Supplementary Appendix to Secrecy versus Patenting

This appendix contains proofs and derivations omitted in the printed version of Secrecy versus Patenting (Kultti, Takalo, and Toikka, 2006).

Proof of Proposition 5. We complete the proof of Proposition 5 by showing that α^* yields a maximum. Let us rewrite (A1) as $g(\alpha_p, j_p(\alpha_p)) f(\alpha_p, j_p(\alpha_p)) = 0$, where

$$g(\alpha_p, j_p(\alpha_p)) \equiv \frac{\left(1 - e^{-\widetilde{\theta}_p}\right)}{4\theta\alpha_p(1-\lambda)\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p} e^{-j_p}\right)},$$

and

$$f(\alpha_p, j_p(\alpha_p)) \equiv \left[4\widetilde{\theta}_p\left(e^{-\widetilde{\theta}_p} e^{-j_p} - 1 + \lambda\right) - \alpha_p\left(1 - e^{-\widetilde{\theta}_p}\right)\right].$$

The second order condition can then be written as

$$\left(\frac{\partial g}{\partial \alpha_p} + \frac{\partial g}{\partial j_p}\frac{dj_p}{d\alpha_p}\right) f(\alpha_p, j_p(\alpha_p)) + g(\alpha_p, j_p(\alpha_p))\left(\frac{\partial f}{\partial \alpha_p} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\alpha_p}\right) < 0.$$

Since $g(\alpha_p, j_p(\alpha_p)) > 0$ and $f(\alpha^*, j_p(\alpha^*)) = 0$, it suffices to show that $\frac{\partial f}{\partial \alpha_p} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\alpha_p} < 0$. Differentiating $f(\alpha_p, j_p(\alpha_p))$ gives

$$e^{-\widetilde{\theta}_p} - 1 + \theta e^{-j_p}\left[4e^{-\widetilde{\theta_p}}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4(1-\lambda) - \alpha_p e^{-\widetilde{\theta}_p}\right]\frac{dj_p}{d\alpha_p}.$$

By substituting $\frac{dj_p}{d\alpha_p}$ from the proof of Lemma 4 and recalling that $\widetilde{\theta}_p = \theta\left(1 - e^{-j_p}\right)$, the second order condition can be rewritten as

$$e^{-\widetilde{\theta}_p} - 1 + \frac{\widetilde{\theta}_p e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p}\right)}{\alpha_p\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p} e^{-j_p}\right)}\left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4(1-\lambda) - \alpha_p e^{-\widetilde{\theta}_p}\right] < 0.$$

Multiplying by $\frac{\alpha_p\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p} e^{-j_p}\right)}{1 - e^{-\widetilde{\theta}_p}}$ and simplifying yields

$$-\alpha_p\left(1 - e^{-\widetilde{\theta}_p}\right) + 4\widetilde{\theta}_p e^{-j_p}\left[e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 1 + \lambda\right] < 0.$$

This can be rewritten as

$$-\alpha_p\left(1 - e^{-\widetilde{\theta}_p}\right) + 4\widetilde{\theta}_p e^{-j_p}\left(e^{-j_p} e^{-\widetilde{\theta}_p} - 1 + \lambda\right) - 4\widetilde{\theta}_p e^{-\widetilde{\theta}_p} e^{-j_p}\left(1 - e^{-j_p} + \widetilde{\theta}_p e^{-j_p}\right) < 0.$$

Evaluating the condition at $\alpha_p = \alpha^*$ (i.e. using the first order condition (A1)) gives

$$-4\widetilde{\theta}_p\left(e^{-\widetilde{\theta}_p} e^{-j_p} - 1 + \lambda\right)\left(1 - e^{-j_p}\right) - 4\widetilde{\theta}_p e^{-\widetilde{\theta}_p} e^{-j_p}\left(1 - e^{-j_p} + \widetilde{\theta}_p e^{-j_p}\right) < 0,$$

which is negative, since $e^{-j_p} e^{-\widetilde{\theta}_p} - 1 + \lambda > 0$ whenever the first order condition (A1) holds. ■

Proof of Proposition 6. We show first that α^* is decreasing in λ, which implies the comparative statics result with respect to the obsolescence rate $1 - \lambda$. Let us show the claim for $\delta = 1$; then it is clear that the result holds for sufficiently large $\delta < 1$ by continuity.

