Investment without Q[☆]

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Abstract

This paper proposes an alternative to standard investment-Q regressions. Policy functions summarize the key predictions of any dynamic investment model, are easy to estimate and, unlike Tobin's Q, account for a large fraction of the variation in corporate investment. As such policy functions are much better suited to evaluate and estimate dynamic investment models. Using this superior characterization of firm investment behavior we use indirect inference methods to estimate deep parameters of a structural model of investment and show that investment adjustment cost parameters are generally better identified from estimated policy function coefficients.

Keywords: Investment, Policy Functions, Indirect Inference

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1. Introduction

Hayashi (1982)'s famous elaboration of Brainard and Tobin's Q-theory has influenced the study of corporate and aggregate investment for nearly three decades and, despite a long-standing consensus about its empirical limitations, Q-type investment regressions still form the basis for most inferences about corporate behaviors.[1] Many papers have been written on the failures of Q theory and several alternative variables have been used to predict investment behavior. However, most of this research has been disjointed and often takes the form of simply proposing augmenting Q regressions with yet another variable.

Instead, our paper starts with a simple observation: in any model, optimal policies are functions of the relevant state variables, which are always true summary statistics. Therefore, if our goal is to estimate these policies, and any deep structural parameters, we should work directly with state variables. It makes little sense to start with Q since only rarely is there a one-to-one mapping between it and the underlying state variables.

Moreover, unlike marginal Q, the state variables we propose are either directly observable or can be readily constructed from observables, under fairly general conditions. This approach is then not only theoretically correct, but also straightforward to implement even under very general assumptions about the nature of markets, production and investment technologies.[2]

We show both in theory and in the data that even a simple low order polynomial approximation in the key state variables provides a good description of corporate investment,

[1]Q regressions, often augmented with ad-hoc variables have been used to, among other purposes, test the importance of financial constraints, the effects of corporate governance, and the efficiency of market signals.

[2]Frictions include market power or decreasing returns to scale in production (Gomes, 2001; Cooper and Ejarque, 2003; Abel and Eberly, 2002), in homogeneous costs of investment (Abel and Eberly, 1994, 1997; Cooper and Haltiwanger, 2006) or of external financing (Hennessy and Whited, 2007). Although he relies on homogeneity, Philippon (2009) also offers another alternative to the use of Tobin's Q.

one that performs far better than standard Q-type regressions. Formally, the covariances between investment and Q, implied by standard regressions, are far less informative about underlying structural parameters, than covariances with key state variables. Moreover, we show that elasticity of regression coefficients to the deep parameters is always significantly higher than those obtained in Q regressions. Altogether this evidence suggests policy function estimates should receive considerably more weight in indirect inference studies.

From a practical standpoint, the main novelty of our approach is to explicitly identify firm size and productivity as key state variables for optimal investment behavior under general assumptions about markets and technology. Surprisingly, given its popularity in other empirical applications, firm size is often ignored in the investment literature, and when used, it usually shows up either as a catch-all variable to account for omitted variables in investment regressions or as a sorting variable for identification of financially constrained firms.[3] Here we formally establish that firm size is naturally an important determinant of investment, with decreasing returns to scale technologies, even in the absence of financial market frictions. Similarly, our approach also clarifies the role of sales and cash flow variables. Contrary to their once popular use in tests of financing constraints, we show that these variables should matter because they capture underlying movements in the state of productivity and demand or in factor prices.[4]

[3]A notable recent exception is Gala and Julio (2016). Exploiting variation across industries, they provide direct empirical evidence that firm size captures technological decreasing returns rather than differences in firms' financing frictions.

[4]Gomes (2001), Cooper and Ejarque (2003) and Abel and Eberly (2002) all argue that cash flow might capture differences between marginal and average Q. Instead, we show that flow variables like sales and/or cash flow, and not Q, should always be the primary determinant of investment, even in the absence of capital market imperfections.

With respect to the use of Tobin's Q, our paper delivers perhaps the most logical conclusion to the influential arguments in Erickson and Whited (2000, 2006, 2011) that "Tobin's Q contains a great deal of measurement error because of a conceptual gap between true investment opportunities and observable measures". Our approach offers a simple way to circumvent the problem by avoiding the use of Q entirely, or, at least, limiting its use.

A possible concern is that current/recent values of measured state variables like sales, capital or leverage, may not perfectly capture all the forward looking information in the true underlying state variables. In these cases firm valuation will naturally capture that information better than any *observed* state variables. Hence, an empirically oriented researcher, mainly concerned in obtaining good empirical description of investment, might continue to use Q as a catch-all that captures (some of) the impact of any omitted variables. Methodologically, however, we believe she is better served by the discipline of writing an explicit model (even without solving it) and thus being specific about the exact state variables. She can then think about measuring them and testing empirically whether they are indeed relevant for investment (or any other policy). Our treatment of leverage in the paper offers a practical example of this disciplined approach.

As with any structural method, specification error remains a concern and this manifests itself in the possibility that the model is specified with the wrong state variables. However, our approach offers a very natural way to address this issue. By projecting the empirical investment policies on a set of candidate state variables, and using variance decomposition techniques, we let the data inform us about the relevant state variables to include in a model. Model specification is thus guided by the data.[5]

[5] In addition, by relying on higher order polynomial approximations our paper also addresses the type of misspecification concerns in Barnett and Sakellaris (1998) and Bustamante (2016) who emphasize the

We believe our paper contributes to the literature in three significant ways. First, and foremost, it provides a robust empirical methodology to characterize firm level investment behavior, that can be applied in many settings, including the study of private firms' investment and to compare it with that of publicly traded corporations[6], because it does not require information about the market value of the firm. Second, direct approximation of investment policy functions delivers many more informative empirical moments for the identification and inference of the underlying structural parameters of the model. Finally, formal variance decomposition exercises proposed in the paper can be used to isolate the contribution of different state variables and distinguish across classes of models. For example, debt will only be an important state variable in models with financial frictions.

The rest of our paper is organized as follows. The next section describes the general model and the implied optimal investment policies. In Section 3 we discuss a number of practical issues regarding the empirical estimation of investment policy functions. Section 4 reports the results from estimating empirical policy functions. Section 5 uses the information from the estimated policy functions to structurally estimate the key parameters. We then conclude with a brief discussion of the role of asset prices in estimating investment.

2. Investment Policy Functions

This section describes our approach in the context of a streamlined dynamic structural model of investment suitable for empirical work on firm level investment. This is a generalized version of Abel and Eberly (1994, 1997) and Caballero and Engel (1999). We allow for a weakly concave production technology and an investment technology featuring both

importance of including higher order terms to address misspecification concerns, albeit in the context of standard Q investment regressions.

[6]Asker, Farre-Mensa, and Ljungqvist (2011) offer an example of the limitations in describing the investment decisions of private firms without data on market values.

non-convex and convex capital adjustment costs which are potentially asymmetric and discontinuous. This environment is flexible enough to ensure the vast majority of investment models in literature can be treated as special cases. The model specification is crucial as it imposes all the necessary discipline on the identification and measurement of relevant state variables for empirical work. For exposition purposes we delay discussion of important features such as financial market imperfections and aggregate shocks to the next section.

2.1. The Benchmark Model

We examine the optimal investment decision of a firm seeking to maximize current shareholder value, V, in the absence of any financing frictions. For simplicity, we assume that the firm is financed entirely by equity and denote the value of periodic distributions net of any securities issuance by D.

The operating cash flows or profits of this (representative) firm are summarized by the function Π defined as sales revenues net of operating costs. We formalize this relation as:

$$\Pi\left(K_t, A_t, W_t\right) = \max_{N_t}\left\{F\left(A_t, K_t, N_t\right) - W_t N_t\right\}. \tag{1}$$

The function $Y_t = F\left(A_t, K_t, N_t\right)$ denotes the value of sales revenues in period t, net of the cost of any materials. Revenues depend on a firm's capital stock and labor input, denoted by K_t and N_t, respectively. The variable A_t captures the exogenous state of demand and/or productivity in which the firm operates. W_t denotes unit labor costs, including wages, taxes and other employee benefits. Both A_t and W_t can vary stochastically over time, thus accommodating any variations to the state of the economy or industry in which a firm operates. We now summarize our main assumptions about revenues and profits.

