Make America Great: Long-Run Impacts of Short-Run Public Investment*

Alexandr Kopytov[†] and Haotian Xiang[‡]

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Abstract

We document S-shaped dynamics of the US economy associated with the construction of the Interstate Highway System in the 1960s. We then propose a business cycle model with two steady states arising due to productive public capital and production non-convexities. Small-scale short-run public investment programs generate transitory responses while large-scale ones can produce long-run impacts. Our quantitative analysis highlights the critical role played by public investment in explaining the economic dynamics around the 1960s. However, it casts doubt on the efficiency of a large public investment expansion in the post-Great Recession era.

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[†]Wharton School of the University of Pennsylvania: akopytov@wharton.upenn.edu

[‡]Wharton School of the University of Pennsylvania: xhaotian@wharton.upenn.edu

1 Introduction

The Great Recession and a subsequent slow recovery have rekindled an enthusiasm for public investment within both the academic world and the policy circle. One of the few approaches to revitalize the economy and return it to a pre-crisis path that both parties in the past presidential campaign agreed on was a massive infrastructure investment. On the Republican side, Donald Trump proposed a $1 trillion infrastructure plan that approximated 5% of the annual US domestic product. On the Democratic side, Hillary Clinton announced a $275 billion plan to rebuild US infrastructure and promised to have the plan passed in her first 100 days in the office.

A clear understanding of the potential outcome of such a large-scale short-run public investment is crucial for these policy discussions. However, existing quantitative studies of public investment consider only small shocks to productive public expenditure and consequent short-run economic responses within the standard business cycle frequency (Baxter and King, 1993 and Leeper, Walker, and Yang, 2010). When the impact of transient public investment is nonlinear, utilizing a close-to-linear RBC framework to analyze large-scale government investment programs can be quantitatively implausible.

Is there a macroeconomic nonlinearity associated with short-run public investment. Does a large-scale public investment shock generate a long-run impact that goes beyond the business cycle frequency?

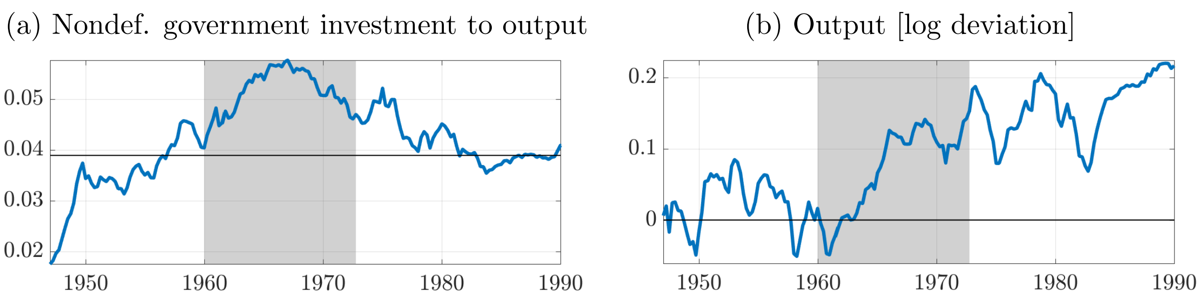

Figure 1: Public Investment and Nonlinear Output Dynamics in the 1960s. *Notes*: For panel (b), log-linear trend is constructed using the data between 1947Q1 and 1959Q4. Details about data construction are provided in Appendix A.

Panel (a) of Figure 1 shows that the period between the late 1950s and the early 1970s is marked by a historically high level of non-defense government investment over the postwar US, largely contributed by the construction of the Interstate Highway System. Interestingly, as presented in panel (b) of Figure 1, aggregate output per capita exhibits S-shaped dynamics around the same time. It stays on the new growth path for decades after public investment has returned to its long-run level.[1] In Section 2, we confirm similar dynamics across states. Moreover, we find that states with relatively larger highway spending booms witnessed more pronounced shifts in the level of per capita income.

In this paper, we analyze the impacts of short-run government investment in light of the US economy's dynamics around the massive public investment expansion of the 1960s. In particular, we propose a business cycle model exhibiting two stable steady states. Small shocks to public investment generate standard economic responses that fade away relatively quickly while large shocks can cause a transition across steady states and thus a long-run impact.

The multiplicity of steady states in our model rests on two main pillars. First, a là Barro (1990), public capital is productive and the government investment rule is procyclical. The government follows a fiscal rule under which its productive expenditure is proportional to the aggregate output. In our economy, an increase in private investment raises the aggregate output and thus leads to a build-up of public capital. An elevated stock of public capital enhances productivity and improves private incentives to invest.

Another key ingredient is production non-convexities. Each period, besides renting capital and hiring labor, individual firms can choose to utilize a productivity enhancing technology by paying a fixed cost (e.g., Durlauf, 1993 and Schaal and Taschereau-Dumouchel, 2015). When production factors are abundant, an increase in productivity is more attractive.

Our stationary economy features two stable steady states with different levels of output, hours, capital stocks as well as technology adoption intensity. In the high steady state, all firms choose to pay the fixed cost and operate the efficient technology. This in turn accelerates both private and public investment and thus helps to sustain capital stocks at high levels. In contrast, the economy is trapped in the low steady state when public and private capital are scarce and firms find it optimal not to adopt the efficient technology. Low aggregate productivity feeds back into low aggregates.

Despite the steady state multiplicity, the dynamic recursive equilibrium of our model

[1] Figure 1 presents the series up to 1990 because the mid 1990s are known to be marked by a structural change in the productivity and output growth rates, related to adoption of computer technologies (Fernald, 2016). Importantly, in the post-1990 sample, the ratio of non-defense government investment to output fluctuates around the level it reached by the mid 1970s. Appendix B.1 shows the series extended up to 2017.

is unique. We are therefore able to precisely understand how the economy responds to the two shocks inherent to our model, namely, public investment rate and productivity shocks. For small transitory disturbances to government investment, impulse responses are similar to what a standard RBC model delivers – their impacts are short-run. Similarly, productivity shocks of small scales generate only high-frequency macroeconomic responses.

Large shocks can generate a long-run impact and highly nonlinear dynamics when transitions across the steady states are involved. A sizable public investment shock significantly raises the marginal productivity in the economy starting at the low steady state. The private sector is encouraged to hire labor, accumulate capital and upgrade the technology. If the private capital stock becomes sufficiently large before the spike in public capital fades away due to depreciation, the private sector's desire to operate under the efficient technology perseveres. In this case, the economy keeps converging to the high steady state. In other words, a successful transition requires a temporary public investment project to be sufficiently large for the private sector to respond aggressively enough within a relatively short period of time.

The timing of a public project matters for a transition. Positive productivity shocks accelerate a transition to the high steady state, while negative ones impede or can even overturn it. A sequence of large productivity shocks is able to generate a transition by themselves. It suggests that a successful public investment action in a decade with good productivity realizations, such as the 1960s, does not necessarily imply a transition at a time when the productivity behaves poorly.

We calibrate our model and conduct two quantitative case studies in different decades under the same parameter choices.[2] First, we investigate whether the structural shift of the US economy pictured in Figure 1 was indeed caused by the large-scale public investment program or simply by positive productivity shocks. We extract public investment shocks during the transition period from the data and construct productivity shocks using the residual approach. Their impacts are then isolated. We find that the massive public investment alone can rationalize the parallel shifts to a large extent. At the same time, the transition was significantly accelerated by favorable productivity realizations. In contrast, productivity shocks by themselves cannot account for the observed macroeconomic

[2]The fact that one set of parameters governing the non-convex technology make it possible for the model to match two different episodes lends certain degree of support to our construction of the model and its quantitative realism.

dynamics.

Next, we take our model to the recent decade with the Great Recession and the slow recovery, when the US economy again witnessed S-shaped dynamics. Similarly, we feed in observed government investment shocks between 2007Q2 and 2017Q2 and retrieve productivity shocks via the residual approach. We investigate the following question: Could a large program of public investment, if successfully implemented right after the crisis, have helped the economy return to the high steady state? Experimenting with a counter-factual scenario where the public investment was increased by 1 trillion 2009 dollars in the post-Great Recession time (2009Q3 to 2017Q2), we arrive at a negative answer.

The reason why large government investment shocks succeeded in triggering a transition in the 1960s but might not be helpful during the slow recovery lies in the difference in exogenous productivity. Although the technology adoption decisions of firms in our model explain a large fraction of the slowdown in *measured* productivity over the past decade, mapping our model to the data reveals that the exogenous productivity has not recovered yet. With the presence of these negative productivity shocks, the experimented stimulus would not be powerful enough to induce the private sector to expand. In contrast, productivity shocks did not counter-affect the role of public investment in the first case.

Our work contributes to the business cycle models with productive government capital (Baxter and King, 1993 and Leeper, Walker, and Yang, 2010).[3] Given the US evidence in the 1960s, by introducing highly nonlinear dynamics, our framework provides a unified laboratory to study the impact of transitory public investment shocks of both small and large scales.[4]

Our model is also related to endogenous growth models with productive public capital. In his seminal work, Barro (1990) constructs an AK model, where the flow of public capital directly enters the production function. Futagami, Morita, and Shibata (1993) extend this framework and incorporate the stock of public capital into the aggregate production function. Turnovsky (2000) enriches framework of Barro (1990) by introducing elastic labor supply. These models, as usual for AK-style frameworks, rely on the knife-edge as-

[3] Aschauer and Greenwood (1985), Aschauer (1988), Barro (1989), Fisher and Turnovsky (1998), among others, analyze similar questions in a neoclassical Ramsey framework.

[4] One interesting example of applying a standard neoclassical framework to study large fiscal shocks is McGrattan and Ohanian (2010), who focus on the World War II. Our model, however, addresses a distinct type of public policy.

sumption of exact constant returns in the accumulatable factors. They imply high values of aggregate output elasticity with respect to private and public capital. In contrast, our calibration is in line with the existing empirical estimates.

Fernald (1999) empirically shows that the construction of the Interstate Highway System significantly contributed to the productivity growth during the 1960s and its completion was partially responsible for a consequent growth slowdown. Particularly, in contrast to the implications of standard growth models, he finds a network effect of highways: the first highway system is extremely productive, while the second one might not be.[5] Furthermore, inconsistent with what a linear RBC model would predict, Röller and Waverman (2001) find that investment in telecommunications significantly affects economic growth only when a critical mass of infrastructure is established. Implications of our model are consistent with both sets of evidence.

The impact of government investment shocks is studied by Perotti (2004), Auerbach and Gorodnichenko (2012), Ilzetzki, Mendoza, and Végh (2013) in a SVAR setting. A different strand of literature tries to estimate the elasticity of the aggregate output with respect to public capital following the seminal work of Aschauer (1989). Results of subsequent research are summarized by Romp and Haan (2007) and Bom and Ligthart (2014) in recent review papers.

Our formulation of a non-convex cost and resulting persistent economic dynamics share similarities with the works of Durlauf (1993) and Schaal and Taschereau-Dumouchel (2015).[6] Different from us, they focus on productivity shocks and the complementarity generated respectively by industry spillovers and demand externality. Cai (2016) constructs a business-cycle model in which collateral constraints give rise to multiple steady states. Similar to Schaal and Taschereau-Dumouchel (2015), his goal is to rationalize the slow recovery after the Great Recession. Long-lasting impacts of transitory shocks in a business-cycle environment bridges this paper with the work of Comin and Gertler (2006) and Anzoategui, Comin, Gertler, and Martinez (2016), who develop a two-sector RBC model in which R&D activities contribute to productivity variations and oscillations over medium-term cycles.

The paper proceeds as follows. Section 2 discusses evidence on the aggregate and state-

[5] Realtedly, Candelon, Colletaz, and Hurlin (2013) find that investment in infrastructure tends to be highly productive only when the initial stock of public capital is not too high.

[6] Cooper (1987) and Murphy, Shleifer, and Vishny (1989) exploit similar settings to analyze multiple equilibria.

level impacts of the infrastructure boom of the 1960s. Section 3 presents our model. We then characterize the model and provide further discussions in Section 4. Quantitative assessments are carried out in Section 5. Two case studies are conducted in Section 6. Section 7 concludes.

2 Motivating facts

To further motivate our model, this section provides additional facts related to the public capital boom in the 1960s. As is well acknowledged, a key event over that decade was the construction of the Interstate Highway System. The construction was authorized by the Federal-Aid Highway Act of 1956 passed under the presidency of Dwight D. Eisenhower and largely completed by 1973 (Fernald, 1999). An immediate growth impact is however unlikely. First, implementation delay of large infrastructure projects is around 3 years (Leeper, Walker, and Yang, 2010). Second, it takes time for private firms to take advantage of the improvement in public capital. Therefore, we use the observations from 1947Q1 to 1959Q4 when constructing pre-expansion trends.

As shown in Figure 1 in the introduction, the public investment boom of the 1960s was accompanied by S-shaped dynamics in the aggregate output per capita. Importantly, similar time-series patterns are observed in both consumption and investment per capita (Figure 2). The consistency across several major economic series, under identical detrending choices, suggests that what we have documented is indeed a systematic feature of the aggregate US economy around this period of time.

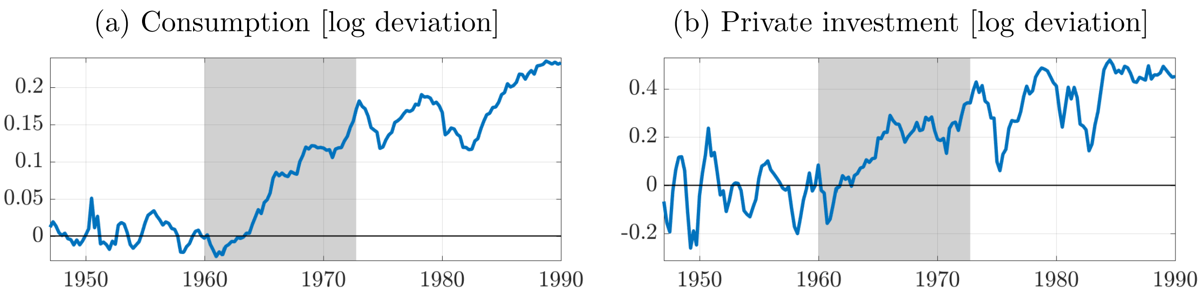

Figure 2: Nonlinear Consumption and Investment Dynamics in the 1960s. *Notes*: log-linear trends are constructed using the data between 1947Q1 and 1959Q4. Details about data construction are provided in Appendix A.

We next turn to the state level. A cross-sectional variation in sizes of public investment expansions shall be associated with differential behaviors of state-level output series. Below we verify that this is indeed the case in the data. We use state-level data on highway spending from the US Census. Since the BEA data on gross state product goes back only to 1963, we utilize state personal income as our output measure. All states, except Alaska, Hawaii and the District of Columbia, are equally split into two portfolios according to the size of their increases in the highway expenditure during the period of interest – more specifically, the difference between the average highway expenditure to state personal income ratio from 1960 to 1972 and that over the rest of time span.

According to Panel (a) of Figure 3, both of our state portfolios experienced a sizable increase in highway spending during the 1960s. Between 1960 and 1972, highway-to-income ratios jump up respectively by 1.00 and 0.44 percentage points.[7] Such increases are economically large: averaged across all states in the postwar sample, highway expenditures account for 1.59% of personal income. Importantly, the difference between the two is both economically and statistically significant (t-value is above 6).

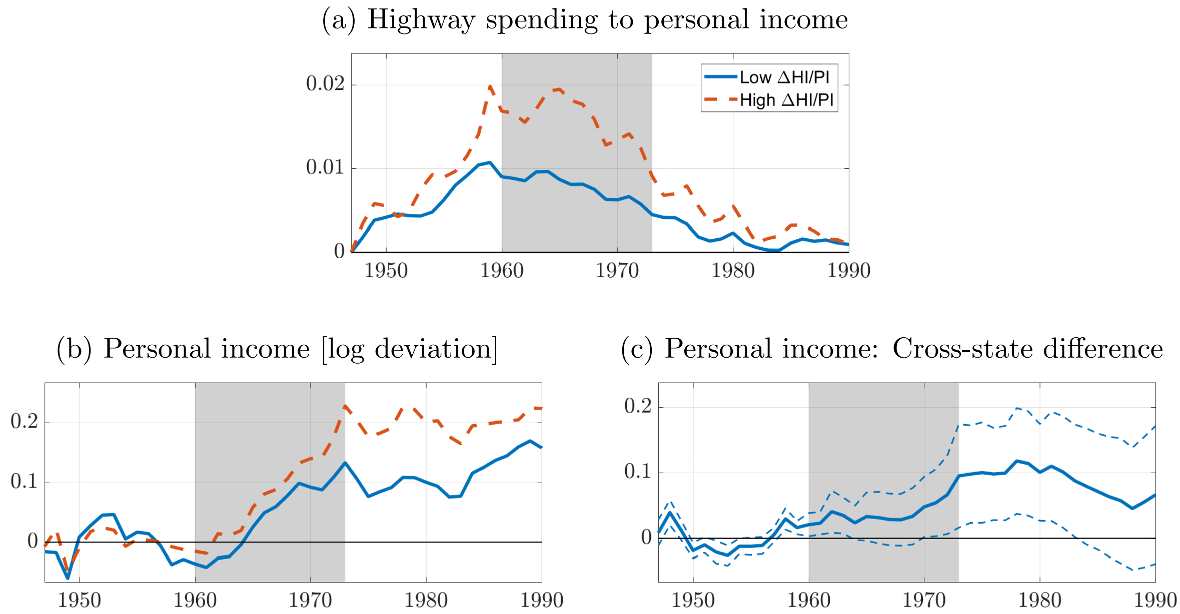

Figure 3: Highway Investment and State Nonlinear Output Dynamics. *Notes*: For panel (b), log-linear trends are constructed using the data between 1947 and 1959. Panel (c): the difference between the series shown in panel (b) (solid line) with 90% confidence bands (dashed lines). Details about data construction are provided in Appendix A.

Panel (b) of Figure 3 shows the averages of detrended real personal income per capita

[7]Notice that highway spending start to rise before 1960, consistent with the fact that the Federal-Aid Highway Act was authorized in 1956. The results are largely unchanged if portfolios are formed based on infrastructure expansion sizes between 1956 and 1972.

across portfolios. Nonlinear dynamics we have established for the aggregate economy are seen in both series. At the same time, on average, personal income in states where highway expenditures increased relatively more substantially witnessed much larger economic booms. Panel (c) shows that the difference between the personal income series is statistically different from zero on 10% level for at least a decade after the completion of the Interstate Highway System construction.

