Safety Transformation and the Structure of the Financial System

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Abstract

This paper studies how a financial system that is organized to efficiently create safe assets responds to macroeconomic shocks. Financial intermediaries face a cost of bearing risk, so they choose the least risky portfolio that backs their issuance of riskless deposits: a diversified pool of nonfinancial firms' debt. Nonfinancial firms choose their capital structure to exploit the resulting segmentation between debt and equity markets. Increased safe asset demand yields larger and riskier intermediaries and more levered firms. Quantitative easing reduces the size and riskiness of intermediaries and can decrease firm leverage, despite reducing borrowing costs at the zero lower bound.

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An important role of financial intermediaries is to issue safe, money-like assets, such as bank deposits and money market fund shares. As an empirical literature has documented (Krishnamurthy and Vissing-Jorgensen (2012), Sunderam (2015), Nagel (2016)), these assets have a low rate of return that is strictly below the risk-free rate they would earn without providing monetary services. Agents who can issue these assets therefore raise financing on attractive terms, capturing the "demand for safe assets" that pushes their cost of borrowing below that of others. This paper presents a general equilibrium model of how the financial system is organized to meet this demand for safe assets and examines how the system as a whole adjusts when the supply or demand for safe assets changes.

In the model, financial intermediaries create safe assets by issuing riskless debt (deposits) backed by a diversified portfolio of risky assets. Financial intermediaries face an agency cost of bearing risk, so they hold the lowest risk portfolio that backs their issuance of riskless debt. The size of the financial sector is determined by the trade-off between the benefit of safe asset creation and this agency cost of bearing risk. This trade-off determines both the composition of intermediary balance sheets and the leverage of the nonfinancial sector. The lowest risk portfolio of any size containing only risky assets consists of all risky debt issued by nonfinancial firms, with the nonfinancial firms' leverage chosen optimally. Risky debt securities are held by intermediaries while riskless assets and equities are held by households. These asset holdings are broadly consistent with the balance sheets of U.S. commercial banks and households presented in Figure 1.[1]

The model provides a framework for analyzing the general equilibrium effects of changes in the supply and demand for safe assets. If the demand for safe assets increases, financial intermediaries increase the quantity of safe assets that they supply to clear the market. To back their increased supply of safe assets, intermediaries therefore buy more debt securities,

[1]Household portfolio holdings are based on the assumption that their mutual funds are 70% equity and 30% debt, consistent with data from the Investment Company Institute's Investment Company Fact Book. Bank assets omits life insurance reserves, foreign direct investment, and miscellaneous assets.

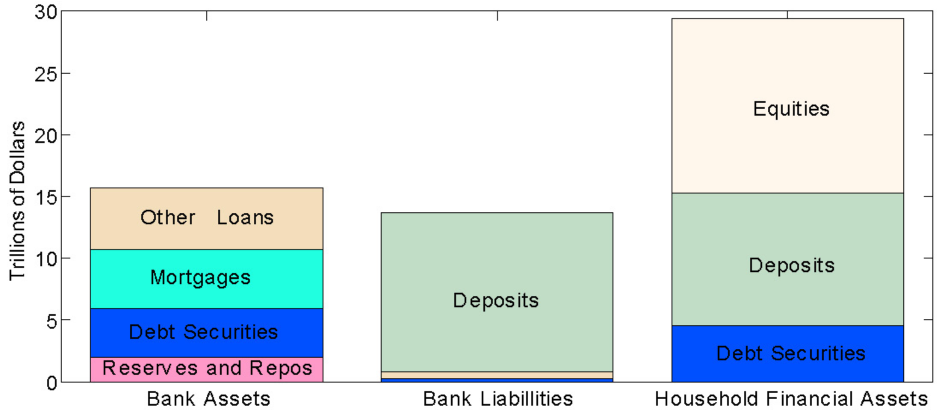

Figure 1. **Household and bank balance sheets (2015 Financial Accounts of the United States).**

which in turn induces the nonfinancial sector to increase its leverage. Since 2002, Caballero and Farhi (2017) document an increased spread between the risk-free rate and their measure of the expected return on risky assets, caused in part by growing foreign demand for U.S. safe assets (Bernanke et al. (2011)). As the model predicts, this growing demand coincided with a large credit boom in the 2000s. Mortgage payments grew from 5.87% of disposable income in 2003 to a peak of 7.21% at the end of 2007, [2] accompanied by growth in the size and riskiness of the financial sector (Bhattacharyya and Purnanandam (2011)). The model shows how a scarcity of safe assets may have fueled the risky lending that occurred in this period, a possible contributor to the financial crisis of 2008.

To stimulate recovery after the crisis, the U.S. Federal Reserve began a quantitative easing (QE) program that increased the supply of safe assets by providing riskless bank reserves to financial institutions. In the model, this supply of reserves crowds out the need for intermediaries to make risky loans to the nonfinancial sector and reduces the cost for intermediaries to issue riskless debt. This increases the supply of safe assets, decreases the risk of assets held by intermediaries, and under some conditions reduces the leverage of the nonfinancial sector. This finding is consistent with empirical evidence that QE reduced the

[2]Board of Governors of the Federal Reserve System (U.S.), Mortgage Debt Service Payments as a Percent of Disposable Personal Income [MDSP]. Retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/MDSP, December 25, 2019.

riskiness of financial institutions (Chodorow-Reich (2014)) and partially mitigates concerns raise by some policymakers (Stein (2012a)) that QE may have induced firms to issue more risky debt. During the implementation of QE, interest rates were stuck at the zero lower bound. With a zero lower bound and nominal rigidities added, the model is also consistent with evidence that QE led to a drop in long-term and risky interest rates (Krishnamurthy and Vissing-Jorgensen (2011)). At the zero lower bound, QE stimulates consumption because it lowers the risk-free rate earned by an asset that provides no monetary services, as documented empirically by van Binsbergen, Diamond, and Grotteria (2020).

In the model, firms produce all resources by managing a continuum of projects with exogenous output (Lucas trees). Firms choose whether to buy a single tree or to act as a financial intermediary that can invest in securities. Each tree-owning nonfinancial firm chooses to issue (i) as much riskless debt as possible, (ii) an additional low-risk debt security, and (iii) a high-risk equity security. Firms are exposed to both aggregate and idiosyncratic risk, and their idiosyncratic risk implies that the quantity of riskless debt that they can issue does not entirely meet households' demand for riskless assets. This gap provides a role for intermediaries, who buy a diversified portfolio of nonfinancial firms' low-risk debt that is safe enough to back a large quantity of riskless deposits with a small buffer of equity capital to bear any systematic risk. Intermediaries do not buy equities, which are too systematically risky to back enough riskless deposits. Intermediaries do not buy riskless debt since doing so would not increase the total supply of riskless assets.

The fact that intermediaries are willing to pay more than households for low systematic risk assets but less for high systematic risk assets implies that asset prices are endogenously segmented. The risk-free rate is strictly lower than that implied by the representative household's consumption Euler equation because households obtain utility directly from holding riskless assets (Caballero and Krishnamurthy, 2009, Krishnamurthy and Vissing-Jorgensen, 2011, Stein, 2012b). The risk-free rate implied by the representative intermediary's pricing kernel is also below that implied by the household's pricing kernel, since the intermediary can use such assets to back the issuance of riskless deposits and borrow at the low risk-free rate. However, the intermediary charges a higher price of systematic risk than the household charges, since the intermediary has an agency cost of bearing systematic risk. As in models with leverage constraints (Black, 1972, Frazzini and Pedersen, 2014), less systematically risky assets owned by the intermediary earn a higher risk-adjusted return than riskier assets owned by the household.

The segmentation in asset prices is exploited by nonfinancial firms when they choose their capital structure, resulting in segmentation specifically between the debt and equity markets. To raise financing on the most attractive terms, each firm issues as much riskless debt as possible and then chooses its leverage so that its additional debt is sufficiently low-risk to sell to intermediaries, while its equity is sufficiently high risk to sell to households. Because the household and intermediary price risk differently, the debt and equity markets are endogenously segmented, consistent with evidence in Baker, Hoeyer, and Wurgler (2020)[3] and Frazzini and Pedersen (2014). In partial equilibrium, nonfinancial firms choose their leverage to exploit asset pricing anomalies. In general equilibrium, these leverage choices provide the intermediary with a portfolio of debt securites that allows it to most efficiently back its issuance of safe assets.

Relation to Literature This paper contributes to the literature on the supply and demand for safe assets and its connection with central bank policies such as quantitative easing. He and Krishnamurthy (2013) demonstrate how equity injections are more potent than asset purchases for boosting the value of intermediaries' assets and present the benchmark framework relating intermediation frictions to asset prices. Moreira and Savov (2017) study the issuance of liabilities of varying degrees of liquidity ("money" and "shadow money"), study asset pricing dynamics, and show how quantitative easing boosts asset prices. Caballero and Krishnamurthy (2009) show how growing demand for the safe liabilities of intermediaries

[3]Baker, Hoeyer, and Wurgler (2020) also show both empirically and theoretically that firms' capital structure decisions are influenced by asset market segmentation.

can increase the value of their risky asset portfolios. These papers assume that intermediaries own all risky assets, and they do not distinguish between the financial and nonfinancial sectors. The analysis in this paper on how intermediary portfolio choices and nonfinancial sector leverage decisions respond to an increased demand for safe assets and to quantitative easing policies is new.[4] See Diamond and Landvoigt (2020) for a quantitative analysis of these indirect effects on the nonfinancial sector.

The paper also contributes to the literature on how financial intermediaries create safe assets. This literature goes back to Gorton and Pennacchi (1990), who show that safe debt avoids an adverse selection problem that makes risky assets illiquid. Dang et al. (2017) demonstrate that when banks hold risky assets, they can make their liabilities safe in the short term by concealing interim information. Bigio and Weill (2016) present a theory in which only assets whose payoffs are not too correlated with aggregate output are liquid, where banks swap illiquid assets for liquid liabilities. DeAngelo and Stulz (2015) is also related, with a demand for safe assets and a cost of bank scale but in partial equilibrium and with complete markets.[5] Unlike my model, diversification and market segmentation have no role in these theories, and none anaylze indirect effects on the nonfinancial sector.[6] Relative to this literature, my results on the composition of household and intermediary balance sheets as well as the endogenous leverage of the nonfinancial sector are new. In addition, because all assets are publicly traded in my model (unlike previous work), it is testable with

[4] A crucial ingredient for my results on borrowing choices is an adaptation of tools related to Geanakoplos (2010) and particularly Simsek (2013) on endogenous leverage. Standard models of leverage constraints following Kiyotaki and Moore (1997) cannot have leverage choices respond to borrowing costs.

[5] Section 6 at the end of DeAngelo and Stulz (2015) discusses how their results would generalize in incomplete markets and their intuition is consistent with the composition of bank balance sheets in my model.

[6] Diversification plays a crucial role in much of financial intermediation theory (Diamond, 1984, Bond, 2004, DeMarzo, 2005), but its role is to reduce the cost of asymmetric information rather than to meet a demand for safe assets as in my model.

asset pricing evidence.

The rest of the paper is organized as follows. Section I presents the baseline model of the financial system, analyzes the allocation chosen by a benevolent social planner, and shows that a competitive equilibrium replicates this allocation. Section II studies how the financial system responds to an increased demand for safe assets and to quantitative easing policies. Section III discusses empirical evidence consistent with the model's asset pricing predictions, and section IV concludes. Technical proofs are relegated to the appendix.

I. Baseline Model

A . Setup

The model has two periods ($t = 1, 2$). Goods Y_1 are available at time 1 and cannot be stored. Output at time 2 is produced by a continuum of projects with exogenous output (Lucas trees) indexed by $i \in [0, 1]$, where tree i produces δ_i. At time 2, a binary aggregate shock is realized to be "good" or "bad," each with probability $\frac{1}{2}$. The realized outputs of the trees are conditionally independent given this aggregate shock. Aggregate and idiosyncratic shocks to each tree's output are the only sources of risk.

Agents There are two classes of agents: households and firm managers. Households have expected utility

$$u(c_1) + E[u(c_2) - T] + v(d), \tag{1}$$

which depends on consumption (c_1, c_2) at times 1 and 2, on a transfer T of utility paid to firm managers at time 2, and directly on the holding d of riskless assets that pay out at time 2. The functions u and v are strictly increasing, strictly concave, twice continuously differentiable, and satisfy Inada conditions. Managers have expected utility equal to the expected transfer $E(T)$ they receive at time 2.

Managers can run two types of firms: nonfinancial firms and financial intermediaries. A nonfinancial firm can own a Lucas trees and issue financial securities backed by the payoff of its tree. Each nonfinancial firm can own only one tree producing δ_i and cannot hold a diversified portfolio of multiple trees. All trees must be held by nonfinancial firms and not directly by households in order to produce output. Financial intermediaries cannot hold Lucas trees but can own a portfolio with payoff δ_I of securities issued by nonfinancial firms. Unlike nonfinancial firms, intermediaries can own a diversified portfolio that consists of securities issued by all nonfinancial firms.

Agency Problem Firm managers face an agency problem. For a firm that owns assets with a payoff of x at time 2, an increasing and (weakly) convex portion $C(x)$ of the output is nonpledgeable and can be seized by firm managers, where $C(0) = 0$, $1 > C' > 0$, and $C'' \geq 0$. As a result, only the payoff $P(x) = x - C(x)$ can be paid to outside investors. For a nonfinancial firm, the payoff seized by management is determined only by the payoff δ_i of the Lucas tree it owns, and the firm's pledgeable cash flows are $\delta_i^* = P(\delta_i)$. Because all trees must be owned by nonfinancial firms, and each tree i owned by a different firm, the payoffs seized by managers of nonfinancial firms are effectively fixed. For the intermediary, the cash flows seized by management depend on the payoff δ_I of its (endogenously chosen) asset portfolio, yielding pledgeable cash flows of $\delta_I^* = P(\delta_I)$. If the payoff δ_I of the intermediary's portfolio increases, more output $C(\delta_I)$ can be seized by its manager.

Utility Transfers After firm managers seize output, they sell it to households at a market price in exchange for direct transfers of utility. If households consume c_2 at time 2 and managers have c_{seized} worth of seized consumption goods, households are willing to make a transfer $T = u'(c_2)c_{seized}$ to recover the seized consumption goods. Because managers get no utility from consuming seized goods (but do value utility transfers from households), all seized consumption goods are sold back to households. This market for utility transfers (instead of having managers consume seized goods) keeps the tractability of an endowment economy with a representative household, because households consume all output. However, households are harmed by managers seizing output, because they must transfer utility to buy the output back.

Securities Issuance and Portfolios All firms issue only debt and equity securities, though firms can issue multiple tranches of debt.[7] In equilibrium, nonfinancial firms issue at most two tranches while intermediaries issue just one, and this is reflected in our notation. Let f_i and F_i respectively be the face value of senior and junior debt issued by the firm owning tree i, which I call firm i. The payoffs of the senior and junior tranches of firm i's debt are $d_i = min(\delta_i^*, f_i)$ and $D_i = min(\delta_i^* - min(\delta_i^*, f_i), F_i)$, respectively. The payoff of firm i's equity is $E_i = max(\delta_i^* - f_i - F_i, 0)$. Let f_I be the face value of the intermediary's debt, so $D_I = min(\delta_I^*, f_I)$ and $E_I = max(\delta_I^* - f_I, 0)$ are the payoffs of its debt and equity, respectively.

