



Housing dynamics: An urban approach



Edward L. Glaeser^{a,b}, Joseph Gyourko^{c,b}, Eduardo Morales^{d,b}, Charles G. Nathanson^{a,*}

^a Department of Economics, Harvard University, United States

^b NBER, United States

^c The Wharton School, University of Pennsylvania, United States

^d Department of Economics, Princeton University, United States

ARTICLE INFO

Article history:

Received 4 January 2013

Revised 30 January 2014

Available online 22 February 2014

Keywords:

Housing supply

Housing demand

Method of moments

ABSTRACT

A dynamic linear rational equilibrium model in the tradition of Alonso, Rosen and Roback is consistent with many outstanding stylized facts of housing markets. These include: (a) that the markets are local in nature; (b) that construction persistence is fully compatible with mean reversion in prices; and (c) that price changes are predictable. Calibration exercises to match moments of the real data have notable successes and failures. The volatility in local income processes as reflected in HMDA mortgage applicant data can account for much of the observed price and construction volatility, except for the most inelastically supplied local markets. The model's biggest failure lies in its inability to match the strong persistence in high frequency price changes from year to year.

© 2014 Elsevier Inc. All rights reserved.

1. Introduction

Can the dynamics of housing markets be explained by a dynamic, rational expectations version of the standard urban real estate models of Alonso (1964), Rosen (1979) and Roback (1982)? In this tradition, housing prices reflect a spatial equilibrium, where prices are determined by local wages and amenities so that local heterogeneity is natural. Our model extends the Alonso–Rosen–Roback framework by focusing on high frequency price dynamics and by incorporating endogenous housing supply.

An urban approach can potentially help address the fact that most variation in housing price changes is local, not national. Less than 8% of the variation in price levels and barely more than one-quarter of the variation in price changes across cities can be accounted for by national, year-specific fixed effects. Clearly, there is much local variation that cannot be accounted for by common macroeconomic variables such as interest rates or national income.

We focus not on the most recent boom and bust, which was extraordinary in many dimensions, but rather on long-term stylized facts about housing markets. One such fact is that price changes are predictable (Case and Shiller, 1989; Cutler et al., 1991). Depending upon the market and specific time period being examined, a \$1 increase in real constant quality house prices in one year is associated with a 60–80 cent increase the next year. However, a \$1 increase in local market prices over the past five years is

associated with strong mean reversion over the next five year period. This raises the question of whether the high frequency momentum and low frequency mean reversion of price changes can be reconciled with a rational market.

Another outstanding feature of housing markets is that the strong mean reversion in price appreciation and strong persistence in housing unit growth across decades shown in Figs. 1 and 2 is at odds with simple demand-driven models in which prices and quantities move symmetrically. This raises the question of what else is needed to generate this pattern.

Third, price changes and construction levels are quite volatile in many markets. The range of standard deviations of three-year real changes in our sample of metropolitan area average house prices runs from about \$6500 in sunbelt markets to over \$30,000 in coastal markets. New construction within markets also can be volatile, with its standard deviation much higher in the sunbelt region. Can this volatility be the result of real shocks to housing markets or must it reflect bubbles or animal spirits?

Section 2 presents our model and its implications. Naturally, the urban approach predicts that housing markets are local, not national, in nature. Predictable housing price changes also are shown to be compatible with a no-arbitrage rational expectations equilibrium. Mean reversion over the medium and longer term results if construction does not respond immediately to shocks and if local income shocks themselves mean revert. High frequency positive serial correlation of housing prices results if there is enough positive serial correlation of labor demand or amenity shocks. Conceptually, a dynamic rational expectations urban model is at least

* Corresponding author.

E-mail address: cnathans@fas.harvard.edu (C.G. Nathanson).

$$V_t = W_t + A_t - (\alpha_W + \alpha_A)N_t - \left(H_t - \frac{E(H_{t+1})}{1+r} \right). \quad (2)$$

To pin down this utility level, we turn to the cross-metropolitan area spatial equilibrium concept introduced by Rosen (1979) and Roback (1982). Consumers are indifferent across cities at all points in time. This indifference condition is a particularly strong version of the standard spatial equilibrium assumption that assumes away moving costs. There is a “reservation” city where housing is completely elastic: $c_0 = c_1 = c_2 = 0$, so that housing prices always equal C .¹ Wages and amenities do not depend on the reservation city population: $\alpha_W = \alpha_A = 0$. If we let \bar{V}_t equal $W_t + A_t$ for the reservation city, then the reservation utility level that holds in this city as well as in all cities is

$$V_t = \bar{V}_t - \frac{rC}{1+r}. \quad (3)$$

The existence of the reservation city makes our calculations considerably easier, and there are places within the United States, especially in the growing areas of the sunbelt, that are marked by elastic labor demand and housing supply (Glaeser et al., 2005).²

Putting together Eqs. (2) and (3) gives the following housing demand equation:

$$H_t - \frac{E(H_{t+1})}{1+r} - \frac{rC}{1+r} = W_t + A_t - (\alpha_W + \alpha_A)N_t - \bar{V}_t. \quad (4)$$

For our estimation, we assume the following functional form:

$$W_t + A_t - \bar{V}_t = \bar{x} + qt + x_t,$$

where x_t is a stochastic term that follows an ARMA(1,1) process:

$$x_t = \delta x_{t-1} + \epsilon_t + \theta \epsilon_{t-1},$$

with $0 < \delta < 1$ and the ϵ_t independently and identically distributed with mean 0 and finite variance σ^2 . The \bar{x} term is a city fixed effect and q is a city-specific drift term. We also define

$$\alpha \equiv \alpha_W + \alpha_A$$

to be the slope of the housing demand curve, and we assume that $\alpha > 0$.