Assumption 1. Sales. The function $F : A \times K \times N \rightarrow R_+$, (i) is increasing in A, and increasing and concave in both K and N; (ii) is twice continuously differentiable; (iii) satisfies $F\left(hA, hK, hN\right) \leq hF\left(A, K, N\right)$ for all $\left(A, K, N\right)$; and (iv) obeys the standard Inada conditions: $\lim_{K\to 0} \partial F/\partial K = \lim_{N\to 0} \partial F/\partial N = \infty$ and $\lim_{K\to\infty} \partial F/\partial K = \lim_{N\to\infty} \partial F/\partial N = 0$

Item (iii) is a departure from the standard linear homogeneous model and explicitly allows for decreasing returns to scale. It is straightforward to show that the function $\Pi(K, A, W)$ is also increasing and weakly concave in K.[7]

Installed capital depreciates at a rate $\delta \geq 0$, and capital accumulation requires investment, I_t. We assume that current investment does not affect the current level of installed capacity and becomes productive only at the beginning of the next period:

$$K_{t+1} = (1 - \delta) K_t + I_t. \tag{2}$$

Moreover, there exist costs to adjusting the stock of capital, $\Phi(\cdot)$, which reduce operating profits. Capital adjustment costs depend on the amount of investment and the current stock of capital. Our assumptions about the adjustment cost function are described below.

Assumption 2. Adjustment Cost. The adjustment cost function $\Phi(\cdot) : I \times K \to R_+$ obeys the following conditions: (i) it is twice continuously differentiable for all I, except potentially $I = I^*(K)$**;** (ii) $\Phi(I^*(K), K) = 0$; (iii) $\Phi_I(\cdot) \times (I - I^*(K)) \geq 0$; (iv) $\Phi_K(\cdot) \leq 0$; and (v) $\Phi_{II}(\cdot) \geq 0$.

Items (ii) and (iii) together imply that adjustment costs are non-negative and minimized at the natural rate of investment $I^*(K)$. In most cases this is assumed to be either 0 or δK depending on whether adjustment costs apply to gross or net capital formation. Item (i) allows for general non-convex and potentially discontinuous adjustment costs.

[7]We could assume either that the technology exhibits decreasing returns or that markets are not perfectly competitive. Either way, sales can be described by the decreasing returns to scale function.

2.2. *The Investment Decision*

We now define the sequence of optimal investment decisions by the firm as the solution to the following dynamic problem:

$$V\left(K_t, A_t, W_t, \Omega_t\right) = \max_{\{I_{t+s}, K_{t+s+1}\}_{s=0}^{\infty}} \mathrm{E}_t \left[\sum_{s=0}^{\infty} M_{t,t+s} D_{t+s} \right] \tag{3}$$

$$\text{s.t.} \qquad D_{t+s} = \Pi\left(K_{t+s}, A_{t+s}, W_{t+s}\right) - \Phi\left(I_{t+s}, K_{t+s}\right) \tag{4}$$

together with the capital accumulation equation (2). $M_{t,t+s}$ is the stochastic discount factor between periods t and $t+s$, and Ω_t denotes the set of *aggregate* state variables summarizing the state of the economy. Aggregate state variables may include shocks to productivity, wages, capital adjustment costs, relative price of investment goods, and representative household preferences.

If $\Phi(\cdot)$ is twice continuously differentiable for all I - standard first-order conditions are sufficient to characterize the solution to (3). The optimal investment policy equates marginal benefit and cost of investment:

$$q_t = \Phi_I\left(I_t, K_t\right) \tag{5}$$

where q_t is the marginal value of installed capital, or *marginal* q, and satisfies the following Euler equation:

$$q_t = \mathrm{E}_t\left[M_{t,t+1}\left(\Pi_K\left(K_{t+1}, A_{t+1}, W_{t+1}\right) + (1-\delta)\, q_{t+1} - \Phi_K\left(I_{t+1}, K_{t+1}\right)\right)\right]. \tag{6}$$

The computation of optimal investment policies requires combining the expressions in (5) and (6). However, under general conditions, there exists no explicit closed form solution. Nevertheless, under the assumption that the marginal cost of investment, Φ_I, is monotone, these policies can be further characterized by inverting the (5) to get:

$$\frac{I_t}{K_t} = \widetilde{G}\left(K_t, q_t\right).$$

2.3. *Our Estimation Approach*

Much of the literature follows Hayashi (1982) and assumes linear homogeneity (in I and K) for the functions $\Pi(\cdot)$ and $\Phi(\cdot)$ to obtain a linear investment policy from (5) under quadratic adjustment costs:

$$\frac{I_t}{K_t} = \alpha_0 + \alpha_1 q_t. \tag{7}$$

Under these assumptions marginal q equals average Q - i.e. ratio of market value to replacement cost of capital - and the investment equation in (7) can be estimated directly from the data. With less restrictive conditions, however, marginal q is no longer directly observable.

Instead, our approach is much more general. It relies only on rational expectations and the recursive nature of process for the stochastic variables. Under these assumptions, the marginal value of installed capital can always be written as $q_t = q(K_t, Z_t)$, where the vector Z denotes all state variables other than capital and captures possible shocks to firm productivity, costs and output demand as well as aggregate state variables, i.e. $Z_t = \{A_t, W_t, \Omega_t\}$.

As a result the optimal rate of investment can always be characterized by the following state variable representation:

$$\frac{I_t}{K_t} = G(K_t, Z_t) \tag{8}$$

The explicit form for the function $G(\cdot)$ depends on the specific functional forms of $\Pi(\cdot)$ and $\Phi(\cdot)$, and may not be readily available in most circumstances. However, given the measurability of investment, it can be directly estimated as a function of its underlying state variables K and Z as long as they are also measurable.[8]

[8] When item (i) of Assumption 2 holds for any level of investment excluding $I^*(K)$, the optimal investment policy may be a discontinuous function. Nonetheless, it still admits the representation in (8), and it can be directly estimated as function of its underlying state variables.

Formally then, our methodology relies on the observation that under general conditions we can approximate the optimal investment policy arbitrarily closely with the following tensor product representation:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_z=0}^{n_z} c_{i_k,i_z} k^{i_k} z^{i_z} + \epsilon_{it} \tag{9}$$

where $z = \ln(Z)$ and $k = \ln(K)$ and ϵ_{it} is the approximation error.[9] Once estimated, the approximation coefficients c_{i_k,i_z} can be used to infer the underlying structural parameters of the model, or at the very least, place restrictions on the nature of technology and adjustment costs. We investigate several parameterizations of the model in the next section.

The choice of the polynomial order can be made according to standard model selection techniques based on a measure of model fit such as adjusted R^2 or Akaike information criterion (AIC). In the next section we show that a second order polynomial is often sufficient, and higher order terms are generally not important to improve the quality of the approximation. The low order of approximation mitigates the need to use orthogonal polynomials, simplifying the interpretation of the estimated coefficients and their relationships with the underlying structural parameters of the model.

2.4. Discussion

The appeal of Tobin's Q lies in the general belief (hope) that it serves as a forward-looking measure of investment opportunities summarizing all information about expected future profitability and discount rates. It is well known, however, that this is true only under some extreme assumptions and in most settings Tobin's Q will fail to capture a significant amount of relevant forward information (e.g. Gomes, 2001, Eberly, Rebelo,

[9] Non-smooth investment policies may require several high order polynomial terms to better capture nonlinearities in investment. More generally, although not pursued in this paper, optimal policies can also be estimated using a full nonparametric approach.

and Vincent, 2011). What is generally correct however, is that *all* relevant current and forward looking information is incorporated in the underlying state variables.

Direct estimation of the policy functions has other important benefits. First, unlike Q-type regressions which are based on an optimality condition where Q and investment are determined simultaneously, state variables are, by construction, pre-determined at the time current investment is chosen. Thus our method represents a distinct improvement over standard Q-regressions. Second, policy function estimation also minimizes the measurement error concerns induced by potential stock market misvaluations (Blanchard, Rhee, and Summers, 1993; Erickson and Whited, 2000), although it is more vulnerable to errors in the measurement state variables.

3. Estimation Issues

We now describe some key issues concerning the practical implementation of our method to construct empirical estimates of optimal investment policies at the firm level.

3.1. Measurement

Empirical implementation of (9) requires measurement of the state variables, most importantly, of the possible components of the exogenous state Z. This can be achieved by imposing the theoretical restrictions implied by the model.

For example, under the common assumption that the sources of uncertainty are in firm technology and demand (i.e. $Z = \{A\}$) we can measure these shocks directly from observed sales by inverting the revenue function $Y = F(Z, K, N)$.[10]

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[10] Alternatively, we could also estimate Z directly (e.g. Olley and Pakes, 1996) and use a two stage approach. However this requires specification of the precise revenue function and adds a number of econometric problems, most significantly, endogeneity. However, since we are interested in characterizing investment, exact knowledge of Z is not required.

