) X_{t+n}^{agg}\right] \\ &= \exp\left\{-nr^f - \gamma\sigma_u^2\frac{1-\phi^n}{1-\phi} + \frac{1}{2}\sigma_u^2\frac{1-\phi^{2n}}{1-\phi^2} + \phi^n x_t^{agg}\right\}. \end{aligned} \tag{16}$$

The realized return on the strip is

$$R_{t+1}^{(n)} = \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}} = \exp\left\{r^f + \gamma\sigma_u^2\phi^{n-1} - \frac{1}{2}\sigma_u^2\phi^{2(n-1)} + \phi^{n-1}u_{t+1}\right\} \tag{17}$$

and the subjective expected return on the strip is

$$E_t^*\left[R_{t+1}^{(n)}\right] = \exp\left\{r^f + \gamma\sigma_u^2\phi^{n-1}\right\}. \tag{18}$$

The first term (r^f) reflects the risk-free rate and the second term ($\gamma\sigma_u^2\phi^{n-1}$) reflects the subjective risk premium, i.e., the compensation agents require for exposure to risk.

Equation (18) shows the main feature of subjective risk premia in this model: longer horizon strips carry a lower annual subjective risk premium $\gamma\sigma_u^2\phi^{n-1}$. Equation (9) shows that aggregate shocks are persistent but not permanent. This means that short horizon cash flows are disproportionately sensitive to the aggregate shock.[33] Because of this, the annual risk premium is higher for short horizon cash flows. This is similar to the mechanism in

[33] Specifically, the covariance of the log cumulative stochastic discount factor $\log\left(\prod_{j=1}^{n} M_{t+j}\right)$ with the aggregate cash flows x_{t+n}^{agg} is $\frac{1-\phi^n}{1-\phi}\gamma\sigma_u^2$, which increases with horizons but not proportionally.

Lettau and Wachter (2007).

Each firm i can be viewed as a collection of strips. Specifically, since shocks to the firm-level component $v_{i,t}$ are uncorrelated with the aggregate shock, we can express the firm's price as

$$P_{i,t} = \sum_{n=1}^{\infty} E_t^* \left[\left(\prod_{j=1}^{n} M_{t+j} \right) X_{t+n}^{agg} \tilde{X}_{i,t+n} \right] = \sum_{n=1}^{\infty} P_t^{(n)} \exp\left\{ \frac{1}{2}\sigma_v^2 + E_t^* \left[\tilde{x}_{i,t+n} \right] \right\}. \quad (19)$$

The subjective expected return on firm i is then simply a weighted average of the subjective expected return on the individual strips,

$$E_t^* \left[R_{i,t+1} \right] = E_t^* \left[\frac{X_{t+1}^{agg} \tilde{X}_{i,t+1} + P_{i,t+1}}{P_{i,t}} \right] = \sum_{n=1}^{\infty} w_{i,t,n} E_t^* \left[R_{t+1}^{(n)} \right] \quad (20)$$

where the weight $w_{i,t,n} = \frac{\exp\{nE_t^*[g_i]\}P_t^{(n)}}{\sum_{n=1}^{\infty} \exp\{nE_t^*[g_i]\}P_t^{(n)}}$ captures how much of the firm's value in equation (19) comes from its horizon n cash flows.

The realized return for firm i is

$$R_{i,t+1} = \frac{X_{t+1}^{agg} \tilde{X}_{i,t+1} + P_{i,t+1}}{P_{i,t}} = \sum_{n=1}^{\infty} w_{i,t,n} R_{t+1}^{(n)} \frac{E_{t+1}^* \left[\tilde{X}_{i,t+n} \right]}{E_t^* \left[\tilde{X}_{i,t+n} \right]}. \quad (21)$$

In addition to depending on a weighted average of realized strip returns $R_{t+1}^{(n)}$, the realized firm return also depends on the change in the expected future firm-level component. From equations (12), (13), and (15), this change in expectations can be expressed entirely in terms of the surprise about one-period growth $\Delta\tilde{x}_{i,t+1} - E_t^* \left[\Delta\tilde{x}_{i,t+1} \right]$, as for $n \geq 2$ we have that

$$\frac{E_{t+1}^* \left[\tilde{X}_{i,t+n} \right]}{E_t^* \left[\tilde{X}_{i,t+n} \right]} = \exp\left\{ n\beta \left(\Delta\tilde{x}_{i,t+1} - E_t^* \left[\Delta\tilde{x}_{i,t+1} \right] \right) \right\}. \quad (22)$$

D. Model implications

Below, we discuss three qualitative implications from the model that are relevant to our empirical findings.

First, increases in $E_t^* \left[g_i \right]$ raise the firm's price in two ways: increasing the expected future cash flows and decreasing the subjective risk premium. From equation (19), a higher expected

g_i naturally increases the value of the firm by increasing the value of future expected cash flows. What is less straightforward is that raising $E_t^*[g_i]$ lowers the subjective risk premium. As shown in equation (18), longer horizon cash flow strips carry a lower risk premium in this model, as their annualized return is less sensitive to the aggregate shock u_{t+1}. A higher value for $E_t^*[g_i]$ means that more of the firm's value comes from its longer horizon cash flows and therefore the weights $w_{i,t,n}$ in equation (20) are more concentrated on the longer horizon $E_t^*\left[R_{t+1}^{(n)}\right]$. In line with the findings in Table I, this means that both higher expected earnings growth and lower expected returns will help to explain high price-earnings ratios.

Second, if the constant-gain parameter β is small, then expectations are "stubborn." After an earnings surprise $\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]$ (which is equivalent to $\Delta\tilde{x}_{i,t+1} - E_t^*[\Delta\tilde{x}_{i,t+1}]$), expectations of future earnings are only slightly revised as shown in equation (22). Because expected future earnings are only slightly revised, the earnings surprise does not translate into a large immediate return, as shown in equation (21). Rather than being highly sensitive to recent surprises, the agent largely attributes the surprise to the noisy level shock $v_{i,t+1}$. This result is in line with the findings in Table III and highlights how the inclusion of temporary shocks substantially changes the model's predictions relative to the models discussed in Section IV.B.

Third, the presence of stubborn errors in expectations increases price volatility. If agents had full-information rational expectations (i.e., agents knew the parameters $g_i = \bar{g}$), then all firms would have the same price and there would be no predictable differences in returns between firms. However, because of errors in earnings expectations, the prices of some firms will rise or fall relative to others due to waves of optimism or pessimism about the firm's underlying growth parameter g_i. Because errors in expectations are stubborn, these deviations in prices away from the FIRE benchmark can be long-lived and a price that is above the FIRE benchmark will predict a gradual sequence of lower returns.

It is important to note that these model implications are the result of two mechanisms: slow updating of future cash flow expectations in response to surprises and a preference for the timing of cash flows. While the model represents these two mechanisms by parameter learning and non-permanent aggregate shocks, there are other mechanisms that could potentially deliver similar implications.

Constant-gain learning about g_i delivers slow updating of future cash flow expectations if the parameter β is small, consistent with the evidence in Table III and prior estimates (Orphanides and Williams, 2005 and Milani, 2007). This approach closely resembles learning from experience, as shown in Malmendier and Nagel (2016), and can also accommodate learning about a latent, time-varying growth component (see Appendix I.2, where we estimate this feature and find similar results). The preference for timing is generated from persistent but non-permanent aggregate shocks, which cause short-term cash flows to be disproportionately discounted relative to long horizon cash flows. Importantly, this result depends on perceived, not objective, risk differences. Using survey-based subjective risk measures (Jensen, 2024), we confirm in Appendix G that low price-earnings portfolios are indeed perceived as riskier than high price-earnings portfolios.

Ultimately, we choose to focus on constant-gain parameter learning and differences in perceived risk, as this provides a parsimonious description of these mechanisms and allows us to estimate the model parameters without targeting any of our empirical results from Section III. As shown in the model estimation in Section VII, we take the gain β from previous work on constant-gain learning and set risk sensitivity γ to match the aggregate equity premium. As a result, we can fully utilize the empirical decomposition results of Table I to evaluate the quantitative realism of our model.

VII. Quantitative performance and full decomposition

A. Estimation

To make a fair comparison to the models mentioned in Section III, we do not use any information from the decomposition results to estimate the model parameters. Instead, we

Table V

Model estimation

This table shows the value of the six parameters of the model. The parameters for the aggregate cash flow process (ϕ, σ_u) are derived directly from the autocorrelation and standard deviation of the S&P 500 annual earnings growth. The firm-level volatility σ_v is derived directly from the standard deviation over time of the portfolio-level annual earnings growth. The risk-free rate r^f and risk sensitivity γ are set to match the average one-year Treasury yield and average aggregate equity return during the sample period. The constant-gain learning parameter β is taken from Malmendier and Nagel (2016). All moments are estimated over the full sample period of 1982 to 2020.

Parameter	Value	Moments
	Cash flow process	
ϕ	0.83	$AC(\Delta x_{t+1}^{agg})$
σ_u	0.34	$\sigma\left(\Delta x_{t+1}^{agg}\right)$
σ_v	0.10	$\sigma\left(\Delta x_{i,t+1}\right)$
	SDF	
r_f	4.6%	Risk-free rate
γ	1.61	Average aggregate return
	Learning	
β	1.8%	Constant-gain learning (Malmendier and Nagel, 2016)

set all of the parameters based on time-series moments and previous estimates of the learning gain β, then evaluate how well the model matches the cross-sectional decompositions as well as a number of other moments.[34]

The model only has six parameters, which are all shown in Table V. The parameters for cash flows $(\phi, \sigma_u, \sigma_v)$ are all estimated directly from realized earnings growth for our full sample of 1982-2020. For the aggregate process, the standard deviation and autocorrelation of S&P 500 earnings growth imply a persistence $\phi = 0.83$ and a volatility $\sigma_u = 0.34$. The volatility of individual shocks $\sigma_v = 0.10$ is obtained from the volatility over time of the portfolio-level earnings growth. Appendix H.1 shows the exact formulas mapping these empirical moments to the model parameters. The constant-gain parameter is obtained from Malmendier and Nagel (2016) as $\beta = 0.018$. Note that this is nearly identical to the gain estimated in Milani (2007) of 0.0183.[35] For the agent's stochastic discount factor, the risk-

[34] The model is simulated yearly over 500 periods for 300 firms. To avoid being impacted by the initial value of the expectations $E_0^*[g_i]$, we calculate all moments after $t = 150$.

[35] These papers estimate the constant-gain parameter on a quarterly frequency. We show in Appendix H.2 that this estimation is quantitatively very similar to an estimation using an annual frequency. Conceptually, we are simply imposing that all updating occurs at the end of the year rather than allowing for small amounts of updating within the year.

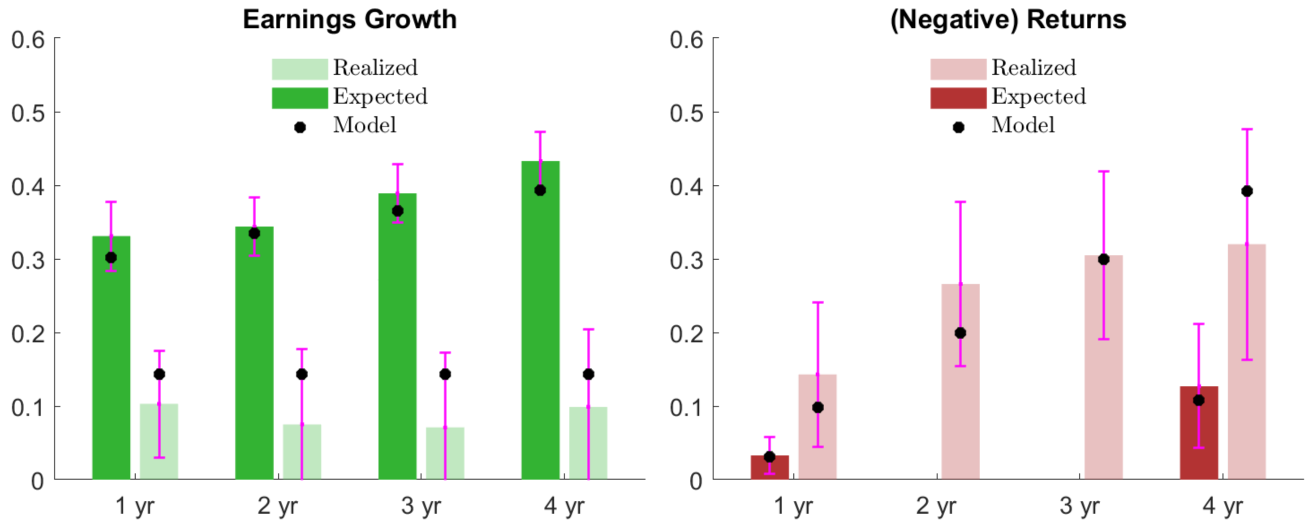


Figure 6. Empirical decomposition and model decomposition. This figure evaluates the decomposition of $\tilde{px}_{i,t}$ dispersion in the model across multiple horizons. The light bars show the contribution of realized earnings growth and realized returns to the dispersion of price-earnings ratios obtained in the *FIRE* columns of Table I. The dark bars show the contribution of expected earnings growth and expected returns to the dispersion of price-earnings ratios obtained in the *Expected* columns of Table I. Each bar shows Driscoll-Kraay 95% confidence intervals. The black dots show the values of both the realized and expected decomposition implied by the model.

free rate $r^f = 4.6\%$ and the sensitivity to risk $\gamma = 1.61$ are set to match the average one-year Treasury yield and average aggregate stock return of 10.5% for 1982-2020.