2.3. Equilibrium

The supply Eq. (1) and the demand Eq. (4) jointly determine equilibrium prices, housing stock, and investment. To obtain a unique solution to our model, we impose a transversality condition

$$\lim_{j \rightarrow \infty} \frac{E_t(H_{t+j})}{(1+r)^j} = 0 \quad (5)$$

for all t . The transversality condition limits the possible role of housing bubbles in accounting for housing dynamics. While we do not discount the possible explanatory power of bubbles, our focus here allows us to learn what aspects of housing dynamics can already be explained by a model in which prices equal the discounted sum of current and future expected rents. The following lemma shows that price, housing stock, and investment converge towards “trend” levels of these variables when the transversality condition is satisfied.

Lemma 1. *When Eq. (5) is satisfied, there exist unique price, stock, and investment functions \hat{H}_t , \hat{N}_t , and \hat{I}_t such that*

$$\lim_{j \rightarrow \infty} E_t(H_{t+j}) - \hat{H}_{t+j} = \lim_{j \rightarrow \infty} E_t(N_{t+j}) - \hat{N}_{t+j} = \lim_{j \rightarrow \infty} E_t(I_{t+j}) - \hat{I}_{t+j} = 0$$

for any H_t, N_t , and I_t that satisfy the supply and demand equations. \hat{H}_t and \hat{N}_t are linear in t and \hat{I}_t is constant.

We call \hat{H}_t, \hat{N}_t , and \hat{I}_t trend prices, population, and investment. Closed-form expressions for these trend variables as well as a proof of the lemma appear in the [technical Appendix](#).

If $x_t = 0$ for all t and $N_t = \hat{N}_t$ for some initial period, then the steady state quantities would fully describe the equilibrium.³ Secular trends in housing prices come from the trend in housing demand as long as $c_2 > 0$, or the trend in construction costs as long as $\alpha > 0$. If $c_2 = 0$, so that construction costs do not increase with total city size, then trends in wages or amenities will impact city size but not housing prices. If $\alpha = 0$ and city size does not decrease wages or amenities, then trends in construction costs will impact city size but not prices.

Lemma 2 then describes housing prices and investment when there are shocks to demand and when $N_t \neq \hat{N}_t$. The proof is in the [technical Appendix](#).

Lemma 2. *At time t , housing prices equal*

$$H_t = \hat{H}_t + x_t + \frac{E_t(x_{t+1})}{\bar{\phi} - \delta} - \frac{\alpha(1+r)}{1+r-\phi} (N_t - \hat{N}_t)$$

and investment equals

$$I_t = \hat{I} + \frac{1+r}{c_1} \frac{E_t(x_{t+1})}{\bar{\phi} - \delta} - (1-\phi)(N_t - \hat{N}_t),$$

where $\bar{\phi} > 1 > \phi > 0$ are parameters that depend on α, c_1, c_2 , and r .⁴

This lemma describes the movement of housing prices and construction around their trend levels. A temporary shock, ϵ_t , will increase housing prices by $(\bar{\phi} + \theta)/(\bar{\phi} - \delta)$ and increase construction by $(1+r)(\delta + \theta)/(c_1(\bar{\phi} - \delta))$. Higher values of δ (i.e., more permanent shocks) will make both of these effects stronger. Higher values of c_1 mute the construction response to shocks and increase the price response to a temporary shock (by reducing the quantity response). These results provide the intuition that places which are quantity constrained should have less construction volatility and more price volatility.

The following proposition provides implications about expected housing price changes.

Proposition 1. *At time t , the expected home price change between time t and $t + j$ is*

$$\begin{aligned} & \hat{H}_{t+j} - \hat{H}_t \\ & + \frac{E_t(x_{t+1})}{\bar{\phi} - \delta} \left(\frac{1+r}{c_1} \frac{\delta^{j-1}((1-\delta)c_1 - c_2) - \phi^{j-1}((1-\phi)c_1 - c_2)}{\phi - \delta} - 1 \right) \\ & - x_t + \left(\frac{\alpha(1+r)}{1+r-\phi} - \phi^{j-1}((1-\phi)c_1 - c_2) \right) (N_t - \hat{N}_t), \end{aligned}$$

the expected change in the city housing stock between time t and time $t + j$ is

³ In this case, the assumption that there is always some construction requires that $q(1+r) > rc_0$.

⁴ The formulas for $\bar{\phi}$ and ϕ are

$$\begin{aligned} \bar{\phi} &= \frac{(1+r)(\alpha+c_1)+c_1-c_2+\sqrt{((1+r)(\alpha+c_1)+c_1-c_2)^2-4(1+r)c_1(c_1-c_2)}}{2c_1}; \\ \phi &= \frac{(1+r)(\alpha+c_1)+c_1-c_2-\sqrt{((1+r)(\alpha+c_1)+c_1-c_2)^2-4(1+r)c_1(c_1-c_2)}}{2c_1}. \end{aligned}$$

¹ While it is possible that prices will deviate around this value because of temporary over- or under-building, we simplify and assume that the price of a house always equals C .

² Van Nieuwerburgh and Weill (2010) present a similar model in their exploration of long run changes in the distribution of income.

$$j\hat{I} + \frac{1+r}{c_1(\bar{\phi}-\delta)} \frac{\phi^j - \delta^j}{\phi - \delta} E_t(x_{t+1}) - (1 - \phi^j)(N_t - \hat{N}_t),$$

and expected time $t + j$ construction is

$$\hat{I} + \frac{1+r}{c_1(\bar{\phi}-\delta)} \left(\frac{\delta^{j-1}(1-\delta) - \phi^{j-1}(1-\phi)}{\phi - \delta} \right) E_t(x_{t+1}) - \phi^{j-1}(1 - \phi^j)(N_t - \hat{N}_t).$$

Proposition 1 delivers the implication that a rational expectations model of housing prices is fully compatible with predictability in housing prices. If utility flows in a city are high today and expected to be low in the future, then housing prices will also be expected to decline over time. Any predictability of wages and construction means that predictability in housing price changes will result in our model.