In this case we can work instead with the polynomial approximation:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} \sum_{i_n=0}^{n_n} g_{i_k,i_y,i_n} k^{i_k} y^{i_y} n^{i_n} + \epsilon_{it}. \tag{10}$$

The investment policy is now represented as a direct function of three observable variables, including capital, sales and labor, and can be readily estimated from the data.[11]

Finally, since the right hand side variables are all in logs, we can - without any loss of generality - scale employment and sales by the capital stock and estimate a version of (10) using $\ln(Y/K)$ and $\ln(N/K)$. This transformation allows us to make our results more directly comparable with the existing literature.

3.2. Firm Fixed Effects

It is natural to expect differences in firms' natural rate of investment, $I^*(K)/K$, mainly due to variations in the depreciation rates on their assets. We can readily capture firm heterogeneity in depreciation rates, i.e. $\delta = \delta_j$, by allowing the constant term in (10) to include a firm-specific component.

3.3. Aggregate Shocks and Time Effects

A complete state-variable representation of investment in (9) also includes some *aggregate* state variables, Ω, as part of the exogenous state Z. The set of *aggregate* state variables can include, among others, *aggregate* shocks to productivity, wages, capital adjustment costs, relative price of investment goods, and investors' discount rates. While the measurement of our firm level state variables, like sales and size, captures part of the variation in these underlying aggregate state variables, there may still be substantial investment variation attributable to omitted variation in these aggregate state variables. For instance,

[11] The coefficients g_{i_k,i_z,i_n} are now convolutions of the structural parameters of the revenue function and the approximation coefficients c's.

aggregate productivity shocks may affect firm investment indirectly through the stochastic discount factor, M, by impacting risk premia.

Given a large enough panel of firms, however, complete knowledge of the aggregate state variables in Ω is not required for the purpose of estimating investment. Instead, we can capture the impact of *all* unobserved aggregate variation by allowing for both time fixed effects *and* time-specific polynomial slope coefficients. The former will capture all unobserved aggregate variation that affects all firms equally, while the latter will account for unobserved variation that impacts them differently.

Formally, allowing for time-specific polynomial coefficients in our baseline firm level state variables, k and y, is equivalent to a tensor product polynomial representation of investment which includes a complete set of time dummies, η, as state variables:

$$\frac{I_{jt+1}}{K_{jt}} \simeq \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} \sum_{i_\eta=0}^{n_\eta} b_{i_k,i_y,i_\eta} \times k_{jt}^{i_k} \times y_{jt}^{i_y} \times \eta_t^{i_\eta} = \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} d_{i_k,i_y,t} \times k_{jt}^{i_k} \times y_{jt}^{i_y} \tag{11}$$

where the equality follows from the fact that $\eta_t^{i_\eta} = \eta_t$ for any $i_\eta \geq 0$, and $d_{i_k,i_y,t} \equiv \eta_t \times \sum_{i_\eta=0}^{n_\eta} b_{i_k,i_y,i_\eta}$.

3.3.1. Example: Time Varying Discount Rates

One obvious example of how aggregate shocks can impact firm level investment decisions is through variation in the firm discount rates, embedded in $M_{jt,t+s} = \frac{1}{R_{j,t+s}}$. Theoretically, however, firm specific discount rates can be constructed as a premium on the aggregate interest rate, R_{ft}. For example, under a simple CAPM model we get that:

$$R_{jt} = R_{ft} + \beta_{j,t}(R_{mt} - R_{ft}) + \epsilon_{jt} \tag{12}$$

where R_{mt} is the return on the aggregate stock market. Importantly, the firm specific factor loading, β_{jt}, captures the time-varying market risk exposures of firm i and is defined by:

$$\beta_{jt} = \frac{Cov(R_{jt}, R_{mt}|\mathrm{I}_t)}{Var(R_{mt}|\mathrm{I}_t)} \tag{13}$$

where I_t is information set at time t. By construction, this information set must also be completely summarized by the current state variables so that we can write $\beta_{jt} = \beta(Z_t)$.[12,13] More generally, with multiple arbitrary risk factors, F_t, we will have $R_{jt} = R_{ft} + B_{jt}F_t + \epsilon_{jt}$ where now $B_{jt} = B(Z_t)$ is a vector of risk loadings.

To illustrate how this case can be handled consider the extreme Jorgensonian case without any adjustment costs and with Cobb-Douglas technology, $Y_t = A_t K_t^{\alpha}$. In this case, the optimal investment policy obeys:

$$1 = \alpha \mathrm{E}_t[M^i_{jt,t+1} A_{j,t+1} K^{\alpha-1}_{j,t+1} + (1 - \delta_j)] \tag{14}$$

Optimal investment be directly calculated in closed form as

$$\ln(K_{j,t+1}) \quad = \quad \frac{1}{1-\alpha}[\ln(\alpha/\delta_j) + \ln \mathrm{E}_t(A_{j,t+1} R_{j,t+1})] \tag{15}$$

where $\ln(\alpha/\delta_j)$ is a firm fixed effect and $\ln(A_{j,t+1})$ is firm j's only state variable (without adjustment costs). To a first order approximation this equals:

$$\begin{aligned}
\ln(K_{j,t+1}) &\approx \frac{1}{1-\alpha}[\ln(\alpha/\delta_j) + \ln \mathrm{E}_t A_{j,t+1} + \ln \mathrm{E}_t R_{j,t+1}] \\
&= \frac{1}{1-\alpha}\Big[\ln(\alpha/\delta_j) + \ln \mathrm{E}_t A_{j,t+1} \\
&+ \ln R_{f,t+1} + \ln \mathrm{E}_t \beta(A_{j,t+1})[R_{m,t+1} - R_{ft+1}]\Big]
\end{aligned} \tag{16}$$

where we ignored the covariance term for clarity and assumed expected returns obey CAPM.

[12]To be precise, the information set *relevant* to the firm is summarized by the current value of its state variables.

[13]Closed form solutions are difficult to obtain in general. Berk, Green, and Naik (1999) and Gomes, Kogan, and Zhang (2003) provide formal derivations of β_{jt} as a function of firm level state variables for specific investment technologies.

Since $A_t = Y_t/K_t^{\alpha}$, it follows that the optimal (log) investment policy can be approximated by a linear function of a firm specific fixed effect, $\ln(\alpha/\delta_j)$, a simple time fixed effect that captures variation in the risk free rate, $\ln R_{f,t+1}$, and a function of firm specific observables Y_t and K_t, with time-specific coefficients summarizing time variation in risk premia, $R_{m,t+1} - R_{ft+1}$.

In the presence of adjustment costs a closed form solution for the optimal policy is generally not available. However, the logic of our argument remains and the state-variable polynomial representation in (11) can be used to deal with many types of aggregate shocks to firms, in particular shocks to the discount rate of firms.

4. Empirical Findings

We now implement our methodology to estimate the empirical policy function. All details concerning the data and the construction of the variables are provided in the Appendix. Table 1 reports the key summary statistics including mean, standard deviation and main percentiles for the primary variables of interest.

[Table 1 about here.]

4.1. Baseline Estimates

Our first goal is to identify a parsimonious polynomial representation both in terms of variables and an order of approximation that provides the best overall fit for investment empirically and can be used to evaluate our structural model.

Table 2 shows the empirical estimates for various polynomial approximations to the investment policy (10). All estimates use time and firm fixed effects to account for potential aggregate shocks and firm differences in average investment rates.

[Table 2 about here.]

Generally, we find that first and second order terms are all strongly statistically significant. However, it is generally the case that adding the employment-to-capital ratio leaves the overall fit of the regression virtually unaffected. Interaction terms among the variables are generally not statistically significant and do not improve much the quality of the approximation as witnessed by the virtually unchanged adjusted R^2. We omit higher order terms in the polynomial representation because they are not statistically significant and are generally not necessary to improve the quality of the approximation.

Overall, while including second order terms improves the approximation regardless of the variables selection, adding the employment-to-capital ratio in the polynomial leaves instead virtually unaffected the overall fit for investment. We conclude that a second order polynomial approximation that uses firm size and the sales-to-capital ratio (column 2) offers the best parsimonious empirical representation of investment.

