Corporate Reputational Dynamics, Private Regulation, and Activist Pressure[1]

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Abstract

We model the interaction between a profit-maximizing firm and an activist using an infinite-horizon dynamic stochastic game. The firm enhances its reputation through "self-regulation": voluntary provision of an abatement activity that reduces a negative externality. We show that in equilibrium the externality-reducing activity is subject to decreasing marginal returns, which can cause the firm to "coast on its reputation," i.e., decrease the level of externality-reducing activity as its reputation grows. The activist, which benefits from increases in the externality-reducing activity, can take two types of action that can harm the firm's reputation: *criticism*, which can impair the firm's reputation on the margin, and *confrontation*, which can trigger a crisis that may severely damage the firm's reputation. The activist changes the reputational dynamics of the game by tending to keep the firm in reputational states in which it is highly motivated to invest in externality-reducing activity. Criticism and confrontational activity are shown to be imperfect substitutes. The more patient the activist or the more passionate it is about externality reduction, the more likely it is to rely on confrontation. The more patient the firm and the more important corporate citizenship is to firm's brand equity, the more likely that it will be targeted by an activist that relies on confrontation.

1 Introduction

The regulation of economic activity is one of the main arenas of political competition. The impetus for changes to regulatory regimes frequently originates with concerned citizens, often motivated by social or ethical concerns. Traditionally, concerned citizens have used public institutions such as legislatures, executive agencies, and courts to advance their agenda. In recent years, however, many activists have concluded that public processes respond too slowly and can be blocked too easily by special interests. In response they have turned to private politics instead. Private politics refers to actions by private interests such as activists and NGOs that target private agents, typically firms, often in the institution of public sentiment (e.g. Baron 2001, Baron 2003, Baron and Diermeier 2007, Feddersen and Gilligan 2001, Ingram, Yue, and Rao, 2010, McDonnell and King 2013, King and Pearce 2010). Issues have included, among others, environmental protection, human rights, discrimination, privacy, safety of employees and customers, endangered species, and animal welfare testing. The activists' explicit or implicit goal is *private regulation*, i.e. the "voluntary" adoption of rules that constrain certain company conduct without the involvement of public agents.[1] Michael Brune, executive director of the Rainforest Action Network (RAN), a leading global activist group, commented that "companies were more responsive to public opinion than certain legislatures were. We felt we could create more democracy in the marketplace than in the government." (Baron and Yurday 2004). If successful, such strategies may lead to alternative, private governance mechanisms. Examples are the Equator Principles or the Sustainable Forestry Initiative.[2] Private regulation is also particularly widely used in cases where public institutions are missing or are underdeveloped. One such example is the attempt to reduce the availability of "conflict diamonds," which are used to fund civil wars in West Africa.

The actions of activist groups center on a corporate campaign, the organizational framework for satisfying activists' goals. In a campaign an activist group tries to affect the business practices of a target firm through a combination of threatened harms and promised rewards (e.g. Baron and Diermeier 2007). Harm usually takes the form of damage to the company's reputation, though more direct actions (e.g. disrupting certain operations) are not uncommon. Similarly, rewards may come in the form of endorsements that enhance a company's reputation.

Activists pursue different goals and use different tactics. While some are quite radical and use confrontational means, others are more moderate and use a combination of criticism and engagement such as letter writing and shareholder resolutions (Baron 2012). One such confrontational approach is to try to create a reputational crisis that has a significant impact on the company's image, as in the confrontation between the activist group Greenpeace and Royal Dutch/Shell over

[1] Maxwell, Lyon, and Hackett (2000) call this "self regulation". Vogel (2010) presents the closely related idea of civil regulation. In some cases (e.g. Maxwell, Lyon, Hackett 2000) self regulation may be motivated by the desire to forestall more demanding public regulation. In our model public regulation is either highly unlikely (perhaps because of effective industry lobbying) or practically infeasible (as in the case of conflict diamonds). See Egorov and Harstad (2012) and Baron (2013) for recent models that investigate the interaction between private and public politics.

[2] For an overview of such governance models see Koppell (2010).

the disposal of the Brent Spar oil storage buoy (Diermeier 2011). In that example Greenpeace occupied the platform in the middle of the North Sea. Shell then decided to clear it using water canons, which led to a media storm and drops in sales of up to 40 percent across Northern Europe.

This paper focuses on modeling a campaign in a dynamic context between a firm that cares (to some extent) about its image as a good citizen and an activist that seeks to tarnish the firm's image to advance its own agenda. The firm can enhance its image by engaging in private regulation, modeled here as voluntary activity aimed at curbing a negative externality, above and beyond that required by public policy. The activist group can target the firm in two ways. It can engage in an effort that counteracts the firm's efforts to improve its reputation through private regulation. Or it can try to trigger a crisis that can drastically harm the firm's reputation.

We model the interaction between the firm and the activist as a discrete-time, infinite-horizon dynamic stochastic game involving a firm and an activist. Though we provide some analytical characterization of the Markov perfect equilibrium of this game, we rely heavily on computational methods because, as we show, the activist's impact on firm behavior is complex and subtle. In particular, we employ the homotopy method utilized in Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) to show how equilibrium behavior is affected by the fundamentals of the model, including the efficacy of the activist, the discount factors, and the returns to corporate citizenship. The efficacy of the activist is affected by two factors: the saliency of the activist's campaign to call attention to the firm's shortcomings (what we call *criticism*) and the newsworthiness of the activist's efforts to provoke a crisis (what we call *confrontation*). Criticism harms the firm's reputation at the margin, while confrontation—if it becomes newsworthy—can drastically hurt the firm's reputation. Increases in salience (holding newsworthiness constant) are shown to induce a substitution, in the long run, of confrontation for criticism, while increases in newsworthiness (holding salience constant) induces substitution of effort in the opposite direction. Thus, fundamentally, criticism and confrontation are substitutes.

We further show that an activist with a higher discount factor — i.e., a more patient activist — tends to rely to a greater extent on confrontation than one with a lower discount factor. Since the discount factor is driven, in part, by the likelihood that the activist persists over time, our model suggests that, all other things equal, crisis provocation is a tool more likely to be used by a secure and well-funded activist than by one who may not be around in the future. We also find that the firm's long-run value declines significantly as the activist's discount factor increases. Moreover, a firm with a higher discount factor is more susceptible to a crisis than a firm with a lower discount factor and that the activist derives more long-run value when the firm has a high discount factor than when it has a low discount factor. Thus, the most dangerous adversary for a firm is a patient activist, and the most inviting target for an activist is a patient firm.

Our paper focuses on the positive implications of the interaction between a firm and an activist.[3] As just noted, the model illustrates circumstances under which an activist would tend to rely

[3] Abito, Besanko and Diermeier (2014) studies the normative aspects of this interaction.

more on confrontation or criticism, and thus it provides a positive theory of activist behavior. In addition, the model helps resolve a puzzle: why do firms like Coca Cola, Cisco Systems, or McDonalds—firms with established brands and multi-billion market capitalizations—devote the same or increasing resources, year after year, to voluntary efforts that address social problems such as carbon emissions or obesity, even when it is hard to imagine how such activity could make their very strong brands even stronger? It seems plausible that this activity would reach a point of diminishing marginal returns, making increased levels of it inconsistent with shareholder value maximization.

One way to resolve this puzzle is to invoke a theory of moral management of the kind developed by Baron (2009a). Another way to resolve it is through agency arguments: senior managers who authorize spending on corporate citizenship do so to burnish their own private reputations, rather than enhance shareholder wealth, and equilibrium contracting may be unable to eliminate this form of perquisite consumption entirely. Our model, by contrast, provides a different explanation for the puzzle that does not require abandoning the assumption of shareholder wealth maximization. Specifically, it suggests that in a modern media and communications environment, corporate reputation is subject to both countervailing pressures and drastic shocks that, at least to some extent, can be triggered by activists. These pressures and shocks boost private regulation by keeping a firm in situations where the accumulation of additional reputational capital has significant value. A potential social value of an activist, then, is to keep the firm well below the point at which diminishing marginal returns would induce it to scale back its voluntary activity.

The organization of the remainder of the paper is as follows. Section 2 describes the model of competition between the firm and the activist. Section 3 presents the general conditions for equilibrium. Section 4 explains the computational approach we employ and the baseline parameters we use in those computations. Section 5 provides some analytical characterization of the Markov perfect equilibrium and also presents the results of our computational analysis. Finally, Section 6 summarizes and concludes. Throughout the paper we distinguish between "Propositions" that are established through formal arguments and "Results," which either establish a possibility through a numerical example or summarize a regularity revealed through a systematic exploration of the parameter space. Proofs of all propositions are in the Appendix.

2 The model

2.1 Model structure

The basic structure of our model is one of competition between a firm and an activist group. Put simply, the firm seeks to enhance its reputation for corporate citizenship, while the activist takes steps to undermine that image.[4] We model this competition as an infinite horizon dynamic game.

[4] In the model activists can only harm the firm's reputation, they cannot improve it, e.g. by endorsing the firm's business practices and products. Much of the empirical literature on activists has pointed out that activists focus

2.1.1 The firm

The firm produces a single product which has a demand curve $q_t = e_t - p_t$, where q_t is quantity at time t, p_t is price, and e_t is the strength of the firm's overall brand equity. We assume that brand equity is given by $e_t = e_0 R_t^{\theta}$, where e_0 is a fixed component (determined by factors such as product performance or design), and R_t is the firm's reputation for corporate citizenship (hereafter referred to as "reputation"). The importance of reputation is captured by the parameter $\theta > 0$.

[Take in Figure 1]

Our model is set in discrete time and has a discrete state space $\mathcal{R} = \{1, \ldots, \overline{R}\}$.[5] The firm's initial reputation is $R_0 \in (1, \overline{R})$.[6] We assume that R_t is influenced by activities of the firm and the activist group and evolves according to the following stochastic process:

$$R_{t+1} = \begin{cases} R_t + \widetilde{F}_t - \widetilde{A}_t & \text{if} \quad \widetilde{\Delta}_t = 0 \\ \in \{1, \ldots, \max\{R_t - 1, 1\}\} & \text{if} \quad \widetilde{\Delta}_t = 1, \end{cases} \tag{1}$$

where, $\widetilde{F}_t$, $\widetilde{A}_t$, and $\widetilde{\Delta}_t$ are random variables taking on values $\{0, 1\}$. As Figure 1 illustrates, the firm's reputation for corporate citizenship evolves in two ways. If $\widetilde{\Delta}_t = 0$, reputation evolves incrementally, moving up or down by one unit depending on the realizations of $\widetilde{A}_t$ and $\widetilde{F}_t$ (both of which we discuss below). By contrast, if $\widetilde{\Delta}_t = 1$, the firm's reputation can drop precipitously. In particular, if $\widetilde{\Delta}_t = 1$, reputation will fall to a particular value between 1 and $\max\{R_t - 1, 1\}$ according to a uniform distribution. We characterize this event as a crisis. A crisis can cause a firm's reputation for citizenship to take a potentially significant one-time "hit," which equals $\frac{R_t}{2}$ on average. This formulation captures a "bigger they are, the harder they fall" property: firms with greater reputations for citizenship will, on average, experience a greater *absolute drop* in reputation as a result of a crisis. This is consistent with the observation that companies with the strongest reputations for citizenship tend to receive the greatest scrutiny by activists and the media, and thus seem to have the greatest vulnerability in the event of a crisis.[7] An alternative view (e.g. Dowling 2002, Alsop 2004, Minor 2010) is that firms that have invested heavily in building a reputation for citizenship may have a "bank account" that can cushion the impact of the reputational shock from the crisis. In the model this is captured by the feature that the *proportionate drop* in a firm's

on inducing harm (e.g. Friedman 1999). While we follow this approach here, future work may enlarge the strategy set for activists to include providing benefits for the firm and then provide an equilibrium analysis to explain the prevalence of harm. For a discussion and a static model with both harm and benefits, see Baron and Diermeier (2007).

[5] The upper bound $\overline{R}$ contributes to diminishing marginal return to investments in reputation building, but as illustrated below it is not the only source of diminishing marginal returns in the model. Diminishing marginal returns to investment in reputation building is supported by empirical evidence; see, for example, Lev, Petrovits, and Radhakrishnan (2006) on the impact of corporate charitable contributions on sales growth.

[6] R_0 does not affect the equilibrium, but it does affect the transient (short-run) dynamics implied by the equilibrium.

[7] See Argenti (2004). Dean (2004) and King and McDonnell (2012) provide empirical evidence that supports this phenomenon.

reputation is independent of R (on average it is 50 percent). Thus, a firm with a strong reputation is cushioned to some extent from the impact of a crisis.[8]

The firm's production process creates a negative externality that is neither taxed, priced in the market, or regulated. The firm can positively affect its reputation by voluntarily abating some of the externality, and we let x_t denote the level of the firm's externality-reducing activity. The situation we are modeling is especially pertinent to firms whose supply chains are located in parts of the world where conventional policy interventions for negative externalities are either ineffective or unavailable. As such, externality-reducing activity occurs in the realm of what Baron (2003) and Baron and Diermeier (2007) refer to as private politics, and we refer to it as private regulation.

Potential consumers do not observe x_t, but x_t generates an imperfect signal $\widetilde{X}_t$ which, if it exceeds a threshold X_0, creates awareness among consumers that the firm is voluntarily taking steps to reduce an externality.[9] From this, the firm gets credit for being a good citizen, which incrementally enhances its reputation, i.e., $\widetilde{F}_t = 1$. We assume $\widetilde{X}_t$ has a logistic distribution with mean $\ln(x_t)$ and scale parameter 1, so

$$\phi_F(x_t) \equiv \Pr(\widetilde{F}_t = 1 | x_t) = \frac{\eta x_t}{1 + \eta x_t},$$

and $\eta \equiv \exp(-X_0) > 0$. The function $\phi_F(\cdot)$ takes on values between 0 and 1 for all positive values of x_t; it is strictly increasing and concave, and approaches 1 as x_t becomes arbitrarily large.

We assume that externality-reducing activity does not affect the performance, quality, or appearance of the firm's products. Therefore, although the activity creates a direct *social* benefit, it has no direct *consumer* benefit and thus does not enter the firm's demand function. The provision of x_t is assumed to increase the firm's total costs, which are given by $cq_t + kx_t$, where $c \in (0, e_0)$ is the marginal cost of output and $k > 0$ is the marginal cost of externality-reducing activity.

2.1.2 The activist

Unlike the firm, the activist internalizes direct benefits from x. The social benefit of x is denoted by $w(x)$ and is given by:

$$w(x) = \begin{cases} w_0 x - \frac{1}{2} w_1 x^2 & x \leq \frac{w_0}{w_1} \\ \frac{w_0^2}{2w_1} & \text{otherwise} \end{cases}.$$

The activist's private utility is given by $u(x) = \psi w(x)$, where $\psi \geq 0$ is a parameter that measures the activist's passion for the social benefits created by x. If $\psi > 1$, the activist is so passionate that it over-internalizes the social benefits of externality-reduction.

Offsetting the firm's efforts to build its reputation, the activist can harm the firm's reputation in two distinct ways. First, the activist group can engage in *criticism,* denoted by z. Criticism

[8] Note that by changing assumptions on the distribution over 1 and $\max\{R-1, 1\}$ in the event of a crisis, we can change the implicit strength of the "bigger they are, harder they fall" property and the "bank account" property.

[9] We do not model the process by which the signal $\widetilde{X}_t$ is generated. This process may involve costs, such as advertising or public relations efforts that are independent of the direct costs of providing x_t.

comprises things such as letters to editors, op-ed pieces, letter-writing campaigns, share-holder resolutions, Facebook groups, and blogs that publicly call attention to the firm's shortcomings and which may, therefore, counteract the firm's attempt to burnish its image through private regulation. In each period, criticism influences a signal $\widetilde{Z}_t$ which can damage the image of the firm if it exceeds a threshold Z_0 beyond which the activist's criticism penetrates the public consciousness. Thus, $\widetilde{A}_t = 1$ if and only if $\widetilde{Z}_t \geq Z_0$. We assume that $\widetilde{Z}_t$ has a logistic distribution with mean $\ln(z_t)$ and scale parameter 1, so

$$\phi_A(z_t) \equiv \Pr(\widetilde{A}_t = 1|z_t) = \frac{\alpha z_t}{1 + a z_t},$$

where $\alpha \equiv \exp(-Z_0) > 0$.

Second, the activist group can engage in *confrontation*, denoted by d. Confrontation is deliberately aimed at provoking a reputational crisis. In each period, confrontation creates a level $\widetilde{D}_t$ of potentially newsworthy negative publicity about the firm's activities. The publicity need not be accurate; what matters is that it is potentially newsworthy enough to attract mass media attention. Once $\widetilde{D}_t$ exceeds a newsworthiness threshold D_0, the publicity "blows up" and develops into a crisis.[10] Thus, $\widetilde{\Delta}_t = 1$ if and only if $\widetilde{D}_t \geq D_0$. We assume that $\widetilde{D}_t$ has a logistic distribution with mean $\ln(d_t)$ and scale parameter 1, so

$$\phi_\Delta(d_t) \equiv \Pr(\widetilde{\Delta}_t = 1|d_t) = \frac{\omega d_t}{1 + \omega d_t},$$

where $\omega \equiv \exp(-D_0) > 0$.

Criticism and confrontation work through different channels. The former is more constructive and intended on changing business practices. It frequently does not generate significant coverage by the mass media. Religious organizations and pensions funds, e.g. TIAA-CREF, commonly pursue this approach, but it is also in the arsenal of many activist groups (e.g. Eesley and Lenox, 2006). By contrast, confrontation is intended to generate significant mass media coverage, critical to the firm. The general idea is to create a spectacle through acts of civil disobedience (e.g. occupying an installation or throwing a pie at the CEO), theatrical protests (e.g. dressing up as a polar bear to protest global warming), and other forms of confrontation (e.g. posting "wanted posters" in the CEO's residential community).[11] In his handbook for activists, San Francisco low-rent-housing advocate Randy Shaw summarized the approach as follows:

> "Ideally, tactical activists should use the media both to generate a scandal and then to demand a specific, concrete result." (p.155).

The parameters α and ω capture the efficacy of each type of activity.[12] The parameter α depends on the salience of the activist's efforts to draw attention to the firm's shortcomings using

[10] For a discussion of the underlying processes see Baron (2009b) and Diermeier (2011), especially Chapters 1-3.
[11] For examples of such tactics see Diermeier (2011), Chapter 3.
[12] For empirical studies of the impact of different forms of activism on firm behavior in the context of environmental pollution see Eesley and Lenox (2005), Eesley and Lenox (2006), and Lenox and Eesley (2009).

non-confrontational tools such as blogs or op-ed pieces. It would thus be a function of the activist's skill in developing a persuasive narrative that counters the firm's efforts to burnish its reputation, as well as its effectiveness in mobilizing a community of followers to disseminate that narrative. It would also depend on the salience of the activist's issue in a given market-place.[13] By contrast, the parameter ω depends on the mass media environment. For example, it would reflect the extent to which mass media outlets are inclined to provide coverage of the actions the activist group takes to provoke a crisis. The likelihood that the media will provide such coverage may depend on many factors such as the issue environment in a given country, the skill of the activists in generating media coverage, and the structure of the media, e.g. the importance of state-owned media.[14] In a world without mass media we have $\omega = 0$. Given the distinction between these parameters, we refer to α as the salience parameter and ω as the newsworthiness parameter.

