Safety Transformation and the Structure of the Financial System

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Abstract

This paper models how the financial system is organized to efficiently create safe assets and its response to changes in safe asset supply and demand. Bank-like financial intermediaries choose the least risky portfolio that backs their issuance of riskless deposits- a diversified pool of non-financial firms' debt. Non-financial firms choose their capital structure to exploit the resulting segmentation between debt and equity markets. Higher demand for safe assets yields larger and riskier intermediaries and more levered firms. Quantitative easing reduces the size and riskiness of intermediaries and can decrease firm leverage, despite reducing borrowing costs at the zero lower bound.

An important role of financial intermediaries is to issue safe, money-like assets, such as bank deposits and money market fund shares. As an empirical literature has documented (Krishnamurthy & Vissing-Jorgensen 2012, Sunderam 2015, Nagel 2016), these assets have a low rate of return, strictly below the risk-free rate they would earn without providing monetary services. Agents who can issue these assets therefore raise financing on attractive terms, capturing the "demand for safe assets" that pushes their cost of borrowing below that of others. This paper presents a general equilibrium model of how the financial

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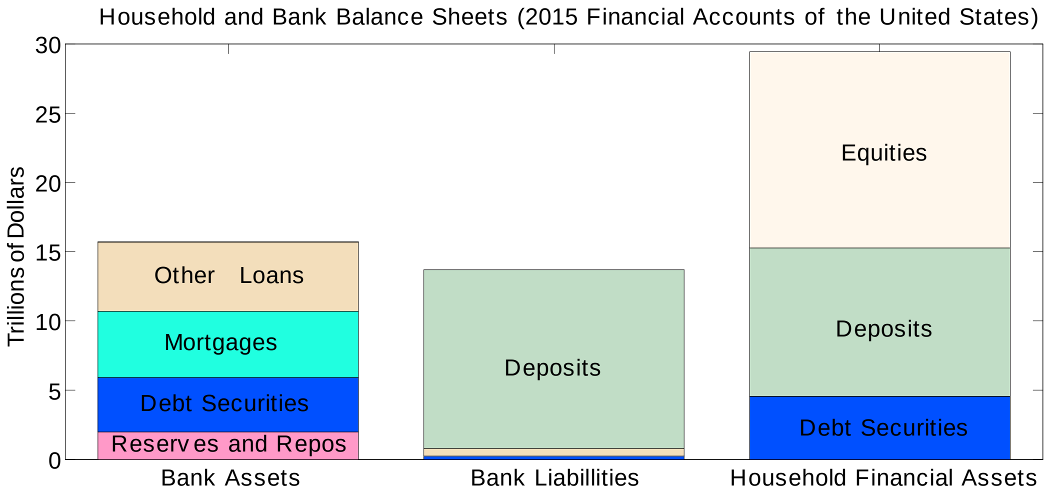
system is organized to meet this demand for safe assets and studies how the system as a whole adjusts when the supply or demand for safe assets changes.



The assets owned by money-creating financial institutions are primarily loans and debt securities issued by firms, households, and governments.[1] Of the \$17.3 trillion of assets owned by depository institutions in the USA in 2015, \$4.8 trillion were mortgages, \$3.9 were debt securities including \$2.1 trillion of agency and GSE backed securities, \$5.0 trillion were non-mortgage loans to firms and households, and \$2.0 trillion were reserves, while only \$100 billion were equities which are held primarily by households.[2] While money creation in the "shadow banking" system is harder to measure, money market funds, securitization vehicles, and broker dealers that play a role here also invest significantly in debt. Two of these stylized facts emerge endogenously in my model's equilbrium. First, debt is held by money-creating financial intermediaries while equities are held by households. Second, financial intermediaries are necessary, since non-financial firms are not able to issue a large enough supply of safe assets on their own.

Three basic ingredients are at the core of the model. First, households obtain utility directly from holding riskless assets, which captures the demand for money-like assets without modeling the frictions

[1] The bank assets listed above are not quite the total \$17.3 trillion. The remainder is almost entirely \$720 billion of "miscellaneous assets", \$240 billion of foreign direct investment, and \$156 billion of life insurance reserves. The \$157 billion of equities and mutual fund shares is too small to label in the figure and plays a negligible role.

[2] Household portfolio holdings are based on the assumption that their mutual funds are 70% equity and 30% debt, consistent with data from the Investment Company Institute's Investment Company Fact Book. 37% of households' direct holdings of debt securities are municipal bonds where they face a tax advantage over other investors.

that make money essential (Stein 2012*b*). This implies that firms have an incentive to issue riskless debt, since households are willing to lend at a low risk free rate. Second, the output of non-financial firms are subject to idiosyncratic risk. This constrains their ability to issue riskless debt and implies that a financial intermediary with a diversified asset portfolio can issue more riskless assets than firms can alone. Third, financial intermediaries have managerial agency costs that increase with their size. This implies that intermediaries choose the smallest asset portfolio that backs a given quantity of additional riskless assets, which is composed of all of the risky debt of the non-financial sector. Equities are too exposed to systematic risk and would back fewer riskless assets in a given size portfolio, so in equilibrium they are held directly by households.

In the model, a continuum of projects with exogenous output (Lucas trees) provide all resources and must be managed by firms. Firms choose whether to buy a single tree or act as a financial intermediary who can invest in securities. Each tree-owning non-financial firm chooses to issue as much riskless debt as possible, an additional low risk debt security, and a high risk equity security. These securities are exposed to both aggregate and tree-specific idiosyncratic risk, and this idiosyncratic risk ensures that non-financial firms' debt cannot entirely meet households' demand for riskless assets. Households cannot commit to make future payments, so they cannot issue safe assets themselves. This provides a role for intermediaries, who buy a diversified portfolio of non-financial debt which is safe enough to back a large quantity of riskless deposits with a small buffer of equity capital to bear any systematic risk. Intermediaries do not buy equity securities because their higher systematic risk implies they back fewer riskless deposits, and they do not buy firms' riskless debt since this does not increase the total supply of riskless assets. As is true empirically, the balance sheet of an intermediary is composed of a pool of debt which it then tranches into deposits and equity. Because non-financial firms' debt has low systematic risk, the intermediary can be highly levered, consistent with Berg & Gider (2017)'s empirical finding that the low asset risk of banks explains their high leverage.

The fact that intermediaries are willing to pay more than households for low systematic risk assets but less for high systematic risk assets implies that asset prices are endogenously segmented. The risk free rate is strictly lower than that implied by the household's consumption preferences, due to the additional utility benefit for households directly from holding riskless assets. The risk free rate at which the intermediary would invest is also below that reflecting the household's consumption preferences, since the intermediary can use such assets to back the issuance of riskless deposits and borrow at the low risk free rate. However, the intermediary charges a higher price of systematic risk than the household, since buying risky assets increases the size of the intermediary's portfolio but not its capacity to issue riskless debt. As in models with leverage constraints (Black 1972, Frazzini & Pedersen 2014), less systematically risky assets owned by the intermediary therefore earn a higher risk-adjusted return than riskier assets owned by the household.

The segmentation in asset prices is exploited by non-financial firms when they choose their capital structure, resulting in segmentation specifically between debt and equity markets.[3] Each firm issues as much riskless debt as possible and then chooses its leverage so that its additional debt is sufficiently low risk to sell to intermediaries, while its equity is sufficiently high risk to sell to households. The firm's total market value is therefore strictly higher than any agent would be willing to pay for all of the firm's cash flows. When each firm chooses its capital structure optimally, all risky debt is low enough risk to be held by the intermediary and all equity is high enough risk to be held by the household. Thus, the segmentation between low and high risk assets coming from household and intermediary portfolio choices results in segmentation between the debt and equity markets. This endogenous leverage choice ensures there is always enough debt for the intermediary to buy and enough equity for the household to buy, so neither investor chooses to invest in the other asset class.

Because the model endogenously determines intermediary and household balance sheets, financial and non-financial capital structure, and segmented pricing of debt and equity securities, it provides a rich framework for studying the financial system's response to changes in the supply and demand for safe assets. I apply the model to understand how the financial system responds to 1. a growing demand for safe assets and 2. the Federal Reserve's quantitative easing policies.

Changes in the supply and demand for safe assets have played a crucial macroeconomic role in the buildup to the 2008 financial crisis and the policy response following it. As noted by Caballero & Farhi (2017), since 2002 there has been an increase in the spread between the risk free rate and their measure of the expected return on equities, reflecting a growing scarcity of safe assets. Caused in part by growing

[3] Baker et al. (2017) present a partial equilibrium theory of corporate capital structure exploiting asset pricing segmentation between the debt and equity market and empirical evidence consistent with it.

foreign demand for U.S. safe assets (Bernanke et al. 2011), this growing scarcity coincided with a large credit boom in the 2000s. Mortgage payments as a percent of disposable income grew from 5.87 % in 2003 to a peak 7.20 % at the end of 2007, as both rich and poor households increased their borrowing (Foote et al. 2016). This was accompanied by growth in the size and riskiness of the financial sector (Bhattacharyya & Purnanandam 2011). My model explains why a growing demand for safe assets causes the financial sector to grow and take more risk and causes the non-financial sector to increase its borrowing.

To stimulate recovery after the crisis, the U.S. Federal Reserve's Quantitative Easing (QE) program issued short maturity and safe assets to purchase long maturity and risky assets held by financial institutions. This policy caused a drop in long term and risky interest rates (Krishnamurthy & Vissing-Jorgensen 2011) and a decrease in the riskiness of financial institutions (Chodorow-Reich 2014). The policy also caused an increase in household borrowing (Kermani et al. 2018), though policymakers (Stein 2012*a*) have worried that this may have caused firms to issue too much risky debt with negative consequences for financial stability. My model explains why QE reduces risk premia in debt markets and causes the financial sector to take less risk. In addition, it shows that firms tend to reduce their debt issuance in response to QE, and it implies that a version of QE that purchases equity securities always reduces firm leverage.

QE was implemented in practice when interest rates were at the zero lower bound, and I compare the effects of QE with and without a zero lower bound constraint. In either case, QE acts by reducing the scarcity of safe assets. Away from the zero lower bound, this causes an increase in the risk free rate, as the price of safe assets fall due to increased supply. This rate increase is under some conditions reflected in higher borrowing costs for firms, who then reduce their leverage. When interest rates are stuck at the zero lower bound, the risk free rate must stay fixed, so the reduced scarcity of safe assets results in a rise in the price of risky assets instead. This is consistent with event study evidence from the zero lower bound period (Krishnamurthy & Vissing-Jorgensen 2011, Neely 2011, Chodorow-Reich 2014) that QE lowers risky interest rates and boosts stock prices, and it implies that firms' borrowing costs fall. Surprisingly, QE has the same effect on firms' leverage and the riskiness of the financial sector with or without the zero lower bound. This addresses the concern of Stein (2012*a*) that quantitative easing can increase firms' leverage and hurt financial stability because it reduces their borrowing costs. At the zero lower bound, firms should borrow less despite having lower borrowing costs, because in equilibrium their cost of equity falls too.

Relation to Literature This paper sits between the literature on the theory of financial intermediation and the literature on the macroeconomics of the supply and demand for safe assets. It also relates to the intermediary asset pricing literature and specifically to the role of leverage constraints in asset prices.

One recent trend in financial intermediation theory is an emphasis on the creation of safe assets. This literature goes back to Gorton & Pennacchi (1990), who show that safe debt avoids an adverse selection problem that makes risky assets illiquid. Dang et al. (2017) demonstrate that when banks hold risky assets, they can make their liabilities safe in the short term by concealing interim information. Assets are held by banks rather than sold in markets if they they are sufficiently opaque. Bigio & Weill (2016) present a theory in which only assets whose payoffs are not too correlated with aggregate output are liquid, where banks swap illiquid assets for liquid liabilities. They demonstrate how banks may invest in a portfolio of assets and issue several liabilities to maximize their total creation of safe/liquid assets. DeAngelo & Stulz (2015) is also similar to my work, featuring a liquidity premium on riskless assets and a cost of bank scale, though they work in complete markets and partial equilibrium.[4] Unlike in my model, diversification and market segmentation play no role in these existing theories.

Relative to this recent work on intermediation theory, my paper makes several contributions. First, my model explains not only why debt is held by intermediaries (a common feature of intermediation theory) but also why equities are held by households. Existing theories which explain why a firm borrows from an intermediary imply that such a firm will not also issue securities in public equity markets, due to agency or information frictions. Second, the model demonstrates how the ability of intermediaries to diversify away idiosyncratic risk makes them better at creating safe assets than non-financial firms, explaining why it is specifically financial intermediaries who create safe assets.[5] Third, because my model features publicly available securities on intermediary balance sheets, it connects with empirical evidence on asset prices and the intermediary asset pricing literature, which often takes the composition of intermediary balance sheets as exogenous (He & Krishnamurthy 2013). Other recent work on the endogenous structure of financial intermediation (Bond 2004, Donaldson et al. 2018) features verification or storage frictions that apply to

[4]Section 6 at the end of DeAngelo & Stulz (2015) discusses how their results would generalize in incomplete markets and their intuition is consistent with the composition of bank balance sheets in my model.

[5]This "risk diversification effect" also appears in the model of DeMarzo (2005) of pooling and tranching in securitization, though the assets which can be securitized are exogenous in that model.

less liquid assets.

The paper also contributes to the literature on the supply and demand for safe assets and its connection with central bank policies such as quantitative easing. He & Krishnamurthy (2013) demonstrate how equity injections are more potent than asset purchases for boosting the value of intermediaries' assets, as well as presenting the benchmark framework relating intermediary funding frictions to asset pricing dynamics. Moreira & Savov (2017) study the issuance of liabilities of varying degrees of liquidity ("money" and "shadow-money"), study asset pricing dynamics, and show how quantitative easing boosts asset prices. Caballero & Krishnamurthy (2009) show how growing demand for the safe liabilities of intermediaries can increase the value of their risky asset portfolios.

Relative to this literature, the model's results on how portfolio choices and leverage decisions respond to changes in the supply and demand for safe assets are new. The above papers assume that intermediaries owns all risky assets, and they do not draw a distinction between the financial and non-financial sectors. Two of my results that require this unique approach are first that a growing demand for safe assets can cause the non-financial sector to increase its leverage (and fuel something like the 2000s subprime boom). Second, my model shows how the non-financial sector may reduce its leverage in response to quantitative easing policies, mitigating the fear that QE could fuel a boom of risky debt issuance as hypothesized by a policy speech (Stein 2012*a*). If quantitative easing featured the purchase of equities rather than debt securities, it would result in even lower leverage for the non-financial sector- a result that relies crucially on the model's joint determination of household and intermediary portfolios unlike the existing literature. With an off-the-shelf zero lower bound constraint added, my model demonstrates how QE can simultaneously cause an empirically realistic reduction in borrowing costs with a reduction in corporate borrowing.[6]

[6] A crucial ingredient for my results on borrowing choices are an adaptation of tools related to Geanakoplos (2010) and particularly Simsek (2013) on endogenous leverage. Standard models of leverage constraints following Kiyotaki & Moore (1997) cannot have leverage choices respond to borrowing costs.

1 Baseline Model

Setup The model has two periods ($t = 1, 2$). Goods C_1 are available at time 1 which cannot be stored. Output at time 2 is produced by a continuum of projects with exogenous output (Lucas trees) indexed by $i \in [0, 1]$, where tree i produces δ_i. At time 2, a binary aggregate shock is realized to be "good" or "bad" with probability $\frac{1}{2}$, and the output of the trees are conditionally independent given this aggregate shock. Aggregate and idiosyncratic shocks to each tree's output are the only sources of risk.

There are two classes of agents: households and firm managers. Households have expected utility

$$u(c_1) + E[u(c_2) - T] + v(d). \tag{1}$$

which depends on its consumption (c_1, c_2) at times 1 and 2, on a transfer T of utility paid to firm managers at time 2, and directly on its holding d of riskless assets that pay out at time 2. u and v are strictly increasing, strictly concave, twice continuously differentiable, and satisfy Inada conditions. Managers have expected utility equal to the expected transfer $E(T)$ they receive from households at time 2.

