How Efficient is Dynamic Competition? The Case of Price as Investment
— Online Appendix —

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The Online Appendix contains additional results and technical details pertaining to the indicated sections of the main paper.

OA1 Model

We establish that our model is equivalent to a model that switches the order of the price-setting and exit-entry phases. During the exit-entry phase, the state changes from $\mathbf{e}$ to $\mathbf{e}'$; during the price-setting phase, the state changes from $\mathbf{e}'$ to $\mathbf{e}''$. Discounting occurs after the price-setting phase. The exit-entry phase determines the value function $\widehat{\mathbf{U}}_n$ along with the policy function $\widehat{\boldsymbol{\phi}}_n$ with typical element $\widehat{U}_n(\mathbf{e}')$, respectively, $\widehat{\phi}_n(\mathbf{e}')$; the price-setting phase determines the value function $\widehat{\mathbf{V}}_n$ along with the policy function $\widehat{\mathbf{p}}_n$ with typical element $\widehat{V}_n(\mathbf{e})$, respectively, $\widehat{p}_n(\mathbf{e})$.

We work backwards from the price-setting phase to the exit-entry phase.

Pricing decision of incumbent firm. We focus on firm 1. In the price-setting phase, the expected NPV of incumbent firm 1 is

$$\widehat{V}_1(\mathbf{e}') = \max_{p_1} D_1(p_1, \widehat{p}_2(\mathbf{e}'))(p_1 - c(e'_1)) + \beta \widehat{U}_1(\mathbf{e}') + \sum_{n=1}^{2} D_n(p_1, \widehat{p}_2(\mathbf{e}'))\beta \left[\widehat{U}_1\left(\mathbf{e}'^{n+}\right) - \widehat{U}_1(\mathbf{e}')\right], \tag{OA1}$$

where $\mathbf{e}'^{1+} = (\min\{e'_1 + 1, M\}, e'_2)$ and $\mathbf{e}'^{2+} = (e'_1, \min\{e'_2 + 1, M\})$. The pricing decision $\widehat{p}_1(\mathbf{e}')$ is uniquely determined by the first-order condition

$$\widehat{p}_1(\mathbf{e}') - \frac{\sigma}{1 - D_1(\widehat{\mathbf{p}}(\mathbf{e}'))} - c(e'_1) + \beta \left[\widehat{U}_1\left(\mathbf{e}'^{1+}\right) - \widehat{U}_1(\mathbf{e}')\right] + \Upsilon(\widehat{p}_2(\mathbf{e}'))\beta \left[\widehat{U}_1(\mathbf{e}') - \widehat{U}_1\left(\mathbf{e}'^{2+}\right)\right] = 0. \tag{OA2}$$

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Exit decision of incumbent firm. In the exit-entry phase, if incumbent firm 1 exits the industry, it receives the scrap value X_1 in the current period and perishes. If it does not exit, its expected NPV is

$$\widehat{\widehat{X}}_1(\mathbf{e}) = \left[\widehat{V}_1(\mathbf{e})(1-\widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(e_1,0)\widehat{\phi}_2(\mathbf{e})\right].$$

The probability of incumbent firm 1 exiting the industry in state $\mathbf{e}$ is therefore $\widehat{\phi}_1(\mathbf{e}) = 1 - F_X(\widehat{\widehat{X}}_1(\mathbf{e}))$ and the expected NPV of incumbent firm 1 in the exit-entry phase is given by the Bellman equation

$$\widehat{U}_1(\mathbf{e}) = (1-\widehat{\phi}_1(\mathbf{e}))\left[\widehat{V}_1(\mathbf{e})(1-\widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(e_1,0)\widehat{\phi}_2(\mathbf{e})\right] + \widehat{\phi}_1(\mathbf{e})E_X\left[X_1|X_1 \geq \widehat{\widehat{X}}_1(\mathbf{e})\right]. \tag{OA3}$$

Entry decision of potential entrant. If a potential entrant does not enter, it perishes. If it enters, it becomes an incumbent firm without prior experience in the subsequent period. Hence, upon entry, the expected NPV of potential entrant 1 is

$$\widehat{\widehat{S}}_1(\mathbf{e}) = \left[\widehat{V}_1(1,e_2)(1-\widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(1,0)\widehat{\phi}_2(\mathbf{e})\right].$$

The probability of potential entrant 1 *not* entering the industry in state $\mathbf{e}$ is therefore $\widehat{\phi}_1(\mathbf{e}) = 1 - F_S(\widehat{\widehat{S}}_1(\mathbf{e}))$ and the expected NPV of potential entrant 1 in the exit-entry phase is given by the Bellman equation

$$\widehat{U}_1(\mathbf{e}) = (1-\widehat{\phi}_1(\mathbf{e}))\Big\{[\widehat{V}_1(1,e_2)(1-\widehat{\phi}_2(\mathbf{e})) + \widehat{V}_1(1,0)\widehat{\phi}_2(\mathbf{e})] - E_S\left[S_1|S_1 \leq \widehat{\widehat{S}}_1(\mathbf{e})\right]\Big\}. \tag{OA4}$$