Let $q_I(d_i)$, $q_I(D_i)$, and $q_I(E_i)$ be the fraction of firm i's senior debt, junior debt, and equity held by the intermediary, with the remainder $q_H(.) = 1 - q_I(.)$ held by the household.[8] The payoff δ_I of the intermediary's portfolio is

$$\delta_I = \int_0^1 min(\delta_i^*, f_i) q_I(d_i) di + \int_0^1 min(\delta_i^* - min(\delta_i^*, f_i), F_i) q_I(D_i) di \\ + \int_0^1 max(\delta_i^* - f_i - F_i, 0) q_I(E_i) di. \quad (2)$$

Conditional on the aggregate state, each of these integrals is a sum of a continuum of independent random variables that satisfies a law of large numbers (Judd (1985), Uhlig (1996)) as shown in section B of the Internet Appendix. The payoff of the intermediary's portfolio equals its conditional expectation given the aggregate state.

Regularity Conditions I impose two regularity conditions on this environment. The first condition implies that a firm's debt is less exposed to systematic risk than its equity, which is the key reason why debt is held by financial intermediaries and equity is held by households in the model. The first condition also determines the quantity of riskless assets that nonfinancial

[7]Internet Appendix section I.A shows that this is the optimal security design when the firm faces frictions similar to those in Innes (1990).

[8]This notation assumes that all securities issued by the intermediary are held by the household, as occurs in equilibrium.

firms can issue without the need for financial intermediation. The second condition ensures that the benefit of the first unit of riskless assets issued by intermediaries exceeds its cost, so that financial intermediaries actually exist.

CONDITION 1 For each $i \in [0,1]$, there is a constant $\bar{\delta}_i \geq 0$ such that (i) $Pr(\delta_i > \bar{\delta}_i|bad) = Pr(\delta_i > \bar{\delta}_i|good) = 1$ and (ii) $\frac{Pr(\delta_i > u|good)}{Pr(\delta_i > u|bad)}$ is continuously differentiable with respect to u on $[\bar{\delta}_i, \infty)$, with the derivative strictly positive on $(\bar{\delta}_i, \infty)$ and $lim_{u\to\infty}\frac{Pr(\delta_i > u|good)}{Pr(\delta_i > u|bad)} = \infty$.

One implication of this condition is that for $u > \bar{\delta}_i$, we have $\frac{Pr(\delta_i > u|good)}{Pr(\delta_i > u|bad)} > 1$ and thus $Pr(\delta_i > u|bad) < 1$. It follows that $\bar{\delta}_i$ is the largest riskless payoff produced by firm i's Lucas tree.[9] Because a portion $C(\bar{\delta}_i)$ of this payoff can be seized by management, the firm is able to issue at most $\bar{\delta}_i^* = P(\bar{\delta}_i) = \bar{\delta}_i - C(\bar{\delta}_i)$ in riskless assets. The total supply of riskless assets created by nonfinancial firms is at most $\mu = \int_0^1 \bar{\delta}_i^* di$. Any additional riskless assets must be issued by financial intermediaries. If $\bar{\delta}_I$ is the lowest possible realization of the payoff δ_I of the intermediary's portfolio, the intermediary can create $\bar{\delta}_I^* = P(\bar{\delta}_I)$ additional safe assets. The condition $\frac{\partial}{\partial u}\frac{Pr(\delta_i > u|good)}{Pr(\delta_i > u|bad)} > 0$ implies that more senior claims on firm i's cash flows have lower systematic risk. This is why the intermediary prefers to hold a nonfinancial firm's debt instead of its equity, which is a more junior security.

CONDITION 2 If the total supply of riskless assets is only the quantity μ created by nonfinancial firms, the marginal cost $E[u'(\int_0^1 \delta_i di)C'(0)]$ of the intermediary increasing the supply of riskless assets is strictly less than the marginal benefit $v'(\mu)$.

This second condition ensures that the intermediary creates a positive supply of safe assets. The intermediary increases welfare by issuing safe assets in addition to the supply μ from nonfinancial firms, boosting the utility of holding safe assets by $v(\mu + \bar{\delta}_I - C(\bar{\delta}_I)) - v(\mu)$. This costs the household an additional utility transfer of $u'(c_2)C(\delta_I)$ to buy back $C(\delta_I)$ of seized goods. For $\bar{\delta}_I$ near zero, the marginal benefit exceeds the marginal cost if and only if Condition 2 holds.

[9] The constant $\bar{\delta}_i$ is unique for each i, since $\frac{\partial[\frac{Pr(\delta_i > u|good)}{Pr(\delta_i > u|bad)}]}{\partial u}$ is not strictly positive for $u < \bar{\delta}_i$.

B . The Social Planner's Problem

To illustrate how the financial system is most efficiently organized to create safe assets, we analyze what a benevolent social planner would choose to maximize the welfare of the representative household. This social planner can choose what securities are issued by what firms as well as whether these securities are held directly by the household or by the financial intermediary. The planner trades off two basic forces. First, the planner wants to maximize the total supply of riskless assets. Second, the planner wants to minimize the amount of resources seized by managers. As a result, the planner wants the intermediary to hold a portfolio that allows it to issue as many safe assets as possible while minimizing the amount of resources its manager seizes.

Statement of Social Planner's Problem The planner maximizes the household's expected utility (given in expression (1)) by choosing its consumption (c_1, c_2), the face values of debt (f_i, F_i, f_I) issued by all firms, and the asset portfolio $q_I(.)$ held by the financial intermediary (with $1 - q_I(.)$ held by the household). The planner's choices for these variables indirectly determine the payoff δ_I of the intermediary's portfolio, the quantity d of riskless assets in the economy, and the size of the utility transfer T households must make to purchase seized consumption resources back from managers.

Given these portfolio and issuance decisions, the payoff $\delta_I(q_I(.), f_i, F_i)$ of the intermediary's portfolio is given by equation (2). The amount of riskless debt the intermediary can issue is $\bar{\delta}_I^*(q_I(.), f_i, F_i) = P[min(\delta_I^{bad}(q_I(.), f_i, F_i), \delta_I^{good}(q_I(.), f_i, F_i))]$, where $(\delta_I^{bad}, \delta_I^{good})$ are the realizations of δ_I in the bad and good aggregate states. This yields a total quantity of riskless debt held by the household of

$$d(q_I(.), f_i, F_i, f_I) = f_I \mathbb{1}\{f_I \leq \bar{\delta}_I^*(q_I(.), f_i, F_i)\} + \int_0^1 (1 - q_I(d_i)) f_i \mathbb{1}\{f_i \leq \bar{\delta}_i^*\} di. \quad (3)$$

To see this, note that the indicator function $\mathbb{1}\{f_i \leq \bar{\delta}_i^*\}$ is defined to equals one if $f_i \leq \delta_i^*$

and zero otherwise, which means that the senior tranche of firm i's debt has a face value no greater than the (exogenous) worst realization $\bar{\delta}_i^*$ of its pledgeable cash flows. This is equivalent to the debt being riskless. Similarly, $\mathbb{1}\{f_I \leq \bar{\delta}_I^*\}$ is equivalent to the intermediary's debt being riskless.[10]

Given the payoff $\delta_I(q_I(.), f_i, F_i)$ of the intermediary's portfolio, $C(\delta_I(q_I(.), f_i, F_i))$ is seized by the intermediary's manager. In addition, $C(\delta_i)$ is seized by the manager of nonfinancial firm i. To repurchase all of these seized resources requires a transfer of utility from the household equal to

$$T(q_I(.), f_i, F_i, c_2) = u'(c_2)[\int_0^1 C(\delta_i)di + C(\delta_I(q_I(.), f_i, F_i))]. \tag{4}$$

The planner's problem can be now be written as

$$\max_{q_I(.), f_I, f_i, F_i, c_1, c_2} u(c_1) + E\left[u(c_2) - T(q_I(.), f_i, F_i, c_2)\right] + v\left(d(q_I(.), f_i, F_i.f_I)\right). \tag{5}$$

subject to $\quad c_1 \leq Y_1, \quad c_2 \leq \int_0^1 \delta_i di, \quad$ and $\quad 0 \leq q_I(.) \leq 1.$

The first two constraints are resource constraints requiring that consumption is no greater than total output, which is Y_1 at time 1 and the sum $\int_0^1 \delta_i di$ of all Lucas tree payoffs at time 2. The third constraint requires that each investor owns a nonnegative share of every financial asset.

The household's expected utility is increasing in the supply $d(q_I(.), f_i, F_i.f_I)$ of riskless assets, but it is decreasing in the utility transfer $T(q_I(.), f_i, F_i, c_2)$ to managers. The planner therefore chooses an allocation that maximizes the quantity of riskless assets subject to the size of the utility transfer to managers.

A Simplified Planner's Problem Taking the intermediary's portfolio as given, the total

[10] Without loss of generality, the junior tranche D_i of firm i's debt is assumed to be risky, because the firm could simply combine the two riskless tranches of debt into a single tranche with a higher face value. In addition, the intermediary is assumed without loss of generality not to hold any securites issued by itself.

supply of riskless assets that the intermediary issues is maximized by setting $f_I = \bar{\delta}_I^*$. In addition, because $P' = 1 - C' < 1$, the supply of riskless assets held by the household is reduced if the intermediary buys a riskless security to issue more riskless debt. Because this purchase would increase the resources seized by the intermediary's manager, the planner chooses that each nonfinancial firm issues as many riskless assets as it can ($f_i = \bar{\delta}_i^*$) and that all riskless assets are held by the household and not the intermediary ($q_I(d_i) = 0$). The household's utility is also increasing in c_1 and c_2, so the planner chooses that all available resources be consumed. The only remaining variables to set are the intermediary's holdings of junior debt $q_I(D_i)$ and equity $q_I(E_i)$ and the face value F_i of junior debt issued by each nonfinancial firm. The planner's problem reduces maximizing the following expression over the choice variables $q_I(D_i)$, $q_I(E_i)$, and F_i:

$$E\left[-\overbrace{u'\left(\int_0^1 \delta_i di\right)\left(C(\int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)q_I(D_i)di + \int_0^1 max(\delta_i^* - \bar{\delta}_i^* - F_i, 0)q_I(E_i)di)\right)}^{\text{utility transferred to intermediary manager}}\right] + \tag{6}$$

$$\overbrace{v\left(P(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)q_I(D_i)di + \int_0^1 E(max(\delta_i^* - \bar{\delta}_i^* - F_i, 0)q_I(E_i)di|bad) + \int_0^1 \bar{\delta}_i^* di)\right)}^{\text{utility of holding safe assets}}$$

subject to $0 \leq q_I(.) \leq 1$.

If we write the planner's objective function (expression (6)) in terms of the payoff δ_I of the intermediary's portfolio rather than the planner's exogenous choice variables, we obtain

$$\begin{aligned} E\left[-u'\left(\int_0^1 \delta_i di\right)C(\delta_I)\right] + v\left(P(\delta_I^{bad}) + \int_0^1 \bar{\delta}_i^* di\right) &= -\frac{1}{2}u'\left(\int_0^1 E(\delta_i|good)di\right)C(\delta_I^{good}) \\ &-\frac{1}{2}u'\left(\int_0^1 E(\delta_i|bad)di\right)C(\delta_I^{bad}) + v\left(P(\delta_I^{bad}) + \int_0^1 \bar{\delta}_i^* di\right). \end{aligned} \tag{7}$$

Expression (7), which the planner wants to maximize, is strictly decreasing in δ_I^{good}, and the only terms in the equation that depend on the planner's remaining choices are δ_I^{good} and δ_I^{bad}. The planner therefore chooses the portfolio ($q_I(D_i)$,$q_I(E_i)$) of the intermediary and the amount F_i of risky debt of the nonfinancial sector to minimize δ_I^{good} subject to the chosen value of δ_I^{bad}. This proves part 1 of the following lemma. Part 2, proved in the Appendix, shows that because part 1 holds, the intermediary's portfolio consists entirely of the risky debt issued by the nonfinancial sector.

Lemma 1 1. The intermediary's portfolio has the lowest systematic risk of all possible portfolios of a given size consisting only of risky assets. That is, if δ_I^{good} and δ_I^{bad} are the payoff of the intermediary's portfolio in the good and bad states, any other possible portfolio consisting only of risky assets with the same bad-state payoff δ_I^{bad} has a good-state payoff weakly greater than δ_I^{good}.

2. The lowest systematic risk portfolio with a given bad-state payoff δ_I^{bad} that consists only of risky assets is a portfolio of all risky debt issued by nonfinancial firms, with the face value of each nonfinancial firm's debt chosen appropriately.

If the planner wants the intermediary to create a given quantity of safe assets, it faces the lowest cost of managerial diversion by giving the nonfinancial firms' risky debt to the intermediary, all riskless assets and all equity to households, and choosing the capital structure of nonfinancial firms appropriately. The following proposition illustrates this result, which is depicted in Figure 2.

Proposition 1 The social planner's optimal allocation satisfies the following conditions. 1) All riskless assets are held by households ($q_I(d_i) = 0$). 2) All risky debt securities are held by the financial intermediary ($q_I(D_i) = 1$). 3) All equity securities are held by the household ($q_I(E_i) = 0$). 4) Each nonfinancial firm issues as much riskless debt as it possibly can. It also issues an additional risky debt security as well as an equity security.