Managers can run two types of firms: non-financial firms and financial intermediaries. Non-financial firms can own Lucas trees and issue financial securities backed by the payoff of their Lucas tree. Each non-financial firm can only own one tree producing δ_i and cannot hold diversified portfolios of trees indexed by different values of i. All Lucas trees must be held by non-financial firms and not directly by households in order to produce output. Financial intermediaries cannot hold Lucas trees but can choose a portfolio of financial securities whose payoff δ_I is endogenous, with no restrictions on their ability to diversify. Financial securities can be held either by households or by financial intermediaries (who themselves are financed by issuing additional financial securities). Managers have no wealth of their own, so all firms must be funded entirely by issuing financial securities.

Following Innes (1990), firms face frictions that require them to issue securities whose payoffs are all increasing in their own cash flows. The appendix describes these frictions and shows that firms optimally issue only debt and equity subject to them. The main paper takes as given that all firms only issue debt and equity securities, though the face value of the debt is endogenous and there may be any number of tranches of debt.

The managers of firms face an agency problem. For a firm whose owns assets with a payoff of x at time 2, a portion $C(x)$ of the output is non-pledgeable and can be seized by firm managers, where $C(0) = 0$, $1 > C' > 0$ and $C'' \geq 0$. As a result, only the payoff $P(x) = x - C(x)$ can be paid out to holders of securities the firm issued. For non-financial firms, this managerial rent is determined only by the payoff of the Lucas tree it owns, and the firm's pledgeable cash flows are $\delta_i^* = P(\delta_i)$. Because all trees must be owned by firms (and each tree i owned by a different non-financial firm), the managerial rents of non-financial firm managers are effectively fixed. For financial intermediaries, the cash flows seized by management depend on the size of its (endogenously chosen) asset portfolio that pays δ_I, yielding pledgeable cash flows of $\delta_I^* = P(\delta_I)$. If the value of the interediary's financial asset portfolio increases, more output can be seized by firm managers.

After firm managers seize output, they trade it back to households at a competitive market price. If households consume c_2 at time 2 and managers have c_{seized} worth of seized consumption goods, households are willing to pay $T = u'(c_2)c_{seized}$ in order to recover the seized consumption goods. Because managers get no utility themselves from consuming seized goods (but do value the utility transfer from households), all consumption goods are sold back to households in this market for transfers. This market for transfers (instead of having managers consume seized goods) allows us to keep the tractability of an endowment economy with exogenous consumption by a representative household while still featuring a cost of assets seized by management.

The set of securities in the economy are the debt and equity of all non-financial firms and the debt and equity of the intermediary. All firms are able to issue multiple tranches of debt. In equilibrium, non-financial firms will issue at most two tranches while intermediaries will issue just one, and this will be reflected in our notation. Let f_i and F_i respectively be the face value of senior and junior debt issed by the firm owning tree i (which I will call firm i). $d_i = min(\delta_i^*, f_i)$ and $D_i = min(\delta_i^* - d_i, F_i)$ are respectively the payoffs of the senior and junior tranche of firm i's debt, and $E_i = max(\delta_i^* - f_i - F_i, 0)$ be the payoff of non-financial firm i's equity. Let F_I be the face value of the intermediary's debt, so $D_I = min(\delta_I^*, F_I)$ and $E_I = max(\delta_I^* - F_I, 0)$ are respectively the payoff of the its debt and equity. Let $q_I(d_i)$, $q_I(D_i)$, and $q_I(E_i)$ respectively be the fraction of firm i's senior and junior debt and equity held by the intermediary, with the remainder $q_H(.) = 1 - q_I(.)$ held by the household. Under this normalization, there is one unit of each firm's debt and equity outstanding. Our notation will assume that all securities issued by the intermediary are held by the household. While the intermediary can issue and then purchase its own securities, this redundant transaction has no benefits but has the cost of increasing the rents of the intermediary's management.

Diversification, Intermediation, and Safe Asset Creation The role of financial intermediaries in this model is to create safe assets, and the special advantage intermediaries have over other firms is that they can hold diversified asset portfolios. If the intermediary owns fractions $q_I(d_i)$, $q_I(D_i)$, and $q_I(E_i)$ of firm i's senior debt, junior debt, and equity, the payoff of the intermediary's portfolio equals

$$\delta_I = \int_0^1 d_i q_I(d_i) di + \int_0^1 D_i q_I(D_i) di + \int_0^1 E_i q_I(E_i) di.$$

These are integrals of a continuum of random variables that are independent given the aggregate state, so following Uhlig (1996) a law of large numbers applies (as shown in the appendix).[7] The payoff of the intermediary's portfolio in the good and bad aggregate states is therefore

$$\begin{aligned} \delta_I^{good} &= \int_0^1 E(d_i|good) q_I(d_i) di + \int_0^1 E(D_i|good) q_I(D_i) di + \int_0^1 E(E_i|good) q_I(E_i) di \\ \delta_I^{bad} &= \int_0^1 E(d_i|bad) q_I(d_i) di + \int_0^1 E(D_i|bad) q_I(D_i) di + \int_0^1 E(E_i|bad) q_I(E_i) di. \end{aligned}$$

The fact that δ_I can be positive in the bad aggregate state, even if each individual security owned by the intermediary may have arbitrarily small payoffs, is why the intermediary is able to issue riskless securities even when buying risky securities issued by non-financial firms. In this sense, diversification allows financial intermediaries to increase the supply of riskless assets.

However, it may also be possible for non-financial firms to issue riskless securities themselves. Even if non-financial firms can create some safe assets, the idiosyncratic risk to which they are exposed constrains their ability to supply safe assets without financial intermediation. We impose the following condition on

[7] The law of large numbers holds so long as the payoffs of each Lucas tree has that $E(\delta_i|good)$ and $E(\delta_i|bad)$ are bounded and continuous in i and $max[E(\delta_i^2|good), E(\delta_i^2|bad)]$ is bounded across all i.

the cash flows of each tree δ_i, which determines how many riskless assets a non-financial firm can issue. The condition also implies that a firm's debt has less systematic risk than its equity, so it will play a key role in determining the composition of the intermediary's portfolio.

Condition 1 *There is a constant $\bar{\delta}_i \geq 0$ such that $Pr(\delta_i > \bar{\delta}_i | bad) = Pr(\delta_i > \bar{\delta}_i | good) = 1$. $\frac{Pr(\delta_i > u | good)}{Pr(\delta_i > u | bad)}$ is continuously differentiable with respect to u on $[\bar{\delta}_i, \infty)$, with the derivative strictly positive on $(\bar{\delta}_i, \infty)$. In addition, $lim_{u \to \infty} \frac{Pr(\delta_i > u | good)}{Pr(\delta_i > u | bad)} = \infty$.*

One implication of this condition is that for $u > \bar{\delta}_i$, $\frac{Pr(\delta_i > u | good)}{Pr(\delta_i > u | bad)} > 1$ and thus $Pr(\delta_i > u | bad) < 1$. $\bar{\delta}_i$ is therefore the largest riskless payoff produced by firm i's Lucas tree. Because a portion $C(\bar{\delta}_i)$ of this payoff will be seized by management, the firm is able to issue at most $\bar{\delta}_i^* = P(\bar{\delta}_i) = \bar{\delta}_i - C(\bar{\delta}_i)$ in riskless assets. The total supply of riskless assets created by non-financial firms is therefore at most $\mu = \int_0^1 \bar{\delta}_i^* di$. However, risky securities sold by non-financial firms can be purchased by the financial intermediary. The condition that $\frac{Pr(\delta_i > u | good)}{Pr(\delta_i > u | bad)}$ means that more senior claims on firm i's cash flows have lower systematic risk. Because the intermediary's can pool assets issued by many firms, it is precisely this systematic risk that concerns the intermediary. This condition is crucial for showing that the intermediary prefers to hold more senior rather than more junior claims on a firm's cash flows (and therefore prefers to buy the firm's debt over the firm's equity).

To ensure that financial intermediaries exist in equilibrium, we must impose a condition that makes it optimal for the intermediary to create a positive supply of safe assets. The intermediary increases welfare by increasing the supply of safe assets from μ which non-financial firms can create alone to some greater supply $\mu + \bar{\delta}_I - C(\bar{\delta}_I)$, boosting the utility of holding safe assets by $v(\mu + \bar{\delta}_I - C(\bar{\delta}_I)) - v(\mu)$. This comes at the expense of managerial diversion, which will cost the household an increased utility transfer of $u'(c_2)C(\delta_I)$ to buy seized goods back from management. The following condition will ensure that the intermediary creates a positive supply of safe assets in equilibrium.

Condition 2 *If we only have the supply μ of riskless assets created by non-financial firms, the marginal cost $E(u'(\int_0^1 \delta_i di) C'(0))$ of the intermediary increasing the supply of riskless assets is strictly less than the marginal benefit $v'(\mu)$.*

Discussion of Assumptions The fundamental assumption here is the utility $v(d)$ households obtain from holding riskless assets, since the financial system is organized to provide these assets. The next important assumption is the agency problem in managing firms, which implies that more resources are seized by management as a firm grows in size. A crucial point here is that even the pledgeable output of non-financial firms can be partially seized when held by the intermediary. Thus, there are additional agency problems generated when an asset is intermediated rather than sold directly to households. The that households can transfer utility T to managers in order to purchase consumption goods seized by the manager is a purely technical one, so that all consumption resources are traded back to households, reducing households' overall utility but not changing their marginal utility of consumption. This provides the tractability of an endowment economy, crucial for the genereal equilibrium applications of the mode. Finally, the fact that non-financial firms own trees exposed to idiosyncratic risk is crucial for intermediaries to play a role by pooling and tranching, since otherwise the non-financial sector could issue all possible safe assets directly to households.

An additional feature of this economy is that households are unable to issue securities. This is because the legal construct of a firm is necessary to enforce contracts in which securities are issued. This means that households are not able to construct riskless securities backed by their own portfolio and obtain utility by holding these riskless securities without facing the agency costs of starting an intermediary firm. While the role of safe assets is in reduced form in the model, this can be interpreted as households being unable to issue money that can circulate due to inability to commit to pay in the future. This also means households cannot short securities, since they are unable to commit to pay the cashflows promised by securities. The intermediary can issue securities backed by its portfolio, but it also faces frictions when shorting assets in the sense that shorting a security does not reduce the agency cost of its asset portfolio despite reducing its size.

The Social Planner's Problem To illustrate how the financial system is most efficiently organized to create safe assets, we study how a benevolent social planner would organize the financial system to maximize the welfare of the household. This social planner can choose what securities are issued by what firms as well as whether these securities are held directly by the household or by the financial intermediary.

The planner trades off two basic forces. First, the planner wants to maximize the total supply of riskless assets available, since the household gets utility from holding riskless assets. Second, the planner wants to minimize the amount of resources seized by the managers of firms, since this reduces the utility of households. The amount of resources seized by a firm's management is increasing in the size of the cash flows its assets generate, so the planner does not want firms to be inefficiently large.

The way the planner trades off these two basic forces is through determining the composition of the intermediary's portfolio. The planner chooses which securities are held by the intermediary and also the capital structures chosen by non-financial firms, which indirectly determine the payoffs of the securities they issue. While there are indeed agency problems in running non-financial firms, their severity is determined by the exogenous size of the non-financial sector, so the planner's only way of changing the amount of resources seized by managers is to determine the size and composition of the intermediary's portfolio. The planner wants the intermediary to hold the smallest possible portfolio that backs the quantity of riskless assets it issues, since this minimizes the amount of resources seized by management.

In order to keep the intermediary's portfolio small while maximizing the amount of riskless assets it can issue, the planner decides that the intermediary should hold all of the risky debt of the non-financial sector. This is for two reasons. First, riskless assets issued by the non-financial sector can themselves meet the household's demand for safe assets, so no extra value is created by having the intermediary hold riskless assets. Second, the intermediary should hold the least risky portfolio it possibly can composed of all the remaining risky assets. The low risk of this portfolio implies that it can back more riskless assets than any other of a given size. Proposition 5 shows that this portfolio is composed of all the risky debt issued by non-financial firms. This is because debt securities have lower systematic risk than equities and therefore back a larger quantity of riskless assets when held in a diversified portfolio. Riskier equities are given directly to the household instead, explaining jointly why intermediaries invest in debt while households invest in equity.

The final choice of the planner is the degree of leverage taken on by all non-financial firms, which provides a single equation (equation 5) that characterizes the choices made by the planner. As non-financial firms increase their leverage, the amount of riskless assets backed by the intermediary's portfolio grows, but so does the risk of this portfolio. Eventually, the riskiness of the non-financial sector's debt reaches a point where the agency cost of including in it in the intermediary's portfolio outweighs the benefits of the intermediary issuing more riskless assets backed by it.

The planner maximizes the household's expected utility

$$u\left(c_1\right) + E\left[u\left(c_2\right) - T\right] + v\left(d\right) \tag{2}$$

The planner faces several constraints. First, there is a resource constraint on consumption ($c_1 \leq C_1$ and $c_2 \leq \int_0^1 \delta_i di$). Second, managers of both financial and non-financial firms will divert all non-pledgeable resources $c_{seized} = \int_0^1 C(\delta_i)di + C(\delta_I)$. Third, any diverted resources will be sold back to consumers at a competitive market price in exchange for utility transfers, $T = u'(c_2)c_{seized}$. Finally, the supply of riskless assets is equal to the face value of riskless debt issued by the financial intermediary and non-financial firms

$$d = F_I \mathbb{1}\{f_I \leq \bar{\delta}_I^*\} + \int_0^1 f_i \mathbb{1}\{f_i \leq \bar{\delta}_i^*\}di. \tag{3}$$

Note that $\mathbb{1}\{f_i \leq \bar{\delta}_i^*\}$ is defined to be a function that equals 1 if $f_i \leq \delta_I^*$, which means that the senior tranche of firm i's debt has face value no greater than the worst realization $\bar{\delta}_i^*$ of its pledgeable cash flows. This is equivalent to the debt being riskless.

Given the amount of consumption resources c_{seized} seized by managers, the planner optimally chooses for households to repurchase them all. If $c_{retained}$ is the amount of resources not seized by managers at time 2, then the planner maximizes $u(c_2) - T = u(c_{retained} + t) - u'(c_{retained} + t)t$ over $t \epsilon [0, c_{seized}]$. This expression is strictly increasing in t, since $\frac{\partial}{\partial t}(u(c_{retained} + t) - u'(c_{retained} + t)t) = -u''(c_{retained} + t)t$. The household's utility net of the transfer to management is therefore strictly increasing in t, so $t = c_{seized}$ is optimal and all resources seized by managers are traded back to households. It follows that the total output at time 2, $\int_0^1 \delta_i di$, is consumed by households.

