SUPPLEMENTARY MATERIAL TO "IRONING WITHOUT CONTROL"

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This document contains omitted proofs and additional results for the manuscript "Ironing without Control." Appendix A contains the proofs for the separable case. Appendix B makes precise the sense in which the construction in the concave case generalizes the separable case. Appendix C adapts the concave case to problems where the set of feasible allocations is discrete. Appendix D uses this setting to construct an example that shows that weak concavity (or more precisely, nonincreasing marginal returns) can not be dispensed with. For notation, please refer to the main text.

APPENDIX A. PROOFS FOR THE SEPARABLE CASE

The following sufficient conditions are useful in verifying that an allocation rule has the pooling property.

Lemma A.1. *A function $\phi : \Theta \to X$ has the pooling property if for all $(\theta, \theta') \in \Theta^2$ at least one of the following holds:*
  1. *If $\Phi(\theta) = \Phi(\theta')$, then $\phi(\theta) = \phi(\theta')$.*
  2. *If $\bar{k}(\theta) = \bar{k}(\theta')$, then $\phi(\theta) = \phi(\theta')$.*

If Φ is single-valued except at countably many points, then all selections from Φ satisfy the above conditions and hence have the pooling property.

Proof. By definitions of $\bar{J}$ and Φ, (1) $\implies$ (2). To show sufficiency of (2), note that if I is an open interval in Θ such that $G(F(\theta)) < H(F(\theta))$ for all $\theta \in I$, then $G(q) < H(q)$ for all $q \in F(I)$. Since G is the convex hull of H, it is affine on $F(I)$. Thus $g = G'$ is constant on $F(I)$, and hence $\bar{k} = g \circ F$ is constant on I. Then (2) implies that ϕ is constant on I.

Assume then that Φ is single-valued except at countably many points. It is enough to show (1). Suppose that $\theta < \theta'$ and $\Phi(\theta) = \Phi(\theta')$. Since $\bar{J}$ has increasing differences in (x, θ) by construction, Topkis's theorem implies that Φ is constant on $[\theta, \theta']$. But $[\theta, \theta']$ is uncountable so that $\Phi(\theta)$ and $\Phi(\theta')$ must be single-valued. Now (1) is trivial. $\square$

Proof of Corollary 3.8. Since $\Phi(\theta)$ is assumed nonempty and compact for all θ, the selections ϕ^* and ϕ_* are well-defined. Since $\bar{J}$ has increasing differences in (x, θ) we have $\phi^* \in \mathcal{M}$ and $\phi_* \in \mathcal{M}$ by Topkis's theorem. By Lemma A.1(1), ϕ^* and ϕ_* have the pooling property. The claim now follows from Theorem 3.7. $\square$

Proof of Corollary 3.9. Let ϕ be a selection from Φ, where Φ is single-valued except at countably many points. By Theorem 3.7 it suffices to show that ϕ is nondecreasing and has the pooling property. The second claim follows from Lemma A.1. As

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for the first one, assume towards a contradiction that $\theta < \theta'$ and $\phi(\theta) > \phi(\theta')$. By Topkis's theorem, $\{\phi(\theta), \phi(\theta')\} \subset \Phi(\tau)$ for all $\tau \in [\theta, \theta']$. But then Φ is multi-valued at uncountably many points, a contradiction. $\square$

Proof of Theorem 3.7. Integrating by parts we have

$$
\begin{aligned}
\int_0^1 \left(J(\phi(\theta), \theta) - \bar{J}(\phi(\theta), \theta)\right) dF(\theta) &= \int_0^1 a(\phi(\theta))(k(\theta) - \bar{k}(\theta)) dF(\theta) \\
&= \int_0^1 a(\phi(\theta)) \left(h(F(\theta)) - g(F(\theta))\right) dF(\theta) \\
&= \left[a(\phi(\theta)) \left(H(F(\theta)) - G(F(\theta))\right)\right]_{\theta=0}^1 \\
&\quad - \int_0^1 \left(H(F(\theta)) - G(F(\theta))\right) da(\phi(\theta)) \\
&= \int_0^1 \left(G(F(\theta)) - H(F(\theta))\right) da(\phi(\theta)) \leq 0,
\end{aligned}
$$

where the last equality follows since G is the convex hull of H on $[0,1]$ and H is continuous, implying that $G(0) = H(0)$ and $G(1) = H(1)$. The inequality follows from the facts that the measure induced by $a \circ \phi$ is nonnegative since a and ϕ are nondecreasing, and $G \leq H$ by construction.