Optimal Nonfinancial Sector Leverage Proposition 1 characterizes the intermediary's optimal portfolio q_I, leaving only the face value F_i of risky debt issued by each nonfinancial firm for the planner to choose. The intermediary's portfolio consisting only of risky debt)

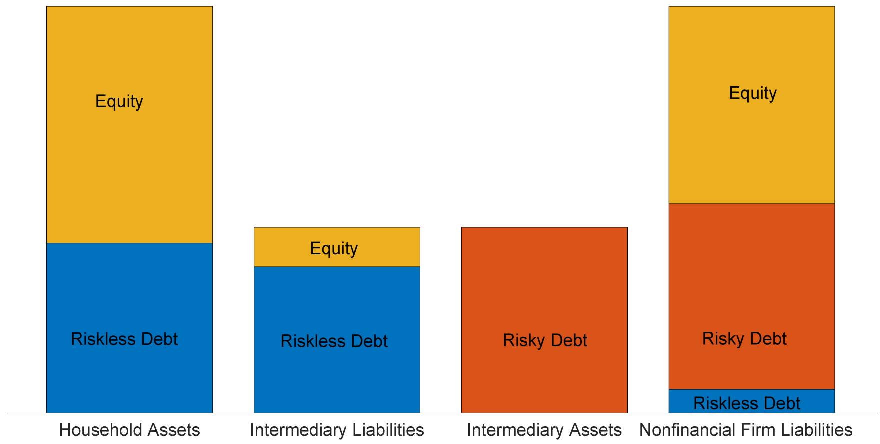

Figure 2. Composition of Optimally Chosen Balance Sheets

pays $\delta_I = \int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)di$. Plugging this into the planner's objective function (equation (6)) yields

$$\max_{F_i} E\left[-u'\left(\int_0^1 \delta_i di\right) C\left(\int_0^1 min[\delta_i^* - \bar{\delta}_i^*, F_i]di\right)\right] + \\ v\left(\int_0^1 \bar{\delta}_i^* di + P\left(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|bad\}di\right)\right). \tag{8}$$

A higher debt face value F_i makes the intermediary's portfolio larger, since there is then more debt for the intermediary to hold. As F_i increases, the riskiness of firm i's debt grows by Condition 1. For each unit of payoff in the bad state provided by firm i's debt (which allows the intermediary to issue more riskless assets), a greater quantity of good-state payoffs is also added to the intermediary's portfolio. This yields an interior optimum for the quantity of debt each firm should issue. The first-order condition for firm i's optimal capital structure is

$$\overbrace{E\left[u'\left(\int_0^1 \delta_i di\right) C'\left(\int_0^1 min[\delta_i^* - \bar{\delta}_i^*, F_i]di\right) \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}\right]}^{\text{agency cost of increasing firm } i\text{'s debt}} = \tag{9}$$

$$\overbrace{v'\left(\mu + P\left(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|bad\}di\right)\right) P'\left(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|bad\}di\right)}^{\text{utility benefit of additional safe assets backed by firm } i\text{'s debt}} *$$

$$\overbrace{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}|bad)}$$

After breaking the expectation in equation (9) into good-state and bad-state payoffs (each weighted by their probability $\frac{1}{2}$ of occurring), the expression can be rearranged to yield

$$\frac{Pr\{\delta_i^* - \bar{\delta}_i^* \geq F_i|good\}}{Pr\{\delta_i^* - \bar{\delta}_i^* \geq F_i|bad\}} = \frac{-u'\left(c_2^{bad}\right) C'(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|bad\}di))}{u'\left(c_2^{good}\right) C'(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|good\}di))} + \tag{10}$$
$$\frac{2v'\left(\mu + P(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|bad\}di)\right) P'(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|bad\}di)}{u'\left(c_2^{good}\right) C'(\int_0^1 E\{min[\delta_i^* - \bar{\delta}_i^*, F_i]|good\}di)}.$$

For each firm's debt, solving this first-order condition sets the ratio $\tau = \frac{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|good\right)}{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|bad\right)}$ equal to some constant τ that is the same across all nonfinancial firms.[11] The ratio of the expected payoff of an asset in the good and bad aggregate states is a natural measure of its systematic risk, so I refer to this ratio τ as the "risk threshold." Assets of systematic risk lower than τ belong on the intermediary's balance sheet, while those of systematic risk higher than τ belong on the household's balance sheet. The risk threshold uniquely determines the face value of each firm's debt, as the ratio $\frac{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|good\right)}{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|bad\right)}$ is strictly increasing in F_i and ranges from one to ∞ as F_i moves from zero to ∞ by Condition 1. The planner now only has to choose a value of the risk threshold τ. We must have that the planner chooses $\tau > 1$ because condition 2 implies that the intermediary should create some positive quantity of safe assets. Equation (9) sets equal an expression that is increasing in τ to one that is decreasing in τ, so only one value of τ solves this equation. The planner's problem therefore

[11] Firms own trees whose cash flows need not be identically distributed. Those for which $\frac{\Pr(\delta_i^* > u|good)}{\Pr\left(\delta_i^* > u|bad\right)}$ is higher at a given value of u choose lower leverage. This prediction that more systematically risky firms have lower leverage is empirically confirmed in Schwert and Strebulaev (2014).

has a unique solution.

Discussion of Social Planner's Problem Three features of the solution to the planner's problem are particularly relevant. First, a financial intermediary exists to produce safe assets that nonfinancial firms cannot produce on their own. This is because nonfinancial firms face idiosyncratic risk, which the intermediary diversifies away by holding a pool of securities issued by all nonfinancial firms. If nonfinancial firms faced no idiosyncratic risk, intermediation would not be necessary. Pooling and tranching to create safe assets is the purpose of intermediation in the model.

Second, the intermediary holds a diversified portfolio of all risky debt securities because this is the least costly way to create riskless assets. The agency cost of diversion by the intermediary's manager increases with the size of the intermediary's portfolio. Proposition 1 shows that a diversified portfolio of risky debt securities is the smallest portfolio that backs a given quantity of riskless assets. This result provides a new explanation for why banks both borrow from depositors and lend to firms. The low risk of investing in debt is ideal for backing riskless deposits.

Third, the nonfinancial sector's leverage is indirectly determined by the demand for safe assets. Risky debt securities in the model are intermediate inputs for the intermediary to create riskless deposits. Because the intermediary holds all risky debt, expanding the intermediary's balance sheet requires an increase in nonfinancial firms' leverage. This results provides an explanation for the nonfinancial sector's leverage that complements theories studying an individual firm in partial equilibrium.

C . Decentralized Market Equilibrium

A competitive equilibrium in this economy, where households maximize their expected utility and firms maximize their profits, yields the same allocation of resources as the planner's problem. The competitive equilibrium allows us to solve for asset prices, with three main asset pricing implications. Riskless assets are held by the household, and the risk-free rate is pushed down by the household's demand for safe assets. Low-risk assets are held by the intermediary, which uses them to back the issuance of riskless assets, and the intermediary's agency problem makes it endogenously risk averse. High-risk assets are held by the household, and the price of risk for these assets is lower than for the assets held by the intermediary, resulting in segmented asset markets. nonfinancial firms exploit this segmentation when choosing what securities to issue, and each nonfinancial firm has a unique optimal capital structure. Nonfinancial firms optimally issue a riskless senior debt security (if they can) that is held by the household, a low-risk junior debt security that is held by the intermediary, and a high-risk equity security that is held by the household.

Setup All financial securities trade at competitive market prices and can be bought by households or financial intermediaries subject to a no-short-sales constraint.[12] The risk-free rate is i_d, and a security paying cash flows x_s trades at a price p_s. At time 2, there is a competitive price $p_{transfer}$ of a utility transfer, stating how much consumption can be purchased by transferring one unit of utility to managers.

Households maximize their expected utility by investing at these competitive market prices. Firms maximize their profits, taking as given how the market prices the securities they issue and the fact that managers seize all nonpledgeable cash flows that they generate. Financial intermediaries choose both the assets that they purchase and the liabilities that they issue to maximize their profits. Nonfinancial firms own Lucas trees whose cash flows δ_i are exogenous, so they only choose which securities to issue.

Household's Problem The household is endowed with wealth W_H and chooses a quantity $q_H(s)$ of each risky asset s, a quantity d of riskless assets, consumption c_1, and utility transfer

[12] The short-sale constraint for households emerges from their inability to commit to pay the promised cash flows of a security. For the intermediary, the short-sale constraint follows from the fact that shorting does not reduce the amount of nonpledgeable output on its balance sheet, even though the equilibrium below is robust to allowing the intermediary to issue any security backed by its portfolio.

T to solve

$$\max_{q_H(.),d,c_1,T} u(c_1) + E\left[u\left(\int q_H(s)x_s ds + d + p_{transfer}T\right) - T\right] + v(d) \tag{11}$$

$$\text{subject to } c_1 + \frac{d}{1+i_d} + \int q_H(s)p_s ds = W_H \text{ (budget constraint)}$$

$$q_H(.) \geq 0 \text{ (short-sale constraint).}$$

If the household puts $q_H(s)$ in each risky asset s, its total risky asset portfolio pays $\int q_H(s)x_s ds$ and sells for a price of $\int q_H(s)p_s ds$. In addition, the transfer T at time 2 buys $p_{transfer}T$ of consumption goods.

The first-order conditions for the quantity of riskless assets d[13], for the quantity $q_H(s)$ of risky asset s, and for the amount of utility T to transfer in exchange for consumption are

$$u'(c_1) = (1+i_d)\left(E\left[u'(c_2)\right] + v'(d)\right), \tag{12}$$

$$p_s \geq E\left[\frac{u'(c_2)}{u'(c_1)}x_s\right], \tag{13}$$

$$u'(c_2)\, p_{transfer} = 1, \tag{14}$$

where inequality (13) must be an equality for any risky asset held in positive quantity by the household.

Intermediary's Problem One risky security that must be held by the household in equilibrium is the equity of the financial intermediary. As a result, inequality (13) must be an equality for this security. If the intermediary pays a dividend of E_I to equityholders at time 2 and raises equity e_I at time 1, then its market value at time 1 is (net of equity issuance)

$$E\left(\frac{u'(c_2)}{u'(c_1)}E_I\right) - e_I. \tag{15}$$

The intermediary maximizes this market value of its equity by choosing to buy a quantity

[13] The first-order condition for the quantity of riskless assets d has an interior solution since $v'(0) = \infty$.

$q_I(s)$ of each risky asset s, to buy a quantity d_I of riskless securities, and to issue quantities e_I of risky equity and D_I of riskless deposits. The intermediary's problem can be written as

$$\max_{q_I(.),d_I,e_I,D_I} E\left[\frac{u'(c_2)}{u'(c_1)}\left\{P\left(\int q_I(s)x_s ds + d_I\right) - D_I\right\}\right] - e_I \tag{16}$$

$$\text{subject to } \int q_I(s)p_s ds + \frac{d_I}{1+i_d} = D_I\frac{1}{1+i_d} + e_I \text{ (budget constraint)},$$

$$D_I \leq P(\int q_I(s)E(x_s|bad)ds + d_I) \text{ (deposit issuance constraint)},$$

$$q_I(.) \geq 0 \text{ (short-sale constraint)}.$$

The intermediary's budget constraint states that the sum of the equity e_I issued and the proceeds $D_I\frac{1}{1+i_d}$ from issuing riskless deposits must equal the price $\int q_I(s)p_s ds$ of the intermediary's risky asset portfolio plus the price $\frac{d_I}{1+i_d}$ of the riskless assets it buys. Of the payoff $\int q_I(s)x_s ds + d_I$ of the intermediary's portfolio, only $P(\int q_I(s)x_s ds + d_I)$ remains after managers seize the nonpledgeable output, so this is what remains to sell to outside investors. The realization of $P(\int q_I(s)E(x_s|bad)ds + d_I)$ in the bad aggregate state is the largest riskless payoff the intermediary can promise to outside investors, so this is the amount of riskless deposits D_I the intermediary is able to issue.

Nonfinancial Firm's Problem Each nonfinancial firm owns a Lucas tree with exogenous payoffs δ_i, and it only chooses which securities to issue. Because firm i's manager can seize $C(\delta_i)$ of Lucas tree i's cash flows, the firm is only able to pay $\delta_i^* = P(\delta_i) = \delta_i - C(\delta_i)$ to outside investors. The firm issues senior debt of face value f_i, junior debt of face value F_i, and an equity security.[14] The firm chooses the face values of its debt securities to maximize the total market value of the securities it issues. The price of each security is the maximum of what the household and intermediary are willing to pay for it. If $p_H(x)$ and $p_I(x)$ are respectively the household's and the intermediary's willingness to pay for a cash flow x, and

[14]Internet Appendix A shows that this is the firm's optimal set of securities to issue, subject to the constraint that all securities have payoffs that are increasing in δ_i and that do not depend explicitly on the aggregate state.

$p_{max}[x] = max(p_H(x), p_I(x))$, the firm's problem can be written as

$$\max_{f_i, F_i} \{p_{max}[min(\delta_i^*, f_i)] + p_{max}[min(\delta_i^* - min(\delta_i^*, f_i), F_i)] + p_{max}[max(\delta_i^* - f_i - F_i, 0)]\}. \quad (17)$$

The payoffs $d_i = min(\delta_i^*, f_i)$ of the firm's senior debt, $D_i = min(\delta_i^* - min(\delta_i^*, f_i), F_i)$ of its junior debt, and $E_i = max(\delta_i^* - f_i - F_i, 0))$ of its equity satisfy $d_i + D_i + E_i = \delta_i^*$. The total cash flow paid out by the firm is independent of the securities it issues. In a frictionless asset market, where a single pricing kernel prices all assets, $p_H(.)$ and $p_I(.)$ would both be equal to some linear function $p(.)$. This would imply that $p(d_i) + p(D_i) + p(E_i) = p(d_i + D_i + E_i) = p(\delta_i^*)$, so the firm's value would be the same regardless of which securities it issues. However, if there are securities for which the household and intermediary are willing to pay different prices, so $p_I \neq p_H$, the firm's value may depend on the securities it issues. The total value of the securities issued by the firm can be strictly greater than either investor would pay for all of the firm's cash flows δ_i. The optimal choices of f_i and F_i are analyzed after $p_H(.)$ and $p_I(.)$ are charecterized below.

Asset Prices and Portfolio Choices This section analyzes how assets are priced and the composition of the household's and intermediary's portfolios. The price of an asset is the maximum of what the household and intermediary are willing to pay for it. The household's portfolio consists of all assets for which it will pay more than the intermediary; all remaining assets are owned by the intermediary.

To see how the intermediary prices assets, note that it chooses to issue as many riskless securities as it can and to not buy any riskless securities, as shown in the Appendix. This is because the household's utility $v(d)$ from holding riskless assets means that it is willing to invest at a lower risk-free rate, which makes it attractive to borrow at the rate and unattractive to lend at it. Taking this as given, and using equation (12) to determine the risk-free rate, the intermediary's problem reduces to

$$\max_{q_I(.)\geq 0} E\left[\frac{u'(c_2)}{u'(c_1)}\{\int q_I(s)x_s ds - C\left(\int q_I(s)x_s ds\right)\}\right] +$$
$$P\left(\int q_I(s)E(x_s|bad)ds\right)\frac{v'(d)}{u'(c_1)} - \int q_I(s)p_s ds. \tag{18}$$

The intermediary's first-order condition for buying risky asset s is thus

$$\overbrace{E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right)}^{\text{household's willlingness to pay}} - \overbrace{E\left(\frac{u'(c_2)}{u'(c_1)}C'(\int q_I(s)x_s ds)x_s\right)}^{\text{agency cost of buying asset}}$$
$$+\overbrace{P'\left(\int q_I(s)E(x_s|bad)ds\right)E(x_s|bad)}^{\text{additional riskless payoff backed by asset}} \overbrace{\frac{(v'(d))}{u'(c_1)}}^{\text{safety premium}} \leq p_s, \tag{19}$$

with equality if the intermediary holds a positive quantity of the asset. The intermediary's willingness to pay for an asset differs from that of the household for two reasons. First, a portion $C(\int q_I(s)x_s ds)$ of the intermediary's portfolio is seized by its manager, and this agency cost grows when the intermediary buys more assets. Second, as part of the intermediary's diversified portfolio, an asset x_s increases the amount of riskless securities that the intermediary can issue by $P'\left(\int q_I(s)E(x_s|bad)ds\right)E(x_s|bad)$. Because the risk-free rate is low (due to the household's demand for safe assets), the intermediary benefits from issuing more riskless securities. The intermediary buys an asset if the agency cost of holding the asset is less than the benefit of the extra riskless securities the asset allows the intermediary to issue. As shown in the Appendix, the intermediary buys all assets for which

$$\frac{2P'\left(\int q_I(s)E(x_s|bad)ds\right)v'(d) - u'(c_2^{bad})C'\left(\int q_I(s)E(x_s|bad)ds\right)}{u'(c_2^{good})C'\left(\int q_I(s)E(x_s|good)ds\right)} \geq \frac{E(x_s|good)}{E(x_s|bad)}. \tag{20}$$

It follows that the intermediary buys all assets whose systematic risk $\frac{E(x_s|good)}{E(x_s|bad)}$ is sufficiently low, while the household buys all assets with higher systematic risk. This proves the following proposition, whose implications are illustrated in Figure 3 below.

