Understanding the Behavior of Distressed Stocks^{*}

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January 16, 2020

Abstract

We construct an asset pricing model with explicit default to develop a risk-based source of the distress anomaly. We show that distress produces sharply countercyclical betas leading to biased estimates of risk premia and alphas. This effect is amplified when earnings growth is mean-reverting, so that distressed stocks also have high expected future earnings. This bias can account for between 39 and 76 percent of the distress anomaly in a calibrated economy that replicates the key characteristics of these stocks.

^{*}We thank Hengjie Ai, Max Croce, Zhi Da, Jan Ericsson, Lorenzo Garlappi, Brent Glover, John Griffin, Jens Hilscher, Lars-Alexander Kuehn, Xiaoji Lin, Ali Ozdagli, and participants at the AEA meetings, CMU (Tepper), HEC-McGill Winter Finance workshop, Minnesota Macro-Asset Pricing conference, Minnesota Junior Finance conference, SFS Cavalcade, University of North Carolina, WFA, and Wharton for several helpful comments.

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1 Introduction

Understanding the behavior of distressed stocks has proved somewhat challenging for standard asset pricing theory. Earlier thought, going back to at least Fama and French (1992), suggested financial distress could be the source of the higher expected returns of value stocks. Most empirical research, however, indicates that portfolios of highly distressed stocks tend to severely underperform those of other stocks.¹ Equally surprising, estimated loadings of distress portfolios on standard risk factors are often large, making it even more difficult to understand returns to distressed stocks with conventional multi-factor models.²

In this paper we develop an equilibrium asset pricing model with explicit default risk to understand the key patterns in the returns of distressed stocks. Our theoretical starting point is the observation that the possibility of default implies that, as cash flows and equity value dwindle, a firm's equity beta becomes more levered, increasing the risk compensation demanded by shareholders. The model in Section 2 is developed around this central insight.

We next establish our second main result. Sharp movements in the betas of distressed firms will, in turn, imply that standard empirical estimates of the expected returns on portfolios of highly distressed firms will be downward biased. In particular, we conclude that an unconditional OLS regression, the literature's standard model of performance evaluation, will produce a biased estimate of alpha for portfolios of highly distressed firms.³

The final, and most novel, contribution of our model, however, is to explicitly link the exact bias in expected returns on distress stocks to the expected growth in corporate earnings. More precisely, our model implies that the *magnitude* of the bias in expected returns increases with the degree of mean reversion in earnings growth. Since distressed firms naturally exhibit high expected earnings growth relative to safer firms

¹Notable examples include Dichev (1998), Griffin and Lemmon (2002), Campbell, Hilscher, and Szilagyi (2008), and Garlappi and Yan (2011).

²For example Friewald, Wagner, and Zechner (2014) find that firms with a high failure probability have high equity beta but low, and even negative, stock returns on average.

³Other studies that entertain payoff nonlinearities in producing biases in returns and model misspecifications are Boguth, Carlson, Fisher, and Simutin (2011) and Korteweg and Nagel (2016).

in our model, the combined effects of countercyclicality in firm beta and mean reversion in earnings become stronger as firms near default. Therefore, portfolios *constructed and ranked* on their likelihood of distress will be increasingly exposed to these phenomena.

We validate the model and these three key predictions in the data in Section 3. We begin by empirically defining the distress anomaly as a measure of ex ante probability of default using the reduced-form logit approach introduced by Shumway (2001) and Campbell, Hilscher, and Szilagyi (2008). Market capitalization, volatility, and market-to-book ratios all play a chief role in predicting default, consistent with our model. Next, in double-sorted portfolios of distress with several measures of valuations we show that the distress anomaly is concentrated in portfolios that have high valuation ratios. Therefore, under rational pricing, these portfolios are expected to have higher-than-average earnings going forward, a fact that we can also confirm empirically. Last, we demonstrate that betas of the most distressed firms change more drastically within a portfolio holding period, confirming that equity risk driven by mean reversion substantially falls after portfolio formation.

Having validated our model, we then provide a quantitative assessment of its main theoretical mechanisms and key implications in Section 4. To accomplish this we first calibrate our model to quantitatively match empirical ex-ante probabilities of default and return volatilities across portfolios as well as quantities of market risk and leverage.

After disciplining the model in this way, we provide a quantitative estimate of the likely magnitude of the biases in distressed portfolios' expected excess returns and alphas. Depending on the considered pricing specification, our bias explains between 39 and 76 percent of the estimated anomaly. The four-factor Carhart (1997) and the five-factor Fama and French (2015) models, which empirically perform very well in explaining stock returns, imply that the bias likely accounts for around 70 percent of the anomaly. We further document how its overall magnitude depends crucially on the degree of mean reversion in returns and, more subtly, on the portfolio rebalancing frequency. Here, we find that frequent rebalancing exposes the investor to significantly more default risk while holding highly distressed stocks which, in turn, exacerbates the perceived distress anomaly.

This section also includes some more direct attempts to validate our model's core insights by comparing some of its predictions with their empirical counterparts. Notably, we show that, in the model, as in the data, the distress anomaly is indeed especially concentrated among stocks with above average rates of earnings growth. Moreover, we also find that the anomaly actually reverses sign for portfolios that are expected to have below average earnings growth rates, a fact that is difficult to explain were it not for the forces in the model.

The empirical literature documenting the distress anomaly is quite extensive. Empirical work on distress risk began by documenting its negative pattern in returns with Dichev (1998) and Griffin and Lemmon (2002), and, more recently, in the work of Campbell, Hilscher, and Szilagyi (2008), Elkamhi, Ericsson, and Parsons (2012), Hackbarth, Haselmann, and Schoenherr (2015), and Gao, Parsons, and Shen (2017). Two exceptions to these findings are Vassalou and Xing (2004) but only to the extent that distressed firms are small, value stocks and are illiquid (Da and Gao (2010)) and Chava and Purnanandam (2010) when using the implied cost of capital developed in Pastor, Sinha, and Swaminathan (2008) as a measure of expected returns.

Friewald, Wagner, and Zechner (2014) explore the link between a firm's equity and credit risk by sorting firms on credit default swap spreads, finding that greater spreads positively correlate with higher expected equity returns. They conclude that CDS spreads uncover risk not captured by physical default expectations alone, which is the risk source we focus on. By linking these features they also broaden the perspective on the distress anomaly by tying it to the vast literature on credit risk (e.g., Collin-Dufresne and Goldstein (2001) and Bai, Collin-Dufresne, Goldstein, and Helwege (2015)). Work by Avramov, Chordia, Jostova, and Philipov (2013) also uncovers more of the anomaly's features, partially associating it to momentum.

Several risk-based theories have been developed in response to this evidence, most of which bear a relation to the classic Leland (1994) model. A partial list includes George and Hwang (2010), O'Doherty (2012), Ozdagli (2013), Conrad, Kapadia, and Xing (2014), Eisdorfer, Goyal, and Zhdanov (2018), Opp (2018) and McQuade (2018). Perhaps closest to us is the theoretical model in Garlappi and Yan (2011), who also allow shareholders to partially recover assets upon default. The key distinction between our work and these papers however is that ultimately they must rely on the fact that equity betas must fall as firms approach distress, thus making them safer, to produce low theoretical returns for distressed stocks.

More recently, Chen, Hackbarth, and Strebulaev (2018) modify our basic model to propose an alternative risk-based explanation driven by procyclical leverage. In their setup, distressed firms optimally delever by selling assets and therefore reduce their equity betas over time. As this is more likely to occur during recessions when risk premia are large, this creates a conditional pricing effect in the spirit of Jagannathan and Wang (1996). Empirically, however, leverage tends to rise during recessions (Halling, Yu, and Zechner (2016)), so their mechanism will in practice contribute to amplify rather than explain the distress puzzle.⁴

By contrast our approach requires distressed firms to actually be riskier but clearly ties the *estimated* underperformance of distressed firms to *high* measures of risk and betas. Empirically this seems more plausible since distressed stocks are very volatile and possess large betas; that is, they observationally appear to be risky. Furthermore, they also appear to move with aggregate market conditions in the way we would expect if investors understand them to be risky (Campbell, Hilscher, and Szilagyi (2008) and Eisdorfer and Misirli (2017)).

We now turn to discuss our methods and findings in more detail.

2 Equilibrium Equity Returns with Default

We begin by developing a partial equilibrium economy with endogenous default that allows us to derive the implied endogenous process for a representative firm's expected stock returns. We next use this framework to characterize analytically the implied theoretical biases in estimated unconditional risk premia and linear factor models.

⁴Deconditioning arguments are also unlikely to quantitatively explain anomalies as argued persuasively in Lewellen and Nagel (2006).

2.1 Discounting and Risk

Since our goal is to understand relative, not absolute, movements in asset returns, we adopt a partial equilibrium perspective and simply posit the representative investor's stochastic discount factor at time t as:

$$\Lambda_t = \exp\left\{-\rho t - \gamma w_t\right\},\tag{1}$$

where $w_t = \log W_t$ denotes log investor wealth, and its dynamics are driven by a constant drift μ and an aggregate Brownian shock z with volatility σ :

$$w_t - w_0 = \mu t + \sigma z_t. \tag{2}$$

Intuitively, we can think of ρ as the rate of time preference and γ as the representative investor's risk aversion. For simplicity we assume all wealth is invested in the stock market, so that its return equals that of the overall market. As a result, the covariances between these returns and those on individual stocks will allow us to construct CAPM betas.

As is well known, given the above assumptions, Ito's lemma can be used to derive the level of the risk-free rate in our economy as:

$$r = \rho + \gamma \mu - \frac{1}{2} \gamma^2 \sigma^2 \tag{3}$$

and, because the wealth portfolio is itself priced, the restriction:

$$\mu - r = \frac{1}{dt} \mathbb{E}_t[dw] - r = -\frac{1}{dt} \mathbb{E}_t\left[\frac{d\Lambda}{\Lambda}dw\right] = \gamma\sigma^2 \tag{4}$$

requires that the equilibrium price of risk satisfies $\lambda \equiv \gamma \sigma = \frac{\mu - r}{\sigma}$.

2.2 Firms

The model economy is populated by a continuum of firms, indexed by the subscript *i*. Each firm generates an instantaneous cash flow (EBITDA) according to a stochastic process that is mean-reverting at rate κ_i to a long-run value \overline{X} and has volatility σ_i .⁵

⁵Raymar (1991) and Garlappi and Yan (2011) also use mean-reverting cash flow environments to explore capital structure and bankruptcy decisions.

Formally, firm cash flows are assumed to follow the Ornstein-Uhlenbeck process:⁶

$$dX_i = \kappa_i (\overline{X} - X_i) dt + \sigma_i \left(\rho_i dz + \sqrt{1 - \rho_i^2} dz_i \right).$$
(5)

In this expression cash flow is uncertain due to both a firm-specific idiosyncratic Brownian shock dz_i and an aggregate shock, dz, with which it has correlation ρ_i .

Operating the firm requires a (constant) flow payment of C_i per unit of time. We will think of C_i as the instantaneous coupon payment on an outstanding consol bond, but a more general interpretation allows for the addition of any operating and depreciation costs. In either case, $X_i - C_i$ represents firm *i*'s (instantaneous) earnings before any taxes. As a result, equity cash flows can become negative, consistent with empirical evidence (e.g., Griffin and Lemmon (2002)).

In principle, the model allows for firms to be potentially heterogeneous in their cost, C_i , rate of mean reversion, κ_i , idiosyncratic volatility, σ_i , and the correlation with the market, ρ_i . We explore some of this heterogeneity in our simulations. All firms are assumed to face a constant marginal tax rate on earnings, τ .

2.3 Equity Valuation

To construct explicit expressions for the value of the firm it is useful to switch from physical to risk neutral probabilities. Girsanov's theorem allows us to do this and rewrite the distribution of the physical cash flow process under the risk-neutral measure, $dz^{\mathbb{Q}}$, as:

$$dX_i = \kappa_i (\overline{X}_i - X_i) dt + \sigma_i \left(\rho_i dz^{\mathbb{Q}} + \sqrt{1 - \rho_i^2} dz_i \right).$$
(6)

where $\overline{X}_i = \overline{X} - \frac{\lambda \rho_i \sigma_i}{\kappa_i}$. Of course, idiosyncratic risk, dz_i , has a zero market price of risk and is therefore always under the risk-neutral measure.