Equivalence. Let $\mathbf{V}_1$, $\mathbf{U}_1$, $\mathbf{p}_1$, and $\boldsymbol{\phi}_1$ solve equations (2), (3), (4), and (5) in the main paper. Define

$$\begin{aligned}
\widehat{\mathbf{V}}_1 &= \beta\mathbf{V}_1,\\
\widehat{\mathbf{U}}_1 &= \frac{1}{\beta}\mathbf{U}_1,\\
\widehat{\mathbf{p}}_1 &= \mathbf{p}_1,\\
\widehat{\boldsymbol{\phi}}_1 &= \boldsymbol{\phi}_1.
\end{aligned}$$

It is straightforward to verify that $\widehat{\mathbf{V}}_1$, $\widehat{\mathbf{U}}_1$, $\widehat{\mathbf{p}}_1$, and $\widehat{\boldsymbol{\phi}}_1$ solve equations (OA1), (OA2), (OA3), and (OA4). This establishes the equivalence between the models.

OA2 Is dynamic competition necessarily fully efficient?

We provide further details on the first-best planner solution and the equilibria mentioned thereafter in Section 4 of the main paper. In addition to Assumptions 1 and 2, we throughout maintain $\kappa > 0$, $\rho \in [0,1]$, and $\beta \in [0,1)$.

$\mathbf{e}$	$p_1^{FB}(\mathbf{e})$	$p_2^{FB}(\mathbf{e})$	$\psi_{0,0}^{FB}(\mathbf{e})$	$\psi_{1,0}^{FB}(\mathbf{e})$	$\psi_{0,1}^{FB}(\mathbf{e})$	$\psi_{1,1}^{FB}(\mathbf{e})$	$V^{FB}(\mathbf{e})$	$U^{FB}(\mathbf{e})$
$(0,0)$	∞	∞	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$v - p_0 + \beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) - \overline{S}$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) - \overline{S}$
$(0,1)$	∞	p_0^-	0	0	1	0	$v - \kappa + \frac{\beta}{1-\beta}(v-\rho\kappa)$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa)$
$(0,2)$	∞	p_0^-	0	0	1	0	$\frac{1}{1-\beta}(v-\rho\kappa)$	$\frac{\beta}{1-\beta}(v-\rho\kappa)$
$(1,0)$	p_0^-	∞	0	1	0	0	$v - \kappa + \frac{\beta}{1-\beta}(v-\rho\kappa)$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa)$
$(1,1)$	p_0^-	p_0^-	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$v - \kappa + \frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) + \overline{X}$
$(1,2)$	p_0	p_0^-	0	0	1	0	$\frac{1}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$
$(2,0)$	p_0^-	∞	0	1	0	0	$\frac{1}{1-\beta}(v-\rho\kappa)$	$\frac{\beta}{1-\beta}(v-\rho\kappa)$
$(2,1)$	p_0^-	p_0	0	1	0	0	$\frac{1}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$
$(2,2)$	p_0^-	p_0^-	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{1-\beta}(v-\rho\kappa) + \overline{X}$	$\frac{\beta}{1-\beta}(v-\rho\kappa) + \overline{X}$

Table OA1: First-best planner solution. Two-step learning curve. In columns labelled $p_n^{FB}(\mathbf{e})$, superscript $-$ indicates that firm n charges just below the price stated.

First-best planner solution. Table OA1 shows the first-best planner solution. Note that while there exist asymmetric solutions, we focus on the symmetric solution. In particular, we set $\psi_{1,0}^{FB}(\mathbf{e}) = \psi_{0,1}^{FB}(\mathbf{e}) = \frac{1}{2}$ in state $\mathbf{e} = (e, e)$. Note also that while we arbitrarily set $p_n(\mathbf{e}) = p_0^-$ in state $\mathbf{e} \geq (0, 0)$, firm n may charge any price below p_0 in accordance with the main paper.

The proof is similar to that of Proposition 1. First, we show that given the policy functions, the value functions solve the Bellman equations (8) and (9) in the main paper. Second, we show that there is no profitable one-shot deviation in any state of the industry.