From the proof of Proposition 5 we know that α^* solves

$$f\left(\alpha_p, \lambda, j_p\left(\alpha_p, \lambda\right)\right) \equiv \left[4\widetilde{\theta}_p\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right) - \alpha_p\left(1 - e^{-\widetilde{\theta}_p}\right)\right] = 0.$$

By the implicit function theorem,

$$\frac{d\alpha^*}{d\lambda} = -\frac{\frac{\partial f}{\partial \lambda} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\lambda}}{\frac{\partial f}{\partial \alpha_p} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\alpha_p}}.$$

Since the denominator is negative by the second order condition (see the proof of Proposition 5), the sign of $\frac{d\alpha^*}{d\lambda}$ is given by the sign of $\frac{\partial f}{\partial \lambda} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\lambda}$. Differentiating $f\left(\alpha_p, \lambda, j_p\left(\alpha_p, \lambda\right)\right)$ with respect to λ gives

$$4\widetilde{\theta}_p + \theta e^{-j_p}\left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4\left(1-\lambda\right) - \alpha_p e^{-\widetilde{\theta}_p}\right]\frac{dj_p}{d\lambda}.$$

Inserting $\frac{dj_p}{d\lambda}$ from the proof of Lemma 4 and letting $\delta = 1$ gives

$$4\widetilde{\theta}_p + \theta e^{-j_p}\left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4\left(1-\lambda\right) - \alpha e^{-\widetilde{\theta}_p}\right]\frac{\left(1 - e^{-j_p}\right)\left(1 - e^{-\widetilde{\theta}_p}\right)}{\left(1-\lambda\right)\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}e^{-j_p}\right)},$$

which, because $\widetilde{\theta}_p = \theta\left(1 - e^{-j_p}\right)$, can be further simplified to

$$\widetilde{\theta}_p\left\{4 + \left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4\left(1-\lambda\right) - \alpha_p e^{-\widetilde{\theta}_p}\right]\frac{e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p}\right)}{\left(1-\lambda\right)\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}e^{-j_p}\right)}\right\}.$$

The sign of $\frac{d\alpha^*}{d\lambda}$ is determined by the sign of the term in the curly brackets, which is negative if

$$\begin{aligned}&4\left(1-\lambda\right)\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}e^{-j_p}\right)\\ &\quad + e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p}\right)\left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4\left(1-\lambda\right) - \alpha_p e^{-\widetilde{\theta}_p}\right] < 0,\end{aligned}$$

or, upon rearranging, if

$$\begin{aligned}&4\left(1-\lambda\right)\left[\left(1 - e^{-\widetilde{\theta}_p}\right)\left(1 - e^{-j_p}\right) - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}e^{-j_p}\right]\\ &\quad + e^{-\widetilde{\theta}_p}e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p}\right)\left[4\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - \alpha_p\right] < 0.\end{aligned}$$

Evaluating the condition at $\alpha_p = \alpha^*$ gives the requirement that

$$\begin{aligned}&4\left(1-\lambda\right)\left[\left(1 - e^{-\widetilde{\theta}_p}\right)\left(1 - e^{-j_p}\right) - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}e^{-j_p}\right]\\ &\quad + 4e^{-\widetilde{\theta}_p}e^{-j_p}\left[\left(1 - e^{-\widetilde{\theta}_p}\right)\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - \widetilde{\theta}_p\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right)\right] < 0.\end{aligned}$$

This can be simplified to

$$(1-\lambda)\left(1-e^{-\widetilde{\theta}_p}\right)\left(1-e^{-j_p}\right)+e^{-\widetilde{\theta}_p}e^{-j_p}\left[\left(1-e^{-\widetilde{\theta}_p}\right)\left(2e^{-j_p}-1\right)-\widetilde{\theta}_p e^{-j_p}\right]<0,$$

which is equivalent to

$$\left(1-\lambda-e^{-j_p}e^{-\widetilde{\theta}_p}\right)\left(1-e^{-j_p}\right)\left(1-e^{-\widetilde{\theta}_p}\right)+e^{-2j_p}e^{-\widetilde{\theta}_p}\left(1-e^{-\widetilde{\theta}_p}-\widetilde{\theta}_p\right)<0.$$

This holds for all $\alpha^*>0$ and $j_p>0$, since then $\widetilde{\theta}_p>0$ so that both $1-\lambda-e^{-j_p}e^{-\widetilde{\theta}_p}<0$ (by equation (A1)) and $1-e^{-\widetilde{\theta}_p}-\widetilde{\theta}_p<0$. Thus α^* is decreasing in λ, that is, it is increasing in the obsolescence rate $1-\lambda$.