4.2. General Cases

4.2.1. Aggregate Shocks and Time-Specific Coefficients

We now discuss the results of expanding the baseline polynomial approximation by adding time-specific coefficients to the baseline regressions. Table 3 reports the estimates of the average partial effects for each firm-level state variable in the polynomial approximation.

[Table 3 about here.]

Essentially, we see that the introduction of time-specific slopes, while allowing for a more flexible investment specification leads to only a very marginal improvement in overall goodness-of-fit. The average coefficient estimates are also all in line with their corresponding estimates in the baseline case without time-specific slopes.[14].

[14]In the online appendix we show most time-specific coefficients on the polynomial terms exhibit sub-

4.2.2. Labor Market Shocks and Cash Flow

Aggregate variation in the price of variable inputs, such as labor, will be captured by adding simple time effects to (9). However, if some of these shocks are firm-specific, the set of state variables, Z, would now need to be expanded to also include the firm level wage rate, W (i.e. $Z = \{A, W\}$). Since direct evidence on firm level labor costs is often sparse it is often best to again use theory to infer these shocks directly from observed cash flow data.

For example, if the production function, $F(A, K, N)$, is Cobb-Douglas, operating profits become $\Pi = ZK^{\theta}$, where Z captures joint information about A and W, and can be directly constructed from:

$$z \equiv \ln Z = \ln \Pi - \theta \ln(K).$$

The investment policy is now be approximated as:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_\pi=0}^{n_\pi} d_{i_k,i_\pi} k^{i_k} \pi^{i_\pi} + \epsilon_{it},$$

using only data on log operating profits, $\pi = \ln \Pi$ and the stock of capital.

Table 4 reports the results of estimating equation (10) using the classic measure of cash flow (earnings before extraordinary items plus depreciation) instead of sales-to-capital ratio. Although there are some differences compared to our main results, levels of significance and goodness of fit are all substantively robust. Overall, we find that these specifications perform slightly less well than the baseline specification which includes firm sales instead of cash flow.

[Table 4 about here.]

stantial variation over time, but with estimates preserving their sign over the sample period

4.3. Capital Market Imperfections and Leverage

Our basic approach can be easily extended to models with financial frictions. Most modifications of the firm problem (3), that allow for frictions such as tax benefits of debt, collateral requirements and costly external financing, also imply that firm debt, B, becomes an additional state variable for the optimal investment policy.[15]

Formally, nearly all structural debt models imply that the optimal investment policy will take the general form:

$$\frac{I}{K} = G\left(K, B, Z\right). \tag{17}$$

In this case we can generalize our procedure by augmenting the approximate policy function (10) with additional terms including (log) corporate debt, $b = \log(B)$:

$$\frac{I}{K} = \sum_{i_k=0}^{n_k} \sum_{i_y=0}^{n_y} \sum_{i_n=0}^{n_n} \sum_{i_b=0}^{n_b} g_{i_k,i_y,i_n,i_b} k^{i_k} y^{i_y} n^{i_n} b^{i_b} + \epsilon_{it}. \tag{18}$$

As is well known, past evidence for financial frictions has often - and incorrectly - rested on excess sensitivity of investment to cash flow variable in standard Q regressions. By contrast, our approach cleanly identifies violations of Modigliani Miller with findings that leverage is a relevant state variable for optimal investment policies and thus will show up as an important empirical determinant of these choices. This is a good example of how we can use empirical evidence to discipline our modeling choices and pin down the relevant state variables.[16]

[15] Examples of models where net financial liabilities represents an additional state variable for the optimal investment policy include Whited (1992), Bond and Meghir (1994), Gilchrist and Himmelberg (1998), Moyen (2004), Hennessy and Whited (2007), Hennessy, Levy, and Whited (2007), Gomes and Schmid (2010), Bustamante (2016), and Bolton, Chen, and Wang (2011), among others.

[16] The finding that leverage is an important state variable is sufficient but not necessary to establish the importance of financial frictions since it is possible to construct some stylized models of frictions where firm leverage is not a state variable.

Panel A of Table 5 shows the results of introducing leverage to our baseline state variable approximation of investment. The first column measures debt as the sum of short-term plus long-term debt, while the second column uses a measure of net leverage, by subtracting cash and short-term investments from debt.[17]

[Table 5 about here.]

Our estimates show that all leverage terms are generally statistically significant confirming that there is indeed some degree of interaction between financing and investment decisions of firms. The negative point estimates in Columns (2) and (4) of Panel A in Table 5 are also generally consistent with theoretical restrictions imposed by most models of financing frictions.

4.4. Alternative Adjustment Costs and Lagged Investment

Policy function estimation can naturally accommodate more detailed investment models with frictions such as time-to-build and complex adjustment cost specifications such as Eberly, Rebelo, and Vincent (2011), by simply including lagged investment as additional state variable in the optimal investment policy, $G(\cdot)$.[18]

We investigate the role of lagged investment in Panel B of Table 5. To address endogeneity issues in dynamic panel data with a lagged dependent variable, we instrument

[17] Using several alternative measures of leverage does not alter the main findings. These results are available upon request.

[18] Eberly, Rebelo, and Vincent (2011) use the following linear-quadratic adjustment cost function:

$$\Phi(I_t, I_{t-1}) = \left[1 - \xi\left(\frac{I_t}{I_{t-1}} - \gamma\right)^2\right] I_t$$

This adjustment cost specification makes lagged investment, I_{t-1}, an additional state variable in the optimal investment policy.

lagged investment with prior two lags of its first-difference. Consistent with the evidence in Eberly, Rebelo, and Vincent (2011), lagged investment enters with a positive and significant coefficient, and increases the overall fit to investment. While lagged investment enters significantly, its inclusion does not affects point estimates or the significance of the baseline state variables. The AIC however decreases substantially from 79,062.98 to 43,296.39.[19]

4.5. Alternative Samples

Capital-intensive manufacturing firms form probably the most reliable panel for this study, but it is nevertheless interesting to examine the usefulness of our methodology across different samples. Accordingly, Panel C of Table 5 reports our findings in three alternative panels of firms. The first column looks at a panel that now includes all firms except those in the financial sector, regulated utilities and public services. The second shows the results for a panel covering only the sub-period between 1982-2010, where many authors often focus. Finally, the third column reports the results for a balanced panel of manufacturing firms during the period 1982-2010.[20]

Adding non-manufacturing firms substantially expands the sample and the statistical significance of our estimates, but it does not affect the overall goodness of fit. On the other hand, eliminating the first ten years of data from our baseline sample slightly reduces overall performance. The main results are still confirmed on the smaller balanced sample, which shows that our findings are not driven by the attrition in database due to entry and exit.

[19]These results are virtually unchanged when leverage is included to the state variable approximation of investment.

[20]Several other subsamples were also examined without noticeable changes in the findings. All results are available upon request.

4.6. Identifying State Variables

Our methodology builds on the idea that a model is described not only by its restrictions on functional forms, but also, and most importantly, by its state variables. Different classes of investment models often lead to different sets of state variables. As we show below the importance of various classes of investment models can be assessed through a statistical variance decomposition of their corresponding state variable representation of investment.

To detect the importance of the various state variables in capturing investment variation we follow the analysis of covariance (ANCOVA) in Lemmon, Roberts, and Zender (2008). To do so we estimate the empirical model of investment:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1} \tag{19}$$

where δ_j is a firm fixed effect and η_t is a year fixed effect. X denotes a vector of explanatory variables that includes various combinations of the possible state variables.

Table 6 investigates this possibility empirically by reporting the results of a covariance analysis (ANCOVA) as in Lemmon, Roberts, and Zender (2008). Specifically, each column in the table corresponds to a different specification for investment. The numbers reported in the table, excluding the adjusted R^2 reported in the last row, correspond to the fraction of the total Type III partial sum of squares for a particular model.[21] That is, we normalize the partial sum of squares for each effect by the aggregate partial sum of squares across all effects in the model, so that each column sums to one. Intuitively, each value in the table is the fraction of model sum squares attributable to a particular effect (i.e. firm, year, Q, cash flow, etc.).

[21] We use Type III sum of squares because (i) the sum of squares is not sensitive to the ordering of the covariates, and (ii) our data is unbalanced (some firms have more observations than others).

[Table 6 about here.]

Theory implies that all long run cross-sectional variation in investment rates will be accounted by differences in the depreciation rate, δ_j. Thus, it is not surprising that firm fixed effects account for a large fraction of the variation in **levels**. However, a decomposition of the variation in investment **changes** shows that our baseline polynomial in firm sales and size now accounts for 94 percent of the total variation.

