Housing Demand During the Boom: The Role of Expectations and Credit Constraints

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Abstract

Optimism about future house price appreciation and loose credit constraints are commonly considered drivers of the recent housing boom. This paper infers both mean and variance of short-run expectations of future house price growth, and home equity requirements from observed household choices. The expectations and credit constraints are implied by a life-cycle portfolio choice model that encompasses home ownership, housing demand, and financing choices. I estimate the parameters of this model using data from the Survey of Consumer Finances from 1992 to 2010. The main results are that (i) expectations of future mean price growth were close to the long-run average, (ii) minimum home equity requirements were lax in the initial phase of boom, but tightened after the bust, and (iii) subjective uncertainty about the house price growth rate was large. Expectations and credit constraints are separately identified due to their differential effects on the intensive and extensive margins of housing demand. The increase in uncertainty about future prices helps to explain the rise in household debt. Given the option to default, greater subjective volatility leads to higher optimal leverage.

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1 Introduction

Low interest rates and loose lending standards are usually considered to be the main drivers of the housing boom of the 2000s. It remains an open question to which extent overly optimistic expectations about future house price appreciation contributed to the run-up[1]. This paper examines the role expectations and credit constraints played in shaping household behavior during the boom period. I do this by inferring short-run beliefs about future house price growth, both for mean and variance, and average home equity requirements from observed household choices.

The goal of this paper is not to determine the cause of the boom, but rather to test whether the choices of the majority of households during the boom can be explained by a rational model with reasonable expectations about future prices. My approach further connects the financing side of observed household choices with the extensive and intensive margins of housing demand, i.e., the decision whether to rent or own, and the amount of housing services consumed. To accomplish this, I solve a life-cyle portfolio choice model with housing, and use the optimal policies to estimate expectations and credit constraints with data from the Survey of Consumer Finances (SCF) for the period 1992 to 2010.

The main finding is that household expectations of mean price growth were relatively close to average long-run house price growth (of 2.5% annually), with slightly higher expectations at the beginning and the end of the boom. Even though the estimated mean expectations are close to the long run average, the subjective volatility for the period from 2004 to 2007 is considerably higher with an estimated standard deviation of house price growth of 25% annually. The estimation also finds low short term home equity constraints between 7.9% and 9.9% (as share of the house value at the time of the purchase) for the 1998 to 2004 period, and a tightening of constraints to 18.6% for the 2007 to 2010 period.

In order to perform this inference one must assume a structure for the path of household beliefs about time-varying parameters. I divide household beliefs into short-term beliefs,

[1]Demyanyk and Hemert (2011) and Mayer, Pence, and Sherlund (2009), among others, provide evidence that the share of newly originated mortgages with little documentation, high initial loan-to-value ratios or negative amortization increased over the period from 2000 to 2005, indicating laxer credit standards. Several authors such as Gerardi, Lehnert, Sherlund, and Willen (2008) or Burnside, Eichenbaum, and Rebelo (2011) have argued that the credit boom was driven by optimistic house price expectations, both on the side of lenders and home buyers.

which dictate expectations for the next three years (one life-cycle period in the model), and long-term beliefs that are based on long-run averages and apply to all subsequent life-cycle periods. This way I can use the model to estimate the short-term expectations for house price growth and home equity requirements, while keeping long-term beliefs about all variables set to long-run averages[2]. This structure of expectations is consistent with households believing in mean reversion.

The model used for the estimation is similar to the partial-equilibrium models developed by Campbell and Cocco (2003), Cocco (2004), and Yao and Zhang (2005). Campbell and Cocco (2015) analyze optimal default in a life-cycle model with more realistic mortgage contracts, but simplified housing choices. The emphasis of these papers is on analyzing optimal household choices in calibrated models. Li, Liu, and Yao (2009) and Bajari, Chan, Krueger, and Miller (2013) perform a structural estimation of a life-cycle model with housing similar to the one in this paper, using data from the PSID. However, their focus is on using the fitted model to conduct experiments and predict future household behavior. While Li, Liu, and Yao (2009) focus on policy experiments about changes in lending conditions, Bajari, Chan, Krueger, and Miller (2013) are predicting the length and depth of the slump in the housing market.

The estimation in this paper needs to identify three groups of parameters, (i) mean expectations, (ii) subjective volatility, and (iii) credit constraints. The estimation is performed using a Simulated-Method-of-Moments (SMM) approach. As data moments, I use the home ownership rate, house value-to-income ratios, and loan-to-value ratios. A contribution of this paper is to clarify which moments are the main source of identification for each group of parameters.

First, credit constraints are mainly identified from the intensive margin of housing demand (house value-to-income ratios). This is because the other two sets of parameters, beliefs about mean and variance of house price growth, have a limited effect on optimal house values. The key feature of the model for understanding this limited effect is the transaction cost of selling houses. The transaction cost causes inertia in the choices of existing

[2]The observed values for several other time-varying model parameters, such as interest rates and rental prices, are fed into the model during estimation.

home owners who are consequently less likely to adjust their housing demand in response to short-term fluctuations in expectations. Hence the identification mainly relies on the choices of young households who are on the margin between renting and owning[3]. These households are financially constrained, and changing their optimism about future house prices, or their subjective uncertainty about price growth, has little effect on the size of the house they are able to afford (the intensive margin). On the other hand, tightening or relaxing the home equity constraint directly impacts the house size for constrained households.

The estimated credit constraints can be understood by comparing data house values to model-predicted house values. Even absent any change in expectations or credit conditions, the model-predicted house values rise almost one-for-one with house prices since transaction costs cause most home owners to hold on to their existing houses despite the large increase in prices. The estimation then finds lax credit constraints for the periods in which house value-to-income ratios rose even more than house prices (1998-2004), and tight credit constraints for the period of declining house value-to-income ratios[4].

On the flip side, the mean expected price growth is mainly identified from the ownership rate. The decision whether to own or rent of young households is directly affected by expectations of future price growth, as it causes households close to the margin to advance or delay ownership. From the perspective of the model, very optimistic expectations would imply a counterfactually large increase in home ownership rates. The only moderately optimistic expectations are hence identified from the only moderate increase in ownership rates during the boom period.

The estimates of subjective house price growth volatility are mainly identified from loan-

[3]This model feature is consistent with recent empirical evidence by Fuster and Zafar (2014) that the housing demand of poorer and more credit-constrained households has a greater sensitivity to the change in credit conditions.

[4]The finding of tighter constraints for the 2007-2010 period during the housing market bust is consistent with evidence on tightening credit conditions during that period. The estimate of the 2004-07 constraint at 15% runs counter the well established notion that credit constraints were relaxed during the boom years. However, it is consistent with the fact that the estimates represent average home equity requirements across different segments of the mortgage market, including conforming prime mortgages and subprime mortgages. Foote, Gerardi, and Willen (2008) show that the median LTV at origination for subprime loans in Massachusetts reached 90% in 2005. Demyanyk and Hemert (2011) report an average LTV of 86% at origination for subprime loans in 2006. The 20% down payment requirement for conforming loans as defined by the GSEs remained constant throughout the boom period. There is also the possibility that the SCF data undersample the population of subprime borrowers.

to-value ratios. Estimated volatilities are clearly above the long run average at the height of the boom. A higher standard deviation of house price growth leads to increased leverage in the model. This is because home owners with defaultable mortgages are effectively holding a call option on their houses, and the value of this option increases as house price volatility rises. Households then consume part of this greater option value through higher debt today. Hence the estimates of greater house price volatility are identified from the increase in household debt[5]. The estimates show that even at times of growing house values and few observed defaults, the possibility of default may affect household choices through second moments.

While greater uncertainty about house prices increases the value of the implicit option, it also increases the probability that the option will be "out of the money", meaning that home owners default on their mortgage. Thus from the perspective of lenders, the larger uncertainty translates into greater expected default rates. An important question is whether observed mortgage credit spreads during the boom are consistent with the higher expected default rates implied by the estimated household beliefs. To address this question I compute the model-implied default and loss rates (both expected and realized). I find that data mortgage spreads do not reflect the increased credit risk implied by greater house price volatility. However, model-predicted *realized* default and loss rates are very close to ex-post realized delinquency and charge-off rates on residential mortgages in the data for the 2007-2010 period[6]. Providing an explanation for the wedge between expected loss rates implied by the estimated beliefs, and expected loss rates reflected in data credit spreads, is beyond the scope of this paper. The fact that the ex-post realized losses are consistent with the estimates of greater uncertainty suggests that household beliefs might have been "correct", and lenders did not properly incorporate the true credit risk in mortgage rates[7].

House prices are exogenous in this analysis, which therefore does not offer an explanation why house prices rose in the first place. It merely says that conditional on the realized

[5]The approximately constant leverage ratios during the boom imply a substantial increase in debt due to the large increase in house values.

[6]The model-implied realized loss rate for the 2007-2010 period is 1.7%, while the charge-off rate on residential mortgages for the same period is 1.9%.

[7]For the privately securitized segment of the mortgage market, Keys, Mukherjee, Seru, and Vig (2010) and Griffin and Maturana (2016), among others, provide evidence that originators' incentives to screen borrowers may have been inadequate. For the GSE-securitized segment of the market, Hurst, Keys, Seru, and Vavra (2015) and Elenev, Landvoigt, and Van Nieuwerburgh (2016) argue that the guarantee fees charged by the GSEs did not reflect the true mortgage credit risk.

path of house prices, interest rates, mortgage spreads, and rent-to-price ratios, household choices are implying expectations of moderate price growth, but high uncertainty. In any competitive equilibrium model that generates the observed path of house prices and interest rates, the conclusions of the demand analysis in this paper are valid. Furthermore, the result of moderate expected growth but high uncertainty (in the sense of disagreement) is consistent with a theory that relies on a small subset of agents who are very optimistic and whose actions drive price growth, such as articulated in Piazzesi and Schneider (2009) or Nathanson and Zwick (2015).

There is a growing literature on the role of expectations in housing markets[8]. Several empirical studies use surveys to elicit expectations about house prices[9]. My approach is complementary in that I infer expectations indirectly from household choices. Survey evidence on return expectations for the US housing market during the boom years is limited. Case, Quigley, and Shiller (2003) performed mail surveys of home *buyers* in 2002. Their point estimates suggest high capital gains expectations among buyers – between 6 and 11 percent per year – for different regions of the US, although they are rather imprecise[10]. Contrary, Piazzesi and Schneider (2009) report based on the 2005 Michigan Survey of Consumers that the large majority of households expressed the view that buying a house is not a good idea, and only 20% of households expected future prices to be high. The estimates in this paper are consistent with both kinds of evidence to the extent that they represent *average* expectations across potentially more optimistic buyers, and less optimistic incumbent owners and renters. Furthermore, my estimates are not pessimistic – they are expectations of continual moderate growth, both at the beginning and at the peak of the boom. They imply that households did not anticipate the bust, but expected past price gains to persist.

While quantitative survey evidence on mean expected house price gains is limited, there

[8]Recent papers on the role of expectation formation in the housing market include Piazzesi and Schneider (2009), Burnside, Eichenbaum, and Rebelo (2011), Glaeser, Gottlieb, and Gyourko (2010), Goetzmann, Peng, and Yen (2012), and Glaeser and Nathanson (2015). These papers propose different theoretical mechanisms by which expectations of future house price gains may feed back to current house prices.

[9]See for example Case, Quigley, and Shiller (2003), Case, Shiller, and Thompson (2012), and Kuchler and Zafar (2015).

[10]Case, Shiller, and Thompson (2012) report additional survey evidence on *buyer* expectations for four US metropolitan areas from 2002 to 2012. They emphasize that long-run expectations during the boom years were even more optimistic than short-run expectations, although some questions remain about the survey design.

is even less quantitative evidence on uncertainty about future house prices during this period. The survey by Case, Quigley, and Shiller (2003) finds greater standard errors for the mean expectation of respondents in 2002 than in 1998, hinting at an increase in dispersion during the boom. Similarly, the numbers reported Piazzesi and Schneider (2009) from the 2005 Michigan Survey of Consumers are indicative of greater disagreement about future house prices during the late boom period. There was certainly a discussion among academic economists and in the media during 2004 and 2005 whether the large run-up in prices constituted a bubble, see for example a special report in the Economist (2005), or studies by Case and Shiller (2005) and Himmelberg, Mayer, and Sinai (2005).