The predictability of construction and prices comes in part from the convergence to trend values. If $x_t = \epsilon_t = 0$ and initial population is above its trend level, then prices and investment are expected to converge on their trend levels from above. If initial population is below its trend level and $x_t = \epsilon_t = 0$, then price and population are expected to converge to their trend levels from below. The rate of convergence is determined by $r, \alpha, c_1,$ and c_2 . Higher levels of c_1 and c_2 cause the rate of convergence to slow by reducing the extent that new construction responds to changes in demand.

The impact of a one-time shock is explored in the next proposition.

Proposition 2. *If $N_t = \hat{N}_t, x_{t-1} = \epsilon_{t-1} = 0, c_2 = 0,$ and $\epsilon_t > 0,$ then investment and housing prices will initially be higher than steady state levels, but there exists a value j^* such that for all $j > j^*,$ time $t + j$ expected values of time $t + j$ construction and housing prices will lie below steady state levels. The situation is symmetric when $\epsilon_t < 0.$*

Proposition 2 highlights that this model not only delivers mean reversion, but overshooting. **Fig. 3** shows the response of population, construction and prices relative to their steady state levels in response to a one time shock. Construction and prices immediately shoot up, but both start to decline from that point. At first, population rises slowly over time, but as the shock wears off, the heightened construction means that the city is too large relative to its steady state level. Eventually, both construction and prices end up below their steady state levels because there is too much housing in the city relative to its wages and amenities. Places with positive shocks will experience mean reversion, with a quick boom in prices and construction, followed by a bust.⁵

Finally, we turn to the puzzling empirical fact that there was strong mean reversion of prices and strong positive serial correlation in population levels across the 1980s and 1990s. We address this by looking at the one period covariance of price and population changes. We focus on one period for simplicity, but we think of this proposition as relating to longer time periods. Since mean reversion dominates over long time periods, we assume $\theta = 0$ to avoid the effects of serial correlation:

Proposition 3. *If $N_0 = \hat{N}_0, \theta = 0, x_0 = \epsilon_0,$ cities differ only in their demand trends q and their shock terms $\epsilon_0, \epsilon_1,$ and $\epsilon_2,$ and these terms are uncorrelated, then if $\delta > 1 - \phi,$ second period population growth will always be positively correlated with first period population growth, while second period price growth will be negatively correlated with first period population growth as long as $\frac{\text{Var}(q)}{\text{Var}(\epsilon_t)}$ is below a bound.*

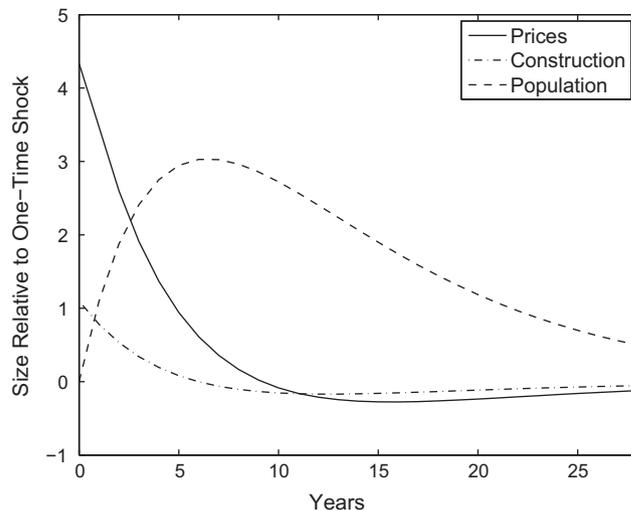


Fig. 3. One-time shock. Notes: We use the parameters estimated for the interior region using HMDA data in this figure: $\delta = 0.88, \theta = 0.20, c_1 = 3.16,$ and $c_2 = 0.12.$

Proposition 3 tells us that, in the model, positive serial correlation of construction levels is quite compatible with negative serial correlation of price changes. The proposition only proves that the reversal occurs when persistence of shocks is high, but in the **technical Appendix**, we show that the persistence can occur when the process is less persistent. The positive correlation of quantities is driven by the heterogeneous trends in demand across urban areas. As long as the variance of these trends is high enough relative to the variance of temporary shocks, there will be positive serial correlation in quantities, as in **Fig. 2.**

Yet these long trends may have little impact on price changes, since the trends are completely anticipated. As discussed above, when c_2 is low, trends will have little impact on steady state price level, although these trends will determine the steady state price level. Instead, price changes will be driven by the temporary shocks, and if these shocks mean revert, then so will prices.

This suggests two requirements for the observed positive correlation of quantities and negative correlation of prices: city-specific trends must differ significantly and the impact of city size on construction costs must be small. Both conditions appear to occur in reality. The extensive heterogeneity in city-specific trends is discussed and documented by **Gyourko et al., 2013** and **Van Nieuwerburgh and Weill (2010).** The literature on housing investment suggests that the impact of city size on construction costs is quite small (**Topel and Rosen, 1988; Gyourko and Saiz, 2006.**)

3. Estimating the model

We now calibrate the model to see whether certain moments of the data are compatible with our framework. We focus on movements in prices and construction intensity around steady state levels. The aim of this exercise is to show how a model which posits that variation in prices and construction levels is solely driven by exogenous shocks to both amenity levels and the demand for labor can fit certain moments of the housing data. As we lack data on the short term fluctuations in the level of amenities, we will identify the parameters of the stochastic process governing these shocks to housing demand only from wage data.⁶ This is not to claim that there are no other shocks that will affect the volatility of both prices

⁵ Overshooting occurs here with no depreciation in the housing stock. The case with depreciation is addressed in **Glaeser and Gyourko (2005).**

⁶ There can still be long run trends in amenities that differ across metropolitan areas, but these will not impact the short term housing price and construction dynamics that are the focus of our simulations.

and construction. There are, but our approach still provides some quantitative measure of how misspecified our housing models would be if we were to ignore these additional shocks.