As well as choosing the household's consumption, the social planner trades off the benefit of creating safe assets against the cost of increasing the amount of goods seized by managers. The only way for the manager to change the amount of goods seized is to change the payoff of the intermediary's portfolio δ_I. As a result, holding fixed the intermediary's portfolio, the planner chooses the maximum possible supply of riskless assets. This means that the intermediary should issue a face value of debt equal to the lowest possible pledgeable payoff of its portfolio, $F_I = \bar{\delta}_I^* = P(\bar{\delta}_I)$. In addition, riskless assets issued by non-financial firms should never be put on the intermediary's balance sheet, since this increases managerial rents without increasing the supply of safe assets. It follows that any non-financial firm for which its worst possible payoff $\bar{\delta}_i^*$ is strictly positive should issue a senior tranche of debt with face value $f_i = \bar{\delta}_i^*$. This riskless asset should be held directly by households and not the intermediary. Let $\mu = \int_0^1 \bar{\delta}_i^* di$ be the supply of riskless assets that can be created directly by non-financial firms. The total supply of riskless assets at the planner's optimal allocation is $\bar{\delta}_I^* + \int_0^1 \bar{\delta}_i^* di$, the sum of the lowest possible cashflow realized by each firm (including the intermediary). Knowing that all available consumption resources are consumed ($c_1 = C_1$ and $c_2 = \int_0^1 \delta_i di$), all nonpledgeable resources are seized by management and sold at their marginal value to the household ($T = u'(c_2)(\int_0^1 C(\delta_i)di + C(\delta_I))$), and all firms issue as much safe debt as their assets allow them to ($d = \bar{\delta}_I^* + \int_0^1 \bar{\delta}_i^* di$),the social planner's objective function (equation 2) becomes

$$u(C_1) + E\left[u\left(\int_0^1 \delta_i di\right) - \overbrace{u'\left(\int_0^1 \delta_i di\right)\left(\int_0^1 C(\delta_i)di + C(\delta_I)\right)}^{\text{utility transferred to managers}}\right] + \overbrace{v\left(\bar{\delta}_I^* + \int_0^1 \bar{\delta}_i^* di\right)}^{\text{utility of holding safe assets}}.$$

The choice that remains for the social planner is the payoff of the intermediary's portfolio, taking as given that the intermediary holds no perfectly riskless assets. The payoff of this portfolio depends both on which assets the intermediary holds as well as the face value of risky debt issued by each non-financial firm, with the agency cost of running non-financial firms $\int_0^1 C(\delta_i)di$ and quantity of riskless assets $\mu = \int_0^1 \bar{\delta}_i^* di$ issued by non-financial firms being exogenous. The intermediary's portfolio diversifies across a continuum of non-financial firms, so its payoff only depends on the aggregate state. The lowest payoff of the intermediary's portfolio occurs in the bad aggregate state, so $\bar{\delta}_I^* = P(\delta_I^{bad})$. This is because every risky security issued by non-financial firms has a lower expected payoff given the bad state than the good state, which follows from condition 1. The planner therefore maximizes a constant term plus

$$E\left[-u'\left(\int_0^1 \delta_i di\right) C(\delta_I)\right] + v\left(P(\delta_I^{bad}) + \mu\right) =$$

$$-\frac{1}{2}u'\left(\int_0^1 E(\delta_i|good)di\right) C(\delta_I^{good}) - \frac{1}{2}u'\left(\int_0^1 E(\delta_i|bad)di\right) C(\delta_I^{bad}) + v\left(P(\delta_I^{bad}) + \mu\right) \tag{4}$$

The planner trades off the benefit of safe asset creation, which is increasing in δ_I^{bad}, with the cost of diversion by managers, which is increasing in δ_I in both the good and bad states of the world (that each occur with probability $\frac{1}{2}$). The intermediary's portfolio payoff only depends on the aggregate state because the idiosyncratic risk of securities issued by each of the continuum of non-financial firms is diversified away. The planner therefore chooses the intermediary's portfolio (and the capital structure of non-financial firms) to make the intermediary's balance sheet as low risk as possible (without holding any perfectly riskless assets), so that δ_I^{good} is not much greater than δ_I^{bad}. This yields our first result about the planner's solution.

Proposition 3 *The intermediary's portfolio has the lowest systematic risk of all possible portfolios of a given size composed only of risky assets. That is, if δ_I^{good} and δ_I^{bad} are the payoff of the intermediary's portfolio in the good and bad states, any other possible portfolio composed of only risky assets with the same bad state payoff δ_I^{bad} has a good state payoff weakly greater than δ_I^{good}.*

This result follows from the tradeoff of two forces. First, a larger intermediary balance sheet is useful because it allows the intermediary to issue a larger quantity of riskless debt. Second, a larger intermediary balance sheet is costly because it increases the agency rents of running the intermediary. The intermediary therefore needs an asset portfolio which backs the largest quantity of riskless debt among all portfolios of the same size, which is a portfolio with low systematic risk.

Proposition 3 alone is enough to determine which securities the intermediary holds. The planner chooses both which securities the intermediary holds as well as the capital structure of the non-financial firms which issue the securities available to the intermediary. Because the planner can make both these portfolio choice and capital structure decisions, the following proposition implies that the intermediary's portfolio should be composed of all of the risky debt of the non-financial sector.

Proposition 4 *Suppose regularity condition 1 holds for each non-financial firm's cash flows. Of all possible portfolios with a given payoff in the bad aggregate state that are composed only of risky securities, the unique portfolio with the lowest payoff in the good aggregate state is composed of the entire supply of every firm's risky debt, with the face value of each non-financial firm's debt chosen appropriately.*

Proof. Appendix. ■

Because of this, if the planner wants the intermediary to create any given quantity of safe assets, it faces the lowest cost of managerial diversion by giving the risky non-financial debt to the intermediary, all riskless assets and all equity to households, and choosing the capital structure of non-financial firms appropriately. The following proposition illustrates this result.

Proposition 5 *In the optimal allocation chosen by the social planner 1. All riskless assets are held by households, to meet the households demand for safe assets. 2. All risky debt securities are held by the financial intermediary. 3. All equity securities are held by the household. 4. Each non-financial firm issues as much riskless debt as it possibly can. It also issues an additional risky debt security as well as an equity security.*

If the face value of each non-financial firm's risky debt is F_i, the intermediary's portfolio therefore pays $\delta_I = \int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)di$, the total payoff of all risky debt securities issued by non-financial firms. Plugging this into the intermediary's objective function (equation 4) yields

$$E\left[-u'\left(\int_0^1 f_i di\right) C(\int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)di)\right] + v\left(\mu + P(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)\right)$$

The planner optimizes this expression by choosing the face value F_i of risky debt issued by each non-financial firm. A larger face value F_i effectively makes the intermediary's portfolio larger, since there is then more debt for the intermediary to hold. This capital structure choice trades off the cost of increasing the amount of riskless assets the intermediary can back with the growing agency rents of expanding the intermediary's portfolio. As F_i increases, the riskiness of the firm i's debt grows, so a larger amount of it is needed to back a given quantity of riskless deposits. Eventually the costs of this growing riskiness outweight the benefits of a greater supply of safe assets, leading to an interior optimum for the quantity of debt each firm should issue. The first order condition for firm i's optimal capital structure is

$$E\overbrace{\left[u'\left(\int_0^1 f_i di\right)C'(\int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)di)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}\right]}^{\text{agency cost of increasing firm } i\text{'s debt}}$$

$$= \overbrace{v'\left(\mu + P(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)\right)P'(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}|bad)}^{\text{utility benefit of additional safe assets backed by firm } i\text{'s debt}}. \quad (5)$$

This can be rearranged to yield, after breaking the expectation in equation 5 into good and bad state payoffs (each weighted by their probability $\frac{1}{2}$ of occurring)

$$\frac{Pr\{\delta_i^* - \bar{\delta}_i^* \geq F_i|good\}}{Pr\{\delta_i^* - \bar{\delta}_i^* \geq F_i|bad\}} = \frac{-u'\left(c_2^{bad}\right)C'(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)}{u'\left(c_2^{good}\right)C'(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|good)di)} +$$

$$\frac{2v'\left(\mu + P(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)\right)P'(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)}{u'\left(c_2^{good}\right)C'(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|good)di)}.$$

For each firm's debt, solving this first order condition sets the ratio $r = \frac{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|good\right)}{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|bad\right)}$ equal to some constant r that is the same across all non-financial firms.[8] The ratio of the expected payoff of an asset in the good and bad aggregate state is a natural measure of its systematic risk, so this ratio r I will call the "risk threshold". Assets of systematic risk lower than r belong on the intermediary's balance sheet, while those of systematic risk higher than r belong on the household's. The risk threshold uniquely determines the face value of each firm's debt, since the ratio $\frac{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|good\right)}{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i|bad\right)}$ is strictly increasing in F_i and ranges from 1 to infinity as F_i moves from 0 to infinity by the condition 1. The planner now only has to choose a value of the risk threshold r. We must have that the planner chooses $r > 1$ because condition 2 implies that the intermediary should create some positive amount of safe assets, which requires the intermediary holding some positive quantity of risky non-financial debt. There is a unique solution to the planner's problem because the optimal capital structure first order condition (equation 5) sets equal an expression that is increasing in r to one that is decreasing in r, so only one value of r can possible make this first

[8]Firms own trees whose cash flows need not be identically distributed. Those for whom $\frac{\Pr(\delta_i^* > u|good)}{\Pr\left(\delta_i^* > u|bad\right)}$ are higher at given value of u choose lower leverage. This prediction that more systematically risky firms is empirically confirmed in Schwert & Strebulaev (2014)

order condition hold for all firms.

Discussion of Social Planner's Problem Three features of the solution to the planner's problem are particularly relevant. First, the fact that an intermediary exists despite its managerial costs in order to produce safe assets that non-financial firms cannot on their own. Second, the fact that the intermediary's optimal portfolio is a diversified portfolio of all of the debt of the non-financial sector. Third, the capital structure of the non-financial sector is indirectly determined by the costs and benefits of safe asset creation by the intermediary. These features are seen most clearly in the social planner's problem, while asset pricing properties of the model are best discussed after decentralizing the allocation.

The planner chooses for a financial intermediary to exist in order to issue safe assets backed by a pool of securities issued by non-financial firms. This is because non-financial firms are exposed to idiosyncratic risk which constrains their ability to create safe assets. While a positive quantity of safe assets can in fact be issued by non-financial firms, the ability of the intermediary to hold a diversified asset portfolio means that it is able to increase the safe asset supply. As part of the intermediary's diversified portfolio, securities can back the issuance of safe assets in proportion to their expected payoff in the bad aggregate state instead of their worst possible realization. Non-financial firms can only issue safe assets up to the worst possible realization of their cash flows, which will be strictly lower than their expected payoff in the bad state as long as they are exposed to idiosyncratic risk. If non-financial firms were not exposed to idiosyncratic risk, there would be no need for financial intermediation and safe assets would all be issued directly by non-financial firms. This ability to pool and tranche in order to create safe assets is the key role of financial intermediaries in the model.

The planner chooses for the intermediary to hold a diversified portfolio of the debt of the non-financial sector because this is the least costly way for intermediaries to create safe assets. The amount of resources managers of the financial intermediary can divert is increasing in the size of the intermediary's portfolio, so the planner wants the intermediary to hold the smallest portfolio that can back a given quantity of safe assets. Any possible portfolio the intermediary can own pays more in the good state of the world than the bad state of the world, and the quantity of safe assets the intermediary can issue is determined by the bad state payoff of its portfolio. As a result, it is optimal to keep the good state payoff of the intermediary's portfolio as low as possible holding fixed the bad state payoff. By theorem X, this optimal portfolio is composed of the debt of the non-financial sector, with the non-financial sector's capital structure chosen appropriately. This provides a new perspective on the synergy between borrowing and lending by financial intermediaries. Intermediaries invest in debt because it is the most cost effective way for them to create safe assets. A debt portfolio minimizes the size of the intermediary and allows them to issue many riskless deposits while maintaining a highly levered capital structure.

The planner's problem presents a novel perspective on corporate capital structure, in which the leverage of non-financial firms is indirectly determined by the demand for safe assets. Because the intermediary's portfolio is composed of all risky debt issued by non-financial firms, the only way for the intermediary's balance sheet to expand is if the non-financial firms issue more debt. As a result, if the demand for safe assets is high or the cost for intermediaries to create safe assets is low, it is optimal to have a highly levered non-financial sector in order to provide a large asset portfolio for the intermediary to hold. Thus, while this model is primarily about financial intermediation and the creation of safe assets, it indirectly provides a new perspective on the determinants of corporate capital structure. This capital structure result can also be interpreted as about a household's mortgage leverage, if δ_i^* is thought of as the flow of services provided by a durable good like a house that a borrower pledges as collateral.

Decenteralized Market Equilibrium The social planner's optimal solution is also the outcome of an appropriately defined competitive equilibrium of the model. As well as demonstrating that the competitive equilibrium is socially optimal, the analysis of the planner's problem presents the model's results about the composition of intermediary balance sheets and firm leverage separately from its asset pricing implications.

Several new results emerge from the equilibrium analysis. First, asset prices feature market segmentation. The risk free rate is low relative to the pricing of risky securities because households directly get utility from holding riskless assets and therefore bid up their price. In addition, risky assets held by the household and the intermediary will feature different risk-return tradeoffs. The risk free rate implied by the intermediary's willingness to pay for assets lies above the true risk free rate, but strictly below the rate implied by the household's willingness to pay for risky assets. In addition, the price the intermediary charges for bearing systematic risk is strictly higher than that of the household. This is because owning risky assets increases the agency rents of the the intermediary's management but does not significantly increase the intermediary's ability to issue riskless debt.

Setup In the model's equilibrium, all financial securities trade at competitive market prices and can be bought either by households or financial intermediaries subject to a no short sales constraint.[9] In addition, all firms (both financial and non-financial) choose the face value of all debt securities they issue. Households invest in financial securities or consume out of their wealth in order to maximize their expected utility. The financial intermediary chooses both its financial portfolio and the face value of debt it issues in order to maximize the market value of its equity, net of cost of raising outside equity financing. Non-financial firms choose the face value of the debt securities they issue also to maximize the value of their equity net of financing costs.

All securities trade in a competitive securities market in which no short sales are permitted. The senior debt, junior debt, and equity of non-financial firm i (which pay d_i, D_i, and E_i) trade at market prices of p_{d_i} , p_{D_i}, p_{E_i} respectively. The intermediary's debt and equity trade at prices p_{D_I} and p_{E_I}. It is also convenient to refer to a risk free rate i_d, since all riskless securities will have to yield the same rate of return in equilibrium. Households and financial intermediaries are both able to purchase securities in this competitive market. There are no differences in the assets available to households and intermediaries or to the payoffs of the asset depending on which agent holds it. In addition, at time 2, there is a competitive price $p_{transfer}$ which measures how many consumption goods the household can purchase by transferring utility to managers that own a stock of diverted consumption goods.

Both intermediaries and non-financial firms are able to issue securities in the market. All firms can choose the face value of each debt security they issue to maximize the value of their equity. Because of short sale constraints, markets can be segmented in equilibrium. Firms will have an optimal capital structure they choose in order to exploit this market segmentation. This capital structure choice is the only decision of non-financial firm i, whose assets are exogenous and pay the output δ_i of the Lucas tree they own. Financial intermediaries instead have to choose both the portfolio of financial securities they

[9] The short sale constraint for households emerges from their inability to commit to pay the promised cash flows of a security. For the intermediary, the short sale constraint follows from the fact that shorting does not reduce the amount of non-pledgeable output on their balance sheet, even though the equilibrium below is robust to allowing the intermediary to issue any security backed by its portfolio.

purchase as well as their capital structure. All firms face the agency problem described in the model's setup, where a firm whose assets pay δ has a portion $C(\delta)$ seized by their management, leaving only a pledgeable portion $P(\delta) = \delta - C(\delta)$ that can be sold to outside investors.

Optimal portfolio choices I first analyze the optimal portfolio choices of the household and financial intermediary, taking as given the set of assets available for them to purchase. When all portfolios are chosen optimally, asset prices will be segmented, with assets held by these two investors reflecting different risk free rates and different prices of systematic risk. I then study the capital structure choices of the non-financial sector, which exploit this market segmentation.

