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An Empirical Bargaining Model with Left-Digit Bias: A Study on Auto Loan Monthly Payments

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Abstract. This paper studies price bargaining when both parties have left-digit bias when processing numbers. The empirical analysis focuses on the auto finance market in the United States, using a large data set of 35 million auto loans. Incorporating left-digit bias in bargaining is motivated by several intriguing observations. The scheduled monthly payments of auto loans bunch at both \$9- and \$0-ending digits, especially over \$100 marks. In addition, \$9-ending loans carry a higher interest rate, and \$0-ending loans have a lower interest rate. We develop a Nash bargaining model that allows for left-digit bias from both consumers and finance managers of auto dealers. Results suggest that both parties are subject to this basic human bias: the perceived difference between \$9- and the next \$0-ending payments is larger than \$1, especially between \$99- and \$00-ending payments. The proposed model can explain the phenomena of payments bunching and differential interest rates for loans with different ending digits. We use counterfactuals to show a nuanced impact of left-digit bias, which can both increase and decrease the payments. Overall, bias from both sides leads to a \$33 increase in average payment per loan compared with a benchmark case with no bias.

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Keywords: bargaining • left-digit bias • auto finance • dealer compensation

1. Introduction

Bargaining is a commonly used price-setting mechanism in many markets, such as automobiles and business-to-business (B-to-B) transactions. In bargaining, final prices vary across transactions instead of being set by one side as fixed posted prices. The two parties in negotiations evaluate the key variable of interest (e.g., price) and reach a bargaining outcome depending on their relative bargaining power. Most of the empirical bargaining literature characterizes the perceived value of the bargaining outcome with a fully rational model (e.g. Draganska et al. 2010). However, people often use simple cognitive shortcuts when processing information, which makes accounting for bounded rationality important in describing economic behaviors (see Conlisk 1996 for a review). In a bargaining setting, decision-makers on both sides are human beings. Behavioral decision researchers have long recognized psychological influence in negotiation, such as status quo bias and reciprocity heuristic (Malhotra and Bazerman 2008) and focal point effects (Pope et al. 2015). Using online bargaining interactions, Backus et al. (2020) find behavioral norms play an important role in the bargaining outcome. Decision-makers may also be subject to bias when evaluating numbers. For example, people have the tendency to focus on the leftmost digit of a number while partially ignoring other digits (Poltrock and Schwartz 1984, Lacetera et al. 2012, Strulov-Shlain 2019). With such left-digit bias, a number with 99-ending (e.g., \$299) may be perceived to be significantly lower than the next round number (e.g., \$300). One consequence of such bias in the marketplace is the ubiquitous 99-cents pricing (Basu 2006, Thomas et al. 2010).

In this paper, we empirically study a bargaining setting where the outcomes are affected by left-digit bias in addition to bargaining power. When *both parties* are influenced by left-digit bias in a negotiation, they will try to push the price toward their favorite side. For example, while buyers deem a price with 99-ending as a lower number, sellers perceive a price a bit higher with 00-ending to be a better deal. This makes the bargaining outcome different from when left-digit bias does not play a role. In this study, we discover several intriguing data patterns using a large data set with 35 million auto loans in the United States.[1] First, the scheduled monthly payments of auto loans bunch at both \$9- and \$0-endings. The bunching pattern is more pronounced over \$100 marks, with more than twice as many loans with \$99-ending and 1.5 times as many loans with \$00-ending, as loans with \$01-ending. Second, while the interest rate for \$9-ending loans is 0.6% higher than the average, the rate for \$0-ending loans is 0.5% lower. These data patterns are difficult to explain by a standard economic model without accounting for the influence of left-digit bias in bargaining. The data patterns are heterogeneous across different consumer groups. For example, consumers from regions with a higher minority proportion (African American or Hispanic) and a lower income are more likely to have \$9-ending loans and pay a higher interest rate than others with a similar credit profile.

The auto finance market provides a perfect setting to study price bargaining. With 107 million Americans carrying an auto loan,[2] the size of the auto finance market makes this study economically important. More importantly, in indirect auto lending, the dealer markup compensation policy leads to negotiations that cause loan payments to vary across transactions. In a standard loan arrangement, banks quote a risk-adjusted interest rate, called the buy rate, based on the consumers' risk profile (e.g., credit score). Auto dealers charge a markup, which represents their compensation for arranging the loan. Unlike the bank buy rate, the markup reflects the relative bargaining power between consumers and finance managers of auto dealers. Thus, loan payments can be viewed as the outcome of price negotiations instead of fixed prices.

We seek to address two main research questions in this study. The first question is to understand how including left-digit bias in bargaining explains the observed bunching and interest rate patterns in the auto finance market. In particular, we show how bias from *both* consumers and finance managers plays a role in the observed patterns. The second question is to evaluate the impact of left-digit bias on the bargaining outcome. This is achieved by building and estimating a bargaining model that incorporates left-digit bias and exploring the effect of bias on bargaining outcomes through a counterfactual analysis.

To answer our research questions, we propose a bargaining model that allows for left-digit bias from both parties and estimate the model using a large-scale auto loan data set. More specifically, given the nature of the dealer compensation policy, we let loan payments arise from a Nash bargaining solution between individual consumers and finance managers. The model allows both parties to have potential left-digit bias when evaluating payment numbers. Note that the bias is not imposed: Depending on the parameter estimates, both parties' perceived payments can reflect the bias or reduce to a standard bargaining model without bias. For model estimation, we use simulated method of moments that match the level of payments and the proportion of payment ending digits between actual and model-simulated data. These moments pin down the parameters associated with the left-digit bias of consumers and finance managers and the bargaining power parameters.

Estimation results suggest that left-digit bias exists not only for consumers but also for finance managers. For consumers, the average perceived difference between \$9- and \$0-ending payments is \$1.22 instead of \$1, and the perceived gap is even higher, at \$2.01, between \$99- and \$00-ending payments. Within each \$10 range, each \$1 increase is perceived to be smaller than \$1. The estimated bias for finance managers is only slightly smaller than that for consumers. Standard economic studies usually assume that companies are fully rational entities in making business decisions. In this setting, however, finance managers who represent auto dealers are also human beings, who can be subject to the same human tendency with numbers. This study thus adds to the existing literature that documents psychological bias among professionals, such as lawyers (Birke and Fox 1999), professional traders (Coval and Shumway 2005), dealers in used-car auctions (Lacetera et al. 2016), and managers in a multinational corporation (Workman 2012). It is worth noting that the bias for consumers and finance managers is exactly the same type of bias, where a \$9-ending number is perceived to be substantially smaller than the next \$0-ending number. They are, however, on the opposite side of bargaining: While consumers prefer to *pay* a lower payment (\$9-ending), finance managers prefer to *receive* a higher payment (\$0-ending).

How does incorporating the left-digit bias from both sides in bargaining contribute to the observed data patterns? Bias from consumers and finance managers creates a discontinuity in perceived payments at every \$10 mark between \$9- and \$0-ending, leading to an excess number of loans at both \$9- and \$0-ending payments. In particular, the consumer bias leads to bunching at \$9-ending, and finance manager bias leads to bunching at \$0-ending. The interest rate patterns are driven by the bargaining power for those with \$9- versus \$0-ending loans. Lower bargaining consumers, who will receive a higher interest rate, are more likely to get \$9-ending payments; in contrast, higher bargaining power consumers are more likely to get \$0-ending payments and have a lower interest rate.

With the model estimates, we explore the impact of left-digit bias on payments using counterfactual analysis. Behavioral biases are typically thought to make people worse off, and researchers propose ways to debias consumers for a better decision-making strategy (Larrick 2004). The impact of left-digit bias in bargaining is nuanced. For example, we find that consumer bias can both increase and decrease the payments compared with a benchmark case with no bias.

Consumer bias induces two types of effects. The first effect comes from the nonstandard perceived payment: The larger perceived difference between \$9- and \$0-ending makes it more difficult to increase payments from a \$9-ending number, which can lead to a lower payment, while the smaller perceived difference within \$10 range makes it easier for finance managers to push up the payments, which can lead to a higher payment. The second is a level effect: The perceived payments are lower with consumer bias (except at the hundreds); therefore, with the same bargaining power, payments become higher to achieve the same division of total surplus from bargaining. The effect of finance manager bias on payments has a similar logic. When both parties have the estimated bias, the loan payments will increase by 0.13%, or \$33 per loan, compared with a benchmark scenario where neither party has the bias.

This paper is related to the literature in bargaining, numerical cognition, and 9-ending prices, as well as studies of the bunching phenomenon. The prior literature uses bargaining models to study automotive sales and auto financing (Chen et al. 2008, Morton et al. 2011, Jiang et al. 2020, Larsen 2020), B-to-B transactions (Draganska et al. 2010, Grennan 2014, Zhang and Chung 2020), and interactions between online sellers and buyers (Zhang et al. 2018, Backus et al. 2019), all of which assume fully rational agents. This paper contributes to the empirical bargaining literature by studying how left-digit bias from both sides influences bargaining outcomes. We show that considering left-digit bias is essential in explaining the reduced-form data patterns in bargaining outcomes. The insights could generalize to other settings where the bargaining happens between two individuals and the bargaining outcomes are numeric.

This paper also draws from the literature on numerical cognition and the marketing literature on 9-ending prices. The numerical cognition literature in psychology primarily focuses on the differences in behavioral perception between round and precise numbers. Past research has shown that buyers may underestimate the magnitude of precise prices (Thomas et al. 2010) and that precise numbers signal sellers' confidence (Jerez-Fernandez et al. 2014). However, offers at round numbers can symbolize completion (Yan and Pena-Marin 2017) and willingness to cut prices (Backus et al. 2019). This paper also relates to the marketing literature that studies the prevalence of 9-ending prices in retail sales (e.g., Monroe 1973, Schindler and Kibarian 1996, Stiving and Winer 1997, Anderson and Simester 2003, Thomas and Morwitz 2005, Strulov-Shlain 2019). Generally, 9-ending prices are found to increase sales, because consumers round down the prices or the prices signal a low-price image. This phenomenon is not limited to prices only. Lacetera et al. (2012) find a discontinuous drop in used car prices when the odometer crosses 10,000 miles, driven by consumers' left-digit bias when processing mileage. In this paper, we study left-digit bias in a bargaining setting with an economically significant purchase. We find that not only consumers but also finance managers are subject to such bias.

This paper is also related to the economic literature that studies the bunching phenomena. Bunching is commonly observed at the level where discontinuities in monetary incentives occur, such as income bunching at the level where the tax rate changes (Saez 2010) and drug demand bunching at the level where insurance payments jump (Einav et al. 2015). Bunching can also be driven by psychological incentives. For example, Allen et al. (2016) find the finishing times of marathon races bunch before hour marks, which serve as a reference point. In the above examples, bunching occurs because consumers make one-sided decisions driven by the incentive discontinuity. This paper studies the bunching phenomenon with consumers and finance managers bargaining on auto loan payments. It leads to payments bunching at both \$9- and \$0-ending digits with systematically different interest rates in the opposite direction.

The rest of the paper is organized as follows. Section 2 introduces the auto finance industry background and presents reduced-form data patterns. We describe the bargaining model incorporating left-digit bias in Section 3 and discuss the model estimation in Section 4. Section 5 presents the estimation results and findings from the counterfactual analysis. Finally, Section 6 concludes.

2. Industry Background and Data

The auto finance market is of high economic significance. With a \$1.2 trillion balance in 2017, auto loans represent the third-largest consumer credit market in the United States.[3] Consumers typically obtain financing through auto dealers (i.e., indirect auto loans). Cohen (2012) shows that about 80% of auto loans are originated at a dealer location following the purchase of a new or used vehicle. Indirect auto loans are a significant source of profit for dealers.

This paper focuses on cases where consumers get auto loans from a traditional bank *through an auto dealer*. In a typical transaction, the consumer first chooses a car and negotiates on the car price itself. After that, she will be brought to the finance manager's office to arrange auto financing. This paper aims to study how the monthly payment number is determined after consumers have selected the loan amount and the loan length. Why is the monthly payment a bargained outcome? This is because finance managers add a discretionary markup on top of the bank buy rate, which serves as the dealer compensation for arranging the loan.[4] Note that, after loan amount and length are determined, the monthly payment and interest rate have a one-to-one relationship—a higher interest rate will imply a higher monthly payment and vice versa.

Because of the markup policy, the finance manager has an incentive to increase the loan payment so that the dealer will receive a higher markup. Yet the consumer can negotiate for a lower payment if she finds the payment too high. A report by the Center for Responsible Lending estimates that the average markup is $714 per transaction using the 2009 auto industry data, and the markup varies across individual consumers. Consistent with these results, we find that auto loans' interest rates vary for consumers with the same credit profile and loan characteristics in our data.

2.1. Data Description

The empirical analysis of this paper leverages anonymized data on individual credit profiles provided by Equifax Inc., one of the three major credit bureaus in the United States. The data sample includes all nonsubprime auto loans originated from banks or credit unions in the United States from 2011 to 2014.[5] We observe the origination date, loan amount, loan length, and scheduled monthly payment for each auto loan in the sample. The annual percentage rate (APR) can be calculated from these loan attributes.[6] To remove potential outliers, we select auto loans with loan lengths from two to eight years, loan amount between $10k and $60k, and APRs above 1.9%. Detailed data-cleaning procedures, including how we calculate the interest rate, are described in Online Appendix A.1. The data sample includes 35 million auto loans. Panel A of Table 1 shows some descriptive statistics of the loan characteristics. The average loan amount is about $23,000, with a $399 monthly payment for about five and a half years, and the average APR is 4.8%.

For consumer characteristics, we observe the credit score and age of each consumer and the five-digit zip code of her living place. The credit score is measured at the month of auto loan origination. We further obtain some zip-code level data, including the average household income and racial composition, from the Census. The racial composition data measure the proportion of the population that is Caucasian, African American, Hispanic, Asian, or others in a zip code. We use the income and racial composition data to proxy for the consumer characteristics based on location. Panel B of Table 1 shows some descriptive statistics of these variables. An average consumer in the data sample is 46 years old, has a 726 credit score, and lives in an area with an average of $83.7k household income, 73.3% Caucasians, 9.7% Hispanics, and 8.9% African Americans.