Plugging in the policy functions, the Bellman equations (8) and (9) in the main paper become:

$$U^{FB}(0,0) = -\overline{S} + \beta\left(\frac{1}{2}V^{FB}(0,1) + \frac{1}{2}V^{FB}(1,0)\right),$$
$$U^{FB}(0,1) = \beta V^{FB}(0,1),$$
$$U^{FB}(0,2) = \beta V^{FB}(0,2),$$
$$U^{FB}(1,0) = \beta V^{FB}(1,0),$$
$$U^{FB}(1,1) = \overline{X} + \beta\left(\frac{1}{2}V^{FB}(0,1) + \frac{1}{2}V^{FB}(1,0)\right),$$
$$U^{FB}(1,2) = \overline{X} + \beta V^{FB}(0,2),$$
$$U^{FB}(2,0) = \beta V^{FB}(2,0),$$
$$U^{FB}(2,1) = \overline{X} + \beta V^{FB}(2,0),$$
$$U^{FB}(2,2) = \overline{X} + \beta\left(\frac{1}{2}V^{FB}(0,2) + \frac{1}{2}V^{FB}(2,0)\right),$$
$$V^{FB}(0,0) = v - p_0 + U^{FB}(0,0),$$
$$V^{FB}(0,1) = v - \kappa + U^{FB}(0,2),$$
$$V^{FB}(0,2) = v - \rho\kappa + U^{FB}(0,2),$$
$$V^{FB}(1,0) = v - \kappa + U^{FB}(2,0),$$
$$V^{FB}(1,1) = v - \kappa + \frac{1}{2}U^{FB}(1,2) + \frac{1}{2}U^{FB}(2,1),$$
$$V^{FB}(1,2) = v - \rho\kappa + U^{FB}(1,2),$$
$$V^{FB}(2,0) = v - \rho\kappa + U^{FB}(2,0),$$
$$V^{FB}(2,1) = v - \rho\kappa + U^{FB}(2,1),$$
$$V^{FB}(2,2) = v - \rho\kappa + U^{FB}(2,2).$$

It is easy but tedious to show that the value functions solve the Bellman equations.

We proceed state-by-state to show that there is no profitable one-shot deviation. It suffices to consider deviations in pure strategies.

1. Exit-entry phase in state $\mathbf{e} = (0,0)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e}) = 1$ yields $\beta V^{FB}(\mathbf{e}) < U^{FB}(\mathbf{e})$ by part (iii) of Assumption 2 because

$$\beta\left(v - p_0 + \beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v - \rho\kappa) - \overline{S}\right) < \beta(v-\kappa) + \frac{\beta^2}{1-\beta}(v-\rho\kappa) - \overline{S}$$

$$\Leftrightarrow (1-\beta)\overline{S} < (1-\beta)\beta(p_0-\kappa) + \beta^2(p_0-\rho\kappa)$$
$$\Leftrightarrow \overline{S} < \beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right).$$

Deviating to $\psi_{1,0}^{FB}(\mathbf{e})=1$ yields $-\overline{S}+\beta V^{FB}(1,0)=U^{FB}(\mathbf{e})$. Deviating to $\psi_{0,1}^{FB}(\mathbf{e})=1$ yields $-\overline{S}+\beta V^{FB}(0,1)=U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e})=1$ yields $-2\overline{S}+\beta V^{FB}(1,1)<U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2 because

$$-2\overline{S}+\beta\left(v-\kappa+\frac{\beta}{1-\beta}(v-\rho\kappa)+\overline{X}\right) < \beta(v-\kappa)+\frac{\beta^2}{1-\beta}(v-\rho\kappa)-\overline{S}$$
$$\Leftrightarrow \beta\overline{X} < \overline{S}.$$

2. Exit-entry phase in state $\mathbf{e}=(0,1)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e})=1$ yields $\overline{X}+\beta V^{FB}(0,0)<U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e})=1$ yields $\overline{X}-\overline{S}+\beta V^{FB}(1,0)<U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2. Deviating to $\psi_{1,1}^{FB}(\mathbf{e})=1$ yields $-\overline{S}+\beta V^{FB}(1,1)<U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2.

3. Exit-entry phase in state $\mathbf{e}=(0,2)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e})=1$ yields $\overline{X}+\beta V^{FB}(0,0)<U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e})=1$ yields $\overline{X}-\overline{S}+\beta V^{FB}(1,0)<U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2. Deviating to $\psi_{1,1}^{FB}(\mathbf{e})=1$ yields $-\overline{S}+\beta V^{FB}(1,2)<U^{FB}(\mathbf{e})$ by part (ii) of Assumption 2.

4. Exit-entry phase in state $\mathbf{e}=(1,0)$: Analogous to exit-entry phase in state $\mathbf{e}=(0,1)$.

5. Exit-entry phase in state $\mathbf{e}=(1,1)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e})=1$ yields $2\overline{X}+\beta V^{FB}(0,0)<U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e})=1$ yields $\overline{X}+\beta V^{FB}(1,0)=U^{FB}(\mathbf{e})$. Deviating to $\psi_{0,1}^{FB}(\mathbf{e})=1$ yields $\overline{X}+\beta V^{FB}(0,1)=U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e})=1$ yields $\beta V^{FB}(\mathbf{e})<U^{FB}(\mathbf{e})$.

6. Exit-entry phase in state $\mathbf{e}=(1,2)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e})=1$ yields $2\overline{X}+\beta V^{FB}(0,0)<U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e})=1$ yields $\overline{X}+\beta V^{FB}(1,0)<U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e})=1$ yields $\beta V^{FB}(1,2)<U^{FB}(\mathbf{e})$.

