The Effect of Government Support on Green Product Design and Environmental Impact

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Abstract

We study competing firms' green product design decisions, and the effect of two common types of government support, namely R&D support and sales subsidies, on the products, firms and the resulting environmental impact. Each firm produces a product that contains a "traditional" quality and an "environmental" quality according to corresponding technology capabilities of the firm and market competition. Our main results are as follows. First, we show that firms will produce greener products and charge higher prices when they embrace greater technology capabilities related to production of the environmental quality, or when consumers become more conscious of the environmental impact. Second, we find that although both the government R&D support and sales subsidies prompt firms to produce greener products, the overall environmental impact is mixed. Specifically, while sales subsidies generally lead to positive environmental benefits, R&D support can have an unanticipated negative overall impact on the environment. Third, we show that firms do not always benefit from either type of the government support.

Keywords: green product development; market competition; government support; environmental impact.

1 Introduction

With the dual pressure of energy conservation and environmental protection, many countries in the world have spared no effort to encourage the development and sales of green(er) products, herein defined as environmentally friendly, sustainable, energy efficient and easily recyclable products. Examples of green products include efficient lighting, energy saving appliances, and environmentally friendly vehicles, among others.

However, despite this general trend, the development and production of green(er) products still faces many obstacles, especially higher costs (*The Telegraph*, 2010; Plambeck, 2013; Yenipazarli and Vakharia, 2015). For example, in 2016, electric vehicles cost about $15,000 more than conventional vehicles on average and are not expected to match the price of conventional vehicles until 2025 (Lu, 2017). Fortunately, governments have realized the issue and introduced various policies to support green products. The different types of government support can be broadly divided into two categories, namely, i) improving the "Industry Commons" and ii) providing financial incentives.

The Industrial Commons refers to "a foundation of knowledge and capabilities (technical, design and operational) that is shared within an industry sector, such as R&D know-how, advanced process development and engineering skills, and manufacturing competencies related to a specific technology" (Pisano and Shih, 2009). In this paper, we will refer to government support that improves the Industrial Commons for green products in a particular industry simply as the *government R&D support*. For example, the German government (Federal Ministry of Education and Research), the U.S. government (Department of Energy) and the Japanese government (Ministry of International Trade and Industry) have long funded the development of LED technology, and key technologies in electric vehicles including lithium-ion batteries and electric powertrain.[1] Such government-supported research projects are often carried out by partner universities and national laboratories, producing technological innovations and advanced knowledge that can be leveraged by the industry.

Comparatively, financial incentives provided by a government to incentivize green products can take various forms, for firms or consumers, such as rebates, tax credits, and tax exemptions (Lu, 2017). Because these financial incentives are provided based on the sales of green products, we

[1] See https://www.bmbf.de/en/the-new-high-tech-strategy-2322.html; https://www.energy.gov/eere/ssl/research-development; https://www.energy.gov/eere/vehicles/vehicle-technologies-office-electric-drive-systems; Åhman (2006)

will address them as the *government sales subsidies*. Examples of government sales subsidies can be found in various markets such as efficient lighting (Harder and Beard, 2016), energy-efficient home appliances (Liu, 2015; Yu et al., 2018) and electric or hybrid vehicles in many different countries (Gibson, 2017). Typically, a greener product will cost more but also receives a higher sales subsidy. For instance, the central government in China offered subsidies up to RMB 30,000 ($4,765) for each plug-in hybrid electric vehicle and up to RMB 55,000 ($8,736) for each battery electric vehicle in 2016 (Lu, 2017). Interestingly, in 2010, the German government announced that it would not provide sales subsidies to electric cars but instead it would only provide R&D support in the area of electric vehicles (*Deutsche Welle*, 2010; Steinhilber et al., 2013), illustrating how governments can choose the form of their policies related to the support of green products.

Driven by these developments of government support for green products, we seek to study their impact on products, firms and the environment in this paper. Specifically, we ask the following research questions: (i) Does the government support, in the form of providing R&D or sales subsidies, promote the development of greener products? (ii) Does government support always lead to a positive environmental impact? (iii) What are the implications of government support on firm profits?

To address these questions, we first consider a base model where two firms compete in a market without any government support. Each firm produces a product that contains a "traditional" quality and an "environmental" quality (Chen, 2001). For example, if the two firms are producing cars, the traditional quality refers to quality related to car design, technical specifications, and safety, while the environmental quality corresponds to fuel economy and green ratings. The firms possess different levels of technology capacities related to production of the traditional and environmental qualities which together determine the cost of producing a product. In addition to product quality decisions, firms also make product pricing decisions. Consumers decide on which product to purchase based on their preferences of the product qualities, prices, and the two brands.

We then study the impact of government support related to green products, which is the focus point of this paper. When the government provides R&D support, it helps to enhance the environmental technologies of the firms. In contrast, when the government provides sales subsidies, the amount of subsidy per product received by each firm increases with the environmental quality of its product. Note that these model setups are consistent with actual government policies observed in practice, e.g., see Åhman (2006), Diamond (2009), Steinhilber et al. (2013) and Lu (2017).

We report the following findings in this paper. First, as environmental technologies advance in many industries, the cost of producing green products have been declining. As a result, some people predict that the price of green products will be driven down accordingly (Temple, 2018). However, even in a competitive market, we find that firms rarely pass on the savings of any production cost to the consumers. Instead, our model predicts that firms will produce greener products and charge higher prices when they embrace greater environmental technologies or when the consumers become more conscious of the environmental impact.

Second, we show that government policy designed for green products, regardless of providing R&D support or sales subsidies, incentivizes the firms to produce greener products. However, different government policies may have different implications for the environment. While sales subsidies generally lead to positive environmental benefits, R&D support can cause an unanticipated negative overall outcome on the environment.

Third, we find that firms do not always benefit from government support, even though the policy works to either reduce firm costs (R&D support) or subsidize its sales (sales subsidies). Moreover, we find that the R&D support of a government tends to provide a greater benefit towards firms that have poorer environmental technologies while the sales subsidies typically reward firms with better environmental technologies. This implies that sales subsidies can better motivate firms to develop their own environmental technologies compared to R&D support. Therefore, sales subsidies rather than direct R&D support may actually be a more effective way for a government to promote the research and development of green technologies in a competitive market.

2 Literature Review

In the literature of sustainable operations (see recent reviews by Girotra and Netessine, 2013; Bouchery et al., 2016; Lee and Tang, 2017; Kalkanci et al., 2018; Agrawal et al., 2019) and socially responsible practices (see, e.g., Guo et al., 2015; Huang et al., 2017; Agrawal and Lee, 2018; Kalkanci and Plambeck, 2018; Kraft et al., 2018a,b and the references therein), our paper is most related to those that study sustainable or green product design problems. Agrawal and Ülkü (2012) study the modular upgradability decision of a product as a green design strategy. Kraft et al. (2013) and Kraft and Raz (2017) study firms' replacement strategies of potentially hazardous substances in their product. Yenipazarli and Vakharia (2015) considers a firm's decision to expand a "brown" product line with a new green product. Agrawal et al. (2015) and Örsdemir et al. (2018) consider product durability decisions as a sustainable product feature. Bellos et al. (2017) find that car sharing can increase green product designs because it is optimal for automakers to increase the fuel efficiency of the vehicles used for car sharing.

While most of the above-mentioned papers focus on the decisions of a monopolist, our model explicitly captures market competition. In our paper, we allow two competing firms to make product design decisions for their products that consist of a "traditional" quality and an "environmental" quality. In particular, a higher environmental quality corresponds to a greener product. As such, our work adds to the literature of product design with multiple quality-type attributes, see, e.g., Chen (2001), Kim and Chhajed (2002), Krishnan and Zhu (2006), Krishnan and Lacourbe (2011) and Huang et al. (2019).

We then apply analytical models to study the role of government policies in the stimulation of green product design and sales. This literature dates back to Fullerton and Wu (1998). In this space, Plambeck and Wang (2009) study the effects of e-waste regulations of a government such as "fee-upon-sale" versus "fee-upon-disposal" on new product introduction timing and expenditure decisions. Many other papers have also considered the impact of Extended Producer Responsibility (EPR)-based take-back legislation on product design and green supply chain, see, e.g., Atasu et al. (2009); Chen and Sheu (2009); Atasu and Subramanian (2012); Atasu and Van Wassenhove (2012); Esenduran and Kemahlıoğlu-Ziya (2015); Gui et al. (2015, 2018); Alev et al. (2018) and Huang et al. (2019). Drake (2018) studies the impact of emissions regulation (i.e., carbon tariffs) on the environment. Murali et al. (2018) study the impact of voluntary ecolabels and mandatory environmental regulation on green product development among competing firms. Chen (2001) (resp. Bellos et al. (2017)) considers the impact of minimum environmental quality regulations (resp. standards) on green product design and the environment. Krishnan and Lacourbe (2011) consider the impact of "fleet" weighted average quality for a car manufacturer. While most of these papers examine the impact of government regulations on green product design, we consider and compare two common types of *supportive* government policies, namely R&D support and sales subsidies, which are widely adopted in many countries (Åhman, 2006; Diamond, 2009; Steinhilber et al., 2013).

Finally, our findings may provide some practical policy implications for the development of electric vehicles in the automotive industry. A relatively small number of papers in the Operations

Management literature have studied particular issues related to electric vehicles, compared to the Transportation literature and the Energy literature, see, e.g., Sioshansi (2012). In particular, Chocteau et al. (2011) study the impact of collaboration on the adoption of electric vehicles among commercial fleets. Mak et al. (2013) and Avci et al. (2014) consider battery-switching stations for electric vehicles. Lim et al. (2014) investigates how customer characteristics such as range and resale anxiety, and leasing of electric vehicle batteries, affect the adoption of electric vehicles.

3 The Base Model

There are two firms, labeled by firm 1 and firm 2.[2] They compete in the same industry by each offering one product that contains two types of attributes, traditional and environmental (Chen, 2001). For example, if the two firms are producing cars, the traditional attributes refer to features related to car design, technical specifications, and safety, while environmental attributes include fuel economy and green features. We assume both types of attributes behave like "qualities," i.e., everything else being equal, consumers prefer higher levels to lower levels for each type of attributes, and call them traditional and environmental qualities in the paper. For the product offered by firm $i \in \{1, 2\}$, or simply product i, we denote its traditional and environmental qualities by q_i and e_i, respectively.

Following Krishnan and Zhu (2006) and Huang et al. (2019), we use the following function form to capture the unit production cost of product i when its quality levels are q_i and e_i:

$$c(q_i, e_i|\theta_{qi}, \theta_{ei}) = (\theta_{qi}q_i)^2 + (\theta_{ei}e_i)^2 + \delta(\theta_{qi}q_i)(\theta_{ei}e_i), \quad i = 1, 2 \tag{1}$$

Here, in (1), θ_{qi} and θ_{ei} are positive cost factors that reflect an increase in the unit production cost as a result of any increase in the traditional or environmental quality. The quadratic forms indicate convex increasing production cost. Moreover, the cost factors are determined by production technologies possessed by each firm. Specifically, if firm i has a higher level of technology capacity associated with producing the traditional (resp. environmental) attributes, the cost factor θ_{qi} (resp. θ_{ei}) is lower and so is the total unit production cost.

The last term in (1) captures the possible interaction between the traditional and environment

[2] Duopoly models are typical in the product development literature, see, e.g., Crampes and Hollander (1995) and Ferrer and Swaminathan (2006).

qualities when determining the unit production cost. The coefficient δ is positive, reflecting the notion that due to the increased complexity, a higher traditional quality makes the production of the same environmental attributes more expensive, and vice versa. Furthermore, following Krishnan and Zhu (2006), we assume $\delta < 2$ so that the unit production cost function $c(q_i, e_i|\theta_{qi}, \theta_{ei})$ is jointly convex in the quality levels q_i and e_i.

Product i's valuation to consumers is given by $v + v_q q_i + v_e e_i$ where $v > 0$ is a base valuation parameter, and $v_q, v_e > 0$ reflect consumers' positive marginal utility with higher product qualities (Chen, 2001). In particular, v_q represents consumers' preference for the traditional attributes. And v_e can be interpreted as their appreciation of the product environmental quality either due to social responsibility or the energy cost savings during use of the product, i.e., a higher value of v_e implies that consumers are more respectful of environmental stewardship and are thus willing to pay more for a product's environmental attributes. In the base model, quality preferences v_q, v_e are the same for all consumers, but we will relax this assumption in §6.1.

In addition, consumers have heterogeneous preference over the two firms/brands, which is captured using the following Hotelling model. We assume that consumers (or consumers' preferences to be exact) are uniformly distributed on a $[0, 1]$ line, with the two firms at the two ends. Denote $x \in [0, 1]$ as the distance from a customer's location to firm 1, and thus $(1 - x)$ is the distance between this customer and firm 2. Then, for this customer, we specify that her utility from purchasing product $i \in \{1, 2\}$, denoted by u_i, is given as follows:

$$u_1 = v + v_q q_1 + v_e e_1 - v_l x - p_1 \tag{2}$$

$$u_2 = v + v_q q_2 + v_e e_2 - v_l (1 - x) - p_2 \tag{3}$$

Here, in (2) and (3), $v_l > 0$ measures the strength of consumers' brand preference (l for loyalty), and p_i denotes the price of product $i \in \{1, 2\}$ which is determined by firm i.

