Implications of investor-focused ESG reporting ... and misreporting

Henry L. Friedman Mirko S. Heinle Irina Luneva

Abstract: Firms and jurisdictions worldwide are adopting ESG reporting in various forms. To better understand potential implications of ESG reporting, we develop a model in which a firm provides ESG and financial reports to investors. Investors price the firm's stock, and stock prices provide both real and reporting incentives to management. We characterize how the introduction of ESG reporting affects ESG performance, expected cash flows, and misreporting. ESG reporting tends to encourage corporate ESG but can discourage ESG when it has significantly negative cash flow implications. For ESG with moderately negative cash flow implications, the firm's price can suffer from the introduction of ESG reporting. Finally, we use comparative statics to show how changes in investor preferences (e.g., concern for ESG) and ESG efforts' cash flow implications (e.g., penalties, subsidies, or physical and transition risk) affect market responses to financial and ESG reports, corporate misreporting, and ESG and cash flow outcomes.

Note: This paper builds on but is largely distinct from the authors' prior work, "A Theoretical Framework for ESG Reporting to Investors" available at SSRN Webpage.

Keywords: ESG reporting, ESG valuation, real effects

JEL Classification: G11, G23, G34, M14, M40

*Henry L. Friedman is at the Anderson School of Management, University of California, Los Angeles, 110 Westwood Plaza, D402, Los Angeles, CA 90095; email: henry.friedman@anderson.ucla.edu. Mirko S. Heinle is at The Wharton School, University of Pennsylvania, 3720 Locust Walk, Suite 1300, Philadelphia, PA 19104; email: miheinle@wharton.upenn.edu. Irina Luneva is at The Wharton School, University of Pennsylvania, 3720 Locust Walk, Suite 1300, Philadelphia, PA 19104; email: iluneva@wharton.upenn.edu. For helpful comments, we thank our colleagues at UCLA and Wharton, Edwige Cheynel (discussant), Mandy Ellison (discussant), Ginger Zhe Jin (discussant), Georg Schneider (discussant), Iman Sheibany (discussant), and workshop participants at UC Riverside, the 14th Workshop in Accounting and Economics, Mannheim, DAR/DART, the Columbia/Georgetown ADP, the FARS Midyear Meeting, the AAA Annual Meeting, the

1 Introduction

Social activists and investors around the world demand that corporations disclose their environmental, social, and governance (ESG) performance.[1] In return, a considerable number of companies voluntarily started providing sustainability reports to their investors.[2] Multiple countries have already adopted mandatory reporting of non-financial performance for some companies, and many governments (including the U.S.) are crafting new sets of standards.[3] Despite growing acceptance of ESG reporting, our understanding of its potential consequences for firms' prices, financial and ESG performance, and misreporting remains limited.

To provide clarity on these important consequences of ESG reporting, we model a firm that generates financial cash flows and ESG impacts, and provides reports about both. The firm's shares are traded on a market where all investors value financial performance and where a fraction of investors also value ESG performance. The firm is led by a manager who, motivated by the firm's stock price, chooses an action (i.e., an effort, investment, or expenditure) that determines the firm's ESG performance and affects its cash flows. While we focus on this ESG action, we include a second cash-flow term that that captures cash flows that are unaffected by the manager's ESG action. For instance an oil company executive might choose how much to incorporate carbon capture and storage into its production process. This costly activity affects the company's ESG performance and cash flows, but there are many other components (such as global oil prices) that determine the firm's profits.

AAA Sustainability, ESG, and Accounting Conference, the Utah Winter Accounting Conference, the NBER Conference on Measuring and Reporting Corporate Carbon Footprints and Climate Risk Exposure, the Stanford Corporate Climate Disclosure Conference, the Accounting and Economics Society 2022 Retreat, and the Miami Winter Warm-up Conference. Friedman thanks the UCLA Anderson Center for Impact for generous financial support of this research. The usual caveat applies.

[1] While use the ESG term, our analysis can be interpreted as capturing aspects of corporate sustainability, corporate social responsibility (CSR), specific dimensions of environmental or social performance, or other externalities potentially valued by a fraction of investors.

[2] See, for example, Deloitte's "US Public Companies Prepare for Increasing Demand for High Quality ESG Disclosures" press release from March 14, 2022 (link).

[3] Krueger et al. (2021) identify 29 countries that introduced mandatory ESG disclosure policies from 2001 to 2016.

In the main specification of the model, the firm issues two types of reports to investors: a financial report that is a noisy measure of cash flows (e.g., the annual earnings report), and an ESG report that is a noisy signal about the company's ESG action (e.g., a report about the level of carbon emissions). Other than choosing the ESG action, the manager can also engage in financial misreporting (i.e., biasing the financial report) and greenwashing (i.e., biasing the ESG report). In addition, the ESG report may contain error due to mismeasurement: the manager's ESG action may have different effects on ESG output and the ESG report, by design. For example, the ESG output that investors value might be the company's total CO_2 emissions, while the ESG report only includes Scope-1 emissions.

We start by analyzing a benchmark case in which the firm only provides the financial report. We show that, even in the absence of ESG reporting, some investors use financial reports to learn about the company's ESG performance. When the financial report is disclosed, all investors update their beliefs about firm's cash flows. ESG-interested investors also update their beliefs about ESG performance because the financial report is a signal about the ESG-productive action. This updating of beliefs propagates through the price response to the report (the earnings response coefficient, ERC) and, in turn, changes the manager's ESG action and financial misreporting in equilibrium. When the ERC is positive, the manager engages in more earnings management and increases (decreases) her ESG effort when the effort positively (negatively) affects the firm's cash flow.

Next, we introduce ESG reporting. The introduction of ESG reporting changes the firm's price through three main channels. First, investors use the additional report to learn about the ESG effort that partly determines the firm's cash flows and ESG output. This learning is reflected in a price response to the ESG report itself. Second, the information conveyed by the ESG report changes the risk premium and the price response to the financial report, since the financial and ESG reports are both informative about the manager's efforts.[4] Finally, the introduction of the ESG report affects price through its effect on the manager's equilibrium

[4]Note that even if the ESG report perfectly revealed the manager's action, investors would still react to the financial report to learn about non-ESG-related components of the firm's cash flows.

choice of ESG action and financial and ESG misreporting. When the market's response to the financial report increases, the manager adds greater bias to her financial report and increases (decreases) her ESG action if the action is cash-flow-improving (cash-flow-reducing). When price increases in the reported ESG performance, the manager engages in greenwashing and increases her ESG action because the action always increases the ESG report. Investors anticipate the ESG report's effects on the manager's efforts and reports and, as a result, change their expectations of cash flows, ESG output, and financial and ESG reports in equilibrium, altering the price.

Many insights regarding the implications of introducing ESG reporting, either at the firm or jurisdiction level, emerge from comparing the benchmark equilibrium to the equilibrium with ESG reporting. Whether price responses, firm's outputs, and misreporting increase or decrease depends on the nature of the manager's ESG action. Specifically, when the manager's ESG action is cash-flow-improving (i.e., ESG and cash flows are complementary), issuing ESG reports in addition to financial reports allows investors to observe two positively correlated signals about the manager's action. This dampens the price response to the financial reports, i.e., lowers the ERC. The reduced price sensitivity to the financial report leads to lower misreporting incentives and less earnings management. For a cash-flow-positive ESG action, price responses to higher reports are unambiguously positive, so the manager chooses a higher action, increasing both expected cash flows and expected ESG compared to the no-ESG-reporting case. This increases the firm's price. A potential downside of introducing ESG reporting when ESG actions are cash-flow-enhancing is the emergence of greenwashing, as the same incentives that motivate the manager to increase ESG efforts also encourage the manager to bias the ESG report.[5]

The implications of ESG reporting initiations are different when the effect of the ESG effort on the firm's cash flows is moderately negative (i.e., there is a moderate trade-off

[5]Of course, the same might be expected to occur for the financial report, as a positive price response to the financial report provides incentives for financial misreporting. However, we show in Section 4.1 that the introduction of an ESG report can lead to divergent changes in expected financial misreporting and financial performance.

between ESG performance and cash flows). By moderately negative we mean that the price response to the ESG report remains positive despite the adverse effect of the ESG action on cash flows because there is a sufficiently-large mass of ESG-interested investors in the market. In this case, the manager's action has a large effect on the ESG output, and ESG-related components of cash flows are not important relative to other components of cash flows. In the scenario of a moderately cash-flow-negative ESG action, after the firm starts providing ESG reports, the ERC increases, providing the manager with stronger incentives to manage earnings. The increase in the ERC occurs because the ESG report is a negative measure of the company's cash flows, and the introduction of this negatively-correlated signal increases the weight on the original, financially oriented signal. The introduction of the ESG report in this setting also stimulates a higher ESG action, increasing expected ESG and reducing expected cash flows. In this parameter region, the introduction of an ESG report can decrease price through its effect on the firm's cash flows.

Finally, when the ESG action is sufficiently negative for cash flows, such that the price response to the ESG report is negative, introducing the ESG report decreases the ERC and earnings management. This occurs because the two signals about the manager's action are negatively correlated, such that the ESG report has a negative weight in the firm's price. Reduced market reaction to the financial report and negative reaction to the ESG report jointly disincentivize the ESG action. The manager chooses a lower level of ESG-improving effort, leading to lower expected ESG but higher expected cash flows. In this parameter region, again, introducing ESG reporting increases the expected stock price. Different from the case with a positive cash flow impact of ESG actions, here the price increase occurs because the manager reduces the effort, economizing on ESG efforts to improve cash flows. In equilibrium, the introduction of the ESG report therefore decreases the firm's expected ESG performance and results in fewer shares being held by ESG investors. Because the ESG report has a negative weight, the manager has brownwashing incentives to signal lower ESG performance but higher cash flows.

We derive further empirical predictions from comparative statics on the equilibrium with both reports. We show that when the fraction of ESG investors increases, the firm's expected ESG performance and the manager's greenwashing incentives increase. That is, more greenwashing may be indicative of higher ESG performance driven by an underlying change in the importance of ESG activities to investors. However, they do not always co-move this way. We derive conditions under which changes in the cash flow implications of ESG activities, the importance of non-ESG cash flows, or the manager's ability and incentives to greenwash can cause ESG and financial performance to move in one direction while financial misreporting and greenwashing move in the other. Additionally, we discuss how variation in the parameters of interest affect price responses to ESG and financial reports, which can be measured using standard event study methodologies.

Our study supports and offers a mechanism to explain existing empirical findings. We demonstrate how investors, even if they do not inherently value ESG, can learn cash-flow relevant information from ESG reports, consistent with Amel-Zadeh and Serafeim (2018)'s evidence that one of the most frequent reasons for investors using ESG data is to improve financial performance. The introduction of value-relevant ESG disclosures in our model can on average cause positive price reactions, as evidenced by Arif et al. (2022). Matsumura et al. (2014) find that investors can "penalize" firms for low ESG performance; this penalty is dampened if companies provide more detailed ESG disclosures.

Second, this paper speaks to real effects of ESG disclosures. A number of studies show that companies subject to mandatory ESG disclosure regimes reduced their emissions (Chen et al. (2018), Grewal et al. (2022), Downar et al. (2021), Yang et al. (2021)) and improved CSR performance (Fiechter et al. (2022), Christensen et al. (2019)), consistent with our prediction that the introduction of ESG reporting can increase expected ESG performance if an ESG effort is not destroying firm's cash flows too much. Yang et al. (2021)'s finding suggests that one of the mechanisms for companies changing their behavior is stock price: the results are stronger for publicly traded firms. Wang (2022) finds similar evidence in

the banking setting: corporate U.S. borrowers of non-U.S. banks that are exposed to ESG disclosure regulations improve their environmental and social performance. Thomas et al. (2022) highlight the trade-off between cash flows and ESG performance that some companies face, and how the pressure to beat a financial reporting benchmark pushes the firms to choose financial performance over non-financial.

Finally, we demonstrate how ESG reporting can lead to an undesired outcome – companies overstating their ESG performance, or greenwashing. Raghunandan and Rajgopal (2022) consider mutual funds and show that funds that self-label as ESG end up holding a portfolio of companies with worse ESG performance but higher ESG scores. The disconnect between how funds' ESG performance is measured (investment portfolio's ESG score) and what this performance actually is makes the ESG-labelled mutual funds maximize the metric they report rather than their real ESG outcome.

2 Literature Review

We address regulators' and researchers' call to improve our understanding of trade-offs related to characteristics of managers' reports about internal and external effects of their ESG actions (Christensen et al., 2019; Grewal and Serafeim, 2020). Our model broadly combines four strands of literature. First, our focus on a setting with ESG and financial reports ties our analysis to prior studies incorporating multiple performance measures (e.g., Holmstrom and Milgrom, 1991; Datar et al., 2001). Second, as in the literature on earnings management (e.g., Dye and Sridhar, 2004; Fischer and Verrecchia, 2000), our focal firm discloses reports that need not be truthful. Third, our model features feedback effects whereby price responses to reports encourage both real activities and reporting choices (e.g., Kanodia, 2007). Finally, some investors who receive the reports incorporate their beliefs about the firm's ESG when forming their demand, similar to Pástor et al. (2020), Friedman and Heinle (2016), and additional studies discussed in greater detail below. Concurrent studies have

also incorporated real effects of capital market responses on firms' ESG choices (see, e.g., Goldstein et al. (2022) and Xue (2022), which feature rational expectations equilibria).

Our model extends the literature on earnings management (e.g., Dye and Sridhar, 2008) by allowing a manager to manipulate the report of firms' ESG, i.e., to engage in greenwashing. This allows us to examine how greenwashing and financial misreporting interact. We analyze the manager's reporting strategies as functions of parameters capturing features related to investor preferences for ESG, misreporting opportunities and incentives, productivity parameters, and the importance of ESG and non-ESG factors to the firm's cash flows.

Several studies provide evidence that individuals value the societal impacts of their investments. For example, the survey in Krueger et al. (2020) suggests that institutional investors recognize the importance of climate risks for their portfolios' cash flows. Similarly, Bauer et al. (2021) survey members of pension funds and find that two-thirds of respondents are willing to sacrifice some financial benefits to invest in companies whose goals are aligned with the United Nations' Sustainable Development Goals (SDG). Barber et al. (2021) and Bolton and Kacperczyk (2021) provide further evidence of tradeoffs between ESG and market performance.

The pricing of companies' non-financial performance has received much recent academic interest. Closely related is Pástor et al. (2020), who show that agents' tastes for green holdings affect asset prices in equilibrium and derive predictions about the returns on a green factor. Zerbib (2020) develops an asset-pricing model where ESG performance is priced due to the impact of two investor groups: those that exclude certain assets from their investment options and those that internalize private costs of externalities in their expected returns. These investors cause two types of premia to occur: taste premia and exclusion premia. Pedersen et al. (2020) analyze an economy where the ESG score contains information related to firm fundamentals and some investors have preferences about firms' non-financial performance. They show that in equilibrium, prices of assets satisfy a four-fund separation theorem incorporating both financial and ESG performance. Chowdhry

et al. (2018), Oehmke and Opp (2019), and Friedman and Heinle (2021) derive conditions for impact investment to improve social outcomes when some investors value impact as well as cash flows.

Most of the literature assumes symmetric information and/or is silent on the source of the information that investors have about firms' non-financial performance. Lyon and Maxwell (2011) provide a model of greenwashing based on discretionary disclosure of favorable signals (e.g., Jung and Kwon, 1988), in contrast to our model of reporting bias with uncertain costs. Despite the relative paucity of theoretical research, there exists rich empirical evidence for firms' greenwashing or providing inappropriate information on their ESG activities (e.g., Bingler et al., 2021; Basu et al., 2021; Delmas and Burbano, 2011; Marquis et al., 2016), as well as numerous examples from the popular and business press (e.g., Brogger and Marsh, 2021; Kowsmann and Brown, 2021).

