Financial Information and Diverging Beliefs

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Abstract: Standard Bayesians' beliefs converge when they receive the same piece of new information. However, when agents initially disagree and have uncertainty about the precision of a signal, their disagreement might instead increase despite receiving the same information. We demonstrate that this divergence of beliefs leads to a unimodal effect of the absolute surprise in the signal on trading volume. We show that this prediction is consistent with the empirical evidence using trading volume around earnings announcements of US firms. We find evidence of elevated volume following moderate surprises and depressed volume following more extreme surprises, a pattern that is more pronounced when investors are more uncertain about earnings' precision. The evidence suggests that investors in the U.S. financial market in general disagree about stocks' expected returns and do not know the quality of accounting earnings reports.

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We, the authors, Christopher S. Armstrong, Mirko S. Heinle, and Irina M. Luneva, have nothing to disclose. We have had no paid or unpaid positions as officer, director, or board member of relevant non-profit organizations or profit-making entities. No partner nor close relative has had a paid or unpaid position as officer, director, or board member of relevant non-profit organizations or profit-making entities. No third party had the right to review any part of this paper, prior to its circulation. This study does not require IRB approval.

1 Introduction

Information plays a key role in financial markets. Conventional wisdom in the finance and accounting literature suggests that when investors receive a common signal, their beliefs converge. For example, for a signal in between two investors' priors, an optimistic investor revises her belief downward while a pessimistic investor updates upward. Despite this perception, there is ample empirical and theoretical evidence (e.g., Gentzkow and Shapiro (2006), Filipowicz et al. (2018), Fryer et al. (2019), Kartik et al. (2021)) that individuals often reject information that does not conform to their priors, believing that the news is not informative. In this case, new information can actually cause their beliefs to diverge more.[1] We theoretically and empirically investigate the effect of information on the beliefs divergence. First we develop a theoretical model of investors in a firm and show that when investors disagree about the firm's value and are uncertain about the precision of a signal about the firm's value, investors' beliefs can diverge more after they observe the signal. Because beliefs cannot be observed in the data directly, we derive predictions about an observable variable – trading volume. We choose it as our research setting because trading volume is "a product of the extent to which investors hold diverse opinions ... and the extent to which these opinions change on average" at the time when a new piece of information arrives (Verrecchia (2001)), in contrast to the price change, which is a function only of the average evolution of investors' beliefs. Finally, we empirically test the predictions from our model using trading volume around quarterly earnings disclosures of publicly traded U.S. firms.

In our model, investors have different expectations about the firm's cash flow, are uncertain about the precision of a signal about the firm, and trade both before and after the signal is disclosed. We first demonstrate why difference-of-beliefs and signal-precision uncertainty are the two key features that explain beliefs divergence after investors receive the same signal. When investors only disagree about the value of the firm but know the precision of the signal about the firm, their beliefs strictly converge following the disclosure. When investors initially agree but do not know the signal's precision, no signal can create a beliefs divergence that did not exist before the signal is realized. However, with signal-precision uncertainty *and* difference-of-beliefs together, investors' beliefs can be driven further away by certain realizations of the disclosed signal.

[1] See also Banerjee and Kremer (2010) and Banerjee et al. (2021) who argue that investors disagree about news.

Investors' beliefs can diverge after investors receive the same signal for the following reason. An investor that is uncertain about the signal's precision rationally assigns a higher precision to signals that are closer to her prior, or believes that information that is consistent with her view of the world is more credible, and thus updates her belief stronger based on that information. Because different investors have different priors, they assign different levels of precision to different signals. As a result, investors' (posterior) beliefs diverge more following some realizations of the signal. This feature makes the model realistic: in everyday life, we often observe how people, even if they are professionals, may disagree more after they receive the same piece of information.[2]

Because investors' beliefs in financial markets are unobservable, to test our model, we derive predictions about an observable statistic – trading volume. We demonstrate that the divergence of beliefs after a signal is released results in trading volume that is an M-shaped function of the signal itself and unimodal in the absolute value of its information content (or "surprise"). In other words, trading volume is, at first, increasing in a signal that produces a moderate surprise, but then decreases as the signal generates a more extreme surprise. This M-shape pattern is more pronounced when investors are more uncertain about the signal's precision.

We next assess the validity of our model by developing several empirical tests of its implications based on firms' quarterly earnings disclosures and systematic patterns in the trading volume surrounding the disclosures. We formulate and test two predictions: (1) trading volume increases for intermediate levels of earnings surprise and dampens for extreme levels, and (2) the M-shape of trading volume is more pronounced under high signal-precision uncertainty. To test our first prediction, we analyze non-parametric and parametric shapes of the trading volume. We show that for the sample of U.S. firms, trading volume's reaction is indeed weakened for extreme levels of an earnings surprise. If we impose a polynomial shape on trading volume, we find that trading volume as a function of the earnings surprise is most closely fitted by fourth-order polynomial regression. The estimated parameters exhibit a clear M-shape. We also estimate quantile regressions and show that trading volume increases for moderate absolute surprise levels, but does not increase for the upper 5% of surprises.

[2] Examples include polarization of justices when new evidence is presented in court ("Supreme Court Justices Differ on Boston Bomber's Death Sentence", Wall Street Journal), historians' different descriptions of a fact depending on historians' reference points ("Meghan at 40 examines two sides of a royal renegade on Channel 5", Financial Times), and government officials coming to contradicting conclusions after a meeting ("Analysts weigh in on Fed's contradictory inflation remarks", Financial Times).

To test our second prediction, we develop a new measure of earnings-announcement-precision uncertainty that allows us to separate precision uncertainty from the overall market uncertainty. Namely, we measure earnings-announcement precision uncertainty as residuals from the regression of analyst forecast spread on the VIX index. Conditional on the VIX index picking up fundamental uncertainty, the residual component of analyst forecast spread is an estimate of the market's uncertainty about how precise is the reported earnings number. We verify our measure by showing that an S-shaped earnings response coefficient (ERC)[3] is more pronounced for observations with high earnings-precision uncertainty. Trading volume gradually gets more inverse U-shaped in the absolute earnings surprise (or more M-shaped in the earnings surprise). This evidence is supported by both non-parametric and parametric tests.

We rely on and aim to extend multiple streams of literature. First, we contribute to the broad literature in Economics and Psychology that demonstrates the evidence and studies the implications of individuals putting more trust in information that confirms their prior beliefs (Gentzkow and Shapiro (2006), Filipowicz et al. (2018), Fryer et al. (2019), Kartik et al. (2021), Martel and Schneemeier (2021)). We demonstrate that such behavior is present in financial markets and leads to unusual patterns in trading volume, which are more pronounced as individuals become more uncertain about the precision of the information they receive. We also propose a way to measure the degree of this signal-precision uncertainty.

Researchers have proposed several mechanisms to explain why investors' views diverge after they receive the same information (e.g., Atmas and Basak (2018), Banerjee and Kremer (2010), Bordalo et al. (2021)). Atmas and Basak (2018) is a dynamic model with a continuum of investors that have different beliefs. Similarly to our paper, the authors obtain the M-shaped trading volume as a function of information surprise when there are two types of investors. In their paper the M-shape arises as a result of assumptions about the distribution of types and evolution of news in the stock market. In our paper, investors' different perceived precision arises endogenously because investors perceive signals that are closer to their priors as more precise. Our second set of tests provides evidence that precision uncertainty plays a role in the M-shaped trading volume. Banerjee and Kremer (2010) develop a difference-of-beliefs model where investors exogenously disagree about

[3] See, for example, Freeman and Tse (1992), Cheng et al. (1992), and Das and Lev (1994). Subramanyam (1996) shows in a theoretical model that because of signal variance uncertainty, ERC dampens for extreme levels of an earnings surprise.

their interpretations of a public signal. This gives rise to "belief-convergence" trade, when investors' beliefs converge after a prior disagreement, and "idiosyncratic" trade, when investors disagree on the interpretation of new information. Banerjee et al. (2021) demonstrate how empirically descriptive deviations from the rational expectations framework arise endogenously when investors use "wishful thinking" – choose subjective beliefs to make themselves happier about the future. In Bordalo et al. (2021), investors overreact to positive news if they observed a stock growth in the past. Our paper differs from Bordalo et al. (2021) and Banerjee and Kremer (2010) in that in our model the differential reaction to signals arises endogenously, as opposed to being exogenously imposed. In our model, investors only underreact (namely when they observe very surprising signals) and never overreact. The greatest divergence of beliefs occurs when one type of investor underreacts and the other type reacts as if there were no signal-precision uncertainty. Another study that is closer to ours is Jia et al. (2017), which provides evidence that market segmentation may increase after an analyst recommendation because of the social connection between analysts and local investors: local investors react stronger to local analysts' recommendations than foreign investors. If local investors' and local analysts' priors are closer or if local investors believe that local analysts are more precise than foreign analysts, the mechanism that we propose in our paper would also explain their evidence of increased market segmentation after analyst recommendations. Our main contribution to this literature is showing how precision uncertainty interacts with the divergence of beliefs and to provide empirical evidence that US investors are uncertain abou thte precision of disclosed earnings.

Our paper extends the theoretical literature on the trading volume effects of public signals (e.g., Karpoff (1986), Kim and Verrecchia (1991), Kondor (2012), Banerjee (2011)). Early work in this area suggests that trading volume is monotonically increasing in a signal's surprise, which is attributable to either differential information or information precision among traders before the signal's release (Kim and Verrecchia (1991)). Subsequent studies introduced differences-in-beliefs models and they typically provide inferences that are generally similar to those of their predecessors (Bamber et al. (2011)). To our knowledge, our study is the first to incorporate uncertainty about public signals' precision and show that it can result in a non-monotonic pattern between the magnitude of a signal's surprise and subsequent trade volume.

Finally, our study contributes to the parallel empirical literature that studies trading volume. Bamber (1987) finds that trading volume is increasing in the magnitude of an earnings surprise, and Choi (2019) shows that this relationship is amplified when markets are more volatile. We build on the latter, which studies uncertainty about the value of the signal (i.e., the first moment) by instead focusing on uncertainty about its precision (i.e., the second moment). Our paper is also a logical extension of antecedent work by Bamber et al. (1997), Irvine and Giannini (2012), Al-Nasseri and Menla Ali (2018), and Booker et al. (2018), who show that abnormal trading volume exhibits a positive relationship with changes in beliefs. Another related study by Giannini et al. (2019) suggests that both convergence and divergence of beliefs lead to increased trading volume around earnings announcements. Our paper differs from these by showing – both theoretically and empirically – a previously unknown effect of precision uncertainty: namely that trading volume exhibits different behavior across environments with low and high signal-precision uncertainty.