The timing of events is as follows. First, both firms simultaneously decide product qualities, q_i and e_i. Then, both firms simultaneously decide product price p_i. This reflects the notion that product design decisions are longer-term decisions compared to price decisions. Note that at this stage, both products have been developed and launched in the market, so everyone (including the firms and the consumers) can observe the qualities of both products. Finally, given the qualities and prices of both products, consumers make purchasing decisions by comparing u_1 and u_2 given by (2) and (3), respectively. We assume consumers' base product valuation, v, is sufficiently large so that the market is fully covered. This assumption is standard in Hotelling models (see, e.g., Shaffer and Zhang (1995); Jain (2008)), and it enables us to focus on the interesting and realistic scenario where both firms are competing for limited market demand. We also assume in the base model that the total market demand is deterministic and, without loss of generality, normalized to 1. We will extend the base model to include uncertain market demand in §6.2.

It is straightforward to derive the total demand for product $i \in \{1, 2\}$, denoted by d_i:

$$d_i(q_i, e_i, p_i | q_j, e_j, p_j) = \frac{v_q(q_i - q_j) + v_e(e_i - e_j) - (p_i - p_j) + v_l}{2v_l} \text{ where } j = 3 - i.$$

As a result, firm i's profit function can be expressed as follows:

$$\pi_i(q_i, e_i, p_i | q_j, e_j, p_j) = [p_i - c(q_i, e_i | \theta_{qi}, \theta_{ei})]\, d_i(q_i, e_i, p_i | q_j, e_j, p_j) \text{ where } j = 3 - i.$$

We develop the subgame perfect equilibrium of the base model in the following proposition. We use superscript $\cdot^*$ to denote the equilibrium outcomes. All proofs are relegated to Appendix B.

Proposition 1. *$q_i^* > 0$, $e_i^* > 0$ and $d_i^* > 0$ for $i = 1, 2$ if and only if $\delta < \min\left\{\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right\}$ and $v_l > \underline{v_l}$ for some $\underline{v_l} \geq 0$. Under these conditions, we have, for $i \in \{1, 2\}$ and $j = 3 - i$,*

(i) $q_i^* = \frac{2v_q\theta_{ei}^2 - v_e\delta\theta_{qi}\theta_{ei}}{4\theta_{qi}^2\theta_{ei}^2 - \delta^2\theta_{qi}^2\theta_{ei}^2}$, $e_i^* = \frac{2v_e\theta_{qi}^2 - v_q\delta\theta_{qi}\theta_{ei}}{4\theta_{qi}^2\theta_{ei}^2 - \delta^2\theta_{qi}^2\theta_{ei}^2}$;

(ii) $p_i^* = \frac{v_q(q_i^* - q_j^*) + v_e(e_i^* - e_j^*) + (\theta_{qj}q_j^*)^2 + (\theta_{ej}e_j^*)^2 + \delta(\theta_{qj}q_j^*)(\theta_{ej}e_j^*) + 2(\theta_{qi}q_i^*)^2 + 2(\theta_{ei}e_i^*)^2 + 2\delta(\theta_{qi}q_i^*)(\theta_{ei}e_i^*)}{3} + v_l$.

To focus on the most interesting and realistic situations in which both firms exist in the market (i.e., $q_i^* > 0$, $e_i^* > 0$ and $d_i^* > 0$ for $i = 1, 2$), we will assume $\delta < \min\left\{\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right\}$ and $v_l > \underline{v_l}$ in the remaining paper. The next proposition summarizes the impact of a firm's cost factor in producing environmental attributes, θ_{ei}, which is negatively correlated to the level of the corresponding technology capacity, and the impact of consumers' engagement in social responsibility, which is measured by v_e, on the product quality and price decisions.

Proposition 2.

(i) $\frac{\partial q_i^*}{\partial \theta_{ei}} > 0$, $\frac{\partial e_i^*}{\partial \theta_{ei}} < 0$, $\frac{\partial q_i^*}{\partial v_e} < 0$, *and* $\frac{\partial e_i^*}{\partial v_e} > 0$;

(ii) $\frac{\partial p_i^*}{\partial \theta_{ei}} < 0$ *while* $\frac{\partial p_i^*}{\partial v_e} > 0$ *if and only if* $e_i^* > e_j^*/4$ *where* $j = 3 - i$.

Proposition 2/(i) shows that when firm i possesses a higher level of technology capacity in terms of producing the environmental attributes (i.e., the corresponding cost factor θ_{ei} is smaller), it is optimal for the firm to incorporate a higher environmental quality into its product. Similarly, if consumers pay more attention to the environmental attributes of a product (i.e., v_e is larger), it will encourage the firm to improve the environmental quality of the product. These results are in line with expectations. On the other hand, when the environmental quality is enhanced, the traditional quality should be reduced, due to the cost trade-offs, to achieve the new optimality.

The impact of better production technologies or stronger consumer preferences, related to the environmental attributes of a product, on product price is more intriguing. For example, the battery cost of electric vehicles has been declining in recent years (Hodges, 2018), which is equivalent to a lower value of θ_{ei} in our model. One might attempt to conclude that electric vehicles will become cheaper consequently (Temple, 2018). On the contrary, Proposition 2/(ii) reveals that when θ_{ei} is smaller, firms hardly pass on the savings of any production cost to the consumers. Instead, they choose to enhance the environmental quality of their products and increase product prices accordingly (i.e., $\frac{\partial p_i^*}{\partial \theta_{ei}} < 0$). Proposition 2/(ii) also points out that as consumers become more supportive of green products (*Sustainable Brands*, 2015; Newport, 2018), i.e., when v_e increases, a firm should increase its product price as long as the environmental quality of its product is comparable or better than that of the competitor's product (i.e., when $e_i^* > e_j^*/4$ which subsumes the case when $e_i^* \approx e_j^*$). Therefore, for the automotive industry, with the continuous advancement of electric vehicle technology and the growing consumer desire for environmental benefits, Proposition 2/(ii) indicates that the price of electric vehicles will rise. This finding is consistent with recent empirical data which reports that the average electric vehicle prices in the U.S. have steadily increased from 2012 to 2016 (International Energy Agency, 2017).

4 The Impact of Government Support

As discussed in the Introduction section, governments in many countries have developed policies to stimulate the development and sale of green products. In this section, we will discuss two types of government support that are commonly used in practice, namely R&D support and sales subsidies. Our goal is to study the impact of these government policies on the products, the firms and the environment. As such, we need to define a metric for measuring total environmental benefits.

For a product that has an environmental quality e_i, we use a general function $b(e_i)$ to characterize the environmental benefits of the product. We assume $\partial b/\partial e_i > 0$ to indicate that a higher environmental quality corresponds to greater environmental benefits (i.e., less negative environmental impact). Then, we define the total environmental benefits, denoted by B, by factoring in the demand for each product, i.e.,

$$B = d_1(q_1, e_1, p_1, q_2, e_2, p_2) \cdot b\,(e_1) + d_2(q_2, e_2, p_2, q_1, e_1, p_1) \cdot b\,(e_2) \tag{4}$$

when product $i \in \{1, 2\}$ has quality specification q_i, e_i and price p_i. We point out that first, such a characterization of the total environmental benefits is consistent with the literature, see, e.g., Agrawal et al. (2012); second, the total environmental benefits are determined by two things: i) the per unit environmental impact $b(e_i)$ for product i, and ii) the demand for product i.

4.1 R&D Support

Research and development support is a common governmental policy that encourages the development of greener products. For example, in the U.S., the Vehicle Technologies Office within the Department of Energy "supports research and development (R&D) to reduce the cost and improve the performance of innovative electric drive devices, components, and systems."[3] Such government-supported research projects are often carried out by partner universities and national laboratories, producing technological innovations and advanced knowledge that can be leveraged by the industry.

Therefore, with government R&D support, we assume that the cost factors of the two firms associated with producing environmental attributes can be reduced to some level $\theta_{ei}^r < \theta_{ei}$ for both $i = 1, 2$.[4] Furthermore, each firm needs to independently incorporates the government R&D support into its own R&D process, and a firm with a better technology prior to the support should still have a better technology after the support; in other words, we assume $\theta_{ei}^r < \theta_{ej}^r$ if $\theta_{ei} < \theta_{ej}$.

The unit production cost function for the two firms becomes the following:

$$c(q_i, e_i|\theta_{qi}, \theta_{ei}^r) = (\theta_{qi} q_i)^2 + (\theta_{ei}^r e_i)^2 + \delta(\theta_{qi} q_i)(\theta_{ei}^r e_i),\ i = 1, 2.$$

[3]Source: https://www.energy.gov/eere/vehicles/vehicle-technologies-office-electric-drive-systems

[4]It is easy to verify that both firms would find it optimal to adopt θ_{ei}^r for $i = 1, 2$.

Consumers' utility functions and decision-making process (i.e., expressions for the demand functions) stay unchanged under the government R&D support compared to the base model, and we solve for the new equilibrium outcomes. The next three propositions summarize the impact of the government R&D support on product environmental qualities, total environmental benefits, and firm profits. We use superscript $\cdot^r$ to denote the equilibrium outcomes under the R&D case.

Proposition 3. *With government R&D support, $e_i^r > e_i^*$ for $i = 1, 2$.*

With better technologies available to produce environmental attributes (making it cheaper to do so), Proposition 3 shows that both firms will improve environmental quality for its product (i.e., $e_i^r > e_i^*$). While this result is as expected, our next result will show that the impact of the government R&D support on the total environmental benefits is not straightforward.

Proposition 4. *With government R&D support, $B^r < B^*$ if the following conditions hold:* (i) $\theta_{e1} < \theta_{e2}$*;* (ii) $\theta_{q1} \geq \theta_{q2} > \underline{\theta_q}$*;* (iii) $\bar{\theta}_e > \theta_{e2}^r > \theta_{e1}^r > \underline{\theta_e}$*; and* (iv) $v_l < \bar{v}_l$ *where* $\underline{\theta_q}$*,* $\bar{\theta}_e$*,* $\underline{\theta_e}$ *and* $\bar{v}_l$ *are thresholds.*

Although R&D support from the government has prompted both firms to produce products with better environmental quality, i.e., $e_i^r > e_i^*$ (see Proposition 3), Proposition 4 surprisingly reveals that the *total* environmental benefits of the products may get *worse* (i.e., $B^r < B^*$). In other words, improving the environmental benefits of each product in the market does not necessarily translate into a positive impact on the total environmental benefits. We can explain this counter-intuitive finding in detail below while interpreting the conditions of Proposition 4.

Consider the scenario in which neither firm simultaneously dominates its competitor in both types of technologies related to production of the traditional attributes and the environmental benefits, i.e., $\theta_{e1} < \theta_{e2}$ while $\theta_{q1} \geq \theta_{q2}$. Then, without the government R&D support, product 1 carries a lower traditional but a higher environmental quality than product 2 in equilibrium. When the firms receive R&D support from the government, we have $e_1^* < e_1^r$ and $e_2^* < e_2^r$ because both firms will leverage the new technology to produce higher environmental attributes for its product. However, given $\theta_{q1} \geq \theta_{q2}$ and $\theta_{e1}^r < \theta_{e2}^r$, we continue to have $q_2^r > q_1^r$ and $e_1^r > e_2^r$ because firm 2's competitive advantage, compared to firm 1, still lies in the production of traditional attributes.

Consequently, when government R&D support does not lead to a very significant improvement in both firms' environmental technology (i.e., when $\theta_{e2}^r > \theta_{e1}^r > \underline{\theta_e}$), we can encounter $e_2^* < e_2^r <$

$e_1^* < e_1^r$ and thus $b(e_2^r) < b(e_1^*)$, i.e., firm 2's product after the government support still has poorer environmental benefits compared to firm 1's product before the government support. On the other hand, firm 2 could seize a bigger market share when the firms receive the government R&D support compared to when they do not (i.e., $d_2^s > d_2^*$), if θ_{e2}^r is not too big (i.e., $\theta_{e2}^r < \bar{\theta}_e$), under which condition the government R&D support could reduce the technology disadvantage of firm 2 relative to firm 1 in producing environmental attributes, that is, the new technology reduces firm 2's cost factor associated with production of the environmental attributes more than it reduces firm 1's. Hence in equilibrium (under the government R&D support), product 2 has a relatively significantly better traditional quality than product 1 while its environmental quality is only relatively slightly worse. This pushes more customers to purchase product 2, especially when consumers do not have strong loyalty preferences (i.e., when $v_l < \bar{v}_l$). As a result, government R&D support, while making both products greener, prompts more consumers to purchase the product that is less environmentally friendly, leading to reduced total environmental benefits for all products on the market.

Next, we put forward a condition in the following proposition to demonstrate that the government's R&D support may actually reduce firm profit.

