

# Secrecy versus patenting

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*We develop an equilibrium search model of innovation with the possibility of multiple independent discovery. We distinguish innovations from ideas, and we view patents as probabilistic property rights that are constrained by the innovators' option to keep the innovation secret. We find that the patent system can simultaneously stimulate innovation, information disclosure and welfare. An optimal patent may provide more or less protection than secrecy, and in many cases, it provides less, suggesting that its main function is information spreading rather than rewarding the costs of the innovative activity.*

## 1. Introduction

■ The standard view about patents and intellectual property rights (IPRs) is that, as they grant the inventor a temporary monopoly, they provide inventors an incentive to innovate. Without such a reward, the innovators would not be willing to invest as much because they would be afraid of someone else expropriating their ideas. The trade-off here is between the inefficiency of the monopoly and the incentive to innovate. This view underlies much of the extensive research on patents that has been accumulated since Nordhaus's seminal work (1969) (for surveys see, e.g., Denicolò, 1996; Langinier and Moschini, 2002). Evidence does not seem to fully support this view, though. There are numerous studies suggesting that secrecy typically offers better protection than patents (see, e.g., Cohen, Nelson, and Walsh, 2000; Arundel, 2001). This raises the question of why firms engage in costly patenting.

Active patenting by firms is even more puzzling because another traditional rationale for the patent system is to stimulate public disclosure of private information. A patent application should contain sufficient information to allow a skilled person to reproduce the particular innovation. It is not clear how the two aims of the patent system, to enhance the incentive to innovate and to disseminate information, can be reconciled; there seems to be an inherent tension between

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them. On one hand, if patenting enhances the incentive to innovate by improving appropriability, how can it simultaneously spread information and thereby the possibilities to imitate the patented innovation? On the other hand, if patent protection stimulates information disclosure rather than investments in innovation, as Gallini (2002) suggests in a survey of the empirical evidence, then why do firms patent?

The aim of this study is to compare the effects of the patent system and secrecy on the incentive to innovate, information spreading and welfare. We provide an explanation why firms patent even if they have the option to resort to secrecy offering better protection. We find that, if a patent system is effective in the sense that innovators patent their innovations rather than keep them secret, it can stimulate both the innovative activity and information disclosure and, as a result, improve welfare. An effective patent system can provide less protection than secrecy. To see this, suppose that the innovators first invest in R&D and then, after finding out whether they succeeded in producing an innovation, but when still unaware of other potential developers of the same innovation, decide whether to patent the innovation or keep it secret. Then under a patent system, it only pays to keep the innovation secret when the probability that a competitor comes up with the same innovation and patents it is sufficiently small. If the probability is large, it pays to apply for the patent even if it confers only weak protection because, otherwise, someone else gets it, and the innovator risks infringement if she tries to capitalize her innovation. In other words, when innovators contemplate patenting, the typical choice is not between patenting or keeping the innovation secret but between patenting or letting the competitors patent.

Critics of strong patent protection argue that the strengthening of patent rights over the past decades has only resulted in an increase in patenting without a corresponding increase in the innovative activity (see, e.g., Jaffe and Lerner, 2004). Our model also incorporates this view. If patent protection is enhanced when some innovators resort to secrecy and some patent, all that happens is that more innovators begin to patent but R&D expenditures remain the same. Only if everyone patents, does stronger patent protection encourage innovation.

Our view, according to which the patent system can both accelerate the pace of innovation and spread information, necessitates that firms frequently make similar innovations. As discussed in Granstrand (2002), the phenomenon of independent or nearly simultaneous discoveries is well documented in 'traditional' industries, but we think that it especially characterizes network industries such as software, Internet, telecommunications and payment media where standardization limits the possible paths for future technologies and, accordingly, firms concentrate their R&D activities on the same fields. Similar views are expressed by Rahnasto (2003) and Varian, Farrell, and Shapiro (2004). Rahnasto (2003), in particular, argues that intellectual property protection should be reconsidered because of increased relevance of simultaneous innovation.

To highlight the intuition of one of our basic results, one can rely on the following simple example: Two firms are engaged in R&D that results either in an innovation or failure. Suppose first that the innovation is protected by secrecy and leaks out with probability  $1 - \alpha$ . When this happens, the innovation is publicly available and production is at the competitive level. If only one firm succeeds in R&D and the innovation does not become public, the firm earns monopoly profit  $\pi^M$ . If both firms succeed and their innovation does not become public, each firm earns duopoly profit  $\pi^D < \pi^M$ . Assuming that the probability of success  $\beta$  is independent across firms, a firm's expected profit is  $\beta(1 - \beta)\alpha\pi^M + \beta^2\alpha\pi^D$ . If the innovation can be protected by a patent, the firms have to decide whether to file for a patent ( $P$ ) or resort to secrecy ( $S$ ). This decision has to be made before learning whether the competitor has succeeded or not. If both firms are successful and file for the patent, each firm obtains it with probability  $1/2$ . Let us measure patent protection by the probability that a patent holder can exclude the competitor from using the innovation. This probability is denoted  $\alpha_p$  below.

The expected payoffs given a competitor's patenting strategy are represented in Table 1, which displays the row player's payoff in each cell. From the table, it is immediate that patenting is a strictly dominant strategy when  $\alpha_p = \alpha$ . Hence, by continuity, there exists an  $\alpha'_p < \alpha$  such that patenting is a strictly dominant strategy even though the protection offered by the patent is weaker than that offered by secrecy. Note that we impose no specific demand structure or

TABLE 1

	<i>S</i>	<i>P</i>
<i>S</i>	$\beta(1 - \beta)\alpha\pi^M + \beta^2\alpha\pi^D$	$\beta(1 - \beta)\alpha\pi^M$
<i>P</i>	$\beta(1 - \beta)\alpha_p\pi^M + \beta^2\alpha_p\pi^M$	$\beta(1 - \beta)\alpha_p\pi^M + .5\beta^2\alpha_p\pi^M$

form of the duopolistic competition. For example, the argument could accommodate a standard decreasing inverse demand function and Cournot or Bertrand competition. We could even assume that the firms can collude in the product market if they are both successful so that  $\pi^D = (1/2)\pi^M$ . Furthermore, the example generalizes readily to the case of  $n$  firms.

The example shows the logic of our result at its simplest. It incorporates some key elements of our model, notably the decision to patent and the probabilistic view of IPRs. Our model is, however, richer in several respects. Because the economics of IPRs is an area where policy considerations are important, we use an equilibrium search model with nontrivial demands so that we can calculate welfare measures. The model naturally yields the possibility that more than one innovator comes up with the same innovation. We assume that there is a large number of innovators as well as ideas, i.e., potential innovations. The relative number of innovators to potential innovations tells roughly how mature and competitive an industry is, and this varies across industries. We also allow for the possibility that the innovations become obsolete. The optimal strength of the patent system depends on these variables, and these effects cannot be addressed in the two-agent example. Moreover, unlike most previous studies of IPRs, we distinguish actual innovations from unknown ideas (for an exception, see O'Donoghue, Scotchmer, and Thisse, 1998). Finally, to study the incentive effects of IPRs, we assume that the innovators can decide how much to invest in R&D. The investment determines the probability of success, and it depends on whether there is a patent system available and the strength of protection it offers.

Although information spreading has been a main purpose of the patent system since its origins, it is relatively little studied in economics (Granstrand, 2002). Since the seminal article by Horstman, MacDonald, and Slivinski (1985), there are only a few studies where secrecy is regarded as a viable option to patenting. The works that are the closest to ours are Anton and Yao (2004) and Denicolò and Franzoni (2004). Like Horstman, MacDonald, and Slivinski (1985), Anton and Yao (2004) build a signalling model to study the strategic disclosure of information through patenting and find that small innovations are fully revealed but large innovations are mainly kept secret. Reminiscent of our findings, they show that small innovations are patented even if patents afford weak protection. In our model, innovators either keep their innovations fully to themselves or they become fully public, and matters of signalling do not arise. Like us, Denicolò and Franzoni (2004) find that patenting is in general socially preferable to secrecy. They, however, focus on the question of prior user rights in a model of sequential innovation. Using logic similar to ours, Arora, Fosfuri and Gambardella (2001) show how the incentive to license is decreasing in the degree of product differentiation of an industry. In their model, licensing is an equilibrium even though it increases information spreading.

The rest of the article is organized as follows. We present the model in the next section. In Sections 3 and 4, we consider an economy without a patent system; in Section 3, we solve for the equilibrium and, in Section 4, we study its welfare properties. Patents are introduced in Section 5. Much of the section is devoted to the analysis of the choice between patenting and secrecy. In Section 6, we study the incentive effects of the patent system. Finally, in Section 7, we consider the welfare effect of patents and optimal patent policy. In Section 8, we present conclusions and discuss the first-inventor defense. The proofs and calculations omitted in the main text are collected in the Appendix and the supplementary Appendix at [www.rje.org/sup-mat.html](http://www.rje.org/sup-mat.html).

## 2. The model

■ We study an infinite horizon, discrete-time economy, where time is discounted using a common discount factor,  $\delta \in (0, 1)$ . There are risk-neutral agents, innovators, who conduct R&D and produce the resulting innovations. The innovation process involves the Schumpeterian distinction between inventions, which we here call ideas, and innovations. The process has two stages, where the innovators first come up with an idea and invest amount  $j$  in it. In the second stage, they find out whether they succeeded in producing an innovation or not. We introduce uncertainty inherent to an innovation process by assuming that, with probability  $1 - e^{-j}$ , an innovator is successful in developing the idea into an innovation.<sup>1</sup> Thus, the probability of success is increasing and concave in the investment. When there are no IPRs, choosing the level of investment,  $j$ , is the only decision the innovators have to make. Once we introduce patent policy in Section 5, the innovators also have to decide whether to file for a patent or resort to secrecy. When there are no IPRs, the form of protection is trivially secrecy.