The market value of equity under risk-neutral pricing can be expressed as:

$$E_{i}(X_{i0}) = \sup_{\tau_{i}^{D}} \mathbb{E}^{\mathbb{Q}} \left[\int_{0}^{\tau_{i}^{D}} e^{-rs} (1-\tau) \left(X_{is} - C_{i} \right) ds + e^{-r\tau_{i}^{D}} \delta \Theta(X_{i}^{D}) \right],$$
(7)

⁶All random variables depend on time. However, to save on notation we avoid using time subscripts unless necessary. Note that the rate of mean reversion of this process is linear in X_i and that the expectation of dX_i/X_i is not defined as the variance term would go to infinity if X_i approached 0.

where $\tau_i^D \equiv \inf \{s : X_{is} \leq X_i^D\}$ is the stopping time corresponding to the firm's (optimal) decision to default which occurs when the value of cash flows hits the (endogenous) threshold X_i^D , to be computed below. Following Garlappi and Yan (2011) we allow for possible (small) deviations from the absolute priority rule by assuming shareholders can recover a fraction $0 \leq \delta < 1$ of the firm's assets in default, $\Theta(X_i^D)$.

It follows again from Ito's lemma that, while in operation, firm i's equity value satisfies the ordinary differential equation:

$$rE_i(X_i) = \frac{1}{2}E_i''(X_i)\sigma_i^2 + E_i'(X_i)\kappa_i(\overline{X}_i - X_i) + (1 - \tau)(X_i - C_i).$$
(8)

Its solution (described in detail in Appendix A) has the form

$$E_i(X_i) = (1 - \tau) \left(\frac{\overline{X}_i - C_i}{r} + \frac{X_i - \overline{X}_i}{r + \kappa_i} \right) + A_i H \left(-\frac{r}{\kappa_i}, -\frac{\kappa(\overline{X}_i - X_i)}{\sqrt{\kappa_i \sigma_i^2}} \right)$$
(9)

where H(n, v) is the generalized Hermite function of order n. The values of $A_i > 0$ and X_i^D must be computed numerically by using the following value matching and smooth pasting conditions associated with optimal decision to default:

$$E_i(X_i^D) = \delta\Theta(X_i^D) = \delta(1-\tau) \left(\frac{\overline{X}}{r} + \frac{X_i^D - \overline{X}}{r+\kappa_i}\right)$$
(10)

$$E_i'(X_i^D) = \delta \frac{1-\tau}{r+\kappa_i}.$$
(11)

Intuitively, equity holders choose to default when $X_i = X_i^D$ because at that point the value of running the firm equals that of defaulting (value matching) and the rates of return on the two options are identical (smooth pasting).

2.4 Returns and Betas

Using the valuation equation (8), it is straightforward to express the stock return under the physical measure as:

$$dR_{i} = \frac{dE_{i} + (1 - \tau)(X_{i} - C_{i})dt}{E_{i}} = \mathbb{E}_{t}[dR_{i}] + \frac{E_{i}'(X_{i})}{E_{i}(X_{i})}\sigma_{i}\left(\rho_{i}dz + \sqrt{1 - \rho_{i}^{2}}dz_{i}\right)$$
(12)
where $\mathbb{E}_{t}[dR_{i}] = \frac{(1 - \tau)(X_{i} - C_{i})}{E_{i}} + \frac{E_{i}'(X_{i})}{E_{i}}\kappa_{i}(\overline{X} - X_{i}) + \frac{1}{2}\frac{E_{i}''(X_{i})}{E_{i}(X_{i})}\sigma_{i}^{2}.$

Firm *i*'s *conditional* CAPM beta can then be constructed by computing the covariance of this return with the return to overall household wealth:

$$\beta_{it} = \frac{\mathbb{E}_t \left[dw dR_i \right]}{\operatorname{var}_t \left[dw \right]} = \frac{E'_i(X_i)}{E_i(X_i)} \frac{\rho_i \sigma_i}{\sigma}.$$
(13)

Although the mean-reverting earnings process prevents us from isolating its properties analytically, firm betas will be driven by the same three separate forces identified in Gomes and Schmid (2010): (i) a firm's unlevered asset beta; (ii) firm leverage through the coupon C_i ; and (iii) endogenous changes in the value of the equity holders' option to default. Crucially, as shown in Garlappi and Yan (2011), a firm's β_{it} will generally be decreasing in X_i , so that firms will become *increasingly risky* as they approach their default threshold unless δ is large.⁷

Taking the conditional variance of (12) we get:

$$\frac{1}{dt} \operatorname{var}_t(dR_i) = \underbrace{\beta_{it}^2 \sigma^2}_{\text{Systematic}} + \underbrace{\beta_{it}^2 \sigma^2 (1 - \rho_i^2) / \rho_i^2}_{\text{Idiosyncratic}} = \beta_{it}^2 \frac{\sigma^2}{\rho_i^2}.$$
(14)

Thus, as β_{it} rises near default, so does the variance of firm level returns. It follows that distressed firms will have a high conditional, and therefore also unconditional, variance of returns. In conclusion, it is *symptomatic* of firms approaching default to exhibit high measures of risk.

This is a key result that distinguishes our paper from many other risk driven explanations of the distress puzzle. In many papers, the observed low excess returns on distress stocks are rationalized by the fact that the expected returns on these stocks are themselves lower since they become less risky as default approaches. Moreover, our model's implication is generally consistent with the evidence in Campbell, Hilscher, and Szilagyi (2008) and Eisdorfer and Misirli (2017) that portfolios of distressed firms move in a way that suggests investors perceive them to be risky.

2.5 Conditional and Unconditional CAPM

By construction, theoretical equity returns are both linear in the underlying risk factor, dz, and conditionally log normal. Hence, the conditional CAPM holds and its

⁷Garlappi and Yan (2011) use this result to motivate the choice of a high value for δ to rationalize the low returns on distress stocks. By contrast, our approach is to choose a very low value of δ .

conditional alpha is zero. Formally, for each firm i, we get that:

$$\alpha_{it} = \left(\frac{1}{dt}\mathbb{E}_t \left[dR_i\right] - r\right) - \beta_{it} \left(\frac{1}{dt}\mathbb{E}_t \left[dw\right] - r\right)$$
$$= \beta_{it}\gamma\sigma^2 - \beta_{it}(\mu - r) = 0, \tag{15}$$

where the last equality holds from the economic restriction on the price of risk.

In practice, however, empirical studies rely on discrete sampling of a local (instantaneous) risk factor model that takes the general form:

$$R_{it}^e = \alpha_i + \beta_i R_{wt}^e + \epsilon_{it}.$$
 (16)

where R_{it}^e is the excess return on stock *i* over the risk free rate, r_t . When the sole risk factor is the excess return on the market portfolio, R_{wt}^e , we can evaluate the CAPM by running the above time series OLS regression, as is common in the distress literature. As usual, the assumed economic restriction in testing the CAPM would be that $\alpha_i = 0$ for all *i* so that pricing errors are zero for every test asset.

However, over a longer enough horizon it is well understood that this discrete sampling of a local (instantaneous) risk factor model can lead to sizable biases in the estimated expected returns series (Longstaff (1989)). In our case, under the model's true dynamics, the unconditional expected excess return for firm i, over a period of arbitrary length T > 0, can be decomposed as

$$\mathbb{E}[R_{iT}^{e}] = \mathbb{E}\left[\int_{0}^{T} (dR_{it} - rdt)\right] = \mathbb{E}\left[\int_{0}^{T} \beta_{it}(dw - rdt)\right]$$
$$= \underbrace{\frac{\mu - r}{\sigma}}_{\text{Discrete CAPM}} + \underbrace{\frac{\mu - r}{\sigma}}_{\equiv \text{Bias}} \mathbb{E}\left[\int_{0}^{T} (\beta_{it} - \beta_{i0})\sigma dt\right].$$
(17)

As previous studies have shown (for example, Jagannathan and Wang (1996) and O'Doherty (2012)), when a firm's market exposure is expected to change over time, standard (unconditional) factor models will generally produce a bias in estimated expected returns which manifests itself in a potentially sizable estimate for the value of α_i .⁸ However, unlike those earlier papers however, the bias in (17) does not depend on the covariance of the firm's beta with the market risk premium.⁹

⁸We interpret β_{i0} as the exposure of firm *i* to the market risk factor at time 0 (today).

⁹This bias is general in the sense that it will be relevant even if there are multiple risk factors. Specifically, the bias would be a linear combination of each risk factor's beta and price of risk.

2.6 Understanding The Bias in Expected Returns

To truly understand the fundamental drivers of the theoretical bias in expected returns, we can take the expectation inside the integral in (17) and apply Ito's lemma to express it as function of the dynamics of β_{it} :

$$\operatorname{Bias} \propto \frac{\mu - r}{\sigma} \int_0^T \int_0^t \Big(\underbrace{\frac{\partial \beta_{is}}{\partial X_{is}} \kappa_i(\overline{X} - X_{is})}_{\operatorname{Earnings Bias}} + \frac{1}{2} \frac{\partial^2 \beta_{is}}{\partial X_{is}^2} \sigma_i^2 \Big) \sigma ds dt.$$
(18)

We can see that the bias depends on: (i) the (negative) slope, $\partial \beta_{is}/\partial X_{is}$; and (ii) and the degree of mean reversion in the firm's earnings process, $\kappa_i(\overline{X} - X_{is})$. The former captures the direct negative impact of systematic shocks on firm β : firms will become increasingly risky as they become more distressed. Equation (18) shows that this is crucial to obtain a negative bias in returns.

The magnitude of the bias however depends on the degree of mean reversion in earnings. This central role of earnings growth is new to our understanding of the puzzling behavior of distressed firms. It will only be large for firms with sizable expected growth rates in earnings. We label it an *earnings-induced* bias. Equation (18) shows that when the combined effects of time varying betas, and mean reversion in earnings are large enough, unconditional estimates of portfolio *returns* may significantly underperform the market return. Section 4 quantifies these effects in our model.

Figure 1 summarizes the key theoretical predictions of our model by decomposing estimated mean excess returns into various components. As discussed, the default probability increases as the firm's earnings, X_{it} , fall towards the default boundary. A firm's beta, $\beta_i(X)$, rises as profitability falls and its equity becomes increasingly more levered. Importantly, as the figure shows, not only does the firm's beta increase, but the slope of the function increasingly becomes negative. Convexity in expected returns means that a firm's beta becomes more sensitive to the aggregate market shocks. This effect interacts with the expected growth rate of earnings to amplify the earnings bias.

The figure also conveys that our model also generates a number of other interesting attributes for these firms. In particular, distressed stocks are small and are expected to have higher-than-average earnings going forward, and therefore should have high market-to-book ratios, under rational pricing. Finally, the bottom-right panel shows that these stocks also exhibit high idiosyncratic return volatility.

3 Empirical Validation

In this section we use the data to validate our model's central predictions by examining the role of mean reversion in earnings growth as well as the properties and temporal behavior of distressed stocks.

Our data covers the period 1950 to 2015, although most of the analysis focuses on the period from 1970 on. We identify a default event with a stock delisting for any type of performance-related reason. Appendix B discusses these events, their classifications and their properties. For the sample period in question our classification yields 5,652 delistings out of over 210,000 firm-year observations.

Detailed firm-level data comes from combining annual and quarterly accounting data from COMPUSTAT with monthly and daily data from CRSP. We prefer annual over quarterly accounting data. Details about the data and our approach to construct the key variables are included in Appendix C.

3.1 Estimating Default Probabilities

A proper quantitative version of our model must first of all match reliable empirical measures of ex-ante default probabilities. We construct these by estimating the probabilities of a stock delisting, or default event, for firm i at time t over the next year, denoted p_{it} .