Proposition 5. *Suppose $\theta_{e1} < \theta_{e2}$ and $\theta_{q1} = \theta_{q2}$. Then government R&D support will decrease firm 1's profit while increasing firm 2's profit, i.e., $\pi_1^r < \pi_1^*$ and $\pi_2^r > \pi_2^*$, if $\theta_{e2}^r < \tilde{\theta}_e$ for some $\tilde{\theta}_e > \theta_{e1}^r$.*

Proposition 5 shows that the government R&D support, despite reducing the cost factor, is not always beneficial to firms in a competitive market. In addition, firms that are getting worse because of the government R&D support which promotes green products, could be the ones with better technologies related to the production of environmental attributes. The reason here is similar to that for Proposition 4, namely, government R&D support could have a greater impact on enhancing technology for firms that have poorer environmental technologies (when $\theta_{e2}^r < \tilde{\theta}_e$), helping them to capture demand from firms that are savvier in the environmental technologies. As a result, firms with better environmental technologies can have their competitive advantage reduced and become worse off when the government provides R&D support to the industry.

4.2 Sales Subsidies

In this section, we examine the scenario in which the government provides sales subsidies for green products. Suppose firm $i \in \{1,2\}$ receives a subsidy $s(e_i)$ from the government for each unit of product sold in the market. Assume $s(e_i)$ is a (weakly) increasing function of the product environmental quality e_i, which is common among existing government subsidy programs on green products (Lu, 2017). Note that although we focus on the case in which the firms receive the sales subsidies (which is oftentimes referred to as upstream incentives (du Can et al., 2014)), it is easy to verify that the equilibrium outcomes are identical in our model when, instead, the consumers receive the same amount of sales subsidies (downstream incentives), because the firms would collect the benefits from consumers through endogenous product price decisions in that case.[5]

Consumers' utility functions and decision-making process (i.e., expressions for the demand functions) stay unchanged under the government sales subsidies compared to the base model. The profit functions for firm $i \in \{1,2\}$ are updated as follows to take into account the sales subsidies:

$$\pi_i(q_i, e_i, p_i | q_j, e_j, p_j) = [p_i - c(q_i, e_i | \theta_{qi}, \theta_{ei}) + s(e_i)]\, d_i(q_i, e_i, p_i | q_j, e_j, p_j) \text{ where } j = 3 - i.$$

We develop the next three propositions to summarize the impact of green product sales subsidies from the government on product environmental qualities, total environmental benefits, and firm profits. We use superscript $\cdot^s$ to denote the equilibrium outcomes associated with the government sales subsidies case.

Proposition 6. *With government sales subsidies , $e_i^s \geq e_i^*$ for $i = 1, 2$.*

Similar to the impact of government R&D support related to environmental technologies (i.e., Proposition 3), Proposition 6 indicates that government sales subsidies for environmental attributes incentivize firms to improve their product environmental quality. This is consistent with the current trends in the automotive industry. In particular, many automakers are "going electric" due to government sales subsidies and other monetary incentives, shifting the focus of new product design from traditional gasoline cars to electric or hybrid models (Frost, 2017).

[5] Upstream incentives are common in both the home appliance industry, see, e.g., §3.3 in du Can et al. (2014), and the automotive industry, see, e.g., Lewis (2010) where it is noted that "Instead of handing out subsidies to consumers directly, the (Chinese) government would allocate the money to carmakers, who would then lower the prices of relevant models accordingly."

Proposition 7. *With government sales subsidies, $B^s \geq B^*$.*

A significant difference of Proposition 7, compared to the impact of government R&D support related to environmental technologies (i.e., Proposition 4), lies in that the impact of government sales subsidies on the total environmental benefits is always positive. This is because the sales subsidies $s(e_i)$ are designed to be increasing in the product environmental quality level e_i. This means that the firm that focuses more on product environmental attributes will receive greater subsidies from the government and is therefore the same firm with an increasing market share. In other words, $e_i^* > e_j^*$ for $j = 3 - i$ implies that $d_i^s > d_i^*$. Under the government sales subsidies, total environmental benefits become greater because not only are all products on the market more environmentally friendly, but more consumers are buying the greener product (also see Eqn. (4)).

Proposition 8. *Suppose $\theta_{e1} < \theta_{e2}$ and $\theta_{q1} = \theta_{q2}$. Then government sales subsidies will increase firm 1's profit while decreasing firm 2's profit, i.e., $\pi_1^s \geq \pi_1^*$ and $\pi_2^s \leq \pi_2^*$.*

Consistent with Proposition 5, Proposition 8 shows that government sales subsidies, despite giving firms money, is not always beneficial to the recipients in a competitive market. However, in contrast to the case of the government R&D support, the government sales subsidies make the firms that have savvy (resp. poor) technologies related to production of environmental attributes better off (resp. worse off). The reason is as follows. Government sales subsidies magnify the impact of the environmental technology gaps between firms, as firms with better environmental technologies tend to have a higher environmental quality in their product and thus receive more subsidies. As a result, firms with large θ_{ei}'s find it even harder to compete for market share under the government sales subsidies, leading to a reduced firm profit.

5 General Discussion

In Section 4, we studied the impact of two types of government policies that are commonly used to encourage the design and sales of green products, namely R&D support and sales subsidies. We now summarize their impact on products, firms and the environment in Table 1, and relate our findings to government policy implications.

When we compare the impact of the government R&D support and sales subsidies, our results indicate that the government should be careful about the R&D support for two reasons.

Table 1: Impact of Government Policies

	R&D Support	Sales Subsidies
Environmental quality for individual products	↑	↑
Total environmental benefits	↑↓	↑
Profit of the firm with poorer environmental technology*	↑↓	↓
Profit of the firm with better environmental technology*	↓↑	↑

* Assume both firms have the same level of traditional technology.

First, the R&D support can have a counterproductive impact on the total environmental benefits (Proposition 4). Second, ceteris paribus, R&D support could hurt firms with better environmental technologies (Proposition 5). This will discourage firms from developing better environmental technologies because they are being penalized for doing so. In contrast, providing sales subsidies is a good strategy for the government because it not only improves total environmental benefits but also rewards firms that have better environmental technologies. In other words, if the government wants to accomplish better green technologies in a particular industry, providing sales subsidies (which increases in the size of sales) may be more effective compared to giving direct R&D support (which reduces cost). We mentioned in the Introduction section that in 2010, the German government announced that it would not provide sales subsidies to electric cars but instead it would only provide R&D support in the area of electric vehicles (*Deutsche Welle*, 2010; Steinhilber et al., 2013). This strategy might have been ineffective, as evidenced by how the German government approved a total of 600 million Euro to use as sales subsidies on electric vehicles in 2016 (*The Gaurdian*, 2016).

Government supportive policies can play important roles in maintaining and improving our environment. Given the various policies that are observed in practice, our findings have the following policy implications. When a government policy is designed to help firms with poor environmental technologies, it can improve the environmental quality of the environmentally dirtiest products on the market (i.e., the "brown" products) but may result in an overall reduction in environmental impact as the dirtier products can capture more market share. On the other hand, if a government policy is designed to reward firms with better environmental technologies, the greener products will capture more market share, increasing the overall environmental impact. These findings suggest that development of government policies to promote green product design requires careful consideration of the current state of environmental technology in the industry. Overall our results favor sales subsidies but we note that doing so could adversely affect some firms, which could impact the competitiveness of the industry.

6 Extensions

In this section, we establish the robustness of our findings by considering two extensions. The detailed analysis and proofs are relegated to the Appendices of the paper.

6.1 Heterogeneous Consumer Preferences for Product Qualities

So far, we have assumed that consumers have homogeneous preferences for product qualities. In this section, we consider an extension where consumers' preferences for the traditional and environmental qualities, denoted as V_q and V_e, follow a joint distribution $\Pr(V_q \leq v_q, V_e \leq v_e) = F(v_q, v_e)$. As a result, consumers are heterogeneous in two dimensions, including the horizontal differentiation/preference for the two brands (i.e., x) and the vertical differentiation/preference for product qualities (i.e., V_q and V_e). For a consumer with preferences (x, V_q, V_e), her utility functions for the products are

$$
\begin{aligned}
u_1 &= v + V_q q_1 + V_e e_1 - v_l x - p_1 &(5)\\
u_2 &= v + V_q q_2 + V_e e_2 - v_l(1-x) - p_2. &(6)
\end{aligned}
$$

Given the qualities (q_i, e_i) and price (p_i) for both products as well as their individual preferences (x, V_q, V_e), consumers decide which product to purchase by comparing u_1 and u_2 in (5) and (6). Consequently, the total demand for product i is

$$d_i(q_i, e_i, p_i | q_j, e_j, p_j) = \iint \frac{v_q(q_i - q_j) + v_e(e_i - e_j) - (p_i - p_j) + v_l}{2v_l} dF\left(v_q, v_e\right)$$

where $j = 3 - i$. It follows that firm i's profit function can be expressed as follows:

$$\pi_i(q_i, e_i, p_i | q_j, e_j, p_j) = [p_i - c(q_i, e_i | \theta_{qi}, \theta_{ei})]\, d_i(q_i, e_i, p_i | q_j, e_j, p_j) \text{ where } j = 3 - i.$$

In Appendix A.1, we present a detailed analysis of this model. We find that all of our main insights continue to hold.

6.2 Uncertain Market Size and Limited Production Capacities

In practice, the total market size may be unpredictable, and firms may have a limited production capacity, resulting in consumers having to wait to get the product. For example, Tesla has been facing a long backlog with its Model 3 vehicle due to its production capacity (Higgins, 2017). In this section, we present an extension to the base model to incorporate uncertain market size and firms' limited production capacities.

Suppose the total market size M is random. Denote firm i's capacity or production rate as k_i, which measures how quickly the firm can produce a product and deliver it to the customers. Suppose all consumers make purchase decisions at the same time and that they receive the product in a random order. Then, consumers' expected waiting time to receive product i, or equivalently the average lead-time of firm i, is equal to $\mathbb{E}D_i/(2k_i)$, where $\mathbb{E}D_i$ is the expected number of consumers who purchase product i. Let w denote consumers' waiting cost per unit of time, then their expected utility functions of buying product $i \in \{1, 2\}$ can be expressed as follows:

$$u_1 = v + v_q q_1 + v_e e_1 - v_l x - p_1 - w\mathbb{E}D_1/(2k_1) \quad (7)$$

$$u_2 = v + v_q q_2 + v_e e_2 - v_l(1-x) - p_2 - w\mathbb{E}D_2/(2k_2) \quad (8)$$

Given the quality levels (q_i, e_i) and price information (p_i) for both products, consumers decide which product to purchase by comparing u_1 and u_2 in (7) and (8). Suppose consumers have the right expectations for the average waiting times, i.e., $\mathbb{E}D_1/(2k_1) = x^*\mathbb{E}M/(2k_1)$ and $\mathbb{E}D_2/(2k_2) = (1-x^*)\mathbb{E}M/(2k_2)$, where x^* is the location of the marginal customers who are indifferent between the two products in equilibrium. Then, the total (random) demand for each firm can be derived as

$$D_i(q_i, e_i, p_i | q_j, e_j, p_j) = \frac{v_q(q_i - q_j) + v_e(e_i - e_j) - (p_i - p_j) + v_l + w\mathbb{E}M/(2k_j)}{2v_l + w\mathbb{E}M/(2k_i) + w\mathbb{E}M/(2k_j)} \cdot M$$

where $j = 3 - i$. As a result, the expected profit function of firm $i \in \{1, 2\}$ can be expressed as follows:

$$\pi_i(q_i, e_i, p_i | q_j, e_j, p_j) = \mathbb{E}\left[p_i - c(q_i, e_i | \theta_{qi}, \theta_{ei})\right] D_i(q_i, e_i, p_i | q_j, e_j, p_j) \text{ where } j = 3 - i.$$

In Appendix A.2, we present a detailed analysis of this model. We find that all of our main insights continue to hold.

7 Conclusions and Future Research

In this paper, we first studied competing firms' green product design decisions and then investigated the impact of government support. We found that firms will produce greener products and charge higher prices when they embrace greater environmental technologies or when the consumers become more conscious of the environmental impact. We also found that although both government R&D support and sales subsidies prompt the firms to produce greener products, their overall environmental impact is mixed. Specifically, while sales subsidies generally lead to positive environmental benefits, R&D support can have an unanticipated negative overall impact on the environment. Interestingly, firms also do not always benefit from either type of policy.

We demonstrated the robustness of our results by considering consumer preference heterogeneity, uncertain market size and limited production capacities for the firms. However, our model is certainly not without limitations. First, given that our research focuses on firms' product design decisions, we assumed exogenous production capacities for the firms. An interesting future research topic is to study firms' joint product design and capacity decisions. Second, our model treated government policies exogenously. A promising extension is to endogenize them. In such a case, the benefits of the policies being studied in this paper are only half of the story, and the costs of them need to be assessed to determine the optimal policy structure for the government, which is an interesting research question to pursue that goes beyond the scope of the current paper. We refer interested readers to a stream of recent work by Cohen et al. (2015), Babich et al. (2017), Murali et al. (2018), Yu et al. (2018) and Cui and Lu (2019) in which the government endogenously determines the optimal structure of its policy in order to maximize either the consumer or the social welfare.