Both z and d are costly to the activist, and the activist's cost function is given by $b_z z + b_d d$, where $b_z > 0$ and $b_d > 0$ are constants. We normalize b_d by assuming that $b_d = b_d(R) = \frac{R}{2} b_z$. This specification ensures that the cost to the activist of obtaining one unit of reputation reduction through criticism is equal to the cost of obtaining, on average, one unit of reputation reduction through confrontation.[15] We adopt this specification so as not to bias the choice between the two activities solely due to differences in their marginal costs. Any difference in the intensity of the activist's critical and confrontational activities will be due to differences in α and ω or to the intrinsically different ways that the two activities affect reputational dynamics.

2.1.3 Comments on the model specification

1. The firm's objective is the maximization of the discounted value of its profits. The firm thus has no intrinsic preference for engaging in externality-reducing activity. It does so only to improve its reputation or to blunt the effort of the activist.

2. The activist receives no intrinsic utility from harming the firm's reputation: it cares only about the level of x provided by the firm. In this respect, the activist is "pragmatic." Its benefit from harming the firm's reputation is to keep the firm motivated to supply higher levels of x. In practice, activists may have ideological interests that translate into a direct utility for hurting firms' reputations or financial conditions. Still, the pragmatic activist specification is plausible because we believe that to be effective an activist must be pragmatic to some extent. The pragmatic activist model is also a useful benchmark because it highlights the role of the activist as a strategic player in the firm's reputation management process. (By contrast, purely ideological activists would merely be "noise.")

[13] For example, it has been hypothesized that a country's concern about animal welfare may be systematically related to its economic growth. See Frank (2008).

[14] See Baron (2009b), Baron and Diermeier (2007) and Diermeier (2011) for details.

[15] Strictly speaking, this would be correct only if $\alpha = \omega$. By normalizing the cost parameters, we ensure that the difference between reputation-impairing and crisis-inducing effort is due either to differences in α and ω, or to fundamental differences in the nature of reputation-impairing activity and crisis-inducing activity.

3. R is assumed to be observable to the activist, and thus the activist can condition its actions on it. This may appear to be a strong assumption. Unlike a firm's physical capital, installed base of customers, or cumulative experience, R is not a standard metric that would be followed by investment analysts. However, the media does give attention to firms' reputations for corporate citizenship (often in the form of rankings). For example, for many years *Fortune* has published a list of "America's Most Admired Corporations," and one component of that ranking (used in empirical work on corporate reputation) is "Responsibility to the community and environment."[16] In addition, effective activists are likely to be skillful at sensing public sentiment about companies and tailoring their efforts to that sentiment. Finally, tools from computational linguistics and computer science provide technologies that enable individuals and groups to perform highly sophisticated content analysis of media and analyst coverage of firms to determine how their public image is being portrayed. In light of these considerations, the assumption that R is observable to the activist strikes us as plausible.

4. The firm cannot take actions *ex ante* to reduce the likelihood of a crisis. All the firm can do is to "plug away" and attempt to build its reputation over time (which, as noted above, provides a cushion in the event of a crisis). A hamburger restaurant chain, for example, can improve its animal welfare standards but cannot give up serving meat entirely without abandoning its business model. The inability of the firm to take actions to reduce the likelihood of a crisis can be motivated by the following view of a crisis. A crisis is primarily a phenomenon that arises within, and plays itself out, in the context of the media. Within that realm, there are notable asymmetries between what activists and firms can do to provoke or prevent a crisis. Activists may be able to draw attention to problems that can provoke media scrutiny, but firms typically have less ability to influence the media "narrative" (Dennis and Merrill, 1996; Bond and Kirshenbaum, 1998). This arises because "good news" that a firm might want to highlight to prevent a crisis (e.g., Toyota solving problems with its accelerators) is typically less newsworthy than "bad news" that an activist might highlight to trigger a crisis (e.g., car crashes traceable to faulty accelerators).

5. The firm and the activist group are assumed to be unable to contract on the provision of x, z, and d.[17] In practice, of course, bargaining between activists and firms sometimes does occur, but there are various reasons why bargaining solutions may be infeasible. For example, some activist groups may be unwilling to strike deals with firms lest their volunteers or donors see them as "selling out". This effect will be particularly pronounced if the activist group competes in a market for donors or volunteers with other groups less willing to compromise. Such competition may also make the enforcement of any agreement between a firm and an

[16] This ranking is now called the "World's Most Admired Companies."

[17] In this respect, our model differs from that of Baron (2003), which assumes that the target firm and the activist can negotitate over the firm's provision of externality-reducing activity and the activist's undertaking of a campaign against the firm.

activist group impossible.[18]

3 Equilibrium conditions

We restrict attention to the Markov perfect equilibrium (MPE) in which the state variable is the firm's reputation R. A MPE is a vector of strategies $\{(x^*(R), z^*(R), d^*(R)), R \in \mathcal{R}\}$ such that:

- For each state $R \in \{1, \ldots, \overline{R}\}$, $x^*(R)$ maximizes the discounted present value of the firm's expected profits, given the activist's strategies $\{(z^*(R), d^*(R)), R \in \mathcal{R}\}$.
- For each state $R \in \{1, \ldots, \overline{R}\}$, $(z^*(R), d^*(R))$ maximizes the discounted present value of the activist's expected utility, given the firm's strategy $\{x^*(R), R \in \mathcal{R}\}$.

3.1 Firm's Bellman equation and Kuhn-Tucker conditions

With linear demand and constant marginal cost, the firm's per-period profit contribution in state R is $\frac{(e_0 R^\theta - c)^2}{4}$. We assume that $\theta \in (0, \frac{1-\frac{c}{2e_0}}{2-\frac{c}{2e_0}})$, which implies that single-period profit is strictly concave in R.

Let $V_F^*(R)$ denote the present value of the firm's expected profit in state R in equilibrium. It is defined by the Bellman equation

$$V_F^*(R) = \frac{(e_0 R^\theta - c)^2}{4} + \max_{x \geq 0} \left\{ -kx + \beta_F \left[\begin{array}{c} [1-\phi_\Delta^*(R)] \left\{ \begin{array}{c} \phi_F(x)\,[1-\phi_A^*(R)]\,V_F^*(R+1) \\ +[1-\phi_F(x)-\phi_A^*(R)+2\phi_F(x)\phi_A^*(R)]\,V_F^*(R) \\ +[1-\phi_F(x)]\,\phi_A^*(R) V_F^*(R-1) \end{array} \right\} \\ +\phi_\Delta^*(R)\frac{\sum_{r=1}^{\max\{R-1,1\}} V_F^*(r)}{R-1} \end{array} \right] \right\}, \quad (2)$$

where $\beta_F \in (0,1)$ is the firm's discount factor, $\phi_A^*(R) \equiv \phi_A(z^*(R))$, and $\phi_\Delta^*(R) \equiv \phi_\Delta(d^*(R))$. In writing this expression, it is understood that in state $R = \overline{R}$, $V_F^*(R+1) = V_F^*(R)$ and in state $R = 1$, $V_F^*(R-1) = V_F^*(R)$.

Straightforward algebra reveals that the firm's continuation value (the term in large square brackets in (2)) is a function of (among other things) $V_F^*(R) - V_F^*(R-1)$ and $V_F^*(R+1) - V_F^*(R)$. Following Cabral and Riordan (1994), we refer to these differences as *prizes*. A prize is the increment to the firm's long-run value due to a one-step increase in its reputation and thus represents the marginal benefit of reputation enhancement to the firm.

The Kuhn-Tucker conditions for the firm are:

$$MB_x \leq k;\ x \geq 0;\ (MB_x - k)\,x = 0, \quad (3)$$

[18] For a discussion of activist commitment see Baron and Diermeier (2007).

where MB_x is the firm's marginal benefit of externality-reducing activity given by:

$$MB_x(x, \mathbf{V}_F^*(\mathbf{R}), \phi_A^*(R), \phi_\Delta^*(R)|R) \equiv \frac{\beta_F(1-\phi_\Delta^*(R))\eta}{(1+\eta x)^2}\left\{\begin{array}{c}(1-\phi_A^*(R))\left[V_F^*(R+1)-V_F^*(R)\right] \\ +\phi_A^*(R)[V_F^*(R)-V_F^*(R-1)]\end{array}\right\}, \tag{4}$$

where $\mathbf{V}_F^*(\mathbf{R}) \equiv (V_F^*(1), \ldots, V_F^*(\overline{R}))$.[19]

The marginal benefit function, which is strictly decreasing in x, is the firm's "demand curve" for externality reduction: it shows how much externality-reducing activity the firm "purchases" at "price" k. The activist shifts this demand curve both directly (through ϕ_Δ and ϕ_A) and indirectly, through the impact of the activist on $\mathbf{V}_F^*(\mathbf{R})$.

3.2 Activist's Bellman equation and Kuhn-Tucker conditions

Analogous to the Bellman equation for the firm, the Bellman equation for the activist gives us the present value of the activist's utility in state R in an equilibrium:

$$V_A(R) = \max_{z\geq 0, d\geq 0}\left\{\begin{array}{c} u(x^*(R)) - b_z z - b_d(R)d \\ +\beta_A\left[\begin{array}{c}[1-\phi_\Delta(d)]\left\{\begin{array}{c}\phi_F^*(R)\left[1-\phi_A(z)\right]V_A^*(R+1) \\ +\left[1-\phi_F^*(R)-\phi_A(z)+2\phi_F^*(R)\phi_A(z)\right]V_A^*(R) \\ +\left[1-\phi_F^*(x)\right]\phi_A(z)V_A^*(R-1)\end{array}\right\} \\ +\phi_\Delta(d)\frac{\sum_{r=1}^{\max\{R-1,1\}}V_A^*(r)}{R-1}\end{array}\right]\end{array}\right\}, \tag{5}$$

where $\beta_A \in (0,1)$ is the activist's discount factor; $\phi_F^*(R) \equiv \phi_F(x^*(R))$, and (analogous to before) it is understood that in state $R = \overline{R}$, $V_A^*(R+1) = V_A^*(R)$ and in state $R = 1$, $V_A^*(R-1) = V_A^*(R)$. The Kuhn-Tucker conditions are:

$$MB_z \leq b_z; z \geq 0; [MB_z - b_z]\, z = 0, \tag{6}$$

$$MB_d \leq b_d(R); d \geq 0; [MB_d - b_d(R)]\, d = 0, \tag{7}$$

where MB_z and MB_d are the marginal benefits of criticism and confrontation, respectively, and are given by:

$$MB_z(z, d, \mathbf{V}_A^*(\mathbf{R}), \phi_F^*(R)|R) = \frac{\beta_A(1-\phi_\Delta(d))\alpha}{(1+\alpha z)^2}\left\{\begin{array}{c}\phi_F^*(R)\left[V_A^*(R)-V_A^*(R+1)\right] \\ +(1-\phi_F^*)\left[V_A^*(R-1)-V_A^*(R)\right]\end{array}\right\}, \tag{8}$$

[19] We condition on R in writing $MB_x(\cdot)$ because MB_x does not depend on the entire vector $\mathbf{V}_F^*(\mathbf{R})$ but rather on just parts of it in a manner specific to the state R.

$$MB_d(d, z, \mathbf{V}_A^*(\mathbf{R}), \phi_F^*(R)|R) \equiv \frac{\beta_A \omega}{(1+\omega d)^2} \left\{ \begin{array}{c} \sum_{r=1}^{R-1} \frac{1}{R-1} \left[V_A^*(r) - V_A^*(R)\right] \\ +\phi_F^*(R)(1-\phi_A(z))[V_A^*(R) - V_A^*(R+1)] \\ -(1-\phi_F^*(R))\phi_A(z) \left[V_A^*(R-1) - V_A^*(R)\right] \end{array} \right\}. \tag{9}$$

Like MB_x, MB_z and MB_d depend on prizes that result from changes in the firm's reputation.

3.3 Equilibrium conditions

A MPE is a collection of five ($\overline{R} \times 1$) vectors ($\mathbf{V}_\mathbf{F}^*, \mathbf{V}_\mathbf{A}^*, \mathbf{x}^*, \mathbf{z}^*, \mathbf{d}^*$) satisfying the five sets of equilibrium conditions[20] for each of the $\overline{R}$ values of R: (2), (3), (5) (6), and (7).[21] The Kuhn-Tucker conditions are complementary slackness conditions, so for the the computational analysis below, it is useful to reformulate these conditions as a system of equations. To illustrate, consider (3). Following Borkovsky, Doraszelski, and Kryukov (2010, p. 1127), we can rewrite (3) as a pair of equations involving two variables, x and ζ_x

$$\begin{aligned} MB_x - k + [\max\{0, \zeta_x\}]^n &= 0, & (10) \\ -x + [\max\{0, -\zeta_x\}]^n &= 0, & (11) \end{aligned}$$

where $n \in \mathbb{N}$ is a large positive integer. The system (10) and (11) can be shown to be equivalent to (3) when

$$\zeta_x = \begin{cases} [-(MB_x - k)]^{\frac{1}{n}} & \text{if} \quad MB_x - k < 0, \\ -[x]^{\frac{1}{n}} & \text{if} \quad x > 0, \\ 0 & \text{if} \quad MB_x - k = x = 0. \end{cases} \tag{12}$$

Moreover, conditions (10) and (11) are $n-1$ continuously differentiable with respect to x and ζ_x. Transforming the other Kuhn-Tucker conditions in this fashion, let $\mathbf{H}(\mathbf{V}_A, \mathbf{V}_F, \mathbf{x}, \mathbf{z}, \mathbf{d}|\Omega)$ denote the system of equations defining a MPE where Ω is a vector of parameters. A MPE ($\mathbf{V}_\mathbf{F}^*, \mathbf{V}_\mathbf{A}^*, \mathbf{x}^*, \mathbf{z}^*, \mathbf{d}^*, \boldsymbol{\zeta}_x^*, \boldsymbol{\zeta}_z^*, \boldsymbol{\zeta}_d$) thus solves

$$\mathbf{H}(\mathbf{V}_\mathbf{F}^*, \mathbf{V}_\mathbf{A}^*, \mathbf{x}^*, \mathbf{z}^*, \mathbf{d}^*, \boldsymbol{\zeta}_x^*, \boldsymbol{\zeta}_z^*, \boldsymbol{\zeta}_d|\Omega) = 0, \tag{13}$$

where $\mathbf{H}(\mathbf{V}_\mathbf{F}^*, \mathbf{V}_\mathbf{A}^*, \mathbf{x}^*, \mathbf{z}^*, \mathbf{d}^*|\Omega) = 0$ is a system of $8\overline{R}$ non-linear equations in $8\overline{R}$ unknowns.[22] Condition (13) forms the basis of the computational analysis below.

4 Computational approach

Our objective is to develop a comprehensive intuition about equilibrium interactions between a forward-looking firm and a forward-looking activist. To do this, we rely on a partial analytical

[20] We check second order conditions in the appendix.

[21] Thus, for example, $\mathbf{V}_\mathbf{F}^* = (V_F^*(1), \ldots, V_F^*(\overline{R}))$, with the other terms in ($\mathbf{V}_\mathbf{F}^*, \mathbf{V}_\mathbf{A}^*, \mathbf{x}^*, \mathbf{z}^*, \mathbf{d}^*$) defined in the same way.

[22] These equations are the two Bellman equations (2) and (5); and six reformulated Kuhn-Tucker conditions, i.e. (10) and (11) applied to (3), (6), and (7), using (12) for ζ_x, ζ_z and ζ_d.

Parameter	Description	Baseline value
R	Size of state space	30
R_0	Firm's initial reputation	15
c	Marginal cost of output	20
k	Marginal cost of externality-reducing investment x	100
w_0	Intercept of marginal social benefit curve for x	125
w_1	Slope of marginal social benefit curve for x	0.5
ψ	Activist's passion	2
e_0	Brand equity for firm with $R = 1$	100
θ	Elasticity of brand equity with respect to reputation for citizenship	0.25
η	Salience parameter: externality-reducing activity	0.20
α	Salience parameter: criticism	0.20
ω	Newsworthiness parameter: confrontation	0.20
b_z	Marginal cost of criticism (note: $b_d(R) = b_z \frac{R}{2}$)	150
β_F	Firm's discount factor	0.95
β_A	Activist's discount factor	0.95

Table 1: Baseline parameter values.

characterization of the MPE, supplemented by computations of the MPE for a large set of parameter values. This section sets the stage for the computational analysis.

4.1 Baseline parameterization

Table 1 shows the baseline parameterization used to compute the "showcase" equilibrium. While the baseline parameterization is not intended to be representative of any particular industry, it is neither entirely unrepresentative nor extreme. To put these parameters in perspective, we note that the growth in the firm's reputation for corporate citizenship can potentially increase the firm's brand equity from $e = (15^{0.25})100 \approx 197$ at its initial value of $R = 15$ to $e = 100(30^{0.25}) \approx 234$, while brand equity could potentially fall to $e = 100$ if $R = 1$. Given the baseline demand and cost parameters, a crisis in the initial state that crashed the firm's reputation by the expected amount would cause its per-period profit contribution to fall by about 30 percent, while the worst possible crisis would cause per-period profit contribution to fall by about 80 percent. To put this in perspective, when Extra Strength Tylenol was implicated in six deaths in suburban Chicago area in 1982, Tylenol's sales dropped by about 87 percent (Lewin 1982). In our model, a shift in the demand curve sufficient to cause an 87 percent decline in sales revenue (given an optimal pricing response pre- and post-shift) would decrease the per-period profit contribution by about 98 percent. Thus, the worst possible crisis under our parameterization would be on a par with a Tylenol-style reputational crisis.[23]

[23] Of course, this effect would be transitory since, in equilibrium, the firm would then take steps to rebuild its reputation, as Johnson and Johnson did during the Tylenol crisis. Activist campaigns can have a similar (short-term) impact on sales. During Greenpeace's 1995 campaign against Shell over the Brent Spar Platform Shell's sales in Germany fell by 40 percent. For a discussion of both cases see Diermeier (2011), Chapters 1 and 3.

The discount factor can be thought of as $\beta_i = \frac{\zeta_i}{1+r}$, where $r > 0$ is the per-period discount rate and $\zeta_i \in (0, 1]$ is the exogenous probability that the agent survives from one period to the next. This interpretation is especially relevant for the activist, who may operate on a very tight budget, and who may suddenly disappear as a result of shocks to its funding. Consequently, our baseline parameterization corresponds to a variety of scenarios that differ in the length of a period. For example, it corresponds to a period length of one year, a yearly discount rate of 5.3 percent, and certain survival. But it also corresponds, e.g., to a period length of one month, a monthly discount rate of 1 percent (which translates into a yearly discount rate of 12.68 percent), and a monthly survival probability of 0.96, which translates into an expected life span of about 26 months.

4.2 Computational analysis

We perform two types of computational analyses. First, to generate insight about possible regularities of the equilibrium, we compute equilibria over a grid $\mathcal{G}$ of parameter values given by:

$$\mathcal{G} = \left\{ \begin{array}{c} (\alpha, \omega, \psi, \theta, \beta_A) | \alpha \in \{0.10, 0.20, 0.30, 0.40\}, \omega \in \{0.10, 0.20, 0.30, 0.4\}, \psi \in \{0.5, 2.0, 4.0\}, \\ \beta_A \in \{0.80, 0.95, 0.99\}, \beta_F \in \{0.80, 0.95, 0.99\}, \theta \in \{0.001, 0.15, 0.25, 0.35, 0.40\} \end{array} \right\},$$

where in defining $\mathcal{G}$ it is understood that all other parameters are fixed at baseline levels. The grid is designed to determine how the equilibrium varies as we vary all the parameters $(\alpha, \omega, \psi, \beta_A)$ that determine the activist's incentives, as well as the parameter θ that determines the returns to the firm from corporate citizenship.