This variance decomposition shows that, in this sample of publicly traded firms, only about 1 percent of the explained variation in investment levels can be accounted by the covariation with firm financial leverage. Similarly, financial leverage accounts only for about 3 percent of the overall explained variation in investment changes, while 90 percent is attributable to our core state variables alone.

The thrust of our argument is that marginal Q should matter a lot more than average Q for investment policies. Theoretically, any information contained in *marginal* Q will be spanned by the state variables characterizing the optimal investment policy. How useful is then Tobin's average Q? It remains true that Tobin's average Q remains an endogenous variable in the model which retains some (but generally far from perfect) correlation with investment behavior. As such is it possible that this variable may isolate additional investment variation due to some omitted state variables.

Column (4) shows that only 1 percent of the overall variation in investment can be attributed to Tobin's Q, while 17 percent is attributable to the state variable polynomial. A similar decomposition of investment rates changes is more stark. Tobin's Q accounts only for 2 percent of the overall explained variation in investment changes, while 92 percent is attributable to our core state variables alone. Overall, it seems that Tobin's Q offers very little additional information beyond the identified state variables of investment.

5. Structural Estimation

The information from the empirical policy functions should be a key input in the structurally estimation of the model and its key parameters. We now use the information from the estimated policy functions to structurally estimate the key adjustment cost parameters using indirect inference. To do this we must first specify functional forms for sales and adjustment cost functions that satisfy Assumptions 1 and 2.

5.1. Model Parameterization

We assume either that the technology exhibits decreasing returns or, that markets are not perfectly competitive. Either way, sales revenues can be described by the decreasing returns to scale function:

$$Y = A\left(K^{\alpha}N^{1-\alpha}\right)^{\gamma}$$

where $\alpha \in (0,1)$ and $\gamma < 1$ captures the degree of returns to scale. The stochastic process for A is of the AR(1) form:

$$\ln A_t = (1-\rho)\ln \bar{A} + \rho \ln A_{t-1} + \sigma\zeta_t$$

where $|\rho| < 1$, $\sigma > 0$ and ζ follows a truncated i.i.d. normal with zero mean and unit variance. We assume that the unit labor cost W is constant and normalized to one.

A general adjustment cost function that satisfies Assumption 2 is:

$$\Phi(I,K) = I + \begin{cases} aK + \frac{b}{v}\left(\frac{I-I^{*}(K)}{K}\right)^{v} K & \text{if } I \neq I^{*}(K) \\ 0 & \text{if } I = I^{*}(K) \end{cases} \tag{20}$$

where a, b are all non-negative, and $v \in \{2,4,6,...\}$. We normalize the relative price of investment to one and assume that adjustment costs apply to net capital formation, $I^{*}(K) = \delta K$. We have non-convex fixed cost of investment when a is positive. Note that standard smooth quadratic adjustment costs are obtained as special case of (20) when $v = 2$ and $a = 0$.

5.2. *Estimation Results*

Several structural parameters can be accurately estimated directly from unconditional moments of variables such as sales and/or profits without resorting to indirect inference methods. We thus fix a number of these auxiliary parameters to what are more or less consensual values in the literature. Specifically we set the degree of decreasing returns, $\gamma = 0.85$, and $\alpha = 0.35$ implying a capital share ($\alpha\gamma$) of 0.30 in line with values used in previous studies (Gomes, 2001). Moreover, values like the average depreciation rate, δ, and discount factor, M, are largely immaterial for our results. We set their values at 0.10 and 0.95, respectively. Throughout our analysis, we also set the persistence and the standard deviation of the technology shocks, ρ and σ, respectively, to 0.80 and 0.10. Although it is straightforward to include these parameters in the structural estimation exercise, they are usually best identified from the variance and persistence of profits or revenues and are not generally crucial to the identification of adjustment costs parameters.

The algorithm for indirect inference is now well understood. First, given a specific set of parameter values, we solve numerically the problem of the firm in (3) using standard value function iteration techniques. We then generate multiple panels of simulated data using the optimal policy and value functions of the model. Next, we estimate the regression coefficients from both standard Q regressions and polynomial approximations to the optimal investment policy in each panel and compare the average estimate to those obtained in the Compustat dataset. The method then picks the model parameters that make the actual and simulated moments as close to each other as possible.[22]

For each parameterization of the adjustment cost function we simulate 100 artificial panels of 500 firms each with 390 years of data. We estimate the investment polynomial regressions using the last 39 years of simulated data, which corresponds to the time span

[22]For a detailed description in a very general setting see Warusawitharana and Whited (2016).

of the actual data sample. We then report the average coefficient estimates and standard errors across artificial panels.

Table 7 shows the estimated parameter values and compares the implied moments in the artificial data with our own empirical estimates. The table shows that a model with quadratic adjustment costs but also a small amount of fixed costs does well in matching all regression coefficients found in the data. This model is able to both generate a weak sensitivity of investment to Q and produce the coefficients from empirical policy function estimates. Crucially, the estimated level of fixed costs implies a large enough inaction region where investment and average Q are uncorrelated.

[Table 7 about here.]

5.3. Moment Elasticities

We next follow Hennessy and Whited (2007) to use the simulated model to measure the elasticity of key theoretical moments with respect to the several parameters.[23] Formally, the elasticity of moment x with respect to parameter κ is computed as:

$$\xi_{x,\kappa} = \frac{x\left(\widehat{\kappa}(1+\varepsilon)\,;\theta\right) - x\left(\widehat{\kappa}(1-\varepsilon)\,;\theta\right)}{2\varepsilon x\left(\widehat{\kappa}\right)}$$

where $\widehat{\kappa}$ is the baseline value of κ, ε is the percent deviation from the baseline, and θ is a vector of the other structural parameters.[24] where we use our parameter estimate as our baseline.

Table 8 reports our findings. For completeness we include also the elasticities with respect to the technology parameters γ and α. The table shows that most coefficients are

[23] Intuitively, if the elasticity of a particular theoretical moment to a particular parameter is low, then that moment is an unreliable guide to inferring the true value of the underlying structural parameter.

[24] We generally use $\varepsilon = 0.1$, except for the curvature of the adjustment costfunction where we use $\varepsilon = 1$ and consider a one sided deviation only.

quite sensitive to the degree of returns to scale, γ. As expected, the capital elasticity α has a larger effect on unconditional moments of the investment distribution.

[Table 8 about here.]

The main conclusion, however, is that investment adjustment cost parameters are generally better identified from estimated policy function coefficients, which exhibit higher elasticities than the coefficient from a standard Q-regression. More generally, we find that the coefficient estimates on Q regressions are quite similar across alternative adjustment cost parameterizations ranging only from a minimum of 0.002 in the specification without adjustment costs to a maximum of about 0.095 across parameterizations. On the other hand, the coefficients on the polynomial approximation exhibit substantial variation. For instance, we found that across the same parameterizations, the coefficients on the linear terms in firm size and sales range from -0.320 to -0.001, and 0.001 to 0.909, respectively.

This suggests that full estimation of a structural model, should primarily target unconditional moments of the investment distribution together with the approximate investment policy function implied by the model. By contrast, the slope of a Q regression is generally less informative about model parameters.

5.4. Replicated Empirical Policies

Given the estimated parameters we can use the structural model as a laboratory, to create an artificial panel and use the simulated data to estimate the approximated investment policy functions. Specifically, given our estimated parameters in Table 7 we create panels of 2,000 firms each with 390 years of data. We run the investment policy regressions using the last 39 years of simulated data. Table 9 reports our estimation results.

[Table 9 about here.]

The approximated investment policy in our simulated data is generally consistent with that estimated in Table 2. The magnitude of all coefficients is generally comparable, except in Column (3) where the signs of $\ln K$ and $\ln \frac{Y}{K}$ flip. More importantly, the simulated model confirms the main argument that the state variable approach to estimate investment policies outperforms the traditional Q approach. Here we do not match the magnitude of adjusted R^2 across regressions. To do do so we introduce measurement error below.

5.5. Measurement Error

It is impossible to directly evaluate how substantial the measurement error might be in Tobin's Q or in the state variables $\ln K$, $\ln \frac{Y}{K}$, etc. However, we can use our artificial panels to assess the quantitative impact of measurement error across these two approaches.

