

# Housing assignment with restrictions: theory and evidence from Stanford campus\*

Tim Landvoigt  
UT Austin

Monika Piazzesi  
Stanford & NBER

Martin Schneider  
Stanford & NBER

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## Abstract

This paper studies housing markets where a subset of houses in a restricted area can be bought only by a subset of "eligible" buyers. In the data, houses on Stanford campus that can only be bought by faculty trade at a substantial discount to similar houses off campus. An assignment model with heterogeneous houses and buyers predicts such discounts if the matchup of quality and buyer pools is sufficiently different inside versus outside the restricted area. The restriction can distort allocations by making eligible buyers choose either higher or lower qualities than ineligible buyers with the same characteristics.

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\*Email addresses: [tim.landvoigt@mcombs.utexas.edu](mailto:tim.landvoigt@mcombs.utexas.edu), [piazzesi@stanford.edu](mailto:piazzesi@stanford.edu), [schneidr@stanford.edu](mailto:schneidr@stanford.edu). We would like to thank trulia.com for access to and help with their housing databases. We would also like to thank Nadr Essabhoy, Carole Feldstein, Bob Hall, Pablo Kurlat, Shari Ornstein, John Taylor, and the Stanford Housing Office.

# 1 Introduction

Within narrow geographic areas, housing markets assign buyers with different characteristics to indivisible houses that differ by quality. This paper studies housing assignment when a subset of *eligible* buyers have exclusive access to a subset of houses that form a *restricted area*. We show that houses in the restricted area can trade at a discount if the matchup of quality and buyer pools is sufficiently different inside versus outside the restricted area. Moreover, the restriction can distort allocations by making eligible buyers choose either higher or lower qualities than ineligible buyers with the same characteristics.

In our leading example, buyers affiliated with Stanford University have exclusive access to houses on campus.<sup>1</sup> We begin by presenting evidence on house prices on and right around Stanford campus over the last decade. Using both a simple comparables approach and nearest neighbor matching, we show that houses on campus trade at a substantial discount to similar properties off campus. The discount is relatively smaller for higher quality houses.

We then study the effect of an access restriction in an assignment model with a continuum of houses in which buyer types differ not only by eligibility but also by the marginal utility of house quality.<sup>2</sup>

Without the access restriction, our model has an efficient equilibrium in which higher types buy higher quality houses. House prices reflect the relative dispersion of house quality and buyer types. The cost of an additional unit of quality depends on the marginal buyer type; it rises at a faster rate if more distinct buyers must be assigned to similar houses.

When there are more eligible buyers than houses in the restricted area, the efficient equilibrium may survive even with the restriction. Arbitrage by eligible buyers across areas equates prices quality-by-quality as long as the dispersion of quality in the restricted area relative to the dispersion of type (that is, marginal utility) among eligible buyers is everywhere sufficiently similar to the relative dispersion in the economy at large.

Once pairs of distributions are sufficiently different, however, arbitrage across areas becomes impossible and houses in the restricted area trade at a discount. We study an example economy in which house quality in the restricted area is relatively low, so eligible buyers who do not buy in the restricted area instead buy higher quality houses outside. The example generates price patterns consistent with those found around Stanford. It also illustrates that a restriction can distort allocations differently at the high and low end of the quality spectrum.

On the one hand, eligible buyers of the best restricted houses buy lower quality houses than non-eligible buyers with the same preferences (and lower quality houses than they would buy if the restriction were lifted). For those high buyer types, the price discount thus provides compensation for compromising on quality inside the restricted area. On the other hand, eligible

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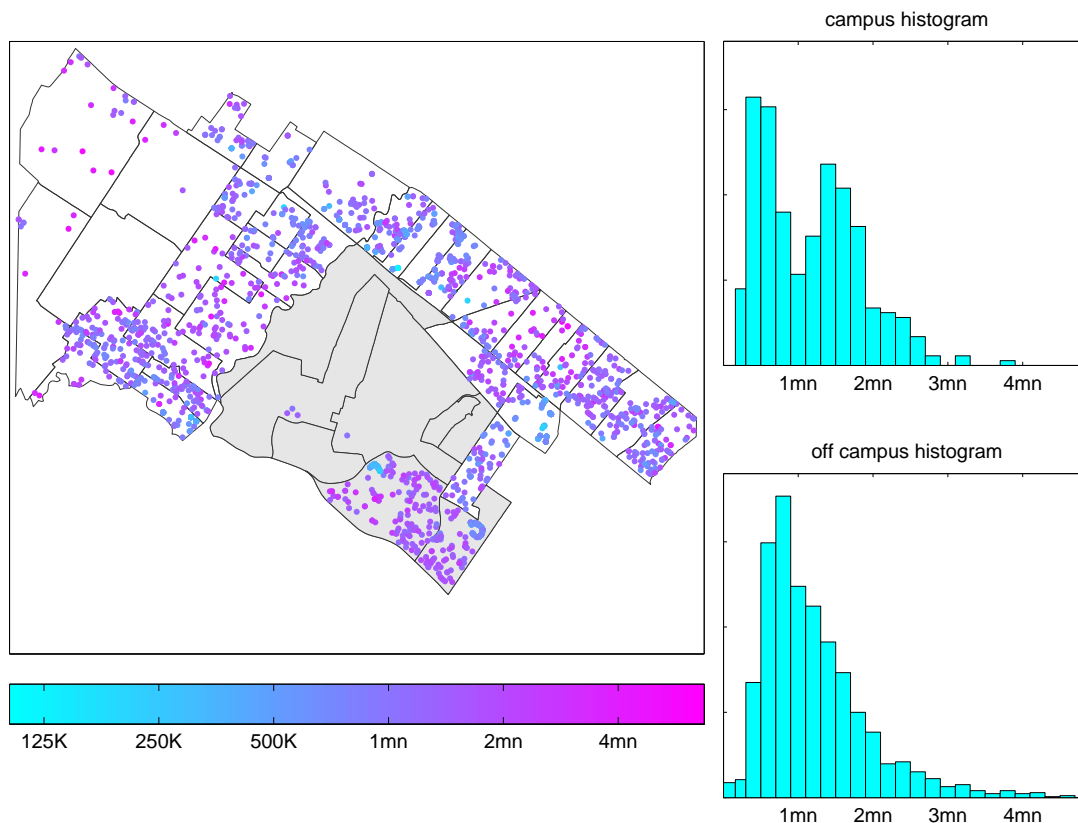
<sup>1</sup>Similar issues arise whenever a subset of buyers receives much lower utility from a subset of houses, for example, families with children may not consider neighborhoods with very bad access to schools.