B. *Model performance*

B.1. Price ratio decomposition

After quantifying the model, we now evaluate the joint dynamics of price-earnings ratios, earnings growth, and returns. Figure 6 shows the price-earnings ratio decomposition results from Figure 1, along with their 95% confidence intervals. For comparison, the black dots show the values implied by our model. Overall, the model successfully matches both the objective decomposition of price-earnings ratio dispersion (i.e., comovement of price ratios with future earnings growth and future returns) and the subjective decomposition (i.e., comovement of price ratios with expected earnings growth and expected returns) at every horizon.

In the model, high price-earnings ratios are associated with significantly higher expected earnings growth and moderately lower expected returns. Figure 6 shows that a one unit increase in $\tilde{px}_{i,t}$ is associated with a 0.30 (0.39) increase in expected one-year (four-year) earnings growth and a 0.03 (0.11) decrease in expected returns. Because of the temporary shocks to the level of earnings $v_{i,t}$, realized one-year future earnings growth is partly predictable. In line with the data, a one unit increase in the model price-earnings ratio predicts a 0.14 increase in realized one-year earnings growth and this coefficient is unchanged as we increase the horizon to four years. The relationship between price-earnings ratios and expected earnings growth is quantitatively much larger than the relationship between price-earnings ratios and realized future earnings growth, meaning that high price-earnings ratios predict disappointment in future earnings growth. As a result, the relationship between price-earnings ratios and realized negative returns is larger than expected, 0.10 (0.39) at the one-year (four-year) horizon. Overall, this parsimonious model is able to closely match all 14 moments from the decomposition.

The fact that the model matches our decomposition results at multiple horizons highlights its success both in terms of magnitudes and in terms of timing. While the difference in expected and realized one-year earnings growth is large, this does not translate into a large difference between expected and realized one-year negative returns. Instead, agents are slow to adjust their beliefs and the disappointment in earnings growth leads to much lower than expected returns at longer horizons.

Because of this slow adjustment of prices, the model is able to simultaneously match the large one-year earnings growth disappointment shown in Figure 6 and the high empirical persistence of $\tilde{px}_{i,t}$, which is 0.77 in the data and 0.76 in the model. In general, these two facts are difficult to match for models in which growth expectations are sensitive to recent realizations (e.g., overstating the persistence of growth or diagnostic expectations of growth), as disappointing earnings growth for high $\tilde{px}_{i,t}$ firms would cause their price-earnings ratios to quickly fall. Further, our model is still consistent with previous cross-sectional evidence of overreaction. Using the Coibion and Gorodnichenko (2015) regression, we find that revisions in expected long-term growth $E_t^*[g_i] - E_{t-1}^*[g_i]$ negatively predict forecast errors $g_i - E_t^*[g_i]$ with a coefficient of -0.5, which is quantitatively similar to the empirical coefficients estimated in Bordalo et al. (2019) of -0.20 to -0.31. However, the model predicts that these revisions in expectations do not lead to large immediate changes in prices.

C. *Price volatility and other moments*

On top of the 14 untargeted moments shown in Figure 6, Table VI shows that the model performs well in matching a wide array of additional asset pricing moments. Perhaps the most surprising of these results is that the model generates realistic differences in returns $\tilde{r}_{i,t}$ across firms (i.e., the cross-sectional standard deviation) and realistic volatility over time in firms' $\tilde{r}_{i,t}$. This is almost entirely driven by errors in agents' expectations. Under a FIRE benchmark in which agents know the parameters $g_i = \bar{g}$, the cross-sectional standard deviation of $\tilde{r}_{i,t}$ and the idiosyncratic volatility of $\tilde{r}_{i,t}$ are both reduced by a factor of 5: from 5.6% to only 0.9% and from 5.5% to only 1.1%.

Beyond evaluating realized returns, the three panels of Table VI test the model's ability to match moments of price-earnings ratios, realized earnings growth, expected earnings growth, and expected returns. First, despite not using any price information in the estimation other than the average aggregate equity return, Panel A shows that the model generates realistic dynamics for the aggregate price-earnings ratio. The unconditional mean, volatility and autocorrelation of the log price-earnings ratio in the model (2.31, 43.2%, and 0.81) are consistent with the observed values (2.98, 42.5%, and 0.74) and the model generates volatile aggregate returns.

Second, while no information on cross-sectional dispersion was used in the estimation, Panel B shows that the model performs well in matching the empirical dispersion of nearly all of our variables.[36] In other words, constant-gain learning with temporary shocks can suc-

[36]The empirical dispersion is measured as the median cross-sectional standard deviation for each variable.

Table VI

Model evaluation

This table evaluates the model by comparing the untargeted aggregate and cross-sectional moments in the model simulations with those observed in the data. Panel A shows the mean, standard deviation and autocorrelation of the aggregate price-earnings ratio as well as the standard deviation of aggregate stock returns. Panel B shows the cross-sectional standard deviations of price-earnings ratios, future earnings growth and returns, and expected earnings growth and returns. Panel C shows the idiosyncratic volatility across time of price-earnings ratios, future earnings growth and returns, and expected earnings growth and returns. All moments in the table are untargeted, except for idiosyncratic realized earnings growth volatility. Aggregate moments are estimated over the full sample period of 1982 to 2020. The cross-sectional standard deviation and idiosyncratic volatility moments are estimated over the main sample of 1999 to 2020 due to data availability.

Panel A: Aggregate value					
	Mean px_t	$\sigma(px_t)$	$AC(px_t)$	$\sigma(r_t)$	
Model	2.31	43.2%	0.81	11.5%	
Data	2.98	42.5%	0.74	15.9%	

Panel B: Cross-sectional standard deviation					
	$\tilde{px}_{i,t}$	$\Delta\tilde{x}_{i,t+1}$	$\tilde{r}_{i,t+1}$	$E_t^*[\Delta\tilde{x}_{i,t+1}]$	$E_t^*[\tilde{r}_{i,t+1}]$
Model	20.9%	14.1%	5.6%	11.9%	0.8%
Data	22.6%	12.6%	5.7%	14.0%	2.6%

Panel C: Idiosyncratic volatility					
	$\tilde{px}_{i,t}$	$\Delta\tilde{x}_{i,t+1}$	$\tilde{r}_{i,t+1}$	$E_t^*[\Delta\tilde{x}_{i,t+1}]$	$E_t^*[\tilde{r}_{i,t+1}]$
Model	19.7%	14.0%	5.5%	11.7%	0.7%
Data	19.0%	16.6%	6.3%	12.3%	7.4%

cessfully generate large differences across firms in price-earnings ratios and realized earnings growth, which have model dispersions of 20.9% and 14.1% respectively. Beyond explaining the realized data, we find that the model also accurately captures the large empirical dispersion in expected earnings growth (11.9%).

We do find that the model understates the cross-sectional dispersion in expected returns. For the sake of parsimony, in the model, subjective discount rates are entirely driven by risk premia related to cash flow timing, as shown in equation (20). Expanding the model to incorporate other risks into discount rates could help to better match this moment. However, as shown in Figure 6, the model still succeeds in matching the covariance of price-earnings ratios with expected returns. Thus, while the model does not capture all cross-sectional differences in expected returns, it does capture the portion that is predictable with price-earnings ratios, i.e., the portion that is useful for generating large differences in price-earnings ratios.

Third, Panel C shows that the model replicates the measured portfolio-level volatilities. Note that these volatilities, which reflect variation in a single portfolio across time, are distinct from our estimates of dispersion, which capture cross-sectional variation across portfolios. The only information about portfolio-level volatility utilized in the estimation is the volatility of realized earnings growth, which means that the model provides an accurate mapping of how volatility in earnings growth translates into volatility in price-earnings ratios, returns, and expected earnings growth. Similar to our results for dispersion, we find that the model understates the volatility of expected returns.

In summary, we find that the model not only successfully matches the untargeted decomposition moments, but also generates realistic aggregate stock market moments as well as realistic cross-sectional dispersion and portfolio-level volatility. This demonstrates that a relatively parsimonious structural model of belief formation can feasibly improve upon FIRE models in terms of quantitatively matching the realized data while also matching the dynamics of empirically observed beliefs.

D. Full role of objective cash flows, cash flow mistakes, and discount rates

Using the quantified model, we can measure the full role of cash flow growth expectations and subjective discount rates in accounting for price-earnings ratio dispersion, the persistence of this dispersion, and dispersion in returns. Table VII Panel C shows the decomposition in equation (3) when we extend to the infinite horizon. Specifically, it shows the cross-sectional dispersion $Var\left(\tilde{px}_{i,t}\right)$ and the two components $Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right)$, $Cov\left(-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right)$. Additionally, Panel C shows the persistence of $\tilde{px}_{i,t}$, which measures whether cross-sectional differences in price-earnings ratios are transitory or long-lived, and the dispersion in returns $\tilde{r}_{i,t}$.

To start, we focus on the final column, which is our main model parameterization. As

Table VII

Infinite-horizon decomposition and counterfactual analysis

Each column shows the decomposition implied by the constant-gain learning model using different key parameter choices. Panel A reports the varying parameters for each specification, with all other parameters fixed at the values from Table V. The specifications include: (1) no learning or risk sensitivity ($\beta = 0$, $\gamma = 0$), (2) no learning ($\beta = 0$), (3) no risk sensitivity ($\gamma = 0$), and (4) the main specification. For specifications with $\gamma = 0$, we set the risk-free rate r_f to 10.5% to maintain consistent average equity returns across all cases. Panel B reports the mean aggregate price-earnings ratio and mean aggregate return for each specification. Panel C presents the cross-sectional results, showing the variance of $\tilde{r}_{i,t}$, the persistence and variance of $\tilde{px}_{i,t}$, and the decomposition of the cross-sectional variance of $\tilde{px}_{i,t}$ using the infinite-horizon version of equation (3):

$$Var\left(\tilde{px}_{i,t}\right) = Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right) + Cov\left(-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right], \tilde{px}_{i,t}\right).$$

The fourth and seventh rows of Panel C show the contributions of expected earnings growth $\sum_{j=1}^{\infty} E_t^*\left[\Delta\tilde{x}_{i,t+j}\right]$ and subjective discount rates $-\sum_{j=1}^{\infty}\rho^{j-1}E_t^*\left[\tilde{r}_{i,t+j}\right]$ in the decomposition. The fifth and eighth rows show the contributions of realized earnings growth and returns, and the sixth and ninth rows show the contribution of earnings growth and return forecast errors. The share of the cross-sectional variance of $\tilde{px}_{i,t}$ is shown in parentheses. For ease of reading, all coefficients in Panel C except for "Persistence $\tilde{px}_{i,t}$" are scaled by 100.

Panel A: Parameter values				
β	0	0	0.018	0.018
γ	0	1.61	0	1.61
r^f	10.5%	4.6%	10.5%	4.6%
Panel B: Levels				
Mean px_t	2.31	2.31	2.31	2.31
Mean r_{t+1}	10.5%	10.5%	10.5%	10.5%
Panel C: Cross section				
Variance $\tilde{r}_{i,t}$	0.01	0.01	0.12	0.31
Persistence $\tilde{px}_{i,t}$	0.00	0.00	0.54	0.76
Variance $\tilde{px}_{i,t}$	0.98	0.98	2.07	4.37
Expected earnings growth	0.98	0.98	2.07	2.87
	(100%)	(100%)	(100%)	(65.7%)
Realized earnings growth	0.98	0.98	0.78	0.65
	(100%)	(100%)	(37.6%)	(14.9%)
Forecast errors	0	0	1.29	2.22
	(0%)	(0%)	(62.4%)	(50.8%)
Subjective discount rates	0	0	0	1.50
	(0%)	(0%)	(0%)	(34.3%)
Negative realized returns	0	0	1.29	3.72
	(0%)	(0%)	(62.4%)	(85.1%)
Negative forecast errors	0	0	-1.29	-2.22
	(0%)	(0%)	(-62.4%)	(-50.8%)

shown in the fourth row of Panel C, the model estimates that differences in expected cash flow growth account for two-thirds (65.7%) of all dispersion in price-earnings ratios.[37] Combined with the aggregate time series findings of Delao and Myers (2021), this means that both time series variation in aggregate price ratios and cross-sectional dispersion in price ratios are both primarily explained by expected cash flow growth. However, unlike the aggregate time series findings, we also estimate a non-trivial role for subjective discount rates in accounting for price-earnings ratio dispersion. The seventh row of Panel C shows that low subjective discount rates for high price-earnings ratio firms accounts for roughly one-third (34.3%) of all dispersion in price-earnings ratios.

Looking at the breakdown of the 65.7% contribution from expected earnings growth, we see that this largely comes from forecast errors. The comovement of price-earnings ratios with realized future earnings growth only accounts for 14.9% of the dispersion, meaning that the remaining 50.8% comes from price-earnings ratios predicting forecast errors for earnings growth. As a result, high price-earnings ratios are largely associated with low future returns, with negative realized returns accounting for 85.1% of all price-earnings ratio dispersion. Note that at the infinite horizon, forecast errors for earnings growth and forecast errors for returns are equal (i.e., the forecast error row for earnings growth and negative returns are exactly opposite). While gradual learning affects how quickly earnings growth surprises are reflected in unexpected returns, eventually all unexpected earnings growth will appear as unexpected returns.