If problem (P) is separable, then given any allocation rule ϕ we can add and subtract the expected generalized virtual surplus $\int_0^1 \bar{J}(\phi(\theta), \theta) dF(\theta)$ from the objective and use the above equalities to write the problem equivalently as

$$
\text{(A.1)} \qquad \sup_{\phi \in \mathcal{M}} \left\{ \int_0^1 \bar{J}(\phi(\theta), \theta) dF(\theta) + \int_0^1 \left(G(F(\theta)) - H(F(\theta))\right) da(\phi(\theta)) \right\}.
$$

"If": Assume that $\phi \in \mathcal{M}$ has the pooling property and $\phi(\theta) \in \Phi(\theta)$ a.e. Then ϕ maximizes the integrand of the first integral in (A.1) a.e. by definition of Φ. Hence ϕ attains the supremum of the first term in (A.1). Consider then the second term. Assume that $G(F(\theta)) < H(F(\theta))$ for some θ. Then by continuity, $G \circ F < H \circ F$ in some open neighborhood I of θ. By the pooling property ϕ is constant on I. But then I has zero measure under $da(\phi(\theta))$. Thus the second term evaluated at ϕ is zero, which is the highest value it can take. Therefore, ϕ attains the supremum in (A.1), and hence also in (P).

"Only if": Assume that ϕ attains the supremum in (P), and hence also in (A.1). I show first that $\phi(\theta) \in \Phi(\theta)$ a.e. Consider maximizing the first term in (A.1) over all functions $\gamma : \Theta \to X$. Since $\bar{J}$ has increasing differences in (x, θ) by construction, it is without loss to assume that $\gamma \in \mathcal{M}$. Furthermore, recalling the definition of $\bar{J}$, it is seen that it entails no loss to assume that γ is constant whenever $\bar{k}$ is constant so that it has the pooling condition by Lemma A.1(2). But in the proof of sufficiency above it was shown that any function that has the pooling property maximizes the second term. Thus the first term in (A.1) can be maximized by a monotone function that also achieves the maximum of the second term. So if ϕ achieves the supremum in (P), and hence in (A.1), it must maximize the integrand of the first term almost everywhere. That is, $\phi(\theta) \in \Phi(\theta)$ a.e.

Suppose then by the way of contradiction that ϕ does not have the pooling property. Then on some open interval $I \subset \Theta$ where $G \circ F < H \circ F$, there exists $\theta' < \theta''$ such that $\phi(\theta') < \phi(\theta'')$. Since a is strictly increasing this implies $a(\phi(\theta')) <$

$a(\phi(\theta''))$. Then $[\theta', \theta''] \subset I$ is assigned strictly positive measure by $da(\phi(\theta))$ so that

$$\int_0^1 (G(F(\theta)) - H(F(\theta)))da(\phi(\theta)) \leq \int_{\theta'}^{\theta''} (G(F(\theta)) - H(F(\theta)))da(\phi(\theta)) < 0.$$

Fix $x \in [\phi(\theta'), \phi(\theta'')]$ and define $\delta \in \mathcal{M}$ by

$$\delta(\theta) := \begin{cases} \phi(\theta) & \text{if } \theta \in \Theta \setminus [\theta', \theta''], \\ x & \text{if } \theta \in [\theta', \theta'']. \end{cases}$$

I show that x can be chosen such that δ is a strict improvement over ϕ. Since δ is constant on $[\theta', \theta'']$, the value of the second term in (A.1) increases strictly going from ϕ to δ for any x. As for the first term, note that $G \circ F < H \circ F$ on I implies $G < H$ on $F(I)$. Since G is the convex hull of H, it is affine on $F(I)$. Thus $g = G'$ is constant on $F(I)$, and hence $\bar{k} = g \circ F$ is constant on I. By definitions of $\bar{J}$ and Φ, this implies that $\Phi(\theta) = \Phi(\theta')$ for all $\theta \in [\theta', \theta''] \subset I$. Since $\theta' < \theta''$ and $\phi(\theta) \in \Phi(\theta)$ a.e., there exists $\tau \in [\theta', \theta'']$ such that

$$\phi(\tau) \in \Phi(\tau) = \Phi(\theta') = \arg\max_{y \in X} \left\{ a(y)\bar{k}(\theta') + b(y) \right\}.$$