<sup>2</sup>Assignment models are surveyed by Sattinger (1993). We consider two-sided assignment with a continuum of houses and multiple dimensions of mover heterogeneity, as in Landvoigt, Piazzesi and Schneider (2013). In such a setting, a change in the characteristics of a subset of movers (a change in credit condition there, reducing eligibility here) has potentially uneven effects on prices across house qualities.

buyers of the worst restricted houses buy higher quality houses than their peers outside. The price discount helps these low buyer types to buy a better house than what they would buy at outside market prices or in the absence of the restriction.

## 2 House prices on and around Stanford campus

We obtain house prices at the property level from deeds data for the years 2002-2012. We match deeds to assessor data that contain house characteristics such as lot size, building size, the number of bathroom and bedrooms. Since we have coordinates of for each house, we can also use the American Community Survey to measure neighborhoods characteristics at the blockgroup level. We restrict attention to a narrow area around Stanford campus. Figure 1 shows the Stanford campus (zipcode 94305) together with the areas of Palo Alto and Menlo Park close to campus (census tracts 5109, 5113, 5114, 5115, 5116, 5130, 6125, 6126, 6127, 6128 and 6129). The campus is the grey shaded area. The campus transactions are mostly in the south-east corner of the grey shaded area.



**Figure 1:** Left: Map of transactions, 2002-2012. The color-coding uses cold colors for cheap and warm colors for expensive houses. The map shows the Stanford campus together with areas of Palo Alto and Menlo Park in close proximity to campus. Right: Campus and off-campus histograms of house prices in these transactions.

The right panels of Figure 1 show a histogram of house prices on and off campus. To make the historical house prices comparable with each other, we use the median of local house prices in each year to construct a price index and convert all transactions to year 2012 dollars. The main point here is that the support of the campus price distribution is narrower than that of the surrounding area. On the one hand, the left tail of the campus distribution does not include a few cheap houses that are available off campus. On the other hand, the upper tail of the campus distribution does not include mansions as large as those in nearby neighborhoods.<sup>3</sup>

Do similar houses trade at different prices on campus? An answer to this question requires estimating the hypothetical price of an on-campus house if it were located off-campus. Figure 2 takes a first crack at this by comparing prices of campus homes to those of their off-campus comparables. Each dot in the figure represents a campus transaction during the years 2002-2012. Condos are light blue whereas single family homes are dark blue. The horizontal axis measures the transaction price for the campus house. The vertical axis measures the median price of comparable off campus houses. We select comparables from transactions that occurred within 180 days based on similarity by building area, lot size as well as the number of bathrooms and bedrooms<sup>4</sup>.

The majority of dots are located above the 45 degree line that would indicate equal pricing on and off campus. Off-campus comparables are thus typically more expensive than the house on campus. This premium is particularly large for condos at the low end of the price distribution. This visual impression is confirmed by an OLS regression though the cloud of dots in Figure 2: the green regression line has an intercept of \$668K and a slope coefficient of 0.91 (which is highly significant, but insignificantly different from one.)

Many expensive homes on campus – say, above two million dollars – have large lots. The homes with lots larger than 2/3 of an acre are indicated with red dots. These transactions are far above the OLS regression line, indicating a particularly large premium in off-campus transactions with similarly sized lots.