There is a continuum of innovators and ideas, and the innovators find ideas randomly. The mass of innovators is normalized to unity, while the mass of ideas is  $I$ . In this setting, the number of innovators who find any particular idea is given by a Poisson distribution with rate  $\theta = 1/I$ .<sup>2</sup>

Despite the practical relevance of independent innovation, assuming that more than one innovator can come up with the same idea is not crucial. All that is needed is the possibility of multiple independent discoveries of similar innovations, such that an innovator—in choosing between secrecy and patenting—has to take into account the potential competition and the possibility that an overlapping patent makes the production of the innovation difficult. We simply take this one step further to gain tractability. Nor is the possibility of several innovators simultaneously making the same innovation a feature that drives the results. The model would work in a similar way if an innovator could create the same innovation that was made earlier by another innovator, but this would be harder to analyze.

We assume that, under secrecy, an innovator can keep the innovation to herself with probability  $\alpha$ . This means that with probability  $1 - \alpha$  the innovation becomes public; this is the spillover effect. If the innovation becomes public, there is free entry to its utilization, which drives production to the competitive level. We take the probability  $\alpha$  to be a measure of the strength of protection under secrecy. As some innovations are complex or involve soft information, whereas some innovations are easy to describe and reproduce, the parameter  $\alpha$  is likely to be industry or even innovation specific (see, e.g., Cohen, Nelson, and Walsh, 2000; Arundel, 2001). For example, process innovations are typically better protected by secrecy than product innovations, which are observable to users. In practice, innovators can invest in the protection technology whereas competitors can invest to circumvent the protection, and policy makers can affect the strength of protection through trade-secret laws. However, for our purposes,  $\alpha$  can be taken as exogenous. We also assume that, at the end of each period, innovations become obsolete with probability  $1 - \lambda$ . This allows us to determine a steady-state equilibrium.

Because there is a possibility of multiple discovery, we need to specify the nature of competition. We assume that, if more than one innovator develops the same innovation, production takes place according to Bertrand competition. Thus, an innovator who makes an innovation alone becomes a monopolist and otherwise production is at the competitive level. Consequently, all innovations in the economy are such that there is production either at the monopoly or at the competitive level. As suggested by the example in the Introduction, the form of the product-market competition is not critical for the results.<sup>3</sup>

We normalize the marginal cost of production to zero for all innovations. For the consumption

<sup>1</sup> Motivation for choosing this particular functional form can be found in Kultti (2003).

<sup>2</sup> In random matching literature, this is known as the urn-ball model (e.g., Lu and McAfee, 1996). One can also think that the innovators simultaneously choose an idea to work on using symmetric mixed strategies.

<sup>3</sup> Cournot competition would yield essentially the same results (see Kultti and Takalo, 2006). The results do not change qualitatively even if innovators collude to produce as a monopoly under secrecy.

side, we assume that there is an inverse demand function,  $p = \sqrt{2} - q$ , corresponding to each idea.<sup>4</sup> We close the model by assuming that the innovators are also the consumers; they are symmetrically and randomly distributed on the demand curves. Because the number of innovators is very large (a continuum), the innovators do not take their own consumption into account in their production decisions.<sup>5</sup>

□ **Timing of the events within a period.** First, the innovators find ideas according to the matching technology and make the investment  $j$ . At this stage, the innovators do not know how many other innovators have found the same idea. Second, the innovators find out whether their investment resulted in a new innovation (this happens with probability  $1 - e^{-j}$ ). A new innovation turns out to be profitable if exactly one innovator comes up with it as she gets a monopoly. If two or more innovators come up with the same innovation or the innovation becomes public (with probability  $1 - \alpha$ ), it is produced at the competitive level and yields zero profits. At the end of the period, innovations (both new and old) either become obsolete (with probability  $1 - \lambda$ ) or remain economically viable.

It is essential that the investment in innovation is made before the innovations may become public, but otherwise the results are robust as to the timing of the events. It is also immaterial whether innovations may become public only once or each period.

□ **Steady-state stocks of innovations.** We focus on a symmetric steady-state equilibrium, which will be defined in the next section. In an equilibrium the investment  $j$  and the stocks of innovations remain the same from one period to another and the investment is the same for all innovators. After accounting for the differences in the total surpluses between the monopoly and competitive innovations, different levels of steady-state stocks correspond to different welfare levels.

Let  $M^t$  denote the period  $t$  stock of innovations that are held by one innovator, i.e., monopoly innovations. The stock is determined at the time of production and it evolves according to the law of motion,

$$M^t = \lambda M^{t-1} + \alpha \Delta_1^t,$$

where  $\Delta_1^t$  gives the new innovations developed by a single innovator at period  $t$ . The period  $t$  stock consists of two terms. The first term is the proportion of the stock from the previous period that does not become obsolete at the end of period  $t - 1$ . The second term is the proportion of the new monopoly innovations that does not become public. In a steady state,  $M^t = M$  and  $\Delta_1^t = \Delta_1$  for all  $t$ , so that

$$M = \frac{\alpha \Delta_1}{1 - \lambda}.$$

That a new innovation is made exactly by one innovator requires first that at least one innovator finds the idea and then that exactly one innovator is successful in developing the idea into an innovation. The inflow to the stock of monopoly innovations  $M$  is thus

$$\Delta_1 = \sum_{h=1}^{\infty} e^{-\theta} \frac{\theta^h}{h!} \binom{h}{1} (1 - e^{-j}) e^{-(h-1)j} I = \tilde{\theta} e^{-\tilde{\theta}} I,$$

where we have defined  $\tilde{\theta} \equiv \theta(1 - e^{-j})$ . The steady-state stock is then

$$M = \frac{\alpha \tilde{\theta} e^{-\tilde{\theta}}}{1 - \lambda} I. \quad (1)$$

<sup>4</sup> We use linear demands for simplicity. As the simple example in the Introduction indicates, the mechanism that drives the results does not hinge on the specific functional form for the demand curve.

<sup>5</sup> Whether the innovators take their own consumption into account or not is inconsequential, as utility from any particular innovation is a negligible part of the total utility of an agent.

The difference between ideas and innovations shows up in the variables  $\theta$  and  $\tilde{\theta}$  above: the Poisson parameter  $\theta = 1/I$  measures roughly the innovative potential of the industry, whereas  $\tilde{\theta} = \theta(1 - e^{-j})$  measures actual R&D intensity, which incorporates the level of the R&D investment.

The period  $t$  stock of innovations for which the production is at the perfectly competitive level is denoted by  $C^t$ . The inflow to the stock of competitive innovations consists of the new innovations developed each period minus the new innovations for which there is a monopoly. The number of new innovations developed exactly by  $k$  innovators each period is

$$\Delta_k^t = \sum_{h=k}^{\infty} e^{-\theta} \frac{\theta^h}{h!} \binom{h}{k} (1 - e^{-j})^k e^{-(h-k)j} I = \frac{\tilde{\theta}^k e^{-\tilde{\theta}}}{k!} I. \quad (2)$$

As (2) shows, for a new innovation to be made by exactly  $k$  innovators, at least  $k$  innovators must first find the idea. Then exactly  $k$  of the innovators must be successful in developing the idea into an innovation. As there can be any number  $k$  of developers, we must sum over  $k$  so that the total number of new innovations is

$$\sum_{k=1}^{\infty} \Delta_k^t = \sum_{k=1}^{\infty} \frac{\tilde{\theta}^k e^{-\tilde{\theta}}}{k!} I = (1 - e^{-\tilde{\theta}}) I.$$

The stock  $C^t$  evolves according to

$$C^t = \lambda C^{t-1} + \sum_{k=1}^{\infty} \Delta_k^t - \alpha \Delta_1^t,$$

where  $\lambda C^{t-1}$  is the proportion of the stock of the previous period that does not become obsolete and  $\sum_{k=1}^{\infty} \Delta_k^t - \alpha \Delta_1^t$  is the inflow of new competitive innovations. Accordingly, in a steady state, the stock of competitive innovations is

$$C = \frac{1 - e^{-\tilde{\theta}} - \alpha \tilde{\theta} e^{-\tilde{\theta}}}{1 - \lambda} I. \quad (3)$$

□ **Revenue and consumer surplus.** Given inverse demand function  $p = \sqrt{2} - q$  and Bertrand behavior with zero marginal costs, the per period revenue from a monopoly innovation is  $\pi = 1/2$ , whereas all other innovations yield zero revenue. The corresponding periodic consumer surpluses are  $\gamma = 1/4$  for monopoly innovations and  $\gamma_c = 1$  for competitive innovations. For further reference, we record the per period aggregate revenue,

$$\Pi \equiv M\pi = \frac{\alpha \tilde{\theta} e^{-\tilde{\theta}}}{2(1 - \lambda)} I, \quad (4)$$

and the aggregate consumer surplus,

$$\Gamma \equiv M\gamma + C\gamma_c,$$

which can be, by using (1) and (3), rewritten as

$$\Gamma = \frac{4(1 - e^{-\tilde{\theta}}) - 3\alpha \tilde{\theta} e^{-\tilde{\theta}}}{4(1 - \lambda)} I. \quad (5)$$

### 3. Equilibrium without patents

■ Next we determine the equilibrium level of innovation in an industry lacking intellectual property rights. Without IPRs, an innovator can do nothing else but invest in R&D and hope that no one else makes the same innovation. If there are other innovators who have found the same idea—corresponding to a situation where many firms are working on similar R&D projects—the innovator can make profits only if she is successful in transforming the idea into an innovation and all others fail. The successful innovator becomes a monopolist with probability  $\alpha$ . If more than one innovator is successful or if the innovation becomes public, competition will drive revenue to zero.