The rest of our paper is as follows. Section 2 develops a new model that incorporates both difference-of-beliefs and signal-precision-uncertainty and shows that the introduction of the latter can cause investors' beliefs to become more divergent after they receive the same signal. We then characterize the resulting equilibrium and derive empirical predictions for trading volume patterns. Section 3 describes our research design and develops multiple empirical tests of the two main predictions implied by our model. Section 4 summarizes our theoretical predictions, empirical evidence, and collective inferences from the synthesis of the two.

2 The Model

In this section, we first introduce the assumptions and timeline of the model that features investors with different beliefs and uncertainty about the precision of the signal that the investors receive. Next, we present two benchmark models: with only difference-of-beliefs and with only signal-precision uncertainty. These benchmarks help us understand how the two main forces of the model work and why both of them are needed to explain the divergence of investors' beliefs after the investors receive the signal. Finally, we discuss the full model and the empirical implications that it yields.

2.1 Model Setup

A continuum of investors competes for shares in two assets. The first asset is riskfree, has infinite supply and a rate of return that is normalized to zero without loss of generality. The second asset — the shares of a firm — is risky and yields a random return of $\tilde{x}$. The entire firm is sold to new investors in the first trading period; supply of the risky asset at date $t = 1$ is 1. We assume that there are two types of investors and denote the type of investor by $i \in \{1, 2\}$. A fraction λ_1 of investors are of type 1 and a fraction $\lambda_2 = 1 - \lambda_1$ are of type 2. We assume that investors are risk-averse and have mean-variance utility over their terminal wealth:

$$U_i = E[W_i] - \frac{1}{2} r_i Var[W_i], \tag{2.1}$$

where W_i and r_i are investor i's terminal wealth and coefficient of risk aversion, respectively.[4] Investors are initially endowed with wealth $W_{i0} = 0$.

There are three time periods:

$t = 1$. **Pre-announcement Period.** Investors trade in anticipation of the disclosure at $t = 2$.

$t = 2$. **Post-announcement Period.** The signal, $\tilde{y} = \tilde{x} + \tilde{u}$, is disclosed and investors trade for the second time.

[4] Note that mean-variance utility implies that investors do not price any moment higher than the second. Investors with, for example, negative exponential utility would price skew and kurtosis that arise from updating over information with uncertain precision, e.g., see Heinle and Smith (2017) or Heinle et al. (2018).

$t = 3$. **Realization Period.** The risky asset's return, x, is realized and investors consume their terminal wealth, which is given by:

$$W_{i3} = d_{i2}x + q_{i2}, \tag{2.2}$$

where d_{i2} and q_{i2} are the amounts of risky and riskless assets held in $t = 2$, respectively.

We assume that investors have heterogeneous prior beliefs about the risky asset's expected return:

$$\tilde{x} \sim N\left(m_i, \frac{1}{\nu}\right),$$

where m_i is investor i's expected return of the risky asset and $\frac{1}{\nu}$ is the variance of the return. Furthermore, we assume that $\tilde{u}$, the noise term of the signal $\tilde{y} = \tilde{x} + \tilde{u}$, is independent of $\tilde{x}$ and normally distributed with zero mean and unknown precision. This implies that investors have heterogeneous prior beliefs about the signal:

$$\tilde{y} \sim N\left(m_i, \frac{1}{\tilde{w}}\right),$$

where $\tilde{w}$ is a random variable. Following Subramanyam (1996), we assume that the true signal-precision, w, follows a truncated gamma distribution with support $[0, \nu]$ and parameters α and β[5].

2.2 Benchmark Cases: Difference-of-Beliefs or Signal-Precision Uncertainty

In this section, we first present the case where investors only disagree about the risky asset's return, $\tilde{x}$, but know the precision of the signal, w. We will see that in this setup, after signal y is revealed, all investors update their beliefs about the risky asset's return and trade. However, different investors' beliefs always converge following the disclosure of the signal.

Proposition 1. *When investors have different prior beliefs about the risky asset's return and know the precision of the signal about the asset's return, the investors' beliefs will always weakly converge after the disclosure of the signal:* $|E_1[x] - E_2[x]| \geq |E_1[x|y] - E_2[x|y]|$

[5]The probability distribution of $\tilde{w}_i$ is given by: $f(\tilde{w}_i) = \frac{\beta^\alpha w_i^{\alpha-1} exp(-\beta w_i)}{\Gamma(\alpha)}$ where $\Gamma(\alpha) = \int_0^\nu \beta^\alpha w_i^{\alpha-1} exp(-\beta w_i)$.

The proposition above suggests that simple disagreement about first moments is not enough to explain the real-world pattern of beliefs divergence even when economic agents receive the same piece of information. When agents disagree about the expected asset return and agree on the precision of the signal about the assets that they receive, the agents' beliefs always converge following the release of the signal.

Next, we analyse the case where investors agree on the first moment of the distribution of the asset's return, but are uncertain about the precision of the signal about the asset's return that they receive. In this benchmark, all investors will always hold the same beliefs, implying no disagreement and no trading volume neither before nor after the disclosure of the signal y, consistent with the traditional framework with homogeneously informed investors (e.g., Kim and Verrecchia (1991)).

Proposition 2. *When investors have the same prior beliefs about the risky asset's return and are uncertain about the precision of the signal about the asset's return, all investors will hold the same beliefs before and after the disclosure of the signal.*

The two benchmarks considered in this section imply that neither different beliefs nor uncertainty about signal-precision alone can generate the phenomenon that we seek to describe: divergence of economic agents' beliefs after they receive the same signal. When the agents disagree but receive the signal with known precision, they move their beliefs closer to each other. When the agents do not disagree in the first place, uncertainty about the second moment of the signal can not make the agents disagree about the first moment. In the next section, we analyze a model where difference-of-beliefs is combined with signal-precision uncertainty.

2.3 Full Model: Convergence and Divergence of Beliefs

This section describes how investors' beliefs evolve in the model where the investors (1) disagree about the expected return of the risky asset, x, and (2) are uncertain how precise is the signal about this asset's return.

Proposition 3. *At $t = 2$, after the investors observe the realization of the signal, y, their updated* *beliefs about the mean and the variance of the risky asset's return are:*

$$E[\tilde{x}|y] = m_i + \hat{w}_i(y - m_i)\nu^{-1} \tag{2.3}$$

$$Var[\tilde{x}|y] = \nu^{-1}\left(1 - \hat{w}_i\nu^{-1}\right) \tag{2.4}$$

where $\hat{w}_i = E[\tilde{w}_i|y]$ *is investor* i*'s estimated precision of the signal, conditional on observing* y*, given by:*

$$\hat{w}_i = \frac{\Gamma(\alpha + 1.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)[\frac{(y-m_i)^2}{2} + \beta]^{-1}}{\Gamma(\alpha + 0.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)} \tag{2.5}$$

Investors' beliefs can diverge after they receive the signal y*, or* $|m_1 - m_2|$ *can be less than* $|m_1 + \hat{w}_1(y - m_1)\nu^{-1}|$*.*

Proposition 3 shows that the combination of different prior beliefs and precision uncertainty is sufficient to yield cases when investors disagree about the risky asset's return more after they receive the same piece of new information. Figure 1a plots investors' *ex ante* beliefs about the mean and variance of the firm's cash flows, $E_i[\tilde{x}]$ and $Var_i[\tilde{x}]$, Figure 1b plots the *ex-post* counterparts of these two moments, conditional on the realization of the signal y, $E_i[\tilde{x}|y]$ and $Var_i[\tilde{x}|y]$, and Figure 1c plots the difference between investors' *ex-ante* and *ex-post* beliefs to illustrate the magnitude and direction in which the signal alters their beliefs. Before the signal is realized (Figure 1a), investors disagree: the first type of investor (call her investor H) expects firm cash flow to be 6, the second type of investor (call her investor L) expects it to be 0. After observing the same piece of information — or public signal — investors' beliefs can become more divergent. The reason is that investors infer that the signal has a low precision when the signal does not correspond to their priors. Consequently, an optimistic investor (type H) will exhibit a stronger response to an optimistic signal than will a pessimistic investor (type L), who will exhibit a more muted response. Figure 1b demonstrates this phenomenon. In this case, for $y = 3$, investors agree more about the firm's expected cash flows after observing the signal. The solid line in Figure 1c is below the dashed line. Here, both investors come to the same estimate of the signal-precision and their beliefs converge towards 3. However, for $y = 10$, the signal increases investors' disagreement: H now expects the firm's cash flows to be around 9, whereas L expects cash flows of roughly 2. The solid line in Figure 1c is above the dashed line. In other words, while both investors update their beliefs towards the realized signal realization, the optimistic investor does so more strongly than does the pessimistic investor, which results in greater divergence.

Investors' beliefs about the variances also diverge for some values of y. Expected variances are equal before the signal (Figure 1a), but after the signal, they may be different (Figure 1b). After the signal is realized, investors revise their expectations in the same direction. However, an investor whose *ex-ante* expectation of cash flow is closer to the signal, moves her beliefs more than another investor. In our example, $y = 9$ is closer to investor H's prior expectation. As a result, the *ex-post* expectations are driven further away.

The model is descriptive of common real-world scenarios. When researchers get a result of a drug test, they may further debate the drug's effectiveness. Specifically, a researcher who does not expect that a drug should work and is uncertain about the conditions of the experiment may dismiss the results altogether. Similarly, political opinions in the news are often dismissed by people with opposing views. Finally, even more generic news in the media can easily be dismissed by appealing to the credibility of the source. In all these cases people disagree even more because they (1) disagree in the first place and (2) are not sure how precise the signal is (Jaynes (2005)). The same logic holds for investors: they have different prior beliefs and do not know exactly how precise the signal by a firm is.