To prove the claim about the existence of a cutoff we will first show that in the limit where $\lambda\to 1$, we have $\alpha^*<\overline{\alpha}$. From the equilibrium condition (20) we see that $j_p\to\infty$ as $\lambda\to 1$ (when $\delta=1$). As a result,

$$\lim_{\lambda\to 1}\alpha^*\equiv\frac{4\widetilde{\theta}_p\left(e^{-\widetilde{\theta}_p}e^{-j_p}-1+\lambda\right)}{1-e^{-\widetilde{\theta}_p}}=0<\overline{\alpha}.$$

By continuity we can find a $\lambda<1$ close enough to one such that the above inequality holds. This shows that the constraint $\alpha_p\geq\overline{\alpha}$ is then binding at the optimum, and that, in particular, α_p^* is then strictly less than α. This implies that for small enough obsolescence rate $1-\lambda$ the optimal patent policy increases spillover.

Let us then consider the case where $\lambda\to 1-\frac{\alpha}{2}$, where $1-\frac{\alpha}{2}$ is the smallest λ satisfying the parameter restrictions of Lemma 1. At any interior solution of the welfare maximization problem the following equations must hold:

$$4\widetilde{\theta}_p\left(e^{-\widetilde{\theta}_p}e^{-j_p}-1+\lambda\right)=\alpha^*\left(1-e^{-\widetilde{\theta}_p}\right),$$

and

$$2(1-\lambda)\widetilde{\theta}_p=\alpha^* e^{-j_p}\left(1-e^{-\widetilde{\theta}_p}\right).$$

The first equation follows from the first order condition to the welfare maximization problem (A1) and the second equation is the equilibrium condition for a patenting equilibrium (20) when $\delta=1$. Dividing the first equation by the latter gives

$$\frac{2\left(e^{-\widetilde{\theta}_p}e^{-j_p}-1+\lambda\right)}{1-\lambda}=\frac{1}{e^{-j_p}}.$$

Inserting this into the equation for α^* (A2) and letting $\lambda=1-\frac{\alpha}{2}$ yields

$$\alpha^*|_{\lambda=1-\frac{\alpha}{2}}=\frac{\widetilde{\theta}_p\alpha}{e^{-j_p}\left(1-e^{-\widetilde{\theta}_p}\right)}.$$

Thus we have that $\alpha^*|_{\lambda=1-\frac{\alpha}{2}}>\alpha$ if $\widetilde{\theta}_p-e^{-j_p}\left(1-e^{-\widetilde{\theta}_p}\right)>0$. This holds for all $j_p>0$, because then $\widetilde{\theta}_p-e^{-j_p}\left(1-e^{-\widetilde{\theta}_p}\right)>\widetilde{\theta}_p-1+e^{-\widetilde{\theta}_p}>0$. Thus $\alpha^*|_{\lambda=1-\frac{\alpha}{2}}>\alpha$ so that for small enough λ (or, equivalently, for high enough $1-\lambda$) the optimal patent protection decreases the spillover. It also immediately follows that, for $\alpha = 1$ and by continuity for some $\alpha < 1$, the constraint $\alpha_p \leq 1$ is binding for small enough λ. Thus for these parameter values we have a corner solution with $\alpha_p^* = 1$.

As α_p^* is monotone in $1-\lambda$, there must thus be a cutoff $\hat{\lambda}$ such that for $1-\lambda < 1-\hat{\lambda}$ the optimal patent policy increases spillovers and for $1-\lambda > 1-\hat{\lambda}$ it decreases the spillovers. ■

Proof of Proposition 7. From the proof of Proposition 5 we know that α^* solves $f(\alpha_p, \theta, j_p(\alpha_p, \theta)) \equiv \left[4\widetilde{\theta}_p\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right) - \alpha_p\left(1 - e^{-\widetilde{\theta}_p}\right)\right] = 0$. By the implicit function theorem,

$$\frac{d\alpha^*}{d\theta} = -\frac{\frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\theta}}{\frac{\partial f}{\partial \alpha} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\alpha_p}}.$$