2.2. Reduced-Form Data Analysis

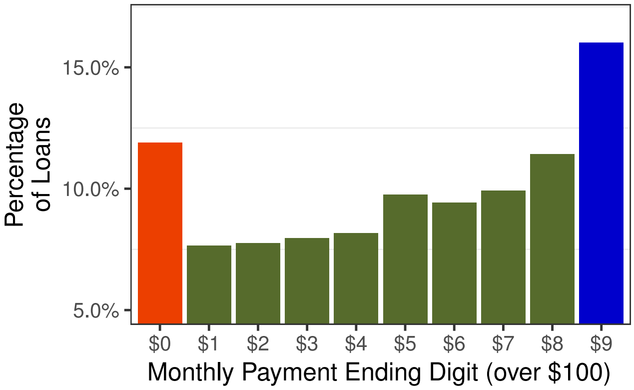
The Bunching Phenomenon. We illustrate the bunching pattern in monthly loan payments. Scheduled monthly payments bunch at both $9- and $0-endings. The bunching pattern is more significant at $100 marks. Moreover, the level of $9-ending bunching varies systematically across different consumer groups.

Figure 1 plots the frequency of the monthly payment ending digit when payments cross $100. Each bar represents the percentage of loans with ending digit from $0 to $9. Instead of a uniform distribution of 10% probability for each number, there are more loans with $9-ending payments as well as $0-ending payments.[7] When payments cross $100 marks, $9-ending payments are more than twice as likely and

Table 1. Summary Statistics

	Mean	25th percentile	Median	75th percentile
Panel A: Loan characteristics				
Loan amount	$22,965	$15,821	$21,161	$28,115
Loan length (years)	5.4	5	5.8	6
Monthly payment	$399	$294	$370	$475
APR	4.8%	3.0%	4.0%	5.5%
Panel B: Consumer characteristics				
Credit score (620–850)	726	674	725	778
Age	46	33	45	56
Income	$83,749	$56,578	$74,659	$101,062
Caucasian	73.3%	60.3%	78.7%	90.4%
Hispanic	9.7%	2.5%	5.6%	13.1%
African American	8.9%	1.4%	4.0%	10.4%
Asian	4.0%	0.9%	2.0%	4.5%
Other	4.1%	0.9%	2.2%	5.4%

Figure 1. (Color online) Frequency of Monthly Payment Ending Digit



$0-ending payments are 1.5 times as likely as payments ending with $01. The bunching pattern is similar, although less pronounced, at other $10 marks, where $9-ending ($0-ending) payments are 30% (12%) more likely than $1-ending payments.[8]

Consumers with $9-ending monthly payments and those with $0-ending payments have systematically different consumer characteristics. Panel A in Table 2 shows the ratio of the number of $99-ending over the next $01-ending loans (e.g., $399/$401) across different consumer groups. The $9-ending bunching is higher among consumers with lower credit scores, older ages, and living in areas with lower incomes and larger minority populations. Panel B in Table 2 shows the ratio of the number of $00-ending loans over the next $01-ending loans (e.g., $400/$401). Opposite to $9-ending, the $0-ending bunching is higher among consumers with higher credit scores.

Interest Rates. We find a systematic difference in loan interest rates with different ending digits. Table 3, Column 2 shows that the average interest rate for $9-ending loans is 0.6% higher than the average, while the interest rate for $0-ending loans is 0.5% lower. The difference in loan characteristics drives part of the interest rate gap. For example, Column 3 shows that consumers with $9-ending payments tend to have lower credit scores than those with $0-ending payments, which will lead to a difference in interest rates. Moreover, there are also systematic differences in other loan characteristics: On average, loans with $9-ending payments have a larger loan amount and a longer loan length compared with those with $0-ending payments (Columns 4 and 5).

To control for the impact of other relevant factors, we use regression analysis to further investigate the difference in interest rates for loans with different ending digits:

$$int_i = \sum_{j=1}^{9} \gamma_j \cdot I\left[d\left(payment_i\right) = j\right] + \boldsymbol{x}_i\boldsymbol{\beta} + \epsilon_i,$$

Table 2. Heterogeneous Levels of Payment Bunching at $99- and $00-Endings

Panel A: The ratio of $99-ending loans to $01-ending loans (overall ratio: 2.08)					
Credit score	620–660	661–700	701–740	741–780	781–850
	2.27	2.21	2.10	2.02	1.90
Age	<30	31–40	41–50	51–60	>60
	2.03	2.08	2.10	2.09	2.13
Income	< $50k	$50k–$70k	$70k–$90k	$90k–$120k	> $120k
(zip-level)	2.23	2.07	2.08	2.07	1.97
Caucasian proportion	<50%	50%–70%	70%–80%	80%–90%	>90%
(zip-level)	2.31	2.14	2.08	2.00	1.98
Hispanic proportion	<2%	2%–5%	5%–10%	10%–20%	>20%
(zip-level)	2.04	2.04	2.05	2.12	2.24
African American proportion	<2%	2%–5%	5%–10%	10%–20%	>20%
(zip-level)	1.93	2.05	2.16	2.17	2.36
Panel B: The ratio of $00-ending loans to $01-ending loans (overall ratio: 1.55)					
Credit score	620–660	661–700	701–740	741–780	781–850
	1.50	1.50	1.54	1.57	1.61
Age	<30	31–40	41–50	51–60	>60
	1.48	1.55	1.56	1.56	1.59
Income	< $50k	$50k–$70k	$70k–$90k	$90k–$120k	> $120k
(zip-level)	1.57	1.56	1.54	1.54	1.52
Caucasian proportion	<50%	50%–70%	70%–80%	80%–90%	>90%
(zip-level)	1.58	1.54	1.52	1.55	1.55
Hispanic proportion	<2%	2%–5%	5%–10%	10%–20%	>20%
(zip-level)	1.56	1.54	1.53	1.54	1.59
African American proportion	<2%	2%–5%	5%–10%	10%–20%	>20%
(zip-level)	1.56	1.55	1.55	1.52	1.53

Table 3. Characteristics for Loans with Different Ending Digits

Ending digits	APR	Credit score	Loan amount ($1,000)	Loan length (years)
(1)	(2)	(3)	(4)	(5)
$5	4.785%	725.52	22.97	5.45
$6	4.778%	725.90	22.90	5.44
$7	4.791%	725.66	22.96	5.45
$8	4.804%	725.46	23.04	5.46
$9	4.847%	724.20	23.34	5.54
$0	4.754%	726.23	22.82	5.42
$1	4.761%	726.29	22.84	5.41
$2	4.770%	726.18	22.84	5.41
$3	4.776%	726.10	22.88	5.42
$4	4.787%	725.86	22.93	5.44

where int_i is the interest rate of loan i, and $I[d(payment_i) = j]$ is an indicator variable that equals 1 if the ending digit of the monthly payment is j (j is from 1 to 9, with 0 as the normalized factor). The variable $\boldsymbol{x}_i$ includes credit score, loan amount, and loan length. We also include date and state fixed effects for each loan. The results are reported in Table 4. To capture the potential nonlinearity of the relationship between the interest rate and covariates $\boldsymbol{x}_i$, Columns 1 and 3 use linear and quadratic terms of these variables, while Columns 2 and 4 categorize them into bins and use bin fixed effects. Columns 3 and 4 also include additional consumer characteristics, including age and zip-code-level income and ethnicity data.

Across different specifications, $9-ending loans consistently carry the highest interest rate, about 0.053% higher than $0-ending loans.[9] To put the numbers in perspective, this difference would result in a $36 higher cost for consumers for a five-year $25,000 loan with 6% APR. As the coefficients for $1-ending to $9-ending are all significant and positive, it implies that $0-ending payments have the lowest interest rate. Figure 2 visually presents the regression results from Column 1, which show a clear gap in the interest rate between $9- and $0-ending loans. As a side note, while not a focus in this paper, we see consistent patterns with $5-ending payments: The interest rate of $4-ending loans is higher than that of $5-ending loans. This is likely due to $5-ending payments being perceived as round numbers, similar to $0-ending payments, and $4-ending payments are just below a round number, similar to $9-ending payments.

Columns 3 and 4 of Table 4 also include consumer demographic variables in the regression. Results suggest that, after controlling for loan characteristics, including credit scores, there exists a systematic relationship between consumer characteristics and loan interest rates. In particular, consumers of older age and those from zip codes with a lower income and a higher minority population are more likely to have a higher interest rate. Column 5 further interacts the $9-ending dummy with the consumer demographics variables, and the results are robust. Since banks do not use these characteristics when deciding the buy rate, the interest rate gap reflects the difference in the discretionary dealer markup.[10] This pattern is consistent with studies that found disadvantaged consumers, such as minority consumers, pay a higher dealer markup than their white counterparts (Hudson et al. 2001, Cohen 2012).

3. Model

We propose a bargaining model that involves consumers and finance managers of auto dealers. Importantly, it allows for left-digit bias from both parties. The proposed model can explain the bunching phenomenon and the differential interest rates discussed in Section 2.2.

3.1. Perceived Payment

With left-digit bias, the perceived payment can be different from the actual payment. We decompose the payment number into the hundreds, tens, and single digits: The hundreds of a payment p is $\lfloor p \rfloor_{100} = \lfloor \frac{p}{100} \rfloor \cdot 100$, the tens of a payment p is $\lfloor p \rfloor_{10} = \lfloor \frac{p - \lfloor p \rfloor_{100}}{10} \rfloor \cdot 10$, and the single digit is $\lfloor p \rfloor_1 = p - \lfloor p \rfloor_{100} - \lfloor p \rfloor_{10}$. For example, for $p = 382$, $\lfloor p \rfloor_{100} = 300$, $\lfloor p \rfloor_{10} = 80$, and $\lfloor p \rfloor_1 = 2$. Following the prior literature on modeling left-digit bias (e.g., Lacetera et al. 2012, Strulov-Shlain 2019), let the perceived payment be

$$\hat{p} = \lfloor p \rfloor_{100} + (1 - \theta_1)\lfloor p \rfloor_{10} + (1 - \theta_1)(1 - \theta_2)\lfloor p \rfloor_1, \quad (1)$$

where θ_1 captures the perceived bias for the tens digit, and θ_2 allows for increased bias for the single digit. The bias parameters are defined between 0 and 1. Note the specification does not impose a bias: When $\theta_1 = \theta_2 = 0$, there is no bias, and the perceived payment is the same as the actual. At the other extreme, all numbers after the hundreds digit are completely ignored when $\theta_1 = 1$, and all the single digits are

Table 4. Interest Rate Regression Results

	Dependent variable:				
	APR (%)				
	(1)	(2)	(3)	(4)	(5)
\$1-ending	0.0116***	0.0131***	0.0130***	0.0143***	0.0130***
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)
\$2-ending	0.0185***	0.0193***	0.0194***	0.0200***	0.0194***
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)
\$3-ending	0.0225***	0.0237***	0.0235***	0.0245***	0.0235***
	(0.0018)	(0.0018)	(0.0018)	(0.0018)	(0.0018)
\$4-ending	0.0290***	0.0310***	0.0290***	0.0309***	0.0290***
	(0.0018)	(0.0018)	(0.0018)	(0.0017)	(0.0018)
\$5-ending	0.0174***	0.0208***	0.0166***	0.0200***	0.0166***
	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)
\$6-ending	0.0187***	0.0219***	0.0191***	0.0221***	0.0191***
	(0.0018)	(0.0017)	(0.0018)	(0.0017)	(0.0018)
\$7-ending	0.0277***	0.0308***	0.0275***	0.0305***	0.0275***
	(0.0018)	(0.0017)	(0.0018)	(0.0017)	(0.0018)
\$8-ending	0.0365***	0.0403***	0.0355***	0.0393***	0.0355***
	(0.0017)	(0.0017)	(0.0017)	(0.0017)	(0.0017)
\$9-ending	0.0523***	0.0614***	0.0478***	0.0571***	0.0422***
	(0.0017)	(0.0017)	(0.0017)	(0.0016)	(0.0054)
Age (in 100)			0.3476***	0.3413***	0.3452***
			(0.0029)	(0.0028)	(0.0030)
Income (in \$1 million)			−3.371***	−3.3371***	−3.3574***
(zip-level)			(0.0110)	(0.0108)	(0.0116)
African American proportion			1.0671***	0.9855***	1.0618***
(zip-level)			(0.0035)	(0.0034)	(0.0037)
Hispanic proportion			1.4828***	1.3884***	1.4791***
(zip-level)			(0.0056)	(0.0055)	(0.0058)
\$9-ending: Age (in 100)					0.0188**
					(0.0081)
\$9-ending: Income (in \$1 million)					−0.1148***
(zip-level)					(0.0317)
\$9-ending: African American proportion					0.0407***
(zip-level)					(0.0091)
\$9-ending: Hispanic proportion					0.0291**
(zip-level)					(0.0118)
Covariates *x*	Quadratic	Categorical	Quadratic	Categorical	Quadratic
Date opened fixed effects	Yes	Yes	Yes	Yes	Yes
State fixed effects	Yes	Yes	Yes	Yes	Yes
Observations	34,760,946	34,760,946	34,760,577	34,760,577	34,760,577
R^2	0.3002	0.3173	0.3083	0.3246	0.3083

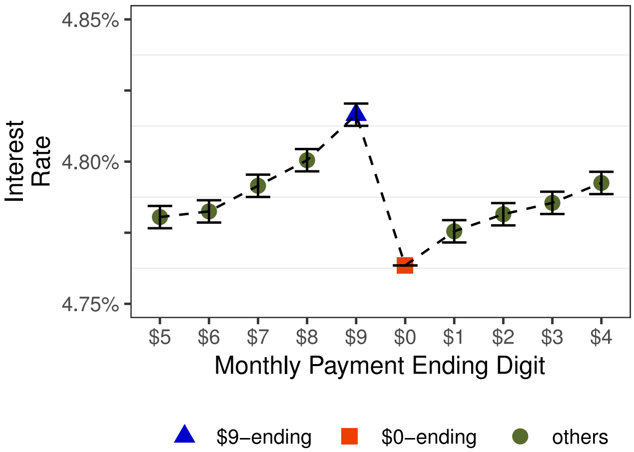
Notes. Covariates X include credit score, loan amount, and loan length. Results from two specifications are shown: Columns 1, 3, and 5 use linear and quadratic terms, and Columns 2 and 4 categorize each covariate into bins and use bin fixed effects.