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A Details of the Extended Models

A.1 Heterogeneous Consumer Preferences for Product Qualities

Suppose consumers have heterogeneous preferences for the traditional and environmental qualities, denoted as V_q and V_e, which follow the joint distribution $\Pr(V_q \leq v_q, V_e \leq v_e) = F(v_q, v_e)$. Here, consumers are heterogeneous in two dimensions, including the horizontal differentiation/preference for the brands (i.e., x) and vertical differentiation/preference for product qualities (i.e., V_q and V_e). For a consumer with preference levels (x, V_q, V_e), her utility functions of purchasing the two products are

$$\begin{aligned} u_1 &= v + V_q q_1 + V_e e_1 - v_l x - p_1; \\ u_2 &= v + V_q q_2 + V_e e_2 - v_l (1-x) - p_2. \end{aligned}$$

The timing of events is as follows. First, both firms simultaneously decide the product qualities, q_i and e_i. Then, both firms engage in a price competition, each deciding the price p_i of its product at the same time. Note that at this stage, both products have been introduced in the market, and thus everyone (including both firms and customers) are able to observe the qualities of both products. Finally, given the qualities (q_i, e_i) and price (p_i) for both products as well as their individual preference levels (x, V_q, V_e), consumers decide which product to purchase by comparing u_1 and u_2. Suppose consumers' base valuation for the product, i.e., v, is large enough so that the market is fully covered. Then, the total demand for product i is

$$d_i(q_i, e_i, p_i | q_j, e_j, p_j) = \iint \frac{v_q(q_i - q_j) + v_e(e_i - e_j) - (p_i - p_j) + v_l}{2v_l} dF\left(v_q, v_e\right)$$

where $j = 3 - i$. It follows that firm i's profit function can be expressed as follows:

$$\pi_i(q_i, e_i, p_i | q_j, e_j, p_j) = \left[p_i - c(q_i, e_i | \theta_{qi}, \theta_{ei})\right] d_i(q_i, e_i, p_i | q_j, e_j, p_j) \text{ where } j = 3 - i.$$

Denote $\mu_q = \mathbb{E}V_q$ and $\mu_e = \mathbb{E}V_e$. The following proposition gives the subgame perfect equilibrium of the game.

Proposition A.1.1. *$q_i^* > 0$, $e_i^* > 0$ and $d_i^* > 0$ for $i = 1, 2$ if and only if* $\delta < \min\left\{\frac{2\mu_q\theta_{ei}}{\mu_e\theta_{qi}}, \frac{2\mu_e\theta_{qi}}{\mu_q\theta_{ei}}\right\}$

and $v_l > \underline{v_l}$ *for some* $\underline{v_l} \geq 0$. *Under these conditions,*

(i) $q_i^* = \frac{2\mu_q\theta_{ei}^2 - \mu_e\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$,

(ii) $e_i^* = \frac{2\mu_e\theta_{qi}^2 - \mu_q\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$,

(iii) $p_i^* = \frac{\mu_q(q_i^* - q_j^*) + \mu_e(e_i^* - e_j^*) + (\theta_{qj}q_j^*)^2 + (\theta_{ej}e_j^*)^2 + \delta(\theta_{qj}q_j^*)(\theta_{ej}e_j^*) + 2(\theta_{qi}q_i^*)^2 + 2(\theta_{ei}e_i^*)^2 + 2\delta(\theta_{qi}q_i^*)(\theta_{ei}e_i^*)}{3} + v_l$ *where* $j = 3 - i$.

Comparing this to the base model, the only difference is that v_q and v_e are replaced by the expected value μ_q and μ_e. So, the structural results in Proposition 2 should continue to hold, which is summarized in Proposition A.1.2. As before, we assume that interior solution exists in Proposition A.1.1, i.e., $\delta < \min\left\{\frac{2\mu_q\theta_{ei}}{\mu_e\theta_{qi}}, \frac{2\mu_e\theta_{qi}}{\mu_q\theta_{ei}}\right\}$ and $v_l > \underline{v_l}$.

Proposition A.1.2.

(i) $\frac{\partial q_i^*}{\partial \theta_{ei}} > 0$, $\frac{\partial e_i^*}{\partial \theta_{ei}} < 0$, $\frac{\partial q_i^*}{\partial \mu_e} < 0$, *and* $\frac{\partial e_i^*}{\partial \mu_e} > 0$*;*

(ii) $\frac{\partial p_i^*}{\partial \theta_{ei}} < 0$ *while* $\frac{\partial p_i^*}{\partial \mu_e} > 0$ *if and only if* $e_i^* > e_j^*/4$ *where* $j = 3 - i$.

The following six propositions confirm the robustness of Propositions 3 to 8.

Proposition A.1.3. *With government R&D support,* $e_i^r > e_i^*$ *for* $i = 1, 2$.

Proposition A.1.4. *With government R&D support,* $B^r < B^*$ *if the following conditions hold:* (i) $\theta_{e1} < \theta_{e2}$*;* (ii) $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$*;* (iii) $\bar{\theta}_e > \theta_{e2}^r \geq \theta_{e1}^r > \underline{\theta_e}$*; and* (iv) $v_l < \bar{v}_l$ *where* $\underline{\theta_q}$, $\bar{\theta}_e$, $\underline{\theta_e}$ *and* $\bar{v}_l$ *are thresholds.*

Proposition A.1.5. *Suppose* $\theta_{e1} < \theta_{e2}$ *and* $\theta_{q1} = \theta_{q2}$*. Then government R&D support will decrease firm 1's profit while increasing firm 2's profit, i.e.,* $\pi_1^r < \pi_1^*$ *and* $\pi_2^r > \pi_2^*$*, if* $\theta_{e2}^r < \tilde{\theta}_e$ *for some* $\tilde{\theta}_e > \theta_{e1}^r$.

Proposition A.1.6. *With government sales incentive,* $e_i^s \geq e_i^*$.

Proposition A.1.7. *With government sales incentive,* $B^s \geq B^*$.

Proposition A.1.8. *Suppose* $\theta_{e1} < \theta_{e2}$ *and* $\theta_{q1} = \theta_{q2}$*. Then government sales subsidies will increase firm 1's profit while decreasing firm 2's profit, i.e.,* $\pi_1^s \geq \pi_1^*$ *and* $\pi_2^s \leq \pi_2^*$.

A.2 Uncertain Market Size and Limited Production Capacities

Suppose the total market size M is random. Denote firm i's capacity or production rate as k_i, which measures how quickly the firm can produce a product and deliver it to the customers. Suppose all consumers make purchase decisions at the same time and that they receive the product in a random order. Then, consumers' expected waiting time to receive product i, or equivalently the average lead-time of firm i, is equal to $\mathbb{E}D_i/(2k_i)$, where $\mathbb{E}D_i$ is the expected number of consumers who purchase product i. Let w denote consumers' waiting cost per unit of time, then their expected utility functions of buying product $i \in \{1, 2\}$ can be expressed as follows:

$$\begin{aligned} u_1 &= v + v_q q_1 + v_e e_1 - v_l x - p_1 - w\mathbb{E}D_1/(2k_1) \\ u_2 &= v + v_q q_2 + v_e e_2 - v_l(1-x) - p_2 - w\mathbb{E}D_2/(2k_2) \end{aligned}$$

The timing of events is as follows. First, both firms simultaneously decide the product qualities, q_i and e_i. Then, both firms engage in a price competition, each deciding the price p_i of its product at the same time. Note that at this stage, both products have been introduced in the market, and thus everyone (including both firms and customers) are able to observe the qualities of both products. Finally, given the product qualities and prices, consumer make purchasing decisions by comparing the corresponding utilities. Suppose consumers have the right expectations for the average waiting times, i.e., $\mathbb{E}D_1/(2k_1) = x^*\mathbb{E}M/(2k_1)$ and $\mathbb{E}D_2/(2k_2) = (1 - x^*)\mathbb{E}M/(2k_2)$, where x^* is the location of the marginal customers who are indifferent between the two products in equilibrium. Then, the total (random) demand for each firm can be derived as

$$D_i(q_i, e_i, p_i|q_j, e_j, p_j) = \frac{v_q(q_i - q_j) + v_e(e_i - e_j) - (p_i - p_j) + v_l + w\mathbb{E}M/(2k_j)}{2v_l + w\mathbb{E}M/(2k_i) + w\mathbb{E}M/(2k_j)} \cdot M$$

where $j = 3 - i$. As a result, the expected profit function of firm $i \in \{1, 2\}$ can be expressed as follows:

$$\pi_i(q_i, e_i, p_i|q_j, e_j, p_j) = \mathbb{E}\left[p_i - c(q_i, e_i|\theta_{qi}, \theta_{ei})\right] D_i(q_i, e_i, p_i|q_j, e_j, p_j) \text{ where } j = 3 - i.$$

Note that the base model in the paper is a special case with $k_1 = k_2 = \infty$ (i.e., both firms have unlimited capacity) and $M \equiv 1$ (i.e., market size is deterministic). The following proposition gives the subgame perfect equilibrium of the game. For notational convenience, denote $\mathbb{E}M = \mu_m$.

Proposition A.2.1. *$q_i^* > 0$, $e_i^* > 0$ and $\mathbb{E}D_i^* > 0$ for $i = 1, 2$ if and only if $\delta < \min\left\{\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right\}$ and $v_l > \underline{v}_l$ for some $\underline{v}_l \geq 0$. Under these conditions,*

(i) $q_i^* = \frac{2v_q\theta_{ei}^2 - v_e\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$,

(ii) $e_i^* = \frac{2v_e\theta_{qi}^2 - v_q\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$,

(iii) $p_i^* = \frac{v_q(q_i^* - q_j^*) + v_e(e_i^* - e_j^*) + w_i\mu_m + 2w_j\mu_m + (\theta_{qj}q_j^*)^2 + (\theta_{ej}e_j^*)^2 + \delta(\theta_{qj}q_j^*)(\theta_{ej}e_j^*) + 2(\theta_{qi}q_i^*)^2 + 2(\theta_{ei}e_i^*)^2 + 2\delta(\theta_{qi}q_i^*)(\theta_{ei}e_i^*)}{3} + v_l$ *where* $j = 3 - i$.

Comparing Proposition A.2.1 to Proposition 1, we find that the product design decisions (q_i^*, e_i^*) remain unchanged, indicating that the production inefficiency has little impact on product development process. However, the product price p_i^* is different under the two scenarios. More interestingly, it is straightforward to verify that the product price is higher when the product capacity is limited (i.e., when $w_i > 0$). The reason is as follows. With limited production capacity, customers have to wait to receive the product and the wait time depends on the total demand. Thus, there are negative externalities among all the buyers; specifically, a higher demand leads to a longer waiting time and thus a lower customer utility. In this case, the firm could charge a higher product price to curb demand, since customers are willing to pay more in expectation of a shorter production lead time.

As before, we assume that interior solution exists in Proposition A.2.1, i.e., $\delta < \min\left\{\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right\}$ and $v_l > \underline{v}_l$.

Proposition A.2.2.

(i) $\frac{\partial q_i^*}{\partial \theta_{ei}} > 0$, $\frac{\partial e_i^*}{\partial \theta_{ei}} < 0$, $\frac{\partial q_i^*}{\partial v_e} < 0$, *and* $\frac{\partial e_i^*}{\partial v_e} > 0$;

(ii) $\frac{\partial p_i^*}{\partial \theta_{ei}} < 0$ *while* $\frac{\partial p_i^*}{\partial v_e} > 0$ *if and only if* $e_i^* > e_j^*/4$ *where* $j = 3 - i$.

Next, we study the impact of government policies. Define the expected environmental benefit as

$$B = \mathbb{E}_M(D_1 b(e_1) + D_2 b(e_2)).$$

We can verify that the following six propositions are carried over from Propositions 3 to 8 in the main model.

Proposition A.2.3. *With government R&D support,* $e_i^r > e_i^*$ *for* $i = 1, 2$.

Proposition A.2.4. *With government R&D support,* $B^r < B^*$ *if the following conditions hold:* (i) $\theta_{e1} < \theta_{e2}$*;* (ii) $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$*;* (iii) $\bar{\theta}_e > \theta_{e2}^r \geq \theta_{e1}^r > \underline{\theta_e}$*; and* (iv) $v_l < \bar{v}_l$ *where* $\underline{\theta_q}$*,* $\bar{\theta}_e$*,* $\underline{\theta_e}$ *and* $\bar{v}_l$ *are thresholds.*

Proposition A.2.5. *Suppose* $\theta_{e1} < \theta_{e2}$ *and* $\theta_{q1} = \theta_{q2}$*. Then government R&D support will decrease firm 1's profit while increasing firm 2's profit, i.e.,* $\pi_1^r < \pi_1^*$ *and* $\pi_2^r > \pi_2^*$*, if* $\theta_{e2}^r < \tilde{\theta}_e$ *for some* $\tilde{\theta}_e > \theta_{e1}^r$.

Proposition A.2.6. *With government sales incentive,* $e_i^s \geq e_i^*$.