A separate literature has focused on the materiality of ESG disclosures (e.g., Khan et al., 2016; Jebe, 2019). Amel-Zadeh and Serafeim (2018) report survey evidence that mainstream investment organizations primarily use ESG information because of its relevance to investment performance, ahead of client demand and ethical considerations. Although Moss et al. (2020) find no evidence of retail investors reacting to ESG press releases, Moss et al. (2022) show that stock prices respond to ESG performance information. Materiality implies "relevant to investor decision-making," and can be evaluated based either on relevance to fundamentals, i.e., future cash flows or discount rates, or based on investor responses to ESG information releases. The Sustainability Accounting Standards Board (SASB) has promulgated industry-specific sustainability standards that focus on materiality, while the SEC encourages disclosure of material ESG information under existing disclosure rules. Our model, by clearly delineating between cash flow relevance, investor response, and effects on ESG allows us to show how focusing on different definitions of materiality in designing ESG reports can affect prices, greenwashing, and corporate ESG efforts.

An issue related to investor-focused materiality is on how trading activity and investor

engagement affect firms' ESG performance. Landier and Lovo (2020) show how the policy of an ESG fund forces companies to internalize (at least partially) their externalities. An ESG fund's optimal strategy is to invest in firms with the strongest capital search frictions and most inefficient externalities. Green and Roth (2021) derive optimal strategies for social investors to maximize social welfare in an environment of competition between commercial and social investors. De Angelis et al. (2020) show how companies' greenhouse gas emissions can be reduced through the increase in the cost of capital for those companies, wherein the cost of capital becomes more sensitive to emissions as the share of green investors and environmental stringency increase.

Our contribution to these streams of literature is through explicitly modeling firms' reporting of ESG activities as well as potential greenwashing. We show how in equilibrium, a firm's price is sensitive to its ESG report and its financial report. We analyze how price and its sensitivity to the reports varies with report characteristics and show how this sensitivity in turn affects managers' real and reporting choices.

3 Model and equilibrium

3.1 Setup

Our model features a firm whose manager makes production and reporting choices and a continuum of investors who allocate their wealth between shares in the firm and a risk-free asset that is assumed to have a gross return of 1. The timeline is such that the manager's choices are made first. These result in reports provided to investors, who trade the firm's shares in a competitive market and thus establish a stock price. Finally, firm performance is realized and all parties consume.

The manager chooses the level of ESG effort, $e \in \Re$, which affects the firm's cash flows and ESG performance. While we use the term 'effort' throughout, it can also be read as an action or investment, and we allow it to be positive, negative, or zero (e.g., level of

net carbon abatement investment, net efforts towards equitable and inclusive hiring, or the degree of anti-corruption controls). In particular, the firm's cash flows are $\tilde{x} = \theta e + \tilde{\varepsilon}_x$, with $\theta \in \Re$ and $\tilde{\varepsilon}_x \sim N(0, \sigma_x^2)$. The firm's ESG performance is $\tilde{y} = \eta e$, with $\eta > 0$. The productivity parameters, θ and η, are constants known by all actors. We restrict $\eta > 0$ so that the action can be interpreted as ESG-improving, or "green." In contrast, θ can be positive or negative, which gives us flexibility to consider cash flow enhancing ESG efforts (e.g., efficiency improvements) as well as cash flow detracting efforts (e.g., costly carbon capture and storage). Put another way, $\theta > 0$ implies complementarity between ESG and cash-producing activities, summarized by the scalar e, while $\theta < 0$ implies activities where cash flow and ESG performance are substitutes. The stochastic $\tilde{\varepsilon}_x$ captures factors that affect cash flows but are not related to the firm's ESG performance, i.e., non-ESG cash flows.

We assume that the continuum of investors has unit mass, and that the supply of shares is fixed at 1. The risk-free asset (money) serves as the numeraire in which investors can borrow or lend. It is supplied elastically, such that its price and gross return – both 1 – are not affected by demand.

Investor are heterogeneous with regard to their preferences over the firm's ESG performance but have homogeneous preferences with regard to cash flows. Specifically, although all investors value cash flows and are risk averse with respect to their cash holdings, a λ-fraction of investors also value the firm's ESG performance.[6] To simplify the analysis, we assume that the ESG-concerned investors are risk-neutral with respect to ESG.[7]

[6]Empirical evidence suggests that some shareholders seem to value non-financial outputs of their holdings (e.g., Krueger et al. (2020), Bonnefon et al. (2022), Heath et al. (2021), Barber et al. (2021)). For example, the survey in Krueger et al. (2020) finds that institutional investors recognize the importance of climate risks for their portfolios' cash flows. Bonnefon et al. (2022) use an experimental setting to provide evidence suggesting that investors obtain "warm glow giving" utility when their investments are aligned with with social values, and Barber et al. (2021) demonstrate this phenomena for venture capital investors.

We treat λ as an exogenous parameter, and discuss comparative statics on λ in Section 4.2.1. Future work might consider the drivers of λ, which could include, for instance, the potential for λ to depend on the availability of information about firms' ESG performance.

[7]There is little evidence on the degree of risk aversion with respect to ESG outcomes. Our assumption of risk neutrality is made here to keep the model tractable and ease interpretation of the effects we identify. It sidesteps issues of ESG risk sharing between investors, as a risk-efficient allocation would have the non-ESG

Let q_i and l_i denote the amount of shares and money, respectively, held by investor i. Denote type-1 investors as those who care only about cash flows. Their utility is $u_1 = -exp[-\rho(q_1\tilde{x} + l_1)]$. Type-2 investors, who also value the firms' ESG performance (in risk-neutral expectation), have utility defined by $u_2 = -exp[-\rho(q_2(\tilde{x} + E[\tilde{y}|\Omega]) + l_2)]$. Ω is the information on which investors condition their expectations.

The firm's manager is interested in maximizing the firm's stock price but bears costs related to her productive and reporting choices. In particular, the manager has a private preference for her ESG effort captured by $\frac{c_e}{2}(e - \phi)^2$, where $\phi \sim N(\bar{\phi}, \sigma_\phi^2)$ captures her ex ante stochastic private ESG preference based on, for instance, personal preferences, social pressure, or compensation incentives (based on measures besides stock price or the modeled reports). Including a privately observed ESG preference implies that the manager's effort choice cannot be perfectly anticipated by investors and, thus, that investors use the provided information to update their beliefs about the manager's choices. We allow $\bar{\phi}$ to be positive, negative, or zero, as one type of manager may, in expectation, prefer emissions reduction while another may equate such efforts with undesirable political posturing.

Crucially, investors do not observe the manager's realized preferences, though the expectation, $\bar{\phi}$, and variance, σ_ϕ^2, are common knowledge, as are the other parameters (e.g., η, θ, λ, ρ, σ_x^2, and other cost and distributional terms introduced below). The manager's private observation of ϕ induces randomness in her effort, e, which makes it impossible for investors to infer her equilibrium efforts with certainty.[8] In equilibrium, investors in our model use

concerned type 1 investors hold all of the ESG risk. Broadly, adding risk aversion in ESG performance would introduce additional tradeoffs that we leave for future work.

[8]The private preference for ESG effort could be affected by unmodeled compensation (see Krueger et al. (2020) and Walker (2022) for empirical evidence), which may only be partially observable or understandable to outside investors. Walker (2022) emphasizes the economic importance of ESG-related capital market incentives relative to ESG-related contractual compensation incentives in practice. Nonetheless, we can interpret $\bar{\phi}$ as capturing compensation incentives known by investors and σ_ϕ^2 as capturing compensation incentives over which investors are uncertain, as long as the incentives are not based on stock price or the financial and ESG reports, as these would introduce feedback effects that would alter the equilibrium. Introducing such incentives is a potentially interesting direction for future work. However, modeling the equilibrium determination of these incentives by, for instance, the board of directors, requires an explicit formulation of the board's objective with respect to financial and ESG performance. Our approach is to take price incentives as given and incorporate managerial preferences as a random action shifter, ϕ. See Bonham and Riggs-Cragun (2022) for a theoretical model of ESG-related compensation.

reports to learn about the manager's effort. We assume that all random variables are independent, including reporting incentive parameters introduced below. Our main model will feature both ESG and financial reports.[9] This allows us to consider ESG reporting in the context of the current landscape of corporate reporting, in which financial reporting is well established.

Both financial and ESG reports are chosen by the manager in consideration of effects on stock price as well as report-specific costs. The financial report, f, provides a noisy measure of the firm's cash flows, x. The manager's reporting cost associated with the financial report is $\frac{c_f}{2}(f - x - \varepsilon_f)^2$, with $c_f > 0$ and $\varepsilon_f \sim N\left(0, \sigma_f^2\right)$. As in Dye and Sridhar (2008), ε_f "reflects idiosyncratic circumstances that influence the manager's misreporting costs and prevents unraveling of the reporting bias effect in pricing," and could alternatively be incorporated via a mechanism as in Fischer and Verrecchia (2000) with uncertain incentives. Unlike pure window-dressing models, the manager does not minimize her reporting cost by issuing unbiased reports. Rather, her costs are minimized when she issues a report that incorporates true cash flows, $x = \theta e + \varepsilon_x$, as well as her idiosyncratic circumstances, summarized by ε_f.

Paralleling the financial report, the ESG report, r, provides a noisy measure of the firm's ESG performance. However, with regard to ESG reporting, we introduce an additional consideration. Specifically, the ESG report captures the ESG effort with a coefficient, $z > 0$. This captures the potential for the ESG report to be designed to capture inputs ($z = 1$), outputs ($z = \eta$), cash flow effects (θ), or a mix of these (e.g., double materiality: $\eta + \theta$).[10] As such, the ESG-related reporting cost the manager faces is $\frac{c_r}{2}(r - ze - \delta\varepsilon_r)^2$. Here z is a commonly-known parameter that represents the effect of the manager's "green" action on

[9] Moss et al. (2022) empirically examine heterogeneity in market responses to ESG performance news depending on whether ESG news is released during an earnings announcement period or at a different time. The joint study of ESG and earnings performance news is germane to our setting. Though we do not specifically address issues of heterogeneous timing of information releases, implications for price responses when one report to come out before the other can be viewed as a mild theoretical extension of results we present in Sections 3.2 and 3.3.

[10] Even when designed to capture environmental effects, it may still be difficult or impossible to design the report perfectly, i.e., to set $z = \eta$. Muller (2021) argues that commonly used measures of carbon dioxide (CO2) emissions can not fully capture broad range of risks that investors of companies with high CO2 emissions face.

the ESG report. For instance, z and η can differ if the ESG performance of interest is tons of CO_2 equivalent, but the ESG report is designed to capture ESG intensity, i.e., tons of CO_2 equivalent per output unit, or emissions measured with a different organizational or operational boundary. As with the financial report, the ESG report is affected by $\varepsilon_r \sim N(0, \sigma_r^2)$, which represents idiosyncratic circumstances that affect the manager's ESG reporting incentives. These could be reputational costs or benefits associated with greenwashing. The commonly-known parameter $\delta > 0$ scales the importance of these idiosyncratic incentives. A greater penalty for greenwashing or requirements to audit the ESG report would be captured by a smaller δ, for instance.

The timeline is as follows: at $t = 0$, the manager observes her private preference, ϕ, and chooses ESG effort, e, to maximize the following utility function, which consists of stock price net of the costs of the ESG action, the financial report, and the ESG report.[11]

$$u_m = p - \frac{c_e}{2}(e - \phi)^2 - \frac{c_f}{2}(f - x - \varepsilon_f)^2 - \frac{c_r}{2}(r - ze - \delta\varepsilon_r)^2. \tag{1}$$

At $t = 1$, the manager observes the non-ESG cash flows, ε_x, and idiosyncratic reporting circumstances, ε_f and ε_r, and chooses financial and ESG reports r and f to maximize u_m. To focus on stock market incentives that operate via investor pricing, we assume that the reports and outcomes are not contractable. Investors observe the financial and ESG reports, then trade in the shares and establish the stock price, p. Specifically, stock price is set to ensure that the market for shares clears in a competitive Walrasian equilibrium with market-clearing condition: $(1 - \lambda) q_1 + \lambda q_2 = 1$. At $t = 2$ cash flows are paid out and ESG performance is revealed.

In the next section, we present the equilibrium with the financial report only, as a benchmark. In the subsequent section, we provide our full equilibrium in which the firm provides

[11] As noted above, we can interpret distributional characteristics of ϕ as a reduced-form representation of non-price compensation incentives. A weight of 1 on price in equation 1 is without loss of generality, relative to other weights, since the cost parameters, c_e, c_f, and c_r, can be viewed as scaling the importance of effort and reporting costs relative to the importance of stock price.

both financial and ESG reports to investors. In both equilibria, investors use the reports to update their beliefs about the relevant outcomes. Type-1 investors use the available information to learn about cash flows, which means learning about the ESG effort, e, and the non-ESG component of cash flows, σ_x^2. Type-2 investors do the same but are additionally interested in the ESG implications of the ESG effort, given by $y = \eta e$.

3.2 Equilibrium with financial reports only

The proposition below summarizes the equilibrium price, action, and financial report in a world in which there is no ESG report, r. This provides a useful baseline that will allow us to generate predictions about the introduction of ESG reporting given pre-existing financial reports. Additionally, this benchmark will show clearly that ESG-interested investors will in equilibrium use financial reports to learn about corporate ESG performance.

Proposition 1 *In the equilibrium with only the financial report (no ESG report), the stock price, manager's effort, and financial report are given by*

$$\begin{aligned} p^\dagger &= E_x^\dagger + \frac{\sigma_{x|f}^2}{\sigma_f^2}\left(f - E_f^\dagger\right) + \lambda\left(E_y^\dagger + \frac{\sigma_{xy|f}}{\sigma_f^2}\left(f - E_f^\dagger\right)\right) - \rho\sigma_{x|f}^2, \\ e^\dagger &= \phi + \frac{\varphi_f^\dagger \theta}{c_e}, \text{ and} \\ f^\dagger &= \frac{\varphi_f^\dagger}{c_f} + \theta e^\dagger + \varepsilon_x + \varepsilon_f, \end{aligned}$$

where $E_j^\dagger = E[j]$ are prior means in equilibrium. The price response to the financial report is

$$\varphi_f^\dagger = \frac{\partial p^\dagger}{\partial f} = \left(\frac{\sigma_{x|f}^2}{\sigma_f^2} + \frac{\lambda\sigma_{xy|f}}{\sigma_f^2}\right) = \frac{\sigma_x^2 + \sigma_\phi^2\left(\theta^2 + \lambda\theta\eta\right)}{\sigma_f^2 + \sigma_x^2 + \theta^2\sigma_\phi^2},$$

and the relevant conditional variance terms are given by

$$\begin{aligned} \sigma_{x|f}^2 &= Var(x|f) = \sigma_f^2 \frac{\sigma_x^2 + \theta^2\sigma_\phi^2}{\sigma_f^2 + \sigma_x^2 + \theta^2\sigma_\phi^2} \text{ and} \\ \sigma_{xy|f} &= Cov(x, y|f) = \frac{\theta\eta\sigma_f^2\sigma_\phi^2}{\sigma_f^2 + \sigma_x^2 + \theta^2\sigma_\phi^2}. \end{aligned}$$

In the equilibrium described by Proposition 1, the price reflects four main components. The first is the prior expectation of cash flows,

$$E_x^\dagger = \theta E\left[e^\dagger\right] + E\left[\varepsilon_x\right] = \theta\bar{\phi} + \theta^2\frac{\varphi_f^\dagger}{c_e}$$

which is increasing in the price response to the financial report, $\varphi_f^\dagger$. The second term reflects investors' use of the financial report to update beliefs about cash flows, $\sigma_{x|f}^2\left(f - E_f\right)/\sigma_f^2$. The third term captures type 2-investors' expectations of ESG performance, conditional on the financial report, $E_y^\dagger + \sigma_{xy|f}\left(f - E_f^\dagger\right)/\sigma_f^2$, which are weighted in price by the fraction of ESG investors present in the market, λ. The fourth term captures the risk premium, $\rho\sigma_{x|f}^2$, or the discount to price due to risk averse investors bearing cash flow risk.