With respect to credit constraints, empirical evidence suggests that easier access to credit mattered for house prices at the regional level[11]. Another set of recent studies embed household life-cycle models in an equilibrium framework of the housing market to assess the importance of cheap credit[12]. They calibrate credit constraints and belief parameters based on external empirical evidence and focus on equilibrium effects. The contribution of this paper is to infer changes in expectations and credit constraints from observed choices. Furthermore, even though my estimates do not indicate a large relaxation of loan-to-value constraints, they are consistent with laxer credit constraints in terms of mortgage debt-to-income ratios, as they match the large rise in mortgage debt relative to incomes during the boom period.

To summarize, the US housing boom of the 2000s was characterized by a large rise in house value-to-income ratios and mortgage debt, with a relatively stable average leverage ratio and home ownership rate. The structural estimation exercise in this paper finds that neither overly optimistic expectations about future house prices nor extremely low home equity requirements are necessary to rationalize average household choices over this period. Rather, low interest rates in combination with underpriced mortgage credit risk are sufficient.

[11] Mian and Sufi (2009), Mian and Sufi (2011), and Maggio and Kermani (2015), among others, relate local house prices to local measures of credit expansion.

[12] They include Kiyotaki, Michaelides, and Nikolov (2011), Landvoigt, Piazzesi, and Schneider (2014), and Favilukis, Ludvigson, and Van Nieuwerburgh (2015). These papers have endogenous house prices and focus on the effect of relaxed credit constraints on house prices. Corbae and Quintin (2015) use an equilibrium model of the mortgage market to show that relaxation of payment-to-income constraints was important to explain the increase in mortgage debt. See Davis and Van Nieuwerburgh (2015) and Piazzesi and Schneider (2016) for surveys on the subject.

2 Model

2.1 Household Problem

A household lives for finite number of discrete life-cycle periods, $a = 0, \ldots, A$, with a probability of survival from period $a-1$ to a of λ_a, and $\lambda_{A+1} = 0$. Calendar time is indexed by t, with periods of the same length as the life-cycle periods; household age in calendar period t is denoted by a_t, such that $a_{t+1} = a_t + 1$. Every period until retirement at age a_R, the household receives labor income $Y_t(a_t)$ that follows an exogenous stochastic process. After retirement, the household receives a constant fraction of its last labor income $Y_t(a_R)$ until death. The household chooses consumption of housing services S_t and other goods C_t (the numéraire) every period to maximize expected lifetime utility. The per-period utility function $u(C_t, S_t)$ is assumed to satisfy the usual properties of being strictly increasing and concave in its two arguments arguments. Lifetime utility at age $a_0 = 0$ is given by

$$\mathrm{E}_t \left\{ \sum_{t=0}^{A} \beta^t \left[\Lambda_{a_t} \lambda_{a_{t+1}} \, u(C_t, S_t) + \Lambda_{a_t} (1 - \lambda_{a_{t+1}}) \, B_t \right] \right\},$$

where B_t is the bequest the household leaves to its children in case it does not survive until period $t+1$, and

$$\Lambda_{a_t} = \prod_{s=0}^{t} \lambda_{a_s}$$

is the unconditional probability that the household is alive in period $t < A$.

Housing has the dual role of an asset that the household can save in, and a durable consumption good that generates housing services. Households can consume housing services in two ways: they can either own or rent a house. The variable $\tau_t \in \{0, 1\}$ represents a household's decision whether to be a home owner or not in period t, with $\tau_t = 1$ indicating ownership. A house of size H_t produces housing services with the linear technology

$$S_t = \Phi(\tau_t, a_t) H_{t,}. \qquad (1)$$

The housing services production coefficient $\Phi(\cdot)$ generally depends on the home ownership status τ_t and age a_t. It captures age-dependent aspects of the preference for ownership that are not directly contained in this model, such as changes in household size and uncertainty about future household size. A unit of the housing asset sells for P_t units of numéraire, and can be rented for P_t^r in the rental market.

The household assumes that labor income and house price follow a Markov process with transition rule

$$[Y_t,\, P_t] = F\left([Y_{t-1},\, P_{t-1}],\, \epsilon_t\right), \tag{2}$$

where ϵ_t is a two-dimensional random vector distributed independently over time. I will specify the exact form of the transition rule below.

The rental price is pegged to the asset price through a deterministic, but potentially time-varying ratio

$$\alpha_t = \frac{P_t^r}{P_t}. \tag{3}$$

In addition to the housing asset, the household can save and borrow the amount L_t in a risk-free bond. By saving one unit of numéraire in the bond at $t-1$, the bond pays out $R_t > 1$ units at t. In order to borrow, the household has to own a house and use part of its value as collateral. In particular, when the household buys a house, it can at most borrow an amount $(1-\delta_t)$ of the house value to finance the purchase, where δ_t is the fraction required as a down payment:

$$L_t \geq -(1-\delta_t)P_t H_t. \tag{4}$$

Furthermore, the interest rate when borrowing is higher by a spread of ζ_t.

The budget constraint and the evolution of household wealth over time are best understood by distinguishing two cases. First, if the household did not own a house at time $t-1$, its liquid resources in period t consist of savings and interest from the previous period and current labor income. The household can use this wealth to consume the numéraire good, buy or rent units of the housing asset, and save in the risk-free asset. If the household decides to buy a house (i.e. purchase a positive amount of the housing asset), it can also borrow in the risk-free asset subject to constraint (4). Since the borrowing rate is higher than the rate for savings, the household will never optimally save and borrow at the same time. Thus it suffices to keep track of the net position L_t in the risk-free asset. This yields the following budget constraint for a household who was renting in period $t-1$

$$R_t L_{t-1} + Y_t = C_t + L_t + P_t H_t[(1-\tau_t)\alpha_t + \tau_t(1+\psi)], \tag{5}$$

subject to the home equity constraint (4), and using the fact that the rental price can be expressed in terms of the house price and the rent-to-price ratio α_t based on equation (3). The coefficient $(1+\psi)$ multiplying the expenditure on the new house in the last term accounts for a proportional maintenance cost $\psi P_t H_t$ that a homeowner must pay each period in order to offset depreciation.

The second case is that of a household who enters period t owning a house. The household may sell its current house in order to buy a new one of different size or rent instead. In this case, the sale requires payment of a transaction cost proportional to the house value, $\nu P_t H_{t-1}$. In general, the homeowner can decide to stay in the current house, and therefore not incur the transaction cost. Hence the home owner's liquid resources consist of savings and labor income as for the renter, plus the value of the house net of mortgage principal, interest, and the sales transactions cost. Denoting the decision whether to sell or keep the house by $\xi_t \in \{0, 1\}$, with 0 indicating keeping the house and 1 selling, the constraint for the home owner is

$$\begin{aligned}(R_t + 1_{[L_{t-1}<0]}\zeta_t)L_{t-1} + Y_t + P_t H_{t-1} = &C_t + L_t + (1-\xi_t)P_t H_{t-1} \\ &\xi_t\{P_t H_t[(1-\tau_t)\alpha_t + \tau_t(1+\psi)] + \nu P_t H_{t-1}\} \qquad (6)\end{aligned}$$

again subject to home equity constraint (4), and with τ_t indicating the ownership decision as in equation (5).

In addition to the decision whether to stay in the current house, sell and rent, or sell and buy, a home owner can also decide to default on its debt. In case of default, mortgage debt and home equity are erased, and the household incurs a cost of default κ that is proportional to its income. In addition, a defaulting household cannot purchase a house for one model period. This assumption reflects that foreclosed-upon previous home owners need time to rebuild their credit eligibility before being able to receive another mortgage[13]. Hence the budget constraint for the defaulting household is

$$(1-\kappa)Y_t = C_t + L_t + \alpha_t P_t H_t,$$

[13]Empirical studies on the cost of default document both monetary and non-monetary components. Guiso, Sapienza, and Zingales (2013) show that the cost of default is increasing in household wealth, but less than proportionally. The model reflects this cost structure.

subject to $L_t \geq 0$. Denote the decision whether or not to default for a home owner by $d_t \in \{0, 1\}$, with $d_t = 1$ indicating default.

Each household has to move with a certain probability every period, independent of all other shocks and previous periods. This shock is only relevant for home owners since it forces them to sell their house and incur the transaction cost. Renters sign period-by-period rental contracts, and thus their problem is unaffected. Let the outcome of this shock be denoted by $M_t \in \{0, 1\}$, with 0 indicating that the household may keep the house and 1 that it must move.

The complete life-cycle optimization problem can be stated recursively using dynamic programming. Denote the vector of state variables at time t by $X_t = [M_t, P_t, a_t, \tau_{t-1}, d_{t-1}, H_{t-1}, Y_t, L_{t-1}]$, and the vector of choice variables $Z_t = [\tau_t, \xi_t, d_t, H_t, C_t, L_t]$. Then the value function at age $a_t = 0, \ldots, A$ is defined as

$$V_{a_t}(X_t) = \lambda_{a_{t+1}} \left\{ \max_{Z_t} u\left(C_t, \Phi(\tau_t, a_t) H_t\right) + \beta \mathrm{E}_t \left[V_{a_{t+1}}(X_{t+1}) \right] \right\} + (1 - \lambda_{a_{t+1}}) B(X_t) \qquad (7)$$

subject to constraints (4), (5), and (6) and the transition equation for income and prices (2).

To close the model, I still need to specify functional forms for the intra-period utility function $u(C_t, S_t)$ and the bequest function $B(X_t)$. For the utility function, I use the conventional Cobb-Douglas form for composite utility from housing services and other goods:

$$u(C_t, S_t) = \frac{\left[C_t^{1-\rho} \left(\Phi(\tau_t, a_t) H_t \right)^{\rho} \right]^{\gamma}}{1 - \gamma}, \qquad (8)$$

where ρ determines the relative weight on housing services and γ is the risk-aversion parameter. The function $\Phi(\tau_t, a_t)$ that governs the age-dependent preference for renting is given by

$$\Phi(\tau_t, a_t) = 1 + (1 - \tau_t) \exp(-\phi a_t).$$

with parameter ϕ. If $\phi > 0$, as will be the empirically relevant case, then the additional utility from renting is decreasing exponentially with age.

To specify bequest utility, it is helpful to first define liquid wealth after the potential sale of the housing asset as

$$W_t = (R_t + 1_{[L_{t-1} < 0]} \zeta_t) L_{t-1} + \tau_{t-1}(1 - \nu) P_t H_{t-1} + Y_t. \qquad (9)$$

Bequest utility depends on liquid wealth in the household's final year and the current house price

$$B(W_t, P_t) = \bar{B}\frac{(W_t/P_t^{\rho})^{1-\gamma}}{1-\gamma}, \tag{10}$$

where $\bar{B}$ is a parameter that governs the strength of the bequest motive[14].

2.2 House Price and Labor Income Processes

Since the empirical analysis will involve cross-sections of households of different age cohorts, I will use the subscript t to index the calendar year, and i to index an individual household. The age of household i in year t will be denoted by a_{it}.

To solve the model, it is necessary to specify a parametric form for the transition rule $F\left([Y_{it-1},\ P_{it-1}],\ \epsilon_{it}\right)$ in equation 2 for income and house prices. First, I assume that the individual house price follows a random walk in logs, i.e. the growth rate of the house price is

$$R_{it}^{H} \equiv \frac{P_{it}}{P_{it-1}} = \exp(m_{t-1} + \epsilon_{it}^{H}), \tag{11}$$

where ϵ_{it}^{H} is a random variable with zero mean, and m_{t-1} is the deterministic drift. As is evident from the subscript, I assume that the drift parameter is common across all houses.