Proposition 2 1. All riskless assets are bought by the household. The risk-free rate i_d is given by

$$u'(c_1) = (1 + i_d)[Eu'(c_2) + v'(d)]. \tag{21}$$

2. For some cutoff value τ, all risky assets whose payoffs x_s have sufficiently low systematic risk ($\frac{E(x_s|good)}{E(x_s|bad)} < \tau$) are bought by the intermediary. The price p_s of such an asset equals

$$E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right) - E\left(\frac{u'(c_2)}{u'(c_1)}C'\left(\int q_I(s)x_s ds\right)x_s\right) + P'\left(\int q_I(s)E(x_s|bad)ds\right)E(x_s|bad)\frac{v'(d)}{u'(c_1)}. \tag{22}$$

3. All risky assets with sufficiently high systematic risk ($\frac{E(x_s|good)}{E(x_s|bad)} > \tau$) are bought by the household. For these assets, the price p_s is given by

$$E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right) = p_s. \tag{23}$$

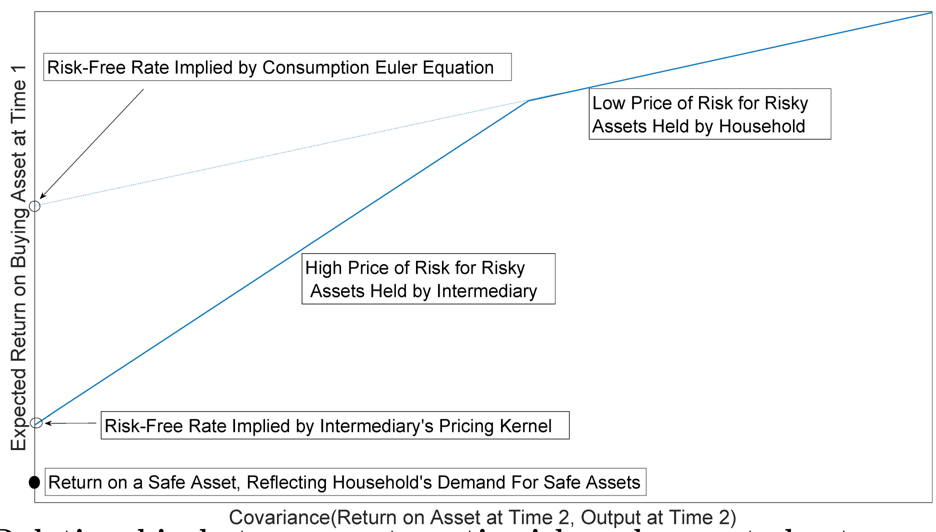

Figure 3. **Relationship between systematic risk and expected return on financial assets.**

To understand Figure 3, note that the intermediary's willingness to pay for cash flows in the good state is strictly less than that of the household. This is because a good-state cash flow has an agency cost for the intermediary but does not increase the intermediary's ability to issue riskless assets. As a result, the intermediary requires a greater risk premium than the household to voluntarily take on systematic risk. This is reflected in the greater slope of the line that represents the intermediary's willingness to pay for assets than the line representing that of the household. Because the intermediary buys a positive quantity of assets, the line representing its pricing kernel must be below the household's line for some positive values on the x-axis. The intermediary's line must therefore have a lower intercept as well as a higher slope than the household's line. This is equivalent to the intermediary's pricing kernel implying a lower risk-free rate than the pricing kernel of the household. These two risk-free rates are given by the white circles in Figure 3. Finally, the return on a safe asset, which reflects the household's demand for safe assets, lies strictly below these two rates, as shown by the black dot in this figure. Any risky asset whose systematic risk $\frac{E(x_s|good)}{E(x_s|bad)}$ is low enough to be left of the "kink" in the picture is held by the intermediary, while systematically riskier assets are held by the household. Asset prices in this model feature endogenous market segmentation, such that not all assets are priced by the same pricing kernel.

Capital Structure Choices of Nonfinancial Firms Because of the segmented asset prices described in Proposition 2, each nonfinancial firm has a unique optimal capital structure that exploits this market segmentation. First, because the yield on a safe asset lies strictly below the risk-free rates implied by both the household's and the intermediary's pricing kernels for risky assets, the nonfinancial firm issues the largest riskless security it possibly can. As a result, its senior debt security has a face value of $\bar{\delta}_i^*$, the largest possible riskless payoff the firm can promise. Second, because the intermediary is willing to pay more than the household for low-systematic-risk securities but less for high-systematic-risk securities, the firm optimally divides its remaining risky cash flows into a low-risk security for the intermediary and a high-risk security for the household. Condition 1 implies that the firm's debt has lower systematic risk than its equity, so the firm sells its risky debt to the intermediary and its equity to the household. As shown in the Appendix, the firm's value can now be written as

$$\max_{F_i} E\left(\frac{u'(c_2)+v'(d)}{u'(c_1)}\bar{\delta}_i^*\right) + \left(\frac{u'(c_2)}{u'(c_1)}(\delta_i^* - \bar{\delta}_i^*)\right) - E\Big[\frac{u'(c_2)}{u'(c_1)}C'\left(\int q_I(s)x_s ds\right) * \tag{24}$$

$$min(\delta_i^* - \bar{\delta}_i^*, F_i)\Big] + P'\left(\int q_I(s)E(x_s|bad)ds\right) E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)\frac{v'(d)}{u'(c_1)}. \tag{25}$$

To compute the optimal face value F_i of risky debt, note that except on an event of probability 0,[15]

$$\frac{\partial min(\delta_i^* - \bar{\delta}_i^*, F_i)}{\partial F_i} = -\frac{\partial max(\delta_i^* - \bar{\delta}_i^* - F_i, 0)}{\partial F_i} = \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}. \tag{26}$$

The first-order condition for the optimal face value F_i of risky debt is therefore

$$E\left(\frac{u'(c_2)}{u'(c_1)}C'(\int q_I(s)x_s ds)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}\right) = P'(\int q_I(s)E(x_s|bad)ds)Pr(\delta_i^* - \bar{\delta}_i^* \geq F_i|bad)\frac{v'(d)}{u'(c_1)}. \tag{27}$$

This first-order condition implies that a security whose payoff is one when the firm pays off its debt and zero when it defaults is of equal value to the intermediary and to the household. Equation (27) implies that the two expressions in Proposition 2 for the household's and the intermediary's willingness to pay for an asset are equal for an asset paying $\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}$. This is because a firm that increases the face value of its debt increases the payment it makes to debtholders precisely in those states of the world in which it does not default, and this increased payment comes out of the dividends that would have been paid to equityholders. At the firm's optimal capital structure, a small increase in leverage would leave the firm's value unchanged, so this additional payment must be of equal value to debt and equity investors.

This capital structure choice also determines the composition of household and intermediary portfolios. When the firm chooses its capital structure optimally, $\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}$ has the level of systematic risk at which the household and the intermediary value it equally,

[15] The function $\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}$ equals *one* if $\delta_i^* - \bar{\delta}_i^* \geq F_i$ and *zero* otherwise.

so $\frac{E\left(\mathbb{1}\{\delta_i^*-\bar{\delta_i^*}\geq F_i\}|good\right)}{E\left(\mathbb{1}\{\delta_i^*-\bar{\delta_i^*}\geq F_i\}|bad\right)}$ equals the risk threshold τ. In Figure 3, the expected return (on the y-axis) and systematic risk (on the x-axis) of $\mathbb{1}\{\delta_i^* - \bar{\delta_i^*} \geq F_i\}$ would locate at the kink point of the two lines representing the pricing kernels of the household and intermediary. At this optimal capital structure, the firm's debt has lower systematic risk than $\mathbb{1}\{\delta_i^* - \bar{\delta_i^*} \geq F_i\}$, and the firm's equity has higher systematic risk (as implied by Condition 1). This implies that all assets whose systematic risk is to the left of the kink point (and therefore are held by the intermediary) in Figure 3 are debt securities, while those to the right of the kink point (and therefore are held by the household) are equity securities. As a result, the composition of household, intermediary, and nonfinancial firm balance sheets are as shown in Figure 2: all risky debt is held by the intermediary, while riskless assets and equities are held by the household.

Equilibrium This section characterizes the model's equilibrium, which imposes resource constraints and market-clearing conditions in addition to the optimizing behavior of the household, intermediary, and nonfinancial firms described above.

DEFINITION 1 An equilibrium is a set of asset prices, portfolio and leverage choices, and consumption allocations that satisfies the following conditions. 1) The household, the intermediary, and nonfinancial firms behave optimally, solving maximization problems (11), (16), and (17). In addition, managers optimize, so they transfer all seized consumption resources in exchange for utility transfers. 2) Resource constraints are satisfied, so $c_1 = Y_1$ and $c_2 = \int_0^1 \delta_i di$. 3) Asset markets clear, so $q_I + q_H = 1$.

Plugging in $c_2 = \int_0^1 \delta_i di$ and using the fact that the intermediary's portfolio consists of all risky debt issued by the nonfinancial sector, so $\delta_I = \int_0^1 min(\delta_i^* - \bar{\delta_i^*}, F_i)di$, equation (27) becomes

$$E\left[u'\left(\int_0^1 \delta_i di\right) C'\left(\int_0^1 min(\delta_i^* - \bar{\delta_i^*}, F_i)di\right) \mathbb{1}\{\delta_i^* - \bar{\delta_i^*} \geq F_i\}\right] =$$
$$v'\left(\mu + P\left(\int_0^1 E(min(\delta_i^* - \bar{\delta_i^*}, F_i)|bad)di\right)\right) P'\left(\int_0^1 E(min(\delta_i^* - \bar{\delta_i^*}, F_i)|bad)di\right) *$$
$$Pr(\{\delta_i^* - \bar{\delta_i^*} \geq F_i\}|bad). \quad (28)$$

This expression is identical to the first-order condition of the social planner's problem (equation (9)), which uniquely determines all leverage and portfolio decisions. It follows that the decentralized equilibrium yields the same allocation as that chosen by the social planner.

II . Application to the Supply and Demand for Safe Assets

The model developed in the previous section can be used to understand the general equilibrium effects of changes in the supply and demand for safe assets. Because the model endogenously determines asset prices, intermediary portfolios and leverage, and the capital structure of the nonfinancial sector, all of these will adjust to clear the market for safe assets.

Although many aspects of the financial system endogenously change in response to changes in the supply or demand for safe assets, the analysis is tractable. A single equation can be used to charecterize both the planner's optimal allocation and the model's competitive equilibrium (equations (9) and (28), which are identical). By equation (10), the leverage of each nonfinancial firm is characterized by a common value of the risk threshold $\tau = \frac{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i | good\right)}{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i | bad\right)}$ across all firms. Given a value of $\tau > 1$, each firm has a unique face value of risky debt $F_i(\tau)$. The payoff of the intermediary's portfolio can be written as $\delta_I(\tau) = \int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i(\tau))di$, where $\delta_I^{good}(\tau)$ and $\delta_I^{bad}(\tau)$ are the payoffs of this portfolio in the two aggregate states. The following lemma provides a tractable expression for studying changes in the supply and demand for safe assets, with a proof in the Appendix.

LEMMA 2 1. Let $M(\tau) = u'(c_2^{good})C'(\delta_I^{good}(\tau))\tau + u'(c_2^{bad})C'(\delta_I^{bad}(\tau))$ $-2v'\left(\mu + P[\delta_I^{bad}(\tau)]\right) P'(\delta_I^{bad}(\tau))$. The equilibrium value of the risk threshold τ satisfies

$$M(\tau) = 0. \tag{29}$$

2. The functions $\delta_I^{good}(\tau)$, $\delta_I^{bad}(\tau)$, $\frac{\delta_I^{good}(\tau)}{\delta_I^{bad}(\tau)}$, and $F_i(\tau)$ (for all i) all have strictly positive derivatives with respect to the risk threshold τ, which implies that $M'(\tau) > 0$.

Safe Asset Demand and the Subprime Boom This section analyzes how the financial system responds to a safe asset shortage, which a macroeconomic literature (e.g., Caballero and Farhi (2017)) presents as one cause of the low real interest rates in recent decades. My model implies that growing demand for safe assets causes something akin to the subprime boom of the 2000s. In particular, the financial sector expands and invests in riskier assets, which leads to an increase in the leverage and default risk of the nonfinancial sector due to a reduction in its cost of borrowing. Relative to the literature, the novelty of my analysis comes from the endogenous choices of portfolios and capital structure, which are often taken as exogenous, and my joint modelling of the financial and nonfinancial sectors. In particular, my results on how the nonfinancial sector's leverage responds to changes in the supply and demand for safe assets are perhaps the most novel part of this analysis. The following proposition (and Figure 4) summarizes the results on the effects of an increase in the demand for safe assets.

Proposition 3 An increase in the demand for safe assets causes: 1) a decrease in the risk-free rate and an increase in the equilibrium quantity of safe assets, 2) an increase in the size and systematic risk of the intermediary's asset portfolio, 3) an increase in the leverage and default risk of all nonfinancial firms, 4) an increase in the intermediary's willingness to pay for all debt securities and for a risk-free asset, and 5) if $C'' > 0$, an increase in the spread between the household's and the intermediary's willingness to pay for a risk-free asset.

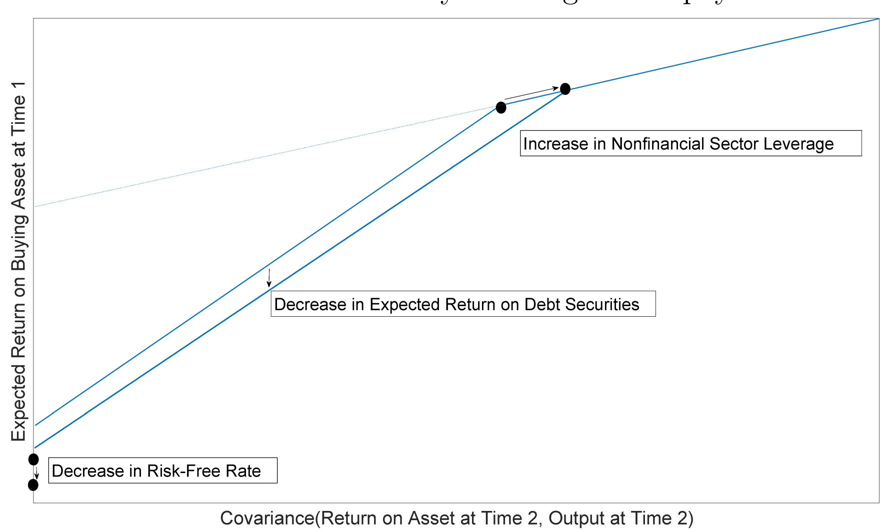

Figure 4. **Effect of a growing demand for safe assets on the financial system**

I model an increase in the demand for safe assets by exogenously increasing the marginal utility v' of holding a safe asset. If we increase the function v' by one unit, the expression in equation (29) decreases by $2P'(\delta_I^{bad}(\tau))$. The economy responds in equilibrium by finding a new value of the risk threshold τ that solves equation (29), and the response $\frac{d\tau}{dv}$ of the risk threshold satisfies

$$M'(\tau)\frac{d\tau}{dv} = 2P'(\delta_I^{bad}(\tau)). \tag{30}$$

Equation (30) implies that $\frac{d\tau}{dv} > 0$, so the quantity of safe assets increases, the size of the intermediary's portfolio rises, and the leverage of the nonfinancial sector rises. The proof of the proposition is in the Appendix.

This result follows from the fact that financial intermediaries buy risky debt securities as an intermediate input for creating riskless assets. As the demand for the final good (riskless assets) increases, the intermediary must purchase more intermediate inputs (debt securities) to meet this growing demand. As a result, the size of the intermediary's portfolio must increase. The nonfinancial sector issues more debt to accommodate the growth in the intermediary's balance sheet. By issuing more debt, the nonfinancial sector takes on a more levered capital structure and therefore is more likely to default on its debt. These changes in portfolio and capital structure choices are responses to changes in asset prices. The increased demand for safe assets pushes down the risk-free rate. Intermediaries are therefore willing to pay more for debt securities, because buying these securities allows the intermediary to issue riskless debt at a low risk-free rate. The nonfinancial sector in turn issues more debt because its cost of borrowing also falls.