Note that the financial report plays multiple roles. First, it provides information about the non-ESG component of cash flows, ε_x. This is useful to all investors, and allows them to update beliefs about expected cash flows and the variance of cash flows. Second, the financial report provides information about the firm's ESG efforts, which are stochastic from investors' perspective because investors do not observe the manager's private preferences, ϕ. Learning about the ESG efforts is useful to all investors as long as $\theta \neq 0$, such that ESG efforts affect cash flows, and is incrementally useful to type-2 investors due to the effects on ESG performance. Fundamentally, investors' response to the financial report is driven by the degree to which it contains information about relevant cash flow and ESG outcomes as opposed to reflecting the manager's idiosyncratic reporting incentives. From the investors' perspective, the effect of these incentives is effectively to add noise to the report, making it less informative about relevant outcomes and thus less relevant to investors' share demand.

The last primary effect of the financial report is what may be called a real effect. The dependence of the stock price on the report and the manager's stock price concern jointly affect the manager's equilibrium effort incentives, which in turn affect real cash flows and ESG performance. This effect is borne out in the $\varphi_f^\dagger\theta/c_e$ term in the manager's equilibrium

effort. Note that this term is not ex ante stochastic. Rather, it depends on the sensitivity of stock price to the financial report, $\varphi_f^\dagger$, which is in turn a known function of model parameters.

An additional effect in our setting is that investors' use of the financial report motivates the manager to engage in misreporting. The model admits two types of misreporting, which can be illustrated by the difference between the financial report and the cash flows it is supposed to capture: $f^\dagger - x = \varepsilon_f + \frac{\varphi_f^\dagger}{c_f}$. The first type is a reflection of the manager's idiosyncratic financial reporting incentives, ε_f. The second type is a reaction to price-based incentives, reflecting the manager's desire to increase stock price, scaled by her aversion to misreporting costs, $\varphi_f^\dagger / c_f$.

Corollary 1 *In the financial report only setting, the sensitivity of price to the financial report*

1. *increases (decreases) in the fraction of type-2 investors, λ, when θ is positive (negative), i.e., $\frac{d\varphi_f^\dagger}{d\lambda} = \frac{\sigma_\phi^2 \theta \eta}{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2} \propto \theta$, and*
2. *increases (decreases) in the cash flow effects of ESG efforts, θ, when $2\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2-\theta^2\sigma_\phi^2\right)$ is positive (negative), i.e., $\frac{d\varphi_f^\dagger}{d\theta} = \sigma_\phi^2 \frac{2\theta\sigma_f^2+\lambda\eta\sigma_f^2+\lambda\eta\sigma_x^2-\theta^2\lambda\eta\sigma_\phi^2}{\left(\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2\right)^2} \propto 2\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2-\theta^2\sigma_\phi^2\right)$.*

Corollary 1 provides two results regarding potential impacts of ESG-relevant features (λ and θ) on price sensitivity to the financial report. These are operationalizable, as empirical measures of each are readily available. For instance, λ could be captured by the fraction of firm-, country-, or market-level funds that are signatories of the United Nations Principles for Responsible Investment (PRI). For θ, the recent Inflation Reduction Act (IRA) in the U.S. made several policy changes including subsidies and tax credits that increased the financial benefits to ESG-positive actions, i.e., increasing θ. Finally, the price sensitivity to the financial report has a clear empirical analogy in the oft-used earnings response coefficient obtained from regressing abnormal returns around an earnings announcement on a measure of the earnings innovation or surprise.

The intuition underlying Corollary 1 is as follows. The parameter θ plays two roles in our model. It determines the marginal impact of the manager's effort on cash flows and,

second, on the financial report. As a result, increasing the magnitude, $|\theta|$, makes it more important for investors to infer the effort because it has a more severe impact on cash flows. Higher θ also makes the financial report more informative about the manager's effort. The latter effect arises because more of the variation in the financial report is due to variation in the manager's effort. When $\theta > 0$ both type-1 and type-2 investors place a positive weight on the financial report. However, type-2 investors, relative to type-1 investors, place a larger positive weight on the report because they also care about the positively correlated ESG performance. In turn, when $\theta < 0$, type-2 investors place a smaller weight on the financial report, because it is negatively correlated with the firm's ESG performance. For sufficiently negative values of θ, type-2 investors may even place a negative weight on the report altogether. As a result, increasing the fraction of type-2 investors reduces the average weight on the report in price.

In Corollary 1, part 2, we investigate changes in θ itself. For $\theta > 0$, while type-1 investors monotonically increase the weight on the financial report as θ grows, type-2 investors will reduce the weight once θ grows very large. The reason is that for sufficiently large values of θ, the financial report places too much weight on the effort, relative to its impact on the ESG performance. For $\theta < 0$, type-1 investors reduce their weight on the report as θ becomes less negative. Because this reduces the negative correlation between the report and ESG performance, type-2 investors may increase their weigh on the report, which explains the non-monotone result.

3.3 Equilibrium with financial and ESG reports

We now present the equilibrium with both ESG and financial reports. ESG reports provide information about the firm's ESG effort, but they are not introduced into an information vacuum. Even with only financial reports, investors are able to make inferences and update their beliefs about the firm's ESG activities, e. As we will see below, making an ESG report available improves learning about the ESG effort, which is relevant to cash flows.

Additionally, the information contained in the ESG report can also allow investors to learn more about the firm's non-ESG cash flows from the financial report.

Proposition 2 *In the equilibrium with ESG and financial reports, the stock price, manager's effort, financial report, and ESG report are given by*

$$\begin{aligned} p^* &= E_x^* + \frac{\hat{\sigma}_x^2}{\sigma_f^2}\left(f - E_f^*\right) + \frac{\hat{\sigma}_{xy}}{\delta^2\sigma_r^2}\frac{z}{\eta}\left(r - E_r^*\right) \\ &\quad +\lambda\left(E_y^* + \frac{\hat{\sigma}_{xy}}{\sigma_f^2}\left(f - E_f^*\right) + \frac{\hat{\sigma}_y^2}{\delta^2\sigma_r^2}\frac{z}{\eta}\left(r - E_r^*\right)\right) \\ &\quad -\rho\hat{\sigma}_x^2, \\ e^* &= \phi + \frac{\varphi_r^* z + \varphi_f^* \theta}{c_e}, \\ f^* &= \frac{\varphi_f^*}{c_f} + \theta e^* + \varepsilon_x + \varepsilon_f, \text{ and} \\ r^* &= \frac{\varphi_r^*}{c_r} + z e^* + \delta\varepsilon_r, \end{aligned}$$

where $E_j = E[j]$ *are prior means,* $\varphi_f^* = \frac{\partial p^*}{\partial f}$ *and* $\varphi_r^* = \frac{\partial p^*}{\partial r}$ *are the price responses to the financial report and ESG report, respectively, and conditional variances are denoted by* $\hat{\sigma}_x^2 = Var[x|r,f]$, $\hat{\sigma}_y^2 = Var[y|r,f]$, *and* $\hat{\sigma}_{xy} = Cov[x,y|r,f]$. *Detailed expressions for these are given in the Appendix.*

In our discussion of the equilibrium in Proposition 2, we focus on differences relative to Proposition 1. Intuitively, the salient differences revolve around the introduction of the ESG report.

First, there is a reporting strategy for the ESG report, r^*, that was absent in Proposition 1. This reporting strategy has a similar functional form as that for the financial report, f^*. First, the ESG report captures the ESG effort, via the ze^* term. Recall that this is allowed to differ from the effect of the ESG effort on ESG performance, ηe, as $z = \eta$ is a knife-edge case not generally assumed. Second, the ESG report responds to the manager's idiosyncratic ESG reporting incentives, ε_r, scaled by the δ parameter. Third, the ESG report is affected by the price-increasing incentives the manager faces, summarized by the price response, φ_r^*, scaled by the reporting cost, c_r.

This breakdown yields two types of deviation between the ESG report, r, and the firm's ESG performance, y. First, there is a bias, often referred to as greenwashing in the ESG (particularly environmental) space, characterized by the price-derived misreporting pressure and the manager's idiosyncratic incentives, $\frac{\varphi_r^*}{c_r} + \delta\varepsilon_r$. Second, we have an inherent measurement error derived from the potential for the ESG report to be designed (i.e., parameterized) to capture the firm's ESG efforts in a manner that differs from the effects of these efforts on performance. This can be represented as $(\eta - z)\, e^*$, a value whose magnitude is increasing both in the measurement divergence, $(\eta - z)$, and in the magnitude of the firm's ESG activities in equilibrium, e^*. Below, we will refer to $\frac{\varphi_r^*}{c_r} + \delta\varepsilon_r$ as ESG misreporting, and $(\eta - z)\, e^*$ as ESG mismeasurement. Note that expected ESG misreporting is proportional to the price response, as $E[\varepsilon_r] = 0$.

Notably, the equlibrium bias in the ESG report can be positive or negative, i.e., we can have either greenwashing or brownwashing. First, if the price response to the ESG report, ϕ_r^*, is negative, which holds when the ESG action is sufficiently detrimental to the company's cash flows ($\theta << 0$) or the fraction of ESG-interested investors (λ) is small, the manager may understate the ESG report in equilibrium. Though corporate greenwashing commands much greater attention than brownwashing, prior studies have documented behavior consistent with brownwashing. For instance, Kim and Lyon (2015) found that many U.S. energy utilities companies understated reductions in their carbon emissions, and attributed their finding to forces related to investors unconcerned with environmental progress. Second, the manager may report a lower level of ESG performance because of idiosyncratic factors (ε_r is negative enough).

Returning to the equilibrium in Proposition 2, the second salient area where it differs from Proposition 1 is in the effect of the ESG report, r, on equilibrium stock price, p^*. As with the financial report, investors use the ESG report to learn about the ESG effort e, which has cash flow effects via θ, and ESG performance effects via η.

Additional effects are embedded in the posterior variances ($\hat{\sigma}_x^2$, $\hat{\sigma}_y^2$, and $\hat{\sigma}_{xy}$), the price

sensitivity to the financial report (φ_f^*), and the prior means (E_x^*, E_y^*, E_f^*, and E_r^*). Posterior variances differ due to the addition of a noisy signal about the manager's effort, e.[12] In turn, the addition of the ESG report changes how investors respond to the financial report. To illustrate this, consider a case in which $\delta = 0$, i.e., institutional features prevent the manager's idiosyncratic preferences from influencing her ESG reporting strategy. In this case, the ESG report would fully reveal e^* and the posterior variance regarding ESG performance, $\hat{\sigma}_y^2$, would go to zero. Investors would here be able to use the financial report as a clear, but still noisy, measure of the firm's non-ESG cash flows, ε_x. Finally, note that the changes in price responses (from $\varphi_f^\dagger$ to φ_f^* and from 0 to φ_r^*) change the equilibrium effort and reporting strategies, which in turn affect prior expectations. That is, even before observing reports, investors know that expected price responses will cause, for example, $E_y^* = \bar{\phi} + \frac{\varphi_r^* z + \varphi_f^* \theta}{c_e}$ to differ from $E_y^\dagger = \bar{\phi} + \frac{\varphi_f^\dagger \theta}{c_e}$. This carries through to expected cash flows and expected financial and ESG report values as well.

For the remainder of the paper, we assume that the parameters are such that the price response to the financial report is positive, i.e., $\varphi_f^* > 0$. This is satisfied if θ is not too negative.[13]

4 Analysis

In this section we present analyses broadly divided into two sets. First, we explore the implications of a firm initiating ESG reporting by comparing equilibria with and without ESG reporting (i.e., comparing equilibria under Propositions 1 and 2). While we do not differentiate between voluntary and mandatory ESG reporting adoption per se, we develop results around expected pricing effects to illustrate when a price-maximizing manager would choose

[12] Ceteris paribus, investors have more information with the addition of the ESG report. However, changes in reporting strategies can leave the financial report less informative or noisier than it was absent the ESG report (as we will show below).

[13] Specifically, $\varphi_f^* > 0 \Leftrightarrow -\frac{1}{\lambda\eta\sigma_\phi^2}\left(\left(1 + \frac{z^2}{S}\sigma_\phi^2\right)\sigma_x^2 + \theta^2\sigma_\phi^2\right) < \theta$, where $S = \left(1 + \frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)$

to adopt ESG reporting.[14] Second, we use comparative statics to generate predictions around the effects of differences in investor preferences (λ), the cash flow implications of ESG (θ), the importance of non-ESG cash flows (σ_x^2), and factors affecting idiosyncratic misreporting incentives (δ and σ_r^2). The first relates to a secular trend of increasing investor interest in ESG. The second captures heterogeneity cross-sectionally and over time in production modes, customer or employee interest, regulatory changes, or fiscal subsidies or penalties. The third, σ_x^2, relates to cross-firm heterogeneity, or the fact that some firms' non-ESG cash flows are proportionately much more important than others', e.g., technology firms' revenues are less directly related to carbon emissions than construction, transportation, and manufacturing firms'. Finally, δ and σ_r^2 capture factors associated with managerial preferences for appearing green (or brown/anti-woke) or institutional constraints on ESG reporting such as assurance requirements. Each of these is also likely to vary cross-sectionally and over time.

4.1 Introducing ESG reports

Recall from the discussion above that introducing ESG reports has three primary effects. First, the ESG report provides additional information about the firm's ESG efforts, which are relevant to cash flows and ESG performance. Second, the ESG report changes what investors learn from the financial report, as they are both noisy signals of the firms' actions and performance. Third, these two effects conspire to change the market pricing of the manager's effort and reporting choices, which in turn change her equilibrium incentives and actions.

To facilitate discussion of the effects of adding an ESG report, we introduce the following notation. The change in the price response to the financial report is defined as $\Delta_\varphi = \varphi_f^* - \varphi_f^\dagger$. For brevity, we will refer to this as the change in the earnings response coefficient, or ERC. The change in financial misreporting is given by $\Delta_b = (x^* - f^*) - \left(x^\dagger - f^\dagger\right) = \frac{\varphi_f^*}{c_f} - \frac{\varphi_f^\dagger}{c_f} = \Delta_\varphi / c_f$. Similarly, the changes in expected cash flows and ESG performance are defined as

[14] Black et al. (2022) provide evidence that greater ESG concerns among a firm's investors are associated with increased ESG disclosures.

$\Delta_x = x^* - x^\dagger$ and $\Delta_y = y^* - y^\dagger$. We also present results for changes in expected share ownership by type-1 and type-2 investors: $\Delta_{q_1} = E[q_1^*] - E\left[q_1^\dagger\right]$ and $\Delta_{q_2} = E[q_2^*] - E\left[q_2^\dagger\right]$. With this nomenclature in hand, we can present our main results of this subsection, given in the following three propositions. Each proposition presents results for a range of θ.

Proposition 3 *When the cash flow effects of ESG effort are positive ($\theta > 0$), the price response to the ESG report is positive ($\varphi_r^* > 0$) and introducing ESG reporting has the following effects:*

1. *the ERC and financial misreporting both decrease, $\Delta_\varphi, \Delta_b < 0$;*
2. *expected cash flows and ESG performance both increase, $\Delta_x, \Delta_y > 0$; and*
3. *ESG-interested investors increase their holdings, $\Delta_{q_2} > 0 > \Delta_{q_1}$.*

When the ESG effort has a positive cash-flow impact, the weight on the ESG report is positive because all investors interpret the ESG report as providing positive news. Given the alignment with the financial report, the response to the the financial report decreases. This fits with natural intuition in a multi-signal game: adding a positively correlated signal reduces the agents' (i.e., investors') reliance on existing signals. The weaker market reaction to the financial report lowers the manager's incentives to engage in costly financial misreporting. Interestingly, the weaker market reaction to the financial report does not lead to lower cash flows, as the manager's incentives to engage in ESG efforts face a net increase from the joint impact of the ESG and financial reports. This increases ESG and financial performance, the latter driven by the positive effect of ESG efforts on cash flows. As a result, financial performance and financial misreporting act as substitutes; when performance increases, misreporting decreases. A downside of the introduction of the ESG report here is positive greenwashing, but this could be seen as a natural response to the ESG-report-based incentives that help motivate increased ESG efforts. For those interested in improving ESG performance, the increase in ESG misreporting is a negative side effect of a positive main effect: stronger incentives to generate positive ESG outcomes. The increase in expected ESG performance in turn shifts the shareholder base in favor of investors who value ESG performance.