The labor income for household i in year t also follows a random walk in logs

$$G_{it}^{Y} \equiv \frac{Y_{it}}{Y_{it-1}} = \exp(f(a_{it}) + g + \epsilon_{it}^{Y}), \tag{12}$$

where $f(a_{it})$ is a deterministic life-cycle trend, g is mean aggregate income growth, and ϵ_{it}^{Y} is a random variable with mean zero. I assume that the vector $\epsilon_{it} = (\epsilon_{it}^{H}, \epsilon_{it}^{Y})$ is independently distributed over time; however, the two components may have a non-zero contemporaneous covariance $\sigma_{HY,t} > 0$ that represents a potential common exposure of housing and income risks at the regional or national level[15]:

$$\text{Var}(\epsilon_{it}) = \begin{bmatrix} \sigma_{H,t}^2 & \sigma_{HY,t} \\ \sigma_{HY,t} & \sigma_{Y,t}^2 \end{bmatrix}. \tag{13}$$

[14] The functional form of the bequest motive ensures that the value function is homogeneous in the house price. It is also sensible since it reflects that at high house prices, a given amount of wealth buys less housing consumption.

[15] Put differently, the ϵ_{it}^{j}, $j = H, Y$, include both aggregate and idiosyncratic innovations to house prices and income growth, respectively. It should be noted that, from the perspective of the optimizing household, the distinction between aggregate and idiosyncratic risk is only important to the extent that aggregate risk may induce a positive correlation between income and house price growth.

I will assume that ϵ_{it} is normally distributed. For the rest of the paper, it will then be convenient to directly write the mean and standard deviation of the log-normal random variable R_{it}^H as

$$\hat{m}_{t-1} = \mathrm{E}_{t-1}[R_{it}^H] - 1, \text{ and}$$
$$\hat{\sigma}_{H,t-1} = \mathrm{Var}_{t-1}[R_{it}^H]^{1/2},$$

respectively[16].

2.3 Time-varying Parameters and Household Beliefs

I allow a subset of parameters to vary over time. These are

- the interest rate $r_t = R_t - 1$,
- the mortgage spread ζ_t,
- the rent-to-price ratio α_t,
- the minimum home equity δ_t,
- mean house price growth $\hat{m}_t$, and
- the volatility of house price growth $\hat{\sigma}_{H,t}$.

The first four parameters are prices or other contractual features observable to households at time t. The latter two parameters represent expectations about future house prices, and I interpret them as subjective beliefs at t. However, one needs to specify a structure for household beliefs for all time-varying parameters for periods $t+1, t+2, \ldots$, since households solve a forward-looking problem for all remaining periods of their life.

The structure of beliefs I adopt is that all time-varying parameters revert to a fixed long-run value in the next model period, but may deviate from the long-run mean in the short-run (i.e. the current period). For the estimation, one model period will correspond to three years. Hence the above assumption is equivalent to households expecting mean

[16] In terms of the parameters m_{t-1} and $\sigma_{H,t}$, one therefore gets

$$\hat{m}_{t-1} + 1 = m_{t-1} + \frac{1}{2}\sigma_{H,t}^2,$$
$$\hat{\sigma}_{H,t-1} = \left[\left(\exp(\sigma_{H,t}^2) - 1\right)\, \exp(2m_{t-1} + \sigma_{H,t}^2)\right]^{1/2},$$

by the usual arithmetic for log-normal random variables.

reversion within three years, which is reasonable for the parameters considered. It is also simple enough to be computationally tractable for solving and estimating the model.

For concreteness, denote the vector of time-varying parameters as $\theta_t = [r_t, \zeta_t, \alpha_t, \delta_t, \hat{m}_t, \hat{\sigma}_{H,t}]$. Then all households share the same beliefs about the sequence of current and future realizations $\{\theta_t, \theta_{t+1}, \theta_{t+2}, \ldots\} = \{\theta_t, \theta_{LR}, \theta_{LR}, \ldots\}$, where $\theta_{LR} = [r_{LR}, \zeta_{LR}, \alpha_{LR}, \delta_{LR}, \hat{m}_{LR}, \hat{\sigma}_{H,LR}]$ is the vector of long-run values and the current realization θ_t is unrestricted.

For the estimation, all long-run values will be set to the long-run means of corresponding data time-series. The short-run values for interest rate, mortgage spread, and rent-to-price ratio will be set to their observed values in the data for each three-year period included in the estimation. The short-run values for home equity requirement, expected mean house price growth, and expected house price volatility will be estimated separately for each three-year period from the cross-section of households in the SCF.

To implement this belief structure for any given date t, one generally needs to solve a separate life-cycle dynamic program for each age cohort, with the current period's parameters given by θ_t, and the parameters for all future periods given by θ_{LR}. However, without time-variation in parameters, the dynamic program is independent of calendar time. Therefore it suffices to once compute a full life-cycle program as specified in equation (7) with the constant long run-parameters to get $\{V_a(X;\, \theta_{LR})\}_{a=0}^{A}$. These value functions can then be used as continuation values to compute the decision problem for each age cohort in period t, yielding a set of short-run value functions $\{V_{a_t}^{SR}(X_t;\, \theta_t)\}$ and corresponding policy functions $\{Z_{a_t}^{SR}\}$, defined by

$$V_{a_t}^{SR}(X_t;\, \theta_t) = \lambda_{a_{t+1}} \left\{ \max_{Z_{a_t}^{SR}} u\left(C_t, \Phi(\tau_t, a_t) H_t\right) + \beta \mathrm{E}_t \left[V_{a_{t+1}}(X_{t+1};\, \theta_{LR}) \right] \right\} + (1 - \lambda_{a_{t+1}}) B(X_t). \tag{14}$$

To compute the dynamic program efficiently in practice, the problem can be transformed to reduce the state space. Further, the computation is best performed in terms of two different value functions (both normalized as above) and the resulting optimal policies: one for households who were renting in the previous period or those who were forced to sell and move due to the exogenous shock, and one for homeowners that have the additional option of staying in their current house. Appendix A contains details on these transformed value functions and the corresponding budget constraints and transition equations for the states.

2.4 Discussion

Several assumptions deserve a brief discussion. First, note that the model specified above yields the optimal demands for housing conditional on age, income, wealth, home ownership status, and the price of the housing asset. I do not explicitly specify the equilibrium in the markets for the housing asset or housing services. However, the goal of this analysis is to infer implied household beliefs about future price growth, and in any competitive equilibrium households will take the house price P_t as given. Therefore, the exercise of inferring implied expectations from observed demands is well-defined without an explicit specification of equilibrium as long as the optimal demand functions are evaluated at realized equilibrium prices[17].

2.4.1 Transaction Cost and Ownership Decision

The most important aspect of the distinction between owning and renting arises from the transaction cost for selling houses. In the absence of the transaction cost, the recursive structure of the problem implies that in addition to the household's age, only the beginning-of-period net worth and income are relevant state variables. However, with the transaction cost homeowners have the option of not selling their house and thus not incurring the cost. This creates inertia in homeowners' adjustments to changes in the economic environment. Hence the quantity of housing owned at the beginning of the period, H_{t-1}, becomes a state variable.

Young households face a life-cycle labor income profile with a deterministic component that is increasing. These households want to frequently adjust the level of housing services as their incomes rise during the early part of their life-cycle. However, if they chose to become home owners, the transaction cost would punish frequent upgrades in house size, and the down payment requirement makes a house that would also be large enough later in life unaffordable to the young household. Thus, the home equity constraint in equation (4) deters young households from becoming home owners too early. Instead, the cash-poor, but

[17] Of course, an implicit restriction on equilibrium results from the assumed time-series properties of house prices as specified in equation (11).

human capital-rich constrained young households rent and save until they have enough cash for the down payment of a house that is large enough. As previous quantitative analyses have found, the life-cycle pattern of ownership induced by borrowing constraints is however not effective in explaining the low rate of ownership among young households who have sufficient funds for their down payment. To account for household mobility observed in the data, the shock M_t is a reduced-form way of modeling that homeowners may have to move and sell their house for reasons exogenous to the model, such as job-related relocations. To exactly match the life-cycle profile of ownership, the model further contains a preference for rental housing that is declining in household age. This preference stands in for non-financial considerations driving the home ownership decision of young household, such as uncertainty about future family size[18].

2.4.2 Leverage and Mortgage Default

Labor income is upward-sloping over the life-cycle, but it is not tradable. The net present value of the non-risky trend part of future labor income for young households is similar to a long position in a safe asset. For realistic parameterizations of the income process and housing returns, it is optimal for young households to offset this position by taking a short position in the actual risk-free bond. Due to the collateral constraint, going short the risk-free bond means taking out a mortgage to finance the purchase of a house, and in this way achieve the optimal portfolio composition of risky and safe assets[19]. As households in the model age, they reduce their leverage and instead hold a positive position of the safe assets. The amount of savings of old households largely depends on the strength of the bequest motive.

The possibility of default on the mortgage interacts with the optimal leverage choice. A defaultable mortgage means that households hold a call option on their house, with leverage taking on the role of the strike price. Exercising the option is equivalent to keeping the house and not defaulting on the mortgage. The net value of the option is decreasing in the

[18]The hazard rate of the mobility shock M_t is decreasing in age, reflecting the greater mobility of young households. However, to fully match the low ownership rate among young households, an additional weak preference for rental housing is required.

[19]See Yao and Zhang (2005) for a detailed discussion of the optimal portfolio composition with labor income and housing as collateral.

cost of default[20]: if the cost was prohibitively large, the optionality would disappear and households would simply hold a long position in the housing asset. Further, the value of the call option is decreasing in the strike price (i.e. leverage), but increasing in the mean m *and* the volatility σ_H of the house price. Any increase in the value of the option makes the household wealthier today. Everything else equal, this leads to higher consumption and greater debt today. For example, if perceived house price volatility goes up, the option becomes more valuable ceteris paribus, and households optimally react by increasing the strike price of the option (the leverage ratio) and consuming some of this option value today. In summary, this means that leverage is increasing in the option value, so any factor that raises the option value also raises optimal leverage.

This analysis of course only considers household demand at given market prices. Any change in the value of the option and consequently leverage will generally also affect the optimal default decision. If lenders rationally anticipate the resulting changes in default rates, this should affect equilibrium mortgage rates and potentially reverse the original effect on household demand.

3 Data and Estimation Procedure

3.1 Data Description

To estimate short-term expectations about house prices and credit constraints over the period of the recent housing boom, I use the cross-sections from years 1992 to 2010 of the Survey of Consumer Finances (SCF). The SCF contains detailed information on the wealth composition and income of a representative sample of U.S. households[21]. Since the data are only available in three-year increments, I set the length of a model period to three years[22].

[20] The default cost consists of the lost income κY and the indirect cost of being excluded from credit markets for one model period (3 years). The total cost is akin to the option premium.

[21] The Federal Reserve conducts the survey every three years. The SCF oversamples rich households who hold the majority of aggregate U.S. wealth, but also provides sampling weights that can be used to calculate statistics based on a representative U.S. sample. This paper only computes statistics from the SCF using the sampling weights.

[22] It would also be possible to set one model to period to one year, and then aggregate the model output into three-year periods when matching to the data. An earlier version of the paper took this approach. The results were similar, but increasing the number of life-cycle periods by factor three of course increases computation time by the same proportion.

For each of the SCF surveys from 1992 to 2010, I use the prepared extract sample of the SCF. I remove all observations with the household head being younger than 25 years of age, which is the starting age of the life-cycle labor income profile I use. I take labor income to be broadly defined as the sum of wage income, income from social security and other retirement funds, income from own businesses, and government transfers. As definition of net worth, I use the pre-generated variable "networth" from the SCF, which is the balance of all household assets and liabilities. For the house value of homeowners, I use the SCF variable "houses", which is the value of the primary residence[23]. As the mortgage principal of homeowners, I use the SCF variable "mrthel", which includes home equity loans and other types of loans that use the primary residence as collateral.

Further, I remove all households with more than 5 million dollars of net worth (in year 2000 dollars) from the sample. The life-cycle income process of these very wealthy households is usually not well described by the one assumed in equation (12), since a large fraction of their income is from dividends and capital gains. These households tend be older, with traditional sources of retirement income only being a very small fraction of their overall income. The removal of these households has the additional advantage of being able to economize on grid points during the estimation. The disadvantage is a loss of about 15% of raw observations for each year, but due to the strong oversampling of wealthy households in the SCF this only amounts to about 1.5% of effective observations after applying the SCF-provided sampling weights.