7. Exit-entry phase in state $\mathbf{e}=(2,0)$: Analogous to exit-entry phase in state $\mathbf{e}=(0,2)$.

8. Exit-entry phase in state $\mathbf{e}=(2,1)$: Analogous to exit-entry phase in state $\mathbf{e}=(1,2)$.

9. Exit-entry phase in state $\mathbf{e}=(2,2)$: Deviating to $\psi_{0,0}^{FB}(\mathbf{e})=1$ yields $2\overline{X}+\beta V^{FB}(0,0)<U^{FB}(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2. Deviating to $\psi_{1,0}^{FB}(\mathbf{e})=1$ yields $\overline{X}+\beta V^{FB}(2,0)=U^{FB}(\mathbf{e})$. Deviating to $\psi_{0,1}^{FB}(\mathbf{e})=1$ yields $\overline{X}+\beta V^{FB}(0,2)=U^{FB}(\mathbf{e})$. Deviating to $\psi_{1,1}^{FB}(\mathbf{e})=1$ yields $\beta V^{FB}(\mathbf{e})<U^{FB}(\mathbf{e})$.

10. Price-setting phase in state $\mathbf{e}=(0,0)$: By default.

11. Price-setting phase in state $\mathbf{e} = (0,1)$: Deviating to firm 2 matching the outside good ($p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \kappa) + \frac{1}{2}U^{FB}(\mathbf{e}) + \frac{1}{2}U^{FB}(0,2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 2 being undercut by the outside good ($p_2^{FB}(\mathbf{e}) > p_0$) yields $v - p_0 + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

12. Price-setting phase in state $\mathbf{e} = (0,2)$: Deviating to firm 2 matching the outside good ($p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 2 being undercut by the outside good ($p_2^{FB}(\mathbf{e}) > p_0$) yields $v - p_0 + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

13. Price-setting phase in state $\mathbf{e} = (1,0)$: Analogous to price-setting phase in state $\mathbf{e} = (0,1)$.

14. Price-setting phase in state $\mathbf{e} = (1,1)$: Deviating to firm 1, say, matching the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \kappa + U^{FB}(1,2) = V^{FB}(\mathbf{e})$. Deviating to firm 1, say, being undercut by the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \kappa + U^{FB}(1,2) = V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{3}(v - p_0) + \frac{2}{3}(v - \kappa) + \frac{1}{3}U^{FB}(\mathbf{e}) + \frac{1}{3}U^{FB}(2,1) + \frac{1}{3}U^{FB}(1,2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1, say, being undercut by the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \kappa) + \frac{1}{2}U^{FB}(\mathbf{e}) + \frac{1}{2}U^{FB}(1,2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $(v - p_0) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

15. Price-setting phase in state $\mathbf{e} = (1,2)$: Deviating to firm 1 undercutting the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) = p_0^-$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields

$$\frac{1}{2}(v - \kappa) + \frac{1}{2}(v - \rho\kappa) + \frac{1}{2}U^{FB}(2,2) + \frac{1}{2}U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e}).$$

Deviating to firm 1 being undercut by the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \rho\kappa + U^{FB}(\mathbf{e}) = V^{FB}(\mathbf{e})$. Deviating to firm 1 undercutting the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0^-$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields $v - \kappa + U^{FB}(2,2) \leq V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{3}(v - p_0) + \frac{1}{3}(v - \kappa) + \frac{1}{3}(v - \rho\kappa) + \frac{2}{3}U^{FB}(\mathbf{e}) + \frac{1}{3}U^{FB}(2, 2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 undercutting the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) = p_0^-$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $v - \kappa + U^{FB}(2, 2) \leq V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \kappa) + \frac{1}{2}U^{FB}(\mathbf{e}) + \frac{1}{2}U^{FB}(2, 2) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $(v - p_0) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

16. Price-setting phase in state $\mathbf{e} = (2, 0)$: Analogous to price-setting phase in state $\mathbf{e} = (0, 2)$.

17. Price-setting phase in state $\mathbf{e} = (2, 1)$: Analogous to price-setting phase in state $\mathbf{e} = (1, 2)$.

18. Price-setting phase in state $\mathbf{e} = (2, 2)$: Deviating to firm 1, say, matching the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \rho\kappa + U^{FB}(\mathbf{e}) = V^{FB}(\mathbf{e})$. Deviating to firm 1, say, being undercut by the outside good and firm 2 undercutting the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0^-$) yields $v - \rho\kappa + U^{FB}(\mathbf{e}) = V^{FB}(\mathbf{e})$. Deviating to firm 1 matching the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) = p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{3}(v - p_0) + \frac{2}{3}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1, say, being undercut by the outside good and firm 2 matching the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(v - p_0) + \frac{1}{2}(v - \rho\kappa) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$$

by part (i) of Assumption 2. Deviating to firm 1 being undercut by the outside good and firm 2 being undercut by the outside good ($p_1^{FB}(\mathbf{e}) > p_0$ and $p_2^{FB}(\mathbf{e}) > p_0$) yields $(v - p_0) + U^{FB}(\mathbf{e}) \leq V^{FB}(\mathbf{e})$ by part (i) of Assumption 2.