A Nash equilibrium of the model is an infinite sequence of investment rules for each innovator such that each sequence of rules is optimal given other innovators' sequences.<sup>6</sup> A steady-state equilibrium is an equilibrium where the stocks of innovations remain at constant levels. The equilibrium investments also remain constant in a steady-state equilibrium. We call a steady-state equilibrium symmetric with respect to investment if all innovators invest the same amount. It turns out that symmetry is a necessary condition for a steady-state equilibrium. A symmetric steady-state equilibrium is thus characterized by the common constant equilibrium investment  $j$ . In what follows, we focus on the symmetric steady-state equilibria, which we simply refer to as equilibria.

In seeking the equilibrium investment level, we assume that all other innovators choose investment  $j$  and the innovator under study chooses  $i$ . Because the stream of revenue from all the previous innovations is just like fixed income without an impact on the investment, we can, without loss of generality, focus on an innovator who has no previous monopoly innovations. The innovator's expected utility is given by the Bellman equation,

$$U^0 = \max_i \{-i + \rho(i)\pi + \Gamma + \delta[\rho(i)\lambda U^1 + (1 - \rho(i)\lambda)U^0]\}. \quad (6)$$

The utility of the current period comes from the cost of investment  $-i$ , the probability of becoming a monopolist  $\rho(i)$  times the monopoly revenue  $\pi$  and the innovator's share of the aggregate consumer surplus,  $\Gamma$ . If the innovator comes up with a new monopoly innovation and if this innovation does not become obsolete at the end of the period, the continuation value is  $U^1$ . In the complementary case, i.e., if there are others that have developed the same innovation or if the innovation becomes public or obsolete, the innovator's continuation value is  $U^0$ .

The probability of becoming a monopolist given investment  $i$  is

$$\rho(i) = (1 - e^{-i})\alpha \sum_{k=0}^{\infty} e^{-\theta} \frac{\theta^k}{k!} (e^{-j})^k = (1 - e^{-i})\alpha e^{-\tilde{\theta}}. \quad (7)$$

Equation (7) tells us that, to become a monopolist, the innovator has to be successful in developing the idea into an innovation and the resulting innovation may not become public. This happens with probability  $(1 - e^{-i})\alpha$ . If there are  $k$  other innovators that have found the same idea, none of them should be successful. This happens with probability  $(e^{-j})^k$ . Because there can be any number  $k$  of other innovators, we sum over  $k$ .

The difference between  $U^0$  and  $U^1$  is the expected stream of revenue from one monopoly innovation. By recalling that  $\pi = 1/2$ , we can hence write

$$U^1 = U^0 + \pi \sum_{t=0}^{\infty} (\delta\lambda)^t = U^0 + \frac{1}{2(1 - \delta\lambda)}. \quad (8)$$

<sup>6</sup> We model the pricing stage only implicitly through payoffs. Requiring subgame perfection in a game where the innovators choose first investment and then prices would yield exactly the same results.

After substituting (8) and (7) for  $U^1$  and  $\rho(i)$ , we can solve (6) for the value function as

$$U^0 = \frac{1}{1-\delta} \max_i \left\{ -i + (1 - e^{-i})e^{-\tilde{\theta}} \frac{\alpha}{2(1-\delta\lambda)} + \Gamma \right\}. \quad (9)$$

The first-order condition to the maximization is

$$-1 + e^{-i} e^{-\tilde{\theta}} \frac{\alpha}{2(1-\delta\lambda)} = 0. \quad (10)$$

It is straightforward to check that the second-order condition holds. As all innovators are *ex ante* identical, their first-order conditions are identical as well. This implies that an equilibrium is necessarily symmetric, i.e.,  $i = j$ . Consequently, (10) implies the equilibrium condition

$$e^{-j} e^{-\tilde{\theta}} = \frac{2(1-\delta\lambda)}{\alpha}. \quad (11)$$

Condition (11) implicitly determines the equilibrium investment  $j$  as a function of the parameters. Because  $\tilde{\theta} = \theta(1 - e^{-j})$ , the lefthand side of (11) is a strictly decreasing function of  $j$  that goes from one to zero as  $j$  varies from zero to infinity. Thus, the equilibrium is unique. The equilibrium investment is positive as long as the righthand side of (11) is strictly between zero and one. The first inequality is always satisfied because  $\delta, \lambda \in (0, 1)$ . For further reference, we record the condition ensuring the second as the following lemma.

*Lemma 1.* The equilibrium investment is strictly positive if and only if  $\delta > (2 - \alpha)/(2\lambda)$ .

Roughly speaking, an interior solution exists if the parameters are large enough; as all  $\alpha$ ,  $\delta$  and  $\lambda$  belong to the unit interval, a necessary condition for the model to be interesting is that  $\lambda > 1/2$ . Because  $\delta$  and  $\lambda$  enter the maximization problem in the innovator's value function (9) symmetrically, we could also define the effective discount factor as  $\delta\lambda$  and rewrite the parameter restriction as  $\delta\lambda + (1/2)\alpha > 1$ . In words, for a given level of the effective discount factor, protection should be strong enough or, for a given level of protection, the innovators should put sufficient weight on future profits. However, the size of the relevant parameter space is of little importance because it depends on the specifics of the demand function. For instance, if we increased the demand intercept, the parameter space would expand. In the sequel, we consider only the part of the parameter space where the solution is interior. We also focus only on the parameters  $\alpha$ ,  $\lambda$  and  $\theta$  in the analysis. This is without loss of generality, as the results concerning  $\delta$  are identical to those concerning  $\lambda$ .

The following comparative statics show that the model behaves nicely.

*Lemma 2.* The equilibrium investment is increasing in the strength of protection,  $\alpha$ . It is decreasing in the obsolescence rate,  $1 - \lambda$ , and in the ratio of innovators to ideas,  $\theta$ .

*Proof.* See the Appendix.

Lemma 2 tells us that the larger is the spillover  $1 - \alpha$  or the obsolescence rate  $1 - \lambda$ , the smaller is the incentive to innovate. Similarly, if the ratio of innovators to ideas,  $\theta$ , is high, as is likely to be the case in mature industries, where possible avenues for product development are all but exhausted, the investment in R&D is low. In contrast, in new high-tech industries, there are more opportunities to innovate ( $\theta$  is low) and a successful innovator is likely to encounter less competition, which naturally stimulates the incentives.

## 4. Welfare without patents

■ As we analyze steady-state equilibria, it is natural to use the periodic aggregate surplus as a welfare measure. The welfare in the economy is given by

$$W \equiv -j + \Pi + \Gamma = -j + \frac{4(1 - e^{-\tilde{\theta}}) - \alpha\tilde{\theta}e^{-\tilde{\theta}}}{4\theta(1 - \lambda)}, \quad (12)$$



where we have used (4) and (5) to substitute for the aggregate profits  $\Pi$  and the aggregate consumer surplus  $\Gamma$ . In the above derivation, we have also used the fact that  $\theta = 1/I$ . Inserting the equilibrium condition (11) into (12) yields the welfare in the market equilibrium with secrecy. Because of the distortions arising from the monopoly pricing, spillovers and duplication of research effort, one expects the market solution to differ from the second best optimum, where investment  $j$  is chosen to maximize  $W$ .<sup>7</sup> The first two effects tend to yield underinvestment in innovation, the third overinvestment. Proposition 1 shows that the effects contributing to underinvestment dominate.

*Proposition 1.* There is too little investment in the market equilibrium compared with the second best optimum.

*Proof.* See the Appendix.

Unlike in standard patent races where the possibility of overinvestment is pervasive, in our model, markets unambiguously deliver too little innovation. A reason is that the duplication of research here is not necessarily wasteful because it leads to greater production and consumer surplus. As a result, Proposition 1 conveys the traditional rationale for technology policy: Because of the incomplete appropriability of the social value of innovation, there is too little investment in R&D as individual innovators do not take into account the benefits of their innovations to consumers. The benefits do not only arise from the new innovations fully owned by their innovators but also from spillovers, which expand consumer surplus as they increase the number of innovations for which the production is at the competitive level. Technology policy tries to boost the incentive to innovate through prizes, contract research, R&D subsidies and various IPRs. In the next section, we turn to the last instruments and, in particular, to patents.

We point out that the underinvestment result can be sensitive to market and cost structure. For example, the result is violated under Cournot competition if there are fixed costs of innovation, ideas are rare and spillovers insignificant.