Second, we change key parameters on a one-at-a-time basis to isolate how each parameter affects the equilibrium. Any parameter not varied is set at its baseline level: The parameters varied are:[24] (1) salience of criticism: $\alpha \in [0, 0.40]$; (2) newsworthiness: $\omega \in [0, 0.04]$; (3) activist's discount factor: $\beta_A \in [0, 0.999]$; (4) firm's discount factor $\beta_F \in [0, 0.999]$; (5) returns to corporate citizenship: $\theta \in [0, 0.40]$; activist passion: $\psi \in [0.5, 4]$. The method used for these computational exercises is described in the Appendix.

5 Analysis

5.1 Equilibrium behavior with no activist

To establish a benchmark, we describe the outcome when there is no activist. This corresponds to the case in which $\alpha = \omega = 0$.

Proposition 1 *In the absence of an activist, the firm's externality-reducing effort and value func-*

[24] We take a finer grid than $\mathcal{G}$ in the comparative statics exercise since equilibria are easier to compute in this case using the homotopy method.

tion $(x^*(R), V_F^*(R))$ *are found by solving the following system of equations for* (V_F, x) *recursively:*

$$\begin{aligned} x^*(\overline{R}) &= 0 \\ V_F^*(\overline{R}) &= \frac{\left[e_0\overline{R}^\theta - c\right]^2}{4(1-\beta_F)}, \end{aligned}$$

and for $R < \overline{R}$

$$x = \max\left[\frac{1}{\eta}\left(\left\{\frac{\beta_F \eta}{k}\left[V_F^*(R+1) - V_F\right]\right\}^{\frac{1}{2}} - 1\right), 0\right] \tag{14}$$

$$V_F = \frac{\left[e_0 R^\theta - c\right]^2}{4(1-\beta_F)} + \frac{k\eta x^2}{(1-\beta_F)}. \tag{15}$$

For any $R \in \{1, \ldots, \overline{R}-1\}$ *the firm's value function is strictly increasing and strictly concave in* R*, i.e.,* $V_F^*(R+1) > V_F^*(R)$ *and* $V_F^*(R+1) - V_F^*(R) < V_F^*(R) - V_F^*(R-1)$*. The firm's level of externality-reducing activity is non-increasing in* R*, i.e.,* $x^*(R+1) \leq x^*(R)$*, and the inequality is strict in any state in which* $x^*(R) > 0$*.*

Proposition 1 implies that in the absence of an activist, reputation enhancement is valuable to the firm, but it is subject to diminishing marginal returns. The firm thus reduces its externality-reducing activity as its reputation grows, i.e., it "coasts" on its reputation. Because Proposition 1 holds for an arbitrary end state, $\overline{R}$, which can be made arbitrarily large, the concavity of the value function is attributable to fundamentals (principally, the concavity of single-period profit in R), not to "end effects" due to a finite $\overline{R}$.

[Take in Figure 2]

Figure 2 shows the equilibrium in the no activist case for the baseline parameter values. In this case, $x^*(R)$ (depicted in the middle panel) decreases monotonically from about 15 in the lowest state of $R = 1$ to 0 in $R = 30$. Given the assumed parameter values, this implies that there is a 0.58 probability of reputation growth in the initial state $R = 15$, but this declines over time as the firm's externality-reducing activity diminishes. This process of reputation growth can be shown to take about 40 periods on average. As the firm's reputation grows from $R = 15$ to $R = 30$, it is able to raise its price by about 17.5 percent (from about 108 to 127, as depicted in the right-hand panel), and its value grows by about 23 percent (from about 186,000 to 229,000, depicted in the left-hand panel). Thus, as the firm's reputation grows, it coasts, and eventually draws its externality-reducing activity down to zero.

5.2 Equilibrium behavior with an activist

5.2.1 The role and impact of the activist: a preliminary cut

The firm's incentive to coast provides the basis for reputation-impairing action by the activist. If the firm did not coast (either globally or locally)—i.e., if $x^*(R)$ monotonically increased in R—the interests of the firm and the activist would be aligned. Both parties would benefit from a growth in the firm's reputation, and the activist would have no reason to block the firm from increasing R. We show, though, that it in equilibrium $x^*(R)$ cannot be monotonically increasing in R. Thus, even when the firm faces an activist, it still coasts on its reputation *to some extent*. This, in turn, provides a potential motivation for the activist to choose positive levels of either criticism or confrontation.

Proposition 2 *The firm's equilibrium level of externality-reducing activity cannot be monotonically increasing in R; i.e., there exists states R and $R+1$ such that $x^*(R+1) \leq x^*(R)$. Thus, the firm (weakly) "coasts" on its reputation in at least some states.*

The activist's behavior has a potentially complex set of effects on the firm's decision to invest in externality reduction. A useful starting point is to perform the following thought experiment. Start with a situation in which the activist's criticism and confrontation are zero and then consider a small exogenous increase in confrontation d in a single focal state R_n. This generates an exogenous perturbation in the probability $\phi_\Delta^n = \phi_\Delta(d(R_n))$ of a crisis in the focal state, but keeps ϕ_Δ and ϕ_A fixed at zero in all other states. As the firm adjusts to this perturbation optimally, it will alter the profile of values, so $\mathbf{V}_F(\mathbf{R}) = \mathbf{V}_F(\mathbf{R}|\phi_\Delta^n) = (V_F(1|\phi_\Delta^n), \ldots, V_F(\overline{R}|\phi_\Delta^n))$. This one-state perturbation, though far simpler than what actually happens in equilibrium, provides a relatively "clean" way to isolate how confrontation shifts the firm's demand curve for externality-reducing activity at various points in the state space.

With ϕ_A held at zero, the marginal benefit of externality reduction in the focal state R_n becomes:

$$MB_x(x, \mathbf{V}_F(\mathbf{R}|\phi_\Delta^n), \phi_\Delta^n|R_n) = \frac{\beta_F \eta}{(1+\eta x)^2}(1-\phi_\Delta^n)\left[V_F(R_n+1|\phi_\Delta^n) - V_F(R_n|\phi_\Delta^n)\right],$$

and thus

$$\frac{dMB_x(x, \mathbf{V}_F(\mathbf{R}|\phi_\Delta^n), \phi_\Delta^n|R_n)}{d\phi_\Delta^n} = \frac{\beta_F \eta}{(1+\eta x)^2}\left\{\begin{array}{c} -\left[V_F(R_n+1|\phi_\Delta^n) - V_F(R_n|\phi_\Delta^n)\right] \\ +(1-\phi_\Delta^n)\frac{\partial\left[V_F(R_n+1|\phi_\Delta^n) - V_F(R_n|\phi_\Delta^n)\right]}{\partial \phi_\Delta^n} \end{array}\right\}. \quad (16)$$

In all other states $R \neq R_n$ the marginal benefit of externality reduction is

$$MB_x(x, \mathbf{V}_F(\mathbf{R})|\phi_\Delta^n), \phi_\Delta^n|R) = \frac{\beta_F \eta}{(1+\eta x)^2}\left[V_F(R+1|\phi_\Delta^n) - V_F(R|\phi_\Delta^n)\right],$$

and thus
$$\frac{dMB_x(x, \mathbf{V}_F(\mathbf{R}|\phi_\Delta^n), \phi_\Delta^n|R)}{d\phi_\Delta^n} = \frac{\beta_F \eta}{(1+\eta x)^2} \frac{\partial \left[V_F(R+1|\phi_\Delta^n) - V_F(R|\phi_\Delta^n)\right]}{\partial \phi_\Delta^n}. \tag{17}$$

In the focal state R_n, the firm's demand curve depends on ϕ_Δ^n in two ways. First, an increase in ϕ_Δ^n directly decreases the marginal benefit of x in state R_n. Holding the prize $[V_F(R_n+1|\phi_\Delta^n) - V_F(R_n|\phi_\Delta^n)]$ fixed, this effect would unambiguously shift the demand curve leftward in state R_n in a manner akin to the impact of *ad valorem* sales tax on a consumer demand curve. We call this the *direct effect* of confrontation on the firm's demand curve for externality reduction.[25] Note that there is *no* direct effect in states other than R_n.

Second, an increase in ϕ_Δ^n affects the prize itself, both in state R_n, as well in other states $R \neq R_n$. We call this the *prize effect* of confrontation, and it is given by the sign of $\frac{\partial\left[V_F(R+1|\phi_\Delta^n) - V_F(R|\phi_\Delta^n)\right]}{\partial\phi_\Delta^n}$, which is not obvious. We gain insight into it by performing a comparative statics analysis on ϕ_Δ^n, allowing ϕ_Δ^n to go to 0 (which places us at the no-activist equilibrium). The following Lemma characterizes the prize effect as the perturbation in confrontation goes to 0 in the limit.

Lemma 1 *Starting from the no-activist equilibrium, consider a one-state perturbation ϕ_Δ^n in the probability of a crisis in state $R_n < \overline{R}$: (a) In states $R \in \{R_n+1, \ldots, \overline{R}\}$, as $\phi_\Delta^n \to 0$, the prize effect for confrontation is zero; (b) In the focal state R_n, as $\phi_\Delta^n \to 0$,the prize effect is strictly positive; (c) in states $R \in \{1, \ldots, R_n - 1\}$, as $\phi_\Delta^n \to 0$, the prize effect is strictly positive.*

Lemma 1, in conjunction with (16) and (17), lead immediately to:

Proposition 3 *Starting from the no-activist equilibrium, consider a one-state perturbation ϕ_Δ^n in the probability of a crisis in state $R_n < \overline{R}$. As $\phi_\Delta^n \to 0$, the impact of the perturbation on the firm's externality-reducing activity is as follows: (a) In states $R > R_n$, the perturbation has no effect; (b) In the focal state $R = R_n$, the perturbation has an ambiguous effect (a negative direct effect may or may not be offset by a positive prize effect); (c) In states $R < R_n$, the perturbation has a positive effect (due to the positive prize effect).*

However, if the perturbation is "large," the direct effect must dominate the prize effect in the focal state.

Proposition 4 *Starting from the no-activist equilibrium, consider a one-state perturbation ϕ_Δ^n in the probability of a crisis in state $R_n < \overline{R}$, and suppose that in this state, the firm would have invested a positive amount in externality-reducing activity. As $\phi_\Delta^n \to 1$, the direct effect in state R_n dominates the prize effect and has an unambiguously negative effect on the firm's externality-reducing activity in state R_n.*

Propositions 3 and 4 hint at the complexity of the activist's role in shaping the firm's behavior. In equilibrium, "single-state perturbations" occur in *all* states simultaneously, and none of these

[25] The direct effect is analogous to the compensated effect of price on quantity demanded in consumer theory.

"perturbations" are necessarily "small" or "large." Moreover, they interact with perturbations in criticism z, which was fixed in our thought experiment. Moreover, in equilibrium the "perturbations" are themselves endogenous, so the firm's equilibrium decisions feed back and effect them. Finally, both the direct and prize effects are static phenomenon in the sense that they relate to the impact of the activist on the intensity of the firm's incentives in particular states. They do not speak to how the presence of the activist will change the dynamics of how the firm's reputation evolves over time.

Still, this analysis, limited though it is, provides a helpful insight: the activist's impact on the firm's externality-reducing activity is not unambiguously positive. That is, the presence of an activist may have (for the activist) the unintended consequence (through the direct effect of confrontation) of suppressing the thing that the activist wants, namely an abundant supply of x.

We can also conduct an analysis of a small perturbation in criticism, yielding to a positive probability ϕ_A^n in focal state R_n. With ϕ_Δ held to 0, the expression for marginal benefit (4) in the focal state is:

$$MB_x(x, \mathbf{V}_F(\mathbf{R})|\phi_A^n), \phi_A^n|R) = \frac{\beta_F \eta}{(1+\eta x)^2}\left[\begin{array}{c} V_F(R+1|\phi_A^n) - V_F(R|\phi_A^n) \\ +\phi_A^n \left\{\begin{array}{c} [V_F(R|\phi_A^n) - V_F(R-1|\phi_A^n)] \\ -[V_F(R+1|\phi_A^n) - V_F(R)|\phi_A^n] \end{array}\right\} \end{array}\right].$$

The direct effect of ϕ_A^n is given by the term $\left\{\begin{array}{c} [V_F(R|\phi_A^n) - V_F(R-1|\phi_A^n)] \\ -[V_F(R+1|\phi_A^n) - V_F(R)|\phi_A^n] \end{array}\right\}$, which depends on the concavity of the firm's value function. Our computations reveal that the firm's equilibrium value function is not necessarily concave when an activist is present. However, as $\phi_A^n \to 0$,

$$\left\{\begin{array}{c} [V_F(R|\phi_A^n) - V_F(R-1|\phi_A^n)] \\ -[V_F(R+1|\phi_A^n) - V_F(R)|\phi_A^n] \end{array}\right\} \to \left\{\begin{array}{c} [V_{F0}^*(R) - V_{F0}^*(R-1)] \\ -[V_{F0}^*(R+1) - V_{F0}^*(R)] \end{array}\right\},$$

and Proposition 1 established that this latter expression is non-negative. This gives us the following result:

Proposition 5 *In the focal state R_n, the direct effect of criticism for small perturbations is non-negative.*

Proposition 5 highlights that criticism and confrontation can have different effects on the firm's marginal benefit of externality reduction. The proposition, to be sure, provides only limited insight (e.g., it says nothing about the prize effect of criticism, or the direct effect for "non-small" changes, both of which appear to be generally ambiguous). But like Propositions 3 and 4, it points to the complex impact that the activist can have on the firm's equilibrium behavior.

[Take in Figure 3]

Figure 3 summarizes the implications of the preceding propositions. Confrontation in a given state may shift the firm's demand curve leftward or rightward, depending on the strength of the direct and prize effects. Confrontation in higher states unambiguously shifts the demand curve in a given state leftward due to the prize effect. Small levels of criticism in a given state have a direct effect of shifting the firm's demand curve rightward in a given state, but an ambiguous prize effect. What happens in equilibrium is a complex amalgam of these various shifts.

5.2.2 Computational results: baseline parameterization

[Take in Figure 4]

Figure 4 shows the value functions (upper panel) and policy functions (lower panel) for this showcase equilibrium.[26] The equilibrium level of externality-reducing activity, $x^*(R)$, generally decreases as the firm's reputation grows, but not everywhere. Thus, there is coasting, but the coasting is not global.

A comparison of Figures 2 and 4 indicates that in any state R, the firm's externality-reducing activity is less when there is an activist than when there is not. The direct effect of confrontation, discussed above, is one driver of this, though it may not be the only one.

However, even though the activist induces a reduction in the intensity of externality reduction state-by-state, it does not follow that over time the firm will invest less when there is an activist. This is because, as can be seen in the lower panel of Figure 4, the activist generally engages in positive amounts of both criticism and confrontation, with the mix of the two activities varying with R. Therefore, unlike the no-activist case, the firm will, in all likelihood, not reach states in which its externality-reducing activity falls to 0.

[Take in Figure 5]

To expand on this point, it is useful to describe the dynamics of the model. The equilibrium actions of the firm and activist generate a Markov process. Given any starting state, this process implies a transient probability distribution over R, $x^*(R)$, $z^*(R)$ and $d^*(R)$ for any time period t. Using these distributions, we can construct expectations over the firm and the activist's equilibrium behavior, as well as the firm and activist's value, for any time $T = t$. Figure 5 illustrates the path of these expectations assuming $R_0 = 15$. The upper panels show how the firm's expected reputation and externality-reducing effort vary over time. For example, by $T = 20$, the firm's expected reputation $\mathbb{E}_{20}[R]$ is approximately equal to 11, and as time passes, reputation is expected to fall to slightly less than 10. Due the activist's efforts to impair the firm's reputation, the firm experiences a gradual decline in reputation from the initial state.[27] However, unlike the no-activist case, the firm

[26] Though we cannot rule out the possibility of multiple equilibria, we were unable to find more than one equilibrium in this case.

[27] Of course, these dynamics are contingent on the starting state. If the starting state had been less than $R = 10$, there would have been a gradual rise in the firm's reputation to about 10.

does not, in the long run, stop investing in externality reduction. Indeed, there is sharp contrast in the time path of the firm's externality-reducing activity without and with an activist. Without an activist, the time path of externality reduction declines over time; with an activist, the firm's expected externality-reducing activity would actually rise over time, settling into an expected level of a little over 7 in the long run.

[Take in Figure 6]

But this expectation actually disguises the fluidity of the firm's situation. As time passes both the firm and the activist continue to invest, which causes small increases and decreases in the firm's reputation, as well as an occasional crisis which causes reputation to fall dramatically. Figure 6 shows the transient distributions over the firm's reputation at three points in time: $T = 4, 8$, and 16. It also shows the limiting distribution over R, which we use to characterize the long-run dynamics of the game. In the long run, the firm's reputation could range from 1 to 30, with (as indicated earlier) an expectation a little less than 10 and a mode of about 2. Thus, the interaction between the firm and the activist gives rise to a dynamic in which the firm occasionally manages to increase its reputation, but because $x^*(R)$ decreases in R throughout most of the state space, each time it increases its reputation, it reduces its externality-reducing activity. From time to time, the activist's criticism reduces the firm's reputation, and sometimes, the activist provokes a severe crisis that causes the firm's reputation to collapse drastically. In the aftermath of these episodes, the firm's motivation to enhance its reputation increases and it steps up its externality-reducing activity.

5.2.3 Computational results: grid search over $\mathcal{G}$

The baseline parameters represent a single point in parameter space. To explore the generalizability of the insights generated by this example, we turn to the grid search over $\mathcal{G}$.[28]

We begin by summarizing the extent to which equilibria have certain properties in common. Table 2 reports percentages of the parameterizations for which various properties were true in particular states, while Table 3 reports the percentage of parameterizations in which the equilibrium had particular dynamic properties.[29] (In the table and throughout the remainder of the paper, the subscript "0" refers to the no-activist case.)

[28] The grid contains 720 unique parameter combinations. We were able to compute equilibria for 641 of these, or about 89 percent of the grid. To reflect the 79 parametrizations for which we could not compute equilibria, we can construct bounds for the computed proportions for each property in the subsequent tables. Specifically, if p is the proportion of computed equilibria satisfying some property, then the corresponding proportion over the entire grid is inside the interval $[0.89p, 0.89p + 0.11]$. Of the 79 cases for which equilibria could not be computed, 60 of these involve $\psi = 4$, while the rest involve $\psi = 2$. For those with $\psi = 2$, there is one case with $\theta = 0.35$ and 18 cases with $\theta = 0.4$. These cases are associated with high returns to reputation for the firm.

[29] At all points in $\mathcal{G}$, we did not identify cases of multiple MPE.