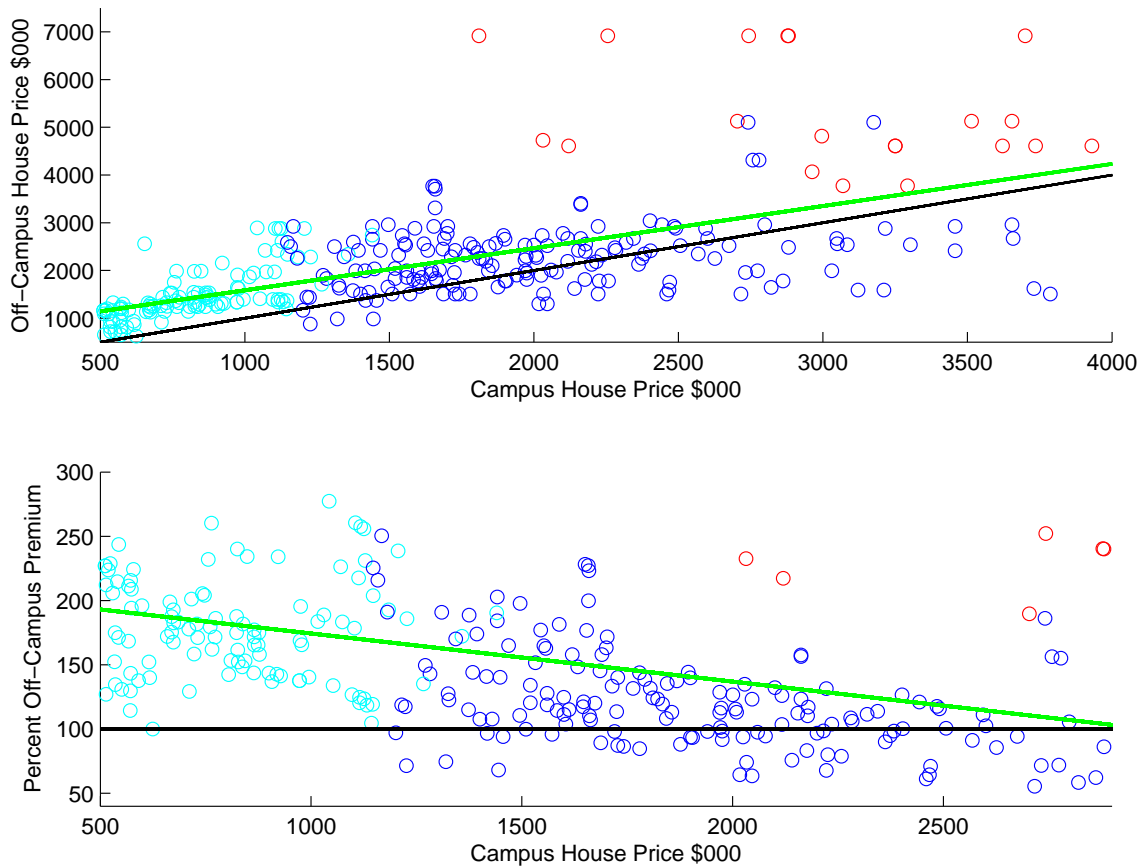
The bottom panel of Figure 2 shows the premium in off-campus transactions as a percentage of the campus house prices. The black horizontal line at 100% indicates equal pricing on and off campus. The green OLS regression line suggests that campus houses are mostly cheaper percentage-wise than their off-campus comparables. The discount is larger for low-end houses on campus. To buy a condo or small house off campus, a faculty member would have to pay almost double as much as on campus. For medium houses in the 1-1.5 million dollar range, a faculty member has to pay roughly 150% more to buy off campus. The campus discount disappears at the high end of the house quality spectrum. Houses worth more than 2 million Dollars on campus are not cheaper than those off campus.

Table 1 reports results from an alternative approach to estimating the hypothetical off-

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<sup>3</sup>For example, the off-campus area on the map in Figure 1 features private residences such as the home of Facebook Co-Founder and CEO Mark Zuckerberg, who bought a \$7 million property in 2011 and added the four surrounding properties for \$30 million in 2013. Other private residences are the \$7 million home of Google Co-Founder and CEO Larry Page or the \$11.2 property that Yahoo! CEO Marissa Mayer bought recently. Such houses are simply not available on Stanford campus.

<sup>4</sup>Appendix A contains a detailed description of the procedure used to find comparable off-campus transactions.



**Figure 2: Top panel:** Campus house transactions measured on the horizontal axis together with the median value of their off-campus comparables on the vertical axis. The light blue color is condos. The black 45 degree line indicates equal pricing on and off campus. The green OLS regression line uses all dotted observations. The data are from years 2002-2012. **Bottom panel:** The percent premium in off-campus transactions, 2002-2012. Light blue dots are condos. The black horizontal line indicates equal pricing on and off campus. The green line is an OLS regression through the dots.

campus value of campus properties. Rather than use median comparable prices, we use predicted values from a nearest neighbor regression as in Caplin et al (2008). The regression is run year by year and regressors include the above characteristics as well as geographic and neighborhood information; in particular, we include latitude and longitude of the property, the shares of units in the census block group that are rented and that are in multi-unit buildings and the share of households in the highest (topcoded) income bracket of the census blockgroup. The latter variables help predict prices in neighborhoods with diverse individual properties.

The quantitative findings based on this alternative approach confirm the visual impressions from Figure 2. The percentage premium for houses outside campus is highest at the low end of the house quality spectrum. A faculty member who wants a house outside of campus comparable to a house in the bottom quartile of the quality distribution on campus pays 163% of what he would pay on campus. This premium declines and reaches zero for high-end houses (in the top quartile of the campus quality distribution, where houses cost more than \$2 million

**Table 1:** How much more expensive are houses outside campus?

Price range of campus houses	Difference between off and on campus			
	in percent	s.e.	in Dollars	s.e.
in lowest quartile $\leq$ \$800K	63.3%	7.0	\$611	128
second quartile \$800K – \$1.5 mio	23.4%	6.1	\$446	144
third quartile \$1.5 mio – \$2 mio	12.9%	6.3	\$380	149
top quartile $\geq$ \$2 mio	1.6%	7.2	\$212	209
# observations	357			

Note: Nearest neighbor matching with small sample standard errors. The imputation of nearest neighbor is done by year and uses information on lot size, building size, # bathrooms and geographical information (longitude, latitude). The imputation controls for the share of housing units, renters and households in the highest income bracket of the census block.

in year 2010 Dollars.) We estimate the absolute dollar premium for a house outside campus to be roughly constant: \$400K across the board.