This result shows how growing demand for safe assets in the early 2000s could have contributed to a concurrent boom in the size of the financial system and in the issuance of debt by the nonfinancial sector. As Bernanke et al. (2011) argue, a growing international demand for safe assets may have pushed risk-free interest rates down during this period. Concurrently, the size of the financial sector (particularly shadow banks) grew substantially

(Pozar et al. (2012)). In addition, households increased their borrowing against their homes (Bhutta and Keys (2016)), and a boom in leveraged buyouts (Shivdasani and Wang (2011)) fueled an increase in corporate debt. This high leverage may have made households and firms vulnerable and contributed to the wave of mortgage defaults in the financial crisis of 2007 to 2008.[16]

Quantitative Easing One of the U.S. Federal Reserve's key policy responses to the 2008 financial crisis was quantitative easing (QE), the purchase of Treasury bonds and agency mortgage-backed securities (MBS) financed by increasing the supply of bank reserves (riskless assets that must be held by financial intermediaries). Treasuries and agency MBS are exposed to duration and prepayment risk, so they are best thought of as risky debt securities in the model. Within the model, I examine the effects of a purchase of risky debt by the central bank in exchange for a special riskless asset that must be held by the financial intermediary. An alternative policy implemented by the Bank of Japan is to issue bank reserves to buy equities (which are not held by the intermediary); I analyze this policy as an intermediate step in the analysis of risky debt purchases.[17] In the model, a central bank that performs QE is an agent that is able to issue securities that it backs with lump sum taxes on households, and it trades these securities with other investors at competitive market prices.

QE that purchases either debt or equity securities both adds riskless assets to the intermediary's balance sheet and removes risky assets from financial markets. For debt QE, these risky assets would have been held by the intermediary, while for equity QE these risky assets would have been held by the household. The effect of equity QE on the intermediary is to simply increase its holdings of riskless assets. Debt QE combines this increase in riskless

[16] See Diamond and Landvoigt (2020) for a richer quantitative analysis of the connection between safe asset demand, the size of the financial sector, the leverage of the nonfinancial sector, and the vulnerability of the economy to financial crises.

[17] Some of the Federal Reserve's interventions to stabilize distressed banks can be thought of as a purchase of bank equity, although my model does not feature bank runs, which these interventions were intended to prevent.

asset holdings with a reduction in risky asset holdings by the intermediary. Both policies reduce the scarcity of safe assets and the riskiness of the intermediary's portfolio.

Quantitative Easing: Equities I first consider the effect of QE policies in which the central bank issues bank reserves in order to purchase equity securities. Before agents have made any decisions, the central bank announces that it will issue bank reserves making a riskless payoff of R at time 2 and use the proceeds of selling these reserves to purchase equity securities. The intermediary pays $Q_R = E\left(\frac{u'(c_2)}{u'(c_1)}(1 - C'(\delta_I))R\right) + P'(\delta_I^{bad})R\frac{v'(d)}{u'(c_1)}$ for these reserves by equation (22). The central bank uses the proceeds of this sale to buy equities from the household.[18] If the intermediary buys a fraction Q_i of the outstanding shares of nonfinancial firm i's equity (that pays E_i), equation (23) implies that $Q_R = \int_0^1 (E\frac{u'(c_2)}{u'(c_1)}E_i)Q_i di$. Because the equities purchased in QE would have been held by the household, while the reserves issued in QE are held by the intermediary, the intermediary's portfolio has an extra riskless payoff of R added by this transaction. To see how the financial system responds to this intervention, we need to see how the risk threshold τ changes when $\delta_I(\tau)$ is increased to $\delta_I(\tau) + R$ for some $R > 0$ by implicitly differentiating equation (29). The derivative $\frac{d\tau}{dQE_{equity}}$ of the equilibrium risk threshold τ with respect to the quantity of reserves R used to purchase equities by the central bank satisfies

$$\begin{aligned} M'(\tau)\frac{d\tau}{dQE_{equity}} &= 2v''(\mu + P(\delta_I^{bad}(\tau))P'(\delta_I^{bad}(\tau))^2 + 2v'(\mu + P[\delta_I^{bad}(\tau)])P''(\delta_I^{bad})(\tau) \\ &\quad -u'(c_2^{bad})C''(\delta_I^{bad}(\tau)) - u'(c_2^{good})C''(\delta_I^{good}(\tau))\tau. \end{aligned} \tag{31}$$

It follows that $\frac{d\tau}{dQE_{equity}} < 0$ since $v'' < 0$, $P'' \leq 0$, and $C'' \leq 0$, so the systematic risk of the intermediary's portfolio declines and the leverage of the nonfinancial sector declines. The change in the amount of bad-state payoffs in the intermediary's portfolio is $1 + (\delta_I^{bad})'(\tau)\frac{d\tau}{dQE_{equity}}$, which is strictly positive as shown in the Appendix. This implies

[18] Equivalently, the intermediary could purchase equities that it would not hold on its balance sheet in order to sell them to the central bank in exchange for reserves.

that the total quantity of riskless assets issued by the intermediary increases and that the risk-free rate rises. The effects of equity QE are summarized in the following proposition, with additional proof details in the Appendix.

Proposition 4 A QE policy in which the central bank issues riskless bank reserves to purchase risky equity securities held by the household leads to 1) a reduction in the risk of the intermediary's asset portfolio, 2) a reduction in the leverage of the nonfinancial sector, 3) an increase in the risk-free rate, and 4) a decrease in the value of all risky bonds.

This result holds because bank reserves are a better intermediate input than the nonfinancial sector's debt for the creation of safe assets. Bank reserves are riskless, so the intermediary can hold a smaller portfolio that backs a given quantity of safe assets by holding them instead of risky debt securities. This reduces the agency rents of the intermediary's management and crowds out the intermediary's holding of risky debt. The nonfinancial sector therefore issues less debt. In addition, the intermediary now creates a greater quantity of riskless assets, and this increased supply reduces the price of a safe asset and raises the risk-free rate. In practice, short-term interest rates stayed fixed at zero during QE, and this counterfactual prediction of a rate increase is fixed in the zero-lower-bound analysis below. Although the risk premium for corporate bonds over the risk-free rate either stays fixed or decreases, the increase in the risk-free rate passes through to an increase in risky bond yields. This increases the nonfinancial sector's cost of borrowing, which explains its rational decision to issue less debt. *Quantitative Easing: Debt* I next consider the effect of a QE policy in which the central bank buys debt securities held by the intermediary instead of equities. If the central bank issues R riskless bank reserves and buys a quantity Q_i of each debt security i (which pays D_i), the transaction must satisfy

$$0 = E\left(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I)(R - \int_0^1 Q_i D_i di)\right) + P'(\delta_I^{bad})(R - \int_0^1 Q_i E(D_i|bad)di)\frac{v'(d)}{u'(c_1)}. \quad (32)$$

For each risky debt security $E(D_i|good) > E(D_i|bad)$, this transaction can be seen as a combination of adding a riskless payoff of $\int_0^1 (R - \int_0^1 Q_i E(D_i|bad)di$ to the intermediary's balance sheet while removing a payoff only in the good state of $\int_0^1 Q_i[E(D_i|good) - E(D_i|bad)]di$. The effect of QE that purchases equities is to simply add a riskless payoff to the intermediary's balance sheet. I also analyze the effect of removing a good-state payoff from the intermediary's balance sheet to see the effects of QE that purchases debt.

If we remove a good-state payoff from the intermediary's balance sheet, the economy adjusts so that equation (29), which characterizes an equilibrium, remains true. The partial derivative of $M(\tau)$ with respect to $\delta_I^{good}(\tau)$ is $u'(c_2^{good})C''(\delta_I^{good}(\tau))\tau$, so

$$M'(\tau)\frac{d\tau}{dgood} = u'(c_2^{good})C''(\delta_I^{good}(\tau))\tau. \tag{33}$$

It follows that $\frac{d\tau}{dgood} > 0$ if $C'' > 0$ and $\frac{d\tau}{dgood} = 0$ if $C'' = 0$. If $C'' > 0$, the amount of bad-state payoff on the intermediary's balance sheet increases, and hence the total supply of riskless assets available to the household increases. This supply increase reduces the price of a riskless asset, so the risk-free rate rises. In addition, the increase in the risk threshold τ also implies that the leverage of all firms increases. The change in the amount of good-state payoff on the intermediary's balance sheet is $(\delta_I^{good})'(\tau)\frac{d\tau}{dgood} - 1$. Because $\frac{(\delta_I^{good})'(\tau)\frac{d\tau}{dgood}}{M'(\tau)} < 1$, equation (33) implies $(\delta_I^{good})'(\tau)\frac{d\tau}{dgood} - 1 < 0$, so the amount of good-state payoff on the intermediary's balance sheet decreases, despite the increase in nonfinancial sector leverage. Adding riskless assets to the intermediary's portfolio and removing good-state payoffs from the intermediary's portfolio move the risk threshold τ in opposite directions. The Appendix shows that τ is reduced by a QE policy that purchases debt if and only if $C''(\delta_I^{good}(\tau))$ is sufficiently large and provides an explicit expression for the change in τ. This yields the following proposition.

PROPOSITION 5 Compared to a QE policy that purchases equities, a QE policy that purchases debt that adds the same quantity of riskless payoffs to the intermediary's portfolio causes 1) a greater decrease in the systematic risk of the intermediary's asset portfolio, and if

$C'' > 0$ causes 2) a greater increase in the total supply of riskless assets held by households, 3) a greater increase in the risk-free rate, and 4) either a smaller decrease in the leverage of the nonfinancial sector if $C''(\delta_I^{good}(\tau))$ is sufficiently small or an increase in the leverage of the nonfinancial sector. If $C'' = 0$, then the change in the quantity of safe assets, the risk-free rate, and the leverage of the nonfinancial sector are identical to the QE policy that purchases equities.[19]

QE increases the supply of riskless bank reserves, which reduces the incentive for the intermediary to bear risk, and directly takes risk off of the intermediary's balance sheet by purchasing risky debt. Both sides of this exchange make it easier for the intermediary to create safe assets, so the quantity of safe assets the and risk-free rate both increase. Both sides of this exchange make the intermediary's asset portfolio safer, although the effect on nonfinancial firm leverage is ambiguous. A priori, it is unclear if the risk reduction that the intermediary chooses is greater or less than the direct partial equilibrium effects of asset purchases on the risk of the intermediary's portfolio. If the purchase of risky debt in QE removes less risk than the intermediary would itself choose, the intermediary sheds risky assets and induces the nonfinancial sector to decrease its leverage. This addresses the concern of Stein (2012a) that QE would increase the leverage of the nonfinancial sector. My results imply that a debt QE policy can increase or decrease the leverage of the nonfinancial sector, while an equity QE policy always reduces the nonfinancial sector's leverage.

Proposition 5 compares a debt QE policy to an equity QE policy that adds the same number of riskless payoffs to the intermediary's portfolio, not one that issues the same quantity of bank reserves. A debt QE policy that issues R units of reserves and buys Q_i^d units of firm i's debt only adds $R - \int_0^1 Q_i^d E(D_i|bad)di$ units of riskless payoff to the intermediary's

[19]Part 3 of this proposition implies that if $C'' > 0$, the expected return on a debt security with sufficiently low systematic risk has a greater increase under such a debt QE policy. Part 4 implies that if $C'' > 0$, a debt security whose systematic risk is sufficiently high has either a lower increase or a decrease in expected return under debt QE. This is proved in the Appendix.

portfolio, since the assets purchased in debt QE are removed from the intermediary's portfolio. The central bank requires $R - \int_0^1 Q_i^d E(D_i|bad)di$ units of tax revenue in the bad state to finance this transaction. However, an equity QE policy that buys Q_i^e units of firm i's equity by issuing R reserves adds R riskless payoffs to the intermediary's balance sheet while only requiring $R - \int_0^1 Q_i^e E(E_i|bad)di$ units of bad-state tax revenue. This is because the purchased equities would have been held by the household. When the government's ability to raise taxes is limited, this lower tax burden is an additional benefit of an equity QE policy over a debt QE policy.

Quantitative Easing at the Zero Lower Bound One counterfactual implication of my analysis of both possible QE policies is that they increase the risk-free rate. This is because such policies increase the supply of safe assets and therefore decrease their price. In practice, interest rates stayed fixed at the zero lower bound throughout the implementaiton of QE. With sticky prices and a zero lower bound (Krugman (1998)) common in the New Keynesian macroeconomics literature added to my model, my model's implications for QE become consistent with empirical stylized facts.

The model with sticky prices is identical to that without sticky prices, except that the output of the economy at time 1 is now endogenous. The labor of households is needed to transform consumption resources into a final consumable good, and final goods prices are perfectly rigid. For tractability, the household costlessly supplies labor. Because prices are perfectly rigid, the nominal and real interest rates are equal, so the central bank chooses the real risk-free rate. The "natural interest rate" that solves equation (12) at the optimal level of consumption c_1 is assumed to be negative, so the central bank optimally pushes nominal rates down to 0. However, consumption at time 1 is strictly less than is socially optimal. At time 2, consumption is assumed to be at its exogenous optimal value. This is true in an infinite horizon model with rigid prices if the natural interest rate is positive after time 1. See Internet Appendix II for a detailed discussion. For the purpose of our analysis we take $c_2 = \int_0^1 \delta_i di$ to be exogenous, and the central bank's choice of i_d determines c_1 by the household's first-order condition[20]

$$u'(c_1) = (1 + i_d)\left(E\left[u'(c_2)\right] + v'(d)\right). \tag{34}$$

Other than the fact that c_1 is now endogenously determined by equation (34) and that the risk-free rate i_d is fixed at zero, nothing changes from the analysis above. None of the variables in equation (29) depend on c_1, so the portfolio choice and capital structure analyses above also hold at the zero lower bound.

Since u' is decreasing, a reduction in either the nominal rate or in the premium $v'(d)$ on safe assets stimulates consumpion at time 1, similar to Caballero and Farhi (2017). Holding fixed $i_d = 0$, we have that

$$u''(c_1)\frac{\partial c_1}{\partial QE} = v''(d)\frac{\partial d}{\partial QE}. \tag{35}$$

Because both forms of QE increase the supply d of riskless assets held by the household, it follows that they increase consumption c_1 if the nominal rate i_d stays at the zero lower bound.