## 5. Equilibria with patents and the choice between patenting and secrecy

■ To facilitate the comparison of patent protection and secrecy, patent policy is encapsulated in a single dimension, the strength of patent protection. It is denoted by  $\alpha_p \in [0, 1]$ , where  $\alpha_p = 1$  means maximum protection, i.e., a patent gives perfect, well-defined property right with no spillovers and  $\alpha_p = 0$  means that the property right immediately disappears and the innovation is made public. As with parameter  $\alpha$  in the case of secrecy, we take  $\alpha_p$  to be the probability that the innovator maintains her monopoly. This is a quite general definition of the strength and suggests that patents represent uncertain property rights. In practice, there are many events that can make a patent valueless before its formal expiration date, such as court challenges to the legal validity of the patent, a rival's success in inventing around the patent and legal malpractice that forfeits the patent through the failure to renew it, observe legal formalities or reserve rights when the technology is disclosed. In other words, we can think of  $\alpha_p$  denoting the probability that a patent holder succeeds to exclude others from using her innovation and with complementary probability  $1 - \alpha_p$  the patent is found invalid or it can be infringed.

The way we model patents means that, once a patent is granted, there is a single randomization that determines whether the innovation becomes public; if not, the innovator gets an infinitely lived, perfect property right. One can think that all uncertainties concerning the property right conferred by the patent during the entire life of the innovation are determined by this single randomization. Any level of protection that can be achieved using an imperfect, finitely lived patent can be achieved using the combination of perfect, infinite patents and an appropriately chosen initial randomization. This is a particularly appealing way to define patent protection in this model as the length of protection under secrecy is also infinite.

<sup>7</sup> Our definition of second best assumes that the social planner is able to adjust investment but is unable to change the market structure.

In practice, the protection under patent policy can be stronger or weaker than under secrecy. We therefore allow  $\alpha_p$  to be greater or smaller than  $\alpha$ , which is the measure of protection under secrecy. As a result, we can consider both the traditional patent policy, where patents are assumed to increase protection, and the view according to which patents actually decrease it because of, say, disclosure rules and costly litigation.

We assume that  $\alpha_p$  is the decision variable of the policy makers and the individual innovators take it as given. In addition to the investment decision, the innovators now have to choose between patenting ( $p$ ) and secrecy ( $s$ ). If an innovation is kept secret, protection is determined by  $\alpha$ , as in the case without a patent system. If it is patented, protection will be determined by  $\alpha_p$ . If two or more innovators apply for the same patent, the patent holder is chosen by a lottery, where each applicant has an equal probability of winning. When an innovator receives a patent on an innovation, she can exclude the other developers of the same innovation from the market with probability  $\alpha_p$ .

The innovators make the patenting decision after finding out whether their own investment resulted in an innovation. At this stage, they do not know how many other innovators have made the same innovation. Otherwise, the timing is as before. Thus, each period, a two-stage extensive form game is played where, at the first stage, the innovators find ideas and simultaneously invest in R&D and, at the second stage, find out about their success and choose patenting or secrecy. In making the patenting decision, a successful innovator knows only her own first-stage investment. Being successful reveals no information about the other innovators' behavior, as the probability of success is independent across innovators.

An equilibrium in the periodic extensive form game ascribes to each innovator an investment and a decision rule that associates with every possible investment level the decision to patent or not. An equilibrium of the infinite horizon game is a sequence of rules for each innovator determining the innovator's behavior in each period as a function of the history of the play. As in the case without patents, we restrict attention to steady-state equilibria, where the stocks of innovations and investment remain constant from one period to another. There are potentially three types of such equilibria: pure strategy equilibria, where all innovations are kept secret; pure strategy equilibria, where all innovations are patented; and mixed equilibria, where some proportion of the innovations is kept secret while others are patented. We will refer to these classes of equilibria as secrecy, patenting, and mixed equilibria. Because there is a continuum of innovators, we model mixed strategies by using the distributional approach. That is, we assume that proportion  $\sigma$  of the innovators use the pure strategy secrecy and proportion  $1 - \sigma$  use the pure strategy patenting for some  $\sigma \in [0, 1]$ . In equilibrium, the proportion  $\sigma$  is chosen to satisfy the indifference requirement between the payoffs from the two pure strategies.

We next establish that an equilibrium with patents is necessarily symmetric with respect to investment.

*Lemma 3.* In an equilibrium with patents, the investment is the same for all innovators.

*Proof.* The formal proof of the claim will be a part of the proof of Proposition 2, but we give a heuristic argument here. Recall that, when choosing between secrecy and patenting, an innovator does not know whether there are others who have made the same innovation. Furthermore, the optimal second-stage decision is independent of the (sunk) first-stage investment. As a result, in an equilibrium, either all successful innovators make the same decision, or, if there is mixing, secrecy and patenting must yield exactly the same expected payoff in the second stage. This in turn implies that the equilibrium expected utility from being successful, as calculated at the first stage, is the same for all innovators. Therefore, also the equilibrium investment is the same for all innovators: the marginal benefit from increasing the investment is the increase in the probability of success times the utility from being successful. The marginal cost is simply the marginal cost of investment. As the utility from being successful and the cost of investment is the same for all innovators, the result follows. *Q.E.D.*

When a patent system is introduced, the stocks of monopoly and competitive innovations are no longer determined by (1) and (3). In particular, the number of new monopoly innovations

depends on whether the innovators choose patenting or secrecy. To account for all possibilities, we determine the stocks as functions of the proportion  $\sigma \in [0, 1]$  of the innovators choosing secrecy. As we are ultimately interested in the equilibrium stocks we can, by Lemma 3, assume that all innovators make the same investment  $j$ . A derivation analogous<sup>8</sup> to that in Section 2 yields the steady-state stock of monopoly innovations,

$$M(\sigma) = \frac{\sigma\alpha\tilde{\theta}e^{-\tilde{\theta}} + \alpha_p(1 - e^{-(1-\sigma)\tilde{\theta}})}{1 - \lambda}I,$$

and the steady-state stock of competitive innovations,

$$C(\sigma) = \frac{1 - e^{-\tilde{\theta}} - \sigma\alpha\tilde{\theta}e^{-\tilde{\theta}} - \alpha_p(1 - e^{-(1-\sigma)\tilde{\theta}})}{1 - \lambda}I.$$

It is easy to verify that  $M(1)$  and  $C(1)$  equal (1) and (3).

The periodic aggregate revenue is now given by

$$\Pi(\sigma) = M(\sigma)\pi = \frac{\sigma\alpha\tilde{\theta}e^{-\tilde{\theta}} + \alpha_p(1 - e^{-(1-\sigma)\tilde{\theta}})}{2(1 - \lambda)}I, \quad (13)$$

and the aggregate consumer surplus is given by

$$\begin{aligned} \Gamma(\sigma) &= M(\sigma)\gamma + C(\sigma)\gamma_c \\ &= \frac{4(1 - e^{-\tilde{\theta}}) - 3\sigma\alpha\tilde{\theta}e^{-\tilde{\theta}} - 3\alpha_p(1 - e^{-(1-\sigma)\tilde{\theta}})}{4(1 - \lambda)}I. \end{aligned} \quad (14)$$

Next we determine the equilibrium in an industry with a patent system. As in Section 3, we can, without loss of generality, focus on an innovator who possesses no monopolies, patented or otherwise. The innovator under study chooses investment  $i$  while all others choose investment  $j$ . When each period proportion  $\sigma$  of the innovators opt for secrecy, the innovator's expected utility is given by the Bellman equation,

$$U_p^0(\sigma) = \max_i \left\{ -i + \Gamma(\sigma) + \max \left\{ \rho_p(i, \sigma)\pi + \delta[\rho_p(i, \sigma)\lambda U_p^1(\sigma) + (1 - \rho_p(i, \sigma)\lambda)U_p^0(\sigma)], \right. \right. \\ \left. \left. \rho(i, \sigma)\pi + \delta[\rho(i, \sigma)\lambda U_p^1(\sigma) + (1 - \rho(i, \sigma)\lambda)U_p^0(\sigma)] \right\} \right\},$$

where  $U_p^1(\sigma)$  is the value function of an innovator who possesses one monopolized innovation. The inner maximization on the righthand side deals with the second-stage decision between patenting and secrecy: the first term gives the expected payoff from patenting and the latter term the expected utility from secrecy. In the above Bellman equation,  $\rho(i, \sigma)$  is the probability of becoming a monopolist when choosing investment  $i$  and secrecy, given that proportion  $\sigma$  of the innovators chooses secrecy. Similarly,  $\rho_p(i, \sigma)$  is the probability of becoming a monopolist when choosing  $i$  and patenting. The interpretation of the other terms is analogous to (6).

Once the innovator holds a monopoly position, it no longer matters whether the position was achieved by patenting or secrecy, as in both cases there is a single, initial randomization determining whether the others can be excluded from using the innovation. Thus, as in (8), the difference between  $U_p^1(\sigma)$  and  $U_p^0(\sigma)$  is just the expected stream of revenue  $1/[2(1 - \delta\lambda)]$  from one monopolized innovation. Analogously to the derivation of (9), we can then substitute  $U_p^0(\sigma) + [1/(2(1 - \delta\lambda))]$  for  $U_p^1(\sigma)$  in the above Bellman equation, and solve it for the value function,

$$U_p^0(\sigma) = \frac{1}{1 - \delta} \max_i \left\{ -i + \Gamma(\sigma) + \max \left\{ \frac{\rho_p(i, \sigma)}{2(1 - \delta\lambda)}, \frac{\rho(i, \sigma)}{2(1 - \delta\lambda)} \right\} \right\}. \quad (15)$$

<sup>8</sup> See the supplementary Appendix for details.