Proposition A.2.7. *With government sales incentive,* $B^s \geq B^*$.

Proposition A.2.8. *Suppose* $\theta_{e1} < \theta_{e2}$ *and* $\theta_{q1} = \theta_{q2}$*. Then government sales subsidies will increase firm 1's profit while decreasing firm 2's profit, i.e.,* $\pi_1^s \geq \pi_1^*$ *and* $\pi_2^s \leq \pi_2^*$.

B Proofs

Proof of Proposition 1: Let us first ignore the constraints that $q_i > 0, e_i > 0, d_i > 0$. We can solve the game using backward deduction: Given (e_1, q_1, e_2, q_2), both firms decide price. The best response functions are:

$$p_1 = \frac{v_e(e_1-e_2)+v_q(q_1-q_2)+p_2+v_l+(\theta_{e1}e_1)^2+(\theta_{q1}q_1)^2+\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)}{2}$$
$$p_2 = \frac{-v_e(e_1-e_2)-v_q(q_1-q_2)+p_1+v_l+(\theta_{e2}e_2)^2+(\theta_{q2}q_2)^2+\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)}{2}$$

Thus, the equilibrium prices in this subgame are

$$p_1^* = \frac{v_e(e_1-e_2)+v_q(q_1-q_2)+(\theta_{e2}e_2)^2+(\theta_{q2}q_2)^2+\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)+2(\theta_{e1}e_1)^2+2(\theta_{q1}q_1)^2+2\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)}{3} + v_l$$
$$p_2^* = \frac{-v_e(e_1-e_2)-v_q(q_1-q_2)+(\theta_{e1}e_1)^2+(\theta_{q1}q_1)^2+\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)+2(\theta_{e2}e_2)^2+2(\theta_{q2}q_2)^2+2\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)}{3} + v_l$$

Back to the stage when the two firms make product design decisions. Given p_1^* and p_2^*, the two profit functions can be expressed as follows:

$$\pi_1 = \frac{(\frac{\Delta_1-\Delta_2}{3}+v_l)^2}{2v_l}$$
$$\pi_2 = \frac{(\frac{\Delta_2-\Delta_1}{3}+v_l)^2}{2v_l}$$

where $\Delta_1 = v_e e_1 + v_q q_1 - (\theta_{e1}e_1)^2 - (\theta_{q1}q_1)^2 - \delta(\theta_{e1}e_1)(\theta_{q1}q_1)$ and $\Delta_2 = v_e e_2 + v_q q_2 - (\theta_{e2}e_2)^2 - (\theta_{q2}q_2)^2 - \delta(\theta_{e2}e_2)(\theta_{q2}q_2)$. Note the two firms' product design decisions are separable. Thus, each solves the following optimization problem separately:

$$\max_{e_i,q_i} v_e e_i + v_q q_i - (\theta_{ei}e_i)^2 - (\theta_{qi}q_i)^2 - \delta(\theta_{ei}e_i)(\theta_{qi}q_i), \text{ for } i = 1, 2$$

which gives the following solution:

$$q_i^* = \frac{2v_q\theta_{ei}^2 - v_e\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$$
$$e_i^* = \frac{2v_e\theta_{qi}^2 - v_q\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$$

As a result, each firm's demand in equilibrium is:

$$d_1^* = \frac{\Delta_1^* - \Delta_2^*}{6v_l} + \frac{1}{2}$$
$$d_2^* = \frac{\Delta_2^* - \Delta_1^*}{6v_l} + \frac{1}{2}$$

where $\Delta_1^* = v_e e_1^* + v_q q_1^* - (\theta_{e1} e_1^*)^2 - (\theta_{q1} q_1^*)^2 - \delta(\theta_{e1} e_1^*)(\theta_{q1} q_1^*)$ and $\Delta_2^* = v_e e_2^* + v_q q_2^* - (\theta_{e2} e_2^*)^2 - (\theta_{q2} q_2^*)^2 - \delta(\theta_{e2} e_2^*)(\theta_{q2} q_2^*)$.

Finally, let's check the constraints. Note $q_i^* > 0, e_i^* > 0 \Leftrightarrow \delta < \min\left(\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right)$. Also, $d_i^* > 0 \Leftrightarrow v_l > \frac{|\Delta_1^* - \Delta_2^*|}{3}$. □

Proof of Proposition 2: For point (i): $\frac{\partial q_i^*}{\partial \theta_{ei}} = \frac{v_e \delta \theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2} > 0$, $\frac{\partial e_i^*}{\partial \theta_{ei}} = \frac{-4v_e\frac{\theta_{qi}^2}{\theta_{ei}} + v_q\delta\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2} < 0$, $\frac{\partial p_i^*}{\partial \theta_{ei}} = \frac{-\frac{10}{3}v_e^2\frac{\theta_{qi}^2}{\theta_{ei}} + \frac{5}{3}v_e v_q \delta\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2} < 0$ where the inequalities are because $\delta < \min\left(\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right)$.

For point (ii): $\frac{\partial q_i^*}{\partial v_e} = \frac{-\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2} < 0$, $\frac{\partial e_i^*}{\partial v_e} = \frac{2\theta_{qi}^2}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2} > 0$. Moreover, note that $p_i^* = -\frac{2\Delta_i^*}{3} - \frac{\Delta_j^*}{3} + v_e e_i^* + v_q q_i^* + v_l$. By Envelope Theorem, we can find $\frac{\partial \Delta_i^*}{\partial v_e} = e_i^*$. Thus, we have $\frac{\partial p_i^*}{\partial v_e} = -\frac{2}{3}e_i^* - \frac{1}{3}e_j^* + 2e_i^* = \frac{4}{3}e_i^* - \frac{1}{3}e_j^*$ and thus $\frac{\partial p_i^*}{\partial v_e} > 0 \Leftrightarrow e_i^* > \frac{e_j^*}{4}$. □

Lemma 1. *With government R&D support, the product qualities in equilibrium are given as follows:*

(i) $q_i^r = \frac{2v_q\theta_{ei}^{r2} - v_e\delta\theta_{ei}^r\theta_{qi}}{4\theta_{ei}^{r2}\theta_{qi}^2 - \delta^2\theta_{ei}^{r2}\theta_{qi}^2}$,

(ii) $e_i^r = \frac{2v_e\theta_{qi}^2 - v_q\delta\theta_{ei}^r\theta_{qi}}{4\theta_{ei}^{r2}\theta_{qi}^2 - \delta^2\theta_{ei}^{r2}\theta_{qi}^2}$,

Proof of Lemma 1: The proof is similar to that of Proposition 1 (simply replacing θ_{ei} with θ_{ei}^r) and thus omitted. □

Proof of Proposition 3: Note $\frac{\partial e_i^*}{\partial \theta_{ei}} < 0$ (see Proposition 2), and $\frac{\partial e_i^*}{\partial \theta_{ej}} = 0$. Since $\theta_{ei}^r < \theta_{ei}$, we have $e_i^r > e_i^*$. □

Proof of Proposition 4: Note

$$\begin{aligned} B^r - B^* &= d_1^r b\left(e_1^r\right) + d_2^r b\left(e_2^r\right) - d_1^* b\left(e_1^*\right) - d_2^* b\left(e_2^*\right) \\ &= \left(\frac{\Delta_1^r - \Delta_2^r}{6v_l} + \frac{1}{2}\right) b\left(e_1^r\right) + \left(\frac{\Delta_2^r - \Delta_1^r}{6v_l} + \frac{1}{2}\right) b\left(e_2^r\right) \\ &\quad - \left(\frac{\Delta_1^* - \Delta_2^*}{6v_l} + \frac{1}{2}\right) b\left(e_1^*\right) + \left(\frac{\Delta_2^* - \Delta_1^*}{6v_l} + \frac{1}{2}\right) b\left(e_2^*\right) \end{aligned}$$

where Δ_1^* and Δ_2^* are defined in the proof of Proposition 1, and $\Delta_1^r = v_e e_1^r + v_q q_1^r - (\theta_{e1}^r e_1^r)^2 - (\theta_{q1} q_1^r)^2 - \delta(\theta_{e2}^r e_1^r)(\theta_{q1} q_1^r)$, $\Delta_2^r = v_e e_2^r + v_q q_2^r - (\theta_{e2}^r e_2^r)^2 - (\theta_{q2} q_2^r)^2 - \delta(\theta_{e2}^r e_2^r)(\theta_{q2} q_2^r)$.

Suppose $\theta_{e1}^r = \theta_{e1}$ and $\theta_{e1} < \theta_{e2}$.

Since $\theta^r_{e1} = \theta_{e1}$, we have $e^*_1 = e^r_1$ and $q^*_1 = q^r_1$, and thus $\Delta^*_1 = \Delta^r_1$. Hence,

$$B^r - B^* = \frac{b(e^r_2) - b(e^*_2)}{2} - \frac{(\Delta^r_2 - \Delta^*_2)b(e^*_1) + (\Delta^*_1 - \Delta^r_2)b(e^r_2) - (\Delta^*_1 - \Delta^*_2)b(e^*_2)}{6v_l}$$

By Envelope Theorem, $\frac{\partial \Delta^*_2}{\partial \theta_{q2}} = -2\theta_{q2}q^{*2}_2 - \delta\theta_{e2}e^*_2 q^*_2 < 0$. Also, because $\theta_{e1} < \theta_{e2}$, if $\theta_{q2} = \theta_{q1}$, then $\Delta^*_2 < \Delta^*_1$. Thus, there exists $\underline{\theta_q} < \theta_{q1}$ such that $\Delta^*_1 - \Delta^*_2 > 0$ if $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$. Also, if $\theta^r_{e2} = \theta_{e1} = \theta^r_{e1}$, we have $e^r_2 < e^r_1 = e^*_1$. Thus, there exists $\bar{\theta}_e > \theta^r_{e1}$ such that $e^r_2 < e^*_1$ if $\theta^r_{e2} < \bar{\theta}_e$.

Therefore, if $v_l = \bar{v}_l = \frac{\Delta^*_1 - \Delta^*_2}{3}$, $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$ and $\theta^r_{e2} < \bar{\theta}_e$, then $B^r - B^* = \frac{\Delta^r_2 - \Delta^*_2}{2(\Delta^*_1 - \Delta^*_2)}(b(e^r_2) - b(e^*_1)) < 0$ where the inequality is because $\Delta^*_1 - \Delta^*_2 > 0$, $\Delta^r_2 - \Delta^*_2 > 0$ and $e^r_2 < e^*_1$.

Finally, since $B^r - B^*$ is continuous in both θ^r_{e1} and v_l, we can conclude the result. □

Proof of Proposition 5: Based on the equilibrium outcomes, we have

$$\pi^*_1 = \frac{(\frac{\Delta^*_1 - \Delta^*_2}{3} + v_l)^2}{2v_l}$$
$$\pi^*_2 = \frac{(\frac{\Delta^*_2 - \Delta^*_1}{3} + v_l)^2}{2v_l}$$
$$\pi^r_1 = \frac{(\frac{\Delta^r_1 - \Delta^r_2}{3} + v_l)^2}{2v_l}$$
$$\pi^r_2 = \frac{(\frac{\Delta^r_2 - \Delta^r_1}{3} + v_l)^2}{2v_l}$$

Since $\theta_{e1} < \theta_{e2}$, $\theta_{q1} = \theta_{q2}$, we have $\Delta^*_1 > \Delta^*_2$. If $\theta^r_{e2} = \theta^r_{e1}$, then $e^r_1 = e^r_2$, $q^r_1 = q^r_2$, thus $\Delta^r_1 = \Delta^r_2$ and $\pi^*_1 > \pi^r_1$ and $\pi^*_2 < \pi^r_2$. Since $\Delta^r_1 - \Delta^r_2$ is continuous in θ^r_{e2}, we can conclude the result. □

Proof of Proposition 6: Using a similar proof as in Proposition 1, we can find that

$$(q^s_i, e^s_i) = \arg\max v_e e_i + v_q q_i - (\theta_{ei}e_i)^2 - (\theta_{qi}q_i)^2 - \delta(\theta_{ei}e_i)(\theta_{qi}q_i) + s(e_i) \tag{9}$$

Suppose $e^s_i < e^*_i$. Then, since $s(e_i)$ is an increasing function, we have

$$\begin{aligned} & v_e e^s_i + v_q q^s_i - (\theta_{ei}e^s_i)^2 - (\theta_{qi}q^s_i)^2 - \delta(\theta_{ei}e^s_i)(\theta_{qi}q^s_i) + s(e^s_i) \\ & \leq v_e e^s_i + v_q q^s_i - (\theta_{ei}e^s_i)^2 - (\theta_{qi}q^s_i)^2 - \delta(\theta_{ei}e^s_i)(\theta_{qi}q^s_i) + s(e^*_i) \\ & < v_e e^*_i + v_q q^*_i - (\theta_{ei}e^*_i)^2 - (\theta_{qi}q^*_i)^2 - \delta(\theta_{ei}e^*_i)(\theta_{qi}q^*_i) + s(e^*_i) \end{aligned} \tag{10}$$

where the last inequality is because

$$(q^*_i, e^*_i) = \arg\max v_e e_i + v_q q_i - (\theta_{ei}e_i)^2 - (\theta_{qi}q_i)^2 - \delta(\theta_{ei}e_i)(\theta_{qi}q_i)$$

Note that the inequality (10) contradicts (9). Thus, we must have $e_i^s \geq e_i^*$. □

Lemma 2. *With government sales incentive, if* $e_2^s < e_1^*$*, then* $\Delta_1^s - \Delta_2^s > \Delta_1^* - \Delta_2^*$*, where* $\Delta_1^s = v_e e_1^s + v_q q_1^s - (\theta_{e1} e_1^s)^2 - (\theta_{q1} q_1^s)^2 - \delta(\theta_{e1} e_1^s)(\theta_{q1} q_1^s) + s(e_1^s)$ *and* $\Delta_2^s = v_e e_2^s + v_q q_2^s - (\theta_{e2} e_2^s)^2 - (\theta_{q2} q_2^s)^2 - \delta(\theta_{e2} e_2^s)(\theta_{q2} q_2^s) + s(e_2^s)$.