To generate predictions from the model, we need to calibrate eight parameters: $(r, \alpha, w, \delta, \theta, \sigma, c_1, c_2)$. The parameters (δ, θ, σ) govern housing demand. Consistent with the spirit of the calibration exercise described in the previous paragraph, we estimate these parameters exclusively using wage data. Identifying the remaining five parameters using only data on deviations of housing prices and construction of new houses from their steady state levels turns out to be infeasible.⁷ Therefore, we borrow estimates of the real interest rate, r , the slope of the inverse housing demand equation, α , and the slope of labor demand, w , from other sources. Finally, we use data on housing prices and quantities to estimate the parameters determining the housing supply, (c_1, c_2) .

We assume that r equals 0.04. This value is higher than standard estimates of the real interest rate because it is also meant to reflect other aspects of the cost of owning such as taxes or maintenance expenses that roughly scale up with the cost of the house. Different values of the real interest rate have little impact on our calibration, as long as it is assumed to be constant.

The value of α reflects the impact that an increase in the housing stock will have on the willingness to pay to live in a locale. If population was fixed, Eq. (2) would imply that the derivative of steady state housing prices with respect to the number of homes equals $-(1+r)\alpha/r$, which can be interpreted as the slope of the housing demand curve. Typically, housing demand relationships are estimated as elasticities, so we must first convert elasticities into the comparable slope in levels and then multiply by $r/(1+r)$. Many housing demand elasticity estimates are around one (or slightly below, in absolute value; see, e.g. Polinsky and Ellwood, 1979 or Saiz, 2003), and there is a wide range in the literature, so we experiment with a range from 0 to 2. To transform the elasticity into slope in levels, we multiply by an average ratio of price to population, and that produces a range of estimates for $(1+r)\alpha/r$ ranging from 0 to 3. Multiplying this range by $r/(1+r)$ yields a range from 0 to 0.15. We use a parameter value of 0.1 in our estimation, which implies that for every 10,000 extra homes sold the marginal purchaser likes living in the area \$1000 less per year.

Lower values do not significantly change our estimates. Even with $\alpha = 0.1$, most of the variation in house prices comes from direct shocks to wages and not from variation in congestion effects. Lemma 2 shows that we can decompose the variation in house prices from trend as

$$H_t - \hat{H}_t = \underbrace{x_t + \frac{E_t(x_{t+1})}{\phi - \delta}}_{\text{wage shocks}} - \underbrace{\frac{\alpha(1+r)}{1+r-\phi}(N_t - \hat{N}_t)}_{\text{congestion effects}} \quad (6)$$

Table 2 lists the volatility of each term using the parameters we estimate for each of the three regions of the United States (calculation details are in the technical Appendix). In all three cases, wage shocks are much more important than variation in congestion effects. The value of α is much more important in determining the steady-state (i.e. trend) size of the city, but this steady-state is not our focus here.

The parameter α combines the impact that extra population has on wage levels with the impact that extra population has on amenities, and we also must use a distinct estimate of the connection between population and wage levels to correct our wage series

⁷ As will be seen in the next Section, in order to identify the parameters of the model, we derive moment conditions from the equation in Lemma 2. More moment conditions than parameters we have to identify are derived. Nevertheless, when we try to simultaneously identify the five parameters (r, α, w, c_1, c_2) , the resulting objective function is relatively flat and identification is very weak.

for the change in population. Given the absence of compelling evidence on the links between population size and amenity levels, and the possibility that the link is actually positive (if access to other people is a consumption amenity), we make the simplifying assumption that the impact of population on amenities is zero, so that the value of α is the same as the value of α_w . While we do not literally believe this, assuming it has little impact on our estimates since it only serves to allow us to infer productivity changes from wage changes by correcting for the changes in population. As year-to-year population changes are relatively modest, different means of correcting for population changes have little impact on the inferred productivity series.

In principle all eight parameters in our model could differ across each metropolitan area, but data limitations make it impossible for us to precisely estimate distinct values for each location. Instead, we assume the calibrated parameters (r, α, w) to be identical for all metropolitan areas and we estimate different values of the parameters $(\delta, \theta, \sigma, c_1, c_2)$ for three different regions of the U.S.⁸ Our three regions are coastal, sunbelt and interior. Metropolitan areas whose centroids are within 50 miles of the Atlantic or Pacific Oceans are defined as coastal. Metropolitan areas more than 50 miles from either coast and which are in the broad swath of southern and western states on the southern border of the country running from Florida through Arizona are defined to be in the sunbelt region. The remainder of our metropolitan areas are defined as being in the interior region of the country.

3.1. Data

For our estimation exercise, we need data on housing prices, construction of new houses, number of households potentially supplying labor, and income per household for a significant number of metropolitan areas.

The housing price data is based on Federal Housing Finance Agency repeat sales indices. Construction data are housing permits reported by the U.S. Census. To estimate annual changes in the number of households, we impute the housing stock based on decadal census estimates of the housing stock and annual permits data. Specifically, we estimate the housing stock at time $t + j$ to be

$$N_t^i + \frac{\sum_{k=0}^{j-1} \text{Permits}_{t+k}^i}{\sum_{k=0}^9 \text{Permits}_{t+k}^i} (N_{t+10}^i - N_t^i),$$

where N_t^i and N_{t+10}^i are the housing stocks measured during the two closest censuses in metropolitan area i . Thus, the change in housing stock is partitioned across years based on the observed permitting activity.

