

# Which Subjective Expectations Explain Asset Prices?

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We present a method for determining whether errors in expectations explain asset pricing puzzles without imposing assumptions about the error mechanism. Using accounting identities and survey forecasts, we find that errors in expected long-term inflation explain price variation, return predictability, and the rejection of the expectations hypothesis for aggregate stock and bond markets. Errors in short-term (long-term) nominal earnings growth expectations explain (do not explain) stock price variation and return predictability. The relevant errors are consistent with mistakes about the persistence of forecasted variables and the response to surprises. A simple framework based on fundamental extrapolation successfully replicates these findings. (*JEL* G40, G12, G14, E71)

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Expectations of future fundamentals are a crucial part of asset pricing, as prices represent discounted expectations of future real cash flows. Under standard Rational Expectations (RE), investors' expectations match the objective probability distribution observed by an econometrician, which has led to many puzzles regarding price movements and future returns. However, RE imposes a huge number of restrictions on expectations and, unsurprisingly, many

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deviations from RE have been documented in subjective expectations.<sup>1</sup> While the existence of these systematic errors is a challenge to the RE framework, their role in resolving asset pricing puzzles is unclear. This raises the question, which deviations from RE help to explain asset prices?

Using accounting identities, we derive a diagnostic test on subjective expectations that (a) provides a necessary and sufficient condition for deviations from RE to explain price movements and return puzzles and (b) is easily implementable. Using survey expectations, we apply the diagnostic test to bond and stock markets, where real cash flows depend on inflation and nominal earnings growth minus inflation, respectively. We document two key results. First, and most importantly, we find a large role for systematic errors in *long-term inflation* expectations for explaining stock price movements and return predictability. These systematic errors also help to explain bond yield movements and violations of the expectations hypothesis. Second, long-term nominal earnings growth expectations fail the diagnostic test for explaining aggregate stock prices, whereas short-term nominal earnings growth expectations pass the diagnostic test.<sup>2</sup> A simple framework based on fundamental extrapolation replicates both key findings.

We first establish a diagnostic test that determines whether deviations from RE in subjective expectations of real cash flows explain asset price movements and return puzzles. The condition is that forecast errors for these expectations must be predicted by current prices. Intuitively, if high prices and subsequent low returns are associated with overly optimistic expectations of future real cash flows, then high prices should be followed by negative forecast errors (i.e., disappointment) for real cash flows. While we focus on stock and bond markets for our empirical analysis, we show that the diagnostic test holds more broadly and can be applied to topics as wide-ranging as UIP violations and corporate yield volatility.

Applying this diagnostic test to aggregate stock prices and survey forecasts of CPI inflation and nominal S&P 500 earnings growth, we find a large role for systematic errors in long-term inflation expectations. This is surprising if one thinks that inflation does not affect real cash flows; that is, nominal earnings growth responds 1-1 to changes in inflation. We find empirically that expected nominal earnings growth comoves relatively little with changes in expected inflation, which shows that increases in expected inflation lower expected real earning growth.<sup>3</sup> Because of this, systematic errors in long-term inflation expectations help to explain stock price movements and return predictability, as

<sup>1</sup> See [Barberis and Thaler \(2003\)](#) for a summary.

<sup>2</sup> We reconcile this finding with [Bordalo et al. \(2023a\)](#) and [Hillenbrand and McCarthy \(2022\)](#), who argue for a large role for long-term expectations. In particular, we show that our finding holds whether prices are measured using smoothed measures, such as the cyclically adjusted price-earnings ratio (CAPE) and the price-dividend ratio, or more volatile measures, such as the price-earnings ratio.

<sup>3</sup> Instead of thinking about real cash flows, one can alternatively think of the stock price as the expectation of future nominal earnings discounted using nominal rates. An increase in expected inflation raises the nominal discount

we find that low stock price ratios are consistently associated with overly high expectations of long-term inflation. Similarly, high long-term bond yields are also associated with overly high long-term inflation expectations, meaning that these systematic errors also help to explain yield movements and violations of the expectations hypothesis.

Importantly, not all deviations from RE satisfy the diagnostic test, meaning that the test serves as a useful tool for disciplining which subjective expectations are most relevant for explaining asset prices and return puzzles. Because the test is derived from accounting identities, it is general and holds for any deviation from RE (e.g., learning, rational inattention, bounded rationality, and behavioral biases). This allows us to identify the relevant errors without imposing assumptions about the mechanism behind the deviations from RE.

This ability to identify the relevant errors leads to our second main result: long-term nominal earnings growth expectations (LTG) fail the diagnostic test for explaining aggregate stock prices and return predictability, whereas short-term nominal earnings growth expectations pass the diagnostic test.<sup>4</sup> Specifically, we show that forecast errors for LTG are uncorrelated with current prices and therefore do not help to explain price variation or the ability of prices to predict future returns. In contrast, current prices strongly predict negative forecast errors for short-term nominal earnings growth, meaning that high current prices can be partly explained by overly high short-term nominal earnings growth expectations. Given that a 10% increase in short-term nominal earnings growth raises the level of all expected future earnings by 10%, these expectations can have a large impact on prices even if expected growth for future horizons remains constant.

Given the importance of this finding for the recent but growing literature on earnings growth expectations (Delao and Myers 2021; Nagel and Xu 2022; Adam and Nagel 2023; Hillenbrand and McCarthy 2022), we show that our results hold for both volatile measures of aggregate prices, such as the price-earnings ratio, and smoothed measures, such as the price-dividend ratio and the ratio of price to 10-year cyclically adjusted earnings (CAPE). While, ex ante, one might expect that long-term growth expectations would matter more for prices than short-term nominal earnings growth expectations, we show that LTG has relatively little variation over time and, more importantly, is uncorrelated with the price-earnings ratio, the price-dividend ratio, and CAPE. In contrast, expectations of short-term nominal earnings (scaled by current earnings, current dividends, and current cyclically adjusted 10-year earnings)

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rate, which we verify in our bond results. Given the weak empirical relationship between expected inflation and expected nominal earnings growth, this decreases the stock price.

<sup>4</sup> The importance of short-term nominal earnings growth expectations is consistent with the previous findings of Delao and Myers (2021). A key result for this paper is the distinction between short-term nominal earnings growth and LTG.

are an order of magnitude more volatile than LTG and are highly correlated with the three price ratios, respectively.

We reconcile our findings about LTG with the findings of [Bordalo et al. \(2023a\)](#), who argue that overreaction in LTG explains aggregate price movements and the ability of prices to predict future returns. We replicate the forecast error test originally proposed by [Coibion and Gorodnichenko \(2015\)](#) and implemented in [Bordalo et al. \(2023a\)](#). The test refutes the presence of overreaction in LTG. The difference between our test results and those in [Bordalo et al. \(2023a\)](#) comes from the fact that they rescale some, but not all, variables before running the test. Specifically, they rescale future earnings growth and expected earnings growth when calculating forecast errors, but do not rescale expected earnings growth when calculating revisions in expectations. This distorts the size of forecast errors relative to the revisions in expectations. Appendix B shows that once this rescaling discrepancy is removed, the test rejects overreaction.

To demonstrate the quantitative importance of these deviations from RE in expectations of inflation and nominal earnings growth, we show that subjective expectations of real cash flows closely match observed aggregate stock prices, bond yields, and return puzzles.<sup>5</sup> We calculate fundamental values for three measures of S&P 500 price ratios (price-earnings ratio, price-dividend ratio, and CAPE) and the 10-year Treasury yield based on the survey forecasts and constant discount rates (i.e., constant expected real returns). Over our 1976–2018 sample, these fundamental prices closely match the observed prices. For stocks, fundamental and observed price ratios predict future returns with nearly identical coefficients and similar  $R^2$ 's. For bonds, holding returns based on observed and fundamental yields both violate the variance bound implied by the expectations hypothesis by a factor of 5.<sup>6</sup>

To reconcile all of these findings, we propose and estimate a parsimonious framework of subjective expectation formation that accurately replicates our results. Since the diagnostic test is based on comovements, we are able to decompose the test results to show that beliefs about the persistence of variables and the response to surprises both play important roles in the survey data. We show that both of these elements can be integrated into a single equation in which agents form their expectations for each variable based on a geometrically weighted average of past realizations and a believed persistence for the variable. We use this as a description of subjective expectations rather than a fully specified psychological model, but the emphasis on forming expectations based on a weighted average of realizations is consistent with

<sup>5</sup> See [Campbell and Shiller \(1988b\)](#) and [Shiller \(1979\)](#) for early summaries on the predictability of stock returns and the rejection of the expectations hypothesis, and [Cochrane \(2011\)](#) and [Gürkaynak and Wright \(2012\)](#) for more recent reviews of the literature.

<sup>6</sup> Under RE, the expectations hypothesis places a bound on the variance of holding returns ([Shiller 1979](#); [Singleton 1980](#)). Even though the expectations hypothesis holds by construction for fundamental yields, variation in errors in expectations increases the volatility of holding returns beyond the RE variance bound.

fundamental extrapolation and our framework conveniently nests both adaptive expectations and the sticky expectations model of [Mankiw and Reis \(2002\)](#). In our setting, agents have constant discount rates and price assets based on their biased expectations of real cash flows. We estimate all parameters from expected and realized inflation and nominal earnings growth.

Despite not using any price information in the parameter estimation, this framework closely replicates the results of our diagnostic test both qualitatively and quantitatively. Prices have large comovements with forecast errors for both long-term inflation and short-term nominal earnings growth. Conversely, forecast errors for short-term inflation and long-term nominal earnings growth have virtually no comovement with prices. The stark difference in the term structure of errors is driven by the difference in the objective persistence of inflation and nominal earnings growth. For a persistent process like inflation, errors due to fundamental extrapolation are concentrated in longer horizon expectations. For a transitory process like nominal earnings growth, errors are concentrated in short-term expectations. Beyond matching the diagnostic test results, our framework is also consistent with the existing evidence on stickiness in expectations by [Coibion and Gorodnichenko \(2015\)](#), as the weighted sum of current and past realizations adjusts slowly over time, and our single-equation summary of expectations closely replicates the time series of inflation and nominal earnings growth expectations over the entire 1976-2018 sample.

A long-standing behavioral finance literature attempts to explain return puzzles through departures from RE.<sup>7</sup> Our empirical analysis guides the models in this literature, informing where errors should and should not arise in expectations. The fact that errors differ with the forecast horizon adds to a growing literature on the term structure of subjective expectations and errors ([d'Arienzo 2020](#); [Wang 2021](#); [Cassella et al. 2023](#); [Delao, Han, and Myers 2024](#)). Importantly, we find that the term structure of errors differs substantially depending on the variable being forecasted, similar to [Afrouzi et al. \(2023\)](#). Our findings can all be summarized by a framework based on fundamental extrapolation, in line with [Barsky and DeLong \(1993\)](#), [Barberis, Shleifer, and Vishny \(1998\)](#), [Hirshleifer, Li, and Yu \(2015\)](#), and [Alti and Tetlock \(2014\)](#).

This paper also adds to a large empirical literature in finance that uses survey expectations to understand aggregate asset price movements.<sup>8</sup> We document new systematic errors in long-term inflation expectations that account for a substantial share of stock price movements. We show that these systematic errors also help to explain bond price movements, which is related to previous

<sup>7</sup> See [Hirshleifer \(2015\)](#) for a survey of the literature using behavioral biases, and see [Collin-Dufresne, Johannes, and Lochstoer \(2017\)](#) for an example using Bayesian learning.

<sup>8</sup> See [Amromin and Sharpe \(2014\)](#), [Bacchetta, Mertens, and Wincoop \(2009\)](#), [Greenwood and Shleifer \(2014\)](#), [Piazzesi, Salomao, and Schneider \(2015\)](#), [Delao and Myers \(2021\)](#), [Nagel and Xu \(2022\)](#), [Giglio et al. \(2021\)](#), [Brunnermeier et al. \(2021\)](#), and [Bordalo et al. \(2023a\)](#).

work by Cieslak and Povala (2015), who use a weighted average of past inflation as a proxy for inflation expectations to explain yields, and Haubrich, Pennacchi, and Ritchken (2012), who use yields, survey expectations, and inflation swaps to estimate a seven-factor model of yield curves in which shifts in the path of inflation play an important role. We build on this previous work by demonstrating that the large role for inflation expectations in bond markets can be attributed to errors in long-term inflation expectations.

More broadly, a general literature has tried to understand the importance of inflation expectations. Focusing on short-term expectations and testing for different forms of errors, Ang, Bekaert, and Wei (2007), Negro and Eusepi (2011), and Chernov and Mueller (2012) do not find any significant errors while Coibion and Gorodnichenko (2015) find significant evidence of sticky expectations. We study long-term expectations in addition to short-term expectations. For short-term expectations, we find that systematic errors play no role in explaining price movements and return puzzles, in line with Cieslak (2018). For long-term expectations, we document significant predictable errors, which play a large role in both stock and bond markets and are largely explained by an overly high expected persistence of inflation. For stocks, these errors have a similar effect as the money illusion hypothesis of Modigliani and Cohn (1979), causing prices to be too low in periods of high expected inflation. These inflation expectations are particularly relevant given the recent increase in inflation and low realized real holding returns on long-term bonds. This is consistent with our high estimate for the believed persistence of inflation and our framework for subjective expectation formation, where after a long period of low inflation investors understate the chance that inflation will rise.

## 1. Price Identities under Subjective Expectations

In this section, we use accounting identities to derive general relationships that connect subjective expectations of real cash flows to asset price variation and return puzzles. Specifically, we look at the role of subjective expectations in asset price movements, stock return predictability, and the excess volatility of bond returns. Because these relationships come from accounting identities, they hold even if subjective expectations are not rational. Appendix A fully derives all of the equations in this section.

### 1.1 Price identities

We start with price identities based on the definition of returns. In terms of notation,  $E_t^*[\cdot]$  denotes subjective expectations, which may or may not align with standard rational expectations. For any variable  $x_t$ , we use  $\tilde{x}_t$  to denote the inflation-adjusted value,  $x_t - \pi_t$ , where  $\pi_t$  is inflation.

For stocks, we use the approximate log-linearized return, which states the one-period real return in terms of real earnings growth  $\Delta \tilde{e}_{t+1}$  and the price-earnings ratio  $pe_{t+1}$ , all in logs:

$$\tilde{r}_{t+1} \approx \kappa + \Delta \tilde{e}_{t+1} - pe_t + \rho pe_{t+1}, \quad (1)$$

where  $\kappa$  is a constant,  $\rho = e^{\bar{p}d} / (1 + e^{\bar{p}d}) < 1$  and  $\bar{p}d$  is the mean value of the log price-dividend ratio. By imposing a no-bubble condition,  $\lim_{T \rightarrow \infty} \rho^T pe_{t+T} = 0$ , we can iterate this equation and apply subjective expectations to get

$$pe_t \approx \frac{1}{1-\rho} \kappa + \sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\Delta \tilde{e}_{t+j}] - \sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\tilde{r}_{t+j}]. \quad (2)$$

This states that an increase in the price-earnings ratio must be associated with an increase in subjective real earnings growth expectations or a decrease in subjective real return expectations. These subjective expectations do not need to be rational. Because Equation (2) is derived from the definition of real returns, it holds under any probability distribution.

For bonds, we use the return from holding an  $n$ -period zero-coupon bond for one period. Expressing all variables in logs, the holding return from  $t$  to  $t+1$  is defined as

$$h_{t+1}^{(n)} = ny_t^{(n)} - (n-1)y_{t+1}^{(n-1)}. \quad (3)$$

Iterating and applying subjective expectations, we find the yield is

$$y_t^{(n)} = \frac{1}{n} \sum_{j=1}^n E_t^* [\pi_{t+j}] + \frac{1}{n} \sum_{j=1}^n E_t^* [\tilde{h}_{t+j}^{(n+1-j)}], \quad (4)$$

where  $\tilde{h}_t^{(n)}$  is the real holding return. Intuitively, an increase in the bond yield must be associated with higher subjective inflation expectations or higher subjective real holding return expectations. This is analogous to Equation (2), where prices are linked to expectations of real cash flows and real returns. For bonds, real cash flows only depend on inflation, as the nominal cash flow is known, whereas for stocks this will depend on inflation and nominal earnings growth  $\Delta e_{t+j}$ . For the zero-coupon bond, there is a single real cash flow at maturity, so inflation in different years all have the same impact on the yield. For stocks, there are real cash flows every period, so real earnings growth that occurs immediately will have a larger impact on prices than real earnings growth that occurs later, a fact reflected in  $\rho^{j-1}$ .

## 1.2 Price movements and return puzzles

Given these equations for the price-earnings ratio and bond yields, we establish three identities that connect errors in expectations to asset prices and return puzzles. Our two return puzzles are the predictability of stock returns using

price ratios, such as the price-earnings ratio or price-dividend ratio, and the rejection of the expectations hypothesis for bond yields. Other than the subjective expectations  $E_t^*[\cdot]$ , all operators use the objective probability distribution. For example,  $Var(\cdot)$  and  $Cov(\cdot, \cdot)$  denote the observable variance or covariance of variables. This allows us to use the subjective expectations observed in the survey data to explain empirical puzzles measured by an econometrician, such as the comovement between current prices and future returns.

First, the comovement of prices with subjective expectations of real cash flows is

$$Cov(pe_t, E_t^*[\Delta \tilde{e}_{t+j}]) = Cov(pe_t, E_t[\Delta \tilde{e}_{t+j}]) - Cov(pe_t, f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}) \quad (5)$$

$$Cov(y_t^{(n)}, E_t^*[\pi_{t+j}]) = Cov(y_t^{(n)}, E_t[\pi_{t+j}]) - Cov(y_t^{(n)}, f_t^{\pi_{t+j}}), \quad (6)$$

where  $E_t[\cdot]$  represents expectations under RE,  $f_t^{\Delta e_{t+j}}$  is the forecast error for nominal earnings growth  $\Delta e_{t+j} - E_t[\Delta e_{t+j}]$ , and  $f_t^{\pi_{t+j}}$  is the forecast error for inflation  $\pi_{t+j} - E_t[\pi_{t+j}]$ . Note that these identities hold regardless of whether subjective expectations  $E_t^*[\cdot]$  are rational. If subjective expectations are not rational and the comovement of subjective expectations with prices differs from the comovement of RE expectations with prices, then there must be an observable comovement between prices and forecast errors.

Second, from Equations (1) and (2), the ability of the price-earnings ratio to predict future real returns is<sup>9</sup>

$$Cov\left(pe_t, \sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}_{t+j}\right) = Cov\left(pe_t, \sum_{j=1}^{\infty} \rho^{j-1} E_t^*[\tilde{r}_{t+j}]\right) + Cov\left(pe_t, \sum_{j=1}^{\infty} \rho^{j-1} [f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}]\right). \quad (7)$$

Note that identities (5) and (7) on the comovement of expectations with prices and the predictability of returns also hold if we replace the price-earnings ratio  $pe_t$  with other price ratios, like the price-dividend ratio  $pd_t$  or CAPE.

Third, we consider the volatility of bond holding returns and the rejection of the expectations hypothesis. The expectations hypothesis posits that expected

<sup>9</sup> Equation (2) holds with and without applying expectations, which gives  $\sum_{j=1}^{\infty} \rho^{j-1} E_t^*[\Delta \tilde{e}_{t+j}] - \sum_{j=1}^{\infty} \rho^{j-1} E_t^*[\tilde{r}_{t+j}] = \sum_{j=1}^{\infty} \rho^{j-1} \Delta \tilde{e}_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}_{t+j}$ . This means that  $\sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}_{t+j} = \sum_{j=1}^{\infty} \rho^{j-1} E_t^*[\tilde{r}_{t+j}] + \sum_{j=1}^{\infty} \rho^{j-1} [f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}]$ , which leads to Equation (7).

term premia  $E_t^*[h_{t+1}^{(n)}] - y_t^{(1)}$  are constant. Using Equations (3) and (4), we generalize the Shiller (1979) variance bound for holding returns under the expectations hypothesis,

$$\text{Var}(h_{t+1}^{(n)}) \leq \eta \text{Var}(y_t^{(1)}) - 2n \text{Cov}\left(y_t^{(n)}, \sum_{j=2}^n f_t^{\pi_{t+j}} + f_t^{\tilde{h}_{t+j}^{(n+1-j)}}\right) \quad (8)$$

where  $\eta = n^2 / (2n - 1)$ .

## 2. Diagnostic Test

In this section, we establish a single necessary and sufficient condition for errors in expectations of real cash flows to explain asset price variation and return puzzles. This single condition provides a simple diagnostic test to determine if deviations from RE account for price movements and return puzzles. Importantly, not all errors are relevant, and we include an intuitive example of a systematic error that fails the diagnostic test.

To provide a benchmark, we first discuss the interpretation of price movements and return puzzles under RE. We then show when deviations from RE in subjective expectations of real cash flows do and do not help to explain price movements and return puzzles. For all three identities, the key condition is that forecast errors must be predictable with current prices. If this condition is not satisfied, then differences between subjective expectations and RE will not explain price variation or the return puzzles. Given the generality of this diagnostic test, we also discuss in Internet Appendix A other applications, such as different asset classes, different measures of return predictability, and bounding finite-horizon returns.

### 2.1 Benchmark under standard rational expectations

Under RE, forecast errors are uncorrelated with current prices. From Equations (5) and (6), this means that the comovement of expectations with current prices must match the comovement of future realized real cash flows with current prices,

$$\text{Cov}(p_{e,t}, E_t[\Delta \tilde{e}_{t+j}]) = \text{Cov}(p_{e,t}, \Delta \tilde{e}_{t+j}) \quad (9)$$

$$\text{Cov}(y_t^{(n)}, E_t[\pi_{t+j}]) = \text{Cov}(y_t^{(n)}, \pi_{t+j}). \quad (10)$$

Empirically, the comovement of prices with future realized real cash flows is too small to explain most variation in stock and bond prices (Ang and Piazzesi 2003; Cochrane 2011). This directly implies that under RE, expectations of real cash flows do not explain most price movements. Using the Campbell and Shiller (1988a) variance decomposition, we find over our sample that RE expectations of 10-year inflation only account for 27% of the variation in the

10-year yield.<sup>10</sup> While it is true that yields are highly correlated with past inflation (Cieslak and Povala 2015), yields have a much lower comovement with future inflation. Thus, RE expectations of inflation do not account for much of the variation in yields. Similarly, RE expectations of 10-year real earnings growth only account for 42% of the variation in the price-earnings ratio.

Under RE, Equations (7) and (8) provide simple tests of time-varying discount rates and the expectations hypothesis. Because forecast errors are unpredictable, the final term in Equation (7) is zero. Therefore, the observed comovement between prices and future returns must be explained by time variation in investors' discount rates. Similarly, Equation (8) reduces to the standard Shiller (1979) bound, which restricts the variance of holding returns to be less than  $\eta \text{Var}(y_t^{(1)})$  if the expectations hypothesis is true. Thus, under RE, the observed excess volatility of holding returns beyond  $\eta \text{Var}(y_t^{(1)})$  rejects the expectations hypothesis (Singleton 1980).

## 2.2 Diagnostic test for deviations

If we allow for more general subjective expectations, then Equations (5)–(8) show when the results using subjective expectations will deviate from the results under RE for explaining asset prices, predictable stock returns, and excess volatility in bond holding returns. Importantly, these phenomena all share a single necessary and sufficient condition, namely, that forecast errors for real cash flows must negatively comove with current prices.<sup>11</sup> In other words, this single condition acts as a useful diagnostic test to determine whether deviations from RE explain asset prices and return puzzles. Additionally, this allows us to measure the relative importance of subjective expectations at different horizons  $j$  by comparing how much forecast errors at different horizons comove with current prices.

If prices negatively comove with forecast errors, then Equations (5) and (6) show that the comovement between subjective expectations and prices will be larger than the comovement under RE. This means that subjective expectations of real cash flows will account for more of the variation in prices than RE would imply. Even in the extreme case in which prices are uncorrelated with RE expectations of real cash flows, subjective expectations of real cash flows will still comove with prices. Similarly, if the price-earnings ratio negatively comoves with forecast errors, then the price-earnings ratio will negatively predict future real returns even if discount rates  $E_t^*[\tilde{r}_{t+j}]$  are constant over

<sup>10</sup> This decomposes  $\text{Var}(y_t^{(n)})$  into  $\frac{1}{n} \sum_{j=1}^n \text{Cov}(y_t^{(n)}, E_t[\pi_{t+j}])$  and  $\frac{1}{n} \sum_{j=1}^n \text{Cov}(y_t^{(n)}, E_t[\tilde{r}_{t+j}^{(n+1-j)}])$  and  $\text{Var}(pe_t)$  into  $\sum_{j=1}^{\infty} \rho^{j-1} \text{Cov}(pe_t, E_t[\Delta \tilde{e}_{t+j}])$  and  $\sum_{j=1}^{\infty} \rho^{j-1} \text{Cov}(pe_t, -E_t[\tilde{r}_{t+j}])$ .

<sup>11</sup> For stocks, the condition translates to  $pe_t$  negatively comoving with forecast errors for real earnings growth. For bonds, this condition translates to yields negatively comoving with forecast errors for inflation.

time.<sup>12</sup> Finally, forecast errors that negatively comove with the yield relax the bound in Equation (8), implying that excess volatility of holding returns beyond  $\eta \text{Var}(y_t^{(1)})$  does not violate the expectations hypothesis. Predictable forecast errors generate excess volatility even if the expectations hypothesis holds, that is, expected term premia  $E_t^* \left[ h_{t+1}^{(n)} \right] - y_t^{(1)}$  are constant.