2.4 Full Model: Equilibrium Prices and Trading Volume

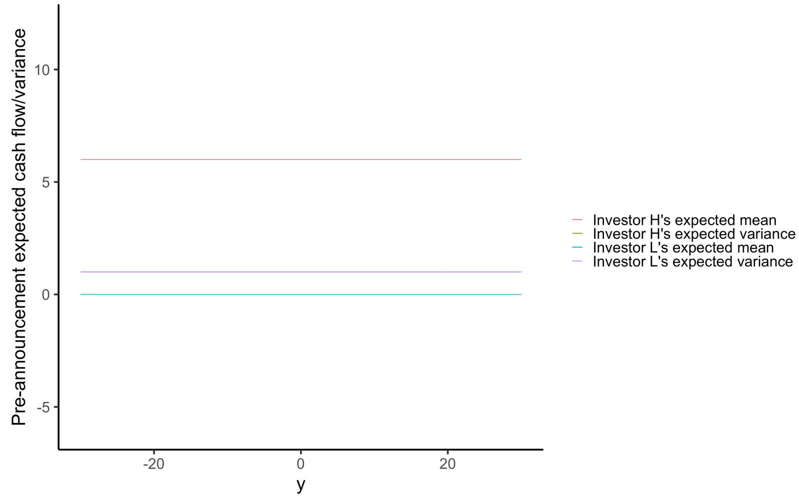
Because market participants' beliefs are not directly observable to researchers, to test the validity of our theory, we need to develop a number of predictions about observed variables. In this section, we solve for equilibrium prices and trading volume in the model.

We solve the model by backward induction. First, we derive investors' demands and the market-clearing price at $t = 2$. Next, anticipating their choices in $t = 2$, we solve for investors' demands and the price at $t = 1$. We measure trading volume as the absolute difference between demands at $t = 1$ and $t = 2$.

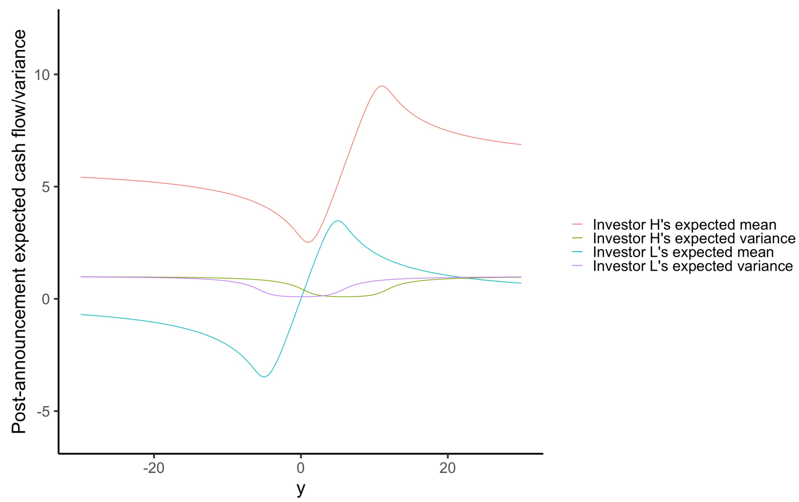
Let P_t denote the price of the risky asset at time t.

Proposition 4. *At $t = 2$, after the signal y is disclosed, the equilibrium price is set at*

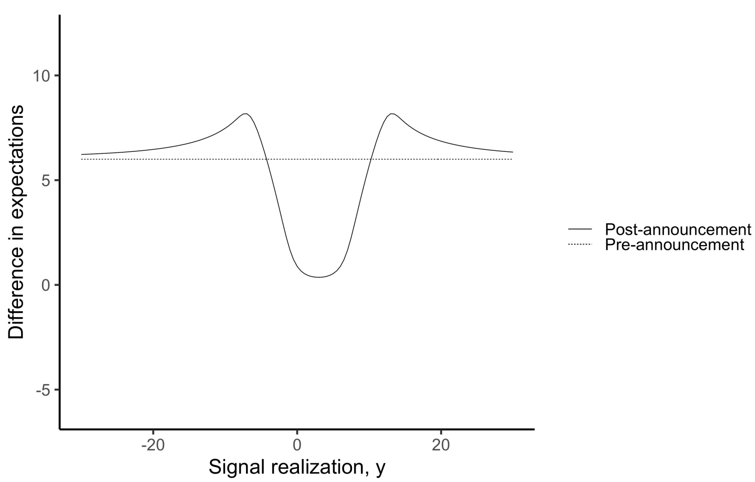
$$P_2^* = [\psi_1(\hat{w}_1) + \psi_2(\hat{w}_2)]^{-1} [E_1[\tilde{x}|y]\psi_1(\hat{w}_1) + E_2[\tilde{x}|y]\psi_2(\hat{w}_2) - 1], \quad (2.6)$$



(a) Investors' expectations of firm cash flow and its variance before y is realized as a function of y.



(b) Investors' expectations of firm cash flow and its variance after y is realized as a function of y.



(c) Differences in investors' expectations of firm cash flow and its variance before and after y is realized as a function of y.

Figure 1: $\lambda_H = \lambda_L = 0.5$, $r_H = r_L = 4$, $\alpha = 10$, $m_H = 6$, $m_L = 0$, $\nu = 1$, $n = 1$.

where $\psi_i(\hat{w}_i) = \frac{\lambda_i \nu}{r_i(1-\frac{\hat{w}_i}{\nu})}$, $E_i[\tilde{x}|y] = m_i + \hat{w}_i(y - m_i)\nu^{-1}$.

Subscript i denotes investor i's expectation or variance, respectively. The function $\psi_i(\hat{w}_i)$ represents an investor i's confidence in the quality of the signal. It increases as an investor perceives the signal as more precise. Because investors are risk averse, a higher value of $\psi_i(\hat{w}_i)$ increases investor i's demand for the risky asset.

Proposition 5. *The equilibrium price at time* $t = 1$ *is:*

$$P_1^* = (\phi_1 + \phi_2)^{-1} \left(E_1[P_2^*]\phi_1 + E_2[P_2^*]\phi_2 - 1 \right), \tag{2.7}$$

where $\phi_i = \frac{\lambda_i}{r_i Var_i[P_2^*]}$

$\phi_i = \frac{\lambda_i}{r_i Var_i[P_2^*]}$ is similar to $\psi_i(\hat{w}_i)$. Because the signal is not yet realized, investors cannot estimate its precision. Instead, they rely on the expected next-period price variance, $Var_i[P_2^*]$.

Following Subramanyam (1996), we plot the return, $\frac{P_2^* - P_1^*}{P_1^*}$, as a function of the signal realization. Our model predicts an "S-shaped" form of returns. Figure 2 shows the return, defined as the difference between prices at times 2 and 1 scaled by price at time 1, as a function of the firm's signal, y, predicted by the model. The figure also shows that the S-shape of the return is less pronounced for higher values of the parameter β. That is, when the precision uncertainty decreases, the S shape in returns becomes less pronounced.

Our model contrasts the traditional framework with homogeneously informed investors (e.g., Kim and Verrecchia (1991)), where there is no trading volume following the disclosure of a signal unless investors disagree about the signal-precision in the first place. In our case, while investors initially have homogeneous beliefs about the signal-precision, they have heterogeneous beliefs following the disclosure. This divergence of beliefs, in turn, generates trade following the disclosure.

Investor i's trading volume is defined as the absolute difference between demands in pre-announcement and post-announcement periods:

$$V_i = |d_{i2}^* - d_{i1}^*| \tag{2.8}$$

In our model with two investor types $V_i = -V_j$ such that total trading volume is given by $2V_i$. Because of the updating over the uncertain precision, it is difficult to analyze V_i analytically.

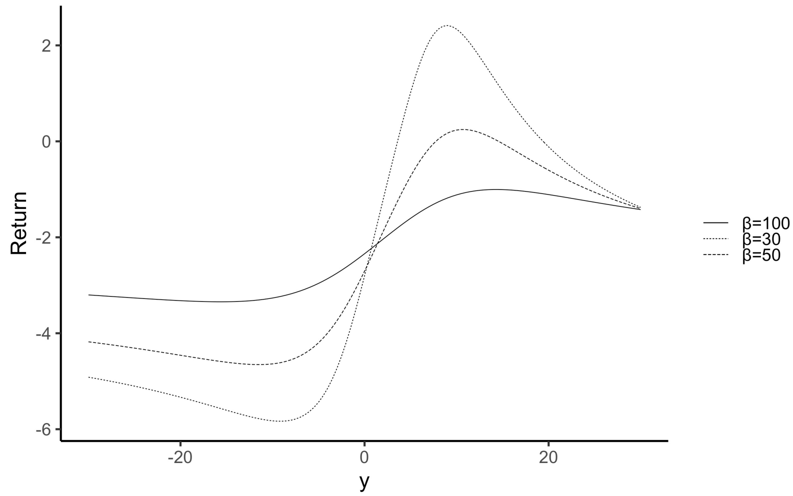
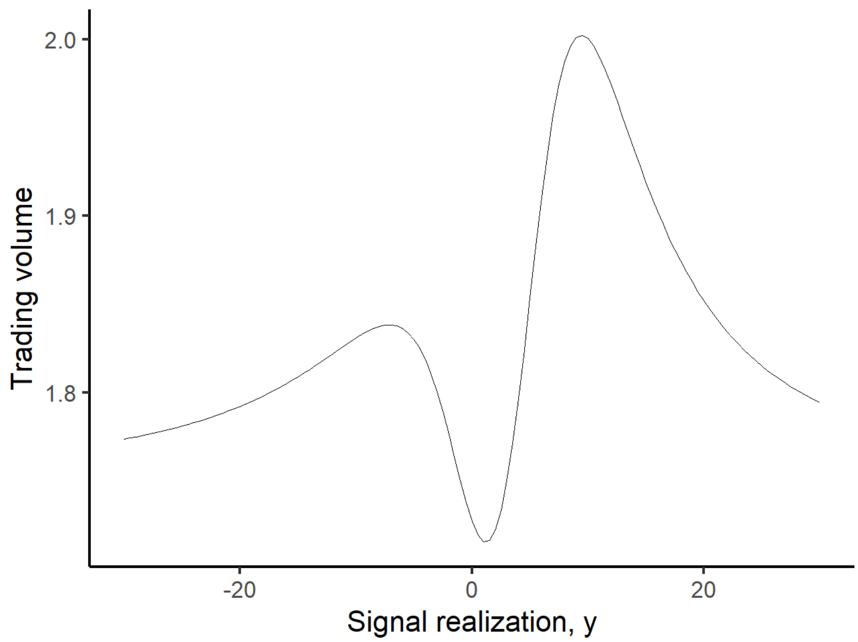


Figure 2: Return, $\frac{P_2^* - P_1^*}{P_1^*}$ as a function of y, for different levels of β.
$\lambda_1 = \lambda_2 = 0.5$, $r_1 = r_2 = 4$, $\alpha = 10$, $m_1 = 6$, $m_2 = 0$, $\nu = 1$, $n = 1$.