Our primary source of income data comes from the Home Mortgage Disclosure Act (HMDA) files on reported income on mortgage applications. We observe all loan applicants, not just successful buyers. The HMDA data extend back to 1990. Since HMDA is essentially a 100% sample of everyone who sought a mortgage, the sample sizes are quite large and we have data for every metropolitan area. Importantly, the HMDA data captures household level income, which is the appropriate level given our model. The disadvantages of using HMDA income data are a relatively short time series, the fact that we do not observe those who searched but did not apply for a mortgage, and that the homebuying decision is endogenous, which can create biases because the selected sample of people who decide to apply for a loan can differ across markets or years.

⁸ Obtaining different estimates of (r, α, w) for each of these three areas is impossible, as the sources from which we borrow those estimates do not provide such detail.

An alternative data source on income is the Bureau of Economic Analysis (BEA) per capita income measure. It is available beginning in 1980 and for all metropolitan areas. However, it suffers from a number of drawbacks. First, it is at the individual, not household, level as its name implies. Households, not individuals, purchase housing units. Hence, in our experimentation with this measure, we translate per capita incomes into household-levels by multiplying by 2.63, which is the average number of people per housing unit in our sample of areas in 1990. It also captures the incomes of many people who were not potential buyers. The incomes earned by permanent renters or people who have been immobile homeowners for many years may not have much to do with the advantage that a location brings to the marginal purchaser. In addition, the incomes of renters are both lower and less volatile than those of owners. Hence, the BEA series is likely to understate the relevant volatility in local incomes, which is critical given our purposes.⁹

While we experimented with both income measures, we believe the advantages of the HMDA series far outweighs its negatives. Hence, we report results using this series and comment on findings with the BEA data where appropriate.

The sample used in the estimation has 21 sunbelt metropolitan areas, 32 coastal metropolitan areas, and 60 interior ones. The data for housing prices, construction, number of households, and borrower income spans the period 1990–2004.

3.2. Methodology

As indicated above, we estimate the parameters $(\delta, \theta, \sigma, c_1, c_2)$ subject to particular values of (r, α, w) . We estimate these five parameters using a sequential two-step Generalized Method of Moments estimator.¹⁰ Our two stage procedure estimates our parameters by first using the population-corrected wage series to estimate the housing demand parameters and then using housing price and construction series to identify the housing supply parameters. More specifically, the parameters (δ, θ, σ) are estimated from an equilibrium equation in the labor market using a two-step GMM estimator. Given these estimates, the parameters (c_1, c_2) are estimated from the equilibrium equations for the housing market in Lemma 1 using again a two-step GMM estimator.

3.2.1. Description of moments

The vector of moments used to estimate (δ, θ, σ) is based on the reduced form relationship between productivity per worker and the equilibrium number of workers: $W_t^i = \tilde{W}_t^i - \alpha_w N_t^i$. The

assumption that x_t^i works entirely through the wage process allows us to write: $W_t^i = w_0^i + w_1^i t + x_t^i - \alpha_w N_t^i$, which allows for a city-specific constant and a region-specific time trend in labor demand.¹¹ Using this expression for wages as well as the assumed value of w , we define our productivity variable, which is wages normalized for changes in the number of workers: $\tilde{W}_t^i = W_t^i + \alpha_w N_t^i$. The resulting equation is: $\tilde{W}_t^i = w_0^i + w_1^i t + x_t^i$, where x_t^i follows an ARMA(1, 1) process. The stochastic process for the shocks is therefore

$$x_t^i = \delta x_{t-1}^i + \epsilon_t^i + \theta \epsilon_{t-1}^i,$$

with ϵ_t^i independently and identically distributed over time with

$$\mathbb{E}[\epsilon_t^i | x_t^i, x_{t-1}^i] = 0, \quad \text{and} \quad \text{var}[\epsilon_t^i | x_t^i, x_{t-1}^i] = \sigma_\epsilon^2.$$

Using these two restrictions on ϵ and data on \tilde{W}_t^i , we identify the parameter vector (δ, θ, σ) through a vector of moments

$$\mathbb{E}[f(\tilde{W}^i; (\delta, \theta, \sigma))] = 0.$$

The exact functional form of the moment function $f(\tilde{W}^i; (\delta, \theta, \sigma))$ is contained in Appendix A. This moment function is based on different moments of the one-period changes in our productivity measure, $\Delta \tilde{W}$, and relies on the ϵ shocks having mean zero, being uncorrelated with lagged values of \tilde{W}^i , and having constant variance.¹²

Given the first stage estimates of the housing demand parameters, $(\delta, \hat{\theta}, \hat{\sigma}^2)$, we use the equilibrium equations in Lemma 1 to build moment conditions that allow us to identify the vector (c_1, c_2) . Identification of these two parameters is performed through the vector of moment conditions:

$$\mathbb{E}[v(H^i, N^i, I^i; (c_1, c_2))] = 0.$$

The exact functional form of the moment function $v(H^i, N^i, I^i; (c_1, c_2))$ also is reported in Appendix A. This moment function is based on different moments of the deviations between the vector of housing prices, construction, and number of households and their steady state levels, $(H - \hat{H}, I - \hat{I}, N - \hat{N})$. The moments defined by the moment function $v(H^i, N^i, I^i; (c_1, c_2))$ rely on the ϵ shocks having mean zero, being uncorrelated with lagged values of N^i , and having constant variance.