Since the denominator is negative by the second order condition (see the proof of Proposition 5), the sign of $\frac{d\alpha^*}{d\theta}$ is given by the sign of $\frac{\partial f}{\partial \theta} + \frac{\partial f}{\partial j_p}\frac{dj_p}{d\theta}$. Differentiating $f(\alpha_p, \theta, j_p(\alpha_p, \theta))$ with respect to θ gives

$$\begin{aligned}&\left(1 - e^{-j_p}\right)\left[4\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right) - e^{-\widetilde{\theta}_p}\left(4\widetilde{\theta}_p e^{-j_p} + \alpha\right)\right] \\ &\qquad + \theta e^{-j_p}\left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4(1-\lambda) - \alpha e^{-\widetilde{\theta}_p}\right]\frac{dj_p}{d\theta},\end{aligned}$$

which, after inserting $\frac{dj}{d\theta}$ from the proof of Lemma 4, can be rewritten as

$$\begin{aligned}&\left(1 - e^{-j_p}\right)\Bigg\{4\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right) - e^{-\widetilde{\theta}_p}\left(4\widetilde{\theta}_p e^{-j_p} + \alpha_p\right) \\ &\quad - e^{-j_p}\left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4(1-\lambda) - \alpha e^{-\widetilde{\theta}_p}\right]\frac{\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}\right)}{\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}e^{-j_p}\right)}\Bigg\}.\end{aligned}$$

The sign of $\frac{d\alpha^*}{d\theta}$ is thus determined by the sign of the term in the curly brackets, which is negative if

$$\begin{aligned}&\left[4\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right) - e^{-\widetilde{\theta}_p}\left(4\widetilde{\theta}_p e^{-j_p} + \alpha\right)\right]\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}e^{-j_p}\right) \\ &\quad - e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}\right)\left[4e^{-\widetilde{\theta}_p}\left(2e^{-j_p} - 1 - \widetilde{\theta}_p e^{-j_p}\right) - 4(1-\lambda) - \alpha_p e^{-\widetilde{\theta}_p}\right] < 0,\end{aligned}$$

or, upon simplifying and dividing by $1 - e^{-j_p}$, if

$$\left[4\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right) - e^{-\widetilde{\theta}_p}\left(4\widetilde{\theta}_p e^{-j_p} + \alpha_p\right)\right]\left(1 - e^{-\widetilde{\theta}_p}\right) + 4e^{-\widetilde{\theta}_p}e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}\right) < 0.$$

After some further simplifications, we can rewrite the condition as

$$\left[4\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right) - e^{-\widetilde{\theta}_p}\alpha_p\right]\left(1 - e^{-\widetilde{\theta}_p}\right) + 4e^{-\widetilde{\theta}_p}e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p\right) < 0.$$

Evaluating the condition at $\alpha_p = \alpha^*$ using (A1) gives the requirement that

$$4\left(e^{-\widetilde{\theta}_p}e^{-j_p} - 1 + \lambda\right)\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p e^{-\widetilde{\theta}_p}\right) + 4e^{-\widetilde{\theta}_p}e^{-j_p}\left(1 - e^{-\widetilde{\theta}_p} - \widetilde{\theta}_p\right) < 0,$$

which is equivalent to

$$e^{-\widetilde{\theta}_p}e^{-j_p}\left[2\left(1-e^{-\widetilde{\theta}_p}\right)-\widetilde{\theta}_p\left(1+e^{-\widetilde{\theta}_p}\right)\right]-(1-\lambda)\left(1-e^{-\widetilde{\theta}_p}-\widetilde{\theta}_pe^{-\widetilde{\theta}_p}\right)<0.$$

This holds for all $j_p>0$, since then $\widetilde{\theta}_p>0$ so that $\left[2\left(1-e^{-\widetilde{\theta}_p}\right)-\widetilde{\theta}_p\left(1+e^{-\widetilde{\theta}_p}\right)\right]<0$ and $1-e^{-\widetilde{\theta}_p}-\widetilde{\theta}_pe^{-\widetilde{\theta}_p}>0$. ■