Conveniently, we can summarize the relative importance of realized future earnings growth, errors in earnings growth expectations, and subjective discount rates. The model estimates that realized earnings growth accounts for roughly 1/6 (14.9%) of price-earnings ratio dispersion, errors in earnings growth expectations account for 1/2 (50.8%), and subjective discount rates account for 1/3 (34.3%). Additionally, besides decomposing differences in price-earnings ratios, the model also decomposes the low realized returns earned by ex-

[37] This is consistent with the empirical results of Table I, where we find that expected earnings growth over just the first four years already accounts for 43.3% of all price-earnings ratio dispersion.

pensive stocks. The estimation implies that 40.3% (34.3/85.1) of the difference in returns between high and low price-earnings ratio stocks reflects subjective discount rates while 59.7% (50.8/85.1) reflects disappointment in earnings growth.

More broadly, by having a structural model, we can investigate the economic role of learning and risk sensitivity in driving the cross-sectional dispersion in price-earnings ratios and returns. The different columns in Table VII Panel C show the dispersion in returns, the persistence in price-earnings ratios, the dispersion in price-earnings ratios, and the decomposition results when β and/or γ are set to 0, i.e., learning and/or risk sensitivity are turned off. In all cases, the initial expected g_i is set to $\bar{g}$ for all firms, which means that the $\beta = 0$ scenarios are FIRE (i.e., the agent always knows the true parameters).[38] Given that we are interested in cross-sectional dispersion rather than aggregate levels, in the two cases where $\gamma = 0$ we also raise the risk-free rate from 4.6% to 10.5%. As shown in Panel B, this ensures that the aggregate level for price-earnings ratios and equity returns are identical across all four cases and it is only the dispersion that changes. Thus, the two cases where $\gamma = 0$ are equivalent to saying that all firms have the same subjective discount rate of 10.5%.

In the first column, both β and γ are set to 0. In this case, the dispersion in price-earnings ratios is less than 1/4 the value in our main specification (0.98 compared to 4.37) and there is almost no dispersion in returns. The dispersion in price-earnings ratios comes entirely from differences in expected earnings growth, as there are no differences in subjective discount rates. Price-earnings ratios do not predict earnings growth forecast errors. Instead, all differences in expected earnings growth are simply due to the noise shocks $v_{i,t}$. Since these shocks are i.i.d., the autocorrelation in expected earnings growth is zero, which explains why the persistence in $\tilde{px}_{i,t}$ is also zero.

In the second column, the model includes risk sensitivity ($\gamma > 0$) but keeps $\beta = 0$. As shown in Panel C, only including risk sensitivity has no effect on the results relative to the

[38] If the prior is not $g_i = \bar{g}$, then $\beta = 0$ is not FIRE and actually represent the most stubborn errors in expectations possible. This produces highly persistent dispersion in price-earnings ratios, as agents start with incorrect expectations of g_i and never update.

first column. This highlights that, in our model, variation across firms in expected cash flow growth and subjective discount rates are both ultimately related to variation across firms in expected g_i. While agents may be sensitive to risk related to cash flow timing, this only matters if firms are expected to differ in the timing of their cash flows.

In comparison, the third column shows that including learning but keeping $\gamma = 0$ does substantially change the results. The dispersion in price-earnings ratios doubles from 0.98 to 2.07. This largely comes from price-earnings ratios now comoving with future earnings growth forecast errors. However, there is also the interesting result that the comovement of price-earnings ratios with realized earnings growth decreases (0.98 to 0.78). The FIRE expectation for future earnings growth is simply $-v_{i,t}$. With learning, expected earnings growth depends on $E_t^*[g_i]$, which comoves positively with $v_{i,t}$, as a positive shock will tend to increase the guess for g_i. Thus, introducing learning means that the price-earnings ratio, which depends on expected earnings growth, will now be less related to future realized earnings growth due to the muted response to shocks $v_{i,t}$. Further, while objective expected earnings growth has 0 persistence over time, subjective expected earnings growth is persistent when agents are learning about g_i. Because of this, cross-sectional differences in price-earnings ratios are moderately long-lived, with a persistence of 0.54. The increased dispersion in price-earnings ratios also leads to a substantial increase in the dispersion of returns, from 0.01 to 0.12.

Finally, the last column shows the interaction from including both risk sensitivity and learning. While risk sensitivity by itself has no effect, once we incorporate learning, increasing γ from 0 to 1.61 more than doubles the dispersion in returns (0.12 to 0.31) and price-earnings ratios (2.07 to 4.37) and makes cross-sectional differences in price-earnings ratios more persistent. Specifically, including risk sensitivity along with learning increases the persistence of $\tilde{px}_{i,t}$ from 0.54 to 0.76, helping the model match the empirical persistence of 0.77. Looking at the contribution of subjective discount rates (the seventh row of Panel C), we clearly see the interaction between risk sensitivity and learning, as dispersion in subjective discount rates now contributes 1.50 (34.3%) to the total dispersion in price-earnings ratios.

More surprisingly, we also find an important interaction between risk sensitivity and learning for the contribution of earnings growth expectations. Given that γ has no impact on equations (12)-(15), changing γ has no effect on expected earnings growth. Thus, the increase in comovement between price-earnings ratios and expected earnings growth (2.07 to 2.87) is entirely due to changes in the price-earnings ratios. Intuitively, incorporating discount rates that depend on expected cash flow timing increases the sensitivity of price-earnings ratios to $E_t^*[g_i]$ and decreases their sensitivity to transitory shocks $v_{i,t}$. The increased sensitivity to $E_t^*[g_i]$ is reflected in the larger comovement of price-earnings ratios with expected earnings growth, and the reduced sensitivity to $v_{i,t}$ is reflected in an even higher persistence of $\tilde{px}_{i,t}$. This logic extends to any model with preferences for the timing of cash flows and shows that while discount rates may not affect expected earnings growth, they can be quantitatively important for driving the comovement of price ratios with expected earnings growth and earnings growth forecast errors.

Overall, the fact that dispersion in price-earnings ratios and returns for the full model (i.e., $\beta > 0, \gamma > 0$) is more than twice as large as any of the other counterfactuals highlights the natural interaction between preferences for the timing of cash flows and learning about cash flow growth. We find that this interaction is quantitatively important for matching the large empirical dispersion in returns and price-earnings ratios and the persistence of cross-sectional differences in price-earnings ratios.

VIII. Conclusion

We find that subjective expectations have substantial potential to explain the cross-section of stock price ratios and shed light on the relative importance of expected future cash flows and discount rates. Using a variance decomposition, we show that cross-sectional dispersion in price-earnings ratios is primarily explained by predictable errors in subjective expectations of earnings growth. Subjective discount rates play a secondary, but non-trivial role. Disappointment in one-year earnings growth does not immediately lead to an equivalent disappointment in one-year returns. Instead, earnings growth surprises are reflected gradually in future returns over time. To understand these findings, we provide a quantitative model which not only outperforms standard FIRE models in matching the dynamics of prices and realized earnings growth and returns, but also outperforms several behavioral models in matching the dynamics of prices and expectations. The model features constant-gain learning about earnings growth and risk premia related to cash flow timing and emphasizes the importance of slow-moving beliefs in order to match the empirical timing of earnings growth expectations and realized returns.

These findings for the cross-section of stock prices are consistent with the aggregate time-series findings of Delao and Myers (2021, 2024) and Bordalo et al. (2023), who emphasize that aggregate stock prices are largely driven by subjective earnings growth expectations and that errors in these expectations play a particularly large role in explaining long-term returns. This harmony between the aggregate time-series and the cross-section indicates that a single mechanism could potentially explain both dimensions of the data and provides a strong motivation for further research understanding how investors form cash flow expectations and discount rates.

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Appendix

A. Model prices and returns

To derive equation (16), we guess and verify a log-affine form for the strip price, $P_t^{(n)} = \exp\left\{A(n) + \phi^n x_t^{agg}\right\}$. The strip price is then pinned down by $P_t^{(0)} = \exp\left\{x_t^{agg}\right\}$ (i.e., $A(0) = 0$) and

$$\begin{aligned} P_t^{(n)} &= E_t^*\left[M_{t+1}P_{t+1}^{(n-1)}\right] \\ &= E_t^*\left[\exp\left\{-r^f - \frac{1}{2}\gamma^2\sigma_u^2 - \gamma u_{t+1} + A(n-1) + \phi^n x_t^{agg} + \phi^{n-1}u_{t+1}\right\}\right] \\ &= \exp\left\{-r^f - \frac{1}{2}\gamma^2\sigma_u^2 + A(n-1) + \phi^n x_t^{agg} + \frac{1}{2}\left(\phi^{n-1} - \gamma\right)^2\sigma_u^2\right\}. \end{aligned} \tag{A1}$$

This gives that

$$\begin{aligned} A(n) &= A(n-1) - r^f - \gamma\phi^{n-1}\sigma_u^2 + \frac{1}{2}\phi^{2(n-1)}\sigma_u^2 \\ &= -nr^f - \gamma\sigma_u^2\frac{1-\phi^n}{1-\phi} + \frac{1}{2}\sigma_u^2\frac{1-\phi^{2n}}{1-\phi^2}. \end{aligned} \tag{A2}$$

The expected and realized strip returns in equations (17)-(18) then simply utilize the formula for $P_t^{(n)}$. The firm price in equation (19) uses the independence of aggregate and idiosyncratic shocks to simplify $E_t^*\left[\left(\prod_{j=1}^n M_{t+j}\right)X_{t+n}^{agg}\tilde{X}_{i,t+n}\right] = E_t^*\left[\left(\prod_{j=1}^n M_{t+j}\right)X_{t+n}^{agg}\right]E_t^*\left[\tilde{X}_{i,t+n}\right] = P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]$.

Given that the firm price is simply a collection of strip prices, the return for a firm is

$$\begin{aligned} R_{i,t+1} &= \frac{\tilde{X}_{i,t+1}X_{t+1}^{agg} + P_{i,t+1}}{P_{i,t}} = \frac{\sum_{n=1}^{\infty} P_{t+1}^{(n-1)}E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^{\infty} P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]} \\ &= \sum_{n=1}^{\infty}\frac{P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^{\infty}P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}\frac{P_{t+1}^{(n-1)}}{P_t^{(n)}}\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} \\ &= \sum_{n=1}^{\infty} w_{i,t,n}R_{t+1}^{(n)}\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} \end{aligned} \tag{A3}$$

where the weight is $w_{i,t,n} = \frac{P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]}{\sum_{n=1}^{\infty}P_t^{(n)}E_t^*\left[\tilde{X}_{i,t+n}\right]} = \frac{\exp\{nE_t^*[g_i]\}P_t^{(n)}}{\sum_{n=1}^{\infty}\exp\{nE_t^*[g_i]\}P_t^{(n)}}$ from equation (15).

Applying expectations, we then get equation (20).

B. Connecting returns, earnings growth, and price-earnings ratios

First, we derive the equation for a firm which has zero dividends. For simplicity, we eliminate the index i in this derivation. In this case, the return is equal to the price growth which after log-linearization becomes an exact relationship

$$r_{t+1} \quad = \quad \Delta x_{t+1} - px_t + px_{t+1}. \tag{A4}$$

A high price-earnings ratio px_t must be followed by low future price growth Δp_{t+1} (returns r_{t+1}), high future earnings growth Δx_{t+1}, or a high future price-earnings ratio px_{t+1}.

Now, we consider the case where dividends are non-zero. We start with the one-year return identity of a portfolio

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{P_{t+1}}{D_{t+1}} + 1\right) \frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}},$$

where P_t and D_t are the current price and dividends. Log-linearizing around $\bar{pd}$, we can represent the price-dividend ratio pd_t in terms of future dividend growth, Δd_{t+1}, future returns, r_{t+1}, and the future price-dividend ratio, pd_{t+1}, all in logs:

$$r_{t+1} \quad \approx \quad \kappa^d + \Delta d_{t+1} - pd_t + \rho pd_{t+1}, \tag{A5}$$

where κ^d is a constant, $\rho = e^{\bar{pd}} / \left(1 + e^{\bar{pd}}\right) < 1$. We can then insert the identity $px_t = pd_t + dx_t$, where dx_t is the log payout ratio, into (A5) to obtain

$$r_{t+1} \quad \approx \quad \kappa + \Delta x_{t+1} - px_t + \rho px_{t+1} \tag{A6}$$

where we approximate $(1 - \rho)\, dx_{t+1}$ as a constant given that $(1 - \rho)$ is very close to 0.[39] Here, $\bar{pd}$ does not need to be the mean price-dividend ratio of this specific stock or portfolio. In order to study cross-sectional variation without resorting to portfolio-specific approximation parameters, we use the average price-dividend ratio of the market for $\bar{pd}$ following Cochrane

[39] The zero dividend relationship in equation (A4) is a special case of equation (A6) as $\bar{pd}$ goes to infinity.

(2011).

While the identity relies on the approximation that $(1-\rho)\,dx_{t+1}$ is close to a constant, empirically equation (A6) holds tightly. For horizons of 1 to 4 years, Table I shows that a one unit increase in px_t is associated with almost exactly a one unit increase in $\sum_{j=1}^{h}\rho^{j-1}\Delta x_{t+j} - \sum_{j=1}^{h}\rho^{j-1}r_{t+j} + \rho^h px_{t+h}$.[40] In other words, the approximation error from ignoring the payout ratio and using a single value for ρ accounts for at most 3.1% of variation in price-earnings ratios in the decomposition of equation (3). For robustness, the next section uses an exact relationship instead of equation (3) to ensure the approximation is not driving our results.