Table 1 provides means and standard deviations for the variables used in the estimation.

3.2 Calibrated Parameters

Table 2 shows the values of the time-varying parameters discussed in section 2.3 that are not estimated. Since the estimation period consists of the years 1998, 2001, 2004 and 2007, the values of these parameters are inputs for the estimation.

Table 3 shows the constant parameters of the model that I do not estimate. The top panel list the long-run means of the time-varying parameters, and the bottom panel reports

[23]This implies that other real estate investments of the household will be included in net worth and hence are counted as savings in the sense of the model.

the remaining parameters.

The last two columns of table 2 contain realized aggregate income and house price growth for each three-year period. Real aggregate income growth is estimated from NIPA disposable household income. House price growth is calculated from the FHFA house price index (deflated by the CPI). These realized growth rates are required for the simulation step of the simulation.

With respect to the time-varying parameters, both short-run realizations and long-run means are measured from the same data sources. The interest rate is computed as the real annualized yield of 3-year treasury bonds. To calculate the rent-to-house-price ratio, I deflate the aggregate FHFA house price index by the CPI for rental prices to obtain a series for the price-to-rent ratio. I then take the value of 5.5% as computed by Davis, Lehnert, and Martin (2008) and extrapolate this number over the sample period by scaling it with the inverse of the FHFA/CPI index growth. Mortgage spreads are computed as the difference between the 30-year fixed mortgage rate reported by Freddie Mac and yields on 20-year Treasuries[24].

Turning to the long-run values of the time-varying parameters to be estimated, I set the minimum home equity constraint to 15%. This number reflects that for the majority of borrowers over the sample period, it was possible to get a mortgage with a down payment amount below the 20% limit that the government-sponsored enterprises set for conforming loans. If we think of the parameter δ as a stand-in for the average ease of access to credit, setting it to the GSE-imposed limit for prime conforming loan is to tight, as high LTV loans were available both in the prime and subprime segments of the market from the beginning of the sample[25].

The expected long-run price growth of the housing asset is set to 2.5%. The underlying assumption is that aggregate house prices are growing at the same rate as aggregate income in the long term. The number is also consistent with average growth rates of regional and national house price indexes, such as the FHFA or the Case-Shiller S&P 500 index.

[24] An alternative way to isolate the mortgage spread would be to compute the difference between 1-year ARM rates and 1-year T-bill yields. However, 1-year ARM are far less common and their pricing may not be representative of the majority of mortgages. The results when using this alternative measure would be similar.

[25] Of course a literal interpretation of δ as the minimum possible home equity for all available loan contracts in the market would imply a number around zero for most of the sample. However, this would not be representative of the typical mortgage options offered to the average borrower.

The volatility of house price growth is set to 18% annually. This number reflects purely idiosyncratic house price risk, which Landvoigt, Piazzesi, and Schneider (2014) document to be between 9% and 11%. In addition, the innovation ϵ_t^Y also includes aggregate housing risk at the regional and national level, which is between 5% and 9% based on MSA house price indexes (see e.g. Flavin and Yamashita (2002)).

The sales transaction cost and the maintenance share are in line with the values used by other studies of the housing market. The transaction cost reflects the actual cost of selling such as realtor's fees and the cost of moving for homeowners (over renters). The maintenance share is the fraction of the house value that homeowners have to spend to offset depreciation. The monetary cost of default κ is set to 10 percent of annual household income. Recall that the total cost of default includes not being able to purchase a home for three years (one model period). Unfortunately, there is little direct empirical evidence on the cost of default. Foote, Gerardi, and Willen (2008), Bhutta, Dokko, and Shan (2010), Bajari, Chu, Nekipelov, and Park (2013), and Guiso, Sapienza, and Zingales (2013) empirically study foreclosure decisions of households and arrive at the conclusion that home owners do not immediately default once their home equity becomes negative. The fact that clearly negative home equity of -30% or more is observed for most households going into foreclosure, suggests a substantial cost that can take the form of exclusion from credit markets, legal recourse, or psychic costs. The cost of default as calibrated implies that similar magnitudes of negative home equity are required to cause optimal default in the model (the exact foreclosure threshold in the model depends on income, house value, and age).

The annual standard deviation of the shock to permanent income growth is set to 13% based on the results of Cocco, Gomes, and Maenhout (2005). The correlation of both shocks is set to 0%, based on the low estimate by Flavin and Yamashita (2002). Other studies have found slightly higher correlations[26].

Finally, I take three sets of parameters from the literature that enter the household problem due to its life-cycle character.

- The deterministic part of labor income growth ($f(a)$ in equation 12) follows a third-

[26] Robustness checks with correlation values of 20% and 40% mainly affected the estimated preference parameters, but had little effect on the expectation estimates.

degree polynomial whose coefficients are taken from Cocco, Gomes, and Maenhout (2005), and thus are consistent with the shock to income growth. Specifically, I use coefficients describing the income profile of high-school graduates estimated by Cocco, Gomes, and Maenhout (2005) using data from the PSID. The life-cyle profile has the common hump-shape.

- The survival probabilities λ_a are computed from the mortality rates reported by the National Center of Health Statistics.

- I estimate the life-cycle profile of mobility (i.e. the probabilities of moving) from 2000 census data as in Landvoigt, Piazzesi, and Schneider (2014). The basic shape of the mobility rate function over the life-cycle is convex and declining in age.

3.3 Estimation and Target Moments

3.3.1 Estimation Procedure

The estimation uses a Simulated Method of Moments (SMM) approach applied to a dynamic model and repeated cross-sections. Indexing households by i, I construct a sample $S_t \equiv \{a_{it}, \tau_{it-1}, W_{it}, P_{it}H_{it-1}, Y_{it}\}_{i=1}^{N_t}$ from the SCF for each model period t, where a_{it} is the household age, τ_{it-1} indicates ownership status (rent vs. own), and the remaining variables denote net worth, house value, and labor income as defined in the model description.

Denote the vector of model parameters to be estimated by θ. Solving the model given all parameters (including θ) yields optimal policies as functions of the state variables. Given the policy function, it is now possible to calculate the optimal choices for each household in the sample, $Z(S_t, \theta) = \{C_{it}, \tau_{it}, L_{it}, H_{it}, d_{it}\}_{i=1}^{N_t}$, with C_{it} denoting numéraire consumption, τ_{it} next period's ownership status, L_{it} the mortgage or savings amount, H_{it} the size of the house being rented or owned in the next period, and d_{it} is the default decision. These year-t choices can in turn be mapped to year-$t+1$ state variables by simulating the house price, income, and mobility shock realizations for each household in the sample, and by applying the realized aggregate price and income growth from t to $t+1$ (the last two columns of table 2). Applying the model policies to sample S_t in this way thus leads to a simulated sample of next year's state variables $\hat{S}_{t+1}(S_t, \theta)$, that is a function of this year's observed state variables and the model parameters.

The estimation procedure essentially entails finding the parameter vector θ that minimizes the distance (in a method-of-moments sense) of the simulated $t+1$-samples $\hat{S}_{t+1}$ constructed in the way outlined above, and the observed $t+1$-samples S_{t+1} for each of the years 1992/1995, 1998, 2001, 2004, and 2007.

There are several practical issues to be dealt with. In particular, the data are repeated cross-sections, but computing standard errors for the estimates requires a panel data structure. I apply a pseudo-panel approach based on age-eduction groups to solve this problem. Appendix B contains a detailed description of the pseudo-panel approach, the implementation of the SMM estimator, and how standard errors are computed.

3.3.2 Target Moments and Estimated Parameters

I estimate (i) several preference parameters, which I restrict to be identical for all periods, (ii) expected house price growth $\{\hat{m}_t\}_t$ and its volatility $\{\hat{\sigma}_{H,t}\}_t$, and (iii) average home equity requirements $\{\delta_t\}_t$, which I allow to take different values for each period.

Mean and volatility of house prices and the credit constraint parameters are estimated for the years 1998, 2001, 2004, and 2007. To estimate the preference parameters, I use additional moments from the survey years 1992 and 1995. For these years, I assume that the economy is in a long-run "steady state", with short term beliefs about house prices and short term credit constraints being equal to their long term values. The reason for this strategy is that I do not want to overuse preference parameters such as risk aversion and discount factor to explain household choices during the extreme boom-bust episode of the years 1998-2010[27].

Table 4 lists the estimated parameters and the target moments. I will discuss the estimates in the next section. As moments in the objective function, I use the average homeownership rate, the value-to-income ratio and the loan-to-value ratio for each of the years 2001, 2004, 2007, and 2010, and for the combined sample 1992/1995. In addition, I include the average LTV ratio among older households (age 58 or older) for the initial 1992/1995 sample. This gives 16 moments and 12 parameters when only the utility parameters, the means of

[27] Since all parameters are jointly estimated, preference parameters are of course partially identified from the moments of the boom years. However, including additional moments of the preceding years stabilizes the estimates to reasonable values.

the house price growth process and the minimum home equity shares are estimated. Four parameters are added when the volatilities of house price growth are estimated in addition. The choice of these moments rests mainly on their natural connection to model quantities. The homeownership rate is calculated as the sample average of households' discrete own-versus-rent decisions. Similarly, the house value-to-income ratio is the sample average of a state variable of the model, and the loan-to-value ratio is the ratio of two choice variables, mortgage principal and house value.

The choice of LTV ratios as target moments deserves some discussion. The model only allows for a positive or negative position in the bond, so when matching it to the data one could proceed in two different ways: (i) one can either ignore bond-like asset holdings and match purely mortgage-based loan-to-value ratios, or (ii) one can match a net position in fixed income assets relative to the house value. This distinction is potentially important, since many home owners with mortgages also have holdings of fixed-income assets. These are usually small balances on savings and checking accounts. To gauge the importance of these simultaneous bond holdings, I compute a net position in fixed-income assets for all households. For the majority of households, this position is negative and close to the mortgage balance. I then use this variable to compute an alternative LTV ratio, which is defined as the ratio of the negative net fixed-income position to house value. Table 1 displays this alternative definition of the loan-to-value ratio in the row "Net Debt LTV". One can see that averages are smaller but close to the purely mortgage-based LTV. For the estimation, however, I decide to use the standard definition of the LTV ratio, since I want changes in the ratio over time to be driven by changes in mortgage debt and house prices only. Using the second kind of leverage ratio would confound mortgage-related changes with changes in overall savings.

4 Results

4.1 Estimation Results

Table 4 shows results of the estimation step. The asymptotic standard errors in parenthesis are calculated using the GMM formula for the case with a constant weighting matrix.

Appendix B contains details on how the standard errors were computed. Specification (1) keeps the short-run volatility of house price expectations fixed at the long-run value of 0.18, while specification (2) also estimates these parameters.

The point estimates of the preference parameters in specification (1) and (2) are very close, suggesting that they are pinned down by average levels of the different moments across all periods. The addition of the volatilities as free parameters does not significantly affect the estimates of mean expectations either. The point estimates are all close to the long-run mean of 2.5%. The estimates for 1998 and 2007 are somewhat higher, while the estimate in 2001 is lower.

The point estimates of the minimum home equity shares do not have a clear pattern in specification (1), and neither of the estimates are significantly different from the long-run value of 15%. When freeing up the volatilities in specification (2), however, the estimated home equity shares exhibit an increasing trend. At the same time, the estimated subjective volatility of house price growth is below the long-run mean in 1998 and 2001, but above the mean in 2001 and 2004.

4.2 Model Fit

The top panel of table 5 compares model and data values of the targeted moments (for specification (2), in bold font). The model is exactly identified and matches the targeted moments almost perfectly[28].

As an "out-of-sample" test for the model, it is useful to evaluate the fit of the model for several outputs that are not target moments. First, one concern with the asset side of the model may be that the only risky asset for savings is housing. As the bottom panel of table 5 shows, the model matches wealth-to-income ratios well. For the base period 1992-1997, the data average cash-to-income ratio is 5.92, while the model generates a ratio of 5.65. The model predicts too little wealth for the period of high stock valuations from 1998 to 2001.