Deriving the steady state stocks of innovations with patents. Consider first the period t stock of monopoly innovations $M^t(\sigma)$. The stock evolves according to

$$M^t(\sigma)=\lambda M^{t-1}(\sigma)+\sigma\alpha\Delta_1^t+\alpha_p\sum_{k=1}^{\infty}\left(1-\sigma^k\right)\Delta_k^t,$$

where the first term on the right hand side is the proportion of the stock of period $t-1$ that does not become obsolete. The other two terms capture the inflow to the stock. The first inflow term $\sigma\alpha\Delta_1^t$ comes from the new monopoly innovations that are kept secret. Such innovations are developed by exactly one innovator who resorts to secrecy, which does not subsequently leak out. The second inflow term $\alpha_p\sum_{k=1}^{\infty}\left(1-\sigma^k\right)\Delta_k^t$ gives the new patented innovations. When exactly k innovators have succeeded in turning an idea into an innovation, the probability that the innovation is patented is $1-\sigma^k$. Then the probability that the patent holder succeeds in excluding others from using the innovation is α_p. By substituting for Δ_k^t from (2) we can solve for the steady state stock

$$M(\sigma)=\frac{\sigma\alpha\widetilde{\theta}e^{-\widetilde{\theta}}+\alpha_p\left(1-e^{-(1-\sigma)\widetilde{\theta}}\right)}{1-\lambda}I.$$

The law of motion for the stock of competitive innovations $C^t(\sigma)$ is

$$C^t(\sigma)=\lambda C^{t-1}(\sigma)+\sum_{k=1}^{\infty}\Delta_k^t-\left[\sigma\alpha\Delta_1^t+\alpha_p\sum_{k=1}^{\infty}\left(1-\sigma^k\right)\Delta_k^t\right],$$

where the second and the third term on the right hand side define the inflow to the stock. It is the difference between all new innovations and new monopoly innovations. At a steady state we have

$$C(\sigma)=\frac{1-e^{-\widetilde{\theta}}-\sigma\alpha\widetilde{\theta}e^{-\widetilde{\theta}}-\alpha_p\left(1-e^{-(1-\sigma)\widetilde{\theta}}\right)}{1-\lambda}I.$$

■

Simplifying the probability of receiving a patent. Suppose first that $\sigma\neq1$. The probability of getting a patent is then

$$\rho_p(i,\sigma)=\left(1-e^{-i}\right)\alpha_p\sum_{k=0}^{\infty}e^{-\theta}\frac{\theta^k}{k!}\sum_{h=0}^{k}\binom{k}{h}\left(1-e^{-j}\right)^h\left(e^{-j}\right)^{k-h}\sum_{l=0}^{h}\binom{h}{l}(1-\sigma)^l\sigma^{h-l}\frac{1}{l+1}.$$

Let us start with the last sum, which becomes

$$\begin{aligned}
\sum_{l=0}^{h}\binom{h}{l}(1-\sigma)^{l}\,\sigma^{h-l}\frac{1}{l+1} &= \sum_{l=0}^{h}\frac{h!}{(h-l)!l!}\frac{1}{l+1}(1-\sigma)^{l}\,\sigma^{h-l}\\
&= \sum_{l=0}^{h}\frac{1}{h+1}\frac{(h+1)!}{[h+1-(l+1)]!\,(l+1)!}(1-\sigma)^{l}\,\sigma^{h-l}\\
&= \frac{1}{h+1}\sum_{l=0}^{h}\binom{h+1}{l+1}(1-\sigma)^{l}\,\sigma^{h-l}\\
&= \frac{1}{(h+1)\,(1-\sigma)}\sum_{l=0}^{h}\binom{h+1}{l+1}(1-\sigma)^{l+1}\,\sigma^{h+1-(l+1)}\\
&= \frac{1}{(h+1)\,(1-\sigma)}\sum_{l=1}^{h+1}\binom{h+1}{l}(1-\sigma)^{l}\,\sigma^{h+1-l}\\
&= \frac{(1-\sigma+\sigma)^{h+1}-\sigma^{h+1}}{(h+1)\,(1-\sigma)}\\
&= \frac{1-\sigma^{h+1}}{(h+1)\,(1-\sigma)}.
\end{aligned}$$

Inserting this back into the expression for the probability yields

$$\rho_p(i,\sigma)=\left(1-e^{-i}\right)\alpha_p\frac{1}{1-\sigma}\sum_{k=0}^{\infty}e^{-\theta}\frac{\theta^k}{k!}\sum_{h=0}^{k}\binom{k}{h}\left(1-e^{-j}\right)^h\left(e^{-j}\right)^{k-h}\frac{1-\sigma^{h+1}}{h+1}.$$