I now analyze the effect of QE at the zero lower bound on risky bond prices. For tractability, I impose that $C''' = 0$, so C' and $P' = 1 - C'$ are positive constants. Using equation (22), in the case $P' = 1 - C'$ is a constant that can be factored out, the price of a security held by the intermediary paying x_s is

$$\begin{aligned} E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right) - E\left(\frac{u'(c_2)}{u'(c_1)}C'\left(\int q_I(s)x_s ds\right)x_s\right) + P'\left(\int q_I(s)E(x_s|bad)ds\right)E(x_s|bad)\frac{v'(d)}{u'(c_1)} \\ = (P')\left(E(x_s|bad)\frac{Eu'(c_2)+v'(d)}{u'(c_1)} + \frac{u'(c_2^{good})}{2u'(c_1)}\left(E(x_s|good) - E(x_s|bad)\right)\right). \end{aligned} \tag{36}$$

Because $\frac{Eu'(c_2)+v'(d)}{u'(c_1)} = 1$ at the zero lower bound and $u'(c_1)$ decreases while $u'(c_2^{good})$ stays

[20] If the central bank could choose i_d so that $c_1 = Y_1$, all available resources would be consumed. When such an i_d is negative and the central bank chooses $i_d = 0$, an increase in consumption demand at time 1 increases household utility at no cost.

fixed, the intermediary's risk-free rate stays fixed, and the prices of all risky debt securities increase (by the right-hand side of equation (36)). This result matches the empirical findings of Krishnamurthy and Vissing-Jorgensen (2011) that QE reduced risky and long-term interest rates while holding the short-term interest rate fixed at zero. In addition, because $u'(c_1)$ decreases as a result of the increase in consumption, equity prices increase as well, since they are priced by the household's consumption Euler equation (equation (23)). This reduced cost of equity financing implies that firms need not borrow more even though their cost of borrowing decreases too. Propositions 4 and 5 imply that all firms reduce their borrowing in response to QE if $C'' = 0$. The following proposition summarizes these results.

Proposition 6 Suppose $C'' = 0$. With nominal rigidities and a binding zero lower bound, quantitative easing 1. reduces firms' borrowing costs and boosts equity prices while holding the risk-free rate fixed, 2. increases consumption at time 1, and 3. has the same effects on portfolio choices and capital structures as without nominal rigidities. In particular, since $C'' = 0$ all firms reduce their leverage in response to quantitative easing despite their reduced borrowing costs. These results hold whether the central bank purchases debt or equity securities to implement QE.

This general equilibrium analysis at the zero lower bound addresses the concern raised by some policymakers (Stein (2012a)) that QE would have negative financial stability implications because lower borrowing costs would induce firms to issue more debt. Away from the zero lower bound, QE would have raised borrowing costs (in the case in which $C'' = 0$ considered here) and induced firms to borrow less. It is only because of the special features of the zero lower bound that the reduction in risk premia caused by QE is reflected in reduced borrowing costs rather than an increase in the risk-free rate. Both at and away from the zero lower bound, if $C'' = 0$, then firms always reduce leverage in response to QE. Nevertheless, the analysis in the previous sections shows that if policymakers are worried about high leverage in the nonfinancial sector, implementing QE by purchasing equities rather than debt is an effective policy response.

III . Empirical Asset Pricing Implications

As shown in Proposition 2 and the corresponding discussion, the model implies that the assets held by the intermediary and the household are priced with different pricing kernels. This results yields two sets of empirical predictions. First, the risk-free rate (reflecting the household's demand for safe assets) lies strictly below the risk-free rates implied by both the household's and intermediary's pricing kernels for risky assets. The intermediary's pricing kernel (which prices debt securities) implies a lower risk-free rate than the household's pricing kernel (which prices equities). Second, the price of risk implied by the intermediary's pricing kernel is strictly higher than the price of risk implied by the household's pricing kernel.

The model's predictions on risk-free rates are consistent with evidence in Frazzini and Pedersen (2014). The authors show that the monthly alpha, that is, thespread in returns above the risk-free rate of a zero beta long-short portfolio (the so-called "betting against beta" strategy), is larger in equities than in debt securities but is positive in both markets. Their zero-beta long-short portfolio in U.S. equities earns a monthly alpha of 0.73. Their analogous betting against beta portfolios in U.S. credit indices, U.S. corporate bonds, and U.S. Treasuries earn alphas of 0.17, 0.57, and 0.16, respectively. These alphas can be interpreted as the spread between the risk-free rate implied by the prices of equity and debt securities and the observed risk-free rate. The positive alpha implied by debt securities but larger alpha implied by equities is precisely what my model predicts.

Evidence in Frazzini and Pedersen (2014) is also consistent with my model's prediction of a higher price of risk in debt markets than in equity markets. For each asset class, the paper reports betas (exposure to risk) and excess returns of 10 portfolios based on sorting assets into the deciles of their beta. Within each asset class, the slope of a regression of excess return on beta measures the price of risk in that asset class. The slope coefficient is 0.0721 for U.S. equities, 0.1914 for credit indices, and 0.0853 for Treasuries. For U.S. corporate bonds, the slope is 0.2549 using their data on bonds of different credit ratings.[21] The evidence is

[21] The authors also include returns on distressed corporate bonds, which are low with a high beta. Including

consistent with a higher price of systematic risk for bonds than for stocks. One difficulty in interpreting this evidence is that betas are computed with respect to a different reference index for each asset class. Baker, Hoeyer, and Wurgler (2020) provide more direct evidence by computing the CAPM betas and expected returns on stock and bond portfolios. They find a strictly lower expected return on low beta bonds than implied by the pricing of risk in equity markets, consistent with the "kinked" securities market line in my model.

My model can also reconcile a large difference in the risk-free rate implied by the pricing kernel for equities with the lower empirical estimates of the risk-free rate implied by risky bond prices. Krishnamurthy and Vissing-Jorgensen (2012) estimate that the spread between annual yields on AAA bonds and Treasuries is roughly 70 bps. This number maps naturally into the spread in my model between the yield on a safe asset and the risk-free rate implied by the intermediary's pricing kernel, since bonds with any credit risk are held by the intermediary. This spread is considerably smaller than the 9.1% annual excess return of a zero beta long-short equity portfolio implied by the estimates of Frazzini and Pedersen (2014).[22] Some of this 9.1% excess return may be compensation for risk. However, my model explains why this spread could be larger than 70 bps even after adjusting for risk exposure. More generally, my model implies that risk-free rates inferred from asset classes traded primarily by intermediaries may differ from those inferred from asset classes held primarily by households (such as equities).

IV . Conclusion

This paper develops a general equilibrium model of an economy organized to efficiently create safe assets and analyzes its response to shocks to the supply and demand for safe as-

this outlier makes the price of risk negative. See Campbell, Hilscher, and Szilagyi (2008) for an examination of the poor returns on distressed securities.

[22] Frazzini and Pedersen (2014) report a monthly alpha of 73 bps on this portfolio. I annualize their number by computing $1.0073^{12} - 1$.

sets. The role played by intermediaries is to issue safe assets backed by a pool of risky debt issued by nonfinancial firms. The debt and equity markets are endogenously segmented, and the nonfinancial sector's optimal capital structure arbitrages these segmented markets. The model shows that growing demand for safe assets can cause a credit boom and provides a framework for understanding the transmission mechanism of QE policies. The joint determination of household and intermediary portfolios as well as the leverage of the financial and nonfinancial sectors allows for a particularly rich analysis of the effects of QE, where the effect of QE depends on whether debt or equity securities are purchased.

Several features of the model suggest a future research agenda. First, the model takes as given the demand for safe, money-like assets. A more fundamental framework in which the demand for money and the role of intermediaries as creators of money are both endogenous may provide additional insights. Second, existing safe assets are typically denominated in a currency. A framework with safe assets in multiple currencies may be useful for understanding the international spillovers of QE and the role of the dollar in the international financial system. The perspective taken in this model, where the demand for safe assets influences the capital structure and portfolio choices of both the financial and nonfinancial sectors, may be a useful and tractable framework for many questions about the role of intermediaries in macroeconomics and finance. Existing work by Scharfstein (2018) on the impact of pension policy on the structure of the financial system and Diamond and Landvoigt (2020) on the impact of intermediaries on household leverage suggest the importance of endogeneous leverage and portfolio choices in applied work.

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Appendix: Proofs of Results

A Proof of Part 2 of Lemma 1

The payoff of firm i's junior debt and equity when the debt has face value F_i and the firm issues a face value $\bar{\delta}_i^*$ of riskless senior debt can be written respectively as

$$min(\delta_i^* - \bar{\delta}_i^*, F_i) = \int_0^{F_i} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du \text{ and} \tag{A.1}$$

$$max(\delta_i^* - \bar{\delta}_i^* - F_i, 0) = \int_{F_i}^{\infty} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du. \tag{A.2}$$

If the intermediary owns a fraction $q_I(D_i)$ of the firm's risky junior debt and $q_I(E_i)$ of the firm's equity, the payoff to the intermediary of assets issued by firm i is

$$q_I(D_i)\int_0^{F_i} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du + q_I(E_i)\int_{F_i}^{\infty} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du, \tag{A.3}$$

which is a special case of the expression $\int_0^\infty q(u)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$, where the image of q is contained in [0,1] in the case $q(u) = q_I(D_i)\mathbb{1}\{F_i \geq u\} + q_I(E_i)(1 - \mathbb{1}\{F_i \geq u\})$. The expected payoff of this portfolio in the good and bad states is $\int_0^\infty q(u)Pr(\delta_i^* - \bar{\delta}_i^* \geq u|good)du$ and $\int_0^\infty q(u)Pr(\delta_i^* - \bar{\delta}_i^* \geq u|bad)du$, respectively. There is some face value F_i^* of debt for which a portfolio that owns all of the firm's debt has the same bad-state payoff, $\int_0^{F_i^*} Pr(\delta_i^* - \bar{\delta}_i^* \geq u|bad)du$. Let $C = \frac{Pr(\delta_i^* - \bar{\delta}_i^* \geq F_i^*|good)}{Pr(\delta_i^* - \bar{\delta}_i^* \geq F_i^*|bad)}$ and $q^*(u) = \mathbb{1}\{F_i^* > u\}$. The difference in the good-state payoffs of the two portfolios is

$$\int_0^\infty (q(u) - q^*(u))(\frac{Pr(\delta_i^* - \bar{\delta}_i^* \geq u|good)}{Pr(\delta_i^* - \bar{\delta}_i^* \geq u|bad)} - C)Pr(\delta_i^* - \bar{\delta}_i^* \geq u|bad)du. \tag{A.4}$$

If q is not almost everywhere equal to q^*, this expression is strictly positive. If $q = q^*$ almost everywhere, then q^* also gives the payoff of holding all of firm i's debt. It follows that a portfolio composed of the whole outstanding stock of firm i's debt minimizes the expected good-state payoff holding fixed the bad-state payoff. Moreover, any other portfolio has a strictly higher good-state payoff if its bad-state payoff is the same. Because this is true for an individual firm, the portfolio of risky assets composed of securities issued by all firms that has the lowest possible good-state payoff for a given bad-state payoff must be composed entirely of risky debt securities.

B Proof of Lemma 2

1. Using the definitions of c_2^{good},c_2^{bad}, $\delta_I^{good}(\tau)$, and $\delta_I^{bad}(\tau)$, equation (28) can be written as

$$E\left[u'\left(c_2\right)C'(\delta_I(\tau))\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}\right] = v'\left(\mu + P[\delta_I^{bad}(\tau)]\right)P'(\delta_I^{bad}(\tau))Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|bad). \tag{B.1}$$

Breaking expectations into realizations in the good and bad state that each occur with probability $\frac{1}{2}$ yields

$$u'(c_2^{good})C'(\delta_I^{good}(\tau))Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|good) + u'\left(c_2^{bad}\right)C'(\delta_I^{bad}(\tau))Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|bad) \tag{B.2}$$

$$= 2v'\left(\mu + P[\delta_I^{bad}(\tau)]\right)P'(\delta_I^{bad}(\tau))Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|bad). \tag{B.3}$$

Since $Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|good) = \tau Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|bad)$ this simplifies to

$$u'(c_2^{good})C'(\delta_I^{good}(\tau))\tau + u'(c_2^{bad})C'(\delta_I^{bad}(\tau)) = 2v'\left(\mu + P[\delta_I^{bad}(\tau)]\right)P'(\delta_I^{bad}(\tau)). \tag{B.4}$$

2. When each firm i chooses its optimal capital structure, we have

$$\frac{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|good)}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|bad)} = \tau \quad \text{and} \tag{B.5}$$

$$\left(\frac{d}{dF_i}\left[\frac{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|good)}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|bad)}\right]\right)F_i'(\tau) = 1. \tag{B.6}$$

Because $\frac{d}{dF_i}[\frac{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|good)}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(\tau)\}|bad)}] > 0$ by Condition (1), it follows that $F_i'(\tau) > 0$. The intermediary's portfolio has a payoff, $\delta_I(\tau) = \int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i(\tau))di$. Denote the payoff of this portfolio in the good state and the bad state as $\delta_I^{good}(\tau)$ and $\delta_I^{bad}(\tau)$, respectively. Note that

$$(\delta_I^{good})'(\tau) = \int_0^1 Pr(\delta_i^* - \bar{\delta}_i^* > F_i(\tau)|good))F_i'(\tau)di > 0 \quad \text{and}$$

$$\begin{aligned}
(\delta_I^{bad})'(\tau) &= \int_0^1 Pr(\delta_i^* - \bar{\delta}_i^* > F_i(\tau)|bad))F_i'(\tau)di \\
&= \int_0^1 \frac{Pr(\delta_i^* - \bar{\delta}_i^* > F_i(\tau)|bad))}{Pr(\delta_i^* - \bar{\delta}_i^* > F_i(\tau)|good))} Pr(\delta_i^* - \bar{\delta}_i^* > F_i(\tau)|good))F_i'(\tau)di \\
&= \tau \int_0^1 Pr(\delta_i^* - \bar{\delta}_i^* > F_i(\tau)|good))F_i'(\tau)di = \tau(\delta_I^{good})'(\tau) > 0.
\end{aligned}$$

Note that $\frac{d}{d\tau}\left(\frac{\delta_I^{good}(\tau)}{\delta_I^{bad}(\tau)}\right) = \frac{(\delta_I^{good})'(\tau)\delta_I^{bad}(\tau) - \delta_I^{good}(\tau)(\delta_I^{bad})'(\tau)}{\delta_I^{bad}(\tau)^2} = \frac{(\delta_I^{bad})'(\tau)}{\delta_I^{bad}(\tau)}[\frac{(\delta_I^{good})'(\tau)}{(\delta_I^{bad})'(\tau)} - \frac{\delta_I^{good}(\tau)}{\delta_I^{bad}(\tau)}]$ as well. Because $\frac{\delta_I^{good}(\tau)}{\delta_I^{bad}(\tau)} < \frac{(\delta_I^{good})'(\tau)}{(\delta_I^{bad})'(\tau)}$ by Condition 1, it follows that $\frac{d}{d\tau}\left(\frac{\delta_I^{good}(\tau)}{\delta_I^{bad}(\tau)}\right) > 0$. Note also that $\frac{d}{d\tau}[\delta_I^{good}(\tau) - \delta_I^{bad}(\tau)] = (\delta_I^{good})'(\tau)(1-\tau) > 0$. Finally, we have that

$$M'(\tau) = u'(c_2^{good})\left[C''(\delta_I^{good}(\tau))(\delta_I^{good})'(\tau)\tau + C'(\delta_I^{good}(\tau))\right] + u'(c_2^{bad})C''(\delta_I^{bad}(\tau))(\delta_I^{bad})'(\tau) \tag{B.7}$$

$$-2v''\left(\mu + P[\delta_I^{bad}(\tau)]\right)P'(\delta_I^{bad}(\tau))^2(\delta_I^{bad})'(\tau) - 2v'\left(\mu + P[\delta_I^{bad}(\tau)]\right)P''(\delta_I^{bad}(\tau))(\delta_I^{bad})'(\tau) > 0. \tag{B.8}$$

C Proof of Proposition 3

As stated in the main text, the risk threshold τ responds to an increased demand for safe assets according to

$$M'(\tau)\frac{d\tau}{dv} = 2P'(\delta_I^{bad}(\tau)). \tag{C.1}$$

The quantity of safe assets issued by the intermediary $P(\delta_I^{bad}(\tau))$, the systematic risk of the intermediary's portfolio $\frac{P(\delta_I^{good}(\tau))}{P(\delta_I^{bad}(\tau))}$, and the leverage of the nonfinancial sector are each an increasing function of the risk threshold τ. Because $\frac{d\tau}{dv} > 0$, these each increase when the demand for safe assets grows.