Properties	**Proportion of equilibria in $\mathcal{G}$ with property in state R**														
	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
1. $x^*(R) \leq x_0^*(R)$	0.95	0.89	0.89	0.85	0.84	0.86	0.87	0.88	0.86	0.90	0.86	0.90	0.85	0.87	0.88
2. $x^*(R) \leq x^*(R+1)$	0.64	0.62	0.44	0.40	0.38	0.38	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37	0.37
3. $x^*(R) < x^*(\frac{R}{2})$ or $x^*(\frac{R-1}{2})$	.	0.36	0.38	0.48	0.53	0.60	0.61	0.62	0.63	0.63	0.64	0.64	0.64	0.64	0.64
4. $V_F^*(R) \leq V_{F0}^*(R)$	0.68	0.68	0.68	0.67	0.67	0.66	0.66	0.65	0.65	0.65	0.65	0.65	0.65	0.65	0.65
5. $V_A^*(R) \geq V_{A0}^*(R) = 0$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6. $V_F^*(R+1) \geq V_F^*(R)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
7. $V_A^*(R) \geq V_A^*(R+1)$	1.00	1.00	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Properties	**Proportion of equilibria in $\mathcal{G}$ with property in state R**														
	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1. $x^*(R) \leq x_0^*(R)$	0.88	0.88	0.88	0.85	0.83	0.84	0.87	0.85	0.85	0.85	0.83	0.82	0.81	0.72	0.86
2. $x^*(R) \leq x^*(R+1)$	0.37	0.37	0.37	0.36	0.37	0.36	0.37	0.37	0.37	0.37	0.37	0.37	0.53	0.36	.
3. $x^*(R) < x^*(\frac{R}{2})$ or $x^*(\frac{R-1}{2})$	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64
4. $V_F^*(R) \leq V_{F0}^*(R)$	0.65	0.65	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.63	0.63
5. $V_A^*(R) \geq V_{A0}^*(R) = 0$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
6. $V_F^*(R+1) \geq V_F^*(R)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.
7. $V_A^*(R) \geq V_A^*(R+1)$	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	.

Table 2: Properties of equilibria, state-by-state, in the parameter grid.

Property	Proportion of equilibria in $\mathcal{G}$
$\mathbb{E}_\infty[R] < \overline{R} = 30$	0.964
$\mathbb{E}_\infty[R] < 20$	0.712
$\mathbb{E}_\infty[R] < 10$	0.297
$Mode_\infty[R] < 30$	0.730
$Mode_\infty[R] < 20$	0.694
$Mode_\infty[R] < 10$	0.321

Table 3: Expected and modal long-run states for equilibria in the parameter grid

From these tables we can draw a number of conclusions. The first property in Table 2 compares the firm's equilibrium level of externality-reducing activity in a given state R to the level that would have prevailed in the absence of an activist:

Result 1 *For each state $R \in \mathcal{R}$, $x^*(R) \leq x_0^*(R)$ for over 70 percent of the equilibria in the grid, and in 29 of 30 states, $x^*(R) \leq x_0^*(R)$ for over 80 percent of the equilibria in the grid. Thus, the presence of the activist is often associated with a reduction in the firm's externality-reducing activity in a given state.*

We also see that, Proposition 2 notwithstanding, Proposition 1 does not extend to the equilibrium with an activist:

Proposition 6 *In contrast to the equilibrium in the absence of an activist, the firm's equilibrium level of externality-reducing activity is not monotonically decreasing in R; i.e., there exists states R and $R+1$ such that $x^*(R+1) > x^*(R)$. Thus, the firm does not globally coast on its reputation.*

On the other hand, coasting does occur to some extent, and it ensures that in the wake of a crisis of expected severity, a firm may often increase its level of externality-reducing activity:

Result 2 *For each state $R \geq 6$, $x^*(R) < x^*(\frac{R}{2})$ or $x^*(\frac{R-1}{2})$ for over 60 percent of the equilibria in the grid. Thus, for a firm with a sufficiently strong reputation, a crisis that reduces the firm's reputation to the expected post-crisis level will tend to motivate the firm to increase its externality-reducing activity.*

We also see from Table 2 that while the firm benefits from enhancing its reputation, the activist's value often declines when R increases.

Result 3 *The firm's value function is increasing in R. By contrast, the activist's value function usually —though not always—is decreasing in R. Thus, the firm benefits from an improvement in its reputation, while the activist often benefits by hurting the firm's reputation.*

Table 3 provides insight into the nature of the dynamics in the model and shows that in contrast to the no-activist case, the firm's reputation would rarely be expected to grow to the maximal attainable level:

Equilibrium type	**Proportion of cases in $\mathcal{G}$**	**Representative parameterization**
Diversified activist	0.063	baseline parameterization
Specialized activist: confrontation	0.059	ω large relative to α
Specialized activist: criticism	0.029	α large relative to ω
Ineffective activist	0.523	low value of θ, ψ, β_A, α and/or ω

Table 4: Taxonomy of equilibria

Equilibrium type	$\mathbb{E}_4[x^*(R)]$	$\mathbb{E}_\infty[x^*(R)]$
Diversified activist	10.29	10.83
Specialized activist: confrontation	9.95	9.51
Specialized activist: criticism	9.17	2.98
Ineffective activist	2.52	0

Table 5: Average level of short-run and long-run externality-reducing investment by type of equilibrium.

Result 4 *The firm rarely (in fewer than 4 percent of computed equilibria) would be expected to attain the maximum possible reputation state R in the long run, and often (more than 2 out of 3 cases) would be expected to attain a long-run reputational state less than $R = 20$.*

We next classify equilibria into three categories: (1) Diversified activist; (2) Specialized activist; (3) Ineffective activist. A diversified activist engages in positive amounts of criticism and confrontation for *long-run relevant* values of R, where "long-run relevant" means that the probability of that state in the limiting distribution is at least 0.10.[30] A specialized activist engages in just criticism or just confrontation in the long-run relevant states. An ineffective activist does not engage in either activity in the long-run relevant states. Table 4 characterizes the equilibria according to this taxonomy, while Table 5 shows how the type of equilibrium correlates with short-run and long-run levels of externality-reducing activity (measured by the average values of $\mathbb{E}_4[x^*(R)]$ and $\mathbb{E}_\infty[x^*(R)]$ over each equilibrium in the category).

About 52 percent of the equilibria in the grid involve ineffective activists. This is because for particularly low values of θ (like $\theta = 0.001$), the firm does not invest in externality reduction because a reputation for corporate citizenship has very limited value, so, in turn, there is limited benefit to activities aimed at compromising the firm's reputation. Ironically, then, firms that are apathetic to building a reputation are not inviting targets for activists. Table 5 indicates that externality-reducing activity is generally highest in both the short and long run when there is a diversified activist and generally the lowest when there is an activist that specializes in criticism. We summarize these results as follows:

Result 5 *The highest levels of externality-reducing activity in both the short run and the long run are generally associated with diversified activists.*

[30] We also used a cutoff of 0.05 to define long-run relevant states. The lower cutoff results in more non-classifiable equilbria, but the breakdown between diversified, specialized, and ineffective activists was about the same.

Results 1-5 provide a strong suggestion of the forces at work in the model and the trade-offs they create. On the one hand, positive levels of criticism and confrontation change the firm's incentives for externality-reducing activity in a given state through the direct and prize effects. In particular, the tendency for $x^*(R)$ to be less than $x_0^*(R)$ in a given state R (identified in Result 1) is a footprint of the negative direct effect of confrontation. On the other hand, the activist's presence changes the evolution of states in the game. The clearest manifestation of this is Result 2, which indicates that in the wake of a crisis the firm would, more often than not, step up its level of externality-reducing activity. The activist thus provides an antidote to coasting: its actions shape the dynamics of the game in a way that tends to keep the firm in states in which it is more motivated to invest in x.

In the introduction we raised the question of why firms with well-established brands (e.g., Coca Cola) seem to devote significant resources to voluntary efforts that address negative externalities or other social problems, even when it is hard to imagine how such activity could make their very strong brands even stronger. Or, put differently, why don't firms with strong reputations seem to reach a point of diminishing returns that makes it worthwhile to coast on their reputations? Our model resolves this puzzle by highlighting the role of activists and the damage they can do to corporate reputations. In particular, the activist tends to keep a targeted firm's reputational capital at levels at which the accumulation of additional reputational capital has significant value. For this to occur, the activist must use confrontation and criticism as more than just threats; there must be a positive probability along the equilibrium path that the activist actually harms the firm's reputation from time to time.[31]

5.3 Comparative statics of long-run outcomes

An equilibrium is a vector in a large multi-dimensional space, so describing in a compact way how it changes with changes in underlying parameters is difficult. To simplify this task, we focus on how changes in parameters affect the long-run equilibrium behavior, summarized by $\mathbb{E}_\infty[x^*(R)], \mathbb{E}_\infty[z^*(R)]$, and $\mathbb{E}_\infty[d^*(R)]$; long-run performance, summarized by $\mathbb{E}_\infty[R]$ and $\mathbb{E}_\infty[p^*(R)] = \mathbb{E}_\infty\left[\frac{e_0 R^\theta + c}{2}\right]$; and long-run value, summarized by $\mathbb{E}_\infty[V_F^*(R)]$ and $\mathbb{E}_\infty[V_A^*(R)]$.

5.3.1 Activist efficacy: variations in α and ω

[Take in Figure 7]

[Take in Figure 8]

[31] This effect could also operate if a firm faced an exogenous probability of either a reputational crisis or a marginal diminution of its reputation. Thus, activists *per se* are not critical to resolving the puzzle set out in the introduction. Still, the forces that enable activists to harm corporate reputations, such as the nature of the mass media environment, are much the same forces that allow exogenous events, such an oil spill or a major product defect, to become newsworthy enough to impair a company's reputation. Moreover, activists are often quick to exploit such exogenous events. The key point of our theory is that periodic destruction of reputational capital, whether activist-induced or exogenous, can keep a firm sufficiently motivated to enhance it, thereby explaining why some companies never seem to reach the point at which it makes sense to coast.

Figures 7 and 8 summarize how expected long-run equilibrium outcomes change as we vary the saliency parameter α, holding all other parameters fixed at baseline levels. (The dotted line identifies the baseline value of the focal parameter, in this case α.)[32] Figure 7 illustrates that as α increases, the activist tends to engage in more criticism and less confrontation in the long run. This suggests that the activist's policy tools are substitutes. If the activist's criticism has salience just a little below the baseline level (which, recall, is where $\alpha = \omega$), then in the long run it stops relying on this instrument and specializes in fomenting crises. Thus, z and d appear to be imperfect but close substitutes.

Figure 8 illustrates that an increase in salience has an ambiguous impact on the firm's long-run reputation and its value. Initially, long-run reputation and value rises with an increase in α. This is because the increase in criticism induces a substitution away from confrontation, which reduces the probability of crises, limiting the frequency with which the firm's reputation crashes. However, as salience increases even more, the firm's expected long-run reputation and value fall. This is associated with an increase in the level of the firm's externality-reducing activity.

[Take in Figure 9]
[Take in Figure 10]

Figures 9 and 10 summarize the impact on the equilibrium of changes in the newsworthiness parameter ω. This analysis reinforces the insight that criticism and confrontation are imperfect but close substitutes. If ω increases just a little above the baseline level, $\mathbb{E}_\infty[z^*(R)]$ falls to 0, while if ω increases just a little below the baseline level, $\mathbb{E}_\infty[d^*(R)]$ falls to 0. An empirical implication is that if otherwise similarly situated activists have different α's or ω's due to idiosyncratic reasons, we would expect to see the activists specialize in one tactic or the other.

Changes in ω have an ambiguous effect on the firm's long-run externality-reducing activity, and that effect differs from the effect of changes in α. If the newsworthiness parameter increases beyond a certain point, the long-run level of x declines. This reflects a sufficiently powerful direct effect of confrontation discussed earlier.

$\mathbb{E}_\infty[d^*(R)]$ is also non-monotonic in ω; as it becomes sufficiently easy to provoke a crisis, the activist's expected confrontation is scaled back in equilibrium.

Finally, the activist's long-run value $\mathbb{E}_\infty[V_A^*(R)]$ may decrease in ω. This is a consequence of the decline in $\mathbb{E}_\infty[x^*(R)]$ for sufficiently large values of ω. If provocative activity is highly newsworthy, the activist is actually hurt. The logic is that in a media environment in which crises are very easy to provoke, a firm simply gives up hope that it can sustain a good reputation for corporate citizenship and scales back the externality-reducing activity that the activist values. We can summarize the results of this analysis as follows:

[32] For each comparative statics analysis in this section, we present two figures. The first shows how long-run equilibrium behavior (x, z, d) varies with the focal parameter, and the second shows how long-run performance (reputation, price, and firm and activists values) vary with the focal parameter.

Result 6 *(i) For the activist, criticism and confrontation are imperfect substitutes; (ii) Increasing the newsworthiness of the activist's efforts to provoke a crisis does not unambiguously increase the firm's long-run externality-reducing activity, nor does it necessarily increase the intensity of the activist's confrontation in the long run; (iii) By contrast, increasing the saliency of the activist's criticisms of the firm does increase the firm's long-run externality-reducing activity (over the range where the activist engages in positive amounts of criticism).*

5.3.2 Activist and firm patience: variations in β_A and β_F

[Take in Figure 11]
[Take in Figure 12]

Figures 11 and 12 summarize the impact on long-run equilibrium outcomes of varying β_A, holding all other parameters fixed at baseline levels. A moderately impatient activist (β_A between about 0.60 and 0.90) is a minor threat to the firm; it engages in small amounts of criticism, but no confrontation in the long run. A highly impatient activist (β_A less than about 0.60) is no threat at all; it engages in no equilibrium activity of any kind. The most dangerous activist, from the firm's perspective, is a patient one. As β_A increases, the activist aggressively substitutes confrontation for criticism (except for the very highest values β_A, at which point the activist increases both activities in tandem).[33]

Recall that the discount factor reflects both the time preference of the activist, as well as its survival probability. The computational results indicate that a well-funded activist with strong survival prospects is more likely to attempt to provoke a crisis, while an activist with a lower survival probability will tend to engage in criticism. This is not because confrontation is less expensive or more efficacious for the well-funded activist (efficacy and cost are being held fixed in this analysis), but rather because the payoff from the activist's two instruments have different dynamic implications. Inducing a crisis that crashes the firm's reputation has a potentially big payoff to the activist since the firm, in the wake of the crisis, significantly increases x to rebuild its image. However, it takes time for the activist to trigger a crisis of sufficient impact to really matter, so a less well-funded, and therefore more impatient, activist may forego crisis provocation altogether and instead seek to motivate the firm by activities that marginally chip away at its reputation. We summarize these insights as follows:

Result 7 *A more patient activist tends to rely on confrontation to a greater degree, and on criticism to a lesser degree, than a less patient activist. Above a threshold value of β_A, the firm's value declines precipitously as β_A increases.*

[Take in Figure 13]
[Take in Figure 14]

[33] Thus, for extremely patient activists, reputation-impairing and crisis-inducing effort become complementary.

Figures 13 and 14 summarize the impact on long-run equilibrium outcomes by varying β_F, holding all other parameters (including β_A) fixed at baseline levels. There are two noteworthy implications of this analysis. First, a more patient firm is more vulnerable to a crisis than a less patient firm. Second, an activist prefers to interact with a more patient firm. These implications arise because a more patient firm derives a bigger prize from building reputation than a less patient firm, which makes the more patient firm more willing to invest in reputation-building. This directly benefits the activist. When a crisis occurs, the more patient firm has a greater motivation to rebuild its reputation than the less patient firm, which makes crisis provocation a particularly attractive strategy against a patient firm. This latter implication suggests that if the activist must choose among potential targets, a financially sound firm would be a more attractive target than a marginal firm. This is consistent with empirical evidence about activist behavior (Eesley and Lenox, 2006). We summarize the insights from this part of the analysis as follows:

Result 8 *A more patient firm is more susceptible to crises than a less patient firm. The activist prefers a more patient target to a less patient target.*

5.3.3 Returns to reputation for corporate citizenship: variations in θ

[Take in Figure 15]
[Take in Figure 16]

Figures 15 and 16 summarize the impact on long-run equilibrium outcomes by varying θ, holding all other parameters fixed at baseline levels. The greater the impact of reputation on brand equity, the more the firm invests in externality-reducing activity. However, the activist's behavior is not monotonic in θ. For $\theta \leq 0.125$, the firm does not invest in externality reduction, and the activist accordingly chooses no activity of either kind. Once θ exceeds 0.125, there is an upward jump in criticism, but as θ increases between 0.125 and 0.25, the activist decreases criticism and substitutes confrontation for it. As θ increases further above 0.25, increases in θ elicit more of both types of activities. Criticism is apparently the more attractive tool for the activist when it faces a firm that has only a modest concern with using corporate citizenship to build brand equity. By contrast, when corporate citizenship has a large effect on brand equity, crisis provocation becomes increasingly attractive. Thus, a firm for whom an image of good corporate citizenship is particularly important would be especially vulnerable to confrontational tactics by an activist.[34] We summarize this part of the analysis as follows:

Result 9 *If θ is below a threshold level, the firm does not engage in externality-reducing activity in the long run, and the activist does not engage in any effort to harm the firm's reputation. Above that threshold, as θ increases, the firm's long-run externality-reducing activity increases, as does the activist's levels of confrontation, increasing the likelihood of crises. The activist's long-run level of criticism initially falls as θ increases above the threshold, but it eventually begins to increase.*

[34] For empirical evidence supporting this claim see, e.g. Eesley and Lenox (2006).

5.3.4 Activist passion: variations in ψ

[Take in Figure 17]
[Take in Figure 18]

Figures 17 and 18 summarize how variations in the activist's passion affect long-run equilibrium behavior and performance. If ψ is slightly less than 1, it is completely ineffective: it engages in no criticism or confrontation. If the activist's objective function is social welfare ($\psi = 1$, indicated by the dotted line in the left of each panel), it engages in positive, but very small, amounts of criticism and confrontation. Thus, an activist that sees itself merely acting on behalf of the general public interest will hardly make a difference in the long run. Only if the activist is sufficiently passionate will it be motivated to take actions that lead to significant amounts of externality-reducing activity in the long run. However, beyond a certain point (ψ slightly less than 2) increases in ψ induce a decline in externality-reducing activity. This is because the passionate activist is very keen to provoke a crisis (long-run confrontation monotonically increases in ψ). However, because of the direct effect of confrontation this will induce the firm to scale back its externality-reducing activity. Increasing the passion of the activist makes the activist a more dangerous adversary for the firm, but may not advance the social interest. We summarize this analysis as follows:

Result 10 *If the activist is insufficiently passionate, it engages in no crisis or confrontation of any kind and is thus ineffective. As the activist's passion increases beyond this threshold, the long-run level of confrontation rises monotonically, while the long-run level of criticism initially decreases but then increases in ψ. The firm's long-run externality-reducing activity increases as the activist's passion increases beyond the threshold, but eventually it falls. The firm's long-run value monotonically decreases as activist passion increases.*

6 Summary and conclusions

We model the interaction between a firm and an activist using a discrete-time, infinite-horizon dynamic stochastic game. The firm is assumed to be profit maximizing while activists care about reducing a socially inefficient externality. The firm can engage in activity that reduces the externality and, with some probability, will receive an improvement in its corporate reputation, which enhances consumer demand. Activists can engage in two forms of costly activity: they can "criticize" the firm, which, with some probability, has a marginally negative impact on the firm's reputation, or they can trigger a "crisis" which crashes the firm's reputation.