In contrast, errors that do not satisfy this condition will have no impact on Equations (5)–(8). To give an intuitive example of a systematic error that does not satisfy the condition, suppose there is a variable  $x_t$  that predicts real earnings growth but investors do not know this. For simplicity, assume that  $x_t$  is independent of the investors' information set  $I_t$ . In this example, an econometrician would find systematic errors in investors' expectations, as  $x_t$  predicts forecast errors. Further, the econometrician would find that  $x_t$  predicts real stock returns. However, these systematic errors do not drive price movements, as prices are only a function of  $I_t$ . As a result, these errors will not explain why prices or price ratios predict future returns.<sup>13</sup>

In [Internet Appendix A](#), we provide several extensions for the diagnostic test. [Internet Appendix A.1](#) connects predictable forecast errors to finite horizon return predictability. [Internet Appendix A.2](#) shows how systematic errors in both inflation expectations and nominal earnings growth expectations can explain the predictability of nominal returns and excess returns. [Internet Appendix A.3](#) discusses how the diagnostic test can be applied to asset classes beyond stocks and bonds. The necessary and sufficient condition is still that forecast errors for real cash flows must negatively comove with current prices. Thus, applying the diagnostic test to different assets simply requires identifying the real cash flows for each asset.

### 2.3 Applicability of the diagnostic test

The single condition identified by the diagnostic test, as illustrated in Equation (7), effectively separates the relative importance of two components: time variation in subjective real discount rates  $E_t^* [\tilde{r}_{t+j}]$  and systematic errors in real cash flows. Excess price variation and return predictability not explained by errors in *real cash flows* must be explained by time variation in *real discount rates* and vice versa. Because this condition is derived from accounting identities, it is general and holds for any deviation from RE, for example, learning, rational inattention, bounded rationality, and behavioral biases. This means that testing this condition is a simple way to quickly determine if

<sup>12</sup> [Internet Appendix A.2](#) shows that these forecast errors also increase the comovement between prices and nominal returns, as well as excess returns.

<sup>13</sup> Empirically, we will apply this test to professional forecasts, thereby raising another reason some errors may pass the diagnostic test and other may not. If there is measurement error (i.e., differences between the reported forecasts and the market expectations), then certain errors in the forecast data may not affect prices. In this case, the diagnostic test would correctly determine that these errors do not help to explain price variation or return puzzles.

systematic errors in subjective expectations help to explain price variation and return puzzles without needing to make assumptions about the primitive cause of the errors. Similarly, this condition does not depend on whether or not the expectation formation process is focused on real cash flows.

For instance, note that under any subjective probability distribution, errors in real return expectations equal errors in real cash flow expectations given Equation (2). This means that as long the return identity (1) is satisfied in investors' minds, any model of errors in real return expectations must also feature errors in real cash flow expectations (e.g., [Barberis et al. 2015](#); [Jin and Sui 2022](#); [Nagel and Xu 2022](#)). Thus, this diagnostic test is valid even in settings that focus on errors in return expectations. Intuitively, given that investors know the current price, agents that are overoptimistic about future real cash flows also will be overoptimistic about future real returns and vice versa.<sup>14</sup> Rephrased, these models still can be tested by examining whether high prices are associated with overly optimistic expectations of real cash flows. This highlights that the diagnostic test determines whether errors in real cash flow expectations *explain* price variation, not whether these errors *cause* price variation.

Another example is when agents form nominal expectations instead of real expectations. One might imagine a world in which investors simply care about nominal earnings and discount them at the nominal discount rate. Why would expectations about real variables still be relevant? Because, even in this world, the identity effectively separates price movements into time-varying *real discount rates* and *errors in real cash flows*. As long as our focus is on explaining return puzzles as an alternative to what can be accounted for by time-varying *real discount rates*, the relevant errors will be errors in *real cash flows*. An analogous argument in nominal terms would be able to separate errors in *nominal cash flows* from time variation in *nominal discount rates*. In this case, only errors in nominal cash flows would be relevant. Here, we focus on subjective real discount rates because they are what is most relevant for models of investor preferences and consumption. For example, consumption-based asset pricing models relate investors' marginal utility to their required real return on risky assets.<sup>15</sup>

Section 4.2 discusses the scenario of investors thinking in nominal terms in more detail. Importantly, it shows that the diagnostic test still holds even in scenarios in which deviations from RE in inflation expectations do not affect prices. In these scenarios, the covariance of inflation forecast errors with current prices would be zero and the diagnostic test would correctly identify

<sup>14</sup> This point is similar to the one made in [Giglio et al. \(2021\)](#) that, because all investors see the current price, disagreement across investors in expected returns must be positively correlated with disagreement in expected cash flows, which they confirm empirically.

<sup>15</sup> More broadly, it is common for asset pricing models with production and/or consumption to deal only with real variables.

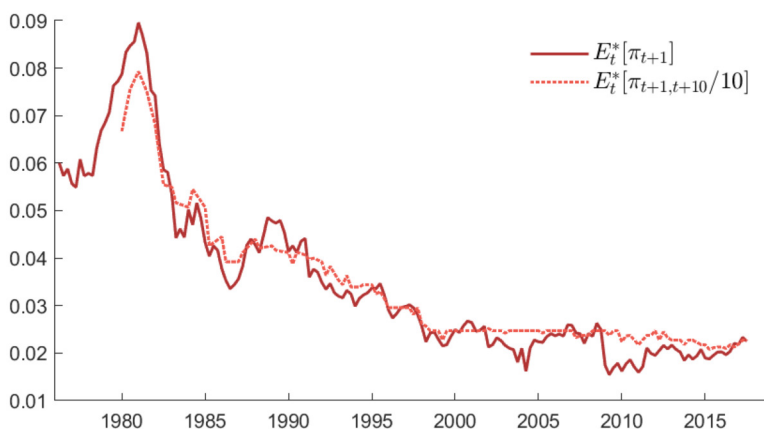
that deviations from RE in inflation expectations do not help to explain price variation and return predictability.

### 3. Data and Diagnostic Test Results

In this section, we apply the diagnostic test to survey expectations of both short-term and long-term inflation and nominal earnings growth. Section 3.1 gives an overview of the survey data and discusses the term structure of subjective expectations. Section 3.2 shows the results of the diagnostic test. We find that long-term inflation expectations satisfy the diagnostic test from Section 2 for explaining asset price movements and return puzzles while long-term nominal earnings growth expectations do not. Section 3.3 discusses the implications for models of expectation formation and further sharpens our findings by showing that the results for long-term inflation expectations are explained by the high believed persistence of inflation and the results for short-term nominal earnings growth expectations are explained by the response to surprises.

#### 3.1 Term structure of subjective expectations

**3.1.1 Inflation expectations** We use quarterly median inflation forecasts from the Survey of Professional Forecasters. To understand the relation between short-term and long-term inflation expectations, we analyze 1-year inflation expectations from 1976Q1 to 2018Q2 and expectations of average inflation over the next 10 years from 1979Q4 to 2018Q2. Figure 1 shows that 1-year expectations, denoted as  $E_t^*[\pi_{t+1}]$ , rise sharply in the late 1970's. These expectations peak at 9% before falling steadily during the Volcker period in the 1980's and reach historic lows after 2000. Importantly, average 10-year



**Figure 1**  
**Subjective inflation expectations**

The figure compares subjective expectations for 1-year inflation (solid) and subjective expectations for annualized 10-year inflation (dotted). All variables are in logs.

expectations, denoted as  $E_t^*[\pi_{t+1,t+10}/10]$ , closely follow 1-year expectation movements. At the beginning of the sample, short-term expectations are high and inflation is expected to remain high for the next 10 years. At the end of the sample, we see the same pattern but in the opposite direction where short-term inflation expectations are low and inflation is expected to stay low for the next 10 years. This relationship suggests that analysts believe movements in expected 1-year inflation will largely persist and be observed in long-term inflation.

To quantify the relationship between expectations of short-term and long-term inflation, we measure the believed annual persistence  $\phi_\pi^*$  from

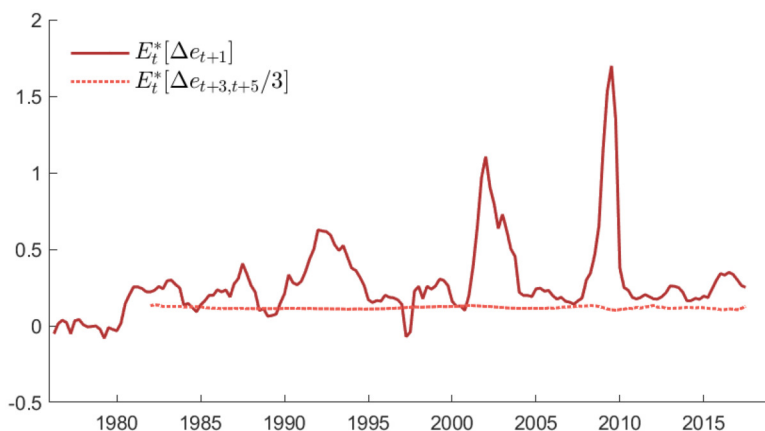
$$E_t^*[\pi_{t+1+j}] = \alpha_{\pi,j} + \phi_\pi^{*j} E_t^*[\pi_{t+1}] + \varepsilon_{t,j}^\pi. \quad (11)$$

An increase in 1-year expectations is associated with a  $\phi_\pi^*$  increase in  $E_t^*[\pi_{t+2}]$ ,  $\phi_\pi^{*2}$  increase in  $E_t^*[\pi_{t+3}]$ , and so on. Using 1-year expectations and expectations of average inflation over the next 10 years  $E_t^*[\pi_{t+1,t+10}/10]$ , we estimate  $\phi_\pi^*$  as 0.96 (0.01). The constants  $\alpha_{\pi,j}$  will not matter for our analysis as they will not affect comovements or variances. Importantly, the believed persistence may differ from the objective persistence of inflation, in line with [Alti and Tetlock \(2014\)](#) and [Afrouzi et al. \(2023\)](#). In this case, long-term expectations will respond more or less to changes in short-term expectations than RE would imply. For example, a believed persistence higher than the objective persistence could cause forecasters to understate the chance of a decline (rise) in inflation after a long period of high (low) inflation, such as the decline in the 1980's or conversely the recent rise in 2020-2022.

We choose the functional form in Equation (11) for three reasons. First, this form nests standard AR(1) processes, making the estimated value for  $\phi_\pi^*$  an easily interpretable measure of the degree to which long-term expectations move in response to a change in short-term expectations.<sup>16</sup> Second, we show in Section 3.2 that this functional form accurately reflects the comovement of long-term expectations with asset prices, that is, long-term inflation expectations and the approximation of long-term inflation expectations in Equation (11) have nearly identical comovements with both stock and bond prices. Third, we show in Section 7 that this form is consistent with a framework based on fundamental extrapolation, which closely replicates the survey data.

**3.1.2 Nominal earnings growth expectations** We construct short-term (1-year) nominal earnings growth expectations for the S&P 500 from the Thomson Reuters I/B/E/S individual firm forecasts following the methodology proposed in [Delao and Myers \(2021\)](#). We also estimate long-term (3- to 5-year) nominal

<sup>16</sup> While this form nests AR(1) processes, it does not require that investors believe inflation is AR(1). For example, if investors believe inflation is ARMA(1,1), their expectations will still satisfy Equation (11).



**Figure 2**

**Subjective nominal earnings growth expectations**

The figure compares subjective expectations of 1-year nominal earnings growth (solid) and the subjective expectations for annualized 3- to 5-year nominal earnings growth (dotted) for the S&P 500. All variables are in logs.

earnings growth expectations using the earnings-weighted average of long-term growth forecasts (LTG) for individual firms.<sup>17</sup> Figure 2 shows 1-year nominal earnings growth expectations from 1976Q1 up to 2018Q2, and average 3- to 5-year nominal earnings growth expectations from 1982Q1 to 2018Q2. Expectations of 1-year nominal earnings growth are volatile, with a standard deviation of 26.4%, rising substantially after the large decline in earnings during financial crisis. In comparison, expectations of 3- to 5-year nominal earnings growth have much lower volatility, with a standard deviation of only 1.2%. For robustness, we show in Internet Appendix C.6 that our results are unchanged if we use the value-weighted average of long-term growth forecasts from Bordalo et al. (2023a) which have a similarly low volatility of only 1.4%.

The high volatility of short-term expectations relative to long-term expectations suggests that movements in nominal earnings growth expectations are primarily concentrated at short horizons. The fact that movements in short-term expectations are not matched by similar movements in long-term expectations indicates that analysts believe movements in expected one-nominal earnings growth will not persist through to long-term nominal earnings growth. We quantify the relationship between expectations of short-term and long-term nominal earnings growth by measuring the believed annual persistence  $\phi_e^*$  from

$$E_t^*[\Delta e_{t+j}] = \alpha_{e,j} + \phi_e^* E_t^*[\Delta e_{t+1}] + \varepsilon_{t,j}^e. \quad (12)$$

Using the 1- and 3- to 5-year expectations, we estimate a small  $\phi_e^*$  of 0.004 (0.075). This sharply contrasts with the large, significant estimate of  $\phi_\pi^*$  of

<sup>17</sup> Full details on the construction can be found in Internet Appendix B.

0.96.<sup>18</sup> Just as with inflation expectations, we show in Section 3.2 that this estimate for  $\phi_e^*$  accurately capture the comovement of long-term nominal earnings growth expectations with stock price ratios and the functional form of Equation (12) is consistent with the framework of Section 7 based on fundamental extrapolation.

### 3.2 Diagnostic test results

Section 2 establishes a diagnostic test for deviations from RE in subjective expectations of real cash flows to explain asset prices, stock return predictability, and excess bond return volatility. The necessary and sufficient condition is that current prices must predict forecast errors for real cash flows. For stocks, this means that the current price-earnings ratio positively predicts forecast errors for inflation and negatively predicts forecast errors for nominal earnings growth. For bonds, this means that the current yield negatively predicts forecast errors for inflation. In this section, we empirically test if subjective expectations satisfy this condition. We find that expectations of long-term inflation and short-term nominal earnings growth pass the test while expectations of short-term inflation and long-term nominal earnings growth do not. Table 1 shows the results.<sup>19</sup>

We first focus on short-term inflation expectations, which have been shown to significantly deviate from RE based on tests of sticky expectations (Coibion and Gorodnichenko 2015). Despite this deviation from RE, we find that forecast errors for short-term inflation expectations do not satisfy the diagnostic test for the price-earnings ratio. The first row of panel A shows the comovement of the price-earnings ratio with expected and realized short-term inflation, as well as the comovement with the forecast error. We see that expected short-term inflation and realized short-term inflation both have significant negative comovements with the price-earnings ratio and that the magnitudes are quite similar. As a result, forecast errors do not significantly comove with the price-earnings ratio, meaning that forecast errors for short-term inflation do not explain movements in the price-earnings ratio or the stock return puzzle.

In contrast, we document new systematic errors in long-term inflation expectations that do satisfy the diagnostic test. Consistent with the findings of Section 3.1, we find a large amount of action in long-term inflation expectations. The second row of panel A shows that long-term inflation

<sup>18</sup> For comparison, the estimated true persistence of inflation is 0.76, and the estimated true persistence of earnings growth is  $-0.20$ .

<sup>19</sup> Table OA.3 in Internet Appendix C.2 scales all of the covariances in Table 1 by the variance of prices. This makes the coefficients equivalent to regression betas.

**Table 1**  
**Diagnostic test for inflation and nominal earnings growth expectations**

| A. Inflation               |                                    |          |          |                |
|----------------------------|------------------------------------|----------|----------|----------------|
|                            | Horizon                            | Expected | Realized | Forecast error |
| $Cov(pe_t, \cdot)$         | Short-term ( $\pi_{t+1}$ )         | -0.66*** | -0.82*** | -0.16          |
|                            | Long-term ( $\pi_{t+1,t+10}$ )     | -3.97*** | -1.66*   | 2.31***        |
| $Cov(y_t^{(10)}, \cdot)$   | Short-term ( $\pi_{t+1}$ )         | 0.04***  | 0.04***  | 0.00           |
|                            | Long-term ( $\pi_{t+1,t+10}$ )     | 0.25***  | 0.13***  | -0.13***       |
| B. Nominal earnings growth |                                    |          |          |                |
| $Cov(pe_t, \cdot)$         | Short-term ( $\Delta e_{t+1}$ )    | 9.44***  | 5.18     | -4.26***       |
|                            | Long-term ( $\Delta e_{t+3,t+5}$ ) | 0.36     | 2.71     | 2.35           |

This table shows the results of the diagnostic test that determines whether differences between subjective expectations and RE explain asset price movements and return puzzles. The condition is that prices must comove with forecast errors. In panel A, the first row shows the covariance of the S&P 500 price-earnings ratio with short-term inflation expectations  $E_t^*[\pi_{t+1}]$ , realized short-term inflation  $\pi_{t+1}$ , and the forecast errors  $\pi_{t+1} - E_t^*[\pi_{t+1}]$  from 1976Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-earnings ratio with long-term inflation expectations  $E_t^*[\pi_{t+1,t+10}]$ , realized long-term inflation  $\pi_{t+1,t+10}$ , and the forecast errors  $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$  using quarterly data from 1979Q4 to 2018Q2. The third and fourth rows show analogous results using the 10-year Treasury yield instead of the S&P 500 price-earnings ratio. In panel B, the first row shows the covariance of the S&P 500 price-earnings ratio with short-term nominal earnings growth expectations  $E_t^*[\Delta e_{t+1}]$ , realized short-term nominal earnings growth  $\Delta e_{t+1}$ , and the forecast errors  $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$  from 1976Q1 to 2018Q2. The second row shows the covariances of the S&P 500 price-earnings ratio with long-term nominal earnings growth expectations  $E_t^*[\Delta e_{t+3,t+5}]$ , realized long-term nominal earnings growth  $\Delta e_{t+3,t+5}$  and the forecast errors  $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$  from 1982Q1 to 2018Q2. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. \* $p < .1$ ; \*\* $p < .05$ ; \*\*\* $p < .01$ .

expectations have a significant negative comovement with the price-earnings ratio of  $-3.97$  that is substantially larger than the comovement of short-term expectations with the price-earnings ratio of  $-0.66$ . This is also significantly larger than the covariance of the price-earnings ratio with future realized long-term inflation of  $-1.66$ , leading to significant forecast errors that are predictable with the current price-earnings ratio. This matches our high believed persistence  $\phi_\pi^*$  of 0.96, as  $E_t^*[\pi_{t+1}](1 - \phi_\pi^{*10}) / (1 - \phi_\pi^*)$  has a nearly identical covariance with the price-earnings ratio of  $-3.85$  over the same sample.

We find the same results when we test the condition using bond yields rather than the price-earnings ratio, meaning that systematic errors in long-term inflation expectations also explain price movements and return puzzles for bonds, while systematic errors in short-term inflation do not. Expected and realized short-term inflation have virtually the same comovement with the 10-year bond yield at 0.04, leading to an insignificant comovement between the yield and forecast errors. Compared to short-term expectations, long-term expectations comove much more with the yield at 0.25, again emphasizing

the importance of movements in longer horizon inflation expectations. This comovement is substantially larger than the comovement of future realized long-term inflation with the yield of 0.13, meaning that forecast errors significantly comove with the yield. Again, this matches our high persistence estimate  $\phi_{\pi}^*$  of 0.96, as  $E_t^*[\pi_{t+1}](1 - \phi_{\pi}^{*10}) / (1 - \phi_{\pi}^*)$  has a covariance of 0.23 with the yield.

The first row of panel B confirms the previous findings of [Delao and Myers \(2021\)](#). Specifically, the price-earnings ratio has a large comovement with short-term nominal earnings growth expectations and negatively comoves with forecast errors for these expectations. Thus, forecast errors for these expectations satisfy the diagnostic next.

More importantly, the final row of panel B shows the new finding that forecast errors for long-term nominal earnings growth fail the diagnostic test. In contrast with short-term expectations, long-term nominal earnings growth expectations have a small and insignificant comovement with the price-earnings ratio of 0.36. Consistent with our low estimate for the believed persistence  $\phi_e^*$  of 0.004, this covariance is nearly two orders of magnitude smaller than the covariance of short-term nominal earnings growth expectations with the price-earnings ratio of 9.44. Importantly, realized long-term nominal earnings growth is also insignificantly related to the price-earnings ratio, meaning that forecast errors do not significantly comove with the price-earnings ratio. While expectations of long-term nominal earnings growth may fail other tests of RE, the inability of the price-earnings ratio to predict the forecast errors implies that these deviations from RE will not help in matching prices or explaining return puzzles. In fact, the lack of comovement between long-term nominal earnings growth expectations and the price-earnings ratio slightly worsens the ability to explain prices and return puzzles, as expected long-term nominal earnings growth comoves less with the price-earnings ratio than realized long-term nominal earnings growth. Section 5.2 provides a detailed discussion on the role of long-term nominal earnings growth and reconciles the lack of importance of errors in long-term nominal earnings growth with the findings of [Bordalo et al. \(2023a\)](#).

We perform multiple robustness checks to confirm the results of Table 1. We show that our diagnostic test results are robust to alternative measures of survey expectations, measures of aggregate prices, and methods for determining significance and rule out that subjective expectations of real cash flows are simply reflecting investors' expectations of returns. [Tables OA.1 and OA.4](#) show that the diagnostic test results are virtually identical if we use consumer inflation expectations from the Michigan survey and the value-weighted long-term nominal earnings growth expectations of [Bordalo et al. \(2023a\)](#). [Table OA.2](#) shows the diagnostic test applied to the model-based inflation

expectations generated by [Haubrich, Pennacchi, and Ritchken \(2012\)](#)<sup>20</sup> and updated regularly by the Federal Reserve of Cleveland, which again gives similar results. Table 3 shows that the results are unchanged if we use the price-dividend ratio or the Shiller CAPE rather than the price-earnings ratio and also shows that the results are unchanged if we use smoothed earnings rather than current earnings when measuring short-term earnings growth expectations. [Internet Appendix C.3](#) accounts for overlapping observations, trends, and potential small-sample biases in long-term inflation expectations using the methodology of [Bauer and Hamilton \(2018\)](#) and shows that our findings are still significant even under the worst-case scenario. Finally, [Internet Appendix C.4](#) shows that the patterns for where the relevant errors do and do not appear are inconsistent with the idea that forecasters are embedding return expectations into their cash flow forecasts.

It is important to note that our results still imply that there are significant errors in expected long-term nominal earnings that are relevant for price variation and return puzzles. Holding growth fixed for all following years, a 10-percentage-point increase in 1-year nominal earnings growth raises the level of all future nominal earnings by 10%. Thus, mistakes about 1-year nominal earnings growth generate mistakes in all expected future nominal earnings. This is why short-term nominal earnings growth expectations can have a large impact on prices, as well as on long-term returns.<sup>21</sup> Mistakes about long-term nominal earnings growth would generate additional errors solely in expected long-term nominal earnings, but not in expected short-term nominal earnings. Our findings show that this latter type of mistake is not correlated with prices. However, mistakes about long-term nominal earnings will still be relevant via the errors in short-term nominal earnings growth expectations.

### 3.3 Implications for models of expectation formation

The results of Table 1 provide several insights for models of deviations from RE that attempt to explain asset prices. First, the diagnostic test reveals that price variation and return puzzles are largely explained by errors in expectations of long-term inflation and short-term nominal earnings growth, while errors in expectations of short-term inflation and long-term nominal earnings growth do not appear to play a role. Thus, the relevant errors differ across horizons and the term structure differs depending on the variable being forecasted. Second, for both long-term inflation and short-term nominal earnings growth, these errors arise because the comovement of subjective expectations with prices is

<sup>20</sup> We thank the authors for sharing the relevant data.

<sup>21</sup> By this same logic, if current prices predicted forecast errors for short-term inflation, then systematic errors in short-term inflation expectations would be important for explaining price variation and return predictability, as mistakes about 1-year expected inflation affect the level of all future real earnings. However, Table 1 demonstrates that forecast errors for short-term inflation do not satisfy this condition (i.e., current prices do not comove with 1-year inflation forecast errors).

substantially larger in magnitude than the value that RE would imply. Section 7 provides an illustrative framework that replicates all of these findings.

In this section, we further sharpen our findings by showing that the large comovement of long-term inflation expectations and short-term nominal earnings growth expectations with prices is mainly explained by the high believed persistence of inflation and the response of short-term nominal earnings growth expectations to surprises. Because the diagnostic test is based on comovements, we can decompose these comovements to better understand the implications for models of expectation formation. In other words, once the diagnostic test has identified which subjective expectations help to explain price variation and return puzzles, we can investigate these specific comovements further to shed more light on possible mechanisms.

For long-term inflation expectations, we find that the high comovement with prices is almost entirely explained by the high believed persistence  $\phi_\pi^*$ . We can express long-term inflation expectations as

$$E_t^*[\pi_{t+1,t+10}] = \alpha_\pi^{LT} + \frac{1 - \phi_\pi^{*10}}{1 - \phi_\pi^*} E_t^*[\pi_{t+1}] + \varepsilon_{t,\pi}^{LT} \quad (13)$$

where  $\frac{1 - \phi_\pi^{*10}}{1 - \phi_\pi^*} E_t^*[\pi_{t+1}]$  represents movements in long-term expectations due to the believed persistence and  $\varepsilon_{t,\pi}^{LT}$  represents all other movements in long-term inflation expectations. By extension, the comovement of prices with  $E_t^*[\pi_{t+1,t+10}]$  can be decomposed into the comovement of prices with  $\frac{1 - \phi_\pi^{*10}}{1 - \phi_\pi^*} E_t^*[\pi_{t+1}]$  and the comovement of prices with  $\varepsilon_{t,\pi}^{LT}$ . As mentioned in Section 3.2 and formally shown in Table 2, panel A, the large comovement between long-term inflation expectations and the current prices is almost entirely attributed to  $\frac{1 - \phi_\pi^{*10}}{1 - \phi_\pi^*} E_t^*[\pi_{t+1}]$ . As a result, shocks or information that only affect long-term inflation expectations  $\varepsilon_{\pi,t}^{LT}$  are not important for explaining this large comovement.