### 3 An assignment model with restricted access

A continuum of houses of measure one has been put up for sale. Houses differ by quality, measured by a one dimensional index  $h$ . A share  $\rho$  of houses are located in a *restricted area* that only a subset of buyers have access to. The distribution of quality inside and outside the restricted area is described by densities  $g_r$  and  $g$ . For much of the exposition, we refer to a specific example, based loosely on the Stanford area, that is depicted in Figure 3. In particular, the second panel of the figure shows the house quality densities. The restricted area offers a subset of qualities, with both the highest and lowest qualities missing.

There is a continuum of buyers of measure one. Everyone buys at most one house. A share  $\eta \geq \rho$  of *eligible buyers* can buy anywhere. The remaining buyers must buy outside the restricted area. Utility from housing does not depend on location: anyone who buys a house of quality  $h$  at price  $p$  receives surplus  $\theta h - p$ .<sup>5</sup> Buyers differ by their marginal utility of house quality  $\theta$ . The distribution of types  $\theta$  for eligible and other buyers is described by densities  $f_e(\theta)$  and  $f(\theta)$ , respectively, plotted in the first panel of Figure 2. The type distribution for eligible buyers is truncated at a point  $\underline{\theta}_e > 0$ .

An equilibrium consists of buyers' house choices  $h$  as well as prices for restricted and unrestricted houses  $p(h)$  and  $p_r(h)$  so that all buyers optimize given prices and markets clear. We consider equilibria such that house quality is strictly increasing in type  $\theta$ . We further require

<sup>5</sup>This assumption serves to zero in on the role of distributions on prices. Allowing eligible agents to obtain higher utility from restricted houses introduces an additional force that works to increase house prices in the restricted area. For the application we consider, this force must be weak enough and is omitted.

that all buyers obtain nonnegative surplus from buying a house, so  $p(0) = 0$ . We also have  $p_r(h) \leq p(h)$  in equilibrium since eligible agents do not lose from buying outside the restricted area.

### 3.1 Equilibrium without an access restriction

The overall distributions of types and houses in the economy, regardless of eligibility, are given by

$$\begin{aligned} g_u(\theta) &= \eta f_e(\theta) + (1 - \eta) f(\theta), \\ g_u(h) &= \rho g_r(h) + (1 - \rho)g(h). \end{aligned}$$

Throughout we denote cdfs by upper case letters. Without an access restriction, buyers are assigned to houses according to the strictly increasing QQ plot of  $F_u$  against  $G_u$ , that is,  $\theta_u(h) = F_u^{-1}(G_u(h))$ . The optimal choice for a buyer of type  $\theta$  satisfies the first order condition  $p'(h) = \theta$ . The marginal buyer at quality  $h$  prefers a slightly higher (lower) quality house if the price schedule increases by less (more) than  $\theta$  at quality  $h$ . Prices follow by integration given the initial condition  $p(0) = 0$ .

The unrestricted assignment is plotted in green in the third panel of Figure 3; it coincides with the blue line near the boundaries. It is steep when the distribution of types is more dispersed than the distribution of house qualities. Indeed, the slope  $\theta'_u(h)$  is given by the density ratio  $\theta'_u(h) = g(h) / f(\theta_u(h))$ . When it is high, there are relatively more similar houses close to  $h$  than there are buyers of similar type close to  $\theta_u(h)$ . Similar houses must thus be assigned to buyer types with rather different marginal utilities. Prices must then increase at a faster rate  $p''(h) = \theta'_u(h)$  near  $h$  to induce those different buyers not to prefer  $h$  itself, as shown in the last panel of Figure 3.

### 3.2 Market clearing with an access restriction

If quality is increasing in type, the assignment must be the same for all buyers of type  $\theta$  who buy outside the restricted area, regardless of whether they are eligible or not. We thus define house quality assignments  $\theta_r(h)$  and  $\theta(h)$  inside and outside the restricted area, respectively. Let  $\tilde{f}_e(\theta)$  denote the (endogenous) density of eligible agents who buy in the restricted area.

Markets must clear at every quality level both inside and outside the restricted area:

$$\begin{aligned} \rho g_r(h) &= \rho \tilde{f}_e(\theta_r(h)) \theta'_r(h), \\ (1 - \rho)g(h) &= (f_u(\theta(h)) - \rho \tilde{f}_e(\theta(h)) \theta'(h)). \end{aligned}$$

Houses for sale in the restricted area at quality  $h$  must be bought by eligible agents who are assigned those houses in the restricted area. Moreover, houses for sale outside the restricted area must be bought by buyers who are not assigned houses in the restricted area.