In Table 10, we report results of estimating the theoretical investment policies with measurement error. Specifically, we add i.i.d. measurement errors to the simulated variables V_{it}, K_{it}, and Y_{it} across firms and years. For K and Y/K that have to take the natural logarithm afterwards, we make its values equal to 10^{-8} if the value drops below zero after adding measurement errors. We pick the standard deviation of measurement error in state variables so that the adjusted R^2 in the second order regression in column (5) can match its empirical counterpart in Table 2. The measurement error in V is then set to ensure that the adjusted R^2 in the standard Q regressions is also comparable to its empirical counterpart.

The Table shows that when measurement error is calibrated to empirically plausible magnitudes the marginal value of Tobin's Q in our state variable regressions drops dramatically. The results in Columns (2) and (3) show that adjusted R^2 barely changes when we add Q to a simple first order state variable representation.

[Table 10 about here.]

Together, Tables 9 and 10 suggest that while Tobin's Q can contain some additional information about investment rates, much of it can be lost when accounting for measurement error. An important caveat however, is that adding measurement error in K induces a mechanical correlation between the dependent variable, I/K and the independent variables on the right hand side, because we scale all relevant variables by K, including Q, in regressions of Table 10. We can see this by looking at the point estimate of the regression coefficients on $\ln K$ in columns (2) and (3) in Table 10 which are higher than the comparable numbers in to Table 9. Similarly, the coefficient of Q is also larger with measurement error (Table 10) than without (Table 9). Nevertheless, while this induced correlation is in itself problematic it does not alter our key findings because it impacts standard Q regressions with equal force.

6. Conclusion

Optimal investment policies must be functions of the state variables alone. These are true summary statistics of the investment behavior. This paper relies on this insight to propose an asset price-free alternative that is easy to implement in practice. Under very general assumptions about the nature of technology and markets, our approach ties investment rates directly to firm size, sales or cash flows, and, in the presence of financial market frictions, measures of net liabilities. Our work offers a theoretical foundation to implement a practical alternative to Q under very general assumptions about the firm's problem. Although Tobin's Q is a sufficient statistic only under extreme cases, we find that it often retains some explanatory power in addition to simple linear quadratic representations of the underlying state variables. Hence, depending on the circumstances, a researcher may decide to rely on our approach, Q theory, or combining them.

Abel, Andrew B. and Janice Eberly, "A Unified Model of Investment Under Uncertainty", *American Economic Review*, 84, 1369-1384, 1994.

Abel, Andrew B. and Janice Eberly, "An Exact Solution for the Investment and Market Value of a Firm Facing Uncertainty, Adjustment Costs, and Irreversibility", *Journal of Economic Dynamics and Control* 21, 831-852, 1997.

Abel, Andrew B. and Janice Eberly, "Investment and Q with Fixed Costs: An Empirical Investigation", unpublished manuscript, University of Pennsylvania, 2002.

Asker, J., J. Farre-Mensa, and A. Ljungqvist, "Comparing the Investment Behavior of Public and Private Firms", Working Paper, 2011.

Barnett, S. A. and P. Sakellaris, "Non-linear Response of Firm Investment to Q: Testing a Model of Convex and Non-Convex Adjustment", *Journal of Monetary Economics*, 42, 261-288, 1998.

Berk, Jonathan B., Richard C. Green, and Vasant Naik, "Optimal Investment, Growth Options, and Security Returns", *Journal of Finance*, 54, 1553-1607, 1999.

Blanchard, Olivier, Changyong Rhee, and Lawrence Summers, "The Stock Market, Profit and Investment", *Quarterly Journal of Economics,* 108, 261-288, 1993.

Bolton Patrick, Hui Chen, and Neng Wang, "A Unified Theory of Tobin's q, Corporate Investment, Financing, and Risk Management", *Journal of Finance* 2011.

Bond, Stephen and Costas Meghir, "Dynamic Investment Models and the Firms' Financial Policy", *Review of Economic Studies* 61, 197-222, 1994.

Bustamante, Maria Cecilia, "How Do Frictions Affect Corporate Investment? A Structural Approach ", Journal of Financial and Quantitative Analysis, 51, 1863-1895, 2016.

Caballero, Ricardo J. and Eduardo M. R. A. Engel, "Explaining Investment Dynamics in U.S. Manufacturing: A Generalized (S,s) Approach", Econometrica, 67, 783-826, 1999.

Cooper, Russell and João Ejarque, "Financial Frictions and Investment: A Requiem in Q", Review of Economic Dynamics, 6, 710–728, 2003.

Cooper, Russell and John Haltiwanger, "On the Nature of Capital Adjustment Costs", *Review of Economic Studies* 73, 611-633, 2006.

Eberly, Janice, Sergio Rebelo, and Nicolas Vincent, "What Explains the Lagged Investment Effect?", *Journal of Monetary Economics*, 59, 370-380, 2011.

Erickson, Timothy and Toni Whited, "Measurement Error and the Relationship Between Investment and q", *Journal of Political Economy*, 108, 1027-57, 2000.

Erickson, Timothy and Toni Whited, "On the Accuracy of Different Measures of Q", *Financial Management,* 35, 5-33, 2006.

Erickson, Timothy and Toni Whited, "Treating Measurement Error in Tobin's q", *Review of Financial Studies*, forthcoming, 2011.

Gala, Vito D. and Brandon Julio, "Convergence in Corporate Investments", Working Paper, The Wharton School, 2016.

Gilchrist, Simon and Charles Himmelberg, 1998, "Investment: Fundamentals and Finance", in NBER Macroeconomics Annual, Ben Bernanke and Julio Rotemberg eds.

Gomes, João F., "Financing Investment", *American Economic Review*, 90, 5, 1263-1285, 2001.

Gomes, João F., Leonid Kogan, and Lu Zhang, "Equilibrium Cross Section of Returns", *Journal of Political Economy*, 111(4), 693-732, 2003.

Gomes, João F., and Lukas Schmid. "Levered Returns", *Journal of Finance*, 65(2), 467-494, 2010.

Hayashi, Fumio, "Tobin's Marginal and Average q: A Neoclassical Interpretation", *Econometrica,* 50, 213-224, 1982.

Hennessy, Christopher and Toni Whited, "How Costly is External Financing? Evidence from a Structural Estimation", *Journal of Finance*, 62, 1705-1745, 2007.

Hennessy, Christopher, Amnon Levy and Toni Whited, "Testing Q Theory with Financing Frictions", *Journal of Financial Economics*, 83, 691-717, 2007.

Lemmon, Michael L., Michael R. Roberts, and Jaime F. Zender, "Back to the Beginning: Persistence and the Cross-section of Corporate Capital Structure", *Journal of Finance*, 63, 1575-1608, 2008.

Moyen, Nathalie, "Investment-Cash Flow Sensitivities: Constrained Versus Unconstrained Firms", *Journal of Finance* 59, 2061-2092, 2004.

Olley, G. S., and Pakes, A., "The Dynamics of Productivity in the Telecommunications Equipment Industry", *Econometrica*, 64, 1263-1297, 1996.

Philippon, Thomas, "The Bond Market's Q", *Quarterly Journal of Economics*, 124, 1011-1056, 2009.

Warusawitharana, Missaka, and Whited, Toni "Equity Market Misvaluation, Financing, and Investment", *Review of Financial Studies*, 29, 603-654, 2016

Whited, Toni, "Debt, Liquidity Constraints, and Corporate Investment: Evidence from Panel Data", *Journal of Finance* 47, 1425-1460, 1992.

Appendix

Our data comes from the combined annual research, full coverage, and industrial COMPUSTAT files. To facilitate comparison with much of the literature our initial sample is made of an unbalanced panel of firms for the years 1972 to 2010, that includes only manufacturing firms (SIC 2000-3999) with at least five years of available accounting data.

We keep only firm-years that have non-missing information required to construct the primary variables of interest, namely: investment, I, firm size, K, employment, N, sales revenues, Y, and Tobin's Q. Firm size, or the capital stock, is defined as net property, plant and equipment. Investment is defined as capital expenditures in property, plant and equipment. Employment is the reported number of employees. Sales are measured by net sales revenues. In our implementation these variables are scaled by the beginning-of-year capital stock. Finally, Tobin's Q is measured by the market value of assets (defined as the book value of assets plus the market value of common stock minus the book value of common stock) scaled by the book value of assets.[25] We use also standard measures of cash flow, CF, defined as earnings before extraordinary items plus depreciation; and net corporate debt, B, computed as the sum of short-term plus long-term debt minus cash and short-term investments.