For short-term nominal earnings growth expectations, we find that the large comovement of subjective expectations with prices is largely explained by the response of short-term expectations to surprises in nominal earnings growth. In line with the earnings growth reversal model of Delao and Myers (2021), we separate movements in short-term nominal earnings growth expectations into a response to the recent earnings growth surprise  $\beta_e (E_{t-1}^*[\Delta e_t] - \Delta e_t)$  and all other movements  $\varepsilon_{t,e}^{ST}$ ,

$$E_t^*[\Delta e_{t+1}] = \alpha_e^{ST} + \beta_e (E_{t-1}^*[\Delta e_t] - \Delta e_t) + \varepsilon_{t,e}^{ST}. \quad (14)$$

Estimating Equation (14) gives a  $\beta_e$  of 0.74 (0.02).<sup>22</sup> As shown in Table 2, panel B, the large comovement between short-term nominal earnings growth

<sup>22</sup> For comparison, if  $\Delta e_{t+1}$  is used as the left-hand side of Equation (14), then the estimated  $\beta_e$  is 0.52. In other words, the earnings growth surprise  $E_{t-1}^*[\Delta e_t] - \Delta e_t$  negatively predicts forecast errors.

**Table 2**  
**Decomposing comovement with price ratios**

| A. Inflation             |                         |                                                          |                            |
|--------------------------|-------------------------|----------------------------------------------------------|----------------------------|
|                          | $E_t^*[\pi_{t+1,t+10}]$ | $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*[\pi_{t+1}]$ | $\varepsilon_{t,\pi}^{LT}$ |
| $Cov(pe_t, \cdot)$       | -3.97***                | -3.85***                                                 | -0.12                      |
| $Cov(y_t^{(10)}, \cdot)$ | 0.25***                 | 0.23***                                                  | 0.02                       |

| B. Nominal earnings growth |                         |                                               |                          |
|----------------------------|-------------------------|-----------------------------------------------|--------------------------|
|                            | $E_t^*[\Delta e_{t+1}]$ | $\beta_e(E_{t-1}^*[\Delta e_t] - \Delta e_t)$ | $\varepsilon_{t,e}^{ST}$ |
| $Cov(pe_t, \cdot)$         | 9.31***                 | 8.46***                                       | 0.85***                  |

This table decomposes the result of the diagnostic test for long-term inflation expectations and short-term nominal earnings growth expectations. It shows the covariance of prices and yields with each element in Equations (13) and (14). In panel A, the first row shows the covariance of the S&P 500 price-earnings ratio with long-term inflation expectations  $E_t^*[\pi_{t+1,t+10}]$ , and with each of its two components,  $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*[\pi_{t+1}]$  and  $\varepsilon_{t,\pi}^{LT}$ . The second row shows analogous results using the 10-year Treasury yield instead of the S&P 500 price-earnings ratio. Panel B shows the covariance of the S&P 500 price-earnings ratio with short-term nominal earnings growth expectations  $E_t^*[\Delta e_{t+1}]$  and with each of its two components,  $\beta_e(E_{t-1}^*[\Delta e_t] - \Delta e_t)$  and  $\varepsilon_{t,e}^{ST}$ . Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. \* $p < .1$ ; \*\* $p < .05$ ; \*\*\* $p < .01$ .

expectations and the current price is almost entirely attributed to the response to the recent surprise  $\beta_e(E_{t-1}^*[\Delta e_t] - \Delta e_t)$ . Section 7 shows that this fact on the role of surprises and our finding about the importance of the high believed persistence of inflation can both be summarized by a single Equation description of subjective expectation formation based on fundamental extrapolation.

#### 4. The Role of Inflation Expectations in Stock Prices

The key novel result from the diagnostic test is the importance of systematic errors in long-term inflation expectations for explaining stock prices and return predictability. This is surprising under the common view that inflation is irrelevant for stock prices. In this section, we discuss two topics that are important for understanding our results.

##### 4.1 Why do inflation expectations matter for stocks?

If people believe that inflation increases nominal earnings growth one-for-one, then changes in expected inflation should cancel out when pricing stocks. For example, if investors price stocks based on discounted expectations of real earnings growth, then increases in expected inflation would be canceled out by increases in expected nominal earnings growth, leaving expected real earnings growth unchanged. Similarly, one might imagine that stock investors simply care about forecasting future *nominal* earnings and then discount these

nominal earnings using Treasury yields plus their required risk premium. In this case, changes in expected inflation would raise both expected nominal earnings growth and the nominal discount rate, which cancel out and leave the price-earnings ratio unchanged.

Despite this common view, Table 1 shows that the price-earnings ratio has a large and significant negative comovement with long-term inflation expectations.<sup>23</sup> The key for explaining this behavior is that expected long-term nominal earnings growth comoves relatively little with expected long-term inflation. A regression of expected annualized long-term nominal earnings growth on expected annualized long-term inflation gives a small and insignificant coefficient of 0.22 (0.33). As shown in Figures 1 and 2, long-term inflation expectations have dropped considerably over time whereas long-term nominal earnings growth expectations have virtually no trend over time.

In a world in which expected nominal earnings growth comoves little with expected inflation, increases in expected inflation will lower the price-earnings ratio. If investors price stocks by discounting expected real earnings growth, then increases in expected inflation will lower expected real earnings growth and lower the price-earnings ratio. If investors price stocks based on expected nominal earnings discounted at Treasury yields plus their required risk premium, then increases in expected inflation will increase the nominal discount rate and lower the price-earnings ratio.<sup>24</sup> By extension, if the price-earnings ratio positively comoves with inflation forecast errors, then a low price-earnings ratio can be explained by overly high inflation expectations.

There are two potential explanations for why long-term nominal earnings growth expectations comove little with long-term inflation expectations. The first is that forecasters are making a mistake by not properly incorporating inflation expectations into their forecasts of long-term nominal earnings growth. This is similar in spirit to [Modigliani and Cohn \(1979\)](#), in the sense that failing to account for expected inflation leads stocks prices to be low during times of high expected inflation. The second is that forecasters are not forgetting to account for inflation, but instead simply believe that increases in inflation do not substantially increase nominal earnings growth. If earnings are nominally rigid, then increases in inflation lower real earnings growth, in line with the empirical findings of [Fang, Liu, and Roussanov \(2022\)](#) on inflation and real dividend growth.

Given that Table 1 shows that the price-earnings ratio does not predict forecast errors for long-term nominal earnings growth, one may be inclined to favor the second explanation. While expected long-term nominal earning

<sup>23</sup> The price-earnings ratio also has a significant negative comovement with expected 1-year inflation, although this comovement is similar in size to the comovement of the price-earnings ratio with realized future 1-year inflation. Because the comovement is not significantly different from what would be predicted under RE, we focus our attention on the comovement of the price-earnings ratio with long-term inflation expectations.

<sup>24</sup> We show in Section 6 that increases in expected long-term inflation are associated with large changes in nominal Treasury yields.

growth did not fall with the decline in long-term inflation expectations, neither did realized long-term nominal earnings growth. Rephrased, long-term inflation expectations do not predict forecast errors for long-term nominal earnings growth. However, this is still consistent with the first explanation. Failing to incorporate long-term inflation expectations into their forecasts of long-term nominal earnings growth means that biases in expected long-term inflation will not bias expected long-term nominal earnings growth. Thus, the price-earnings ratio could predict forecast errors for long-term inflation while not predicting forecast errors for long-term nominal earnings growth. For our purposes, the two explanations (failing to account for inflation or believing inflation does not affect nominal earnings growth) are observationally equivalent, as they imply the same inflation expectations, nominal earnings growth expectations, and price-earnings ratios.

## 4.2 Does the diagnostic test require that investors think in real terms?

An important benefit of the diagnostic test is that it does not require that investors think in real terms. Again, consider the hypothetical scenario in which investors price stocks based on expected nominal earnings growth and nominal discount rates and both variables move 1-1 with expected inflation. In this scenario, the diagnostic test would correctly determine that deviations from RE in inflation expectations do not help to explain price variation or return predictability. Any deviation from RE in inflation expectations ( $E_t[\pi_{t+j}] - E_t^*[\pi_{t+j}]$ ) would raise both expected nominal earnings growth and the nominal discount rates. These effects would cancel out when pricing stocks, meaning that the price-earnings ratio would not comove with  $E_t[\pi_{t+j}] - E_t^*[\pi_{t+j}]$ . As a result, inflation expectations would fail the diagnostic test,  $Cov(pe_t, f_t^{\pi_{t+j}}) = Cov(pe_t, E_t[\pi_{t+j}] - E_t^*[\pi_{t+j}]) = 0$ .<sup>25</sup> In this scenario, inflation forecast errors drop out of Equations (5) and (7) and only systematic errors in nominal earnings growth expectations would be relevant (so long as they comove with  $pe_t$ ).

For comparison, consider the scenario in which investors still price stocks based on expected nominal earnings growth and nominal discount rates, but, in line with our results, expected nominal earnings growth comoves little with expected inflation. Even though investors price stocks using nominal variables rather than real variables, overly high inflation expectations would increase the nominal discount rate and decrease the price-earnings ratio. This would lead to a negative  $Cov(pe_t, -f_t^{\pi_{t+j}})$ , satisfying the diagnostic test. Thus, the diagnostic test would still correctly identify that systematic errors in inflation

<sup>25</sup> One could similarly consider the scenario in which neither expected nominal earnings growth nor the nominal discount rate moves with expected inflation. Again, the diagnostic test would correctly identify that deviations from RE in inflation expectations do not explain price variation or return predictability.

expectations help to explain price movements and the ability of prices to predict future returns.

## 5. Connection to Recent Literature

### 5.1 Robustness to other price ratios

A recent conjecture first stated in Adam and Nagel (2023) and later in Hillenbrand and McCarthy (2022) is that the importance of short-term nominal earnings growth expectations for the price-earnings ratio is due to the volatility of earnings. They posit that using a more smooth denominator for the price ratio, such as the price-dividend ratio, would lead to long-term nominal earnings growth expectations playing a large role in price ratio movements rather than short-term nominal earnings growth expectations. We show that this is not the case using both the price-dividend ratio  $pd_t$  and CAPE  $pe_t^{CA}$ , which is the ratio of the price to the cyclically adjusted 10-year average earnings.

Table 3 repeats our diagnostic test using the price-earnings ratio, the price-dividend ratio, and CAPE. Panel A shows that the results of the diagnostic test for inflation expectations are virtually identical across all three price ratios. For all three price ratios, forecast errors for short-term inflation have virtually no comovement with the price ratios while forecast errors for long-term inflation have large and significant comovements with the price ratios. More importantly, the first three rows and the last three rows of panel B show the results of the diagnostic test for short-term and long-term nominal earnings growth expectations. While the covariance with expected and realized short-term nominal earnings growth differs depending on the price ratio, we find that all three price ratios significantly negatively comove with short-term nominal earnings growth forecast errors, with values of  $-4.26^{***}$ ,  $-3.43^{***}$ , and  $-3.65^{***}$  for the price-earnings ratio, the price-dividend ratio, and CAPE, respectively. In comparison, none of the price ratios significantly comoves with long-term nominal earnings growth forecast errors and these comovements are, if anything, positive (2.35, 0.25, 1.39, respectively).

In short, we consistently find that high price ratios are associated with overly optimistic short-term nominal earnings growth expectations (i.e., they negatively predict forecast errors). Conversely, we find no evidence that high price ratios are associated with overly optimistic long-term nominal earnings growth expectations. The diagnostic test results are not only qualitatively similar across price ratios but also quantitatively similar.

As further robustness, we show that changing the denominator in our short-term nominal cash flow growth expectations does not alter our findings. For example, there may be concern that movements in short-term earnings growth expectations mainly reflect changes in current realized earnings  $e_t$  rather than changes in expected future earnings  $E_t^*[e_{t+1}]$ . To ensure that it is changes in expected future cash flows that drive the comovements, we repeat our

**Table 3**  
Diagnostic test for inflation and nominal earnings growth expectations

| A. Inflation                       |                                      |          |          |          |
|------------------------------------|--------------------------------------|----------|----------|----------|
| Horizon                            | Price ratio                          | Expected | Realized | FE       |
| Short-term ( $\pi_{t+1}$ )         | $Cov(pe_t, \cdot)$                   | -0.66*** | -0.82*** | -0.16    |
|                                    | $Cov(pd_t, \cdot)$                   | -0.70*** | -0.77*** | -0.08    |
|                                    | $Cov(pe_t^{CA}, \cdot)$              | -0.69*** | -0.74**  | -0.06    |
| Long-term ( $\pi_{t+1,t+10}$ )     | $Cov(pe_t, \cdot)$                   | -3.97*** | -1.66*   | 2.31***  |
|                                    | $Cov(pd_t, \cdot)$                   | -5.13*** | -2.25*** | 2.88***  |
|                                    | $Cov(pe_t^{CA}, \cdot)$              | -5.02*** | -2.24*** | 2.79***  |
| B. Nominal earnings growth         |                                      |          |          |          |
| Short-term ( $\Delta e_{t+1}$ )    | $Cov(pe_t, \cdot)$                   | 9.44***  | 5.18     | -4.26*** |
|                                    | $Cov(pd_t, \cdot)$                   | 2.52     | -0.90    | -3.43**  |
|                                    | $Cov(pe_t^{CA}, \cdot)$              | 2.25     | -1.40    | -3.65**  |
| Short-term ( $e_{t+1} - x_t$ )     | $Cov(pe_t, e_{t+1} - e_t)$           | 9.44***  | 5.18     | -4.26*** |
|                                    | $Cov(pd_t, e_{t+1} - d_t)$           | 7.00***  | 3.57*    | -3.43**  |
|                                    | $Cov(pe_t^{CA}, e_{t+1} - e_t^{CA})$ | 6.45***  | 2.80*    | -3.65**  |
| Long-term ( $\Delta e_{t+3,t+5}$ ) | $Cov(pe_t, \cdot)$                   | 0.36     | 2.71     | 2.35     |
|                                    | $Cov(pd_t, \cdot)$                   | 0.59     | 0.84     | 0.25     |
|                                    | $Cov(pe_t^{CA}, \cdot)$              | 0.56     | 1.95     | 1.39     |

This table shows the results of the diagnostic test under different stock price ratios. The condition is that prices must comove with forecast errors and the table shows the results for the S&P 500 price-earnings ratio ( $pe_t$ ), the price-dividend ratio ( $pd_t$ ), and the cyclically adjusted price-earnings ratio ( $pe_t^{CA}$ ). The first three rows of panel A show the covariance of the price ratios with short-term inflation expectations  $E_t^*[\pi_{t+1}]$ , realized short-term inflation  $\pi_{t+1}$ , and the forecast errors  $\pi_{t+1} - E_t^*[\pi_{t+1}]$  from 1976Q1 to 2018Q2. The next three rows show the covariance of the price ratios with long-term inflation expectations  $E_t^*[\pi_{t+1,t+10}]$ , realized long-term inflation  $\pi_{t+1,t+10}$ , and the forecast errors  $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$  using quarterly data from 1979Q4 to 2018Q2. In panel B, the first three rows show the covariance of the price ratios with short-term nominal earnings growth expectations  $E_t^*[\Delta e_{t+1}]$ , realized short-term nominal earnings growth  $\Delta e_{t+1}$ , and the forecast errors  $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$  from 1976Q1 to 2018Q2. The next rows repeat the diagnostic test using  $E_t^*[e_{t+1} - d_t]$  and  $E_t^*[e_{t+1} - e_t^{CA}]$  as the measure of short-term cash flow growth for the covariances with  $pd_t$  and  $pe_t^{CA}$ , respectively. The final three rows show the covariance of the price ratios with long-term nominal earnings growth expectations  $E_t^*[\Delta e_{t+3,t+5}]$ , realized long-term nominal earnings growth  $\Delta e_{t+3,t+5}$  and the forecast errors  $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$  from 1982Q1 to 2018Q2. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. \*  $p < .1$ ; \*\*  $p < .05$ ; \*\*\*  $p < .01$ .

diagnostic test using  $E_t^*[e_{t+1} - d_t]$  and  $E_t^*[e_{t+1} - e_t^{CA}]$  as our measure of short-term cash flow growth for the price-dividend ratio and CAPE, respectively. The middle three rows of panel B show the results. Across all three price ratios, the results are virtually identical. All three price ratios have large and significant comovements with short-term nominal cash flow growth expectations, with values of 9.44\*\*\*, 7.00\*\*\*, and 6.45\*\*\* for the price-earnings ratio, the price-dividend ratio, and CAPE, respectively. In comparison, long-term nominal earnings growth expectations have virtually no comovement with the price ratios (0.36, 0.59, 0.56, respectively).<sup>26</sup> These short-term nominal cash flow

<sup>26</sup> In their appendix, Hillenbrand and McCarthy (2022) find similar results. Using a different methodology, they estimate that short-term expectations  $E_t^*[e_{t+1} - d_t]$  and  $E_t^*[e_{t+1} - e_t^{CA}]$  explain 63% and 67% of all variation in

growth expectations are consistently overoptimistic when price ratios are high, leading price ratios to have a negative comovement with the forecast errors ( $-4.26^{***}$ ,  $-3.43^{***}$ ,  $-3.65^{***}$ ), while we find no relation between price ratios and forecast errors for long-term nominal earnings growth expectations.<sup>27</sup>

In other words, the results are remarkably consistent across different denominators. Even if we replace current earnings with more smooth variables, such as dividends or 10-year cyclically adjusted earnings, we find that high price ratios are strongly associated with higher short-term nominal cash flow growth expectations, whereas long-term nominal earnings growth expectations have almost no comovement with the price ratios. This highlights that the choice of denominator is not driving the results. Instead, it is the connection between prices and expected short-term nominal earnings.

Initially, this robustness may seem surprising, as Table 2 shows that the excessive comovement of short-term nominal earnings growth expectations with the price-earnings ratio is largely explained by the response to earnings growth surprises  $\Delta e_t - E_{t-1}^*[\Delta e_t]$ . If there is a sudden drop in earnings, then the price-earnings ratio and expected earnings growth  $E_t^*[\Delta e_{t+1}]$  will both increase substantially, but the price-dividend ratio and CAPE would have a much smaller change. The key to understanding the robustness of our findings is that earnings growth surprises ( $\Delta e_t - E_{t-1}^*[\Delta e_t]$ ) occur even if there is no objective shock to earnings ( $\Delta e_t - E_{t-1}[\Delta e_t]$ ) because investors' expectations deviate from RE. For example, suppose there is no objective shock and no change in earnings,  $E_t[\Delta e_t] = \Delta e_t = 0$ . Investors may still perceive a negative earnings growth surprise  $\Delta e_t - E_{t-1}^*[\Delta e_t]$ , a scenario that leads them to increase the price-earnings ratio based on their expected future earnings growth. Given that there was no change in earnings, this increase in the price-earnings ratio would also appear 1-1 as an increase in the price-dividend ratio and CAPE. Thus, all three ratios would predict the subsequent forecast error. [Internet Appendix D](#) studies the robustness to other price ratios formally in a setting in which expectations respond to the perceived surprise rather than the objective shock.

## 5.2 The role of expected long-term earnings growth

Our diagnostic test identifies which subjective expectations contain deviations from RE that are relevant for explaining price movements and the ability of

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the price-dividend ratio and CAPE, respectively. In comparison, they estimate that expected long-term nominal earnings growth accounts for  $-1\%$  and  $0\%$  of price-dividend ratio and CAPE variation, respectively. For their benchmark specification, [Hillenbrand and McCarthy \(2022\)](#) choose to scale by 3-year earnings, as they argue that it has less of a trend than 10-year cyclically adjusted earnings.

<sup>27</sup> Note that the forecast errors for  $E_t^*[e_{t+1} - d_t]$  and  $E_t^*[e_{t+1} - e_t^{CA}]$  are identical to the forecast errors for  $E_t^*[\Delta e_{t+1}]$ . Thus, the comovement with forecast errors in the middle three rows of panel B are identical to the comovement with forecast errors in the first three rows of panel B. In other words, the diagnostic test on forecast errors is invariant to changes in the denominator for short-term nominal cash flow growth, which highlights the generality of our findings on short-term nominal cash flow growth errors.

prices to predict future returns. A key benefit of this diagnostic test is that it does not assume a specific mechanism and holds for any deviation from RE, for example, learning, rational inattention, bounded rationality, and behavioral biases. This allows us to relate our diagnostic test results to recent papers that focus on a single mechanism, such as Nagel and Xu (2022), who study biased learning, and Bordalo et al. (2023a), who study overreaction.

Nagel and Xu (2022) emphasize the importance of expected long-term real earnings growth, which is consistent with our findings. We split real earnings growth into inflation and nominal earnings growth to align with the available survey data. As shown in Table 3, while forecasts errors for long-term nominal earnings growth fail the diagnostic test, we find that forecast errors for long-term inflation pass the diagnostic test. Thus, our results indicate that systematic errors in expected long-term *real* earnings growth are relevant.

Interestingly, splitting expected real earnings growth into expected nominal earnings growth and expected inflation helps to connect the model of Nagel and Xu (2022) to previous evidence on biased learning in inflation expectations (Malmendier and Nagel 2016). Our results indicate that expected long-term inflation plays a large role in expected real long-term earnings growth. Thus, the model of Nagel and Xu (2022), in which expected real long-term earnings growth is based on past experiences with fading memory, is closely linked to the empirical micro-evidence that lifetime experiences influence inflation expectations. Rephrased, our results indicate that biased expectations of real earnings growth based on past experiences and biased expectations of inflation based on past experiences may be two manifestations of the same inflation learning problem.

In comparison, Bordalo et al. (2023a) argue for a large role for long-term nominal earnings growth expectations (*LTG*). They find that revisions in *LTG* negatively predict returns. They propose that high *LTG* leads to “an inflated stock price and a future price reversal when over-optimism is disappointed.” However, our diagnostic test in Table 3 shows that there is no evidence that high price ratios are related to high *LTG* or that high price ratios are associated with disappointment in future long-term nominal earnings growth. Put simply, our diagnostic test rejects the idea that systematic errors in long-term nominal earnings growth expectations explain variation in price ratios or explain why high price ratios are followed by low future returns. For robustness, we repeat our analysis using the value-weighted long-term nominal earnings growth expectations of Bordalo et al. (2023a) in Internet Appendix C.6 and find the same results.<sup>28</sup> Note that changing the horizon for *LTG* (e.g., treating it as expected earnings growth from  $t$  to  $t+5$  or from  $t$  to  $t+10$ ) will not change the basic fact that *LTG* is uncorrelated with all three price ratios.

<sup>28</sup> We thank Andrei Shleifer and Rafael La Porta for sharing these expectations.

How can we reconcile our findings with those of [Bordalo et al. \(2023a\)](#)? The explanation appears to be that increases in *LTG* are actually associated with *decreases* in future nominal earnings growth.<sup>29</sup> Thus, an increase in *LTG* will predict lower future returns even if there is no change in current prices, as the increase in *LTG* is an objectively negative signal about future fundamentals. Rephrased, *LTG* appears to predict future returns because *LTG* negatively predicts future cash flows and not because increases in *LTG* cause prices to increase. As discussed in Section 2.2, there may be systematic errors that predict future earnings growth and also predict future returns, but these systematic errors will not explain price ratio movements or the ability of price ratios to predict returns if the systematic errors do not comove with price ratios. Thus, the finding that revisions in *LTG* predict future returns does not overturn our finding that systematic errors in *LTG* do not help to explain price ratio movements or why high price ratios are followed by low returns.<sup>30</sup>

Appendix B provides more details on the connection to [Bordalo et al. \(2023a\)](#) as well as a related paper ([Bordalo et al. 2023b](#)). First, we show in Appendix B.1 that there is no overreaction in *LTG*. The model and empirical test of [Bordalo et al. \(2023a\)](#) assume agents understand whether new information is objectively good or bad for future earnings growth and can only make mistakes by overreacting or underreacting to this new information.<sup>31</sup> However, as mentioned above, increases in *LTG* are associated with *decreases* in future long-term nominal earnings growth, violating this key assumption. This directly affects their test for overreaction, as we show that their decision to rescale variables when calculating forecast errors, but not to rescale variables when calculating revisions in expectations, is not problematic if this assumption holds; however, if this assumption is violated, the results of the test are substantially altered. This highlights the benefits of having a diagnostic test with minimal modeling assumptions.

Second, in a related paper [Bordalo et al. \(2023b\)](#) posit that *LTG* is important for explaining the level of the S&P 500 price index and excess volatility in prices. The reason is that a synthetic price index constructed with realized earnings and both short-term and long-term nominal earnings growth expectations matches the observed S&P 500 index far better than a rational measure based on future earnings. Appendix B.2 shows that, consistent with [Delao and Myers \(2021\)](#), the expectations responsible for matching the S&P 500 index and the excess volatility of prices are the short-term earnings growth expectations. Removing *LTG* from the synthetic price index changes the  $R^2$  for matching the S&P 500 price index by less than .01 (.88 compared to .89),

<sup>29</sup> This is shown in detail in Appendix B.1.

<sup>30</sup> Appendix B.3 provides more results on return predictability and shows that both short-term cash flow growth expectations and *LTG* negatively predict returns.

<sup>31</sup> In their terminology, they “assume  $\theta > -1$ , which ensures that good news are not viewed as bad and vice versa” ([Bordalo et al. 2023a](#)).

a fact that is not surprising given the low volatility of *LTG* and its lack of correlation with price ratios. Similarly, excess volatility in the synthetic price index compared to the rational measure mainly comes from the short-term earnings growth expectations, rather than *LTG*.

## 6. Asset Prices and Return Puzzles

In this section, we show that subjective expectations of inflation and nominal earnings growth accurately match observed asset prices and return puzzles. As discussed in Section 3, this is because of errors in long-term inflation expectations and short-term nominal earnings growth expectations. At first glance, it may be surprising that errors in short-term nominal earnings growth can have a large role in price movements and long-term returns. However, as discussed in Section 3.2, errors in short-term nominal earnings growth expectations affect the forecasted level of all future nominal earnings. Because it affects the level of cash flows at all horizons, errors in short-term nominal earnings growth have a substantial impact on stock prices and can lead to high (low) returns for many years as realized nominal earnings are consistently above (below) what investors originally forecasted.