Proposition 1. The proof proceeds in two steps. First, we show that given the policy functions, the value functions solve the Bellman equations (2), (3), and (4) in the main paper. Second, we show that there is no profitable one-shot deviation in any state of the industry.

Plugging in the policy functions, the Bellman equations (2), (3), and (4) in the main paper become:

$$U_1(0,0) = \left(1 - \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)$$

$$\times\left(-\overline{S} + \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)\right),$$

$$U_1(0,1) = 0,$$
$$U_1(0,2) = 0,$$
$$U_1(1,0) = \beta V_1(1,0),$$

$$U_1(1,1) = \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\overline{X} + \left(1 - \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)$$

$$\times\left(\frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)\right),$$

$$U_1(1,2) = \overline{X},$$
$$U_1(2,0) = \beta V_1(2,0),$$
$$U_1(2,1) = \beta V_1(2,0),$$

$$U_1(2,2) = \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\overline{X} + \left(1 - \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\right)$$

$$\times\left(\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\beta V_1(2,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\right)\beta V_1(2,2)\right),$$

$$V_1(1,0) = p_0 - \kappa + U_1(2,0),$$
$$V_1(1,1) = -\frac{1}{2}\left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right) + \frac{1}{2}U_1(1,2) + \frac{1}{2}U_1(2,1),$$
$$V_1(1,2) = U_1(1,2),$$
$$V_1(2,0) = p_0 - \rho\kappa + U_1(2,0),$$
$$V_1(2,1) = \kappa(1-\rho) + U_1(2,1),$$
$$V_1(2,2) = U_1(2,2),$$

where we omit the Bellman equation (4) for state $\mathbf{e}$ if $e_1 = 0$. Recall that the firm that sets the lowest price makes the sale for sure and that, if there is more than one such firm, each of them makes the sale with equal probability. It is easy but tedious to show that the value functions solve the Bellman equations.

We proceed state-by-state to show that there is no profitable one-shot deviation. It suffices to consider deviations in pure strategies.[1]

1. Exit-entry phase in state $\mathbf{e} = (0,0)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields

$$-\overline{S} + \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{\overline{S} - \beta\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)$$
$$= 0 = U_1(\mathbf{e}).$$

   Deviating to $\phi_1(\mathbf{e}) = 1$ yields $0 = U_1(\mathbf{e})$.

2. Exit-entry phase in state $\mathbf{e} = (0,1)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields $-\overline{S} + \beta V_1(1,1) < U_1(\mathbf{e})$ by part (ii) of Assumption 2.

3. Exit-entry phase in state $\mathbf{e} = (0,2)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields $-\overline{S} + \beta V_1(1,2) < U_1(\mathbf{e})$ by part (ii) of Assumption 2.

4. Exit-entry phase in state $\mathbf{e} = (1,0)$: Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} < U_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

5. Exit-entry phase in state $\mathbf{e} = (1,1)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields

$$\frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\beta V_1(1,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\beta\left(p_0 - \kappa + \frac{\beta}{1-\beta}(p_0 - \rho\kappa)\right) - \beta\overline{X}}\right)\beta V_1(1,1)$$
$$= \overline{X} = U_1(\mathbf{e}).$$

   Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} = U_1(\mathbf{e})$.

6. Exit-entry phase in state $\mathbf{e} = (1,2)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields $\beta V_1(\mathbf{e}) = \beta\overline{X} < \overline{X} = U_1(\mathbf{e})$.

7. Exit-entry phase in state $\mathbf{e} = (2,0)$: Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} < U_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

8. Exit-entry phase in state $\mathbf{e} = (2,1)$: Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} < U_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

9. Exit-entry phase in state $\mathbf{e} = (2,2)$: Deviating to $\phi_1(\mathbf{e}) = 0$ yields

$$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\beta V_1(2,0) + \left(1 - \frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \beta\overline{X}}\right)\beta V_1(2,2)$$
$$= \overline{X} = U_1(\mathbf{e}).$$

   Deviating to $\phi_1(\mathbf{e}) = 1$ yields $\overline{X} = U_1(\mathbf{e})$.

[1] Note that in the price-setting phase in state $\mathbf{e} > (0,0)$, the outside good remains priced out of the market even after a deviation by parts (i), (ii), and (iii) of Assumption 2.