The household maximizes its expected utility given a risk free rate i_d, prices p_s of securities s which pay cash flows x_s in period 2, and its wealth W_H, equal to the value of the Lucas trees it initially sold to non-financial firms plus the period 1 output C_1. The index s denotes any traded security including all debt and equity issued by all firms. Given the rate i_d, the price of one riskless deposit at time 1 is $\frac{1}{1+i_d}$. Consumption at period 2 is the sum of the payoff of the household's portfolio of risky assets δ_H^{risky}, the quantity d of riskless assets owned by the household, and consumption $p_{transfer}T$ obtained by transferring utility T to managers. The household's holding $q_H(s)$ of asset s cannot be negative, since short selling is forbidden by the household's inability to commit to make future payments. The initial wealth the household does not consume is spent on the price $p_H = \frac{d}{1+i_d} + p_H^{risky}$ of its asset portfolio, where p_H^{risky} is the sum of the price of all risky assets bought by the household, and $\frac{d}{1+i_d}$ is the price of the riskless assets the household buys. The household's problem can be written as

$$\begin{aligned} &\max_{q_H,d,c_1,T} u(c_1) + E\left[u\left(\delta_H^{risky} + d + p_{transfer}T\right) - T\right] + v(d) \qquad (6)\\ &\text{subject to } c_1 + \frac{d}{1+i_d} + p_H^{risky} = W_H \text{ (budget constraint)},\\ &q_H(.) \geq 0 \text{ (short sale constraint)} \end{aligned}$$

The first order conditions for the quantity of riskless assets d (which has an interior solution since $v'(0) = \infty$), for the quantity $q_H(s)$ to purchase of security s, and for the amount of utility T to transfer in exchange for consumption are

$$u'(c_1) = (1 + i_d)\left(E\left[u'(c_2)\right] + v'(d)\right) \tag{7}$$

$$p_s \geq E\left[\frac{u'(c_2)}{u'(c_1)} x_s\right] \tag{8}$$

$$u'(c_2)\, p_{transfer} = 1 \tag{9}$$

where inequality 8 must be an equality for any risky security held in positive quantity by the household.

One risky security which must be held in equilibrium by the household is the equity of the financial intermediary. As a result, inequality 8 must be an equality for this security. If the intermediary's equity makes a payout of E_I at time 2 and raises equity e_I at time 1, then its market value at time 1 is (net of equity issuance)

$$E(\frac{u'(c_2)}{u'(c_1)} E_I) - e_I.$$

The intermediary maximizes this market value of its equity. It does so by choosing a portfolio of assets to purchase, as well as the face value of debt that it issues. The intermediary takes as given that all securities it issues in equilibrium must be owned by the household. This implies that all risky assets issued by the intermediary are priced by the household's consumption based pricing kernel (solving inequality 8 with equality). As a result, we can assume the intermediary only issues equity and riskless debt, since only the intermediary's face value of riskless debt has implications for the firm's value. Because the intermediary operates in a competitive market, the risk free rate and prices of risky assets are taken as given by the intermediary when it chooses its assets and liabilities.

If the intermediary owns a portfolio which pays δ_I, it has pledgeable cash flows $P(\delta_I) = \delta_I - C(\delta_I)$

after management has diverted $C(\delta_I)$. If the intermediary's debt pays a riskless cash flow D_I, it can sell this debt for $D_I \frac{1}{1+i_d}$ since it borrows at the risk free rate. The cost p_{δ_I} of the intermediary's asset portfolio must equal the funds it raises from deposits and equity issuance $D_I \frac{1}{1+i_d} + e_I$. The intermediary's problem can be written as

$$\begin{aligned}
\max_{q_I, e_I, D_I} E(\frac{u'(c_2)}{u'(c_1)}(P(\delta_I) - D_I) - e_I \\
\text{subject to } p_{\delta_I} = D_I \frac{1}{1+i_d} + e_I \text{ (budget constraint)} \\
D_I \leq P(\bar{\delta}_I) \text{ (deposit issuance constraint)} \\
q_I(.) \geq 0 \text{(short sale constraint)}
\end{aligned}$$

Because the risk free rate i_d lies strictly below the rate implied by the household's consumption based pricing kernel, it is optimal to issue the maximum quantity of riskless deposits. If the deposit issuance constraint binds, and the budget constraint is used to solve for $e_{I,1}$, the intermediary's problem reduces to

$$\max_{q_I} E(\frac{u'(c_2)}{u'(c_1)}(P(\delta_I) - P(\bar{\delta}_I)) + D_I \frac{1}{1+i_d} - p_{\delta_I}$$

subject to its short sale constraint. Because there are a continuum of atomic non-financial firms, any portfolio the intermediary could buy diversifies away all idiosyncratic risk. The portfolio's payoff only depends on the good or bad aggregate state. In addition, its payoff in the good state δ_I^{good} must be larger than its payoff in the bad state δ_I^{bad}. The intermediary therefore equals riskless debt of face value $P(\delta_I^{bad})$. This riskless payoff can be sold at a price of $P(\delta_I^{bad}) \frac{(E[u'(c_2)] + v'(d))}{u'(c_1)}$, since the risk free rate solves the household's first order condition(equation 7). This implies that the intermediary maximizes over the choice of its portfolio

$$E\left(\frac{u'(c_2)}{u'(c_1)}(P(\delta_I) - P(\delta_I^{bad}))\right) - p_{\delta_I} + P(\delta_I^{bad}) \frac{(E\left[u'\left(c_2\right)\right] + v'\left(d\right))}{u'(c_1)}$$

This can be rewritten as

$$E\left(\frac{u'(c_2)}{u'(c_1)}P(\delta_I)\right) - p_{\delta_I} + P(\delta_I^{bad})\frac{(v'(d))}{u'(c_1)}$$

If the intermediary buys an asset paying x_s for a price p_s, δ_I increases by x_s while the price of the intermediary's portfolio p_{δ_I} increases by p_s. The first order condition for buying such an asset is thus

$$E\left(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I)x_s\right) + P'(\delta_I^{bad})E(x_s|bad)\frac{(v'(d))}{u'(c_1)} \le p_s \tag{10}$$

with equality if the intermediary holds a positive quantity of the asset. These portfolio choice results are summarized in the following proposition.

Proposition 6 *The risk free rate i_d is determined by equation 7. Any risky asset held by the household satisfies inequality 8 with equality. Any risky asset held by the intermediary satisfies inequality 10 with equality. Because every asset must be held by either the household or the intermediary, this charecterizes all asset prices.*

Composition of household and intermediary portfolios The household's preference for safe assets yields two different expressions that reflect its demand for assets- one for riskless assets (equation 7) and one for risky assets (inequality 8). In addition, inequality 10 reflects the intermediary's willingness to pay for any asset, safe or risky. The household's demand for safe assets pushes down the risk free rate at which it is willing to lend relative to what it would pay for a risky asset, since it obtains utility directly from holding riskless assets. The intermediary's willingness to pay for an asset reflects the fact that it can use its portfolio to issue riskless assets demanded by the household (note that household's marginal utility of safe asset holding $v'(d)$ appears in inequality 10, in proportion to the quantity of riskless assets $P'(\delta_I^{bad})E(x_s|bad)$ that can be issued after the intermediary buys asset s). However, the intermediary's managerial agency problem reduces its willingness to pay for assets, since it can only issue securities paying $P'(\delta_I)x_s = (1 - C'(\delta_I))x_s < x_s$ if it adds a payoff x_s to its asset portfolio. This is because $C'(\delta_I))x_s$ will be seized by the intermediary's management and cannot be passed to outside investors.

Once we know the willingness to pay of each agent for assets, we can determine the composition of each agent's portfolio. Whoever is willing to pay the most for an asset will own it in equilibrium. For a riskless asset (whose payoff is normalized to 1), the difference between the intermediary and household's willingness to pay is simply the difference between what equation 7 and inequality 10 imply for the price of the asset. This equals

$$E\left(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I)\right)+P'(\delta_I^{bad})\frac{(v'(d))}{u'(c_1)}-E\left(\frac{u'(c_2)+v'(d)}{u'(c_1)}\right)$$
$$-\left(E\left(\frac{u'(c_2)}{u'(c_1)}C'(\delta_I)\right)+C'(\delta_I^{bad})\frac{(v'(d))}{u'(c_1)}\right)<0$$

This implies that the household is willing to pay more for riskless assets than the intermediary, so the household holds all riskless assets in equilibrium. For assets that are not perfectly riskless, the intermediary holds the asset if the implied price of the asset in inequality 10 reflecting the intermediary's willingness to pay is greater than that in inequality 8, which reflects the households willingness to pay. The intermediary holds the asset if

$$E\left(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I)x_s\right)+P'(\delta_I^{bad})E(x_s|bad)\frac{(v'(d))}{u'(c_1)}>E\left(\frac{u'(c_2)}{u'(c_1)}x_s\right)$$

$$P'(\delta_I^{bad})E(x_s|bad)\left(2v'(d)-u'(c_2^{bad})C'(\delta_I^{bad})\right)>u'(c_2^{good})C'(\delta_I^{good})E(x_s|good)$$

$$\frac{2P'(\delta_I^{bad})v'(d)-u'(c_2^{bad})C'(\delta_I^{bad})}{u'(c_2^{good})C'(\delta_I^{good})}>\frac{E(x_s|good)}{E(x_s|bad)}$$

It follows that for a risky asset with payoff x_s, the intermediary will buy the asset if its systematic risk $\frac{E(x_s|good)}{E(x_s|bad)}$ is sufficiently low. Otherwise the household will buy it. Moreover, by condition 2, the intermediary will hold some assets for which $\frac{E(x_s|good)}{E(x_s|bad)}>1$, because otherwise the intermediary would not exist in equilibrium. This is summarized in the following proposition.

Proposition 7 *1. All riskless assets are bought by the household. The risk free rate is determined by the household's willingness to pay for a riskless asset, equation 7. 2. For some cutoff value $k>1$, all risky assets whose payoff x_s has sufficiently low systematic risk ($\frac{E(x_s|good)}{E(x_s|bad)}<k$) are bought by the intermediary.*

For these assets, the price p_s of the asset solves 10 with equality. 3. All risky assets with sufficiently high systematic risk ($\frac{E(x_s|good)}{E(x_s|bad)} > k$) are bought by the household. For these assets, the price p_s of the asset solves 8 with equality.

Just like in the planner's problem, the intermediary holds no perfectly riskless assets but holds all of the risky assets whose systematic risk is sufficiently low. The intuition presented when discussion the planner's problem explains this. Putting riskless assets on the intermediary's balance sheet would not increase the total supply of riskless assets. Of the remaining risky assets, the intermediary can issue the most riskless deposits while keeping the size of its asset portfolio fixed by holding assets with the least systematic risk.

Capital structure choices of non-financial firms Taking this market segmentation as given, the non-financial firms pick their capital structure to exploit in the interest of their shareholders. Issuing a security which they can sell for a price strictly higher than its value to the shareholders increases the value of the firm's equity, so firms maximize the total market value of the securities they issue. Firms optimally issue the as much riskless senior debt as they can, some junior risky debt to sell to the intermediary, and equity to sell to the household. The debt must be sufficiently low systematic risk to be held by the intermediary while the equity must be sufficiently high systematic risk to be held by the household. If this is the case, the value of the non-financial firm is (by proposition 7's charecterization of asset prices)

$$E\left(\frac{u'(c_2)+v'(d)}{u'(c_1)}d_i\right)+E\left(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I)D_i\right)+P'(\delta_I^{bad})E(D_i|bad)\frac{v'(d)}{u'(c_1)}+E\left(\frac{u'(c_2)}{u'(c_1)}E_i\right)$$

Because the household is willing to pay strictly more than the intermediary for riskless cash flows, it is optimal to set the face value of senior debt f_i equal to the maximum riskless payoff backed by the firm's assets, $\bar{\delta}_i^*$. To compute the optimal face value of junior debt, note that except on an event of probability 0, $\frac{\partial D_i}{\partial F_i} = -\frac{\partial E_i}{\partial F_i} = \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}$[10]. The first order condition for an optimal face value of risky junior debt is that the derivative of the firm's value with respect to this face value is 0. It follows that at the optimal face value of junior debt,

[10] $\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}$ is a function that equals 1 if $\delta_i^* - \bar{\delta}_i^* \geq F_i$ and 0 otherwise.

$$E\left(\frac{u'(c_2)}{u'(c_1)}C'(\delta_I)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}\right) = P'(\delta_I^{bad})Pr(\delta_i^* - \bar{\delta}_i^* \geq F_i|bad)\frac{v'(d)}{u'(c_1)} \tag{11}$$

Equilibrium The optimal behaviour of the household, the intermediary, and non-financial firms describe above all take as given asset prices. Together with resource constraints and market clearing conditions, they charecterize the model's equilibrium. The resource constraints on consumption are as in the planner's problem $c_1 = C_1$ and $c_2 = \int_0^1 \delta_i di - c_{seized} + p_{transfer}T$. Because the managers optimally transfer back all seized consumption resources at any positive price $p_{transfer} > 0$, this implies $c_2 = \int_0^1 \delta_i di$.

Definition 8 *An equilibrium is a set of asset prices, portfolio and leverage choices, and consumption allocations such that 1. The household, the intermediary, and non-financial firms behave optimally, so equations 7,8, 9, 10, and 11 all hold. In addition, managers optimize, so they transfer all seized consumption resources in exchange for utility transfers. 2. Resource constraints are satisfied and the market for utility transfers clears, so $c_1 = C_1$ and $c_2 = \int_0^1 \delta_i di - c_{seized} + p_{transfer}T$. This implies $c_2 = \int_0^1 \delta_i di$. 3. Asset markets clear, so $q_I + q_H = 1$.*

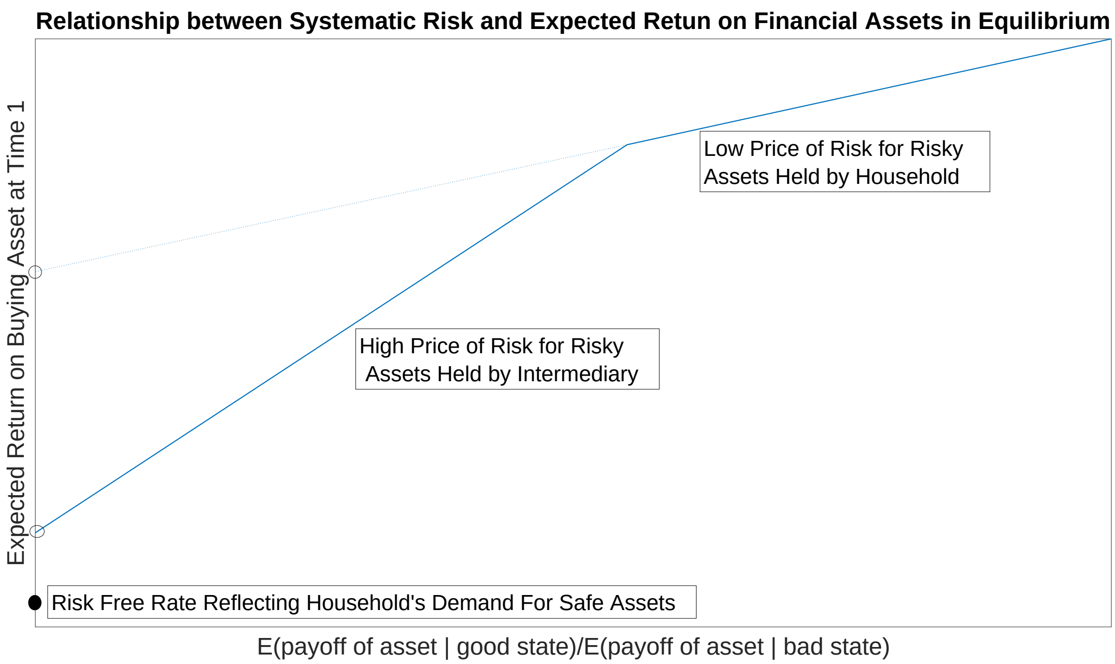
Plugging in that $c_2 = \int_0^1 \delta_i di$ and that the intermediary's portfolio is composed of all risky debt issued by the non-financial sector, so $\delta_I = \int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)di$, equation 11 becomes

$$E\left[u'\left(\int_0^1 \delta_i di\right)C'(\int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)di)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}\right]$$
$$= v'\left(\mu + P(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)\right)P'(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}|bad).$$

This is identical to the first order condition of the social planner's problem (equation 5), which uniquely determines all leverage and portfolio decisions. It follows that the decentralized equilibrium yields the same allocation as that chosen by the social planner.