The model generates slightly too low housing expenditure for renting households, with the ratio of annual rental payments to income in the data being 16.2% for the base periods,

[28]The estimation achieves a SMM objective of approximately zero. The small deviations are due to the model's nonlinearities and the discreteness of the approximation grid.

and 13.1% in the model. The only households renting in the model are young households who are saving up for eventual home ownership. In the data, renters most likely also include older households that rent for a different reason. The model correctly captures the rise in rental expenditure during the boom observed in the data.

Furthermore, the model also generates realistic consumption-to-income ratios. For the base period, the model produces a ratio of 93.9%. The aggregate savings rate for the U.S. for this period implies a ratio a consumption-to-income ratio of 92.8%.

Finally, the model also produces realistic levels of persistence in household decisions. The row labeled "Fraction Stayers" compares the fraction of home owners in the data who bought their house more than 3 years ago, to the fraction of home owners in the model-generated sample who kept their previous house (i.e., who did not adjust their house size at the beginning of the three-year model period). For the base period, this number is 73.3% in the data and 68.7% in the model, demonstrating that the model contains the right magnitude of transaction costs.

I further examine model's cross-sectional fit to understand how well the model's mechanisms capture the actual heterogeneity in choices in the data, despite only targeting aggregate moments. Table 6 demonstrates that the inference about the estimated parameters does not come at the expense of counterfactual cross-sectional implications. The table shows the three main model outcomes, broken down by age and net worth, and comparing the data to the model-generated sample. For all three choice margins, the model is able to qualitatively match the pattern in the data.

With respect to the home ownership rate, the pattern in the data is best described by the statement that young/poor households rent, whereas old/wealthy households own their homes. While the model generally matches this pattern, the model is not able to explain the steep increase of the ownership rate in net worth for the oldest group of households. The credit constraint in the model mainly affects young households in connection with their upward-sloping labor income profile and the transaction cost. Unlike in the data, where most poor old households rent, the optimal choice for these households according to the model is to own a small house[29].

[29] Some other studies impose a minimum house size for owner occupancy to deal with this issue. While

The model matches the cross-sectional distribution of house value-to-income ratios reasonably well, again with the exception of the data values for poor and old households, who report very low ratios in the data.

Similar to the ownership rate, the model generates the overall pattern for leverage across age and wealth groups that we observe in the data. However, it somewhat overstates the leverage of young households, and it undershoots for older households. The reason for the too low LTV ratios of old households is the lack of other assets that can function as savings devices in the model.

Overall, the model describes the cross-sectional distribution of ownership, house values, and leverage reasonably well. It should hence provide a good basis for estimation.

4.3 Identification

In the following, I will explore the sources of identification for the results. Most of the identifying variation comes from young households who are on the margin of becoming home owners. The transaction cost creates an inactivity region that prevents older existing homeowners from adjusting their house size or selling their house in response to small changes in expectations or credit constraints. Finally, I will discuss how expected house price growth, borrowing constraints and the volatility of house price growth are separately identified.

4.3.1 Preference Parameters

The chief source of identification for the preference parameters are the four moments in the base year (1992/1995) of the estimation, for which beliefs and credit constraint are set to their long-run values. The Cobb-Douglas weight ρ is the most important determinant of value-to-income ratios in the model, and thus is identified from the mean of this ratio in the data. The discount factor β determines model leverage and is hence identified from average LTV ratios in the data. The estimated value of 0.804 is for one model period of three years, implying an annual discount factor of 0.93. The discount factor interacts with the bequest motive to determine the effective age-dependent discount factor in the model. The parameter $\bar{B}$ that governs the strength of the bequest motive is therefore identified from the leverage

such a restriction would improve the cross-sectional fit of this model as well, it would not significantly alter the estimation results based on targeting data averages.

ratio of older households[30]. The parameter of the rental preference factor, ϕ, regulates the home ownership rate among young households. Since both in model and data renters are mostly young households, it is identified from the average home ownership rate in the data.

4.3.2 Expectations, Down Payment Constraints, and Volatility

Main Sources of Identification Generally each parameter simultaneously affects all three choice margins for a given year. Therefore describing the identification amounts to understanding which moment is quantitatively most important for each type of parameter.

The estimates for home equity constraints are mainly identified from the intensive margin of housing demand, i.e., house value-to-income ratios. This is because the two other parameters governing expected house price gains (mean and volatility) are less powerful in determining model-predicted house values. There are two reasons for the relatively weak effect of short-term expectations on optimal house values. First, given the large transaction cost of selling a house, existing home owners will not adjust their house size in response to a moderate short term change in expected house prices. Secondly, new home *buyers* (previously renters) are mostly at their leverage constraint. Hence changing these buyers' degree of optimism has little effect on their optimal house values.

However, changing the minimum down payment for these constrained buyers directly affects their optimal house value choice. Loosely speaking, the much stronger impact of the collateral constraints on model-implied house values relative to the other parameters forces the estimation to "use" variation in the constraints to match data house values. This means in turn that the estimates of the home equity constraints are effectively pinned down by the house value-to-income ratios in the data.

Even though expectations have a limited effect on the intensive margin, they do have a large effect on the extensive margin – they decision whether to own or rent. Again home buyers who enter the period as renters are the source of identification. Their short term benefit of owning versus renting is directly affected by expected house price gains. Thus households on the margin of buying will decide to advance (delay) their purchase of a house in response to a positive (negative) change in expected price growth.

[30]The combined estimates of β and $\bar{B}$ imply a reasonable life-cycle profile of effective discount factors when combined with the survival probabilities.

The third set of parameters to be estimated – price growth volatilities – are mainly identified from the third set of moments, which are the loan-to-value ratios. The main effect of an increase in subjective house price risk is greater optimal leverage, through the call option channel discussed in section 2.4 above. Since debt is frictionlessly adjustable in the model, households consume the additional future wealth from the increased value of the call option by borrowing more[31]. The effects on the intensive and extensive margins of housing demand are smaller, and they are ambiguous. A rise in the option value makes owning a house more attractive, but from a portfolio perspective, higher house price risk makes housing less attractive as an investment.

Time-varying House Prices and Interest Rates It is useful to break the identification argument in several steps using graphical representations of the objective function. Each step involves comparing data moments to model-implied moments for a different set of model inputs, while the utility function parameters are set to their estimated values from table 4. First, I will consider the hypothetical case that the only variation in model inputs over time is the estimation sample. In other words, all model parameters, including the realized aggregate price and income growth, are set to their long-run values for each year. The top row of figure 1 shows data and model-generated moments for this case. The model almost perfectly matches ownership rates over time, but significantly misses data value-to-income ratios and loan-to-value ratios.

The next step is to feed in the non-estimated time-varying parameters from table 2. These are interest rates, rent-to-price ratios, mortgage spreads, and realized price and income growth for each 3-year period. The resulting model-generated moments are shown in the middle row of figure 1 by the solid red line. Simulating the model using the realized price gains and low interest rates and spreads in 2004 to 2010 drives value-to-income ratios up significantly. For the 2001-2004 period, interest rates are already lower than the long-term average, but rent-to-price ratios are still close to the long-term average, which results in a model-predicted ownership rate that is too high. From 2004 to 2010, the drop in interest

[31]It turns out that household optimization keeps the option value roughly constant. When volatility increases and the option value rises holding everything else equal, households choose higher leverage which is equivalent to choosing a higher strike price of the call option. This reduces the option value and increases current consumption.

rates is counteracted by a simultaneous drop in rent-to-price ratios, keeping the ownership rate stable. Leverage is too low for the 2001-2007 period. Note that in 2001 and 2004, even though households substantially increase their mortgage debt, leverage stays roughly constant due to the large realized rise in house values. In the last model period, the large realized drop in house prices raises leverage to the data value.

Time-varying Estimates We can think of the solid red line in the middle row of figure 1 as the starting point for the estimation of the time-varying parameters, $\hat{m}_t$, δ_t, and $\hat{\sigma}_{H,t}$. The estimation procedure can choose these parameters for each period to make the model-implied moments match the data.

The third row of figure 1 shows the model fit if the mean expectation and home equity parameters are set to their estimates from specification (1), but volatilities are held fixed at their long run values (the solid blue line). The estimation matches home ownership rates and house value-to-income ratios closely just by varying these two sets of parameters. Low expected house price growth for the 2001-04 period lowers the model-generated ownership rate in 2004 to the data value, while at the same time low estimated home equity requirements prevent house values and leverage from falling below the respective data values. However, it is apparent that the model has difficulty matching LTV ratios.

Freeing up the volatility parameters allows the estimation to also match LTV ratios by choosing a combination of credit constraints and volatilities that fits leverage and house value-to-income ratios simultaneously, as can be seen from the solid black line. This requires laxer credit constraints (to increase house values) and lower volatility (to decrease leverage) from 1998-2004, and tighter credit constraints (to decrease house values) and higher volatility (to increase leverage) from 2004-2010, which is what we see in specification (2) in table 4.

To summarize, the estimates of mean expected house price growth are mainly identified from the extensive margin – the home ownership rate – in the data. The estimated values of the credit constraints and volatilities, on the other hand, are mainly identified from jointly matching the intensive margin – house value-to-income ratios – and leverage.

The key take-away from the estimates is that on average, household choices during the boom are not consistent with very optimistic expectations about mean house price growth.

The reason is that home ownership in the data increased only moderately over the boom period. From the perspective of the model, optimistic expectations would imply a counterfactually large increase in home ownership. However, the estimation finds large subjective uncertainty about house price growth during the boom. This rise in uncertainty is identified from the large increase in mortgage debt: despite large realized increases in house prices, home owners even slightly increased leverage. The model explains this rise in debt through an increased value in the implicit call option given by the combination of house and defaultable mortgage.

4.4 Subjective Volatility and Expected Losses

The volatility estimates are generated in a model of household demand. Equilibrium interest rates and mortgage credit spreads are taken as given, and are set to their observed data values. As households' subjective beliefs change over time, so will their implied optimal default decisions. For example, an increase in the subjective volatility of house price growth will imply a greater probability of default. From the perspective of mortgage lenders, this means greater expected credit losses. A key question is then if the expected losses given household beliefs are consistent with observed credit spreads.

To address this question, I compute two kinds of model-generated default and loss rates. First, I compute *expected* default and loss rates conditional on current parameters, using only expected price and income growth. For each period and each household in the sample, I compute the anticipated default decision at the beginning of the next model period, given current optimal policies with respect to house and mortgage. I can then compute a state-contingent default rate for the sample of households who choose to have a mortgage (for each future state), and an expected default rate by applying the probabilities implied by the estimated beliefs. To get expected loss rates, I additionally compute the state-contingent loss given default from the perspective of lenders who repossess the house through foreclosure. For household i optimizing at date t, I calculate this loss rate given default as

$$1 - \frac{\text{House Value}_{i,t+1}}{\text{Mortgage Balance}_{i,t+1}},$$

for each future state for which default is optimal. Note that in the model households only default when their net worth becomes negative, and this can only occur because of negative home equity. The default threshold is approximately at the net worth to income ratio of -14%. In the data, a few households have positive home equity, but negative net worth, probably because of other debts that are not included in the mortgage variable. Hence the best way to summarize the wealth of households who optimally default according to the model, is to report the net worth to income ratio of these households.

In addition, I compute a *realized* model-implied default and loss rate for each period. These are defined in the same way as the expected rates, with the only difference that for each household I impose realized aggregate income and house price growth over that period (instead of the estimated mean house price growth or the constant expected aggregate income growth). However, as volatility of the simulated shocks I still use the estimates from table 4.

The results of these calculations are displayed in table 7. The model-implied expected default rates are moderately higher than the average delinquency rate in the data with a model-implied average of 4.7% for the steady-state period 1992/95[32]. Model-implied expected loss rates are close to the mortgage spreads observed in the data that are fed into the model during estimation. The model produces a long-run expected loss rate of 0.9%, compared to an average mortgage spread in the data of 1.1%. To the extent that the data mortgage spread represents expected credit losses, the model produces a time-varying wedge between losses implied by model beliefs and observed expected losses. For example, for the 1998/2001 period with low estimated volatility the model only implies 0.3% losses, whereas the credit spread during this period was 1.4%. However, during the 2004/07 period with high estimated volatility the spread was only 0.9% in the data, but the model-implied expected loss rate is 1.4%.