Of course, our suggestive evidence, both on the aggregate and state levels, do not establish a causal relation between the public investment boom and observed nonlinear economic dynamics. To answer whether such nonlinearity was indeed driven by the public capital expansion or other factors that happened to contribute to productivity, we turn to our quantitative model.

3 Model

In this section, we lay out a simple general equilibrium model consisting of a representative household, a government and a cross-section of static firms. In Section 3.5, we discuss some key assumptions in our model.

3.1 Households

The economy is populated by a single representative family with GHH preferences (Greenwood, Hercowitz, and Huffman, 1988) maximizing its life-time utility,

$$V = \max_{\{c_t, l_t, k_{t+1}\}_{t=0}^{\infty}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(c_t - \frac{1}{1+\nu} l_t^{1+\nu} \right)^{1-\gamma}, \tag{1}$$

in which the discount factor, the inverse Frisch elasticity and the risk aversion are denoted respectively by β, ν and γ. The representative household decides on the inter-temporal capital accumulation and the intra-period labor supply. It collects capital income $R_t k_t$, labor income $W_t l_t$, and firm profits π_t. All sources of income are subject to a uniform tax rate τ. The household's budget constraint is given by

$$c_t + k_{t+1} = (1-\tau)(W_t l_t + R_t k_t + \Pi_t) + (1-\delta_k) l_t - T_t, \tag{2}$$

where δ_k denotes the depreciation rate of private capital and T_t represents lump-sum taxes (negative values of T_t correspond to government transfers).

3.2 Firms

A continuum of identical firms operate in this economy, each of which is equipped with a Cobb-Douglas production technology,

$$y_t^i = A_t(K_t^G)^\alpha (k_t^i)^{\theta_k} (l_t^i)^{\theta_l} o_t^i,$$

where θ_k and θ_l represent the output elasticities with respect to private capital and labor. The stock of public capital K_t^G enters the production function and thus directly affects marginal productivity. The output elasticity with respect to public capital is α. Aggregate productivity evolves exogenously as

$$\ln A_{t+1} = (1-\rho_A)\ln \bar{A} + \rho_A \ln A_t + \sigma_A \epsilon_{t+1}^A, \;\; \epsilon^A \sim N(0,1).$$

In addition to renting capital k_t^i and hiring labor l_t^i from competitive capital and labor markets every period, firm i chooses whether to raise its productivity by a factor of $\omega > 1$ or not, i.e. $o_t^i \in \{\omega, 1\}$. If the former option is chosen, a fixed transfer cost f is incurred. It stands for expenses incurred when firms adopt a better technology to enhance production efficiency – for example hiring consulting firms and employee-training companies. Hereafter firms with $o_t^i = \omega/1$ will be referred to as operating with the high/low technology, respectively.

Firm i solves the following static profit maximization problem:

$$\pi_t^i = \max\Bigg\{ \underbrace{\max_{k_t^i, l_t^i} A_t(K_t^G)^\alpha (k_t^i)^{\theta_k} (l_t^i)^{\theta_l}\omega - W_t l_t^i - R_t k_t^i - f}_{\equiv \pi_t^H}, \\ \underbrace{\max_{k_t^i, l_t^i} A_t(K_t^G)^\alpha (k_t^i)^{\theta_k} (l_t^i)^{\theta_l} - W_t l_t^i - R_t k_t^i}_{\equiv \pi_t^L} \Bigg\}. \quad (3)$$

Conditional on the technology choice, firms adopt identical hiring and renting policies. We therefore denote optimal capital and labor choices of firms with the high/low technology by superscripts H/L.

Public good provision – an expansion in K_t^G – enhances marginal productivity of capital and labor. It also raises the marginal benefit of technology adoption, and as a result, a weakly larger number of firms will find it profitable to scale up their productivity. As will become clear in Section 4, this is the key mechanism that supports the steady state multiplicity and therefore gives rise to the nonlinearity of the model. Notice that productivity shock A_t also influences the capital and labor choices as well as the technology adoption.

3.3 Government

Government behaviors are summarized by a set of exogenous rules. Public capital depreciates at the rate δ_g and is accumulated through public investment G_t^I,

$$K_{t+1}^G = (1 - \delta_g)K_t^G + G_t^I,$$

where the public investment to output ratio $g_t^I \equiv G_t^I / Y_t$ ($Y_t = \int y_t^i di$), is controlled by a mean reverting spending rule,

$$g_{t+1}^I = (1 - \rho_g)\bar{g}^I + \rho_g g_t^I + \sigma_g \epsilon_{t+1}^g, \ \epsilon^g \sim N(0, 1). \tag{4}$$

Besides investment in public goods, government expenditures also include consumption G_t^C. Without aiming for a welfare analysis, we assume that the government consumption G_t^C does not enter the household's utility function. It contains, for example, wage bills of authorities, national defense spending, etc. Since our focus is public investment, we do not incorporate shocks to government consumption in order to keep the model and its solution simple. Specifically, government consumption purchases account for a fixed fraction of the total output,

$$\frac{G_t^C}{Y_t} = \bar{g}^C.$$

To finance its spending on consumption and investment, the government utilizes both lump-sum and distortionary taxes. The distortionary tax rate τ is fixed and, as a result, lump-sum taxes balance the government budget:[8]

$$G_t^I + G_t^C = \tau Y_t + T_t.$$

3.4 Equilibrium

The model has four state variables: the stocks of private capital and public capital, the public investment to output ratio, and productivity, $\Omega = (K, K^G, g^I, A)$. A recursive competitive equilibrium is characterized by i) a value function $V(k,\Omega)$ and policy functions $c(k,\Omega)$, $k'(k,\Omega)$ and $l(k,\Omega)$ for the household; ii) individual firm i's decisions $o^i(\Omega) \in \{\omega, 1\}$, $k^i(\Omega)$, $l^i(\Omega)$, $y^i(\Omega)$ and implied by them profit $\pi^i(\Omega)$ for all $i \in [0,1]$; iii) a set of exogenous fiscal rules $G^I(\Omega)$, $G^C(\Omega)$ and $T(\Omega)$; iv) pricing functions $R(\Omega)$ and $W(\Omega)$; v) laws of motion for private and public capital stocks, $K'(\Omega)$ and $K^{G'}(\Omega)$; vi) mass of firms adopting the high technology $m(\Omega)$; vii) aggregate variables $Y(\Omega)$, $\Pi(\Omega)$, such that:

1. $V(k,\Omega)$, $c(k,\Omega)$, $k'(k,\Omega)$ and $l(k,\Omega)$ solve (1) subject to (2), taking prices $W(\Omega)$, $R(\Omega)$, profits $\Pi(\Omega)$, transfers $T(\Omega)$, and the evolution $\Omega'(\Omega)$ as given.

2. $o^i(\Omega) \in \{\omega, 1\}$, $k^i(\Omega)$, $l^i(\Omega)$, $y^i(\Omega)$ and $\pi^i(\Omega)$ solve the problem (3), taking prices $W(\Omega)$, $R(\Omega)$ as given. Moreover, $k^i(\Omega) = k^L(\Omega)$, $l^i(\Omega) = l^L(\Omega)$ and $y^i(\Omega) = y^L(\Omega) \equiv A\left[K^G\right]^\alpha \left[k^L(\Omega)\right]^{\theta_k} \left[l^L(\Omega)\right]^{\theta_l}$ if firm i operates the low technology and $k^i(\Omega) = k^H(\Omega)$, $l^i(\Omega) = l^H(\Omega)$ $y^i(\Omega) = y^H(\Omega) \equiv A\left[K^G\right]^\alpha \left[k^H(\Omega)\right]^{\theta_k} \left[l^H(\Omega)\right]^{\theta_l} \omega$ otherwise.

3. The mass of firms adopting the high technology is $m(\Omega) = \int_{o^i(\Omega)=\omega} di$.

4. Individual decisions are consistent with the aggregate dynamics $k'(K,\Omega) = K'(\Omega)$.

5. Aggregate variables $Y(\Omega)$ and $\Pi(\Omega)$ are given by $Y(\Omega) = m(\Omega)y^H(\Omega) + (1 - m(\Omega))y^L(\Omega)$ and $\Pi(\Omega) = m(\Omega)(\pi^H(\Omega) + f) + (1 - m(\Omega))\pi^L(\Omega)$.[9]

[8] In Appendix D.1, we numerically investigate the case where the shocks to the government budget are partially absorbed by adjustments in the distortionary tax rate.

[9] Recall that f is assumed to be a transfer from firms to the representative household. Hence, the aggregate profit received by the household is adjusted by total fixed costs paid by firms operating the high technology. Alternatively, f can be modeled as a resource cost. Multiple steady states and highly nonlinear dynamics still arise under this setting.

6. Markets for labor, capital and consumption goods clear: $l(K,\Omega) = m(\Omega)l^H(\Omega) + (1-m(\Omega))l^L(\Omega)$, $k(K,\Omega) = m(\Omega)k^H(\Omega) + (1-m(\Omega))k^L(\Omega)$, and $c(K,\Omega)+K'(\Omega) = Y(\Omega) - T(\Omega) - G^C(\Omega) - G^I(\omega)$.

7. The government budget is balanced, $G^I(\Omega) + G^C(\Omega) = \tau Y(\Omega) + T(\Omega)$, where $G^I(\Omega) = g^I Y(\Omega)$ and $G^C(\Omega) = \bar{g}^C Y(\Omega)$.

8. The stock of public capital evolves according to $K^{G'}(\Omega) = (1-\delta_g)K^G + G^I(\Omega)$.

3.5 Discussions of assumptions

Before proceeding to the model characterizations, we discuss several key assumptions in the model.

3.5.1 Production non-convexities

The key innovation in our model, compared to a standard RBC model with public capital, is the non-convexity in the firms' production choice. In our model, the non-convexity takes a simple form of a binary technology choice. The main results of the paper are unchanged if a continuous non-convex technology choice is considered.

Voluminous literature documents non-convexities in production adjustments on the micro level. Examples of such non-convexities might include capital adjustment (Cooper and Haltiwanger, 2006), labor adjustment (Caballero, Engel, and Haltiwanger, 1997), production process adjustment (Bresnahan and Ramey, 1994, Hall, 2000), and marketing costs (Spence, 1976).

On the industry level, Cooper and Haltiwanger (1990) argue that the observed statistics of the automobile sector is consistent with a model featuring non-convexity in capital replacement. Ramey (1991) provides evidence of cost functions' non-convexities for seven industries.

In our setting, micro-level non-convexities have important impacts on the aggregate economic dynamics. Consistent with this view, Hansen (1985) shows the importance of labor indivisibility on aggregate fluctuations. Hansen and Prescott (2005) demonstrate that occasionally binding capacity constraints can help explain business cycle asymmetries. Durlauf (1991), Durlauf (1994) and Schaal and Taschereau-Dumouchel (2015) argue that a binary technology choice and complementarities between firms can give rise to highly persistent path-dependent responses of macro variables to productivity shocks, as suggested by the data. A series of works emphasize that firm-level non-convexities can explain economy-wide fluctuations without aggregate shocks (Bak, Chen, Scheinkman, and Woodford, 1993, Nirei, 2006 and Nirei, 2015).[10] In the growth literature, non-convexities are utilized to explain club convergence and poverty traps (see Durlauf, 1993 and Galor, 1996 for a literature overview).

3.5.2 Fiscal rules

Our analyses of the impacts of public investment shocks are positive. Therefore, instead of fully specifying the government's problem, we assume that the government follows an exogenous fiscal rule. This strategy has been widely adopted by the existing literature that tries to quantify the impact of fiscal policies (e.g., Leeper, Plante, and Traum, 2010, Leeper, Walker, and Yang, 2010, Fernández-Villaverde, Guerrón-Quintana, Kuester, and Rubio-Ramírez, 2015).

Since the model requires a global solution, we try to minimize the number of state variables not essential to our study. We assume that government investment follows a stochastic process, while government consumption constitutes a fixed fraction of output. Moreover, instead of introducing government debt and associated bond smoothing rules, we assume that all government investment shocks are financed through adjustments of lump-sum taxes. In our model and case studies, the tax rate τ is fixed and thus Ricardian equivalence holds.

3.5.3 Productive public capital

We assume that the stock of public capital K^G enters production function of private firms. Our way of modeling public capital goes in line with Arrow and Kurz (1970), Futagami, Morita, and Shibata (1993), Baxter and King (1993) and Leeper, Walker, and

[10] Thomas (2002) argues that lumpy plant-level investment is not relevant for business cycle fluctuations in general equilibrium. However, as pointed out by Nirei (2015), her "...model features a continuum of firms... This choice precludes the possibility that interactions of 'granular' firms give rise to aggregate fluctuations..." Importance of this granularity for aggregate fluctuations is underscored by Gabaix (2011). In the setting similar to Thomas (2002), Bachmann, Caballero, and Engel (2013) demonstrate that lumpy microeconomic capital adjustments are important to generate procyclical aggregate investment sensitivity to shocks.

Yang (2010) among others. An alternative modeling approach is to put a flow of public spending in the production function, as in Barro (1990). However, many public goods, including highways, are stock variables in nature. Empirical research, starting from at least Aschauer (1989), generally investigates how stock of public capital affects aggregate productivity. Working with stock variables helps us to link our model, particularly the elasticity α, tightly with empirical estimates.

4 Characterizations

In this section, we establish some properties of the model and outline key intuitions behind. Under certain restrictions on parameters, the model exhibits two locally stable steady states and has a unique recursive equilibrium. Multiplicity of the steady states is addressed in Section 4.1. We consider a special case of a deterministic environment where we can analytically characterize the two steady states. In Section 4.2, we establish the existence and uniqueness of the recursive equilibrium in a stochastic environment. Lastly, in Section 4.3 we discuss the dynamic behavior of the model. All derivations and proofs are provided in Appendix E.

4.1 Two stable steady states

In this section, our goal is to investigate non-stochastic steady states of the model. Time subscripts are therefore omitted.

Our departure from existing RBC models with productive government capital lies in the binary technology adoption choice. To see how such a formulation alters the environment, we calculate the capital and labor decisions of the H and L firms given prices R and W, as well as the mass of firms operating under the high technology m. We then express the prices as functions of the aggregate state variables via market clearing conditions. After plugging them back into the firms' optimal choices, we arrive at the following aggregate production function,

$$Y = \hat{A}(A,m)\left(K^G\right)^{\alpha} K^{\theta_k} L^{\theta_l}, \tag{5}$$
$$\hat{A}(A,m) = A\left[1 + m(\omega^{\frac{1}{1-\theta_l-\theta_k}} - 1)\right]^{1-\theta_l-\theta_k},$$

where we call $\hat{A}(A, m)$ the measured TFP. It depends on the exogenous productivity A and the mass of firms operating under the high technology m, the latter of which is determined endogenously by aggregating individual firms' binary decisions. K and L, respectively, stand for aggregate capital and labor. Notice that for a fixed m, our model reduces to a standard real business cycle model with productive public capital.

To understand how m is determined, consider the technology choice given in Equation (3). We calculate the difference between profits π^H and π^L for a given m,

$$\begin{aligned}
&\Delta\pi(m; K, K^G, A) \equiv \pi^H - \pi^L \\
&= \zeta A^{\frac{1+\nu}{1-\theta_l+\nu}} \left(K^G\right)^{\frac{\alpha(1+\nu)}{1-\theta_l+\nu}} \left[1 + m(\omega^{\frac{1}{1-\theta_l-\theta_k}} - 1)\right]^{\frac{\theta_l-(\theta_l+\theta_k)(1+\nu)}{1-\theta_l+\nu}} K^{\frac{\theta_k(1+\nu)}{1-\theta_l+\nu}} - f,
\end{aligned} \tag{6}$$

where scaler $\zeta = (1 - \theta_l - \theta_k)(\omega^{\frac{1}{1-\theta_l-\theta_k}} - 1)[(1 - \tau)\theta_l]^{\frac{\theta_l}{1-\theta_l+\nu}} > 0$.

The gain of adopting a new technology $\Delta\pi$ is strictly increasing in the private and public capital stocks K and K^G, together with productivity A. However, since $\theta_l < (\theta_k + \theta_l)(1+\nu)$, it is decreasing in m. Factor competition lies behind this within-period substitution effect. Due to their higher marginal productivity, H firms optimally choose higher capital and labor in the competitive markets than L firms. Given a predetermined capital stock and the household's disutility from working, a larger m drives up the demand for capital and labor and thus the prices R and W. The benefit of the high technology utilization declines accordingly.[11]

Equilibrium m is

$$m(K, K^G, A) = \begin{cases} 1, & \Delta\pi(1; K, K^G, A) > 0, \\ m^*(K, K^G, A) \in (0, 1), & \Delta\pi(m^*(K, K^G, A); K, K^G, A) = 0, \\ 0, & \Delta\pi(0; K, K^G, A) < 0. \end{cases}$$

In the second case, all firms are indifferent between upgrading or not. The result can be interpreted as the outcome of a mixed-strategy equilibrium where each firm operates under the H technology with probability m^*. It is easy to see that equilibrium m is (weakly) increasing in K, K^G and A.

[11] Schaal and Taschereau-Dumouchel (2015, 2016) show that if the economy features demand externality, $\Delta\pi(m)$ can become increasing or non-monotone in m. In their setting, firms' technology choice is subject to coordination problem and multiple equilibria can arise. In our model, each firm is strictly worse off when more firms are utilizing the high technology, hence equilibrium choice of m is unique given the state variables.

Now inter-temporal complementarity can be clearly seen. Larger capital stocks K and K^G enhance marginal productivities through encouraging the technology adoption and generating a larger m. As m elevates marginal productivity, total output and private investment increase. On the other hand, following the spending rule specified in Equation (4), K^G also expands at a faster pace.

In the long run, endogenous state variables – K and K^G – fully adjust. In a deterministic world, shocks are held at their long-run means. Write the steady state mass of H firms as a function of the endogenous state variables, $m^{ss}(K, K^G) \equiv m(K, K^G, \bar{A})$, and define $\Delta\pi^{ss}(K, K^G) \equiv \Delta\pi(m^{ss}(K, K^G); K, K^G, \bar{A})$. The following proposition shows the importance of our two key ingredients: productive public capital and non-convexities.

Proposition 1 *The model exhibits two stable deterministic steady states:* $\{K_H,\ K_H^G,\ m^{ss}(K_H, K_H^G) = 1\}$ *and* $\{K_L,\ K_L^G,\ m^{ss}(K_L, K_L^G) = 0\}$ *such that* $K_H > K_L$ *and* $K_H^G > K_L^G$ *if*
i) $\dfrac{\nu}{1+\nu}\theta_l < \alpha < 1 - \theta_k - \dfrac{\theta_l}{1+\nu}$,
ii) $\Delta\pi^{ss}(K_L, K_L^G) < 0 < \Delta\pi^{ss}(K_H, K_H^G)$.