We forecast delisting events using an updated version of the reduced-form logistic model proposed by Campbell, Hilscher, and Szilagyi (2008) with one significant modification. Specifically, these authors use monthly regressions and focus on predicting the probability of defaulting 12 months ahead, *conditional* on no default occurring in the 11^{th} month. Instead, we use annual rolling logit regressions that can be interpreted as estimating the probability of defaulting, at any time *within the next year*, given the information available at the beginning of the year. More precisely, we estimate these rolling regressions on an annual basis to avoid any look-ahead bias. We use maximum likelihood methods to estimate a logistic function on eight explanatory variables in a pooled estimation across all firm-years. Formally, we define $p_{it} = 1/(1 + \exp(-y_{it}))$, where y_{it} can be approximated by the following empirical specification:

$$y_{it} = \gamma_0 + \gamma_{EXRETAVG} EXRETAVG_{it} + \gamma_{SIGMA} SIGMA_{it} + \gamma_{PRICE} PRICE_{it} + \gamma_{NIMTAAVG} NIMTAAVG_{it} + \gamma_{TLMTA} TLMTA_{it} + \gamma_{CASHMTA} CASHMTA_{it} + \gamma_{RSIZE} RSIZE_{it} + \gamma_{MB} MB_{it}$$
(19)

where $EXRETAVG_{it}$ is a measure of average excess returns over the S&P500 index, $SIGMA_{it}$ is the volatility of equity returns, MB_{it} is the market-to-book ratio, $NIMTAAVG_{it}$ is a measure of profitability, $TLMTA_{it}$ is a measure of firm leverage, $CASHMTA_{it}$ is a measure of cash holdings, $RSIZE_{it}$ is the relative size of the firm, and $PRICE_{it}$ is the log stock price, capped at \$15.

The full sample logistic regression results do not differ materially from those in Campbell, Hilscher, and Szilagyi (2008).¹⁰ The McFadden pseudo R-squared for these firm level estimates is 40% and all of these financial and accounting ratios are immensely significant. An important observation made by Campbell, Hilscher, and Szilagyi (2008) is that long-horizon predictions of default are largely driven by three predictors: relative market capitalization (RSIZE), which enters negatively; and volatility (SIGMA) and market-to-book ratio (MB), which both enter positively. The fact that these three variables play an important role in distress is precisely as predicted by the theoretical analysis developed above.

3.2 Delisting Portfolios

Based on the estimated firm-level probabilities \hat{p}_{it} , each firm is then ranked and assigned a percentile on a scale of zero to one-hundred in this empirical distribution. We then form nine portfolios, j = 1, 2, ..., 9, in December of every year and place each firm in its appropriate percentile portfolio. We emphasize that our choice of annual rebalancing is important to accurately estimate the premium investors receive for holding distressed

¹⁰The regression coefficients are included in the Online Appendix.

stocks; otherwise, a monthly rebalancing strategy risks mixing the distress anomaly with mechanical effects known to reduce returns such as turnover costs and the wider bid-ask spreads and lower liquidity of small stocks (Campbell, Hilscher, and Szilagyi (2008) and Da and Gao (2010)).

In univariate analysis, these portfolios are then ranked in a symmetric and increasing order as follows:¹¹

- Portfolio 1: Percentiles between 0% and 5%
- Portfolio 2: Percentiles between 5% and 10%
- Portfolio 3: Percentiles between 10% and 20%
- Portfolio 4: Percentiles between 20% and 40%
- Portfolio 5: Percentiles between 40% and 60%
- Portfolio 6: Percentiles between 60% and 80%
- Portfolio 7: Percentiles between 80% and 90%
- Portfolio 8: Percentiles between 90% and 95%
- Portfolio 9: Percentiles between 95% and 100%

When considering double-sorts, we create 30th and 70th percentile breakpoints to ensure a sufficient number of firms in each double-sorted portfolio. Although portfolio composition is fixed over the course of a calendar year, both the probabilities and the value weights on each stock are allowed to fluctuate monthly over the year with the change in each firm's accounting variables and returns, respectively.

Portfolio returns are constructed using value weights. As in Campbell, Hilscher, and Szilagyi (2008), a stock's delisting return is incorporated by simply using the CRSP delisting return when available, or its lagged monthly return otherwise.

Average ex-ante delisting probabilities for each portfolio, denoted \hat{p}_{jt} , are computed using equal weights. Formally, the year-to-year average predicted probability of default

¹¹As usual there is a degree of arbitrariness about these classifications. In practice, virtually all delistings come from stocks in the higher percentiles so the breakdowns for the first five or six portfolios are not particularly important. It is sometimes useful to create finer portfolios for the upper percentiles but there is also a concern that the number of firms in each of them will become quite low, particularly as so many are then delisted over the calendar year.

for portfolio j is given by

$$\hat{p}_{jt} = \sum_{i \in j} \hat{p}_{it} / N_{jt} \tag{20}$$

where N_{jt} is the number of stocks in portfolio j at time t.

As we show below in Section 5, our measure of ex ante propensities of default offers a very good forecast of delisting events over this period, at least at the portfolio level. This displays the accuracy of our logistic regression and confirms that our constructed portfolios do in fact reflect the risk premia attached directly to the distress anomaly.

Table I documents the basic patterns of delisting probabilities, stock returns and other characteristics across the nine delisting portfolios. As we can see, average delisting probabilities are quite low for the first five portfolios. Average excess returns (over the market) are negligible for the first six portfolios but turn increasingly negative for those with high average delisting probabilities. Return volatility and skewness are also much higher for these stocks. The sharp increase in return skewness is consistent with our view of delistings as highly non-linear events.

3.3 Distinguishing Evidence

We now turn our attention to the key empirical patterns in these distressed portfolios that help distinguish our theory. In particular, we examine detailed evidence on the (i) importance of mean reversion in earnings and (ii) shifts in portfolios' risk profiles over a holding period.

3.3.1 Mean Reversion in Earnings

Table II shows that, as implied by our model, firms in distress are smaller and exhibit higher market-to-book ratios. They are also unprofitable. A central feature of our theory, however, is the requirement that distressed firms have high expected earnings growth going forward. Recall that the bias partly arises from an earnings-induced component that, in our model, is driven by the degree of mean reversion.

Table II reports the results of using earnings data to estimate the mean reversion parameter κ in (5) by running monthly regressions of each portfolio's operating profitability, OIMTAAVG, on its lag. We then convert the monthly discrete-time autoregressive coefficient estimate $(\hat{\varphi})$ to an annual parameter in a continuous-time model with the formula $\hat{\kappa} = -\log(\hat{\varphi})/(1/12)$. As we can see there is substantial evidence of mean reversion in earnings across almost all portfolios with annual estimates of κ clustered around 0.1, the value used in Garlappi and Yan (2011). Interestingly, the rates of mean reversion also appear higher for the more risky portfolios.

Direct estimates of rates of mean reversion in earnings can be imprecise but information about them should also be captured in valuation ratios. Under rational pricing, a high market-to-book ratio signals that a stock is expected to have either higher-than-average earnings or lower returns going forward.

Table III offers an empirical counterpart to this prediction. It reports both realized mean excess returns and forward earnings growth rates for portfolios double-sorted on the alternative valuation ratios and distress. In all panels we rank firms on these measures and place them into three portfolios (Low, Medium, and High) separated by the 30th and 70th percentile breakpoints across distress and each valuation ratio.

Panel A shows that, consistent with our model's key predictions, low average returns are concentrated in the high distress/high M/B portfolio. Additionally across portfolios of increasingly distressed firms, greater valuation ratios imply lower (more negative) excess returns. This is because, holding distress probabilities fixed, high valuation ratios imply higher expected earnings growth. The subpanel on the right confirms this conjecture by showing that M/B portfolios correlate strongly with higher-thanaverage earnings growth, here defined as the two-year-ahead annual growth rate in earnings, $\log(X_{t+24}/X_t)$ for the median firm in each portfolio. These earnings and valuation patterns are consistent with rational pricing and also prior empirical evidence of mean reversion in earnings among distressed firms from Griffin and Lemmon (2002).

The market-to-book ratio is generally preferred by most empiricists, as both cash flows and operating earnings can become negative. For completeness, however, Panel B also reports the mean excess returns for portfolios formed using these alternative ratios. To avoid the impact of negative values in the denominator we replace each flow variable F (cash flow or operating earnings per share) with $\exp(F/A)$, where A is book assets.¹² The alternative ratios used in these double-sorts produce very similar results.

These results suggest that when a firm enters a distressed state, it does so only transiently. To confirm this conjecture we directly examine the expected duration of being in a particular distress portfolio. Specifically, in Table IV we report the transition probability matrix for all nine portfolios as well as their ex post propensity to delist. From this estimated matrix, we can compute the conditional probability that a firm will remain in its current state or improve it the following period, $\mathbb{P}(\text{State}(t+1) \leq \text{State}(t)|\text{State}(t))$. For the five riskiest portfolios, 4060 through 9500, these probabilities are respectively 72, 74, 47, 71, and 85 percent. Thus, firms in the distressed state generally tend to become healthy rather than further deteriorate, providing evidence that such state is temporary.

3.3.2 Beta Sensitivity

Another important component of the bias in returns is the sensitivity of a stock's beta to its earnings. Because empirical betas need to be estimated using a rolling window, we test for the magnitude of this effect by studying how betas evolve after portfolio formation. As equation (17) suggests, the bias in returns requires that betas change more among distressed firms than otherwise healthy firms.

Our method to estimate beta sensitivity is as follows. For each portfolio we run rolling factor regressions for every week using the previous 26 weeks of data for the year following portfolio formation.¹³ After this, we calculate the difference in average estimated weekly betas in the fourth quarter and that in the first quarter within every year and portfolio in our sample. Finally, we average these differences across years to get the estimated decline in a portfolio's factor exposure across our one year holding horizon.

Portfolios that have substantial sensitivity will exhibit large changes in their betas between the first and fourth quarters. In Figure 2 we depict these estimated averages

 $^{^{12}}$ This adjusts for size and keeps firms in the correct tail of the valuation distribution.

 $^{^{13}}$ Betas need to be estimated within a narrow window to accurately measure its current sensitivity, while still allowing for a reasonable number of observations to estimate the coefficient. We prefer weekly data to daily because of the illiquidity of distressed firms (Campbell, Hilscher, and Szilagyi (2008), Da and Gao (2010)).

for various betas in a general Carhart four-factor model. It is apparent from the figure that there is a decline in beta sensitivity over time. As a portfolio's riskiness rises, the magnitude of the change in betas grows. The sign of these changes for each factor is consistent with a general betterment of prospects for risky firms: over the course of the year, market betas fall, and firms grow bigger and become more likely winners.

3.3.3 Summary

To recap, equation (18) serves as a guide towards understanding the behavior of distressed stocks. It predicts the existence of a bias in measured returns that is induced by two forces: mean reversion and beta sensitivity. We find novel evidence of both of these forces in the data, providing a new perspective on the distress anomaly. In the next section, we calibrate and use our model to quantify the degree of bias that is present in the data. We then show that it can also replicate the key empirical findings documented in this section.

4 Quantitative Analysis

4.1 Constructing an Artificial Panel

We now use the empirical default probabilities estimated above to quantify the magnitude of the predicted theoretical biases in equity returns. To accomplish this, we use our theoretical model to construct 100 artificial panels of firms that resemble the one obtained from the CRSP/COMPUSTAT dataset. Specifically, we use the stochastic process governing firm-level cash flow dynamics X_t described in (5) to generate panels of 5,000 individual firms across a period of 480 months. We then rely on the theoretical results derived in Section 2 to compute the corresponding time series for stock returns, (12), and one-year probabilities of default at the monthly frequency. At any point in time, t, the default probability of firm i up to time horizon T can be computed from:

$$p_i(T, X_{it}) \equiv \int_t^T g_i(X_{is} = X_i^D, s | X_{it}, t) ds, \qquad (21)$$

where $g_i(X_{is} = X_i^D, s | X_{it}, t)$ is the probability density that the first hitting time is at time s given X_{it} , which depends on the hitting-time density of our Ornstein-Uhlenbeck process. This is constructed using the method proposed by Collin-Dufresne and Goldstein (2001) and described in Appendix A.¹⁴

Next, we sort our firms at the beginning of each calendar year based on their (estimated) default likelihood, and form nine delisting portfolios using the same methodology as in our empirical analysis. To keep the sample size constant we assume that each defaulting stock is replaced by a new one but only at the beginning of the next rebalancing period. All entering firms start with an initial value of cash flow $X_0 = 0$.