Proposition 4 *When the cash flow effects of ESG effort are moderately negative ($0 > \theta > -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$), the price response to the ESG report is positive ($\varphi_r^* > 0$) and introducing ESG reporting has the following effects:*

1. *the ERC and financial misreporting both increase, $\Delta_\varphi, \Delta_b > 0$; and*
2. *expected cash flows decrease while expected ESG performance increases, $\Delta_x < 0 < \Delta_y$.*

When the action has a moderately negative cash-flow impact, the weight on the ESG report is positive despite investors interpreting a more positive ESG report as bad news for cash flows. Essentially, the market response to the ESG report remains positive because the cash flow effects are not too negative given a sufficiently large fraction of ESG-interested investors (λ), a large effect of ESG efforts on ESG performance (η), and non-ESG cash flows that are important relative to the manager's idiosyncratic financial reporting incentives $\left(\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2\right)$.

Interestingly, the introduction of the ESG report in this region of the parameter space causes investors' response to the financial report to increase. This results from the ESG report effectively being a negative measure of expected cash flows. Investors giving it positive weight allows them to increase their response to the financial report, which is a direct measure of cash flows. The increase in the ERC motivates additional financial misreporting.

Turning to the manager's efforts, the net effect of adding the ESG report given $0 > \theta > -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$ is to increase incentives to engage in ESG efforts. This increases ESG performance but, due to $\theta < 0$, decreases expected cash flows.

Whether the holdings by ESG investors increase or not depends on parameter values. Note that the change in shareholdings by ESG-interested green investors is proportional to $\Phi = \sigma_x^2\left(\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)\right) + \theta^2\sigma_f^2\sigma_\phi^2\left(\theta + \lambda\eta\right)$. The assumption of $\theta > -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$ in Proposition 4 implies that the first term is positive. For $-\lambda\eta < \theta < 0$ the second term is positive, and ESG-interested investor holdings increase. For $\theta < -\lambda\eta < 0$ the second term is negative. In this situation, a sufficiently low value of σ_ϕ^2 ensures that $\Phi > 0$, whereas for sufficiently high values of σ_ϕ^2 we will have $\Phi < 0$. Thus, in this region of parameter space, we would expect ESG investors to increase their holdings when the manager's ESG

efforts are relatively predictable, ex ante, while more uncertainty about the manager's efforts would cause the introduction of ESG reports to tilt the shareholder base towards traditional investors.

Proposition 5 *When the cash flow effects of ESG effort are sufficiently negative* ($\theta < -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$)*, the price response to the ESG report is negative* ($\varphi_r^* < 0$) *and introducing ESG reporting has the following effects:*

1. *the ERC and financial misreporting both decrease,* $\Delta_\varphi, \Delta_b < 0$*;*
2. *expected cash flows increase while expected ESG performance decreases,* $\Delta_x > 0 > \Delta_y$*; and*
3. *ESG-interested investors decrease their holdings,* $\Delta_{q_2} < 0 < \Delta_{q_1}$.

When the ESG effort has a sufficiently negative cash-flow impact, the weight on the ESG report is negative. This occurs because investors interpret the ESG report as a negative indicator of the firm's expected cash flows, and this interpretation outweighs the reaction to the ESG performance information by the ESG-interested investors. In addition, the weight on the financial report declines, as now we have a negatively-correlated signal, r, that is given negative weight in equilibrium.

Note that for evaluating cash flows, the financial report always receives a positive weight. The reason is that the weight on the action in the report is also θ. When $\theta < 0$ the report measures this accurately and, thus, receives a positive weight. However, when $\theta < 0$ and $\eta > 0$, gleaning the ESG impact from the financial report requires a negative weight. Similarly, gleaning the financial impact from the ESG report requires a negative weight. As in Proposition 3, the addition of the ESG report allows investors to reduce the weight on the financial report. Because of the decrease in the ERC, financial misreporting goes down.

The net effect of the negative response to the ESG report and the lower ERC is less positive ESG incentives. This reduces ESG performance and, because of the negative effect of ESG efforts on cash flows, leads to an increase in expected cash flows. Because investors react negatively to the ESG report, the manager effectively has brownwashing incentives,

i.e., incentives to show that it is doing less ESG to reduce the negative implied effect on cash flows.

As a result of the reduced ESG performance, fewer shares are held by green investors. For firms with sufficiently negative cash flow effects of ESG efforts, the additional information about their ESG performance can discourage ESG efforts, improve expected cash flows, and tilt the shareholder base in favor of traditional type-1 investors. Thus, introducing ESG reporting need not imply an increase in shares held by ESG-conscious investors.

Before moving on to consider voluntary ESG reporting choices driven by expected price maximization, we comment briefly on a theme that emerges across Propositions 3, 4, and 5. Financial performance and financial misreporting behave as substitutes. The economic forces that lead to increases or decreases in expected financial performance flow through incentive effects related to the ESG effort. These tend to work in the opposite direction of the incentives for financial misreporting driven by the price response to the financial report. Figure 1 provides an illustration summarizing the regions represented in Propositions 3, 4, and 5.

The following proposition characterizes the expected change in price driven by the introduction of ESG reporting.

Proposition 6 *Introducing ESG reporting yields an expected price change of*

$$\begin{aligned}
\Delta_p &= E\left[p^*\right] - E\left[p^\dagger\right] \\
&= (\theta + \lambda\eta)\frac{z}{c_e}\varphi_r^* && (2) \\
&\quad + (\theta + \lambda\eta)\frac{\theta}{c_e}\left(\varphi_f^* - \varphi_f^\dagger\right) && (3) \\
&\quad + \rho\left(\sigma_{x|f}^2 - \hat{\sigma}_x^2\right).
\end{aligned}$$

The expected price is affected by the introduction of ESG reporting in three ways. First there are effort incentives provided by the price response to the ESG report. The shift in expected efforts given by $\frac{z}{c_e}\varphi_r^*$ is capitalized into price based on the sum of the cash flow effect of ESG efforts, θ, and the ESG effect multiplied by the fraction of investors concerned

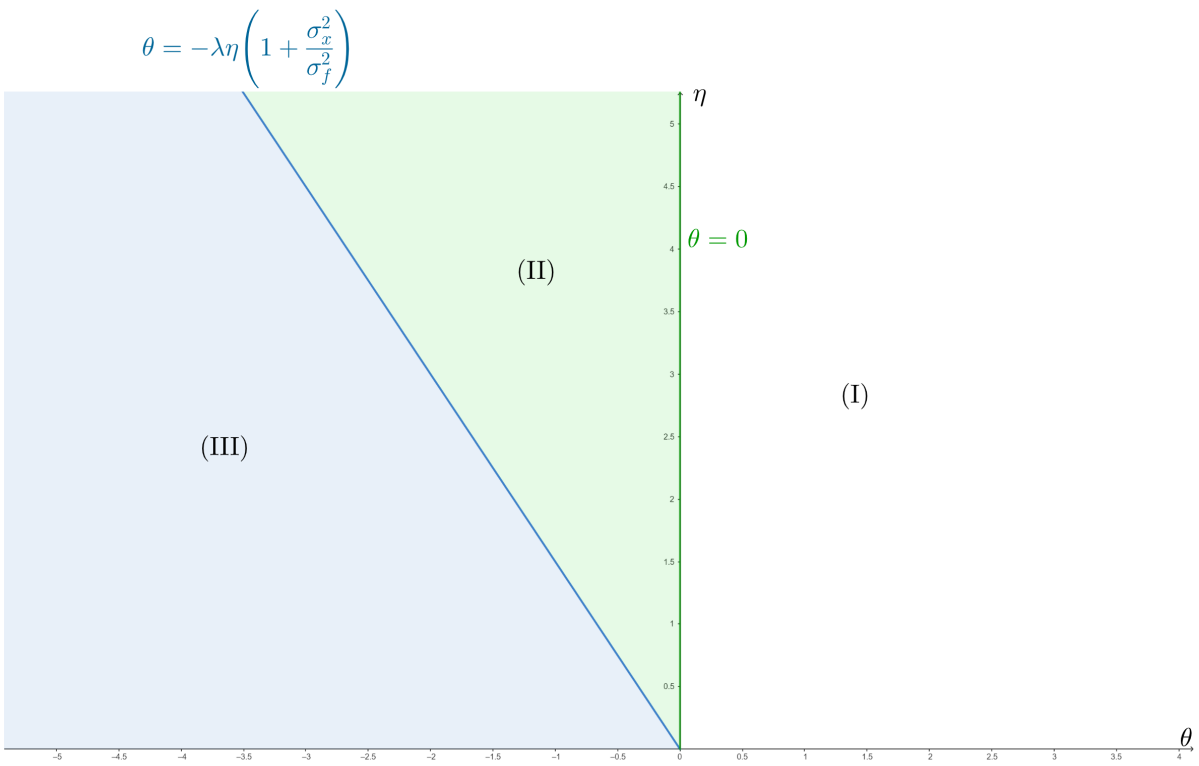

Figure 1: Regions of θ and η where the effects of introducing ESG reporting on misreporting, expected cash flows and ESG performance are different. $\lambda = \frac{1}{3}, \sigma_x^2 = 1,$ and $\sigma_f^2 = 1$. In region (I), after the introduction of ESG reporting, the ERC and financial misreporting decrease, and expected cash flows and ESG performance increase. In region (II), after the introduction of ESG reporting, the ERC and financial misreporting increase, expected cash flows decrease, and expected ESG performance increases. In region (III), the ERC and financial misreporting decrease, expected cash flows increase, and expected ESG performance decreases.

about ESG performance, $\lambda\eta$. Second, there is a shift in effort incentives provided by the financial report, since the introduction of the ESG report changes how investors respond to the financial report. The shift is captured by $\frac{\theta}{c_e}\left(\varphi_f^* - \varphi_f^\dagger\right)$ and is capitalized into price in the same manner as the shift driven by the price response to the ESG report. Finally, there is a reduction in cash flow risk, which reduces the risk premium proportionally to investors' risk aversion parameter, ρ. The net impact of these effects is not necessarily positive, given the variation in potential effects on response coefficients, φ_r and φ_f, the potential for cash flow effects of ESG efforts, θ, to be positive or negative, and the incremental effects of risk

aversion that did not affect the results of Propositions 3, 4, and 5 above. The following corollary establishes a necessary condition under which the introduction of ESG reporting can reduce the firm's price by changing the effort incentives.

Corollary 2 *Introducing ESG reporting can reduce the firm's price only when* $-\lambda\eta\left(1+\frac{\sigma_x^2}{\sigma_f^2}\right) < \theta < -\lambda\eta$.

When the firm's risk premium is not affected by idiosyncratic risks and when the ESG report is not informative about systematic risks, introducing the ESG report does not change the firm's risk premium. In this situation, the condition in Corollary 2 is both a necessary and a sufficient condition for the firm's price to decrease. In effect, there are four regions for the parameter values that determine the effect of ESG reporting on price in lines (2) and (3) in Proposition 6, above. First, when $\theta > 0$, introducing the ESG report increases effort incentives, which increases both expected cash flows and expected ESG performance. As a result, the firm's price increases. Second, when $0 > \theta > -\lambda\eta$, introducing the ESG report increases effort incentives, which increases expected ESG performance but decreases cash flows. However, the reduction in cash flows is sufficiently small and, again, the firm's price increases. Third, when $-\lambda\eta\left(1+\frac{\sigma_x^2}{\sigma_f^2}\right) < \theta < -\lambda\eta$ (as in Corollary 2), introducing the ESG report still increases effort incentives, which increases the expected ESG performance but hurts the expected cash flows. However, in this setting, the average investor does not benefit from an increase in effort and, therefore, the firm's price can decrease. Finally, when $-\lambda\eta\left(1+\frac{\sigma_x^2}{\sigma_f^2}\right) < \theta$, introducing the ESG report reduces the effort incentives. This increases cash flows (because the manager reduces activities that benefit ESG performance at a sufficiently large cost to cash flows) but decreases the firm's ESG performance. This trade-off is valuable to the average investor and, thus, the firm's price increases.

Figure 2 illustrates the effects of introducing ESG reporting on prices, ESG and financial performance, and misreporting. Table 1 summarizes the effects of parameter changes.

Table 1: Impacts of introducing ESG reporting

Effect on \ Parameter region	$\theta > 0$	$0 > \theta > -\lambda\eta$	$0 > \theta > -\lambda\eta\left(1+\frac{\sigma_x^2}{\sigma_f^2}\right)$	$-\lambda\eta\left(1+\frac{\sigma_x^2}{\sigma_f^2}\right) > \theta$
ERC (Δ_ϕ)	-	+	+	-
Price (Δ_P)	+	+/-	+/-	+
Expected cash flows (Δ_x)	+	-	-	+
Expected ESG performance (Δ_y)	+	+	+	-
Financial misreporting (Δ_b)	-	+	+	-
ESG investors' holdings (Δ_{q_2})	+	+	+/-	-
Traditional investors' holdings (Δ_{q_1})	-	-	+/-	+

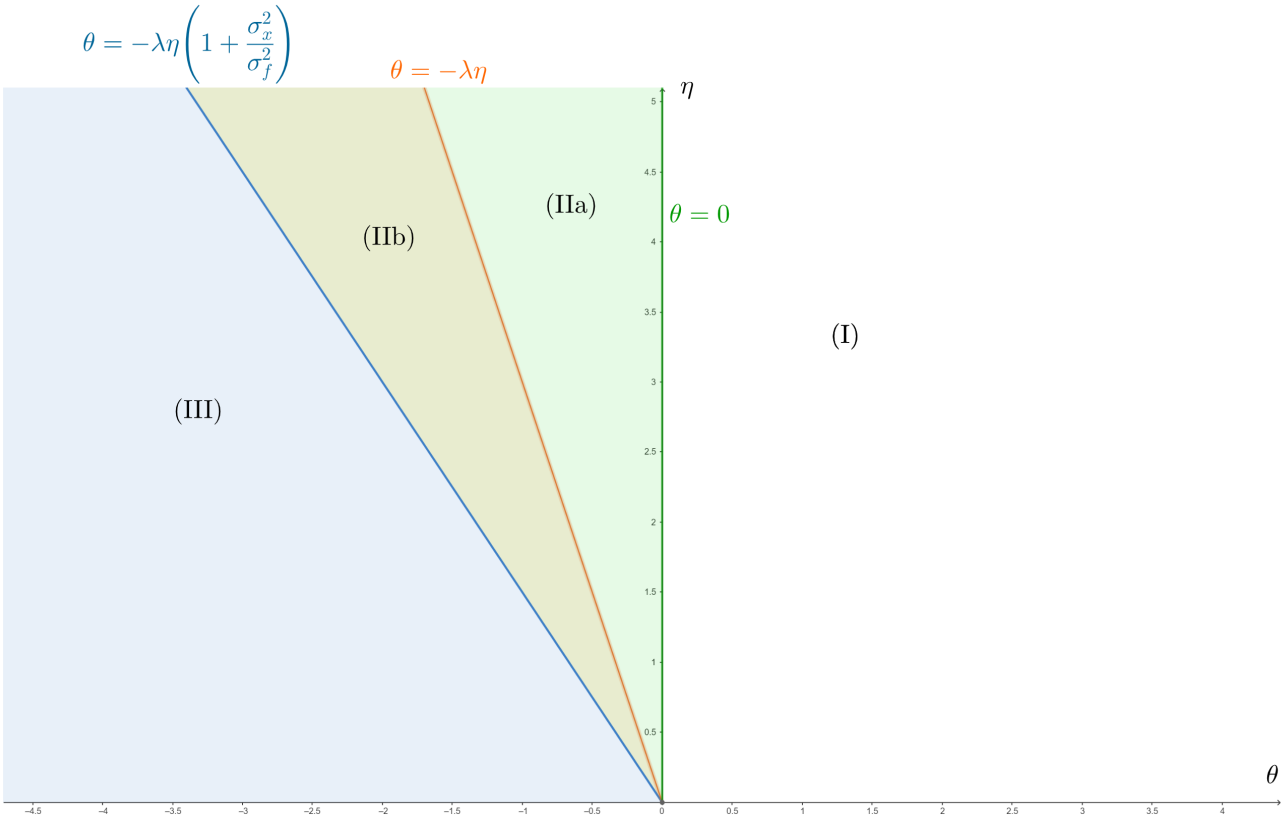

Figure 2: Regions of θ and η where the effects of introducing ESG reporting on misreporting, expected cash flows and ESG performance, and prices are different. $\lambda = \frac{1}{3}, \sigma_x^2 = 1,$ and $\sigma_f^2 = 1$. The plots assume no impact of ESG reporting on risk premium. In region (I), after the introduction of ESG reporting, the firm's price increases, the ERC and financial misreporting decrease, and expected cash flows and ESG performance increase. In region (IIa), the firm's price increases, the ERC and financial misreporting increase, expected cash flows decrease, and expected ESG performance increases. In region (IIb), the firm's price decreases, the ERC and financial misreporting increase, expected cash flows decrease, and expected ESG performance increases. In region (III), the firm's price increases, the ERC and financial misreporting decrease, expected cash flows increase, and expected ESG performance decreases.