10. Price-setting phase in state $\mathbf{e} = (0, 0)$: By default.

11. Price-setting phase in state $\mathbf{e} = (0, 1)$: By default.

12. Price-setting phase in state $\mathbf{e} = (0, 2)$: By default.

13. Price-setting phase in state $\mathbf{e} = (1, 0)$: Deviating to match the outside good ($p_1(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(p_0 - \kappa) + \frac{1}{2}U_1(2, 0) + \frac{1}{2}U_1(\mathbf{e}) < V_1(\mathbf{e}).$$

Deviating to be undercut by the outside good ($p_1(\mathbf{e}) > p_0$) yields $U_1(\mathbf{e}) < V_1(\mathbf{e})$.

14. Price-setting phase in state $\mathbf{e} = (1, 1)$: Deviating to undercut firm 2 ($p_1(\mathbf{e}) = \left(\kappa - \left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right)\right)^-$ yields

$$-\left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right) + U_1(2, 1) = V_1(\mathbf{e}).$$

Deviating to be undercut by firm 2 ($p_1(\mathbf{e}) > \kappa - \left(\frac{\beta}{1-\beta}(p_0 - \rho\kappa) - \overline{X}\right)$) yields $U_1(1, 2) = V_1(\mathbf{e})$.

15. Price-setting phase in state $\mathbf{e} = (1, 2)$: Deviating to match firm 2 ($p_1(\mathbf{e}) = \kappa^-$) yields

$$\frac{1}{2}U_1(\mathbf{e}) + \frac{1}{2}U_1(2, 2) = V_1(\mathbf{e}).$$

Deviating to undercut firm 2 ($p_1(\mathbf{e}) = \kappa^{--}$, where κ^{--} is the price just below κ^-) yields $U_1(2, 2) = V_1(\mathbf{e})$.

16. Price-setting phase in state $\mathbf{e} = (2, 0)$: Deviating to match the outside good ($p_1(\mathbf{e}) = p_0$) yields

$$\frac{1}{2}(p_0 - \rho\kappa) + U_1(2, 0) < V_1(\mathbf{e}).$$

Deviating to be undercut by the outside good ($p_1(\mathbf{e}) > p_0$) yields $U_1(\mathbf{e}) < V_1(\mathbf{e})$.

17. Price-setting phase in state $\mathbf{e} = (2, 1)$: Deviating to match firm 2 ($p_1(\mathbf{e}) = \kappa$) yields

$$\frac{1}{2}(1 - \rho)\kappa + \frac{1}{2}U_1(\mathbf{e}) + \frac{1}{2}U_1(2, 2) < V_1(\mathbf{e})$$

by parts (ii) and (iii) of Assumption 2. Deviating to be undercut by firm 2 ($p_1(\mathbf{e}) > \kappa$) yields $U_1(2, 2) < V_1(\mathbf{e})$ by parts (ii) and (iii) of Assumption 2.

18. Price-setting phase in state $\mathbf{e} = (2, 2)$: Deviating to undercut firm 2 ($p_1(\mathbf{e}) = \rho\kappa^-$) yields $U_1(\mathbf{e}) = V_1(\mathbf{e})$. Deviating to be undercut by firm 2 ($p_1(\mathbf{e}) > \rho\kappa$) yields $U_1(\mathbf{e}) = V_1(\mathbf{e})$.

$\mathbf{e}$	$p_1(\mathbf{e})$	$\phi_1(\mathbf{e})$	$V_1(\mathbf{e})$	$U_1(\mathbf{e})$
$(0,0)$	∞	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	–	0
$(0,1)$	∞	0	–	$\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\overline{S}$
$(0,2)$	∞	1	–	0
$(1,0)$	p_0^-	1	$p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\overline{X}$
$(1,1)$	$\kappa-\left(\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\overline{X}\right)$	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$
$(1,2)$	κ	1	$\overline{X}$	$\overline{X}$
$(2,0)$	p_0^-	0	$\frac{p_0-\rho\kappa}{1-\beta}$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,1)$	κ^-	0	$(1-\rho)\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,2)$	$\rho\kappa$	$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$

Table OA2: Additional equilibrium 1. Two-step learning curve. In column labelled $p_1(\mathbf{e})$, superscript $-$ indicates that firm 1 charges just below the price stated.

$\mathbf{e}$	$p_1(\mathbf{e})$	$\phi_1(\mathbf{e})$	$V_1(\mathbf{e})$	$U_1(\mathbf{e})$
$(0,0)$	∞	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	–	0
$(0,1)$	∞	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	–	0
$(0,2)$	∞	1	–	0
$(1,0)$	p_0^-	$\frac{\overline{S}-\beta\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\overline{X}$
$(1,1)$	$\kappa-\left(\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\overline{X}\right)$	$\frac{(1-\beta)\overline{X}}{\beta\left(p_0-\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)\right)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$
$(1,2)$	κ	1	$\overline{X}$	$\overline{X}$
$(2,0)$	p_0^-	0	$\frac{p_0-\rho\kappa}{1-\beta}$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,1)$	κ^-	0	$(1-\rho)\kappa+\frac{\beta}{1-\beta}(p_0-\rho\kappa)$	$\frac{\beta}{1-\beta}(p_0-\rho\kappa)$
$(2,2)$	$\rho\kappa$	$\frac{(1-\beta)\overline{X}}{\frac{\beta}{1-\beta}(p_0-\rho\kappa)-\beta\overline{X}}$	$\overline{X}$	$\overline{X}$

Table OA3: Additional equilibrium 2. Two-step learning curve. In column labelled $p_1(\mathbf{e})$, superscript $-$ indicates that firm 1 charges just below the price stated.