In addition, the safe asset premium, $v'(\mu+P[\delta_I^{bad}(\tau)])$ changes as $1+\frac{d\tau}{dv}\frac{d}{d\tau}v'(\mu+P[\delta_I^{bad}(\tau)])$. This is because it exogenously increases by 1 due to growing demand, but the growing quantity of safe assets supplied by the financial intermediary acts to reduce this increase. Note that $\frac{d}{d\tau}v'(\mu+P[\delta_I^{bad}(\tau)])$ equals

$$v''\left(\mu+P[\delta_I^{bad}(\tau)]\right)P'(\delta_I^{bad}(\tau))^2+v'\left(\mu+P[\delta_I^{bad}(\tau)]\right)P''(\delta_I^{bad}(\tau))(\delta_I^{bad})'(\tau)>-\frac{M'(\tau)}{2(\delta_I^{bad})'(\tau)}. \tag{C.2}$$

It follows that

$$1+\frac{d\tau}{dv}\frac{d}{d\tau}v'(\mu+P[\delta_I^{bad}(\tau)])>1-\frac{M'(\tau)}{2(\delta_I^{bad})'(\tau)}\frac{d\tau}{dv}=0. \tag{C.3}$$

This implies that the risk-free rate decreases.

The intermediary's willingness to pay for an asset with payoff x_s can be written as

$$E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right)-E\left(\frac{u'(c_2)}{u'(c_1)}C'(\delta_I(\tau))x_s\right)+P'(\delta_I^{bad}(\tau))E(x_s|bad)\frac{v'(d)}{u'(c_1)}. \tag{C.4}$$

This equals a constant term plus

$$\frac{E(x_s|bad)}{2u'(c_1)}\left[u'\left(c_2^{good}\right)C'(\delta_I^{good}(\tau))\frac{E(x_s|good)}{E(x_s|bad)}+u'\left(c_2^{bad}\right)C'(\delta_I^{bad}(\tau))-2v'\left(\mu+P[\delta_I^{bad}(\tau)]\right)P'(\delta_I^{bad}(\tau))\right]. \tag{C.5}$$

For $\frac{E(x_s|good)}{E(x_s|bad)}=\tau$, this must be 0 in order for $M(\tau)=0$. Because $\frac{d\tau}{dv}>0$, it follows that this expression is strictly increasing holding $\frac{E(x_s|good)}{E(x_s|bad)}$ fixed at the value τ. Because the intermediary's willingness to pay for a good-state payoff is nonincreasing, it follows that the intermediary's willingness to pay for any debt security (which must satisfy $\frac{E(x_s|good)}{E(x_s|bad)}\leq\tau$) or any riskless asset strictly increases.

However, the difference between the equilibrium price of a riskless payoff and the intermediary's willingness to pay is equal to

$$E\left(\frac{u'(c_2)}{u'(c_1)}C'(\delta_I(\tau))\right) + \frac{v'\left(\mu + P[\delta_I^{bad}(\tau)]\right)C'(\delta_I^{bad}(\tau))}{u'(c_1)}. \tag{C.6}$$

This difference also increases, since τ (and hence $C'(\delta_I(\tau))$, in both states of the world) weakly increases and $v'\left(\mu + P[\delta_I^{bad}(\tau)]\right)$ strictly increases.

D Proof of Proposition 4

As stated in the main text just below equation (31), $\frac{d\tau}{dQE_{equity}} < 0$. This decrease in the risk threshold τ reduces the systematic risk of the intermediary's portfolio and the leverage of the nonfinancial sector since these are increasing in τ. The change in the quantity of riskless payoffs on the intermediary's portfolio is $1 + (\delta_I^{bad})'(\tau)\frac{d\tau}{dQE_{equity}}$. Recall that

$$M'(\tau)\frac{d\tau}{dQE_{equity}} = 2v''(\mu + P[\delta_I^{bad}(\tau)])P'(\delta_I^{bad}(\tau))^2 + 2v'(\mu + P[\delta_I^{bad}(\tau)])P''(\delta_I^{bad}(\tau)) \tag{D.1}$$

$$-u'(c_2^{bad})C''(\delta_I^{bad}(\tau)) - u'(c_2^{good})C''(\delta_I^{good}(\tau))\tau. \tag{D.2}$$

All four terms of the left hand side of this equation are (weakly) negative. The first two terms times $(\delta_I^{bad})'(\tau)$ plus the third and fourth terms times $(\delta_I^{good})'(\tau)$,which is greater than $(\delta_I^{bad})'(\tau)$, equals $M'(\tau)$. It follows that $(\delta_I^{bad})'(\tau)\frac{d\tau}{dQE_{equity}} > -1$ and thus $1+(\delta_I^{bad})'(\tau)\frac{d\tau}{dQE_{equity}} > 0$. This implies that the quantity of bad-state payoffs on the intermediary's balance sheet increases, and thus the quantity of riskless debt it can issue also increases. This in turn causes the risk-free rate to increase.

Because $\frac{d\tau}{dQE_{equity}} < 0$, the intermediary's willingness to pay for an asset of systematic risk equal to the risk threshold τ also decreases. Because the systematic risk of every debt security lies below the risk threshold τ, the systematic component of its payoff can be written as that of an asset whose systematic risk is τ plus a payoff in the bad state. The prices of all debt securities therefore decrease, since the intermediary's willingness to pay for a bad-state cash flow decreases.

E Proof of Proposition 5

Parts 1-3 of the proposition are demonstrated in the main text. We now characterize how the leverage of the nonfinancial sector is impacted by a QE policy that buys debt securities (or debt QE). Because the nonfinancial sector's leverage is a strictly increasing function of the risk threshold τ, we need to compute $\frac{d\tau}{dQE_{debt}}$. Recall that the transaction must satisfy $0 = E\left(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I)[R - \int_0^1 Q_i D_i di]\right) + P'(\delta_I^{bad})[R - \int_0^1 Q_i E(D_i|bad)di]\frac{v'(d)}{u'(c_1)}$.

This can be written as

$$\{[u'(c_2^{bad}) + 2v'(d)]P'(\delta_I^{bad}) + u'(c_2^{good})P'(\delta_I^{good})\}\left(R - \int_0^1 Q_i E(D_i|bad)di\right) \tag{E.1}$$

$$= u'(c_2^{good})P'(\delta_I^{good})\left(\int_0^1 Q_i E(D_i|good)di - \int_0^1 Q_i E(D_i|good)di\right). \tag{E.2}$$

If we then let $L = \frac{\{[u'(c_2^{bad})+2v'(d)]P'(\delta_I^{bad})+u'(c_2^{good})P'(\delta_I^{good})\}}{u'(c_2^{good})P'(\delta_I^{good})}$, it follows that L good-state payoffs are removed by this transaction for every riskless payoff that is added to the intermediary's portfolio. The response $\frac{d\tau}{dQE_{debt}}$ of the risk threshold to debt QE therefore equals

$$\frac{d\tau}{dQE_{equity}} + L\frac{d\tau}{dgood} = \frac{1}{M'(\tau)}\Big[2v''(\mu + P[\delta_I^{bad}(\tau)]P'(\delta_I^{bad}(\tau))^2 + 2v'(\mu + P[\delta_I^{bad}(\tau)])P''(\delta_I^{bad}(\tau))$$
$$-u'(c_2^{bad})C''(\delta_I^{bad}(\tau)) + \left[\frac{[u'(c_2^{bad}) + 2v'(d)]P'(\delta_I^{bad}) + u'(c_2^{good})P'(\delta_I^{good})}{u'(c_2^{good})P'(\delta_I^{good})} - 1\right]u'(c_2^{good})C''(\delta_I^{good}(\tau))\tau\Big]. \tag{E.3}$$

This expression is positive if $C''(\delta_I^{good}(\tau))$ is positive and sufficiently large and is otherwise negative.

The risk threshold τ increases with a QE policy if and only if the intermediary's willingness to pay for a security of systematic risk equal to τ increases. The expected return on a firm's debt (holding fixed its leverage) whose systematic risk is sufficiently close to τ increases if and only if $\frac{d\tau}{dQE_{debt}} < 0$. Since $\frac{d\tau}{dgood} > 0$, a firm with sufficiently high systematic risk debt always has a smaller increase in borrowing cost from a debt QE policy than an equity QE policy that adds the same quantity of riskless payoffs to the intermediary's portfolio. Because the risk free rate increases more under such a debt QE policy than such an equity QE policy, the expected return on debt securities of sufficiently low systematic risk increases with either QE policy but more under the debt QE policy.

F Additional Proof Details

LEMMA 3 The intermediary chooses to issue as many riskless securities as it can and to not buy any riskless securities.

Proof. Suppose the intermediary owns a portfolio paying δ_I, so that it can pledge $P(\delta_I)$ to outside investors, and $P(\delta_I^{bad})$ worth of riskless payoffs. If the intermediary issues n riskless assets where $n < P(\delta_I^{bad})$, the value of the intermediary's riskless debt plus its equity equals $E\frac{u'(c_2)}{u'(c_1)}(P(\delta_I) - n) + nE\frac{u'(c_2)+v'(d)}{u'(c_1)} = E\frac{u'(c_2)}{u'(c_1)}P(\delta_I) + nE\frac{v'(d)}{u'(c_1)}$, which is strictly increasing in n. The intermediary therefore chooses to issue $n = P(\delta_I^{bad})$ of riskless debt. Finally, if the intermediary both buys and issues riskless securities, both transactions are at the same risk-free rate. If the intermediary buys a riskless asset, $C'(\delta_I)$ of the asset's payoff is seized, so the quantity of riskless debt issued by the intermediary goes up by strictly less than the payoff of the purchased asset. The transaction reduces the value of the intermediary's equity. □

LEMMA 4 The intermediary buys all assets for which

$$\frac{2P'(\int q_I(s)E(x_s|bad)ds)v'(d) - u'(c_2^{bad})C'(\int q_I(s)E(x_s|bad)ds)}{u'(c_2^{good})C'(\int q_I(s)E(x_s|good)ds)} \geq \frac{E(x_s|good)}{E(x_s|bad)}. \qquad \text{(F.1)}$$

Proof. The intermediary's willingness to pay for a risky asset is greater than that of the household if

$$E\left(\frac{u'(c_2)}{u'(c_1)}C'(\int q_I(s)x_s ds)x_s\right) - P'(\int q_I(s)E(x_s|bad)ds)E(x_s|bad)\frac{(v'(d))}{u'(c_1)} \leq 0. \qquad \text{(F.2)}$$

Breaking the expectation into good-state and bad-state realizations (each weighted by probability $\frac{1}{2}$) yields

$$u'(c_2^{good})C'(\int q_I(s)E(x_s|good)ds)E(x_s|good) + u'(c_2^{bad})C'(\int q_I(s)E(x_s|bad)ds)E(x_s|bad) \tag{F.3}$$

$$-2P'(\int q_I(s)E(x_s|bad)ds)E(x_s|bad)\,(v'\,(d)) \leq 0 \qquad \text{and thus} \tag{F.4}$$

$$\frac{E(x_s|good)}{E(x_s|bad)} \leq \frac{2P'(\int q_I(s)E(x_s|bad)ds)v'\,(d) - u'(c_2^{bad})C'(\int q_I(s)E(x_s|bad)ds)}{u'(c_2^{good})C'(\int q_I(s)E(x_s|good)ds)}, \tag{F.5}$$

as desired. □

Lemma 5 nonfinancial firm i's value is equal to

$$max_{F_i}E\left(\frac{u'(c_2)+v'(d)}{u'(c_1)}\bar{\delta}_i^*\right) + \left(\frac{u'(c_2)}{u'(c_1)}(\delta_i^* - \bar{\delta}_i^*)\right) - E\left(\frac{u'(c_2)}{u'(c_1)}C'(\int q_I(s)x_s ds)min(\delta_i^* - \bar{\delta}_i^*, F_i)\right)$$
$$+ P'(\int q_I(s)E(x_s|bad)ds)E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)\frac{v'(d)}{u'(c_1)}.$$

Proof. Recall that the nonfinancial firm's problem can be stated as

$$\max_{f_i,F_i}(max[p_H(d_i), p_I(d_i)] + max[p_H(D_i), p_I(D_i)] + max[p_H(E_i), p_I(E_i)], \tag{F.6}$$

$$\text{where } d_i = min(\delta_i^*, f_i), \quad D_i = min(\delta_i^* - d_i, F_i), \quad E_i = max(\delta_i^* - f_i - F_i, 0). \tag{F.7}$$

As shown in the main text, the household is willing to pay more for a riskless payoff than the intermediary, so the firm issues the largest riskless security it can, paying $\bar{\delta}_i^*$. The household will pay $E\left(\frac{u'(c_2)+v'(d)}{u'(c_1)}\bar{\delta}_i^*\right)$ for this security. Condition 1 implies that the firm's debt has lower systematic risk than its equity, and the intermediary is willing to pay more than the household only for low systematic risk assets. The firm therefore optimally chooses the face value of its junior debt so that it is bought by the intermediary, while its equity is bought by the household. Plugging in the expressions in equations (22) and (23) for the intermediary's and the household's willingness to pay for risky assets yields

$$\max_{F_i} E\left(\frac{u'(c_2)+v'(d)}{u'(c_1)}\bar{\delta}_i^*\right) + E\left(\frac{u'(c_2)}{u'(c_1)}P'(\int q_I(s)x_s ds) min(\delta_i^* - \bar{\delta}_i^*, F_i)\right) \tag{F.8}$$

$$+P'(\int q_I(s)E(x_s|bad)ds)E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)\frac{v'(d)}{u'(c_1)} + E\frac{u'(c_2)}{u'(c_1)}max(\delta_i^* - \bar{\delta}_i^* - F_i, 0). \tag{F.9}$$

This yields the desired expression, since $max(\delta_i^* - \bar{\delta}_i^* - F_i, 0) + min(\delta_i^* - \bar{\delta}_i^*, F_i) = \delta_i^* - \bar{\delta}_i^*$ and $P'(\int q_I(s)x_s ds) = 1 - C'(\int q_I(s)x_s ds)$. $\square$

Internet Appendix for "Safety Transformation and the Structure of the Financial System"

William Diamond *

This document contains two internet appendices. The first appendix contains additional results related to the model in the main text but that are not needed to derive results in the main paper. This includes two results. The first result states that it is optimal for firms to issue only debt and equity securities even when they are able to issue any security whose payoff is a monotone increasing function of their own cash flow. The second result is a proof an exact law of large numbers for a continuum of independent random variables that is assumed to hold in the main text. The second appendix presents an infinite horizon model with nominal rigidities. Because of these nominal rigidities, there is a role for a central banker to choose the nominal interest rate. Under the optimal interest rate policy in this model, the first two periods of the model are identical to the two-period model with nominal rigidities presented in the main text that I use to analyze QE at the zero lower bound.