C. Robustness Exercises

C.1. Exact decomposition results

In this section, we derive all the main results using an exact decomposition of price-earnings ratios based on price growth, rather than the approximate decomposition based on returns. For any stock or portfolio of stocks i, the price-earnings ratio $px_{i,t}$ can be expressed in terms of the one-year ahead log price growth $\Delta p_{i,t+1}$, the future earnings growth $\Delta x_{i,t+1}$, and the future price-earnings ratio:

$$px_{i,t} = \Delta x_{i,t+1} + \Delta p_{i,t+1} + px_{i,t+1}. \tag{A7}$$

This equation is exact and does not contain a log-linearization constant ρ. Applying subjective expectations $E_t^*[\cdot]$, we see that a higher than average price-earnings ratio must be explained by higher than average expected earnings growth, lower than average expected price growth, or a higher than average expected future price-earnings ratio,

$$\tilde{px}_{i,t} = \sum_{j=1}^{h} E_t^*[\Delta\tilde{x}_{i,t+j}] - \sum_{j=1}^{h} E_t^*[\Delta\tilde{p}_{i,t+j}] + E_t^*\left[\tilde{px}_{i,t+h}\right]. \tag{A8}$$

Just like the main decomposition, this equation holds under any subjective probability

[40] For example, at the one-year horizon, a one unit increase in px_t is associated with a 0.103 increase in Δx_{t+1}, a 0.143 increase in $-r_{t+1}$, and a 0.746 increase in ρpx_{t+1}. At the four-year horizon, a one unit increase in px_t is associated with a with a 0.099 increase in $\sum_{j=1}^{4}\rho^{j-1}\Delta x_{t+j}$, a 0.320 increase in $-\sum_{j=1}^{4}\rho^{j-1}r_{t+j}$, and a 0.550 increase in $\rho^4 px_{t+4}$.

distribution and we can decompose the variance of $\tilde{px}_{i,t}$ into three components:

$$1 = \underbrace{\frac{Cov\left(\sum_{j=1}^{h} E_t^*\left[\Delta\tilde{x}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{CF_h} + \underbrace{\frac{Cov\left(-\sum_{j=1}^{h} E_t^*\left[\Delta\tilde{p}_{i,t+j}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{PG_h} + \underbrace{\frac{Cov\left(E_t^*\left[\tilde{px}_{i,t+h}\right], \tilde{px}_{i,t}\right)}{Var\left(\tilde{px}_{i,t}\right)}}_{FPX_h}. \tag{A9}$$

The coefficients CF_h and PG_h give a quantitative measure of how much dispersion in price-earnings ratios is accounted for by dispersion in expected earnings growth and how much is accounted for by dispersion in expected price growth. We can now estimate this equation using the exact expectations of price growth without an approximation.

Table AI shows that the results of this exact decomposition are very similar to the main decomposition results in Table I. We find that 10.3% of dispersion in price-earnings ratios is accounted for by differences in one-year future earnings growth and 13.2% is accounted for by differences in one-year price growth. Just as in the main decomposition, differences in earnings growth are overestimated, with expected earnings growth accounting for nearly a third (33.1%) of all dispersion in price-earnings ratios. Differences in price growth are underestimated, with expected price growth accounting for only 3.3% of all dispersion in price-earnings ratios. A similar pattern can be observed at longer horizons. Overall, all the coefficients closely align with those reported in Table I.

We can also estimate an exact version of the unexpected anomaly return decomposition (7). Just as in the main identity, we normalize all anomalies $\tilde{a}_{i,t}$ so that they have variance 1 and positively comove with future price growth. From equation (A7), we have the identity

$$\begin{aligned} \underbrace{Cov\left(\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,p}} &= \underbrace{Cov\left(\tilde{x}_{i,t+1} - E_t^*\left[\tilde{x}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,x}} \\ &+ \underbrace{Cov\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right], \tilde{a}_{i,t}\right)}_{\sigma_{a,px}}. \end{aligned} \tag{A10}$$

Here, the values for $\sigma_{a,x}$ and $\sigma_{a,px}$ indicate how much the predictable price growth forecast

Table AI

Decomposition of dispersion in price-earnings ratios (exact decomposition)

This table decomposes the variance of price-earnings ratios using the exact decomposition (A9) at multiple horizons. The *FIRE* columns report the elements CF_h, PG_h and FPX_h of the decomposition using future earnings growth, future price growth and future price-earnings ratios. The *Expected* columns report the elements of the decomposition using expected earnings growth, expected price growth and expected price-earnings ratios. The *Error* columns report the contribution of the forecast errors. For instance, for year $h = 1$, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the *FIRE* column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decomposition estimated over the longer sample period of 1982-2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	$h=1$			$h=2$			$h=3$			$h=4$		
	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error
Main Sample: 1999-2020												
CF_h	0.103***	0.331***	−0.228***	0.075	0.344***	−0.269***	0.070	0.391***	−0.321***	0.100*	0.439***	−0.340***
	[0.037]	[0.024]	[0.032]	[0.052]	[0.020]	[0.046]	[0.053]	[0.020]	[0.049]	[0.057]	[0.020]	[0.055]
	[0.052]	[0.027]	[0.044]	[0.069]	[0.022]	[0.065]	[0.073]	[0.022]	[0.071]	[0.080]	[0.022]	[0.077]
DR_h	0.132**	0.033***	0.100*	0.250***			0.284***			0.292***	0.135***	0.157*
	[0.052]	[0.013]	[0.054]	[0.060]			[0.062]			[0.088]	[0.046]	[0.089]
	[0.051]	[0.014]	[0.054]	[0.075]			[0.084]			[0.120]	[0.047]	[0.106]
FPX_h	0.765***	0.636***	0.129**	0.675***			0.646***			0.608***	0.426***	0.182***
	[0.051]	[0.020]	[0.058]	[0.036]			[0.042]			[0.063]	[0.029]	[0.069]
	[0.041]	[0.024]	[0.053]	[0.043]			[0.052]			[0.078]	[0.031]	[0.082]
Long Sample: 1982-2020												
CF_h	0.137***	0.312***	−0.175***	0.102**	0.335***	−0.234***	0.107**	0.402***	−0.296***	0.149***	0.471***	−0.322***
	[0.026]	[0.021]	[0.027]	[0.049]	[0.023]	[0.042]	[0.044]	[0.025]	[0.035]	[0.041]	[0.028]	[0.035]
	[0.026]	[0.021]	[0.028]	[0.050]	[0.023]	[0.042]	[0.045]	[0.026]	[0.035]	[0.043]	[0.029]	[0.037]

Table AII

Unexpected anomaly price growth (exact decomposition)

This table measures and decomposes unexpected anomaly price growth. The Representative Anomaly is the average ranking of each stock across 20 different anomalies, and $\tilde{px}_{i,t}$ is the demeaned price-earnings ratio. For each anomaly variable, we sort stocks into five equal-value portfolios based on the anomaly variable. Each column shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. Both anomaly variables are scaled to have unit variance and to positively comove with future price growth. The three dependent variables are the unexpected price growth $\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right]$, the earnings forecast errors $\tilde{x}_{i,t+1} - E_t^*\left[\tilde{x}_{i,t+1}\right]$, and the price-earnings ratio forecast errors $\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]$. The sample period is 1999 to 2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	Representative Anomaly	$\tilde{px}_{i,t}$
$\Delta\tilde{p}_{i,t+1} - E_t^*\left[\Delta\tilde{p}_{i,t+1}\right]$	0.031^{**}	0.030^{*}
	[0.013]	[0.016]
	[0.013]	[0.015]
$\tilde{x}_{i,t+1} - E_t^*\left[\tilde{x}_{i,t+1}\right]$	0.064^{***}	0.069^{***}
	[0.020]	[0.010]
	[0.020]	[0.014]
$\left(\tilde{px}_{i,t+1} - E_t^*\left[\tilde{px}_{i,t+1}\right]\right)$	-0.033^{***}	-0.039^{**}
	[0.009]	[0.018]
	[0.009]	[0.015]

errors are explained by predictable errors in next-year earnings expectations and expectations of the future price-earnings ratio. Table AII shows the results for the representative anomaly studied in Section IV.C. For nearly every anomaly, we estimate a positive value of $\sigma_{a,p}$, meaning that investors do not fully anticipate the high growth on high $\tilde{a}_{i,t}$ stocks. The predictable errors in one-year earnings growth expectations are more than large enough to account for the unexpected one-year price growth (i.e., $\sigma_{a,x}$ is greater than $\sigma_{a,p}$). In Appendix D Table AXI we show the exact decomposition results for each of the individual anomalies.

C.2. Overlapping observations and Bauer and Hamilton (2018)

Overlapping forecast horizons can increase the persistence of residuals in Table I. Because of this, we use Driscoll-Kraay and block-bootstrap standard errors to account for any autocorrelation. For additional robustness, in this section we also directly calculate the significance of each coefficient under the worst-case scenario for overlapping observations.

We do this following the methodology proposed by Bauer and Hamilton (2018). Specifically, for expected and realized earnings growth and returns, we run simulations to measure how often we spuriously find a coefficient as large as what we observe in the data. For clarity, we discuss the simulation for the regression of earnings forecast errors $\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]$ on price-earnings ratios $\tilde{px}_{i,t}$, however, the methodology is identical for the other left hand side variables.

We specify the price-earnings ratio of each portfolio i as an AR(1) process,

$$\tilde{px}_{i,t} = \mu_i + \left(\tilde{px}_{i,t-1} - \mu_i\right) + \sigma_i \varepsilon_{i,t}.$$

The mean, persistence, and variance are set equal to the observed values over our sample. Additionally, the initial value of the simulated price-earnings ratio for portfolio i is set equal to the initial value observed in our data to account for any drift back to the mean which may generate trends in price-earnings ratios over the sample. For example, if the price-earnings ratio for the Growth portfolio is substantially above its mean at the beginning of the sample, then reversion to the mean will create a downward trend in the price-earnings ratio for this portfolio over time. We then simulate one-period forecast errors under the null hypothesis that forecast errors are unpredictable.

If subjective expectations change over time, then there will be little overlap in longer horizon forecast errors. For example, if $E_t^*\left[\Delta\tilde{x}_{i,t+2}\right]$ is very different from $E_{t+1}^*\left[\Delta\tilde{x}_{i,t+2}\right]$, then there is little similarity between the second term of $\sum_{j=1}^{h}\rho^{j-1}\left(\Delta\tilde{x}_{i,t+j} - E_t^*\left[\Delta\tilde{x}_{i,t+j}\right]\right)$ and the first term of $\sum_{j=1}^{h}\rho^{j-1}\left(\Delta\tilde{x}_{i,t+1+j} - E_{t+1}^*\left[\Delta\tilde{x}_{i,t+1+j}\right]\right)$. However, in the worst-case scenario in which expectations do not change at all over time, then $\sum_{j=1}^{h}\rho^{j-1}\left(\Delta\tilde{x}_{i,t+j} - E_t^*\left[\Delta\tilde{x}_{i,t+j}\right]\right)$ will be an MA$(h-1)$ process. This will cause the forecast errors at longer horizons to be persistent which increases the probability of spuriously finding a large coefficient between price-earnings ratios and four-year forecast errors.

For our simulations, we push this worst-case scenario even further by making each period

Table AIII

Worst-case decomposition of dispersion in price-earnings ratios

This table decomposes the variance of price-earnings ratios using equation (3) at multiple horizons. The *FIRE* columns report the elements CF_h and DR_h of the decomposition using future earnings growth and future negative returns. The *Expected* columns report the elements of the decomposition using expected earnings growth and expected returns. The *Error* columns report the contribution of the forecast errors. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decomposition estimated over the longer sample period of 1982-2020. Worst-case p-values using the Bauer and Hamilton (2018) procedure are reported in parentheses. Superscripts indicate significance at the 1% (***), 5% (**), and 10% (*) level.

	$h=1$			$h=2$			$h=3$			$h=4$		
	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error
	Main Sample: 1999-2020											
CF_h	0.103***	0.331***	−0.228***	0.075	0.344***	−0.268***	0.071	0.389***	−0.317***	0.099	0.433***	−0.335***
	(0.006)	(0.000)	(0.000)	(0.117)	(0.000)	(0.001)	(0.174)	(0.000)	(0.002)	(0.124)	(0.000)	(0.001)
DR_h	0.143***	0.033***	0.110***	0.266***			0.305***			0.320***	0.127***	0.192**
	(0.000)	(0.008)	(0.002)	(0.000)			(0.000)			(0.004)	(0.008)	(0.047)
FPX_h	0.746***	0.620***	0.126**	0.642***			0.599***			0.550***	0.385***	0.165**
	(0.000)	(0.008)	(0.010)	(0.000)			(0.000)			(0.000)	(0.000)	(0.038)
	Long Sample: 1982-2020											
CF_h	0.137***	0.312***	−0.175***	0.102**	0.335***	−0.232***	0.107**	0.399***	−0.291***	0.147**	0.462***	−0.316***
	(0.000)	(0.000)	(0.000)	(0.018)	(0.000)	(0.000)	(0.029)	(0.000)	(0.000)	(0.016)	(0.000)	(0.000)

one month instead of one year. This means that $\sum_{j=1}^{h} \rho^{j-1} \left(\Delta \tilde{x}_{i,t+j} - E_t^* \left[\Delta \tilde{x}_{i,t+j}\right]\right)$ will be MA$(12h-1)$ which dramatically increases the persistence. For all of the left-hand side variables in Table I, we find that this worst-case scenario substantially overstates the observed variable persistence. We then set the variance of the monthly forecast errors to match the observed variance of $\sum_{j=1}^{h} \rho^{j-1} \left(\Delta \tilde{x}_{i,t+j} - E_t^* \left[\Delta \tilde{x}_{i,t+j}\right]\right)$. We then run 10,000 simulations and report the probability of spuriously finding a coefficient as large as what we observe in the data.