The model does well with respect to realized default rates, especially during the bust period 2007/10. Realized losses in the data are charge-off rates on single-family residential mortgages. Especially for the loss rate, the model predicts low losses during the boom when high price growth made foreclosures less likely: for the 2001/04 period, the data value is 19

[32]The average delinquency rate for single-family residential mortgages for the longest available sample from 1991-2014 is 3.99% according to the data series published by the Federal Reserve Board.

basis points and the model value is 27 basis points. For the 2007/09 period, when house prices dropped sharply, the model produces a 1.7% loss rate, while the corresponding data value is 1.9%.

The bottom two rows provides further insight into the model-generated default policies. The average net worth to income ratio for defaulting households is always less than negative 15%. The negative net worth stems from negative home equity for most of these households. The income and credit market exclusion penalties in the model make default optimal only for significantly negative net worth. The expected default rate is higher than the realized default rate in all periods but the last. The difference between expected and realized default rates is mainly due to realized house price growth that deviates from expectations.

There are two general potential explanations for the wedge between the expected losses to lenders implied by household beliefs and the expected losses reflected in mortgage spreads: (i) households and lenders are in disagreement, with households perceiving greater uncertainty during the boom, or (ii) both groups had the same beliefs, but lenders did not properly incorporate mortgage risk in mortgage interest rates. The second potential explanation seems plausible with respect to the recent housing boom. For the privately securitized share of the mortgage market, the evidence suggests that originators during the boom did not have incentives to properly screen borrowers (see e.g. Keys, Mukherjee, Seru, and Vig (2010)), and investors in mortgage-backed securities may have been misinformed about the true riskiness of the loans (see e.g. Griffin and Maturana (2016)). For the large conforming share of the market with mortgage insurance provided by the government-sponsored enterprises (GSEs), it is further questionable whether the insurance fee charged by the GSEs properly reflected the level or the time-variation in aggregate default risk (Elenev, Landvoigt, and Van Nieuwerburgh (2016)), or the regional variation in risk (Hurst, Keys, Seru, and Vavra (2015)). Therefore, rates were potentially insensitive to changes in risk in both market segments.

5 Conclusion

This paper estimates expectations of house price appreciation. The inference is based on structural estimation of a life-cycle dynamic program that encompasses housing demand choices at the extensive and intensive margin. The methodology I develop allows for short-term expectations that are unrestricted in the time series and identified from the cross-section of household choices in a given time period.

The main result is that model-implied aggregate expectations of future price growth were very close to the long-term average throughout the period from 1998 to 2010, with slightly higher expectations at the beginning and the end of the boom period. The stability in expected house price gains is needed to explain the stability in the ownership rate – with very high expectations, the model would predict a counterfactually large rise in the ownership rate.

The estimation further finds that high uncertainty about future house prices during the boom years may have contributed to the increase in mortgage debt. Everything else equal, higher uncertainty leads to an increase in the value of the call option on housing that is implied by the combination of owning a house with a defaultable mortgage. If debt is more easily adjusted than housing, households will optimally consume part of this higher option value through higher debt. The higher volatility estimates also imply higher mortgage default rates. In fact, the model predicts realistically small default rates for the boom period and realistically large default rates for the bust. It also implies higher ex-ante expected losses for mortgage lenders that are not reflected in data mortgage spreads during the boom.

Overall, the quantitative results are consistent with households beliefs in mean-reverting house price growth. The results also imply a high sensitivity of the housing demand of credit-constrained home buyers to changes in credit conditions and expectations, and a low sensitivity of the housing demand of existing home owners.

The results of this paper do not contradict the notion that the recent housing boom was partly fueled by overly optimistic expectations of house price appreciation. Only a relatively small share of the housing stock gets traded per year, and these transactions form the basis for price measurement. It is hence possible that few very optimistic households caused the price spike by increasing short-term demand in local housing markets. Aggregate beliefs, however, are identified from observing the majority of households who did not substantially adjust their housing choices during the boom.

It would be an interesting extension to allow heterogeneous beliefs across age or income groups, and for different geographic areas, to see whether the approach taken here could identify those subgroups with optimistic beliefs that possibly were the driving force behind the strong price movement.

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Table 1: Summary Statistics

	1992/95	1998	2001	2004	2007	2010
Home Owner[a]	0.69	0.71	0.73	0.74	0.74	0.72
	0.46	0.45	0.45	0.44	0.44	0.45
Age[a]	49.01	49.47	50.22	51.01	51.47	52.32
	16.38	16.10	16.14	16.32	16.34	16.27
Income[a]	63.90	68.46	72.85	74.98	73.91	72.30
	52.28	55.52	60.73	62.49	64.73	68.96
Net worth[a]	221.59	266.11	339.11	357.27	376.80	349.89
	152.35	148.76	190.22	258.58	275.02	248.60
House value[b]	168.52	180.25	212.77	271.21	294.63	249.17
	152.35	148.76	190.22	258.58	275.02	248.60
Mortgage[b]	56.97	70.29	76.51	101.23	110.56	104.88
	80.98	86.34	101.55	121.53	139.11	143.02
Mortgage LTV[b]	0.32	0.38	0.36	0.39	0.38	0.44
	0.33	0.35	0.34	0.34	0.33	0.40
Net debt LTV[b]	0.27	0.29	0.26	0.30	0.30	0.36
	0.36	0.40	0.37	0.39	0.38	0.45
Annual rent[c]	4.68	5.93	6.62	7.32	7.74	9.09
	3.22	3.72	4.25	5.12	6.04	7.19
# obs	27770	14717	15533	15868	15354	24741

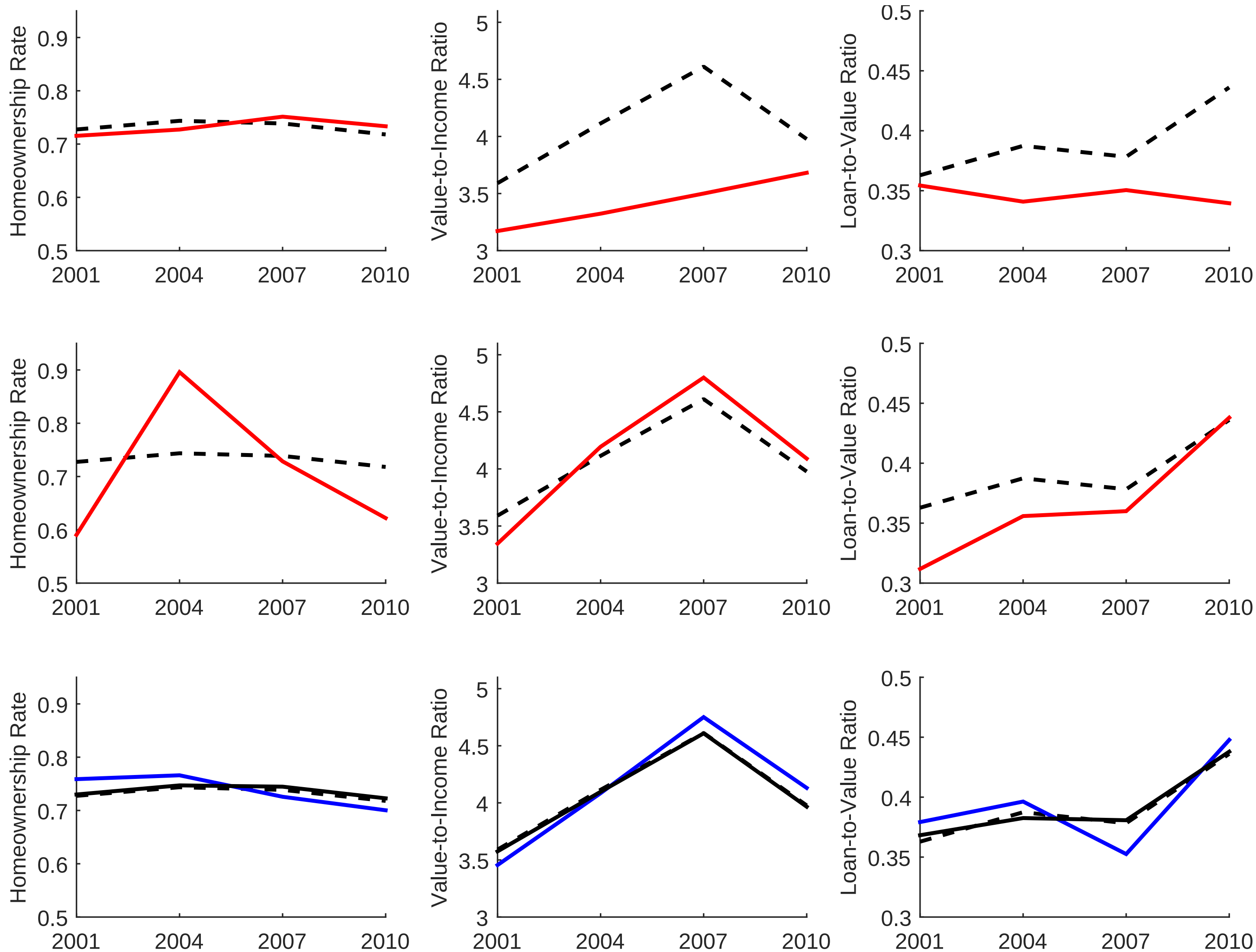
Table reports means (top row) and standard deviations (bottom row) of the variables from the SCF used during estimation. All observations with net worth greater than $5 million (in 2004 dollars) have been removed from the sample, resulting in a loss of approximately 1.5% of the sample-weighted observations. See text for details.

[a] Home ownership status, age, income, and net worth are reported for all households in the estimation sample.

[b] House value, mortgage amount, and the two LTV ratios are reported for home owners only. LTV is the ratio of mortgage balance to house value. Net debt LTV is the ratio the negative net fixed income position to house value.

[c] Annual rent is only reported for renters.

Figure 1: Target and model-generated moments



The figure compares model-generated (solid lines) to data moments (dashed line). **Top row:** The only model input varying over time are the SCF samples that are fed into the model as distribution of households' state variables. **Middle row:** The only model input varying over time are the SCF samples and realized rent-to-price ratios, mortgage spreads, interest rates, and price and income growth rates. **Bottom row, blue line:** Expected growth rate of house prices and down payment requirements are set to their estimated values. **Bottom row, black line:** In addition, price volatilities are set to their estimated values.

Table 2: Short-run Model Inputs

Year t	r_t	ζ_t	α_t	ΔP_{t+1}	ΔY_{t+1}
1998	3.24	1.43	5.40	10.0	8.0
2001	0.86	1.10	5.19	14.0	6.0
2004	0.73	0.93	4.02	16.0	5.0
2007	0.73	1.18	3.30	−19.0	−2.0

All values annual in percent. r_t is real average annualized interest rate over the three-year period based on 3-year treasury yields. α_t is rent-to-price ratio calculated by rescaling 1992 base value of 0.06 over time. ζ_t is calculated as the difference of the 30-year fixed conventional mortgage rate and 20-year treasury yields.
The last two columns contain realized aggregate house price and income growth from t to $t+1$.

Table 3: Calibrated Parameters

Parameter	Value
Interest rate r_{LR}	3%
Rent-to-price ratio α_{LR}	5.5%
Mortgage spread ζ_{LR}	1.5%
Minimum down payment δ_{LR}	15%
Expected price growth $\hat{m}_{LR}$	2.5%
Volatility of price growth $\hat{\sigma}_{H,LR}$	17%
Risk aversion γ	3
Sales transaction cost ν	10%
Maintenance share ψ	2%
Std.Dev.(ϵ_{it}^Y)	13%
Corr($\epsilon_{it}^Y, \epsilon_{it}^H$)	0%
Income growth $\hat{g}$	2.5%
Cost of default κ	10%

The top panel reports the long-run means of the time-varying parameters listed in section 2.3. The bottom panel lists values of other constant parameters. All parameters are annual.