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

ignored when $\theta_2 = 1$. With this specification, the perceived payment is smaller than the actual, except when payments are exactly at \$100 marks, for example, \$400, and the perceived and actual payments are the same.

With the bias, the perceived difference for \$1 within the \$10 range (when $\lfloor p \rfloor_{10}$ does not change) is $(1 - \theta_1)(1 - \theta_2)$. When payments cross each \$10 mark ($\lfloor p \rfloor_{10}$ changes), there is a discontinuous change with the perceived difference for \$1 between \$9- and \$0-ending at $(1 - \theta_1)(1 + 9\theta_2)$. When payments cross \$100 marks ($\lfloor p \rfloor_{100}$ changes), the perceived difference for \$1 change is even larger. This is illustrated graphically in Figure 3. The blue lines plot the perceived payment, which has a kink each time payments cross \$10 marks, with a larger gap at \$100. The discontinuity in perceived payments is larger if the agent is more biased. This is illustrated by Figure 3, where the left chart

Figure 2. (Color online) Interest Rate for Loans with Different Ending Digits



assumes a lower bias ($\boldsymbol{\theta} = 0.1$) with smaller gaps between \$9- and \$0-ending numbers than the right chart ($\boldsymbol{\theta} = 0.15$).

The level of left-digit bias can be different for consumers and finance managers, which is captured by a separate bias term $\boldsymbol{\theta}_c$ for consumers and $\boldsymbol{\theta}_f$ for finance managers. Moreover, the bias can be heterogeneous across different consumer groups. For example, minority or lower-income consumers are more likely to get \$9-ending loans than others (see Table 2), which could be driven by the level of bias. Note that the level of \$9-ending bunching is jointly determined by bias and bargaining power. We discuss how to separately identify heterogeneous consumer bias and bargaining power in Section 4.1.

Let the consumer left-digit bias be a function of the observed consumer characteristics:

$$\boldsymbol{\theta}_{c,i} = \mathbb{L}(\boldsymbol{x}_i \boldsymbol{\gamma}). \tag{2}$$

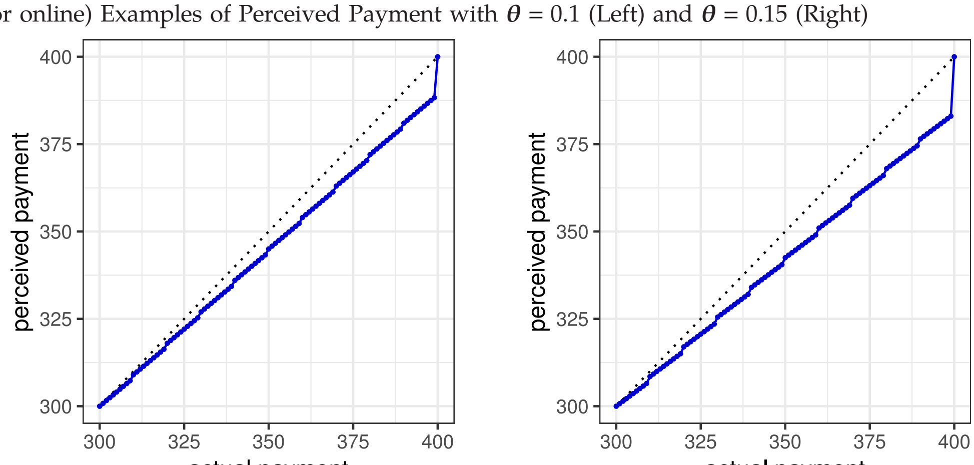
Here $\mathbb{L}$ is the logistic function $\mathbb{L}(x) \equiv e^x/(1+e^x)$ which ensures the bias term $\boldsymbol{\theta}_c$ to be between 0 and 1, and $\boldsymbol{x}_i$ includes a constant term and a vector of consumer characteristics: credit score, age, zip-code-level household income, and the proportion of African Americans and Hispanics in the population. We allow the constant term to be different for $\theta_{c,1}$ and $\theta_{c,2}$, and the rest of the $\boldsymbol{\gamma}$ parameters are assumed to be the same for the two bias terms. Such specification allows the level of bias toward tens and single digits to be different, but if some consumers are more biased toward the tens digit, they are also more likely to be biased toward the single digit.

3.2. Nash Bargaining

We treat the realized monthly payments as the bargaining outcome between consumers and finance managers. In the auto finance market, there is considerable variation in price conditional on observed credit profile. Such price dispersion is consistent with the discretionary dealer markup compensation (see the industry background in Section 2). We use the Nash bargaining model to describe how the outcome arises based on the relative bargaining power between two parties.[11] The Nash bargaining solution is a convenient way to characterize the outcome when we do not observe information on the actual bargaining process or cases where the bargaining failed.

We assume that, by the time of discussing the loan arrangement, consumers have already chosen the car they want to buy and have agreed on the car price with the dealer. Besides, consumers have decided on the loan amount and loan length. What is left for bargaining is the monthly payment, which will determine the loan interest rate.[12] More specifically, we let the monthly payment be the Nash bargaining outcome between consumers and finance managers for each loan, conditional on loan amount and length. Note that, given loan amount and length, loans with a higher monthly payment will also have a higher interest rate. We assume that monthly payments instead

Figure 3. (Color online) Examples of Perceived Payment with $\boldsymbol{\theta} = 0.1$ (Left) and $\boldsymbol{\theta} = 0.15$ (Right)



of interest rates are the focal point of bargaining because consumers tend to focus on the monthly payment number, much more so than the interest rate, when considering auto loans (e.g., Attanasio et al. 2008, Karlan and Zinman 2008, Argyle et al. 2020a). Moreover, doing so also allows us to specify bias on the monthly payment numbers to explain the reduced-form data patterns discussed in Section 2.2.

The key assumption behind the Nash bargaining solution concept is that, for auto loan i, the monthly payment p_i observed from the data will maximize a joint-value function as follows:

$$v(p_i) = u_c(p_i)^{\omega_i} \cdot u_f(p_i)^{1-\omega_i}, \tag{3}$$

where ω_i is the consumer's relative bargaining power, ranging from 0 to 1, and the finance manager's bargaining power is $1-\omega_i$. Here $u_c(p_i)$ represents the surplus for the consumer:

$$u_c(p_i) = r_{i,c} - \hat{p}_i(\boldsymbol{\theta}_c), \tag{4}$$

where $r_{i,c}$ is the consumer's reservation value and $\hat{p}_i(\boldsymbol{\theta}_c)$ is the consumer's perceived payment as defined in Section 3.1. Then $u_f(p_i)$ is the surplus for the finance manager:

$$u_f(p_i) = \hat{p}_i(\boldsymbol{\theta}_f) - r_{i,f}, \tag{5}$$

where $\hat{p}_i(\boldsymbol{\theta}_f)$ is the finance manager's perceived payment and $r_{i,f}$ is the finance manager's reservation value. To maximize the joint-value function, p_i has to be within the range where both consumers and finance managers enjoy a positive surplus. The larger the bargaining power of one party, the larger the surplus they will gain from the bargaining.

To model the reservation prices, we use the institutional detail that a bank will offer the finance managers a bank buy rate based on the loan and consumer characteristics $\boldsymbol{x}_i$ (e.g., credit score). This bank buy rate determines a monthly payment $\underline{p}(\boldsymbol{x}_i)$. We assume that finance managers will not accept a monthly payment lower than $\underline{p}(\boldsymbol{x}_i)$,[13] and therefore,

$$r_{i,f} = \underline{p}(\boldsymbol{x}_i). \tag{6}$$

Next, we assume there is a maximum interest rate that the consumer can obtain from outside sources (e.g., a personal loan), which determines a monthly payment $\bar{p}(\boldsymbol{x}_i)$. The consumer will not accept a monthly payment higher than $\bar{p}(\boldsymbol{x}_i)$. Therefore, the reservation price for consumers is

$$r_{i,c} = \bar{p}(\boldsymbol{x}_i). \tag{7}$$

Finally, the relative bargaining power ω_i in Equation (3) can be heterogeneous across different consumer groups. For example, minority or lower-income consumers may be more likely to have lower bargaining power and tend to pay more than others for the same type of loan. The consumer bargaining power is specified as

$$\omega_i = \mathbb{L}(\boldsymbol{x}_i\boldsymbol{\beta} + \epsilon_i). \tag{8}$$

Here $\mathbb{L}$ is the logistic function defined the same as above, and $\boldsymbol{x}_i$ includes a constant term and a vector of consumer characteristics, including credit score, age, zip-code-level household income, and the proportion of African Americans and Hispanics in the population. The stochastic component ϵ_i captures the heterogeneity in bargaining power beyond what is explained by $\boldsymbol{x}_i$. We assume that it follows a normal distribution, that is, $\epsilon_i \sim N(0, \sigma_\epsilon^2)$. The parameters $\boldsymbol{\beta}$ and σ_ϵ govern the distribution of bargaining power in the consumer population.[14]

3.3. Nash Bargaining Solution with Left-Digit Bias

We present the Nash bargaining solution with left-digit bias. With the bias, the perceived payment $\hat{p}$ (hence, the joint value function $v(p)$) is discontinuous when payment crosses each \$10 mark. With the discontinuity in the objective function, the solution has a similar flavor as that of Strulov-Shlain (2019), who solves the firm's optimal pricing problem when consumers have left-digit bias in a supermarket setting. For ease of notation, we omit the consumer subscript i in describing the Nash solution concept.

Proposition 1. *Conditional on bargaining power ω, biases $\boldsymbol{\theta}_c$ and $\boldsymbol{\theta}_f$, and reservation values r_c and r_f, find the appropriate* zero-ending number $Q = \lfloor Q \rfloor_{100} + \lfloor Q \rfloor_{10}$ *such that $\omega \in (\bar{\omega}_{Q+9}, \bar{\omega}_{Q-1}]$. Then the Nash bargaining solution (for $\boldsymbol{\theta}$'s not too large) is*

$$p = \begin{cases} Q-1 & \omega \in [\bar{\omega}_Q, \bar{\omega}_{Q-1}], \\ Q & \omega \in (\bar{\omega}_{Q+1}, \bar{\omega}_Q), \\ p^\star & \omega \in (\bar{\omega}_{Q+9}, \bar{\omega}_{Q+1}], \end{cases} \tag{9}$$

where $p^\star$ is the interior solution when the payment that maximizes the joint value function lands within a \$10 *range (i.e., the perceived payment is continuous):*

$$\begin{aligned} p^\star = {} & \frac{(1-\omega)r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} + \frac{\omega r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} \\ & + \lfloor p \rfloor_{100}\left(1 - \frac{1-\omega}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{\omega}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) \\ & + \lfloor p \rfloor_{10}\left(1 - \frac{1-\omega}{1-\theta_{c,2}} - \frac{\omega}{1-\theta_{f,2}}\right). \end{aligned}$$

The range of bargaining power $(\bar{\omega}_{Q+9}, \bar{\omega}_{Q-1}]$ corresponds to the Nash bargaining solution belonging to

$[Q-1, Q+9)$, where Q is a zero-ending number. We can calculate the cutoff points defining the bound $\bar{\omega}_{Q-1}$ and $\bar{\omega}_{Q+9}$ (as well as $\bar{\omega}_{Q+1}$ in Equation (9)) as

$$\bar{\omega}_p = \frac{\frac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \lfloor p \rfloor_{100}\left(\frac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - 1\right) - \lfloor p \rfloor_{10}\left(\frac{1}{1-\theta_{c,2}} - 1\right) - p}{\frac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} - \lfloor p \rfloor_{100}\left(\frac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{1}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) - \lfloor p \rfloor_{10}\left(\frac{1}{1-\theta_{c,2}} - \frac{1}{1-\theta_{f,2}}\right)}$$

for payment p. These are the bargaining power points where the interior solution $p^{\star} = p$. Detailed derivations appear in Appendix A.1.

Next we solve for $\bar{\omega}_Q$, which is the bargaining power cutoff where the payment will switch from $Q-1$ to a point in the segment $[Q, Q+9)$. We cannot use the first-order condition to solve $\bar{\omega}_Q$ because of the utility discontinuity when the payment crosses \$10 marks from $Q-1$ to Q. The cutoff point $\bar{\omega}_Q$ is the lowest bargaining power that payment will end up at $Q-1$:

$$\bar{\omega}_Q = \arg\min_{\omega} v(p^{Q-}) \geq v(p^{Q+}),$$

where p^{Q+} is the payment from the range $[Q, Q+9)$ that has the highest joint value function, and p^{Q-} is the payment from the range $[Q-1, Q)$ that has the highest joint value function. Note that both p^{Q+} and p^{Q-} can be either an interior solution or a corner solution.

When the bias is very large, there could be missing numbers. For example, in an extreme case, if consumers completely disregard the ending digit so that Q is perceived to be the same as any number from $Q+1$ to $Q+9$, then it is always optimal to end up with a 9-ending payment. This is because finance managers gain from payment moving Q to $Q+9$, and consumers are indifferent. Similarly, when the finance manager bias is very large, there could be missing payments on the large ending digits. This is because finance managers do not suffer a loss moving from $Q+9$ to Q, but consumers prefer a lower payment. The adjustment for the Nash bargaining solution with large bias appears in Appendix A.1.