If the innovator decides to patent, her probability of obtaining a monopoly position is

$$\rho_p(i, \sigma) = (1 - e^{-i})\alpha_p \sum_{k=0}^{\infty} e^{-\theta} \frac{\theta^k}{k!} \sum_{h=0}^k \binom{k}{h} (1 - e^{-j})^h (e^{-j})^{k-h} \sum_{l=0}^h \binom{h}{l} (1 - \sigma)^l \sigma^{h-l} \frac{1}{l+1}.$$

The factors in front of the summations on the righthand side capture the probabilities that the innovator is successful in transforming an idea into an innovation and, if she obtains a patent, that the others can be excluded from using the innovation. The summations determine the probability of obtaining the patent. The first sum is over the probabilities that  $k$  other innovators find the same idea as the innovator we study. The middle summation gives the probabilities that  $h$  of the  $k$  other innovators succeed in developing the idea into an innovation. The last summation takes into account the probabilities that the innovator we study receives the patent when there are  $l$  other patent applicants among the  $h$  successful innovators. Straightforward but tedious calculations<sup>9</sup> simplify the above expression to

$$\rho_p(i, \sigma) = \begin{cases} (1 - e^{-i})\alpha_p \frac{1 - e^{-(1-\sigma)\tilde{\theta}}}{(1-\sigma)\tilde{\theta}} & \text{if } \sigma \in [0, 1), \\ (1 - e^{-i})\alpha_p & \text{if } \sigma = 1. \end{cases} \quad (16)$$

In the limit where  $\sigma = 1$ , the innovator is the only patent applicant, so she receives the patent for sure, provided that she has succeeded in developing the idea into an innovation.

If the innovator chooses secrecy, her probability of getting a monopoly is

$$\rho(i, \sigma) = (1 - e^{-i})\alpha \sum_{k=0}^{\infty} e^{-\theta} \frac{\theta^k}{k!} (e^{-j})^k = (1 - e^{-i})\alpha e^{-\tilde{\theta}}, \quad (17)$$

which is the same as (7). In particular,  $\rho$  does not depend directly on  $\sigma$ , as the only case where the innovator can obtain a monopoly position when choosing secrecy is when no other innovator is successful with the same idea.

As (15) shows, the optimal investment, given the behavior of the other innovators, maximizes  $U_p^0(\sigma)$  with respect to  $i$ . The first-order condition depends on the outcome of the innovator's patenting decision. In case the innovator opts for secrecy, we have

$$-1 + e^{-i} e^{-\tilde{\theta}} \frac{\alpha}{2(1 - \delta\lambda)} = 0,$$

which is the same as the first-order condition without patents (10). In case the innovator patents, the first-order condition is given by

$$-1 + e^{-i} \frac{1 - e^{-(1-\sigma)\tilde{\theta}}}{(1-\sigma)\tilde{\theta}} \frac{\alpha_p}{2(1 - \delta\lambda)} = 0, \quad (18)$$

which, in the limit when no one else patents (i.e., when  $\sigma = 1$ ), simplifies to

$$-1 + e^{-i} \frac{\alpha_p}{2(1 - \delta\lambda)} = 0. \quad (19)$$

It is straightforward to verify that the second-order condition holds in each of the above cases.

In the following proposition, we show that all three different types of equilibria, characterized by the innovators' second-stage decisions, exist for some values of parameters. For given values

<sup>9</sup> Details can be found in the supplementary Appendix.

of the other parameters, the type of equilibria is determined by the strength of the patent protection  $\alpha_p$ . Because the proof of the proposition forms the core of our analysis, we present it in the main text despite its length.

*Proposition 2.* Suppose that the parameter restrictions in Lemma 1 hold so that the equilibrium investment without patents is positive.<sup>10</sup> Then for each patent strength  $\alpha_p$ , there is a unique steady-state equilibrium. Furthermore, there exist patent strengths  $\underline{\alpha}$  and  $\bar{\alpha}$  such that the equilibrium is

- (i) secrecy if  $\alpha_p \in [0, \underline{\alpha}]$ ;
- (ii) mixed if  $\alpha_p \in (\underline{\alpha}, \bar{\alpha})$ ;
- (iii) patenting if  $\alpha_p \in [\bar{\alpha}, 1]$ .

The cutoff patent strengths satisfy  $0 < \underline{\alpha} < \bar{\alpha} < \alpha$ , where  $\alpha$  is the protection under secrecy.

*Proof.* Let us start with the patenting equilibria (iii). In this case,  $\sigma = 0$ , as all innovators patent by definition. The equilibrium investment must satisfy the first-order condition (18) for  $\sigma = 0$ . By symmetry, we see that the equilibrium investment must then be the same for all innovators, confirming Lemma 3 for the patenting case. We denote this investment by  $j_p$ . It is determined by (18), reproduced here as

$$e^{-j_p} \frac{1 - e^{-\tilde{\theta}}}{\tilde{\theta}} = \frac{2(1 - \delta\lambda)}{\alpha_p}, \quad (20)$$

where  $\tilde{\theta} = \theta(1 - e^{-j_p})$ . The lefthand side is a strictly decreasing function of  $j_p$  that attains values from one to zero as  $j_p$  ranges from zero to infinity. Hence, the equilibrium investment is unique. Comparing the equilibrium condition to equation (11), we see that the parameters on the righthand side must satisfy restrictions analogous to Lemma 1 for the equilibrium investment to be positive.

Let us then turn to the decision to patent. By the one-shot deviation property, it is sufficient to consider only deviations in a single period. From the inner maximization problem in the value function  $U_p^0(0)$  in (15), we see that a deviation to secrecy is unprofitable iff

$$\frac{\rho_p(i, 0)}{2(1 - \delta\lambda)} \geq \frac{\rho(i, 0)}{2(1 - \delta\lambda)},$$

where  $i$  is the innovator's sunk investment. After the substitution of (16) and (17) for  $\rho_p(i, 0)$  and  $\rho(i, 0)$ , the condition can be simplified to

$$\alpha_p \geq \frac{\tilde{\theta}e^{-\tilde{\theta}}}{1 - e^{-\tilde{\theta}}}\alpha, \quad (21)$$

where the righthand side is also a function of  $\alpha_p$  as  $\tilde{\theta} = \theta(1 - e^{-j_p})$  and the equilibrium investment  $j_p$  depends on  $\alpha_p$  through the first-order condition (18). The derivative of the right-hand side of (21) with respect to  $\alpha_p$  is

$$\frac{d}{dj_p} \left( \frac{\tilde{\theta}e^{-\tilde{\theta}}}{1 - e^{-\tilde{\theta}}}\alpha \right) \frac{dj_p}{d\alpha_p} < 0,$$

where the inequality follows from the facts that  $dj_p/d\alpha_p > 0$  by Lemma 4 below and  $d[\tilde{\theta}e^{-\tilde{\theta}}/(1 - e^{-\tilde{\theta}})]\alpha/dj_p < 0$ . Thus, the righthand side of (21) is decreasing and the lefthand side increasing in  $\alpha_p$ . At  $\alpha_p = 0$ , the lefthand side of (21) is zero and the righthand side is equal to  $\alpha \leq 1$ , whereas at  $\alpha_p = 1$ , the righthand side is less than  $\alpha \leq 1$  because it is strictly decreasing

<sup>10</sup> If the parameter restrictions do not hold, then there is no investment without patents, but it may be possible to induce positive investment by introducing strong enough patent protection. In this case, the successful innovators always patent.

in  $\alpha_p$ . Therefore, there exists a unique cutoff patent strength  $\bar{\alpha}$  such that the deviation to secrecy is unprofitable iff  $\alpha_p \geq \bar{\alpha}$ . This cutoff is implicitly given by

$$\bar{\alpha} = \frac{\tilde{\theta} e^{-\tilde{\theta}}}{1 - e^{-\tilde{\theta}}} \alpha, \quad (22)$$

where  $j_p$  in  $\tilde{\theta}$  is such that it satisfies the first-order condition (18) for  $\alpha_p = \bar{\alpha}$ . To show that  $\bar{\alpha} < \alpha$ , notice first that  $1 - e^{-\tilde{\theta}} > \tilde{\theta} e^{-\tilde{\theta}}$  for all  $\tilde{\theta} > 0$ . Hence, it suffices to argue that the equilibrium investment  $j_p$  is positive at  $\alpha_p = \bar{\alpha}$ , as then  $\tilde{\theta} = \theta(1 - e^{-j_p}) > 0$ . We do this below by showing that the equilibrium investment with patents at  $\alpha_p = \bar{\alpha}$  is actually equal to the equilibrium investment without patents, which in turn is positive by assumption. Hence, there exists a cutoff  $\bar{\alpha} < \alpha$  such that there exists a unique patenting equilibrium iff  $\alpha_p \geq \bar{\alpha}$ .

Consider then the secrecy equilibria (i). In a secrecy equilibrium,  $\sigma = 1$  by definition. As all innovations are kept secret, the equilibrium is characterized by the same conditions as in Section 3, where we studied an industry without patent protection. This implies that secrecy equilibria are unique and symmetric with respect to investment, confirming Lemma 3 for the secrecy case. However, the introduction of the patent system requires us to check that no innovator can profitably deviate to patenting. From (15), we see that such a deviation is unprofitable iff

$$\frac{\rho_p(i, 1)}{2(1 - \delta\lambda)} \leq \frac{\rho(i, 1)}{2(1 - \delta\lambda)}.$$

Substituting for  $\rho_p(i, 1)$  and  $\rho(i, 1)$  from (16) and (17) and simplifying, the condition can be written as

$$\alpha_p \leq e^{-\tilde{\theta}} \alpha.$$

On the righthand side, the investment in  $\tilde{\theta}$  is the equilibrium investment without patents determined by the equilibrium condition (11), which is independent of  $\alpha_p$ . Thus, the deviation to patenting is unprofitable iff  $\alpha_p \leq e^{-\tilde{\theta}} \alpha \equiv \underline{\alpha}$ , establishing the existence of the cutoff  $\underline{\alpha}$ . Furthermore, the parameter restrictions of Lemma 1 imply  $\tilde{\theta}, \alpha > 0$ , so that  $\underline{\alpha} > 0$ .