Additional equilibria. Tables OA2 and OA3 show the two other equilibria that exist in addition to the one in Table 1 in the main paper. The proof is similar to that of Proposition 1 and therefore omitted.

Equilibrium with cost-inefficient exit. Table 2 in the main paper shows an equilibrium with cost-inefficient exit. The proof is similar to that of Proposition 1 and therefore omitted.

OA3 Decomposition

Per-period, avoidable fixed costs. We establish that our model with scrap values is equivalent to a model with per-period, avoidable fixed costs but without scrap values. For simplicity, we focus on the special case of mixed exit and entry strategies: $\Delta_X = \Delta_S = 0$.

First consider incumbent firm 1. In the exit-entry phase in state $\mathbf{e}'$ with $e'_1 > 0$, the Bellman equation (2) in the main paper becomes

$$U_1(\mathbf{e}') = \max\left\{\widehat{X}_1(\mathbf{e}'), \overline{X}\right\}, \tag{OA5}$$

where

$$\widehat{X}_1(\mathbf{e}') = \beta\left[V_1(\mathbf{e}')(1-\phi_2(\mathbf{e}')) + V_1(e'_1, 0)\phi_2(\mathbf{e}')\right].$$

In the price-setting phase in state $\mathbf{e}$ with $e_1 > 0$, the Bellman equation (4) in the main paper becomes

$$V_1(\mathbf{e}) = \max_{p_1} D_1(p_1, p_2(\mathbf{e}))(p_1 - c(e_1)) - \overline{F} + U_1(\mathbf{e}) + \sum_{n=1}^{2} D_n(p_1, p_2(\mathbf{e}))\left[U_1\left(\mathbf{e}^{n+}\right) - U_1(\mathbf{e})\right], \tag{OA6}$$

where $\overline{F} \geq 0$ is per-period, avoidable fixed costs. Note that incumbent firm 1 can avoid the fixed costs for the subsequent period by deciding to exit the industry in the current period. Next consider potential entrant 1. In the exit-entry phase in state $\mathbf{e}'$ with $e'_1 = 0$, the Bellman equation (3) in the main paper becomes

$$U_1(\mathbf{e}') = \max\left\{\widehat{S}_1(\mathbf{e}') - \overline{S}, 0\right\}, \tag{OA7}$$

where

$$\widehat{S}_1(\mathbf{e}') = \beta[V_1(1, e'_2)(1-\phi_2(\mathbf{e}')) + V_1(1,0)\phi_2(\mathbf{e}')].$$

Let $\mathbf{V}_1^{(\overline{X},\overline{S},\overline{F})}$, $\mathbf{U}_1^{(\overline{X},\overline{S},\overline{F})}$, $\mathbf{p}_1^{(\overline{X},\overline{S},\overline{F})}$, and $\boldsymbol{\phi}_1^{(\overline{X},\overline{S},\overline{F})}$ denote the value and policy functions of firm 1 in a symmetric equilibrium for given values of $\left(\overline{X}, \overline{S}, \overline{F}\right)$; these solve the Bellman equations (OA5), (OA6), and (OA7) along with the corresponding optimality conditions.

Our model sets $\overline{X} \geq 0$ and $\overline{F} = 0$. We show that our model is equivalent to an alternative model that sets $\overline{X}' = 0$ and $\overline{F}' \geq 0$. To this end, we show that if $\mathbf{V}_1^{(\overline{X},\overline{S},0)}$, $\mathbf{U}_1^{(\overline{X},\overline{S},0)}$, $\mathbf{p}_1^{(\overline{X},\overline{S},0)}$, and $\boldsymbol{\phi}_1^{(\overline{X},\overline{S},0)}$ solve the Bellman equations (OA5), (OA6), and (OA7) given $\left(\overline{X}, \overline{S}, 0\right)$, then

$$\mathbf{V}_1^{(0,\overline{S}',\overline{F}')} = \mathbf{V}_1^{(\overline{X},\overline{S},0)} - \frac{\overline{X}}{\beta},$$