Payment Bunching. The bargaining model with left-digit bias can rationalize the data pattern of payment bunching. The intuition is that, with a large perceived difference between \$9- and \$0-ending payments for consumers, it is hard for finance managers to increase payments from \$9- to \$0-ending or beyond. Therefore, there are more loans with payments bunched at \$9-ending. Likewise, a large perceived difference for the finance managers from \$0 to \$9 makes it hard for consumers to bargain down from \$0-ending payments. This will lead to more payments to bunch at \$0-ending. Only when both consumers and finance managers have the left-digit bias, payments can bunch at both \$9- and \$0-ending. Intuitively, this is because the bias from both sides creates friction when payments go from \$9- to \$0-ending, as well as from \$0- to \$9-ending. Furthermore, it is easy to see that, all else being equal, the number of \$9-ending payments increases as the consumer bias becomes larger. And similarly, the number of \$0-ending payments increases as the finance manager bias becomes larger.

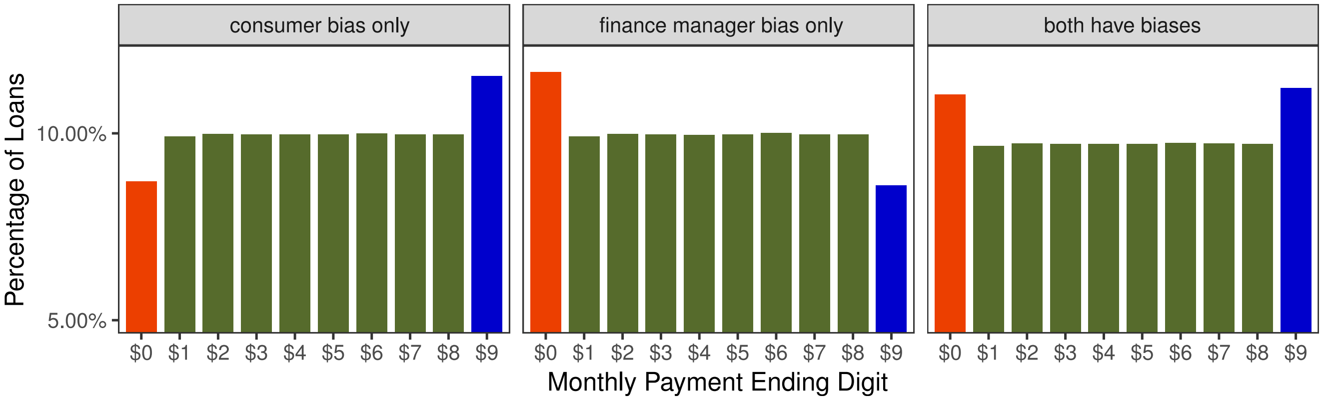
We show the connection of left-digit bias and payment bunching with simulation. First, we simulate the case where only consumers have left-digit bias, $\boldsymbol{\theta}_c > 0$. Payments are more likely to bunch at \$9-ending, which is shown in the left chart in Figure 4. Similarly, when only finance managers have left-digit bias, $\boldsymbol{\theta}_f > 0$, payments are more likely to bunch at \$0-ending (middle chart). Finally, when both consumers and finance managers have left-digit bias, $\boldsymbol{\theta}_c > 0$ and $\boldsymbol{\theta}_f > 0$, payments can bunch at both \$9- and \$0-endings (right chart).

Differential Interest Rate. The systematic interest rate difference between \$9- and \$0-ending loans reflects the bargaining power difference for consumers with these loans. Note the bargaining power and left-digit bias jointly determine the final monthly payment. A consumer with a large bargaining power ω_i can push the monthly payment closer to the lower bound $\underline{p}(\boldsymbol{x}_i)$ and away from the upper bound $\bar{p}(\boldsymbol{x}_i)$. In addition to the impact of bargaining power, left-digit bias creates discontinuities in the joint value function and makes the final monthly payment different from when the bias does not exist.

Similar to the intuition for bunching, bias will lead to the average bargaining power systematically different for loans with \$9- and \$0-ending payments. In theory, the model could generate patterns where \$9-ending payments have either a higher or a lower bargaining power than \$0-ending payments. The exact relationship depends on the bargaining power and the extent of the bias for consumers and finance managers.

We use simulation to illustrate the bargaining power patterns for those who end up with \$9- and \$0-ending payments. First, when the consumer bias is large (relative to the finance manager bias), Panel A of Figure 5 plots the average bargaining power of simulated payments with \$9- and \$0-endings. When the overall bargaining power is high among consumers (i.e., their ω's are in

Figure 4. (Color online) Frequency of Monthly Payment Ending Digit



the region of 0.5–1), the left figure of Panel A shows that the average bargaining power for $9-ending loans is lower and the bargaining power for $0-ending payments is higher. Given that interest rates are negatively related to the bargaining power, these results suggest that, when the consumer bargaining power is high in general, those with $9-ending payments are more likely to pay a higher interest rate and those with $0-ending payments are more likely to pay a lower interest rate. In the region where the consumer bargaining power is low overall (i.e., their ω's are in the region of 0–0.5). We see the opposite pattern. Consumers with $9-ending payments are associated with higher bargaining power, hence pay a lower interest rate, and those with $0-ending payments are more likely to have lower bargaining power and a higher interest rate.

In contrast, when the finance manager bias is large (relative to the consumer bias), Panel B of Figure 5 shows a different pattern. When the consumer bargaining power is high (i.e., their ω's are in the region of 0.5–1), consumers with $9-ending ($0-ending) payments are more likely to pay a lower (higher) interest rate. When the overall consumer bargaining power is low (i.e., their ω's are in the region of 0–0.5), we see the opposite pattern.

To conclude, the relationship between the $9- and $0-ending loans and their interest rates depends on the bargaining power and the extent of the biases from both sides. Note the model is flexible enough to describe not only the relationship we observe in the data but also when the relationship is the opposite.

4. Estimation

The data that we use to estimate the proposed model include the monthly payment p_i and the loan and consumer characteristics $\boldsymbol{x}_i$. The set of model parameters is $\boldsymbol{\Theta} = \{\boldsymbol{\gamma}, \boldsymbol{\theta}_f; \boldsymbol{\beta}, \sigma_\epsilon\}$, where $\boldsymbol{\gamma}$ governs the heterogeneous left-digit bias for consumers, $\boldsymbol{\theta}_f$ captures the bias for finance managers, and $\boldsymbol{\beta}$ and σ_ϵ determine the bargaining power distribution. In this section, we discuss the estimation strategy and present a Monte Carlo study.

4.1. Moment Conditions

We use the simulated method of moments (SMM) for model estimation since it is challenging to derive a likelihood function with the stochastic term ϵ_i entering the joint value function nonlinearly. Another advantage of using SMM is that consistent estimates can be obtained with a finite number of simulations to construct the moment conditions.

In the estimation, we draw the unobserved stochastic component of the bargaining power ϵ_i^{sim} for every loan from the distribution $N(0, \sigma_\epsilon^2)$, where $sim = 1, \ldots, NS$. Given ϵ_i^{sim}, we simulate the monthly payment $p_i^{sim}(\boldsymbol{\Theta})$ based on the observed covariates $\boldsymbol{x}_i$ and model parameters $\boldsymbol{\Theta}$ according to the Nash bargaining solution (Equation (9)).[15] Let $\boldsymbol{\Theta}^0$ be the true parameters.

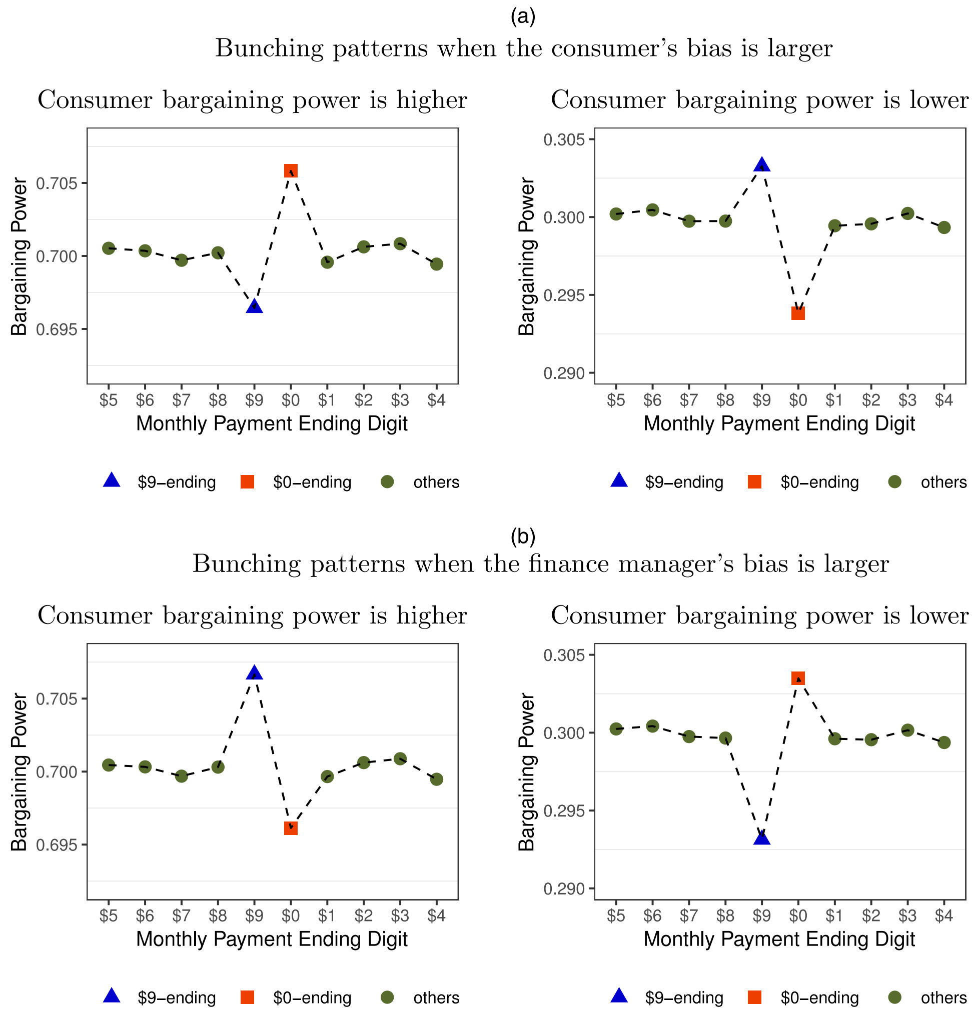
We utilize two sets of moments to identify left-digit bias and bargaining power parameters. The first set of moments map the distribution of loans with different ending digits among simulated payments to that among observed payments:

$$\mathbb{E}\left[\mathbb{I}(\lfloor p_i \rfloor_1 = d_1) - \frac{1}{NS}\sum_{sim=1}^{NS} \mathbb{I}(\lfloor p_i^{sim}(\boldsymbol{\Theta}^0)\rfloor_1 = d_1) \Big| \boldsymbol{x}_i\right] = 0,$$

$$\mathbb{E}\Bigg[\mathbb{I}(\lfloor p_i \rfloor_{10} + \lfloor p_i \rfloor_1 = d_{10}) - \frac{1}{NS}\sum_{sim=1}^{NS} \mathbb{I}(\lfloor p_i^{sim}(\boldsymbol{\Theta}^0)\rfloor_{10} + \lfloor p_i^{sim}(\boldsymbol{\Theta}^0)\rfloor_1 = d_{10}) \Big| \boldsymbol{x}_i\Bigg] = 0, \quad (10)$$

where $\mathbb{I}(\cdot)$ is an indicator function that equals 1 if the logical expression is true, and 0 otherwise.

Figure 5. (Color online) Bunching at \$9- and \$0-Ending Payments and Bargaining Power



Notes. (a) Bunching patterns when the consumer's bias is larger, and consumer bargaining power is higher (left) and consumer bargaining power is lower (right). (b) Bunching patterns when the finance manager's bias is larger, and consumer bargaining power is higher (left) and consumer bargaining power is lower (right).

The first line in Equation (10) sets the proportion of single digit at d_1 to be the same among the observed and simulated payments. Let $d_1 = 9$ and 0, respectively. Similarly, the second line sets the proportion of the tens and single digits at d_{10} to be the same among the observed and simulated payments. Let $d_{10} = 99$ and 0, respectively. The residual terms are orthogonal to the observed attributes $\boldsymbol{x}_i$, including a constant term, credit score, age, income, and African American and Hispanic proportions, at true model parameters $\boldsymbol{\Theta}^0$. These moment conditions correspond to the level of bunching at \$9- and \$0-endings at \$10 marks as well as \$100 marks across different consumer groups and help pin down the bias parameters.

The second set of moment conditions map the level of payments between the observed and simulated payments:

$$\mathbb{E}\left[p_i - \frac{1}{NS}\sum_{sim=1}^{NS} p_i^{sim}(\boldsymbol{\Theta}^0)\middle|\boldsymbol{x}_i\right] = 0,$$
$$\mathbb{E}\left[(p_i - \bar{p})^2 - \frac{1}{NS}\sum_{sim=1}^{NS}\left(p_i^{sim}(\boldsymbol{\Theta}^0) - \bar{p}^{sim}(\boldsymbol{\Theta}^0)\right)^2\right] = 0, \quad (11)$$

where p_i is the observed payment. The first line in Equation (11) matches the level of observed and simulated payments, where the residual term is orthogonal to the observed attributes $\boldsymbol{x}_i$, including a constant term, age, income, and African American and Hispanic proportions, at true model parameters $\boldsymbol{\Theta}^0$. This moment condition helps identify bargaining power parameters $\boldsymbol{\beta}$. The second line matches the observed payments' variance to that of the simulated payments, where $\bar{p}$ and $\bar{p}^{sim}$ are the average monthly payments for the observed and simulated data, respectively. The second-order moment helps estimate the variance of the unobserved bargaining power σ_ϵ.