4.2 Model Calibration

To calibrate the model we need to assign values to 10 parameters. The values of μ , σ , and r capture collectively the market price of risk. We fix the average market return, μ , to 8 percent, the market volatility, σ , to 14 percent, and the risk-free rate, r, to 2.5 percent.

Our two institutional parameters are the effective tax rate on corporate income, τ , and the recovery rate upon default, δ . We set τ to 0.3, which is close to the US statutory corporate income tax rate. The recovery parameter, δ , is calibrated to target an equal-weighted average delisting return of -28%, which corresponds to the empirical moment tabulated over the period from 1971 until 2015.¹⁵

Given the central role of mean reversion in our model, we explicitly allow firms to be heterogeneous in their rates of mean reversion in earnings, κ_i . Specifically for our baseline calibration we assume that κ_i is uniformly distributed across firms with a mean value of $\bar{\kappa}_i = 0.08$, in line with our empirical estimates and Garlappi and Yan (2011). Later we also report results when $\bar{\kappa}_i$ is set to 0.06 and 0.10.¹⁶ We also allow firm-specific cash flow volatility σ_i to be heterogeneous and uniformly distributed

¹⁴While we rely on the model-implied default measure to sort our portfolio throughout our simulation results, we also show in the Online Appendix that such measure is highly correlated to the its empirical counterpart constructed based on the Campbell, Hilscher, and Szilagyi (2008) methodology, and that sorting firms based on the latter will generate qualitatively and quantitatively similar results.

¹⁵The model-based delisting return is defined as the annualized return observed over the month immediately preceding a firm default, consistent with the empirical definition.

¹⁶As we later show, our baseline calibration is conservative in the sense that it generates a smaller resolution of the distress anomaly implied by the estimation bias relative to the $\bar{\kappa}_i = 0.10$ case, all else equal. In our Online Appendix we also report results obtained for an alternative calibration allowing for a wider distribution of κ_i and show that they are overall consistent.

across firms with mean value $\bar{\sigma}_i = 0.30$, and use this parameter to target portfolio-level volatilities of weighted-average returns in excess of the risk-free rate.¹⁷

We target volatilities as our model has only one source of systematic risk that is driven by variation in the mean-variance frontier, dw. For both model and data, total portfolio return volatility is an appropriate target as it summarizes variation due to an arbitrary number of factors. In practice, multiple risk factors may price equity returns and the linear combinations of these factors approximate the true exposure to the meanvariance frontier. Recall that our bias term in (17) will continue to hold even in the presence of multiple factors. Thus, to better bridge our theoretical predictions with the data we follow the empirical literature and measure over- or under-performance with both alphas and mean excess returns.

To keep the main calibration exercise straightforward we abstract from other potential sources of ex-ante firm heterogeneity and assume the parameters governing the dynamics of the firm-specific cash flow process, \overline{X} and ρ , and the periodic coupon payment on debt C, to be identical across firms. The long-run mean of cash flows \overline{X} is a scaling variable arbitrarily set to 1. The other two parameters are chosen to match the estimated delisting probabilities of highly distressed stocks and to produce a cross-sectional average value of market leverage of 23 percent, consistent with the evidence in Halling, Yu, and Zechner (2016).¹⁸

Table V summarizes our parameter choices. Tables VI and VII show the annual default probabilities, return volatilities, and other targeted moments for both the model and the data.

4.3 Results

4.3.1 Model Implied Returns on Distress Portfolios

Table VIII contains our main result: it reports the mean excess returns over the market across delisting portfolios in our artificial dataset and compares them with the data.

¹⁷While heterogeneity in the rate of mean reversion and firm-specific cash flow volatility is unnecessary for most of our simulation results, it is however important in our double-sort tests as it guarantees that firms are reasonably well distributed across the two sorting dimensions.

¹⁸The model-equivalent market leverage at the firm level is defined as the ratio of debt over total firm value: $(C_i/r)/(C_i/r + E_i(X_i))$.

We can see that the excess returns across the various default portfolios implied by our baseline quantitative model can be sizable. Notably, our model predicts very substantial negative excess returns for the last four portfolios where delisting probabilities are also large.

The correct exercise here is to directly compare mean excess returns in both model and data as these are independent of the true factor structure of returns. An estimate of the price of distress risk in the model is -5.31 - 0.47 = -5.78 percent while its data counterpart is -6.68 - 0.92 = -7.60 percent. Thus, our implied theoretical bias is equal to 5.78/7.60 = 76 percent of the observed anomaly.¹⁹

However, as many studies have shown the presence of multiple sources of risk in the data, controlling for these factors in regressions may better uncover the true performance of these portfolios. For the Carhart and five-factor models, which otherwise perform very well empirically in explaining stock returns, our estimated bias accounts for 70 and 74 percent of the anomaly, respectively. If we use the three-factor Fama and French (1992) specification however, our model-implied bias accounts for only 39 percent of the spread between low and high distress portfolios estimated alpha.

We next examine the roles played by mean reversion and beta sensitivity in generating this result.

4.3.2 The Role of Mean Reversion

Figure 3 illustrates the theoretical relationship between bias and distress probability for alternative degrees of mean reversion in earnings growth. It shows that the magnitude of the bias is amplified for a given distress probability as the mean reversion parameter, κ , is increased. Intuitively, a larger drift, $\kappa(\bar{X} - X_i)$, lowers mean excess returns for highly distressed stocks. Betas fall more rapidly and thus exacerbate the estimation bias in returns.

Table IX shows the quantitative impact of using different values for average mean

¹⁹While these results rely on portfolio sorts based on the theoretical default probability, (21), we show in our Online Appendix that such measure is highly correlated to the model-equivalent Campbell, Hilscher, and Szilagyi (2008) specification, both at the firm and portfolio levels. Furthermore, we show that portfolio sorts based on either default measure generate similar results, both qualitatively and quantitatively.

reversion, $\bar{\kappa}_i$, on the model-implied mean excess returns across distress-sorted portfolios.²⁰ We see that when we raise $\bar{\kappa}_i$ to 0.1, the implied bias in returns for the most distressed portfolio increases by about 0.9% relative to our baseline estimate.

As before, we can also examine the link between expected future growth in earnings and the distress anomaly by looking at valuation ratios instead. Expectations of future earnings growth rates are closely tied to valuation ratios and, empirically, these are far easier to compute. Table X reports for our baseline calibration the implied return spreads for independent double sorts based on distress probability and either the drift, $\kappa_i(\bar{X} - X_i)$, or the price-to-cash flow ratio, $E_i(X_i)/\exp(X_i)$.²¹

The top two subpanels of Panel A corroborate our empirical findings: the distress anomaly is concentrated in stocks exhibiting both high distress risk and large rates of drift. Drift rates show up directly in the bias equation (18) and provide the cleanest evidence of our mechanism at play. But we can also look at patterns generated by valuation ratios as we show in the bottom two subpanels of Panel A. In general, these double sorts show that the anomaly is concentrated in the high distress and large valuation portfolios. Our price-to-cash flow measure is clearly less than perfect. However, when fixing distress and reducing drift rates, as in the top panels, we see that the bias is clearly concentrated among high distress and high valuation stocks.

4.3.3 The Role of Beta Sensitivity

The other main force in generating the earnings bias in (18) is the sensitivity of beta to changes in earnings, $\partial\beta/\partial X$. This sensitivity is an important part of our risk-based argument that differentiates it from most alternative explanations of the distress puzzle. As discussed earlier, our model implies that betas will be higher near default. High betas then interact with strong mean reversion to produce drastic changes in betas during the portfolio holding period.

To quantify the rate of change of betas over time we use our model to replicate

²⁰Higher mean reversion also moves the optimal default threshold, X_i^D , to the left, lowering default frequencies. To ensure these comparisons are appropriate we adjust idiosyncratic volatility, σ_i , so that distressed probabilities for portfolio 9500 are nearly unchanged.

²¹As before in the data, we use $\exp(X_i)$ to ensure that the denominator stays positive.

the empirical exercise in Section 3. Specifically, we run rolling regressions over the annual portfolio holding period in our artificial panels. Moreover, the model allows us to also change the estimation window for the rolling regressions to see how it might impact estimated betas. To do this we compare betas estimated at three different frequencies: instantaneously, β_{inst} , as in (13); and using either six or twelve months of (artificial) data, β_{6M} and β_{12M} . While we cannot observe instantaneous betas in the real applications, this exercise lets us gauge how wider rolling windows could affect our empirical estimates.

Table XI reports the results. Similar to the data, the most distressed portfolios display the most sensitive betas, as seen by the sharp declines over the one year holding period. Interestingly, the measured decline in betas becomes significantly smaller when we use wider rolling windows, and does so monotonically within portfolios. Intuitively, a wider window creates a broader average that dulls the precision of the estimated betas. This suggests that our earlier empirical estimates likely underestimated the true sensitivity of betas for the most distressed stocks.

5 Additional Implications

In this section we investigate two additional interesting implications of our model regarding the importance of portfolio rebalancing and the connection between momentum and distress stocks.

5.1 Adjusting Rebalancing Frequencies

Equation (18) shows that the theoretical bias also depends on the time horizon between observations, T. Intuitively, the earnings bias can only matter when there are sizable gaps between current and expected future cash flows. Formally, since the conditional expectation of cash flows, assuming no default between times t and T for firm i:

$$\mathbb{E}[X_{iT}|X_{it}] = \overline{X} + [X_{it} - \overline{X}]e^{-\kappa_i(T-t)},$$
(22)

is monotonically increasing in T, where $X_{it} < \overline{X}$ (the distressed stocks), the magnitude of the bias should increase with the rebalancing horizon.²²

Panel A of Table XII examines the effects of rebalancing on the size of the distress anomaly in the model. Specifically we report the mean excess returns for a variety of double sorted portfolios, using two rebalancing frequencies: our benchmark annual rebalancing and more frequent quarterly rebalancing. As we can see, the model implies that a more frequent rebalancing leads to the greater sampling of highly distressed firms that generates a larger estimated price of distress risk.

Two distinct, but opposite, forces drive this finding. First, there is a direct effect that works to accentuate the earnings bias as the holding period increases since the cash flows of initially distressed firms eventually converge back to their long-run mean. However, more frequent rebalancing also means more frequent resorting of highly distressed firms with very sensitive betas and high expected rates of mean reversion. This, in turn, raises the size of the bias estimated in the unconditional returns. Table XII shows that the effect of mechanical rebalancing dominates in our simulated model.

In Panel B, we report the effects of using alternative portfolio rebalancing frequencies on our empirical estimates of the mean excess returns on distressed stocks. Consistent with the model, more frequent rebalancing also exacerbates the empirical distress puzzle so that the bias appears more pronounced at the quarterly frequency.

5.2 Momentum and Distress

Momentum and distress are often linked in empirical work. However, our model implies that the source of bias in distressed stock returns is only partially connected to momentum. This is consistent with what we find in the data in table 8 where controlling for momentum in the Carhart factors does not explain away the distress anomaly. In this section we further separate distress from momentum.

We first compare the ability to predict ex-post defaults, p_{jt} using both distress and momentum. Columns 2 and 3 of Table XIII report the results of regressing ex-post delisting frequencies, p_{jt} , on the ex-ante average predicted probabilities, $\hat{p}_{j,t-1}$ across

²²Importantly, however, default probabilities must remain constant across the different time horizons.

each distress portfolios. We can see that, for the high distress portfolios, where ex-post default is concentrated, the fit is extremely accurate with estimated R-squareds close to 90 percent and estimated coefficients very close to 1 as we would expect.²³

The time series of p_{jt} for the four highest-risk distress portfolios are also shown in Figure 4 and all exhibit significant variation over time. Visually, the predicted probabilities, $\hat{p}_{j,t-1}$, track the realized series remarkably well, confirming again that our logit-based probability model captures well the realized delisting frequencies.