$$
\begin{aligned}
\mathbf{U}_1^{(0,\overline{S}',\overline{F}')}(\mathbf{e}) &= \begin{cases} \mathbf{U}_1^{(\overline{X},\overline{S},0)}(\mathbf{e}) & \text{if} \quad e_1 = 0, \\ \mathbf{U}_1^{(\overline{X},\overline{S},0)}(\mathbf{e}) - \overline{X} & \text{if} \quad e_1 > 0, \end{cases} \\
\mathbf{p}_1^{(0,\overline{S}',\overline{F}')} &= \mathbf{p}_1^{(\overline{X},\overline{S},0)}, \\
\boldsymbol{\phi}_1^{(0,\overline{S}',\overline{F}')} &= \boldsymbol{\phi}_1^{(\overline{X},\overline{S},0)}
\end{aligned}
$$

solve these equations given $\left(0, \overline{S}' = \overline{S} - \overline{X}, \overline{F}' = \frac{(1-\beta)\overline{X}}{\beta}\right)$.

Starting with incumbent firm 1, plugging in the Bellman equations (OA5) and (OA6) given $\left(0, \overline{S}' = \overline{S} - \overline{X}, \overline{F}' = \frac{(1-\beta)\overline{X}}{\beta}\right)$ reduce to those under $\left(\overline{X}, \overline{S}, 0\right)$. Turning to potential entrant 1, the Bellman equation (OA7) given $\left(0, \overline{S}' = \overline{S} - \overline{X}, \overline{F}' = \frac{(1-\beta)\overline{X}}{\beta}\right)$ similarly reduces to that under $\left(\overline{X}, \overline{S}, 0\right)$.

OA4 Why is the best equilibrium so good?

Linear demand. Consider a representative consumer who allocates her income I among the inside goods that are offered by the incumbent firms at prices $\mathbf{p} = (p_1, p_2)$, an outside good at an exogenously given price p_0, and a numeraire good. Substituting the budget constraint into the utility function, the maximization problem of the representative consumer is

$$
\max_{Q_0,Q_1,Q_2} \sum_{n=0}^{2} a_n Q_n - \frac{b}{2} \sum_{n=0}^{2} Q_n^2 - \theta b \left(Q_0 Q_1 + Q_0 Q_2 + Q_1 Q_2\right) + I - \sum_{n=0}^{2} p_n Q_n,
$$

where $a_0 > 0$, $a_1 > 0$, $a_2 > 0$, $b > 0$, and $\theta \in [0, 1)$ are parameters. The parameter θ governs the degree of product differentiation, with higher values of θ corresponding to weaker product differentiation.

The first-order conditions in matrix form are:

$$
\begin{bmatrix} 1 & \theta & \theta \\ \theta & 1 & \theta \\ \theta & \theta & 1 \end{bmatrix} \begin{bmatrix} Q_0 \\ Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} \frac{a_0 - p_0}{b} \\ \frac{a_1 - p_1}{b} \\ \frac{a_2 - p_2}{b} \end{bmatrix}.
$$

Solving yields the demand functions

$$
\begin{aligned}
Q_0 &= D_0(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)} \left(\left(1+\theta\right) a_0 - \theta a_1 - \theta a_2 - (1+\theta) p_0 + \theta p_1 + \theta p_2\right), \\
Q_1 &= D_1(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)} \left(-\theta a_0 + \left(1+\theta\right) a_1 - \theta a_2 + \theta p_0 - (1+\theta) p_1 + \theta p_2\right), \\
Q_2 &= D_2(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)} \left(-\theta a_0 - \theta a_1 + \left(1+\theta\right) a_2 + \theta p_0 + \theta p_1 - (1+\theta) p_2\right).
\end{aligned}
$$

The aggregate demand for the inside goods is

$$
D_T(\mathbf{p}) = \sum_{n=1}^{2} D_n(\mathbf{p}) = \frac{1}{b(2\theta+1)(1-\theta)} \left[-2\theta a_0 + a_1 + a_2 + 2\theta p_0 - (p_1 + p_2)\right].
$$

To prevent $D_T(\mathbf{p}) < 0$, we maintain $-2\theta a_0 + a_1 + a_2 + 2\theta p_0 > 0$. We compute the price elasticity of aggregate demand as the percentage change in aggregate demand $D_T(\mathbf{p})$ that from results a one-percent change in prices $\mathbf{p}$:

$$\eta_T(\mathbf{p}) = \left.\frac{\partial D_T(\lambda \mathbf{p})}{\partial \lambda} \frac{\lambda}{D_T(\lambda \mathbf{p})}\right|_{\lambda=1} = \frac{-(p_1 + p_2)}{-2\theta a_0 + a_1 + a_2 + 2\theta p_0 - (p_1 + p_2)}.$$

Note that the absolute value $|\eta_T(\mathbf{p})|$ of this price elasticity increases in $p_1 + p_2$. Moreover, the quantity of the outside good demanded $D_0(\mathbf{p})$ increases in $p_1 + p_2$. Thus, as the prices of the inside goods decrease, the aggregate demand for the inside goods becomes less price elastic, and at the same time, the quantity of the outside good demanded decreases.