The estimated $\hat{\boldsymbol{\Theta}}$ set the sample analogue of moments as close as possible to zero. We use a two-step feasible generalized method of moments (GMM) estimation method. In step 1, we let the weighting matrix W be the identity matrix and compute estimate $\hat{\boldsymbol{\Theta}}_{(1)}$. In step 2, we calculate the optimal weighting matrix

$$\hat{\Sigma} = \left(\frac{1}{N}\sum_{i=1}^{N} g\left(p_i, \boldsymbol{x}_i, \hat{\Theta}_{(1)}\right)^T \cdot g\left(p_i, \boldsymbol{x}_i, \hat{\Theta}_{(1)}\right)\right)^{-1},$$

where $g(p_i, \boldsymbol{x}_i, \hat{\boldsymbol{\Theta}}_{(1)})$ is an $N \times K$ matrix that represents the sample moments (N is the number of loans, and K is the number of moments). The optimal weighting matrix takes account of the variances and covariance between the moment conditions. Model estimates $\hat{\boldsymbol{\Theta}}$ are recomputed with the updated weighting matrix.

After laying out the moment conditions and the estimation procedure, the next few paragraphs offer an intuitive explanation of how the parameters are estimated from the above moment conditions. In particular, we discuss how the heterogeneous left-digit bias and bargaining power can be separately identified.

In the estimation, we compute $\bar{p}(\boldsymbol{x}_i)$ and $\underline{p}(\boldsymbol{x}_i)$ in the first step (details are in Section 5.1). Then parameters associated with the bargaining power determine how close the realized monthly payment p_i is to $\bar{p}(\boldsymbol{x}_i)$ relative to $\underline{p}(\boldsymbol{x}_i)$. If the average payment of consumers with attributes $\boldsymbol{x}_i$ is closer to $\underline{p}(\boldsymbol{x}_i)$ than other consumers to their reservation values, this implies a larger bargaining power for consumers with attributes $\boldsymbol{x}_i$. Furthermore, a larger variance of the unobserved bargaining power σ_ϵ^2 will lead to a larger variation of monthly payments from consumers conditional on observed characteristics.

The identification of the left-digit bias parameters comes from the distribution of the number of loans ending in different digits, in particular, the bunching at \$9- and \$0-endings. As illustrated in Figure 4, consumer left-digit bias leads to \$9-ending bunching, while finance manager bias leads to \$0-ending bunching. Moreover, if the level of \$9-ending bunching for consumers with specific $\boldsymbol{x}_i$ is much higher than others, it implies that these consumers are likely to be more biased, and vice versa. In particular, the level of \$9-ending bunching interacted with attributes $\boldsymbol{x}_i$ identifies γ, which specifies the heterogeneous level of left-digit bias for consumers.

Even though the bargaining power and left-digit bias jointly determine the final payments, we can separately identify the two sets of parameters. This is because we can rely on two types of data patterns—namely, payment level and bunching—for estimation. For example, suppose Hispanic consumers have a higher monthly payment than others, all else being equal, but a low level of \$9-ending bunching; then we would infer that Hispanic consumers have a lower bargaining power but are not very biased. On the other hand, suppose older consumers have a lower monthly payment than others, all else being equal, but have a high level of \$9-ending bunching; then we would infer that older consumers have a higher bargaining power but are very biased. It is important to model the bargaining power and left-digit bias together in one framework. This is because bargaining power influences bunching patterns together with the level of bias, and bias influences the final payments together with the bargaining power.

This identification argument is consistent with observed heterogeneity in the bias parameters. If one allows for a flexible unobserved heterogeneity in the bias term, bargaining power and left-digit bias cannot be separately identified. For example, consider a consumer who pays \$399, much higher than others all else being equal. There are numerous ways to explain the payment level that are observationally equivalent. This consumer may have a high bargaining power but is very biased by perceiving \$399 as \$300. Alternatively, this consumer may have no bias but have a low bargaining power, which leads to a high \$399 payment. The consumers may also be somewhat biased by perceiving \$399 as a lower payment and have moderate bargaining power. In fact, there are numerous combinations of bargaining power and left-digit bias that would rationalize the observed payment if one were to allow a flexible unobserved bias term. Following prior literature (e.g., Lacetera et al. 2012, Strulov-Shlain 2019), we do not model unobserved heterogeneity in the left-digit bias.

4.2. Monte Carlo Study

We use a Monte Carlo study to show that the proposed estimation strategy can successfully recover the true parameters. We simulate 100,000 loans by drawing the attributes according to their respective distributions from the observed data. For model estimation, we include two attributes, African American proportion and Hispanic proportion, to represent the attributes that influence both the level of bias and bargaining power. The goal is to demonstrate that the proposed model can successfully identify the heterogeneous effects of bias and bargaining power separately. Extending to more attributes in the model is straightforward.

We simulate the monthly payment for each loan using the "true" parameter values (Table 5, Column 1) according to the Nash bargaining solution (Equation (9)). With the simulated data, we estimate the model using the estimation strategy as described in Section 4.1 with the number of simulations $NS = 10$. The parameter estimates and standard errors are reported in Columns 2 and 3 of Table 5. The parameter estimates are all close to the true values, with small standard errors, showing that the true model parameters can be recovered with the proposed estimation strategy.

5. Results

In this section, we first describe how we calculate reservation values for consumers and finance managers. Then we discuss model estimation results for left-digit bias and bargaining power parameters. For ease of computation, the model is estimated from a randomly selected sample of 1 million loans. Using the model estimates, we conduct counterfactuals to evaluate the impact of left-digit bias on the bargaining outcome. Finally, we discuss several alternative explanations for the observed data patterns.

5.1. Reservation Values

In order to estimate the proposed bargaining model, we need to know the reservation values for the consumers and finance managers (Equations (6) and (7)). As is typical in empirical bargaining applications, reservation values are not directly observed in the data. For finance managers, their reservation values are determined by the bank buy rate, which is the cost of the loan for the dealer. We approximate the bank buy rate by the 2.5*th* percentile of the interest rates

Table 5. Monte Carlo Simulation

	True values	Estimates	S.E.
	(1)	(2)	(3)
Consumer bias $\boldsymbol{\gamma}$			
Constant for $\theta_{c,1}$	−5.0000	−5.0322	(0.0977)
Constant for $\theta_{c,2}$	−3.5000	−3.5028	(0.1502)
African American proportion	0.0500	0.0493	(0.0149)
Hispanic proportion	0.1000	0.1057	(0.0158)
Finance manager bias $\boldsymbol{\theta}_f$			
$\theta_{f,1}$	0.0067	0.0064	(0.0007)
$\theta_{f,2}$	0.0296	0.0297	(0.0047)
Bargaining power $\boldsymbol{\beta}$			
Constant	−0.7500	−0.7674	(0.0129)
African American proportion	−1.0000	−1.0249	(0.0296)
Hispanic proportion	−0.5000	−0.5090	(0.0280)
Bargaining power sd: $\log(\sigma_e)$	−0.2000	−0.1138	(0.0595)

Note. S.E., standard errors.

for a given type of loan. We use quantile regression to estimate quantiles of the interest rate distribution as a function of relevant covariates. We include credit score, loan amount, and loan length, and their square terms, as well as year-month fixed effects, as covariates. These attributes are used by banks to determine the risk-adjusted bank buy rates. In contrast, demographic variables such as age and ethnicity are not included since banks cannot use them for loan pricing.[16]

Quantile regression minimizes a sum of asymmetrically weighted absolute residuals by giving different weights to positive and negative residuals (see Koenker and Hallock 2001):

$$\min_\lambda \sum_i \rho_\tau(y_i - x_i\lambda),$$

where $\rho_\tau(u) = \tau \cdot \max(u, 0) + (1 - \tau) \cdot \max(-u, 0)$. The $\rho_\tau(u)$ function weights the positive residual ($u > 0$) by τ and the negative residual ($u < 0$) by $1 - \tau$. The estimated τth quantile of interest rates for loans with characteristic x_i is $Q(\tau) = x_i\hat{\lambda}(\tau)$. After getting the $Q(\underline{\tau} = 2.5\%)$ to represent the bank buy rate, we calculate the corresponding payment as the finance manager reservation value.[17] We do robustness analysis by using other thresholds. Details are in Online Appendix A.3.

Similar to the finance manager reservation, we assume that the consumer reservation value varies based on the loan characteristics x_i only. In particular, we assume that the consumer reservation values do not differ by demographics. This is because reservation value and bargaining power cannot be separately identified based on the final price alone. For example, if minority consumers receive higher prices all else being equal, the higher price can be explained by minority consumers having a higher reservation value, which can depend on other sources of funds, or by minority consumers having a lower bargaining power. Any potential errors in measuring reservation values will be attributed to the bargaining power, which is a function of the demographics. We use the 97.5*th* percentile rate $Q(\bar{\tau} = 97.5\%)$ to calculate the consumer reservation values based on observed loan characteristics x_i. We then calculate the corresponding monthly payments based on the observed loan amount and loan length (see endnote 17). Online Appendix A.3 shows robustness analysis using other thresholds.

5.2. Estimation Results

We proceed to estimate the main bargaining model with left-digit bias with the calculated reservation values for both consumers and finance managers.[18] Model estimation results are reported in Table 6. The table first shows the left-digit bias for consumers. The constant terms are allowed to differ for the consumer bias terms $\theta_{c,1}$ and $\theta_{c,2}$. After transforming

Table 6. Estimation Results

	Estimates	S.E.
Consumer bias γ		
Constant for $\theta_{c,1}$	−4.8431	(0.0251)
Constant for $\theta_{c,2}$	−3.6735	(0.0241)
Credit score (in 100)	0.0033	(0.0005)
Age (in 100)	0.0043	(0.0017)
Income (in $1 million)	−0.0215	(0.0065)
African American proportion	0.0078	(0.0019)
Hispanic proportion	0.0048	(0.0025)
Finance manager bias $\boldsymbol{\theta}_f$		
$\theta_{f,1}$	0.0079	(0.0002)
$\theta_{f,2}$	0.0250	(0.0006)
Bargaining power $\boldsymbol{\beta}$		
Constant	0.9868	(0.0068)
Age (in 100)	−0.2123	(0.0084)
Income (in $1 million)	1.8947	(0.0349)
African American proportion	−0.3993	(0.0098)
Hispanic proportion	−0.5338	(0.0125)
Bargaining power sd: $\log(\sigma_\epsilon)$	−0.5913	(0.0344)

Note. S.E., standard errors.

the estimates of the heterogeneous bias using the logistic function (Equation (2)), we find the average $\theta_{c,1} = 0.0079$ and $\theta_{c,2} = 0.0257$ across all consumers.[19] To better interpret these parameters, we calculate the average perceived difference for $1 between $9- and the next $0-ending payment to be $1.222, larger than $1. The perceived difference for $1 is even larger, at $2.012, between $99- and the next $00-ending payment. Within a $10 range, the average perceived difference for $1 is $0.967. The left-digit bias is heterogeneous among different consumer groups. We find a larger bias for consumers with lower credit scores, older age, and living in areas with a lower income and higher African American and Hispanic proportions. These are consistent with the bunching patterns shown in Table 2.

The table then shows the left-digit bias for finance managers. We calculate the perceived payment using the estimated bias parameters. For finance managers, the perceived difference for $1 is $1.215 between $9- and $0-ending payments and $2.001 between $99- and $00-ending payments. Both are only slightly smaller than the corresponding perceived gaps for consumers. Results suggest that, even for finance managers who have rich experience in negotiations, they are still prone to basic human bias in the same way as consumers. This result adds to the existing literature that documents how behavioral factors can influence professionals or experts in high-stakes decision making, such as lawyers, professional traders, used-car dealers, and managers in a multinational corporation (Birke and Fox 1999, Coval and Shumway 2005, Workman 2012, Lacetera et al. 2016; see Goldfarb et al. 2012 for a review of the behavioral models on managerial decision making).

It is worth noting that the biases for consumers and finance managers are exactly the same type of bias—they all think that a $9-ending number is substantially smaller than the next $0. The bias manifests differently in the bunching patterns only because consumers and finance managers are on the opposite sides of bargaining. While consumers prefer to *pay* a lower payment ($9-ending), finance managers prefer to *receive* a higher payment ($0-ending). Consumer bias contributes to $9-ending bunching since consumers are "reluctant" to go up to the next $10 range. Finance managers perceive a large drop moving from $0- to $9-ending, leading to $0-ending bunching (see Section 3.3 for a detailed discussion).

The rest of the parameters in Table 6 govern the distribution of bargaining power among consumers. The range of ω_i is between 0 and 1. After the logistics transformation (Equation (8)), the average bargaining power for consumers is 0.71. That is, the overall bargaining power of consumers is larger than that of finance managers. One possible reason is that, if the negotiation breaks down, the dealer will lose not only the markup compensation but also the profit from selling the vehicle and other follow-up services. Another way to interpret the bargaining power results is that the monthly payments are distributed more densely toward the lower bound of the range between $\underline{p}(\boldsymbol{x}_i)$ and $\bar{p}(\boldsymbol{x}_i)$.

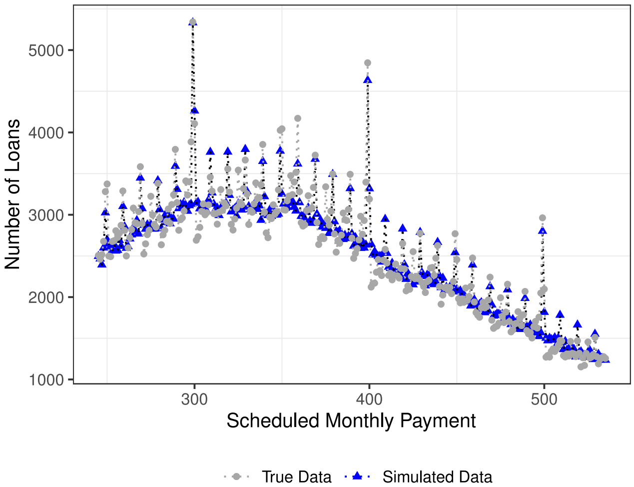
The bargaining power is heterogeneous across different consumer groups: It tends to be lower among older consumers and those living in areas with lower income and higher minority representation. The results for minority consumers have a strong policy implication. To better interpret the parameters, we compare the predicted payments for an African American and a Hispanic consumer with that for a Caucasian consumer while holding the other variables at the sample average.[20] Results show that the African American consumer pays $442 (1.7%) higher total payment, and the Hispanic consumer pays $603 (2.3%) higher total payment than the Caucasian consumer, all else being equal. These numbers are close to that documented in Cohen (2012), who used class action litigation data from five lenders to show African Americans paid between $347 and $508 more in markup than Caucasians. One possible reason for the higher markup among African Americans and Hispanics is that they are less resourceful or less informed about alternative financing sources. The higher markup can also be due to the propensity to discriminate against minority consumers among finance managers. While we cannot pin down the mechanism, the results confirm a significant payment gap for consumers of different races under the discretionary dealer markup practice.