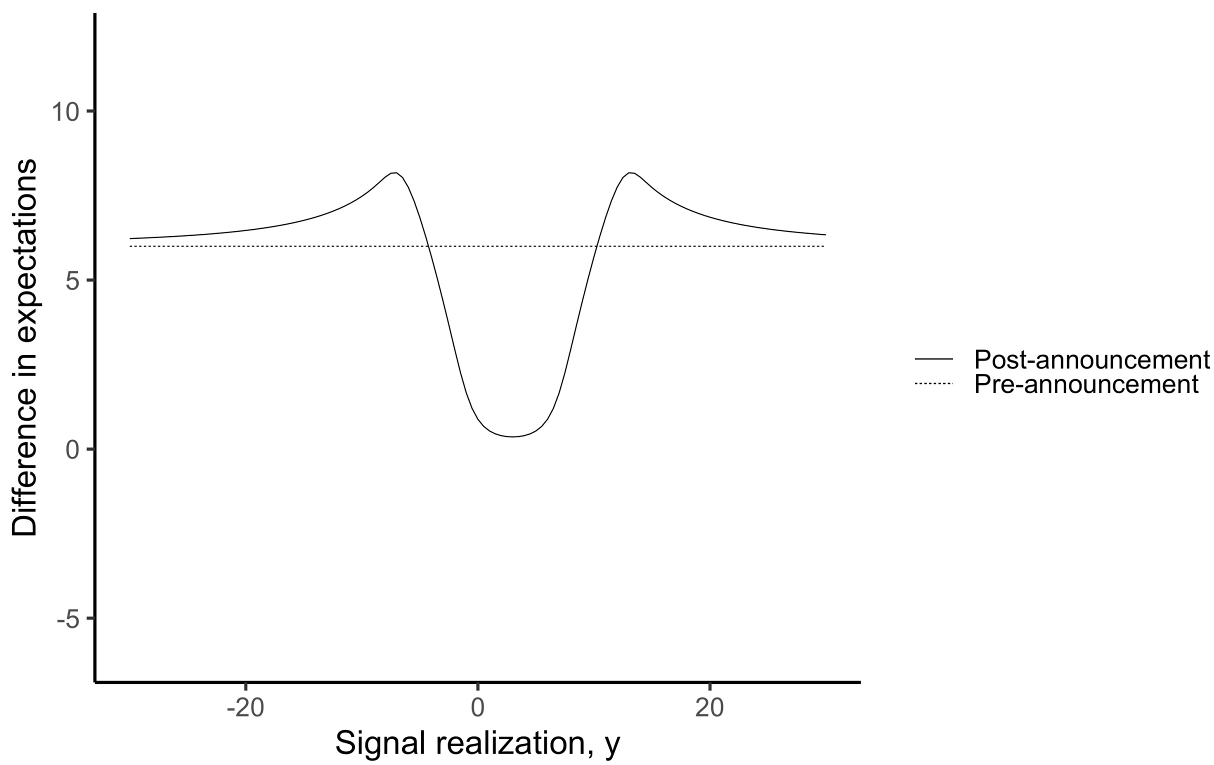
To develop some intuition, Figure 3a displays a graph of trading volume, V_i, as a function of the signal realization y. As the figure shows, trading volume is an M-shaped function of the signal. Specifically, trading volume first increases as the signal moves away from zero but as the signal gets extreme, trading volume starts to decrease.

To get intuition for the shape of the trading volume, we plot the difference in investors' beliefs before and after the signal realization in Figure 3b on the same scale. Post-announcement trading volume is the greatest when the divergence in investors' post-announcement beliefs increases. In contrast, trading volume is smallest when investors' beliefs converge after the disclosure. The forces that drive the M-shape in trading volume are the heterogeneous priors in combination with uncertainty about the signal-precision. As a result, even when investors receive the same public signal, disagreement about the firm's future cash flow may increase, which leads to more trade. Next, we analyze how the parameters of the distribution of signal-precision, α and β, affect the non-linearity of trading volume (see Figure 4). As α increases (β decreases), the non-linearity of trading volume is more pronounced and for sufficiently low levels of α (high levels of β), the humps disappear altogether. To understand this finding, note that the expectation and variance of signal-precision are proportional to α and inversely proportional to β[6]. On the one hand, as α increases (β

[6]The mean and variance of $\tilde{w}$ are $E[w] = \frac{\Gamma(\alpha+1,\beta\nu)}{\beta\Gamma(\alpha,\beta\nu)}$ and $Var[w] = \frac{\Gamma(\alpha+2,\beta\nu)\Gamma(\alpha,\beta\nu)-\Gamma(\alpha+1,\beta\nu)^2}{\beta^2\Gamma(\alpha,\beta\nu)}$. As $\nu \to \infty$, $E[w] \to \frac{\alpha}{\beta}$ and $Var[w] \to \frac{\alpha}{\beta^2}$.



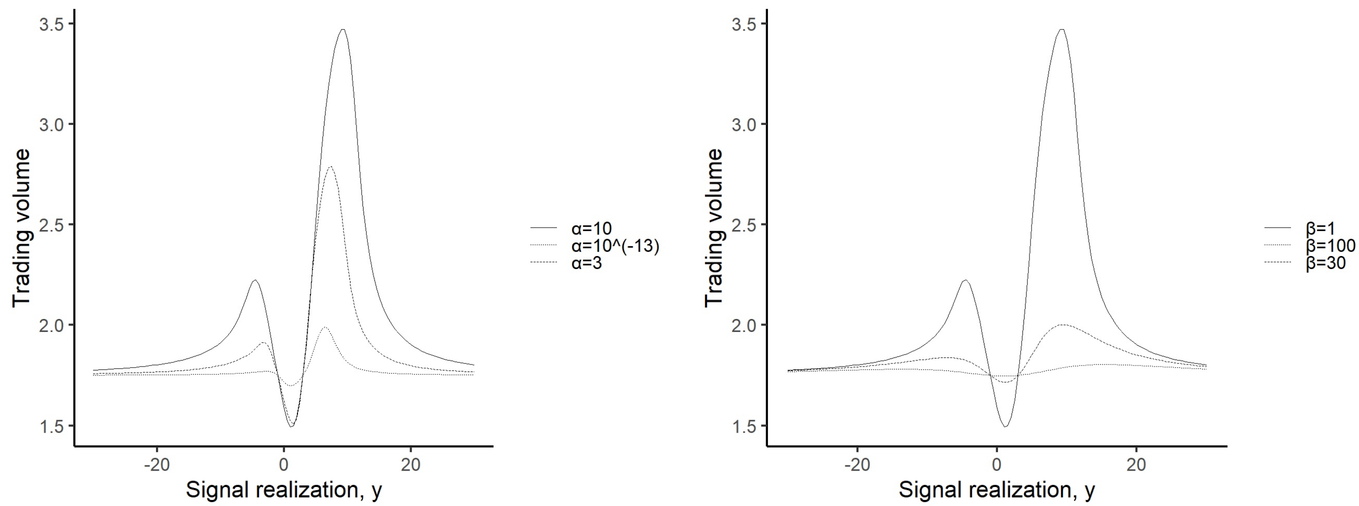
(a) A type-1 investor's trading volume as a function of the public signal, y.



(b) Difference in investors' expectations of the firm's cash flow before and after disclosure as functions of y. $\lambda_1 = \lambda_2 = 0.5$, $r_1 = r_2 = 4$, $\alpha = 10$, $m_1 = 6$, $m_2 = 0$, $\nu = 1$, $n = 1$.

Figure 3

decreases), the expected precision increases. If the signal is perceived as more precise on average, the market reacts to the signal stronger: the increase in trading volume is more pronounced. On the other hand, as α increases (β decreases), the market is also more uncertain about the precision. Similar to Subramanyam (1996), a higher signal-precision uncertainty causes investors to use the signal more to update their beliefs about the signal-precision. This implies that, for example, a positive signal is more easily dismissed by a pessimistic investor and, therefore, the humps in trading volume are more pronounced.



(a) $\beta = 1$. (b) $\alpha = 10$.

Figure 4: A type-1 investor's trading volume as a function of the public signal, y.
$\lambda_1 = \lambda_2 = 0.5$, $r_1 = r_2 = 4$, $m_1 = 6$, $m_2 = 0$, $\nu = 1$, $n = 1$.

3 Empirical Testing

3.1 Model Predictions

In this section, we empirically test the results of the model. While we cannot observe investors' beliefs, we can observe a model outcome: trading volume. We choose earnings announcements as the signal that investors receive and test two model predictions. The first prediction is about the general functional form of trading volume as a function of an earnings surprise:

Prediction 1. Trading volume is an M-shaped function of the earnings surprise.

The second prediction is linked to how trading volume's form changes with the parameters of the signal-precision uncertainty. As discussed in the previous section, when the variance of the signal precision increases, the M-shaped pattern for trading volume becomes more pronounced. Interpreting a high variance of the signal precision as a high uncertainty about this precision, we formulate the second empirical prediction as follows:

Prediction 2. The M-shape in trading volume is more pronounced when there is greater uncertainty about the precision of an earnings announcement.

3.2 Data and Measurement

To test our theoretical predictions, we gather data on quarterly earnings announcements of US firms from the first quarter of 1990 to the fourth quarter of 2019. We obtain data on released earnings, analyst EPS forecasts, and analyst price targets from the IBES database; prices and trading volume from the CRSP database; and firm characteristics from the Compustat database. Because prior research has shown that negative earnings have negligible information content (Hayn (1995), Lipe et al. (1998)), we delete observations with negative actual EPS.[7] We keep only firms with prices at the end of a previous fiscal quarter greater than \$5 to minimize the effect of market frictions (Ball et al. (1995)).