By contrast, column 5 of Table XIII shows that momentum, as commonly defined based on the (12-2) returns, is a very inferior predictor of default. As we can see, loser portfolios, in the lower tails of the momentum factor distribution, exhibit fairly modest R-squareds when compared with the distress portfolios. Moreover, the correlation coefficient of ex ante default probability with the (12-2) return across all firm-months is only -0.21.

To confirm this we next construct independently double-sorted portfolios based on distress and momentum both in the data and in our artificial panels. Table XIV tabulates the mean excess returns on these portfolios using the 30th and 70th percentile breakpoints.

We can see that, for both model and data, the distress anomaly is concentrated in the losers portfolio. Importantly, however, within a specific momentum portfolio, the high distress portfolio always performs worse than a low distress one. Still, whereas the model highlights that the distress anomaly survives within our simulation results even after controlling for momentum, in the data the distress anomaly concentrates only in losers.

We conclude that the two phenomena are only partially linked. We see the distress anomaly as unique and only mechanically correlated with momentum through the construction of the logistic regression for $\hat{p}_{j,t-1}$.²⁴

²³Although the quality of fit appears statistically poor for the first four portfolios there is virtually no variation in the dependent variable (defaults) here.

²⁴In particular the fact that our logistic regressions have EXRETAVG as a significant predictor of default.

6 Conclusion

This paper shows how time variation in expected returns and mean reversion in earnings induced by endogenous default can affect the inference about the behavior of delisting stocks. Financial distress naturally produces sharply nonlinear and countercyclical movements in betas that lead to biases in standard estimates of risk premia and alphas. These movements are greatly amplified when earnings growth is mean reverting so that distressed stocks are also those with high expected future earnings.

We show that these effects are sizable in a calibrated economy that replicates the key characteristics of distressed stocks: they have high betas, volatilities, and market-to-book ratios consistent with a rational forecast of mean-reversion in earnings. Our analysis suggests that the bias can explain between 39 and 74 percent of the distress anomaly, with a greater likelihood centered around 70 percent. In conclusion, our study cautions against the use of linear performance models when assets likely feature short-lived, nonlinear movements in expected returns.

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A Appendix: Derivations

Solution to Equity Value

The total value of the firm is the sum of three parts. The first part is the unlevered value of the firm, one that has neither debt nor operating costs. Its value is simply

$$\mathbb{E}^{\mathbb{Q}}\left[\int_{0}^{\infty} e^{-rt}(1-\tau)X_{it}dt\right] = (1-\tau)\left(\frac{\overline{X}_{i}}{r} + \frac{X_{i0} - \overline{X}_{i}}{r+\kappa_{i}}\right).$$
 (A1)

The second part is simply (minus) the present value of operating and fixed costs; namely $-(1-\tau)C_i/r$. Finally, the value of the default option $D(X_i)$ solves the ordinary differential equation

$$rD_i(X_i) = D'_i(X_i)\kappa_i(\overline{X}_i - X_i) + \frac{1}{2}\sigma_i^2 D''_i(X_i).$$
 (A2)

This is a Hermite differential equation whose solution takes the form

$$D_i(X_i) = A_i H\left(-\frac{r}{\kappa_i}, -\frac{\kappa(\overline{X}_i - X_i)}{\sqrt{\kappa\sigma_i^2}}\right) + B_{i1}F_1\left(\frac{r}{2\kappa_i}, \frac{1}{2}, \frac{\kappa_i(\overline{X}_i - X_i)^2}{\sigma_i^2}\right), \quad (A3)$$

where H(v, x) is the generalized Hermite function of order v and ${}_{1}F_{1}(a, b, x)$ is the Kummer confluent hypergeometric function of parameters a and b. Imposing the boundary $\lim_{X_{i}\to\infty} D_{i}(X_{i}) = 0$ allows us to set B_{i} to zero. Putting these three pieces together gives (9).

Default Probability

This follows the derivation in Collin-Dufresne and Goldstein (2001) but allows for a non-zero default boundary. Define $\gamma_i(X_{iT}, T|X_{it}, t)$ as the free (unabsorbed) transition density for a continuous Markov process and $g_i(X_{it} = X_i^D, t|X_{i0}, 0)$ as the density of the first passage time through a constant boundary X_i^D occurring at time t. A formula provided by Fortet (1943) allows us to implicitly define $g_i(\cdot)$ in terms of $\gamma_i(\cdot)$ as

$$\gamma_i(X_{iT}, T | X_{i0}, 0) = \int_0^T \gamma_i(X_{iT}, T | X_{it} = X_i^D, t) g_i(X_{it} = X_i^D, t | X_{i0}, 0) dt, \quad \text{for } X_{iT} < X_i^D < X_{i0},$$
(A4)

which can be interpreted as saying that X_{i0} must pass through X_i^D to eventually get to X_{iT} .

Since the dynamics of (5) are Gaussian we can construct the terms

$$M_i(T) \equiv \mathbb{E}[X_{iT}|X_{i0}] = X_{i0}e^{-\kappa_i T} + \overline{X}(1 - e^{-\kappa_i T}), \tag{A5}$$

$$L_{i}(T-t) \equiv \mathbb{E}[X_{iT}|X_{it} = X_{i}^{D}] = X_{i}^{D}e^{-\kappa_{i}(T-t)} + \overline{X}(1 - e^{-\kappa_{i}(T-t)}), \text{ and}$$
(A6)

$$S_i^2(T-t) \equiv \operatorname{var}_t(X_{iT}) = \frac{\sigma_i^2}{2\kappa_i} (1 - e^{-2\kappa_i(T-t)}).$$
(A7)

Integrating (A4) by $\int_{-\infty}^{X_i^D} dX_{it}$ gives

$$\mathcal{N}\left(\frac{X_{i}^{D} - M_{i}(T)}{S_{i}(T)}\right) = \int_{0}^{T} \mathcal{N}\left(\frac{X_{i}^{D} - L_{i}(T-t)}{S_{i}(T-t)}\right) g_{i}(X_{it} = X_{i}^{D}, t | X_{i0}, 0) dt, \quad (A8)$$

where $\mathcal{N}(\cdot)$ is the cumulative standard normal distribution.

To solve for the first passage density, we construct N equal time intervals such that $T = N\Delta t$ and approximate the integral by estimating values at the midpoints of these intervals. Defining $m_n = (X_i^D - M(n\Delta t))/S(n\Delta t)$, $l_n = (X_i^D - L(n\Delta t))/S(n\Delta t)$, and $g_{in} = g_i(X_{i(n-1/2)\Delta t} = X_i^D, (n-1/2)\Delta t | X_{i0}, 0)\Delta t$ we get the recursion

$$\mathcal{N}(m_1) = \mathcal{N}(l_{1/2})g_{i1}$$

$$\mathcal{N}(m_2) = \mathcal{N}(l_{3/2})g_{i1} + \mathcal{N}(l_{1/2})g_{i2}$$

$$\vdots$$

Continuing up to the N midpoints gives a system of N equations of the N unknowns $g_{in}, n = 1, ..., N$. The probability of default over the horizon T is then computed as

$$p_i(T, X_{i0}) = \sum_{n=1}^{N} g_{in}.$$
 (A9)

B Appendix: Delistings

We use the following performance-related delisting codes:²⁵

- 500 Issue stopped trading on exchange reason unavailable
- 550 Delisted by current exchange insufficient number of market makers
- 552 Delisted by current exchange price fell below acceptable level
- 560 Delisted by current exchange insufficient capital, surplus, and/or equity
- 561 Delisted by current exchange insufficient (or non-compliance with rules of) float or assets
- 574 Delisted by current exchange bankruptcy, declared insolvent
- 580 Delisted by current exchange delinquent in filing, non-payment of fees
- 584 Delisted by current exchange does not meet exchange's financial guidelines for continued listing

We remove all delisting returns greater than positive 100%. Less than 1% of the delisting returns, out of a total 5,652 delisting observations, are missing across the whole sample period.

²⁵Before 1987, all performance-related and stock-exchange-related delistings were coded 5. After 1987, CRSP started a more refined breakdown. The original code 5 delistings were initially given 500, and are considered to be mainly performance-related delistings (there is only a small number of exchange-related delistings). The 572 delisting code (liquidation at company request), is now discontinued and is replaced by the 400 delisting series. The average delisting returns on the 400 series is slightly positive, which may suggest that it does not really reflect negative company performance.

C Appendix: Firm Level Data and Variables

This appendix describes in detail how our the variables used in the analysis are constructed. All variables codes are for the COMPUSTAT annual file. We use all industrial, standard format, consolidated accounts of USA headquartered firms in COMPU-STAT. From the CRSP monthly and daily file we use all stocks in NYSE, AMEX, and NASDAQ. The S&P500 index comes from the annual MSI file and data on the Fama and French and momentum risk factors come from Ken French's website. We follow Campbell, Hilscher, and Szilagyi (2008) and align each company's fiscal year with that of the calendar year, and then lag the accounting data by two months. Our measure of book equity follows Davis, Fama, and French (2005).

Our variable definitions are as follows:

• Relative size

$$RSIZE_{it} = \log(SIZE_{it}/TOTVAL_t \times 1000)$$

where $TOTVAL_t$ is total dollar value of CRSP's value-weighted portfolio VWRETD and

$$SIZE_{it} = PRC_{it} \times SHROUT_{it}/1000$$

• Leverage

$$TLMTA_{it} = LT_{it}/(SIZE_{it} + LT_{it})$$

• Relative cash holdings

$$CASHMTA_{it} = CHE_{it}/(SIZE_{it} + LT_{it})$$

• Market to book ratio

$$MB_{it} = SIZE_{it} / ADJBE_{it}$$

• Adjusted book equity (observation set to one if negative)

$$ADJBE_{it} = BE_{it} + 0.1 * (SIZE_{it} - BE_{it})$$

• Stock price

$$PRICE_{it} = \log(\min\{PRC_{it}, 15\})$$

• Excess returns

$$EXRETAVG_{it} = (1 - \psi)/(1 - \psi^{12}) \times (EXRET_{it} + ... + \psi^{11}EXRET_{it-11})$$

where

$$EXRET_{it} = \log(1 + R_{it}) - \log(1 + VWRETD_t)$$

and *VWRETD* is CRSP's value-weighted total return. Because of the need for an uninterrupted series any missing variables are set equal to their cross-sectional means.

• Return on assets, or profitability

$$NIMTAAVG_{it} = (1 - \psi^3) / (1 - \psi^{12}) * (NIMTA_{it,t-2} + \psi^3 NIMTA_{it-3,t-5} + \psi^6 NIMTA_{it-6,t-8} + \psi^9 NIMTA_{it-9,t-11})$$

where we use $\psi = 2^{-1/3}$ and

$$NIMTA_{it} = NI_{it}/(SIZE_{it} + LT_{it})$$
$$NIMTA_{it-x,t-x-2} = (NIMTA_{it-x} + NIMTA_{it-x-1} + NIMTA_{it-x-2})/3$$

Because of the need for an uninterrupted series any missing variables are set equal to their cross-sectional means.

• Operating profitability

We also construct a variable, $OIMTAAVG_{it}$, of operating profitability. We do this by repeating the exercise above for profitability (NIMTAAVG) but use $EBITDA_{it}$ in place of NI_{it} for the construction of OIMTA, where EBITDA is Compustat's measure of earnings before before interest and depreciation. EBITDA is closer to our measure of X in the model.

• Return volatility

$$SIGMA_{it} = \sqrt{\frac{252}{N-1}\sum R_{it}^2}$$

where the summation is of daily returns over the past three months and missing SIGMA observations (when N < 5) are replaced with the cross-sectional mean.

Each one of these variables is also winsorized at the fifth and ninety-fifth percentiles each year and all observations with missing size, profitability, leverage, or excess return data are dropped. The Online Appendix reports a table of summary statistics for the variables used in our regressions.