In addition, the number of eligible agents who locate outside the restricted area must be nonnegative, that is, for all  $\theta \in [\underline{\theta}_e, \bar{\theta}]$

$$0 \leq \rho \tilde{f}_e(\theta) \leq \eta f^e(\theta). \quad (1)$$

If  $p_r(h) < p(h)$ , then the right hand condition holds with equality at  $\theta = \theta_r(h)$ . All eligible buyers buy in the restricted area when quality is strictly cheaper there. In contrast, if prices are the same across areas at some quality, then eligible buyers are indifferent between areas.

### 3.3 Equilibrium with equal prices

We first ask whether the restriction is binding, that is, whether it makes the unrestricted equilibrium infeasible. Suppose that prices are the same across areas for all quality levels. The equilibrium assignment  $\theta_u$  implies a unique density  $\tilde{f}_e$  that clears the market. The question is whether there are always enough eligible agents to buy the restricted houses at every quality level.

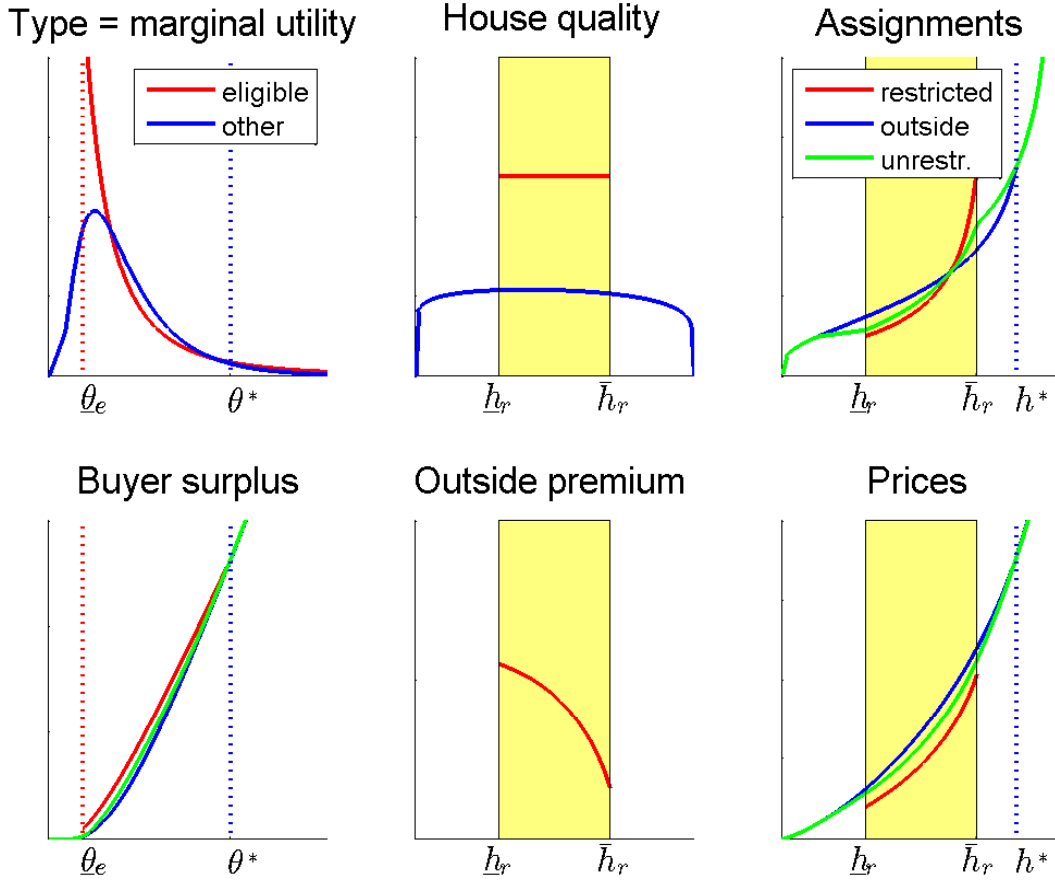
Condition (1) now restricts the slope of the assignment so  $\theta'_u(h) \geq \rho g_r(h) / \eta f_e(\theta_u(h))$ . Since  $\rho \leq \eta$ , the condition is always satisfied if the distributions of houses and buyers are identical. If  $\rho = \eta$ , it says that the density ratios  $g_r(h) / f_e(\theta)$  and  $g(h) / f(\theta)$  must be equal across areas. This is the knife edge condition that implies equal prices if the two areas were completely segmented markets.

With  $\rho < \eta$ , the predictions of the model differ from one with segmented markets: an equal price equilibrium may also exist when the density ratios are different. Indeed, arbitrage by eligible agents can work to equate prices. For example, suppose the house quality densities are the same. Consider a quality range around  $h$  with many more eligible than ineligible agents. With segmented markets, prices rise less with  $h$  in the restricted area since the relative demand for more expensive houses is lower there. In the present model, some eligible agents can move out of the restricted area and thus equate the relative demands.

### 3.4 Price discounts in the restricted area

We now investigate why houses in the restricted area can be strictly cheaper for all quality levels. In this case, if a quality level is available in the restricted area, no eligible buyer will buy it outside. The  $\eta - \rho$  eligible buyers who nevertheless buy outside the restricted area thus choose qualities that are not available inside. The example in Figure 3 has been set up so all that there is a positive mass of eligible buyers who move outside the restricted area, all of whom buy higher quality houses than those available inside.