Following Landsman and Maydew (2002), Truong (2012), and Choi (2019), we measure abnormal trading volume in [0,1] days event window around the earnings announcement as:

$$AVOL_{i,q} = \sum_{t=0}^{t=1} \frac{VOL_{i,q,t} - mVOL_{i,q}}{\sigma(VOL)_{i,q}}, \tag{3.1}$$

where $VOL_{i,q,t}$ is the trading volume, $mVOL_{i,q}$ and $\sigma(VOL)_{i,q}$ are the mean and the standard deviation of the daily trading volume in $[-240, -5]$ days before the earnings announcement (Truong (2012), Choi (2019)); i, q and t denote the firm, the quarter and the day after the earnings announcement, respectively.

We measure the earnings surprise similar to Conrad et al. (2002) and Choi (2019) in the following way:

$$Surp_{i,q} = \frac{\text{Actual EPS}_{i,q} - \text{Med. forecast}_{i,q}}{PRC_{i,q-1}}, \tag{3.2}$$

[7] In untabulated analyses, we confirm that keeping firms with negative earnings does not change our empirical findings.

where Actual $\text{EPS}_{i,q}$ is the actual value announced by the firm, Med. $\text{forecast}_{i,q}$ is the median analyst forecast of firm's EPS, $PRC_{i,q-1}$ is the firm's price at the end of a previous fiscal quarter. We use only the most recent forecasts by each analyst to calculate the median.

We include firm size and market-to-book ratio to control for differences in risk that are not already captured by the excess return (Fama and French (1992, 1993)). We measure firm size as the natural logarithm of the market value of equity and the market-to-book ratio as the market value of equity divided by the book value of equity. Because our model predicts the M-shaped trading volume for a given level of ex-ante disagreement, we also include analyst forecast dispersion before the announcement as a control variable. We control for market-wide liquidity levels by including Pástor and Stambaugh (2003) liquidity factor. We include an indicator for a firm having a Big-4 auditor to control for the mean signal-precision uncertainty: we assume that ex-ante, firms with a Big-4 auditor firm are perceived by investors as having more precise earnings numbers. Finally, we include year fixed effects to control for potential correlation of earnings surprises with business cycles. To minimize the effect of outliers, we truncate the earnings surprise variable at the 5% level and all the other variables at the 1% level. As a result, we have a dataset of 87,944 firm-years from the first quarter of 1990 to the fourth quarter of 2019. The total number of firms in the sample is 4739. Table 1 describes how the sample size changed at each stage.

We present summary statistics in Table 2. Mean abnormal trading volume equals 0.75 with the standard deviation 1.15. Earnings surprises are 0.001 on average and vary from -0.01 to 0.01.

[Insert Tables 1 and 2 around here]

3.3 Functional Form of Trading Volume

The model predicts that trading volume is an M-shaped function of the earnings surprise. Trading volume increases for medium levels of surprises and decreases for extreme levels. We test this result in three ways. First, we look at scatterplots of trading volume as a function of earnings surprise with fitted non-parametric curves. Second, we estimate polynomial regressions of abnormal trading volume on earnings surprise and use an analysis-of-variance statistical test to choose the model that fits the data in the best possible way. Third, we estimate quantile regressions for different quantiles of an absolute earnings surprise.

3.3.1 Non-parametric Analysis

We begin our analysis of the functional form of trading volume by looking at scatterplots of trading volume as a function of earnings surprise with locally estimated scatterplot smoothing (LOESS) curves. LOESS is a local regression method that combines elements of simple linear least squares regression with elements of nonlinear regression. The method builds simple models for localized subsets of the data and then combines them into a function describing full data support. The advantage of this approach is that it does not require a researcher to pre-specify any functional form.

To control for other important factors that might affect trading volume, we first run a regression of abnormal trading volume on the set of control variables:

$$\begin{aligned} AVOL_{i,q} = a_0 + a_1 \times Size_{i,q-1} + a_2 \times Market/Book_{i,q-1} + a_3 \times Dispersion_{i,m-1} \\ + a_4 \times PSLiquidity_{i,q-1} + a_5 \times Big4 + a_6 \times Year\ FE, \end{aligned} \tag{3.3}$$

Next, we analyze scatterplots of the residuals of the regression above, $(AVOL_{i,q} - A\hat{V}OL_{i,q})$. This procedure allows us to concentrate on the part of trading volume that is orthogonal to other factors.

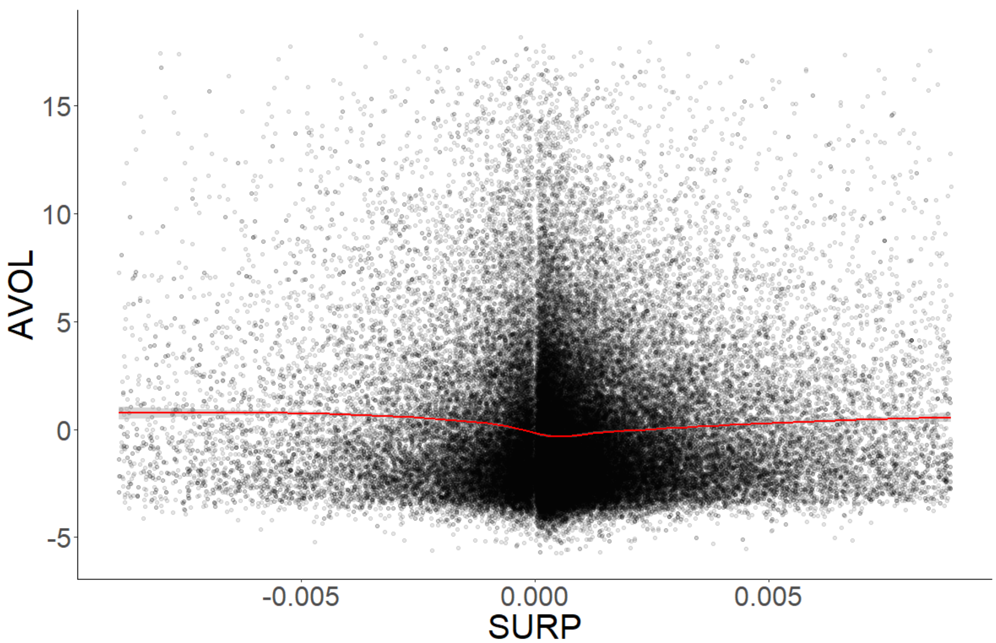


Figure 5: Scatterplot of residuals of abnormal trading volume with LOESS smoother

Figure 5 demonstrates the scatterplot of the residuals of abnormal trading volume as a function of an earnings surprise. While the classic V-shape is pronounced for intermediate levels of earnings surprises, trading volume does not increase but rather stays flat for more extreme surprises. The non-parametric analysis provides preliminary evidence that trading volume reactions are modestly increasing in earnings surprises.

3.3.2 Parametric Analysis: Polynomial Regression

For our second test, we estimate the following regression:

$$ln(AVOL_{i,q}) = a_0 + B' \times poly(Surp_{i,q}) + C'Controls, \tag{3.4}$$

where $poly(Surp_{i,q})$ denotes the polynomials of $Surp_{i,q}$ from first to fifth order,[8] B' is a vector of the polynomial's coefficients, C' is a vector of coefficients in front of control variables.

The results of the estimation of the polynomials up to fifth order are shown in Table 3. To choose the model that most closely describes the relation between trading volume and the earnings surprise, we conduct an analysis-of-variance test. The statistics are shown in Table 4. Adding the third and the fifth order summands to the polynomial does not improve the predictive power, whereas the second and the fourth order summands improve the fit of the model to the data significantly. We conclude that the best model is the one that includes first-, second-, and fourth-order summands.

[Insert Tables 3 and 4 around here]

While the fourth-order polynomial suggests a non-linear effect of the earnings surprise on trading volume, the M-shape is not obvious. To illustrate the shape of the polynomial, we run a regression of abnormal trading volume on all the control variables, take residuals from this regression, and regress the residuals on the earnings surprise. We plot the equation from the regression of the trading volume's residuals after controlling for various factors on the earnings surprise in Figure 6. The plotted curve has two humps and a pronounced M-shape, similar to the model plots in Figure 3a.

[8] Adding higher order polynomials does not significantly increase the predictive power of the model. The results are not reported in the paper to economize the space.

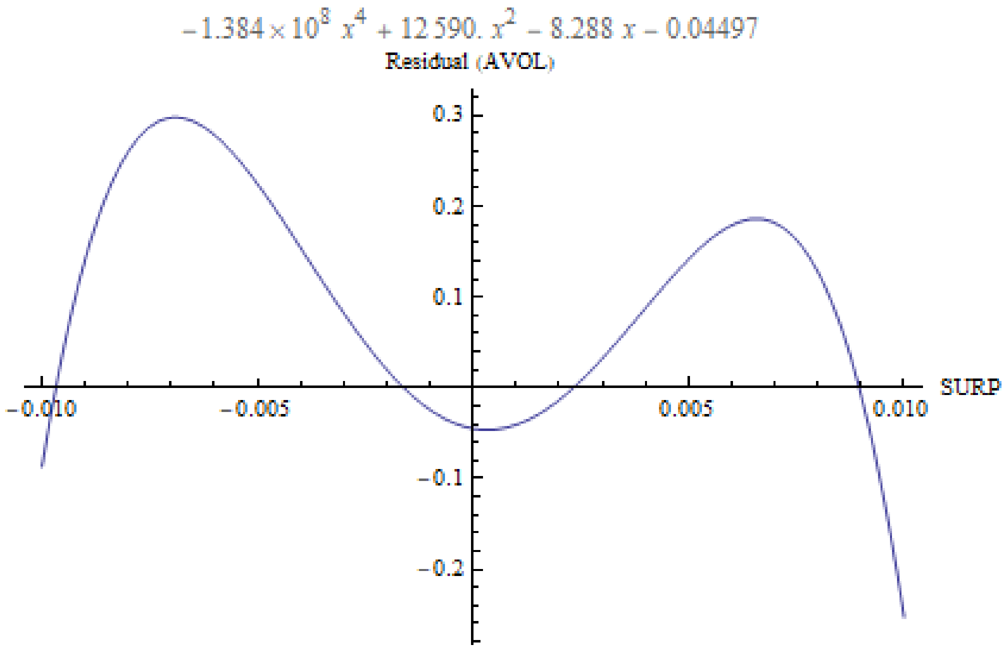


Figure 6: Functional form of the equation from the regression of residual abnormal trading volume on the earnings surprise

3.3.3 Parametric Analysis: Quantile Regression

As an alternative way to show the non-monotonicity in trading volume, we estimate quantile regressions. Specifically, we estimate the following regression:

$$ln(AVOL_{i,q}) = a_0 + b_1 \times |Surp_{i,q}| + C'Controls, \tag{3.5}$$

separately for the upper 5% of *Surp* variable and the rest of the sample. Table 5 shows the results. Earnings surprise is positively associated with the trading volume for the lower 95% of the absolute earnings surprise. For the upper 5%, i.e. for the 2.5% of the lowest and 2.5% of the highest non-absolute earnings surprise, the association disappears. Extreme levels of earnings surprise do not invoke high reactions of trading volume, as predicted by the model.