Empirical Asset Pricing Implications Asset prices in this model feature endogenous market segmentation, since debt and equity are endogenously owned by different agents with different preferences. The risk free rate satisfies $1 + i_d = \frac{Eu'(c_2)+v'(d)}{u'(c_1)}$ by equation 7. The intermediary's willingness to pay for a riskless asset is strictly lower and would imply a strictly higher rate $i_{d,Int}$ where

$1 + i_{d,Int} = E\left(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I)\right) + P'(\delta_I^{bad})\frac{(v'(d))}{u'(c_1)}$ by inequality 10. The fact that $P' < 1$, reflecting that only some share $P(\delta_I)$ of the intermediary's assets are not seized by management, reduces the intermediary's willingness to pay for assets. The rate in turn is even lower than the rate at which the household wold invest without its special demand for safe assets, $E\frac{u'(c_2)}{u'(c_1)} - 1$. In addition, the intermediary's willingness to pay for cash flows knowing that the good state occurs is $\frac{u'(c_2^{good})P'(\delta_I^{good})}{u'(c_1)}$, strictly less than the household's willingness to pay for such a payoff $\frac{u'(c_2^{good})P'(\delta_I^{good})}{u'(c_1)}$. It follows that the intermediary charges a strictly higher price for exposure to systematic risk, since shifting an asset's payoffs from the bad to good state reduces the amount of riskless deposits the intermediary can issue while still keeping the agency rents of the intermediary's management high. The figure below summarizes asset prices in the model, consistent with proposition



7.

The above figure relies only on the fact that the intermediary holds low systematic risk assets while the household holds high systematic risk assets. In the equilibrium, firms choose their capital structure optimally to exploit the "kink" in the market price of risk plotted above. As a result, all debt securities are sufficiently low risk that they are held by the intermediary while all equities are sufficiently high risk that they are hold by household. This implies that the price of systematic risk is strictly higher in the bond market than the stock market, and that the risk free rate in the bond market's pricing kernel lives strictly below that in the equity market's pricing kernel. One empirical implication of this is that a zero systematic risk long-short portfolio composed of bonds and a zero systematic risk long-short portfolio composed of stocks earn expected returns equal to the risk free rates implied respectively by the intermediary's and household's pricing kernels. Because the intermediary own's debt in equilibrium and the household holds equity, this can be mapped into evidence on the relative performance of such a portfolio in the debt or equity market.

This is broadly consistent with evidence presented in Frazzini & Pedersen (2014). First, they show that the alpha (spread in returns above a risk free rate) of a zero beta (their measure of systematic risk) portfolio is larger in equities than in fixed income. Their long-short U.S. equities portfolio with zero beta earns a monthly alpha of .73. Their zero beta long-short portfolio in U.S. credit indices earns an alpha of .17, while such a portfolio composed of individual corporate bonds earns an alpha of .57, and such a portfolio composed of U.S. treasuries earns an alpha of .16. While the specific estimate of alpha varies across the specific fixed income asset class, it is broadly true that the alpha of an equities betting against beta strategy is larger than any of these possible estimates, consistent with the fact that in my model the equity pricing kernel implies a higher risk free rate than the bond pricing kernel. These two implied risk free rates like above the model's true risk free rate, consistent with the positive alphas found across asset classes for a betting against beta strategy.

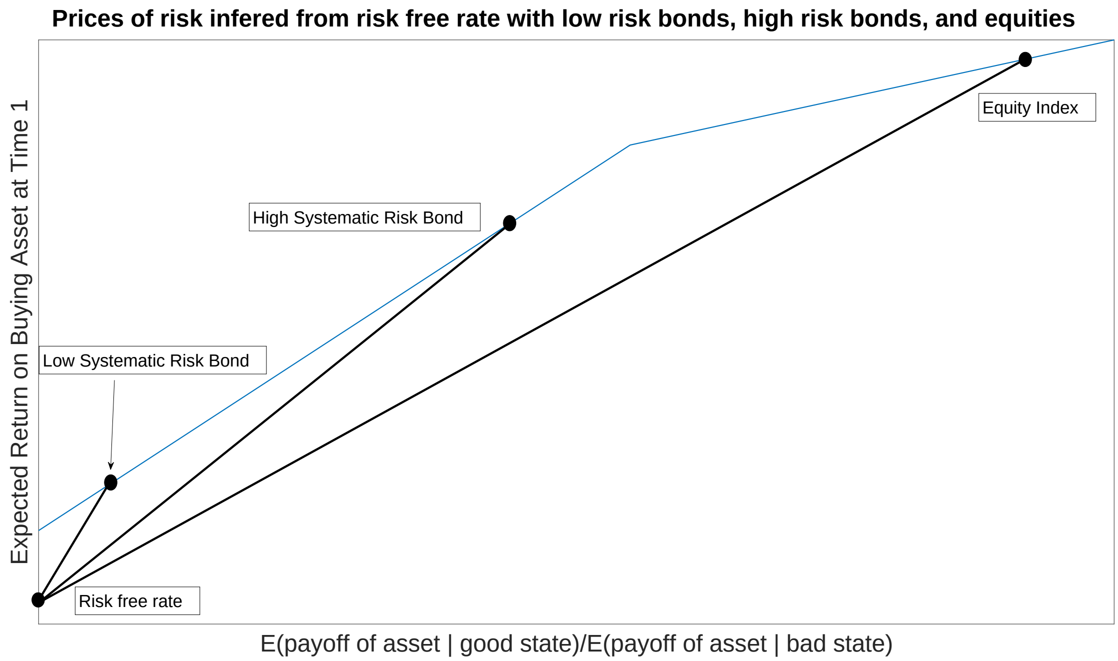
An additional implication which can be compared to Frazzini & Pedersen (2014) is that the pricing kernel in bonds implies a higher price of risk than that in equities. For each asset class, the paper reports the betas and the excess returns of 10 portfolios based on sorting assets into the deciles of their beta. Within each asset class, I regress (with an intercept) the excess returns of these portfolios on their ex ante betas. This yields a slope coefficient of .0721 for U.S. equities, .1914 for credit indices, and .0853 for treasuries. For U.S. corporate bonds, I estimate a slope of .2549 using their data on bonds of different credit ratings.[11] The evidence is consistent with a higher price of systematic risk in the bond market than in the stock market. One difficulty in interpreting this evidence is that betas are computed with respect to a different reference index for each asset class. Baker et al. (2017) provide more direct evidence by computing the CAPM beta and expected returns on various stock and bond portfolios. They find a strictly lower expected return on low beta bonds than implied by the pricing of risk in equity markets,

[11]The authors also include the returns on distressed corporate bonds, which are very low on average and have a very high beta with overall bond returns. Including this outlier makes the slope of this line negative. See Campbell for an examination of the puzzlingly poor behavior of distressed securities.

consistent with the “kink” in the securities market line in my model.

An additional piece of evidence presented inFrazzini & Pedersen (2014) is the Sharpe ratio of betting against beta strategies. In my baseline model, the Sharpe ratio on all of these strategies is infinite since there is no volatility in the return on a riskless investment. The appendix discusses a way to extend the model in order to generate finite Sharpe ratios in order to qualitatively speak to this evidence.

The pricing of debt and equity securities in my model is consistent with evidence on the credit spread puzzle. Huang & Huang (2012) document the yield spread and default probabilities on corporate bonds across credit ratings. While higher rating bonds have both lower default probabilities and lower yield spreads, the ratio of yield spread to default probability is highest for the safest bonds. This is consistent with the fact that in the picture below, a line drawn between the return on a risk free asset and on an equity index implies a strictly lower expected return on bonds than those which occur in equilibrium. In addition, they find that a smaller fraction of the yield spread on safer bonds is explained by credit risk in quantitative models. Albagli et al. (2014) report a finding which maps more directly into my model's predictions- the Sharpe ratio of the return on risky corporate debt is decreasing in the bond's credit rating. If corporate default risk is primarily systematic (or if the a bond's idiosyncratic default risk is positively correlated with its systematic risk), this is precisely what my model would imply, as can be seen in the picture below. From the risk free rate and any other asset's expected return and systematic risk, we can draw a line whose slope reflects an implied price of systematic risk. This implied price (which is proportional to the Sharpe ratio of the risky asset used to construct the line if the asset is not exposed to idiosyncratic risk) is higher when inferred from a lower risk asset.



Another implication follows from the low risk free rate together with a "kinked" securities market line. The model can reconcile a large difference in the riskless rate implied by the pricing kernel in equities markets with the somewhat more moderate empirical estimates of the convenience yield on safe assets. Krishnamurthy & Vissing-Jorgensen (2012) estimate that this convenience yield (inferred from the spread between AAA bonds and treasuries) is roughly 70 basis points, and this number maps naturally into the spread between the risk free rate in the model and the strictly rate at which the intermediary would lend. This is because AAA bonds are low risk (and hence held in the model by the intermediary), but not riskless and thus do not meet the household's demand for safe assets. This 70 basis point spread is considerably smaller than the rougly 6.4 % annual CAPM alpha of a zero systematic risk equity portfolio . Some of this 6.4 % return is likely due to not including all risk factors. However, if the remaining spread after additional risk factors are included is greater than 70 basis points, my model provides a natural interpretation of this finding which relies crucially on having 3 different risk free rates in the model. More generally, my model implies that convenience yields inferred from asset classes traded primarily by intermediaries may be different than those inferred from assets primarily held by households (such as equities).

Finally, the model is consistent with evidence that the credit spread puzzle, low beta anomaly in equities, and convenience yield on safe assets are larger during periods of financial turmoil. The excess bond premium of Gilchrist & Zakrajsek (2012), which is a measure of the severity of the credit spread puzzle, is particularly large in the financial crisis and co-moves with survey measures of risk appetite of bankers. Frazzini & Pedersen (2014) show that the TED spread, which measures intermediaries' default risk, predicts the return on their zero beta portfolio of equities. van Binsbergen et al. (2019) report that the convenience yield on safe assets spikes during the 2008 crisis and particularly the day of Lehman Brothers' default. In the context of my model, financial distress can be interpreted as the function C' being made larger, so that it is more costly for intermediaries to hold assets. This will decrease the equilibrium supply of safe assets and increase the price of risk in the intermediary's pricing kernel. This makes the spread between the zero beta rate in the equity pricing kernel and the risk free rate particularly large. It also makes the price of risk implied by a risk free rate and the return on a bond portfolio particularly large as well. This is consistent with these findings.

2 Application to the Supply and Demand for Safe Assets

The model developed in the previous section can be used to understand the general equilibrium effects of changes in the supply and demand for safe assets. Because the model endogenously determines asset prices, intermediary portfolios and leverage, and the capital structure of the non-financial sector, all of these will adjust in order to clear the market for safe assets.

Although many aspects of the financial system endogenously change in response to shocks to the supply or demand for safe assets, the analysis is quite tractable. As noted above, a single equation can be used to charecterize the model's equilibrium, equation 5 which I repeat below.

$$E\left[u'\left(\int_0^1 \delta_i di\right) C'(\int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i)di)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}\right]$$
$$= v'\left(\mu + P(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)\right) P'(\int_0^1 E(min(\delta_i^* - \bar{\delta}_i^*, F_i)|bad)di)Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i\}|bad).$$

The left hand side of this expression is the marginal increase in agency costs from increasing the leverage of firm i (and thereby growing the portfolio of the intermediary, who owns firm i's debt), while the right hand side is marginal benefit coming from the resulting increase in the intermediary's ability to issue riskless assets. As shown above, when this equation holds for all firms i, the leverage of each firm is characterized by solving $r = \frac{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i | good\right)}{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i | bad\right)}$, where r is the risk threshold such that any asset with systematic risk above r is held by the household. Given a value of r, each firm has a unique face value of risky debt $F_i(r)$, and the payoff of the intermediary's portfolio can be written as $\delta_I(r) = \int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i(r))di$. The following proposition summarizes key properties of these functions, with a proof in the appendix.

Proposition 9 *$\delta_I^{good}(r)$, $\delta_I^{bad}(r)$, $\frac{\delta_I^{good}(r)}{\delta_I^{bad}(r)}$, and $F_i(r)$ (for all i) have strictly positive derivatives with respect to the risk threshold r.*

The equilibrium can now be characterized as

$$\frac{1}{2}u'(c_2^{good})C'(\delta_I^{good}(r))Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|good) + \frac{1}{2}u'(c_2^{bad})C'(\delta_I^{bad}(r))Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|bad)$$
$$= v'\left(\mu + P(\delta_I^{bad}(r))\right) P'(\delta_I^{bad}(r))Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|bad). \quad (12)$$

The expression $s_{good}(r) = u'(c_2^{good})C'(\delta_I^{good}(r))$ is the marginal agency cost of adding a good state payoff to the intermediary's portfolio, in terms of how much utility the household would have to transfer to repurchase it. This marginal agency cost in the bad state is $s_{bad}(r) = u'(c_2^{bad})C'(\delta_I^{bad}(r))$. $m(r) = v'\left(\mu + P(\delta_I^{bad}(r))\right) P'(\delta_I^{bad}(r))$ is the marginal benefit of adding a bad state payoff to the intermediary's balance sheet, in terms of how many additional riskless assets it can back (with $P'(\delta_I^{bad}(r))$ being the share of the payoff not seized by the intermediary's management).

The equilibrium can now be characterized as

$$s_{good}(r)r + s_{bad}(r) = 2m(r) \quad (13)$$

This equation implies that for an asset of systematic risk r, (paying r in the good state and 1 in the bad state in expectation), the agency cost of adding it to the intermediary's balance sheet equals the benefit of additional riskless assets being issued. This follows from 12 since each firm's optimal capital structure decision ensures that $r = \frac{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i | good\right)}{\Pr\left(\delta_i^* - \bar{\delta}_i^* > F_i | bad\right)}$.

Safe asset demand and the subprime boom This provides a framework for understanding how the financial system responds to a safe asset shortage, which a macroeconomic literature (e.g. Caballero Farhi 2017) argues has been a key driving force behind the low real interest rates in recent decades. My model implies that a growing demand for safe assets causes something akin to the subprime boom of the 2000s. In particular, the financial sector expands and invests in riskier assets than it previously did, which leads to an increase in the leverage of the non-financial sector due to a reduction in its cost of borrowing. Relative to the literature, the novelty of my analysis comes from the endogenous choices of portfolios and capital structure, which are often taken as exogenous, and my joint modelling of the financial and non-financial sectors. In particular, my results on how the non-financial sector's leverage responds to changes in the supply and demand for safe assets, which may be a key part of how the real economy is impacted, are perhaps the most novel part of this analysis. The following proposition summarizes the results on the effects of an increase in the demand for safe assets, which are demonstrated below.