The first condition provides a boundary on the elasticity of aggregate output with respect to public capital. The lower bound is determined by the conflict between the public capital induced complementarity and the factor competition – the former makes the benefit of technology adoption an increasing function of m while the latter works in the opposite direction. When the government capital is productive (α is large), the spillover effect of increase in m on accumulation of private capital through the exogenous fiscal rule is strong. If, on the other hand, labor is not responsive (ν is large) and constitutes a large share of output (θ_l is large), high m also induces a significant increase in wages and thus drives down the benefit of the high technology utilization. If $\dfrac{\nu}{1+\nu}\theta_l < \alpha$ then the former force dominates and the multiple steady states arise. It is worth noting that the parameters associated with capital do not show up because capital is fully adjustable across the steady states. The upper bound, $\alpha < 1 - \theta_k - \dfrac{\theta_l}{1+\nu}$, guarantees the boundedness of the policies and prevents explosive dynamics of the economy.

When the household utility function is not of the GHH form, a positive labor response to an increase in wages is mitigated by the wealth effect. In this case, high m is associated with a stronger factor competition. The existence of multiple steady states requires a larger value of α.[12]

However, even if public good externality dominates the factor competition, the existence of the multiple stable steady states is not guaranteed. Consider, for example, the case where a technology upgrade is extremely costly, i.e. $f \to \infty$. It will then never be optimal for any firm to adopt the new technology and therefore m stays at zero. As argued before, in this case the model degenerates to a standard RBC model with a unique steady state. In contrast, when $f \to 0$, m is kept at one and again we only have a unique steady state. The second condition of Proposition 1 makes sure that the fixed cost is mild so that in the high steady state all firms would like to be equipped with the high technology, while at the low steady states no firm wants to.

Stochastic steady states are different from the steady states in a deterministic world due to risk adjustments. We verify numerically that our model preserves the steady state multiplicity in a stochastic environment.

4.2 Equilibrium uniqueness

Though the model exhibits two stable steady states, the recursive equilibrium, characterized by a set of policy and pricing functions, exists and is unique.

Proposition 2 *Under mild conditions, outlined in Appendix E.2, there exists a unique dynamic recursive equilibrium.*

Equilibrium uniqueness is important for policy quantification. In this regard, the model is distinct from previous models with externalities creating social increasing returns to scale and leading to indeterminacy of equilibria (e.g., Benhabib and Farmer, 1994 and Farmer and Guo, 1994). Proofs of the equilibrium existence and uniqueness are nontrivial since our model features aggregate non-convexities and externalities. We extend the monotone operator and lattice-theoretic technique developed by Coleman (1991) to a multi-dimensional endogenous state space.[13]

[12] As argued by Jaimovich and Rebelo (2009) and Schmitt-Grohé and Uribe (2012), the wealth effect on labor supply is almost absent at the business cycle frequencies.

[13] See also Coleman (2000), Datta, Mirman, and Reffett (2002), Morand and Reffett (2003) and Schaal and Taschereau-Dumouchel (2015).

4.3 Model dynamics

Steady state multiplicity implies highly nonlinear dynamics of the economy. In this section, we illustrate the implications of our model and the behavior of our economy in response to public investment and productivity shocks.

The dynamics of the model can be illustrated by the phase diagram in Figure 4. The economy features two basins of attraction. In the low steady state, private and public capital are scarce. Firms optimally choose not to utilize the high technology, $m = 0$. On the contrary, private and public capital are abundant in the high steady state, and all firms operate under the high technology, $m = 1$. Both steady states are stable. Thus, relatively small shocks cause only temporary changes. Large shocks can lead to long-lasting consequences, associated with steady state transitions.

Consider, for example, the economy at the low steady state. A massive public capital expansion incentivizes the private sector to upgrade the technology and accumulate more capital. A large increase in total output and thus the tax revenue further pushes up the amount of public investment. This positive feedback loop is counteracted by depreciation of public capital. For a large enough public investment program, the former force dominates and the economy reaches the high steady state's basin of attraction. This steady state transition is illustrated by the dashed blue line in Figure 4.

In contrast, following a short-run investment project of an insufficient magnitude, the transition is not achieved. In those cases, private and public capital have not yet arrived at large enough levels by the moment depreciation starts to produce an impact. A decline in government capital stock suppresses marginal benefit of private investment and technology upgrade. Eventually, the economy returns back to the low steady state, as illustrated by the dot-dashed red line in Figure 4.

Similarly, insufficient government capital creation during an economic crisis or a devastating destruction of productive public resources in a war or a natural disaster can result in unintended long-lasting consequences, since the economy might slide down from the high to low steady state.

Responses to productivity shocks inherit such nonlinearity. Similar to a government investment shock, a productivity shock alters the private sector's incentive to accumulate capital and adopt the efficient technology and thus is able to trigger a steady-state transition. In a depressed economy with productivity growth sufficiently below its long-run

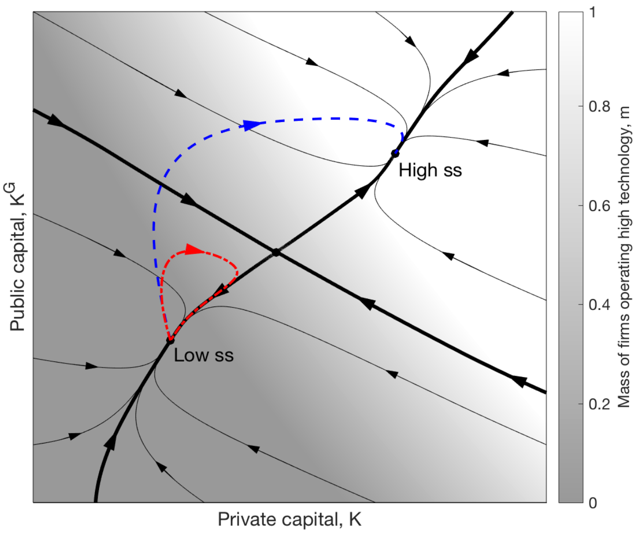

Figure 4: Two Stable Steady States. *Notes*: Phase diagram with two basins of attraction. Productivity A is at its long-run level $\bar{A}$. Dot-dashed red and dashed blue lines represent the paths of the economy starting at the low steady state ($m = 0$) after two positive shocks to public investment of small and large sizes, respectively.

trend, the private sector lacks a desire to invest. The economy starting at the high steady state might slide down to the low steady state unless government intervenes through fiscal actions that can effectively prevent the total factor productivity from a considerable drop.

5 Quantitative assessments

From this section, we take our model to quantitative analyses. We describe our parametrization in Section 5.1. In Section 5.2, we investigate the quantitative performance of the model.

5.1 Calibration

The period of the model is one quarter. Table 1 lists the parameters. Sources of the data used for calibration are outlined in Appendix A.

Parameter	Value	Source/Target
Preferences		
Risk aversion	$\gamma = 1.0$	Log utility
Frisch labor elasticity	$1/\nu = 3.33$	Higher end of macro estimates
Time discounting	$\beta = 0.95^{1/4}$	0.95 annually
Production function		
Labor elasticity	$\theta_l = 0.56$	Basu and Fernald (1997)
Private capital elasticity	$\theta_k = 0.24$	Basu and Fernald (1997)
Public capital elasticity	$\alpha = 0.15$	Bom and Ligthart (2014)
Depreciation rates		
Private capital	$\delta_k = 1 - 0.9^{1/4}$	10% annually
Public capital	$\delta_g = 1 - 0.92^{1/4}$	Leeper, Walker, and Yang (2010)
Fiscal rule		
Government consumption	$\bar{g}^C = 0.235$	Postwar US data
Transfers	$\bar{z} = 0.060$	Postwar US data
Government investment	$\bar{g}^I = 0.041$	Postwar US data
Marginal tax rate	$\tau = 0.336$	$\tau \equiv \bar{g}^I + \bar{g}^C + \bar{z}$
GI shocks		
Persistence	$\rho_g = 0.967$	Postwar US data
Standard deviation of shocks	$\sigma_g = 0.0011$	Postwar US data
Productivity		
Mean	$\bar{A} = 0.764$	Normalization
Persistence	$\rho_A = 0.94$	Output persistence
Standard deviation of shocks	$\sigma_A = 0.008$	Output volatility
Technology adoption		
Fixed cost	$f = 0.0051$	See text
Scaling up parameter	$\omega = 1.02$	Distance between steady states

Table 1: Parametrization.

5.1.1 Standard parameters

A few parameters are chosen using standard values adopted in the literature. We set the time discounting to $\beta = 0.987$. The depreciation rate of private capital is $\delta_k = 0.026$. The elasticities of output with respect to private capital and labor are set to $\theta_k = 0.24$ and $\theta_l = 0.56$, respectively. They together imply a returns-to-scale parameter of 0.8, which lies within a range of industry-wide estimates of Basu and Fernald (1997). For the preferences of the representative household, we adopt the log utility, i.e. $\gamma = 1$. We set $\nu = 0.3$, which implies a Frisch elasticity of labor supply of 3.33 and is consistent with existing macro estimates (Chetty, Guren, Manoli, and Weber, 2011).[14]

5.1.2 Stochastic processes

We calibrate ρ_A and σ_A to match the autocorrelation and volatility of the medium-term cycle component (0-200 quarters) of output. In Section 5.2, we describe how we measure these statistics. The resulting values are $\rho_A = 0.94$ and $\sigma_A = 0.008$. The mean $\bar{A} = 0.764$ is set to normalize capital in the low steady state to 1.

The public investment process is estimated using the postwar US data. We find the mean and persistence of the government investment to output ratio (GI ratio hereafter) to be $\bar{g}^I = 0.041$ and $\rho_g = 0.967$. The standard deviation of GI shocks is $\sigma_g = 0.0011$. The fractions of output spent on government consumption and transfers are $\bar{g}^C = 0.235$ and $\bar{z} = 0.060$. Corresponding income tax rate is $\tau \equiv \bar{g}^I + \bar{g}^C + \bar{z} = 0.336$. The quarterly depreciation rate of public capital is set to $\delta_g = 0.0206$, similar to Baxter and King (1993) and Leeper, Walker, and Yang (2010).[15]

5.1.3 Non-convexity

There are two key parameters for us: α and ω. There is not much consensus on the value of α. Early studies find very high elasticities – for example Aschauer (1989) estimates α to be 0.39 – while more recent estimates tend to be lower on average, although the variability is huge. We set $\alpha = 0.15$, which is close to the average value reported by Bom and Ligthart (2014) in their survey paper. In appendix B.3, we experiment with an α of 0.1, resulting in the model's quantitative performances largely unchanged.

Our calibration of ω rests on the assumption that the US economy was in the low steady state before the start of the massive public spending of the 1960s and had largely converged to the high steady state by the early 1970s. We set $\omega = 1.02$ to match the distance between the steady state levels of output to the difference in the levels of the pre-1960 and

[14]A robustness analysis with a higher value of $\nu = 0.6$ are provided in Appendix B.2.

[15]In Appendix B.4, we demonstrate that a standard RBC model with public capital, even with a fairly low value of $\delta_g = 0.0127$, cannot rationalize the behavior of the US economy around the 1960s.

post-1973 detrended per capita GDP. It is important to notice that we do not re-calibrate ω when taking this model to the post-2007 period. The fact that the model still provides a reasonable match of empirical series lends some support to our calibration strategy.

Being quantitatively less important, the fixed cost f governs the frequency of transitions across the two steady states. Realized transitions are rare, therefore the postwar US data might not be very informative in this regard. Instead, we pick $f = 0.0051$ in order to match frequencies of extreme output growth events in the model and in the data. In our framework, a change in the mass of firms using the high technology m is associated with a larger change in output. If f is too high or low then m is unlikely to change because it is either too costly to operate the high technology (high f, $m = 0$), or the scale-up is cheap and all firms use it (low f, $m = 1$). In this case, large fluctuations in output are less frequent since the amplification through change in m does not take place. For our choice of f, the model economy generates a similar-to-the-data number of large output growth events. Resulting $f = 0.0051$ corresponds to 2% of average aggregate output. Appendix C describes our approach in more details.

5.2 Model assessments

Before applying our framework to two case studies and conducting corresponding counterfactual analyses, it is important to examine whether our calibration is quantitatively realistic. Section 5.2.1 compares the data and model implied second moments of the major macroeconomic series. Section 5.2.2 describes the impulse response functions to the government investment and productivity shocks.

5.2.1 Unconditional moments

As shown in Table 2, our model reproduces both high and medium frequency fluctuations of the US economy.[16] Following Comin and Gertler (2006), we assume that the high frequency component of a medium run cycle includes fluctuations at frequencies below 32 quarters, while frequencies between 32 and 200 quarters correspond to the medium frequency component. We use the band pass filter to separate these components (Baxter

[16]TFP is measured using standard Solow residual approach. Aggregate production technology is given in equation (5). Recall that TFP is endogenous and depends on the mass of firms utilizing the high technology.

	Medium-term cycle, 0-200 qtr		High frequency component, 0-32 qtr		Medium frequency component, 32-200 qtr	
	Data	Model	Data	Model	Data	Model
Output	4.23	4.46 (2.99,6.46)	2.30	2.05 (1.59,2.61)	3.61	3.91 (2.28,6.05)
Consumption	3.34	3.62 (2.36,5.37)	1.30	1.43 (1.11,1.80)	3.12	3.28 (1.92,5.20)
Hours	3.75	3.42 (2.31,4.87)	1.78	1.57 (1.23,1.97)	3.41	3.00 (1.76,4.59)
Investment	12.35	10.61 (7.24,15.64)	8.05	5.86 (4.33,7.92)	9.35	8.65 (5.09,14.24)
TFP	2.46	2.17 (1.57,2.44)	1.52	1.16 (0.93,1.44)	1.95	1.80 (1.10,2.65)

Table 2: Macroeconomic Fluctuations – Model and Data. *Notes*: Standard deviations of macro variables in the model and in the postwar US data. The model is simulated for 280 quarters for 10,000 times. 95% confidence intervals are given between parentheses. Details about data construction are provided in Appendix A.

and King, 1999 and Christiano and Fitzgerald, 2003).

For high frequency fluctuations, standard deviations of output, consumption, hours implied by the model are in a good correspondence with their data counterparts. Private investment and TFP appears to be slightly more volatile in the data. The model under our calibration also generates sizable medium-run oscillations.

5.2.2 Impulse responses

Government investment shocks. We start by shocking the economy located at the low steady state by GI shocks of different sizes. Corresponding impulse responses are depicted in Figure 5. For the solid blue lines, the GI ratio goes up by 0.33 p.p.. For the dashed red lines the shock is 0.67 p.p.. Finally, for the dot-dashed yellow lines the shock is 1 p.p..

A small shock does not induce any firms to start operating with the high technology, and m stays at zero. The impact of the shock is transitory and similar to what is delivered by existing RBC models with public capital. During the early stages, consumption and private investment are depressed due to an increase in lump-sum taxation. Later on, when

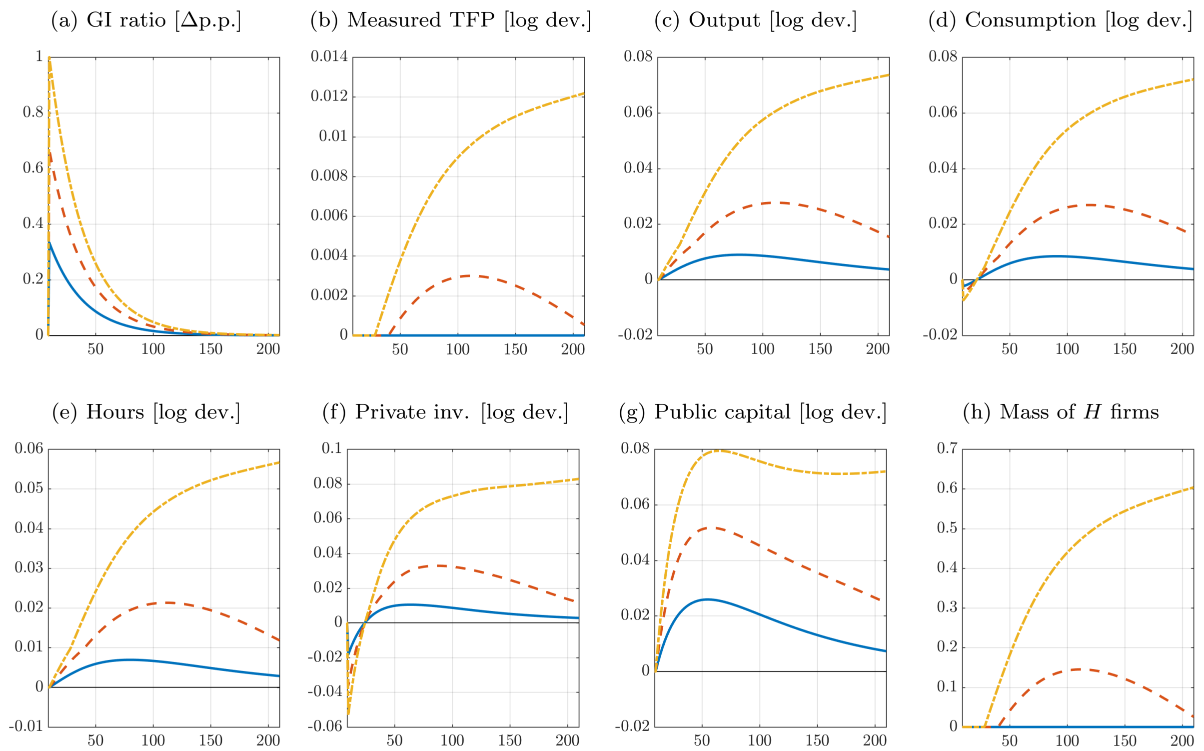

Figure 5: Impacts of GI Shocks. *Notes*: Impulse responses of the economy starting at the low steady state to GI shocks of various sizes. X-axis is the number of quarters.

the stock of public capital goes up, output, consumption and investment increase. GHH preferences, with zero wealth effect on labor supply, imply that hours do not respond at the moment of the shock but later go up significantly, even despite an increase in consumption.

Larger shocks produce a qualitatively different impact on the economy. A large enough increase in the stock of public capital induces some firms to switch to the high technology, which pushes up firms' demand for capital and labor. Consequently, hours and private investment increase significantly. Resulting high output can support larger public investment and hence induce more firms to adopt the new technology. As suggested by Expression (5), a GI shock resulting in an increase in m is also associated with a growth of measured TFP $\hat{A}(A, m)$.