[Insert Table 5 around here]

Our evidence from the three empirical tests speak in favor of the first model's prediction. When we do not impose any restrictions on the functional form, abnormal trading volume increases for small earnings surprises, but less so for larger surprises. Imposing a particular functional form further corroborates this finding: trading volume as a function of the earnings surprise is most closely approximated by the fourth-order polynomial, which is M-shaped. The estimated polynomial pattern is similar to its theoretical counterpart: trading volume is around zero for no earnings surprise, increases for modest surprises, and then decreases for extreme surprises.

3.4 Cross-sectional Analysis

Our second prediction is the M-shaped pattern becomes more pronounced when uncertainty about the earnings precision is higher. To test this implication, we need to measure earnings-announcement-precision uncertainty in the sample. It is difficult to separate the earnings-announcement-precision uncertainty from the overall market uncertainty. We cannot use standard market uncertainty measures, such as VIX index, analyst forecast volatility, or market returns volatility because they may include both fundamental uncertainty and uncertainty about the earnings-announcement-precision.

3.4.1 Measure of Earnings-Precision Uncertainty

We base our earnings-precision uncertainty measure on the following logic. When analysts are not certain about how precise is an earnings number announced by a firm, the range for each analyst's forecasts is wider, and different analysts' forecasts are more distant from each other. At the same time, the range of forecasts may be wide because of market-wide uncertainty, which is not specific to the firm. It is important to distinguish between these two different sources of uncertainty. To do so, we regress a measure that includes both earnings precision and fundamental uncertainty – analyst forecast spread, divided by stock price in prior quarter for normalization, – on a proxy of fundamental uncertainty – VIX index, and take the residuals from this regression as our measure of earnings-announcement-precision uncertainty. We construct the measure in the following steps. First, we run the regression:

$$Analyst \quad fore\hat{c}ast \quad spread_{i,q} = \gamma_0 + \gamma_1 \times VIX_{m-1}, \tag{3.6}$$

where $Analyst \quad forecast \quad spread_{i,q}$ is the difference between the highest and the lowest analyst forecasts of the EPS of the firm i for the quarter q. VIX_{m-1} is the average monthly VIX from the daily data from the Chicago Board of Exchange website. The index is taken in the month before the disclosure month (Choi (2019)). Next, we take the residuals of this regression as the measure of earnings-announcement-precision uncertainty:

$$PrecUnc_{i,q} = Analyst\quad forecast\quad spread_{i,q} - Analyst\quad fo\hat{r}ecast\quad spread_{i,q} \tag{3.7}$$

To validate our measure, we ask ourselves whether the common S-shaped ERC (e.g., Freeman and Tse (1992), Cheng et al. (1992), Das and Lev (1994)) is more pronounced for high levels of our measure, which would be consistent with the theory by Subramanyam (1996) (see Figure 2(b)). We calculate cumulative abnormal returns in the $[0, 1]$ event window around earnings announcements in the following way:

$$CAR_{i,t} = \sum_{t=0}^{1} AR_{i,t}, \tag{3.8}$$

where $AR_{i,t}$ is the abnormal return calculated from the Fama-French three factor model:

$$\hat{R}_{i,t} = \gamma_0 + R_{f,t} + \gamma_1 \times (R_{m,t} - R_{f,t}) + \gamma_2 \times SMB_t + \gamma_3 \times HML_t, \tag{3.9}$$

$$AR_{i,t} = R_{i,t} - \hat{R}_{i,t} \tag{3.10}$$

where $R_{i,t}$ is daily return on a stock, $R_{f,t}$ is a risk-free interest rate, $R_{m,t}$ is a market rate of return, SMB_t is excess returns of small capitalization firms over large capitalization firms, and HML_t is excess returns of value stocks over growth stocks on day t. To estimate ”normal” levels of returns, we use stock returns for 100 days 50 days before the announcement date. We require at least 70 days of returns to be available for the stock to remain in the sample.

First, we confirm (untabulated) that the S-shaped functional form of earnings response coefficient holds for our sample of quarterly earnings announcements. If our measure is indeed capturing the underlying theoretical constract – earnings-precision uncertainty – we expect the S-shape to be more concave for higher levels of our measure. For a quadratic function, the level of concavity can be measured by looking at the coefficient in front of a quadratic term. We run the following regression:

$$CAR_{i,t} = a_0 + b_1 \times Surp_{i,t} + b_2 \times Surp^2_{i,t} + c_1 \times PrecUnc_{i,q} + c_2 \times PrecUnc_{i,q} \times Surp^2_{i,t}, \tag{3.11}$$

separately for positive and negative values of earnings surprises, $Surp_{i,t}$. For positive (negative) levels of earnings surprise, we expect $c_2 < 0$ ($c_2 > 0$). Table 6 presents the results. For positive (negative) levels of earnings surprise, the coefficient in front of the quadratic term ($Surp^2$) is more negative (positive) for higher levels of earnings precision uncertainty, suggesting that our signal-precision uncertainty measure is capturing the theoretical construct to a certain extent.

[Insert Table 6 around here]

Having validated the measure, we proceed to estimate the functional form of trading volume for different levels of the earnings-announcement-precision uncertainty.

3.4.2 Non-parametric Cross-Sectional Analysis of Trading Volume

If the second model prediction is true and the measure of the earnings-announcement-precision uncertainty that we develop accurately captures the underlying theoretical construct, then trading volume's M-shape should be more pronounced for observations with high $PrecUnc$. We conduct both non-parametric and parametric tests of this prediction.

For our non-parametric analysis, we partition our sample into four groups based on the quartiles of our earnings-announcement-precision uncertainty measure. The scatterplots of the residuals of abnormal trading volume as a function of the earnings surprise for different quartiles of the earnings-announcement-precision uncertainty are presented in Figures 7-10. The pictures demonstrate a transition from the typical V-shape to the M-shape as we move from the first to the third quartile of the earnings-announcement-precision uncertainty, however, in the fourth quartile the M-shape is not pronounced well. The evidence largely supports our theoretical prediction: the depressed trading volume at the extremes is more pronounced when there is greater market uncertainty about the signal precision.

3.4.3 Parametric Cross-Sectional Analysis of Trading Volume

Finally, we parametrically test whether the more pronounced M-shape of trading volume is associated with higher signal-precision uncertainty in the financial market. Since measuring the concavity of a fourth-order polynomial is non-trivial, we use absolute values of earnings surprises

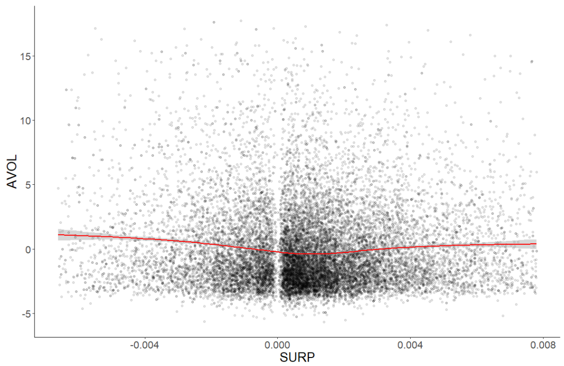


Figure 7: First quartile of $PrecUnc$

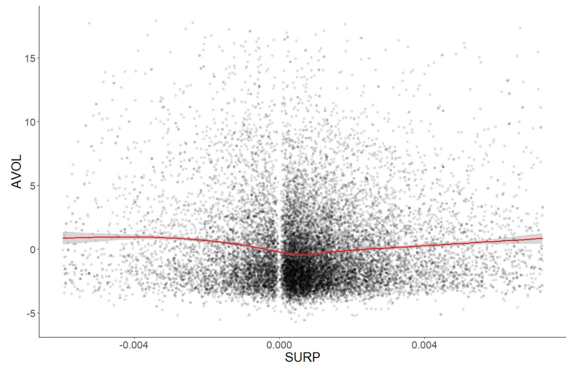


Figure 8: Second quartile of $PrecUnc$

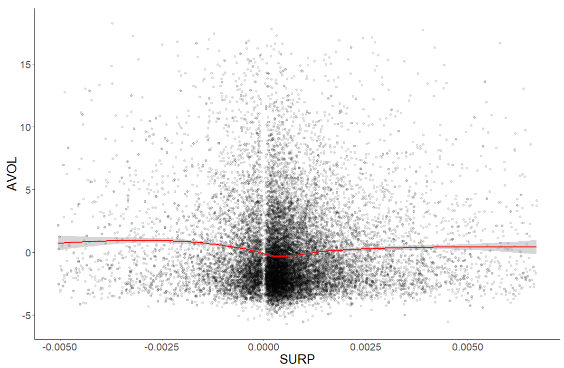


Figure 9: Third quartile of $PrecUnc$

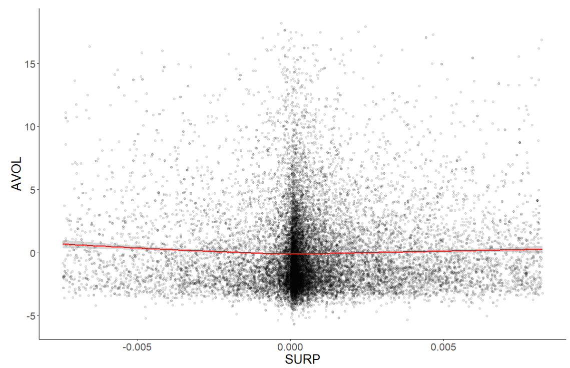


Figure 10: Fourth quartile of $PrecUnc$

and run the following regression:

$$ln(AVOL_{i,q}) = a_0 + b_1 \times |Surp_{i,q}| + b_2 \times Surp_{i,q}^2 + b_3 \times |Surp_{i,q}|^2 \times PrecUnc_{i,q}$$
$$+ b_4 \times PrecUnc_{i,q} + B'Controls \qquad (3.12)$$

We interpret the coefficient in front of the interaction of the signal-precision uncertainty measure and the quadratic term, b_3, as the change in the concavity of absolute trading volume with a 1 unit increase of signal-precision uncertainty. We present the estimates in Table 7.