Proposition 10 *An increase in the demand for safe assets causes 1. A decrease in the risk free rate and an increase in the equilibrium quantity of safe assets. 2. An increase in the size and systematic risk of the intermediary's asset portfolio. 3. An increase in the leverage of all non-financial firms. 4. An increase in the intermediary's willingness to pay for all debt securities, and for a risk free asset. 5. An increase in the spread between the household and intermediary's willingness to pay for a risk free asset, if $C'' > 0$.*

If we increase the demand for safe assets the equilibrium responds by making the function v' larger by one unit, $m(r) = v'\left(\mu + P(\delta_I^{bad}(r))\right) P'(\delta_I^{bad}(r))$ changes by the amount $P'(\delta_I^{bad}(r))$. The change in the risk threshold r follows by implicitly differentiating equation 13

$$\left[\overbrace{s'_{good}(r)r + s'_{bad}(r)}^{\text{Agency cost of growing intermediary portfolio}} + \overbrace{s_{good}(r)}^{\text{direct effect of change in r}} - \overbrace{2m'(r)}^{\text{Expansion of supply of safe assets}}\right]\frac{dr}{dv} = 2P'(\delta_I^{bad}(r)).$$

Note that every term in this expression is positive, so $\frac{dr}{dv} > 0$. This has several implications. First, the quantity of safe assets, $\mu + P(\delta_I^{bad}(r))$ increases to meet the growing demand, since $P(\delta_I^{bad}(r))$ is increasing in the risk threshold r. In addition, the safe asset premium, $v'(\mu + P(\delta_I^{bad}(r)))$ changes as $1 + \frac{dr}{dv}\frac{d}{dr}v'(\mu + P(\delta_I^{bad}(r)))$. This is because it is exogenously increased by 1 by growing demand, but the growing quantity of safe assets supplied by the financial intermediary acts to reduce this increase. Note that $m'(r) = (\frac{d}{dr}v'(\mu + P(\delta_I^{bad}(r))))P'(\delta_I^{bad}(r)) + v'(\mu + P(\delta_I^{bad}(r)))\frac{dP'(\delta_I^{bad}(r)}{dr}$. The total change in the safe asset premium equal to (using equation the above expression for $\frac{dr}{dv}$)

$$1 > \frac{1}{2P'(\delta_I^{bad}(r))}\left[s'_{good}(r)r + s'_{bad}(r) + s_{good}(r) + v'(\mu + P(\delta_I^{bad}(r))))\frac{dP'(\delta_I^{bad}(r))}{dr}\right]\frac{dr}{dv} > 0$$

It follows that the spread between the true risk free rate and that implied by the consumption Euler equation increases. Consistent with supply-and-demand intuition, an increase in the demand for safe assets raises both the equilibrium price and quantity.

In addition, the payoff of the intermediary's portfolio $\delta_I(r)$ is increasing in the risk threshold r in both states the world, so the size of the intermediary's asset portfolio increases. Proposition 9 implies $\frac{d}{dr}\left(\frac{\delta_I^{good}(r)}{\delta_I^{bad}(r)}\right) > 0$, so the systematic risk of the intermediary's asset portfolio increases as well. A growing demand for safe assets therefore causes the financial system to be larger and to invest in riskier assets.

The intermediary's willingness to pay for risky assets also responds to changes in the demand for safe assets. The intermediary's willingness to pay for an asset with payoffs x_s can be written as

$$\begin{aligned} & E(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I(r)x_s)) + E(x_s|bad)\frac{m(r)}{u'(c_1)} \\ = E(\frac{u'(c_2)}{u'(c_1)}x_s) - \frac{s_{good}(r)}{2u'(c_1)}E(x_s|good) - E(x_s|bad)\frac{s_{bad}(r)}{2u'(c_1)} + E(x_s|bad)\frac{m(r)}{u'(c_1)} \end{aligned}$$

The first term in this expression- the household's willingness to pay for the risky payoff x_s is constant. The change in this willingness to pay per unit of expected payoff is therefore the change in

$$\frac{E(x_s|bad)}{2u'(c_1)}\left(2m(r) - s_{good}(r)\frac{E(x_s|good)}{E(x_s|bad)} - s_{bad}(r)\right)$$

The expression inside the parentheses is for $\frac{E(x_s|good)}{E(x_s|bad)} = r$ precisely equation 13, which must equal 0 in equilibrium. The risk threshold r must increase to ensure equation 13 remains true after an increase in safe asset demand, and this increase alone decreases the intermediary's willingness to pay for an asset. The intermediary's willingness to pay for an asset of fixed risk $\frac{E(x_s|good)}{E(x_s|bad)} = r$ therefore increases. In addition, for assets of lower systematic risk (for which $\frac{E(x_s|good)}{E(x_s|bad)} < r$) the increase in the intermediary's willingness to pay is even larger, since $s_{good}(r)$ also increases (which reduces the value of riskier assets). All firms have debt with systematic risk lower than the risk threshold r, so the prices of all debt securities increase. This increase is larger for firms whose debt has lower systematic risk. The intermediary's willingness to pay for a risk free asset increases as well.

However, the difference between the equilibrium price of a riskless payoff and the intermediary's willingness to pay is equal to

$$E\left(\frac{u'(c_2)}{u'(c_1)}C'(\delta_I(r))\right) + \frac{v'\left(\mu + P(\delta_I^{bad}(r))\right)C'(\delta_I^{bad}(r))}{u'(c_1)}$$

It follows that as long as C" is strictly positive, this difference also increases, since r (and hence $\delta_I(r)$, in both states of the world) increases. These results are summarized in the proposition stated above, which is now demonstrated.

Quantitative easing One of the U.S. Federal Reserve's key policy responses to the 2008 financial crisis was quantitative easing, the purchase of treasury bonds and agency mortgage backed securities financed by increasing the supply of bank reserves (riskless assets which must be held by financial intermediaries). Treasuries and agency MBS are exposed to duration and prepayment risk, so they are best thought of as risky debt securities in the context of the model. Within the model, I therefore analyze the effects of a purchase of risky debt held by the intermediary by the government in exchange for a special riskless asset which must be held by the financial intermediary[12] This asset purchase simultaneously adds riskless assets to the intermediary's balance sheet while removing risky assets. This reduces the scarcity of safe assets and the riskiness of the intermediary's portfolio. If the government bought equity securities held by the household instead , the only direct effect on intermediary balance sheets would be an increase in the quantity of riskless assets held, since the risky assets would be bought from the household's portfolio.[13]

I find that a QE policy that purchases equities reduces the scarcity of safe assets, the riskiness of the

[12]For the government to be able to fund their asset purchases, they must be able to pay the investors who hold the bank reserves they issue. I assume the government can levy lump sum taxes to do so.

[13]Some of the Federal Reserve's interventions to stabilize distressed banks can be thought of as a purchase of bank equity, though my model does not feature bank runs which these interventions were intended to prevent.

intermediary's portfolio, and the leverage of the non-financial sector. The intuition is that the increased supply of riskless bank reserves is a more efficient input for creating riskess bank deby than the risky debt of non-financial firms. This crowds out the need for banks to hold firms' risky debt, so the intermediary holds less debt and firms issue less debt. A QE policy that purchased debt instead of equity directly reduces the risk of the intermediary's portfolio, and it is ambiguous if this is greater or smaller than the amount of risk reduction the intermediary would choose. It follows that the intermediary could either buy more or less debt issued by firms, and hence the leverage of firms could either decrease or increase. A QE policy that buys equity rather than debt leads to less firm leverage, but it creates fewer safe assets and causes less reduction in the risk of the intermediary's portfolio.

I first consider the effect of quantitative easing policies in which the central bank issues bank reserves in order to purchase equity securities. The impact of this on the intermediary's balance sheet is to simply increase the quantity of riskless securities available for it to hold, since the risky assets purchased come from the household's portfolio. I therefore consider how the financial system evolves when a single riskless payoff is added to the intermediary's portfolio. The financial system responds according to

$$\left[s'_{good}(r)r + s'_{bad}(r) + s_{good}(r) - 2m(r)\right]\frac{dr}{dQE_{equity}} = \frac{2m'(r)}{(\delta_I^{bad})'(r)} - u'(c_2^{bad})C''(\delta_I^{bad}(r)) - u'(c_2^{good})C''(\delta_I^{good}(r))r.$$

It follows that $\frac{dr}{dQE_{equity}} < 0$, so the systematic risk of the intermediary's portfolio declines and the leverage of the non-financial sector declines. The change in the amount of bad state payoffs in the intermediary's portfolio is $1 + (\delta_I^{bad})'(r)\frac{dr}{dQE_{equity}}$ Because $\frac{2m'(r)}{(\delta_I^{bad})'(r)} + 2m'(r)\frac{dr}{dQE_{equity}} = \frac{2m'(r)}{(\delta_I^{bad})'(r)}(1 + (\delta_I^{bad})'(r)\frac{dr}{dQE_{equity}})$, the above expression that charecterizes $\frac{dr}{dQE_{equity}}$ implies that the change in the bad state payoff of the intermediary's portfolio equals

$$\frac{(\delta_I^{bad})'(r)}{2m'(r)}\left(\left[s'_{good}(r)r + s'_{bad}(r) + s_{good}(r)\right]\frac{dr}{dQE_{equity}} + u'(c_2^{bad})C''(\delta_I^{bad}(r)) + u'(c_2^{good})C''(\delta_I^{good}(r))r\right) > 0$$

This increase in the amount of bad state payoffs on the intermediary's balance sheet means that the quantity of riskless assets available to the household increases and thus the risk free rate rises. Because $\frac{dr}{dQE_{equity}} < 0$, the intermediary's willingness to pay for an asset of systematic risk equal to the risk threshold

r also decreases. Because the systematic risk of every debt security lies below the risk threshold r, the systematic component of its payoff can be written as that of an asset whose systematic risk is r plus a payoff in the bad state. The prices of all debt securities therefore decrease, since the intermediary's willingness to pay for bad state cashflows decreases. These results are summarized in the following proposition.

Proposition 11 *A quantitative easing policy in which the central bank issues riskless bank reserves to purchase risky equity securities held by the household causes 1. A reduction in the risk of the intermediary's asset portfolio. 2. A reduction in the leverage of the non-financial sector. 3. An increase in the risk free rate 4. A decrease in the value of all risky bonds*

Suppose instead that the government issues a riskless asset in order to buy a risky asset with payoff x_s satisfying $1 < \frac{E(x_s|good)}{E(x_s|bad)} < r$, so that it is held by the intermediary. We will infer the size of this purchase from the fact that it trades at a market clearing price. The assets on both sides of the transaction are held by the intermediary, so the assets bought and sold have the same value to the intermediary. This implies if an asset paying x_s is swapped for a riskless asset paying 1 in both states of the world,

$$E(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I(r))x_s) + E(x_s|bad)\frac{m(r)}{u'(c_1)} = E(\frac{u'(c_2)}{u'(c_1)}P'(\delta_I(r))) + \frac{m(r)}{u'(c_1)}$$
$$(u'(c_2^{good})P'(\delta_I^{good}(r))(E(x_s|good) - 1) = (1 - E(x_s|bad))(2m(r) + u'(c_2^{bad})P'(\delta_I^{bad}(r)))$$
$$(u'(c_2^{good}) + s_{good}(r))(E(x_s|good) - 1) = (1 - E(x_s|bad))(2m(r) + u'(c_2^{bad}) + s_{bad}(r)))$$

This equation implies that quantitative easing removes more good state payoffs from the intermediary's balance sheet than it adds in bad state payoffs. This follows because $u'(c_2^{bad}) > u'(c_2^{good})$ and $2m(r) + s_{bad}(r) > s_{good}(r)$ by equation 13, so $E(x_s|good) - 1 > 1 - E(x_s|bad)$. The transaction can therefore be analyzed as combining the addition of riskless payoff of $(1 - E(x_s|bad))$ to the intermediary's balance sheet while removing a good state payoff of $E(x_s|good) - E(x_s|bad) = (E(x_s|good) - 1) + (1 - E(x_s|bad))$. The ratio between the amount of good state payoff removed to the amount of riskless payoff added is

$$\frac{E(x_s|good) - E(x_s|bad)}{(1 - E(x_s|bad))} = \frac{E(x_s|good) - 1}{(1 - E(x_s|bad))} + 1 = \frac{(2m(r) + u'(c_2^{bad}) + s_{bad}(r))}{u'(c_2^{good}) + s_{good}(r)} + 1 \tag{14}$$

which does not depend on the specific asset purchased. To analyze this transaction, I analyze seperately the effects of adding riskless payoffs and removing good state payoffs from the intermediary's balance sheet.

If we remove a good state payoff from the intermediary's balance sheet, the economy adjusts so that equation 13 which charecterizes an equilibrium remains true, so

$$\left[s'_{good}(r)r + s'_{bad}(r) + s_{good}(r) - 2m'(r)\right] \frac{dr}{dgood} = u'(c_2^{good})C''(\delta_I^{good}(r))r. \tag{15}$$

It follows that $\frac{dr}{dgood} > 0$ if $C'' > 0$. If $C'' = 0$ then removing good state payoffs from the intermediary's balance sheet has no effect. If $C'' > 0$, the amount of bad state payoff on the intermediary's balance sheet increases, and hence the total supply of riskless assets available to the household increases. Because the quantity of riskless assets held by the household increases and the household has decreasing marginal utility of holding safe assets, the price of a riskless asset falls and thus the risk free rate rises. In addition, the increase in the risk threshold r also implies that the leverage of all firms increases. The change in the amount of good state payoff on the intermediary's balance sheet is $(\delta_I^{good})'(r)\frac{dr}{dgood} - 1$. Because $s'_{good}(r) = u'(c_2^{good})C''(\delta_I^{good}(r))(\delta_I^{good})'(r)$, equation 15 implies

$$(\delta_I^{good})'(r)\frac{dr}{dgood} - 1 = \frac{-1}{u'(c_2^{good})C''(\delta_I^{good}(r))r}\left[s'_{bad}(r) + s_{good}(r) - 2m'(r)\right]\frac{dr}{dgood} < 0$$

so the amount of good state payoff on the intermediary's balance sheet decreases, despite the increase in non-financial sector leverage which makes the intermediary's debt portfolio riskier.

A quantitative easing policy that purchases equities only adds riskless payoffs to the intermediary's portfolio, while such a policy that purchases debt securites also removes good state payoffs from the intermediary's portfolio. This result therefore allows us to compare the effects of these two policies. A cost to a government of performing quantitative easing is that it requires fiscal backing in order to ensure the bank reserves it issues are indeed safe assets. If the government buys systematically risky assets, this fiscal backing is in terms of the amount of revenue that must be raised in the bad aggregate state, since in the good state the assets bought by the government pay more than the reserves it issues. For a debt QE program and equity QE program that require equal amounts of fiscal backing, the debt program's effect combines that of the equity with a positive amount of good state payoffs removed from the intermediary's portfolio. For any variable whose response to taking good state payoffs off the intermediary's balance sheet has the same sign as its response to an equity QE program, the effect of the debt QE program is strictly larger. This proves the following proposition.

Proposition 12 *Suppose $C'' > 0$. A quantitative easing policy that purchases debt securities held by the intermediary causes 1. a greater increase in the total supply of riskless assets held by households 2. a greater increase in the risk free rate 3. a greater decrease in the systematic risk of the intermediary's asset portfolio than a quantitative easing policy purchasing equities that requires an equal amount of government fiscal capacity. If $C'' = 0$ the two versions of quantitative easing have identical effects on these variables.*

The effects of quantitative easing that buys debt on the leverage of the non-financial sector is ambiguous, since $\frac{dr}{dgood}$ and $\frac{dr}{dQE_{equity}}$ have opposite signs. By equation 14 , quantitative easing that purchases debt removes $\frac{(2m(r)+u'(c_2^{bad})+s_{bad}(r))}{u'(c_2^{good})+s_{good}(r)}+1$ units of good state payoff from the intermediary's balance sheet per riskless security added, since only at this ratio is the transaction consistent with the intermediary's willingness to pay for good and bad state payoffs. It follows that

$$\frac{dr}{dQE_{debt}} = \frac{dr}{dQE_{equity}} + \left(\frac{(2m(r) + u'(c_2^{bad}) + s_{bad}(r))}{u'(c_2^{good}) + s_{good}(r)} + 1\right)\frac{dr}{dgood}$$

Writing this out in terms of the expressions for $\frac{dr}{dQE_{equity}}$ and $\frac{dr}{dgood}$ derived above yields

$$\begin{aligned}\frac{dr}{dQE_{debt}} = \frac{1}{s'_{good}(r)r + s'_{bad}(r) + s_{good}(r) - 2m'(r)} \Bigg[&\frac{2m'(r)}{(\delta_I^{bad})'(r)} - u'(c_2^{bad})C''(\delta_I^{bad}(r)) \\ &+ \frac{(2m(r) + u'(c_2^{bad}) + s_{bad}(r))}{u'(c_2^{good}) + s_{good}(r)}[u'(c_2^{good})C''(\delta_I^{good}(r))r] \Bigg]\end{aligned} \tag{16}$$

Because $m(r)$ depends only on the household's marginal utility of holding risky assets, we can see that this expression is effectively increasing in $C''(\delta_I^{good}(r))$. For a functional form for which this is sufficiently positive and large, it follows that $\frac{dr}{dQE_{debt}} > 0$. If $C''(\delta_I^{good}(r))$ is sufficiently small (for example if $C'' = 0$), then $\frac{dr}{dQE_{debt}} < 0$. The sign of $\frac{dr}{dQE_{debt}}$ is the same as the sign in the change in the intermediary's willingness to pay for an asset whose systematic risk equals the risk threshold r. Because each firm's leverage is increasing in the value of the risk threshold r, this yields the following proposition.