When firms start to operate with the high technology, the responses to shocks become more persistent, in case of the 0.67 p.p. shock, or even permanent, for the 1 p.p. shock. As mentioned earlier, a successful transition requires a transitory GI shock to be large enough – private capital should arrive at a sufficiently high level before depreciation of public capital weakens the private sector's incentives to invest and upgrade.

Another important feature of our model is that the impact of public investment programs is state dependent. At the high steady state, all firms operate at the full capacity, $m = 1$, and even large GI shocks cannot induce additional firms' switching (see Appendix D.2 for corresponding impulse responses). Hence, keep investing in public infrastructure is not a way for the government to pursue a higher growth rate in the long run.[17]

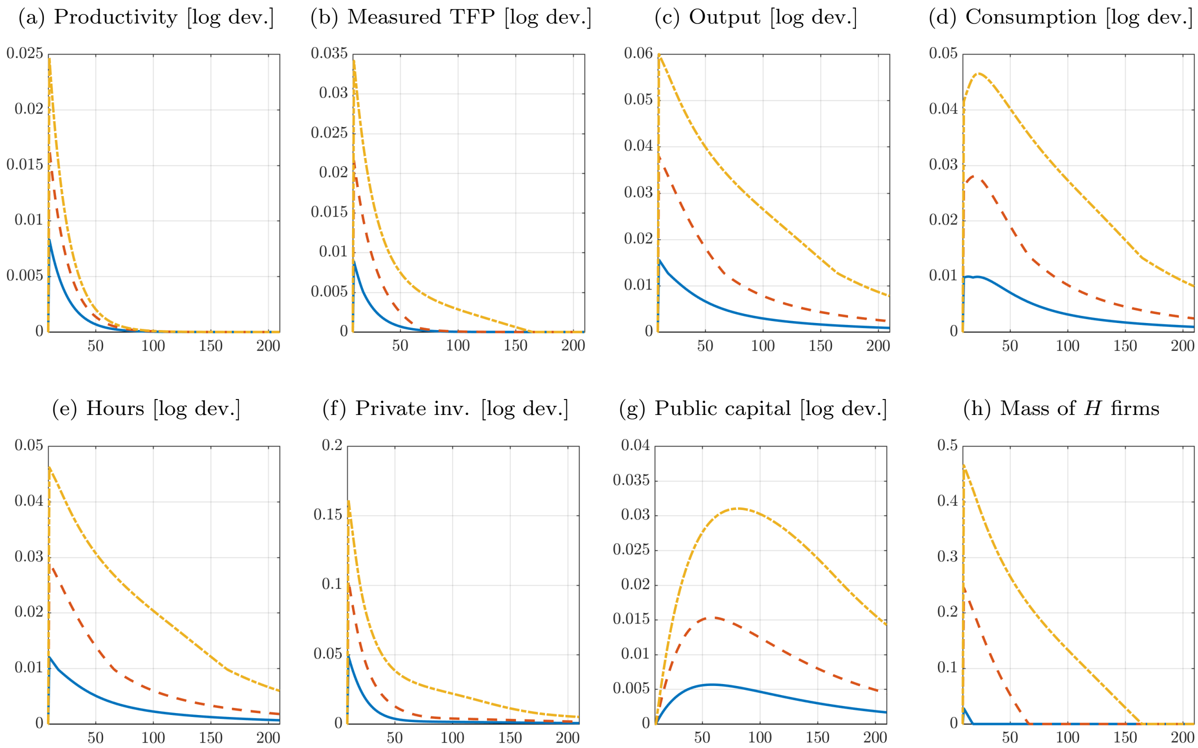

Figure 6: Impacts of Productivity Shocks. *Notes*: Impulse responses of the economy starting at the low steady state to productivity shocks of various sizes. X-axis is the number of quarters.

Productivity shocks. Figure 6 plots the impulse response functions associated with A shocks of 0.83% (solid blue), 1.67% (dashed red) and 2.5% (dot-dashed yellow). As in standard RBC models, investment, consumption, hours and output jump up upon the arrival of a positive productivity shock. The abundance of factors and a high productivity lead to a sharp increase in m and in measured TFP (for example, in response to a 2.5% A shock, measured TFP grows by almost 3.5%). Public investment goes up due to a higher

[17] This is reminiscent of the idea that many types of infrastructure exhibit network properties. Fernald (1999), for example, finds that while investment in highways was very productive during the period of active construction of the Interstate Highway System, afterwards additional dollar spent on roads is unlikely to generate exceptional return.

output.

However, the resulting increase in the stock of public capital is fairly small even under a large A shock, and thus cannot sustain an elevated level of m after productivity returns to its normal levels. A huge one-time or a sequence of large A shocks are required to get a successful transition. We find that a quarterly A shock of at least 6.1% (more than 8.2% in measured TFP) is required to trigger the transition from the low to high steady state.

Notice that the responses to productivity shocks are much less persistent compared to those to public investment shocks. The difference mainly lies in the slow-moving nature of public capital stock, which is of course propagated by responses in m.

6 Counterfactual Experiments

We apply our model to two quantitative case studies. In Section 6.1, we look into the US economy around the 1960s. In Section 6.2, we consider the Great Recession and a consequent slow recovery. Particularly, we are interested whether a large-scale public investment program could have helped the economy to return to its pre-Recession path.

6.1 Public investment in the 1960s

We first turn our attention to the 1960s. Our goal here is to study whether the public investment boom observed in the 1960s was indeed the main driver underlying the S-shaped dynamics of the US economy before 1990, documented in Figure 1. We extract GI shocks between 1960Q1 and 1972Q4 from the data and back out A shocks by matching measured TFP within the same period. In addition to a public investment boom, the model suggests that exogenous productivity A was highly favorable during the 1960s. This might be due to low oil prices or low interest rates.

As presented in Figure 7, the model offers a reasonable match of consumption and output series before 1990. However, it undershoots the investment change. In the data, the difference between the pre-1960 and post-1973 levels is almost the same for consumption and output but much larger for private investment. The model falls short on this margin because output, investment and consumption are proportional to each other across the two steady states.

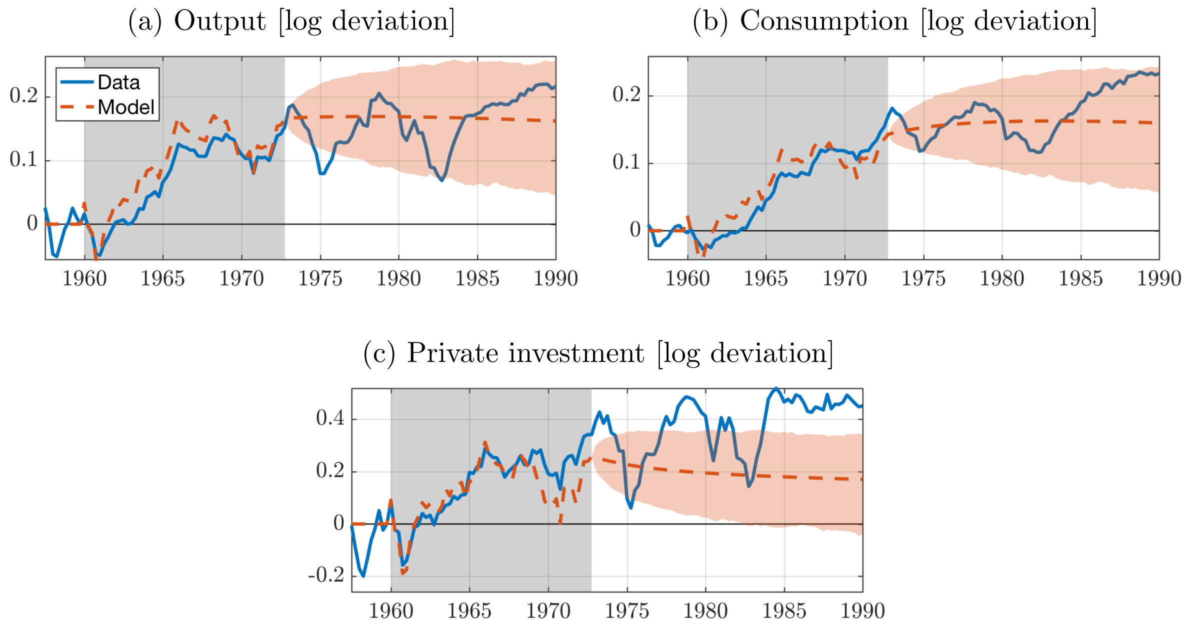

Figure 7: Nonlinear Dynamics in the 1960s – Model and Data. *Notes*: Log-linear trend is constructed using the data between 1947Q1 and 1959Q4. The red shaded areas are 80% confidence intervals. Details about data construction are provided in Appendix A.

We are now ready to address the relative importance of the GI and A shocks for the S-shaped dynamics via a counter-factual test. First, we keep productivity shocks while turn off GI shocks. We then repeat the exercise with GI shocks only. Figure 8 shows our results. During the early stage, the dashed red (A shocks only) and the solid blue lines (both GI and A shocks) are pretty close to each other. The dot-dashed yellow lines (GI shocks only) for all series but public capital stock stay around zero. With the accumulation of government capital, however, the impact of the GI shocks becomes pronounced: the red and blue lines start to diverge. The impulse response functions, likewise, suggest that the impact of productivity shocks is immediate, while that of GI shocks unfolds gradually.

The economy switches to the high steady state in all three cases. However, the transition speeds vary dramatically. With GI surprises shut down, the transition is very prolonged, and the model significantly undershoots the responses of the major macroeconomic variables before 1990. In the absence of productivity innovations (but with the GI shocks extracted from the data), the economy converges to the high regime much faster. We conclude that the GI shocks played a more important role in the transition between the

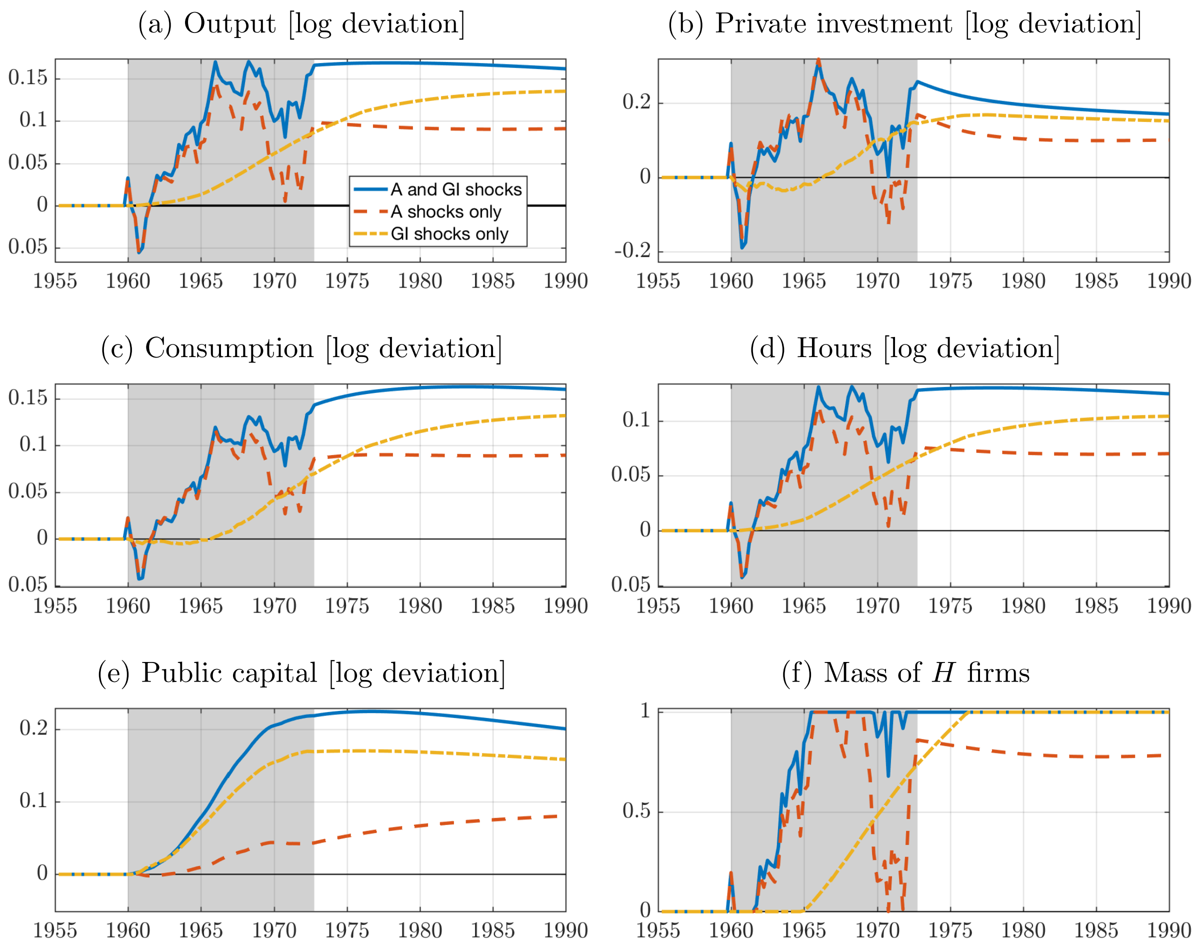

Figure 8: Nonlinear Dynamics in the 1960s – Counterfactual Analyses. *Notes*: Roles of GI and productivity shocks are isolated.

steady states around the 1960s.

6.2 Great Recession and slow recovery

The US economy after 2007 also exhibits a structural shift. The blue solid lines in Figure 9 show detrended output, consumption and private investment per capita. All series plummeted during the Great Recession and their subsequent recoveries have been either weak (for investment) or absent (for output and consumption). Figure **??** shows that the GI ratio gradually decreased to the lowest since the 1950s level after the Great Recession. In this section, we first evaluate the importance of such a decline for the slow recovery. We then investigate whether a large program of public investment would have helped the

US economy to return to the pre-Recession path.

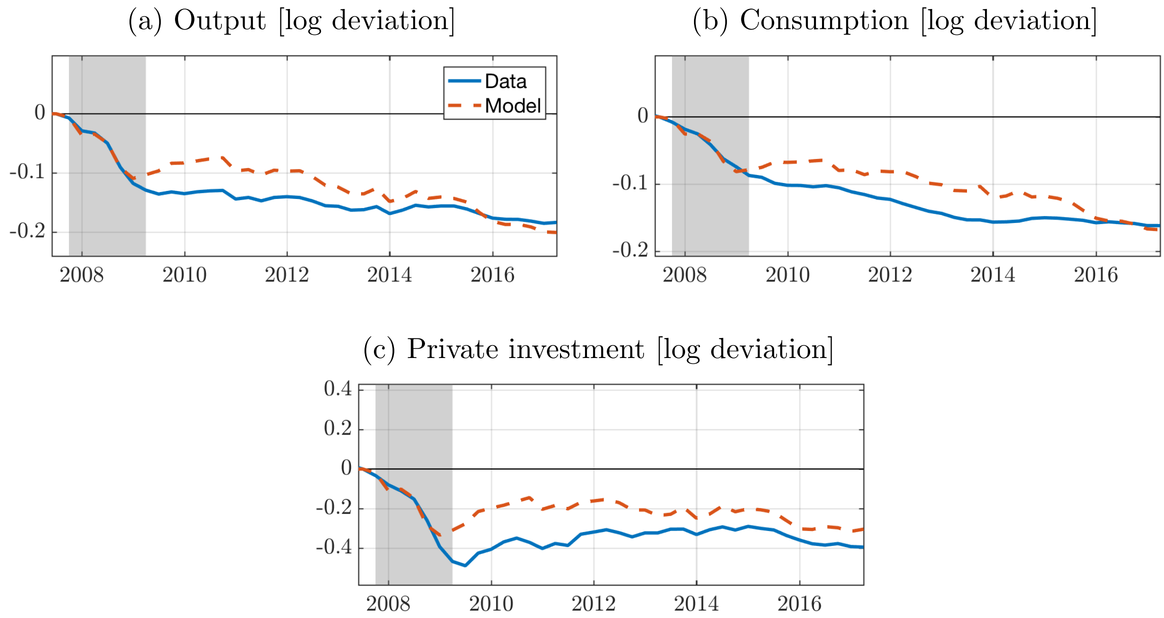

Figure 9: Nonlinear Dynamics after the Great Recession – Model and Data. *Notes*: Log-linear trend is constructed using the data between 1990Q1 and 2007Q3. Details about data construction are provided in Appendix A.

We assume that the economy was at the high steady state right before the Great Recession. Similar to the previous case study, we feed the GI shocks between 2007Q4 and 2017Q2 and back out A shocks with residual approach. Without recalibrating ω, the model reasonably matches aggregate quantities, as can be seen in Figure 9 (red dashed lines).[18,19]

How did the drop in government investment after 2010 contribute to the slow recovery? The solid blue lines in Figure 10 depict the model implied series under both productivity and GI shocks. The dashed red lines illustrate the model's behavior without GI movement. It turns out that these two sets of lines almost coincide, which suggests that the role GI shocks have played during the slow recovery is minor.

Interestingly, even a large program of government investment would not have helped the US economy to return to the high steady state. We consider a GI shock of 1.75 percentage points right after the Great Recession (2009Q3). Being highly persistent, this program

[18] It turns out that these shocks push the economy to the low steady state. If we set shocks to zero after 2017Q2, the economy does not return to the pre-recession levels.

[19] The model undershoots the investment drop. In the data, a large fraction of investment drop is driven by a huge decrease in the residential investment, which is outside of our model.

implies a spending of about 1 trillion 2009 dollars between 2009Q3 and 2017Q2. Impacts of this program are shown by the dot-dashed lines in Figure 10. We find that such a gigantic increase in public investment would produce only limited aggregate impacts within the post-Great Recession decade. The economy is not going to return to the high steady state because of this public investment boom.