4.2 Comparative statics

In this section we consider comparative statics in the equilibrium with both ESG and financial reports. These comparative statics provide empirical predictions on the one hand, and a lens through which to interpret observed empirical relations on the other.

4.2.1 Changes in investor preferences (λ)

Several business press articles and academic studies have noted the secular increase in investor concerns over ESG, as exemplified in the massive run-up in ESG-related assets under management, the increase in PRI signatories, and increases in survey respondents who indicate ESG-related preferences (e.g., Hong and Shore, 2022; Kim and Yoon, 2022; Pastor et al., 2021). As well, different jurisdictions or markets are likely to differ in the degree to which their investors display ESG concerns. The implications of such temporal and cross-sectional patterns are borne out in the following corollary.

Corollary 3 *An increase in the fraction of investors who value the firm's ESG performance, λ, leads to:*

1. *increases in the price response to the ESG report, expected ESG performance, and ESG misreporting, i.e., greenwashing ($\frac{d\varphi_r^*}{d\lambda}, \frac{dE[y^*]}{d\lambda}, \frac{d(\varphi_r^*/c_r)}{d\lambda} > 0$); and*
2. *effects on the price response to the financial report, expected cash flows, and expected financial misreporting that have the same sign as the effect of ESG effort on cash flows ($\frac{d\varphi_f^*}{d\lambda}, \frac{dE[x^*]}{d\lambda}, \frac{dE[f^*-x^*]}{d\lambda} \propto \theta$).*

Generally, an increase in the fraction of investors who value the firm's ESG performance implies that ESG performance becomes more valuable, on average, to the firm's shareholders. The market responds more strongly to the ESG report, which motivates more positive ESG efforts as well as additional ESG-related misreporting. The increase in ESG efforts translates into greater expected ESG performance.

For financial outcomes, the effect of an increase in the proportion of ESG-concerned investors depends on the effects of ESG efforts on cash flows, i.e., θ. When these effects are positive, i.e., $\theta > 0$, investors react more positively to the financial report, as a more positive financial report implies greater ESG efforts and greater expected ESG performance. This motivates additional financial misreporting, while complementing the effort-increasing effect of the increased responsiveness to the ESG report. In contrast, if $\theta < 0$, a higher financial report implies lower ESG efforts, which attenuates investors' response to the financial report

and provides less motivation for financial misreporting. Additionally, $\theta < 0$ means that the increase in ESG efforts in equilibrium leads to lower cash flows in expectation.

4.2.2 Changes in cash flow implications of ESG efforts (θ)

While some firms can effectively use their ESG related actions as advertising and increase their cash flows (such that $\theta > 0$), for other firms any efforts to improve their ESG performance likely lead to costs that cannot be recouped ($\theta < 0$). In this section, we investigate the implications of increasing the benefit to undertaking ESG related activities, which could also be effected by pro-ESG policies such as subsidies or taxes. Corollary 1 shows (absent the ESG report) that the response coefficient on the financial report can increase or decrease when θ increases because investors use the report to update on two output dimensions. This intuition also applies to our setting with two reports: the price sensitivity to either report can increase or decrease in θ.

When $\theta > 0$, a further increase in the cash flow implications of ESG efforts increases both expected cash flows and expected ESG performance. In this situation, updating on both dimensions of the firm's output goes hand-in-hand. As the financial report becomes more informative about the firm's ESG efforts, the manager has incentives to increase these efforts and, thus, to increase the firm's outputs. However, while cash flow and ESG performance increase, the manager's misreporting in both reports can either increase or decrease. We denote $S = \left(1 + \frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)$ and use it to summarize these comparative statics.

Corollary 4 *When the firm provides both reports*

1. *expected cash flows and expected ESG performance increase in θ when $\theta > 0$.*
2. *expected financial misreporting increases (decreases) in θ when $2S\theta\sigma_f^2 + \lambda\eta\left(S\left(\sigma_f^2 + \sigma_x^2\right) - \theta^2\sigma_\phi^2\right)$ is positive (negative);*
3. *expected greenwashing increases (decreases) in θ when $\sigma_f^2\left(S\left(\sigma_f^2 + \sigma_x^2\right) - \theta^2\sigma_\phi^2\right) - 2\theta\lambda\eta\sigma_\phi^2\left(\sigma_f^2 + \sigma_x^2\right)$ is positive (negative),*

where $S = \left(1 + \frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2 + \sigma_x^2\right)$.

Corollary 4 shows that output and misreporting of both financial and ESG performance are complements (all increase in θ) when θ is moderately positive whereas they are substitutes (outputs increase in θ and misreporting decreases) when θ is sufficiently positive. Note that a larger fraction of type-2 investors (or a bigger impact of the efforts on the expected ESG performance) increases the parameter region for which output and misreporting are substitutes.

When ESG efforts have negative cash flow implications, an increase in θ is associated with a smaller signal-to-noise ratio in the financial report. That is, as a negative θ becomes less negative, less of the variation in the financial reports comes from the manager's effort choice, and investors glean less information about the effort-related outputs. This tends to reduce the effort incentives, which increases (decreases) expected cash flows (expected ESG performance) when the expected effort is positive and has the opposite effect when the expected effort is negative. Financial misreporting and greenwashing decrease in θ when θ is sufficiently negative (note the negative coefficient on θ^2 in both comparative static results) but both increase in θ when it is sufficiently close to zero.

4.2.3 Changes in the importance of non-ESG cash flows (σ_x^2)

The next set of comparative static results concerns the extent of cash flow components that are not related to ESG efforts. In particular, as σ_x^2 increases, more of the variation in cash flows comes from factors that are not based on the manager's choice of effort. These factors could be effort choices that have no significant impact on the firm's ESG performance or factors outside of the manager's control. Because the financial report captures these non-ESG related cash flows, increasing σ_x^2 increases the information content of the financial report about the firm's cash flows. The financial report, however, becomes less informative about the firm's ESG performance. As a result, the price response to the financial report decreases in σ_x^2 when learning about ESG is sufficiently important to investors. The following corollary presents this result.

Corollary 5 *When the firm provides both reports,*

1. *the price response to the financial report increases (decreases) in* σ_x^2 *when* $\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2$ *is positive (negative); and*

2. *the price response to the ESG report decreases (increases) in* σ_x^2 *when* $\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2$ *is positive (negative).*

The conditions in Corollary 5 show that when the ESG effort has a negative effect on cash flows ($\theta < 0$), an increase in the importance of non-ESG cash flows increases the price response to the financial report and decreases the price response to the ESG report (because $\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2 > 0$ for $\theta < 0$). However, when $\theta > 0$ and when the importance of ESG performance is sufficiently high ($\lambda\eta > \left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\sigma_f^2/\theta\sigma_\phi^2$), the result is reversed: reducing the ability to infer ESG performance from the financial report decreases the price response to this report but increases the price response to the ESG report.

The change in price responses to both reports affects the manager's effort and misreporting incentives. The following corollary summarizes the effect of increasing σ_x^2 on the firm's expected outputs and the manager's misreporting choices.

Corollary 6 *When the firm provides both reports,*

1. *expected cash flows and financial misreporting are complements in* σ_x^2*, and expected ESG performance and greenwashing are substitutes in* σ_x^2*, i.e.,* $\frac{dE_x^*}{d\sigma_x^2} \propto \frac{dE[f^*-x^*]}{d\sigma_x^2} \propto (1+Q)\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2$, $\frac{dE_y^*}{d\sigma_x^2} \propto -\frac{d(\varphi_r^*/c_r)}{d\sigma_x^2} \propto \theta\left((1+Q)\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2\right)$;

2. *when* $\theta < 0$*,*

   (a) *expected cash flows, financial misreporting, and greenwashing increase in* σ_x^2 *(i.e.,* $\frac{dE_x^*}{d\sigma_x^2} \propto \frac{dE[f^*-x^*]}{d\sigma_x^2} \propto \frac{d(\varphi_r^*/c_r)}{d\sigma_x^2} > 0$*); and*

   (b) *expected ESG performance decreases in* σ_x^2 *(i.e.,* $\frac{dE_y^*}{d\sigma_x^2} < 0$*);*

3. *when* $\theta > 0$ *and* $\lambda\eta < \frac{(1+Q)\sigma_f^2}{\theta\sigma_\phi^2}$*,*

   (a) *expected cash flows, expected ESG performance, and financial misreporting increase in* σ_x^2 *(i.e.,* $\frac{dE_x^*}{d\sigma_x^2} \propto \sigma_x^2\frac{dE_y^*}{d\sigma_x^2} \propto \frac{dE[f^*-x^*]}{d\sigma_x^2} > 0$*); and*

   (b) *greenwashing decreases in* σ_x^2 *(i.e.,* $\frac{d(\varphi_r^*/c_r)}{d\sigma_x^2} < 0$*);*

4. *when* $\theta > 0$ *and* $\lambda\eta > \frac{(1+Q)\sigma_f^2}{\theta\sigma_\phi^2}$,

   (a) *expected cash flows, expected ESG performance, and financial misreporting decrease in* σ_x^2
   (i.e., $\frac{dE_x^*}{d\sigma_x^2} \propto \sigma_x^2 \frac{dE_y^*}{d\sigma_x^2} \propto \frac{dE[f^*-x^*]}{d\sigma_x^2} < 0$*); and*

   (b) *greenwashing increases in* σ_x^2 *(i.e.,* $\frac{d(\varphi_r^*/c_r)}{d\sigma_x^2} > 0$*);*

   where $Q = \frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}$.

As Corollary 5 shows, for the conditions in Corollary 6.2 and 6.3, the parameters are such that the price response to the financial report increases and that to the ESG report decreases in σ_x^2. For the conditions in Corollary 6.4, the opposite is true. When σ_x^2 increases, the importance of learning about the firm's expected cash flows increases for all investors. Corollary 6.1 shows that changes in σ_x^2, irrespective of the parameter conditions, affects effort incentives and greenwashing incentives in the opposite direction. For example, when the ESG effort increases both expected cash flows and ESG performance, and when learning about the ESG performance is sufficiently important (i.e., the conditions in Corollary 6.4 hold), the price sensitivity to the financial report (ESG report) decreases (increases) in σ_x^2. The reason is that investors learn less about the effort from the financial report and, thus, rely more on the ESG report. This reduces financial misreporting but increases the extent of greenwashing. In aggregate, investors learn less about the effort from the two reports, which, in turn, reduces the effort incentives, reducing both expected cash flows and ESG performance.

When, in turn, the conditions in Corollary 6.3 hold, learning about the ESG performance is not important enough to the average investor. Because the financial report is not primarily used to make an inference about the ESG effort, the price response to the financial report increases in σ_x^2. Because investors mainly use the ESG report to learn about cash flows, the price response to the ESG report decreases modestly and effort incentives increase. Finally, when the ESG effort decreases expected cash flows (Corollary 6.2 holds) effort incentives again decrease. However, as $\theta < 0$, this implies higher expected cash flows and lower expected ESG performance. Firms partly offset the decreased ESG performance with increased

greenwashing. Counter to intuition in public media, the reason is not that the manager has a higher incentive to signal a higher ESG performance. Instead, with $\theta < 0$, the manager can signal higher cash flows by brownwashing (negative ESG misreporting). This is balanced by an incentive to signal a higher ESG performance when $\lambda\eta > 0$. When σ_x^2 increases, the desire to signal higher cash flows by reducing the reported ESG performance becomes less important (because the ESG effort is relatively less important for cash flows), such that the manager's brownwashing incentives decrease. As a result, greenwashing increases. A similar effect is present for financial misreporting. With $\theta < 0$, the manager has incentives to understate cash flows, in order to signal higher ESG performance. As σ_x^2 increases, this becomes less important, which leads to more financial misreporting. In addition, Corollary 5 shows that, with $\theta < 0$, the price response to the financial report increases in σ_x^2. These two forces reinforce each other and financial misreporting increases in σ_x^2

4.2.4 Changes in idiosyncratic ESG reporting incentives (σ_r^2 or δ)

In our final set of comparative statics, we investigate a change in the informativeness of the ESG report. In particular, when either σ_r^2 or δ increase, the amount of noise in the ESG report increases, and, thus, the report becomes less informative to investors.

Because green investors and type-1 investors infer different information from the ESG report, the signs of the comparative statics depend on the correlation between cash flows and ESG performance. The following corollary summarizes the results.

Corollary 7 *When the firm provides both reports, increasing δ or σ_r^2 has the following effects.*

1. *When the cash flow effects of ESG effort are positive ($\theta > 0$)*
   (a) *the price response to the financial (ESG) report increases (decreases); and*
   (b) *expected cash flows, ESG performance, and greenwashing decrease while expected financial misreporting increases;*
2. *when the cash flow effects of ESG effort are moderately negative ($0 > \theta > -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$)*
   (a) *the price responses to the financial and ESG reports decrease; and*

(b) expected cash flows increase, while expected ESG performance, financial misreporting decreases, and greenwashing decrease. and

3. *when the cash flow effects of ESG effort are sufficiently negative* ($\theta < -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$)
   (a) the price responses to the financial and ESG reports increase; and
   (b) expected cash flows decrease, while expected ESG performance, financial misreporting, and greenwashing increase.

For all parameter values, expected ESG performance and expected greenwashing are complements. That is, increases in greenwashing go hand-in-hand with increases in ESG performance. However, expected cash flows and financial misreporting are substitutes: when cash flows decrease, the manager increases the extent of financial misreporting.

Similar to our results in Propositions (3-5), when $\theta > 0$, green investors and traditional investors both value an increase in the manager's effort. In this situation, an increase in the noise of the ESG signal reduces the weight on that signal and increases the weight on the financial report. As a result, increasing the noise in the ESG report leads to more financial misreporting but less greenwashing. Because investors learn less about the manager's effort, this effort is reduced and both types of output suffer.

In turn, when θ is moderately negative ($0 > \theta > -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$), green investors place a positive weight on the ESG report whereas traditional investors place a negative weight on it. In aggregate, however, the weight remains positive. An increase in the noise in the ESG report continues to reduce the price response to the ESG report (as before). However, in this parameter region the weight on the financial report also decreases. The reason is green investors' learning about the ESG activities. Reducing the (positive) weight on the ESG report goes hand in hand with increasing the negative weight on the financial report. This reduces the weight on the financial report and, with it, financial misreporting.