Proof of Lemma 2: Note that $\Delta_1^* + s(e_2^s) - \Delta_2^s = \Delta_1^* - (\Delta_2^s - s(e_2^s)) > \Delta_1^* - \Delta_2^*$, where the inequality is because $(q_2^*, e_2^*) = \arg\max v_e e_2 + v_q q_2 - (\theta_{e2} e_2)^2 - (\theta_{q2} q_2)^2 - \delta(\theta_{e2} e_2)(\theta_{q2} q_2)$. Since $e_2^s < e_1^*$, we have $s(e_1^*) \geq s(e_2^s)$. Since $(q_1^s, e_1^s) = \arg\max v_e e_1 + v_q q_1 - (\theta_{e1} e_1)^2 - (\theta_{q1} q_1)^2 - \delta(\theta_{e1} e_1)(\theta_{q1} q_1) + s(e_1)$, we have $\Delta_1^s > \Delta_1^* + s(e_1^*)$. Then, $\Delta_1^s > \Delta_1^* + s(e_2^s)$ and thus $\Delta_1^s - \Delta_2^s > \Delta_1^* - \Delta_2^*$. □

Proof of Proposition 7: Without loss of generality, suppose $e_1^* \geq e_2^*$.

If $e_2^s < e_1^*$: $B^s - B^* = (b(e_1^s) - b(e_1^*))\, d_1^* + (b(e_2^s) - b(e_2^*))(1 - d_1^*) + (b(e_1^s) - b(e_2^s))(d_1^s - d_1^*)$. Note that $d_1^* = \frac{\Delta_1^* - \Delta_2^*}{6v_l} + \frac{1}{2}$ and $d_1^s = \frac{\Delta_1^s - \Delta_2^s}{6v_l} + \frac{1}{2}$. Then, by Lemma 2, we have $d_1^s > d_1^*$. By Proposition 6, we have $e_1^s \geq e_1^*$, $e_2^s \geq e_2^*$ and thus $e_1^s \geq e_2^s$. Since $b(e_i)$ is an increasing function, we have $B^s - B^* \geq 0$.

If $e_2^s \geq e_1^*$: Since $e_2^s \geq e_2^*$, we have $\min(e_1^s, e_2^s) \geq \max(e_1^*, e_2^*)$. Thus, $B^s \geq b(\min(e_1^s, e_2^s)) \geq b(\max(e_1^*, e_2^*)) \geq B^*$. □

Proof of Proposition 8: By the Envelope Theorem, we have $\frac{\partial(\Delta_i^s - \Delta_i^*)}{\partial \theta_{ei}} = -\frac{v_e}{\theta_{ei}}(e_i^s - e_i^*) \leq 0$, where the inequality is due to Proposition 6. Thus, if $\theta_{e1} < \theta_{e2}$, $\theta_{q1} = \theta_{q2}$ and $\delta = \delta$, we have $\Delta_2^s - \Delta_2^* \leq \Delta_1^s - \Delta_1^*$, i.e., $\Delta_1^s - \Delta_2^s \geq \Delta_1^* - \Delta_2^*$. It is easy to verify that $\Delta_1^* > \Delta_2^*$. Thus, we have $\Delta_1^s - \Delta_2^s \geq \Delta_1^* - \Delta_2^* \geq 0$. Note that $\pi_i^* = \frac{(\frac{\Delta_i^* - \Delta_j^*}{3} + v_l)^2}{2v_l}$ and $\pi_i^s = \frac{(\frac{\Delta_i^s - \Delta_j^s}{3} + v_l)^2}{2v_l}$. Thus, we can conclude the result. □

Proof of Proposition A.1.1: Let's first ignore the constraints that $q_i > 0, e_i > 0, d_i > 0$. We can solve the game using backward deduction: Given (e_1, q_1, e_2, q_2), both firms decide price. The best response functions are:

$$p_1 = \frac{\mu_e(e_1 - e_2) + \mu_q(q_1 - q_2) + p_2 + v_l + (\theta_{e1} e_1)^2 + (\theta_{q1} q_1)^2 + \delta_1(\theta_{e1} e_1)(\theta_{q1} q_1)}{2}$$
$$p_2 = \frac{-\mu_e(e_1 - e_2) - \mu_q(q_1 - q_2) + p_1 + v_l + (\theta_{e2} e_2)^2 + (\theta_{q2} q_2)^2 + \delta_2(\theta_{e2} e_2)(\theta_{q2} q_2)}{2}$$

Thus, the equilibrium prices in this subgame are

$$p_1^* = \frac{\mu_e(e_1-e_2)+\mu_q(q_1-q_2)+(\theta_{e2}e_2)^2+(\theta_{q2}q_2)^2+\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)+2(\theta_{e1}e_1)^2+2(\theta_{q1}q_1)^2+2\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)}{3} + v_l$$
$$p_2^* = \frac{-\mu_e(e_1-e_2)-\mu_q(q_1-q_2)+(\theta_{e1}e_1)^2+(\theta_{q1}q_1)^2+\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)+2(\theta_{e2}e_2)^2+2(\theta_{q2}q_2)^2+2\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)}{3} + v_l$$

Back to the stage when the two firms make product design decisions. Given p_1^* and p_2^*, the two profit functions can be expressed as follows:

$$\pi_1 = \frac{(\frac{\Delta_1-\Delta_2}{3}+v_l)^2}{2v_l}$$
$$\pi_2 = \frac{(\frac{\Delta_2-\Delta_1}{3}+v_l)^2}{2v_l}$$

where $\Delta_1 = \mu_e e_1 + \mu_q q_1 - (\theta_{e1}e_1)^2 - (\theta_{q1}q_1)^2 - \delta(\theta_{e1}e_1)(\theta_{q1}q_1)$ and $\Delta_2 = \mu_e e_2 + \mu_q q_2 - (\theta_{e2}e_2)^2 - (\theta_{q2}q_2)^2 - \delta(\theta_{e2}e_2)(\theta_{q2}q_2)$. Note the two firms' product design decisions are separable. Thus, each solves the following optimization problem separately:

$$\max_{e_i,q_i} \mu_e e_i + \mu_q q_i - (\theta_{ei}e_i)^2 - (\theta_{qi}q_i)^2 - \delta(\theta_{ei}e_i)(\theta_{qi}q_i), \text{ for } i = 1, 2$$

which gives the following solution:

$$q_i^* = \frac{2\mu_q\theta_{ei}^2 - \mu_e\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$$
$$e_i^* = \frac{2\mu_e\theta_{qi}^2 - \mu_q\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$$

As a result, each firm's demand in equilibrium is:

$$d_1^* = \frac{\Delta_1^*-\Delta_2^*}{6v_l} + \frac{1}{2}$$
$$d_2^* = \frac{\Delta_2^*-\Delta_1^*}{6v_l} + \frac{1}{2}$$

where $\Delta_1^* = \mu_e e_1^* + \mu_q q_1^* - (\theta_{e1}e_1^*)^2 - (\theta_{q1}q_1^*)^2 - \delta(\theta_{e1}e_1^*)(\theta_{q1}q_1^*)$ and $\Delta_2^* = \mu_e e_2^* + \mu_q q_2^* - (\theta_{e2}e_2^*)^2 - (\theta_{q2}q_2^*)^2 - \delta(\theta_{e2}e_2^*)(\theta_{q2}q_2^*)$.

Finally, let's check the constraints. Note $q_i^* > 0, e_i^* > 0 \Leftrightarrow \delta < \min\left(\frac{2\mu_q\theta_{ei}}{\mu_e\theta_{qi}}, \frac{2\mu_e\theta_{qi}}{\mu_q\theta_{ei}}\right)$. Also, $d_i^* > 0 \Leftrightarrow v_l > \frac{|\Delta_1^*-\Delta_2^*|}{3}$. □

Proof of Proposition A.1.3: Note $\frac{\partial e_i^*}{\partial \theta_{ei}} < 0$, and $\frac{\partial e_i^*}{\partial \theta_{ej}} = 0$. Since $\theta_{eg} < \theta_{ei}$, we have $e_i^r > e_i^*$. □

Proof of Proposition A.1.4: Note

$$\begin{aligned} B^r - B^* &= d_1^r b(e_1^r) + d_2^r b(e_2^r) - d_1^* b(e_1^*) - d_2^* b(e_2^*) \\ &= \left(\frac{\Delta_1^r - \Delta_2^r}{6v_l} + \frac{1}{2}\right) b(e_1^r) + \left(\frac{\Delta_2^r - \Delta_1^r}{6v_l} + \frac{1}{2}\right) b(e_2^r) \\ &\quad - \left(\frac{\Delta_1^* - \Delta_2^*}{6v_l} + \frac{1}{2}\right) b(e_1^*) + \left(\frac{\Delta_2^* - \Delta_1^*}{6v_l} + \frac{1}{2}\right) b(e_2^*) \end{aligned}$$

where Δ_1^* and Δ_2^* are defined in the proof of Proposition A.1.1, and $\Delta_1^r = \mu_e e_1^r + \mu_q q_1^r - (\theta_{e1}^r e_1^r)^2 - (\theta_{q1} q_1^r)^2 - \delta(\theta_{e2}^r e_1^r)(\theta_{q1} q_1^r)$, $\Delta_2^r = \mu_e e_2^r + \mu_q q_2^r - (\theta_{e2}^r e_2^r)^2 - (\theta_{q2} q_2^r)^2 - \delta(\theta_{e2}^r e_2^r)(\theta_{q2} q_2^r)$.

Suppose $\theta_{e1}^r = \theta_{e1}$ and $\theta_{e1} < \theta_{e2}$.

Since $\theta_{e1}^r = \theta_{e1}$, we have $e_1^* = e_1^r$ and $q_1^* = q_1^r$, and thus $\Delta_1^* = \Delta_1^r$. Hence,

$$B^r - B^* = \frac{b(e_2^r) - b(e_2^*)}{2} - \frac{(\Delta_2^r - \Delta_2^*) b(e_1^*) + (\Delta_1^* - \Delta_2^r) b(e_2^r) - (\Delta_1^* - \Delta_2^*) b(e_2^*)}{6v_l}$$

By Envelope Theorem, $\frac{\partial \Delta_2^*}{\partial \theta_{q2}} = -2\theta_{q2} q_2^{*2} - \delta \theta_{e2} e_2^* q_2^* < 0$. Also, because $\theta_{e1} < \theta_{e2}$, if $\theta_{q2} = \theta_{q1}$, then $\Delta_2^* < \Delta_1^*$. Thus, there exists $\underline{\theta_q} < \theta_{q1}$ such that $\Delta_1^* - \Delta_2^* > 0$ if $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$. Also, if $\theta_{e2}^r = \theta_{e1} = \theta_{e1}^r$, we have $e_2^r < e_1^r = e_1^*$. Thus, there exists $\bar{\theta}_e > \theta_{e1}^r$ such that $e_2^r < e_1^*$ if $\theta_{e2}^r < \bar{\theta}_e$.

Therefore, if $v_l = \bar{v}_l = \frac{\Delta_1^* - \Delta_2^*}{3}$, $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$ and $\theta_{e2}^r < \bar{\theta}_e$, then $B^r - B^* = \frac{\Delta_2^r - \Delta_2^*}{2(\Delta_1^* - \Delta_2^*)}(b(e_2^r) - b(e_1^*)) < 0$ where the inequality is because $\Delta_1^* - \Delta_2^* > 0$, $\Delta_2^r - \Delta_2^* > 0$ and $e_2^r < e_1^*$.