[Insert Table 7 around here]

The coefficients in front of the quadratic term, $Surp_{i,q}^2$, and the interaction term, $Surp_{i,q}^2 \times PrecUnc_{i,q}$, are negative and significant. First, this implies that trading volume is an inverse U-shaped function of absolute earnings surprise, confirming the M-shaped function of earnings surprise found earlier. Second, as signal-precision uncertainty increases, the U-shape of trading volume becomes more concave, suggesting that the M-shape is more pronounced when investors are more uncertain about how precise a firm's signal is.

4 Conclusion

We provide initial evidence that investors' beliefs can further diverge even when they receive the same public signal. The source of this phenomenon lies in uncertainty about the precision of the financial information that investors receive, coupled with differential beliefs that they hold. We develop a model where investors with different beliefs about a firm's future cash flow trade in the firm's shares before and after the realization of a public signal. The novelty in the model is that investors are uncertain about the precision of this signal. Because of this uncertainty, investors' beliefs further diverge for some signal realizations. As a result of investors' posterior beliefs, trading volume is increasing for intermediate levels of signal surprise but is dampened for extreme levels. The "M-shape" of trading volume is more pronounced when the uncertainty about the signal precision is higher.

We test the predictions of our model using trading volume around quarterly earnings announcements of public U.S. firms. As a starting point in our empirical tests, we non-parametrically and parametrically show that total trading volume is described by a function that increases for the intermediate levels of the absolute earnings surprise and decreases (or stays flat) for the extreme levels.

We next develop a novel measure of the earnings-announcement-precision uncertainty and validate it by showing how the commonly-known S-shape of an earnings response coefficient changes with the different levels of our measure. As theory suggests, the S-shape is more pronounced for the high levels of earnings-announcement-precision uncertainty.

As a final step, we non-parametrically and parametrically assess the functional form of trading volume for different levels of the earnings-announcement-precision uncertainty. The evidence further supports the model that we develop: as we move from low to high earnings-announcement-precision uncertainty, the M-shape, or declining trading volume for the extreme surprises, gets more pronounced.

Overall, we believe our paper strongly suggests that US investors are uncertain about the accounting quality underlying firms' earnings reports. Our paper provides a small yet significant piece of evidence that investors' beliefs diverge due to this uncertainty about the quality of financial information.

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Appendix

A.1 Proof of Proposition 1

The difference in investors' beliefs before the disclosure of the signal y is $|m_1 - m_2|$. After the signal is released, the difference in investors' beliefs is

$$|E_1[x|y] - E_2[x|y]| = |m_1 - m_2|\frac{w-\nu}{w} \leq |m_1 - m_2| \tag{A1}$$

A.2 Proof of Proposition 3

This section is based on Subramanyam (1996). Conditional on precision w, $\tilde{y}$ is normally distributed, $\tilde{y} \sim N(m_i, w^{-1})$. Then,

$$h(y|w) = \sqrt{\frac{w}{2\pi}} exp\left[-\frac{w(y-m_i)^2}{2}\right] \tag{A2}$$

Compute the conditional expectation of $\tilde{w}$:

$$E[\tilde{w}|y] = \int wh(w|y)dw \tag{A3}$$

$$= \int \frac{1}{f(y)} wh_1(y|w)f(w)dw \tag{A4}$$

$$= \frac{\int wh_1(y|w)f(w)dw}{\int h_1(y|w)f(w)dw} \tag{A5}$$

$$= \frac{\int w^{1.5}(2\pi)^{-0.5} exp[-\frac{w(y-m_i)^2}{2}]f(w)dw}{\int w^{0.5}(2\pi)^{-0.5} exp[-\frac{w(y-m_i)^2}{2}]f(w)dw} \tag{A6}$$

Recall that $\tilde{w} \in [0, \nu]$ and substitute for $f(\tilde{w})$ to get:

$$E[\tilde{w}|y] = \frac{\Gamma(\alpha + 1.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)[\frac{(y-m_i)^2}{2} + \beta]^{-1}}{\Gamma(\alpha + 0.5, [\frac{(y-m_i)^2}{2} + \beta]\nu)} \tag{A7}$$

A.3 Post-announcement Period Equilibrium: Proof of Proposition 4

Investor i's budget constraint at time $t = 2$ is:

$$P_2 d_{i2} + q_{i2} = q^*_{i1} + P_2 d^*_{i1}, \tag{A8}$$

where q^*_{i1} and d^*_{i1} are the amounts of riskless and risky assets hold in equilibrium in $t=1$, respectively. q_{i2} and d_{i2} are the amounts of riskless and risky assets hold in $t=2$. Investor i solves:

$$max_{d_{i2},q_{i2}} \quad E_i[\tilde{x}d_{i2}+q_{i2}|y]-\frac{1}{2}r_iVar_i[\tilde{x}d_{i2}+q_{i2}|y] \tag{A9}$$

subject to (A9). The only random variable in the investor's utility is the return of the risky asset, $\tilde{x}$. Therefore, one can write $E_i[\tilde{x}d_{i2}+q_{i2}|y]=E_i[\tilde{x}|y]d_{i2}+q_{i2}$, $Var_i[\tilde{x}d_{i2}+q_{i2}|y]=Var_i[\tilde{x}|y]d^2_{i2}$. Rewrite the problem:

$$max_{d_{i2},q_{i2}}E_i[\tilde{x}|y]d_{i2}+q_{i2}-\frac{1}{2}r_iVar_i[\tilde{x}|y]d^2_{i2} \tag{A10}$$

$$\text{s.t. } P_2d_{i2}+q_{i2}=q^*_{i1}+P_2d^*_{i1} \tag{A11}$$

Using the budget constraint (A11), express q_{i2}:

$$q_{i2}=q^*_{i1}+P_2d^*_{i1}-P_2d_{i2} \tag{A12}$$

Plug this expression into (A10):

$$max_{d_{i2}}E_i[\tilde{x}|y]d_{i2}+q^*_{i1}+P_2d^*_{i1}-P_2d_{i2}-\frac{1}{2}r_iVar_i[\tilde{x}]|yd^2_{i2} \tag{A13}$$

q^*_{i1} and d^*_{i1} are chosen in $t=1$ and are constant in our problem. Take the derivative of the (A13) with respect to d_{i2} and set it equal to zero:

$$E_i[\tilde{x}|y]-P_2-r_iVar_i[\tilde{x}|y]d_{i2}=0 \tag{A14}$$

Express d_{i2}:

$$d_{i2}=\frac{E_i[\tilde{x}|y]-P_2}{r_iVar_i[\tilde{x}|y]} \tag{A15}$$

or

$$d_{i2}=\frac{m_i+\hat{w}_i(y-m_i)\nu^{-1}-P_2}{r_i\frac{1}{\nu}(1-\frac{\hat{w}_i}{\nu})} \tag{A16}$$

Use the market clearing condition to find an equilibrium price:

$$\lambda_1 d_{12} + \lambda_2 d_{22} = 1 \tag{A17}$$

$$\lambda_1 \frac{m_1 + \hat{w}_1(y - m_1)\nu^{-1} - P_2}{r_1 \frac{1}{\nu}(1 - \frac{\hat{w}_1}{\nu})} + \lambda_2 \frac{m_2 + \hat{w}_2(y - m_2)\nu^{-1} - P_2}{r_2 \frac{1}{\nu}(1 - \frac{\hat{w}_2}{\nu})} = 1 \tag{A18}$$

Solve for the price:

$$P_2^* = \left[\frac{\lambda_1 \nu}{r_1(1 - \frac{\hat{w}_1}{\nu})} + \frac{\lambda_2 \nu}{r_2(1 - \frac{\hat{w}_2}{\nu})} \right]^{-1}$$
$$\times \left[(m_1 + \hat{w}_1(y - m_1)\nu^{-1}) \frac{\lambda_1 \nu}{r_1(1 - \frac{\hat{w}_1}{\nu})} + (m_2 + \hat{w}_2(y - m_2)\nu^{-1}) \frac{\lambda_2 \nu}{r_2(1 - \frac{\hat{w}_2}{\nu})} - 1 \right] \tag{A19}$$

The investor i's demand in equilibrium:

$$d_{i2}^* = \frac{\psi_i(\hat{w}_i)}{\lambda_i} \left[E_i[\tilde{x}|y] - c\left(E_i[\tilde{x}|y]\psi_i(\hat{w}_i) + E_j[\tilde{x}|y]\psi_j(\hat{w}_j) - (\psi_i(\hat{w}_i))^{-1} \right) \right], \tag{A20}$$

where $c = \frac{\psi_i(\hat{w}_i)}{\psi_i(\hat{w}_i) + \psi_j(\hat{w}_j)}$.