So setting $x = \phi(\tau)$ we have

$$\int_0^1 \bar{J}(\delta(\theta), \theta)dF(\theta) - \int_0^1 \bar{J}(\phi(\theta), \theta)dF(\theta)$$

$$= \int_{\theta'}^{\theta''} \left[a(\phi(\tau))\bar{k}(\theta') + b(\phi(\tau)) - \left(a(\phi(\theta))\bar{k}(\theta') + b(\phi(\theta)) \right) \right] dF(\theta) \geq 0.$$

Thus the value of the first term in (A.1) evaluated at δ is no less than at ϕ. But then δ improves strictly on ϕ, which contradicts the fact that ϕ attains the supremum in (P). So ϕ must have the pooling property. $\qquad\square$

Appendix B. Separable case as a special case

I now state and prove the claim in Remark 4.3. It implies that Theorem 3.7 remains true if Φ is replaced with Ψ in the statement. Hence, the construction of Section 4 is a proper generalization of the separable case.

Lemma B.1. *Let J be continuously differentiable. If J is separable (but not necessarily weakly concave in x), then $\Psi = \Phi$.*

Proof. Normalize F to be uniform. Suppose J is separable and let $(x, \theta) \in X \times \Theta$. Following the construction of Section 4 we have

$$h(x, \theta) = J_1(x, \theta) = a'(x)k(\theta) + b'(x) = a'(x)h(\theta) + b'(x).$$

Thus we get

$$H(x, \theta) = \int_0^\theta h(x, r)dr = \int_0^\theta \left(a'(x)h(r) + b'(x) \right) dr = a'(x)H(\theta) + b'(x)\theta,$$

which implies

$$\begin{aligned} G(x, \theta) &= \text{conv}_\theta H(x, \theta) \\ &= \text{conv}_\theta \left(a'(x)H(\theta) + b'(x)\theta \right) \\ &= a'(x)\text{conv}_\theta H(\theta) + b'(x)\theta \\ &= a'(x)G(\theta) + b'(x)\theta. \end{aligned}$$

This gives

$$g(x,\theta) = G_2(x,\theta) = a'(x)G'(\theta) + b'(x) = a'(x)g(\theta) + b'(x) = a'(x)\bar{k}(\theta) + b'(x),$$

where the last equality uses the normalization of F. Therefore, the generalized virtual surplus becomes

$$\bar{J}(x,\theta) = J(0,\theta) + \int_0^x \left(a'(s)\bar{k}(\theta) + b'(s)\right) ds$$
$$= a(x)\bar{k}(\theta) + b(x) + l(\theta) + a(0)\left(k(\theta) - \bar{k}(\theta)\right),$$

where the second line coincides with the definition of the generalized virtual surplus in the separable case up to the term $a(0)\left(k(\theta) - \bar{k}(\theta)\right)$ which is constant in x. The claim now follows by definitions of Φ and Ψ.   □

Appendix C. Discrete allocations and continuous types

This appendix adapts the approach from the concave case to problems where the set of feasible allocations, X, is discrete. To this end, let $X = \{0, 1, \ldots, \bar{x}\}$ for some $\bar{x} < \infty$ and assume that J is continuous. It is convenient to set $J(-1,\theta) \equiv 0$ for all θ.

The definitions are as follows. For all $(x,\theta) \in X \times \Theta$, let

$$(C.1) \qquad\qquad h(x,\theta) := J(x,\theta) - J(x-1,\theta),$$

and let

$$(C.2) \qquad\qquad H(x,\theta) := \int_0^\theta h(x,r)dr.$$

For all $x \in X$, let

$$(C.3) \qquad\qquad G(x,\cdot) := \text{conv} H(x,\cdot).$$

Then $G(x,\cdot)$ is continuously differentiable on $(0,1)$. For all $\theta \in (0,1)$, let

$$(C.4) \qquad\qquad g(x,\theta) := G_2(x,\theta),$$

and extend $g(x,\cdot)$ to $[0,1]$ by continuity.