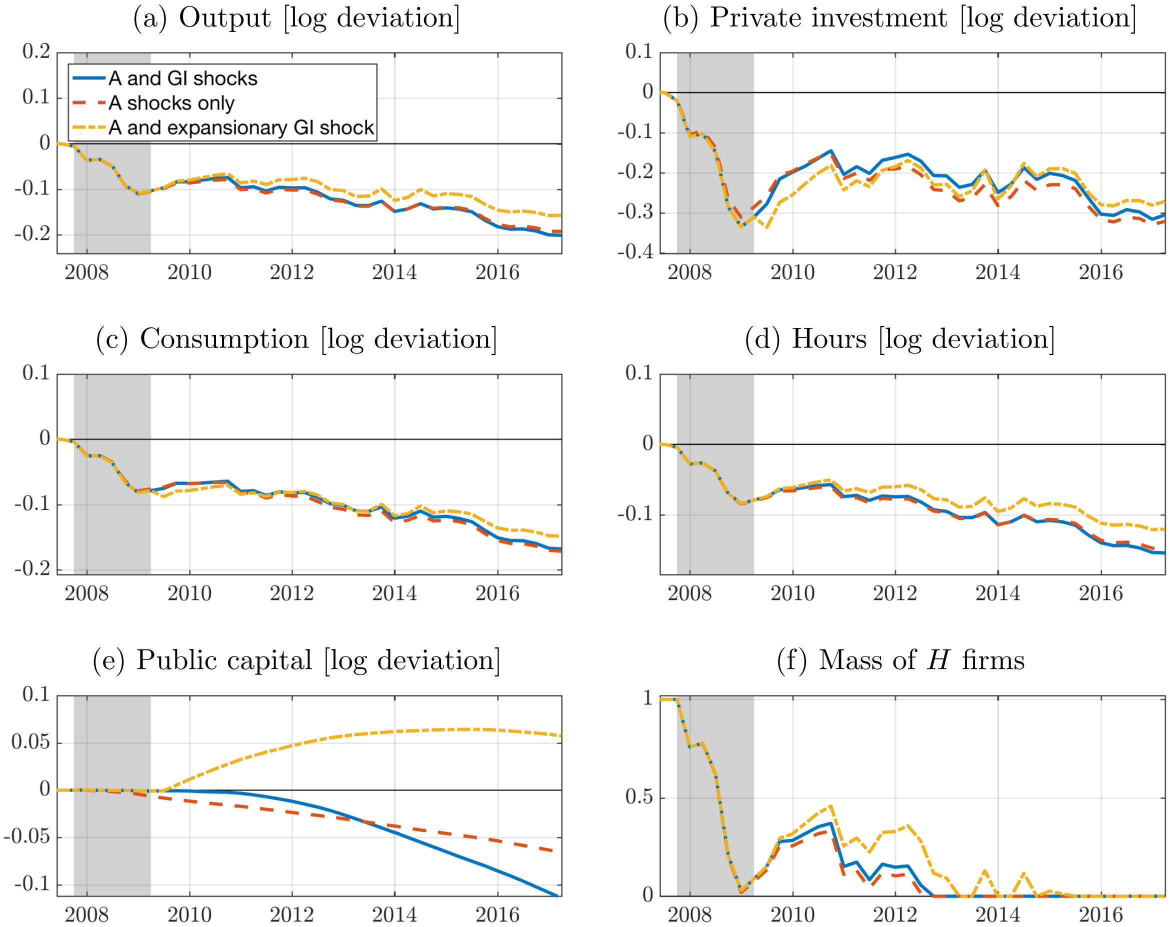

Figure 10: Nonlinear Dynamics after the Great Recession – Counterfactual Analyses. *Notes*: The role of productivity shocks is isolated. An expansionary government investment is experimented.

Why did a large public investment in the 1960s successfully push the economy to the high steady state, while a comparable project was unlikely to help in the aftermath of the Great Recession? It turns out that the model implied true productivity A is still significantly below its trend. There is a force outside of the model that prevents A from the recovery.[20] Under these circumstances, public investment might be inefficient because

[20]Some potential candidates include but are not limited to a sequence of adverse financial shocks

such an undesirable aggregate productivity keeps private investment depressed. On the contrary, not only government investment was at an unprecedented level during the 1960s, but also A shocks were highly favorable.[21]

7 Conclusion

In this paper, we document structural shifts of the US economy associated with a large public investment boom in the 1960s. We then build a business cycle model that can account for such highly nonlinear dynamics. Complementarity introduced by public capital and a non-convex cost associated with utilizing a more efficient technology together give rise to multiple stable steady states and the model's nonlinearity.

The economic dynamics caused by short-run government investment programs crucially depend on their magnitudes. On the one hand, large-scale transitory shocks to public investment can cause parallel shifts in the levels of macroeconomic variables through triggering a transition across steady states. Small-scale disturbances, on the other hand, generate standard short-run economic responses.

Somewhat surprisingly, although our model highlights the merit of the large public investment in the 1960s, it casts doubt on a similar initiative in the post-Great Recession era. Our analysis suggests that the effectiveness of such a program would be severely discounted by the exogenous productivity that is currently highly unfavorable compared to the 1960s.

This paper focuses on a positive analysis of the impact of transitory public investment shocks and thus formulates government's decisions by a set of exogenous rules. An interesting extension would be to conduct a normative investigation and examine how optimal fiscal policy in this nonlinear world looks differently compared to a standard model. We leave it to future work.

(Dominguez and Shapiro, 2013, Cai, 2016), insufficient R&D investment (Anzoategui, Comin, Gertler, and Martinez, 2016), demand complementarities (Schaal and Taschereau-Dumouchel, 2015), or uncertainty (Fajgelbaum, Schaal, and Taschereau-Dumouchel, 2017).

[21] It does not mean, however, that government investment is always useless in post-recession periods. Our model rather suggests that productive public spending can be very efficient if A is recovering after a negative shock, which does not seem to be the case for the Great Recession. We elaborate on this point in Appendix D.3.

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Appendix

A Data

A.1 Aggregate level

All time series are quarterly between 1947Q1 and 2017Q2 and expressed in per capita terms. Following Fernald (2014), output is 'Real gross value added: Business: Nonfarm' from FRED. Private investment is 'Gross private domestic investment' from FRED. Consumption is 'Personal consumption expenditures' from FRED. Hours is 'Nonfarm business sector: Hours of all persons' from FRED. To calculate per capita statistics, we get population as 'Population (midperiod)' from FRED.[22] Nondefense government investment is the sum of 'Federal nondefense gross investment' and 'State and local gross investment' from BEA. Government consumption is the sum of 'Federal national defense consumption expenditures', 'Federal national defense gross investment', 'Federal nondefense consumption expenditures' and 'State and local consumption expenditures' from BEA. Government transfers are constructed as in Leeper, Walker, and Yang (2010).

A.2 State level

State personal income and population are from Table SQ1 of BEA. We use 'Gross Domestic Product: Implicit Price Deflator' from FRED to adjust for inflation. State-level highway expenditures are from the State Government Finances of the US Census.

B Robustness

B.1 Extending the sample

In Figure A1, we extend the sample up to 2017Q2. We find that the long-run growth trend seems to change in the 1990s, consistent with Fernald (2016). In line with what we have presented in Figure 10, when we restart the linear trend since 1990Q1, the Great Recession still exhibits a parallel shift.

[22] Consistent with the approach of BEA, we use total population. First baby boom cohorts entered labor force around 1970, causing a large increase in civilian non-institutional population, which was not accompanied by a comparable increase in output (Feyrer, 2007 and Feyrer, 2011). The driving forces of the phenomenon lie outside of our model, so we prefer to use total population for calculation of per capita values.

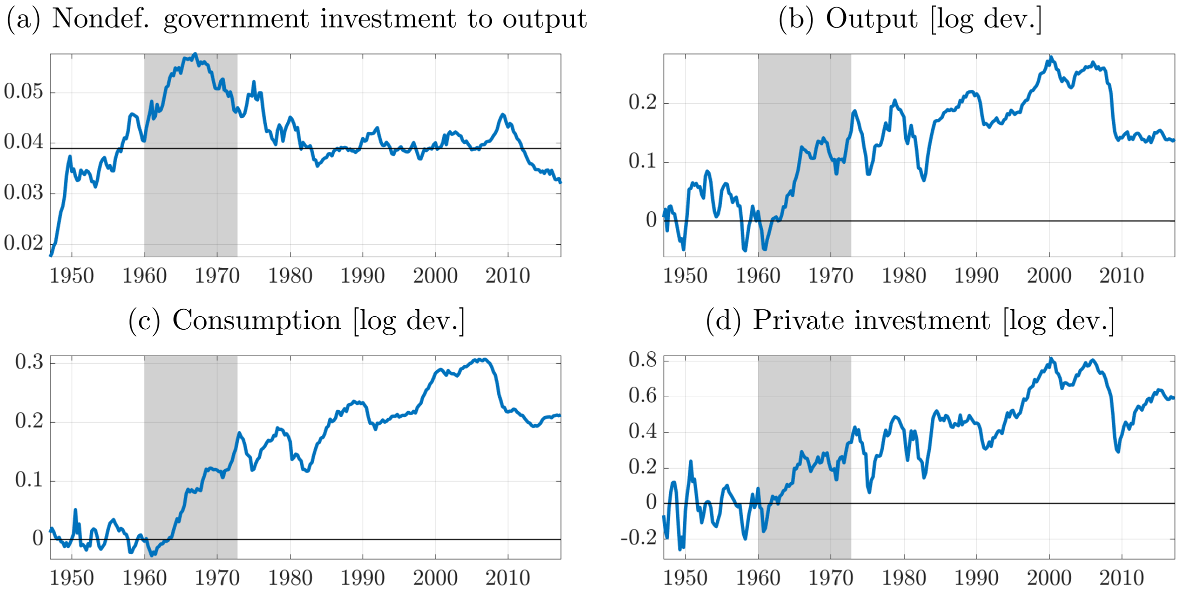

Figure A1: Public Investment and Nonlinear Dynamics – the Full Sample. *Notes*: Log-linear trend is constructed using the data between 1947Q1 and 1959Q4.

B.2 Frisch elasticity of labor supply

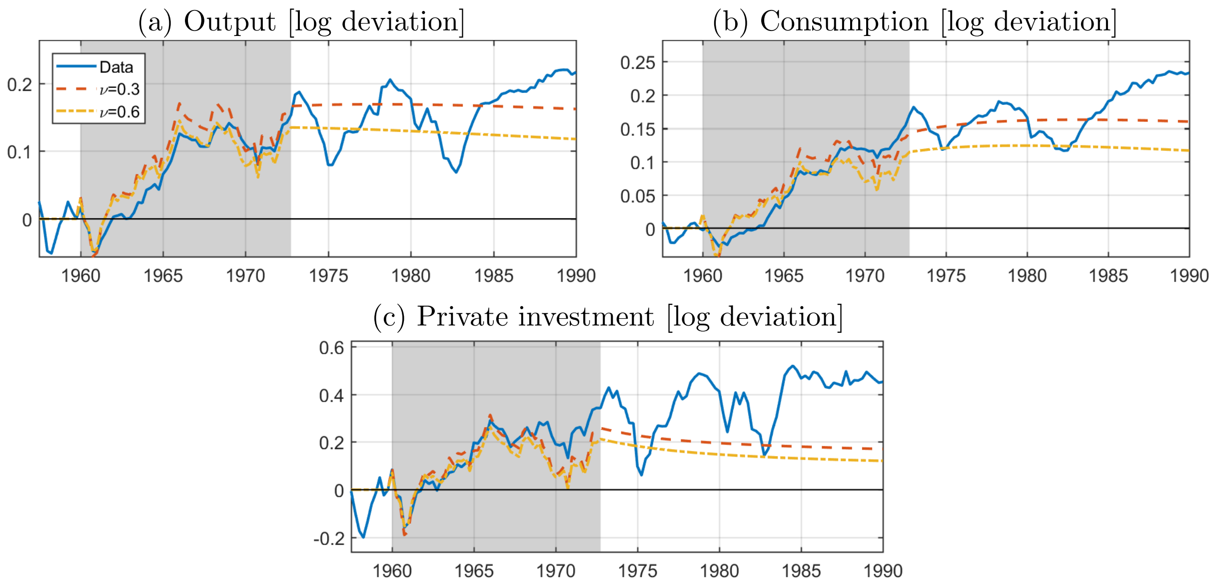

Figure A2: Counterfactual Analyses for the 1960s – Alternative ν's. *Notes*: Log deviations of real output, consumption and private investment per capita in the data, in the benchmark model ($\nu = 0.3$) and in the alternative model ($\nu = 0.6$). Log-linear trends are constructed using the data between 1947Q1 and 1959Q4.

In the benchmark calibration, $\nu = 0.3$, implying the inverse Frisch elasticity of labor supply of 3.33. We experiment with less elastic labor by setting $\nu = 0.6$ as in Greenwood, Hercowitz, and Huffman (1988) and repeating the same exercise. The model features a unique steady state with $\nu = 0.6$. We reparametrize f to guarantee $m = 0$ in the unique steady state of the model. Figure A2 depicts the results. The model still exhibits large persistence. At the same time, the levels of the post-1973 macro series are now lower than in the benchmark case and in the data. Lower ν implies smaller response of working hours and thus of output, private investment and consumption. This problem can be addressed by increasing the scale-up parameter ω.

B.3 Output elasticity with respect to public capital

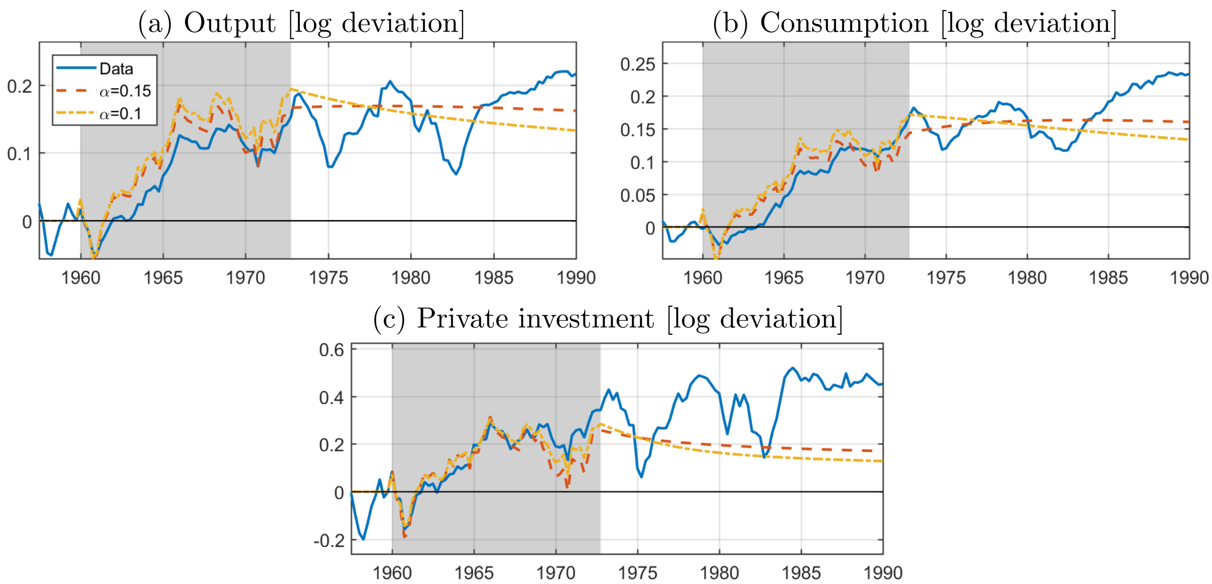

Figure A3: Counterfactual Analyses for the 1960s – Alternative α's. *Notes*: Log deviations of real output, consumption and private investment per capita in the data, in the benchmark model ($\alpha = 0.15$) and in the alternative model ($\alpha = 0.1$). Log-linear trends are constructed using the data between 1947Q1 and 1959Q4.

In our benchmark calibration, the elasticity of output with respect to government capital is $\alpha = 0.15$. In this section, we investigate the model behavior with $\alpha = 0.1$. It turns out that with this value of α the model features a unique steady state (recall Proposition 1).[23] However, since m can adjust, the impact of shocks can be highly persistent. We redo the 1960s case study and plot the results in Figure A3. Even in absence of multiple

[23] We reparametrize f to guarantee $m = 0$ in the unique steady state of the model.

steady states, the model is able to describe the data reasonably well. Despite output, consumption and private investment are all declining starting from 1973, the speed is relatively low.

Since our benchmark counter-factual analyses do not lend support to a GI expansion in the post-Great Recession period, adopting a lower α will make it even less favorable.

B.4 Depreciation of public capital

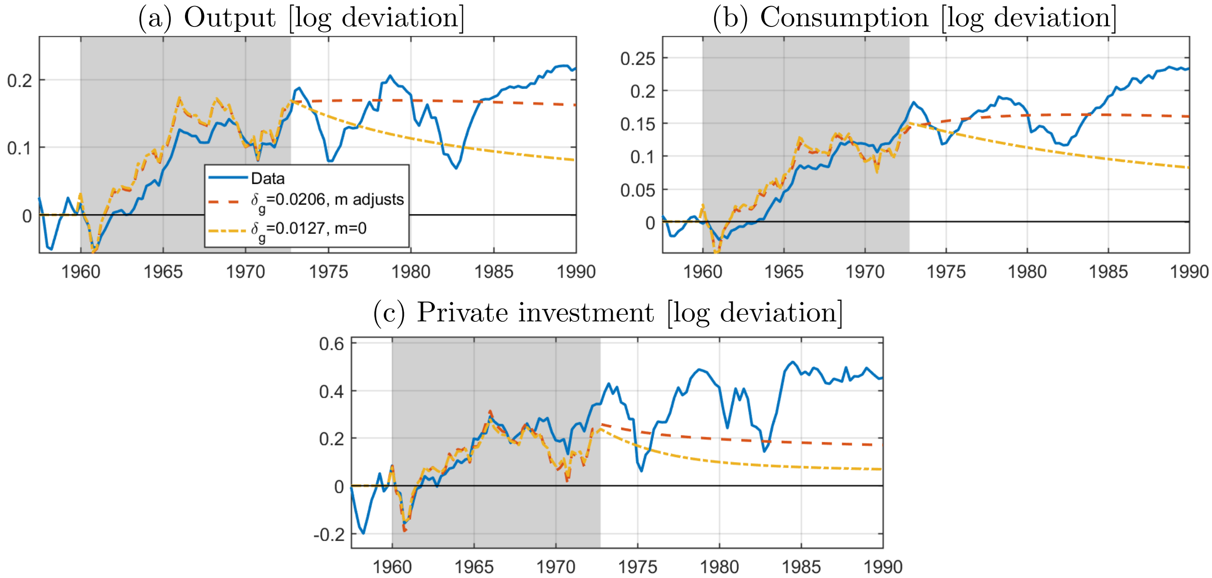

Figure A4: Counterfactual Analyses for the 1960s – Alternative δ_g's. *Notes*: Log deviations of real output, consumption and private investment per capita in the data, in the benchmark model ($\delta_g = 0.0206$, m adjusts) and in the alternative model ($\delta_g = 0.0127$, $m = 0$). Log-linear trends are constructed using the data between 1947Q1 and 1959Q4.

Our model features two steady states and thus can rationalize long-run impacts of large short-run public investment programs. In this section, we investigate whether a standard neoclassical model with low public capital depreciation rate can deliver similar results. A smaller depreciation rate can lead to more persistent effects of transitory GI shocks as public capital remains functional for longer. We consider an alternative model where $\delta_g = 0.0127$ (5% annually) and m is restricted to stay at 0.[24]

As in the main text, we evaluate whether this alternative model can rationalize the S-shaped dynamics of the US economy after the massive public investment of the 1960s.