Table 4: Estimated Parameters and Target Moments

Estimated Parameters						Target Moments		
		$\sigma = 0.18$		σ flexible				
Housing weight	ρ	0.128	(0.006)	0.128	(0.006)	Home Ownership	1992/95	0.708
Discount factor	β	0.804	(0.011)	0.804	(0.017)	Value-To-Income[a]	1992/95	3.174
Bequest motive	$\bar{B}$	2.640	(0.852)	2.443	(0.219)	Loan-To-Value[b]	1992/95	0.369
Rent utility	ϕ	0.819	(0.150)	0.880	(0.252)	Loan-To-Value (>58)[c]	1992/95	0.148
Down payment	δ_{1998}	0.155	(0.162)	0.079	(0.033)	Home Ownership	2001	0.728
	δ_{2001}	0.109	(0.024)	0.099	(0.027)		2004	0.744
	δ_{2004}	0.158	(0.067)	0.158	(0.041)		2007	0.739
	δ_{2007}	0.162	(0.040)	0.186	(0.072)		2010	0.718
Expected price growth	μ_{1998}	0.034	(0.001)	0.034	(0.001)	Value-To-Income	2001	4.197
	μ_{2001}	0.008	(0.006)	0.006	(0.003)		2004	4.371
	μ_{2004}	0.026	(0.007)	0.027	(0.005)		2007	4.537
	μ_{2007}	0.032	(0.005)	0.035	(0.004)		2010	4.326
Price Volatility	σ_{1998}			0.088	(0.034)	Loan-To-Value	2001	0.363
	σ_{2001}			0.153	(0.056)		2004	0.387
	σ_{2004}			0.236	(0.091)		2007	0.378
	σ_{2007}			0.206	(0.064)		2010	0.436

Preference parameters are constant across all estimation years. Down payment requirement as fraction of house value, expected price growth, and (subjective) price volatility are estimated separately for each of 1998, 2001, 2004, and 2007 for three year periods.

Target moments are sample means from the SCF data described in section 3.1.

[a] Value-to-Income is the ratio of the value of the primary residence to annual income.

[b] Loan-to-Value is the ratio of all mortgage loans secured by the primary residence to the value of the primary residence.

[c] Loan-to-Value (>58) is the average LTV ratio among households with household head of age 58 or older.

Table 5: Model Fit

	1992/95		1998/2001		2001/04		2004/07		2007/10	
	D	M	D	M	D	M	D	M	D	M
	Target Moments[a]									
Home Ownership	**0.708**	**0.713**	**0.727**	**0.730**	**0.744**	**0.747**	**0.739**	**0.745**	**0.718**	**0.723**
Value-To-Income	**3.174**	**3.180**	**3.591**	**3.575**	**4.114**	**4.097**	**4.612**	**4.609**	**3.977**	**3.966**
Loan-To-Value	**0.369**	**0.366**	**0.363**	**0.368**	**0.387**	**0.383**	**0.378**	**0.381**	**0.436**	**0.438**
Loan-To-Value (>58)	**0.148**	**0.150**	0.157	0.094	0.178	0.115	0.204	0.201	0.257	0.254
	Other Model Quantities									
Fraction Stayers	0.733	0.687	0.729	0.678	0.702	0.656	0.735	0.638	0.802	0.674
Cash-To-Income	5.926	5.651	6.695	5.793	6.530	6.403	6.840	6.334	6.228	5.740
Rent-To-Income	0.162	0.131	0.188	0.139	0.203	0.173	0.221	0.223	0.269	0.183
Consumption-To-Income[b]	0.928	0.939	0.951	0.939	0.953	0.997	0.965	0.982	0.953	1.062

Comparison of data and model-generated moments.

[a] Bold entries are moments included in objective function of estimation (i.e. "targeted" moments).

[b] Data consumption-to-income ratio is computed as $1-$ the personal savings rate of U.S. households.

Table 6: Cross-sectional model fit in base period (1992/1995)

	Data			Model		
	Net worth			Net worth		
	≤ 30	$> 30 \leq 150$	> 150	≤ 30	$> 30 \leq 150$	> 150
	Home Ownership Rate					
Age ≤ 40	0.296	0.681	0.868	0.323	0.628	0.690
$> 40 \leq 61$	0.358	0.817	0.944	0.823	0.901	0.888
> 61	0.385	0.871	0.954	0.588	0.751	0.780
	Value-to-Income Ratio					
Age ≤ 40	2.161	2.015	2.331	2.487	2.381	2.559
$> 40 \leq 61$	2.087	2.426	2.680	2.609	2.724	2.787
> 61	2.036	3.932	5.718	2.706	4.321	5.357
	Loan-to-Value Ratio					
Age ≤ 40	0.674	0.658	0.520	0.799	0.756	0.470
$> 40 \leq 61$	0.507	0.483	0.366	0.737	0.470	0.151
> 61	0.261	0.137	0.077	0.562	0.096	0.018

All averages computed using SCF sampling weights.

[a] Net worth is the SCF variable with such name measured in thousands of 2001 dollars.

Table 7: Credit Risk and Expected Losses

	1992/95	1998/2001	2001/04	2004/07	2007/10
			Data		
Credit Spread[a]	0.011	0.014	0.011	0.009	0.012
Realized Loss Rate[b]	0.002	0.001	0.002	0.001	0.019
Realized Default Rate[b]	0.033	0.022	0.021	0.016	0.061
			Model		
Expected Loss Rate[a]	0.009	0.003	0.011	0.014	0.006
Realized Loss Rate[c]	0.005	0.001	0.003	0.003	0.017
Realized Default Rate[c]	0.032	0.006	0.026	0.027	0.059
Average Net Worth of Defaulters[d] (percent of income)	-15.4	-15.4	-17.0	-17.1	-15.1
Expected Default Rate[e]	0.047	0.029	0.056	0.054	0.039

[a] Data credit spread is mortgage spread used for estimation (spread of 1-year adjustable mortgage rate over one-year Tbill yield). Model expected loss rate is sample average of model-predicted probability of default times expected loss rate given default (for subsample of households that have a mortgage). Loss rate given default is computed as $1 - 1/\tilde{\mathbf{LTV}}_i$ for each default state.
[b] Data realized default rate is average delinquency rate for residential real estate loans secured by primary residence, held by commercial banks. Data loss rate is average charge-off rate on these loans (data source: Federal Reserve Board).
[c] Model realized default rate is sample average of simulated default decisions for subsample of households that have a mortgage, given realized aggregate income and price growth over the period; model realized loss rate is corresponding loss rate defined analogous to expected loss rate, but with realized income and price growth.
[d] Average net worth of defaulters is the average ratio of net worth to income for the households in the sample that optimally default according to the model-implied decision rule, in percent.
[e] Model expected default rate is sample average of model-predicted probability of default for subsample of households that have a mortgage.

A Dynamic Programming Solution

A.1 Transformed Problem

The state and the choice variables of the dynamic program given by equation 7 can be redefined to allow for a more efficient computational solution. These transformations are also the basis for the mapping of model quantities to observables.

First, after omitting i subscripts for notational simplicity, we can normalize all model quantities by current income Y_t, which is equivalent to normalization by permanent income due to the i.i.d. nature of the innovations to income growth. Specifically, define $w_t = W_t/Y_t$ and $\bar{h}_{t-1} = P_t H_{t-1}/Y_t$ for the endogenous state variables and $c_t = C_t/Y_t$, $l_t = L_t/Y_t$ and $h_t = P_t H_t/Y_t$ with respect to choices. All housing related quantities are expressed in terms of expenditure since this is what we observe in the data. I reduce the choices of both owner and renter to the value of the occupied house h_t, which is possible due to the linearity of housing services production from the housing asset. Thus letting the vector of transformed state variables be given by $x_t = [M_t, \tau_{t-1}, d_{t-1}, w_t, \bar{h}_{t-1}, l_{t-1}]$ and the vector of choice variable by $z_t = [\tau_t, d_t, \xi_t, d_t, h_t, c_t, l_t]$, one can define the normalized value function $v_t(x_t) = V_t(X_t)/(Y_t P_t^{-\rho})^{1-\gamma}$ to get

$$\begin{aligned} v_{a_t}(x_t) =& \lambda_{a_{t+1}} \left\{ \max_{z_t} \frac{\left[c_t^{1-\rho}\left(h_t \Phi(\tau_t, a_t)\right)^{\rho}\right]^{1-\gamma}}{1-\gamma} + \beta \, \mathrm{E}_t \left[v_{a_{t+1}}(x_{t+1}) \ \left(G_{t+1}^{Y} (R_{t+1}^{H})^{-\rho}\right)^{1-\gamma} \right] \right\} \\ & + (1-\lambda_{a_{t+1}}) b(w_t) \end{aligned} \tag{15}$$

subject to conformably rewritten budget and home equity constraints given in section A.2, and where G^Y and R^H are the growth rates of income and house price as defined in equations (12) and (11). It becomes apparent from equation (15) that the normalization of the value function eliminates two exogenous state variables for computational purposes, which are income and the house price.

A.2 Value Functions of Movers and Stayers

First, denote the value function of the household who did not own a house or has sold its house as $v_t^M(w_t)$, where $w_t = W_t/Y_t$ and W_t is defined as in equation 9 in the main text. $v_t^M(\cdot)$ is further defined as the value conditional on survival until age a_{t+1}, and after all shocks are realized. Thus one gets

$$v_t^M(w_t) = \max_{c_t, l_t, \tau_t, h_t} u(c_t, h_t) + \beta \, \mathrm{E}_t \left[v_{t+1}(x_{t+1}) \; \left(G_{t+1}^Y (R_{t+1}^H)^{-\rho} \right)^{1-\gamma} \right] \tag{16}$$

subject to

$$w_t = c_t + l_t + (1 - \tau_t)\alpha_t h_t + \tau_t (1 + \psi) h_t, \tag{17}$$

$$l_t \geq -\tau_t \, (1 - \delta_t) h_t. \tag{18}$$

where $v_t(x_t)$ is defined as in equation (15), the constraints (17) and (18) are obtained by normalizing the budget constraint (5) and the downpayment constraint (4) by income Y_t, and the utility function $u(c, h)$ is defined in equation (8). Secondly, denote the value function of a home owner who is *forced* to stay in the same house as $v_t^S(w_t, \bar{h}_{t-1})$. Again, I define $v_t^S(\cdot)$ as the value conditional on survival until a_{t+1}, and after realization of the mobility shock. This yields

$$v_t^S(w_t, \bar{h}_{t-1}) = \max_{c_t, l_t} u(c_t, \bar{h}_{t-1}) + \beta \, \mathrm{E}_t \left[v_{t+1}(x_{t+1}) \; \left(G_{t+1}^Y (R_{t+1}^H)^{-\rho} \right)^{1-\gamma} \right] \tag{19}$$

subject to

$$w_t + \nu \bar{h}_{t-1} = c_t + l_t + (1 + \psi) \bar{h}_{t-1}. \tag{20}$$

$$l_t \geq -(1 - \delta_t) \bar{h}_{t-1}. \tag{21}$$

Due to the definition of w_t as including the house value net of the transaction cost, the household that stays in the same house receives this cost back on the LHS of budget constraint 20. Thus the house value $\bar{h}_{t-1}$ cancels on both sides of the constraint, i.e. the constraint becomes

$$1 + (R_t + 1_{[l_{t-1} < 0]} \zeta_t) l_{t-1} = c_t + l_t + \psi \bar{h}_{t-1}.$$

To deal with the possibility of default, we further define the value function of a household who can only rent as

$$v_t^R(w_t) = \max_{c_t, l_t, h_t} u(c_t, h_t) + \beta \, \mathrm{E}_t \left[v_{t+1}(x_{t+1}) \; \left(G_{t+1}^Y (R_{t+1}^H)^{-\rho} \right)^{1-\gamma} \right] \tag{22}$$

subject to

$$w_t = c_t + l_t + \alpha_t h_t. \tag{23}$$

$$l_t \geq 0. \tag{24}$$

One can now express the normalized value function $v_t(\cdot)$ in terms of $v_t^M(\cdot)$ and $v_t^S(\cdot)$

$$
\begin{aligned}
v_t(\tau_{t-1}, M_t, w_t, \bar{h}_{t-1}) = \lambda_{t+1} \big[& \tau_{t-1}(1 - M_t) \max \left\{ v_t^R(1-\kappa), v_t^M(w_t), v_t^S(w_t, \bar{h}_{t-1}) \right\} \\
& + (1 - \tau_{t-1})\, v_t^M(w_t) \\
& + \tau_{t-1} M_t \max \left\{ v_t^R(1-\kappa), v_t^M(w_t) \right\} \big] \\
& + (1 - \lambda_{t+1})\, b(w_t).
\end{aligned}
\tag{25}
$$

The first term in square brackets represents the choices of the household who enters the period owning a house ($\tau_{t-1} = 1$) and does not have to move ($M_t = 0$). This household can either stay in the same house ($v^S(\cdot)$), sell the current house and face the same optimization problem as a renter ($v^M(w_t)$), or default on its mortgage and face the problem of a renting household ($v^R(\cdot)$) with cash equal to $1 - \kappa$ percent of its income.