Define the *generalized virtual surplus* $\bar{J} : X \times \Theta \to \mathbb{R}$ by

$$(C.5) \qquad\qquad \bar{J}(x,\theta) := \sum_{s=0}^x g(s,\theta),$$

and define the correspondence $\Xi : \Theta \to \mathcal{P}(X)$ by

$$\Xi(\theta) := \left\{ x \in X \mid \bar{J}(x,\theta) = \sup_{y \in X} \bar{J}(y,\theta) \right\}.$$

Definition C.1. A function $J : X \times \Theta \to \mathbb{R}$ has *nonincreasing marginal returns* if for all $x \in X \setminus \{0, \bar{x}\}$ and all $\theta \in \Theta$,

$$J(x,\theta) - J(x-1,\theta) \geq J(x+1,\theta) - J(x,\theta).$$

Theorem C.2. *If J has nonincreasing marginal returns, then*

$$\sup_{\phi \in \mathcal{M}} \left\{ \int_0^1 J(\phi(\theta),\theta)d\theta \right\} = \sup_{\phi \in \Gamma} \left\{ \int_0^1 \bar{J}(\phi(\theta),\theta)d\theta \right\}.$$

Theorem C.3. *Let J have nonincreasing marginal returns. Then ϕ^* and ϕ_* attain the supremum in (P). Furthermore, if $\phi \in \mathcal{M}$ attains the supremum in (P), then $\phi_*(\theta) \le \phi(\theta) \le \phi^*(\theta)$ and $\phi(\theta) \in \Xi(\theta)$ a.e.*

As in the proof of the concave case, I start by noting the key properties of the newly defined functions. The function g is trivially continuous in x as X is discrete, and it is nondecreasing and continuous in θ by construction. As a result, the generalized virtual surplus $\bar{J}$ is continuous and has increasing differences in (x, θ). Thus, the correspondence Ξ is nonempty- and compact-valued, and upper hemi-continuous. So the extremal selections are well-defined and nondecreasing. Finally, by nonincreasing marginal returns $h(\cdot, \theta)$ is nonincreasing for all $\theta \in \Theta$ and the argument from the proof of Lemma 4.11 then implies that $g(\cdot, \theta)$ is nonincreasing for all $\theta \in \Theta$. Thus $\bar{J}$ has nonincreasing marginal returns.

Given the definition of $\bar{J}$ in (C.5), the difference between the expected virtual surplus and the expected generalized virtual surplus becomes

$$\Delta(\phi) = \int_0^1 \left(J(\phi(\theta), \theta) - \bar{J}(\phi(\theta), \theta) \right) d\theta$$

$$= \int_0^1 \sum_{s=0}^{\phi(\theta)} (h(s, \theta) - g(s, \theta)) \, d\theta$$

(C.6)

$$= \sum_{x=0}^{\bar{x}} \int_{\phi^{-1}(x)}^1 (h(x, \theta) - g(x, \theta)) \, d\theta$$

$$= \sum_{x=0}^{\bar{x}} \left(G(x, \phi^{-1}(x)) - H(x, \phi^{-1}(x)) \right) \le 0.$$

Obviously $\Delta(0) = 0$ so Lemma 4.12 applies. Analogous to Lemma 4.13 we have

Lemma C.4. *If J has nonincreasing marginal returns, then $\Delta(\phi^*) = \Delta(\phi_*) = 0$.*

Proof. Consider first ϕ^*. Assume towards a contradiction that there exists $x \in X$ such that

$$G(x, \phi^{*-1}(x)) - H(x, \phi^{*-1}(x)) < 0.$$

Denote $\theta_x := \phi^{*-1}(x)$. As $G(\cdot, 0) = H(\cdot, 0)$ and $G(\cdot, 1) = H(\cdot, 1)$, we have $\theta_x \in (0, 1)$. This implies $x > 0$, since $\phi^{-1}(0) = 0$ for any $\phi \in \mathcal{M}$. Note then that nonincreasing marginal returns of $\bar{J}$ imply that Ξ is "convex-valued": That is, if $y \in \Xi(\theta_x)$, $y' \in \Xi(\theta_x)$ and $y < z < y'$, then $z \in \Xi(\theta_x)$. As in the proof of the claim after Lemma 4.13, Ξ being upper hemi-continuous and "convex-valued" implies $x \in \Xi(\theta_x)$. Thus $\bar{J}(\cdot, \theta_x)$ has a maximum at $x > 0$ and hence the marginal return from the last unit is nonnegative. That is, $g(x, \theta_x) \ge 0$. By continuity of $G(x, \cdot)$ and $H(x, \cdot)$, there exists an open neighborhood U of θ_x such that $G(x, \theta) - H(x, \theta) < 0$ for all $\theta \in U$. Hence $g(x, \cdot)$ is constant on U so that $g(x, \theta) = g(x, \theta_x) \ge 0$ for all $\theta \in U$. Nonincreasing marginal returns then imply $\phi^*(\theta) = \max \Xi(\theta) \ge x$ for all $\theta \in U$. But this is the desired contradiction since by definition of ϕ^{*-1} we have $\phi^*(\theta) < x$ for all $\theta < \theta_x$.