[24]For example, roads depreciate at the rate of 5% per year, as suggested by the 'Capitalization and Depreciation of Infrastructure' report, prepared by the Mississippi Office of the State Auditor.

Figure A4 illustrates our results. When $m = 0$, measured TFP coincides with productivity, $\hat{A}(A, m) = A$. The alternative model is therefore shocked by relatively more positive A shocks than the benchmark model (recall that in the benchmark model measured TFP is partially explained by m, which was generally increasing between 1960Q1 and 1972Q4). Consequently, the benchmark and alternative models perform similarly during the period of massive infrastructure investment. However, after 1973 the benchmark model describes the data significantly better. In particular, the alternative model predicts a nearly 8% drop in detrended output and consumption by 1985, while the benchmark model predicts that they should stay almost unchanged, as in the data.

C Calibration of f

It this section, we describe our calibration strategy for the fixed cost f. As discussed in Section 5.1.3, f governs the frequency of extreme changes in macroeconomic variables. In each quarter, we construct a year-to-year output growth rate. We then demean the resulting time series and compute the fraction of quarters when the growth rate was above 5% and 7% in absolute terms.

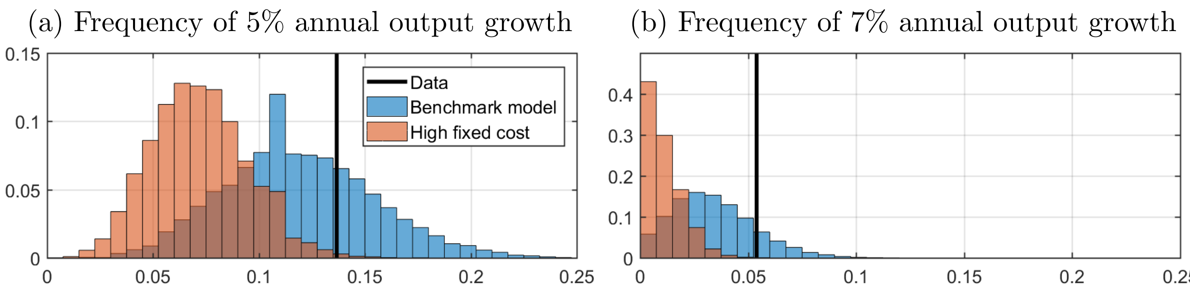

Figure A5: Probability of Extreme Output Growths – Role of Fixed Cost. *Notes*: The model economies are simulated for 280 quarters for 10,000 times.

In Figure A5, black vertical lines show the outcomes. We then calculate the same statistics for our benchmark economy with $f = 0.0051$ and for the economy where $f \gg 1$ and m never deviates from 0. We find that in the latter case, the model cannot generate enough number of episodes of extreme output growths. At the same time, our benchmark economy matches the data decently.[25]

[25]Consistently, under our benchmark calibration kurtosis of annual output growth is 3.35. If $f \gg 1$ and m never deviates from 0, it is 3. In the postwar US data, annual output growth exhibits kurtosis of 3.70.

D Additional results

D.1 IRFs: Alternative financing approaches

In the main text, we assume that shocks in government investment policies are financed by adjusting transfers, while the marginal tax rate τ is fixed. Figure A6 shows how a different way of financing affects the impact of a large 1 p.p. GI shock on the economy starting at the low steady state.

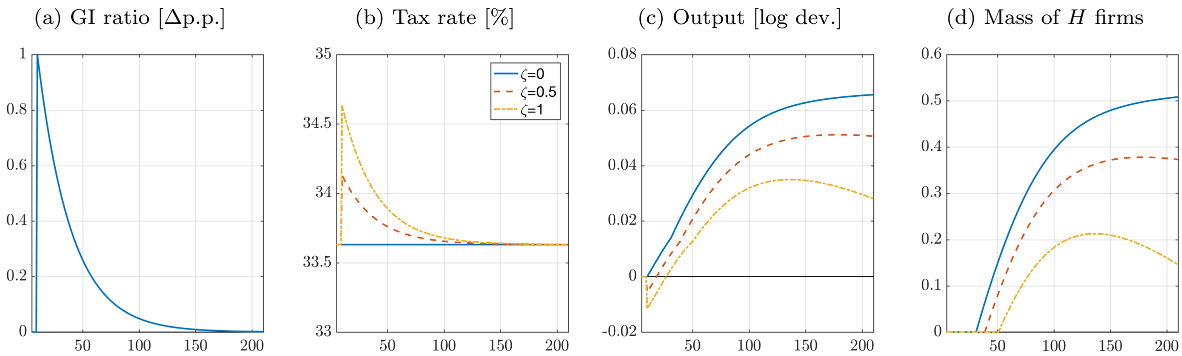

Figure A6: Role of Financing. *Notes*: Impulse responses of the economy starting at the low steady state to a large GI shock under different financing strategies. $\zeta \in [0, 1]$ is the fraction of the shock financed by distortionary taxation. X-axis is the number of quarters.

We assume that fraction $\zeta \in [0, 1]$ of the shock is financed through the adjustment in the distortionary income tax rate and $1 - \zeta$ is still financed by lump-sum taxes. An increase in the distortionary tax rate suppresses private incentives to work and invest. Thus, as long as $\zeta > 0$, investment, hours, output and consumption all go down at the moment of the shock.[26] Moreover, if only lump-sum taxes are used ($\zeta = 0$, the blue solid lines), the economy eventually switches to the high regime, while for distortionary tax financing ($\zeta = 1$, the dot-dashed yellow lines) the economy returns to the low steady state.

D.2 IRFs: GI shocks at the high steady state

One of the important predictions of the model is that the impact of public investment programs is state dependent. A large public spending program might drive the economy from the low to high steady state. However, if the economy starts at the high steady state, the same program is inefficient. Figure A7 plots the response of the economy starting at

[26] It is important to note that our formulation does not lead to equilibrium indeterminacy in the spirit of Schmitt-Grohe and Uribe (1997). In our setting, public spending changes as a fraction of output leading to unambiguous adjustment of the tax rate.

the high steady state on GI shocks the same sizes as in Figure 5. Now the shock size does not qualitatively affect the shapes of the impulse response functions.

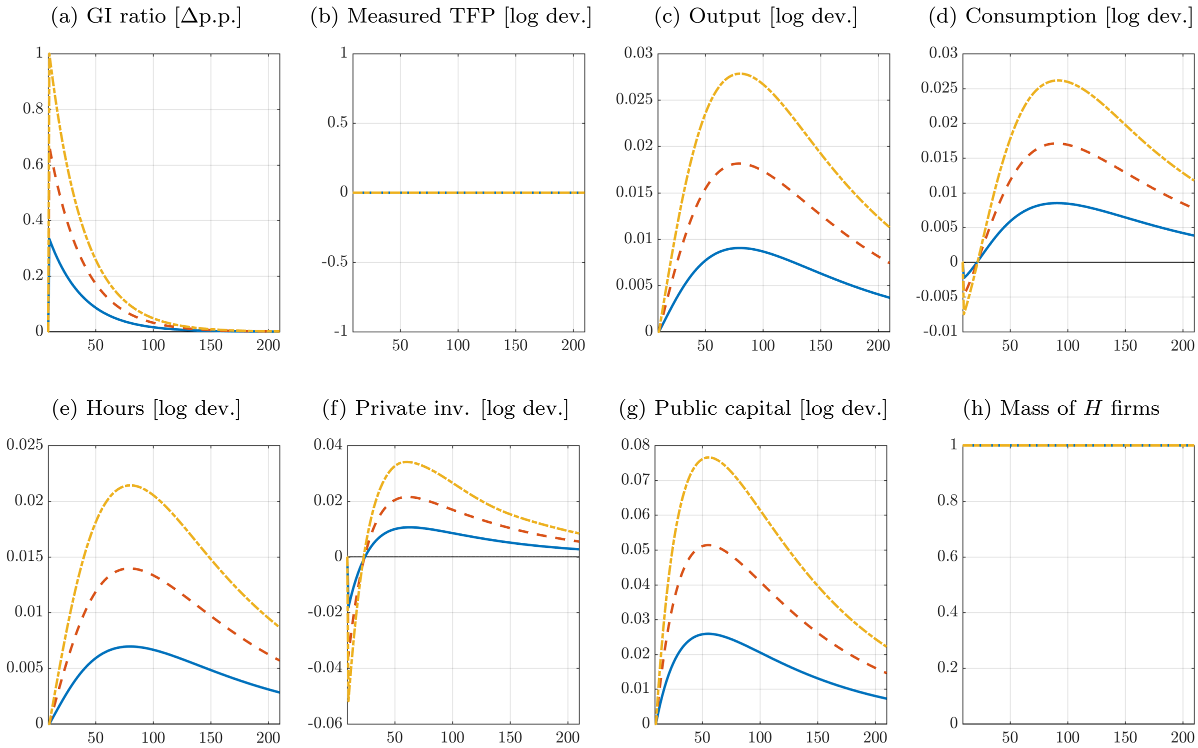

Figure A7: State-Dependent Impacts of GI shocks. *Notes*: Impulse responses of the economy starting at the high steady state to GI shocks of various sizes. X-axis is the number of quarters.

D.3 The Great Recession

In the main text (Section 6.2), we argue that an expansionary government investment program would not have helped the US economy to return to the high steady state. The left column of Figure A8 reproduces these results. In the model, a drop in measured TFP $\hat{A}(A, m)$ (panel 1.b, solid blue line) can be explained either by a decrease in A or m. It turns out that the decrease in m can only partially explain the drop in measured TFP. True productivity A is still significantly below its trend, as shown in panel (1.a). As a result, the expansionary government investment policy from Section 6.2 is unsuccessful in pushing the economy back to the high regime.

The right column of Figure A8 considers a counter-factual economy, where productivity A starts to recover in 2009Q3 (panel 2.a). The model still generates a slow recovery in measured TFP and output (panels 2.b and 2.c, solid blue lines). For example, output is

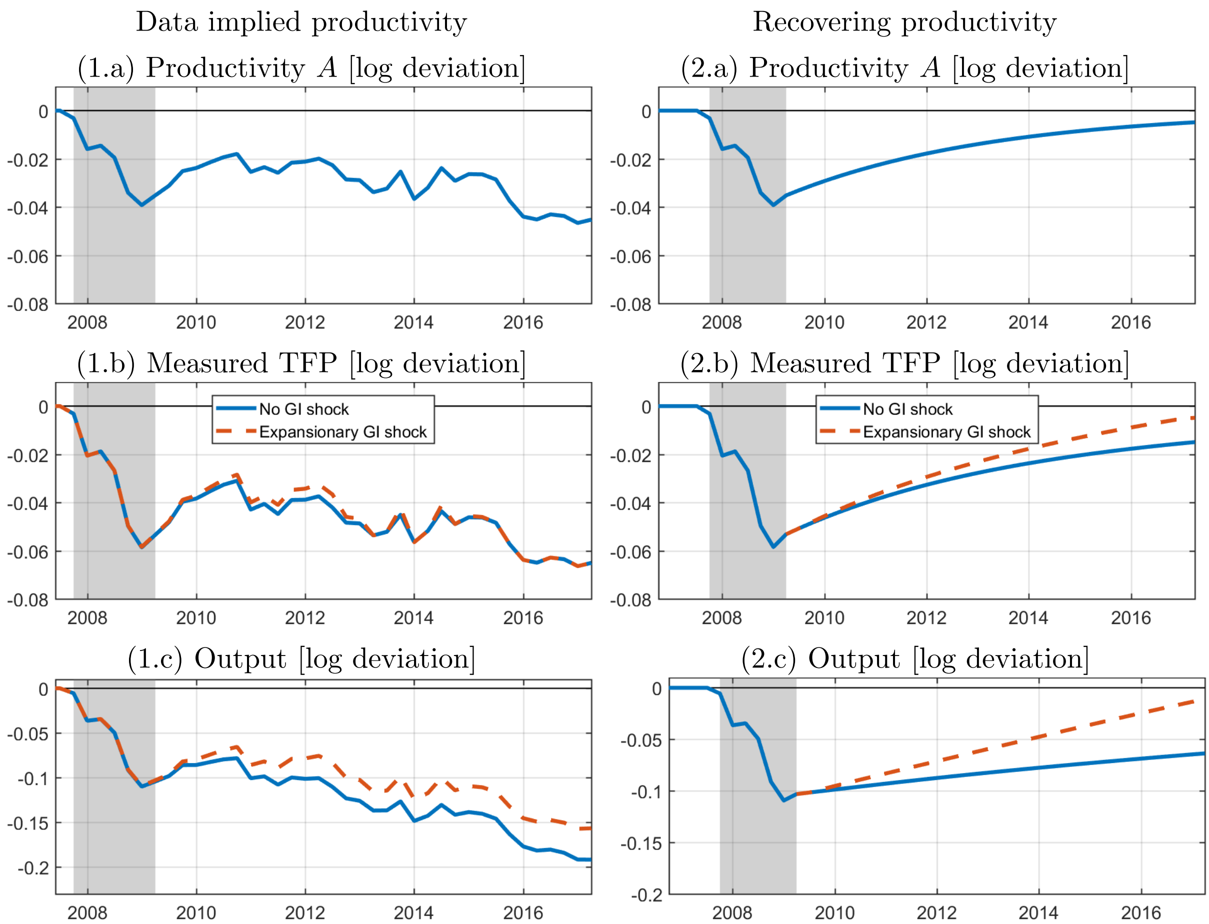

Figure A8: Expansionary Government Investment after the Great Recession – A Counterfactual Scenario. In the left column, data implied productivity process is fed in the model. In the right column, productivity is assumed to recover after the Great Recession. Log-linear trend is constructed using the data between 1990Q1 and 2007Q3.

6% below the pre-Recession trend in 2017Q2. Public investment program of the same size is more efficient in this case (dashed red lines in panels 2.b and 2.c). The distance between the dashed red and solid blue lines is larger in panels (2.b) and (2.c) than in (1.b) and (1.c), respectively.

E Proofs

E.1 Multiplicity of deterministic steady states

E.1.1 First order conditions

Households

The representative household solves

$$\max \ \mathbb{E}\sum_t \beta^t \frac{1}{1-\gamma}\left(C_t - \frac{L_t^{1+\nu}}{1+\nu}\right)^{1-\gamma},$$
$$\text{s.t. } C_t + K_{t+1} = (1-\tau)\left(R_t K_t + W_t L_t + \Pi_t\right) + (1-\delta)K_t - T_t,$$

from which we get the wage equation as one first order condition,

$$W_t(1-\tau) = L_t^{\nu}. \tag{A1}$$

Firms

Since firms are identical when making decisions, we now drop superscript i when no confusion is caused. Consider a firm with the H technology choice. It solves the following problem:

$$\pi_t^H = \max_{k_t, l_t} A_t (K_t^G)^{\alpha}(k_t)^{\theta_k}(l_t)^{\theta_l}\omega - W_t l_t - R_t k_t - f,$$

where k_t^H and l_t^H denote optimal choices. First order conditions are given by

$$\theta_l A_t (K_t^G)^{\alpha}(k_t^H)^{\theta_k}(l_t^H)^{\theta_l - 1}\omega = W_t, \tag{A2}$$
$$\theta_k A_t (K_t^G)^{\alpha}(k_t^H)^{\theta_k - 1}(l_t^H)^{\theta_l}\omega = R_t,$$

from which we get

$$\pi_t^H = (1-\theta_k-\theta_l)A_t(K_t^G)^{\alpha}(k_t^H)^{\theta_k}(l_t^H)^{\theta_l}\omega - f$$

and

$$\frac{\theta_l}{\theta_k}\frac{k_t^H}{l_t^H} = \frac{W_t}{R_t}.$$

We define k_t^L and l_t^L similarly and obtain

$$\frac{k_t^L}{l_t^L} = \frac{k_t^H}{l_t^H}.$$

Plugging back to the first order conditions (A2) results in

$$l_t^H = \omega^{\frac{1}{1-\theta_l-\theta_k}} l_t^L > l_t^L \qquad \text{and} \qquad k_t^H = \omega^{\frac{1}{1-\theta_l-\theta_k}} k_t^L > k_t^L.$$

Given fraction m_t of firms operating the high technology, labor and capital market clearing imply

$$m_t l_t^H + (1-m_t) l_t^L = L_t \Rightarrow l_t^L = \frac{L_t}{g(m_t)}, \quad l_t^H = \frac{L_t}{g(m_t)} \omega^{\frac{1}{1-\theta_l-\theta_k}}$$

and

$$m k_t^H + (1-m) k_t^L = K \Rightarrow k_t^L = \frac{K_t}{g(m_t)}, \quad k_t^H = \frac{K_t}{g(m_t)} \omega^{\frac{1}{1-\theta_l-\theta_k}},$$

where $g(m_t) = 1 + m_t(\omega^{\frac{1}{1-\theta_l-\theta_k}} - 1)$.

Combine these results with the wage equation (A1) and the first order conditions (A2):

$$\begin{aligned} L_t^{\nu} &= (1-\tau)\theta_l A_t \left(K_t^G\right)^{\alpha} \frac{K^{\theta_k}}{g(m_t)^{\theta_k}} \frac{L_t^{\theta_l - 1}}{g(m)^{\theta_l - 1}} \\ \Rightarrow L_t &= \left[\frac{(1-\tau)\theta_l A_t \left(K_t^G\right)^{\alpha} K_t^{\theta_k}}{g(m_t)^{\theta_k + \theta_l - 1}} \right]^{\frac{1}{1-\theta_l+\nu}}. \end{aligned} \tag{A3}$$

Aggregate output Y_t can be written as

$$\begin{aligned} Y_t &= A_t (K_t^G)^{\alpha} (k_t^H)^{\theta_k} (l_t^H)^{\theta_l} \omega m_t + A_t (K_t^G)^{\alpha} (k_t^L)^{\theta_k} (l_t^L)^{\theta_l} (1-m_t) \\ &= A_t \left(K_t^G\right)^{\alpha} g(m_t)^{1-\theta_l-\theta_k} K_t^{\theta_k} L_t^{\theta_l}. \end{aligned} \tag{A4}$$

This is the aggregate production function expressed in Equation (5) of the main text.