Table AIII shows the coefficients for realized and expected earnings growth and returns, along with their associated p-values from the simulations. As in Table I, the FIRE CF_2 and CF_3 are not significant at any level. Additionally, under this worst-case scenario, the FIRE CF_4 is not significant at any level. All other coefficients are significant at the 5% level, which again aligns with the results of Table I. Even after accounting for persistence in price-earnings ratios, trends, and a worst-case assumption for overlapping observations, the probability of spuriously generating coefficients as large as what we find in the data is quite small.

C.3. Smoothed earnings

To show that our decomposition results are not influenced by fluctuations in earnings in the denominator of price-earnings ratio, we repeat our analysis normalizing prices with a three-year-smoothed measure of earnings. AIV shows that the results are very similar to the main decomposition results in Table I. We find that 37.6% of dispersion in price–to-smoothed-earnings ratios is accounted for by differences in expected four-year earnings growth and 12.6% is accounted for by differences in four-year returns. Just as our main results, differences in cash flow growth are overestimated, with errors in expected cash flows accounting for nearly a third (31.0%) of all dispersion in price–to-smoothed-earnings ratios, and differences in return are underestimated.

Table AIV

Decomposition of dispersion in price-earnings ratios using smoothed earnings

This table shows the three components of the right hand side of equation (3) using three-year smoothed earnings instead of annual earnings to form the valuation ratios. Let s_t be the three-year smoothed average of earnings. For each period, we form the price-to-smoothed-earnings ratio $\tilde{ps}_{i,t}$. The *FIRE* columns report the components CF_h, DR_h and FPE_h of equation (3) using future earnings growth, future returns and future price-earnings ratios. The *Expected* columns report the elements of the equation (3) using expected earnings growth, expected returns and expected price-earnings ratios. The *Error* columns report the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{ps}_{i,t}\right)/Var\left(\tilde{ps}_{i,t}\right)$ is shown in the *FIRE* column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{ps}_{i,t}\right)/Var\left(\tilde{ps}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{ps}_{i,t}\right)/Var\left(\tilde{ps}_{i,t}\right)$. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decomposition estimated over the longer sample period of 1982-2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	$h=1$			$h=2$			$h=3$			$h=4$		
	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error
	Main Sample: 1999-2020											
CF_h	0.090**	0.290***	−0.200***	0.053	0.292***	−0.238***	0.045	0.334***	−0.289***	0.066	0.376***	−0.310***
	[0.036]	[0.021]	[0.037]	[0.047]	[0.023]	[0.046]	[0.049]	[0.023]	[0.045]	[0.053]	[0.023]	[0.050]
	[0.047]	[0.024]	[0.053]	[0.067]	[0.031]	[0.062]	[0.065]	[0.029]	[0.046]	[0.069]	[0.028]	[0.066]
DR_h	0.131***	0.032***	0.099**	0.240***			0.276***			0.291***	0.126***	0.165**
	[0.046]	[0.011]	[0.048]	[0.055]			[0.058]			[0.079]	[0.038]	[0.082]
	[0.045]	[0.012]	[0.047]	[0.066]			[0.074]			[0.095]	[0.038]	[0.098]
FPX_h	0.681***	0.573***	0.108**	0.601***			0.565***			0.523***	0.357***	0.166***
	[0.048]	[0.025]	[0.051]	[0.030]			[0.034]			[0.051]	[0.024]	[0.057]
	[0.048]	[0.031]	[0.049]	[0.036]			[0.038]			[0.057]	[0.027]	[0.068]
	Long Sample: 1982-2020											
CF_h	0.120***	0.264***	−0.143***	0.079*	0.275***	−0.196***	0.078**	0.333***	−0.256***	0.109***	0.392***	−0.283***
	[0.025]	[0.018]	[0.029]	[0.042]	[0.022]	[0.040]	[0.038]	[0.023]	[0.033]	[0.038]	[0.024]	[0.032]
	[0.025]	[0.018]	[0.029]	[0.042]	[0.022]	[0.042]	[0.039]	[0.024]	[0.033]	[0.039]	[0.025]	[0.032]

C.4. Delisting firms

As explained in Section V, we require in the main analysis that firms have an observed future price and future earnings. This allows us to calculate direct forecast errors for the subjective expectations. To test whether survivorship bias is impacting the results, we repeat our analysis without the requirement that firms must have a future observed price and observed earnings. Instead, when firms exit the sample, we measure the delisting return and reinvest those funds into the remaining firms in the portfolio. We then calculate earnings growth and returns under this reinvestment strategy.

As shown in Table AV, the results are quite close to our main estimation in Table I. We find that 42.7% (4.6%) of dispersion in price-earnings ratios is explained expected (realized) four-year earnings growth and 16.7% (31.8%) is explained by expected (realized) four-year returns.

C.5. Removing stale forecasts

The possibility that the observed stubbornness in expectations is mechanically driven by stale analyst forecasts is not supported by the evidence in Table III Panel B which shows a similar gradual adjustment in market returns. However, in Section V, we replicate the main stubbornness results of Figures 2 and 3 using only actively updated forecasts within the current quarter to form the consensus expectations. Tables AVI and AVII show the detailed results for Figure 5. Moreover, in Table AVIII we also replicate the main decomposition results from Table I using this subset of actively updated forecasts.

Our findings remain consistent: as in Table I, a substantial share (46.2%) of the dispersion in price–earnings ratios is explained by differences in expected four-year earnings growth, while 16.7% is attributed to differences in four-year returns. Similar to our main results, errors in expected cash flows account for a significant portion of the dispersion, with one-third (36.2%) of the total dispersion reflecting overestimation of differences in cash flow growth. Conversely, differences in returns are underestimated.

Table AV

Decomposition of dispersion in price-earnings ratios including exiting firms

This table decomposes the variance of price-earnings ratios including firms that may exit after portfolio formation. To account for these firms, we reinvest the delisting returns of exiting firms in the corresponding portfolio. The *FIRE* columns report the elements CF_h, DR_h and FPX_h of the decomposition using future earnings growth, future negative returns and future price-earnings ratios. The *Expected* columns report the elements of the decomposition using expected earnings growth, expected returns and expected price-earnings ratios. The *Errors* columns report the contribution of the forecast errors of each element. For instance, $CF_1 = Cov\left(\Delta\tilde{x}_{i,t+1}, \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ is shown in the *FIRE* column. This component can be split into its expected component $Cov\left(E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$ and its error component $Cov\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right], \tilde{px}_{i,t}\right)/Var\left(\tilde{px}_{i,t}\right)$. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decomposition estimated over the longer sample period of 1982-2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	$h=1$			$h=2$			$h=3$			$h=4$		
	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error
						Main Sample: 1999-2020						
CF_h	0.100**	0.306***	−0.196***	0.060	0.317***	−0.255***	0.040	0.370***	−0.330***	0.046	0.427***	−0.374***
	[0.044]	[0.026]	[0.046]	[0.071]	[0.020]	[0.070]	[0.075]	[0.023]	[0.069]	[0.073]	[0.031]	[0.070]
	[0.052]	[0.032]	[0.056]	[0.087]	[0.023]	[0.088]	[0.095]	[0.023]	[0.094]	[0.103]	[0.029]	[0.100]
DR_h	0.079	0.046***	0.034	0.200**			0.263***			0.318***	0.167***	0.172**
	[0.071]	[0.009]	[0.070]	[0.088]			[0.083]			[0.084]	[0.028]	[0.078]
	[0.067]	[0.010]	[0.066]	[0.098]			[0.093]			[0.107]	[0.032]	[0.105]
FPX_h	0.808***	0.632***	0.165***	0.704***			0.649***			0.578***	0.352***	0.199***
	[0.064]	[0.023]	[0.060]	[0.061]			[0.059]			[0.061]	[0.032]	[0.065]
	[0.062]	[0.028]	[0.057]	[0.067]			[0.066]			[0.071]	[0.027]	[0.083]
						Long Sample: 1982-2020						
CF_h	0.140***	0.290***	−0.143***	0.089	0.314***	−0.223***	0.078	0.380***	−0.304***	0.089	0.447***	−0.359***
	[0.033]	[0.019]	[0.035]	[0.066]	[0.019]	[0.062]	[0.062]	[0.022]	[0.055]	[0.057]	[0.027]	[0.050]
	[0.032]	[0.018]	[0.035]	[0.067]	[0.019]	[0.062]	[0.061]	[0.022]	[0.055]	[0.059]	[0.026]	[0.052]

Table AVI

Gradual adjustment of expectations using only actively updated forecasts

Panel A shows the gradual adjustment of expectations about future earnings $\tilde{x}_{i,t+h}$ after an earnings surprise at $t+1$, i.e., the coefficients $\gamma^*_{h,j}$ estimated using equation (5). For example, the first row shows $\gamma^*_{2,1}$, the effect of an earnings surprise $\tilde{x}_{i,t+1} - E^*_t[\tilde{x}_{i,t+1}]$ on the revisions to two-year earnings $E^*_{t+1}[\tilde{x}_{i,t+2}] - E^*_t[\tilde{x}_{i,t+2}]$. The second row shows $\gamma^*_{3,1}$, and $\gamma^*_{3,2}$, the effect of an earnings surprise $\tilde{x}_{i,t+1} - E^*_t[\tilde{x}_{i,t+1}]$ on revisions about $\tilde{x}_{i,t+3}$ occurring in years $t+1$ and $t+2$. Panel B shows the estimated coefficient θ_j from equation (6) which estimates the reaction of returns $\tilde{r}_{i,t+1}, \tilde{r}_{i,t+2}$, and $\tilde{r}_{i,t+3}$ after an earnings surprise at $t+1$. To eliminate the possibility that the decomposition results are driven by stale forecasts, consensus expectations are constructed exclusively from those individual earnings forecasts that were actively updated each quarter. The sample period is 1999 to 2020. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

Panel A: Earnings revisions			
	$j=1$	$j=2$	$j=3$
$\left(E^*_{t+j} - E^*_{t+j-1}\right)[\tilde{x}_{i,t+2}]$	0.13**		
	(0.06)		
$\left(E^*_{t+j} - E^*_{t+j-1}\right)[\tilde{x}_{i,t+3}]$	0.11**	0.11***	
	(0.04)	(0.04)	
$\left(E^*_{t+j} - E^*_{t+j-1}\right)[\tilde{x}_{i,t+4}]$	0.11***	0.08***	0.11**
	(0.04)	(0.03)	(0.04)
Panel B: Returns			
$\tilde{r}_{i,t+j}$	0.10***	0.14***	0.09***
	(0.04)	(0.04)	(0.03)

Table AVII

Revisions in expectations using only actively updated forecasts

This table shows the effect of earnings surprises on revisions using only actively updated forecasts. To eliminate the possibility that the decomposition results are driven by stale forecasts, consensus expectations are constructed exclusively from those individual earnings forecasts that were actively updated each quarter. For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. Each row shows the coefficient from regressing the revision in future earnings $E^*_{t+1}[\tilde{x}_{i,t+2}] - E^*_t[\tilde{x}_{i,t+2}]$ on the earnings surprise $x_{i,t+1} - E^*_t[x_{i,t+1}]$. The Representative Anomaly (RA) is the average ranking of each stock across 20 different anomalies. P/X shows the results for the main portfolios used to study cross-sectional variation in price-earnings ratios. Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to positively comove with future returns. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are calculated for each coefficient using a lag equal to the equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

$\tilde{a}_{i,t}$	Coefficient	$\tilde{a}_{i,t}$	Coefficient
rdm	-0.058	noa	0.159***
	[0.060]		[0.060]
bm	0.202***	oaa	0.177***
	[0.060]		[0.057]
cfp	0.243***	ol	0.156**
	[0.052]		[0.061]
adm	0.049	pia	0.212***
	[0.040]		[0.075]
nop	0.079	poa	0.134**
	[0.049]		[0.067]
ia	0.101***	pta	0.043
	[0.026]		[0.052]
gp	0.258**	occ	0.308***
	[0.104]		[0.07]
ivc	0.013	dur	0.178***
	[0.035]		[0.063]
ivg	0.144***	cei	0.104
	[0.051]		[0.068]
ig	0.113***	P/X	0.131**
	[0.044]		[0.063]
nsi	0.202***	RA	0.221***
	[0.043]		[0.053]

Table AVIII

Decomposition of dispersion in price-earnings ratios using only actively updated forecasts

This table decomposes the variance of price-earnings ratios when the expectations are constructed using only actively updated forecasts. The *FIRE* columns report the elements CF_h, DR_h and FPX_h of the decomposition using future earnings growth, future negative returns and future price-earnings ratios. The *Expected* columns report the elements of the decomposition using expected earnings growth, expected returns and expected price-earnings ratios. To eliminate the possibility that the decomposition results are driven by stale forecasts, consensus expectations are constructed exclusively from those individual forecasts that were actively updated each quarter. The *Errors* columns report the contribution of the forecast errors of each element. The main sample period is 1999 to 2020. The fourth row shows the element CF_h of the decomposition estimated over the longer sample period of 1982-2020. Driscoll-Kraay standard errors and block-bootstrap standard errors are calculated for each coefficient. In both cases, we use a lag equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	$h=1$			$h=2$			$h=3$			$h=4$		
	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error	FIRE	Expected	Error
						Main Sample: 1999-2020						
CF_h	0.109**	0.356***	−0.247***	0.083	0.374***	−0.291***	0.063	0.420***	−0.357***	0.100**	0.462***	−0.362***
	[0.040]	[0.027]	[0.034]	[0.055]	[0.019]	[0.051]	[0.054]	[0.017]	[0.053]	[0.050]	[0.016]	[0.054]
	[0.033]	[0.027]	[0.029]	[0.045]	[0.021]	[0.043]	[0.043]	[0.020]	[0.042]	[0.038]	[0.018]	[0.040]
DR_h	0.157**	−0.002	0.159**	0.293***			0.327***			0.319***	0.167***	0.152
	[0.063]	[0.010]	[0.066]	[0.064]			[0.065]			[0.088]	[0.036]	[0.099]
	[0.060]	[0.011]	[0.067]	[0.058]			[0.051]			[0.060]	[0.031]	[0.077]
FPX_h	0.726***	0.630***	0.096	0.609***			0.588***			0.552***	0.317***	0.234***
	[0.058]	[0.026]	[0.068]	[0.037]			[0.038]			[0.059]	[0.030]	[0.074]
	[0.057]	[0.026]	[0.073]	[0.037]			[0.032]			[0.043]	[0.027]	[0.059]
						Long Sample: 1982-2020						
CF_h	0.143***	0.355***	−0.211***	0.112**	0.378***	−0.266***	0.108**	0.442***	−0.334***	0.152***	0.502***	−0.350***
	[0.027]	[0.028]	[0.025]	[0.052]	[0.029]	[0.039]	[0.048]	[0.031]	[0.034]	[0.042]	[0.032]	[0.031]
	[0.020]	[0.021]	[0.018]	[0.036]	[0.022]	[0.028]	[0.031]	[0.021]	[0.024]	[0.026]	[0.023]	[0.022]

D. *Extended anomaly results*

In Tables AIX, AX, and AXI, we show the detailed results for each of the individual anomalies. Table AIX shows that positive earnings surprises are associated with only a moderate increase in expected next period earnings. Table AX shows the decomposition of unexpected returns into earnings surprises and unexpected future price-earnings ratios. Table AXI shows an exact decomposition of anomaly price growth rather than anomaly returns.