**Figure 3:** Illustrative example. *Top left:* type densities; red for eligible buyers  $f_e$ , blue for other buyers  $f$ . *Top middle:* house densities; red for restricted houses  $g_r$ , blue for other houses  $g$ . *Top right:* assignments; green for  $\theta_u(h)$  (unrestricted), red for  $\theta_r(h)$  (with restriction, inside restricted area), blue for  $\theta(h)$  (with restriction, outside). *Bottom left:* buyer surplus; green: unrestricted, red: with restriction, inside restricted area, blue: with restriction, outside. *Bottom middle:* outside premium  $(p(h) - p_r(h))/p_r(h)$ , *Bottom right:* prices, same legend as bottom left.

The assignment of restricted houses to eligible buyers follows

$$\theta_r(h) = F_e^{-1}(\rho G_r(h)/\eta). \quad (2)$$

In particular, there is a highest type  $\theta^* = \theta_r(\bar{h}_r) = F_e^{-1}(\rho/\eta)$  who is indifferent between buying the highest restricted house  $\bar{h}_r$  at price  $p_r(\bar{h}_r)$  and buying a higher quality  $h^* > \bar{h}_r$  outside the restricted area.

For all types higher than  $\theta^*$ , the restriction does not bite and the assignment is given by  $\theta_u(h)$ . Below the house quality  $h^* = \theta_u^{-1}(\theta^*)$ , outside houses are assigned to ineligible buyers according to

$$\theta(h) = F^{-1}((1 - \rho)G(h)/(1 - \eta)). \quad (3)$$

Since  $h^* > \bar{h}_r$  an equilibrium with equal prices cannot exist. Indeed, since assignments are monotonic we must have  $\theta_r(\bar{h}_r) > \theta_u(\bar{h}_r)$  which is incompatible with (1). With the distri-

butions assumed here, the unrestricted assignment asks relatively low types to move into the restricted area. However, not enough of those types are eligible to support an equilibrium with equal prices.

Equilibrium assignments are shown in the third panel of Figure 3. Eligible buyers at the upper end of the restricted area buy lower quality houses than ineligible buyers with the same preferences; for the same threshold marginal utility  $\theta^*$ , for example, eligible buyers buy  $\bar{h}_r$  while ineligible buyers buy  $h^* > \bar{h}_r$ . In contrast, eligible buyers at the lower end of the restricted area buy higher quality houses than ineligible buyers with the same type. Comparison with the unrestricted assignment shows that the highest (lowest) eligible buyers would buy higher (lower) quality houses if the restriction were lifted.

The assignment is brought about by price discounts, as shown in the fifth panel of Figure 3. First order conditions equating the price change to the marginal buyer type hold both inside and outside the restricted area. At quality levels available in the restricted area, prices are found by integration using the indifference of type  $\theta^*$  between  $\bar{h}_r$  and  $h^*$ :

$$\begin{aligned} p(h) &= \int_0^h \theta(\tilde{h}) d\tilde{h}, \\ p_r(h) &= p(h) - \int_h^{h^*} (\theta_r(\tilde{h}) - \theta(\tilde{h})) d\tilde{h}. \end{aligned} \quad (4)$$

A price discount exists at  $h$  in the restricted area as long as the average assignment between  $h$  and  $h^*$  is higher there. At high qualities, low prices entice relatively high eligible types to buy the relatively low quality houses inside the restricted area.<sup>6</sup> At low qualities, low prices help low eligible types buy relatively high quality houses that are better than those bought by their ineligible counterparts and that they would not buy if the restriction were lifted. Comparison with the unrestricted price shows that the restriction not only lowers price in the restricted area, but also raises them outside, including at qualities available in the restricted area itself.

The bottom left panel shows the surplus earned by different buyers in equilibrium. The restriction favors eligible buyers who must be at least as well off as their ineligible counterparts. At qualities available inside the restricted area, equilibrium surplus is

$$\theta_r(h)h - p_r(h) = \int_0^h (\theta_r(h) - \theta_r(\tilde{h})) d\tilde{h} + \int_0^{h^*} (\theta_r(\tilde{h}) - \theta(\tilde{h})) d\tilde{h}.$$

The first term takes the form that typically obtains in a simple assignment of eligible agents to restricted houses. Surplus is higher the further away a buyer is from the lowest type. Here the interaction with ineligible buyers implies that eligible buyers receive an additional rent represented by the second term.

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<sup>6</sup>To establish that the resulting prices support an equilibrium, we also need to show that eligible types optimally choose their area. Appendix B provides sufficient conditions for the existence of an equilibrium.

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# Appendix

## A Identifying off-campus comparables

This section describes how we select similar properties outside of Stanford campus that we use to estimate off-campus prices for houses sold on campus. The results of these procedures are presented in figure 2 and in table 1.

The pool of transactions from which comparables are drawn comprises all house sales between 2002 and 2012 in the area shown in figure 1. In addition, we include all sales during that period in the town of Atherton, CA, which is just north of the area shown in the map. This area consists of census tracts 5109, 5113, 5114, 5115, 5116, 5130, 6114, 6125, 6126, 6127, 6128 and 6129.

Table 2 displays summary statistics for the campus and off-campus samples, respectively.

We select the comparables for figure 2 by finding, for each campus transaction in a given year, a number of off-campus transactions whose characteristics are sufficiently close to those of the house sold on campus.