Let us then show that  $\underline{\alpha} < \bar{\alpha} < \alpha$ . When  $\alpha_p = \bar{\alpha}$ , the innovator is indifferent between patenting and secrecy given that all others patent. Thus, her investment  $j_p$  could just as well be determined by the first-order condition (10), reproduced here as

$$e^{-j_p} e^{-\theta(1 - e^{-j_p})} = \frac{2(1 - \delta\lambda)}{\alpha}. \quad (23)$$

This has to be satisfied for the innovator's choice  $j_p$  when also all others invest  $j_p$ . On the other hand, the equilibrium investment without patents  $j$  is determined by the equilibrium condition (11). Comparing (11) with (23), we see that  $j_p = j > 0$ . As discussed above, this implies that  $\bar{\alpha} < \alpha$ . Furthermore, because the equilibrium investments are equal, we have

$$\underline{\alpha} < \bar{\alpha} \quad \text{iff} \quad e^{-\tilde{\theta}} \alpha < \frac{\tilde{\theta} e^{-\tilde{\theta}}}{1 - e^{-\tilde{\theta}}} \alpha \quad \text{iff} \quad 1 - e^{-\tilde{\theta}} - \tilde{\theta} < 0.$$

The last inequality  $1 - e^{-\tilde{\theta}} - \tilde{\theta} < 0$  holds for all  $\tilde{\theta} > 0$ . Thus,  $0 < \underline{\alpha} < \bar{\alpha} < \alpha$ .

Consider finally the mixed equilibria (ii). In this case,  $\sigma \in (0, 1)$ . For an innovator to be indifferent between patenting and secrecy, we must have

$$\frac{\rho_p(i, \sigma)}{2(1 - \delta\lambda)} = \frac{\rho(i, \sigma)}{2(1 - \delta\lambda)}.$$

Substituting for  $\rho_p(i, \sigma)$  and  $\rho(i, \sigma)$  from (16) and (17) and simplifying, this becomes

$$\alpha_p \frac{1 - e^{-(1-\sigma)\tilde{\theta}}}{(1 - \sigma)\tilde{\theta}} = \alpha e^{-\tilde{\theta}}. \quad (24)$$

There is a unique  $\sigma$ , denoted by  $\hat{\sigma}$ , that solves (24) iff  $\alpha_p \in [\underline{\alpha}, \bar{\alpha}]$ . It is straightforward to verify that  $\hat{\sigma}$  is a decreasing, continuous function of  $\alpha_p$ . Furthermore, if  $\alpha_p = \underline{\alpha}$ , then  $\hat{\sigma} = 1$ , and if  $\alpha_p = \bar{\alpha}$ , then  $\hat{\sigma} = 0$ . Thus, the proportion of the innovators that choose secrecy declines continuously from one to zero when the patent strength increases over the range  $[\underline{\alpha}, \bar{\alpha}]$ . The equilibrium investment for a given  $\hat{\sigma}$  must simultaneously satisfy the first-order conditions (10) and (18). As the innovators are indifferent between secrecy and patenting, the investment must be the same for all innovators, confirming Lemma 3 for the mixing case.

Let us then establish uniqueness. It is clear from the above that there can be no patenting or mixed equilibria for  $\alpha_p \in [0, \underline{\alpha}]$ . Hence, the equilibrium must be a secrecy equilibrium. But we argued above that, in this case, the secrecy equilibrium is unique. Similarly, for  $\alpha_p \in (\underline{\alpha}, \bar{\alpha})$ , there can be no secrecy or patenting equilibria and there is a unique  $\hat{\sigma}$  corresponding to each  $\alpha_p$ . In the last case, where  $\alpha_p \in [\underline{\alpha}, 1]$ , there can be no secrecy or mixed equilibria and the patenting equilibrium is unique. *Q.E.D.*

Proposition 2 characterizes equilibria in terms of the innovators' second-stage decision. It shows that the propensity to patent is an increasing function of patent strength. If patents afford too weak protection compared with secrecy (i.e., if  $\alpha_p \leq \underline{\alpha}$ ), the innovators simply choose to keep their innovations secret. This might be the case, e.g., for process innovations for which reverse engineering is difficult. Patenting the process innovation would require disclosing information about the innovation to an extent that allows a skilled person to replicate the innovation. However, enforcing the patent would be difficult for the very same reason, as reverse engineering: it would be hard to know from a competitor's end product whether the patent has been infringed. In other words, for a process innovation, the protection under secrecy  $\alpha$  is likely to be high whereas the protection granted by the patent  $\alpha_p$  is likely to be low.

If patent protection is sufficiently strong relative to secrecy (i.e., if  $\alpha_p \geq \bar{\alpha}$ ), all innovations are patented. An extreme example in this direction is pharmaceutical innovations. Introducing a new drug requires disclosing so much information that, without patents, the innovator would be left with practically no protection.

For intermediate levels of patent protection,  $\alpha_p \in [\underline{\alpha}, \bar{\alpha}]$ , the model predicts a mixed equilibrium where both secrecy and patenting coexist. To the best of our knowledge, this is the first model where identical innovations are both kept secret and patented in equilibrium. The proportion of patented innovations is strictly increasing in the patent strength  $\alpha_p$  in the interval  $[\underline{\alpha}, \bar{\alpha}]$ .

It is not surprising that innovators waive patent protection when it is sufficiently weak, nor is it surprising that they hold onto it when the protection is sufficiently strong. However, the cutoff level of patent protection is strictly less than protection under secrecy (i.e.,  $\bar{\alpha} < \alpha$ ). This is due to the prisoner's dilemma-like situation created by the patent system as demonstrated by the example in the Introduction. If the innovator trusts secrecy, she gets positive revenue only when no other innovator is successful with the same idea. In contrast, if the innovator patents, she not only gets positive revenue when she is alone but also has a positive probability of receiving the patent when there are others who have developed the same innovation. Thus, patenting is strictly better than secrecy when they offer equal protection (i.e., when  $\alpha_p = \alpha$ ). It is then clear that we can decrease the patent protection, at least marginally, and still preserve the innovators' choices.

## 6. Incentive to innovate

■ In this section, we discuss the implications of the choice between secrecy and patenting for the incentive to invest in R&D. The comparative statics of the equilibrium investment are given in the following lemma.

*Lemma 4.* The equilibrium investment is increasing in the patent strength  $\alpha_p$  in the patenting equilibria. It is constant in  $\alpha_p$  in the secrecy and mixed equilibria. The investment is decreasing in the obsolescence rate  $1 - \lambda$  and in the ratio of innovators to ideas  $\theta$  in all three types of equilibria.

*Proof.* See the Appendix.

The first property in Lemma 4 shows that strengthening patent protection increases the incentive to innovate because it reduces information spreading. This constitutes the standard justification for patent policy. The explanations for the obsolescence rate and the ratio of innovators to ideas are the same as in the case of Lemma 2.

Whereas Lemma 4 describes the changes in the investment, the next proposition compares the levels of investment in the industries with and without a patent system. Recall from Proposition 2 that  $\bar{\alpha}$  is the cutoff patent strength above which a patenting equilibrium prevails.

*Proposition 3.* If  $\alpha_p > \bar{\alpha}$ , the equilibrium investment with patents is higher than the equilibrium investment without a patent system. If  $\alpha_p \leq \bar{\alpha}$ , the equilibrium investment is the same with and without patents.

*Proof.* The equilibrium investment without a patent system must be the same as the equilibrium investment with patents in the secrecy equilibria, which prevails for  $\alpha_p \leq \underline{\alpha}$  by Proposition 2. Moreover, we show in the proof of Lemma 4 that the equilibrium investment is at the same level in the mixed equilibria for all  $\alpha_p \in [\underline{\alpha}, \bar{\alpha}]$ . Hence, the equilibrium investments with and without patents are the same for  $\alpha_p \leq \bar{\alpha}$ . By Lemma 4, the equilibrium investment is increasing in  $\alpha_p$  in the patenting equilibria, whereas it is constant in  $\alpha_p$  in an industry without patents. This implies the result for  $\alpha_p > \bar{\alpha}$ . *Q.E.D.*

This result shows that, whenever it is optimal for all innovators to patent their innovations, the patent system encourages R&D effort. Because the innovators choose the form of protection that provides the highest expected profits and because they invest more the higher the benefit, whenever all innovators actually prefer patenting to secrecy, the investments will increase. However, the strengthening of patent rights does not necessarily improve the incentive to innovate. At the intermediate level of protection (i.e.,  $\alpha_p \in [\underline{\alpha}, \bar{\alpha}]$ ), the increase in the incentive to innovate resulting from stronger patent protection is exactly offset by the increase in the number of innovators who switch from secrecy to patenting. The key message of Proposition 3 arises from the observation that the cutoff patent protection for increasing the investment is strictly less than protection under secrecy (i.e.,  $\bar{\alpha} < \alpha$ ). Thus, the patent system can simultaneously increase the spillovers and enhance the incentive to invest in R&D.