It is worth noting that, beyond the heterogeneous consumer bias, the bargaining model itself also contributes to the heterogeneous bunching patterns among different consumer groups due to the heterogeneous bargaining power. As shown in Figure 5, when the consumer's relative bargaining power is high, which is the case in our empirical application, lower bargaining power consumers are more likely to get \$9-ending loans. Since African American and Hispanic consumers have lower bargaining power, they are more likely to get \$9-ending loans even with the same level of bias. Higher income and higher credit score consumers, on the other hand, have higher bargaining power and are less likely to get \$9-ending loans. Therefore, it is important to model the left-digit bias and bargaining power together in one framework.

The same mechanism can explain the systematic interest rate difference for \$9- and \$0-ending loans. Given that low bargaining power consumers are more likely to get \$9-ending payments, the average interest rate for the \$9-ending loans will be higher than others. Similarly, consumers with \$0-ending loans have higher bargaining power, and therefore, these \$0-ending loans have a lower interest rate on average. We simulate the monthly payment using the model estimates and calculate the implied interest rate (see Online Appendix A.1). The average interest rate for \$9-ending loans is 0.107% higher than that for \$0-ending loans in the simulated data.

With the simulated monthly payments using the estimation results, Figure 6 plots the number of loans at each payment level in the actual and simulated data. The two distributions match well on the level of payments. In particular, simulated payments also bunch at \$9- and \$0-endings. And the levels of bunching are more significant around \$100 marks with \$99- and \$00-endings. Actual payments exhibit a slight increasing pattern from \$1- to \$8-ending payments, which could be accommodated with a perceived payment function that allows curvature within the \$10 range. However, this model specification is not grounded in prior literature in behavioral economics and psychology and may appear ad hoc. Therefore, we use the established model as in Lacetera et al. (2012) as the main model to capture left-digit bias. We estimate a version of the bargaining model with an alternative perceived payment specification in Appendix A.2.

Figure 6. (Color online) Distribution of Monthly Payments for Actual and Simulated Data



5.3. Counterfactual

We use the model estimates to explore the impact of left-digit bias on monthly payments using counterfactual analysis. Doing so allows us to evaluate the potential changes in loan payments if left-digit bias no longer plays a role in bargaining. While there has been ample literature documenting the consumer response to 9-ending prices (e.g., Anderson and Simester 2003), far fewer have dealt with how having the bias influences the equilibrium outcome.[21] We quantify the changes in payments under several scenarios: when only consumers or finance managers have left-digit bias or when both are biased.

To explore the impact of left-digit bias, we construct a counterfactual scenario where neither consumers nor finance managers are biased and use it as the benchmark case. This is done by setting both $\boldsymbol{\theta}_c$ and $\boldsymbol{\theta}_f$ to 0.[22] The perceived payment functions are linear and continuous with the perceived payment being the same as the actual. We draw a set of bargaining power unobservables ϵ_i and simulate the monthly payments under the benchmark scenario with no bias. Using the same bargaining power draws, we simulate the payments when only consumers or finance managers have the estimated bias and when both are biased.

We first discuss the impact of consumer bias. One may view bias as a negative factor in the bargaining process by intuition and conclude that removing such bias should always benefit consumers. Such intuition has some ground from the level effect—since the perceived payments are lower with consumer bias (except at the hundreds), the payments become higher to achieve the same division of total surplus with the same bargaining power. The impact of bias, however, is more nuanced beyond the level effect. We show that consumer bias can both increase and decrease the payments because of the nonstandard perceived payment.

Why can having the left-digit bias lead to a lower payment for consumers? This is because consumers perceive a large difference when payments increase from a \$9-ending number with the bias, especially over \$100. The large perceived gap makes it more difficult for finance managers to increase the payments from a \$9-ending number. Intuitively, the bias creates a psychological hurdle for consumers so that they are more resistant to payments crossing the hurdle. Without the bias, some consumers with $9-ending payments could have received a higher payment with $0-, $1-, or higher-endings.

On the other hand, the bias can also lead to a higher payment. Because of the bias, consumers perceive each $1 to be smaller than actual when the change is within the $10 range. The smaller perceived gap makes it easier for finance managers to push up the payments to a higher number. Therefore, with consumer bias, some payments would increase to a higher number, for example, $7- to $9-ending. Since consumer bias can both increase and decrease the payments, the total effect depends on which of the two effects prevails.

Similarly, the left-digit bias for finance managers can also both increase and decrease the payments. Same as the consumer bias, the level effect (the perceived payments becoming lower with the bias) leads to a higher payment to achieve the same division of surplus. Besides the level effect, payments will also change because of the nonstandard perceived payment. Within the $10 range, the perceived value for $1 becomes lower for finance managers so that it is easier for consumers to bargain down the payments within the range, which can lead to a lower payment. In contrast, the perceived gap from $9- to $0-ending payments becomes larger; therefore, it is more difficult for consumers to bargain down below a $0-ending number, which can lead to a higher payment.

When both sides have the left-digit bias, they all perceive the payments to be lower than actual. With the same intuition as scenarios with consumer or finance manager bias only, the level effect leads to an increase in payments. Beyond the level effect, all the effects from the nonstandard perceived payments discussed above can occur. Since the impact of left-digit bias can go in either direction, we quantify the overall change in payments with the estimated parameters. To do so, we compare the payments when only consumers or only finance managers or both have the estimated bias to the benchmark case with no bias. The changes in payments, which represent the impact of the bias, are shown in Table 7.

Column 1 shows the change in payments when consumers are biased. In the aggregate, biased consumers pay 0.049% more, with total payments increased by $446 million (or $12.8 per loan). As discussed above, the impact of bias can go in either direction. About 46% of the loans have an increase in payments, with an average change of 0.26% or $71.6 per loan. About 18% have a decrease in payments, with an average change of 0.42% or $115.1 per loan. Column 2 shows the impact on payments when finance managers are biased. Overall, dealers will

Table 7. Effect of Left-Digit Bias on Payments

	Consumer bias only	Finance manager bias only	Bias from both sides
	(1)	(2)	(3)
Overall			
Total payment ($million)	446.09	729.06	1,146.85
Average payment per loan ($)	12.83	20.97	32.99
Percentage change	0.049%	0.080%	0.126%
Among the increased			
Average payment per loan ($)	71.65	117.66	65.44
Percentage change	0.264%	0.444%	0.250%
Among the decreased			
Average payment per loan ($)	−115.14	−71.43	−80.0
Percentage change	−0.416%	−0.234%	−0.116%
By bargaining power			
First quartile			
Average payment ($)	17.84	15.40	33.19
Percentage change	0.066%	0.057%	0.123%
Second quartile			
Average payment ($)	13.89	19.75	32.97
Percentage change	0.053%	0.075%	0.126%
Third quartile			
Average payment ($)	11.30	22.75	32.96
Percentage change	0.044%	0.088%	0.128%
Fourth quartile			
Average payment ($)	8.31	25.99	32.85
Percentage change	0.033%	0.102%	0.129%

receive 0.08% more, which amounts to a total of $729 million or $21 per loan with the finance manager bias. About 28% of the loans increase in payments, with an average change of 0.44% or $117.7 per loan. Another 16% decrease in payments, with an average of 0.23% or $71.4 per loan.

When both parties are biased, Column 3 shows that the average payment will increase by 0.13%. Overall, consumers will pay $1146.8 million more in total or $33 per loan than the benchmark case with no bias. About half of the loans get an increase in payments, and very few have a decrease. This is primarily driven by the level effect, where both the consumers and finance managers perceive the payments to be lower than actual. With the same bargaining power, the level effect will lead to an increase in the actual payment to achieve the same division of surplus.

The effect of left-digit bias is systematically different for consumers of different bargaining power. We find that the increase in payments for biased consumers is more significant among lower bargaining power consumers. For example, among those with the first quantile of bargaining power (0%–25%), the payment increase is 0.07% compared with the 0.03% increase among those in the fourth quantile of bargaining power (75%–100%). In fact, the lower bargaining power consumers are both more likely to experience an increase as well as a decrease in payments due to the bias in our empirical setting, but the payment increase effect prevails. For the finance manager bias, we see the opposite pattern: Higher bargaining power consumers are more likely to get an increase in payments than those with lower bargaining power. When both parties have the left-digit bias, the increases in payments are comparable, with only a slightly higher change among higher bargaining power consumers.

5.4. Alternative Explanations

The proposed model is built upon the assumptions that consumers and finance managers negotiate monthly payments and that both parties have left-digit bias. We have shown how the model can explain the reduced-form data patterns. However, there may be other explanations that can also rationalize the patterns. In this subsection, we will discuss several alternative explanations.

Promotional Effect. Auto dealers may run promotions with advertised payments ending at $99 or $00. This may explain why the bunching phenomenon exists. However, if this is the reason, there should be no systematic difference in the interest rates for these two types of loans. In particular, the interest rate for $99-ending loans should not be higher than other loans. Moreover, promotional loans are much more likely to happen with manufacturer financing, which we do not include in the sample (see endnote 1). In addition, we find from data that the bunching phenomenon is quite stable over time. This is in contrast with auto dealer promotion activities that are periodic in nature.

Consumer Bias Only. One may attempt to come up with a more flexible function of consumer perceived payment in lieu of finance manager bias to explain the data patterns. Suppose the function of consumer perceived payment has a large drop from $9 to $0 as well as from $0 to $1; payments can bunch at both $9- and $0-endings. However, such specification would imply that the average interest rate for $0-ending loans should be higher than $1-ending loans, with a similar logic as the $9-ending loans. This is inconsistent with the empirical evidence, as $0-ending loans actually have the lowest interest rate. Without allowing finance managers to have the left-digit bias, it is difficult to rationalize the systematic interest rate difference between $9- and $0-ending loans and the payment bunching.

Focal Point Effect. Alternatively, one may attribute the $0-ending bunching to a focal point effect. Based on this explanation, the roundedness of the payments may facilitate negotiations, which will lead to a higher number of loans with $0-endings. However, this explanation cannot explain bunching at $9-ending digits. It is also not apparent why the focal point effect would systematically lower interest rates for $0-ending loans.

Installment Loan Effect. Finally, one may wonder if the data patterns are a result of the installment loan setting.[23] To rule out this potential explanation, we run a "placebo" test using data from the mortgage market (also an installment loan) provided by Equifax Inc. The data set consists of 7.3 million mortgages originated in 2014 across the United States. Mortgage loans are typically provided by banks or credit unions directly and are subject to much tighter regulations. Therefore, mortgage monthly payments do not come from a bargaining setting where left-digit bias could play a role. We find no evidence for the bunching phenomenon from the mortgage data: The proportions of monthly payments with both $0- and $9-endings are exactly 10%. Moreover, there is no difference in the average interest rate for loans with $0- or $9-ending payments. These results suggest that the data patterns are not due to the installment loan setting and are unique to the auto lending industry, where monthly payments are a bargaining outcome instead of a fixed price.

6. Conclusion and Discussion

This paper investigates how left-digit bias affects bargaining outcomes in the auto finance market. The proposed model relaxes the fully rational assumption by incorporating the well-established left-digit bias in a bargaining setting. Although the empirical study focuses on the auto finance market, the model framework can be applied to other settings where the bargaining happens between two individuals, and the bargaining outcomes are numeric.

We use a large data set of 35 million auto loans in this study. We find that not only consumers but also finance managers are subject to left-digit bias. For consumers, the average perceived difference for \$1 is \$1.22 between \$9- and \$0-endings and \$2.01 between \$99- and \$00-ending payments. The estimated level of bias for finance managers is only slightly smaller. The two-sided bias in bargaining can explain several interesting data patterns. In particular, the consumer bias contributes to the bunching at \$9-ending payments, and the finance manager bias contributes to the bunching at \$0-ending payments. In addition, lower bargaining consumers, who get a higher interest rate, are more likely to get \$9-ending payments; in contrast, higher bargaining power consumers are more likely to get \$0-ending loans and have a lower interest rate.

Using counterfactual analysis, we find a nuanced impact of left-digit bias on the bargaining outcome. For example, contrary to common intuition, consumer bias can actually decrease the payments for some consumers. This is because the larger perceived difference between \$9- and \$0-ending makes it more difficult to increase payments from a \$9-ending number, which can lead to a lower payment. In other words, the bias acts as a psychological hurdle for consumers so that they are more resistant to payments crossing the hurdle. The focus on payments being just below a round number could come from an actual or mental budget that consumers allocate for car payments each month. We argue that such a response is behavioral because there should not be a sharp difference between \$9- and the next \$0-ending payments in terms of consumers' ability to meet the payment obligation each month.

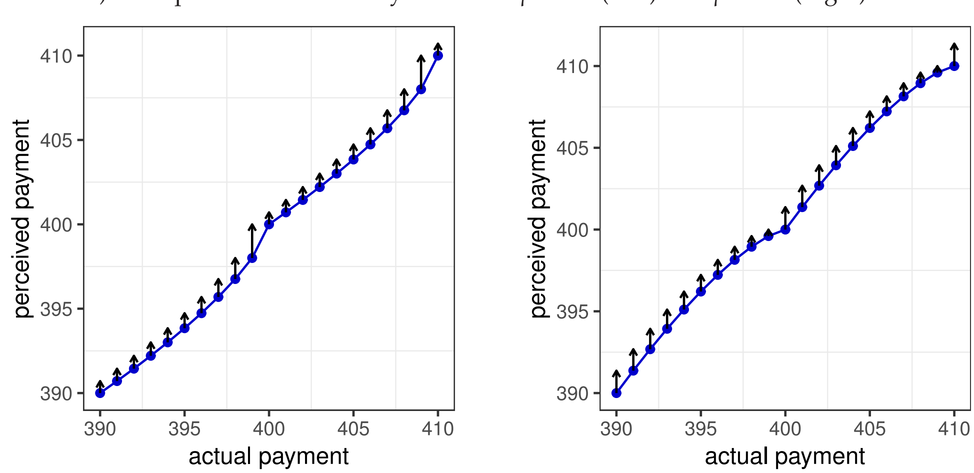
The insights from this study have broad implications beyond the auto finance market. The study suggests that left-digit bias exists not only among consumers but also among employees. This can be useful for firms to better understand what factors drive negotiated prices in many other settings, including estate sales, auto sales, online retail platforms (e.g., Taobao.com in China), and B-to-B environments where price negotiations are common. The result that consumers' perceived value has a large drop when crossing a threshold suggests that \$9-ending prices are stickier than other digits in most retail environments. Consumers' sensitivity toward price change and demand elasticity may vary across different ending digits in the price.