While the firm has an incentive to invest in externality-reducing activity without the existence of an activist, this effort is subject to decreasing marginal returns in equilibrium. The incentive to "coast" when reputational equity is high creates a conflict between firm and activists. To prevent the firm from coasting, activists engage in a combination of criticism and crisis inducing behavior.

The activist's efforts prevents the firm from coasting. That said, state-by-state the activist's presence functions like a tax and *depresses* firm's incentives to engage in corporate citizenship, which serves neither the firm's nor the activist's interests. However, the activists' activities keep the firm motivated to supply externality reduction even in the long-run. In the case where the activist is effective, the trade-off between these incentives leads to a welfare enhancement in most cases, though these improvements are never first-best.

However, to the extent that such forms of private politics present an alternative regulatory mechanism any welfare comparisons need to be discussed in a broader context (Abito, Besanko and Diermeier, 2014). On the one hand, we can consider such mechanisms in cases where traditional conditions for public regulation do not hold. For example, Pigouvian taxes or subsidies may be infeasible; collective action and information problems may make Coasian bargaining impractical; or governance problems may undermine public regulation. Such a perspective may be especially appropriate for globally operating firms with business operations in countries with weak or non-existent regulatory mechanisms. In that case activist pressure would serve as a (partial) substitute for public regulation. That said, activists also operate in mature economies with fully developed legal, political, and regulatory institutions. Still, activists increasingly have resorted to directly targeting firms to change business practice. Moreover, activists have stated publicly that such private politics campaigns are more effective than the traditional channel of pressuring elected officials (e.g. Baron and Diermeier 2007). To assess such claims a proper comparison would move beyond a traditional welfare analysis and compare mechanisms based on private politics with political economy models of public regulation, where public policy is the consequence of competition among politicians, interest groups, and voters in public arenas.

From a positive point of view we can investigate how the nature of the equilibrium depends on the parameters of the model such as the relative effectiveness of the two activist activities, the returns to corporate citizenship, the discount parameters and so forth. For example, companies for whom corporate citizenship has a higher value are more inviting targets for activists. Moreover, activists are more effective when they both criticize and try to trigger crises. More patient activists rely more on confrontation, and more patient firms are more vulnerable to crises. Criticism and confrontation, however, are imperfect substitutes, and only in the case of criticism does effectiveness of that activity necessarily increases long-run externality-reducing activity by the firm.

The analysis generates a variety of empirical implications:

1. NGOs will target companies even if such companies engage in corporate social responsibility and self-regulation.

2. Large, well-financed, highly visible firms are better targets for activists and are more vulnerable to crises.

3. Boycotts and other similar forms of NGO activities negatively impact share-holder value. First, the impact of criticism or a reputational crisis leads to a decrease in the reputation

of the company and thus lower profits. Second, once targeted the company will invest more heavily in costly self-regulation. Both activities negatively impact share-holder value in the short-run.

4. Diversified activists (i.e. NGOs that use both criticism and confrontation) are most successful in inducing firms to chose high levels of externality reducing activities in both the short and the long-run. Better financed activists tend to rely more on confrontation.

5. For activists, criticism and confrontation are imperfect substitutes. Increasing the salience of an issue increases the firm's long-run investment in abatement activities, while increased newsworthiness has mixed effects: in intermediate ranges increasing newsworthiness increases abatement, but once newsworthiness reaches a critical value activists are hurt, since firms simply "give up" if a crisis can be triggered too easily.

6. Firms for whom an image of good corporate citizenship is particularly valuable engage in higher levels of abatement, but are also more frequent targets for confrontational tactics.

7. Only activists that are more passionate than the general public have any impact on the long-run behavior of firms. If effective activists are also more successful in obtaining resources in a market for donation and volunteers this implies that *ceteris paribus* activists should have more extreme preferences that the public. Yet, by the same logic, activists will not be too extreme either, since very passionate activists are less effectively, as they tend to engage too much in confrontation which reduces the incentives for firms to invest in abatement activities in the long-run.

Our implications are consistent with various findings in the empirical literature. First, negative media coverage and activist targeting has a negative impact on firm financial performance. Karpoff, Lott, and Wehrly (2005), Konar and Cohen (1996), and Beatty and Shimshack (2010) show that bad news about a company's environmental performance reduces their share-price. King and Soule (2007) and Vasi and King (2012) provide evidence that protests and boycotts lower corporate financial performance. King (2008) shows that the financial impact of boycotts varies with media attention as captured in our newsworthiness parameter ω.

Second, various studies have found that CSR activities lack a positive financial impact on firms and may have even have a (moderately) negative impact (Fisher-Vanden and Thornburgh 2011, Jacobs, Singal, and Subramanian 2010). Such findings have been viewed as puzzling as we may expect corporate social responsibility to be positively correlated with profits, e.g. by appealing to socially conscious customers, employees and share-holders. But our model points out that CSR and self-regulation may be necessary as a defensive tactic against activist threats. The benefit to the firm then results in risk mitigation. This is consistent with recent evidence. Kotchen and Moon (2012) show that companies tend to engage in more reputation-enhancing activities if they had suffered more reputational damage in the past. McDonnell and King (2013) as well as McDonnell, King and Soule (N.D.) show that firms that were targeted by a boycott respond with increased pro-social activities to mitigate negative reputational impact.

Third, there is considerable evidence that activists select firms strategically. In the context of environmental issues. Eesley and Lenox (2006) show that firm characteristics such as cash-flow, assets, and advertising intensity are positively correlated with an increased likelihood of being targeted in a boycott. Such measures partially capture our notions of newsworthiness, salience and firm patience. Eesley and Lenox also show that companies with higher absolute and relative emissions are more likely to be targeted. This captures our social benefit parameter w. And if selecting among a broad set of firms, *ceteris paribus*, activists will select those firms with the most social impact.

While the broad alignment with the existing empirical literature is encouraging, many of the empirical implications of the model have not been tested. For example, we are not aware of studies that assess whether confrontation and criticism are indeed imperfect substitutes. Similarly, the model points out that newsworthiness has a non-linear effect on activist success, while existing empirical studies tend to focus on linear influence. More generally, the interaction between firms and activities is rich in strategic complexity that should be captured in empirical studies. This suggests that there is an important role for statistical approaches that explicitly capture strategic interaction such as structural estimation.[35]

Our analysis also has some practical implications for managers and activists. In the Introduction we stated the puzzle why well-known companies such as McDonald's or Coca Cola continue to invest in corporate social responsibility activities and self-regulation even though their brand equity is unlikely to be improved by such activities. Our model implies that activists will continue to target such companies to reduce coasting, which creates incentives for companies to continue to invest in self-regulation. Companies will continue to experience criticism and confrontation despite adopting responsible business practices, and from time to time such activities will negatively impact the company's reputation. Following Argenti (2004) we can call this phenomenon the "Starbucks Paradox": a company that engages in CSR activities continues to be targeted by activists asking management to do ever more. Managers are often puzzled by such findings and expect that they should be rewarded for their good deeds by lowering the likelihood of an activist campaign. Such a mind-set rests on a static notion of preemption. That is, self-regulation works to forestall activist actions. But a dynamic view of corporate reputation shows that this reasoning is incomplete. Decreasing dynamic returns to reputation create incentives for companies to coast which activists can counter by engaging in sporadic campaigns. Such campaigns, if successful, destroy some of a firm's reputational capital which keeps the firm hungry, i.e. incentivized to continue to invest in self-regulation.

Even high levels of self-regulation and corporate social responsibility do not forestall NGO activity. Passionate activists always want the company to do more. Companies thus will experience

[35] See Baron, Harjoto and Ho (2011) for the use of a simulataneous equation model to address these concerns.

episodes of criticism and confrontation and rarely achieve the maximal reputational state and, despite ongoing investments in CSR, will hover around a moderate level of reputation. The presence of criticism and confrontation by activists and NGOs is not an aberration, but should be viewed as a normal and expected part of a company's business environment and should be managed as such. This is especially true for well-known companies where maintaining a strong reputation is highly valuable.

Our approach left out many of the complexities regarding the interaction between firms and activists. For example, activists were limited to inflict harm on the firm, an assumption that, while empirically supported, ideally would be derived in equilibrium of a richer model. Correspondingly, it would be worthwhile to consider a socially motivated firm. Other natural extensions would allow for bargaining between firms and activists and consider multiple, competing firms and activists. That said, even in the simple model, the dynamic interactions between the firm and the activist proved surprisingly rich. We hope that more complex approaches can be developed on its foundation.

7 Appendix

7.1 Second order conditions

Second order conditions generally depend on x, z, and d, and thus we can only check that the second order sufficient conditions are satisfied locally. Showing this for the firm's problem is straightforward. However if the activist uses both instruments, then we have to check numerically if these conditions are satisfied. These sufficient conditions are satisfied for all parametrizations in the grid, including our baseline case. In what follows, we derive the second order conditions assuming the nonnegativity constraints do not bind.

7.1.1 Firm's problem

Differentiating the first order condition of the firm's problem with respect to x yields

$$-\frac{2\eta}{(1+\eta x)} MB_x.$$

Since $MB_x = k > 0$, then

$$-\frac{2\eta}{(1+\eta x)} MB_x < 0.$$

7.1.2 Activist's problem

The Hessian of the activist's problem is given by

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

where

$$
\begin{aligned}
M_{11} &= -\frac{2\eta}{(1+\alpha z)} M B_z < 0 \\
M_{12} &= -\frac{2\omega}{(1+\omega d)} M B_d < 0 \\
M_{12} &= M_{21} = \frac{-\beta_A \alpha \omega}{(1+\alpha z)^2 (1+\omega d)^2} \left\{ \begin{array}{c} \phi_F^*(R)\left[V_A^*(R) - V_A^*(R+1)\right] \\ +(1-\phi_F^*)\left[V_A^*(R-1) - V_A^*(R)\right] \end{array} \right\}.
\end{aligned}
$$

The Hessian is negative definite if $M_{11} < 0$ and $M_{11}M_{22} - M_{12}M_{21} > 0$. The condition on the determinant can be rewritten as

$$
4\,(1+\alpha z)\,\Omega_1 > \Omega_2
$$

where

$$
\begin{aligned}
\Omega_1 &= \left\{ \begin{array}{c} \sum_{r=1}^{R-1} \frac{1}{R-1}\left[V_A^*(r) - V_A^*(R)\right] \\ +\phi_F^*(R)(1-\phi_A(z))[V_A^*(R) - V_A^*(R+1)] \\ -(1-\phi_F^*(R))\phi_A(z)\left[V_A^*(R-1) - V_A^*(R)\right] \end{array} \right\} > 0 \\
\Omega_2 &= \left\{ \begin{array}{c} \phi_F^*(R)\left[V_A^*(R) - V_A^*(R+1)\right] \\ +(1-\phi_F^*)\left[V_A^*(R-1) - V_A^*(R)\right] \end{array} \right\} > 0.
\end{aligned}
$$

While $M_{11} < 0$ can be readily seen, we have to compute the condition on the determinant. We check whether $4\,(1+\alpha z)\,\Omega_1 > \Omega_2$ holds at the equilibrium solution for every parametrization in the grid. Our numerical results show that this is true for all parametrizations in the grid.

7.2 Computational method

For each analytical experiment, we explore the graph of the equilibrium correspondence as the relevant parameters vary using the homotopy algorithm discussed in Besanko, Doraszelski, Kryukov, and Satterthwaite (2010). This natural-parameter homotopy gives us a convenient method for computing equilibria *and* succinctly summarizing how the equilibrium correspondence varies as we change the parameters of the model (i.e. comparative statics). To explain the algorithm, let $\mathbf{X}^* \equiv (\mathbf{V}_\mathbf{F}^*, \mathbf{V}_\mathbf{A}^*, \mathbf{x}^*, \mathbf{z}^*, \mathbf{d}^*)$ denote the equilibrium vector, and let $\mathbf{H}^{-1} = \{\mathbf{X}^* | \mathbf{H}(\mathbf{X}^*|\Omega) = 0\}$ denote the equilibrium correspondence. To explore this correspondence, we follow "paths" along the surface by varying a single parameter, such as τ. The parameter that is varied is known as the homotopy parameter. The homotopy method starts with a pair of functions $(\mathbf{X}^*(s), \tau(s)) \in \mathbf{H}^{-1}$ given parametrically by a scalar s, which implies $\mathbf{H}(\mathbf{X}^*(s)|\tau(s), \Omega/\tau) = 0$, where Ω/τ is the set of all parameters remaining fixed as τ varies. To remain on an equilibrium path, it is necessary that:

$$
\frac{\partial \mathbf{H}(\mathbf{X}^*(s)|\tau(s), \Omega/\tau)}{\partial \mathbf{x}} \frac{\mathbf{d}\mathbf{X}^*(s)}{\mathbf{d}s} + \frac{\partial \mathbf{H}(\mathbf{X}^*(s)|\tau(s), \Omega/\tau)}{\partial \tau} \tau'(s) = \mathbf{0}, \tag{18}
$$

where $\frac{\partial \mathbf{H}(\mathbf{X}^*(s)|\tau(s),\Omega/\tau)}{\partial \mathbf{x}}$ is the $(8\overline{R} \times 8\overline{R})$ Jacobian, $\frac{\mathbf{d}\mathbf{X}^*(s)}{\mathbf{d}s}$ and $\frac{\partial \mathbf{H}(\mathbf{X}^*(s)|\tau(s),\Omega/\tau)}{\partial \tau}$ are $(8\overline{R} \times 1)$ vectors, and $\tau'(s)$ is scalar valued. This is a system of $8\overline{R} + 1$ differential equations that must be solved in order to identify a path.

The homotopy algorithm is not guaranteed to find all the MPE. This is because $\mathbf{H}^{-1}$ may contain equilibria that are off the main path. To identify additional equilibria, we use the Pakes-McGuire algorithm at a variety of different starting values. In addition, we can choose other parameters besides τ to be the homotopy parameter. By using, for example, κ as the homotopy parameter for a fixed value of τ, we can "crisscross" the parameter space by using equilibria on the τ paths to generate paths with respect to κ. A κ path must either intersect with all τ paths, or they will lead us to additional equilibria that in turn can give us an initial condition to generate an additional τ path.

To compute equilibria, we use Hompack written in Fortran 90. Our programs are available upon request.

7.3 Proofs of propositions

Proof of Proposition 1:

Characterization of the solution to the firm's problem and proof that the solution is unique:

When there is no activist, (2) simplifies to

$$(1-\beta_F)V_F^*(R) = \frac{\left(e_0\overline{R}^\theta - c\right)^2}{4} + \max_{x\geq 0}\left\{-kx + \frac{\eta x \beta_F}{1+\eta x}\left[V_F^*(R+1) - V_F^*(R)\right]\right\}, \tag{19}$$

and the Kuhn-Tucker condition becomes

$$\frac{\beta_F \eta}{(1+\eta x)^2}\left\{V_F^*(R+1) - V_F^*(R)\right\} \leq k, \tag{20}$$

which holds with equality if $x^*(R) > 0$. Now, at $R = \overline{R}$, $V_F^*(\overline{R}+1) - V_F^*(\overline{R})$ so $x^*(\overline{R}) = 0$. Substituting $x = 0$ into (19) implies $V_F^*(\overline{R}) = \frac{\left(e_0\overline{R}^\theta - c\right)^2}{4(1-\beta_F)}$.

The derivation of (14) and (15), and the proof that the MPE is unique, is by induction. Consider an arbitrary $R < \overline{R}$, and suppose $x^*(R+1)$ and $V_F^*(\overline{R}+1)$ are the unique (x, V_F) satisfying satisfy (14) and (15). Now, for state R, consider the maximization problem in (19). The solution to this maximization problem is:

$$x = \begin{cases} 0 & \text{if } \frac{\beta_F\eta}{k}\left[V_F^*(R+1) - V_F\right] < 1 \\ \frac{1}{\eta}\left(\left\{\frac{\beta_F\eta}{k}\left[V_F^*(R+1) - V_F\right]\right\}^{\frac{1}{2}} - 1\right) & \text{if } \frac{\beta_F\eta}{k}\left[V_F^*(R+1) - V_F\right] \geq 1. \end{cases}, \tag{21}$$

but this expression is equivalent to (14). If we substitute (21) into (19) and rearrange terms, we get (15).

To prove that the solution to (14) and (15) is unique, notice that (21)—which, recall, is equivalent to (14)—traces out a locus in (x, V_F) space. This locus has two pieces. For $V_F > V_F^*(R+1) - \frac{k}{\eta\beta_F}$, this locus coincides with the vertical axis. For $V_F \leq V_F^*(R+1) - \frac{k}{\eta\beta_F}$, the locus is traced out by the equation $V_F^{\#}(x) = V_F^*(R+1) - \frac{k(1+\eta x)^2}{\beta_F\eta}$, which is strictly decreasing in x when $x \geq 0$. Condition (15) also traces out a locus in (x, V_F) space, and this locus, denoted by $\widehat{V}_F(x) = \frac{(e_0 R^\theta - c)^2}{4(1-\beta_F)} + \frac{k\eta x^2}{(1-\beta_F)}$, is monotone increasing in x and takes on a value of $\frac{(e_0 R^\theta - c)^2}{4(1-\beta_F)}$ when $x = 0$. There are two possibilities. If $\frac{(e_0 R^\theta - c)^2}{4(1-\beta_F)} > V_F^*(R+1) - \frac{k}{\eta\beta_F}$, the intersection of the two loci occurs at $x = 0$ and $V_F = \frac{(e_0 R^\theta - c)^2}{4(1-\beta_F)}$, so $x^*(R) = 0$ and $V_F^*(R) = \frac{(e_0 R^\theta - c)^2}{4(1-\beta_F)}$. If $\frac{(e_0 R^\theta - c)^2}{4(1-\beta_F)} < V_F^*(R+1) - \frac{k}{\eta\beta_F}$, the intersection of the two loci occurs at x such that where $\widehat{V}_F(x) - V_F^{\#}(x) = \frac{\left(e_0 R^\theta - c\right)^2}{4(1-\beta_F)} + \frac{k\eta x^2}{(1-\beta_F)} + \frac{k(1+\eta x)^2}{\beta_F\eta} - V_F^*(R+1) = 0$. It is straightforward to establish that this quadratic equation has a unique positive root, $x^*(R)$. This, in turn, implies that $V_F^*(R)$ is unique.