Finally, when θ is sufficiently negative ($\theta < -\lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)/\sigma_f^2$), the average investor places a negative weight on the ESG report. That is, for investors in aggregate, the increase in ESG performance that a high ESG report signals is not worth the cost to cash flows. For these parameter values, the manager has brownwashing incentives (to signal higher cash

flows) and decreasing the informativeness of the ESG report dampens these incentives. As a result, the expected ESG performance increases and the expected greenwashing becomes less negative. Table 2 summarizes the comparative statics results.

5 Conclusion

In response to growing interest in ESG reporting, we develop a model where a firm discloses financial and ESG reports to investors. The firm's manager chooses an effort that affects the firm's cash flows and ESG performance, and the manager can also choose to misreport both types of performance. We analyze the impact of introducing the ESG report by comparing the model's equilibrium where both reports are issued with the benchmark equilibrium where only the financial report is issued. We show how introducing the ESG report affects the manager's incentives, prices, reporting strategies, and investor holdings. When the ESG effort has a positive (sufficiently negative) impact on the firm's cash flows, introducing the ESG report increases prices and cash flows by increasing (decreasing) the manager's effort incentives. However, when the effort has a moderately negative impact on cash flows, ESG reporting can increase the incentives to reduce cash flows and, in turn, reduce the firm's price. In the equilibrium with both reports, we derive conditions under which firms with lower expected ESG performance have higher greenwashing incentives, but we also show that incentives to increase ESG performance can go hand-in-hand with greenwashing incentives. That is, ESG improvement and greenwashing can be complements or substitutes.

Overall, our paper contributes to the emerging literature on ESG reporting, providing a framework for developing and interpreting empirical results in a capital market setting. We view welfare implications and contracting as important but outside the scope of our endeavor. Considering these is likely to result in additional interesting implications of the expansion of ESG reporting and the secular increase in investors' interests in firms' ESG performance.

Table 2: Impacts of increases in parameters when the firm provides both reports

Effect on \ Parameter region	**Fraction of investors who value ESG performance (λ)**	**Cash flow implications of ESG efforts (θ)**	**Importance of non-ESG cash flow (σ_x^2)**	**Idiosyncratic ESG reporting incentives (δ or σ_r^2)**
Price response to ESG report (ϕ_r^*)	+	+/-	$sign(-Z_3)$	+ for $\theta < Z_4$, - otherwise
Expected ESG performance ($E[y^*]$)	+	$sign(\theta)$	$sign(\theta Z_3)$	+ for $\theta < Z_4$, - otherwise
Expected Greenwashing / ESG misreporting ($\frac{\phi_r^*}{c_r}$)	+	$sign(Z_2)$	$sign(-\theta Z_3)$	+ for $\theta < Z_4$, - otherwise
ERC (ϕ_f^*)	$sign(\theta)$	+/-	$sign(Z_3)$	+ for $\theta > 0$ or $\theta < Z_4$, - otherwise
Expected cash flows ($E[x^*]$)	$sign(\theta)$	$sign(\theta)$	$sign(Z_3)$	+ for $0 > \theta > Z_4$, - otherwise
Expected financial misreporting ($f^* - x^*$)	$sign(\theta)$	$sign(Z_1)$	$sign(Z_3)$	+ for $\theta > 0$ or $\theta < Z_4$, - otherwise

$Z_1 = 2S\theta\sigma_f^2 + \lambda\eta\left(S\left(\sigma_f^2+\sigma_x^2\right) - \theta^2\sigma_\phi^2\right), Z_2 = \sigma_f^2\left(S\left(\sigma_f^2+\sigma_x^2\right) - \theta^2\sigma_\phi^2\right) - 2\theta\lambda\eta\sigma_\phi^2\left(\sigma_f^2+\sigma_x^2\right), Z_3 = (1+S)\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2, Z_4 = -\lambda\eta\frac{\sigma_f^2+\sigma_x^2}{\sigma_f^2}, S = 1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}.$

References

Amel-Zadeh, A. and G. Serafeim (2018). Why and how investors use esg information: Evidence from a global survey. *Financial Analysts Journal 74*(3), 87–103.

Arif, S., Y. S. J. Yoon, and H. H. Zhang (2022). The information content of mandatory human capital disclosures-initial evidence. Working paper.

Barber, B. M., A. Morse, and A. Yasuda (2021). Impact investing. *Journal of Financial Economics 139*(1), 162–185.

Basu, S., J. Vitanza, W. Wang, and X. R. Zhu (2021). Walking the walk? bank esg disclosures and home mortgage lending. Working paper.

Bauer, R., T. Ruof, and P. Smeets (2021). Get Real! Individuals Prefer More Sustainable Investments. *Review of Financial Studies 34*(8).

Bingler, J. A., M. Kraus, and M. Leippold (2021). Cheap talk and cherry-picking: What climatebert has to say on corporate climate risk disclosures.

Black, D. E., H. Isidro, and A. C. Marques (2022). Investor commitment to responsible investing and firm esg disclosure. Working paper.

Bolton, P. and M. Kacperczyk (2021). Do investors care about carbon risk? *Journal of Financial Economics 142*.

Bonham, J. and A. Riggs-Cragun (2022). An accounting framework for ESG reporting. Working paper.

Bonnefon, J.-F., A. Landier, P. R. Sastry, and D. Thesmar (2022). The moral preferences of investors: Experimental evidence. Working paper.

Brogger, T. H. and A. Marsh (2021, August). Fund managers' ESG claims face credibility test as gaps found. *Bloomberg*. Accessed at https://www.bloomberg.com/news/articles/2021-08-02/fund-managers-esg-claims-face-credibility-test-as-gaps-found on August 12, 2021.

Chen, Y.-C., M. Hung, and Y. Wang (2018). The effect of mandatory csr disclosure on firm profitability and social externalities: Evidence from china. *Journal of accounting and economics 65*(1), 169–190.

Chowdhry, B., S. W. Davies, and B. Waters (2018). Investing for impact. *The Review of Financial Studies 32*(3), 864–904.

Christensen, H. B., L. Hail, and C. Leuz (2019). Adoption of csr and sustainability reporting standards: Economic analysis and review. European Corporate Governance Institute-Finance Working Paper.

Datar, S., S. C. Kulp, and R. A. Lambert (2001). Balancing performance measures. *Journal of Accounting Research 39*(1), 75–92.

De Angelis, T., P. Tankov, and O. D. Zerbib (2020). Environmental impact investing.

Delmas, M. A. and V. C. Burbano (2011). The drivers of greenwashing. *California management review 54*(1), 64–87.

Downar, B., J. Ernstberger, S. Reichelstein, S. Schwenen, and A. Zaklan (2021). The impact of carbon disclosure mandates on emissions and financial operating performance. *Review of Accounting Studies 26*(3), 1137–1175.

Dye, R. and S. Sridhar (2004). Reliability-relevance trade-offs and the efficiency of aggregation. *Journal of Accounting Research 42*(1), 51–88.

Dye, R. A. and S. S. Sridhar (2008). A positive theory of flexibility in accounting standards. *Journal of Accounting and Economics 46*(2-3).

Fiechter, P., J.-M. Hitz, and N. Lehmann (2022). Real effects of a widespread csr reporting mandate: Evidence from the european union's csr directive. *Journal of Accounting Research 60*(4).

Fischer, P. and R. Verrecchia (2000). Reporting bias. *The Accounting Review 75*(2), 229–245.

Friedman, H. L. and M. S. Heinle (2016). Taste, information, and asset prices: Implications for the valuation of csr. *Review of Accounting Studies 21*(3), 740–767.

Friedman, H. L. and M. S. Heinle (2021). Interested Investors and Intermediaries: When do ESG Concerns Lead to ESG Performance? Jacobs Levy Equity Management Center for Quantitative Financial Research Paper.

Goldstein, I., A. Kopytov, L. Shen, and H. Xiang (2022). On esg investing: Heterogeneous preferences, information, and asset prices. Working paper.

Green, D. and B. Roth (2021). The allocation of socially responsible capital. Working paper.

Grewal, J., G. D. Richardson, and J. Wang (2022). The effect of mandatory carbon reporting on greenwashing. Working paper.

Grewal, J. and G. Serafeim (2020). Research on corporate sustainability: Review and directions for future research. *Foundations and Trends in Accounting 14*(2), 73–127.

Heath, D., D. Macciocchi, R. Michaely, and M. Ringgenberg (2021). Does socially responsible investing change firm behavior? Working paper.

Holmstrom, B. and P. Milgrom (1991). Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law, Economics & Organization 7*, 24.

Hong, H. G. and E. P. Shore (2022). Corporate social responsibility. *Annual Review of Financial Economics*.

Jebe, R. (2019). The convergence of financial and esg materiality: Taking sustainability mainstream. *American Business Law Journal 56*(3), 645–702.

Jung, W.-O. and Y. K. Kwon (1988). Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research 60*(1), 146–153.

Kanodia, C. (2007). Accounting disclosure and real effects. *Foundations and Trends® in Accounting 1*(3), 167–258.

Khan, M., G. Serafeim, and A. Yoon (2016). Corporate sustainability: First evidence on materiality. *Accounting Review 91*(6), 1697–1724.

Kim, E.-H. and T. P. Lyon (2015). Greenwash vs. brownwash: Exaggeration and undue modesty in corporate sustainability disclosure. *Organization Science 26*(3), 705–723.

Kim, S. and A. Yoon (2022). Analyzing active fund managers' commitment to esg: Evidence from the united nations principles for responsible investment. *Management Science*.

Kowsmann, P. and K. Brown (2021, August). Fired executive says Deutsche Bank's DWS overstated sustainable-investing efforts. *Wall Street Journal*. Accessed at https://www.wsj.com/articles/fired-executive-says-deutsche-banks-dws-overstated-sustainable-investing-efforts-11627810380 on August 12, 2021.

Krueger, P., Z. Sautner, and L. T. Starks (2020). The importance of climate risks for institutional investors. *Review of Financial Studies 33*(3), 1067–1111.

Krueger, P., Z. Sautner, D. Y. Tang, and R. Zhong (2021). The effects of mandatory esg disclosure around the world. European Corporate Governance Institute - Finance Working Paper No. 754/2021, Swiss Finance Institute Research Paper No. 21-44.

Landier, A. and S. Lovo (2020). Esg investing: How to optimize impact? HEC Paris Research Paper No. FIN-2020-1363.

Lyon, T. P. and J. W. Maxwell (2011). Greenwash: Corporate environmental disclosure under threat of audit. *Journal of Economics & Management Strategy 20*(1), 3–41.

Marquis, C., M. W. Toffel, and Y. Zhou (2016). Scrutiny, norms, and selective disclosure: A global study of greenwashing. *Organization Science 27*(2), 483–504.

Matsumura, E. M., R. Prakash, and S. C. Vera-Munoz (2014). Firm-value effects of carbon emissions and carbon disclosures. *The Accounting Review 89*(2), 695–724.

Moss, A., J. P. Naughton, C. Wang, and I. Yeung (2022). Bigger fish to fry: The interdependence of earnings and esg news in investor screening. Working paper.

Moss, A., N. J. P., and C. Wang (2020). The irrelevance of ESG disclosure to retail investors: Evidence from Robinhood. Working paper.

Muller, N. Z. (2021). Measuring firm environmental performance to inform asset management and standardized disclosure. Working paper.

Oehmke, M. and M. Opp (2019). A theory of socially responsible investment. Working paper.

Pástor, L., R. F. Stambaugh, and L. A. Taylor (2020). Sustainable investing in equilibrium. *Journal of Financial Economics 142*(2).

Pastor, L., R. F. Stambaugh, and L. A. Taylor (2021). Dissecting green returns. Working paper.

Pedersen, L. H., S. Fitzgibbons, and L. Pomorski (2020). Responsible investing: The esg-efficient frontier. *Journal of Financial Economics 142*(2).

Raghunandan, A. and S. Rajgopal (2022). Do ESG funds make stakeholder-friendly investments? *Review of Accounting Studies 27*(3), 822–863.

Thomas, J., W. Yao, F. Zhang, and W. Zhu (2022). Meet, beat, and pollute. *Review of Accounting Studies 27*(3), 1038–1078.

Walker, D. I. (2022). The economic (in) significance of executive pay ESG incentives. Boston Univ. School of Law Research Paper.

Wang, L. L. (2022). Transmission effects of esg disclosure regulations through bank lending networks. Working paper.

Xue, H. (2022). ESG disclosure, market forces, and investment efficiency. Working paper.

Yang, L., N. Z. Muller, and P. J. Liang (2021). The real effects of mandatory csr disclosure on emissions: Evidence from the greenhouse gas reporting program. Working paper.

Zerbib, O. D. (2020). A sustainable capital asset pricing model (s-capm): Evidence from green investing and sin stock exclusion.

Appendix

Proof of Propositions 1 and 2 and Corollary 1

We solve the model with both reports (Proposition 2) via backward induction then derive the equilibrium in Proposition 1 without the ESG report , r, as a special case.

Given the CARA-normal setting, share demands are given by $q_1 = \frac{E[x|r,f]-p}{\rho Var[x|r,f]}$ and $q_2 = \frac{E[x+y|r,f]-p}{\rho Var[x|r,f]}$. Substituting demands into the market clearing condition $1 = \lambda q_2 + (1-\lambda) q_1$ gives the price function as

$$\begin{aligned} 1 &= \lambda \frac{E[x+y|r,f]-p}{\rho Var[x|r,f]} + (1-\lambda)\frac{E[x|r,f]-p}{\rho Var[x|r,f]} \\ \Leftrightarrow p &= E[x|r,f] + \lambda E[y|r,f] - \rho Var[x|r,f]. \end{aligned} \tag{4}$$

Joint normality implies

$$Var\begin{bmatrix} x \\ y \\ r \\ f \end{bmatrix} = \begin{bmatrix} \sigma_x^2+\theta^2\sigma_\phi^2 & \theta\eta\sigma_\phi^2 & \theta z\sigma_\phi^2 & \sigma_x^2+\theta^2\sigma_\phi^2 \\ \theta\eta\sigma_\phi^2 & \eta^2\sigma_\phi^2 & \eta z\sigma_\phi^2 & \eta\theta\sigma_\phi^2 \\ \theta z\sigma_\phi^2 & \eta z\sigma_\phi^2 & z^2\sigma_\phi^2+\delta^2\sigma_r^2 & \theta z\sigma_\phi^2 \\ \sigma_x^2+\theta^2\sigma_\phi^2 & \eta\theta\sigma_\phi^2 & \theta z\sigma_\phi^2 & \sigma_x^2+\theta^2\sigma_\phi^2+\sigma_f^2 \end{bmatrix}$$

such that

$$\begin{aligned} Var\begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} \Big| r,f \Big] &= \begin{bmatrix} \sigma_x^2+\theta^2\sigma_\phi^2 & \theta\eta\sigma_\phi^2 \\ \theta\eta\sigma_\phi^2 & \eta^2\sigma_\phi^2 \end{bmatrix} \\ &\quad - \begin{bmatrix} \theta z\sigma_\phi^2 & \sigma_x^2+\theta^2\sigma_\phi^2 \\ \eta z\sigma_\phi^2 & \eta\theta\sigma_\phi^2 \end{bmatrix} \begin{bmatrix} z^2\sigma_\phi^2+\delta^2\sigma_r^2 & \theta z\sigma_\phi^2 \\ \theta z\sigma_\phi^2 & \sigma_x^2+\theta^2\sigma_\phi^2+\sigma_f^2 \end{bmatrix}^{-1} \begin{bmatrix} \theta z\sigma_\phi^2 & \eta z\sigma_\phi^2 \\ \sigma_x^2+\theta^2\sigma_\phi^2 & \eta\theta\sigma_\phi^2 \end{bmatrix}. \end{aligned}$$