Our sample is filtered to exclude observations where total capital, book value of assets, and sales are either zero or negative. To ensure that our measure of investment captures the purchase of property, plant and equipment, we eliminate any firm-year observation in which a firm made an acquisition. Finally, all primary variables are trimmed at the 1st and 99th percentiles of their distributions to reduce the influence of any outliers, which are

[25]Erickson and Whited (2006) show that using a perpetual inventory algorithm to estimate the replacement cost of capital and/or a recursive algorithm to estimate the market value of debt barely improves the measurement quality of the various proxies for Q.

common in accounting ratios. This procedure yields a base sample of 79,361 firm-years observations.

List of Tables

Table 1: **Summary Statistics**

This table reports summary statistics for the primary variables of interest from Compustat over the period 1972-2010. The investment rate, I/K, is defined as capital expenditures in property, plant and equipment scaled by the beginning-of-year capital stock. The capital stock, K, is defined as net property, plant and equipment. Firm size, $\ln(K)$, is the natural logarithm of the beginning-of-year capital stock. The sales-to-capital ratio, $\ln(Y/K)$, is computed as the natural logarithm of end-of-year sales scaled by the beginning-of-year capital stock. The employment-to-capital ratio, $\ln(N/K)$, is defined as the natural logarithm of the number of employees scaled by the capital stock. The cash flow rate, CF/K, is calculated as the sum of end-of-year earnings and depreciation scaled by the beginning-of-year capital stock. Tobin's Q is defined as the end of year market value of assets scaled by the book value of assets.

	Obs	Mean	Std. Dev.	25th	50th	75th
I/K	79,361	0.367	0.537	0.114	0.209	0.383
$\ln K$	79,361	2.623	2.552	0.880	2.495	4.269
$\ln \frac{Y}{K}$	79,361	1.688	1.071	1.125	1.690	2.277
$\ln \frac{N}{K}$	79,361	-2.971	1.192	-3.717	-2.931	-2.145
Q	79,361	2.033	2.336	0.942	1.274	2.068

Table 2: **Empirical Investment Policies**

This table reports empirical estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of-year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including average Q, cash flow, CF, firm size, lnK, sales-to-capital ratio, $ln(Y/K)$, and employment-to-capital ratio, $ln(N/K)$. Standard errors are clustered by firm and reported in parenthesis. adj. R^2 denoted the adjusted R^2 and AIC is the adjusted Akaike Information Criterion. The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Q	0.036	0.009	0.013	0.010				
	(0.003)	(0.003)	(0.003)	(0.002)				
CF	0.000	0.000	0.000	0.000				
	(0.000)	(0.000)	(0.000)	(0.000)				
$\ln K$		-0.239	-0.147	-0.066	-0.149	-0.177	-0.066	-0.077
		(0.006)	(0.006)	(0.006)	(0.006)	(0.008)	(0.006)	(0.008)
$\ln \frac{Y}{K}$			0.200	0.068	0.201	0.067	0.068	-0.008
			(0.007)	(0.008)	(0.007)	(0.008)	(0.008)	(0.009)
$\ln \frac{N}{K}$				0.288			0.290	0.502
				(0.010)			(0.010)	(0.024)
$(\ln K)^2$						0.017		0.010
						(0.001)		(0.001)
$(\ln \frac{Y}{K})^2$						0.045		0.028
						(0.003)		(0.003)
$(\ln \frac{N}{K})^2$								0.038
								(0.003)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
$\overline{R}^2$	0.206	0.317	0.356	0.391	0.353	0.388	0.389	0.421
AIC	99,894.19	87,933.13	83,245.25	78,764.64	83,614.94	79,227.30	79,062.98	74,871.73
Obs	79,361	79,361	79,361	79,361	79,361	79,361	79,361	79,361

Table 3: **Empirical Investment Policies with Time-varying Coefficients**

This table reports empirical estimates from the investment regression specification with time-varying coefficients:

$$\frac{I_{jt+1}}{K_{jt}} = \beta_t X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including firm size, $\ln K$, sales-to-capital ratio, $\ln(Y/K)$, and employment-to-capital ratio, $\ln(N/K)$. In every specification, we report the average partial effects across time for each variable. Standard errors are clustered by firm and are reported by taking average across time in parenthesis. $\overline{R}^2$ denotes the adjusted R^2 and AIC is the adjusted Akaike Information Criterion. The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)
$\ln K$	-0.154	-0.177	-0.074	-0.072
	(0.008)	(0.020)	(0.008)	(0.019)
$\ln \frac{Y}{K}$	0.207	0.039	0.078	-0.023
	(0.019)	(0.035)	(0.025)	(0.042)
$\ln \frac{N}{K}$			0.278	0.579
			(0.024)	(0.087)
$(\ln K)^2$		0.016		0.007
		(0.002)		(0.002)
$(\ln \frac{Y}{K})^2$		0.054		0.031
		(0.012)		(0.014)
$(\ln \frac{N}{K})^2$				0.054
				(0.012)
Firm FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
$\overline{R}^2$	0.368	0.405	0.405	0.441
AIC	81,860.17	77,186.84	77,045.19	72,277.41
Obs	79,361	79,361	79,361	79,361

Table 4: **Empirical Investment Policies with Cash Flow**

This table reports empirical estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including firm size, $\ln K$, cash flow, CF/K, and employment-to-capital ratio, $\ln(N/K)$. Standard errors are clustered by firm and are reported in parenthesis. $\overline{R}^2$ denotes the adjusted R^2 and AIC is the adjusted Akaike Information Criterion. The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)
$\ln K$	-0.243	-0.309	-0.077	-0.094
	(0.005)	(0.007)	(0.006)	(0.009)
$\frac{CF}{K}$	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
$\ln \frac{N}{K}$			0.336	0.637
			(0.008)	(0.022)
$(\ln K)^2$		0.022		0.010
		(0.001)		(0.001)
$(\frac{CF}{K})^2$		0.000		0.000
		(0.000)		(0.000)
$(\ln \frac{N}{K})^2$				0.051
				(0.003)
Firm FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
$\overline{R}^2$	0.316	0.341	0.388	0.414
AIC	88,005.82	85,114.29	79,226.82	75,759.52
Obs	79,361	79,361	79,361	79,361

Table 5: **Other Robustness Tests**

This table reports empirical estimates from the investment regression specification:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables:

(i) In Panel A, X includes firm size, $\ln K$, cash flow, $CashFlow$, employment-to-capital ratio, $\ln(N/K)$, and two versions of firm leverage, $Leverage$ in which debt as the sum of short term plus long-term debt, and $NetLeverage$ in which we subtracting cash and short-term investments from debt.

(ii) In Panel B, X includes firm size, $\ln K$, sales-to-capital ratio, $\ln(Y/K)$, and employment-to-capital ratio, $\ln(N/K)$, and $\frac{I_{jt}}{K_{jt-1}}$ denotes the lagged investment. In the 2SLS, we instrument lagged investment with prior two lags of its first-difference.

(iii) In Panel C, X denotes a set of explanatory variables including firm size, lnK, sales-to-capital ratio, $ln(Y/K)$, and employment-to-capital ratio. Column (1) uses a sample that include all firms except those in financial sector, regulated utilities, and public services. Column (2) restricts the panel from Column (1) by focusing on the period between 1982-2010. Column (3) looks at the panel we use in our main regressions while restricting to the period between 1982-2010.

In all above regressions, standard errors are clustered by firm and are reported in parenthesis. $\overline{R}^2$ denotes the adjusted R^2 and AIC is the adjusted Akaike Information Criterion. The sample period is 1972 to 2010.