Table I: Returns on Distress Portfolios

This table reports summary statistics for the portfolios constructed using the estimated delisting probabilities using the logistic regression (19). Mean excess returns, MER, are in excess of CRSP's value-weighted total returns, VWRETD. The data are monthly and cover the period from January 1971 until December 2015. Some denoted quantities are annualized. Distress probability \hat{p} , excess returns (MER) and standard deviation are expressed in percentage terms.

Portfolio	Annual \hat{p}	Annual MER	Standard Deviation	Skewness
0005	0.04	0.92	1.49	-0.04
0510	0.06	-0.05	1.26	0.06
1020	0.09	0.21	1.47	-0.24
2040	0.20	0.94	2.19	-0.46
4060	0.50	0.70	2.90	0.71
6080	1.48	-0.69	4.14	1.29
8090	3.97	-2.72	5.83	1.89
9095	7.33	-6.29	7.20	2.25
9500	14.05	-6.68	8.71	2.59

Table II: Properties of Distress Portfolios

This table reports summary statistics for the portfolios constructed using the estimated delisting probabilities using the logistic regression (19). Definitions of relative size, M/B, and profitability are in Appendix C. Annual κ is the estimate of the mean reversion parameter in (5), obtained by running regressions of each portfolio's operating profitability, on its lag, and then converting the monthly discrete-time autoregressive coefficient estimate ($\hat{\varphi}$) to an annual parameter in continuous-time with the formula $\hat{\kappa} = -\log(\hat{\varphi})/(1/12)$. These monthly data are from January 1971 until December 2015. Some denoted quantities are annualized. Profitability is expressed in percentage terms.

Portfolio	Relative Size	Market- to-Book	Annual κ	Annual Profitability
$ \begin{array}{r} 0005\\0510\\1020\\2040\\4060\\6020 \end{array} $	-7.30 -7.41 -7.73 -8.67 -9.69	$2.36 \\ 2.45 \\ 2.46 \\ 2.33 \\ 2.40 \\ 2.50$	$\begin{array}{c} 0.036\\ 0.068\\ 0.117\\ 0.090\\ 0.112\\ 0.124\end{array}$	$ \begin{array}{c} 1.16\\ 0.87\\ 0.69\\ 0.59\\ 0.29\\ 0.29\\ 0.22 \end{array} $
6080 8090 9095 9500	-10.50 -11.21 -11.63 -11.87	$2.59 \\ 2.93 \\ 3.26 \\ 3.80$	$\begin{array}{c} 0.124 \\ 0.170 \\ 0.132 \\ 0.215 \end{array}$	-0.23 -1.01 -1.64 -2.30

Table III: Distress and Expected Earnings Growth: Data

This table reports empirical time series averages of independently double-sorted portfolios. Portfolios are formed on distress probabilities and other stock characteristics: market to book (M/B), price-to-cash flow (P/CF), price to operating earnings (P/EBITDA). Both P/CF and P/EBITDA are constructed as $P/\exp(X/A)$ where P is the price per share, X is the relevant flow per share, and A is book assets. Cash flow is net income plus depreciation and operating earnings are EBITDA. Low, Medium, and High are separated using the 30th and 70th percentile breakpoints across each characteristic. Mean Excess Return are annualized value-weighted monthly returns over CRSP's value-weighted total return. Forward Earnings Growth is the 24-month forward earnings growth rate, $\log(X_{t+24}/X_t)$, of the median firm in the portfolio. Portfolios are annually rebalanced. All statistics are in percent.

	PANEL A: M/B, Returns and Earnings Growth					
	M/B			M/B		
	L	М	Н	L	М	Н
Distress	Mean Excess Return			Forward	l Earnings	Growth
L	2.36	1.58	-0.43	-3.29	-1.16	1.42
Μ	3.08	2.06	-0.96	-4.72	-5.01	1.63
Н	1.70	-1.46	-6.54	-7.70	-2.55	3.27

PANEL B: Other Valuation Ratios

	P/CF			1	P/EBITDA		
	L	М	Н	L	М	Н	
Distress	Mean Excess Return			Mean Excess Return			
L	7.39	5.02	0.28	0.50	5.30	0.28	
М	7.53	3.20	-0.97	3.97	3.21	-0.96	
Н	0.88	-3.50	-3.53	0.73	-3.55	-5.56	

annually rebalanced. All probabilities are in percent.											
						State	(t + 1)				
		0005	0510	1020	2040	4060	6080	8090	9095	9500	Delist
	0005	62.61	22.28	10.12	3.90	0.74	0.25	0.06	0.01	0.00	0.03
	0510	23.56	35.84	28.58	9.35	1.96	0.54	0.10	0.01	0.01	0.05
	1020	5.63	16.07	43.58	28.34	4.58	1.36	0.26	0.09	0.06	0.03
	2040	1.14	2.70	16.41	51.96	21.02	5.12	1.03	0.31	0.19	0.11
State (t)	4060	0.23	0.43	2.61	23.62	44.96	21.71	4.18	1.17	0.78	0.32
	6080	0.06	0.10	0.65	5.40	22.62	45.89	16.03	5.06	3.33	0.86
	8090	2.65	0.01	0.02	0.21	1.80	8.31	33.76	34.34	16.25	2.65
	9095	0.00	0.00	0.09	0.78	3.66	16.88	27.49	22.41	23.23	5.46
	9500	0.00	0.04	0.09	0.62	2.24	9.45	17.55	19.56	35.77	14.68

Table IV: Portfolio Transition Matrix: Data

This table reports estimates of portfolio transition probabilities. Actual (ex post) delistings are listed in the last column. Sample period runs from 1971 until 2015. Portfolios are

Table V: Calibration

This table reports the parameter choices for our model. These choices are described in detail in Section 4.2. The model is simulated at a monthly frequency and the parameters below are annualized.

Parameter	Value	Description
		Market
μ	0.08	Market return
σ	0.14	Market volatility
r	0.025	Risk-free rate
	In	stitutions
au	0.3	Tax rate
δ	0.015	Recovery rate
		Firms
\overline{X}	1	Level of long-run cash flows
κ_i	U([0.04, 0.12])	Rate of mean reversion
σ_i	U([0.2, 0.4])	Firm cash flow volatility
ho	0.7	Correlation with aggregate shock
C	0.05	Dollar coupon

Table VI: Actual and Simulated Default Frequencies and Volatility Targets

This table reports equal-weighted averages of annual ex-ante default probabilities at the portfolio level for both actual and simulated data from our calibrated model described in Section 4.2. It also reports annual average volatilities of portfolio excess returns (relative to the risk-free rate) for both actual and simulated data. Portfolios in the data are constructed using the estimated probabilities from the logistic regression in (19). The sample period runs monthly from 1971 until 2015. Each portfolio in the model is ranked according to the default probability given in (21). Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. Portfolios in both model and data are rebalanced annually.

Portfolio	p_j^{data}	p_j^{model}	σ_j^{data}	σ_j^{model}
0005	0.04	0.00	5.12	9.94
0510	0.06	0.00	4.32	7.85
1020	0.09	0.00	5.07	5.67
2040	0.20	0.01	7.46	2.42
4060	0.50	0.14	10.09	3.35
6080	1.48	0.96	14.39	9.71
8090	3.97	3.42	20.25	18.05
9095	7.33	7.29	25.07	25.08
9500	14.05	14.05	30.31	30.84

Table VII: Other Targeted Moments

This table reports targeted moments from the model calibration, described in Section 4.2. Delisting returns are tabulated as equal-weighted averages. In the data, we simply use the CRSP delisting returns when available and the lagged monthly returns otherwise. Modelbased delisting returns are defined as the annualized returns observed over the month immediately preceding a firm default. Similarly, average market leverage is computed on an equal-weighted basis. The targeted moment is taken from Table 1 in Halling, Yu, and Zechner (2016) (US-only sample statistics); in the model, it is defined at the firm level as the ratio of debt over total firm value: $\frac{C}{r} / (\frac{C}{r} + E_i(X_i))$. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months.

Moment	Data	Model
Average Market Leverage	0.23	0.24
Average Delisting Return	-0.28	-0.20

Table VIII: Excess Returns Across Distressed Portfolios

This table reports portfolio's mean excess return (MER) over the market for the model as well as measures in the data. The data are tabulated over five specifications: the simple MER as well as alphas from four empirical models; CAPM, three-factor Fama and French (1992) model, four-factor Carhart (1997) specification, and the five-factor Fama and French (2015) regression. In the data, each portfolio is constructed using the estimated distress probabilities from the logistic regression (19). Sample period runs monthly from January 1971 until December 2015. Standard errors are OLS. In the model, portfolios are constructed using the probability of default given in (21) using the parameters summarized in Table V. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. Portfolios in both data and model are rebalanced annually.

				Data		
	Model		CAPM	3-factor	Carhart	5-factor
Portfolio	MER	MER	Alpha	Alpha	Alpha	Alpha
0005	0.47	0.92	1.69^{**}	2.33***	0.49	1.69^{**}
0510	0.58	-0.05	0.31	0.48	0.92	-0.40
1020	0.72	0.21	-0.31	-0.77	0.65	-0.49
2040	0.72	0.94	-0.18	-0.87	0.49	0.65
4060	0.08	0.70	-0.84	-2.35***	-0.45	-0.88
6080	-1.45	-0.69	-2.99	-4.82***	-1.81*	-2.96**
8090	-3.52	-2.72	-5.87**	-7.77***	-4.05**	-4.04**
9095	-4.35	-6.29*	-9.61***	-11.38***	-6.33**	-6.19**
9500	-5.31	-6.68	-10.26**	-12.39***	-7.36**	-6.57**

Table IA. The impact of mean fieversion in Darnings

This table reports time series averages of annually rebalanced, distressed-sorted portfolios in the model for cash flow mean reversion rates κ_i that are uniformly distributed with a mean value $\bar{\kappa}_i = \{0.06, 0.08, 0.10\}$. Distress probabilities p_j are computed on an equalweighted basis and reported in percent. Portfolios are constructed using the probability of default given in (21). The column MER tabulates raw annualized value-weighted mean excess returns (MER) over the market portfolio returns expressed in percent. Mean idiosyncratic volatility, $\bar{\sigma}_i$, is recalibrated to ensure probabilities for the most distressed portfolio remain (approximately) constant. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months.

		Mean Reversion						
	$\bar{\kappa_i}$	= 0.06	$ar{\kappa_i}$	= 0.08	$ar{\kappa_i}$	= 0.10		
Portfolio	p_j	MER	p_j	MER	p_j	MER		
0005	0.00	-0.43	0.00	0.47	0.00	1.19		
0510	0.00	-0.23	0.00	0.58	0.00	1.18		
1020	0.00	-0.06	0.00	0.72	0.00	1.20		
2040	0.00	0.44	0.01	0.72	0.04	0.89		
4060	0.03	0.47	0.14	0.08	0.28	-0.07		
6080	0.51	-0.48	0.96	-1.45	1.33	-1.99		
8090	2.60	-2.32	3.42	-3.52	4.02	-4.24		
9095	6.32	-3.00	7.29	-4.35	7.93	-5.37		
9500	14.20	-4.26	14.05	-5.31	14.07	-6.18		

Table X: Distress and Expected Earnings Growth: Model

Panel A in this table reports time series averages of mean excess returns (MER) for annually rebalanced, independently double-sorted portfolios in the model. The portfolios are formed based on distress probability and drift, defined as $\kappa(\bar{X} - X)$, and the price-to-cash flow ratio (P/CF), $E/\exp(X)$. MERs are annualized value-weighted monthly returns over the market and expressed in percent. Ex-ante distress probabilities and cash flow drifts are computed on an equal-weighted basis. All results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table V. Low, Medium, and High are separated using the 30th and 70th percentile breakpoints across each characteristic.