Conditional variances can be rewritten as

$$\begin{aligned}
Var\left[x|r,f\right] &= \hat{\sigma}_x^2 = \sigma_{x|r,f}^2 = \sigma_f^2 \frac{\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\sigma_x^2 + \theta^2\sigma_\phi^2}{\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}, \\
Var\left[y|r,f\right] &= \hat{\sigma}_y^2 = \sigma_{y|r,f}^2 = \eta^2\sigma_\phi^2 \frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}, \text{ and} \\
Cov\left[x,y|r,f\right] &= \hat{\sigma}_{xy} = \sigma_{xy|r,f} = \theta\eta\sigma_f^2 \frac{\sigma_\phi^2}{\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}.
\end{aligned}$$

Conditional means are given by

$$E\left[\begin{matrix}\tilde{x}\\ \tilde{y}\end{matrix}\middle| r,f\right] = \begin{bmatrix}E_x\\ E_y\end{bmatrix} + \begin{bmatrix}\theta z\sigma_\phi^2\sigma_x^2+\theta^2\sigma_\phi^2 \\ \eta z\sigma_\phi^2\eta\theta\sigma_\phi^2\end{bmatrix}\begin{bmatrix}z^2\sigma_\phi^2+\delta^2\sigma_r^2\theta z\sigma_\phi^2 \\ \theta z\sigma_\phi^2\sigma_x^2+\theta^2\sigma_\phi^2+\sigma_f^2\end{bmatrix}$$

where E_x and E_y are prior expectations. Exploiting the expressions above for conditional variances and covariances, we have

$$\begin{aligned}
E\left[x|r,f\right] &= E_x + \frac{\left(\sigma_x^2\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)+\theta^2\sigma_\phi^2\right)(f-E_f)}{\left(\sigma_f^2+\sigma_x^2\right)\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)+\theta^2\sigma_\phi^2} \\
&\quad + \frac{\theta\left(\frac{z}{\delta^2\sigma_r^2}\sigma_f^2\sigma_\phi^2\right)(r-E_r)}{\left(\sigma_f^2+\sigma_x^2\right)\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)+\theta^2\sigma_\phi^2} \\
&= E_x + \frac{\hat{\sigma}_x^2}{\sigma_f^2}(f-E_f) + \frac{\hat{\sigma}_{xy}}{\delta^2\sigma_r^2}\frac{z}{\eta}(r-E_r)
\end{aligned}$$

and

$$\begin{aligned}
E\left[y|r,f\right] &= E_y + \frac{\theta\eta\sigma_\phi^2(f-E_f)}{\left(\sigma_f^2+\sigma_x^2\right)\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)+\theta^2\sigma_\phi^2} \\
&\quad + \frac{\frac{z}{\delta^2\sigma_r^2}\eta\sigma_\phi^2\left(\sigma_f^2+\sigma_x^2\right)(r-E_r)}{\left(\sigma_f^2+\sigma_x^2\right)\left(1+\frac{z^2}{\delta^2\sigma_r^2}\sigma_\phi^2\right)+\theta^2\sigma_\phi^2} \\
&= E_y + \frac{\hat{\sigma}_{xy}}{\sigma_f^2}(f-E_f) + \frac{\hat{\sigma}_y^2}{\delta^2\sigma_r^2}\frac{z}{\eta}(r-E_r).
\end{aligned}$$

Substituting these into equation (4) yields the price in Proposition 2. The price responses to the reports are given by

$$\begin{aligned}
\frac{\partial p^*}{\partial f} &= \varphi_f^* = \frac{\hat{\sigma}_x^2}{\sigma_f^2} + \lambda \frac{\hat{\sigma}_{xy}}{\sigma_f^2} \\
&= \frac{\left(1 + \frac{z^2}{\delta^2 \sigma_r^2} \sigma_\phi^2\right) \sigma_x^2 + \theta^2 \sigma_\phi^2}{\left(1 + \frac{z^2}{\delta^2 \sigma_r^2} \sigma_\phi^2\right) \left(\sigma_f^2 + \sigma_x^2\right) + \theta^2 \sigma_\phi^2} + \lambda \theta \eta \frac{\sigma_\phi^2}{\left(1 + \frac{z^2}{\delta^2 \sigma_r^2} \sigma_\phi^2\right) \left(\sigma_f^2 + \sigma_x^2\right) + \theta^2 \sigma_\phi^2} \text{ and} \\
\frac{\partial p^*}{\partial r} &= \varphi_r^* = \frac{\hat{\sigma}_{xy}}{\delta^2 \sigma_r^2} \frac{z}{\eta} + \lambda \frac{\hat{\sigma}_y^2}{\delta^2 \sigma_r^2} \frac{z}{\eta} \\
&= \frac{z \theta \sigma_\phi^2}{\delta^2 \sigma_r^2} \frac{\sigma_f^2}{\left(1 + \frac{z^2}{\delta^2 \sigma_r^2} \sigma_\phi^2\right) \left(\sigma_f^2 + \sigma_x^2\right) + \theta^2 \sigma_\phi^2} + \lambda \frac{z \eta \sigma_\phi^2}{\delta^2 \sigma_r^2} \frac{\sigma_f^2 + \sigma_x^2}{\left(1 + \frac{z^2}{\delta^2 \sigma_r^2} \sigma_\phi^2\right) \left(\sigma_f^2 + \sigma_x^2\right) + \theta^2 \sigma_\phi^2}
\end{aligned}$$

Equilibrium reporting strategies, f^* and r^*, can be derived from substituting the price into the manager's utility (equation 1), and maximizing with respect to f and r, taking e, ϕ, ε_x, ε_f, and ε_r as given or known. Substituting these in addition into the manager's utility and maximizing with respect to the ESG effort, e, yields e^*. With this, equilibrium cash flows and ESG performance are $x^* = \theta e^* + \varepsilon_x$ and $y^* = \eta e^*$. Prior means, E_j^*, $j \in \{x, y, r, f\}$, can be derived by substituting in equilibrium efforts and reports and taking expectations.

Proposition 1 can be derived by setting $z = \delta = 0$. Corollary 1 follows from straightforward differentiation of equilibrium expressions in Proposition 1.

Proof of Propositions 3, 4, and 5

Denote $S_r = \delta^2\sigma_r^2$ and recall from Proposition 1 that $\phi_f^\dagger = \frac{\partial p^\dagger}{\partial f} = \frac{\sigma_x^2+\sigma_\phi^2\left(\theta^2+\lambda\theta\eta\right)}{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2}$ is the ERC before the ESG report is introduced.

$$
\begin{aligned}
\Delta_\varphi &= \varphi_f^* - \varphi_f^\dagger \\
&= \left(\frac{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\sigma_x^2+\theta^2\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\theta\eta\frac{\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)-\frac{\sigma_x^2+\sigma_\phi^2\left(\theta^2+\lambda\theta\eta\right)}{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2} \\
&= \frac{\frac{z^2}{S_r}\sigma_\phi^2\sigma_x^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2} \\
&\quad -\left(\sigma_x^2+\sigma_\phi^2\theta^2+\lambda\theta\eta\sigma_\phi^2\right)\left(\frac{1}{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2}-\frac{1}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right) \\
&= \frac{\frac{z^2}{S_r}\sigma_\phi^2\sigma_x^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2} \\
&\quad -\left(\sigma_x^2+\sigma_\phi^2\theta^2+\lambda\theta\eta\sigma_\phi^2\right)\frac{\sigma_\phi^2\frac{z^2}{S_r}\left(\sigma_f^2+\sigma_x^2\right)}{\left(\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2\right)} \\
&= -\theta\sigma_\phi^2 * \frac{\theta\sigma_f^2+\lambda\eta\sigma_f^2+\lambda\eta\sigma_x^2}{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2} * \frac{\frac{z^2}{S_r}\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}.
\end{aligned}
$$

Thus the sign of Δ_φ is the same as the sign of $-\theta\left(\theta\sigma_f^2+\lambda\eta\sigma_f^2+\lambda\eta\sigma_x^2\right)$. When $\theta > 0$, $-\theta\left(\theta\sigma_f^2+\lambda\eta\sigma_f^2+\lambda\eta\sigma_x^2\right) < 0 \Rightarrow \Delta_\varphi < 0$.

When $\theta < 0$,

$$
\begin{aligned}
-\theta\left(\theta\sigma_f^2+\lambda\eta\sigma_f^2+\lambda\eta\sigma_x^2\right) > 0 &\iff \theta\sigma_f^2+\lambda\eta\sigma_f^2+\lambda\eta\sigma_x^2 > 0 \\
&\iff \theta > -\lambda\eta\left(1+\frac{\sigma_x^2}{\sigma_f^2}\right)
\end{aligned}
$$

The change in financial misreporting is $\Delta_b = \frac{\varphi_f^*}{c_f} - \frac{\varphi_f^\dagger}{c_f} = \frac{\Delta_\varphi}{c_f}$. Thus, financial misreporting increases (decreases) whenever the ERC increases (decreases).

The cash flow when only financial report is issued is $x^{\dagger} = \theta\left(\phi + \frac{\varphi_f^{\dagger}\theta}{c_e}\right) + \varepsilon_x$, and when both reports are issued is $x^{*} = \theta\left(\phi + \frac{\varphi_r^{*}z + \varphi_f^{*}\theta}{c_e}\right) + \varepsilon_x$. The change in expected cash flows is

$$\begin{aligned}
\Delta_x = E[x^{*} - x^{\dagger}] &= (\theta e_x^{*}) - \theta\left(\phi + \frac{\varphi_f^{\dagger}\theta}{c_e}\right) \\
&= \frac{\theta}{c_e}\left(\begin{array}{c} \left(1 - \frac{\sigma_f^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right) \\ + \left(1 - \frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\lambda\eta - \frac{\sigma_x^2+\sigma_\phi^2\left(\theta^2+\lambda\theta\eta\right)}{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2}\theta \end{array}\right) \\
&= \frac{\theta}{c_e}\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\frac{\left(\sigma_f^2+\sigma_x^2\right)\left(\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2\right)\right)}{\left(\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2\right)\left(\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)},
\end{aligned}$$

such that Δ_x has the same sign as $\theta\left(\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2+\sigma_x^2\right)\right)$. Thus, $\theta > 0 \Rightarrow \Delta_x > 0$. Furthermore,

$$\begin{aligned}
\theta\left(\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2+\sigma_x^2\right)\right) > 0 &\iff \theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2+\sigma_x^2\right) < 0 \\
&\iff \theta < -\lambda\eta\left(1 + \frac{\sigma_x^2}{\sigma_f^2}\right).
\end{aligned}$$

Expected ESG performance with only the financial report is $y^{\dagger} = \eta\left(\phi + \frac{\varphi_f^{\dagger}\theta}{c_e}\right)$, and with

both reports is $y^* = \eta\left(\phi + \frac{\varphi_r^* z + \varphi_f^* \theta}{c_e}\right)$. The change in expected ESG performance is

$$\begin{aligned}
\Delta_y = y^* - y^\dagger &= \eta e^* - \eta\left(\phi + \frac{\varphi_f^\dagger \theta}{c_e}\right) \\
&= \frac{\eta}{c_e}\left(\begin{array}{c}\left(1 - \frac{\sigma_f^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\theta \\ + \left(1 - \frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\lambda\eta - \frac{\sigma_x^2+\sigma_\phi^2\left(\theta^2+\lambda\theta\eta\right)}{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2}\theta\end{array}\right) \\
&= \frac{\eta}{c_e}\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\left(\sigma_f^2+\sigma_x^2\right)\frac{\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2+\sigma_x^2\right)}{\left(\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2\right)\left(\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)},
\end{aligned}$$

such that the sign of Δ_y is the same as the sign of $\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2+\sigma_x^2\right)$. Thus, $\theta > 0 \Rightarrow \Delta_y > 0$ and

$$\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2+\sigma_x^2\right) > 0 \iff \theta > -\lambda\eta\left(1 + \frac{\sigma_x^2}{\sigma_f^2}\right)$$

Holdings of type-1 and type-2 investors, respectively, with only the financial report are

$$\begin{aligned}
E\left[q_1^\dagger\right] &= 1 - \lambda\frac{\theta\eta}{\rho\sigma_f^2 c_e}\left(1 + \frac{\theta\lambda\eta\sigma_\phi^2}{\sigma_x^2+\theta^2\sigma_\phi^2}\right) \text{ and} \\
E\left[q_2^\dagger\right] &= 1 + (1-\lambda)\,\theta\eta\frac{\sigma_x^2+\theta^2\sigma_\phi^2+\theta\lambda\eta\sigma_\phi^2}{\rho\sigma_f^2 c_e\left(\sigma_x^2+\theta^2\sigma_\phi^2\right)}.
\end{aligned}$$

With both reports, they are

$$\begin{aligned}
E\left[q_1^*\right] &= 1 - \lambda\frac{\eta}{\rho\sigma_f^2 c_e}\frac{Q\sigma_\phi^2\left(\sigma_f^2+\sigma_x^2\right)(\theta+\lambda\eta) + \theta\left(\sigma_x^2+\theta^2\sigma_\phi^2+\theta\lambda\eta\sigma_\phi^2\right)}{\sigma_x^2+\theta^2\sigma_\phi^2+Q\sigma_x^2\sigma_\phi^2} \text{ and} \\
E\left[q_2^*\right] &= 1 + (1-\lambda)\frac{\eta}{\rho\sigma_f^2 c_e}\frac{Q\sigma_\phi^2\left(\sigma_f^2+\sigma_x^2\right)(\theta+\lambda\eta) + \theta\left(\sigma_x^2+\theta^2\sigma_\phi^2+\theta\lambda\eta\sigma_\phi^2\right)}{\sigma_x^2+\theta^2\sigma_\phi^2+Q\sigma_x^2\sigma_\phi^2}.
\end{aligned}$$

where $Q = \frac{z^2}{\delta^2 \sigma_r^2}$. Thus, the changes in holdings are given by

$$\begin{aligned}
\Delta_{q1} &= E\left[q_1^*\right] - E\left[q_1^\dagger\right] \\
&= \lambda \frac{\eta}{\rho \sigma_f^2 c_e} \left(\theta \left(1 + \frac{\theta \lambda \eta \sigma_\phi^2}{\sigma_x^2 + \theta^2 \sigma_\phi^2} \right) - \frac{Q \sigma_\phi^2 \left(\sigma_f^2 + \sigma_x^2\right)(\theta + \lambda\eta) + \theta\left(\sigma_x^2 + \theta^2 \sigma_\phi^2 + \theta\lambda\eta\sigma_\phi^2\right)}{\sigma_x^2 + \theta^2\sigma_\phi^2 + Q\sigma_x^2\sigma_\phi^2} \right) \\
&= -\lambda \frac{\eta}{\rho \sigma_f^2 c_e} Q \sigma_\phi^2 \frac{\lambda\eta\sigma_x^4 + \lambda\eta\sigma_f^2\sigma_x^2 + \theta^2\lambda\eta\sigma_f^2\sigma_\phi^2 + \theta\sigma_f^2\sigma_x^2 + \theta^3\sigma_f^2\sigma_\phi^2}{\left(\sigma_x^2 + \theta^2\sigma_\phi^2\right)\left(\sigma_x^2 + \theta^2\sigma_\phi^2 + Q\sigma_x^2\sigma_\phi^2\right)} \text{ and} \\
\Delta_{q2} &= E\left[q_2^*\right] - E\left[q_2^\dagger\right] \\
&= (1-\lambda) \frac{\eta}{\rho \sigma_f^2 c_e} \left(\frac{Q \sigma_\phi^2 \left(\sigma_f^2 + \sigma_x^2\right)(\theta + \lambda\eta) + \theta\left(\sigma_x^2 + \theta^2 \sigma_\phi^2 + \theta\lambda\eta\sigma_\phi^2\right)}{\sigma_x^2 + \theta^2\sigma_\phi^2 + Q\sigma_x^2\sigma_\phi^2} - \theta\left(1 + \frac{\theta\lambda\eta\sigma_\phi^2}{\sigma_x^2 + \theta^2\sigma_\phi^2}\right) \right) \\
&= \lambda \frac{\eta}{\rho \sigma_f^2 c_e} Q \sigma_\phi^2 \frac{\lambda\eta\sigma_x^4 + \lambda\eta\sigma_f^2\sigma_x^2 + \theta^2\lambda\eta\sigma_f^2\sigma_\phi^2 + \theta\sigma_f^2\sigma_x^2 + \theta^3\sigma_f^2\sigma_\phi^2}{\left(\sigma_x^2 + \theta^2\sigma_\phi^2\right)\left(\sigma_x^2 + \theta^2\sigma_\phi^2 + Q\sigma_x^2\sigma_\phi^2\right)}.
\end{aligned}$$

Therefore, when $\theta > 0$, $\Delta_{q2} > 0$ and $\Delta_{q1} < 0$. We can express Δ_{q2} as:

$$\Delta_{q2} = \lambda \frac{\eta}{\rho \sigma_f^2 c_e} Q \sigma_\phi^2 \frac{\sigma_x^2\left(\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)\right) + \theta^2\sigma_f^2\sigma_\phi^2(\theta + \lambda\eta)}{\left(\sigma_x^2 + \theta^2\sigma_\phi^2\right)\left(\sigma_x^2 + \theta^2\sigma_\phi^2 + Q\sigma_x^2\sigma_\phi^2\right)}$$

From here, we can see that $\varphi_r^* < 0$ implies that $\Delta_{q2} < 0$. The reason is that

$$\begin{aligned}
\varphi_r^* < 0 \quad &\iff \quad \sigma_x^2\left(\theta\sigma_f^2 + \lambda\eta\left(\sigma_f^2 + \sigma_x^2\right)\right) < 0 \\
&\iff \quad \theta < -\lambda\eta \frac{\sigma_f^2 + \sigma_x^2}{\sigma_f^2} < -\lambda\eta.
\end{aligned}$$

Therefore, when $\varphi_r^* < 0$, then the first term in the numerator of Δ_{q2} is negative and whenever the first term is negative, then the second term is negative as well. In other words, when the action has a sufficiently negative impact on cash flows (such that $\varphi_r^* < 0$), adding an ESG report reduces the fraction of shares held by ESG investors. This implies that for $\theta < 0$ and $\varphi_r^* < 0$, adding an ESG report pushes away holdings from green investors.