Consider then ϕ_*. (Much of the argument is the same as for ϕ^*.) Assume towards a contradiction that there exists $y \in X$ such that

$$G(y, \phi_*^{-1}(y)) - H(y, \phi_*^{-1}(y)) < 0.$$

Denote $\theta_y := \phi_*^{-1}(y)$. As above, we have $y > 0$ and $y \in \Xi(\theta_y)$. Thus $g(y, \theta_y) \geq 0$. Furthermore, as above, there exists an open neighborhood V of θ_y such that $g(y, \theta) = g(y, \theta_y) \geq 0$ for all $\theta \in V$. But by the definition of the generalized inverse we have $\phi_*(\theta) < y$ for all $\theta < \theta_y$ so that by nonincreasing marginal returns $g(y, \theta) \leq 0$ for all $\theta < \theta_y$. Combining the two inequalities gives $g(y, \theta) = g(y, \theta_y) = 0$ for all $\theta \in V$. But then the marginal return from going from $y - 1$ to y is zero for all types in V. Thus, by nonincreasing marginal returns, $y - 1 \in \Xi(\theta)$ for all $\theta \in V$. In particular, $\phi_*(\theta) = \min \Xi(\theta) \leq y - 1$ for all $\theta \in V$. But $\theta_y = \phi_*^{-1}(y)$ implies $\phi_*(\theta) \geq y$ for all $\theta > \theta_y$, a contradiction. $\qquad\square$

With the above lemma in place, the proofs of the theorems are completed exactly as the ones for Theorems 4.4 and 4.5.

Appendix D. The role of concavity

This appendix presents a counterexample intended to convince the reader that Theorems 4.4 and 4.5 do not apply to problems where the virtual surplus is not weakly concave in the allocation. For simplicity, I do this in the context of the model of Appendix C where the set of feasible allocations is discrete. The example shows that if the virtual surplus function does not have nonincreasing marginal returns (as defined in Definition C.1), then Theorems C.2 and C.3—the discrete analogs of Theorems 4.4 and 4.5—do not apply. It should be conceptually straightforward, albeit notationally more cumbersome, to construct a similar example for the case where X is an interval.

Let $X := \{0, 1, 2\}$ and let F be the cdf of the uniform distribution on $\Theta = [0, 1]$. Define the virtual surplus function by

$$
J(x, \theta) := \begin{cases} 0 & \text{if } x = 0, \\ 1 - 15\theta^2 & \text{if } x = 1, \\ 2\theta - 1 & \text{if } x = 2. \end{cases}
$$

The function J does not have nonincreasing marginal returns. (For example, we have $J(2, 1) - J(1, 1) = 15 > -14 = J(1, 1) - J(0, 1)$.) Furthermore, maximizing virtual surplus $J(x, \theta)$ pointwise gives the candidate solution

$$
\phi(\theta) = \begin{cases} 1 & \text{if } \theta < \frac{\sqrt{15}}{15}, \\ 0 & \text{if } \frac{\sqrt{15}}{15} < \theta < \frac{1}{2}, \\ 2 & \text{if } \theta > \frac{1}{2}, \end{cases}
$$

which is nonmonotonic. Thus ironing is needed.