Proposition 13 *The response of the non-financial sector's leverage to a quantitative easing policy that purchases debt is ambiguous. If the expression in equation 16 is negative (positive), then leverage decreases (increases). Leverage decreases if $C'' = 0$ or if $C''(\delta_I^{good}(r))$ is sufficiently small. In any case, if $C'' > 0$ the leverage of the non-financial sector is higher after a quantitative easing policy that buys debt than such a policy buying equity that requires an equal amount of government fiscal capacity. If leverage increases, then firms with sufficiently risky debt have a reduction in their borrowing cost, while firms with sufficiently safe debt have an increases. If leverage decreases, then all firms have an increased borrowing cost.*

The intuition for this result is that quantitative easing both increases the supply of riskless assets, which reduces the incentive for the intermediary to bear risk, while simultaneously directly taking risk off of the intermediary's balance sheet by purchasing debt. A priori, it is unclear if the amount of risk reduction the intermediary would choose is greater or less than the direct, partial equilibrium effects of asset purchases on the riskiness of its balance sheet. This addresses the concern of Stein (2012*a*) that quantitative easing would increase the leverage of the non-financial sector, with negative consequences for financial stability. While my results do not imply that this is unambiguously the case for any policy, they do suggest that a quantiative easing policy that purchases equities rather than debt would never have this issue. However, when using an equal amount of government fiscal capacity, a quantiative easing policy that purchases equities instead of debt would do less to reduce the scarcity of safe assets and less to reduce the riskiness of the intermediary's asset porfolio.

Quantitative easing at the zero lower bound One counterfactual implication of the above analysis of both possible quantitative easing policies is that they increase the risk free rate. This is because such policies reduce the scarcity of safe assets, and this scarcity reduces the risk free rate. In practice, quantitative easing was implemented when interest rates were up against the zero lower bound. By imposing an off-the-shelf model of sticky prices (Krugman 1998) common in the New Keynesian macroeconomics literature, the model is able to deliver an analysis of quantitative easing that is consistent with observed stylized facts. The model will be identical to that above, except that the output of the economy at time 1 is now endogenous and produced by the labor of the household and that goods prices are perfectly rigid. The household's utility is reduced by its labor supply l, and the amount of period 1 consumption available is equal to $Y(l)$. In addition, the nominal price P of buying goods is perfectly sticky between times 1 and 2. This leads to disequilibrium in the goods market at time 1. If the central bank imposes a nominal interest rate of i_d, it also equals the real interest rate because there is no inflation. Time 1 consumption is therefore determined by the household's first order condition, taking the nominal interest rate i_d as exogenous.

$$u'\left(c_1\right) = \left(1+i_d\right)\left(E\left[u'\left(c_2\right)\right] + v'\left(d\right)\right)$$

Other than the fact that c_1 is now endogenous and determined by the demand for consumption at time 1 and that the risk free rate i_d is fixed at 0, nothing changes from the analysis above. Because none of the variables in equation 13 depend on c_1, the portfolio choice and capital structure analysis above continues to hold at the zero lower bound.

Since u' is decreasing, it follows that either a reduction in the nominal rate or in the premium $v'(d)$ on safe assets stimulates consumpion at time 1, similar to Caballero & Farhi (2017). Holding fixed the interest rate $i_d = 0$ we have that

$$u''(c_1)\frac{\partial c_1}{\partial QE} = v''(d)\frac{\partial d}{\partial QE}$$

Because both forms of quantitative easing increase the supply d of riskless assets held by the household, it follows that they stimulate consumption at the zero lower bound. I now analyze the effect of quantitative easing on borrowing costs. For tractability, in this section I impose that $C'' = 0$, so C' and $P' = 1 - C'$ are positive constants. The intermediary's willingness to pay for a security paying x_s is

$$\frac{Eu'(c_2)P'(\delta_I)x_s + P'(\delta_I^{bad})v'(d)E(x_s|bad)}{u'(c_1)} =$$
$$(P')\left(E(x_s|bad)\frac{Eu'(c_2)+v'(d)}{u'(c_1)} + \frac{u'(c_2^{good})}{2u'(c_1)}(E(x_s|good) - E(x_s|bad)\right)$$

Because $\frac{Eu'(c_2)+v'(d)}{u'(c_1)} = 1$ at the zero lower bound and $u'(c_1)$ decreases, the intermediary's risk free rate stays fixed and the intermediary's willingness to pay for risky debt securities increases. It follows that the risk premium on all risky debt securities goes down, which with a constant risk free rate implies that all firms have a reduction in borrowing costs. This matches the empirical findings of Krishnamurthy &

Vissing-Jorgensen (2011) that quantitative easing reduced risky and long term interest rates, while holding the short term interest rate fixed at 0. In addition, because $u'(c_1)$ decreases as a result of the increase in consumption, equity prices increase as well, since they are priced by the household's marginal utility of consumption. This reduction in the cost of equity financing implies that firms need not borrow more in respose to a reduction in borrowing costs, and the analysis above when $C'' = 0$ implies that all firms reduce their leverage in response to quantitative easing. The following proposition summarizes these results.

Proposition 14 *Suppose $C'' = 0$. With nominal rigidities and a binding zero lower bound, quantitative easing 1.reduces firms' borrowing costs and boosts equity prices while holding the risk free rate fixed 2. increases consumption at time 1 and 3. has the same effects on portfolio choices and capital structures as without nominal rigidities. In particular, since $C'' = 0$ all firms reduce their leverage in response to quantitative easing despite their reduced borrowing costs. These results hold whether the central bank purchases debt or equity securities to implement quantitative easing.*

This general equilibrium analysis at the zero lower bound speaks to the worries some policymakers (Stein 2012*a*) had that quantitative easing would have negative financial stability implications, because lower borrowing costs would lead firms to issue more risky debt. Away from the zero lower bound, quantitative easing would have raised borrowing costs (at in the case $C'' = 0$ considered here) and induced firms to borrow less. It is only because of the special features of the zero lower bound that the reduction in risk premia caused by quantitative easing is reflected in reduced borrowing costs rather than an increase in the risk free rate. Both at and away from the zero lower bound, if $C'' = 0$, firms always reduce leverage in response to quantitative easing. Nevertheless, the analysis in previous sections shows that if policymakers are worried about high leverage in the non-financial sector, purchasing equities rather than debt to implement quantitative easing is an effective policy response.

Conclusion This paper develops a general equilibrium model of how the financial system is organized to meet a demand for safe assets and applies it to understand recent macroeconomic issues related to the supply and demand for safe assets. The role played by intermediaries is to pool the debt of non-financial firms, who cannot issue enough riskless assets because of idiosyncratic risk, and issue riskless securities and a risky equity tranche backed by this debt portfolio. The debt and equity markets are endogenously segmented, and the non-financial sector's optimal capital structure arbitrages these segmented markets. The model shows how a growing demand for safe assets causes a subprime boom and provides a framework for understanding the transmission mechanism of quantitative easing policies and their implications for financial stability. The joint determination of household and intermediary portfolios as well as the leverage of the financial and non-financial sector provides a particularly rich analysis of the effects of quantitative easing, where buying debt and equity securities has different macroeconomic impacts.

Several features of the model suggest a future research agenda. First, the model takes as given the demand for safe, money-like assets. A more fundamental framework where the demand for money and the role of intermediaries as creators of money are both endogenous may provide additional insights. Second, existing safe assets are typically denominated in a currency. A framework with safe assets in multiple currencies may be useful for understanding the international spillovers of quantitative easing and the role of the dollar in the international financial system. The perspective taken in this model, where the demand for liabilities issued by intermediaries determines their asset portfolio and leverage is endogenous, may be a useful and tractable framework for many questions about the role of intermediaries in macroeconomics and finance. Existing work by Scharfstein (2018) on the impact of pension policy on the structure of the financial system and Diamond & Landvoigt (2018) on the impact of intermediaries on household leverage suggest the importance of endogeneous leverage and portfolio choices in applied work.

References

Adrian, T., Etula, E. & Muir, T. (2014), 'Financial intermediaries and the cross-section of asset returns', *Journal of Finance* **69**(6), 2557– 2596.

Albagli, E., Hellwig, C. & Tsyvinski, A. (2014), 'Dynamic dispersed information and the credit spread puzzle', *working paper* .

Allen, F. & Gale, D. (1988), 'Optimal security design', *Review of Financial Studies* **1**(3), 229–263.

Baker, M., Bradley, B. & Taliaferro, R. (2014), 'The low-risk anomaly: A decomposition into micro and macro effects', *Financial Analysts Journal* **70**(2), 43–58.

Baker, M., Hoeyer, M. F. & Wurgler, J. (2017), 'The risk anomaly tradeoff of leverage', *working paper* .

Bansal, R. & Coleman, W. J. (1996), 'A monetary explanation of the equity premium, term premium, and risk-free rate puzzles', *Journal of Political Economy* **104**(6), 1135–1171.

Berg, T. & Gider, J. (2017), 'What explains the difference in leverage between banks and non-banks?', *Journal of Financial and Quantitative Analysis* **52**(6), 2677–2702.

Bernanke, B., Bertaut, C. C., Demarco, L. P. & Kamin, S. B. (2011), 'International capital flows and the return to safe assets in the united states, 2003-2007', *Financial Stability Review* **15**, 13–26.

Bhattacharyya, S. & Purnanandam, A. (2011), 'Risk-taking by banks: What did we know and when did we know it?', *working paper* .

Bigio, S. & Weill, P.-O. (2016), 'A theory of bank balance sheets', *working paper* .

Black, F. (1972), 'Capital market equilibrium with restricted borrowing', *Journal of Business* **45**(3), 444–455.

Bond, P. (2004), 'Bank and nonbank financial intermediation', *Journal of Finance* **59**(6), 2489–2529.

Caballero, R. J. & Farhi, E. (2017), 'The safety trap', *Review of Economic Studies* **85**(1), 223–274.

Caballero, R. J. & Krishnamurthy, A. (2009), 'Global imbalances and financial fragility', *American Economic Review Papers and Proceedings* **99**(2), 584–588.

Chodorow-Reich, G. (2014), 'Effects of unconventional monetary policy on financial institutions', *Brookings Papers on Economic Activity* pp. 155–204.

Dang, T. V., Gorton, G. & Holmstrom, B. (2013), 'Ignorance, debt and financial crises', *working paper* .

Dang, T. V., Gorton, G., Holmstrom, B. & Ordonez, G. (2017), 'Banks as secret keepers', *American Economic Review* **107**(4), 1005–1029.

DeAngelo, H. & Stulz, R. M. (2015), 'Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks', *Journal of Financial Economics* **116**(2), 219–236.

DeMarzo, P. (2005), ‘The pooling and tranching of securities: A model of informed intermediation’, *Review of Financial Studies* **18**(1), 1– 35.

DeMarzo, P. & Duffie, D. (1999), ‘A liquidity-based model of security design’, *Econometrica* **67**(1), 65–99.

Diamond, D. (1984), ‘Financial intermediation and delegated monitoring’, *Review of Economic Studies* **51**(3), 393–414.

Diamond, D. & Dybvig, P. (1983), ‘Bank runs, deposit insurance, and liquidity’, *Journal of Political Economy* **91**(3), 401–19.

Diamond, D. & Rajan, R. (2001), ‘Liquidity risk, liquidity creation, and financial fragility: A theory of banking’, *Journal of Political Economy* **109**(2), 287–327.

Diamond, W. & Landvoigt, T. (2018), ‘Credit cycles with market based household leverage’, *working paper* .

Donaldson, J. R., Piacentino, G. & Thakor, A. (2018), ‘Warehouse banking’, *Journal of Financial Economics* **129**(2), 250–267.

Foote, C., Loewenstein, L. & Willen, P. (2016), ‘Cross-sectional patterns of mortgage debt during the housing boom: Evidence and implications’, *working paper* .

Frazzini, A. & Pedersen, L. (2014), ‘Betting against beta’, *Journal of Financial Economics* **111**(1), 1–25.

Geanakoplos, J. (2010), ‘The leverage cycle’, *NBER Macroeconomics Annual* **24**(1), 1–66.

Gilchrist, S. & Zakrajsek, E. (2012), ‘Credit spreads and business cycle fluctuations’, *American Economic Review* **102**(4), 1692–1270.

Goldstein, I. & Pauzner, A. (2005), ‘Demand-deposit contracts and the probability of bank runs’, *Journal of Finance* **60**(3), 1293–1327.

Gorton, G. & Pennacchi, G. (1990), ‘Financial intermediaries and liquidity creation’, *Journal of Finance* **45**(1), 49–71.

Hanson, S. G., Shleifer, A., Stein, J. & Vishny, R. W. (2015), 'Banks as patient fixed-income investors', *Journal of Financial* **117**(3), 449–469.

He, Z. & Krishnamurthy, A. (2013), 'Intermediary asset pricing', *American Economic Review* **103**(2), 732–70.

Huang, J.-Z. & Huang, M. (2012), 'How much of the corporate-treasury yield spread is due to credit risk?', *Review of Asset Pricing Studies* **2**(2), 153–202.

Innes, R. (1990), 'Limited liability and incentive contracting with ex-ante action choices', *Journal of Economic Theory* **52**(1), 45–67.

Kashyap, A., Rajan, R. & Stein, J. C. (2002), 'Banks as liquidity providers: An explanation for the coexistence of lending and deposit-taking', *Journal of Finance* **57**(1), 33–73.

Kermani, A., Maggio, M. D. & Palmer, C. (2018), 'How quantitative easing works: Evidence on the refinancing channel', *working paper* .

Kiyotaki, N. & Moore, J. (1997), 'Credit cycles', *Journal of Political Economy* **105**(2), 211–248.

Krishnamurthy, A. & Vissing-Jorgensen, A. (2011), 'The effects of quantitative easing on interest rates: Channels and implications for policy', *Brookings Papers on Economic Activity* pp. 215–287.

Krishnamurthy, A. & Vissing-Jorgensen, A. (2012), 'The aggregate demand for treasury debt', *Journal of Political Economy* **120**(2), 233–267.

Krugman, P. (1998), 'It's baaack: Japan's slump and the return of the liquidity trap', *Brookings Papers on Economic Activity* **29**(1998-2), 137–206.

Lester, B., Postlewaite, A. & Wright, R. (2012), 'Information, liquidity, asset prices, and monetary policy', *Review of Economic Studies* **79**(3), 1209–38.

Moreira, A. & Savov, A. (2017), 'The macroeconomics of shadow banking', *Journal of Finance* **72**(6), 2381–2432.

Nagel, S. (2016), 'The liquidity premium of near-money assets', *Quarterly Journal of Economics* **131**(4), 1927–1971.

Neely, C. (2011), 'The large-scale asset purchases had large international effects', *working paper* .

Scharfstein, D. (2018), 'Presidential address: Pension policy and the financial system', *Journal of Finance* **73**(4), 1463–1512.

Schwert, M. & Strebulaev, I. (2014), 'Capital structure and systematic risk', *working paper* .

Simsek, A. (2013), 'Belief disagreements and collateral constraints', *Econometrica* **81**(1), 1–53.

Stein, J. C. (2012*a*), 'Evaluating large scale asset purchases', *Speech delivered at the Brookings Institution, Washington, October 11.* .

Stein, J. C. (2012*b*), 'Monetary policy as financial-stability regulation', *Quarterly Journal of Economics* **127**(1), 57–95.