E. *FIRE model simulations*

For each model, we simulate the cross-section of firms. We set the number of firms based on the original calculations in each paper. Specifically, we use 50, 5,000, and 200 firms for Berk et al. (1999), Zhang (2005), and Lettau and Wachter (2007), respectively. We set every sample to a length of 20 years and we run 1,000 simulations for each model. All parameter values are taken from the original papers.

For Berk et al. (1999) and Zhang (2005), we sort firms into five portfolios based on their price-book ratios. For Berk et al. (1999), we treat profits as our measure of earnings and for Zhang (2005), we treat profits after the cost of new capital and adjustment costs as our measure of earnings.[41] For Lettau and Wachter (2007), the only firm variables are price and dividends, so we treat dividends as our measure of earnings and sort firms into five portfolios based on their price-dividend ratios. We then estimate the finite-horizon and infinite horizon decomposition in equation (3) for each model.

E.1. Berk, Green, and Naik 1999

Each firm has some existing projects which generate cash flows. Each period, the firm draws a new potential project, which it can pay a fixed cost to undertake. The value of the firm comes from its existing projects as well as the option to undertake future projects ("growth options"). As the term "growth options" implies, future earnings growth plays a key role in

[41] We find nearly identical results if we use profits as our measure of earnings for Zhang (2005).

Table AIX

Revisions in expectations

This table shows the effect of earnings surprises on revisions. For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. Each row shows the coefficient from regressing the revision in future earnings $E^*_{t+1}[\tilde{x}_{i,t+2}] - E^*_t[\tilde{x}_{i,t+2}]$ on the earnings surprise $x_{i,t+1} - E^*_t[x_{i,t+1}]$. The Representative Anomaly (RA) is the average ranking of each stock across 20 different anomalies. P/X shows the results for the main portfolios used to study cross-sectional variation in price-earnings ratios. Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to positively comove with future returns. The first and third columns show the result of the regressions using the main sample period of 1999 to 2020. The second and fourth columns show the result of the regressions using the long sample period of 1982 to 2020. Driscoll-Kraay standard errors are calculated for each coefficient using a lag equal to the equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

$\tilde{a}_{i,t}$	Main Sample 1999-2020	Long Sample 1982-2020	$\tilde{a}_{i,t}$	Main Sample 1999-2020	Long Sample 1982-2020
rdm	-0.059	-0.010	noa	0.127**	0.122***
	[0.046]	[0.041]		[0.056]	[0.039]
bm	0.194***	0.179***	oaa	0.188***	0.167***
	[0.060]	[0.046]		[0.053]	[0.035]
cfp	0.218***	0.196***	ol	0.121**	0.140***
	[0.046]	[0.039]		[0.058]	[0.037]
adm	0.043	0.083***	pia	0.144***	0.113***
	[0.029]	[0.025]		[0.053]	[0.036]
nop	0.073**	0.070**	poa	0.127*	0.141***
	[0.036]	[0.030]		[0.067]	[0.045]
ia	0.104***	0.137***	pta	-0.002	0.016
	[0.027]	[0.024]		[0.028]	[0.024]
gp	0.257**	0.223***	occ	0.274***	0.251***
	[0.111]	[0.074]		[0.081]	[0.057]
ivc	-0.019	0.053*	dur	0.479***	0.443***
	[0.032]	[0.031]		[0.071]	[0.063]
ivg	0.130***	0.134***	cei	0.082	0.114**
	[0.044]	[0.033]		[0.067]	[0.047]
ig	0.062	0.068***	P/X	0.136*	0.214**
	[0.040]	[0.025]		[0.072]	[0.091]
nsi	0.168***	0.143***	RA	0.189***	0.182***
	[0.030]	[0.031]		[0.049]	[0.060]

Table AX

Unexpected anomaly returns

This table measures and decomposes unexpected anomaly returns using equation (7). For each anomaly $\tilde{a}_{i,t}$, we sort stocks into five equal-value portfolios based on the anomaly variable. The table shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. The three dependent variables are the unexpected return $\tilde{r}_{i,t+1} - E_t^*[\tilde{r}_{i,t+1}]$, the earnings forecast errors $\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]$, and the price-earnings ratio forecast errors $\rho\left(\tilde{px}_{i,t+1} - E_t^*[\tilde{px}_{i,t+1}]\right)$. The Representative Anomaly (RA) is the average ranking of each stock across 20 different anomalies. P/X shows the results for the main portfolios used to study cross-sectional variation in price-earnings ratios. Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to positively comove with future returns. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are calculated for each coefficient using a lag equal to the equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	Decomposition				Decomposition		
$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$	$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$
rdm	0.009	0.055***	-0.044***	noa	0.019***	-0.039***	0.057***
	[0.006]	[0.013]	[0.013]		[0.007]	[0.011]	[0.008]
bm	0.013	0.052**	-0.041***	oaa	0.022	0.079***	-0.059**
	[0.014]	[0.023]	[0.016]		[0.018]	[0.029]	[0.024]
cfp	0.014	0.011	-0.002	ol	0.015***	0.070***	-0.051***
	[0.014]	[0.019]	[0.012]		[0.005]	[0.014]	[0.014]
adm	0.018**	0.123***	-0.104***	pia	-0.006	-0.011	0.007
	[0.007]	[0.010]	[0.010]		[0.011]	[0.013]	[0.014]
nop	0.018**	-0.045***	0.057***	poa	0.002	0.038***	-0.035***
	[0.008]	[0.015]	[0.013]		[0.007]	[0.012]	[0.010]
ia	0.046***	0.140***	-0.096***	pta	0.007	-0.003	0.008
	[0.016]	[0.030]	[0.019]		[0.007]	[0.008]	[0.008]
gp	0.008	0.009	0.002	occ	0.019**	0.054***	-0.034***
	[0.010]	[0.016]	[0.015]		[0.009]	[0.016]	[0.012]
ivc	0.011	-0.033***	0.040***	dur	0.062***	0.074**	-0.013
	[0.007]	[0.012]	[0.009]		[0.016]	[0.033]	[0.020]
ivg	0.013	0.023	-0.012	cei	0.048**	0.028	0.016
	[0.010]	[0.015]	[0.012]		[0.021]	[0.031]	[0.015]
ig	0.019	0.035**	-0.018**	P/X	0.033**	0.069***	-0.038**
	[0.012]	[0.014]	[0.008]		[0.016]	[0.010]	[0.017]
nsi	0.026**	0.029	-0.004	RA	0.034**	0.064***	-0.032***
	[0.012]	[0.018]	[0.009]		[0.013]	[0.020]	[0.009]

Table AXI

Individual unexpected anomaly price growth (exact decomposition)

This table measures and decomposes unexpected anomaly price growth for each of the individual anomalies using equation (A10). For each anomaly variable, we sort stocks into five equal-value portfolios based on the anomaly variable. The table shows the coefficients of regressing each of the dependent variables on a specific anomaly variable. The three dependent variables are the unexpected price growth $\Delta\tilde{p}_{i,t+1} - E_t^*[\Delta\tilde{p}_{i,t+1}]$, the earnings forecast errors $\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]$, and the price-earnings ratio forecast errors $\tilde{px}_{i,t+1} - E_t^*[\tilde{px}_{i,t+1}]$. The Representative Anomaly (RA) is the average ranking of each stock across 20 different anomalies. P/X shows the results for the main portfolios used to study cross-sectional variation in price-earnings ratios. Each anomaly variable $\tilde{a}_{i,t}$ is scaled to have unit variance and to positively comove with future price growth. The sample period is 1999 to 2020. Driscoll-Kraay standard errors are calculated for each coefficient using a lag equal to the equal to the maximum lag with any significant residual autocorrelation. Superscripts indicate Driscoll-Kraay significance at the 1% (***), 5% (**), and 10% (*) level.

	Decomposition				Decomposition		
$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$	$\tilde{a}_{i,t}$	$\sigma_{a,r}$	$\sigma_{a,x}$	$\sigma_{a,px}$
rdm	0.010*	0.055***	-0.045***	noa	0.019***	-0.039***	0.058***
	[0.006]	[0.013]	[0.014]		[0.007]	[0.011]	[0.009]
bm	0.011	0.052**	-0.042***	oaa	0.019	0.079***	-0.060**
	[0.014]	[0.023]	[0.016]		[0.018]	[0.029]	[0.024]
cfp	0.009	0.011	-0.002	ol	0.017***	0.070***	-0.053***
	[0.015]	[0.019]	[0.012]		[0.005]	[0.014]	[0.014]
adm	0.017**	0.123***	-0.106***	pia	-0.004	-0.011	0.007
	[0.007]	[0.010]	[0.011]		[0.011]	[0.013]	[0.014]
nop	0.014*	-0.045***	0.059***	poa	0.002	0.038***	-0.036***
	[0.008]	[0.015]	[0.013]		[0.007]	[0.012]	[0.010]
ia	0.043***	0.140***	-0.098***	pta	0.005	-0.003	0.009
	[0.017]	[0.030]	[0.020]		[0.007]	[0.008]	[0.009]
gp	0.011	0.009	0.002	occ	0.020**	0.054***	-0.035***
	[0.010]	[0.016]	[0.015]		[0.009]	[0.016]	[0.013]
ivc	0.008	-0.033***	0.041***	dur	0.062***	0.074**	-0.013
	[0.007]	[0.012]	[0.010]		[0.016]	[0.033]	[0.021]
ivg	0.011	0.023	-0.012	cei	0.045**	0.028	0.016
	[0.011]	[0.015]	[0.012]		[0.021]	[0.031]	[0.016]
ig	0.017	0.035**	-0.019**	P/X	0.030*	0.069***	-0.039**
	[0.012]	[0.014]	[0.008]		[0.016]	[0.010]	[0.018]
nsi	0.024*	0.029	-0.004	RA	0.031**	0.064***	-0.033***
	[0.012]	[0.018]	[0.009]		[0.013]	[0.020]	[0.009]

this model. The ratio of the firm's price to its current earnings reflects how much of the firm's value comes from existing projects versus growth options. Firms with high price-earnings ratios derive most of their value from their expected future projects rather than existing projects, and future earnings growth accounts for most dispersion in price-earnings ratios ($CF_4 = 0.85$).

The model features a time-varying risk-free rate which also generates differences in risk premia.[42] Compared to existing projects, the value of growth options is less sensitive to changes in the risk-free rate, as the firm can endogenously change its decision to exercise the option (i.e., it only undertakes the potential project if the risk-free rate is low). Because of this, the agent requires a higher risk premium for firms with low price-earnings ratios. Quantitatively, the difference in risk premia is only a small part of the dispersion in price-earnings ratios ($DR_4 = 0.03$, $DR_\infty = 0.04$).

E.2. Zhang 2005

In this model, firm earnings are

$$X_{i,t} \quad = \quad e^{x_t+z_{i,t}+p_t} k_{i,t}^{\alpha} - f - i_{i,t} - h\left(i_{i,t}, k_{i,t}\right)$$

where x_t is aggregate productivity, $z_{i,t}$ is idiosyncratic productivity, p_t is the aggregate price level, $k_{i,t}$ is firm-level capital, f is a fixed cost, $i_{i,t}$ is investment in capital, and $h\left(i_{i,t}, k_{i,t}\right)$ is an adjustment cost. Differences across firms are due to differences in their sequence of idiosyncratic productivity $\{z_{i,\tau}\}_{\tau=0}^{t}$. Because idiosyncratic productivity is AR(1), future earnings growth is partly predictable and dispersion in price-earnings ratios largely predicts differences in future earnings growth ($CF_4 = 0.69$).