## 7. Welfare effect of patents and optimal patent policy

■ In this section, we consider the design of the optimal patent policy. Total welfare in the patenting equilibria is, analogous to (12), given by

$$W_p(\alpha_p) = -j_p + \Pi(0) + \Gamma(0) = -j_p + \frac{(4 - \alpha_p)(1 - e^{-\tilde{\theta}_p})}{4\theta(1 - \lambda)}, \quad (25)$$

where we have used (13) and (14) to substitute in for the aggregate profits and the aggregate consumer surplus. To find the optimal patent policy, we must maximize (25) with respect to patent strength  $\alpha_p$ , taking into account that the strength must be large enough for the innovators to patent rather than opt for secrecy, i.e., that  $\alpha_p \geq \bar{\alpha}$ . The reason why we require the equilibrium to be a pure patenting equilibrium rather than a mixed equilibrium is that the welfare in a mixed equilibrium is the same as in the secrecy equilibria: the equilibrium investment is the same in the two types of equilibria as shown in Proposition 3. For the investments to be the same, it must be that the expected profit must be the same. But an innovator can get positive profits only if she has a monopoly. Thus, the expected number of monopolies must be the same in the two cases. Given that the investment is the same and the expected number of monopolies is the same, also the expected number of competitive innovations must be the same. This implies that the sum of the aggregate profits and the aggregate consumer surplus, i.e., welfare, is the same as well.

Because we assume that, in the absence of patent policy, the spillover cannot be affected, we first compare the welfare with and without the patent system. If the welfare without patents is larger, the optimal patent strength will be so low that no one patents. Because in the absence



of patents markets generate too little investment in R&D by Proposition 1, it looks like patent policy necessarily increases welfare at least when it boosts investments in R&D. This turns out to be the case in the sense that the optimal patent policy can always at least replicate the welfare without patents. However, we cannot jump to the conclusion, as patent policy involves familiar negative welfare effects, that granting patents leads to duplication of research effort, creates monopoly distortions, and increasing patent strength decreases the consumer surplus through reduced spillovers. The next result shows that we can establish a patent policy where the negative effects are offset by the positive incentive effects.

*Proposition 4.* There exists a patent strength such that everyone patents and social welfare is the same as in the equilibrium without patents.

*Proof.* See the Appendix.

Proposition 4 implies that the optimal patent policy where all innovators patent yields (weakly) higher welfare than the market equilibrium without intellectual property protection. In the proof of the proposition, we show that, if patent strength is chosen such that the investment in R&D is the same with and without patent policy (i.e., if  $\alpha_p = \bar{\alpha}$ ), then the welfare under the two regimes is the same as well. This means that, even though monopoly distortions reduce welfare, patent policy spreads information and increases the stock of competitive innovations and consumer surplus just enough to offset the reduction. The spreading of information is captured here by the fact that  $\alpha_p = \bar{\alpha} < \alpha$ .

Next we derive the optimal patent policy  $\alpha_p^*$ , which satisfies

$$\alpha_p^* = \arg \max_{\alpha_p \in [\bar{\alpha}, 1]} W_p(\alpha_p).$$

In this task, we need to take into account that the investment  $j_p$  depends on  $\alpha_p$  through the equilibrium condition (20).

*Proposition 5.* The optimal patent strength is given by  $\alpha_p^* = \max\{\bar{\alpha}, \min\{\alpha^*, 1\}\}$ , where  $\alpha^* = [4\tilde{\theta}_p(e^{-\tilde{\theta}_p}e^{-j_p} - 1 + \lambda)]/(1 - e^{-\tilde{\theta}_p})$  is the solution to the unrestricted program  $\max_{\alpha_p} W_p(\alpha_p)$ .

*Proof.* See the Appendix.

This result shows that the optimal patent policy may be constrained by the innovators' option to choose secrecy over patenting. Somewhat surprisingly also the constraint  $\alpha_p \leq 1$  may be binding at the optimum. Roughly speaking, constraint  $\alpha_p \leq 1$  binds when there would be little investments in R&D without patent protection, e.g., when obsolescence rate is very high. Then setting perfect patent protection ( $\alpha_p^* = 1$ ) is optimal despite the involved monopoly distortions, as there is a strong need to enhance the incentive to innovate. Note that the patent protection at an interior solution,  $\alpha^*$ , is given in implicit form as the investment,  $j_p$ , and the realized R&D-intensity,  $\tilde{\theta}_p = \theta(1 - e^{-j_p})$ , on the right-hand side are functions of  $\alpha^*$ .

Finally, we determine how the design of the optimal patent policy depends on obsolescence rate and the R&D potential of the industry.

*Proposition 6.* When the innovators are sufficiently patient (i.e., when  $\delta$  is sufficiently high) the optimal patent strength  $\alpha_p^*$  is increasing in the obsolescence rate  $1 - \lambda$ . Furthermore, then there exists a cutoff obsolescence rate such that the optimal patent policy increases the spillover ( $\alpha_p^* < \alpha$ ) for obsolescence rates below the cutoff and decreases the spillover ( $\alpha_p^* > \alpha$ ) for obsolescence rates above the cutoff.

*Proof.* See the supplementary Appendix.

As explained after Lemma 1, parameters  $\delta$  and  $\lambda$  are practically equivalent from the innovators' point of view. When obsolescence rate is low and the innovators are patient, they value future profits. As a result, they are eager to innovate, and patent protection can be made weak to increase consumer surplus. In such circumstances, the disclosure theory of patents is justified:

optimal patents spread information by conferring weaker protection than secrecy. In contrast, if obsolescence rate is high (or the innovators heavily discount future), strong patents are warranted to restore the incentive to innovate, providing the traditional justification for patent policy.

The effect of the R&D potential of the industry on the design of optimal patent policy is more ambiguous from the outset because an increase in  $\theta$  has several effects on social welfare. It decreases the incentive to innovate by Lemma 4, decreases the number of ideas found, and increases the duplication of research effort. The first two effects suggest a stronger, the last one a weaker patent protection. Proposition 7, however, shows that the duplication of research effect unambiguously dominates.

*Proposition 7.* The optimal patent strength  $\alpha_p^*$  is decreasing in the ratio of innovators to ideas  $\theta$ .

*Proof.* See the supplementary Appendix.

Proposition 7 contrasts with the underinvestment result obtained without patents (Proposition 1). By ignoring the rights of independent innovators, the patent system inherently makes duplication of research effort more wasteful. In other words, under the patent system, only one innovator receives the right to produce regardless of the number of innovators that have independently made the same innovation; but without patents, duplication of research only renders innovations more competitive. Therefore, the optimal patent strength is sensitive to the probability of independent duplication of innovations. In industries where the innovative potential is high ( $\theta$  is low), there is little risk of independent duplication, and thus the patent protection can be strong to maximize the number of innovations. In contrast, in industries where innovative potential is low ( $\theta$  is high), it would be better to dilute the incentive to innovate and thereby reduce overlapping R&D projects. Combining Proposition 7 with Proposition 6 suggests that the range of parameter values where the optimal patent policy enlarges the spillover is increasing in  $\theta$ .

## 8. Conclusion

■ Our aim is to compare the effects of the patent system and secrecy on the incentive to innovate, information spreading, and welfare. To compare an economy with intellectual property rights to an economy without them, we need welfare measures. This is achieved by assuming that there are many innovators and ideas and that to each idea corresponds a demand curve. The innovators produce as in the standard Bertrand competition and, accordingly, as soon as at least two innovators come up with the same innovation the consumers reap the maximum surplus, while the innovators make zero revenue. If just one innovator comes up with an innovation, she becomes a monopolist. This means that there are both revenue and consumer surplus in the economy, and also that duplicative research is not necessarily harmful because it increases production and consumer surplus.

We formalize the widespread idea that patents are perhaps more important in disseminating information than in providing incentives. Innovators have an option to keep their innovations secret, and this constrains the feasible patent policies. Because the innovators can opt for secrecy, the patent policy must provide them with some minimum level of protection. But this protection can be less than the protection provided by secrecy because, in the equilibrium, it is always better to patent than to keep the innovation secret; each innovator fears that other innovators have made the same innovation, and then opting for secrecy would be profitable only if an innovator knew that she were the sole innovator. Even the optimal patent policy does not necessarily provide more protection than secrecy, contradicting the traditional view about optimal patents providing strong protection and incentives to innovate. This observation may provide a rationale for a weak patent institution: an effective patent system, in the sense that innovators patent their innovations, both encourages innovation and spreads information and, accordingly, improves welfare.

Our model is overly simplistic in many respects. Given our strong result and its implications on the heated debate on the pros and cons of the patent system and the expansion of patent subject matter, it would be desirable to enrich the model in future research. For example, we implicitly assume that, in the case of multiple innovators of the same innovation, the first to file

an application gets the patent and can exclude the other innovators. As pointed out by Scotchmer and Green (1990), such first-to-file rules may induce innovators to patent rather than choose secrecy. The other rule of priority dispute resolution, which is used only in the United States, is based on the first-to-invent principle, which tends to make secrecy more attractive. However, the testable prediction concerning the difference in the propensity to patent caused by the first-to-file and first-to-invent rules is moot because the first-to-invent rule sometimes allows patenting by a later innovator if the first innovator has relied on secrecy and the effect of the first-to-file is often counterbalanced by the prior-user rights. If prior-user rights allow all independent innovators to use the innovation, it is clear that incorporating the prior-user rights into our model would weaken the incentive to patent. Therefore, as in Denicolò and Franzoni (2004), our analysis suggests that removing prior-user rights could improve welfare.