		Earnings Drift	
Distress	L	M	Н
L	0.67	0.63	0.72
М	0.44	0.03	-0.58
Η	-1.42	-2.35	-3.86
		P/CF	
Distress	L	Μ	Н
L	0.77	0.65	0.63
Μ	-0.16	0.09	-0.04
Η	-3.68	-2.89	-3.14

Table XI: Sensitivity of Beta by Portfolio

This table reports the estimated differential in market betas of portfolios that are sorted by ex-ante default probabilities. Portfolio betas are tabulated as value-weighted averages of firm-level betas constructed as $\beta_{inst} = \frac{\rho \sigma_i}{\sigma} \frac{E'_i}{E}$, as given in (13), or $\beta = \frac{\text{COV}(R_i^e, R_m^e)}{\text{Var}(R_m^e)}$, for 6and 12-month rolling windows, and with R^e and R_m^e in excess of the risk-free rate. Beta differentials are reported as the average difference between fourth quarter and first quarter betas across all observations within a quarter. Returns are annualized and in percent. In the model, portfolios are constructed using the probability of default given in (21). Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. Portfolios are rebalanced annually and all parameter values used to solve the model are shown in Table V.

			I	$\mathbb{E}[\beta_{4\mathrm{th~qtr}}]$	$-\beta_{1 \mathrm{st qt}}$	r			
	0005	0510	1020	2040	4060	6080	8090	9095	9500
β_{inst}	0.00	0.00	0.00	0.01	-0.00	-0.03	-0.11	-0.23	-0.35
β_{6M}	0.01	0.01	0.02	0.02	0.02	-0.00	-0.06	-0.15	-0.07
β_{12M}	0.01	0.01	0.01	0.01	0.01	0.00	-0.02	-0.06	-0.06

Table XII: The Impact of Rebalancing

Panel A in this table reports time series averages of mean excess returns (MER) for quarterly and annually rebalanced, independently double-sorted portfolios in the model. The portfolios are formed based on distress probability and drift, defined as $\kappa(\bar{X}-X)$, and the price-to-cash flow ratio (P/CF), $E/\exp(X)$. MERs are annualized value-weighted monthly returns over the market and expressed in percent. Ex-ante distress probabilities and cash flow drifts are computed on an equal-weighted basis. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table V. Panel B reports empirical time series averages for independently double-sorted portfolios. These portfolios are formed on distress probabilities and market to book (M/B), defined in Appendix C. MERs are annualized valueweighted monthly returns over CRSP's value-weighted total return. For both model and data, Low, Medium, and High are separated using the 30th and 70th percentile breakpoints across each characteristic.

	PANEL A: Model									
	Quarter	y Rebalan	ced Drift	Annuall	Annually Rebalanced Drift					
	L	М	Н	L	М	Н				
Distress	Mean	n Excess R	eturn	Mear	n Excess R	eturn				
L	0.67	0.68	0.66	0.67	0.63	0.72				
Μ	0.47	0.01	-0.37	0.44	0.03	-0.58				
Η	-1.81	-2.54	-4.08	-1.42	-2.35	-3.86				
	Quarterl	y Rebalanc	ced P/CF	Annually Rebalanced P/CF						
	L	М	Н	L	М	Н				
Distress	Mean	n Excess R	eturn	Mear	n Excess R	eturn				
L	0.77	0.69	0.61	0.77	0.65	0.63				
Μ	-0.23	0.13	0.08	-0.16	0.09	-0.04				
Н	-4.13	-3.11	-3.82	-3.68	-2.89	-3.14				

PANEL B: Data

	Quarterl	y Rebalano	ced M/B	Biennially Rebalanced M/			
	L M H			L	М	Н	
Distress	Mear	n Excess Re	eturn	Mean Excess Return			
L	2.31	1.38	-0.16	1.58	1.75	-0.51	
Μ	1.03	0.85	-1.24	3.93	2.78	-0.91	
Н	-3.85	-5.24	-10.6	4.05	2.04	-5.21	

Table XIII: Actual and Estimated Delisting Frequencies

This table reports R^2 and slope coefficients associated with regressing ex-post observed delisting frequencies on the average estimated probabilities for nine portfolios, indexed by subscript j:

$$p_{jt} = b_j \hat{p}_{j,t-1} + \epsilon_{jt}$$

Each distress portfolio is constructed using the estimated default probabilities using the logistic regression (19). Each momentum portfolio is constructed by using the cumulative return realized over the previous year excluding the most recent month. The predicted probability estimate over the entire following calendar year is paired with the year's corresponding realized delisting frequency. These annual data are from 1970 until 2015.

	Dist	ress	Mom	entum
Portfolio	\hat{b}_j	R^2	\hat{b}_j	R^2
0005	0.68	0.07	1.09	0.59
0510	0.45	0.00	0.70	0.56
1020	0.37	0.07	0.61	0.47
2040	0.80	0.49	0.55	0.63
4060	0.99	0.67	0.60	0.70
6080	0.91	0.79	0.45	0.62
8090	0.96	0.89	0.31	0.29
9095	1.00	0.88	0.41	0.36
9500	0.98	0.87	0.45	0.46

Table XIV: Distress and Momentum

This table reports time series averages of annually rebalanced, independently double-sorted portfolios in the model. The portfolios are formed on distress probability and momentum, defined as the cumulative return realized over the previous year excluding the most recent month. Low, Medium, and High for distress are separated using the 30th and 70th percentile breakpoints, as are Losers, Medium, and Winners are for momentum. Mean Excess Returns are annualized value-weighted monthly returns over the market and expressed in percent. Ex-ante distress probabilities are computed on an equal-weighted basis. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table V.

PANEL A: Data									
	Momentum								
	L	L M W							
Distress	Me	ean Excess Ret	urn						
L	-0.21	0.49	2.15						
Μ	-2.89	2.94	2.57						
Н	-5.79	0.04	1.71						

PANEL B: Model

	Momentum							
	L	W						
Distress		Mean Excess Return						
L	0.33	0.61	0.86					
М	-0.16	0.10	0.21					
Н	-3.43	-2.92	-2.70					



This figure depicts the key properties of our theoretical model, as a function of the current level of a firm's cash flows, X. The top two panels show the physical default probability of the firm, $p_i(T, X_{i0})$, as well as the firm's beta (minus the market), $\beta_{it} - 1$. The middle-left panel plots the earnings bias as defined in (18). Earnings drift, plotted in the middleright panel, is the time t conditional expectation of the earnings process specified in (5): $\mathbb{E}_t[dX] = \kappa(\bar{X} - X)$. The firm's relative size is calculated as the log ratio of the firm's size to average size: $RSIZE = \log(E(X)/E(\bar{X}))$. Idiosyncratic volatility is defined in (14) and is the portion of the firm's return volatility unrelated to systematic risk ($\beta\sigma$) and is shown at an annual rate. The rate of mean reversion κ is set to 0.1. All remaining parameter values are shown in Table V.





This figure displays the average change of a portfolio's factor beta across the fourth and first quarters following portfolio formation. For the following year after portfolio formation, we run weekly rolling factor regressions for each portfolio using the previous 26 weeks of data. We then calculate the difference between all weeks in the fourth quarter's estimate and in the first quarter's estimate within every year, portfolio, and factor in our sample. Finally, we average these differences across years to get the estimated decline in a portfolio's factor exposure across our one year holding horizon. This factor model is the Carhart four-factor model of market (MKT), size (SMB), value (HML), and momentum (WML). The sample period runs January 1971 until December 2015.





This figure shows the relationship between bias and distress for three mean reversion parameters: (i) $\kappa = 0.06$, (ii) $\kappa = 0.08$, (iii) $\kappa = 0.10$. Both distress probability and bias are expressed in percent. The remaining parameter values are shown in Table V.



Figure 4: Realized Default Frequencies and Estimated Probabilities from the Logistic Model

This figure shows the ex post delisting frequencies, p_{jt} , and estimated delisting probabilities, \hat{p}_{jt} , for the four highrisk portfolios: 6080, 8090, 9095, 9500. The estimated delisting probabilities are formed by using the logistic model in (19) and the details of the estimation are described in Section 3.1. The ex-post delisting frequencies are based on performance-based delistings, described in Appendix B, and are calculated over the annual period from 1970 until 2015.

D Internet Appendix: Additional TablesD.1 Model-Implied and CHS Default Probabilities

In this section, we further deepen the connection between the model and data by comparing numerically our model-implied default probability to the one implemented empirically based on Campbell, Hilscher, and Szilagyi (2008) and coefficients reported in Table A-I. The summary statistics of the variables used in the logistic model are in Table A-II. In the context of our model, we omit PRICE and CASHMTA from the logistic specification as these are not identified and define the remaining variables as follows:

• Relative size

$$RSIZE_{it} = \log\left(E_{it} / \left(\sum_{i} E_{it}\right)\right)$$

• Leverage

$$TLMTA_{it} = \frac{c/r}{c/r + E_{it}}$$

• Market to book ratio

$$MB_{it} = \frac{E_{it}}{ADJBE_{it}},$$

where $ADJBE_{it} = 0.1E_{it} + \max(0.9BE_{it}, 0)$ and $BE_{it} = (1 - \tau)(\overline{X}/r + \frac{X_{it} - \overline{X}}{r + \kappa_i} - c/r)$

• Excess returns

$$EXRET_{it} = r_{it} - r_{mt}$$

• Profitability

$$NIMTA_{it} = \frac{(1-\tau)(X_{it}-c)}{c/r + E_{it}}$$

• Return volatility, SIGMA, is based on 12 monthly log return observations, instead of daily raw returns

$$SIGMA_{it} = \sqrt{\sum_{j=0}^{11} r_{i,t-j}^2}$$

Eventually, NIMTAAVG and EXRETAVG are tabulated based on coefficient $\psi = 2^{-1/3}$ and each one of these variables is similarly winsorized, as in the data.

Table A-III gives the comparison. Overall, the theory-based measure of default probability and that obtained from the Campbell, Hilscher, and Szilagyi (2008) specification are consistent and highly correlated, both at the firm and portfolio levels. Indeed, the average correlation at the firm-month level is 66 percent and that across the most distressed portfolios ranges between 87 and 97 percent based on annual frequency.

In Table A-IV we further show that there is not much difference in using either default measure for the implications regarding portfolio return performance.

Finally, in Tables A-V and A-VI we show the impact of changing market correlation on our model's ability to generate the distress anomaly and the model's estimated transition matrix on earnings that mimics what we see in the data.

Table A-I: Logistic Regression Estimates

This table reports the estimated coefficients from the logistic regression (19)

$$y_{it} = \gamma_0 + \gamma_{EXRETAVG} EXRETAVG_{it} + \gamma_{SIGMA} SIGMA_{it} + \gamma_{PRICE} PRICE_{it} + \gamma_{NIMTAAVG} NIMTAAVG_{it} + \gamma_{TLMTA} TLMTA_{it} + \gamma_{CASHMTA} CASHMTA_{it} + \gamma_{RSIZE} RSIZE_{it} + \gamma_{MB} MB_{it}$$

over the annual period from 1970 until 2015. We describe the estimation procedure in Section 3.1 and the variable definitions are in Appendix C.

Coefficient	Estimate
CONSTANT	-9.802
	$(0.230)^{***}$
EXRETAVG	-7.767
CTCD (A	$(0.283)^{***}$
SIGMA	0.402
MD	$(0.041)^{++++}$
MID	(0.008)***
NIMTA AVG	-12.1
	$(0.531)^{***}$
TLMTA	1.309
	$(0.059)^{***}$
CASHMTA	-1.429
	$(0.139)^{***}$
RSIZE	-0.433
DDIGE	$(0.017)^{***}$
PRICE	-0.664
Observations	$(0.021)^{4000}$
Dolisting	219,802
Pseudo- R^2	0.40
*** $n < 0.01$ ** n	$\frac{0.10}{-0.05 * n - 0.1}$
p < 0.01, p	< 0.00, p < 0.1

Table A-II: Summary Statistics

This table reports summary statistics for the core variables used in the logistic regressions. The data are at monthly frequency over the period 1950 to 2015. The variable definitions are listed in Appendix C.