The second term represents the choices of the households entering the period without owning a home ($\tau_{t-1} = 0$). The third term is the choice set of the home owner who has to move ($\tau_{t-1} M_t = 1$). This household also has the choice of defaulting.

The two endogenous state variables of the model are w_t, and $\bar{h}_{t-1}$. To express their transition laws, it is useful to define the discrete choice variable $d \in \{0, 1\}$, with $d = 1$ indicating that the household defaults on its mortgage. Then the transitions for w_t and $\bar{h}_{t-1}$ are

$$
w_{t+1} = (1 - d)[(R_{t+1} + 1_{[l_t < 0]} \zeta_{t+1}) l_t + \tau_t (1 - \nu) h_t \, R_{t+1}^H] \, \frac{1}{G_{t+1}^Y} + 1 - d\kappa, \tag{26}
$$

$$
\bar{h}_t = (1 - d) \tau_t h_t \, \frac{R_{t+1}^H}{G_{t+1}^Y}, \tag{27}
$$

where l_t, h_t, and τ_t denote the optimal savings and housing policies for period t.

The dynamic program specified by equations (16) to (27) can be solved recursively starting in period T, where

$$
v_T(x_T) = b(w_T)
$$

since $\lambda_{T+1} = 0$. To compute the value functions $v_t^M(\cdot)$, $v_t^R(\cdot)$ and $v_t^S(\cdot)$ in practice, I discretize the continuous state variables w_t and $\bar{h}_{t-1}$ on grids with 80 points each. The spacing of the grid points for w_t and $\bar{h}_{t-1}$ is chosen with the goal of estimation in mind such that the points are denser on intervals where more households in the SCF sample are located. Further, the boundaries are chosen such that almost all observations fall within the state space. The innovations to the income an house price processes ϵ_{t+1}^{Y} and ϵ_{t+1}^{H} are assumed to be jointly normally distributed, and are discretized using the method of Tauchen (1986). I use 3 nodes for the income innovation and 7 nodes for the house price innovation. Since for house price growth, the variance is also estimated, it is important to have enough nodes in the discretization. Increasing the number of nodes above 7 did not affect the estimation results. The shock forcing a home owner to move M_{t+1} is independent of both the house price and income shocks, and distributed as a $\{0,1\}$-Bernoulli random variable with $\Pr(M_{it+1} = 1 \mid a_{it})$. I use linear interpolation to compute the continuation value in case the the next period state variables do not lie on the grid.

B Estimation Procedure

B.1 Pseudo-Panel Structure

As objective function for the estimation step I use a weighted sum of squared deviations of a set of data averages from averages of the simulated sample. Since the data are repeated cross-sections and the model is dynamic in nature, a pseudo-panel approach is needed to apply the SMM approach described above.

The basic methodology follows Browning, Deaton, and Irish (1985). Using the same notation as above, let $\hat{S}_t$ and S_t denote the simulated and the data samples for year t, respectively. Since the sample $\hat{S}_t$ was generated by applying the model solution to the year-t data sample S_{t-1}, these samples generally consist of different individual households, so it is not possible to state moment conditions at the level of an individual observation. However, one can divide each sample into Q cells based on observed characteristics that are stable between times t and $t+1$, which here is a three-year period between two consecutive SCF samples. Index cells by $q = 1, \ldots, Q$, and let $g_{qt} = g(q, S_t)$ denote a K-vector of sample means for cell q in year t, where in the application the elements of g_{qt} are the average homeownership rate, the value-to-income ratio and the loan-to-value ratio (i.e. $K = 3$). In practice, I use seven birth cohorts and three education groups to get a total of $Q = 21$ cells. Let the vector $\hat{g}_{qt}(\theta) \equiv g(q, \hat{S}_t; \theta^{LR}, \theta_t)$ denote the vector of sample means for the same variables, but computed from the simulated sample. By treating each cell q as an observation with variables taking on the values of cell means, I can hence create a pseudo-panel with Q observations.

Let g_q and $\hat{g}_q(\theta)$ denote the TK-vectors of the stacked cell means for all T years. Then the TK sample moment conditions are

$$\frac{1}{Q}\sum_{q=1}^{Q} g_q - \hat{g}_q(\theta) \equiv G_Q - \hat{G}_Q(\theta) = 0, \tag{28}$$

and for the case of fewer than TK parameters in θ, the Generalized Method of Moments (GMM) objective function to be minimized in θ is

$$(G_Q - \hat{G}_Q(\theta))' D (G_Q - \hat{G}_Q(\theta)), \tag{29}$$

where D is a positive definite weighting matrix. I use the inverse variance-covariance matrix of the data moments for D, i.e. $D = \hat{\mathrm{Cov}}(g_q)^{-1}$. Note that G_Q and $\hat{G}_Q$ are simply the aggregate sample means in the real and simulated data, for all K variables and T years. However, for the computation of the estimated variance-covariance matrix of the moment conditions, it is necessary to have the pseudo-panel structure and a well-defined notion of an observation.

Equation (29) is a conventional GMM objective function with a constant weighting matrix, and the asymptotic standard errors can generally be obtained in the well-known way (see e.g. Wooldridge (2002)). Since this is a simulation estimator, the estimated covariance matrix of the moment conditions needs to be adjusted by a factor taking into account the number of simulations. Appendix B contains details on how the standard errors are calculated, drawing on the econometric results of Pakes and Pollard (1989).

B.2 Implementation of the Simulation Estimator

Define the year-t sample of variables from the SCF that correspond to the model's normalized state variables

$$s_t = \left\{a_{it}, \tau_{it-1}, w_{it}, \bar{h}_{it-1}\right\}_{i=1}^{N_t},$$

which are as defined in section 3. The goal is to create the model-implied sample $\hat{s}_{t+1}(s_t, \theta)$ of simulated year-$t+1$ state variables. From the transition equations for the state variables (26) and (27) it is clear that the required ingredients for this step are

- the model policy functions for housing, bonds, and home ownership related choices,
- simulated random variables for the move shock M_t and the shocks to income and house price growth $(\epsilon^Y_{t+1}, \epsilon^H_{t+1})$ for each observation,
- and the realized aggregate return to housing ΔP_{t+1} and realized aggregate income growth ΔY_{t+1}.

In the following, I will state the algorithm to be applied to each observation of sample s_t in order to construct the simulate sample $\hat{s}_{t+1}$.

1. Take observation i from sample s_t. Dropping the observation and time subscripts, denote age by a, liquid funds by w, homeownership status by τ, and the value of the house owned or rented by $\bar{h}$. Denote the realized aggregate growth in house prices and income from t to $t+1$ as g^H and g^Y.

2. If $\tau = 1$, that is the current observation is a home owner, draw a uniform random variable u_1 and simulate move shock $M = 1[u_1 < \Pr(M = 1\,|a)]$, where $1[\cdot]$ denotes the indicator function.

3. Using the optimal policies from the model's computational solution with parameter vector θ, calculate the model-implied household choices
$$\begin{aligned} l &= \hat{l}_a(M, w, \tau, \bar{h}; \theta) \\ \tau' &= \hat{\tau}_a(M, w, \tau, \bar{h}; \theta) \\ d &= \hat{d}_a(M, w, \tau, \bar{h}; \theta) \\ h &= \hat{h}_a(M, w, \tau, \bar{h}; \theta). \end{aligned}$$

4. Draw a pair of normally distributed random variables (ϵ^Y, ϵ^H) for the innovations to house price and income growth.

5. Apply the transition equation for the state variables to get next period's implied states, i.e. compute
$$\begin{aligned} w' &= (1-d)[Rl + \tau'(1-\nu)\exp(g^H + \epsilon^H)\, h]\exp(-(f(a+1) + g^Y + \epsilon^Y)) + 1 - d\kappa \\ \bar{h}' &= (1-d)\tau'\, h\exp(g^H + \epsilon^H - f(a+1) - g^Y - \epsilon^Y). \end{aligned}$$

6. Set $a_{it+1} = a + 1$, $\tau_{it} = \tau'$, $w_{it+1} = w'$, and $\bar{h}_{it+1} = \bar{h}'$ to obtain the simulated state variables for this observation for year $t + 1$.

By applying this algorithm to each observation in the sample s_t once, one obtains the simulated sample $\hat{s}_{t+1}(s_t, \theta)$. After repeating this procedure for each pair of consecutive years, one has the complete set of data and simulated samples, s_{t+1} and $\hat{s}_{t+1}(s_t, \theta)$.

Given these samples, the computation of moments and the construction of the GMM objective function proceeds as outlined in the main text. Denote by g_q the TK-vector of data means for birth cohort-education cell q, and by $\hat{g}_q$ the corresponding vector of simulated means. The aggregate moment conditions are computed as in equation 28, and the estimator for θ is defined based on equation 29 as

$$\hat{\theta} = \text{argmin}_{\theta}(G_Q - \hat{G}_Q(\theta))' D (G_Q - \hat{G}_Q(\theta)). \tag{30}$$

To minimize this distance function, I employ a global pattern search algorithm over a range of model parameters for which the dynamic programming solution is well-behaved. This algorithm is essentially an intelligent grid search that only uses direct function evaluation and does not compute any numerical derivatives. Once the search seems close to a minimum, I employ a simplex search algorithm until convergence. In the case of the exactly identified model (with free volatility parameters) the search succeeds to find a local minimum as it manages to reduce the objective function to a value very close to zero.

The objective is sufficiently smooth to be numerically differentiable using bi-directional differentiation at a delta of 1%. This should be sufficient to calculate approximate gradients for standard errors once the minimum is found.

Based on the results of Pakes and Pollard (1989), the SMM estimator's asymptotic distribution is normal with

$$\sqrt{Q}(\hat{\theta} - \theta^*) \xrightarrow{d} N(0, \Lambda_1^{-1}\Lambda_2\Lambda_1^{-1}).$$

To write the expressions for Λ_1 and Λ_2, first define the Jacobian matrix of the population moments with respect to the parameters evaluated at $\theta*$

$$A = \mathrm{E}\left[\nabla_{\theta}(g_q - \hat{g}_q(\theta^*))\right].$$

Then one gets

$$\Lambda_1 = A'DA,$$

and

$$\Lambda_2 = A'D\Omega DA,$$

where Ω is the variance-covariance matrix of the population moments

$$\Omega = \mathrm{E}\left[(g_q - \hat{g}_q(\theta^*))(g_q - \hat{g}_q(\theta^*))'\right].$$

A consistent estimator for the variance-covariance matrix of $\hat{\theta}$ is thus

$$\frac{1}{Q}\,(\hat{A}'D\hat{A})^{-1}\,(\hat{A}'D\hat{\Omega}D\hat{A})\,(\hat{A}'D\hat{A})^{-1},$$

where $\hat{A}$ and $\hat{\Omega}$ are consistent estimators of A and Ω and are calculated as

$$\hat{A} = \nabla_\theta(G_Q - \hat{G}_Q(\hat{\theta})) \text{ and}$$
$$\hat{\Omega} = \frac{1}{Q}\sum_{q=1}^{Q}(g_q - \hat{g}_q(\hat{\theta}))(g_q - \hat{g}_q(\hat{\theta}))'.$$