The model also features differences in discount rates. Because of adjustment costs, it is costly for firms to lower their capital to the new optimal level after a negative shock to aggregate productivity x_t. Therefore, the agent requires a higher risk premium for firms with high capital relative to total firm value, as they are more sensitive to negative aggregate

[42]The risk-free rate is closely tied to the agent's stochastic discount factor.

shocks. Quantitatively, these differences in risk premia are small relative to the dispersion in price-earnings ratios ($DR_4 = -0.03$).[43]

In order to calculate CF_h and DR_h, we have to address the issue that model earnings are often negative, even at the portfolio level, which is not compatible with the Campbell-Shiller decomposition.[44] To use the decomposition, we want to think about an investor that makes a one-time payment to buy a claim to the company, never pays anything more in the future, and receives some cash flows in the future. Thus, we will think of an investor that holds some share $\theta_{i,t}$ of the company. When the company has positive cash flows, the investor does not change her share in the company and receives these cash flows. When the company has negative cash flows, we assume the investor sells a part of her stake in the company to cover this. Specifically, this investor receives cash flows $\hat{X}_{i,t} \equiv \theta_{i,t} \max\{X_{i,t}, 0\}$, where $\theta_{i,t} = \theta_{i,t-1}(1 + \min\{X_{i,t}, 0\}/P_{i,t})$. Intuitively, rather than receiving a negative cash flow, this investor dilutes her claim to the future (on average positive) cash flows. This investor receives the same return as someone who owned the entire firm and received the negative cash flows, $\frac{\theta_{i,t}P_{i,t}+\hat{X}_{i,t}}{\theta_{i,t-1}P_{i,t-1}} \equiv \frac{P_{i,t}+X_{i,t}}{P_{i,t-1}}$.

E.3. Lettau and Wachter 2007

In this model, each firm receives some share $s_{i,t}$ of the aggregate earnings. The value of $s_{i,t}$ goes through a fixed cycle, increasing from $\underline{s}$ to a peak value of $\bar{s}$ and then decreasing back to $\underline{s}$. The cross-section of firms is populated with firms at different points in this share cycle. Because all firms receive a share of the same aggregate earnings, the cross-sectionally demeaned log earnings growth $\Delta\tilde{x}_{i,t}$ is simply the log share growth $\log(s_{i,t}) - \log(s_{i,t-1})$.

In the model, the stochastic discount factor is exposed to shocks that are partly reversed

[43] In the model, high price-earnings ratio firms have *low* price-capital ratios. A 1% increase in $e^{z_{i,t}}$ does not change the current capital ($k_{i,t}$), increases the current earnings by 1%, and increases the current price by less than 1% since the increase in productivity is persistent but not permanent. Thus, an increase in $z_{i,t}$ raises the price-capital ratio and lowers the price-earnings ratio. This is why the discount rate component DR_h is slightly negative, as the model predicts that high price-capital ratio firms will have lower future returns, which means that high price-earnings ratio firms will have *higher* future returns.

[44] After a large aggregate shock, nearly all firms will substantially change their capital which requires paying large adjustment costs.

over time, which means that the agent requires a lower risk premium for longer horizon cash flows. Because of this, firms with high price-earnings ratios (i.e., firms with a low current share $s_{i,t}$) earn slightly lower returns for the first few years ($DR_4 = 0.06$). However, the quantitatively larger component is that high price-earnings ratio firms experience higher earnings growth as their share increases ($CF_4 = 0.24$). Over time, the firms with low current $s_{i,t}$ eventually become the firms with high $s_{i,t+h}$ and require a higher risk premium, as their cash flows are now front-loaded. Thus, the discount rate component is small and ambiguous in terms of sign at long horizons, $DR_\infty = -0.04$ (0.08).

F. *Behavioral and learning model predictions*

In this section, we discuss how our findings relate to several behavioral and learning models in which subjective discount rates are constant and the comovement of current price ratios with future returns is due to non-FIRE beliefs about cash flow growth. When subjective expected returns are constant, $E_t^*[\tilde{r}_{i,t+j}] = \bar{r}$, equations (1) and (2) imply that realized returns are

$$\tilde{r}_{i,t+1} - \bar{r} \approx \left(\Delta\tilde{x}_{i,t+1} - E_t^*[\Delta\tilde{x}_{i,t+1}]\right) + \sum_{j=2}^{\infty} \rho^{j-1}\left(E_{t+1}^*[\Delta\tilde{x}_{i,t+j}] - E_t^*[\Delta\tilde{x}_{i,t+j}]\right).$$

If one-period cash flow surprises $\tilde{x}_{i,t+1} - E_t^*[\tilde{x}_{i,t+1}]$ (which are equivalent to $\Delta\tilde{x}_{i,t+1} - E_t^*[\Delta\tilde{x}_{i,t+1}]$) are positively related to revisions in expected future growth, then one-period cash flow surprises will impact unexpected returns more than 1-1. Empirically, we find that one-period cash flow surprises are negatively related to revisions in expected future growth, as earnings are largely expected to return to their previous levels, and thus translate less than 1-1 into unexpected returns.

Consider the case where agents overstate the persistence of growth. Specifically, the true persistence of growth is ϕ but agents believe the persistence is $\phi^* > \phi$. So long as $\phi^* > 0$, then a positive surprise $\Delta\tilde{x}_{i,t+1} - E_t^*[\Delta\tilde{x}_{i,t+1}]$ will raise expected future growth and translate more than 1-1 into realized returns.

Further, we can consider learning about the mean of an i.i.d. growth process, such as Nagel and Xu (2022). In this setting, agents form expectations of growth based on a weighted average of past realized growth. A higher than expected realization for cash flow growth causes the agent to positively revise her beliefs about mean growth. Because there are no temporary shocks to the level of cash flows, this increase in expected mean growth causes the agent to raise her expectations of all future growth. Specifically, Nagel and Xu (2022) show that the realized unexpected return is

$$r_{t+1} - E_t^* \left[r_{t+1} \right] = \left(1 + \frac{\rho v}{1-\rho} \right) \left(\Delta x_{t+1} - \tilde{\mu}_t \right)$$

where $\tilde{\mu}_t$ is the agent's current expectation of growth and v is the learning gain parameter. Given that $v > 0$, it is immediate that cash flow growth surprises translate more than 1-1 into realized returns.

Finally, we discuss the case of diagnostic growth expectations, as in Bordalo et al. (2024a). This model proposes that earnings growth is impacted by tangible news τ_{t+1} and intangible news η_t. Specifically, the process for earnings growth is

$$\Delta x_{t+1} = \mu \Delta x_t + \eta_t + \tau_{t+1}.$$

Subjective expectations of growth are

$$\begin{aligned} E_t^* \left[\Delta x_{t+j} \right] &= \mu^{j-1} \left(\mu \Delta x_t + \eta_t \right) + \mu^{j-1} \epsilon_t \\ \epsilon_t &= \phi \epsilon_{t-1} + \theta \left(\mu \tau_t + \eta_t \right) \end{aligned}$$

where ϵ_t captures biases in expectations and ϕ is assumed to be less than μ. As stated in the paper, realized unexpected returns are

$$r_{t+1} - \bar{r} = \Delta x_{t+1} - E_t^* \left[\Delta x_{t+1} \right] + \sum_{j=2}^{\infty} \rho^{j-1} \left(E_{t+1}^* \left[\Delta x_{t+j} \right] - E_t^* \left[\Delta x_{t+j} \right] \right).$$

The covariance of the price-earnings ratio with future earnings growth surprises and future unexpected returns is then

$$\begin{aligned}
Cov\left(px_t, \Delta x_{t+1} - E_t^*\left[\Delta x_{t+1}\right]\right) &= Cov\left(px_t, -\epsilon_t\right) \\
Cov\left(px_t, r_{t+1} - \bar{r}\right) &= Cov\left(px_t, -\left[1 + \frac{\rho}{1-\rho\mu}\left(\mu - \phi\right)\right]\epsilon_t\right) \\
&= \left[1 + \frac{\rho}{1-\rho\mu}\left(\mu - \phi\right)\right] Cov\left(px_t, \Delta x_{t+1} - E_t^*\left[\Delta x_{t+1}\right]\right).
\end{aligned}$$

Given the paper's assumption that $\mu > \phi$, this means that the covariance of price-earnings ratios with unexpected returns must be a magnified version of the covariance of price-earnings ratios with earnings growth surprises. In fact, the model implies that for any time t variable, the comovement of that variable with unexpected returns will be $1 + \frac{\rho}{1-\rho\mu}\left(\mu - \phi\right)$ times the comovement of that variable with earnings growth surprises. Thus, the model cannot match our finding that the comovement of price-earnings ratios with one-year unexpected returns is *smaller* than the comovement of price-earnings ratios with earnings growth surprises.

As an extension, we also consider the model of diagnostic expectations of earnings *levels* in Bordalo et al. (2019). In this model, the difference between subjective and objective expectations of the level of log earnings is

$$E_t^*\left[x_{t+j}\right] - E_t\left[x_{t+j}\right] = a^j \frac{1 - (b/a)^j}{1 - b/a}\left(\hat{f}_t^\theta - \hat{f}_t\right)$$

where $a, b > 0$, $\hat{f}_t$ is an objective inference of an underlying component of earnings, and $\hat{f}_t^\theta$ is the biased inference of this component. Simulating the model using the paper's parameter values, we find that high price-earnings ratio stocks have *lower* subjective expected one-year earnings growth. Further, we find that price-earnings ratios are negatively related to current $\hat{f}_t^\theta - \hat{f}_t$, meaning that high price-earnings ratio stocks have pessimistic expectations of earnings at all horizons. These predictions do not align with our empirical findings that high price-earnings ratio stocks have high subjective expected one-year earnings growth and that high price-earnings ratios are associated with overoptimism in subjective expected earnings growth.

G. Subjective risk for our portfolios

One of the main components of our model is that investors perceive lower risk for the high $\tilde{px}_{i,t}$ firms. While the relation is supported by our main evidence in Table I that high price-earnings ratios are associated with lower subjective expected returns, we can also look at more direct measures of subjective risk. In this section we explore the relation between our portfolios and two subjective risk measures: the absolute risk index assigned to firms by Value Line, and a cross-sectionally standardized risk index created by Jensen (2024).

The first measure of risk is the "Safety Rank," directly taken from Value Line. This measure ranges from 1 to 5, where 5 denotes a high perceived risk, and it equals the average of the analyst score for price stability and financial strength, two perceived characteristics for each firm. We take a value–weighted average of this measure across all firms in each portfolio to obtain our first subjective risk measure. To account for time effects, we also construct a second standardized measure of risk following Jensen (2024). For each firm, we define the subjective risk as the average cross-sectional rank of price stability and financial strength, and then we standardize this measure every period (i.e., we rescale the measure so that the cross-sectional mean and cross-sectional standard deviation are 0 and 1 in every period).

Figure A1 shows that our high $\tilde{px}_{i,t}$ portfolios indeed have lower subjective expected risk using both the direct and the standardized measures of expected risk. The direct measure of risk is on average 2.90 for the lowest $\tilde{px}_{i,t}$ portfolio and 2.69 for the highest $\tilde{px}_{i,t}$ portfolio, while the standardized measure is 0.11 for the lowest $\tilde{px}_{i,t}$ portfolio and -0.12 for the highest $\tilde{px}_{i,t}$ portfolio. As shown by the 95% confidence interval bars, these differences are highly significant for both measures.

H. Model estimation

This section derives the cash flow parameters ϕ, σ_u, and σ_v from the standard deviation and autocorrelation of aggregate earnings growth $\sigma\left(\Delta x_t^{agg}\right)$ and $AC\left(\Delta x_t^{agg}\right)$ and the average

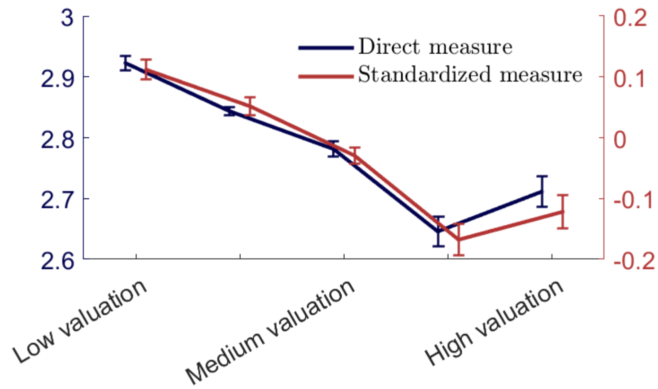

Figure A1. Subjective risk across portfolios. This figure plots the average subjective risk for each of the five main portfolios. The direct measure in blue is the "Safety Rank" measure from Value Line. This measure ranges from 1 to 5, where 5 is the highest perceived risk. The standardized measure in red takes the average of the 'Price Stability' and 'Financial Strength' measures and it is cross-sectionally rescaled to have mean zero and unit standard deviation. This measure increases with perceived risk. Each portfolio shows the 95% confidence intervals. The 5 portfolios are shown in ascending order of price-earnings ratios.

across portfolios of the standard deviation over time of earnings growth $\sigma\left(\Delta\tilde{x}_{i,t+1}\right)$. We also relate the constant-gain learning from annual observations used in our model to the evidence on belief updating from quarterly observations.