Sunderam, A. (2015), 'Money creation and the shadow banking system', *Reveiw of Financial Studies* **28**(4), 939–977.

Uhlig, H. (1996), 'A law of large numbers for large economies', *Economic Theory* **8**(1), 41–50.

van Binsbergen, J., Diamond, W. & Grotteria, M. (2019), 'Risk free interest rates', *working paper* .

Appendix: Proof of proposition 5 Note that the payoff of the firm's debt and equity when the debt has face value F_i can respectively be written as $min(\delta_i^* - \bar{\delta}_i^*, F_i) = \int_0^{F_i} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$ and $max(\delta_i^* - \bar{\delta}_i^* - F_i, 0) = \int_{F_i}^{\infty} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$. For a given face value of debt and intermediary portfolio, the payoff to the intermediary of assets issued by firm i is $q_I(D_i)\int_0^{F_i} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du + q_I(E_i)\int_{F_i}^{\infty} \mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$ which is a special case of the expression $\int_0^{\infty} q(u)\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$ where the image of q is contained in [0,1], in the case $q(u) = q_I(D_i)\mathbb{1}\{F_i \geq u\} + q_I(E_i)(1 - \mathbb{1}\{F_i \geq u\})$. The expected payoff of this portfolio in the good and bad states are $\int_0^{\infty} q(u)Pr(\delta_i^* - \bar{\delta}_i^* \geq u|bad)du$. There is some face value F_i^* of debt for which a portfolio owning all of the firm's debt has the same bad state payoff, $\int_0^{F_i^*} Pr(\delta_i^* - \bar{\delta}_i^* \geq$

$u|bad)du$. Let $C = \frac{Pr(\delta_i^* - \bar{\delta}_i^* \geq F_i^* | good)}{Pr(\delta_i^* - \bar{\delta}_i^* \geq F_i^* | bad)}$. The difference in the good state payoffs of the two portfolios is $\int_0^\infty (q(u) - q^*(u))(\frac{Pr(\delta_i^* - \bar{\delta}_i^* \geq u|good)}{Pr(\delta_i^* - \bar{\delta}_i^* \geq u|bad)} - C)Pr(\delta_i^* - \bar{\delta}_i^* \geq u|bad)du$. If q is not almost everywhere equal to q* (in which case they both are the payoff of holding all of firm i's debt), this expression is strictly positive. It follows that this portfolio composed of the whole outstanding stock of firm i's debt minimizes the expected good state payoff holding fixed the bad state payoff. Moreover, any other portfolio has a strictly higher good state payoff if its bad state payoff is the same.

Appendix: Proof of proposition 9 When all firms choose their optimal captial structure, for each firm i we have

$$\frac{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|good)}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|bad)} = r\frac{d}{dF_i}(\frac{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|good)}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|bad)})F_i'(r) = 1$$

Because $\frac{d}{dF_i}(\frac{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|good)}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq F_i(r)\}|bad)}) > 0$ by regularity condition 1, it follows that $F_i'(r) > 0$. The intermediary's portfolio has a payoff, $\delta_I(r) = \int_0^1 min(\delta_i^* - \bar{\delta}_i^*, F_i(r))di$. Denote the payoff of this portfolio in the good and bad state as $\delta_I^{good}(r)$ and $\delta_I^{bad}(r)$. Note that

$$\begin{aligned}(\delta_I^{good})'(r) &= \int_0^1 Pr(\delta_i^* - \bar{\delta}_i^* > F_i(r)|good))F_i'(r)di > 0 \\ (\delta_I^{bad})'(r) &= \int_0^1 Pr(\delta_i^* - \bar{\delta}_i^* > F_i(r)|bad))F_i'(r)di \\ = \int_0^1 \frac{Pr(\delta_i^* - \bar{\delta}_i^* > F_i(r)|bad))}{Pr(\delta_i^* - \bar{\delta}_i^* > F_i(r)|good))} & Pr(\delta_i^* - \bar{\delta}_i^* > F_i(r)|good))F_i'(r)di \\ = r\int_0^1 Pr(\delta_i^* - \bar{\delta}_i^* > F_i(r)|good))F_i'(r)di &= r(\delta_I^{good})'(r) > 0\end{aligned}$$

Note that $\frac{d}{dr}\left(\frac{\delta_I^{good}(r)}{\delta_I^{bad}(r)}\right) = \frac{(\delta_I^{good})'(r)\delta_I^{bad}(r) - \delta_I^{good}(r)(\delta_I^{bad})'(r)}{\delta_I^{bad}(r)^2} = \frac{(\delta_I^{bad})'(r)}{\delta_I^{bad}(r)}(\frac{(\delta_I^{good})'(r)}{(\delta_I^{bad})'(r)} - \frac{\delta_I^{good}(r)}{\delta_I^{bad}(r)})$ as well. Because $\frac{\delta_I^{good}(r)}{\delta_I^{bad}(r)} < \frac{(\delta_I^{good})'(r)}{(\delta_I^{bad})'(r)}$ by condition 1, it follows that $\frac{d}{dr}\left(\frac{\delta_I^{good}(r)}{\delta_I^{bad}(r)}\right) > 0$.

Appendix: Extension with a finite Sharpe ratio One asset pricing implication that is not realistic is that the Sharpe ratio on bond market and stock market betting against beta strategies are both infinite. This however, can easy be remedied in a small extension in which the demand for safe assets and cost of managerial rents are both stochastic. In the current model, the excess returns on the stock market and bond market betting aginst beta strategies are

$$\frac{\frac{u'(c_1)}{Eu'(c_2)} - \frac{u'(c_1)}{Eu'(c_2)+v'(d)}}{\frac{u'(c_1)}{Eu'(c_2)(1-C'(\delta_I)+[1-C'(\delta_I^{bad})]v'(d)} - \frac{u'(c_1)}{Eu'(c_2)+v'(d)}}$$

Under the approximation that v' and C' are both small, we can consider the Taylor expansions

$$\frac{v'(d)\frac{u'(c_1)}{(Eu'(c_2))^2}}{\left(Eu'(c_2)C'(\delta_I)+C'(\delta_I^{bad})v'(d)\right)\frac{u'(c_1)}{(Eu'(c_2))^2}}$$

The equilibrium of the model is characterized by

$$u'(c_2^{good})C'(\delta_I^{good}(r))r + u'(c_2^{bad})C'(\delta_I^{bad}(r)) - 2P'(\delta_I^{bad})v'(\mu + P(\delta_I^{bad}(r)))) = 0$$

The value of r which characterizes equilibrium is true for any

$$\lambda([u'(c_2^{good})C'(\delta_I^{good}(r))r\gamma_1 + u'(c_2^{bad})C'(\delta_I^{bad}(r))\gamma_2] - 2P'(\delta_I^{bad})v'(\mu + P(\delta_I^{bad}(r)))) = 0$$

where $\gamma_1 = 1 + (1-\gamma_2)\frac{u'(c_2^{bad})C'(\delta_I^{bad}(r))}{u'(c_2^{good})C'(\delta_I^{good}(r))r}$

An increase in γ_2 leaves the excess return on equities unchanged. In addition, it holds fixed the intermediary's willingness to pay for an asset of systematic risk r unchanged by increasing the intermediary's willingness to pay for bad state payoffs and decreasing its willingness to pay for good state payoffs. As a result, the intermediary's willingness to pay for assets of systematic risk lower than r increases, reducing the excess return on the bond's betting against beta portfolio. Similarly, an increase in λ will increase the excess return on both equities and bonds. Combining an increases in λ with an appropriately sized decrease in γ_2 can therefore lead to a volatile excess return on the equity betting against beta portfolio while leaving the excess return on the betting against beta bond portfolio approximately riskless. This argument demonstrates qualitatively that with appropriate shocks (so that the model does not mechanically imply an infinite Sharpe ratio for all betting against beta portfolios), the model can produce a higher Sharpe ratio on the bond betting against beta portfolio than the stock portfolio.

Appendix:Optimality of Debt This appendix shows why it is optimal for non-financial firms to issue only debt and equity securities. These firms face cannot issue securities whose cash flows depend explicitly on the aggregate state. If firm i's Lucas tree yields a pledgeable payoff of δ_i^*, the payoff of every security issued by this firm must be a function only of δ_i^* . In addition, these firms face contracting frictions similar to Innes (1990). At time 2, the managers of the firms act in the interest of equityholders and can either destroy the firm's value or take out a loan and then repay it in the same period if either would increase the payoff to equity. Let $s(\delta_i^*)$ be the sum of all cash flows promised to investors other than equity, so that equity gets the residual claim $\delta_i^* - s(\delta_i^*)$. As shown by Innes (1990), these contracting frictions imply that the firm must issue securities such that both $s(\delta_i^*)$ and $\delta_i^* - s(\delta_i^*)$ are weakly increasing in δ_i^*.

The firm issues securities subject to these frictions in a segmented asset market like that described in the paper above. Like in the paper above, the investor can sell to two investors with stochastic discount factors $m_1(\omega)$ and $m_2(\omega)$ that depend on a scalar state variable ω that takes on lowest value $\bar{\omega}$. The investor also can sell risk free assets at a risk free rate i_d strictly below that of either pricing kernel. I assume that $m_1(\omega) - m_2(\omega)$ is strictly increasing in ω. In addition, I generalize condition 1 above to assume that for $\omega_1 > \omega_2$ that $\frac{d}{du}\frac{Pr(\delta_i^*>u|\omega_1)}{Pr(\delta_i^*>u|\omega_2)} > 0$. If there are any risk free cash flows backed by $s(\delta_i^*)$, it is optimal to tranche these cashflows into the largest possible riskless payoff and a residual $s(\delta_i^*) - min[s(\delta_i^*)]$ in order to borrow at the low risk free rate i_d. I therefore study how to divide the remaining cash flows to sell to the two investors with different pricing kernels. These claims can be written as a payoff $s_e(\delta_i^* - \bar{\delta}_i^*)$ and $s_d(\delta_i^* - \bar{\delta}_i^*)$ where both funtions must be nonnegative and weakly monotone and $s_e(\delta_i^* - \bar{\delta}_i^*) + s_d(\delta_i^* - \bar{\delta}_i^*) = \delta_i^* - \bar{\delta}_i^*$. In particular, both functions are Lipschitz continuous so by the fundamental theorem of calculus they can be written as $\int_0^{\delta_i^* - \bar{\delta}_i^*} (s_e)'(u)du = \int_0^{\infty} (s_e)'(u))\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$ and $\int_0^{\delta_i^* - \bar{\delta}_i^*} (s_d)'(u)du = \int_0^{\infty} (s_d)'(u))\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}du$, where $(s_e)'$ and $(s_d)'$ must be nonnegative and sum to 1.

The difference in the value of a claim $\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}$ according to the two investors' pricing kernels is

$Em_1\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\} - Em_2\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}$. This equals

$$E([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))$$

The deriavative of this with respect to u equals

$$E([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))\frac{\frac{d}{du}Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}$$

Because $log(Pr(\delta_i^* > u|\omega_1)) = log(Pr(\delta_i^* > u|\omega_1)) + log(\frac{Pr(\delta_i^* > u|\omega_1)}{Pr(\delta_i^* > u|\omega_2)})$, we have that $\frac{\frac{d}{du}Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega_1))}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega_1))} = \frac{\frac{d}{du}Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega_2))}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega_2))} + \frac{d}{du}log(\frac{Pr(\delta_i^* > u|\omega_1)}{Pr(\delta_i^* > u|\omega_2)})$. Because $\frac{d}{du}\frac{Pr(\delta_i^* > u|\omega_1)}{Pr(\delta_i^* > u|\omega_2)} > 0$ if $\omega_1 > \omega_2$, it follows that $\frac{\frac{d}{du}Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}$ is strictly increasing in ω. Because $([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))$ is also strictly increasing in ω, $cov([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega), \frac{\frac{d}{du}Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}) > 0$ and thus $E([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))\frac{\frac{d}{du}Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))} > E([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))E\frac{\frac{d}{du}Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}{Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))}$ It follows that if $E([m_1(\omega) - m_2(\omega)]Pr(\{\delta_i^* - \bar{\delta}_i^* \geq u\}|\omega))$ is positive, its derivative with respect to u is also positive. Because this for any value of u for which the payoff $\mathbb{1}\{\delta_i^* - \bar{\delta}_i^* \geq u\}$ is more valued by investor 1 than investor 2, investor 1 also values such a claim more for any higher value of u. It follows that there exists some u^* for which $u > u^*$ implies that this claim is valued more by investor 1, while if $u < u^*$ the claim is valued more by investor 2. The optimal security issued by our non-financial firm therefore bundles all such claims for sufficiently low values of u into 1 security, so $(s_d)'(u) = 1$ for $u < u^*$ and $(s_d)'(u) = 0$ for $u > u^*$. The payoffs of the two securities issued by the firm are therefore $\int_0^{\delta_i^* - \bar{\delta}_i^*} \mathbb{1}\{u < u^*\}du = min(\delta_i^* - \bar{\delta}_i^*, u^*)$ and $\int_0^{\delta_i^* - \bar{\delta}_i^*} \mathbb{1}\{u > u^*\}du = max(\delta_i^* - \bar{\delta}_i^* - u^*, 0)$. These are the payoffs of a debt and equity security, proving that these are the optimal securities for the firm to issue.

Appendix: Diversification and the Continuum Law of Large Numbers In the main text, I make the claim that the payoff of the intermediary's portfolio

$$\delta_I = \int_0^1 d_i q_I(d_i)di + \int_0^1 D_i q_I(D_i)di + \int_0^1 E_i q_I(E_i)di.$$

is equal to in the good and bad aggregate states

$$\delta_I^{good} = \int_0^1 E(d_i|good)q_I(d_i)di + \int_0^1 E(D_i|good)q_I(D_i)di + \int_0^1 E(E_i|good)q_I(E_i)di$$
$$\delta_I^{bad} = \int_0^1 E(d_i|bad)q_I(d_i)di + \int_0^1 E(D_i|bad)q_I(D_i)di + \int_0^1 E(E_i|bad)q_I(E_i)di.$$

Conditional on the aggregate state, the cash flows of all non-financial firms are independent of each other. This result is therefore equivalent to a continuum law of large numbers for indepedent but not identically distributed random variables. Following Uhlig (1996), I define these integrals as the limit of a sequence of Riemann sums, where the limit is taken in the L^2 norm. Each integral is of the form $\int_0^1 r_i q_I(r_i)di$ where the r_i are a continuum of independent random variables with bounded, continous means and bounded variances across $\epsilon \in [0, 1]$ and q_I is a bounded, Riemann integrable function. Pick a grid $r_{i(1)} \dots r_{i(1)}$ in the unit interval and note that

$$(E(\sum_{j=2}^{n} r_i q_I(r_i)(i(j) - i(j-1)) - \int_0^1 E(r_i)q_I(r_i)di))^2)^{.5} \leq$$
$$(E(\sum_{j=2}^{n} r_i q_I(r_i)(i(j) - i(j-1)) - \sum_{j=2}^{n} E(r_i)q_I(r_i)(i(j) - i(j-1)))^2)^{.5}$$
$$+|\sum_{j=2}^{n} E(r_i)q_I(r_i)(i(j) - i(j-1)) - \int_0^1 E(r_i)q_I(r_i)di)|$$

Because $E(r_i)$ is bounded and continuous in i, the second term converges to 0 for any Riemann integrable function q_I as the mesh of our grid converges to 0. To compute the first term, note that it is the variance of the sum of independent random variables, so that it equals the sum of their variances

$$\sum_{j=2}^{n}(i(j) - i(j-1))^2 q_I(r_i)^2 Var(r_i) \leq sup_i Var(r_i) sup_j |i(j) - i(j-1)|.$$

Because we assume that the r_i have uniformly bounded variance, this converges to 0 with the mesh of our grid. This proves that our integrals are well defined, and that the expressions claimed in the main text are valid, if they are interpreted as a Riemann integral computed with an L^2 notion of convergence. Similar results are in Uhlig (1996).