Proof of Corollaries 3-7

Recall from Proposition 2 that

$$\begin{aligned}
\varphi_f^* &= \frac{dp^*}{df} = \frac{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\sigma_x^2+\theta^2\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\theta\eta\frac{\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2} \text{ and} \\
\varphi_r^* &= \frac{dp^*}{dr} = \frac{z\theta\sigma_\phi^2}{S_r}\frac{\sigma_f^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\frac{z\eta\sigma_\phi^2}{S_r}\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2},
\end{aligned}$$

where $S_r = \delta^2\sigma_r^2$. Additionally, prior expectations of financial and ESG performance are:

$$\begin{aligned}
E[x^*] &= \theta\frac{1}{c_e}\left(\left(1-\frac{\sigma_f^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\theta+\left(1-\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\lambda\eta\right) \text{ and} \\
E[y^*] &= \eta\frac{1}{c_e}\left(\left(1-\frac{\sigma_f^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\theta+\left(1-\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\lambda\eta\right).
\end{aligned}$$

Expected financial misreporting is

$$E[f^*-x^*] = \frac{1}{c_f}\left(\frac{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\sigma_x^2+\theta^2\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\theta\eta\frac{\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)$$

Derivatives with respect to λ are:

$$\begin{aligned}
\frac{d\varphi_f^*}{d\lambda} &= \theta\eta\frac{\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2} \propto \theta\eta, \\
\frac{d\varphi_r^*}{d\lambda} &= \frac{z\eta\sigma_\phi^2}{S_r}\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2} \propto z\eta,
\end{aligned}$$

$$\begin{aligned}
\frac{d}{d\lambda}E\left[x\right] &= \frac{1}{c_e}\left(1-\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\theta\eta \propto \theta\eta, \\
\frac{d}{d\lambda}E\left[y\right] &= \frac{1}{c_e}\left(1-\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\lambda\eta^2 > 0, \text{ and} \\
\frac{d}{d\lambda}E\left[f^*-x^*\right] &= \frac{1}{c_f}\frac{\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\theta\eta \propto \theta\eta.
\end{aligned}$$

Using

$$S=\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right),$$

which is independent of θ, we present expressions for derivatives with respect to θ. First, for expected cash flows,

$$\begin{aligned}
\frac{d}{d\theta}E\left[x^*\right] &= \frac{1}{c_e}\left(2\left(1-\frac{\sigma_f^2}{S+\theta^2\sigma_\phi^2}\right)\theta+\left(1-\frac{\sigma_f^2+\sigma_x^2}{S+\theta^2\sigma_\phi^2}\right)\lambda\eta\right)+\frac{2}{c_e}\theta^2\sigma_\phi^2\frac{\theta\sigma_f^2+\lambda\eta\sigma_f^2+\lambda\eta\sigma_x^2}{\left(S+\theta^2\sigma_\phi^2\right)^2} \\
&= \frac{2\theta\left(\left(S+\theta^2\sigma_\phi^2\right)^2-S\sigma_f^2\right)+\left(\left(S+\theta^2\sigma_\phi^2\right)^2-S\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\left(\sigma_f^2+\sigma_x^2\right)\right)\lambda\eta}{c_e\left(S+\theta^2\sigma_\phi^2\right)^2}.
\end{aligned}$$

Cash flows increase in the cash flow productivity for $\theta>0$ because $S=\left(1+\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}\right)\left(\sigma_f^2+\sigma_x^2\right) \Rightarrow \left(S+\theta^2\sigma_\phi^2\right)^2-S\sigma_f^2>0$, and $\left(S+\theta^2\sigma_\phi^2\right)^2-S\left(\sigma_f^2+\sigma_x^2\right)>0$.

For expected ESG performance,

$$\frac{d}{d\theta}E\left[y^*\right]=\eta\frac{\left(S+\theta^2\sigma_\phi^2\right)^2-S\sigma_f^2+\theta^2\sigma_f^2\sigma_\phi^2+2\lambda\theta\eta\sigma_\phi^2\left(\sigma_f^2+\sigma_x^2\right)}{c_e\left(S+\theta^2\sigma_\phi^2\right)^2}.$$

For $\theta>0$ ESG increases in θ because, just as above, $\left(S+\theta^2\sigma_\phi^2\right)^2-S\sigma_f^2>0$.

For expected financial misreporting,

$$\frac{d}{d\theta}E\left[f^*-x^*\right]=\sigma_\phi^2\frac{2S\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2\right)\left(S-\theta^2\sigma_\phi^2\right)}{c_f\left(\sigma_f^2+\sigma_x^2\right)\left(S+\theta^2\sigma_\phi^2\right)^2}.$$

We have $\frac{d}{d\theta}E\left[f^*-x^*\right]>0$ when $2S\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2\right)\left(S-\theta^2\sigma_\phi^2\right)>0$.

For expected ESG misreporting,

$$\begin{aligned}\frac{d}{d\theta}E\left[r^*-y^*\right] &= \frac{1}{c_r}\frac{z\sigma_\phi^2}{\delta^2\sigma_r^2}\left(\sigma_f^2\frac{S-\theta^2\sigma_\phi^2}{\left(S+\theta^2\sigma_\phi^2\right)^2}-2\theta\lambda\eta\sigma_\phi^2\frac{\sigma_f^2+\sigma_x^2}{\left(S+\theta^2\sigma_\phi^2\right)^2}\right)\\ &\quad+\left(z-\eta\right)\frac{1}{c_e}\frac{1}{\left(S+\theta^2\sigma_\phi^2\right)^2}\left(\left(S+\theta^2\sigma_\phi^2\right)^2-S\sigma_f^2+\theta^2\sigma_f^2\sigma_\phi^2+2\theta\lambda\eta\sigma_\phi^2\left(\sigma_f^2+\sigma_x^2\right)\right)\end{aligned}$$

Thus, $\frac{d}{d\theta}E\left[r^*-y^*\right]$ is positive whenever $\sigma_f^2\left(S(\sigma_f^2+\sigma_x^2)-\theta^2\sigma_\varphi^2\right)-2\theta\lambda\eta\sigma_\varphi^2\left(\sigma_f^2+\sigma_x^2\right)>0$.

Sensitivities of equilibrium price responses to the reports are affected by changes in σ_x^2 as follows:

$$\begin{aligned}\frac{d\varphi_f^*}{d\sigma_x^2} &= \frac{d}{d\sigma_x^2}\left(\frac{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\sigma_x^2+\theta^2\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\theta\eta\frac{\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\\ &= \left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\frac{\sigma_f^2+\frac{z^2}{S_r}\sigma_f^2\sigma_\phi^2-\theta\lambda\eta\sigma_\phi^2}{\left(\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)^2}\text{ and}\\ \frac{d\varphi_r^*}{d\sigma_x^2} &= \frac{d}{d\sigma_x^2}\left(\frac{z\theta\sigma_\phi^2}{S_r}\frac{\sigma_f^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\frac{z\eta\sigma_\phi^2}{S_r}\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\\ &= -\frac{z}{S_r}\theta\sigma_\phi^2\frac{\sigma_f^2+\frac{z^2}{S_r}\sigma_f^2\sigma_\phi^2-\theta\lambda\eta\sigma_\phi^2}{\left(\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)^2}.\end{aligned}$$

Thus,

$$\begin{aligned}\frac{d\varphi_f^*}{d\sigma_x^2} &\propto \sigma_f^2\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)-\theta\lambda\eta\sigma_\phi^2\text{ and}\\ \frac{d\varphi_r^*}{d\sigma_x^2} &\propto -\left(\sigma_f^2\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)-\theta\lambda\eta\sigma_\phi^2\right),\end{aligned}$$

where "$\propto$" can be read as "is proportional to" and implies "has the same sign as."

Denote $Q=\frac{z^2\sigma_\phi^2}{\delta^2\sigma_r^2}>0$. Then we can express the remaining comparative statics with

respect to σ_x^2 as:

$$\begin{aligned}
\frac{\partial}{\partial\sigma_x^2}E\left[x^*\right] &= \frac{1}{c_e}\theta^2\frac{\left(1+Q\right)\sigma_f^2-\lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2},\\
\frac{\partial}{\partial\sigma_x^2}E\left[y^*\right] &= \frac{1}{c_e}\theta\eta\frac{\left(1+Q\right)\sigma_f^2-\theta\lambda\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2},\\
\frac{\partial}{\partial\sigma_x^2}E\left[f^*-x^*\right] &= \frac{1+Q}{c_f}\frac{\left(Q+1\right)\sigma_f^2-\lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2}, \text{ and}\\
\frac{\partial}{\partial\sigma_x^2}E\left[r^*-y^*\right] &= -\theta\frac{Q}{c_r\,z}\frac{\left(1+S\right)\sigma_f^2-\theta\lambda\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2}\\
&\quad+\frac{z-\eta}{c_e}\theta\frac{\left(1+Q\right)\sigma_f^2-\theta\lambda\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+S\sigma_x^2\right)^2}.
\end{aligned}$$

For $\theta<0$, we have:

$$\begin{aligned}
\frac{\partial}{\partial\sigma_x^2}E\left[x^*\right] &= \frac{1}{c_e}\theta^2\frac{\left(1+Q\right)\sigma_f^2-\lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2}>0,\\
\frac{\partial}{\partial\sigma_x^2}E\left[y^*\right] &= \frac{1}{c_e}\theta\eta\frac{\left(1+Q\right)\sigma_f^2-\theta\lambda\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2}<0, \text{ and}\\
\frac{\partial}{\partial\sigma_x^2}E\left[f^*-x^*\right] &= \frac{1+Q}{c_f}\frac{\left(Q+1\right)\sigma_f^2-\lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2}>0.
\end{aligned}$$

For $\theta>0$ and $\left(1+Q\right)\sigma_f^2-\lambda\theta\eta\sigma_\phi^2>0$:

$$\begin{aligned}
\frac{\partial}{\partial\sigma_x^2}E\left[x\right] &= \frac{1}{c_e}\theta^2\frac{\left(1+Q\right)\sigma_f^2-\lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+S\sigma_f^2+Q\sigma_x^2\right)^2}>0,\\
\frac{\partial}{\partial\sigma_x^2}E\left[y\right] &= \frac{1}{c_e}\theta\eta\frac{\left(1+Q\right)\sigma_f^2-\theta\lambda\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2}>0, \text{ and}\\
\frac{\partial}{\partial\sigma_x^2}E\left[f^*-x^*\right] &= \frac{1+Q}{c_f}\frac{\left(Q+1\right)\sigma_f^2-\lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2}>0.
\end{aligned}$$

For $\theta > 0$ and $(1+Q)\,\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2 < 0$:

$$\begin{aligned}
\frac{\partial}{\partial\sigma_x^2}E\left[x\right] &= \frac{1}{c_e}\theta^2\frac{(1+Q)\,\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2} < 0,\\
\frac{\partial}{\partial\sigma_x^2}E\left[y\right] &= \frac{1}{c_e}\theta\eta\frac{(1+Q)\,\sigma_f^2 - \theta\lambda\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2} < 0, \text{ and}\\
\frac{\partial}{\partial\sigma_x^2}E\left[f^*-x^*\right] &= \frac{1+Q}{c_f}\frac{(Q+1)\,\sigma_f^2 - \lambda\theta\eta\sigma_\phi^2}{\left(\sigma_x^2+\sigma_f^2+\theta^2\sigma_\phi^2+Q\sigma_f^2+Q\sigma_x^2\right)^2} < 0.
\end{aligned}$$

$\delta^2\sigma_r^2$ always appears as a product in the expressions together. That is why instead of taking derivatives with respect to δ^2 and σ_r^2 separately, we take the derivatives with respect to $S_r = \delta^2\sigma_r^2$. Since both δ^2 and σ_r^2 are positive, the chain rule implies that derivatives with respect to δ^2 and σ_r^2 individually will be proportional to the derivatives with respect to S_r.

$$\begin{aligned}
\frac{d\varphi_f^*}{dS_r} &= \frac{d}{dS_r}\left(\frac{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\sigma_x^2+\theta^2\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\theta\eta\frac{\sigma_\phi^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\\
&= \frac{\frac{\sigma_\phi^2}{S_r}}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}z\theta\varphi_r^*.\\
\frac{d\varphi_r^*}{dS_r} &= \frac{d}{dS_r}\left(\frac{z\theta\sigma_\phi^2}{S_r}\frac{\sigma_f^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}+\lambda\frac{z\eta\sigma_\phi^2}{S_r}\frac{\sigma_f^2+\sigma_x^2}{\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\right)\\
&= -\frac{\sigma_f^2+\sigma_x^2+\theta^2\sigma_\phi^2}{S_r\left(1+\frac{z^2}{S_r}\sigma_\phi^2\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2}\varphi_r^*.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
\frac{d}{dS_r}E\left[x\right] &= -\theta\frac{1}{c_e}\frac{z^2\sigma_\phi^2}{S_r^2}\left(\sigma_f^2+\sigma_x^2\right)\frac{\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2\right)}{\left(\left(1+\frac{z^2\sigma_\phi^2}{S_r}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)^2} \propto -\theta\varphi_r^*,\\
\frac{d}{dS_r}E\left[y\right] &= -\eta\frac{z^2\sigma_\phi^2}{S_r^2}\left(\sigma_f^2+\sigma_x^2\right)\frac{\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2\right)}{c_e\left(\left(1+\frac{z^2\sigma_\phi^2}{S_r}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)^2} \propto -\varphi_r^*, \text{ and}\\
\frac{d}{dS_r}E\left[f^*-x^*\right] &= \theta\frac{z^2\sigma_\phi^4}{S_r^2}\frac{\theta\sigma_f^2+\lambda\eta\left(\sigma_f^2+\sigma_x^2\right)}{c_f\left(\left(1+\frac{z^2\sigma_\phi^2}{S_r}\right)\left(\sigma_f^2+\sigma_x^2\right)+\theta^2\sigma_\phi^2\right)^2} \propto \theta\varphi_r^*
\end{aligned}$$