In order to build the sample analogues of

$$\mathbb{E}[f(\tilde{W}^i; (\delta, \theta, \sigma))] = 0,$$

$$\mathbb{E}[v(H^i, N^i, I^i; (c_1, c_2))] = 0,$$

we use sample moment conditions that pool all the observations across metropolitan areas and time periods which we assume share the same values of the parameter vector $(\delta, \theta, \sigma, c_1, c_2)$. Specifically, we build the sample analogue of the moment conditions aggregating across metropolitan areas within regions and over our entire sample period. We pool observations across metropolitan areas, instead of splitting them across different moment conditions, to increase our sample size. After all, GMM estimators have optimal statistical properties only when the number of observations used in each moment condition goes to infinity, and the standard errors of our GMM estimates are valid only asymptotically.

⁹ Based on data from the New York City Housing and Vacancy Surveys (NYCHVS) from 1978 to 2002, the income of recent homebuyers increases by \$1.29 for every dollar increase in BEA-reported per capita income, while that for renters only rises by \$0.47. The NYCHVS only covers one city, but it highlights that the volatility of BEA per capita income is lowered by its incorporation of renter income.

¹⁰ The details of this estimation method are provided in Appendix A. Hansen (1982) proves consistency and asymptotic normality for the standard two-step GMM estimator, in which all parameters are simultaneously estimated. Newey (1984) expands these results and provides the correct formula for the asymptotic variance of the two-step GMM estimator of a subvector of parameters, when the moments are a function of previous GMM estimates of a different subvector of parameters. Finally, Newey and McFadden (1994) show that the sequential GMM estimators belong to the more general family of *extremum* estimators. These results guarantee that the sequential two-step GMM estimator we use is consistent, asymptotically normal and has the asymptotic variances described in Appendix A. In principle, we could estimate all of our parameters simultaneously, using information on wages, construction levels and housing prices, but, as indicated above, this would contradict the spirit of the exercise we want to perform. If we were to use data on deviations of housing prices and construction levels with respect to their steady state in order to identify the parameters (δ, θ, σ) , then our estimates of the stochastic process governing housing demand would capture not only the income process (as the model indicates should be the case) but also the stochastic process governing any other unobservable variable or shock that might affect the equilibrium in the housing market.

¹¹ We have tried to allow for city-specific time trends but, given the short length of the time series available for estimation, this impedes the identification of the remaining parameters of the wage equation.

¹² As a robustness check, we have also estimated (δ, θ, σ) using a multiple-step estimation procedure. In the first step, we use the Arellano–Bond estimator to obtain estimates of delta (Arellano and Bond., 1991). Given this estimate of δ , we use a Classical Minimum Distance estimator for θ based on the first and second order temporal autocorrelation. Finally, using our estimates of (δ, θ) , we estimate σ from the residual variance. The results are very similar to the ones based on the simultaneous estimation of (δ, θ, σ) using the moment function $f(\tilde{W}^i; (\delta, \theta, \sigma))$ and are available upon request.

Table 1
Estimated demand and supply parameters HMDA income data, 1990–2004.

	Coastal	Sunbelt	Interior
<i>Demand</i>			
δ	0.88 (0.11)	0.89 (0.13)	0.88 (0.10)
θ	0.82 (0.62)	0.13 (0.13)	0.20 (0.10)
σ_ϵ	\$1700 (500)	\$1300 (200)	\$1300 (100)
<i>Supply</i>			
c_1	10.62 (0.58)	1.47 (0.14)	3.16 (0.25)
c_2	4.08 (0.77)	0.34 (0.08)	0.12 (0.11)

Notes: δ , θ , and σ_ϵ are the autocorrelation parameter, moving average parameter and residual variance of an ARMA(1,1) estimated for the component of wages that is not explained by a linear time trend and a metropolitan area-specific constant. c_1 denotes the derivative of expected future housing prices with respect to current investment in housing construction; and c_2 denote the derivative of the physical capital cost of building a home with respect to the stock of houses. The standard errors for the demand parameters are efficient two-step GMM standard errors. The ones for the supply parameters account for error coming from the demand estimates.

3.3. Estimation results

Table 1 reports our estimated parameters. The estimates of the labor demand shocks persistence parameter, δ , are 0.88 in the interior and coastal areas and 0.89 in the sunbelt. While the similarity of these estimates is striking, they are still somewhat imprecise. We cannot reject the possibility that income shocks follow a random walk (i.e., the persistence parameter equals one) and we also cannot reject much more significant mean reversion.

The estimates of the moving average parameter θ are statistically indistinguishable from zero in the sunbelt and coastal regions. In the interior region, this moving average component estimate is 0.2 and is marginally significantly different from zero. The productivity shock estimates range from \$1300 in the sunbelt and interior to \$1700 on the coast. Our estimates of the housing supply parameters reported in the bottom panel of Table 1 indicate a value for c_1 of 10.62 in the coastal region. This implies that a 1000 unit increase in the number of building permits in a given year raises the cost of supplying a home by \$10,620. We estimate a value of c_2 in that region of 4.08, meaning that as the number of units in a metropolitan area increases by 10,000 the cost of supplying a home increases by more than \$40,000. The estimates of c_1 are much lower in the sunbelt and interior regions, at 1.47 and 3.16, respectively. In these two regions, the estimates of c_2 are 0.34 and 0.12, respectively. Housing supply does appear to be far more elastic in those regions.¹³

These latter findings can be compared with the housing supply estimates reported by Topel and Rosen (1988), who use aggregate national data to estimate an elasticity of housing supply with