Finally, since $B^r - B^*$ is continuous in both θ_{e1}^r and v_l, we can conclude the result. □

Proof of Proposition A.1.5: Based on the equilibrium outcomes, we have

$$\begin{aligned} \pi_1^* &= \frac{(\frac{\Delta_1^* - \Delta_2^*}{3} + v_l)^2}{2v_l} \\ \pi_2^* &= \frac{(\frac{\Delta_2^* - \Delta_1^*}{3} + v_l)^2}{2v_l} \\ \pi_1^r &= \frac{(\frac{\Delta_1^r - \Delta_2^r}{3} + v_l)^2}{2v_l} \\ \pi_2^r &= \frac{(\frac{\Delta_2^r - \Delta_1^r}{3} + v_l)^2}{2v_l} \end{aligned}$$

Since $\theta_{e1} < \theta_{e2}$, $\theta_{q1} = \theta_{q2}$, we have $\Delta_1^* > \Delta_2^*$. If $\theta_{e2}^r = \theta_{e1}^r$, then $e_1^r = e_2^r$, $q_1^r = q_2^r$, thus $\Delta_1^r = \Delta_2^r$ and $\pi_1^* > \pi_1^r$ and $\pi_2^* < \pi_2^r$. Since $\Delta_1^r - \Delta_2^r$ is continuous in θ_{e2}^r, we can conclude the result. □

Proof of Proposition A.1.6: Using a similar proof as in Proposition A.1.1, we can find that

$$(q_i^s, e_i^s) = \arg\max \mu_e e_i + \mu_q q_i - (\theta_{ei} e_i)^2 - (\theta_{qi} q_i)^2 - \delta(\theta_{ei} e_i)(\theta_{qi} q_i) + s(e_i) \tag{11}$$

Suppose $e_i^s < e_i^*$. Then, since $s(e_i)$ is an increasing function, we have

$$\begin{aligned} &\mu_e e_i^s + \mu_q q_i^s - (\theta_{ei} e_i^s)^2 - (\theta_{qi} q_i^s)^2 - \delta(\theta_{ei} e_i^s)(\theta_{qi} q_i^s) + s(e_i^s) \\ &\leq \mu_e e_i^s + \mu_q q_i^s - (\theta_{ei} e_i^s)^2 - (\theta_{qi} q_i^s)^2 - \delta(\theta_{ei} e_i^s)(\theta_{qi} q_i^s) + s(e_i^*) \\ &< \mu_e e_i^* + \mu_q q_i^* - (\theta_{ei} e_i^*)^2 - (\theta_{qi} q_i^*)^2 - \delta(\theta_{ei} e_i^*)(\theta_{qi} q_i^*) + s(e_i^*) \end{aligned} \tag{12}$$

where the last inequality is because

$$(q_i^*, e_i^*) = \arg\max \mu_e e_i + \mu_q q_i - (\theta_{ei} e_i)^2 - (\theta_{qi} q_i)^2 - \delta(\theta_{ei} e_i)(\theta_{qi} q_i)$$

Note that the inequality (12) contradicts (11). Thus, we must have $e_i^s \geq e_i^*$. □

Lemma 3. *With government sales incentive, if* $e_2^s < e_1^*$*, then* $\Delta_1^s - \Delta_2^s > \Delta_1^* - \Delta_2^*$*, where* $\Delta_1^s = \mu_e e_1^s + \mu_q q_1^s - (\theta_{e1} e_1^s)^2 - (\theta_{q1} q_1^s)^2 - \delta(\theta_{e1} e_1^s)(\theta_{q1} q_1^s) + s(e_1^s)$ *and* $\Delta_2^s = \mu_e e_2^s + \mu_q q_2^s - (\theta_{e2} e_2^s)^2 - (\theta_{q2} q_2^s)^2 - \delta(\theta_{e2} e_2^s)(\theta_{q2} q_2^s) + s(e_2^s)$.

Proof of Lemma 3: Note that $\Delta_1^* + s(e_2^s) - \Delta_2^s = \Delta_1^* - (\Delta_2^s - s(e_2^s)) > \Delta_1^* - \Delta_2^*$, where the inequality is because $(q_2^*, e_2^*) = \arg\max \mu_e e_2 + \mu_q q_2 - (\theta_{e2} e_2)^2 - (\theta_{q2} q_2)^2 - \delta(\theta_{e2} e_2)(\theta_{q2} q_2)$. Since $e_2^s < e_1^*$, we have $s(e_1^*) \geq s(e_2^s)$. Since $(q_1^s, e_1^s) = \arg\max \mu_e e_1 + \mu_q q_1 - (\theta_{e1} e_1)^2 - (\theta_{q1} q_1)^2 - \delta(\theta_{e1} e_1)(\theta_{q1} q_1) + s(e_1)$, we have $\Delta_1^s > \Delta_1^* + s(e_1^*)$. Then, $\Delta_1^s > \Delta_1^* + s(e_2^s)$ and thus $\Delta_1^s - \Delta_2^s > \Delta_1^* - \Delta_2^*$. □

Proof of Proposition A.1.7: Without loss of generality, suppose $e_1^* \geq e_2^*$.

If $e_2^s < e_1^*$: $B^s - B^* = (b(e_1^s) - b(e_1^*)) d_1^* + (b(e_2^s) - b(e_2^*))(1 - d_1^*) + (b(e_1^s) - b(e_2^s))(d_1^s - d_1^*)$. Note that $d_1^* = \frac{\Delta_1^* - \Delta_2^*}{6v_l} + \frac{1}{2}$ and $d_1^s = \frac{\Delta_1^s - \Delta_2^s}{6v_l} + \frac{1}{2}$. Then, by Lemma 3, we have $d_1^s > d_1^*$. By Proposition A.1.6, we have $e_1^s \geq e_1^*$, $e_2^s \geq e_2^*$ and thus $e_1^s \geq e_2^s$. Since $b(e_i)$ is an increasing function, we have $B^s - B^* \geq 0$.

If $e_2^s \geq e_1^*$: Since $e_2^s \geq e_2^*$, we have $\min(e_1^s, e_2^s) \geq \max(e_1^*, e_2^*)$. Thus, $B^s \geq b(\min(e_1^s, e_2^s)) \geq b(\max(e_1^*, e_2^*)) \geq B^*$. □

Proof of Proposition A.1.8: By Envelope Theorem, we have $\frac{\partial(\Delta_i^s - \Delta_i^*)}{\partial\theta_{ei}} = -\frac{v_e}{\theta_{ei}}(e_i^s - e_i^*) \leq 0$, where the inequality is due to Proposition A.1.6. Thus, if $\theta_{e1} < \theta_{e2}$, $\theta_{q1} = \theta_{q2}$ and $\delta = \delta$, we have

$\Delta_2^s - \Delta_2^* \leq \Delta_1^s - \Delta_1^*$, i.e., $\Delta_1^s - \Delta_2^s \geq \Delta_1^* - \Delta_2^*$. It is easy to verify that $\Delta_1^* > \Delta_2^*$. Thus, we have $\Delta_1^s - \Delta_2^s \geq \Delta_1^* - \Delta_2^* \geq 0$. Note that $\pi_i^* = \frac{(\frac{\Delta_i^* - \Delta_j^*}{3} + v_l)^2}{2v_l}$ and $\pi_i^s = \frac{(\frac{\Delta_i^s - \Delta_j^s}{3} + v_l)^2}{2v_l}$. Thus, we can conclude the result. $\square$

Proof of Proposition A.2.1: Let's first ignore the constraints that $q_i > 0, e_i > 0, \mathbb{E}D_i > 0$. We can solve the game using backward deduction: Given (e_1, q_1, e_2, q_2), both firms decide price. The best response functions are:

$$p_1 = \frac{v_e(e_1-e_2)+v_q(q_1-q_2)+p_2+w_2\mu_m+v_l+(\theta_{e1}e_1)^2+(\theta_{q1}q_1)^2+\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)}{2}$$
$$p_2 = \frac{-v_e(e_1-e_2)-v_q(q_1-q_2)+p_1+w_1\mu_m+v_l+(\theta_{e2}e_2)^2+(\theta_{q2}q_2)^2+\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)}{2}$$

Thus, the equilibrium prices in this subgame are

$$p_1^* = \frac{v_e(e_1-e_2)+v_q(q_1-q_2)+w_1\mu_m+2w_2\mu_m+(\theta_{e2}e_2)^2+(\theta_{q2}q_2)^2+\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)+2(\theta_{e1}e_1)^2+2(\theta_{q1}q_1)^2+2\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)}{3} + v_l$$
$$p_2^* = \frac{-v_e(e_1-e_2)-v_q(q_1-q_2)+w_2\mu_m+2w_1\mu_m+(\theta_{e1}e_1)^2+(\theta_{q1}q_1)^2+\delta_1(\theta_{e1}e_1)(\theta_{q1}q_1)+2(\theta_{e2}e_2)^2+2(\theta_{q2}q_2)^2+2\delta_2(\theta_{e2}e_2)(\theta_{q2}q_2)}{3} + v_l$$

Back to the stage when the two firms make product design decisions. Given p_1^* and p_2^*, the two profit functions can be expressed as follows:

$$\pi_1 = \frac{(\frac{\Delta_1-\Delta_2}{3} + \frac{w_1\mu_m+2w_2\mu_m}{3}+v_l)^2}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m$$
$$\pi_2 = \frac{(\frac{\Delta_2-\Delta_1}{3} + \frac{w_2\mu_m+2w_1\mu_m}{3}+v_l)^2}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m$$

where $\Delta_1 = v_e e_1 + v_q q_1 - (\theta_{e1}e_1)^2 - (\theta_{q1}q_1)^2 - \delta(\theta_{e1}e_1)(\theta_{q1}q_1)$ and $\Delta_2 = v_e e_2 + v_q q_2 - (\theta_{e2}e_2)^2 - (\theta_{q2}q_2)^2 - \delta(\theta_{e2}e_2)(\theta_{q2}q_2)$. Note the two firms' product design decisions are separable. Thus, each just need to solve the following optimization problem separately:

$$\max_{e_i, q_i} v_e e_i + v_q q_i - (\theta_{ei}e_i)^2 - (\theta_{qi}q_i)^2 - \delta(\theta_{ei}e_i)(\theta_{qi}q_i), \text{ for } i = 1, 2$$

which gives the following solution:

$$q_i^* = \frac{2v_q\theta_{ei}^2 - v_e\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$$
$$e_i^* = \frac{2v_e\theta_{qi}^2 - v_q\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2 - \delta^2\theta_{ei}^2\theta_{qi}^2}$$

As a result, each firm's expected demand in equilibrium is:

$$\mathbb{E}D_1^* = \frac{\frac{\Delta_1^*-\Delta_2^*}{3}+\frac{w_1\mu_m+2w_2\mu_m}{3}+v_l}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m$$
$$\mathbb{E}D_2^* = \frac{\frac{\Delta_2^*-\Delta_1^*}{3}+\frac{w_2\mu_m+2w_1\mu_m}{3}+v_l}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m$$

where $\Delta_1^* = v_e e_1^* + v_q q_1^* - (\theta_{e1}e_1^*)^2 - (\theta_{q1}q_1^*)^2 - \delta(\theta_{e1}e_1^*)(\theta_{q1}q_1^*)$ and $\Delta_2^* = v_e e_2^* + v_q q_2^* - (\theta_{e2}e_2^*)^2 - (\theta_{q2}q_2^*)^2 - \delta(\theta_{e2}e_2^*)(\theta_{q2}q_2^*)$.

Finally, let's check the constraints. Note $q_i^* > 0, e_i^* > 0 \Leftrightarrow \delta < \min\left(\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right)$. Also, $\mathbb{E}D_i^* > 0 \Leftrightarrow v_l > \max(\frac{\Delta_2^*-\Delta_1^*-w_1\mu_m-2w_2\mu_m}{3}, \frac{\Delta_1^*-\Delta_2^*-w_2\mu_m-2w_1\mu_m}{3})$. □

Proof of Proposition A.2.2: For point (i): $\frac{\partial q_i^*}{\partial \theta_{ei}} = \frac{v_e\delta\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2-\delta^2\theta_{ei}^2\theta_{qi}^2} > 0$, $\frac{\partial e_i^*}{\partial \theta_{ei}} = \frac{-4v_e\frac{\theta_{qi}^2}{\theta_{ei}}+v_q\delta\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2-\delta^2\theta_{ei}^2\theta_{qi}^2} < 0$, $\frac{\partial p_i^*}{\partial \theta_{ei}} = \frac{-\frac{10}{3}v_e^2\frac{\theta_{qi}^2}{\theta_{ei}}+\frac{5}{3}v_e v_q\delta\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2-\delta^2\theta_{ei}^2\theta_{qi}^2} < 0$ where the inequalities are because $\delta < \min\left(\frac{2v_q\theta_{ei}}{v_e\theta_{qi}}, \frac{2v_e\theta_{qi}}{v_q\theta_{ei}}\right)$.