We construct fundamental stock and bond prices based on Equations (2) and (4) using subjective expectations of real cash flows and constant discount rates (i.e., constant expected real returns). We focus on aggregate stock and bond prices, specifically the S&P 500 and the 10-year zero-coupon Treasury yield. Fundamental prices closely match the observed prices. The quantitative results imply that time-varying discount rates play almost no role in explaining aggregate stock price variation and play only a secondary role in explaining aggregate bond yield variation.

We then test whether these fundamental prices replicate two key puzzles in the asset pricing literature on stocks and bonds: the predictability of stock returns and the rejection of the expectations hypothesis for bonds. Specifically, we study the comovement of future stock returns with current price ratios and the excess volatility of bond holding returns. Under RE, the magnitude of these two puzzles is evidence that discount rates must vary substantially over time. Under subjective expectations, the observed puzzles are closely matched by expectations of real cash flows and constant discount rates.

### 6.1 Price movements

Movements in observed asset prices are closely matched by subjective expectations of real cash flows. To construct a fundamental price-earnings ratio, we first set the discount rate to a constant. Second, we use the believed persistence for inflation and nominal earnings growth  $\phi_\pi^*$  and  $\phi_e^*$ , which are shown in Section 3.2 to accurately capture the comovement of long-term inflation and nominal earnings growth expectations with the price-earnings

ratio. The fundamental price-earnings ratio is then

$$pe_t^{fun} = c + \frac{1}{1 - \rho\phi_e^*} E_t^*[\Delta e_{t+1}] - \frac{1}{1 - \rho\phi_\pi^*} E_t^*[\pi_{t+1}] \quad (15)$$

where the discount rate simply falls into the constant  $c$ .<sup>32</sup>

Note that the fundamental price-earnings ratio is simply a linear function of two time series measured from survey forecasts,  $E_t^*[\Delta e_{t+1}]$  and  $E_t^*[\pi_{t+1}]$ , and the parameters  $\phi_e^*$  and  $\phi_\pi^*$  are also estimated directly from the survey data.<sup>33</sup> The mean price-earnings ratio is used to set  $c$ , but no other information about the observed price-earnings ratio is used when constructing the fundamental price-earnings ratio. By extension, the fundamental price-dividend ratio and CAPE are

$$pd_t^{fun} = pe_t^{fun} + e_t - d_t, \quad (16)$$

$$pe_t^{CA, fun} = pe_t^{fun} + e_t - e_t^{CA}. \quad (17)$$

Similarly, using a constant discount rate, the fundamental 10-year yield is

$$y_t^{(10), fun} = c^y + \frac{1}{10} \frac{1 - \phi_\pi^{*10}}{1 - \phi_\pi^*} E_t^*[\pi_{t+1}], \quad (18)$$

where the second term represents the average expectation of inflation over the next 10 years. The results are almost identical if we use the survey 10-year inflation expectations  $E_t^*[\pi_{t+1, t+10}]$ . This is because the correlation between 1- and 10-year inflation expectations is 0.98 and  $\phi_\pi^*$  is estimated to match the 10-year expectations. The constant  $c^y$  is set to match the mean 10-year yield, but no other information about the observed yield is used in the fundamental yield.

Table 4 shows regressions of the observed stock price ratios and bond yield on the fundamental price ratios and bond yield. In short, subjective expectations of real cash flows explain virtually all movements in stock price ratios and two-thirds of movements in bond yields. For the price-earnings ratio, the price-dividend ratio, and CAPE, the observed values move almost 1-1 with the fundamental values with coefficients of 0.96, 0.96, and 0.98 and high  $R^2$ 's of .81, .79, and .79 respectively. Even for bond yields, where the fundamental value is simply expected inflation, we find a significant coefficient of 1.55 and an  $R^2$  of .66. Additionally, we cannot reject that the observed prices and yields are equal to the fundamental prices and yields plus noise, that is,  $\alpha=0$  and

<sup>32</sup> The term  $c$  conveniently condenses all of the constants. Given the constant discount rate  $\bar{r}$ , the full expressions is  $c = \frac{\bar{r} - \bar{e}}{1 - \rho} + \sum_{j=1}^{\infty} \rho^{j-1} (\alpha_{e,j} - \alpha_{\pi,j})$ .

<sup>33</sup> In Internet Appendix C.5, we also estimate a generalization of Equations (11) and (12) where expectations of inflation can affect expectations of nominal earnings growth. When estimated on the survey expectations, we find no significant interaction and all of our results are robust to including this interaction.

**Table 4**  
**Explaining price movements**

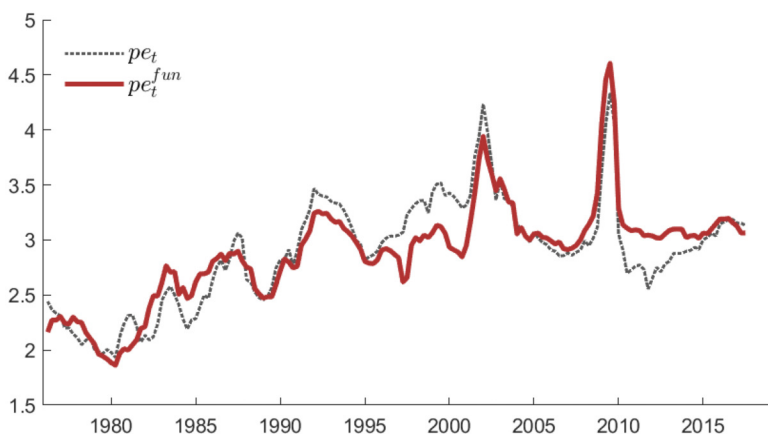
| Regression                                        | <i>a</i>        | <i>b</i>       | <i>R</i> <sup>2</sup> | Constrained <i>R</i> <sup>2</sup><br>( <i>a</i> = 0, <i>b</i> = 1) |
|---------------------------------------------------|-----------------|----------------|-----------------------|--------------------------------------------------------------------|
| $pe_t = a + bpe_t^{fun} + \varepsilon_t$          | 0.13<br>(0.21)  | 0.96<br>(0.09) | .81                   | .81                                                                |
| $pd_t = a + bpd_t^{fun} + \varepsilon_t$          | 0.16<br>(0.39)  | 0.96<br>(0.12) | .79                   | .79                                                                |
| $pe_t^{CA} = a + bpe_t^{CA, fun} + \varepsilon_t$ | 0.06<br>(0.31)  | 0.98<br>(0.12) | .79                   | .79                                                                |
| $y_t = a + by_t^{fun} + \varepsilon_t$            | -0.03<br>(0.02) | 1.55<br>(0.30) | .66                   | .58                                                                |
| $pe_t^{LR} = a + bpe_t^{fun, LR} + \varepsilon_t$ | 0.03<br>(0.14)  | 0.99<br>(0.06) | .82                   | .82                                                                |
| $pe_t^{SR} = a + bpe_t^{fun, SR} + \varepsilon_t$ | 0.00<br>(0.01)  | 0.90<br>(0.05) | .81                   | .80                                                                |

This table shows a comparison between observed stock and bond prices and the fundamental prices constructed with the subjective expectations of inflation and nominal earnings growth. The table shows linear regressions of observed prices on fundamental prices and reports the intercept *a*, the slope *b*, and the *R*<sup>2</sup> of the regression. Additionally, we report the constrained *R*<sup>2</sup> obtained from the residuals of the constrained regression where the intercept is forced to be zero *a* = 0 and the slope is forced to be one *b* = 1. The first three rows show the results for the S&P 500 price-earnings ratio, price-dividend ratio, and CAPE. The fourth row shows the results for the 10-year Treasury yield. The fifth and sixth rows show the results for the long-run component  $pe_t^{LR}$  and the short-run component  $pe_t^{SR}$  of the S&P 500 price-earnings ratio. The short-run and long-run component were obtained by applying a Hodrick-Prescott filter on both the observed and the fundamental price-earnings ratio with a smoothing parameter of 1,600. All calculations use quarterly data. Small-sample adjusted Newey-West standard errors in parentheses.

*b* = 1. In this constrained case of *a* = 0 and *b* = 1, we continue to find high *R*<sup>2</sup>'s of roughly .80 for the stock price ratios and .58 for the bond yield.

The results of Table 4 leave little room for large variation in discount rates. The high *R*<sup>2</sup> values demonstrate that the majority of price movements for both stocks and bonds are attributed to movements in subjective expectations of real cash flows. Even if we ascribe all of the residual variation to movements in discount rates, the discount rate variation would play almost no role in stock prices and only a secondary role in bond yields. Inflation expectations play a particularly important role as they account for the entirety of the bond results and substantially improve the stock results relative to only using nominal earnings growth expectations. To measure this, we reestimate the first three rows of Table 4 holding inflation expectations constant and only find a restricted *R*<sup>2</sup> of .51, .47, and .46, respectively.

Figure 3 shows that the fundamental price-earnings ratio  $pe_t^{fun}$  accurately tracks the observed price-earnings ratio  $pe_t$ . To emphasize how closely subjective expectations of real cash flows match observed stock prices, we calculate two additional measures and test how well the fundamental prices explain long-run changes in the price-earnings ratio as well as short-run



**Figure 3**  
**Fundamental and observed price-earnings ratio**

The figure compares the observed price-earnings ratio for the S&P 500 (dotted black) with the fundamental price-earnings ratio (solid red). The fundamental price-earnings ratio is the value of subjective expectations of real earnings growth plus a constant. All variables are in logs.

business cycle fluctuations. Using a standard Hodrick-Prescott filter, we calculate the long-run and short-run components of the observed price-earnings ratio,  $pe_t^{LR}$  and  $pe_t^{SR}$ , and the fundamental price-earnings ratio,  $pe_t^{fun,LR}$  and  $pe_t^{fun,SR}$ . The final two rows of Table 4 show that the fundamental values continue to move almost 1-1 with the observed values, with coefficients of 0.99 and 0.90 and high  $R^2$ 's of .82 and .81 for the long-run and short-run components, respectively.

## 6.2 Stock return predictability

A central argument for time-varying discount rates is the empirical fact that future stock returns comove significantly with current price ratios. As shown in Equation (7), under RE this comovement must be explained by substantial variation in discount rates. In Table 5, we test whether this comovement is matched by fundamental price ratios. Intuitively, if predictable returns are driven by errors in subjective expectations of real cash flows, then fundamental prices with constant discount rates will also predict returns.

The first column of Table 5 shows the results of univariate regressions of future nominal returns for the next 10 years on current price ratios. Changes in the observed price ratios are almost completely reflected in future returns. All variables are in logs, so a 1% increase in the price ratios is associated with a 0.78%, 0.88%, and 0.87% decrease in future 10-year returns, respectively. Importantly, increases in the fundamental price ratios are also reflected in future returns, with nearly identical coefficients of  $-0.77$ ,  $-0.92$ , and  $-0.95$ , respectively. The fourth column shows the analogous results using real returns for the next 10 years. Again, the coefficients for the observed price

**Table 5**  
**Stock return predictability**

|           |             | Nominal returns |        |       | Real returns |        |       |
|-----------|-------------|-----------------|--------|-------|--------------|--------|-------|
|           |             | $\beta$         | SE     | $R^2$ | $\beta$      | SE     | $R^2$ |
| P/E ratio | Observed    | -0.78           | (0.11) | .60   | -0.59        | (0.14) | .45   |
|           | Fundamental | -0.77           | (0.09) | .42   | -0.54        | (0.13) | .26   |
| P/D ratio | Observed    | -0.88           | (0.11) | .81   | -0.69        | (0.15) | .65   |
|           | Fundamental | -0.92           | (0.17) | .64   | -0.69        | (0.19) | .46   |
| CAPE      | Observed    | -0.87           | (0.14) | .77   | -0.69        | (0.17) | .61   |
|           | Fundamental | -0.95           | (0.20) | .62   | -0.71        | (0.22) | .44   |

This table shows the ability of real cash flow expectations to predict stock returns. The first, third, and fifth rows show the coefficients from univariate linear regressions of 10-year nominal  $\sum_{j=1}^{10} r_{t+j}$  and real stock returns  $\sum_{j=1}^{10} \tilde{r}_{t+j}$  on the observed price-earnings ratio ( $pe_t$ ), price-dividend ratio ( $pd_t$ ), and the ratio of price to the cyclically adjusted 10-year earnings ( $pe_t^{CA}$ ). The second, fourth, and sixth rows show the coefficients from univariate linear regressions of 10-year nominal and real stock returns on the fundamental price-earnings ratio ( $pe_t^{fun}$ ), price-dividend ratio ( $pd_t^{fun}$ ), and the ratio of price to cyclically adjusted 10-year earnings ( $pe_t^{CA, fun}$ ), which are constructed using real cash flow expectations and constant discount rates. For each regression, the table reports the slope coefficient  $\beta$ , the small-sample adjusted Newey-West standard error for the slope coefficient, and the  $R^2$ .

ratios,  $-0.59$ ,  $-0.69$ , and  $-0.69$  are nearly identical to the coefficients for the fundamental price ratios,  $-0.54$ ,  $-0.69$ , and  $-0.71$  and all coefficients are significant. In all cases, the differences between the coefficients for the observed price ratios and the fundamental price ratios are small and statistically insignificant.

Further, looking at the  $R^2$  values, we see that the majority of the  $R^2$  for the observed price ratios is generated by the fundamental price ratios. For example, the observed price-dividend ratio explains 81% of variation in future returns and the fundamental price-dividend ratio explains 64%. Thus, even if we attribute the entire difference to time-variation in discount rates, subjective expectations of real cash flows would still be the primary factor driving return predictability.

Combined, the results of Tables 4 and 5 imply that statistical forecasts of future returns based on observed price ratios are nearly identical to statistical forecasts based on fundamental price ratios. Fundamental price ratios move nearly 1-1 with observed price ratios with high  $R^2$ 's and have virtually identical coefficients for predicting future returns. In other words, an econometrician using observed price ratios and an econometrician using subjective expectations of real cash flows produce almost the same forecasts for future returns. This means that movements in statistical forecasts of returns primarily reflect predictable errors in subjective expectations of inflation and nominal earnings growth.

### 6.3 Rejection of expectations hypothesis

The expectations hypothesis posits that the  $n$ -period yield  $y_t^{(n)}$  is the expectation of the average one-period yield  $y_{t+j}^{(1)}$  over the next  $n$  periods, plus a constant term

**Table 6**  
**Rejection of expectations hypothesis**

|                                           | Observed yields | Fundamental yields |
|-------------------------------------------|-----------------|--------------------|
| $Var(h_{t+1}^{(10)})/\eta Var(y_t^{(1)})$ | 5.60            | 5.22               |
| F-test                                    | 0.00001         | 0.00046            |

This table compares the excess volatility of holding returns for the observed yields and the fundamental yields. The variance ratio measures the variance of one-period holding returns on a 10-year bond divided by the variance of the one-period yield multiplied by a constant  $\eta=10^2/(20-1)$ . Under RE and the expectations hypothesis, this ratio has an upper bound of one. The first column shows this ratio in the observed detrended data. The second column shows the variance ratio based on fundamental yields constructed using detrended inflation expectations and constant discount rates. The second row shows the significance of the one-sided F-test for equality of variance adjusted for autocorrelation of samples as in Priestley (1981).

premium.<sup>34</sup> Under RE, the expectations hypothesis implies that movements in yields must ultimately be related to movements in  $y_{t+j}^{(1)}$  and the volatility of holding returns is constrained by the volatility of movements in the one-period yield. Specifically, Equation (8) shows that under RE and the expectations hypothesis, the variance of holdings returns  $h_{t+1}^{(n)}$  is bounded by  $Var(y_t^{(1)})$  scaled by a constant  $\eta$ . The first column of Table 6 shows that this bound is decisively violated in the data by a factor of 5.6. Under RE, these large movements in  $h_{t+1}^{(n)}$  soundly reject the expectations hypothesis.

In this section, we show that subjective expectations of inflation can quantitatively match this violation without using time-varying term premia. The fundamental yield  $y_t^{(n),fun}$  is simply equal to the expected average inflation over the next  $n$ -periods plus a constant, meaning that the expectations hypothesis holds by definition. The holding return using fundamental yields is then measured from Equation (3).<sup>35</sup> The second column of Table 6 shows that fundamental yields violate this bound by a factor of 5.22, almost exactly matching the violation in the observed data. Using a one-sided F-test for equality of variance, we confidently reject that  $Var(h_{t+1}^{(10)})$  is below the bound for both the observed yields and the fundamental yields. We cannot reject that the two variance ratios are equal.

Equation (8) demonstrates how fundamental yields can violate this bound even though they satisfy the expectations hypothesis. Forecast errors that negatively comove with yields relax the bound, implying that the variance of holding returns under the expectations hypothesis can exceed  $\eta Var(y_t^{(1)})$ . Intuitively, if long-term inflation expectations are too high when yields are high, then these inflation expectations will have to be revised downwards as

<sup>34</sup> Rejecting the expectations hypothesis is equivalent to stating that term premia  $E_t^*[h_{t+1}^{(n)}] - y_t^{(1)}$  vary over time.

<sup>35</sup> Using the believed persistence  $\phi_\pi^*$ , the holding return reduces to a constant plus  $\frac{1-\phi_\pi^*10}{1-\phi_\pi^*} E_t^*[\pi_{t+1}] - \frac{1-\phi_\pi^*9}{1-\phi_\pi^*} E_{t+1}^*[\pi_{t+2}]$ .

investors realize their mistake. This downward revision in long-term inflation expectations lowers yields, which produces a high holding return following Equation (3). To an econometrician assuming RE, it appears as if the high yield and the high holding return are due to investors demanding a high term premium. Time variation in these predictable inflation errors similarly makes it appear as if term premia are time-varying.

The results of Table 6 indicate that the excess volatility of holding returns can be explained as a rejection of RE rather than a rejection of the expectations hypothesis. This is consistent with the results of Piazzesi, Salomao, and Schneider (2015), who find that survey expectations of the term premia  $E_t^*[h_{t+1}^{(n)}] - y_t^{(1)}$  are relatively flat compared to statistical measures of term premia. This is also consistent with Barr and Campbell (1997), who find that holding returns on inflation protected bonds are much less volatile than holding returns on nominal bonds. These findings complement previous work by Haubrich, Pennacchi, and Ritchken (2012), who present an affine model in which the level of yields is driven almost entirely by expectations of inflation and the real rate while violations of the expectations hypothesis are driven by separate volatility risk factors. Rather than using separate factors, we show that both yield movements and violations of the expectations hypothesis can be explained by biased inflation expectations.

## 7. Fundamental Extrapolation Framework

In this section, we provide a parsimonious model based on fundamental extrapolation that accurately replicates the results of our diagnostic test. Rather than proposing a fully-fledged psychological model, we use our model as a succinct mathematical summary of subjective expectation formation that reconciles our findings. Importantly, the impact of fundamental extrapolation depends on the objective persistence of the variable being forecasted. Because of this, the model successfully reproduces the stark differences in the term structure of subjective expectations and errors for inflation and nominal earnings growth documented in Section 3. For a persistent variable, such as inflation, errors are concentrated in long-term expectations. For a transitory variable, such as nominal earnings growth, errors are concentrated in short-term expectations. Despite not using any price information in the estimation, the model is also quantitatively successful in matching the comovements of expectations and realizations with bond yields and the price-earnings ratio. As further support, we find that the model accurately matches the evolution of survey expectations over time. Given the time series of realized inflation and nominal earnings growth for 1976-2018, the model closely replicates the time series of both short-term and long-term expectations for inflation and nominal earnings growth.

## 7.1 Model

Given that we are only interested in variances and covariances, we demean all variables without loss of generality. Inflation and nominal earnings growth are objectively AR(1) processes,

$$\pi_t = \phi_\pi \pi_{t-1} + v_{t,\pi} \quad (19)$$

$$\Delta e_t = \phi_e \Delta e_{t-1} + v_{t,e} \quad (20)$$

with variances  $\sigma_\pi^2, \sigma_e^2$  and covariance  $\sigma_{\pi,e}$ .

Agents are extrapolative and form their beliefs based on an inflation sentiment variable, defined as the weighted sum of current and past realizations,

$$S_{t,\pi} = \sum_{j=0}^{\infty} \beta_\pi^j \pi_{t-j}, \quad (21)$$

where  $\beta_\pi$  determines the relative weight placed on older and more recent observations. Agents believe that inflation depends on sentiment,

$$\pi_{t+1} = \theta_\pi S_{t,\pi} + v_{t+1,\pi}^*. \quad (22)$$

This is equivalent to saying that agents believe that sentiment is AR(1) with persistence  $\phi_\pi^* \equiv \theta_\pi + \beta_\pi$ ,

$$S_{t+1,\pi} = \phi_\pi^* S_{t,\pi} + v_{t+1,\pi}^*. \quad (23)$$

Under rational expectations,  $\beta_\pi = 0$  and  $\phi_\pi^* = \phi_\pi$ .

Equation (22) serves as a succinct, single-equation description of agents' subjective beliefs. From this equation, agents' short-term and long-term expectations are

$$E_t^*[\pi_{t+1}] = \phi_\pi^* \pi_t + \beta_\pi (E_{t-1}^*[\pi_t] - \pi_t) \quad (24)$$

$$E_t^*[\pi_{t+1+j}] = \phi_\pi^{*j} E_t^*[\pi_{t+1}]. \quad (25)$$

In other words, the believed persistence of sentiment  $\phi_\pi^*$  is also the believed persistence of future inflation as defined in Section 3.

We use an analogous framework for nominal earnings growth expectations that leads agents to believe that

$$S_{t,e} = \sum_{j=0}^{\infty} \beta_e^j \Delta e_{t-j}, \quad (26)$$

$$\Delta e_{t+1} = \theta_e S_{t,e} + v_{t+1,e}^*. \quad (27)$$

Here, the believed persistence and weight parameters are  $\phi_e^* \equiv \theta_e + \beta_e$  and  $\beta_e$ , and the agents' expectations can be expressed as

$$E_t^*[\Delta e_{t+1}] = \phi_e^* \Delta e_t + \beta_e (E_{t-1}^*[\Delta e_t] - \Delta e_t), \quad (28)$$

$$E_t^*[\Delta e_{t+1+j}] = \phi_e^{*j} E_t^*[\Delta e_{t+1}]. \quad (29)$$

**Table 7**  
Estimation of model parameters

| $\phi_\pi$     | $\phi_e$        | $\sigma_\pi$     | $\sigma_e$     | $\sigma_{\pi,e}$ | $\phi_\pi^*$   | $\phi_e^*$       | $\beta_\pi$    | $\beta_e$      |
|----------------|-----------------|------------------|----------------|------------------|----------------|------------------|----------------|----------------|
| 0.76<br>(0.14) | -0.20<br>(0.14) | 0.027<br>(0.002) | 0.38<br>(0.05) | 0.001<br>(0.001) | 0.96<br>(0.01) | 0.004<br>(0.075) | 0.69<br>(0.03) | 0.74<br>(0.02) |

This table shows the estimated model parameters. The objective parameters  $\phi_\pi, \phi_e, \sigma_\pi, \sigma_e$  and  $\sigma_{\pi,e}$  are estimated from realized inflation  $\pi_t$  and nominal earnings growth  $\Delta e_t$  over our 1976-2018 sample. The subjective parameters  $\phi_\pi^*, \phi_e^*$  are estimated from Equations (25) and (29) using both short-term and long-term survey expectations. After estimating  $\phi_\pi^*, \phi_e^*$ , we calculate  $\beta_\pi$  and  $\beta_e$  by regressing  $E_t^*[\pi_{t+1}] - \phi_\pi^* \pi_t$  on the surprise  $E_{t-1}^*[\pi_t] - \pi_t$  and regressing  $E_t^*[\Delta e_{t+1}] - \phi_e^* \Delta e_t$  on the surprise  $E_{t-1}^*[\Delta e_t] - \Delta e_t$ . Small-sample adjusted Newey-West standard errors in parentheses.

Last, agents have constant discount rates (i.e., constant expected real returns), which means that bond yields and the stock price-earnings ratio are simply

$$y_t^{(n)} = \frac{1}{n} \frac{1 - \phi_\pi^{*n}}{1 - \phi_\pi^*} E_t^*[\pi_{t+1}], \quad (30)$$

$$pe_t = \frac{1}{1 - \rho \phi_e^*} E_t^*[\Delta e_{t+1}] - \frac{1}{1 - \rho \phi_\pi^*} E_t^*[\pi_{t+1}]. \quad (31)$$

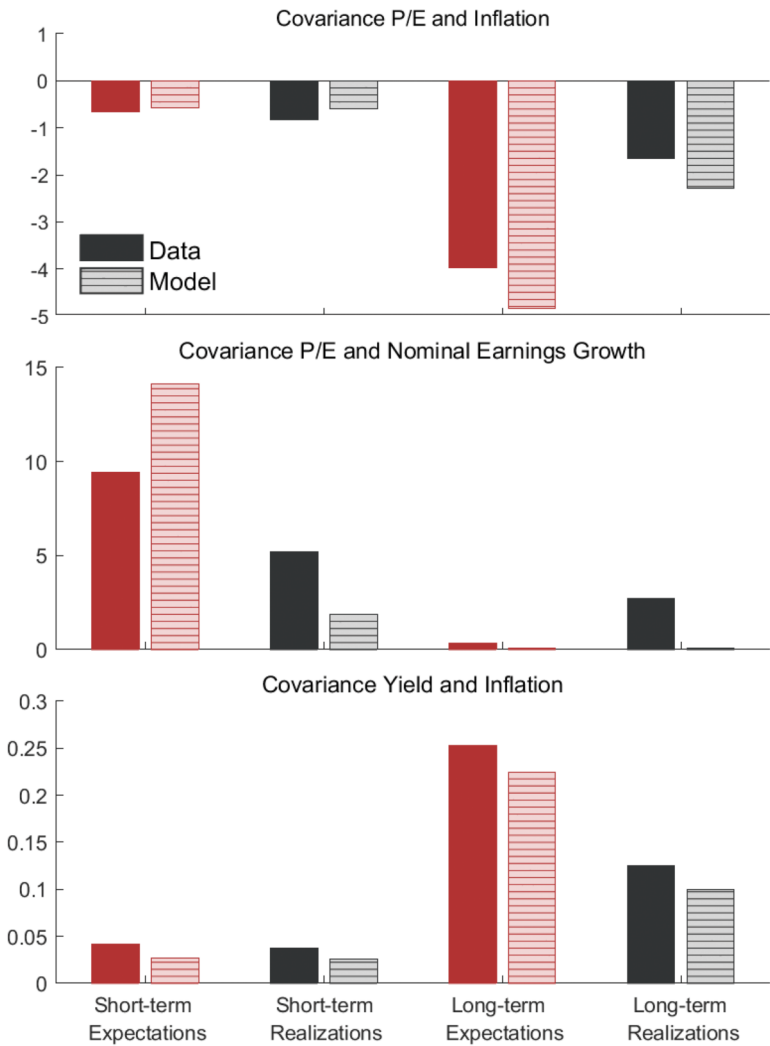
## 7.2 Diagnostic test

Notably, without using any information on observed prices, the model successfully replicates the comovement of bond yields and the price-earnings ratio with both expected and realized inflation and nominal earnings growth at both short and long horizons. Importantly, the model not only matches the qualitative features of these comovements, but quantitatively replicates their magnitudes.