¹³ As noted above, we generated separate estimates using BEA per capita income data in lieu of HMDA data. This has the advantage of including years back to 1980, but we also suspect it might grossly underestimate income volatility, which is critical for our purposes. In fact, estimates of the productivity shocks are much lower, with the largest estimate of \$1200 for coastal region markets being smaller than reported above for sunbelt and interior markets using HMDA data. The moving average parameters are somewhat smaller across all regions, but they are also imprecisely estimated, as was the case with the estimates based on HMDA. The BEA data imply greater differences across regions in the demand shock persistence parameter, δ , with estimates ranging from 0.73 in the interior (and we can reject that coefficient equals one at standard confidence levels) to 0.8 in coastal areas and 0.9 in the sunbelt region. Estimates of supply parameters using BEA per capita income show a very similar pattern to those reported above, albeit with small point estimates. The coastal c_1 is 6.1 and its c_2 is 1.9; those for the interior and sunbelt regions are much closer to zero. See Appendix Table A1 for the analogue to Table 1 based on using BEA per capita income in lieu of HMDA-based income.

Table 2
Relative volatility of terms in house price equation.

	Coastal	Sunbelt	Interior
Wage shocks	44,000	12,000	13,000
Congestion effects	4000	5000	7000

Notes: The house price equation is decomposed in Eq. (6). The volatilities are computed using the estimates in Table 1. Details on the computation are provided in the technical Appendix.

respect to price that is between 1 and 3. In our model, that supply elasticity equals $H_t/(c_1 I_t)$. In 1990, average prices were about \$130,000. Average construction levels in a metropolitan area is approximately 8350 units, as measured by building permits issued. If we take the Topel and Rosen (1988) elasticity to be 3, then this implies a value of c_1 of 5, which lies in the middle of our estimates.

4. Matching the data and discussion

The model presented in Section 2 implies a particular stochastic process for housing prices and for the construction of new houses. If shocks are known as they occur, then it is straightforward to show that our model implies the following ARMA(2,3) process for housing prices, with the parameter vector restricted as outlined in the appendix:

$$\Delta H_t^i = a_0^i + a_1 \Delta H_{t-1}^i + a_2 \Delta H_{t-2}^i + b_0 \epsilon_t^i + b_1 \epsilon_{t-1}^i + b_2 \epsilon_{t-2}^i + b_3 \epsilon_{t-3}^i.$$

Analogously, the model implies the following ARMA(2,1) process for the construction of new homes, with the parameter vector restricted as shown in the appendix:

$$I_t^i = d_0^i + d_1 I_{t-1}^i + d_2 I_{t-2}^i + e_0 \epsilon_{t-1}^i + e_1 \epsilon_{t-2}^i.$$

We then use these two ARMA processes, together with the estimated values of the supply and demand parameters, to derive various predictions of the model over different time horizons. Certain moments directly estimated from the data are compared to those analytically derived. In doing so, we focus on a particular set of moments of these stochastic processes: serial correlations and variances at the one, three and five year horizons. We do not focus on any contemporaneous or lagged correlations between prices and quantities for the reasons discussed next, even though much research in urban and real estate economics uses results from regressions of high frequency prices (or price changes) on demand factors such as income (or income changes).

4.1. The impact of information on the predictions of the model

The model discussed above assumes that shocks are observed as they occur, but we are far from confident that they are not known ahead of time. And, the results of contemporaneous correlations are sensitive to what one assumes about the underlying information structure (i.e., whether information about the change in income becomes known ahead of time or only contemporaneously with its public release). In contrast, *auto* correlations of price and construction series are much less sensitive to information timing as we now demonstrate by comparing the predictions of the model with our assumed information structure and the predictions if shocks are known one period ahead of time.

For this exercise, we use parameter estimates from the coastal region: $r = 0.04$, $\alpha = 0.1$, $c_1 = 10.62$, $c_2 = 4.08$, $\theta = 0.82$, $\delta = 0.88$, and $\sigma = \$1700$. The first column in Table 3 reports our model's predictions for a number of variables presuming such contemporaneous knowledge.¹⁴ The second column represents our model's predictions

¹⁴ For any j year interval, these predictions reflect the relationship between what happened between time t and $t-j$ and what happened between time t and $t+j$.

Table 3
Sensitivity of predictions to different information structures.

Horizon	Contemporaneous knowledge	Knowledge one year ahead
<i>Serial correlation of construction</i>		
1 year	0.51	0.56
3 year	0.18	0.19
5 year	−0.04	−0.03
<i>Volatility of construction (units)</i>		
1 year	1800	2000
3 year	4300	4800
5 year	6000	6700
<i>Serial correlation of house price changes</i>		
1 year	−0.00	0.09
3 year	−0.16	−0.10
5 year	−0.24	−0.21
<i>Volatility of house price changes (\$)</i>		
1 year	18,000	17,000
3 year	30,000	31,000
5 year	37,000	39,000
<i>Correlation of income changes and house price changes</i>		
1 year	0.80	0.08
3 year	0.93	0.61
5 year	0.95	0.75

Notes: The parameter values estimated for the coastal region using HMDA wage data are assumed here: $\delta = 0.88$, $\theta = 0.82$, $\sigma_\epsilon = \$1700$, $c_1 = 10.62$, and $c_2 = 4.08$.

when individuals learn about the income shock one period before it actually impacts wages.

Advance knowledge slightly increases construction volatility and adds some momentum to house price changes. Otherwise the autocorrelations are essentially unchanged. Therefore, the predictions of our model for these moments are robust to a possible misspecification of the information structure and a potential lag between the time the income shocks are known to the agents and when they are made public.

In stark contrast, the impact of the information structure on the contemporaneous correlation between changes in prices and changes in income is enormous. The bottom panel of Table 3 shows that if knowledge is contemporaneous to the shock, then the correlation of price and income changes over short horizons is 0.80. If individuals acquire knowledge one year ahead, then the predicted correlation is only 0.08. The correlation is only somewhat more stable at lower frequencies.