We limit the search to transactions that occurred no more than 180 days before or after the campus transactions. For condos, we identify comparables along the dimensions of building area and the number of bathrooms. For single family homes, we use lot size in addition.

For each campus transaction and each search characteristic, we define an interval around that characteristic’s value of the campus house. We then define as “comparable” those off-campus properties for which all characteristics lie in the specified intervals. We set the range of the filter intervals to the smallest possible size such that we find at least three comparables for each campus transaction.

We use the same samples of transactions on and off campus for the nearest neighbor prediction results in table 1. The “nearest neighbor” transaction is identified through predictive mean matching separately for each year<sup>7</sup>. On the level of the individual property, the regression contains an indicator whether a property is a condo, building area, number of bathrooms, lot size (for single family homes), and the exact geographical location (longitude and latitude). On the level of the neighborhood, the regression includes the average number of units in a structure, the share of households in the highest census income bracket, and the share of rental units.

We then draw five imputations from the posterior distribution centered around the price of the closest off campus house according to the predictive mean criterion. For each year, we divide the campus transactions into quartiles based on price, and estimate the mean difference between the observed campus price and imputed off-campus price by quartile across all years. The estimation takes into account the sampling error resulting from imputation (including an adjustment for small samples; see the documentation of Stata command “mi estimate” for details).

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<sup>7</sup>Pooling all observations and including transaction date as another prediction variable leads to similar results.

**Table 2:** Summary statistics

Year	# Obs	Fraction Condos	Sales price (\$1000s)	Building area (sqft)	# bedr.	# bathr.	Lot size (sqft)
Campus							
2002	29	0.55	1,553.8	1,992.8	3.03	2.59	7,727.6
2003	31	0.35	1,453.9	2,224.6	3.23	2.74	12,073.0
2004	45	0.40	1,316.1	2,018.5	3.09	2.34	10,751.6
2005	43	0.44	1,387.4	2,214.1	3.30	2.51	9,907.3
2006	37	0.35	2,121.7	2,219.5	3.41	2.62	12,657.6
2007	32	0.50	1,752.6	1,996.1	3.28	2.43	10,644.8
2008	26	0.46	1,805.5	2,055.2	3.19	2.54	12,377.4
2009	20	0.55	1,431.0	1,836.3	3.00	2.33	8,515.9
2010	16	0.38	1,412.7	1,955.2	3.50	2.31	9,065.8
2011	27	0.44	1,175.4	1,980.5	3.15	2.30	9,954.1
2012	26	0.23	1,603.5	2,604.2	4.19	3.04	14,592.5
2013	25	0.48	1,389.6	2,002.1	3.48	2.40	10,349.7
Off campus							
2002	214	0.36	1,611.8	1,742.5	2.82	2.27	6,568.0
2003	230	0.28	1,392.3	1,780.6	2.90	2.14	6,631.4
2004	202	0.42	1,542.5	1,804.9	2.82	2.28	6,050.6
2005	156	0.42	1,564.6	1,774.1	2.83	2.30	6,506.9
2006	126	0.42	2,017.5	1,735.8	2.90	2.29	6,386.9
2007	113	0.39	2,042.6	1,667.5	2.82	2.21	6,520.8
2008	50	0.44	1,886.1	1,621.5	2.76	2.07	5,623.1
2009	56	0.29	1,472.9	1,559.8	2.75	1.91	6,203.3
2010	154	0.19	1,523.2	1,673.2	2.84	2.09	5,737.7
2011	149	0.23	1,373.0	1,643.1	2.80	2.06	5,401.9
2012	155	0.21	1,508.3	1,719.1	2.85	2.05	5,713.9
2013	121	0.19	1,771.0	1,706.4	2.91	2.07	5,846.8

Note: Table reports sample means for all variables. Prices are reported in 2012 dollars. Lot size averages exclude condos.

## B Sufficient conditions for existence

Here we provide sufficient conditions for the existence of an equilibrium with unequal prices of the type presented in the text.

**Proposition.** Let  $h^* = \theta^{-1}(\theta_r(\bar{h}_r))$  and suppose that for all  $h \in [0, \bar{h}_r]$ ,

$$\int_h^{h^*} (\theta_r(\tilde{h}) - \theta(\tilde{h})) d\tilde{h} > 0. \quad (\text{B-1})$$

1. There is an equilibrium such that the assignment  $\theta_r$  is given by (2) and the assignment  $\theta$  is given by (3) for  $h \leq h^*$  and  $\theta(h) = \theta_u(h)$  for  $h > h^*$ . Moreover, eligible buyers buy outside the restricted area if and only if  $\theta > \theta^*$ .
2. The equilibrium price functions outside of the restricted area,  $p : [0, \bar{h}] \rightarrow \mathbb{R}_+$ , and inside the restricted area,  $p_r : [\underline{h}_r, \bar{h}_r] \rightarrow \mathbb{R}_+$ , are given by (4).

**Proof.** We first show that price functions are as in part 2 of the proposition if the equilibrium is as characterized in part 1.

The first order conditions for agents choosing to live outside of the restricted area,  $p'(h) = \theta(h)$ , in combination with the initial condition  $p(0) = 0$ , give the result for the price function  $p$ .