For point (ii): $\frac{\partial q_i^*}{\partial v_e} = \frac{-\delta\theta_{ei}\theta_{qi}}{4\theta_{ei}^2\theta_{qi}^2-\delta^2\theta_{ei}^2\theta_{qi}^2} < 0$, $\frac{\partial e_i^*}{\partial v_e} = \frac{2\theta_{qi}^2}{4\theta_{ei}^2\theta_{qi}^2-\delta^2\theta_{ei}^2\theta_{qi}^2} > 0$. Moreover, note that $p_i^* = -\frac{2\Delta_i^*}{3} - \frac{\Delta_j^*}{3} + v_e e_i^* + v_q q_i^* + \frac{w_i\mu_m+2w_j\mu_m}{3} + v_l$. By Envelope Theorem, we can find $\frac{\partial \Delta_i^*}{\partial v_e} = e_i^*$. Thus, we have $\frac{\partial p_i^*}{\partial v_e} = -\frac{2}{3}e_i^* - \frac{1}{3}e_j^* + 2e_i^* = \frac{4}{3}e_i^* - \frac{1}{3}e_j^*$ and thus $\frac{\partial p_i^*}{\partial v_e} > 0 \Leftrightarrow e_i^* > \frac{e_j^*}{4}$. □

Lemma 4. *With government R&D support, the product qualities in equilibrium are given as follows:*

(i) $q_i^r = \frac{2v_q\theta_{eg}^2-v_e\delta\theta_{eg}\theta_{qi}}{4\theta_{eg}^2\theta_{qi}^2-\delta^2\theta_{eg}^2\theta_{qi}^2}$,

(ii) $e_i^r = \frac{2v_e\theta_{qi}^2-v_q\delta\theta_{eg}\theta_{qi}}{4\theta_{eg}^2\theta_{qi}^2-\delta^2\theta_{eg}^2\theta_{qi}^2}$,

Proof of Lemma 4: The proof is similar to that of Proposition A.2.1 and thus omitted. □

Proof of Proposition A.2.3: Note $\frac{\partial e_i^*}{\partial \theta_{ei}} < 0$ and $\frac{\partial e_i^*}{\partial \theta_{ej}} = 0$. Since $\theta_{eg} < \theta_{ei}$, we have $e_i^r > e_i^*$. □

Proof of Proposition A.2.4: Note

$$\begin{aligned} B^r - B^* &= d_1^r b(e_1^r) + d_2^r b(e_2^r) - d_1^* b(e_1^*) - d_2^* b(e_2^*) \\ &= \left(\frac{\frac{\Delta_1^r-\Delta_2^r}{3}+\frac{w_1\mu_m+2w_2\mu_m}{3}+v_l}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m\right) b(e_1^r) + \left(\frac{\frac{\Delta_2^r-\Delta_1^r}{3}+\frac{w_2\mu_m+2w_1\mu_m}{3}+v_l}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m\right) b(e_2^r) \\ &\quad - \left(\frac{\frac{\Delta_1^*-\Delta_2^*}{3}+\frac{w_1\mu_m+2w_2\mu_m}{3}+v_l}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m\right) b(e_1^*) + \left(\frac{\frac{\Delta_2^*-\Delta_1^*}{3}+\frac{w_2\mu_m+2w_1\mu_m}{3}+v_l}{2v_l+w_1\mu_m+w_2\mu_m}\mu_m\right) b(e_2^*) \end{aligned}$$

where Δ_1^* and Δ_2^* are defined in the proof of Proposition A.2.1, and $\Delta_1^r = v_e e_1^r + v_q q_1^r - (\theta_{e1}^r e_1^r)^2 - (\theta_{q1} q_1^r)^2 - \delta(\theta_{e2}^r e_1^r)(\theta_{q1} q_1^r)$, $\Delta_2^r = v_e e_2^r + v_q q_2^r - (\theta_{e2}^r e_2^r)^2 - (\theta_{q2} q_2^r)^2 - \delta(\theta_{e2}^r e_2^r)(\theta_{q2} q_2^r)$.

Suppose $\theta_{e1}^r = \theta_{e1}$ and $\theta_{e1} < \theta_{e2}$.

Since $\theta_{e1}^r = \theta_{e1}$, we have $e_1^* = e_1^r$ and $q_1^* = q_1^r$, and thus $\Delta_1^* = \Delta_1^r$. Hence,

$$B^r - B^* = \frac{(w_2\mu_m + 2w_1\mu_m + 3t)(b(e_2^r) - b(e_2^*))}{6v_l + 3w_1\mu_m + 3w_2\mu_m}\mu_m - \frac{(\Delta_2^r - \Delta_2^*)b(e_1^*) + (\Delta_1^* - \Delta_2^r)b(e_2^r) - (\Delta_1^* - \Delta_2^*)b(e_2^*)}{6v_l + 3w_1\mu_m + 3w_2\mu_m}\mu_m$$

By Envelope Theorem, $\frac{\partial \Delta_2^*}{\partial \theta_{q2}} = -2\theta_{q2} q_2^{*2} - \delta\theta_{e2} e_2^* q_2^* < 0$. Also, because $\theta_{e1} < \theta_{e2}$, if $\theta_{q2} = \theta_{q1}$, then $\Delta_2^* < \Delta_1^*$. Thus, there exists $\underline{\theta_q} < \theta_{q1}$ such that $\Delta_1^* - \Delta_2^* > 0$ if $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$. Also, if $\theta_{e2}^r = \theta_{e1} = \theta_{e1}^r$, we have $e_2^r < e_1^r = e_1^*$. Thus, there exists $\bar{\theta}_e > \theta_{e1}^r$ such that $e_2^r < e_1^*$ if $\theta_{e2}^r < \bar{\theta}_e$.

Therefore, if $v_l = \bar{v}_l = \frac{\Delta_1^* - \Delta_2^* - w_2\mu_m - 2w_1\mu_m}{3}$, $\theta_{q1} > \theta_{q2} > \underline{\theta_q}$ and $\theta_{e2}^r < \bar{\theta}_e$, then $B^r - B^* = \frac{(w_2\mu_m + 2w_1\mu_m + 3t)(\Delta_2^r - \Delta_2^*)}{(6v_l + 3w_1\mu_m + 3w_2\mu_m)(\Delta_1^* - \Delta_2^*)}(b(e_2^r) - b(e_1^*))\mu_m < 0$ where the inequality is because $\Delta_1^* - \Delta_2^* > 0$, $\Delta_2^r - \Delta_2^* > 0$ and $e_2^r < e_1^*$.

Finally, since $B^r - B^*$ is continuous in both θ_{e1}^r and v_l, we can conclude the result. □

Proof of Proposition A.2.5: Based on the equilibrium outcomes, we have

$$\begin{aligned}
\pi_1^* &= \frac{(\frac{\Delta_1^* - \Delta_2^*}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l)^2}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m \\
\pi_2^* &= \frac{(\frac{\Delta_2^* - \Delta_1^*}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l)^2}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m \\
\pi_1^r &= \frac{(\frac{\Delta_1^r - \Delta_2^r}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l)^2}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m \\
\pi_2^r &= \frac{(\frac{\Delta_2^r - \Delta_1^r}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l)^2}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m
\end{aligned}$$

Since $\theta_{e1} < \theta_{e2}$, $\theta_{q1} = \theta_{q2}$, we have $\Delta_1^* > \Delta_2^*$. If $\theta_{e2}^r = \theta_{e1}^r$, then $e_1^r = e_2^r$, $q_1^r = q_2^r$, thus $\Delta_1^r = \Delta_2^r$ and $\pi_1^* > \pi_1^r$ and $\pi_2^* < \pi_2^r$. Since $\Delta_1^r - \Delta_2^r$ is continuous in θ_{e2}^r, we can conclude the result. □

Proof of Proposition A.2.6: Using a similar proof as in Proposition A.2.1, we can find that

$$(q_i^s, e_i^s) = \arg\max v_e e_i + v_q q_i - (\theta_{ei} e_i)^2 - (\theta_{qi} q_i)^2 - \delta(\theta_{ei} e_i)(\theta_{qi} q_i) + s(e_i) \tag{13}$$

Suppose $e_i^s < e_i^*$. Then, since s is an increasing function, we have

$$\begin{aligned}
& v_e e_i^s + v_q q_i^s - (\theta_{ei} e_i^s)^2 - (\theta_{qi} q_i^s)^2 - \delta(\theta_{ei} e_i^s)(\theta_{qi} q_i^s) + s(e_i^s) \\
& \leq v_e e_i^s + v_q q_i^s - (\theta_{ei} e_i^s)^2 - (\theta_{qi} q_i^s)^2 - \delta(\theta_{ei} e_i^s)(\theta_{qi} q_i^s) + s(e_i^*) \\
& < v_e e_i^* + v_q q_i^* - (\theta_{ei} e_i^*)^2 - (\theta_{qi} q_i^*)^2 - \delta(\theta_{ei} e_i^*)(\theta_{qi} q_i^*) + s(e_i^*)
\end{aligned} \tag{14}$$

where the last inequality is because

$$(q_i^*, e_i^*) = \arg\max v_e e_i + v_q q_i - (\theta_{ei} e_i)^2 - (\theta_{qi} q_i)^2 - \delta(\theta_{ei} e_i)(\theta_{qi} q_i)$$

Note that the inequality (14) contradicts (13). Thus, we must have $e_i^s \geq e_i^*$. □

Lemma 5. *With government sales incentive, if $e_2^s < e_1^*$, then $\Delta_1^s - \Delta_2^s > \Delta_1^* - \Delta_2^*$, where $\Delta_1^s = v_e e_1^s + v_q q_1^s - (\theta_{e1} e_1^s)^2 - (\theta_{q1} q_1^s)^2 - \delta(\theta_{e1} e_1^s)(\theta_{q1} q_1^s) + s(e_1^s)$ and $\Delta_2^s = v_e e_2^s + v_q q_2^s - (\theta_{e2} e_2^s)^2 - (\theta_{q2} q_2^s)^2 - \delta(\theta_{e2} e_2^s)(\theta_{q2} q_2^s) + s(e_2^s)$.*

Proof of Lemma 5: Note that $\Delta_1^* + s(e_2^s) - \Delta_2^s = \Delta_1^* - (\Delta_2^s - s(e_2^s)) > \Delta_1^* - \Delta_2^*$, where the inequality is because $(q_2^*, e_2^*) = \arg\max v_e e_2 + v_q q_2 - (\theta_{e2} e_2)^2 - (\theta_{q2} q_2)^2 - \delta(\theta_{e2} e_2)(\theta_{q2} q_2)$. Since $e_2^s < e_1^*$, we have $s(e_1^*) \geq s(e_2^s)$. Since $(q_1^s, e_1^s) = \arg\max v_e e_1 + v_q q_1 - (\theta_{e1} e_1)^2 - (\theta_{q1} q_1)^2 - \delta(\theta_{e1} e_1)(\theta_{q1} q_1) + s(e_1)$, we have $\Delta_1^s > \Delta_1^* + s(e_1^*)$. Then, $\Delta_1^s > \Delta_1^* + s(e_2^s)$ and thus $\Delta_1^s - \Delta_2^s > \Delta_1^* - \Delta_2^*$. □

Proof of Proposition A.2.7: Without loss of generality, suppose $e_1^* \geq e_2^*$.

If $e_2^s < e_1^*$: $B^s - B^* = (b(e_1^s) - b(e_1^*))\,\mathbb{E}D_1^* + (b(e_2^s) - b(e_2^*))(\mu_m - \mathbb{E}D_1^*) + (b(e_1^s) - b(e_2^s))(\mathbb{E}D_1^s - \mathbb{E}D_1^*)$. Note that $\mathbb{E}D_1^* = \frac{\frac{\Delta_1^* - \Delta_2^*}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m$ and $\mathbb{E}D_1^s = \frac{\frac{\Delta_1^s - \Delta_2^s}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m$. Then, by Lemma 5, we have $\mathbb{E}D_1^s > \mathbb{E}D_1^*$. By Proposition A.2.6, we have $e_1^s \geq e_1^*$, $e_2^s \geq e_2^*$ and thus $e_1^s \geq e_2^s$. Since $b(e_i)$ is an increasing function, we have $B^s - B^* \geq 0$.

If $e_2^s \geq e_1^*$: Since $e_2^s \geq e_2^*$, we have $\min(e_1^s, e_2^s) \geq \max(e_1^*, e_2^*)$. Thus, $B^s \geq b(\min(e_1^s, e_2^s))\mu_m \geq b(\max(e_1^*, e_2^*))\mu_m \geq B^*$. □

Proof of Proposition A.2.8: By Envelope Theorem, we have $\frac{\partial(\Delta_i^s - \Delta_i^*)}{\partial\theta_{ei}} = -\frac{v_e}{\theta_{ei}}(e_i^s - e_i^*) \leq 0$, where the inequality is due to Proposition A.2.6. Thus, if $\theta_{e1} < \theta_{e2}$, $\theta_{q1} = \theta_{q2}$ and $\delta = \delta$, we have $\Delta_2^s - \Delta_2^* \leq \Delta_1^s - \Delta_1^*$, i.e., $\Delta_1^s - \Delta_2^s \geq \Delta_1^* - \Delta_2^*$. It is easy to verify that $\Delta_1^* > \Delta_2^*$. Thus, we have $\Delta_1^s - \Delta_2^s \geq \Delta_1^* - \Delta_2^* \geq 0$. Note that $\pi_i^* = \frac{(\frac{\Delta_i^* - \Delta_j^*}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l)^2}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m$ and $\pi_i^s = \frac{(\frac{\Delta_i^s - \Delta_j^s}{3} + \frac{w_1\mu_m + 2w_2\mu_m}{3} + v_l)^2}{2v_l + w_1\mu_m + w_2\mu_m}\mu_m$. Result then follows. □