All parameters are estimated from real cash flow data. The objective persistences  $\phi_\pi, \phi_e$ , variances  $\sigma_\pi^2, \sigma_e^2$ , and covariance  $\sigma_{\pi,e}$  are estimated from realized inflation and nominal earnings growth. The believed persistences  $\phi_\pi^*, \phi_e^*$  are estimated from Equations (25) and (29) using short-term and long-term survey expectations, just as in Section 3. Finally, the weights  $\beta_\pi$  and  $\beta_e$  are estimated from Equations (24) and (28) using short-term survey expectations and realized forecast errors  $\pi_t - E_{t-1}^*[\pi_t]$ ,  $\Delta e_t - E_{t-1}^*[\Delta e_t]$ . Table 7 shows the parameter values.

Figure 4 shows the comovement of prices with expected and realized inflation and nominal earnings growth. The dark bars represent the values measured in the data, and the light bars represent the values in the model. Comparing the dark and light bars side-by-side emphasizes that the model qualitatively and quantitatively matches the size of these comovements. These comovements represent twelve untargeted moments and, in every case, we cannot reject that the value measured in the data exactly equals the model prediction.

The top panel of Figure 4 shows the comovement of the price-earnings ratio with inflation. In the model, expected and realized short-term inflation have small, negative comovements with the



**Figure 4**  
**Matching diagnostic test**

This figure shows the covariance of prices with expected and realized real cash flows in the data and in the estimated model. The empirical covariances are represented by solid bars and the model covariances are represented by striped bars. The top and middle panels show the covariances of the S&P 500 price-earnings ratio with expected and realized inflation and nominal earnings growth at different horizons. The bottom panel shows the covariances of the 10-year Treasury yield with expected and realized inflation at different horizons.

price-earnings ratio that are virtually identical. For expected and realized long-term inflation, these comovements are scaled by  $(1 - \phi_{\pi}^{*10}) / (1 - \phi_{\pi}^*)$  and  $(1 - \phi_{\pi}^{10}) / (1 - \phi_{\pi})$ , respectively. Because the believed persistence is higher than the objective persistence, expected long-term inflation comoves much

more with the price-earnings ratio than realized long-term inflation, matching the large error that we observe in the data. The bottom panel shows the same pattern for the comovement of the 10-year yield with inflation. Importantly, despite not using any information about the variance of yields relative to the variance of the price-earnings ratio in the estimation, the model closely replicates the magnitudes of the comovements with both prices.

The middle panel of Figure 4 shows the comovement of the price-earnings ratio with expected and realized nominal earnings growth. Consistent with the data, the model produces a large comovement between short-term expectations and the price-earnings ratio and almost no comovement between long-term expectations and the price-earnings ratio. This is driven by the low believed persistence  $\phi_e^*$ . While the model understates the comovement of realized short-term and long-term nominal earnings growth with the price-earnings ratio, these comovements are both statistically insignificant in data. As a result, the model still accurately captures the fact that in the data there is no significant comovement between the price-earnings ratio and forecast errors for long-term nominal earnings growth. Instead, in both the model and the data, errors in expectations are concentrated in short-term expectations that comove much more with the price-earnings ratio than realized short-term nominal earnings growth does.

The model also matches the results of Section 3.3 and Table 2, which emphasize the importance of the high believed persistence in long-term inflation expectations and the response of short-term nominal earnings growth expectations to surprises. As shown in Equation (25), movements in model long-term inflation expectations are driven by changes in  $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*} E_t^*[\pi_{t+1}]$  where  $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi^*}$  is more than double the objective value  $\frac{1-\phi_\pi^{*10}}{1-\phi_\pi}$ .<sup>36</sup> Similarly, given that  $\phi_e^*$  is near zero, Equation (28) shows that movements in model short-term nominal earnings growth expectations are due to the response to surprises  $\beta_e(E_{t-1}^*[\Delta e_t] - \Delta e_t)$ . Thus, fundamental extrapolation gives an intuitive framework that captures both of these forces. For comparison, sticky or diagnostic expectations would allow subjective expectations to depend on surprises but do not explain the high believed persistence for inflation as they imply that the believed persistence equals the objective persistence, that is,  $E_t^*[\pi_{t+1+j}] = \phi_\pi^j E_t^*[\pi_{t+1}]$ . This is shown formally in Internet Appendix E.1. Similarly, a model where agents make mistakes about persistences, such as [Alti and Tetlock \(2014\)](#), would allow for a high  $\phi_\pi^*$  but would not allow short-term nominal earnings growth expectations to depend on surprises.

<sup>36</sup> The differences between the objective and believed persistence are particularly pronounced at longer horizons. When estimated at a 1-year horizon, the persistence of inflation is 0.76 (0.14), which could overlap with the believed persistence  $\phi_\pi^*$  of 0.96. However, when estimated using average inflation over the next 10 years, the annual persistence of inflation is 0.76 (0.05), which is significantly lower than the believed persistence. The estimated value for the persistence is consistent across horizons, but the standard error of the estimate is much smaller when longer horizons are used.

While our model allows for a high believed persistence for inflation, it is still consistent with the findings on sticky expectations of Coibion and Gorodnichenko (2015). Looking at Equation (24), the parameter  $\beta_\pi$  causes agents' expectations to depend less on current inflation  $\pi_t$  and more on their past expectations  $E_{t-1}^*[\pi_t]$ . As a result, the parameter  $\beta_\pi$  maps almost exactly to the stickiness coefficient of Mankiw and Reis (2002), which controls how quickly expectations adjust. A regression of forecast errors  $\pi_{t+1} - E_t^*[\pi_{t+1}]$  on revisions  $E_t^*[\pi_{t+1}] - E_{t-1}^*[\pi_{t+1}]$  in the model implies a stickiness coefficient of 0.65.<sup>37</sup> Further, we cannot reject that our model value of 0.65 matches the stickiness coefficient of 0.54 (0.10) measured in Coibion and Gorodnichenko (2015). As shown in Internet Appendix E.2, our model nests sticky expectations, meaning that there is no tension between our findings and previous evidence of stickiness. While we find that errors in model short-term inflation expectations do not contribute to price variation, these expectations are still consistent with important previous evidence on stickiness.

### 7.3 Intuition for term structure of errors

A key result of the model is that it replicates our findings on the term structure of errors in subjective expectations. Namely, current prices mainly comove with errors in long-term rather than short-term inflation expectations and comove with errors in short-term rather than long-term nominal earnings growth expectations.

To avoid repetition, let  $x$  be a general variable that represents inflation or nominal earnings growth. In the model, rational expectations are

$$E_t[x_{t+j}] = \phi_x^j x_t. \quad (32)$$

Agents' subjective expectations are rational when  $\phi_x^* - \phi_x$  and  $\beta_x$  are both zero. As shown in Table 7, the parameters for subjective expectation biases are almost identical for inflation and nominal earnings growth. For both inflation and nominal earnings growth,  $\phi_x^* - \phi_x$  is 0.20 and  $\beta_x$  is very similar at 0.69 for inflation and 0.72 for nominal earnings growth. As a result, the stark difference in the term structure of errors for inflation and nominal earnings growth expectations is not due to a difference in biases. Instead, both inflation and nominal earnings growth expectations demonstrate virtually identical biases and the difference in the term structure of errors is driven by the difference in the objective persistence  $\phi_x$ . For a persistent process like inflation, these biases generate errors that are concentrated in long-term expectations and for a transitory process like nominal earnings growth, these biases generate errors that are concentrated in short-term expectations.

<sup>37</sup> As established in Coibion and Gorodnichenko (2015), the regression coefficient is  $\frac{\lambda}{1-\lambda}$ , where  $\lambda$  is the stickiness coefficient.

For intuition, we consider each effect in isolation. When  $\phi_x^* - \phi_x$  is zero, errors in subjective expectations are

$$E_t^*[x_{t+j}] - E_t[x_{t+j}] = \phi_x^{j-1} \beta_x (E_{t-1}^*[x_t] - x_t). \quad (33)$$

If the objective persistence  $\phi_x$  is small, then the error is primarily concentrated in short-term expectations as the  $\phi_x^{j-1}$  coefficient quickly goes to zero as the horizon increases. Conversely, if the objective persistence is large, then the error is mainly concentrated in longer horizon expectations, as errors in short-term expectations comprise only a small portion of the total  $\frac{1}{1-\phi_x} \beta_x (E_{t-1}^*[x_t] - x_t)$  error across all horizons. The second isolated effect occurs when  $\beta_x$  is zero, in which case errors in subjective expectations are

$$E_t^*[x_{t+j}] - E_t[x_{t+j}] = (\phi_x^{*j} - \phi_x^j) x_t. \quad (34)$$

Once again, if the process is objectively transitory, then a small bias in  $\phi_x^* - \phi_x$  primarily generates errors in short-term expectations as the  $\phi_x^{*j} - \phi_x^j$  coefficient quickly goes to zero as the horizon increases. If the process is objectively persistent, then the error is mainly concentrated in longer horizon expectations. For example, given our estimated values of 0.96 and 0.76,  $\phi_\pi^* - \phi_\pi$  is 0.2 and  $\phi_\pi^{*10} - \phi_\pi^{10}$  is 0.6. In short, the difference between a persistent process and a very persistent process only becomes clear at long horizons.

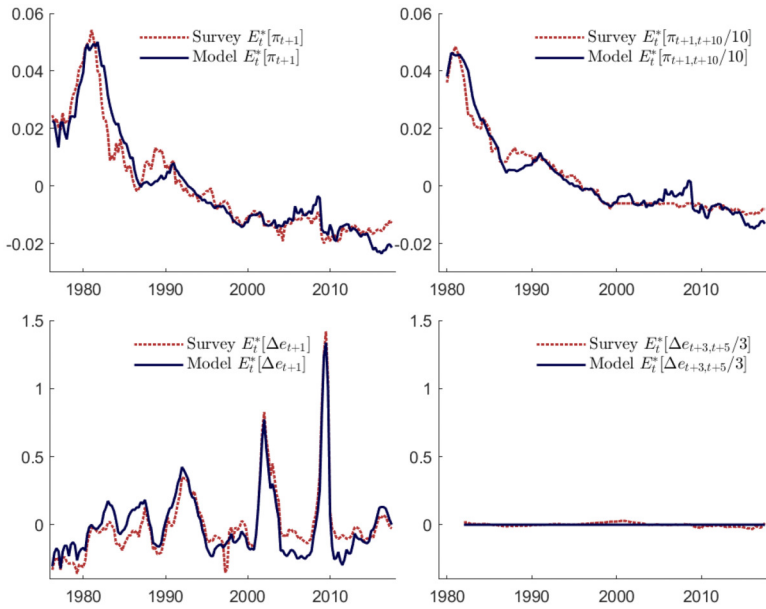
More generally, when  $\phi_x^* - \phi_x$  and  $\beta_x$  are both potentially nonzero, errors in subjective expectations are

$$E_t^*[x_{t+1+j}] - E_t[x_{t+1+j}] = \phi_x^j (E_t^*[x_{t+1}] - E_t[x_{t+1}]) + (\phi_x^{*j} - \phi_x^j) E_t^*[x_{t+1}]. \quad (35)$$

Again, this directly shows that if  $\phi_x$  and  $\phi_x^*$  are small, then errors in expectations will quickly disappear as the horizon increases. Conversely, for a high  $\phi_x$  and  $\phi_x^*$ , long-term expectations can have large errors even if there are no errors in 1-year expectations. Even if 1-year subjective expectations  $E_t^*[x_{t+1}]$  are close or even equal to rational expectations  $E_t[x_{t+1}]$ , errors in the believed persistence will still cause longer horizon subjective expectations  $\phi_x^{*j-1} E_t^*[x_{t+1}]$  to deviate noticeably from the rational  $\phi_x^{j-1} E_t[x_{t+1}]$ .

#### 7.4 Replicating survey expectations

In addition to matching the results of the diagnostic test, we also find that our model of subjective expectation formation accurately replicates the dynamics of survey expectations over time. Specifically, Equations (24)–(25) and (28)–(29) closely match the observed evolution in both short-term and long-term survey expectations for inflation and nominal earnings growth over the last 40 years. Importantly, these equations not only qualitatively match movements in survey expectations but also quantitatively match the magnitude of these changes both across time and across forecast horizons.



**Figure 5**  
**Replicating survey expectations**

The top panels compare the survey expectations (red) for inflation to the model expectations (blue) calculated from realized inflation and  $\phi_\pi^*, \beta_\pi$ . The bottom panels shows the same comparison for survey nominal earnings growth expectations (red) and the model expectations (blue) calculated from realized nominal earnings growth and  $\phi_e^*, \beta_e$ . Short-term expectations are shown on the left and long-term expectations are shown on the right. We demean all series without loss of generality.

We first consider survey and model inflation expectations. Using just two parameters ( $\phi_\pi^*, \beta_\pi$ ), we can closely replicate survey inflation expectations solely from the realized inflation series. We set the initial model surprise  $E_{t-1}^*[\pi_t] - \pi_t$  to match survey expectations in 1976 and then construct the model 1-year inflation expectations using realized inflation  $\pi_t$  and Equation (24).<sup>38</sup> Model expectations for average 10-year inflation are then given by Equation (25). The top panels of Figure 5 compare survey expectations for short-term and long-term inflation to the model expectations. The model accurately replicates the large changes in both short-term and long-term inflation expectations, including the spike from the late 1970's to the early 1980's, the gradual decline from the 1980's to the late 1990's, and the leveling off of expectations after 2000.

Similarly, the two parameters ( $\phi_e^*, \beta_e$ ) provide a parsimonious description of survey nominal earnings growth expectations. Repeating the procedure used for inflation expectations, we replicate the survey expectations solely using realized nominal earnings growth. The initial model surprise  $E_{t-1}^*[\Delta e_t] - \Delta e_t$

<sup>38</sup> We focus on the recursive Equation (24) since the infinite sum in Equation (22) is not observable.

is set to match survey expectations in 1976 and model 1-year expectations are constructed using Equation (28) and realized nominal earnings growth. Model long-term expectations are constructed from Equation (29). The bottom-left panel of Figure 5 shows that the model short-term expectations match almost all variation in survey short-term expectations, including the massive spikes in expected earnings growth during the dot-com bust and the financial crisis. Additionally, the model expectations accurately capture that most movements in nominal earnings growth expectations occur in short-term expectations, with long-term expectations being relatively flat compared to short-term expectations.

## 8. Conclusion

In this paper, we demonstrate when deviations from standard rational expectations (RE) help to explain price variation and return puzzles. Using accounting identities, we show that the necessary and sufficient condition is that forecast errors for subjective expectations must comove with current prices. This provides a simple and easily implementable diagnostic test to determine where the relevant deviations from RE do and do not occur in subjective expectations.

Applying this diagnostic test to survey expectations, we find a large role for systematic errors in expectations of long-term inflation and short-term nominal earnings growth in explaining price variation and return puzzles for aggregate bond and stock markets. In contrast, expectations of short-term inflation and long-term nominal earnings growth do not appear to play a role in either market despite containing systematic deviations from RE. These results are all consistent with a framework based on fundamental extrapolation in which agents price assets using constant discount rates and biased expectations of future real cash flows. In this framework, biases in expectations lead to errors primarily in long-term expectations for an objectively persistent process, such as inflation, and lead to errors primarily in short-term expectations for an objectively transitory process, such as nominal earnings growth.

### Code Availability

<https://doi.org/10.7910/DVN/PJRTEQ>.

## Appendix

### A Derivation of Formulas

To arrive at Equation (1), we start with the 1-year return identity

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\left(\frac{P_{t+1}}{D_{t+1}} + 1\right) \frac{D_{t+1}}{D_t}}{\frac{P_t}{D_t}},$$

where  $P_t$  and  $D_t$  represent the current nominal price and nominal dividends of the S&P 500 index. Log-linearizing around a long-term average of  $P/D$ , we can state the price-dividend ratio  $pd_t$  in

terms of future dividend growth,  $\Delta d_{t+1}$ , future returns,  $r_{t+1}$ , and the future price-dividend ratio,  $pd_{t+1}$ , all in logs:

$$r_{t+1} = \kappa^d + \Delta d_{t+1} - pd_t + \rho pd_{t+1}, \quad (\text{A.1})$$

where  $\kappa^d$  is a constant,  $\rho = e^{\bar{p}d} / (1 + e^{\bar{p}d}) < 1$  and  $\bar{p}d$  is the mean value of the log price-dividend ratio. Using the log payout ratio  $de_t$ , we can insert the identity  $pe_t = pd_t + de_t$ <sup>39</sup> into (A.1) to obtain

$$\tilde{r}_{t+1} \approx \kappa + \Delta \tilde{e}_{t+1} - pe_t + \rho pe_{t+1}, \quad (\text{A.2})$$

where we approximate  $(1 - \rho)de_{t+1}$  as zero given that  $1 - \rho$  is close to zero.

To establish identities (5) and (6), we just need to use the fact that forecast errors are unpredictable with information at time  $t$  under RE, meaning that

$$\text{Cov}(pe_t, \Delta \tilde{e}_{t+j}) = \text{Cov}(pe_t, E_t[\Delta \tilde{e}_{t+j}]). \quad (\text{A.3})$$

Under subjective expectations, the predictability of forecast errors with the price-earnings ratio is:

$$\text{Cov}(pe_t, f^{\Delta \tilde{e}_{t+j}}) = \text{Cov}(pe_t, \Delta \tilde{e}_{t+j}) - \text{Cov}(pe_t, E_t^*[\Delta \tilde{e}_{t+j}]). \quad (\text{A.4})$$

Equation (A.3) and (A.4) lead to Equation (5) which expresses the comovement of price-earnings ratio with subjective real earnings growth expectations in terms of its comovement with expectations under RE and the predictability of forecast errors for real earnings growth. In a similar fashion, the covariance of the 10-year bond yield with subjective inflation expectations can be expressed in terms of its covariance with expectations under RE and the predictability of forecast errors for inflation as in Equation (6).

To establish identity (7), we start from the fact that Equation (2) is satisfied with and without aggregate expectations. Hence

$$\sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} E_t^*[\tilde{r}_{t+j}] = \sum_{j=1}^{\infty} \rho^{j-1} \Delta \tilde{e}_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} E_t^*[\Delta \tilde{e}_{t+j}].$$

By covarying both sides with the price-earnings ratio we arrive to Equation (7). Finally, to establish the inequality in Equation (8) we generalize the Shiller (1979) variance bound. From Equation (3) we have that the variance of one-period holding returns can be expressed as

$$\text{Var}(h_{t+1}^{(n)}) = n^2 \text{Var}(y_t^{(n)}) + (n-1)^2 \text{Var}(y_{t+1}^{(n-1)}) - 2n(n-1) \text{Cov}(y_t^{(n)}, y_{t+1}^{(n-1)}).$$

The covariance term in the above equation can be expressed as

$$(n-1) \text{Cov}(y_{t+1}^{(n-1)}, y_t^{(n)}) = n \text{Var}(y_t^{(n)}) - \text{Cov}(y_t^{(1)}, y_t^{(n)}) - \text{Cov}(h_{t+1}^{(n)} - y_t^{(1)}, y_t^{(n)}),$$

and  $\text{Cov}(h_{t+1}^{(n)} - y_t^{(1)}, y_t^{(n)})$  can be expressed as

$$\begin{aligned} \text{Cov}(h_{t+1}^{(n)} - y_t^{(1)}, y_t^{(n)}) &= \text{Cov}(E_t^*[h_{t+1}^{(n)}] - y_t^{(1)}, y_t^{(n)}) + \text{Cov}\left(f_t^{h_{t+1}^{(n)}}, y_t^{(n)}\right) \\ &= \text{Cov}(E_t^*[h_{t+1}^{(n)}] - y_t^{(1)}, y_t^{(n)}) - \text{Cov}\left(\sum_{j=2}^n f_t^{h_{t+j}^{(n+1-j)}}, y_t^{(n)}\right). \end{aligned}$$

<sup>39</sup> Because we are using the aggregate S&P 500, we do not need to worry about very small or negative values for earnings.

We plug these two values back into the variance of  $h_{t+1}^{(n)}$  to get

$$\begin{aligned} \text{Var}\left(h_{t+1}^{(n)}\right) = & -n^2 \text{Var}\left(y_t^{(n)}\right) + (n-1)^2 \text{Var}\left(y_{t+1}^{(n-1)}\right) + 2n \text{Cov}\left(y_t^{(1)}, y_t^{(n)}\right) \\ & + 2n \text{Cov}\left(E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}, y_t^{(n)}\right) - 2n \text{Cov}\left(\sum_{j=2}^n f_t^{h_{t+j}^{(n+1-j)}}, y_t^{(n)}\right). \end{aligned} \quad (\text{A.5})$$

Under RE and the expectations hypothesis, the fourth and fifth terms of the equation would be zero. The hypothetical variance under these conditions, denoted as  $\overline{\text{Var}}\left(h_{t+1}^{(n)}\right)$  is

$$\overline{\text{Var}}\left(h_{t+1}^{(n)}\right) = -n^2 \text{Var}\left(y_t^{(n)}\right) + (n-1)^2 \text{Var}\left(y_{t+1}^{(n-1)}\right) + 2n \text{Cov}\left(y_t^{(1)}, y_t^{(n)}\right).$$

From Shiller (1979), we know that under these conditions the variance has an upper bound defined by

$$\overline{\text{Var}}\left(h_{t+1}^{(n)}\right) \leq \frac{n^2}{2n-1} \text{Var}\left(y_t^{(1)}\right).$$

This means that the generalized bound from (A.5) is

$$\text{Var}\left(h_{t+1}^{(n)}\right) \leq \frac{n^2}{2n-1} \text{Var}\left(y_t^{(1)}\right) + 2n \text{Cov}\left(E_t^*\left[h_{t+1}^{(n)}\right] - y_t^{(1)}, y_t^{(n)}\right) - 2n \text{Cov}\left(\sum_{j=2}^n f_t^{h_{t+j}^{(n+1-j)}}, y_t^{(n)}\right)$$

which reduces to Equation (8) under the expectations hypothesis as expected term premia are constant.

## B The Role of LTG

**B.1 Testing for Overreaction in LTG.** The results of Bordalo et al. (2023a) focus on revisions in LTG. Using the methodology of Coibion and Gorodnichenko (2015), they focus on the regression

$$FUT_{t+5} - LTG_t = \alpha + \beta_1 (LTG_t - LTG_{t-1}) + \beta_2 LTG_{t-1} + \varepsilon_{t+5}, \quad (\text{B.1})$$

where  $FUT_{t+5}$  is the future average long-term earnings growth  $\frac{1}{5} \sum_{j=1}^5 \Delta e_{t+j}$ . This is equivalent to the regression

$$FUT_{t+5} = \alpha + \gamma_1 LTG_t + \gamma_2 LTG_{t-1} + \varepsilon_{t+5} \quad (\text{B.2})$$

where  $\gamma_1 \equiv 1 + \beta_1$  and  $\gamma_2 \equiv \beta_2 - \beta_1$ .

A coefficient  $\beta_1 > 0$  ( $\gamma_1 > 1$ ) indicates underreaction, as the expectations are positively related to future earnings growth but an econometrician would want to magnify the subjective expectations ( $\gamma_1 > 1$ ) to best predict future earnings growth. A coefficient  $-1 < \beta_1 < 0$  ( $0 < \gamma_1 < 1$ ) indicates overreaction, as the expectations are positively related to future earnings growth but an econometrician would want to dampen the subjective expectations to best predict future earnings growth. Under the assumptions of Bordalo et al. (2023a), these are the only two possibilities. However, there is a third possibility. A coefficient of  $\beta_1 < -1$  ( $\gamma_1 < 0$ ) indicates that expectations are not overreacting or underreacting, but instead are moving in the wrong direction.