Benefit of the technology adoption is given by

$$\begin{aligned} &\Delta\pi(m_t; K_t, K_t^G, A_t) \\ &= \pi_t^H - \pi_t^L \\ &= (1-\theta_k-\theta_l) A_t (K_t^G)^{\alpha} [(k_t^H)^{\theta_k} (l_t^H)^{\theta_l} \omega - (k_t^L)^{\theta_k} (l_t^L)^{\theta_l}] - f \\ &= \zeta A_t^{\frac{1+\nu}{1-\theta_l+\nu}} \left(K_t^G\right)^{\frac{\alpha(1+\nu)}{1-\theta_l+\nu}} g(m_t)^{\frac{\theta_l - (\theta_l+\theta_k)(1+\nu)}{1-\theta_l+\nu}} K_t^{\frac{\theta_k(1+\nu)}{1-\theta_l+\nu}} - f, \end{aligned} \tag{A5}$$

where scaler $\zeta = (1-\theta_l-\theta_k)(\omega^{\frac{1}{1-\theta_l-\theta_k}}-1)[(1-\tau)\theta_l]^{\frac{\theta_l}{1-\theta_l+\nu}} > 0$. This is what we have shown as Equation (6).

$\Delta\pi(m_t; K_t, K_t^G, A_t)$ captures individual firms' incentive to adopt the new technology, and it is straightforward to see that

Lemma A1 *Equilibrium technology choice $m(K_t, K_t^G, A_t)$ is weakly increasing in A_t, K_t^G and K_t.*

E.1.2 Deterministic steady states

We drop all time subscripts t in the following analysis. In a non-stochastic environment, shocks are held at their long-run values. By definition,

$$\Delta\pi^{ss}(K, K^G) \equiv \Delta\pi(m^{ss}(K, K^G); K, K^G, \bar{A}).$$

We are now ready to prove Proposition 1. In steady state, government investment equals depreciation of public capital and as a result

$$G^I = \delta_g K^G = \bar{g}^I Y \Rightarrow K^G = \frac{\bar{g}^I}{\delta_g} Y. \tag{A6}$$

First-order condition of the household with respect to investment delivers

$$(1-\tau)R = \frac{1}{\beta} - 1 + \delta = (1-\tau)\theta_k \frac{Y}{K} \Rightarrow K = \frac{(1-\tau)\theta_k}{\frac{1}{\beta}-1+\delta} Y. \tag{A7}$$

From the government budget constraint, i.e. $(\bar{g}_I + \bar{g}_C)Y = \tau Y - T$, we know that $T \propto Y$. Define $T = \bar{z}Y$. Similarly, from the household budget constraint, $C \propto Y$. Tax rate is constant,

$$\tau Y = \bar{g}^C Y + \bar{g}^I Y + \bar{z} Y \Rightarrow \tau = \bar{g}^C + \bar{g}^I + \bar{z}.$$

From Equation (A3), we get the relationship between labor and output:

$$L \propto \left[\frac{Y^{\alpha+\theta_k}}{g(m)^{\theta_k+\theta_l-1}}\right]^{\frac{1}{1-\theta_l+\nu}}. \tag{A8}$$

Consider aggregate production and utilize Equations (A6), (A7) and (A8):

$$
\begin{aligned}
Y &= AK^{\theta_k}L^{\theta_l}g(m)^{1-\theta_l-\theta_k}\left(\frac{\bar{g}^I}{\delta_g}Y\right)^{\alpha} \\
\Rightarrow Y &\propto Y^{\theta_k}\left[\frac{Y^{\alpha+\theta_k}}{g(m)^{\theta_k+\theta_l-1}}\right]^{\frac{\theta_l}{1-\theta_l+\nu}} g(m)^{1-\theta_k-\theta_l}Y^{\alpha} \\
\Rightarrow Y &\propto g(m)^{\frac{(1-\theta_k-\theta_l)(1+\nu)}{(1-\theta_k-\alpha)(1+\nu)-\theta_l}}.
\end{aligned}
$$

The equation above states that Y is proportional to a nonlinear function of m. Given we have established that K and K_G are proportional to Y, we can therefore rewrite Equation (A5) in the following way:

$$
\begin{aligned}
\Delta\pi^{ss} &= \zeta\left(K^G\right)^{\frac{\alpha(1+\nu)}{1-\theta_l+\nu}} g(m)^{\frac{\theta_l-(\theta_l+\theta_k)(1+\nu)}{1-\theta_l+\nu}} K^{\frac{\theta_k(1+\nu)}{1-\theta_l+\nu}} - f \\
\Rightarrow \Delta\pi^{ss} + f &\propto g(m)^{\frac{\alpha(1+\nu)-\nu\theta_l}{(1-\theta_k-\alpha)(1+\nu)-\theta_l}}.
\end{aligned}
$$

In summary, we know that in steady state high levels of K and K_G correspond to high m whenever $1-\theta_k-\frac{\theta_l}{1+\nu} > \alpha$. Moreover, $\Delta\pi^{ss}$ is increasing in m when

$$
1-\theta_k-\frac{\theta_l}{1+\nu} > \alpha > \frac{\nu}{1+\nu}\theta_l.
$$

It has been established above that for $K_H > K_L$, we have $K_H^G > K_L^G$ and $m(K_H, K_H^G) > m(K_L, K_L^G)$ whenever $1-\theta_k-\frac{\theta_l}{1+\nu} > \alpha$. If in addition $\alpha > \frac{\nu}{1+\nu}\theta_l$, then $\pi^{ss}(K_H, K_H^G) > \pi^{ss}(K_L, K_L^G)$. Notice that $1-\theta_k-\frac{\theta_l}{1+\nu} > \alpha > \frac{\nu}{1+\nu}\theta_l$ is exactly condition i) of Proposition 1.

If condition ii) is satisfied, so that $\pi^{ss}(K_H, K_H^G) > 0 > \pi^{ss}(K_L, K_L^G)$, there is no deviation incentive for firms in the neighborhoods of the high and low steady states. Any small change of (K, K_G) from their steady state values will not change mass of firms operating the high technology by continuity of $\Delta\pi^{ss}(K, K^G)$. Locally, the economy around the high and low steady states has a familiar dynamics of standard neoclassical models (as in, for example, Baxter and King, 1993), which are known to feature a unique stable steady state. This finishes the proof of Proposition 1.

E.2 Existence and uniqueness of the recursive equilibrium

In this section, we prove that the recursive equilibrium defined in the main text exists and is unique. The proof builds on Coleman (1991), Coleman (2000), Datta, Mirman, and Reffett (2002), Morand and Reffett (2003), and Schaal and Taschereau-Dumouchel (2015). We extend the approach outlined in these papers to our setting. The most important novelty is that our model has public capital as a state variable affected by actions of private agents. To the best of our knowledge, existing theoretical proofs are carried out for economies with only one endogenous state variable.

The existence proof is based on the Tarski's fixed point theorem (Tarski, 1955):

Theorem A1 (Tarksi, 1955) *Suppose that* $(X, \geq)$ *is a nonempty complete lattice and* $T : X \to X$ *is an increasing mapping. Then the set of fixed points of* T *is a nonempty complete lattice.*

Under additional assumptions outlined in Krasnosel'skii and Zabreiko (1975) and first used by Coleman (1991) for an economic problem, uniqueness of a fixed point can be established.

E.2.1 Notation

We start by introducing several variables which will be actively used in the proof. We omit time subscripts and instead denote next period variables by the prime symbols. We re-denote public capital by K_G and the government investment to output ratio by g_I.

Recall that at each period labor is a static variable and can be expressed as in Equation (A3). Plugging it into the expression for aggregate output given in Equation (A4), we get

$$Y(K,K_G,A) = [\theta_l(1-\tau)]^{\frac{\theta_l}{1-\theta_l+\nu}} A^{\frac{1+\nu}{1-\theta_l+\nu}} g(m(K,K_G,A))^{\frac{(1-\theta_k-\theta_l)(1+\nu)}{1-\theta_l+\nu}} (K^g)^{\frac{\alpha(1+\nu)}{1-\theta_l+\nu}} K^{\frac{\theta_k(1+\nu)}{1-\theta_l+\nu}}. \quad \text{(A9)}$$

Notice that $Y(K, K_G, A)$ is increasing in all arguments, since $m(K, K_G, A)$ is increasing in all arguments.

Define total funds available to spend on consumption and investment by

$$y(K, K_G, g_I, A) \equiv (1 - g_I - \bar{g}_C - \bar{z})Y(K, K_G, A) + (1 - \delta_k)K.$$

Again, $y(K, K_G, g_I, A)$ is increasing in K, K_G and A.

Define also total funds net of the lowest possible consumption level $\frac{L(K,K_G,A)^{1+\nu}}{1+\nu}$, which guarantees that utility is above $-\infty$,

$$y^d(K, K_G, g_I, A) \equiv y(K, K_G, g_I, A) - \frac{L(K, K_G, A)^{1+\nu}}{1+\nu} = \left((1-\tau)\frac{1+\nu-\theta_l}{1+\nu} + \bar{z} + \bar{g}_I - g_I\right)Y(K, K_G, A) + (1 - \delta_k)K.$$

We are going to require that $y^d(K, K_G, g_I, A)$ is increasing in K, K_G and A.

Assumption A1 *$a(g_I) \equiv (1-\tau)\frac{1+\nu-\theta_l}{1+\nu} + \bar{z} + \bar{g}_I - g_I > 0$ for any g_I.*

Notice that this assumption holds in our calibration and is not restrictive.

Finally, define net interest rate

$$r(K, K_G, A) \equiv \theta_k \frac{Y(K, K_G, A)}{K}(1-\tau) + 1 - \delta_k.$$

Apparently, $r(K, K_G, A)$ is increasing in K_G and A. Below we show that $r(K, K_G, A)$ decreases in K. When all firms operate the high or low technology, so that $m = 1$ or $m = 0$, it is easy to see that $\frac{\partial r}{\partial K} < 0$, since $\frac{\theta_k(1+\nu)}{1-\theta_l+\nu} < 1$. Assume now that $m \in (0, 1)$, so a small change of K also affects m. Using (A5), we get

$$\Delta\pi = 0 \Rightarrow g(m)^{-1} \propto K^{\frac{\theta_k(1+\nu)}{\theta_l-(\theta_l+\theta_k)(1+\nu)}} K_G^{\frac{\alpha(1+\nu)}{\theta_l-(\theta_l+\theta_k)(1+\nu)}}$$

Plugging this into Equation (A9) and omitting K_G term, we find that $Y \propto K^{\frac{\theta_k(1+\nu)}{(\theta_l+\theta_k)(1+\nu)-\theta_l}}$. Since $\frac{\theta_k(1+\nu)}{(\theta_l+\theta_k)(1+\nu)-\theta_l} < 1$, we conclude that $r(K, K_G, A)$ is decreasing in K.

E.2.2 Assumptions

The proof requires all policies to be bounded. We thus restrict exogenous shocks to have bounded support. Moreover, as in Coleman (1991), we assume that they can take only finite number of values, which is consistent with our numerical solution.

Assumption A2 *$A \in \mathcal{A}$, where $\mathcal{A}$ is finite. The shocks evolve according to the Markovian probabilities with a transition matrix P_A.*

Assumption A3 *$g_I \in \mathcal{G}^{\mathcal{I}}$, where $\mathcal{G}^{\mathcal{I}}$ is finite. The shocks evolve according to the Markovian probabilities with a transition matrix P_g.*

All expectations hereafter are conditional on current values of g_I and A. For brevity however we will omit conditioning.

For sufficiently large K and K_G all firms operate high technology, $m = 1$, and the model degenerates to a usual neoclassical model augmented with public capital. We assume

Assumption A4 $\alpha < 1 - \theta_k - \frac{\theta_l}{1+\nu}$.

This assumption guarantees that $Y(K, K_G, A)$ exhibits decreasing returns in (K, K_G), at least for sufficiently large (K, K_G). Notice that this condition coincides with the upper bound on α in Proposition 1. This assumption guarantees that there exist $\bar{K} < \infty$ and $\bar{K}_G < \infty$ such that

$$y^d(\bar{K}, \bar{K}_G, g_I, A) - \frac{L(\bar{K}, \bar{K}_G, A)^{1+\nu}}{1+\nu} < \bar{K},$$
$$(1-\delta_g)\bar{K}_G + g_I Y(\bar{K}, \bar{K}_G, A) < \bar{K}_G,$$

for any $g_I \in \mathcal{G}^{\mathcal{I}}$ and $A \in \mathcal{A}$. We will denote possible values of private capital by $K \in \mathcal{K}$, where $\mathcal{K} \equiv [0, \bar{K}]$. For public capital we additionally restrict possible values to lie above some arbitrary small value $\underline{K}^G$, $K_G \in \mathcal{K}_{\mathcal{G}}$, where $\mathcal{K}_G \equiv [\underline{K}_G, \bar{K}_G]$.[27]

Assumption A5 *There exists $K^- > 0$ such that $y(K^-, K_G, g_I, A) - \frac{L(K^-, K_G, A)^{1+\nu}}{1+\nu} > K^-$ and $\beta \, \mathbb{E}\left[r(K^-, K_G, A', g_I')\right] \leq 1$ for any $K_G \in \mathcal{K}_{\mathcal{G}}$, $g_I \in \mathcal{G}^{\mathcal{I}}$, $A \in \mathcal{A}$.*

This assumption is an analogue of Assumption 5 in Coleman (1991). It is required to show the existence of a non-zero equilibrium. Notice that it requires K^- to be small in order to guarantee that private capital accumulates for K^-, at least when consumption is held at its lowest possible level. At the same time, K^- cannot be too small for the second part to be true.

[27] We thus assume that $K_G'(K, K_G, g_I, A) = \max\left[\underline{K}_G; (1-\delta_g)K_G + g_I Y(K, K_G, A)\right]$. This assumption is innocuous since $\underline{K}_G$ can be arbitrary small.

E.2.3 Definitions

The proof establishes the uniqueness of a solution to the Euler equation. It turns out to be important to represent the Euler equation in the space of inverse marginal utilities. In this space, the operator corresponding to the Euler equation is pseudo-concave, which is a property necessary to establish uniqueness.

Definition A1 *Let* $\mathcal{P} = \Big\{ p(K, K_G, g_I, A) | p : \mathcal{K} \times \mathcal{K}_{\mathcal{G}} \times \mathcal{G}^{\mathcal{I}} \times \mathcal{A} \to \mathbb{R}^+$ *such that*

1. $0 \leq p(K, K_G, g_I, A) \leq U_C(y(K, K_G, g_I, A), L(K, K_G, A))^{-1}$ *for* $(K, K_G, g_I, A) \in \mathcal{K} \times \mathcal{K}_{\mathcal{G}} \times \mathcal{G}^{\mathcal{I}} \times \mathcal{A}$*;*
2. $p(K, K_G, g_I, A)$ *is increasing in* K*;*
3. *For all* $g_I \in \mathcal{G}^{\mathcal{I}}$ *and* $A \in \mathcal{A}$*,* $p(K, K_G, g_I, A)$ *is increasing along a critical direction* $(1, \xi_{crit}(K, K_G))$*, where* $\xi_{crit}(K, K_G)$ *is defined below in* (A13).

The above definition describes the set of possible inverse marginal utilities. We now define the mapping corresponding to the Euler equation.

Definition A2 1. *The mapping from marginal utility value to consumption is*

$$C : \mathbb{R}^+ \times \mathcal{K} \times \mathcal{K}_{\mathcal{G}} \times \mathcal{A} \to \mathbb{R}^+ \text{ so that } C(p, K, K_G, A) = p^{\frac{1}{\gamma}} + \frac{L(K, K_G, A)^{1+\nu}}{1+\nu}.$$

2. *The Euler equation mapping is*

$$Z : \mathbb{R}^+ \times \mathcal{P} \times \mathcal{K} \times \mathcal{K}_{\mathcal{G}} \times \mathcal{G}^{\mathcal{I}} \times \mathcal{A} \to \mathbb{R} \cup \{-\infty, \infty\} :$$

$$Z(p, P, K, K_G, g_I, A) = \begin{cases} 0, \textit{ if } p=0 \textit{ and } \Big(K=0 \textit{ or } P(y(K,K_G,g_I,A)-C(0,K,K_G,A),K'_G(K,K_G,g_I,A),g_I{}',A')=0\Big) \\ \frac{1}{p} - \beta\, \mathbb{E}\left[\frac{r\left(y(K,K^G,g^I,A)-C(p,K,K^G,A);K'_G(K,K_G,g_I,A);A'\right)}{P\left(y(K,K^G,g^I,A)-C(p,K,K^G,A);K'_G(K,K_G,g_I,A);g_I{}';A'\right)} \right], \textit{ otherwise.} \end{cases}$$

3. The operator providing the solution to the Euler is

$$T(P) = \{p \in \mathcal{P} | Z(p(K, K_G, g_I, A), P, K, K_G, g_I, A) = 0 \\ \text{for } K \in \mathcal{K}, K_G \in \mathcal{K}_{\mathcal{G}}, g_I \in \mathcal{G}^{\mathcal{I}}, A \in \mathcal{A}\}.$$

E.2.4 Existence

It is easy to see that $(\mathcal{P}, \geq)$, where $\geq$ is a usual pointwise partial order, is a complete lattice.[28] We then need to prove that T has a fixed point in the space of inverse marginal utilities $\mathcal{P}$. First of all, we will show that the operator T is a well-defined monotone self-map on $\mathcal{P}$.

The main complication in this step is to prove that T is a self-map. Our existence and uniqueness proof crucially relies on monotonicity properties of $p \in \mathcal{P}$, outlined in Definition (A1). As it will become clear later, we need to establish some monotonicity properties of the next period net interest rate r' with respect to *current* period K and K_G, holding current inverse marginal utility fixed. If K increases, K' goes up (recall that p is fixed). That pushes r' down. However, higher K also implies higher output Y and thus more public capital next period. Net interest rate is increasing in K_G, thus the total impact of increase in K on r' becomes unclear. We therefore find a *critical* direction in the (K, K_G) space, along which r' is guaranteed to decrease. We then make several assumptions (we verify numerically that they hold in our setting) to show that any element of $\mathcal{P}$ is mapped on itself by T.

Let's formalize the intuition lied out above. Fix $K > 0$, K_G, g_I and A. Fix $p = p(K, K_G, g_I, A) \in \mathcal{P}$. Consider an increase of (K, K_G) by $(dK, dK_G) = (1, \xi)dK$, where $dK > 0$ and $\xi \geq 0$.[29] Then

$$\frac{dK'}{dK} = a(g_I)\frac{\partial Y}{\partial K} + a(g_I)\frac{\partial Y}{\partial K_G}\xi + (1 - \delta_k) > 0, \tag{A10}$$

$$\frac{dK'_G}{dK} = g_I\frac{\partial Y}{\partial K} + g_I\frac{\partial Y}{\partial K_G}\xi + (1 - \delta_g)\xi > 0. \tag{A11}$$

[28] The proof directly follows Lemma A3 in Schaal and Taschereau-Dumouchel (2015), so we omit it.