Proof that the firm's value function is strictly increasing in R: Suppose, to the contrary, that $V_F^*(R+1) \leq V_F^*(R)$. From the Kuhn-Tucker conditions in (20), it would follow that $x^*(R) = 0$, and from (15) in state R, we would have:

$$V_F^*(R) = \frac{\left[e_0 R^\theta - c\right]^2}{4(1-\beta_F)}. \tag{22}$$

Now, since $x^*(R+1) \geq 0$, (15) in state $R+1$ implies that

$$V_F^*(R+1) \geq \frac{\left[e_0\left(R+1\right)^\theta - c\right]^2}{4(1-\beta_F)}. \tag{23}$$

Comparing (22) and (23), we have $V_F^*(R+1) > V_F^*(R)$, a contradiction.■

Proof that the firm's externality-reducing activity is non-increasing in R:

The proof is by induction. Note that $0 = x^*(\overline{R}) \leq x^*(\overline{R}-1)$, establishing the result at $R = \overline{R} - 1$. Assume, then, $x^*(R+2) \leq x^*(R+1)$. There are two cases to consider: $x^*(R) > 0$ and $x^*(R) = 0$. Consider the first case, $x^*(R) > 0$. In this case, we want to establish that $x^*(R+1) < x^*(R)$. Suppose, contrary to what we want to prove, that $x^*(R+1) \geq x^*(R)$. This, then, implies $x^*(R+1) > 0$, so $x^*(R+1)$ must therefore satisfy (20) with equality in state $R+1$:

$$\frac{\beta_F \eta}{\left[1+\eta x^*(R+1)\right]^2}\left[V_F^*(R+2) - V_F^*(R+1)\right] = k, \tag{24}$$

Similarly, since $x^*(R) > 0$:

$$\frac{\beta_F \eta}{\left[1+\eta x^*(R)\right]^2}\left[V_F^*(R+1) - V_F^*(R)\right] = k. \tag{25}$$

Now, from (15) we have

$$V_F^*(R+2) - V_F^*(R+1) = \frac{\left[e_0(R+2)^\theta - c\right]^2}{4(1-\beta_F)} - \frac{\left[e_0\left(R+1\right)^\theta - c\right]^2}{4(1-\beta_F)} + \frac{\eta k\left[x^*(R+2)\right]^2}{1-\beta_F} - \frac{\eta k\left[x^*(R+1)\right]^2}{1-\beta_F}. \tag{26}$$

$$V_F^*(R+1) - V_F^*(R) = \frac{\left[e_0\left(R+1\right)^\theta - c\right]^2}{4(1-\beta_F)} - \frac{\left[e_0 R^\theta - c\right]^2}{4(1-\beta_F)} + \frac{\eta k\left[x^*(R+1)\right]^2}{1-\beta_F} - \frac{\eta k\left[x^*(R)\right]^2}{1-\beta_F.}. \tag{27}$$

Substitute (26) into the left-hand side of (24) and (27) into the left-hand side of (25), equate the resulting expressions, and rearrange terms to get:

$$\left[\begin{array}{c}[1+\eta x^*(R+1)]^{-2}\left\{\begin{array}{c}\left[e_0(R+2)^\theta-c\right]^2\\-\left[e_0\left(R+1\right)^\theta-c\right]^2\end{array}\right\}\\-\left[1+\eta x^*(R)\right]^{-2}\left\{\begin{array}{c}\left[e_0\left(R+1\right)^\theta-c\right]^2\\-\left[e_0R^\theta-c\right]^2\end{array}\right\}\end{array}\right]=4\eta k\left[\begin{array}{c}[1+\eta x^*(R)]^{-2}\left(\begin{array}{c}[x^*(R+1)]^2-\\ {[x^*(R)]^2}\end{array}\right)\\-\left[1+\eta x^*(R+1)\right]^{-2}\left(\begin{array}{c}[x^*(R+2)]^2\\-[x^*(R+1)]^2\end{array}\right)\end{array}\right]. \tag{28}$$

Now, by assumption, $x^*(R+1)\geq x^*(R)$, so $[1+\eta x^*(R+1)]^{-2}\leq[1+\eta x^*(R)]^{-2}$. Moreover, given our parameter conditions, $(e_0R^\theta-c)^2$ is an increasing, strictly concave function of R, so

$$0<\left\{\begin{array}{c}\left[e_0(R+2)^\theta-c\right]^2\\-\left[e_0\left(R+1\right)^\theta-c\right]^2\end{array}\right\}<\left\{\begin{array}{c}\left[e_0\left(R+1\right)^\theta-c\right]^2\\-\left[e_0R^\theta-c\right]^2\end{array}\right\}$$

Thus, the left-hand side of (28) is strictly negative, so

$$[1+\eta x^*(R)]^{-2}\left(\begin{array}{c}[x^*(R+1)]^2-\\ {[x^*(R)]^2}\end{array}\right)<[1+\eta x^*(R+1)]^{-2}\left(\begin{array}{c}[x^*(R+2)]^2\\-[x^*(R+1)]^2\end{array}\right)\leq 0, \tag{29}$$

where the second inequality in (29) follows because, by the induction hypothesis, $x^*(R+2)\leq x^*(R+1)$. But (29) implies $x^*(R+1)<x^*(R)$, which contradicts the assumption that $x^*(R+1)\geq x^*(R)$. Thus, it must be the case that $x^*(R+1)<x^*(R)$ for the case of $x^*(R)>0$.

Consider, now, the second case: $x^*(R)=0$. In this case, we want to establish that $x^*(R+1)\leq x^*(R)$, which could only hold if $x^*(R+1)=0$. So, suppose, to the contrary, that $x^*(R+1)>0$. Since $x^*(R)=0$, it follows from (15) that $V_F^*(R)=\frac{\left[e_0R^\theta-c\right]^2}{4(1-\beta_F)}$. Moreover, $V_F^*(R+1)=\frac{\left[e_0[R+1]^\theta-c\right]^2}{4(1-\beta_F)}+\frac{k\eta[x^*(R+1)]^2}{(1-\beta_F)}$. Thus, we get the following chain of implications:

$$\beta_F\eta\left[V_F^*(R+1)-V_F^*(R)\right]=\beta_F\eta\left\{\frac{\left[e_0\left[R+1\right]^\theta-c\right]^2}{4(1-\beta_F)}+\frac{k\eta\left[x^*(R+1)\right]^2}{(1-\beta_F)}-\frac{\left[e_0R^\theta-c\right]^2}{4(1-\beta_F)}\right\}$$

$$\geq\beta_F\eta\left\{\frac{\left[e_0\left[R+1\right]^\theta-c\right]^2}{4(1-\beta_F)}+\frac{k\eta\left[x^*(R+2)\right]^2}{(1-\beta_F)}-\frac{\left[e_0R^\theta-c\right]^2}{4(1-\beta_F)}\right\}$$

$$>\frac{\beta_F\eta}{1+\eta x^*(R+1)}\left\{\frac{\left[e_0\left[R+1\right]^\theta-c\right]^2}{4(1-\beta_F)}+\frac{k\eta\left[x^*(R+2)\right]^2}{(1-\beta_F)}-\frac{\left[e_0R^\theta-c\right]^2}{4(1-\beta_F)}\right\}$$

$$>\frac{\beta_F\eta}{1+\eta x^*(R+1)}\left\{\frac{\left[e_0\left[R+2\right]^\theta-c\right]^2}{4(1-\beta_F)}+\frac{k\eta\left[x^*(R+2)\right]^2}{(1-\beta_F)}-\frac{\left[e_0\left[R+1\right]^\theta-c\right]^2}{4(1-\beta_F)}\right\}$$

$$>\frac{\beta_F\eta}{1+\eta x^*(R+1)}\left\{\begin{array}{c}\frac{\left[e_0[R+2]^\theta-c\right]^2}{4(1-\beta_F)}+\frac{k\eta[x^*(R+2)]^2}{(1-\beta_F)}\\-\frac{\left[e_0[R+1]^\theta-c\right]^2}{4(1-\beta_F)}-\frac{k\eta[x^*(R+1)]^2}{(1-\beta_F)}\end{array}\right\}$$

$$= \frac{\beta_F \eta}{1+\eta x^*(R+1)} \left[V_F^*(R+2) - V_F^*(R+1)\right]$$
$$= k$$

The inequality in the second line follows from the induction hypothesis that $x^*(R+1) \geq x^*(R+2)$. The inequality in the third line follows because $\beta_F \eta > \frac{\beta_F \eta}{1+\eta x^*(R+1)}$, since $x^*(R+1) > 0$ by assumption. The inequality in the fourth line follows because $\left[e_0 R^\theta - c\right]^2$ is strictly concave in R. The inequality in the fifth line follows because $x^*(R+1) > 0$. The equality in the sixth line follows from (15), while the equality in the last line follows from the first-order condition for x in state $R+1$. But the implication of this chain of inequalities is that $\beta_F \eta \left[V_F^*(R+1) - V_F^*(R)\right] > k$, which contradicts the fact that when $x^*(R) = 0$, the Kuhn-Tucker condition would imply $\beta_F \eta \left[V_F^*(R+1) - V_F^*(R)\right] \leq k$. Summarizing, we have shown that if $x^*(R) > 0$, then $x^*(R) > x^*(R+1)$, and if $x^*(R) = 0$, then $x^*(R+1) = 0$. This is what we wanted to prove.■

Proof that the firm's value function is strictly concave in R

From the Kuhn-Tucker condition (20)

$$V_F^*(R+1) - V_F^*(R) \leq \frac{k\left(1+\eta x^*(R)\right)^2}{\beta_F \eta} \tag{30}$$

$$V_F^*(R) - V_F^*(R-1) \leq \frac{k\left(1+\eta x^*(R-1)\right)^2}{\beta_F \eta}. \tag{31}$$

There are three cases to consider. First, suppose $x^*(R-1)$ and $x^*(R)$ are both positive. Then, the above conditions hold with equality. We proved above that $x^*(R-1) > x^*(R)$, which immediately implies $V_F^*(R) - V_F^*(R-1) > V_F^*(R+1) - V_F^*(R)$.

Second, suppose that $x^*(R) = 0$, but $x^*(R-1) > 0$. Then, (31) holds with equality, while (30) holds with inequality. This implies $V_F^*(R) - V_F^*(R-1) = \frac{k(1+\eta x^*(R-1))^2}{\beta_F \eta} > \frac{k(1+\eta x^*(R))^2}{\beta_F \eta} \geq V_F^*(R+1) - V_F^*(R)$.

Third, suppose that $x^*(R-1) = x^*(R) = 0$. We have established that $x^*(\cdot)$ is non-decreasing, so it would follow that $x^*(R-1) = x^*(R) = x^*(R+1) = \ldots = x^*(\overline{R}) = 0$. In this case, then from condition (15), we have $V_F^*(R-1) = \frac{(e_0[R-1]^\theta - c)^2}{4(1-\beta_F)}$, $V_F^*(R) = \frac{(e_0 R^\theta - c)^2}{4(1-\beta_F)}$, and $V_F^*(R+1) = \frac{(e_0[R+1]^\theta - c)^2}{4(1-\beta_F)}$. Given our assumptions on θ, $(e_0 R^\theta - c)^2$ is a strictly concave function in R, so $V_F^*(R+1) - V_F^*(R) < V_F^*(R) - V_F^*(R-1)$ in this case as well.■

Proof of Proposition 2:

Suppose, to the contrary, that $x^*(R)$ is strictly increasing in R for all R. We will show that the solution to this problem is $z = d = 0$ in all states, which, in turn, will imply that $x^*(R)$ could not be strictly increasing in R. We begin by noting that if the activist sets $z = d = 0$ in all states, then (5) implies that the activist's value, denoted by $V_A^0(R)$, is given by the recursion:

$$V_A^0(R) = u(x^*(R)) + \beta_A \left[\phi_F(x^*(R)) V_A^0(R+1) + (1 - \phi_F(x^*(R)) V_A^0(R)\right], \tag{32}$$

At $R = \overline{R}$,

$$V_A^0(\overline{R}) = \frac{u(x^*(\overline{R}))}{1-\beta_A}, \tag{33}$$

since $V_A^0(\overline{R}+1) = V_A^0(\overline{R})$. Now, in state $\overline{R}-1$ the recursion in (32) is given by:

$$V_A^0(\overline{R}-1) = u(x^*(\overline{R}-1)) + \beta_A \left[\phi_F(x^*(\overline{R}-1)) V_A^0(\overline{R}) + (1 - \phi_F(x^*(\overline{R}-1)) V_A^0(\overline{R}-1)\right].$$

Rearranging terms gives us:

$$V_A^0(\overline{R}-1) = \frac{u(x^*(\overline{R}-1))}{\left[1-\beta_A(1-\phi_F(x^*(\overline{R}-1))\right]} + \frac{\beta_A \phi_F(x^*(\overline{R}-1)) V_A^0(\overline{R})}{\left[1-\beta_A(1-\phi_F(x^*(\overline{R}-1))\right]}.$$

Substituting (33) in place of $V_A^0(\overline{R})$ in the above expression, and rearranging terms, gives us:

$$V_A^0(\overline{R}-1) = \left(\frac{1}{1-\beta_A}\right)\left\{\tau_1^*(R) u(x^*(\overline{R}-1)) + (1-\tau_1^*(R)) u(x^*(\overline{R}))\right\}, \tag{34}$$

where $\tau_1^*(R) \equiv \frac{1-\beta_A}{1-\beta_A+\beta_A\phi_F(x^*(\overline{R}-1))} \in (0,1)$. Since $u(x)$ is non-decreasing and because we have assumed $x^*(R-1) < x^*(R)$, (34) implies $V_A^0(\overline{R}-1) \leq \frac{u(x^*(\overline{R}))}{1-\beta_A} = V_A^0(\overline{R})$.

Consider, now, the recursion for $V_A^0(\cdot)$ in state $\overline{R}-2$:

$$V_A^0(\overline{R}-2) = u(x^*(\overline{R}-2)) + \beta_A\left[\phi_F(x^*(\overline{R}-2)) V_A^0(\overline{R}-1) + (1-\phi_F(x^*(\overline{R}-2)) V_A^0(\overline{R}-2)\right].$$

Rearranging terms gives us

$$V_A^0(\overline{R}-2) = \left(\frac{1}{1-\beta_A}\right)\left\{\tau_2^*(R) u(x^*(\overline{R}-2)) + (1-\tau_2^*(R)) V_A^0(\overline{R}-1)\right\},$$

where $\tau_2^*(R) \equiv \frac{1-\beta_A}{1-\beta_A+\beta_A\phi_F(x^*(\overline{R}-2))}$. Substituting (34) into the above expression for $V_A^0(\overline{R}-2)$ yields

$$V_A^0(\overline{R}-2) = \left(\frac{1}{1-\beta_A}\right)\left[\begin{array}{c} \tau_2^*(R) u(x^*(\overline{R}-2)) \\ +(1-\tau_2^*(R))\left\{\begin{array}{c} \tau_1^*(R) u(x^*(\overline{R}-1)) \\ +(1+\tau_1^*(R)) u(x^*(\overline{R})) \end{array}\right\} \end{array}\right]. \tag{35}$$

Since $u(x^*(\overline{R}-2)) \leq u(x^*(\overline{R}-1)) \leq u(x^*(\overline{R}))$, (34) and (35) imply $V_A^0(\overline{R}-2) \leq V_A^0(\overline{R}-1)$. Reasoning inductively in this fashion for all R tells us that when $z = d = 0$ for all R,

$$V_A^0(1) \leq \ldots \leq V_A^0(\overline{R}).$$

Given this, along with equations (8) and (9), the activist's marginal benefit for z is non-positive for all $z > 0$, and the activist's marginal benefit for d is also non-positive for all $d > 0$. This implies that $z^*(R) = d^*(R) = 0$, for all R. Thus, if $x^*(R)$ is strictly increasing, the activist will not engage in criticism or confrontation in any state.

However, if the activist sets $z = d = 0$ in all states, the firm's maximization problem is solved by choosing the level of externality-reducing activity as in the no-activist case. By Proposition 1, we have seen that $x^*(R)$ in that case is non-increasing, which contradicts our assumption that $x^*(R)$ is monotone increasing in R.■

Proof of Lemma 1:

With $z(R) = 0$ and an exogenous perturbation $\phi_\Delta^n > 0$ in state R_n, the firm's optimization problem in state R_n can be written as

$$\begin{aligned} V_F(R_n) &= \max_{x \geq 0} \frac{(e_0 R_n^\theta - c)^2}{4} - kx + \beta_F(1-\phi_\Delta^n) V_F(R_n) \\ &\quad + \beta_F(1-\phi_\Delta^n)\phi_F(x)\left[V_F(R_n+1) - V_F(R_n)\right] + \beta_F \frac{\phi_\Delta^n}{R_n - 1}\sum_{r=1}^{R_n-1} V_F(r). \end{aligned} \tag{36}$$

The firm's optimization in a non-focal state $R \in \{1, \ldots, R_{n-1}, R_n + 1, \ldots, \overline{R} - 1\}$ is:

$$V_F(R) = \max_{x \geq 0} \frac{(e_0 R^\theta - c)^2}{4} - kx + \beta_F V_F(R) + \beta_F \phi_F(x) \left[V_F(R+1) - V_F(R)\right]. \tag{37}$$

The firm's optimization in state $\overline{R}$ is

$$V_F(\overline{R}) = \max_{x \geq 0} \frac{(e_0 \overline{R}^\theta - c)^2}{4} - kx + \beta_F V_F(\overline{R}). \tag{38}$$

The solution in state $\overline{R}$ is $x(\overline{R}) = 0$, and from this it follows that $V_F(\overline{R}) = \frac{(e_0 \overline{R}^\theta - c)^2}{4(1-\beta_F)}$, which is independent of ϕ_Δ^n. Thus, $\frac{\partial V_F(\overline{R})}{\partial \phi_\Delta^n} = 0$.[36]

Now consider the firm's optimization in a non-focal states $R > R_n$ and $R + 1 > R_n$. In both states, differentiate condition (37) with respect to ϕ_Δ^n, and evaluate at $\phi_\Delta^n = 0$. Utilizing the envelope theorem and rearranging terms gives us:

$$\left.\frac{\partial V_F(R)}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} = \frac{\beta_F \phi_F(x_0^*(R))}{1 - \beta_F} \left.\frac{\partial \left[V_F(R+1) - V_F(R)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0}$$

$$\left.\frac{\partial V_F(R+1)}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} = \frac{\beta_F \phi_F(x_0^*(R+1))}{1 - \beta_F} \left.\frac{\partial \left[V_F(R+2) - V_F(R+1)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0},$$

where $x_0^*(R)$ is the level of externality-reducing activity chosen by the firm when there is no activist. Subtracting these expressions and rearranging terms gives us:

$$\left.\frac{\partial \left[V_F(R+1) - V_F(R)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} = \frac{\beta_F \phi_F(x_0^*(R+1))}{1 - \left[1 - \phi_F(x_0^*(R))\right] \beta_F} \left.\frac{\partial \left[V_F(R+2) - V_F(R+1)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0}. \tag{39}$$

Evaluating this at $R = \overline{R} - 1$ gives us

$$\left.\frac{\partial \left[V_F(\overline{R}) - V_F(\overline{R}-1)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} = \frac{\beta_F \phi_F(x_0^*(\overline{R}))}{1 - \left[1 - \phi_F(x_0^*(\overline{R}-1))\right] \beta_F} \left.\frac{\partial \left[V_F(\overline{R}+1) - V_F(\overline{R})\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} = 0,$$

where the second equality follows from the fact that $x_0^*(\overline{R}) = 0$. Using (39), we can reason recursively and deduce that $\left.\frac{\partial [V_F(R+1) - V_F(R)]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} = 0$ for all $R > R_n$. This establishes part (a) of the lemma.

Now in the focal state R_n, differentiate (36) with respect to ϕ_Δ^n, utilize the envelope theorem, and evaluate at $\phi_\Delta^n = 0$ to get:

$$\left.\frac{\partial V_F(R_n)}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} = \beta_F \left\{ \begin{array}{c} -\left[V_{F0}^*(R_n) - \frac{\sum_{r=1}^{R_n - 1} V_{F0}^*(r)}{R_n - 1}\right] + \left.\frac{\partial V_F(R_n)}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} \\ -\phi_F(x_0^*(R_n)) \left[V_F(R_n+1) - V_F(R_n)\right] + \phi_F(x_0^*(R_n)) \left.\frac{\partial [V_F(R_n+1) - V_F(R_n)]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n = 0} \end{array} \right\} \tag{40}$$

[36] Throughout the proof, we suppress dependence of $V_F(\cdot)$ on ϕ_Δ^n where there is no ambiguity.