### Appendix

■ Proofs of Lemmas 2 and 4 and Propositions 1, 4, and 5 follow.

*Proof of Lemma 2.* Totally differentiating (11) yields the derivatives

$$\frac{dj}{d\alpha} = \frac{1}{\alpha(1 + \theta e^{-j})} > 0,$$

$$\frac{dj}{d\lambda} = \frac{\delta}{(1 - \delta\lambda)(1 + \theta e^{-j})} > 0 \Rightarrow \frac{dj}{d(1 - \lambda)} < 0$$

and

$$\frac{dj}{d\theta} = -\frac{1 - e^{-j}}{1 + \theta e^{-j}} < 0.$$

*Q.E.D.*

*Proof of Proposition 1.* Denote the second best investment by  $j^*$  and the market-equilibrium investment by  $\hat{j}$ . We will show that  $\hat{j} < j^*$ . Differentiating the righthand side of (12) with respect to  $j$  gives, after simplifying, the first-order condition

$$-1 + \frac{e^{-j} e^{-\tilde{\theta}}}{4(1 - \lambda)}(4 - \alpha + \alpha\tilde{\theta}) = 0.$$

It is straightforward to check that the second-order condition holds globally so that the above equation determines a global maximum. Now define a function  $F$  by setting

$$F(j) = -1 + \frac{e^{-j} e^{-\tilde{\theta}}}{4(1 - \lambda)}(4 - \alpha + \alpha\tilde{\theta}).$$

Thus,  $F(j^*) = 0$ . Notice that, because  $\delta, \lambda \in [0, 1]$  and  $\frac{d}{d\delta}(\frac{1}{1-\delta\lambda}) > 0$ , we have, for all  $j > 0$ ,

$$F(j) > -1 + \frac{e^{-j} e^{-\tilde{\theta}}}{4(1 - \delta\lambda)}(4 - \alpha + \alpha\tilde{\theta}).$$

In particular for  $j = \hat{j}$ , we have

$$F(\hat{j}) > -1 + \frac{e^{-\hat{j}} e^{-\tilde{\theta}}}{4(1 - \delta\lambda)}(4 - \alpha + \alpha\tilde{\theta}).$$

Using the equilibrium condition (11) to substitute in for the term  $(e^{-\hat{j}} e^{-\tilde{\theta}})/[2(1 - \delta\lambda)]$  on the right-hand side, we have

$$F(\hat{j}) > -1 + \frac{1}{2\alpha}(4 - \alpha + \alpha\tilde{\theta}) = \frac{1}{2\alpha}(4 - 3\alpha + \alpha\tilde{\theta}) > 0,$$

where the second inequality follows from the facts that  $\alpha \in [0, 1]$  and  $\tilde{\theta} = \theta(1 - e^{-\hat{j}}) > 0$ . Thus, it must be that  $\hat{j} < j^*$ . *Q.E.D.*

*Proof of Lemma 4.* In the patenting equilibria, the equilibrium investment  $j_p$  is determined by the equilibrium condition (20). Totally differentiating this gives the derivatives

$$\begin{aligned} \frac{dj_p}{d\alpha_p} &= \frac{(1 - e^{-j_p})(1 - e^{-\tilde{\theta}})}{\alpha_p(1 - e^{-\tilde{\theta}} - \tilde{\theta}e^{-\tilde{\theta}}e^{-j_p})} > 0, \\ \frac{dj_p}{d\lambda} &= \frac{\delta(1 - e^{-j_p})(1 - e^{-\tilde{\theta}})}{(1 - \delta\lambda)(1 - e^{-\tilde{\theta}} - \tilde{\theta}e^{-\tilde{\theta}}e^{-j_p})} > 0, \end{aligned}$$

which implies that  $dj_p/d(1 - \lambda) < 0$ , and

$$\frac{dj_p}{d\theta} = -\frac{(1 - e^{-j_p})(1 - e^{-\tilde{\theta}} - \tilde{\theta}e^{-\tilde{\theta}})}{\theta(1 - e^{-\tilde{\theta}} - \tilde{\theta}e^{-\tilde{\theta}}e^{-j_p})} < 0.$$

The equilibrium investment is obviously constant in  $\alpha_p$  in the secrecy equilibria. Other properties of the investment in the secrecy equilibria follow readily from Lemma 2.

To see that the equilibrium investment is also constant in the mixed equilibria, recall from the proof of Proposition 2 that, in a mixed equilibrium, the equilibrium investment must simultaneously satisfy the first-order conditions (10) and (18) for a given  $\sigma$ . Using (18), the investment  $j$  is determined by

$$-1 + e^{-j} \frac{1 - e^{-(1-\hat{\sigma})\tilde{\theta}}}{(1 - \hat{\sigma})\tilde{\theta}} \frac{\alpha_p}{2(1 - \delta\lambda)} = 0.$$

This equation can be rewritten, by using the indifference requirement (24), as

$$e^{-j} e^{-\theta(1-e^{-j})} = \frac{2(1 - \delta\lambda)}{\alpha},$$

which is independent of  $\alpha_p$ . Thus, the equilibrium investment remains constant for all values of  $\alpha_p$  over the range  $[\underline{\alpha}, \bar{\alpha}]$ . For the other properties, we can use the fact that the equilibrium investment in a mixed equilibrium must satisfy the equilibrium condition with secrecy (11), so that the results follow from Lemma 2. *Q.E.D.*

*Proof of Proposition 4.* By Proposition 2, we know that all innovators patent iff  $\alpha_p \geq \bar{\alpha}$ . We claim that

$$W_p(\bar{\alpha}) = W,$$

or, using (12) and (25), that

$$-j_p + \frac{(4 - \bar{\alpha})(1 - e^{-\tilde{\theta}_p})}{4\theta(1 - \lambda)} = -j + \frac{4(1 - e^{-\tilde{\theta}}) - \alpha\tilde{\theta}e^{-\tilde{\theta}}}{4\theta(1 - \lambda)},$$

where, on the righthand side we have the welfare in the market equilibrium without patents so that the investment  $j$  is determined by the equilibrium condition (11). Proposition 3 implies that  $j_p = j$  and hence  $\tilde{\theta}_p = \tilde{\theta}$ . Thus, the above equality holds iff

$$\bar{\alpha} = \frac{\tilde{\theta}e^{-\tilde{\theta}}}{1 - e^{-\tilde{\theta}}}\alpha.$$

Recalling (22), we see that this is exactly the definition of  $\bar{\alpha}$ . *Q.E.D.*

*Proof of Proposition 5.* Let us first study the unconstrained maximization problem,

$$\max_{\alpha_p} W_p(\alpha_p),$$

where  $W_p(\alpha_p)$  is given by (25). The first-order condition is

$$-\frac{1 - e^{-\tilde{\theta}_p}}{4\theta(1 - \lambda)} + \left[ -1 + \frac{(4 - \alpha_p)e^{-\tilde{\theta}_p}e^{-j_p}}{4(1 - \lambda)} \right] \frac{dj_p}{d\alpha_p} = 0.$$

Inserting  $dj_p/d\alpha_p$  from the proof of Lemma 4 and regrouping terms, we have

$$\frac{1 - e^{-\tilde{\theta}_p}}{4\theta(1 - \lambda)} \left\{ -1 + \left[ -4(1 - \lambda) + (4 - \alpha_p)e^{-\tilde{\theta}_p}e^{-j_p} \right] \frac{\theta(1 - e^{-j_p})}{\alpha_p(1 - e^{-\tilde{\theta}_p} - \tilde{\theta}_p e^{-\tilde{\theta}_p}e^{-j_p})} \right\} = 0.$$

Using the fact that  $\tilde{\theta}_p = \theta(1 - e^{-j_p})$  and rearranging, this can be further simplified to

$$\frac{1 - e^{-\tilde{\theta}_p}}{4\theta\alpha_p(1 - \lambda)(1 - e^{-\tilde{\theta}_p} - \tilde{\theta}_p e^{-\tilde{\theta}_p} e^{-j_p})} \left[ 4\tilde{\theta}_p(e^{-\tilde{\theta}_p} e^{-j_p} - 1 + \lambda) - \alpha_p(1 - e^{-\tilde{\theta}_p}) \right] = 0. \quad (A1)$$

Because the first term is positive, the first-order condition is satisfied if the term in the square brackets is zero, i.e., if

$$\alpha_p = \alpha^* \equiv \frac{4\tilde{\theta}_p(e^{-\tilde{\theta}_p} e^{-j_p} - 1 + \lambda)}{1 - e^{-\tilde{\theta}_p}}. \quad (A2)$$

The solution to (A2) may be smaller than  $\bar{\alpha}$  or larger than unity. If it is smaller than  $\bar{\alpha}$ , the patenting condition  $\alpha_p \geq \bar{\alpha}$  binds at the optimum. If it is larger than unity, condition  $\alpha_p \leq 1$  binds. For this reason, the optimal strength of the patent is given by

$$\alpha_p^* = \max\{\bar{\alpha}, \min\{\alpha^*, 1\}\}.$$

This establishes necessity. It can be shown that a second-order condition holds so that the above condition gives a maximum. We omit the details. See the supplementary Appendix for a proof. *Q.E.D.*

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