H.1. Parameter values

According to equation (9), we can express aggregate earnings growth as:

$$\Delta x_t^{agg} = \phi\Delta x_{t-1}^{agg} - u_{t-1} + u_t. \tag{A11}$$

Taking covariance of equation (A11) with current earnings growth on both sides results in:

$$\begin{aligned} Cov(\Delta x_t^{agg}, \Delta x_{t-1}^{agg}) &= \phi Var(\Delta x_{t-1}^{agg}) - \sigma_u^2 \\ AC(\Delta x_t^{agg}) &= \phi - \frac{\sigma_u^2}{Var(\Delta x_t^{agg})}. \end{aligned} \tag{A12}$$

Taking the variance of equation (A11) on both sides gives:

$$\begin{aligned} Var(\Delta x_t^{agg}) &= \phi^2 Var(\Delta x_t^{agg}) + 2\sigma_u^2 - 2\phi\sigma_u^2 \\ Var(\Delta x_t^{agg}) &= \frac{2\sigma_u^2}{1+\phi}. \end{aligned} \tag{A13}$$

From equations (A12) and (A13), we have:

$$\begin{aligned} \phi &= 1 + 2AC\left(\Delta x_t^{agg}\right) \\ \sigma_u &= \left(\frac{1+\phi}{2}\right)^{1/2} \sigma\left(\Delta x_t^{agg}\right). \end{aligned}$$

Finally, to estimate the individual variance, we use equation (10) to obtain the value for σ_v in terms of idiosyncratic earnings growth:

$$\sigma_v = \frac{\sigma\left(\Delta \tilde{x}_{i,t}\right)}{\sqrt{2}}.$$

From the empirical values over the 1982-2020 sample of $\sigma\left(\Delta x_t^{agg}\right) = 0.353$, $AC\left(\Delta x_t^{agg}\right) = -0.086$ and a median portfolio volatility of $\sigma\left(\Delta \tilde{x}_{i,t}\right) = 0.140$ we infer $\phi = 0.828$, $\sigma_u = 0.337$ and $\sigma_v = 0.099$

H.2. Constant-gain parameter

We set our constant-gain parameter based on previous estimates of belief updating from Malmendier and Nagel (2016). In this section, we show that for small gains β, annual updating of beliefs based on annual surprises is quite close to quarterly updating of beliefs based on quarterly surprises.

For intuition, first consider the case of semi-annual updating. In this scenario, the agent is attempting to learn the parameter μ from a semi-annual variable x_t using the updating rule

$$E_t^*[\mu] = E_{t-1/2}^*[\mu] + \beta\left(x_t - E_{t-1/2}^*[\mu]\right). \tag{A14}$$

Iterating this equation, we have

$$\begin{aligned} E_t^*[\mu] &= E_{t-1}^*[\mu] + \beta\left(x_t - E_{t-1/2}^*[\mu]\right) + \beta\left(x_{t-1/2} - E_{t-1}^*[\mu]\right) && \text{(A15)} \\ &= E_{t-1}^*[\mu] + \beta\left(x_{t-1/2} + x_t - 2E_{t-1/2}^*[\mu]\right) - \beta^2\left(x_{t-1/2} - E_{t-1}^*[\mu]\right) && \text{(A16)} \\ &\approx E_{t-1}^*[\mu] + \beta\left(x_{t-1/2} + x_t - 2E_{t-1/2}^*[\mu]\right). && \text{(A17)} \end{aligned}$$

Equation (A17) shows that for small values of β, this semi-annual updating rule is closely approximated by an annual updating rule using the same gain β and the annual surprise $x_{t-1/2} + x_t - 2E^*_{t-1/2}[\mu]$. This is because the effect of within-year updating depends on β^2, which in our case would be quite small at 0.0003. In other words, the adjustment for within-year updating is second order compared to the first order change in beliefs $\beta\left(x_{t-1/2} + x_t - 2E^*_{t-1/2}[\mu]\right)$.

Now, we consider the scenario of quarterly updating. The agent is attempting to learn the parameter μ from a quarterly variable x_t. The agent's updating rule is

$$E^*_t[\mu] = E^*_{t-1/4}[\mu] + \beta\left(x_t - E^*_{t-1/4}[\mu]\right) \tag{A18}$$

$$= E^*_{t-1}[\mu] + \beta\left(x_{t-3/4} + x_{t-1/2} + x_{t-1/4} + x_t - 4E^*_{t-1}[\mu]\right)$$
$$-\beta^2\left[3\left(x_{t-3/4} - E^*_{t-1}[\mu]\right) + 2\left(x_{t-1/2} - E^*_{t-3/4}[\mu]\right) + x_{t-1/4} - E^*_{t-1/2}[\mu]\right]. \tag{A19}$$

Once again, this is approximately equal to an annual updating rule based on the annual surprise $x_{t-3/4} + x_{t-1/2} + x_{t-1/4} + x_t - 4E^*_{t-1}[\mu]$. The adjustments for within-year updating are all scaled by β^2.

I. *Model extensions*

I.1. Stubborn errors and rational arbitrageurs

This extension shows that in the presence of rational arbitrageurs with shorting costs, a lower value of β limits the scope for arbitrageurs to intervene, allowing errors in expectations to contribute more significantly to price dispersion.

First, consider the model of Section VI without any modifications. From equation (16), strip prices are simply functions of the aggregate state variable x^{agg}_t. If agents had FIRE beliefs (i.e., agents knew the true parameters), then all firms would have the same price, $P^{FIRE}_t \equiv \exp\left\{\frac{1}{2}\sigma^2_v\right\}\sum_{n=1}^{\infty} P^{(n)}_t$. Due to errors in expectations $\left(E^*_t[g_i], \tilde{x}_{i,t} - E^*_t[v_{i,t}]\right)$, prices deviate from this FIRE benchmark,

$$\frac{P_{i,t}}{P_t^{FIRE}} = \exp\left\{\tilde{x}_{i,t} - E_t^*\left[v_{i,t}\right]\right\}\left(\sum_{n=1}^{\infty} P_t^{(n)} \exp\left\{nE_t^*\left[g_i\right]\right\}\right) / \sum_{n=1}^{\infty} P_t^{(n)}$$
$$= F\left(x_t^{agg}, E_t^*\left[g_i\right], \tilde{x}_{i,t} - E_t^*\left[v_{i,t}\right]\right). \quad \text{(A20)}$$

Let $b_{i,t} \equiv E_t\left[\Delta\tilde{x}_{i,t+1}\right] - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right] = -E_t^*\left[g_i\right] - \tilde{x}_{i,t} + E_t^*\left[v_{i,t}\right]$ be the bias in one-period cash flow expectations. The objective expected return is

$$E_t\left[R_{i,t+1}\right] = \sum_{n=1}^{\infty} w_{i,t,n} E_t\left[R_{t+1}^{(n)}\right] E_t\left[\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]}\right]$$
$$= w_{i,t,1}\exp\left\{b_{i,t} + r^f\right\} + \sum_{n=2}^{\infty} w_{i,t,n}\exp\left\{n\beta b_{i,t} + \frac{1}{2}\left(n\beta\right)^2\sigma_v^2 + r^f + \gamma\sigma_u^2\phi^{n-1}\right\}$$
$$= H\left(x_t^{agg}, b_{i,t}\right). \quad \text{(A21)}$$

Importantly, the function $H\left(x_t^{agg}, b_{i,t}\right)$ depends on the parameter β, whereas the function $F\left(x_t^{agg}, E_t^*\left[g_i\right], \tilde{x}_{i,t} - E_t^*\left[v_{i,t}\right]\right)$ does not. In an economy without rational arbitrageurs, changing the stubbornness of errors in expectations (i.e., changing β) does not affect how $E_t^*\left[g_i\right]$ and $\tilde{x}_{i,t} - E_t^*\left[v_{i,t}\right]$ impact the level of prices but does affect how much these errors in expectations translate into predictable one-period returns.

We now consider risk-neutral rational arbitrageurs. These agents know the true parameters and can construct long-short portfolios to profit on mispricing.[45] We assume these agents face a linear shorting cost with parameter η. This means that, in equilibrium, there cannot be two assets i, j such that $E_t\left[R_{i,t+1}\right] - E_t\left[R_{j,t+1}\right] > \eta$, otherwise the arbitrageurs would take an infinite long position in i and an infinite short-position in j. It can be shown that this implies in equilibrium that $\left|E_t\left[\tilde{R}_{i,t+1}\right]\right| \leq \eta/2$ for all firms.[46]

Because of the shorting cost, these rational arbitrageurs do not correct all mispricing when they are added to the model. From equation (A21), we see that there are threshold

[45] In the model, the only tradable assets are shares of the firms. Strip prices are simply mathematical objects to help describe firm prices.

[46] Given that we have a continuum of firms, there will always be some firm with expected demeaned return exactly equal to $\eta/2$ or $-\eta/2$. This means that if any other firm had a demeaned expected return outside $[-\eta/2, \eta/2]$ then it would immediately imply that there is a pair of firms with $E_t\left[R_{i,t+1}\right] - E_t\left[R_{j,t+1}\right] > \eta$.

biases such that the arbitrageurs do not trade any firms with $b_{i,t} \in \left[\underline{b}\left(x_t^{agg}\right), \bar{b}\left(x_t^{agg}\right)\right]$. The key takeaway is that these thresholds depend on β. For lower values of β, objective expected one-period returns are less sensitive to $b_{i,t}$ and the range of biases where arbitrageurs do not intervene $\left[\underline{b}\left(x_t^{agg}\right), \bar{b}\left(x_t^{agg}\right)\right]$ is wider. As mentioned above, changing β does not alter the function $F\left(x_t^{agg}, E_t^*\left[g_i\right], \tilde{x}_{i,t} - E_t^*\left[v_{i,t}\right]\right)$, so these errors in expectations will still have a large effect on the level of prices even though they do not lead to large enough objective expected one-period returns for arbitrageurs to intervene.

As a final point, we can examine the role of the temporary level shocks. Relative to the belief-formation models of Lewellen and Shanken (2002) and Nagel and Xu (2022), our non-rational agent attributes part of the earnings growth surprise to temporary shocks to the level of earnings. Without this feature, her expectations follow

$$E_t^*\left[g_i\right] = E_{t-1}^*\left[g_i\right] + \beta\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*\left[\Delta\tilde{x}_{i,t}\right]\right) \tag{A22}$$

$$E_t^*\left[\tilde{x}_{i,t+n}\right] = \tilde{x}_{i,t} + nE_t^*\left[g_i\right] \tag{A23}$$

$$\frac{E_{t+1}^*\left[\tilde{X}_{i,t+n}\right]}{E_t^*\left[\tilde{X}_{i,t+n}\right]} = \exp\left\{\left(1 - \beta + n\beta\right)\left(\Delta\tilde{x}_{i,t+1} - E_t^*\left[\Delta\tilde{x}_{i,t+1}\right]\right)\right\}. \tag{A24}$$

Comparing equations (22) and (A24), revisions would be much more sensitive to surprises, as $n\beta$ is changed to $1 - \beta + n\beta$. Given equation (21), this means that predictable errors in expectations would translate much more strongly into predictable one-year returns.

I.2. Time-varying latent component of growth

In this extension, we consider a model in which each firm's underlying growth is time-varying rather than a fixed parameter. Specifically, the firm-specific component of earnings growth is

$$\tilde{x}_{i,t} = z_{i,t} + v_{i,t}$$

$$\Delta z_{i,t} - \mu = \phi_z\left(\Delta z_{i,t-1} - \mu\right) + \varepsilon_{i,t}$$

where the shocks are independent and have variances σ_v^2 and σ_ε^2, respectively. The agent attempts to infer the underlying $\Delta z_{i,t}$ using constant-gain learning,

$$E_t^*[\Delta z_{i,t} - \mu] = \phi_z E_{t-1}^*[\Delta z_{i,t-1} - \mu] + \beta\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right) \tag{A25}$$

$$E_t^*[v_{i,t}] = (1-\beta)\left(\Delta\tilde{x}_{i,t} - E_{t-1}^*[\Delta\tilde{x}_{i,t}]\right). \tag{A26}$$

Her expectation for the future growth of the firm-level component is then

$$E_t^*[\Delta\tilde{x}_{i,t+1}] = \mu + \phi_z E_t^*[\Delta z_{i,t} - \mu] - E_t^*[v_{i,t}]. \tag{A27}$$

Her expectation for the future level of the firm-level component is

$$E_t^*[\tilde{x}_{i,t+n}] = \tilde{x}_{i,t} + n\mu + \phi_z \frac{1-\phi_z^n}{1-\phi_z} E_t^*[\Delta z_{i,t} - \mu] - E_t^*[v_{i,t}]. \tag{A28}$$

Since we are considering price-earnings ratios, we will normalize μ to 0 for simplicity. In the case of $\phi_z = 1$ and $\sigma_\varepsilon = 0$, this extended model collapses back to the main model of Section VI.

Note that the introduction of a time-varying component of firm-level growth does not affect the pricing of aggregate strips in equations (16)-(18). Further, given these new definitions for expected cash flows, equations (19)-(21) still hold for describing firm prices, expected returns, and realized returns.

Qualitatively, this model matches the main implications of our main model in Section VI. A higher expectation of $\Delta z_{i,t}$ will raise a firm's price by increasing the expected future cash flows and by lowering the subjective risk premium. If the constant-gain β is small, then earnings surprises will only moderately affect expected future earnings, as shown by equations (A28)-(A28). Similarly, if the constant-gain β is small, then disappointment in one-year earnings will only have a small immediate impact on returns, as shown by equations (21), (A25), (A26), and (A28).

Quantitatively, we find that this extension does not noticeably alter our results. Compared to our main model, this extension provides two additional parameters $(\phi_z, \sigma_\varepsilon)$. We estimate these parameters to best match the decomposition targets shown in Figure 6. The result is $\phi_z = 0.9999$ and $\sigma_\varepsilon = 0$, meaning that the extended model is virtually identical to the fixed parameter model of Section VI. In other words, for the purposes of explaining our empirical findings, the main model of Section VI and the extended model with a time-varying latent component of growth perform equally well. Because of this, we focus on the more parsimonious fixed-parameter model for our main analysis.