Variable	Mean	Median	Std. Dev.	Minimum	Maximum
NIMTA	0.001	0.005	0.025	-0.214	0.044
TLMTA	0.434	0.401	0.276	0.014	0.969
EXRET	-0.010	-0.008	0.116	-0.458	0.346
RSIZE	-10.693	-10.896	1.952	-14.749	-4.918
SIGMA	0.525	0.441	0.339	0.105	1.963
CASHMTA	0.087	0.048	0.102	0.001	0.681
MB	2.062	1.552	1.666	0.237	102.69
PRICE	2.050	2.507	0.907	-1.407	2.708

Firm-month observations = 2,832,518

Table A-III: Relationship between Theoretical and CHS Default Measures: Model

This table reports time series averages of annually rebalanced portfolios in the model. The portfolios are formed based on the theoretical default measure, given in (21). The reported default probabilities are constructed based on (i) data, (ii) the theoretical model, and (iii) Campbell, Hilscher, and Szilagyi (2008) specification, and are all computed on an equal-weighted basis. The CHS default measure is constructed based on the regression coefficients in Table A-I and section D.1. The table also reports the correlations between model and CHS default probabilities at the portfolio level and in percentage terms. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table V.

Portfolio	p_j^{data}	$p_j^{model-Th}$	$p_j^{model-CHS}$	$corr(p_j^{model-Th}, p_j^{model-CHS})$
0005	0.04	0.00	0.16	1.12
0510	0.06	0.00	0.17	20.21
1020	0.09	0.00	0.18	52.61
2040	0.20	0.01	0.28	75.19
4060	0.50	0.14	0.76	86.70
6080	1.48	0.96	2.29	93.52
8090	3.97	3.42	5.21	96.44
9095	7.33	7.29	7.86	96.93
9500	14.05	14.05	8.39	95.38

Table A-IV: Portfolio Sort Results Based on Theoretical and CHS Default Measures: Model

This table reports time series averages of annually rebalanced portfolios in the model. The portfolios are formed based on (i) the theoretical default measure, and (ii) the Campbell, Hilscher, and Szilagyi (2008) default measure constructed based on the regression coefficients in Table A-I and section D.1. Mean Excess Returns are annualized value-weighted monthly returns over the market and expressed in percent. Ex-ante distress probabilities are computed on an equal-weighted basis. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table V.

Portfolio	$p_j^{model-Theory}$	MER	$p_j^{model-CHS}$	MER
0005	0.00	0.47	0.13	0.67
0510	0.00	0.58	0.15	0.84
1020	0.00	0.72	0.18	0.72
2040	0.01	0.72	0.29	0.57
4060	0.14	0.08	0.78	0.01
6080	0.96	-1.45	2.28	-1.31
8090	3.42	-3.52	5.10	-3.31
9095	7.29	-4.35	7.54	-4.65
9500	14.05	-5.31	8.92	-4.85

Table A-V: The Impact of Market Correlation

This table reports time series averages of annually rebalanced, distressed-sorted portfolios in the model for correlation levels $\rho = \{0.6, 0.7, 0.8\}$. Distress probabilities p_j are computed on an equal-weighted basis and reported in percent. Portfolios are constructed using the probability of default given in (21). The column MER tabulates raw annualized value-weighted mean excess returns (MER) over the market portfolio returns expressed in percent. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table V.

		Mean Reversion									
		$\rho = 0.6$	I	o = 0.7		$\rho = 0.8$					
Portfolio	p_j	MER	p_j	MER	p_j	MER					
$\begin{array}{c} 0005\\ 0510\\ 1020\\ 2040\\ 4060\\ 0000\\ \end{array}$	0.00 0.00 0.00 0.00 0.02 0.22	0.64 0.72 0.80 0.81 0.32	$0.00 \\ 0.00 \\ 0.00 \\ 0.01 \\ 0.14 \\ 0.00$	$\begin{array}{c} 0.47 \\ 0.58 \\ 0.72 \\ 0.72 \\ 0.08 \\ 1.48 \end{array}$	$0.00 \\ 0.00 \\ 0.01 \\ 0.09 \\ 0.54 \\ 0.01$	$\begin{array}{c} 0.14 \\ 0.39 \\ 0.55 \\ 0.50 \\ -0.11 \\ 1.05 \end{array}$					
6080 8090 9095 9500	$\begin{array}{c} 0.32 \\ 1.88 \\ 5.30 \\ 13.78 \end{array}$	-1.22 -4.38 -7.10 -9.09	$\begin{array}{c} 0.96 \\ 3.42 \\ 7.29 \\ 14.05 \end{array}$	-1.48 -3.52 -4.35 -5.31	$2.01 \\ 4.91 \\ 8.28 \\ 15.01$	-1.05 -1.61 -1.34 -2.28					

Table A-VI: Portfolio Transition Matrix: Model

This table reports estimates of portfolio transition probabilities obtained from the model simulation. Ex post delistings are listed in the last column. Portfolios are annually rebalanced. All probabilities are in percent. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table V.

			State $(t+1)$								
		0005	0510	1020	2040	4060	6080	8090	9095	9500	Delist
	0005	88.73	11.20	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0510	11.20	67.36	21.28	0.15	0.00	0.00	0.00	0.00	0.00	0.00
	1020	0.03	10.65	67.02	22.14	0.16	0.00	0.00	0.00	0.00	0.00
	2040	0.00	0.03	11.07	68.68	19.26	0.94	0.01	0.00	0.00	0.00
State (t)	4060	0.00	0.00	0.07	19.25	57.77	21.95	0.83	0.04	0.00	0.08
	6080	0.00	0.00	0.00	0.83	21.63	58.29	16.29	2.04	0.24	0.69
	8090	0.00	0.00	0.01	0.12	1.63	31.23	43.18	16.82	4.23	2.78
	9095	0.00	0.00	0.01	0.19	0.68	7.54	31.90	33.07	20.34	6.28
	9500	0.00	0.00	0.00	0.12	1.09	4.49	10.53	20.49	50.96	12.31

D.2 An Alternative Calibration

We present below an alternative model calibration that allows for a wider range of the mean reversion parameter κ_i , namely [0.04 - 0.16], to further match our empirical mean reversion rate estimates and check the robustness of our results. We show below the results for single and double sorts, which are overall in line with those obtained for the benchmark calibration reported in the core paper.

Table A-VII: Alternative Model Calibration

This table reports an alternative calibration for our model. The model is simulated at a monthly frequency and the parameters below are annualized.

Parameter	Value	Description
		Market
μ	0.08	Market return
σ	0.15	Market volatility
r	0.025	Risk-free rate
	In	stitutions
au	0.3	Tax rate
δ	0.015	Recovery rate
		Firms
\overline{X}	1	Level of long-run cash flows
κ_i	U([0.04, 0.16])	Rate of mean reversion
σ_i	U([0.3, 0.4])	Firm cash flow volatility
ho	0.8	Correlation with aggregate shock
C	0.045	Dollar coupon

Table A-VIII: Actual and Simulated Default Frequencies and Volatility Targets

This table reports equal-weighted averages of annual ex-ante default probabilities at the portfolio level for both actual and simulated data from our calibrated model (based on our alternative calibration in Table A-VII). It also reports annual average volatilities of portfolio excess returns (relative to the risk-free rate) for both actual and simulated data. Portfolios in the data are constructed using the estimated probabilities from the logistic regression in (19). The sample period runs monthly from 1971 until 2015. Each portfolio in the model is ranked according to the default probability given in (21). Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. Portfolios in both model and data are rebalanced annually.

Portfolio	p_j^{data}	p_j^{model}	σ_j^{data}	σ_j^{model}
0005	0.04	0.00	5.12	11.09
0510	0.06	0.00	4.32	8.94
1020	0.09	0.00	5.07	6.60
2040	0.20	0.06	7.46	3.01
4060	0.50	0.41	10.09	4.15
6080	1.48	1.56	14.39	11.31
8090	3.97	4.09	20.25	19.81
9095	7.33	7.32	25.07	27.26
9500	14.05	13.99	30.31	34.34

Table A-IX: Other Targeted Moments

This table reports targeted moments from the model calibration (based on our alternative calibration in Table A-VII). Delisting returns are tabulated as equal-weighted averages. In the data, we simply use the CRSP delisting returns when available and the lagged monthly returns otherwise. Model-based delisting returns are defined as the annualized returns observed over the month immediately preceding a firm default. Similarly, average market leverage is computed on an equal-weighted basis. The targeted moment is taken from Table 1 in Halling, Yu, and Zechner (2016) (US-only sample statistics); in the model, it is defined at the firm level as the ratio of debt over total firm value: $\frac{C}{r} / (\frac{C}{r} + E_i(X_i))$. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months.

Moment	Data	Model
Average Market Leverage	0.23	0.24
Average Delisting Return	-0.28	-0.20

Table A-A. Excess fielding Across Distressed for toriono	Table A-X: E	Excess Returns	Across Dis	stressed Portfol	ios
----------------------------------------------------------	--------------	-----------------------	------------	------------------	-----

This table reports portfolio's mean excess return (MER) over the market for the model as well as measures in the data. The data are tabulated over five specifications: the simple MER as well as alphas from four empirical models; CAPM, three-factor Fama and French (1992) model, four-factor Carhart (1997) specification, and the five-factor Fama and French (2015) regression. In the data, each portfolio is constructed using the estimated distress probabilities from the logistic regression (19). Sample period runs monthly from January 1971 until December 2015. Standard errors are OLS. In the model, portfolios are constructed using the probability of default given in (21) using the parameters summarized in Table A-VII. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. Portfolios in both data and model are rebalanced annually.

				Data		
	Model		CAPM	3-factor	Carhart	5-factor
Portfolio	MER	MER	Alpha	Alpha	Alpha	Alpha
0005	0.65	0.92	1.69**	2.33***	0.49	1.69**
0510	0.68	-0.05	0.31	0.48	0.92	-0.40
1020	0.83	0.21	-0.31	-0.77	0.65	-0.49
2040	0.51	0.94	-0.18	-0.87	0.49	0.65
4060	-0.09	0.70	-0.84	-2.35***	-0.45	-0.88
6080	-1.21	-0.69	-2.99	-4.82***	-1.81*	-2.96**
8090	-2.23	-2.72	-5.87**	-7.77***	-4.05**	-4.04**
9095	-2.76	-6.29*	-9.61***	-11.38***	-6.33**	-6.19**
9500	-4.57	-6.68	-10.26**	-12.39***	-7.36**	-6.57**

Table A-XI: Distress and Expected Earnings Growth: Model

Panel A in this table reports time series averages of mean excess returns (MER) for annually rebalanced, independently double-sorted portfolios in the model. The portfolios are formed based on distress probability and drift, defined as $\kappa(\bar{X} - X)$, and the price-to-cash flow ratio (P/CF), $E/\exp(X)$. MERs are annualized value-weighted monthly returns over the market and expressed in percent. Ex-ante distress probabilities and cash flow drifts are computed on an equal-weighted basis. All results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table A-VII. Low, Medium, and High are separated using the 30th and 70th percentile breakpoints across each characteristic.

		Earnings Drift	
Distress	L	M	Н
L	0.84	0.65	0.51
М	0.26	-0.14	-0.61
Η	-0.47	-1.73	-3.30
		P/CF	
Distress	L	Μ	Η
L	0.98	0.74	0.58
Μ	0.09	-0.14	-0.33
Η	-2.80	-2.55	-3.61

Table A-XII: Distress and Momentum

This table reports time series averages of annually rebalanced, independently double-sorted portfolios in the model. The portfolios are formed on distress probability and momentum, defined as the cumulative return realized over the previous year excluding the most recent month. Low, Medium, and High for distress are separated using the 30th and 70th percentile breakpoints, as are Losers, Medium, and Winners are for momentum. Mean Excess Returns are annualized value-weighted monthly returns over the market and expressed in percent. Ex-ante distress probabilities are computed on an equal-weighted basis. Model results are tabulated based on moment averages across 100 simulations, each generating an artificial panel of 5,000 firms over a period of 480 months. All parameter values are shown in Table A-VII.

	PANEL A: Data				
	Momentum				
	L M W				
Distress	Me	ean Excess Ret	urn		
L	-0.21	0.49	2.15		
Μ	-2.89	2.94	2.57		
Н	-5.79	0.04	1.71		

PANEL B: Model

	Momentum			
	L	М	W	
Distress		Mean Excess Return		
L	0.53	0.62	0.86	
Μ	-0.48	-0.09	0.03	
Н	-2.60	-2.39	-2.32	