Consider then the summation over h. To simplify the notation, we write $a=e^{-j}$. The sum becomes

$$\sum_{h=0}^{k}\binom{k}{h}(1-a)^h\,a^{k-h}\frac{1-\sigma^{h+1}}{h+1}=\sum_{h=0}^{k}\binom{k}{h}(1-a)^h\,a^{k-h}\frac{1}{h+1}-\sum_{h=0}^{k}\binom{k}{h}(1-a)^h\,a^{k-h}\frac{\sigma^{h+1}}{h+1}.$$

The first sum on the right hand side is of the same form as the one we simplified above. Thus we have

$$
\begin{aligned}
\sum_{h=0}^{k}\binom{k}{h}(1-a)^h a^{k-h}\frac{1-\sigma^{h+1}}{h+1} &= \frac{1-a^{k+1}}{(k+1)(1-a)}-\sum_{h=0}^{k}\binom{k}{h}(1-a)^h a^{k-h}\frac{\sigma^{h+1}}{h+1} \\
&= \frac{1-a^{k+1}}{(k+1)(1-a)}-\sigma\sum_{h=0}^{k}\binom{k}{h}(1-a)^h \sigma^h a^{k-h}\frac{1}{h+1} \\
&= \frac{1-a^{k+1}}{(k+1)(1-a)}-\sigma\sum_{h=0}^{k}\frac{k!}{(k-h)!\,(h+1)!}(1-a)^h \sigma^h a^{k-h} \\
&= \frac{1-a^{k+1}}{(k+1)(1-a)}-\frac{\sigma}{k+1}\sum_{h=0}^{k}\binom{k+1}{h+1}(1-a)^h \sigma^h a^{k-h} \\
&= \frac{1-a^{k+1}}{(k+1)(1-a)}-\frac{1}{(k+1)(1-a)}\sum_{h=1}^{k+1}\binom{k+1}{h}(1-a)^h \sigma^h a^{k+1-h} \\
&= \frac{1-a^{k+1}}{(k+1)(1-a)}-\frac{[(1-a)\sigma+a]^{k+1}-a^{k+1}}{(k+1)(1-a)} \\
&= \frac{1-[(1-a)\sigma+a]^{k+1}}{(k+1)(1-a)}.
\end{aligned}
$$

Inserting this back into the expression for the probability we are left with

$$
\begin{aligned}
\rho_p(i,\sigma) &= \left(1-e^{-i}\right)\alpha_p\frac{1}{1-\sigma}\sum_{k=0}^{\infty}e^{-\theta}\frac{\theta^k}{k!}\frac{1-[(1-a)\sigma+a]^{k+1}}{(k+1)(1-a)} \\
&= \frac{\left(1-e^{-i}\right)\alpha_p e^{-\theta}}{\theta(1-a)(1-\sigma)}\left[\sum_{k=0}^{\infty}\frac{\theta^{k+1}}{(k+1)!}-\sum_{k=0}^{\infty}\frac{\theta^{k+1}[(1-a)\sigma+a]^{k+1}}{(k+1)!}\right] \\
&= \frac{\left(1-e^{-i}\right)\alpha_p e^{-\theta}}{\theta(1-a)(1-\sigma)}\left[\sum_{k=1}^{\infty}\frac{\theta^{k}}{k!}-\sum_{k=1}^{\infty}\frac{\theta^{k}[(1-a)\sigma+a]^{k}}{k!}\right] \\
&= \frac{\left(1-e^{-i}\right)\alpha_p e^{-\theta}}{\theta(1-a)(1-\sigma)}\left[e^{\theta}-1-e^{\theta[(1-a)\sigma+a]}+1\right] \\
&= \frac{\left(1-e^{-i}\right)\alpha_p}{\theta(1-a)(1-\sigma)}\left(1-e^{-\theta(1-a)(1-\sigma)}\right).
\end{aligned}
$$

Recalling that $a=e^{-j}$ and $\widetilde{\theta}=\theta\left(1-e^{-j}\right)$ we can write this as

$$
\rho_p(i,\sigma)=\left(1-e^{-i}\right)\alpha_p\frac{1-e^{-(1-\sigma)\widetilde{\theta}}}{(1-\sigma)\widetilde{\theta}},
$$

which is the same as (16) for the case $\sigma \neq 1$. Suppose then that $\sigma = 1$. In this case all others choose secrecy so that the innovator will get the patent whenever she applies for it. Thus all that is needed is that the innovator is successful $\left(1-e^{-j}\right)$ and that the innovation does not become public (α_p). This yields the result for the case $\sigma = 1$. ■