Running the regression (B.1), we find the coefficient  $\beta_1$  is  $-7.06$  with a standard error of  $1.93$ , which means  $\gamma_1$  is  $-6.06$ . In other words, an increase in  $LTG_t$  is actually associated with lower future earnings growth. The fact that  $\beta_1$  is clearly below  $-1$  means we can confidently reject that LTG overreacts. Further, the magnitude of  $\gamma_1$  indicates that LTG is *less volatile* than rational expectations would imply. To best predict future earnings growth, an econometrician would not

**Table B1**  
**Fitting the price level**

| Coefficients            | (1)            | (2)    | (3)            | (4)    |
|-------------------------|----------------|--------|----------------|--------|
| $e_t$                   | 1<br>-         | 1<br>- | 1<br>-         | 1<br>- |
| $E_t^*[\Delta e_{t+1}]$ | 1<br>-         | 1<br>- |                |        |
| $LTG_t$                 | 7.56<br>(6.97) |        | 6.47<br>(8.19) |        |
| $R^2$                   | .89            | .88    | .76            | .76    |

This table shows how well the log S&P 500 price index  $p_t$  can be fit by current earnings ( $e_t$ ), short-term nominal earnings growth expectations ( $E_t^*[\Delta e_{t+1}]$ ), and long-term nominal earnings growth expectations ( $LTG_t$ ). Column 1 shows the results when all variables are included. Column 2 shows the result when just  $LTG_t$  is removed. Column 3 shows the results when just short-term nominal earnings growth expectations are removed. Column 4 shows the results when both  $LTG_t$  and short-term nominal earnings growth expectations are removed. The first three rows show the coefficients for  $e_t$ ,  $E_t^*[\Delta e_{t+1}]$ , and  $LTG_t$  and the fourth row shows the  $R^2$ . In all columns, the coefficient for current earnings is restricted to one following the present value identity. In any cases in which short-term nominal earnings growth expectations are included (columns 1 and 2), the coefficient for  $E_t^*[\Delta e_{t+1}]$  is restricted to be one following the present value identity. In any cases in which  $LTG_t$  is included, we estimate the optimal coefficient for  $LTG_t$  for fitting  $p_t$ , which means we are showing an upper bound on how much  $LTG_t$  helps in fitting prices. Small-sample adjusted Newey-West standard errors reported in parentheses.

only want to flip the sign of  $LTG$  but also want to magnify the movements in  $LTG$  by a factor of 6 ( $\gamma_1 = -6.06$ ). This is consistent with our finding that  $LTG$  has low volatility.

It's important to note that the coefficient in table 4 of [Bordalo et al. \(2023a\)](#) is not equivalent to  $\beta_1$  because they rescale the dependent variable  $FUT_{t+5} - LTG_t$  to have a variance of 1% but do not rescale  $LTG_t - LTG_{t-1}$  when they run the regression (B.1). In other words, their reported coefficient is  $\beta_1 f$  where  $f$  is the rescaling factor for the dependent variable. We confirm that doing a similar rescaling would push our regression coefficient  $\beta_1 f$  to be between  $-1$  and  $0$ , but this is simply the effect of rescaling rather than evidence of overreaction.

**B.2 The Role of  $LTG$  in Matching Prices and Excess Volatility.** In this section we evaluate the importance of  $LTG$  in matching the S&P 500 price level and the volatility of S&P 500 price changes. [Bordalo et al. \(2023b\)](#) show that a synthetic price index  $\tilde{p}_t$  constructed with realized earnings and both short-term and long-term ( $LTG$ ) nominal earnings growth expectations matches the observed S&P 500 index  $p_t$  quite well. They also show in table 1 that the variance of synthetic price changes  $\Delta \tilde{p}_t$  closely matches the variance of observed price changes  $\Delta p_t$ , while the variance of changes in a rational measure of prices  $\Delta p_t^r$  is quite low. We replicate these findings and highlight that it is the inclusion of short-term earnings growth expectations, rather than  $LTG$ , which drives the bulk of these results.

To properly isolate the importance of  $LTG$ , we construct a synthetic short-term price index  $\tilde{p}_t^{ST}$  using realized earnings and short-term earnings growth expectations. The synthetic price index  $\tilde{p}_t$  and the short-term price index  $\tilde{p}_t^{ST}$  are defined as:

$$\tilde{p}_t = \alpha + e_t + E_t^*[\Delta e_{t+1}] + bLTG_t, \quad (\text{B.3})$$

$$\tilde{p}_t^{ST} = \alpha + e_t + E_t^*[\Delta e_{t+1}], \quad (\text{B.4})$$

where  $b$  is the coefficient obtained from running a constrained regression of prices  $p_t$  on the right-hand side of Equation (B.3) where the coefficients for  $e_t$  and  $E_t^*[\Delta e_{t+1}]$  are constrained to be equal to one. Column 1 of Table B1 shows that the synthetic price index fits the observed price fairly well with an  $R^2$  of .89. However, column 2 shows that the fit comes almost entirely from  $e_t$  and  $E_t^*[\Delta e_{t+1}]$ , since  $\tilde{p}_t^{ST}$  achieves an  $R^2$  of 0.88. Columns 3 and 4 show that the absence of short-term

**Table B2**  
Price growth volatility

| Coefficients       | $\Delta p_t$ | $\Delta p_t^r$ | $\Delta \bar{p}_t$ | $\Delta \bar{p}_t^{ST}$ |
|--------------------|--------------|----------------|--------------------|-------------------------|
| Standard deviation | 16.5%        | 0.8%           | 15.4%              | 11.6%                   |

This table shows the volatility of log price growth for the observed S&P 500 price index ( $\Delta p_t$ ), a rational index for prices ( $\Delta p_t^r$ ), a synthetic price index using current earnings and both short-term and long-term nominal earnings growth expectations ( $\Delta \bar{p}_t$ ), and a synthetic price index using current earnings and short-term nominal earnings growth expectations ( $\Delta \bar{p}_t^{ST}$ ).

earnings growth expectations does affect the amount of variation in  $p_t$  one can explain, with  $R^2$ 's of only .76. Across all of the tests, the inclusion or exclusion of  $LTG_t$  has only a minor impact on the ability to explain  $p_t$ .

To test for excess volatility in prices, [Bordalo et al. \(2023b\)](#) calculate the volatility of price growth for the observed S&P 500 index, the synthetic index  $\bar{p}_t$ , and a rational measure based on future realized earnings and the observed terminal price-earnings ratio,

$$p_t^r = e_t + \frac{1 - \rho^{T-t}}{1 - \rho} (\kappa - \bar{r}) + \sum_{j=1}^{T-t} \rho^{j-1} \Delta e_{t+j} + \rho^{T-t} p e_T.$$

The constant  $\bar{r}$  is set equal to the average return over the sample,  $T$  indicates the last observed period in the sample, and  $p e_T$  is the observed terminal price-earnings ratio. Table B2 not only shows this same calculation but also includes the volatility of price growth using only the short-term earnings growth expectations  $\bar{p}_t^{ST}$ . This is analogous to table 1 in [Bordalo et al. \(2023b\)](#).

We see that the rational measure  $\Delta p_t^r$  has very little volatility compared to observed prices  $\Delta p_t$  (0.8% compared to 16.5%), that is, observed prices are excessively volatile. Using the synthetic price rather than the rational price generates much more price volatility. Looking at the final column of Table B2, we see that the vast majority of this increase in volatility comes from the inclusion of short-term nominal earnings growth expectations (11.6% compared to 0.8%).

**B.3 Predicting Returns.** Table B3 shows the results of regressing future nominal returns on measures of expected short-term earnings growth and  $LTG$ . As in [Bordalo et al. \(2023a\)](#), we find that  $LTG$  does negatively predict future returns. However, it requires an enormous coefficient of  $-38.09$ . This is consistent with our finding that  $LTG$  has relatively little volatility, as this indicates that  $LTG$  can only explain returns if it is substantially scaled up.

Additionally, the large coefficient for  $LTG$  supports our finding that the ability of  $LTG$  to predict returns is not due to  $LTG$  increasing prices. Following Equation (2), a 1-p.p. increase expected earnings growth over the next 3-5 years (i.e.,  $LTG$ ) should raise current prices by 3-5p.p. In the extreme case in which there is no increase in realized 3- to 5-year earnings growth, then this price increase will show up as a 3-5p.p. decrease in future returns. A regression coefficient substantially beyond  $-3$  to  $-5$  would indicate that the increase in  $LTG$  was actually followed by a *decrease* in future earnings growth, which allows  $LTG$  to negatively predict returns (even without an increase in prices) by being an objectively negative signal about future fundamentals. The empirically observed coefficient of  $-38$  strongly supports this second scenario.

For comparison, the third and fourth columns of Table B3 show the  $R^2$  when the coefficient for  $LTG$  is allowed to be  $-38.09$  and the  $R^2$  when the coefficient is constrained to be only  $-5$  (i.e., the lowest value consistent with  $LTG$  predicting returns through its effect on current prices). We see that the  $R^2$  drops sharply from .54 to .13 when the coefficient is constrained.

B3 juxtaposes these results for  $LTG$  with the results for all three measures of short-term earnings growth expectations. All three measures of short-term earnings growth expectations negatively predict future returns with significant coefficients at the 1% level. From Equation (2), a 1-p.p. increase in 1-year earnings growth expectations should increase current prices by 1-p.p. If this

**Table B3**  
**Stock return predictability**

|                           | Nominal returns $\sum_{j=1}^{10} r_{t+j}$ |        |       |                   |
|---------------------------|-------------------------------------------|--------|-------|-------------------|
|                           | $\beta$                                   | SE     | $R^2$ | Constrained $R^2$ |
| $E_t^*[e_{t+1}-e_t]$      | -0.79                                     | (0.27) | .11   | .10               |
| $E_t^*[e_{t+1}-d_t]$      | -1.90                                     | (0.36) | .55   | .43               |
| $E_t^*[e_{t+1}-e_t^{CA}]$ | -2.14                                     | (0.41) | .53   | .38               |
| $LTG_t$                   | -38.09                                    | (7.61) | .54   | .13               |

This table shows the ability of nominal cash flow growth expectations to predict nominal stock returns. The four rows correspond to univariate regressions of 10-year nominal returns  $\sum_{j=1}^{10} r_{t+j}$  on expected short-term earnings relative to current earnings ( $E_t^*[e_{t+1}-e_t]$ ), expected short-term earnings relative to current dividends ( $E_t^*[e_{t+1}-d_t]$ ), expected short-term earnings relative to current 10-year cyclically adjusted earnings ( $E_t^*[e_{t+1}-e_t^{CA}]$ ), and expected 3- to 5-year nominal earnings growth ( $LTG_t$ ). For each regression, the table reports the slope coefficient  $\beta$ , the small-sample adjusted Newey-West standard error for the slope coefficient, and the  $R^2$ . Additionally, we report the constrained  $R^2$  obtained from the residuals of the constrained regression. For the first three rows, the constrained regression forces the intercept to be zero and the slope to be  $\beta=-1$ . For the fourth row, the constrained regression forces the intercept to be zero and the slope to be  $\beta=-5$ .

increase in price is completely reversed, then future returns will decrease by 1-p.p. To gauge how well the regression results fit with this prediction, the third and fourth columns show the  $R^2$  when the coefficient is unconstrained and when the coefficient is constrained to be  $-1$ . Unlike the results for  $LTG$ , we find that the constrained  $R^2$  for short-term earnings growth expectations is fairly close to the unconstrained  $R^2$ .

**References**

Adam, K., and S. Nagel. 2023. Expectations data in asset pricing. In *Handbook of economic expectations*, eds. R. Bachmann, G. Topa, and W. van der Klaauw, 477–506. Cambridge, MA: Academic Press.

Afrouzi, H., S. Kwon, A. Landier, Y. Ma, and D. Thesmar. 2023. Overreaction in expectations: Evidence and theory. *Quarterly Journal of Economics* 138:1713–64.

Alti, A., and P. C. Tetlock. 2014. Biased beliefs, asset prices, and investment: A structural approach. *Journal of Finance* 69:325–61.

Amromin, G., and S. A. Sharpe. 2014. From the horse’s mouth: Economic conditions and investor expectations of risk and return. *Management Science* 60:845–66.

Ang, A., G. Bekaert, and M. Wei. 2007. Do macro variables, asset markets, or surveys forecast inflation better? *Journal of Monetary Economics* 54:1163–212.

Ang, A., and M. Piazzesi. 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. *Journal of Monetary Economics* 50:745–87.

Bacchetta, P., E. Mertens, and E. V. Wincoop. 2009. Predictability in financial markets: What do survey expectations tell us? *Journal of International Money and Finance* 28:406–26.

Barberis, N., R. Greenwood, L. Jin, and A. Shleifer. 2015. X-capm: An extrapolative capital asset pricing model. *Journal of Financial Economics* 115:1–24.

Barberis, N., A. Shleifer, and R. Vishny. 1998. A model of investor sentiment. *Journal of Financial Economics* 49:307–43.

Barberis, N., and R. Thaler. 2003. A survey of behavioral finance. In *Handbook of the economics of finance*, eds. G. Constantinides, M. Harris, and R. Stulz, vol. 1, 1053–124. Amsterdam, the Netherlands: North-Holland.

- Barr, D. G., and J. Y. Campbell. 1997. Inflation, real interest rates, and the bond market: A study of uk nominal and index-linked government bond prices. *Journal of Monetary Economics* 39:361–83.
- Barsky, R. B., and J. B. DeLong. 1993. Why does the stock market fluctuate? *Quarterly Journal of Economics* 108:291–311.
- Bauer, M. D., and J. D. Hamilton. 2018. Robust bond risk premia. *Review of Financial Studies* 31:399–448.
- Bordalo, P., N. Gennaioli, R. La Porta, and A. Shleifer. 2023a. Belief overreaction and stock market puzzles. *Journal of Political Economy*. Advance Access published August 8, 2023, 10.1086/727713.
- Bordalo, P., N. G. Rafael, R. La Porta, M. Obrien, and A. Shleifer. 2023b. Long term expectations and aggregate fluctuations. *NBER Macroeconomics Annual* 38.
- Brunnermeier, M., E. Farhi, R. S. J. Kojien, A. Krishnamurthy, S. C. Ludvigson, H. Lustig, S. Nagel, and M. Piazzesi. 2021. Review article: Perspectives on the future of asset pricing. *Review of Financial Studies* 34:2126–60.
- Campbell, J. Y., and R. J. Shiller. 1988a. The dividend-price ratio and expectations of future dividends and discount factors. *Review of Financial Studies* 1:195–228.
- . 1988b. Stock prices, earnings, and expected dividends. *Journal of Finance* 43:661–76.
- Cassella, S., B. Golez, H. Gulen, and P. Kelly. 2023. Horizon bias and the term structure of equity returns. *Review of Financial Studies* 36:1253–88.
- Chernov, M., and P. Mueller. 2012. The term structure of inflation expectations. *Journal of Financial Economics* 106:367–94.
- Cieslak, A. 2018. Short-rate expectations and unexpected returns in treasury bonds. *Review of Financial Studies* 31:3265–306.
- Cieslak, A., and P. Povala. 2015. Expected returns in treasury bonds. *Review of Financial Studies* 28:2859–901.
- Cochrane, J. H. 2011. Presidential address: Discount rates. *Journal of Finance* 66:1047–108.
- Coibion, O., and Y. Gorodnichenko. 2015. Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review* 105:2644–78.
- Collin-Dufresne, P., M. Johannes, and L. A. Lochstoer. 2017. Asset pricing when 'this time is different'. *Review of Financial Studies* 30:505–38.
- d'Arienzo, D. 2020. Maturity increasing overreaction and bond market puzzles. Working Paper, Nova School of Business and Economics.
- Delao, R., X. Han, and S. Myers. 2024. The cross-section of subjective expectations: Understanding prices and anomalies. Working Paper, University of Southern California.
- Delao, R., and S. Myers. 2021. Subjective cash flow and discount rate expectations. *Journal of Finance* 76:1339–87.
- Fang, X., Y. Liu, and N. Roussanov. 2022. Getting to the core: Inflation risks within and across asset classes. Working Paper, University of Hong Kong.
- Giglio, S., M. Maggiori, J. Stroebel, and S. Utkus. 2021. Five facts about beliefs and portfolios. *American Economic Review* 111:1481–522.
- Greenwood, R., and A. Shleifer. 2014. Expectations of returns and expected returns. *Review of Financial Studies* 27:714–46.
- Gürkaynak, R. S., and J. H. Wright. 2012. Macroeconomics and the term structure. *Journal of Economic Literature* 50:331–67.
- Haubrich, J., G. Pennacchi, and P. Ritchken. 2012. Inflation expectations, real rates, and risk premia: Evidence from inflation swaps. *Review of Financial Studies* 25:1588–629.

- Hillenbrand, S., and O. McCarthy. 2022. Heterogeneous beliefs and stock market fluctuations. Working Paper, Harvard Business School.
- Hirshleifer, D. 2015. Behavioral finance. *Annual Review of Financial Economics* 7:133–59.
- Hirshleifer, D., J. Li, and J. Yu. 2015. Asset pricing in production economies with extrapolative expectations. *Journal of Monetary Economics* 76:87–106.
- Jin, L. J., and P. Sui. 2022. Asset pricing with return extrapolation. *Journal of Financial Economics* 145:273–95.
- Malmendier, U., and S. Nagel. 2016. Learning from inflation experiences. *Quarterly Journal of Economics* 131:53–87.
- Mankiw, N. G., and R. Reis. 2002. Sticky information versus sticky prices: A proposal to replace the new keynesian phillips curve. *Quarterly Journal of Economics* 117:1295–328.
- Modigliani, F., and R. A. Cohn. 1979. Inflation, rational valuation and the market. *Financial Analysts Journal* 35:24–44.
- Nagel, S., and Z. Xu. 2022. Asset pricing with fading memory. *Review of Financial Studies* 35:2190–245.
- Negro, M. D., and S. Eusepi. 2011. Fitting observed inflation expectations. *Journal of Economic Dynamics and Control* 35:2105–31.
- Piazzesi, M., J. Salomao, and M. Schneider. 2015. Trend and cycle in bond premia. Working Paper, Stanford University.
- Priestley, M. B. 1981. *Spectral analysis and time series*. Cambridge, MA: Academic Press.
- Shiller, R. J. 1979. The volatility of long-term interest rates and expectations models of the term structure. *Journal of Political Economy* 87:1190–219.
- Singleton, K. J. 1980. Expectations models of the term structure and implied variance bounds. *Journal of Political Economy* 88:1159–76.
- Wang, C. 2021. Under- and overreaction in yield curve expectations. Working Paper, University of Notre Dame.

# Online Appendix of

## “Which Subjective Expectations Explain Asset Prices?”

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### A. Extensions of diagnostic test

#### *A.1. Finite horizon return predictability*

Equation (7) describes the predictability of the infinite sum of returns. This infinite sum is a reasonable approximation of cumulative long-run returns. However, it is possible to obtain a bound for return predictability at finite horizons.

Equations (2) and (7) yield

$$\begin{aligned}
 Cov \left( pe_t, \sum_{j=1}^{\tau} \rho^{j-1} \tilde{r}_{t+j} \right) &= Cov \left( pe_t, \sum_{j=1}^{\tau} \rho^{j-1} E_t^* [\tilde{r}_{t+j}] \right) \\
 &+ Cov \left( pe_t, \sum_{j=1}^{\tau} \rho^{j-1} \left[ f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}} \right] \right) \\
 &+ \xi_{\tau}^* \rho^{\tau} Cov (pe_t, pe_{t+\tau})
 \end{aligned} \tag{OA.1}$$

where  $\xi_{\tau}^* = 1 - Cov (pe_t, E_t^* [pe_{t+\tau}]) / Cov (pe_t, pe_{t+\tau})$ . The final paragraph of this section gives the full derivation. In this equation,  $\xi_{\tau}^*$  summarizes errors about the persistence of

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the price-earnings ratio. At finite horizons, return predictability can be explained by time-varying discount rates, errors in finite horizon real earnings growth, or errors about the persistence.

Importantly, the distinction between RE or subjective expectations can impact the first two RHS terms of equation (OA.1), as well as  $\xi_\tau^*$ , but will not impact  $\rho^\tau Cov(pe_t, pe_{t+\tau})$  as this is simply measured from realized prices. At short horizons  $\tau$ , the third term can be potentially important as  $\rho^\tau Cov(pe_t, pe_{t+\tau})$  is nontrivial. However, as we increase the horizon  $\tau$ ,  $\rho^\tau Cov(pe_t, pe_{t+\tau})$  decreases which makes the effect of errors about the persistence less and less impactful. This imposes important restrictions for asset pricing models or empirical structural estimations. For instance, if a model suggests that ten-year return predictability differs substantially from the first two RHS terms of equation (OA.1), then it requires very large mistakes about the persistence  $\xi_{10}^*$ , as  $\rho^{10} Cov(pe_t, pe_{t+10})$  is an order of magnitude smaller than  $Cov\left(pe_t, \sum_{j=1}^{10} \rho^{j-1} \tilde{r}_{t+j}\right)$ .

Conveniently, we can bound finite-horizon return predictability. Under the mild assumption that  $Cov(pe_t, E_t^*[pe_{t+\tau}]) \geq 0$  (i.e., investors expect a positive persistence), we have

$$\begin{aligned} \alpha^{(\tau)} Cov\left(pe_t, \sum_{j=1}^{\tau} \rho^{j-1} \tilde{r}_{t+j}\right) &\leq Cov\left(pe_t, E_t^*\left[\sum_{j=1}^{\tau} \rho^{j-1} \tilde{r}_{t+j}\right]\right) \\ &+ Cov\left(pe_t, \sum_{j=1}^{\tau} \rho^{j-1} \left[f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}\right]\right) \end{aligned} \quad (\text{OA.2})$$

where  $\alpha^{(\tau)} = 1 + \rho^h \frac{Cov(pe_t, pe_{t+\tau})}{-Cov\left(pe_t, \sum_{j=1}^{\tau} \rho^{j-1} \tilde{r}_{t+j}\right)}$ .<sup>1</sup> Importantly,  $\alpha^{(\tau)}$  depends only on the empirical covariances of realized variables, which means it is unaffected by the distinction between RE and subjective expectations.

The empirical value for  $\alpha^{(\tau)}$  tells us how much our infinite horizon equation (7) needs to be adjusted to ensure it holds for finite horizons. At the ten-year horizon,  $\alpha^{(10)}$  is close to one (1.136), meaning that the adjustment is quite small. Even if discount rates are con-

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<sup>1</sup>This assumption is only made to establish the extension (OA.2). We do not require this assumption for any of our reported results. However, this assumption is empirically supported, as Delao and Myers (2021) use subjective return and cash flow expectations and find that the subjective expected persistence for the price-dividend ratio and price-earnings ratio are both positive.

stant, negative comovement between the price-earnings ratio and ten-year real cash flow forecast errors will lead the price-earnings ratio to negatively predict ten-year returns. Further, evidence that real cash flow forecast errors negatively comove with  $pe_t$  will limit the amount that discount rates can comove with  $pe_t$ . It cannot be that both RHS terms of equation (OA.2) are large and negative, as their sum is bounded by the empirically observable  $\alpha^{(\tau)} Cov\left(pe_t, \sum_{j=1}^{\tau} \rho^{j-1} r_{t+j}\right)$ .

The full derivation is the following. Equation (2) holds with and without applying expectations, which means that

$$\sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\Delta \tilde{e}_{t+j}] - \sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\tilde{r}_{t+j}] = \sum_{j=1}^{\infty} \rho^{j-1} \Delta \tilde{e}_{t+j} - \sum_{j=1}^{\infty} \rho^{j-1} \tilde{r}_{t+j}$$

which further implies

$$\begin{aligned} \sum_{j=1}^{\tau} \rho^{j-1} E_t^* [\Delta \tilde{e}_{t+j}] - \sum_{j=1}^{\tau} \rho^{j-1} E_t^* [\tilde{r}_{t+j}] + \rho^{\tau} E_t^* [pe_{t+\tau}] &= \sum_{j=1}^{\tau} \rho^{j-1} \Delta \tilde{e}_{t+j} \\ &- \sum_{j=1}^{\tau} \rho^{j-1} \tilde{r}_{t+j} + \rho^{\tau} pe_{t+\tau}. \end{aligned}$$

Rearranging gives

$$\sum_{j=1}^{\tau} \rho^{j-1} \tilde{r}_{t+j} = \sum_{j=1}^{\tau} \rho^{j-1} E_t^* [\tilde{r}_{t+j}] + \sum_{j=1}^{\tau} \rho^{j-1} f_t^{\Delta \tilde{e}_{t+j}} + \rho^{\tau} pe_{t+\tau} - \rho^{\tau} E_t^* [pe_{t+\tau}]. \quad (\text{OA.3})$$

Covarying both sides with  $pe_t$  then gives equation (OA.1).

## A.2. Other forms of return predictability

While equation (7) spells out the diagnostic test for real return predictability, we can also easily extend the analysis to study nominal return predictability and excess return predictability. For nominal returns, we have that

$$\begin{aligned} Cov\left(pe_t, \sum_{j=1}^{\infty} \rho^{j-1} r_{t+j}\right) &= Cov\left(pe_t, \sum_{j=1}^{\infty} \rho^{j-1} \pi_{t+j}\right) + Cov\left(pe_t, \sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\tilde{r}_{t+j}]\right) \\ &+ Cov\left(pe_t, \sum_{j=1}^{\infty} \rho^{j-1} \left[f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}\right]\right). \end{aligned} \quad (\text{OA.4})$$

Note that the first RHS term of equation (OA.4) is an empirical covariance that does not change whether we assume RE or allow for more general subjective expectations. Under RE, the final term of equation (OA.4) is 0 and all nominal return predictability that is not explained by empirical comovement between the price-earnings ratio and future inflation must be explained by time-variation in expected real returns. If we allow for more general subjective expectations, then negative comovement between the price-earnings ratio and forecast errors for real earnings growth will contribute to nominal return predictability.

Importantly, equation (OA.4) highlights that systematic errors in inflation expectations that satisfy the diagnostic test help to explain nominal return predictability as well as real return predictability. It is not the case that systematic errors in inflation expectations are only relevant for real return predictability. Changing the measure of returns (e.g., real returns, nominal returns, excess returns, etc.) on the LHS of equation (7) does not affect whether inflation forecast errors appear on the RHS of the equation.