PANEL A: Empirical Investment Policies with Leverage

	(1)	(2)	(3)	(4)
$\ln K$	-0.060	-0.051	-0.076	-0.051
	(0.006)	(0.006)	(0.008)	(0.008)
$\ln \frac{Y}{K}$	0.063	0.071	-0.008	-0.004
	(0.008)	(0.008)	(0.009)	(0.009)
$\ln \frac{N}{K}$	0.281	0.272	0.495	0.495
	(0.010)	(0.010)	(0.024)	(0.024)
Leverage	0.010		0.012	
	(0.002)		(0.003)	
Net Leverage		- 0.011		-0.008
		(0.001)		(0.001)
$(\ln K)^2$			0.010	0.007
			(0.001)	(0.001)
$(\ln \frac{Y}{K})^2$			0.027	0.027
			(0.003)	(0.003)
$(\ln \frac{N}{K})^2$			0.038	0.039
			(0.003)	(0.003)
$(\text{Leverage})^2$			-0.000	
			(0.000)	
$(\text{Net Leverage})^2$				0.000
				(0.000)
Firm FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes
$\overline{R}^2$	0.392	0.407	0.388	0.434
AIC	78,664.49	76,771.03	74,775.30	72,982.21
Obs	79,361	79,361	79,361	79,361

PANEL B: Empirical Investment Policies with Lagged Investment

	OLS	2SLS
$\ln K$	-0.047	-0.034
	(0.005)	(0.005)
$\ln \frac{Y}{K}$	0.084	0.092
	(0.008)	(0.008)
$\ln \frac{N}{K}$	0.249	0.220
	(0.010)	(0.010)
$\frac{I_t}{K_{t-1}}$	0.116	0.084
	(0.006)	(0.010)
Firm FE	Yes	Yes
Year FE	Yes	Yes
$\overline{R}^2$	0.374	·
AIC	59,009.21	43,296.39
Obs	75,414	68,673

PANEL C: Empirical Investment Policies with Alternative Samples

	(1)	(2)	(3)
$\ln K$	-0.215	-0.232	-0.195
	(0.007)	(0.008)	(0.010)
$\ln \frac{Y}{K}$	0.112	0.104	0.072
	0.006	0.007	(0.009)
$(\ln K)^2$	0.018	0.019	0.019
	0.001	0.001	0.001
$(\ln \frac{Y}{K})^2$	0.031	0.034	0.043
	0.002	0.002	0.003
Firm FE	Yes	Yes	Yes
Year FE	Yes	Yes	Yes
$\overline{R}^2$	0.397	0.390	0.380
AIC	186,791.3	166,757.6	72,196.23
Obs	147,783	115,050	59,504

Table 6: **Empirical Variance Decompositions**

This table presents a variance decomposition of several polynomial specifications for both the levels (Panel A) and changes (Panel B) in investment. We compute the Type III partial sum of squares for each effect in the model and then normalize each estimate by the sum across the effects, forcing each column to sum to one. For example, in specification (4) of Panel A, 1% of the explained sum of squares captured by the included covariates can be attributed to Tobin's Q. Similarly, in specification (4) of Panel B, 2% of the explained investment changes can be attributed to changes in Tobin's Q. Firm FE are firm fixed effects. Year FE are calendar year fixed effects. Q denotes Tobin's Q. "Size" denotes the second order polynomial in firm size, $\ln(K)$, and "Sales" denote sales-to-capital ratio, $\ln(Y/K)$. "Cash Flow" denotes a second order polynomial in firm cash flow-to-capital ratio, CF/K. "Leverage" denotes a second order polynomial in firm net leverage, B/K. $\overline{R}^2$ denotes adjusted R^2. The sample period is 1972 to 2010.

	(1)	(2)	(3)	(4)
A: Investment Levels (I/K)				
Firm FE	0.77	0.78	0.79	0.77
Year FE	0.05	0.05	0.06	0.05
Size	0.13	0.12	0.09	0.12
Sales	0.05	0.05	0.05	0.05
Cash Flow		0.00		
Leverage			0.01	
Q				0.01
$\overline{R}^2$	0.39	0.39	0.40	0.39
B: Investment Changes ($\Delta I/K$)				
Year FE	0.07	0.06	0.07	0.06
Size	0.72	0.72	0.66	0.72
Sales	0.22	0.21	0.24	0.20
Cash Flow		0.00		
Leverage			0.02	
Q				0.02
$\overline{R}^2$	0.34	0.35	0.35	0.35

Table 7: **Estimated Moments and Parameters**

This table reports results from estimating the baseline model using investment regressions from simulations using 100 artificial panels of 500 firms each with 39, which corresponds to the time span of the actual data sample from Compustat. The top panel reports the average regression coefficient estimates and standard errors for the data and across artificial panels. The bottom panel reports the estimated parameter values as well as the implied χ^2 statistic.

PANEL A

	Data Moments		Simulated Moments	
Q	0.036		0.064	
	(0.003)		(0.005)	
$\ln \frac{Y}{K}$		0.067		0.045
		(0.008)		(0.004)
$\ln K$		-0.177		-0.159
		(0.008)		(0.008)
$(\ln \frac{Y}{K})^2$		0.045		0.031
		(0.003)		(0.006)
$(\ln K)^2$		0.017		0.038
		(0.001)		(0.020)

PANEL B

Estimated Parameters			
a	b	v	χ^2
0.08	0.03	2	0.00020127

Table 8: **Sensitivity of Model Moments to Parameters**

This table presents elasticities of model moments with respect to key model parameters. The parameters values are those estimated in Section 5. The set of moments include: (1) the coefficient estimate from a standard Q-type investment regression; (2) the coefficient estimates from the investment policy function approximation; (3) moments of the investment distribution such standard deviation (Std) and autocorrelation (AR).

	Moments	γ	α	a	b
1	Q	16.317	-38.859	0.206	-7.924
2	$\ln \frac{Y}{K}$	7.043	12.384	-1.211	-8.134
	$\ln K$	4.386	-0.241	0.162	0.033
	$(\ln \frac{Y}{K})^2$	2.013	43.517	-1.775	-9.006
	$(\ln K)^2$	17.375	-12.653	1.200	-4.932
3	Std I/K	10.000	-33.204	1.075	-8.342
	AR I/K	0.384	-0.628	0.923	-4.893

Table 9: **Investment Policies in Simulated Data**

This table reports empirical estimates from the investment regression specification by using simulated data from the model in Section 5:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of-year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including average Q, firm size, lnK, and sales-to-capital ratio, $ln(Y/K)$. Specifically, given our estimated parameters in Table 7 we simulate a panels of 2,000 firms each with 390 years of data. We run the investment policy regressions using the last 39 years of simulated data. Standard errors are clustered by firm and reported in parenthesis. adj. R^2 denoted the adjusted R^2 and AIC is the adjusted Akaike Information Criterion.

	(1)	(2)	(3)	(4)	(5)
Q	0.066	0.055	0.275		
	(0.002)	(0.002)	(0.014)		
$\ln K$		-0.108	0.099	-0.161	-0.157
		(0.005)	(0.013)	(0.004)	(0.004)
$\ln \frac{Y}{K}$			-0.219	0.046	0.119
			(0.013)	(0.002)	(0.007)
$(\ln K)^2$					0.056
					(0.008)
$(\ln \frac{Y}{K})^2$					0.050
					(0.004)
Firm FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
$\overline{R}^2$	0.100	0.128	0.164	0.109	0.130
AIC	-204, 304.60	-206, 784.00	-210, 051.00	-205, 163.10	-206, 966.40
Obs	78,000	78,000	78,000	78,000	78,000

Table 10: **Robustness Test: Investment Policies in Simulated Data**

This table reports empirical estimates from the investment regression specification by using simulated data from the model in Section 5:

$$\frac{I_{jt+1}}{K_{jt}} = \beta X_{jt} + \delta_j + \eta_t + \varepsilon_{jt+1}$$

where the left-hand-side is end-of-year capital expenditures scaled by beginning-of-year property, plant and equipment, δ_j is a firm fixed effect, η_t is a year fixed effect, and X denotes a set of explanatory variables including average Q, firm size, $\ln K$, and sales-to-capital ratio, $\ln(Y/K)$. Specifically, given our estimated parameters in Table 7 we simulate a panels of 2,000 firms each with 390 years of data. We then add i.i.d. measurement error to the simulated variables V, K, and Y and rerun the investment policy regressions using the last 39 years of simulated data. The standard deviation of the measurement errors in K and Y is picked so that the adjusted R^2 in the regression of Column (5) can match its empirical counterpart. The standard deviation of the error in V is picked in order to match the Q regression of Column (1) with its empirical counterpart. Standard errors are clustered by firm and reported in parenthesis. adj. R^2 denoted the adjusted R^2 and AIC is the adjusted Akaike Information Criterion.

	(1)	(2)	(3)	(4)	(5)
Q	0.147	0.054	0.052		
	(0.002)	(0.003)	(0.003)		
$\ln K$		-0.660	-0.664	-0.779	-0.700
		(0.013)	(0.013)	(0.011)	(0.011)
$\ln \frac{Y}{K}$			0.005	0.009	0.048
			(0.001)	(0.001)	(0.002)
$(\ln K)^2$					0.262
					(0.029)
$(\ln \frac{Y}{K})^2$					0.002
					(0.000)
Firm FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes
$\overline{R}^2$	0.202	0.392	0.393	0.375	0.396
AIC	-22, 657.22	-43, 945.74	-44, 067.08	-41, 772.76	-44, 387.06
Obs	78,000	78,000	78,000	78,000	78,000