[29] Somewhat abusing notation, we assume that if $\xi = \infty$ then $(dK, dK_G) = (0, dK_G)$ with $dK_G > 0$.

Next period net interest rate will then change in the following way:

$$\frac{dr'}{dK} = \frac{\partial r'}{\partial K'}\left(a(g_I)\frac{\partial Y}{\partial K} + a(g_I)\frac{\partial Y}{\partial K_G}\xi + (1-\delta_k)\right) + \frac{\partial r'}{\partial K_G{}'}\left(g_I\frac{\partial Y}{\partial K} + g_I\frac{\partial Y}{\partial K_G}\xi + (1-\delta_g)\xi\right). \tag{A12}$$

Assumption A6 $\dfrac{dr'}{dK} < 0$ *if* $\xi = 0$ *for all* $K > 0, K^G, g^I, A, A'$ *and* $K' \leq (1-\delta_k)K + a(g_I)Y(K, K^G, A)$*,* $K'_G = K'_G(K, K_G, g_I, A)$*.*

The assumption states that the net interest rate next period r' goes down when K goes up ceteris paribus. Violation of the assumption would imply that increase in K can push up incentives to save. The assumption holds if GI shocks are sufficiently small and $\underline{K}_G$ is not too small. We verify numerically that it holds for our calibration and grids choice.

Notice that $\frac{dr'}{dK}$ is monotone in ξ. By Assumption (A6), $\left.\frac{dr'}{dK}\right|_{\xi=0} < 0$. We define a *critical* direction as

$$\xi_{crit}(K, K_G) \equiv \begin{cases} \infty, \text{ if } \frac{dr'}{dK} \text{ decreases in } \xi \text{ for all } g_I, A, A', 0<K'\leq(1-\delta_k)K+a(g_I)Y, K'_G = K'_G(K,K_G,g_I,A), \\ \min\limits_{\substack{g_I, A, A' \\ 0<K'\leq(1-\delta_k)K+aY \\ K'_G = K'_G(K,K_G,g_I,A)}} -\dfrac{\frac{\partial r'}{\partial K_G{}'}\times g_I\frac{\partial Y}{\partial K} + \frac{\partial r'}{\partial K'}\times\left(a(g_I)\frac{\partial Y}{\partial K}+1-\delta_k\right)}{\frac{\partial r'}{\partial K_G{}'}\times\left(g_I\frac{\partial Y}{\partial K_G}+1-\delta_g\right)+\frac{\partial r'}{\partial K'}\times a(g_I)\frac{\partial Y}{\partial K_G}}. \end{cases} \tag{A13}$$

It is trivial to see that the following useful lemma holds:

Lemma A2 *For a fixed inverse marginal utility* $p \in \mathcal{P}$*, given* $K \in \mathcal{K}\backslash\{0\}$ *and* $K_G \in \mathcal{K}_\mathcal{G}$*, for all* $g_I \in \mathcal{G}^I$ *and* $A \in \mathcal{A}$*, the net interest rate next period,* r'*, is decreasing along all directions* $\xi \in [0, \xi_{crit}(K, K_G)]$*, where* $\xi_{crit}(K, K_G)$ *is defined by* (A13)*.*

Importantly, $\xi_{crit}(K, K_G)$ grows unboundedly when K approaches 0 due to the term $\frac{\partial r'}{\partial K'} \times \frac{\partial Y}{\partial K}$ in the numerator of expression (A13). This result will be used in the uniqueness proof.

Consider now any direction $\xi \geq 0$ along which (K, K_G) are perturbed. Using equations (A10) and (A11), we get

$$\xi' \equiv \frac{dK'_G}{dK'} = \frac{g_I\frac{\partial Y}{\partial K} + g_I\frac{\partial Y}{\partial K_G}\xi + (1-\delta_g)\xi}{a(g_I)\frac{\partial Y}{\partial K} + a(g_I)\frac{\partial Y}{\partial K_G}\xi + (1-\delta_k)}. \tag{A14}$$

It is straightforward to show that $\xi'(\xi)$ is strictly increasing.

Assumption A7 *Fix an inverse marginal utility* $p \in \mathcal{P}$*. Define* $\xi'(\xi)$ *as in* (A14)*. Then* $\xi'(\xi_{crit}(K, K_G)) < \xi_{crit}(K', K'_G)$ *for any* $K > 0$*,* $K_G \in \mathcal{K}_G$ *and any* $0 < K' \leq (1 - \delta_k)K + a(g_I)Y(K, K_G, A)$ *and* $K'_G = K'_G(K, K_G, g_I, A)$*.*[30]

This assumption guarantees that perturbation along a critical direction leads to such an adjustment (dK', dK'_G) that $\frac{dK'_G}{dK'}$ lies below next period critical direction $\xi_{crit}(K', K'_G)$. Intuitively, we require that perturbation along critical direction leads not only to decrease in r' but also in r''. Thus, perturbations along critical direction not only decrease incentives to save today but also next period. We again verify numerically that it is true for our calibration.

We are now ready to prove that T is a self-map. The following lemma establishes the result of interest.

Lemma A3 *T is a well-defined self-map on* $\mathcal{P}$*.*

Proof: We need to check that any $P \in \mathcal{P}$ is uniquely mapped onto $\mathcal{P}$.

i) Fix $K > 0$, K_G, g_I and A. By definition, $Z(p, P, K, K_G, g_I, A) \to \infty$ when $p \to 0$ and $Z(p, P, K, K_G, g_I, A) \to -\infty$ when $p \to U_C(y(K, K_G, g_I, A), L(K, K_G, A))^{-1}$. Recall that $C(p, K, K_G, A)$ is increasing in p, while $r(K, K_G, A)$ is decreasing in K. Thus, $Z(p, P, K, K_G, g_I, A)$ is increasing in p, and there exists a unique $0 < p(K, K_G, g_I, A) < U_C(y(K, K_G, g_I, A), L(K, K_G, A))^{-1}$ solving $Z(p, P, K, K_G, g_I, A) = 0$.

ii) We now need to show that $p(K, K_G, g_I, A) \in P$ or, equivalently, that $p(K, K_G, g_I, A)$ satisfies the three conditions of Definition (A1). Notice that in the previous step we verified that the first condition holds. Let's now show that $p(K, K_G, g_I, A)$ has required monotonicity properties.

[30] The purpose of this assumption is to establish that $p(K, K_G, g_I, A)$ is increasing in K_G, at least for some small K. This is a nontrivial problem since r' tend to grow in K_G. Consider (A12) when $\xi = \infty$. Unless K is small, $\left.\frac{dr'}{dK}\right|_{\xi=\infty}$ tend to be positive due to $1 - \delta_g$ term. However, for a sufficiently small K, $\frac{\partial r'}{\partial K} \times a(g_I)\frac{\partial Y}{\partial K_G}$ dominates and $\left.\frac{dr'}{dK}\right|_{\xi=\infty}$ becomes negative.

ii.a) Denote $p = T(P)$. By definition, $Z(p(K, K_G, g_I, A), P, K, K_G, g_I, A) = 0$. Evaluate Z at $p(K, K_G, g_I, A)$ and $K + dK$

$$Z(p(K,K_G, g_I, A), P, K + dK, K_G, g_I, A) = \frac{1}{p(K, K_G, g_I, A)} - \beta\,\mathbb{E}\left[\frac{r\big(y(K+dK,K_G,g_I,A)-C(p(K,K_G,g_I,A),K+dK,K_G,A);K'_G(K+dK,K_G,g_I,A);A'\big)}{P\Big(y(K+dK,K_G,g_I,A)-C(p(K,K_G,g_I,A),K+dK,K_G,A);K'_G(K+dK,K_G,g_I,A);{g_I}';A'\Big)}\right].$$

In this case, $\xi = 0$ and r' decreases by Assumption (A6). Moreover, ξ', defined by (A14), is increasing in ξ. Therefore, by Assumption (A7), $\xi'(0) < \xi_{crit}(K', K'_G)$. Given that P is increasing in the first argument and along the corresponding critical direction, it also increases for any direction in between. We conclude that $Z(p(K, K_G, g_I, A), P, K + dK, K_G, g_I, A) > 0$. Since $Z(p, P, K, K_G, g_I, A)$ is decreasing in p, it must be that $p(K, K_G, g_I, A) < p(K + dK, K_G, g_I, A)$, which establishes the second required property of Definition (A1).

ii.b) Let's now perturb (K, K_G) along a critical direction $\xi_{crit}(K, K_G)$. By definition, r' will decrease. By Assumption (A7), $\xi'(\xi_{crit}(K, K_G)) < \xi_{crit}(K', K'_G)$ and thus P goes up. Using the same logic as in ii.a), we conclude that p is increasing along a critical direction $\xi_{crit}(K, K_G)$, which finishes the proof that T is a well-defined self-map.

$\blacksquare$

The following lemma directly follows Lemma A5 of Schaal and Taschereau-Dumouchel (2015), so the proof is omitted.[31]

Lemma A4 *T is continuous and monotone.*

Proposition A1 *There exists a strictly positive fixed point $p^* \in \mathcal{P}$.*

Proof: The existence of a fixed point is guaranteed by Tarski (1955). Here we construct a strictly positive fixed point in a sense that $p^*(K, K_G, g_I, A) > 0$ for $K > 0$.

i) Define the sequence $\{p_n\}_{n\in\mathbb{N}\cup\{0\}}$ such that $p_n = T^n p_0$ and the initial value is $p_0 = U_C(y(K, K_G, g_I, A), L(K, K_G, A))^{-1}$. Apparently, $p_1 \leq p_0$ by definition of $\mathcal{P}$. By monotonicity of T, $\{p_n\}_{n\in\mathbb{N}\cup\{0\}}$ is a weakly decreasing sequence. $p^* = \inf_n p_n$ and due to completeness of $\mathcal{P}$, $p^* \in \mathcal{P}$. By continuity of T, $p^* = Tp^*$, hence, p^* is a fixed point of T.

[31]Notice that inverse marginal utility p does not enter the expression for $K'_G = (1 - \delta_g)K_G + g_I Y(K, K_G, A)$, so their proof goes through.

ii) Let's first show that p^* is not zero. By Assumption (A5), there exists K^- such that $y(K^-, K_G, g_I, A) - \frac{L(K^-,K_G,A)^{1+\nu}}{1+\nu} > K^-$ and $\beta\,\mathbb{E}\left[r(K^-, K_G, A', g_I')\right] \leq 1$ for all $K_G \in \mathcal{K}_{\mathcal{G}}$, $g_I \in \mathcal{G}^{\mathcal{I}}$, $A \in \mathcal{A}$. Pick an $\zeta > 0$ such that $0 < \zeta^{\frac{1}{\gamma}} < y(K^-, K_G, g_I, A) - \frac{L(K^-,K_G,A)^{1+\nu}}{1+\nu} - K^-$. Take any $p \in \mathcal{P}$ such that $p(K^-, K_G, g_I, A) \geq \zeta$ for all plausible K_G, g_I and A. We want to show that $Tp(K^-, K_G, g_I, A) > \zeta$:

$$Z(\zeta, p, K^-, K_G, g_I, A) = \frac{1}{\zeta} - \beta\,\mathbb{E}\left[\frac{r\left(y(K^-, K_G, g_I, A) - C(\zeta, K^-, K_G, A); K_G'; A'\right)}{p\left(y(K^-, K_G, g_I, A) - C(\zeta, K^-, K_G, A); K_G'; {g_I}'; A'\right)}\right] \geq$$

$$\frac{1}{\zeta} - \frac{\beta}{\zeta}\,\mathbb{E}\, r\left(y(K^-, K_G, g_I, A) - C(\zeta, K^-, K_G, A); K_G'; A'\right) > 0$$

Therefore, $Tp(K^-, K_G, g_I, A) > \zeta$. Notice that our iterations start from $p_0(K^-, K_G, g_I, A) > \zeta$, hence, $p^*(K^-, K_G, g_I, A) > 0$.

iii) Finally, let's show that p^* is strictly positive for any $K > 0$. Assume not. Then there exist $K > 0$, K_G, g_I and A such that $p^*(K, K_G, g_I, A) = 0$. Since p^* is increasing in the first argument, $p^*(\hat{K}, K_G, g_I, A) = 0$ for all $\hat{K} < K$. Define

$$\tilde{K} = \sup_{K \leq K^-} \left\{p^*(K, K_G, g_I, A) = 0 \text{ for at least one triple } (K_G, g_I, A)\right\}.$$

Notice that $\tilde{K} < K^-$ since $p^*(K^-, K_G, g_I, A) > 0$ for any K_G, g_I and A. By our assumption, however, $0 < \tilde{K} < K^-$. Hence, $y(\tilde{K}, K_G, g_I, A) - \frac{L(\tilde{K},K_G,A)^{1+\nu}}{1+\nu} > \tilde{K}$. Then the RHS of the Euler equation is

$$\beta\,\mathbb{E}\left[\frac{r\left(y(\tilde{K}, K_G, g_I, A) - C(0, \tilde{K}, K_G, A); K_G'; A'\right)}{p^*\left(y(\tilde{K}, K_G, g_I, A) - C(0, \tilde{K}, K_G, A); K_G'; {g_I}'; A'\right)}\right] \leq$$

$$\beta\,\mathbb{E}\left[\frac{r\left(\tilde{K}; K_G'; A'\right)}{p^*\left(y(\tilde{K}, K_G, g_I, A) - C(0, \tilde{K}, K_G, A); K_G'; {g_I}'; A'\right)}\right].$$

Since $y(\tilde{K}, K_G, g_I, A) - \frac{L(\tilde{K},K_G,A)^{1+\nu}}{1+\nu} > \tilde{K}$, $p^*\left(y(\tilde{K}, K_G, g_I, A) - C(0, \tilde{K}, K_G, A); K_G'; {g_I}'; A'\right) > 0$ and the expression above is bounded. Thus, $p^*(\tilde{K}, K_G, g_I, A) \neq 0$.

$\blacksquare$

E.2.5 Uniqueness

In order to establish uniqueness of the strictly positive fixed point constructed above, we need to show that T is pseudo-concave and K_0-monotone. Pseudo-concavity can be proven in exactly the same way as in Schaal and Taschereau-Dumouchel (2015), so we omit the proof of the following lemma:

Lemma A5 *T is pseudo-concave. That is, for any strictly positive $p \in \mathcal{P}$ and $t \in (0,1)$, $T(tp)(K, K_G, g_I, A) > tTp(K, K_G, g_I, A)$ for any $K > 0$, K_G, g_I and A.*

The proof of K_0-monotonicity is provided below.

Lemma A6 *T is K_0-monotone. That is, for any strictly positive $p \in \mathcal{P}$, there exists $K_0 > 0$ such that for any $0 \leq K_1 \leq K_0$ and any $p \in \mathcal{P}$ such that $p(K, K_G, g_I, A) \leq p^*(K, K_G, g_I, A)$ for any $K \geq K_1$, $K_G > 0$, g_I and A*

$$p^*(K, K_G, g_I, A) \geq Tp(K, K_G, g_I, A) \text{ for any } K \geq K_1, K_G > 0, g_I, A.$$

Proof: Let's show that there exists K_0 such that $y(K, K_G, g_I, A) - C(p^*(K, K_G, g_I, A), K, K_G, A) \geq K$ for any $K \leq K_0$, K_G, g_I and A. Assume not. Then for any $K_0 > 0$ there exists $K \leq K_0$ so that $y(K, K_G, g_I, A) - C(p^*(K, K_G, g_I, A), K, K_G, A) < K$ for some K_G, g_I and A. Then

$$\frac{1}{p^*(K, K_G, g_I, A)} = \beta\,\mathbb{E}\left[\frac{r\left(y(K, K_G, g_I, A) - C(p^*(K, K_G, g_I, A), K, K_G, A); K'_G; A'\right)}{p^*\left(y(K, K_G, g_I, A) - C(p^*(K, K_G, g_I, A), K, K_G, A); K'_G; g_I'; A'\right)}\right] >$$

$$\beta\,\Pr(g'_I = g_I; A' = A | g_I, A)\frac{r(K, K'_G, A)}{p^*(K, K'_G, g_I, A)}.$$

Equivalently,

$$\frac{p^*(K, K'_G, g_I, A)}{p^*(K, K_G, g_I, A)} > \beta\,\Pr(g'_I = g_I; A' = A | g_I, A) r(K, K'_G, A)$$

Take a very small positive K_0. Then K is even smaller but still positive. For sufficiently small K, $K'_G \leq K_G$. At the same time, $\xi_{crit}(K, K_G)$ grows unboundedly, implying that any $p(K, K_G, g_I, A) \in \mathcal{P}$ increases in K_G.[32] Hence,

$$1 = \frac{p^*(K, K_G, g_I, A)}{p^*(K, K_G, g_I, A)} > \frac{p^*(K, K'_G, g_I, A)}{p^*(K, K_G, g_I, A)} > \beta \Pr(g'_I = g_I; A' = A | g_I, A) r(K, K'_G, A)$$

However, for a small enough K the RHS of the inequality above is unbounded. Hence, we have a contradiction and there exists K_0 such that $y(K, K_G, g_I, A) - C(p^*(K, K_G, g_I, A), K, K_G, A) \geq K$ for any $K \leq K_0$, $K_G > 0$, g_I and A. Pick a $K_1 \leq K_0$ and a p such that $p(K, K_G, g_I, A) \leq p^*(K, K_G, g_I, A)$ for any $K \geq K_1$, $K_G > 0$, g_I and A. For any $K \geq K_1$, $K_G > 0$, g_I and A we have

$$\begin{aligned} &p(y(K, K_G, g_I, A) - C(p^*(K, K_G, g_I, A), K, K_G, A)) \leq \\ &\qquad\qquad p^*(y(K, K_G, g_I, A) - C(p^*(K, K_G, g_I, A), K, K_G, A)). \end{aligned}$$

It is, therefore, easy to see that $0 = Z(p^*(K, K_G, g_I, A), p^*(K, K_G, g_I, A), K_G, g_I, A) \geq Z(p^*(K, K_G, g_I, A), p(K, K_G, g_I, A), K_G, g_I, A) \Rightarrow Tp(K, K_G, g_I, A) \leq p^*(K, K_G, g_I, A)$.

■

Making minor notational changes in Proposition A3 from Schaal and Taschereau-Dumouchel (2015), we can prove the uniqueness result:

Proposition A2 *A strictly positive fixed point $p \in \mathcal{P}$ is unique.*

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[32] Assumption (A7) suffices to guarantee that $p(K, K_G, g_I, A)$ is not decreasing in K_G.

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