I. Additional Results

A. Optimality of Debt

This appendix presents an enviornment in which it is optimal for nonfinancial firms to issue only debt and equity securities. We assume that firms cannot issue securities whose cash flows depend explicitly on the aggregate state. If firm i's Lucas tree yields a pledgeable payoff of δ_i^*, then the payoff of every security issued by this firm must be a function only of δ_i^*. In addition, these firms face contracting frictions similar to Innes (1990). At time 2, the managers of the firms act in the interest of equityholders and can either destroy the

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firm's value or take out a loan and repay it in the same period if either would increase the payoff to equity. Let $s(\delta_i^*)$ be the sum of all cash flows promised to nonequity investors, so that equity gets the residual claim $\delta_i^* - s(\delta_i^*)$. As shown by Innes (1990), these contracting frictions imply that the firm must issue securities such that both $s(\delta_i^*)$ and $\delta_i^* - s(\delta_i^*)$ are weakly increasing in δ_i^*.

The firm issues securities subject to these frictions in a segmented asset market like that described in the main text above. As in the main text, the investor can sell to two investors with stochastic discount factors $m_1(\omega)$ and $m_2(\omega)$ that depend on a scalar state variable ω that takes on lowest value $\bar{\omega}$. The investor also can sell risk-free assets at a risk-free rate i_d strictly below that implied by either pricing kernel. I assume that $m_1(\omega) - m_2(\omega)$ is strictly increasing in ω. This holds in the main text, since the household will pay more than the intermediary for good-state but not bad-state payoffs. In addition, I generalize Condition 1 above to assume that for $\omega_1 > \omega_2$, $\frac{d}{du}\frac{Pr(\delta_i^* > u|\omega_1)}{Pr(\delta_i^* > u|\omega_2)} > 0$. If ω had binary support, this would be a restatement of Condition 1. If there are any risk-free cash flows backed by $s(\delta_i^*)$, it is optimal to tranche these cash flows into the largest possible riskless payoff and a residual $s(\delta_i^*) - min[s(\delta_i^*)]$ in order to borrow at the low risk-free rate i_d. I examine how to divide the remaining cash flows to sell to the two investors with different pricing kernels. These claims can be written as a payoff $s_e(\delta_i^* - \bar{\delta}_i^*)$ and $s_d(\delta_i^* - \bar{\delta}_i^*)$, where both funtions must be nonnegative and weakly monotone and $s_e(\delta_i^* - \bar{\delta}_i^*) + s_d(\delta_i^* - \bar{\delta}_i^*) = \delta_i^* - \bar{\delta}_i^*$. In particular, both functions are Lipschitz continuous so by the fundamental theorem of calculus they can be written as $\int_0^{\delta_i^* - \bar{\delta}_i^*}(s_e)'(u)du = \int_0^{\infty}(s_e)'(u))\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$ and $\int_0^{\delta_i^* - \bar{\delta}_i^*}(s_d)'(u)du = \int_0^{\infty}(s_d)'(u)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$, where $(s_e)'$ and $(s_d)'$ must be nonnegative and sum to one.

The difference in the value of a claim $\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}$ according to the two investors' pricing kernels is $Em_1\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\} - Em_2\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}$. This equals

$$E\left([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega)\right). \tag{IA.1}$$

The derivative of this expression with respect to u equals

$$E\left([m_1(\omega)-m_2(\omega)]Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)\frac{\frac{d}{du}Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}{Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}\right). \tag{IA.2}$$

Because

$$log(Pr(\delta_i^*>u|\omega_1))=log(Pr(\delta_i^*>u|\omega_1))+log\left[\frac{Pr(\delta_i^*>u|\omega_1)}{Pr(\delta_i^*>u|\omega_2)}\right], \tag{IA.3}$$

we have that

$$\frac{\frac{d}{du}Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega_1)}{Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega_1)}=\frac{\frac{d}{du}Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega_2)}{Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega_2)}+\frac{d}{du}log\left[\frac{Pr(\delta_i^*>u|\omega_1)}{Pr(\delta_i^*>u|\omega_2)}\right]. \tag{IA.4}$$

Because $\frac{d}{du}\frac{Pr(\delta_i^*>u|\omega_1)}{Pr(\delta_i^*>u|\omega_2)}>0$ and thus $\frac{d}{du}log\left[\frac{Pr(\delta_i^*>u|\omega_1)}{Pr(\delta_i^*>u|\omega_2)}\right]>0$, if $\omega_1>\omega_2$, then it follows that $\frac{\frac{d}{du}Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}{Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}$ is strictly increasing in ω. Because $([m_1(\omega)-m_2(\omega)]Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega))$ is also strictly increasing in ω, $cov([m_1(\omega)-m_2(\omega)]Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega), \frac{\frac{d}{du}Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega))}{Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega))})>0$. Thus,

$$E\left[[m_1(\omega)-m_2(\omega)]Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)\frac{\frac{d}{du}Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}{Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}\right] \tag{IA.5}$$

$$>E\left([m_1(\omega)-m_2(\omega)]Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)\right)E\left(\frac{\frac{d}{du}Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}{Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega)}\right). \tag{IA.6}$$

It follows that if $E([m_1(\omega)-m_2(\omega)]Pr(\{\delta_i^*-\bar{\delta}_i^*\geq u\}|\omega))$ is positive, its derivative with respect to u is also positive. Thus, for any value of u for which the payoff $\mathbb{1}\{\delta_i^*-\bar{\delta}_i^*\geq u\}$ is more valued by investor 1 than investor 2, investor 1 also values such a claim more for any higher value of u. It follows that there exists some u^* for which $u>u^*$ implies that this claim is valued more by investor 1, while if $u<u^*$ the claim is valued more by investor 2. The optimal security issued by our nonfinancial firm therefore bundles all such claims for sufficiently low values of u into one security, so $(s_d)'(u)=1$ for $u<u^*$ and $(s_d)'(u)=0$ for $u>u^*$. The payoffs of the two securities issued by the firm are therefore $\int_0^{\delta_i^*-\bar{\delta}_i^*}\mathbb{1}\{u<u^*\}du=min(\delta_i^*-\bar{\delta}_i^*,u^*)$ and $\int_0^{\delta_i^*-\bar{\delta}_i^*}\mathbb{1}\{u>u^*\}du=max(\delta_i^*-\bar{\delta}_i^*-u^*,0)$. These are the payoffs of a debt and an equity security, proving that these are the optimal securities for the firm to issue.

B. Diversification and the Continuum Law of Large Numbers

In the main text, I make the claim that the payoff of the intermediary's portfolio

$$\delta_I = \int_0^1 d_i q_I(d_i) di + \int_0^1 D_i q_I(D_i) di + \int_0^1 E_i q_I(E_i) di, \tag{IA.1}$$

in the good and bad aggregate states is equal to

$$\delta_I^{good} = \int_0^1 E(d_i|good) q_I(d_i) di + \int_0^1 E(D_i|good) q_I(D_i) di + \int_0^1 E(E_i|good) q_I(E_i) di \tag{IA.2}$$

$$\delta_I^{bad} = \int_0^1 E(d_i|bad) q_I(d_i) di + \int_0^1 E(D_i|bad) q_I(D_i) di + \int_0^1 E(E_i|bad) q_I(E_i) di. \tag{IA.3}$$

Conditional on the aggregate state, the cash flows of all nonfinancial firms are independent of each other. This result is therefore equivalent to a continuum law of large numbers for indepedent but not identically distributed random variables. Following Uhlig (1996), I define these integrals as the limit of a sequence of Riemann sums, where the limit is taken in the L^2 norm. Each integral is of the form $\int_0^1 r_i q_I(r_i) di$, where the r_i are a continuum of independent random variables with bounded continous means and bounded variances across $\epsilon \in [0,1]$ and q_I is a bounded Riemann integrable function. Pick a grid $r_{i(1)} \dots r_{i(1)}$ in the unit interval and note that

$$\left[E\left(\sum_{j=2}^{n} r_i q_I(r_i)[i(j) - i(j-1)] - \int_0^1 E(r_i) q_I(r_i) di \right)^2 \right]^{.5} \leq \tag{IA.4}$$

$$\left[E\left(\sum_{j=2}^{n} r_i q_I(r_i)[i(j) - i(j-1)] - \sum_{j=2}^{n} E(r_i) q_I(r_i)[i(j) - i(j-1)] \right)^2 \right]^{.5} \tag{IA.5}$$

$$+ |\sum_{j=2}^{n} E(r_i) q_I(r_i)[i(j) - i(j-1)] - \int_0^1 E(r_i) q_I(r_i) di|. \tag{IA.6}$$

Because $E(r_i)$ is bounded and continuous in i, the second term converges to zero for any Riemann integrable function q_I as the mesh of our grid converges to zero. To compute the first term, note that it is the variance of the sum of independent random variables, so it equals the sum of their variances

$$\sum_{j=2}^{n}(i(j)-i(j-1))^2 q_I(r_i)^2 Var(r_i) \leq sup_i Var(r_i) sup_j |i(j)-i(j-1)|. \tag{IA.7}$$

Because the r_i have uniformly bounded variance, this converges to zero with the mesh of our grid. This proves that our integrals are well defined, and that the expressions in the main text are valid, if they are interpreted as Riemann integrals with an L^2 notion of convergence. Uhlig (1996) obtains similar results.

II. Model with Zero Lower Bound and Nominal Rigidities

A. Infinite-Horizon Model with Nominal Rigidities

I embed the two-period model analyzed in the main paper in an infinite-horizon setting with nominal rigidities. At each time $t > 1$, there are a continuum of Lucas trees indexed by $[0, 1]$ that yield an output $\delta_{i,t}$. The output of all trees indexed by i is the same for every period t, so the aggregate and idiosyncratic shocks to output at $t = 2$ are the only sources of uncertainty. Unlike in the main text, the output of each Lucas tree is now an intermediate good. To produce one unit of final consumption goods, one unit of intermediate goods and one unit of labor are required. Finally, consumption goods sell at a nominal price P that is perfectly rigid over time. In addition to the assets available at time 1 that now pay off at time 2, there is now a risk-free rate $i_{d,t}$ at each time t. For investments made at $t > 1$, this is the only relevant asset price because there is no remaining uncertainty. *Households* At $t = 1$, households are endowed with all trees that pay off at time 2 and consumption resources that provide Y_1 of intermediate goods at time 1. At any time $t = 2$, households are endowed with all Lucas trees that pay off at times $t > 2$. This division ensures that trees paying off after time 2 cannot be used to back assets held by the intermediary at time 1, preserving the results in the main text. Households maximize their expected utility

$$u(c_1) + v(d) + \sum_{t=2}^{\infty} \beta^{t-2} E u(c_t), \tag{IA.1}$$

subject to

$$c_1 - w_1 l_1 + \frac{d}{1+i_d} + \int q_H(s) p_s ds = W_{H,1} \tag{IA.2}$$

$$q_H(.) \geq 0 \tag{IA.3}$$

$$W_{H,2} + d + \int q_H(s) x_s ds = \sum_{t=2}^{\infty} \prod_{\tau=2}^{t-1} \frac{1}{1+i_{d,\tau}} (c_t - w_t l_t). \tag{IA.4}$$

The discount factor β is assumed to be positive and strictly less than one. In addition to the first-order conditions in the main text, maximizing this problem also leads to the first-order conditions

$$\frac{w_t}{P} = 0, \tag{IA.5}$$

and for $t \geq 2$,

$$u'(c_t) = \beta(1 + i_{d,t}) u'(c_{t+1}). \tag{IA.6}$$

The assumption that labor is costless to supply and earns a wage of zero retains the result in the paper that the output of Lucas trees determines consumption. The fact that households discount the future after the first period (but not between periods 1 and 2) preserves the first-order conditions from the main text while ensuring that the zero lower bound does not bind in the future. For tractability, households only have a demand for safe assets in period 1 that pay in period 2, so there is no need for intermediation in future periods. *Nonfinancial Firms* Each nonfinancial firm at time t owns a Lucas tree paying $\delta_{i,t}$. Each unit of output from their tree requires one unit of labor to produce a final good that can be consumed. Nonfinancial firms sell the final goods they produce at the rigid nominal price p, subject to a rationing constraint. If total consumer demand c_t is strictly less than the economy's output

$\int_0^1 \delta_i di$, then each firm is rationed to sell the same fraction of its output as all other firms. Given this rationing rule, the firm is able to sell $c_{i,t}$ units of final output, and at wage 0, it hires $l_{i,t} = c_{i,t}$ units of labor in order to produce this final output.

After $t = 1$, because there are no intermediaries, the capital structure decisions of firms are trivial. At $t = 1$, the security issuance decisions of firms are identical to those in the main text. Similarly, intermediaries are identical to those in the main text.

Implementing the First Best with Interest Rate Policy in t>1 This section shows how the central bank can implement the first best allocation after the first period if the natural rate is positive, as long as there is backing from fiscal authorities to ensure consumption is bounded above zero. Without nominal rigidities, consumption would be $\int_0^1 \delta_i di$ at each time $t = 1$. This would imply a natural interest rate r^* of $i_{d,t} = r^* = \frac{1}{\beta} - 1$. If the central bank simply set the nominal rate equal to this natural rate r^*, the consumption Euler equation would imply that c_t is constant over time without pinning down its level.

To uniquely implement the first-best, the central bank can set an interest rate rule $r_t = r^*$ if $c_t = c^*$ and $r_t = 0$ if $c_t < c^*$ together with an off-equilibrium-path promise to use fiscal policy. Note that we can never have $c_t > c^*$ since there are not enough Lucas trees to produce that much output. If $c_t = c^*$, it follows that $u'(c_t) = \beta(1 + r^*)u'(c_{t+1})$ so $c_{t+1} = c^*$. It also follows that if $c_1 = c^*$, then $c_t = c^*$ for all t, by induction. If $c_t < c^*$, then $u'(c_t) = \beta u'(c_{t+1})$ and $c_{t+1} < c_t < c^*$ as well. By induction, if $c_1 < c^*$, we have that $u'(c_t) = \frac{1}{\beta^t}u'(c_1)$. This implies that c_t converges to zero as t goes to infinity. To rule out such equilibria, suppose that the government is able to produce output with a function $G(\delta_t, l_t)$ that depends on owning Lucas trees that pay δ_t and labor l_t. If the government commits to hiring workers in order to produce a quantity of output $\epsilon > 0$, consumption c_t is bounded away from zero. This uniquely selects the equilibrium that implements the first-best. Because this first-best yields the optimal consumption and labor allocations in each period, it is time-consistent for the government to follow this policy without commitment.

Optimal Interest Rate Policy in the First Period $\boldsymbol{t = 1}$ This section studies interest rate policy at time 1, taking as given that consumption at $t = 2$ will equal the total output that can be produced, $\int_0^1 \delta_i di$. Consumption at time 2 is fixed at its optimal level because the central bank will implement an optimal interest rate policy in the future (when the zero-lower-bound constraint no longer binds). As noted in the main text, the leverage and portfolio decisions of agents in the model are independent of the nominal interest rate chosen at $t = 1$. As a result, only the level of consumption c_1 responds to a change in nominal interest rates. At time $t = 1$, the household's consumption satisfies the first-order condition

$$u'(c_1) = (1 + i_d)\left(E\left[u'\left(\int_0^1 \delta_i di\right)\right] + v'(d)\right). \tag{IA.7}$$

Because the household's consumption at time 2 and the quantity d of riskless assets are not impacted by the choice of the interest rate i_d, the central bank's choice of i_d simply pins down c_1. If $u'(Y_1) > Eu'\left(\int_0^1 \delta_i di\right)$, then the natural interest rate that would induce the household to consume all potential output Y_1 at time 1 is negative. Subject to the zero lower bound, the central bank maximizes household welfare by choosing $i_d = 0$, which is taken as given in the main text. The analysis in the main text of QE at the zero lower bound then follows under this negative natural rate condition.

References for Internet Appendix

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