A. Appendix

A.1. Proof of Proposition 1

In this appendix, we show the proof of the Nash bargaining solution with left-digit bias. We derive all the components in Equation (9), including the interior solution $p^{\star}$ and the bargaining power cutoff points $\bar{\omega}_{Q-1}, \bar{\omega}_{Q}, \bar{\omega}_{Q+1}, \bar{\omega}_{Q+9}$. We then discuss the adjustment to the Nash bargaining solution when the level of bias is large and there are missing payments.

Figure A.1. (Color online) Examples of Perceived Payment with $\rho = 0.7$ (Left) and $\rho = 1.4$ (Right)



Notes. δ is assumed to be 0. A larger (smaller) δ leads to a larger (smaller) extra gap over \$100.

Interior Solution $p^\star$. When the payment that maximizes the joint value function lands within the \$10 range where the perceived payment is continuous, one can calculate the Nash bargaining solution with the first-order condition. We rewrite the joint value function (Equation (3)) with the perceived payment (Equation (1)). Subscript i is omitted for simplicity. We have

$$
\begin{aligned}
v(p) = &\left(r_c - \lfloor p \rfloor_{100} - (1-\theta_{c,1})\lfloor p \rfloor_{10} - (1-\theta_{c,1})(1-\theta_{c,2})\lfloor p \rfloor_{1}\right)^{\omega} \times \\
&\left(\lfloor p \rfloor_{100} + (1-\theta_{f,1})\lfloor p \rfloor_{10} + (1-\theta_{f,1})(1-\theta_{f,2})\lfloor p \rfloor_{1} - r_f\right)^{1-\omega}.
\end{aligned}
$$

We take the first-order derivative:

$$
\begin{aligned}
\frac{\partial v(p)}{\partial p} = &-\omega\big(r_c - \lfloor p \rfloor_{100} - (1-\theta_{c,1})\lfloor p \rfloor_{10} - (1-\theta_{c,1}) \\
&\times (1-\theta_{c,2})\lfloor p \rfloor_{1}\big)^{\omega-1}(1-\theta_{c,1})(1-\theta_{c,2}) \times \\
&\left(\lfloor p \rfloor_{100} + (1-\theta_{f,1})\lfloor p \rfloor_{10} + (1-\theta_{f,1})(1-\theta_{f,2})\lfloor p \rfloor_{1} - r_f\right)^{1-\omega} \\
&+ \left(r_c - \lfloor p \rfloor_{100} - (1-\theta_{c,1})\lfloor p \rfloor_{10} - (1-\theta_{c,1})(1-\theta_{c,2})\lfloor p \rfloor_{1}\right)^{\omega} \times \\
&(1-\omega)\big(\lfloor p \rfloor_{100} + (1-\theta_{f,1})\lfloor p \rfloor_{10} + (1-\theta_{f,1}) \\
&\times (1-\theta_{f,2})\lfloor p \rfloor_{1} - r_f\big)^{-\omega}(1-\theta_{f,1})(1-\theta_{f,2}) \\
= &-\omega(1-\theta_{c,1})(1-\theta_{c,2})\big(\lfloor p \rfloor_{100} + (1-\theta_{f,1}) \\
&\times \lfloor p \rfloor_{10} + (1-\theta_{f,1})(1-\theta_{f,2})\lfloor p \rfloor_{1} - r_f\big) \\
&+ \big(r_c - \lfloor p \rfloor_{100} - (1-\theta_{c,1})\lfloor p \rfloor_{10} - (1-\theta_{c,1}) \\
&\times (1-\theta_{c,2})\lfloor p \rfloor_{1}\big)(1-\omega)(1-\theta_{f,1})(1-\theta_{f,2}).
\end{aligned}
$$

Note that $\lfloor p \rfloor_1 = p - \lfloor p \rfloor_{100} - \lfloor p \rfloor_{10}$. We substitute in $\lfloor p \rfloor_1$, and the equation above becomes

$$
\begin{aligned}
\frac{\partial v(p)}{\partial p} = &-\omega(1-\theta_{c,1})(1-\theta_{c,2})\big(\lfloor p \rfloor_{100} + (1-\theta_{f,1}) \\
&\times \lfloor p \rfloor_{10} + (1-\theta_{f,1})(1-\theta_{f,2}) \\
&\times (p - \lfloor p \rfloor_{100} - \lfloor p \rfloor_{10}) - r_f\big) \\
&+ \big(r_c - \lfloor p \rfloor_{100} - (1-\theta_{c,1})\lfloor p \rfloor_{10} - (1-\theta_{c,1}) \\
&\times (1-\theta_{c,2})(p - \lfloor p \rfloor_{100} - \lfloor p \rfloor_{10})\big) \\
&\times (1-\omega)(1-\theta_{f,1})(1-\theta_{f,2}).
\end{aligned}
$$

We rearrange terms and solve for the $p^\star$ that makes $\frac{\partial v(p)}{\partial p} = 0$:

$$
\begin{aligned}
p^\star = &\frac{(1-\omega)r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} + \frac{\omega r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} + \\
&\lfloor p \rfloor_{100}\left(1 - \frac{1-\omega}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{\omega}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) \\
&+ \lfloor p \rfloor_{10}\left(1 - \frac{1-\omega}{1-\theta_{c,2}} - \frac{\omega}{1-\theta_{f,2}}\right).
\end{aligned}
$$

Bargaining Power Cutoffs $\bar{\omega}_{Q-1}, \bar{\omega}_{Q+1}, \bar{\omega}_{Q+9}$. Using the first-order condition, we can solve all the bargaining power cutoffs except for $\bar{\omega}_Q$ in Equation (9). Rearranging terms from the $p^\star$ equation above, we can get the corresponding bargaining power as a function of payment p and the corresponding $\lfloor p \rfloor_{100}$ and $\lfloor p \rfloor_{10}$ as follows:

$$
\bar{\omega}_p = \frac{\dfrac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \lfloor p \rfloor_{100}\left(\dfrac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - 1\right) - \lfloor p \rfloor_{10}\left(\dfrac{1}{1-\theta_{c,2}} - 1\right) - p}{\dfrac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \dfrac{r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} - \lfloor p \rfloor_{100}\left(\dfrac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - \dfrac{1}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) - \lfloor p \rfloor_{10}\left(\dfrac{1}{1-\theta_{c,2}} - \dfrac{1}{1-\theta_{f,2}}\right)}.
$$

The first cutoff $\bar{\omega}_{Q-1}$ can be solved as the bargaining power where the interior solution equals $Q-1$. The solution depends on whether Q is at the \$100 marks or not. When $\lfloor Q \rfloor_{10} = 0$ (i.e. Q is exactly at the \$100 marks), $Q-1$ has a different hundreds digit: $\lfloor Q-1 \rfloor_{100} = \lfloor Q \rfloor_{100} - 100$ and the tens digit is 9: $\lfloor Q-1 \rfloor_{10} = 90$. For example, $\lfloor 300 \rfloor_{100} = 300$, and $\lfloor 299 \rfloor_{100} = 200$, and $\lfloor 299 \rfloor_{100} = 90$. When $\lfloor Q \rfloor_{10} \neq 0$, $Q-1$ has a lower tens digit but the same hundreds digit: $\lfloor Q-1 \rfloor_{100} = \lfloor Q \rfloor_{100}$, $\lfloor Q-1 \rfloor_{10} = \lfloor Q \rfloor_{10} - 10$. Let $p = Q-1$. We get the $\bar{\omega}_{Q-1}$ as

$$
\bar{\omega}_{Q-1} = \begin{cases}
\dfrac{\dfrac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \lfloor Q \rfloor_{100}\left(\dfrac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - 1\right) - (\lfloor Q \rfloor_{10} - 10)\left(\dfrac{1}{1-\theta_{c,2}} - 1\right) - (Q-1)}{\dfrac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \dfrac{r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} - \lfloor Q \rfloor_{100}\left(\dfrac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - \dfrac{1}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) - (\lfloor Q \rfloor_{10} - 10)\left(\dfrac{1}{1-\theta_{c,2}} - \dfrac{1}{1-\theta_{f,2}}\right)} & \text{if } \lfloor Q \rfloor_{10} \neq 0, \\[2ex]
\dfrac{\dfrac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - (\lfloor Q \rfloor_{100} - 100) \times \left(\dfrac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - 1\right) - 90\left(\dfrac{1}{1-\theta_{c,2}} - 1\right) - (Q-1)}{\dfrac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \dfrac{r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} - (\lfloor Q \rfloor_{100} - 100)\left(\dfrac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - \dfrac{1}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) - 90\left(\dfrac{1}{1-\theta_{c,2}} - \dfrac{1}{1-\theta_{f,2}}\right)} & \text{if } \lfloor Q \rfloor_{10} = 0.
\end{cases}
$$

Similarly, we can solve for $\bar{\omega}_{Q+1}$ and $\bar{\omega}_{Q+9}$ using the interior solution. Since Q is a zero-ending number, both $Q+1$ and $Q+9$ have the same hundreds and tens digits as Q: $\lfloor Q+1 \rfloor_{100} = \lfloor Q \rfloor_{100}$, $\lfloor Q+1 \rfloor_{10} = \lfloor Q \rfloor_{10}$, and $\lfloor Q+9 \rfloor_{100} = \lfloor Q \rfloor_{100}$,

$\lfloor Q+9\rfloor_{10} = \lfloor Q\rfloor_{10}$. Let p be $Q+1$ and $Q+9$, respectively. We get the corresponding bargaining power cutoff points as

$$\bar{\omega}_{Q+1} = \frac{\frac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \lfloor Q\rfloor_{100}\left(\frac{1}{(1-\theta_{c,1})(1-\theta_{c,2})}-1\right) - \lfloor Q\rfloor_{10}\left(\frac{1}{1-\theta_{c,2}}-1\right) - (Q+1)}{\frac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} - \lfloor Q\rfloor_{100}\left(\frac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{1}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) - \lfloor Q\rfloor_{10}\left(\frac{1}{1-\theta_{c,2}} - \frac{1}{1-\theta_{f,2}}\right)},$$

$$\bar{\omega}_{Q+9} = \frac{\frac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \lfloor Q\rfloor_{100}\left(\frac{1}{(1-\theta_c)^2}-1\right) - \lfloor Q\rfloor_{10}\left(\frac{1}{1-\theta_{c,2}}-1\right) - (Q+9)}{\frac{r_c}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{r_f}{(1-\theta_{f,1})(1-\theta_{f,2})} - \lfloor Q\rfloor_{100}\left(\frac{1}{(1-\theta_{c,1})(1-\theta_{c,2})} - \frac{1}{(1-\theta_{f,1})(1-\theta_{f,2})}\right) - \lfloor Q\rfloor_{10}\left(\frac{1}{1-\theta_{c,2}} - \frac{1}{1-\theta_{f,2}}\right)}.$$

Bargaining Power Cutoff $\bar{\omega}_Q$. Finally, we solve for $\bar{\omega}_Q$, which is the bargaining power cutoff for payments to go from $Q-1$ to a point in the segment $[Q, Q+9]$. Conditional on the bias, this is the lowest bargaining power that payment will end up at $Q-1$. Because of the utility discontinuity from $Q-1$ to Q, the bound $\bar{\omega}_Q$ does not have a closed-form solution. It is easy to simulate by comparing the joint value for payments larger than Q and payments smaller than Q, which we detail below.

First, we compute the highest joint value when payment is at least Q. To accomplish this, we find the point p^{Q+} from the range $[Q, Q+9)$ that has the highest joint value. The point p^{Q+} could be either an internal solution or a corner solution at Q. Let $v(p^{Q+})$ be the value of the joint value function at p^{Q+}. In simulations, one can compute the joint value function at these points: the best internal solution using the first-order condition and the corner solution at Q. $v(p^{Q+})$ is the higher of the two values.

Then, we calculate the highest joint value when payment is below Q. Similarly, we find the point p^{Q-} from the range $[Q-1, Q)$ that has the highest joint value. The point p^{Q-} can be an internal solution or corner solutions at $Q-1$ or at $p \to Q^-$, which can happen when the joint value increases as p increases, until when it hits Q with a discontinuous change in the perceived payment. Let $v(p^{Q-})$ be the value of the joint value function at p^{Q-}. In simulations, one can compute the joint value function at each of these points: the best internal solution using the first-order condition, the corner solution at $Q-1$, as well as the corner solution at $p \to Q^-$. $v(p^{Q-})$ is the highest of the three values.

We compare $v(p^{Q+})$ and $v(p^{Q-})$, the highest joint values for payments larger than Q and smaller than Q, respectively. If $v(p^{Q-})$ is higher, then the Nash bargaining solution is $Q-1$; otherwise, the solution belongs to a point in $[Q, Q+9)$. The cutoff point $\bar{\omega}_Q$ is the lowest bargaining power that the Nash bargaining solution will end up at $Q-1$:

$$\bar{\omega}_Q = \operatorname{argmin}_\omega v(p^{Q-}) \geq v(p^{Q+}).$$

Nash Bargaining Solution with Large Bias. When the level of bias is large, there could be missing payments on some digits. As an intuitive example, when consumer bias is very large, it is less likely for the Nash bargaining solution to end up at a zero-ending number Q since it hurts the consumers' perceived value a lot. Under a large consumer bias, it may be optimal to arrive at either $Q-1$, which the consumers perceive as a much lower number, or $Q+i$ where $i \geq 1$ so that there is enough gain for the finance managers to "make up" for the large perceived loss for consumers in the joint value function. In such cases, there will be missing payments between Q and $Q+i-1$.