The reason inflation forecast errors continue to appear is because the identity separates the relative importance of time-variation in subjective real return expectations  $E_t^*[\tilde{r}_{t+j}]$  and systematic errors. So long as we focus on subjective expectations of *real* returns, the relevant errors will be errors in *real* earnings growth, regardless if we are attempting to explain real, nominal, or excess empirical return predictability. We focus on subjective expectations of real returns because they are what is most relevant for models of investor preferences and consumption. For example, consumption-based asset pricing models relate investors' marginal utility to their required real return on risky assets.<sup>2</sup>

Similarly, we can connect forecast errors for nominal earnings growth and inflation to

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<sup>2</sup>More broadly, it is common for asset pricing models with production and/or consumption to deal only with real variables.

excess return predictability. For excess return predictability, we have that

$$\begin{aligned} Cov \left( pe_t, \sum_{j=1}^{\infty} \rho^{j-1} [r_{t+j} - y_{t+j-1}^{(1)}] \right) &= Cov \left( pe_t, - \sum_{j=1}^{\infty} \rho^{j-1} \tilde{y}_{t+j}^{(1)} \right) + Cov \left( pe_t, \sum_{j=1}^{\infty} \rho^{j-1} E_t^* [\tilde{r}_{t+j}] \right) \\ &+ Cov \left( pe_t, \sum_{j=1}^{\infty} \rho^{j-1} [f_t^{\Delta e_{t+j}} - f_t^{\pi_{t+j}}] \right). \end{aligned} \quad (OA.5)$$

Again the distinction between RE and subjective expectations only matters if forecast errors for real earnings growth negatively comove with the price-earnings ratio. Forecast errors that satisfy the diagnostic test will contribute to excess return predictability.

### A.3. Applying diagnostic test to other assets

While this paper focuses on the application of the diagnostic test to stock and bond puzzles, the intuition of this test can be applied to explain puzzles of other assets or investment strategies. For all other assets, the same necessary and sufficient condition remains: forecast errors for real cash flows must negatively comove with current prices. The key element is to identify what the real cash flows are for each of the analyzed assets. As long as the econometrician can observe subjective expectations of those real cash flows, this condition can be tested. We illustrate this using two examples: the violation of uncovered interest parity in exchange rate markets (Hodrick 1987; Engel 1996) and the excess volatility of corporate bond yields (Collin-Dufresne, Goldstein, and Martin 2001; Campbell and Taksler 2003).

**Uncovered Interest Parity** Let  $s_t$  be the nominal exchange rate, and  $\tilde{r}_{t+1}^{fx}$  be the real gross return on buying foreign currency today and selling it in the next period. The Uncovered Interest Parity condition states that on average, the future exchange rate appreciation  $\Delta s_{t+1}$  equals the current interest rate spread between the two countries ( $\delta_t$ ). Empirically, this condition is clearly violated as  $\Delta s_{t+1}$  negatively comoves with current spreads  $\delta_t$ .

The exchange rate can be expressed as

$$\begin{aligned}\tilde{r}_{t+1}^{fx} &= s_{t+1} - s_t - \pi_{t+1} \\ s_t &= E_t^* [s_{t+1} - \pi_{t+1}] - E_t^* [\tilde{r}_{t+1}^{fx}].\end{aligned}$$

If the price of a foreign currency is high, then it must be that the expected real cash flows  $E_t^* [s_{t+1} - \pi_{t+1}]$  are high or the expected real returns  $E_t^* [\tilde{r}_{t+1}^{fx}]$  are low.

If  $\delta_t$  is the difference in interest between currencies, then we can express the covariance of current spreads with future changes in exchange rates as:

$$\begin{aligned}s_{t+1} - s_t &= E_t^* [\tilde{r}_{t+1}^{fx}] + s_{t+1} - E_t^* [s_{t+1} - \pi_{t+1}] \\ s_{t+1} - s_t &= \pi_{t+1} + E_t^* [\tilde{r}_{t+1}^{fx}] + s_{t+1} - \pi_{t+1} - E_t^* [s_{t+1} - \pi_{t+1}] \\ Cov(s_{t+1} - s_t, \delta_t) &= Cov(\pi_{t+1}, \delta_t) + Cov(E_t^* [\tilde{r}_{t+1}^{fx}], \delta_t) + Cov(f_t^{s_{t+1}} - f_t^{\pi_{t+1}}, \delta_t)\end{aligned}$$

If we can observe the subjective expectations of exchange rates and inflation, then errors in expectations of exchange rates and inflation can help the violation of UIP if and only if they are predictable by interest rate spreads.<sup>3</sup> This is analogous to equation (7). Under RE, the final term is 0, and any negative comovement between  $\Delta s_{t+1}$  and  $\delta_t$  that is not explained by realized inflation must be explained by time-variation in expected real FX returns. Under more general subjective expectations, the negative comovement between  $\Delta s_{t+1}$  and  $\delta_t$  can be explained by spreads negatively comoving with forecasts errors for real cash flows in the FX market (i.e., the real value of the foreign currency).

**Corporate Bond Puzzle** The same test can be applied to corporate bond puzzles. Credit bond yields are significantly more volatile than corporate default probabilities and risk-free rates would indicate (i.e., corporate bond spreads are excessively volatile). For this asset, the real cash flow is the real repayment  $\tilde{R}ec_{t+n}$ , and, as derived below, the diagnostic condition tells us that errors in default probabilities and inflation can explain corporate yield volatility if and only if these forecast errors comove with corporate yields.

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<sup>3</sup>See Bacchetta, Mertens, and Wincoop (2009), which show results that suggest this is the case.

Let  $c_t^{(n)}$  be the zero-coupon corporate bond yield for a maturity of  $n$ , and  $Rec_{t+n}$  be the log recovery payment at maturity (i.e., payment after accounting for default). The one-year holding return at time  $t + 1$  is defined as

$$h_{t+1}^{c,(n)} = nc_t^{(n)} - (n-1)c_{t+1}^{(n-1)} \quad (\text{OA.6})$$

whereas the holding-period return for the final period is

$$h_{t+n}^{c,(n)} = c_{t+n-1}^{(n)} + Rec_{t+n}. \quad (\text{OA.7})$$

Iterating (OA.6) and applying subjective expectations until the final period (OA.7), the yield is

$$c_t^{(n)} = -\tilde{Rec}_{t+n} + \frac{1}{n} \sum_{j=1}^n \tilde{h}_{t+j}^{c,(n+1-j)}.$$

where  $\tilde{Rec}_{t+n} = Rec_{t+n} - \frac{1}{n} \sum_{j=1}^n \pi_{t+j}$  is the real recovery rate. Collin-Dufresne, Goldstein, and Martin (2001) and Campbell and Taksler (2003) document that credit bond yields are significantly more volatile than corporate default probabilities and risk-free rates would indicate. In other words,  $Cov\left(c_t^{(n)}, \sum_{j=1}^n \tilde{h}_{t+j}^{c,(n+1-j)}\right) > 0$ . Analogous to equation (6), we can express the covariance of corporate yields and subjective expectations of real cash flows as

$$\begin{aligned} Cov\left(c_t^{(n)}, E_t^* \left[ \tilde{Rec}_{t+n} \right] \right) &= Cov\left(c_t^{(n)}, \tilde{Rec}_{t+n}\right) \\ &- Cov\left(c_t^{(n)}, f^{Rec_{t+n}} - \sum_{j=1}^n f^{\pi_{t+j}}\right). \end{aligned} \quad (\text{OA.8})$$

## B. Data sources for Inflation and Nominal Earnings

### Growth Expectations

#### B.1. Inflation Expectations

Our main source for inflation expectations is the Survey of Professional Forecasters (SPF). The SPF contains quarterly median inflation forecasts of one-year inflation expectations from

1976Q1-2018Q2 and average ten-year inflation expectations from 1991Q1-2018Q2. As suggested by the technical documentation in the SPF, the average ten-year inflation expectation series is complemented backwards to 1979Q3 with the average ten-year inflation expectations from the Philadelphia Fed’s Livingston Survey and from the Blue Chip Economic Indicators Survey.

For robustness, we also analyze two alternative measures of inflation expectations. The first one is obtained from the Michigan Survey of Consumers, which has both one-year inflation expectations and five-to-ten-year average inflation expectations. We utilize the same 1976Q1-2018Q2 sample that is studied for the SPF expectations and find very similar results both qualitatively and quantitatively. The second one is a model-based measure from Haubrich, Pennacchi, and Ritchken (2012) for long-term inflation expectations. Specifically, they construct a measure of expected inflation using Treasury yields, inflation data, inflation swaps, and survey-based measures of inflation expectations. Importantly, this is a market-based measure, as some of the expectations are inferred so that the affine model perfectly matches the observed 5-year yield.

## *B.2. Nominal Earnings Growth Expectations*

We construct one-year cash flow expectations for the S&P 500 index following the aggregating procedure in Delao and Myers (2021). The Summary Statistics of the Thomson Reuters I/B/E/S database contains the median analyst forecasts for EPS (earnings per share) since 1976.<sup>4</sup> Using these individual forecasts, we measure aggregate earnings expectations using the constituents of the S&P 500 at each point in time. Expected earnings growth for the S&P 500 is then simply measured from the expectation of one-year aggregate earnings for the S&P 500 and the current earnings of the S&P 500. The online appendix of Delao and Myers (2021) shows in detail the tests of using this methodology for the I/B/E/S database.

For longer horizons, analysts forecast growth rather than the future level of earnings.

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<sup>4</sup>Using the mean forecasts does not change the results in any noticeable way.

For any set of firms, the growth of total earnings for the entire set is equal to the earnings-weighted average of individual firm earnings growth. Thus, following Nagel and Xu (2022), we measure the expected long-term earnings growth of the S&P 500 using the earnings-weighted average of the individual firm forecasts, dropping firms with negative weights. To ensure few firms are dropped, we weight by 5-year average earnings as these are rarely negative. However, we do not find any noticeable change in our results if we weight by forecasted earnings for the end of the fiscal year as in Nagel and Xu (2022).

## C. Robustness checks

### *C.1. Diagnostic test with alternative inflation expectations*

Our findings are virtually identical using short-term and long-term inflation expectations from the Michigan survey rather than the Survey of Professional Forecasters. As shown in Figure 1, short-term and long-term expectations from the SPF have similar volatilities with standard deviations of 1.8% and 1.2%, respectively. The results are virtually identical for the Michigan survey, where short-term and long-term expectations have standard deviations of 1.8% and 1.4%, respectively. Further, from equation (11), we estimate a believed persistence of  $\phi_\pi^*$  of 0.95 (0.02) which almost exactly matches our estimate of 0.96 (0.01) from the SPF expectations.

Most importantly, we find the same results for our diagnostic test using the Michigan expectations. Table OA.1 shows the results. The SPF results are shown in Table 1. As further robustness, we also consider the market-based long-term inflation expectations from Haubrich, Pennacchi, and Ritchken (2012), published regularly by the Federal Reserve of Cleveland, which gives similar results in Table OA.2.

Table OA.1

**Diagnostic test for Michigan inflation expectations**

This table shows the results of the diagnostic test which determines if differences between subjective expectations and RE explain asset price movements and return puzzles. The condition is that prices must comove with forecast errors. The first row shows the covariance of the S&P 500 price-earnings ratio with Michigan short-term inflation expectations, realized short-term inflation  $\pi_{t+1}$ , and the forecast errors from 1978Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-earnings ratio with Michigan long-term inflation expectations, realized long-term inflation  $\pi_{t+1,t+10}$ , and the forecast errors using quarterly data from 1979Q1 to 2018Q2. The fourth and fifth rows show analogous results using the ten-year Treasury yield instead of the S&P 500 price-earnings ratio. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) level.

|                          | Horizon                        | Expected | Realized | Forecast Error |
|--------------------------|--------------------------------|----------|----------|----------------|
| $Cov(pe_t, \cdot)$       | Short-term ( $\pi_{t+1}$ )     | -0.61*** | -0.74**  | -0.13          |
|                          | Long-term ( $\pi_{t+1,t+10}$ ) | -4.45**  | -1.83**  | 2.62***        |
| $Cov(y_t^{(10)}, \cdot)$ | Short-term ( $\pi_{t+1}$ )     | 0.03***  | 0.04***  | 0.01           |
|                          | Long-term ( $\pi_{t+1,t+10}$ ) | 0.28***  | 0.13***  | -0.14***       |

Table OA.2

**Diagnostic test for market-based inflation expectations**

This table shows the results of the diagnostic test which determines if differences between subjective expectations and RE explain asset price movements and return puzzles. The condition is that prices must comove with forecast errors. The first row shows the covariance of the S&P 500 price-earnings ratio with Haubrich, Pennacchi, and Ritchken (2012) one-year inflation expectations, realized short-term inflation  $\pi_{t+1}$ , and the forecast errors from 1978Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-earnings ratio with Haubrich, Pennacchi, and Ritchken 2012 cumulative ten-year inflation expectations, realized long-term inflation  $\pi_{t+1,t+10}$ , and the forecast errors using quarterly data from 1979Q1 to 2018Q2. The fourth and fifth rows show analogous results using the ten-year Treasury yield instead of the S&P 500 price-earnings ratio. Expectations expressed in percentages. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) level.

|                          | Horizon                        | Expected | Realized | Forecast Error |
|--------------------------|--------------------------------|----------|----------|----------------|
| $Cov(pe_t, \cdot)$       | Short-term ( $\pi_{t+1}$ )     | -0.28**  | -0.16*** | -0.12          |
|                          | Long-term ( $\pi_{t+1,t+10}$ ) | -3.23*** | -1.43*** | 1.80***        |
| $Cov(y_t^{(10)}, \cdot)$ | Short-term ( $\pi_{t+1}$ )     | 0.03***  | 0.02***  | -0.01***       |
|                          | Long-term ( $\pi_{t+1,t+10}$ ) | 0.24***  | 0.12***  | -0.12***       |

Table OA.3

**Diagnostic test normalized by variance of prices**

This table shows the results of the diagnostic test where all covariances are normalized by the variance of price ratios and yields. The condition is that prices must comove with forecast errors. In *Panel A*, the first row shows the covariance of the S&P 500 price-earnings ratio with short-term inflation expectations  $E_t^*[\pi_{t+1}]$ , realized short-term inflation  $\pi_{t+1}$ , and the forecast errors  $\pi_{t+1} - E_t^*[\pi_{t+1}]$  from 1976Q1 to 2018Q2. The second row shows the covariance of the S&P 500 price-earnings ratio with long-term inflation expectations  $E_t^*[\pi_{t+1,t+10}]$ , realized long-term inflation  $\pi_{t+1,t+10}$ , and the forecast errors  $\pi_{t+1,t+10} - E_t^*[\pi_{t+1,t+10}]$  using quarterly data from 1979Q4 to 2018Q2. The third and fourth rows show analogous results using the ten-year Treasury yield instead of the S&P 500 price-earnings ratio. In *Panel B*, the first row shows the covariance of the S&P 500 price-earnings ratio with short-term nominal earnings growth expectations  $E_t^*[\Delta e_{t+1}]$ , realized short-term nominal earnings growth  $\Delta e_{t+1}$ , and the forecast errors  $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$  from 1976Q1 to 2018Q2. The second row shows the covariances of the S&P 500 price-earnings ratio with long-term nominal earnings growth expectations  $E_t^*[\Delta e_{t+3,t+5}]$ , realized long-term nominal earnings growth  $\Delta e_{t+3,t+5}$  and the forecast errors  $\Delta e_{t+3,t+5} - E_t^*[\Delta e_{t+3,t+5}]$  from 1982Q1 to 2018Q2. All covariances are normalized by the variance of price ratios and yields. We use small-sample adjusted Newey-West standard errors. Superscripts indicate significance at the 1% (\*\*\*), 5% (\*\*), and 10% (\*) level.

| Panel A: Inflation                               |                                    |          |          |                |
|--------------------------------------------------|------------------------------------|----------|----------|----------------|
|                                                  | Horizon                            | Expected | Realized | Forecast Error |
| $\frac{Cov(pe_t, \cdot)}{Var(pe)}$               | Short-term ( $\pi_{t+1}$ )         | -0.03*** | -0.03*** | -0.01          |
|                                                  | Long-term ( $\pi_{t+1,t+10}$ )     | -0.21*** | -0.09*   | 0.13***        |
| $\frac{Cov(y_t^{(10)}, \cdot)}{Var(y_t^{(10)})}$ | Short-term ( $\pi_{t+1}$ )         | 0.51***  | 0.46***  | -0.05          |
|                                                  | Long-term ( $\pi_{t+1,t+10}$ )     | 5.17***  | 2.56***  | -2.60***       |
| Panel B: Nominal Earnings Growth                 |                                    |          |          |                |
| $\frac{Cov(pe_t, \cdot)}{Var(pe)}$               | Short-term ( $\Delta e_{t+1}$ )    | 0.40***  | 0.22     | -0.18***       |
|                                                  | Long-term ( $\Delta e_{t+3,t+5}$ ) | 0.02     | 0.14     | 0.12           |

*C.2. Diagnostic test normalized by prices*

Table OA.3 is an alternative way to express the diagnostic test results of Table 1. Panel A shows the covariance of forecast errors of inflation expectations with the price-earnings ratio and the ten-year yield divided by the corresponding variances of prices. Panel B shows the covariance of forecast errors of nominal earning growth expectations with the price-earnings ratio divided by the variance of the price-earnings ratio.

### *C.3. Overlapping observations and Bauer and Hamilton (2018)*

Overlapping forecast horizons can increase the persistence of forecast errors for long-term inflation. Because of this, we use Newey-West standard errors to calculate significance in Table 1 to account for any autocorrelation. For additional robustness, in this section we also directly calculate the significance under the worst-case scenario for overlapping observations. We do this following the methodology proposed by Bauer and Hamilton (2018). The general concept is that we run simulations to measure how often we spuriously find a comovement between forecast errors and current prices as large as what we observe in the data. Below, we describe the process for the price-earnings ratio. We then repeat the same process for the ten-year yield.

The price-earnings ratio is simulated as an AR(1) process, where the persistence is set equal to the observed persistence over our sample. The variance of shocks to the price-earnings ratio is set to match the observed variance of the price-earnings ratio. Additionally, the initial value of the simulated price-earnings ratio is set equal to the initial value observed in our data to account for any drift back to the mean which may generate trends in the price-earnings ratio over the sample. For example, if the price-earnings ratio is substantially below its mean at the beginning of the sample, then reversion to the mean will create an upward trend in the price-earnings ratio over time. We then simulate one-period forecast errors under the null hypothesis that forecast errors are unpredictable, i.e.  $\pi_{t+j} - E_t^*[\pi_{t+j}]$  is white noise. Forecast errors for long-term inflation are then

$$\begin{aligned} f_{t+n} &\equiv \pi_{t+1,t+n} - E_t^*[\pi_{t+1,t+n}] \\ &= \sum_{j=1}^n (\pi_{t+j} - E_t^*[\pi_{t+j}]). \end{aligned}$$

If subjective expectations change over time, then there will be little overlap in the long-term forecast errors. For example, if  $E_t^*[\pi_{t+2}]$  is very different from  $E_{t+1}^*[\pi_{t+2}]$ , then there is little similarity between the second term of  $f_{t+n}$ , which is  $\pi_{t+2} - E_t^*[\pi_{t+2}]$ , and the first term of  $f_{t+1+n}$ , which is  $\pi_{t+2} - E_{t+1}^*[\pi_{t+2}]$ . However, in the worst-case scenario where expectations do

not change, then there is perfect overlap. Formally, when  $\pi_{t+j} - E_t^*[\pi_{t+j}] = \pi_{t+j} - E_{t+k}^*[\pi_{t+j}]$  for all  $k < j$ , then  $f_{t+n}$  is an  $\text{MA}(n-1)$  process. This causes  $f_{t+n}$  to be persistent which increases the probability of spuriously finding a large comovement between the price-earnings ratio and  $f_{t+n}$ .

For our analysis, we consider the worst-case scenario, where there is perfect overlap and where each period is only a quarter rather than a year. This second assumption means that  $f_{t+n}$  is  $\text{MA}(39)$  rather than  $\text{MA}(9)$  for ten-year inflation, which substantially raises the persistence. We set the variance of forecast errors to match the variance observed in the data. We run 100,000 simulations to estimate the probability of spuriously observing a covariance between the price-earnings ratio and long-term forecast errors as large as what we observe in the data.

For the price-earnings ratio, we estimate a p-value of 0.015. Repeating the process for the ten-year yield gives a p-value of 0.014. As a comparison, the p-values for the price-earnings ratio in our benchmark specification using HAC errors is  $2.9 \times 10^{-5}$  and the p-value for the ten-year yield is  $5 \times 10^{-6}$ . This demonstrates that accounting for small-samples, trends, and the worst-case scenario for overlapping observations does appreciably increase the p-values relative to using HAC errors. However, even after incorporating all of these elements, the likelihood of spuriously finding comovements as large as what we observe in the data is quite low.

#### *C.4. Embedding stock return expectations*

Previous papers (Delao and Myers 2021; Adam, Matveev, and Nagel 2021) have explicitly tested and rejected that forecasters use risk-neutral probabilities.<sup>5</sup> However, there still may be concern that forecasters are embedding return expectations into their nominal earnings growth forecasts. For example, suppose that nominal earnings growth forecasters have ratio-

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<sup>5</sup>The central feature of risk-neutral probabilities is that they overweight bad outcomes, which means forecasts for earnings growth and returns using risk-neutral probability should be consistently below the realized values. Delao and Myers 2021 and Adam, Matveev, and Nagel, 2021 test and reject this prediction.

nal expectations but, instead of reporting their true nominal earnings growth expectations, they include their return expectations in their reported forecasts for nominal earnings growth. Rational expectations of returns are significantly correlated with current price ratios (e.g., price-earnings ratio, price-dividend ratio, and CAPE), so this would cause forecast errors to be correlated with current price ratios.

Fortunately, the results of our diagnostic test do not support this type of story. First, if forecasters are embedding return expectations into their forecasts of nominal earnings growth, then both short-term and long-term nominal earnings growth forecasts should pass the diagnostic test. In other words, forecast errors for all horizons should be correlated with current price ratios. Instead, Table 3 shows that only short-term nominal earnings growth forecasts satisfy the diagnostic test. We find that forecast errors for long-term nominal earnings growth are uncorrelated with all three price ratios.

Second, this story would imply that the comovement of forecast errors with current price ratios should be larger in magnitude for long-term nominal earnings growth expectations than for short-term nominal earnings growth expectations. This is because current price ratios predict long-term returns more strongly than they predict short-term returns.<sup>6</sup> Once again, this is inconsistent with the empirical finding from Table 3 that the comovement of price ratios with short-term nominal earnings growth forecast errors is large in magnitude and significant, while the comovement of price ratios with long-term nominal earnings growth forecast errors is smaller in magnitude, insignificant, and the wrong sign for explaining price movements.

These results highlight the benefit of applying the diagnostic test to multiple horizons. The patterns in where the relevant errors do and do not arise are powerful for ruling out or supporting potential explanations for these errors.

As a further note, while we find significant deviations from RE in analysts' forecasts of short-term nominal earnings growth, it has been well-documented that analysts exert sig-

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<sup>6</sup>Empirically, the comovement of price ratios with negative cumulative 3-5 year returns is significantly larger than the comovement of price ratios with negative one-year returns.

nificant effort and have strong incentives to provide accurate forecasts. This points against stories where analysts would knowingly bias their forecasts. Specifically, accuracy in earnings forecasts is important for tenure, compensation, and career outcomes,<sup>7</sup> financial firms' trades are consistent with their own analysts' forecasts (Bradshaw 2004; Chan, Chang, and Wang 2009), analysts' forecasts provide valuable information for forecasting earnings even relative to a machine-learning algorithm using an extensive set of publicly available variables (van Binsbergen, Han, and Lopez-Lira 2022), and analysts exert significant effort collecting information, e.g., asking questions during earnings calls, having meetings with firm management and institutional investors, etc. (Grennan and Michaely 2020). Surveying the literature, Kothari, So, and Verdi (2016) conclude “these studies suggest that analysts are sufficiently concerned with their reputation as credible information intermediaries to be motivated to issue unbiased, accurate forecasts.”

### C.5. Generalized persistence structure

We calculate a more generalized version of equations (11) and (12) that allows inflation expectations to impact nominal earnings growth expectations,

$$\begin{pmatrix} E_t^* [\pi_{t+1+j}] \\ E_t^* [\Delta e_{t+1+j}] \end{pmatrix} = \begin{pmatrix} \alpha_{\pi,j} \\ \alpha_{e,j} \end{pmatrix} + \begin{pmatrix} \phi_{\pi} & 0 \\ \phi_{\pi,e} & \phi_e \end{pmatrix}^j \begin{pmatrix} E_t^* [\pi_{t+1}] \\ E_t^* [\Delta e_{t+1}] \end{pmatrix} + \begin{pmatrix} \varepsilon_{t,j}^{\pi} \\ \varepsilon_{t,j}^e \end{pmatrix} \quad (\text{OA.1})$$

Similar to how equation (11) nests investors believing inflation is AR(1) and investors believing inflation is ARMA(1,1), equation (OA.1) nests investors believing inflation and earnings growth are VAR(1) or VARMA(1,1). This generalization has no impact on our estimate of  $\phi_{\pi}$  of 0.96 (0.01). Using short-term and long-term inflation expectations and nominal earnings growth expectations, we estimate  $\phi_e$  as 0.001 (0.088) which is almost identical to our estimate from equation (12) of 0.004 (0.075). In other words, including this interaction does not impact our estimate of  $\phi_e$ .

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<sup>7</sup>Mikhail, Walther, and Willis (1999); Hong, Kubik, and Solomon (2000); Cooper, Day, and Lewis (2001); Groyberg, Healy, and Maber (2011)

Importantly, when we estimate the interaction term  $\phi_{\pi,e}$ , we find a small and insignificant value of 0.22 (0.28). As discussed in Section 4.1, the large decline in inflation expectations did not lead to a noticeable decline in nominal earnings growth expectations. If we include this interaction term, the fundamental price-earnings ratio would be

$$pe_t^{fun} = \frac{1}{1 - \rho\phi_e} E_t^* [\Delta e_{t+1}] - \frac{1}{1 - \rho\phi_\pi} \left( 1 - \frac{\rho\phi_{\pi,e}}{1 - \rho\phi_e} \right) E_t^* [\pi_{t+1}]. \quad (\text{OA.2})$$

Note that this is identical to equation (15), except for the  $\left( 1 - \frac{\rho\phi_{\pi,e}}{1 - \rho\phi_e} \right)$  scaling on inflation expectations. Given that  $\phi_{\pi,e}$  and  $\phi_e$  are both near 0 and insignificant, this scaling term is close to 1 at 0.79 and is not statistically significantly different from 1.