A.4 Pre-announcement Period Equilibrium: Proof of Proposition 5

Investor i's problem:

$$max E_i[W_{i3}] - \frac{1}{2} r_i Var_i[W_{i3}] \tag{A21}$$
$$\text{s.t. } W_{i3} = x d_{i2}^* + q_{i2}^* \tag{A22}$$

From the budget constraint of the announcement period problem:

$$q_{i2}^* = q_{i1}^* + P_2^* d_{i1}^* - P_2^* d_{i2}^* \tag{A23}$$

The budget constraint in $t = 1$ is

$$P_1 d_{i1} + q_{i1} = W_{i0} = 0, \tag{A24}$$

where q_{i1} and d_{i1} are the amounts of riskless and risky assets hold in $t = 1$. Plug (A23) and (A24) into (A22), express the terminal wealth:

$$W_{i3} = (P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^* \tag{A25}$$

Rewrite the problem of investor i:

$$max_{d_{i1}} E_i[W_{i3}] - \frac{1}{2} r_i Var_i[W_{i3}] \tag{A26}$$

$$\text{s.t. } W_{i3} = (P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^* \tag{A27}$$

Plug (A27) into (A26):

$$max_{d_{i1}} E_i[(P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^*] - \frac{1}{2} r_i Var_i[(P_2^* - P_1)d_{i1} + (x - P_2^*)d_{i2}^*] \tag{A28}$$

Take the derivative with respect to d_{i1} and set it equal to zero:

$$E_i[P_2^*] - P_1 - r_i Var_i[P_2^*]d_{i1} = 0 \tag{A29}$$

Express d_{i1}:

$$d_{i1} = \frac{E_i[P_2^*] - P_1}{r_i Var_i[P_2^*]} \tag{A30}$$

Use the market clearing condition to find an equilibrium price:

$$\lambda_1 d_{11} + \lambda_2 d_{21} = 1 \tag{A31}$$

$$\lambda_1 \frac{E_1[P_2^*] - P_1}{r_1 Var_1[P_2^*]} + \lambda_2 \frac{E_2[P_2^*] - P_1}{r_2 Var_2[P_2^*]} = 1 \tag{A32}$$

The equilibrium price is:

$$P_1^* = \frac{E_1[P_2^*]\frac{\lambda_1}{r_1 Var_1[P_2^*]}}{\frac{\lambda_1}{r_1 Var_1[P_2^*]} + \frac{\lambda_2}{r_2 Var_2[P_2^*]}} + \frac{E_2[P_2^*]\frac{\lambda_2}{r_2 Var_2[P_2^*]}}{\frac{\lambda_1}{r_1 Var_1[P_2^*]} + \frac{\lambda_2}{r_2 Var_2[P_2^*]}} - \frac{1}{\frac{\lambda_1}{r_1 Var_1[P_2^*]} + \frac{\lambda_2}{r_2 Var_2[P_2^*]}}, \tag{A33}$$

where investor i's expectation and variance of P_2^*, $E_i[P_2^*]$ and $Var_i[P_2^*]$, are of the following form:

$$E_i[P_2^*] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\frac{\nu+n}{\nu n}}} exp(-\frac{1}{2\frac{\nu+n}{\nu n}}(y-m_i)^2)P_2^* dy \tag{A34}$$

$$Var_i[P_2^*] = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\frac{\nu+n}{\nu n}}} exp(-\frac{1}{2\frac{\nu+n}{\nu n}}(y-m_i)^2)[P_2^*]^2 dy - [E_i[P_2^*]]^2 \tag{A35}$$

The investor i's demand in equilibrium:

$$d_{i1}^* = \frac{E_i[P_2^*] - P_1^*}{r_i Var_i[P_2^*]} \tag{A36}$$

Table 1: Sample selection procedure

Sample reduction reason	**Sample size**
Initial sample, price more than $5.00 and non-zero EPS	189,248
Firms that are in I/B/E/S, CRSP and Compustat	177,981
Firms with enough data to compute analyst dispersion	156,002
Firms with non-missing data on common/ordinary equity	153,017
Firms with non-missing data on auditors	137,714
Winsorize earnings surprise at 5% level and all the other variables at 1% level	78,218

Table 2: Summary statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Abnormal Trading Volume (logs)	78,218	0.744	1.148	−3.271	0.086	1.566	3.042
Surp	78,218	0.001	0.002	−0.009	−0.0002	0.002	0.009
Control variables							
Size	78,218	7.538	1.459	4.252	6.474	8.484	11.571
Market-to-book ratio	78,218	3.003	2.490	0.620	1.510	3.516	20.009
Dispersion	78,218	0.035	0.049	0.0001	0.010	0.039	0.441
PS liquidity level	78,218	−0.058	0.106	−0.390	−0.112	0.018	0.103
Big 4 auditor	78,218	0.817	0.387	0	1	1	1

Table 3: Polynomial regressions of abnormal trading volume on the earnings surprise

	Dependent variable:					
	AVOL					
	(1)	(2)	(3)	(4)	(5)	(6)
Surp	1.038	-7.490^{***}	-8.472^{***}	-8.965^{***}	-16.327^{***}	-15.754^{***}
	(1.705)	(1.771)	(2.931)	(1.777)	(3.031)	(4.169)
Surp^2		$6.314\times10^{3***}$	$6.274\times10^{3***}$	$14.764\times10^{3***}$	$15.173\times10^{3***}$	$15.226\times10^{3***}$
		(363.391)	(375.547)	(951.596)	(961.291)	(997.252)
Surp^3			33.032×10^{3}		$2.435\times10^{5***}$	1.874×10^{5}
			(7.855×10^{4})		(8.124×10^{4})	(2.913×10^{5})
Surp^4				$-1.638\times10^{8***}$	$-1.774\times10^{8***}$	$-1.783\times10^{8***}$
				(1.705×10^{7})	(1.764×10^{7})	(1.824×10^{7})
Surp^5						8.070×10^{8}
						(4.027×10^{9})
Size	0.048^{***}	0.057^{***}	0.057^{***}	0.061^{***}	0.061^{***}	0.061^{***}
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
Market-to-book	0.046^{***}	0.049^{***}	0.049^{***}	0.050^{***}	0.050^{***}	0.050^{***}
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
Dispersion	-0.422^{***}	-0.750^{***}	-0.749^{***}	-0.835^{***}	-0.839^{***}	-0.839^{***}
	(0.089)	(0.091)	(0.091)	(0.092)	(0.092)	(0.092)
PS liquidity level	-0.256^{***}	-0.246^{***}	-0.246^{***}	-0.245^{***}	-0.246^{***}	-0.246^{***}
	(0.051)	(0.051)	(0.051)	(0.051)	(0.051)	(0.051)
Big 4 auditor	0.213^{***}	0.212^{***}	0.212^{***}	0.211^{***}	0.211^{***}	0.211^{***}
	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)	(0.011)
Constant	0.190	0.139	0.139	0.119	0.120	0.120
	(0.645)	(0.644)	(0.644)	(0.644)	(0.644)	(0.644)
Year FE	Y	Y	Y	Y	Y	Y
Observations	78,218	78,218	78,218	78,218	78,218	78,218
R^2	0.053	0.056	0.056	0.058	0.058	0.058
Adjusted R^2	0.052	0.056	0.056	0.057	0.057	0.057
Residual Std. Error	1.118 (df = 78181)	1.115 (df = 78180)	1.115 (df = 78179)	1.115 (df = 78179)	1.115 (df = 78178)	1.115 (df = 78177)
F Statistic	120.961^{***} (df = 36; 78181)	126.304^{***} (df = 37; 78180)	122.984^{***} (df = 38; 78179)	125.553^{***} (df = 38; 78179)	122.577^{***} (df = 39; 78178)	119.512^{***} (df = 40; 78177)

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$

Table 4: Analysis of variance: comparison of polynomial models

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1st order summand only	78181	97637				
2nd and 1st order summands	78180	97262	1	375.60	302.2899	¡2e-16***
3rd, 2nd, and 1st order summands	78189	97262	1	0.22	0.1771	0.6739
4th, 3rd, 2nd, and 1st order summands	78178	97136	1	125.63	101.1072	¡2e-16***
5th, 4th, 3rd, 2nd, and 1st order summands	78177	97136	1	0.05	0.0402	0.8412
Note:	*The analysis is performed by ANOVA statistical package*					

	Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
4th, 2nd, and 1 st order summands	78179	97147				
4th, 3rd, 2nd, and 1st order summands	78179	97262	0	-114.46		
Note:	*The analysis is performed by ANOVA statistical package*					

Table 5: Quantile regressions of abnormal trading volume for different levels of earnings surprise

	Dependent variable:	
	AVOL	
	Sample without upper 5%	Upper 5%
Surp (absolute)	65.176***	10.970
	(2.848)	(15.309)
Size	0.074***	0.045***
	(0.003)	(0.013)
Market-to-book	0.043***	0.027***
	(0.002)	(0.009)
Dispersion	−0.407***	−0.674***
	(0.075)	(0.194)
PS liquidity level	0.144***	0.428***
	(0.037)	(0.155)
Constant	−0.028	0.481***
	(0.023)	(0.159)
Observations	83,546	4,398
R^2	0.022	0.008
Adjusted R^2	0.022	0.007
Residual Std. Error	1.141 (df = 83540)	1.153 (df = 4392)
F Statistic	372.653*** (df = 5; 83540)	6.852*** (df = 5; 4392)
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 6: Validation of earnings-precision uncertainty measure: S-shaped ERC interacted with the earnings-precision uncertainty measure

	Dependent variable:	
	CAR	
	Positive earnings surprises, $Surp \geq 0$	Negative earnings surprises, $Surp < 0$
$Surp$	11.582^{***}	9.766^{***}
	(0.464)	(0.697)
$Surp^2$	-839.396^{***}	625.341^{***}
	(69.882)	(105.921)
$PrecUnc$	-0.158^{**}	0.279^{***}
	(0.074)	(0.086)
$PrecUnc \times Surp^2$	$-12{,}504.380^{***}$	$19{,}732.760^{***}$
	(3,377.737)	(5,020.077)
Constant	0.005^{***}	-0.015^{***}
	(0.001)	(0.001)
Observations	55,883	23,141
R^2	0.027	0.023
Adjusted R^2	0.027	0.023
Residual Std. Error	0.068 (df = 55878)	0.066 (df = 23136)
F Statistic	387.392^{***} (df = 4; 55878)	137.852^{***} (df = 4; 23136)
Note:		*p<0.1; **p<0.05; ***p<0.01

Table 7: Regression of abnormal trading volume on the earnings surprise interacted with the earnings-precision uncertainty measure

	Dependent variable:
	AVOL
|Surp|	88.473***
	(6.761)
$Surp^2$	−5,161.639***
	(982.086)
PrecUnc	8.608***
	(1.008)
PrecUnc×$Surp^2$	−174,643.400***
	(48,198.190)
Size	0.065***
	(0.003)
Market-to-book	0.049***
	(0.002)
Dispersion	−1.055***
	(0.094)
PS liquidity level	−0.242***
	(0.049)
Big 4 auditor	0.205***
	(0.011)
Constant	0.040
	(0.643)
Year FE	Y
Observations	79,087
R^2	0.058
Adjusted R^2	0.057
Residual Std. Error	1.113 (df = 79047)
F Statistic	124.432*** (df = 39; 79047)
Note:	*p<0.1; **p<0.05; ***p<0.01