Now, do the same for state $R_n + 1$ and rearrange terms:

$$\left.\frac{\partial V_F(R_n+1)}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} = \frac{\beta_F \phi_F(x_0^*(R_n+1))}{1-\beta_F} \left.\frac{\partial\left[V_F(R_n+2)-V_F(R_n+1)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} = 0,$$

where the equality to zero follows from the earlier result that $\frac{\partial[V_F(R+1)-V_F(R)]}{\partial \phi_\Delta^n} = 0$ for $R > R_n$. Using this, and rearranging terms in (40) implies

$$\left.\frac{\partial V_F(R_n)}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} = -\frac{\beta_F\left[\phi_F(x_0^*(R_n))V_F(R_n+1) + \left[1-\phi_F(x_0^*(R_n)\right]V_{F0}^*(R_n) - \frac{\sum_{r=1}^{R_n-1} V_{F0}^*(r)}{R_n-1}\right]}{1-\beta_F[1-\phi_F(x_0^*(R_n)]} < 0,$$

since $\phi_F(x_0^*(R_n))V_F(R_n+1) + [1-\phi_F(x_0^*(R_n)]V_{F0}^*(R_n) - \frac{\sum_{r=1}^{R_n-1} V_{F0}^*(r)}{R_n-1} > 0$ from the result in Proposition 1 that $V_{F0}^*(R_n)$ is monotone increasing in R. Thus, $\left.\frac{\partial[V_F(R_n+1)-V_F(R_n)]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} > 0$, establishing part (b) of the lemma.

Finally, consider states $R < R_n$. Differentiating (37) with respect to ϕ_Δ^n in these states and using the envelope theorem gives us, as before,

$$\left.\frac{\partial\left[V_F(R+1)-V_F(R)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} = \frac{\beta_F \phi_F(x_0^*(R+1))}{1-\left[1-\phi_F(x_0^*(R))\right]\beta_F} \left.\frac{\partial\left[V_F(R+2)-V_F(R+1)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0}.$$

Evaluating this at $R = R_n - 1$ yields

$$\left.\frac{\partial\left[V_F(R_n)-V_F(R_n-1)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} = \frac{\beta_F \phi_F(x_0^*(R_n))}{1-\left[1-\phi_F(x_0^*(R_n-1))\right]\beta_F} \left.\frac{\partial\left[V_F(R_n+1)-V_F(R_n)\right]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} > 0,$$

since we have already established that $\left.\frac{\partial[V_F(R_n+1)-V_F(R_n)]}{\partial \phi_\Delta^n}\right|_{\phi_\Delta^n=0} > 0$. Recursively applying this in all states below R_n establishes part (c) of the lemma.■

Proof of Proposition 4: Note that if $\phi_\Delta^n = 1$, then $MB_x(x, \mathbf{V}_F(\mathbf{R}|\phi_\Delta^n),\phi_\Delta^n|R_n) < 0$. This implies $x^*(R_n) = 0$. Since, by assumption, $x_0^*(R) > 0$, the perturbation unambiguously reduces the firm's choice of x in this state.■

Proof of Proposition 6: This result follows directly from the data in Property 2 of Table 2.■

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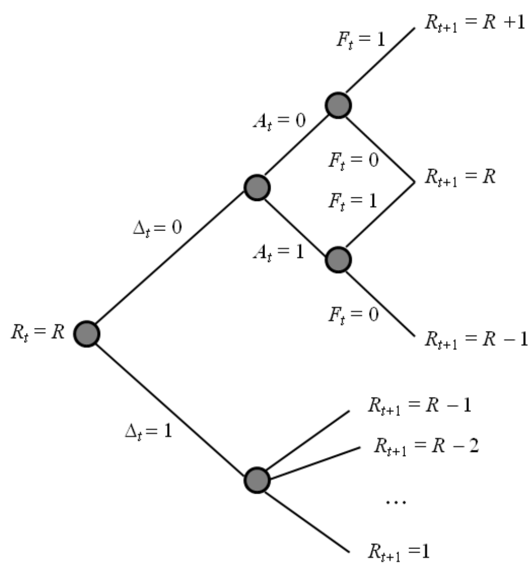

Figure 1: Stochastic process for $\widetilde{R}$

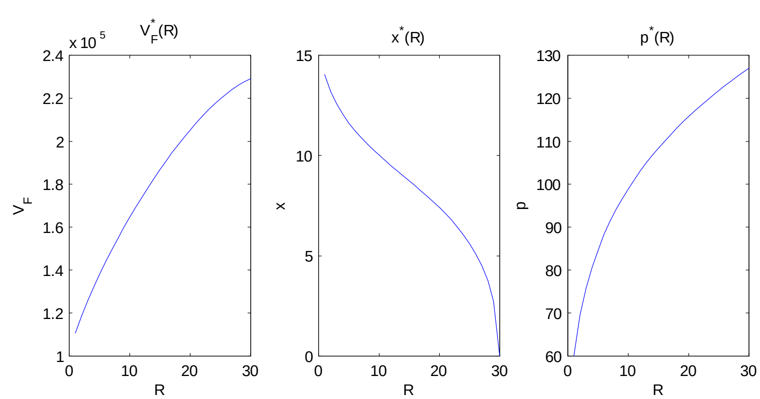

Figure 2: No activist equilibrium: baseline parameter values. The three figures show the equilibrium value function of the firm, its externality-reducing activity, and product price as functions of firm's reputation when there is no activist.

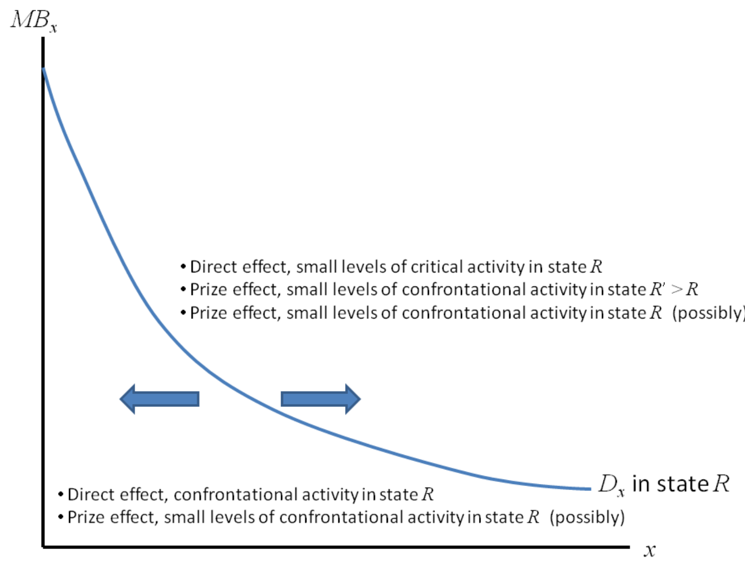

Figure 3: How the effects of activist behavior can shift the firm's "demand curve" for externality-reducing investment.

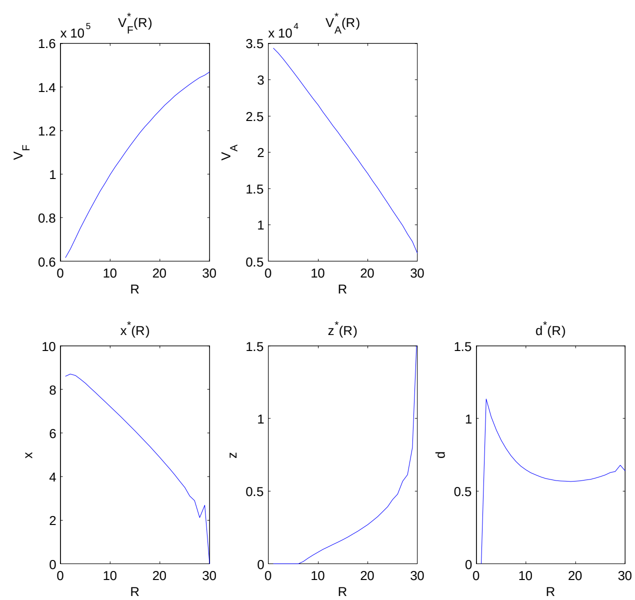

Figure 4: Equilibrium policy and value functions with an activist: baseline parameters. These graphs shows equilibrium values and policies as functions of firm's reputation. The top figures show the equilibrium value functions of the firm and the activist. The bottom three figures show the externality-reducing activity of the firm, and the activist's choice of criticism and confrontation.

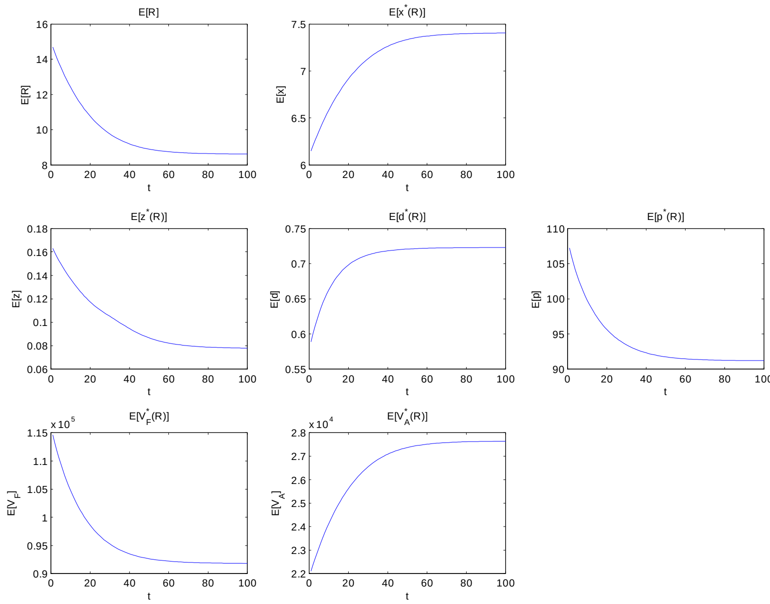

Figure 5: Equilibrium dynamics with an activist: baseline parameters. These figures plot the expected reputation, policies, price, and values, where expectations are taken with respect to the time t distribution.

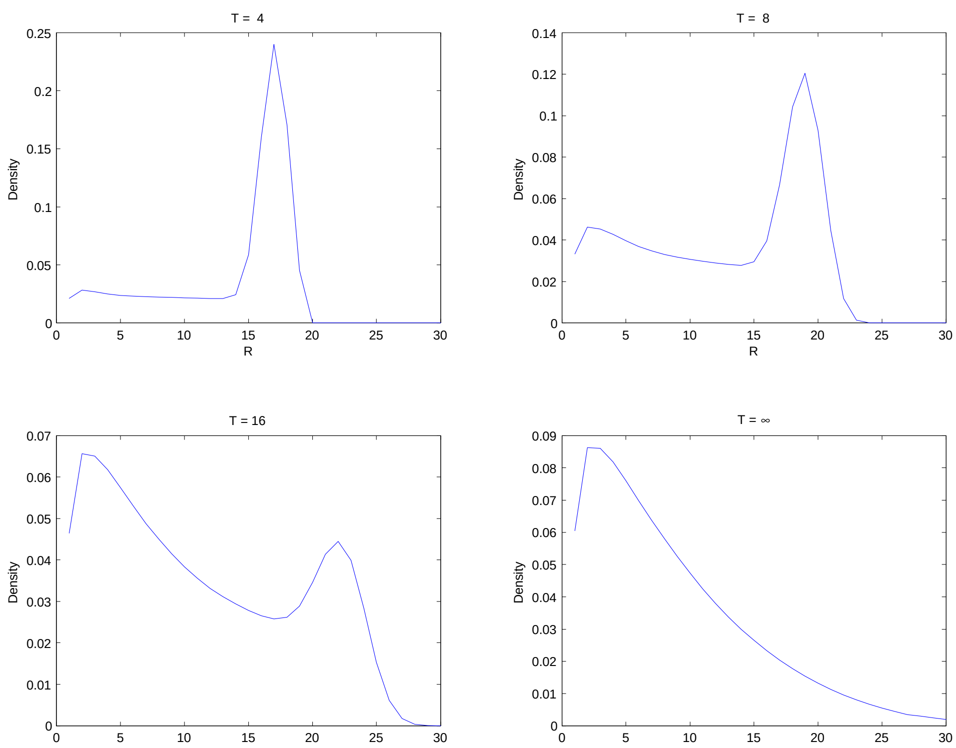

Figure 6: Transient distributions over the firm's reputation R: baseline parameters. These figures show the distribution of reputation as the system evolves.

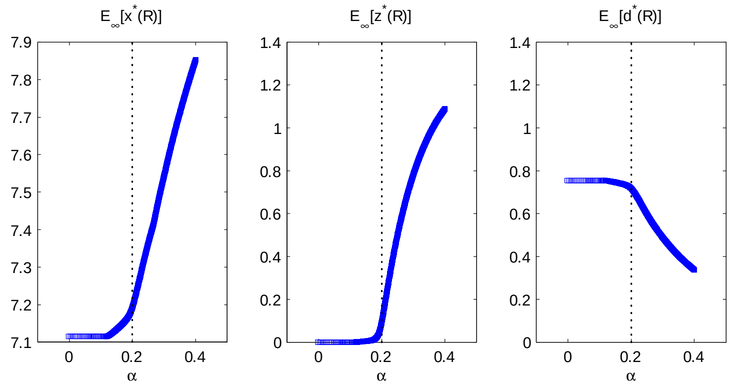

Figure 7: How $\mathbb{E}_\infty[x^*(R)]$, $\mathbb{E}_\infty[z^*(R)]$, and $\mathbb{E}_\infty[d^*(R)]$ vary with α. Figures show how expected equilibrium policies (at the limiting distribution) vary with salience of activist criticism. The dotted line indicates the baseline parameter value.

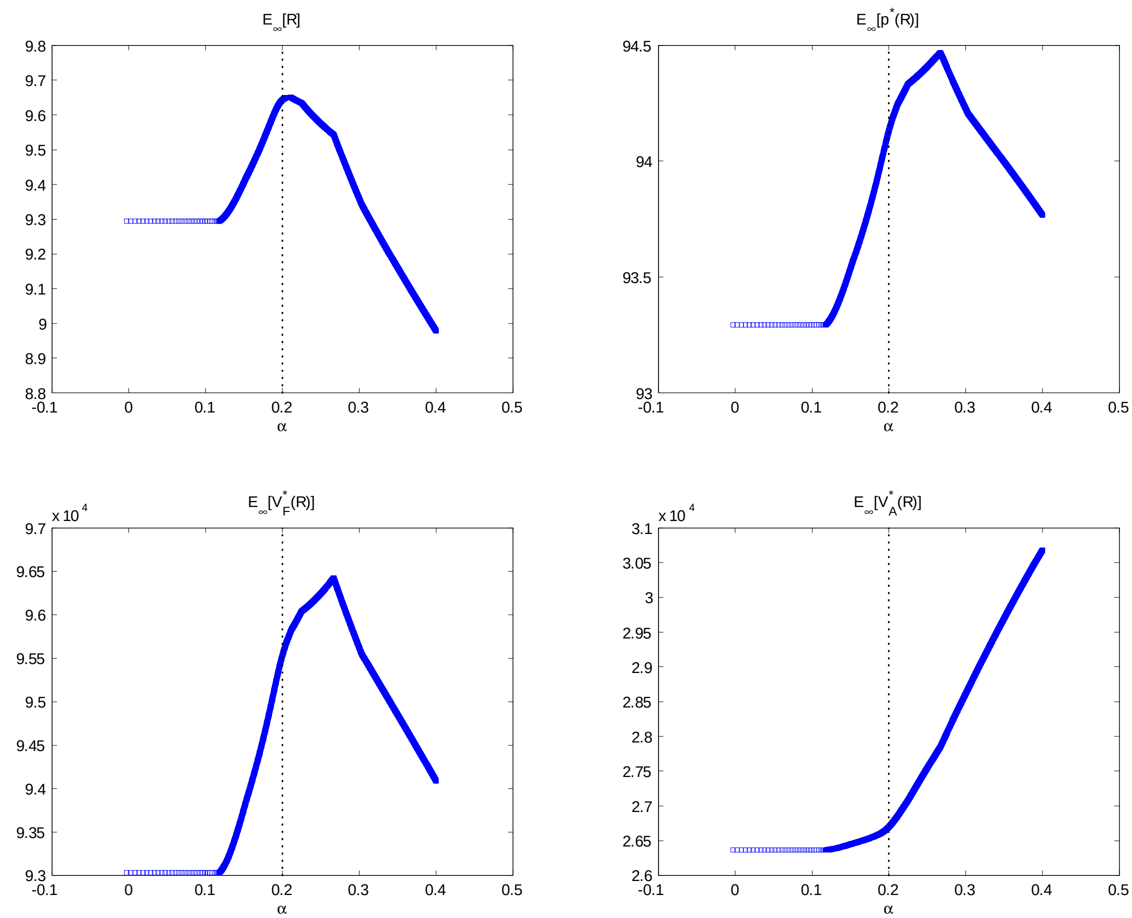

Figure 8: How $\mathbb{E}_\infty[R)]$, $\mathbb{E}_\infty[p^*(R)]$, $\mathbb{E}_\infty[V_F^*(R)]$, and $\mathbb{E}_\infty[V_F^*(R)]$ vary with α. Figures show how expected equilibrium reputation, prices and values (at the limiting distribution) vary with salience of activist criticism. The dotted line indicates the baseline parameter value.

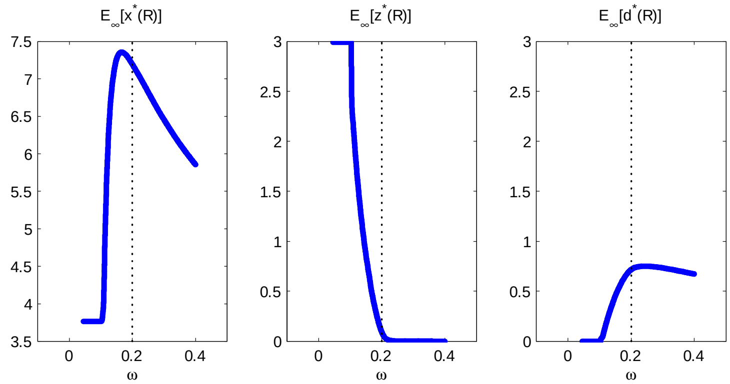

Figure 9: How $\mathbb{E}_\infty[x^*(R)]$, $\mathbb{E}_\infty[z^*(R)]$, and $\mathbb{E}_\infty[d^*(R)]$ vary with ω. Figures show how expected equilibrium policies (at the limiting distribution) vary with newsworthiness of activist confrontation. The dotted line indicates the baseline parameter value.