The Nash bargaining solution needs to be adjusted when the bias is large. We can identify and represent missing payments using the bounds of bargaining power. For example, there are missing payments at Q when $\bar{\omega}_{Q+1} > \bar{\omega}_Q$, where $\bar{\omega}_{Q+1}$ and $\bar{\omega}_Q$ are calculated as above. This is because there exists no bargaining power ω that falls in the range for a payment level at Q, $\omega \in (\bar{\omega}_{Q+1}, \bar{\omega}_Q)$. In such cases, the Nash bargaining solution can be adjusted as follows:

$$p = \begin{cases} Q-1 & \omega \in \left[\bar{\omega}_Q, \bar{\omega}_{Q-1}\right], \\ Q+i & \omega \in \left(\bar{\omega}_{Q+1+i}, \bar{\omega}_Q\right), \\ p^\star & \omega \in \left(\bar{\omega}_{Q+9}, \bar{\omega}_{Q+1+i}\right], \end{cases}$$

where $i = \operatorname{argmin}_i(\bar{\omega}_{Q+1+i} < \bar{\omega}_Q)$, $i = 0, 1, \ldots, 8$. The bargaining power bounds for each payment level are defined the same as above.

When $i = 0$, there are no missing payments, and the solution reduces to the case as in Equation (9). When $i \geq 1$, the bargaining power cutoffs from $\bar{\omega}_{Q+1}$ to $\bar{\omega}_{Q+i}$ are larger than $\bar{\omega}_Q$, so that there exists no bargaining power ω that falls in the range for the payment to be at Q to $Q+i-1$, which are the missing payments.

When the finance manager bias is large, one needs to allow for missing payments on large-ending digits. For example, if the finance manager bias is very large, it is less likely for the Nash bargaining solution to end up at $Q-1$, since it hurts the perceived value for finance managers a lot. The logic to adjust the Nash bargaining solution is the same as above. We do not repeat it here.

There are no missing payments on either the \$9-ending or \$0-ending payments in our empirical analysis. In other words, the biases for consumers and finance managers are not so large to induce missing payments.

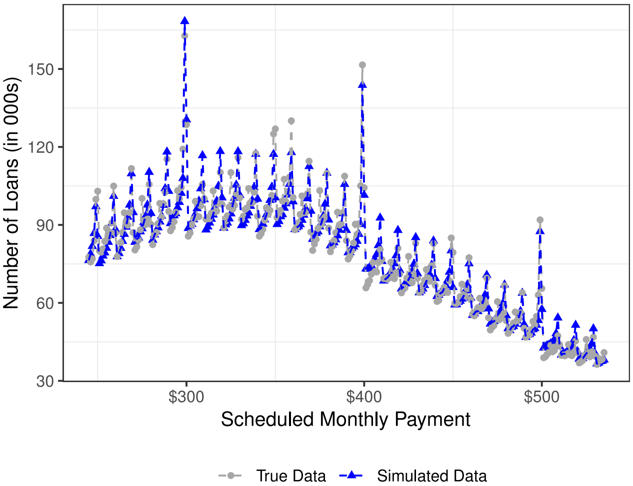
A.2. Alternative Specification for Perceived Payment

In this appendix, we present an alternative specification for the perceived payment and discuss the results. In the main model, the perceived payment functional form assumes a linear relationship within each \$10 range such that the perceived difference from \$1 to \$2 is the same as from \$2 to \$3, and so on. The empirical observation of an increasing number of loans from \$1- to \$9-endings motivates a functional form to allow for curvature within the \$10 range. The curvature allows the perceived \$1 change in payment to differ when the payment increases to the next \$10 level.

Let the perceived payment be:

$$\hat{p} = \lceil p \rceil_{10} - 10^{1-\rho} \cdot \left(\lceil p \rceil_{10} - p\right)^\rho - \delta \cdot \left[h(r_c) - h(p)\right].$$

Figure A.2. (Color online) Monthly Payments for Simulated Data with Alternative Curvature Specification



Let $\lceil p \rceil_{10}$ be the number that rounds up payment p to the next \$10 level. For example, $\lceil \$389 \rceil_{10} = \lceil \$390 \rceil_{10} = \$390$. The first part $\lceil p \rceil_{10} - 10^{1-\rho} \cdot (\lceil p \rceil_{10} - p)^{\rho}$ allows for curvature in the perceived payment within the \$10 range where parameter ρ captures the level of the bias. The functional form ensures that this bias goes away when p ends in \$0 and that the perceived payment is monotonic in p.[24] Depending on the parameter ρ, the perceived payment function can be convex ($0 < \rho < 1$), concave ($\rho > 1$), or linear ($\rho = 1$) within the \$10 range where parameter δ captures the discontinuous change in perceived payment every time payment p crosses \$100 marks. The second part $\delta \cdot [h(r_c) - h(p)]$ allows for additional discontinuity in the perceived value when payments cross \$100 marks, where $h(p)$ denotes the hundreds digit for payment p, for example, $h(\$400) = 4$ and $h(\$399) = 3$. The two bias parameters ρ and δ can vary for consumers and finance managers.

To better illustrate the effect of the curvature on the perceived payment, we plot two examples in Figure A.1. The length of arrows represents the difference in perceived value for a \$1 change in payment. The curvature of the perceived value within the \$10 range is determined by ρ. When $\rho = 0.7$, the perceived difference for \$1 increases as the payment increases to the next \$10 level (Figure A.1, left chart), with the largest perceived difference between \$9- and \$0-ending payments. The right chart of Figure A.1 shows that, with $\rho = 1.4$, the perceived value for \$1 decreases as payment increases to the next \$10 level.

Model estimates under this alternative model specification suggest that the consumers are more sensitive to each \$1 increase as the payment gets closer to the next \$10 level. For consumers, the estimated $\rho_c = 0.8963$, which translates to a \$1.27 perceived difference between \$9- and \$0-ending payments. There is a further discontinuity when payments cross \$100 marks: the perceived difference for \$1 between \$99- and \$00-ending loans is \$2.05 for consumers. The implied perceived payment gaps are very close to the estimates from the main model. The difference for this alternative specification is the curvature within the \$10 range: the perceived difference for each \$1 monotonically increases at larger ending digits, and it is the smallest from \$0- to \$1-ending payments at \$0.90. Such curvature can explain the increasing pattern from \$1- to \$8-ending digits within the \$10 range. The level of bias is estimated to be similar for finance managers, consistent with results from the main model.

We simulate the monthly payment for each loan using the estimation results. Figure A.2 presents the numbers of loans with each level of simulated payments. Compared with Figure 6, this model can reproduce the increasing pattern from \$1 to \$9 and the bunching at \$0- and \$9-ending payments. However, as discussed in the main text, the curvature model specification is not grounded in prior literature in behavioral economics and psychology and may appear ad hoc. Therefore, we follow prior literature in modeling the left-digit bias as the main model and only discuss this as an alternative model specification. It is reassuring that both models imply very similar perceived differences between \$9- and \$0-ending payments and between \$99- and \$00-ending payments. We leave it to future research to experimentally study the potentially different sensitivity to \$1 change within the \$10 range and the psychological explanations underpinning the phenomenon.

Endnotes

[1] The data set excludes auto loans from manufacturing financing (e.g., Toyota Financial). Therefore, the data patterns to be described are not influenced by manufacturer or dealer promotions with bundled packages including certain vehicles and financing options.

[2] Federal Reserve Bank of New York, Quarterly Report on Household Debt and Credit, May 2017 Q1: https://www.newyorkfed.org/medialibrary/interactives/householdcredit/data/pdf/HHDC2017Q1.pdf (accessed February 2018).

[3] Federal Reserve Bank of New York, Quarterly Report on Household Debt and Credit, May 2017 Q1: https://www.newyorkfed.org/medialibrary/interactives/householdcredit/data/pdf/HHDC2017Q1.pdf (accessed February 2018).

[4] Unlike the bank buy rate, which is determined by the consumer's credit profile, the markup is at the dealer's discretion and is not tied to the credit risk. See Jiang et al. (2020) who study the dealer compensation schemes.

[5] Non-subprime consumers refer to those with at least 620 credit score at the time of auto loan origination. Subprime lending typically involves additional required information, such as verified employment and income through providing pay stubs or tax return documents, beyond the standard credit profile. This information can lead to additional variation in interest rates. As the required additional information is unobserved in our data, we exclude subprime consumers in the analysis to avoid potential bias in the analysis (e.g., a high loan payment can be due to the consumer being unemployed and not because of her low bargaining power).

[6] We use APR and interest rate interchangeably in the paper.

[7] The data sample includes all auto loans from banks and credit unions. Some loans may be originated directly from banks or credit unions and are not subject to the typical markup process in indirect auto lending. We expect the bunching pattern to be more significant for loans originated at the dealer location. See Online Appendix A.1 for details.

[8] The number of loans with \$5-ending also tend to be higher, especially for payments ending at \$25 or \$75. This is likely driven by consumers and finance managers perceiving these \$5-ending payments as "round numbers."

[9] For robustness, we have also implemented a machine learning method, using XGBoost, to predict APR for loans with different ending digits, and the results are very similar (see Online Appendix A.2 for details).

[10] Note that these interest rate differences are conditional on the credit score and relevant loan characteristics. Unlike age and ethnicity, income may be used by banks when setting the bank buy rate. Therefore, lower income consumers may be getting a higher interest rate due to either a higher bank buy rate or a lower bargaining power. In this paper, we attribute the difference to the bargaining power difference since we do not directly observe the bank buy rate.

[11] Consumers may or may not actually engage in back-and-forth bargaining with the finance managers. Price dispersion can come from a form of third-degree price discrimination where the finance managers will make different offers to different types of consumers. Price dispersion can also arise from other mechanisms such as search cost (e.g., Argyle et al. 2020b). See endnote 14 for the interpretation of bargaining power.

[12] Consumers could instead face a menu of loan schedules, each with a unique combination of loan amount, loan length, and monthly payment. Even in this case, there can still be room for negotiation on the actual monthly payments, after consumers have selected the loan amount and length. The identification issue will be further discussed in Section 4.

[13] This assumption can be violated if the auto dealer is willing to take a loss from financing so that it can gain from selling the car and add-on services. In the data sample, we exclude loans with APRs lower than 1.9% (see the discussion in Online Appendix A.1) to avoid misspecifying the reservation prices for those loans.

[14] We use bargaining power as a "reduced-form" way to measure the dispersion in final prices conditional on loan characteristics. In practice, the heterogeneous bargaining power among different consumer groups can come from many sources, such as difference in reservation price, search cost, or other potential funding sources.

Dealer attributes, such as the dealership for different car manufacturers or the size of the dealer, could also affect the relative bargaining power. These attributes are not observed in the data and, thus, will be reflected in the stochastic term ϵ_i.

[15] One needs to know the reservation values for both the consumers and finance managers, $\bar{p}(x_i)$ and $\underline{p}(x_i)$, to simulate the monthly payment. We describe the procedure to calibrate the reservation values in Section 4.

[16] The Equal Credit Opportunity Act (ECOA) prohibits creditors from discriminating against credit applicants on the basis of race, color, religion, national origin, sex, marital status, and so on: https://www.justice.gov/crt/equal-credit-opportunity-act-3 (accessed April 2018).

[17] Monthly payment can be calculated as a function of loan amount $amount_i$, loan length n_i, and interest rate r_i: $\frac{amount_i \cdot r_i \cdot (1+r_i)^{n_i}}{(1+r_i)^{n_i}-1}$.

[18] The reservation values, once calculated, are treated as data in the bargaining model estimation. Although the first step quantile estimation uses a large sample, close to 35 million loans, with small estimation errors, conceptually the errors in estimating reservation values should be accounted for in the bargaining model estimation. To do so, one can bootstrap both steps together. However, doing so is very computationally intensive. We acknowledge this as a limitation in the paper.

[19] Note that the estimated bias can be thought of as a lower bound of the actual bias to the extent that some loans may not be an outcome of the bargaining process where bias plays a role (see Online Appendix A.1).

[20] Since the ethnicity data are based on zip codes. We calculate the payment for the African American (Hispanic) consumer by fixing the African American (Hispanic) proportion variable to 1. The payment for the Caucasian consumer is calculated by fixing both African American and Hispanic proportion variables to 0.

[21] An exception is Strulov-Shlain (2019), who studies the equilibrium price with consumer bias in a grocery store setting.

[22] One might be worried that the counterfactual exercise relies on the model specification of the left-digit bias. In particular, if the left-digit bias actually increased the perceived payment, the counterfactual results may not hold. To address this concern, we conduct a robustness check where we set $\boldsymbol{\theta}$ to 0 for only larger-ending digits (7, 8, 9), where the perceived prices are likely to be lower, and the $\boldsymbol{\theta}$ are kept as is for the other lower-ending digits. The results are similar. With consumer bias only, the total payment increase is \$445.27 million; with finance manager bias only, the total payment increase is \$778.56 million; with bias from both sides, the total payment increase is \$413.34 million.

[23] An installment loan has a fixed loan amount, loan length, monthly payment, and APR. Typical installment loans include mortgage, auto loan, and personal loan. This is in contrast to a revolving loan, which does not have a fixed loan amount or length, such as a credit card.

[24] A more general specification is $\lceil p \rceil_{10} - \lambda \cdot (\lceil p \rceil_{10} - p)^{\rho}$. We choose $\lambda = 10^{1-\rho}$ to satisfy the following conditions: (1) When $\rho = 1$, the perceived payment function is linear and $\lambda = 1$. (2) To ensure payoff monotonicity (i.e., the perceived payment must increase (or not decrease) as the actual payment increases), λ needs to satisfy $0 < \lambda < 10 \cdot 9^{-\rho}$.

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