In order to (unwarrantably) apply the results for discrete choice sets (Theorems C.2 and C.3), the generalized virtual surplus $\bar{J}$ needs to be constructed. To this end, using equations (C.1)–(C.4) the following functions are constructed. The marginal returns are given by

$$
h(x, \theta) = \begin{cases} 0 & \text{if } x = 0, \\ 1 - 15\theta^2 & \text{if } x = 1, \\ 15\theta^2 + 2\theta - 2 & \text{if } x = 2, \end{cases}
$$

so that

$$H(x, \theta) = \begin{cases} 0 & \text{if } x = 0, \\ \theta - 5\theta^3 & \text{if } x = 1, \\ 5\theta^3 + \theta^2 - 2\theta & \text{if } x = 2. \end{cases}$$

Since $H(0, \cdot)$ and $H(2, \cdot)$ are convex, we have $G(0, \cdot) = H(0, \cdot)$ and $G(2, \cdot) = H(2, \cdot)$. As $H(1, \cdot)$ is strictly concave, its convex hull is simply the line segment given by $G(1, \theta) = -4\theta$ for $\theta \in [0, 1]$. Taking derivatives gives

$$g(x, \theta) = \begin{cases} 0 & \text{if } x = 0, \\ -4 & \text{if } x = 1, \\ 15\theta^2 + 2\theta - 2 & \text{if } x = 2. \end{cases}$$

So now the generalized virtual surplus of type θ defined in (C.5) is

$$\bar{J}(x, \theta) := \begin{cases} 0 & \text{if } x = 0, \\ -4 & \text{if } x = 1, \\ 15\theta^2 + 2\theta - 6 & \text{if } x = 2. \end{cases}$$

It is then straightforward to see that

$$\phi^*(\theta) = \begin{cases} 0 & \text{if } \theta < \frac{\sqrt{91}-1}{15}, \\ 2 & \text{if } \theta \geq \frac{\sqrt{91}-1}{15}. \end{cases}$$

The value of the objective in (P) evaluated at ϕ^* defined above is

$$\int_{\frac{\sqrt{91}-1}{15}}^{1} (2\theta - 1)d\theta = \frac{17\sqrt{91} - 107}{225} \approx 0.245.$$

Consider the monotone (and hence feasible) allocation rule

$$\delta(\theta) := \begin{cases} 1 & \text{if } \theta < \frac{\sqrt{31}-1}{15}, \\ 2 & \text{if } \theta \geq \frac{\sqrt{31}-1}{15}. \end{cases}$$

Evaluating the objective at δ gives

$$\int_{0}^{\frac{\sqrt{31}-1}{15}} (1 - 15\theta^2)d\theta + \int_{\frac{\sqrt{31}-1}{15}}^{1} (2\theta - 1)d\theta = \frac{62\sqrt{31} - 92}{675} \approx 0.375,$$

contradicting the optimality of ϕ^*. Thus the assumption about nonincreasing marginal returns cannot be dropped in Theorems C.2 and C.3.

Heuristically, what goes wrong in this example is that the averaging done by convexification effectively assumes that if the monotonicity constrained causes the allocation of some type θ to be distorted upwards compared to the solution to pointwise maximization of virtual surplus (i.e., the candidate solution ϕ above), then the allocation assigned to type θ is the smallest allocation consistent with monotonicity. While this is certainly true for problems where the virtual surplus has nonincreasing marginal returns (which are implied by J being weakly concave in x), it is not true in general. Indeed, here $J(1, \theta) < J(2, \theta)$ for all $\theta \in (\frac{\sqrt{31}-1}{15}, 1]$. So if any types in $(\frac{\sqrt{31}-1}{15}, \frac{1}{2})$ for whom $x = 0$ is myopically optimal are distorted upwards, it is best to distort them all the way up to $x = 2$. The allocation rule δ—which is in fact fully optimal—does exactly this. In contrast, the construction

leading to ϕ^* implicitly assumes that if a type θ in $[0, \frac{1}{2})$ is given one unit, then so are all types in $[\theta, 1]$.

To see this in greater detail, note that the contribution of the first unit to the generalized virtual surplus of type θ, $g(1, \theta)$, is the derivative of the convex hull of $H(1, \cdot)$ at θ. By definition, $H(1, \theta)$ is the sum of marginal contributions (of the first unit) from all types $[0, \theta]$. However, the value of the convex hull at θ, $G(1, \theta)$, depends here on all types in $[0, 1]$ since $H(1, \cdot)$ is concave on $[0, 1]$. Effectively the convex hull picks up the fact that allocating one unit for the high types is a really bad idea given the form of $J(1, \cdot)$. If J were concave in x, it would be an even worse idea to give them two units, so that the convex hull would lead us to the right conclusion. Here, however, going up to two units is not so bad for the intermediate types and is great for the high types, so that the negative marginal contribution of the first unit can be more than compensated.

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