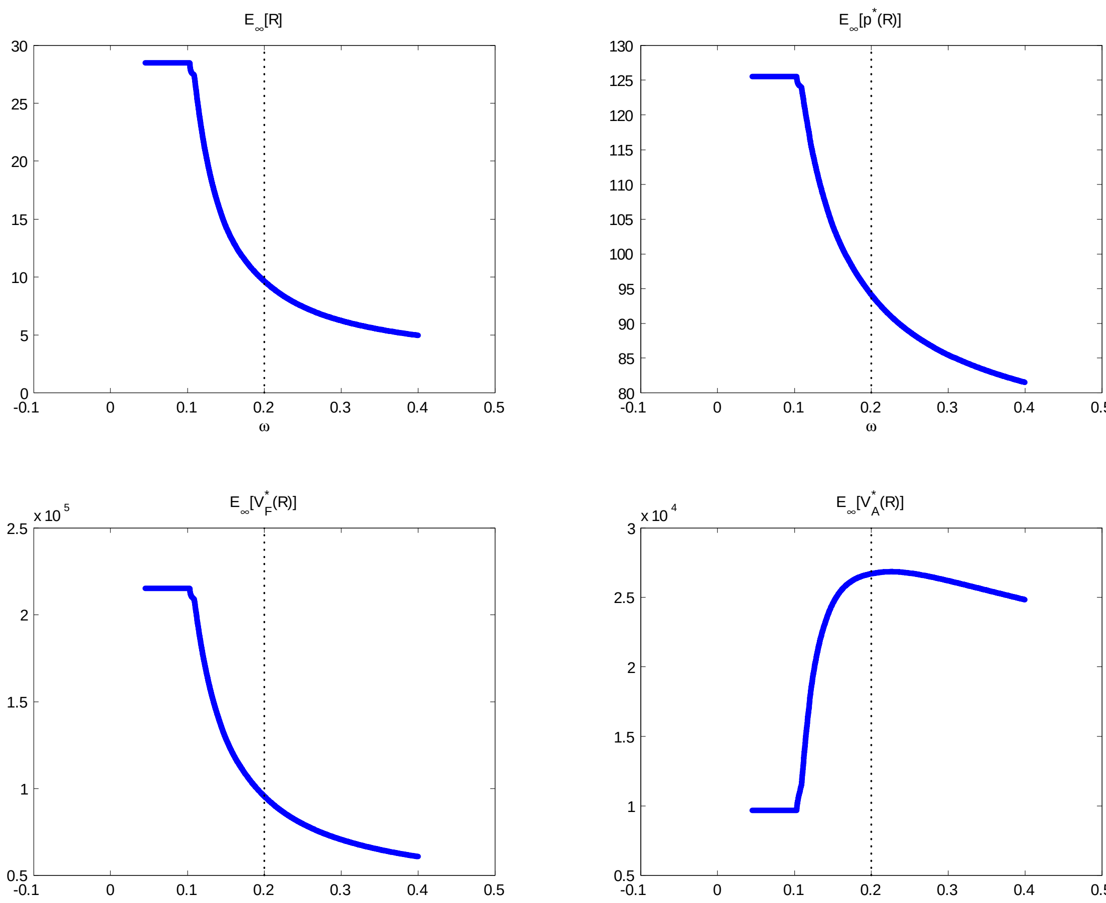

Figure 10: How $\mathbb{E}_\infty[R)]$, $\mathbb{E}_\infty[p^*(R)]$, $\mathbb{E}_\infty[V_F^*(R)]$, and $\mathbb{E}_\infty[V_F^*(R)]$ vary with ω. Figures show how expected equilibrium reputation, prices and values (at the limiting distribution) vary with newsworthiness of activist confrontation. The dotted line indicates the baseline parameter value.

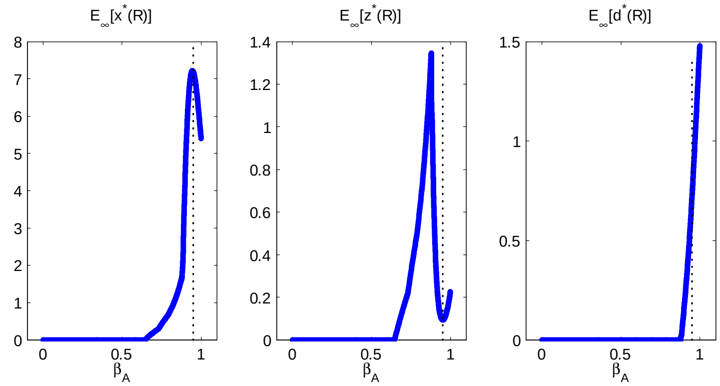

Figure 11: How $\mathbb{E}_\infty[x^*(R)]$, $\mathbb{E}_\infty[z^*(R)]$, and $\mathbb{E}_\infty[d^*(R)]$ vary with β_A. Figures show how expected equilibrium policies (at the limiting distribution) vary with the activist's discount factor. The dotted line indicates the baseline parameter value.

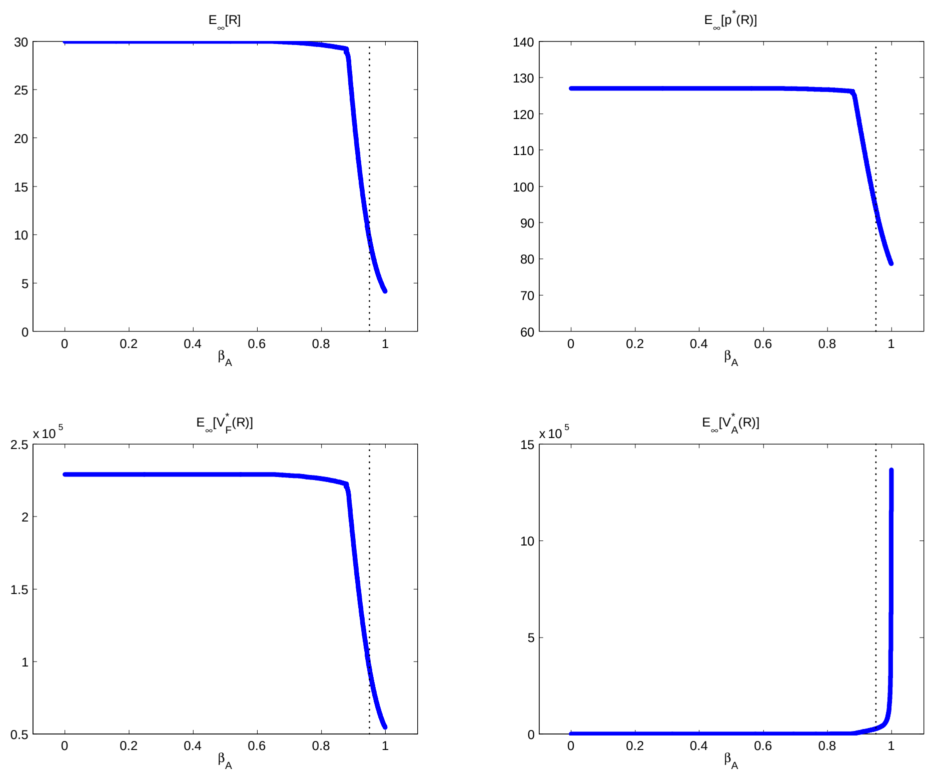

Figure 12: How $\mathbb{E}_\infty[R)]$, $\mathbb{E}_\infty[p^*(R)]$, $\mathbb{E}_\infty[V_F^*(R)]$, and $\mathbb{E}_\infty[V_F^*(R)]$ vary with β_A. Figures show how expected equilibrium reputation, prices and values (at the limiting distribution) vary with the activist's discount factor. The dotted line indicates the baseline parameter value.

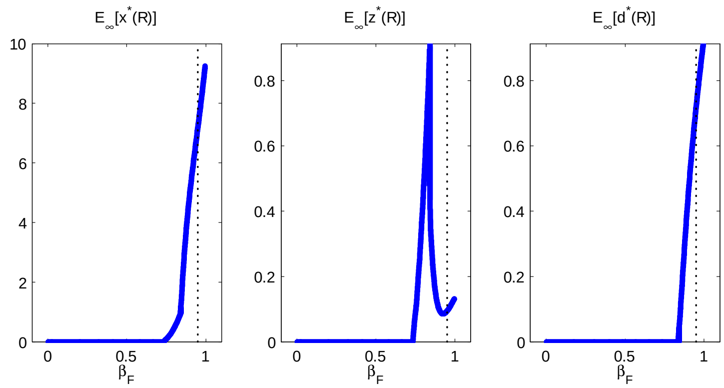

Figure 13: How $\mathbb{E}_\infty[x^*(R)]$, $\mathbb{E}_\infty[z^*(R)]$, and $\mathbb{E}_\infty[d^*(R)]$ vary with β_F. Figures show how expected equilibrium policies (at the limiting distribution) vary with the firm's discount factor. The dotted line indicates the baseline parameter value.

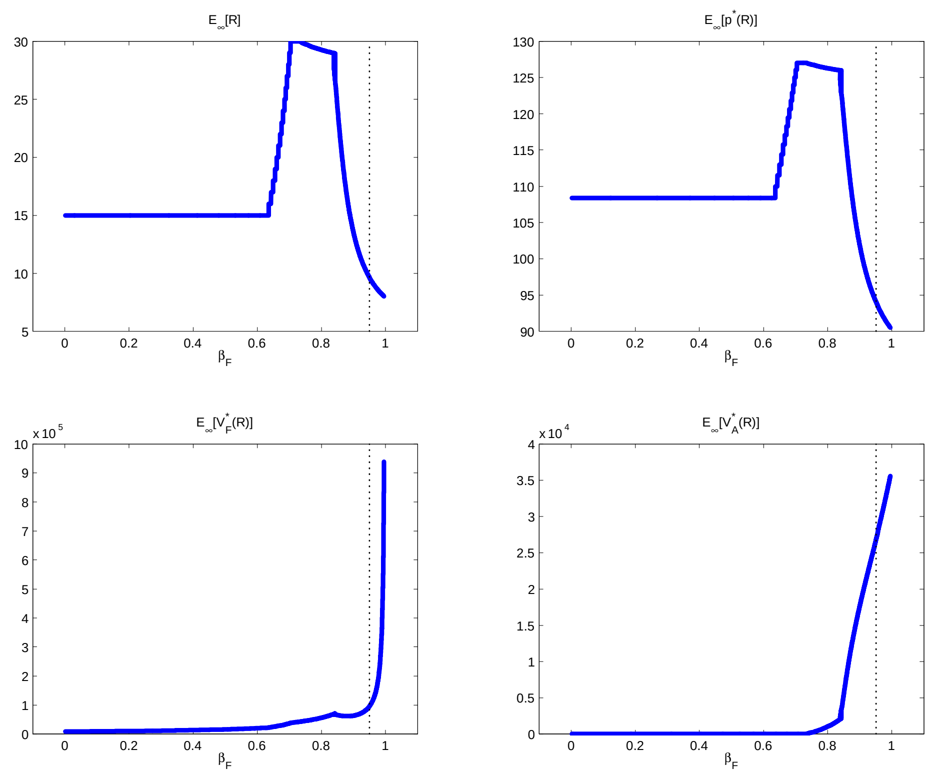

Figure 14: How $\mathbb{E}_\infty[R)]$, $\mathbb{E}_\infty[p^*(R)]$, $\mathbb{E}_\infty[V_F^*(R)]$, and $\mathbb{E}_\infty[V_F^*(R)]$ vary with β_F. Figures show how expected equilibrium reputation, prices and values (at the limiting distribution) vary with the firm's discount factor. The dotted line indicates the baseline parameter value.

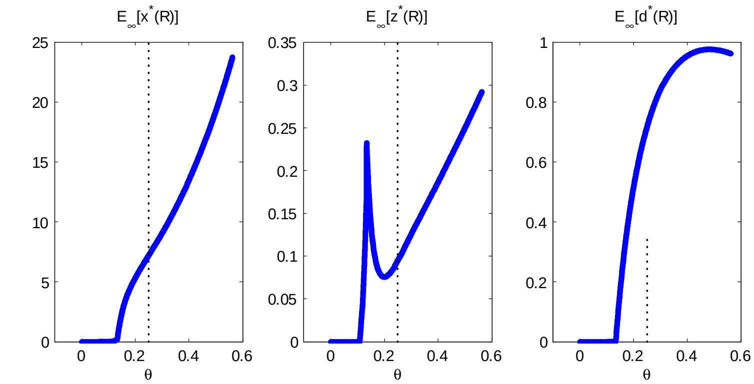

Figure 15: How $\mathbb{E}_\infty[x^*(R)]$, $\mathbb{E}_\infty[z^*(R)]$, and $\mathbb{E}_\infty[d^*(R)]$ vary with θ. Figures show how expected equilibrium policies (at the limiting distribution) vary with the elasticity of brand equity with respect to reputation. The dotted line indicates the baseline parameter value.

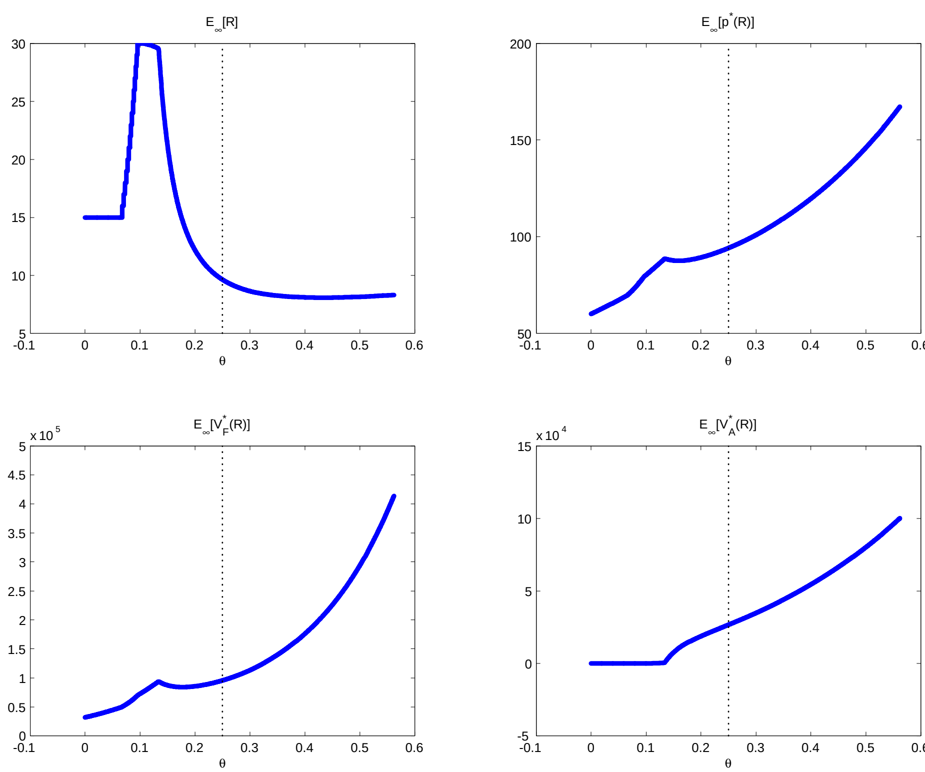

Figure 16: How $\mathbb{E}_\infty[R)]$, $\mathbb{E}_\infty[p^*(R)]$, $\mathbb{E}_\infty[V_F^*(R)]$, and $\mathbb{E}_\infty[V_F^*(R)]$ vary with θ. Figures show how expected equilibrium reputation, prices and values (at the limiting distribution) vary with the elasticity of brand equity with respect to reputation. The dotted line indicates the baseline parameter value.

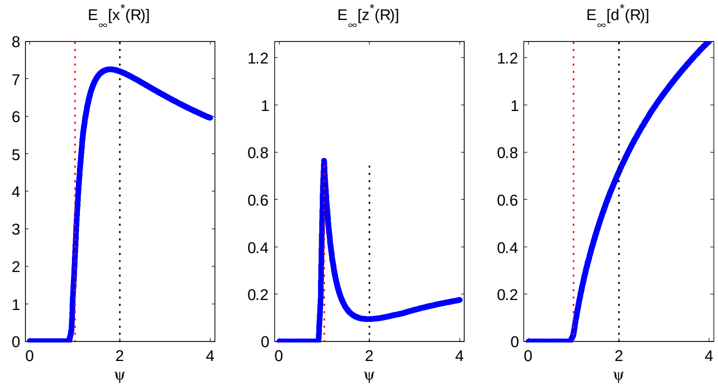

Figure 17: How $\mathbb{E}_\infty[x^*(R)]$, $\mathbb{E}_\infty[z^*(R)]$, and $\mathbb{E}_\infty[d^*(R)]$ vary with ψ. Figures show how expected equilibrium policies (at the limiting distribution) vary with the activist's passion. The dotted line at $\psi = 2$ indicates the baseline parameter value, while the value $\psi = 1$ represents perfect alignment of the acitvist's passion with social welfare.

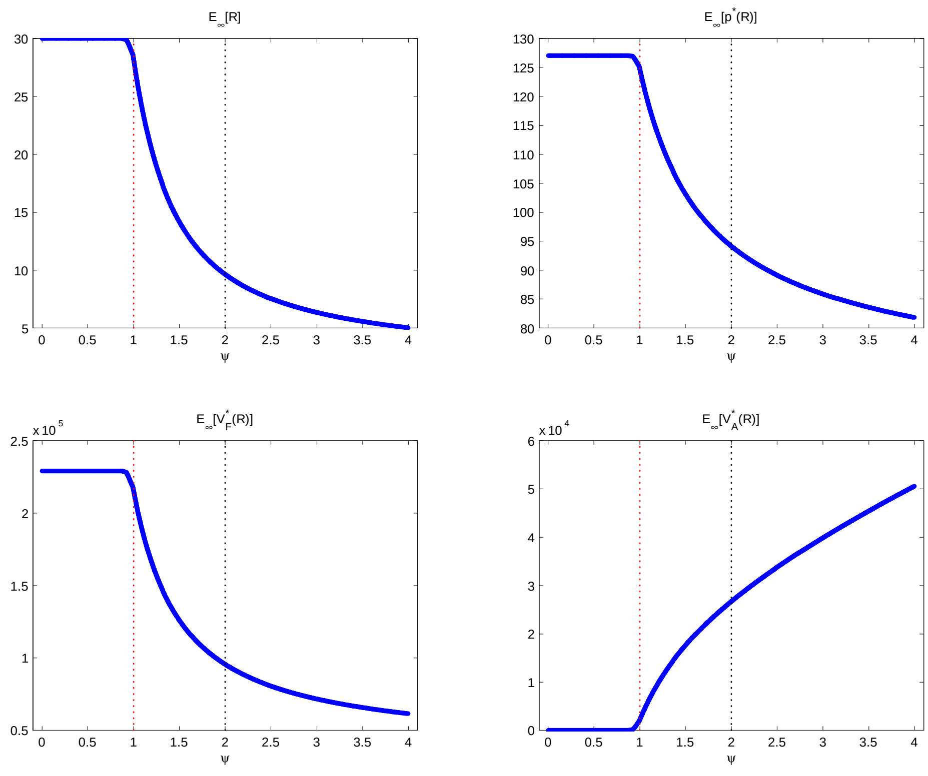

Figure 18: How $\mathbb{E}_\infty[R)]$, $\mathbb{E}_\infty[p^*(R)]$, $\mathbb{E}_\infty[V_F^*(R)]$, and $\mathbb{E}_\infty[V_F^*(R)]$ vary with ψ. Figures show how expected equilibrium reputation, prices and values (at the limiting distribution) vary with the activist's passion. The dotted line at $\psi = 2$ indicates the baseline parameter value, while the value $\psi = 1$ represents perfect alignment of the acitvist's passion with social welfare.