The first order conditions for eligible agents choosing to live in the restricted area,  $p'_r(h) = \theta_r(h)$ , imply

$$p_r(h) = \bar{P}_r + \int_{\underline{h}_r}^h \theta_r(\tilde{h}) d\tilde{h}. \quad (\text{B-2})$$

To determine the value of the constant  $\bar{P}_r$ , we use the fact that the marginal eligible type  $\theta^*$  is indifferent between buying the house of quality  $h^*$  outside of the restricted area and the house of quality  $\bar{h}_r$  within the restricted area:

$$\theta^* \bar{h}_r - p_r(\bar{h}_r) = \theta^* h^* - p(h^*),$$

which yields

$$\begin{aligned} \bar{P}_r &= \int_0^{h^*} \theta(\tilde{h}) d\tilde{h} - \int_{\underline{h}_r}^{\bar{h}_r} \theta_r(\tilde{h}) d\tilde{h} - \theta^*(h^* - \bar{h}_r) \\ &= \int_0^{h^*} \theta(\tilde{h}) d\tilde{h} - \int_{\underline{h}_r}^{h^*} \theta_r(\tilde{h}) d\tilde{h}. \end{aligned} \quad (\text{B-3})$$

The second equality follows since we have that  $\theta_r(h) = F_e^{-1}(\rho/\eta) = \theta^*$  for  $h \geq \bar{h}_r$ , and therefore  $\theta^*(h^* - \bar{h}_r) = \int_{\bar{h}_r}^{h^*} \theta_r(\tilde{h}) d\tilde{h}$ . Inserting the expression for  $\bar{P}_r$  in (B-3) into equation (B-2) yields the result for the price function  $p_r$ .

Now we turn to part 1 of the proposition. Markets clear for all qualities below  $h^*$  by construction of  $\theta$  and  $\theta_r$ . Markets clearing above  $h^*$  follows because  $h^* > \bar{h}_r$  (so there are no restricted houses above  $h^*$ ) and  $\theta = \theta_u$ .

For noneligible agents, the first order condition is necessary and sufficient for optimality given differentiable and convex price functions.

For eligible agents, the first order condition  $p'_r(h) = \theta_r(h)$  says that  $h$  is the best house in the restricted area. We must also establish that they choose the area optimally. Consider first an eligible buyer  $\theta_r(h)$  who buys quality  $h$  in the restricted area. He must prefer this quality to any house  $\tilde{h}$  outside the restricted area:

$$\theta_r(h)h - p_r(h) \geq \theta_r(h)\tilde{h} - p(\tilde{h}).$$

Consider first  $\tilde{h} \leq h^*$ . To see that the inequality holds in this range under condition (B-1), first note that using the integrals for the price functions in equation (4), we can write for any house  $h$  in the restricted area and any house  $\tilde{h}$  outside

$$\begin{aligned} p(\tilde{h}) - p_r(h) &= p(\tilde{h}) - p(h) + \int_h^{h^*} (\theta_r(\hat{h}) - \theta(\hat{h})) d\hat{h} \\ &= \int_{\tilde{h}}^{h^*} (\theta_r(\hat{h}) - \theta(\hat{h})) d\hat{h} + \int_h^{\tilde{h}} \theta_r(\hat{h}) d\hat{h}. \end{aligned}$$

Using this expression, we can write the inequality as

$$\int_{\tilde{h}}^{h^*} (\theta_r(\hat{h}) - \theta(\hat{h})) d\hat{h} \geq \theta_r(h)(\tilde{h} - h) - \int_h^{\tilde{h}} \theta_r(\hat{h}) d\hat{h}. \quad (\text{B-4})$$

By condition (B-1) the LHS is positive for all  $\tilde{h} \leq h^*$ . Since  $\theta_r(h)$  is increasing on the whole interval  $[0, h^*]$ , the RHS is weakly negative for any combination of  $h$  and  $\tilde{h}$ . It follows that (B-4) is true for any  $\tilde{h} \leq h^*$ .

For  $\tilde{h} > h^*$ , we have

$$\begin{aligned} \theta_r(h)h - p_r(h) &\geq \theta_r(h)\bar{h}_r - p_r(h) \\ &= \theta_r(h)\bar{h}_r + \theta^*(h^* - \bar{h}_r) - p(h^*) \\ &\geq \theta_r(h)\bar{h}_r + \theta^*(\tilde{h} - \bar{h}_r) - p(\tilde{h}) \\ &> \theta_r(h)\tilde{h} - p(\tilde{h}) \end{aligned}$$

where the first line follows because  $\theta_r(h)$ 's first order condition holds, the equality follows from the indifference condition for  $\theta^*$ , and the second inequality follows because ineligible buyers of type  $\theta^*$  choose optimally.

It remains to show that eligible buyers  $\theta(h)$  who choose  $h$  outside the restricted area do not prefer  $\tilde{h}$  inside the restricted area. By a similar argument to above, we have

$$\begin{aligned}
\theta(h)h - p(h) &\geq \theta(h)h^* - p(h^*) \\
&= \theta(h)h^* + \theta^*(\bar{h}_r - h^*) - p_r(\bar{h}_r) \\
&\geq \theta(h)h^* + \theta^*(\tilde{h} - h^*) - p(\tilde{h}) \\
&> \theta(h)\tilde{h} - p(\tilde{h}).
\end{aligned}$$

■