Because the scaling term is near 1, this interaction term has almost no impact on the asset pricing results and by extension the return predictability results. Regressing the observed price-earnings ratio on the fundamental price-earnings ratio from equation (OA.2) produces a coefficient of almost exactly 1 at 1.07 (0.11) with an  $R^2$  of 0.81. This is virtually identical to the results of Table 4 where the regression coefficient is 0.96 (0.09) and the  $R^2$  is 0.81.

Regressing future 10-year nominal returns on the fundamental price-earnings ratio from (OA.2) produces a coefficient of  $-0.85$  (0.07) with an  $R^2$  of 0.39. Similarly, regressing 10-year real returns on the fundamental price-earnings ratio from (OA.2) produces a coefficient of  $-0.59$  (0.12) with an  $R^2$  of 0.24. These are almost identical to the results in Table 5.

### C.6. Value-weighted expectations

For robustness, we repeat our analysis using the value-weighted long-term nominal growth expectations of Bordalo et al. (2023). For any set of firms, such as the S&P 500, the growth of total earnings is equal to the earnings-weighted average of individual firm earnings growth. Therefore, we use the earnings-weighted average for our main analysis. However, we find that the value-weighted average of long-term nominal earnings growth expectations from Bordalo et al. (2023) do not change our results in any noticeable way. Specifically, we continue to find that long-term nominal earnings growth expectations (i) have low volatility compared

Table OA.4

**Value-weighted long-term nominal earnings growth expectations**

This table compares the earnings-weighted long-term nominal earnings growth expectation to the value-weighted long-term nominal earnings growth expectation. The first row compares the standard deviation of the series. The second row reports the believed persistence estimated from equation (12) using the short-term and long-term nominal earnings growth expectations. The third, fourth, and fifth rows show the covariance of long-term nominal earnings growth expectations  $E_t^* [\Delta e_{t+3,t+5}]$  with the S&P 500 price-earnings ratio, the price-dividend ratio, and CAPE. The sixth, seventh, and eighth rows show the covariance of forecast errors  $\Delta e_{t+3,t+5} - E_t^* [\Delta e_{t+3,t+5}]$  with the price-earnings ratio, price-dividend ratio, and CAPE. All estimates are calculated using a sample of 1982Q1 to 2018Q2. Small-sample adjusted Newey-West standard errors reported in parenthesis.

|                                                | Earnings-weighted | Value-weighted   |
|------------------------------------------------|-------------------|------------------|
| Standard deviation                             | 1.2%              | 1.4%             |
| Believed persistence $\phi_e^*$                | 0.004<br>(0.075)  | 0.008<br>(0.049) |
| Comovement with $pe_t$                         | 0.36<br>(0.40)    | 0.55<br>(0.48)   |
| Comovement with $pd_t$                         | 0.59<br>(0.42)    | 0.90<br>(0.57)   |
| Comovement with $pe_t^{CA}$                    | 0.56<br>(0.47)    | 0.88<br>(0.65)   |
| Comovement of forecast errors with $pe_t$      | 2.35<br>(1.65)    | 2.16<br>(1.57)   |
| Comovement of forecast errors with $pd_t$      | 0.25<br>(2.33)    | -0.06<br>(2.33)  |
| Comovement of forecast errors with $pe_t^{CA}$ | 1.39<br>(1.96)    | 1.06<br>(1.90)   |

to short-term nominal earnings growth expectations, (ii) imply the believed persistence of nominal earnings growth is small, and most importantly (iii) fail the diagnostic test.

Table OA.4 shows the results. The standard deviations for the earnings-weighted and value-weighted expectations closely align at 1.2% and 1.4%, respectively, and are both substantially lower than the 26.4% standard deviation of short-term nominal earnings growth expectations. As a consequence, the believed persistence is nearly 0 as shown in the second row. The third row shows that the comovement of value-weighted expectations with the

price-earnings ratio is small and insignificant at 0.55 (0.48), matching our findings using the earnings-weighted expectations of 0.36 (0.40). The fourth and fifth rows show that the comovement of long-term nominal earnings growth expectations with other price ratios are similarly small.

Crucially, the final three rows of Table OA.4 show that the forecast errors for both the value-weighted expectations and the earnings-weighted expectations have no significant comovement with the price-earnings ratio, the price-dividend ratio, or CAPE. Further, the comovements of 2.16,  $-0.06$ , and  $1.06$  for the value-weighted expectations are either of the wrong sign or far too small to explain price ratio movements and return predictability compared to the comovement of short-term nominal earnings growth forecast errors with the three price ratios of  $-4.26^{***}$ ,  $-3.43^{***}$ ,  $-3.65^{***}$  from Table 3.

## D. Responding to perceived surprises

This section provides intuition for why smoothed and even lagged price ratios could predict forecast errors for one-year earnings growth, even if systematic errors in  $E_t^*[\Delta e_{t+1}]$  only depend on the most recent surprise  $\Delta e_t - E_{t-1}^*[\Delta e_t]$ .<sup>8</sup> At first glance, this may seem surprising. If the systematic error in expectations only depends on the current surprise, then one might conjecture that alternative price ratios that are unaffected by recent changes in earnings will not be correlated with the systematic errors (i.e., these price ratios will not predict forecast errors for one-year earnings growth).<sup>9</sup> A key focus of this section is the distinction between agents responding to the perceived surprise  $\Delta e_t - E_{t-1}^*[\Delta e_t]$  and agents responding to the objective shock  $\Delta e_t - E_t[\Delta e_t]$ .

To make the setting as transparent as possible and to allow for straightforward closed-

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<sup>8</sup>For brevity, this section condenses “nominal earnings growth” to “earnings growth” given that the focus is entirely on nominal earnings growth expectations.

<sup>9</sup>Section 5.1 uses the price-dividend ratio and CAPE as measures of price ratios that are less affected by recent changes in earnings than the price-earnings ratio. In this section, we use smoothed and lagged versions of the price-earnings ratio as our measures of price ratios that are less affected by recent changes in earnings, as this allows for tractable closed-form solutions.

form solutions, we assume that earnings growth is objectively i.i.d. and agents price stocks based on expected one-year earnings growth,  $pe_t = E_t^* [\Delta e_{t+1}]$ . In the two subsections below, we focus on two expectation-formation processes for  $E_t^* [\Delta e_{t+1}]$ , one in which agents respond to the *objective shock* and one in which they respond to the *perceived surprise*. Note that the case of responding to the perceived surprise is simply a special case of the framework in Section 7 in which  $\phi_e = \phi_e^* = \beta_\pi = \phi_\pi^* = 0$ .

### D.1. Responding to the objective shock

Agents have expectations

$$E_t^* [\Delta e_{t+1}] = \gamma_e (E_{t-1} [\Delta e_t] - \Delta e_t) \quad (\text{OA.1})$$

and the price-earnings ratio is  $pe_t = E_t^* [\Delta e_{t+1}]$ . We create price ratios that are less sensitive to recent changes in earnings by considering the smoothed moving average

$$pe_t^{(n)} \equiv \frac{1}{n} \sum_{j=0}^{n-1} pe_{t-j} \quad (\text{OA.2})$$

and also the lag  $pe_{t-1}^{(n)}$ .

For any price ratio  $pe_t^{(n)}$ , the forecast error  $f_{t+1} \equiv \Delta e_{t+1} - E_t^* [\Delta e_{t+1}]$  can always be expressed as

$$f_{t+1} = -b^{(n)} pe_t^{(n)} + \varepsilon_{t+1}^{(n)} \quad (\text{OA.3})$$

where  $\varepsilon_{t+1}^{(n)}$  is orthogonal to  $pe_t^{(n)}$ . Given equations (OA.1)-(OA.2), the coefficient  $b^{(n)}$  is simply

$$b^{(n)} = \frac{1}{n}. \quad (\text{OA.4})$$

In other words, while the current price-earnings ratio  $pe_t \equiv pe_t^{(1)}$  will have a large negative relationship with future forecast errors, the magnitude of this relationship will shrink quickly to 0 as we smooth the price ratio (increasing  $n$ ). This is intuitive. In this model, the systematic error in expectations,  $E_t^* [\Delta e_{t+1}] - E_t [\Delta e_{t+1}] = -\gamma_e \Delta e_t$ , only depends on the

most recent earnings growth. Because of this, the relationship between price ratios and forecast errors only depends on the sensitivity of the price ratio to the most recent shock, which is large for  $pe_t$  but small for smoothed ratios such as  $pe_t^{(10)}$ .

Even more starkly, the lagged price ratio will have no relationship with future forecast errors. Specifically, we have

$$f_{t+1} = -b_{lag}^{(n)} pe_{t-1}^{(n)} + \varepsilon_{lag,t+1}^{(n)} \quad (\text{OA.5})$$

where  $\varepsilon_{lag,t+1}^{(n)}$  is orthogonal to  $pe_{t-1}^{(n)}$  and the coefficient is

$$b_{lag}^{(n)} = 0. \quad (\text{OA.6})$$

The objective shock  $E_{t-1}[\Delta e_t] - \Delta e_t$  is completely unpredictable at time  $t - 1$ . Thus, if the bias in expectations is due to the response to the objective shock, then the bias in expectations is unrelated to any variables before time  $t$ .

## D.2. Responding to the perceived surprise

Compared to the previous subsection, we only make one change. We assume that agents respond to the perceived surprise  $\Delta e_t - E_{t-1}^*[\Delta e_t]$  rather than the objective shock. Intuitively, if agents have biased expectations, then they should not know the objective shock  $\Delta e_t - E_{t-1}[\Delta e_t]$ , given that they do not know the objective expectation  $E_{t-1}[\Delta e_t]$ . Instead, they respond to their perception of whether earnings growth was higher or lower than expected,

$$E_t^*[\Delta e_{t+1}] = \beta_e (E_{t-1}^*[\Delta e_t] - \Delta e_t). \quad (\text{OA.7})$$

This is simply a special case of the framework in Section 7.

Given that  $\Delta e_t$  is objectively i.i.d., equation (OA.7) shows that one-year earnings growth expectations ( $E_t^*[\Delta e_{t+1}]$ ) are objectively AR(1) with persistence  $\beta_e$ . By extension, the systematic error in expectations  $E_t^*[\Delta e_{t+1}] - E_t[\Delta e_{t+1}]$  is also objectively AR(1) with persistence  $\beta_e$ . This is the key difference between agents responding to the objective shock and agents responding to the perceived surprise. In the previous subsection D.1, the systematic

error in expectations had zero persistence, which meant it had only a small relationship with smooth price ratios and no relationship with lagged price ratios. When agents respond to the perceived surprise, the systematic error is persistent and it can be strongly related with both smoothed and lagged price ratios. Biased expectations  $E_{t-1}^*[\Delta e_t]$  will cause a perceived surprise  $(E_{t-1}^*[\Delta e_t] - \Delta e_t)$ , even if there was no objective shock to earnings growth  $(E_{t-1}[\Delta e_t] - \Delta e_t = 0)$ . Thus, even in the absence of any objective shock, previous biases in  $E_{t-1}^*[\Delta e_t]$  will bias  $E_t^*[\Delta e_{t+1}]$ .

As in subsection D.1, the price-earnings ratio is  $pe_t = E_t^*[\Delta e_{t+1}]$  and we create price ratios that are less sensitive to recent changes in earnings by considering the smoothed moving average  $pe_t^{(n)}$  and the lag  $pe_{t-1}^{(n)}$ ,

$$pe_t^{(n)} \equiv \frac{1}{n} \sum_{j=0}^{n-1} pe_{t-j} \quad (\text{OA.8})$$

The forecast error  $f_{t+1} \equiv \Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$  can always be expressed as

$$f_{t+1} = -b^{(n)} pe_t^{(n)} + \varepsilon_{t+1}^{(n)} \quad (\text{OA.9})$$

where  $\varepsilon_{t+1}^{(n)}$  is orthogonal to  $pe_t^{(n)}$ . Given equations (OA.7)-(OA.8), the coefficient  $b^{(n)}$  is

$$\begin{aligned} b^{(n)} &= \frac{\frac{1}{n} \sum_{j=0}^{n-1} \beta_e^j}{\frac{1}{n^2} \sum_{j=0}^{n-1} \sum_{i=0}^{n-1} \beta_e^{|j-i|}} \\ &= \left[ \frac{1 + \beta_e}{1 - \beta_e^n} - \frac{1}{n} \frac{2\beta_e}{1 - \beta_e} \right]^{-1}. \end{aligned} \quad (\text{OA.10})$$

The predictability of forecast errors  $b^{(n)}$  now depends on the persistence  $\beta_e$ . The dotted green line of Figure OA.1 shows the value of  $b^{(n)}$  for  $n = 1, \dots, 10$  using the estimated  $\beta_e = 0.74$  from Section 3.3 and Section 7. For comparison, the solid blue line shows the value of  $b^{(n)}$  when agents respond to the objective shock. While the value of  $b^{(n)}$  does decline as  $n$  increases, the decline is much smaller when agents respond to the perceived surprise than when they respond to the objective shock. In fact, at  $n = 10$ , the coefficient is still 0.79 compared to the coefficient for the unsmoothed ratio of  $b^{(1)} = 1$ .

We can also consider the predictability of forecast errors with lagged price ratios. We

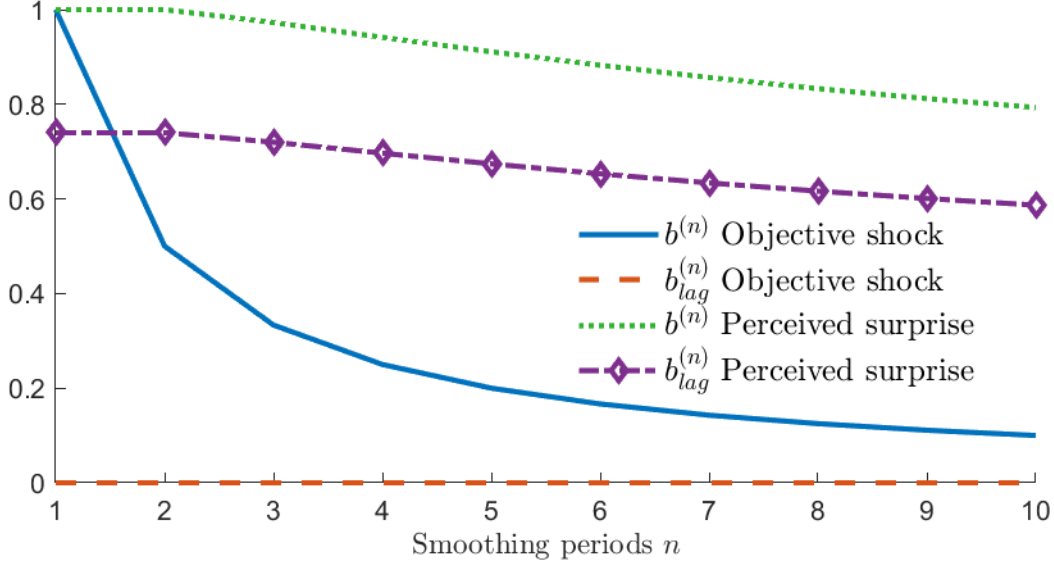


Figure OA.1

**Forecast error predictability.**

This figure shows the magnitude of the relationship between one-year earnings growth forecast errors and smoothed price ratios from equations (OA.4), (OA.6), (OA.10), and (OA.12). For comparison, the results are shown when agents response to the objective shock  $\Delta e_t - E_{t-1}[\Delta e_t]$  and when agents respond to the perceived surprise  $\Delta e_t - E_{t-1}^*[\Delta e_t]$ . The x-axis shows the number of periods used in smoothing the price ratio  $pe_t^{(n)} \equiv \frac{1}{n} \sum_{j=0}^{n-1} pe_{t-j}$ . The parameter  $\beta_e$  is set to 0.74.

have

$$f_{t+1} = -b_{lag}^{(n)} pe_{t-1}^{(n)} + \varepsilon_{lag,t+1}^{(n)} \quad (\text{OA.11})$$

where  $\varepsilon_{lag,t+1}^{(n)}$  is orthogonal to  $pe_{t-1}^{(n)}$  and the coefficient is

$$b_{lag}^{(n)} = \beta_e b^{(n)}. \quad (\text{OA.12})$$

Unlike in subsection D.1, lagging the price ratio does not eliminate the ability to predict forecast errors. The purple diamond line of Figure OA.1 shows the values for  $b_{lag}^{(n)}$  using  $\beta_e = 0.74$ . This contrasts sharply with the dashed red line, which shows  $b_{lag}^{(n)}$  when agents respond to the objective shock.

To connect this to the empirical results, univariate regressions of one-year earnings growth forecast errors  $\Delta e_{t+1} - E_t^*[\Delta e_{t+1}]$  on price ratios shows that the regression coefficient decreases in magnitude by a factor of 0.87 when the forecast errors are regressed on the price-dividend ratio instead of the price-earnings ratio. Similarly, the regression coefficient

decreases in magnitude by a factor of 0.94 when regressed on CAPE instead of the price-earnings ratio. This factor of 0.87 to 0.94 roughly corresponds to the range of  $b^{(4)}$  to  $b^{(7)}$  shown in Figure OA.1 when agents respond to perceived surprises.

## E. Connection to other models of expectation formation

### E.1. Believed persistence and response to surprises

To reduce repetition, let  $x$  be a general variable representing both inflation and nominal earnings growth. First, we show that under diagnostic expectations and sticky expectations, the believed persistence would need to equal the objective persistence. As shown in Bordalo, Gennaioli, and Shleifer (2018), for an objectively AR(1) variable, diagnostic expectations follow

$$\begin{aligned} E_t^* [x_{t+j}] &= E_t [x_{t+j}] + \theta (E_t [x_{t+j}] - E_{t-1} [x_{t+j}]) \\ &= \phi_x^j [x_t + \theta (x_t - \phi_x x_{t-1})] \end{aligned} \tag{OA.1}$$

where  $\theta$  is the parameter controlling the degree of representativeness bias. From equation (OA.1), the relationship between short-term and long-term expectations is

$$E_t^* [x_{t+1+j}] = \phi_x^j E_t^* [x_{t+1}]. \tag{OA.2}$$

In other words, the believed persistence equals the objective persistence.

Under stickiness, subjective expectations are

$$\begin{aligned} E_t^* [x_{t+j}] &= \lambda E_{t-1}^* [x_{t+j}] + (1 - \lambda) E_t [x_{t+j}] \\ &= (1 - \lambda) \sum_{k=0}^{\infty} \lambda^k E_{t-k} [x_{t+j}] \\ &= \phi_x^j (1 - \lambda) \sum_{k=0}^{\infty} (\lambda \phi_x)^k x_{t-k} \end{aligned} \tag{OA.3}$$

where  $1 - \lambda$  represents the probability of updating information. From equation (OA.3), the

relationship between long-term and short-term expectations is

$$E_t^* [x_{t+1+j}] = \phi_x^j E_t^* [x_{t+1}]. \quad (\text{OA.4})$$

Once again, the believed persistence is equal to the objective persistence.

Second, we show that a model that only features bias in the persistence of variables would not allow short-term nominal earnings growth expectations to depend on recent surprises. Consider a model where agents believe the persistence of the variable is  $\phi_x^*$  rather than  $\phi_x$ . In this model, subjective expectations would be

$$\begin{aligned} E_t^* [x_{t+1}] &= \phi_x^* x_t \\ E_t^* [x_{t+1+j}] &= \phi_x^{*j} E_t^* [x_{t+1}]. \end{aligned}$$

By design, this model would allow the believed persistence of inflation to match the values we estimate from the survey data  $(\phi_\pi^*, \phi_\epsilon^*)$  rather than requiring that the believed persistence matches the objective persistence. For nominal earnings growth, survey short-term and long-term expectations imply a believed persistence of nearly 0,  $\phi_\epsilon^* = 0.004$ . In this model, this low believed persistence would imply that short-term expectations are virtually constant. Thus, this model would not match the large response of survey short-term nominal earnings growth expectations to surprises.

## *E.2. Fundamental extrapolation nests sticky and adaptive expectations*

Sticky expectations and adaptive expectations are both special cases of the fundamental extrapolation model where  $\phi_x^*$  equals  $\phi_x, 1$  respectively. Under stickiness, short-term expectations are

$$\begin{aligned} E_t^* [x_{t+1}] &= \lambda E_{t-1}^* [x_{t+1}] + (1 - \lambda) E_t [x_{t+1}] \\ &= \lambda E_{t-1}^* [x_{t+1}] + (1 - \lambda) \phi_x x_t. \end{aligned} \quad (\text{OA.5})$$

As shown in subsection E.1, long-term expectations are

$$E_t^* [x_{t+1+j}] = \phi_x^j E_t^* [x_{t+1}]. \quad (\text{OA.6})$$

Combining equations (OA.5) and (OA.6), gives

$$E_t^* [x_{t+1}] = \phi_x x_t + \lambda \phi_x (E_{t-1}^* [x_t] - x_t). \quad (\text{OA.7})$$

The description of short-term and long-term expectations given by equations (OA.6)-(OA.7) is identical to the fundamental extrapolation model equations (24)-(25) when  $\phi_x^* = \phi_x$  and  $\beta = \lambda \phi_x$ . Intuitively, rational expectations at time  $t$  only depend on the realization  $x_t$ . Sticky expectations are a weighted sum of current and past rational expectations, so sticky expectations depend on a weighted sum of current and past realizations. This can also be seen directly in equation (OA.3).

Under adaptive expectations, subjective expectations are

$$E_t^* [x_{t+j}] = E_{t-1}^* [x_{t+j}] + \gamma (x_t - E_{t-1}^* [x_t]) \quad (\text{OA.8})$$

where  $\gamma$  is the constant gain coefficient. This is equivalent to

$$E_t^* [x_{t+1}] = x_t + (1 - \gamma) (E_{t-1}^* [x_t] - x_t) \quad (\text{OA.9})$$

$$E_t^* [x_{t+j}] = E_t^* [x_{t+1}]. \quad (\text{OA.10})$$

Equations (OA.9)-(OA.10) are identical to the fundamental extrapolation model equations (24)-(25) when  $\phi_x^* = 1$  and  $\beta = 1 - \gamma$ . Expectations that adapt quickly, i.e. high  $\gamma$ , translate to a low weight  $\beta$  for fundamental extrapolation as expectations mainly depend on recent realizations rather than older realizations.

## References

- Adam, K., D. Matveev, and S. Nagel. 2021. Do survey expectations of stock returns reflect risk adjustments? *Journal of Monetary Economics* 117:723–40.
- Bacchetta, P., E. Mertens, and E. V. Wincoop. 2009. Predictability in financial markets: What do survey expectations tell us? *Journal of International Money and Finance* 28:406–26.
- Bauer, M. D., and J. D. Hamilton. 2018. Robust bond risk premia. *Review of Financial Studies* 31:399–448.
- Bordalo, P., N. Gennaioli, R. La Porta, and A. Shleifer. 2023. Belief overreaction and stock market puzzles. *Journal of Political Economy*. Advance Access published August 8, 2023, 10.1086/727713.
- Bordalo, P., N. Gennaioli, and A. Shleifer. 2018. Diagnostic expectations and credit cycles. *Journal of Finance* 73:199–227.
- Bradshaw, M. T. 2004. How do analysts use their earnings forecasts in generating stock recommendations? *The Accounting Review* 79:25–50.
- Campbell, J. Y., and G. B. Taksler. 2003. Equity volatility and corporate bond yields. *The Journal of Finance* 58:2321–50.
- Chan, K. W., C. Chang, and A. Wang. 2009. Put your money where your mouth is: Do financial firms follow their own recommendations? *Quarterly Review of Economics and Finance* 49:1095–112.
- Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin. 2001. The determinants of credit spread changes. *Journal of Finance* 56:2177–207.
- Cooper, R. A., T. E. Day, and C. M. Lewis. 2001. Following the leader: A study of individual analysts' earnings forecasts. *Journal of Financial Economics* 61:383–416.

- Delao, R., and S. Myers. 2021. Subjective cash flow and discount rate expectations. *Journal of Finance* 76:1339–87.
- Engel, C. 1996. The forward discount anomaly and the risk premium: A survey of recent evidence. *Journal of Empirical Finance* 3:123–92.
- Grennan, J., and R. Michaely. 2020. Artificial intelligence and high-skilled work: Evidence from analysts. *Working Paper*, University of California Haas School of Business.
- Groysberg, B., P. M. Healy, and D. A. Maber. 2011. What drives sell-side analyst compensation at high-status investment banks? *Journal of Accounting Research* 49:969–1000.
- Haubrich, J., G. Pennacchi, and P. Ritchken. 2012. Inflation expectations, real rates, and risk premia: Evidence from inflation swaps. *Review of Financial Studies* 25:1588–629.
- Hodrick, R. J. 1987. *The empirical evidence on the efficiency of forward and futures foreign exchange markets*. Harwood Academic Publishers.
- Hong, H., J. D. Kubik, and A. Solomon. 2000. Security analysts' career concerns and herding of earnings forecasts. *The RAND Journal of Economics* 31:121–44.
- Kothari, S. P., E. So, and R. Verdi. 2016. Analysts' forecasts and asset pricing: A survey. *Annual Review of Financial Economics* 8:197–219.
- Mikhail, M. B., B. R. Walther, and R. H. Willis. 1999. Does forecast accuracy matter to analysts? *The Accounting Review* 74:185–200.
- Nagel, S., and Z. Xu. 2022. Asset pricing with fading memory. *Review of Financial Studies* 35:2190–245.
- van Binsbergen, J. H., X. Han, and A. Lopez-Lira. 2022. Man versus machine learning: The term structure of earnings expectations and conditional biases. *The Review of Financial Studies* 36:2361–96.