Because these correlations are so sensitive to small changes in the underlying information conditions, we focus our analysis on the serial correlation properties and volatility of price changes and construction activity.¹⁵

4.2. Volatility and serial correlation in house prices

Table 4 documents how well the model matches the data by comparing the model's predictions of short- and long-run volatility and serial correlation in house price changes and new construction with the actual moments from the data. Standard deviations and serial correlation coefficients from the underlying data over this time period are reported in columns adjacent to our model predictions.

4.2.1. Volatility in house prices

The model generally overpredicts price volatility except in the coastal region at 3- and 5-year horizons. One explanation for this excess predicted volatility is that the HMDA data may be

¹⁵ Over longer horizons, a one-year shift in when information becomes known is less important, so it certainly can make good sense to explore various longer-run relationships with price changes. Because our interest is in higher frequency changes, we do not do that here.

Table 4
Volatility and serial correlation in house prices and construction: HMDA income data, 1990–2004.

Horizon	Coastal		Sunbelt		Interior	
	Model	Data	Model	Data	Model	Data
<i>Volatility of house price changes (\$)</i>						
1 year	18,000	13,300	5000	2000	6000	3600
3 year	30,000	34,100	8000	4400	10,000	8400
5 year	37,000	48,300	9000	5400	12,000	10,700
<i>Serial correlation of house price changes</i>						
1 year	−0.00	0.84	−0.12	0.64	−0.07	0.73
3 year	−0.16	0.32	−0.28	−0.09	−0.25	0.10
5 year	−0.24	−0.80	−0.35	−0.73	−0.36	−0.72
<i>Volatility of construction (units)</i>						
1 year	1800	1900	3600	5300	2000	1600
3 year	4200	4600	9000	12,000	5700	3800
5 year	5900	6300	12,000	15,500	8600	5000
<i>Serial correlation of construction</i>						
1 year	0.50	0.75	0.56	0.82	0.72	0.74
3 year	0.17	0.18	0.25	0.23	0.47	0.25
5 year	−0.04	−0.79	0.03	−0.60	0.25	−0.72

Notes: The moments computed from the data allows the mean of housing price changes and construction to vary across metropolitan areas. The moments generated from the model use the estimates in Table 1.

overestimating the actual volatility in local labor demand. Predicted volatility is closer to the data in both absolute and percentage terms over longer horizons in the interior regions. Those differences are within \$2000. And, the model captures the sharply rising volatility in price changes over longer horizons in coastal markets,¹⁶ but it never matches the very high price volatility seen in those areas over 3- and 5-year horizons. Except in coastal markets, there appears to be more than enough volatility in local income processes to account for house price volatility.¹⁷

4.2.2. Serial correlation in house prices

Turning now to the model predictions about the serial correlation of house price changes over 1, 3 and 5 year horizons reported in the second panel of Table 4, the model predicts very modest autocorrelation of one- year price changes, ranging from zero in the coastal region to −0.12 for the sunbelt region. Comparing these predictions with the actual data reveals a glaring mismatch between the model and reality. In the real world, as Case and Shiller (1989) documented long ago, there is strong positive serial correlation at one-year frequencies. A one dollar increase in prices during one year is associated with between a 64 and 84 cent increase in prices during the next period, depending upon region.

There is no reasonable calibration of the model that can match the strong positive serial correlation of prices at high frequencies. One possible explanation lies in the microfoundations of the housing market. If there is a learning process at work, whereby people gradually infer the state of demand from prices, then this can generate serial correlation. An alternative explanation is less rational: people see past price changes and infer future price growth (as in Glaeser et al., 2008). Neither idea is captured in our model. In our model, individuals are fully rational and they know the parameters that govern the stochastic process for housing prices and construction of new houses.

¹⁶ This is due to the higher underlying volatility in the local income process (σ is 30% higher in the coastal metropolitan areas), as well as higher moving average component θ .

¹⁷ The results are far different if the BEA income series is used. In that case, the model grossly underpredicts price change variation, by 50–75% or more. See Appendix Table A2 for the analogue to Table 4 based on BEA per capita income. Thus, if one disagrees with our conclusion that the HMDA-based income series is superior and that per capita income better reflects reality, then local housing markets are far too volatile given their (income) fundamentals.

At three year periods, the model and the data continue to diverge. The model continues to predict mean reversion in prices, with the implied serial correlation coefficient ranging from -0.16 for the coastal region to -0.28 for the sunbelt region. The real data shows at least mild positive serial correlation for all but the sunbelt region. Once again, price changes are too positively correlated to match the model.

At 5-year time horizons, the model correctly predicts that price changes mean revert, which is an important stylized fact about local housing markets. However, the point estimates are well below the amount of mean reversion apparent in the data. This is one case in which we are skeptical of the data because our procedures for detrending, which involve subtracting the metro area means, probably induce some spurious mean reversion given the limited fifteen year time series.

While part of the reason for the magnitude mismatch may be due to this factor, that does not provide a complete explanation. If we lengthen the price change time series and include the 1980s, computed mean reversion is lower, but is still higher than our estimates in Table 4. For example, the serial correlation in five year price changes falls from -0.80 to -0.57 in the coastal region. That still is more than double the -0.24 estimate yielded by our model (Table 4). And, using BEA per capita income over the longer time period dating back to 1980 does not yield a perfect (or close to perfect) match either.¹⁸ Hence, the model should be viewed as successful in capturing the fact that there is mean reversion in price changes over long horizons, but it fails to match the strength of that pattern.

4.3. Volatility and serial correlation in construction

4.3.1. Volatility in construction

The model matches the volatility of construction activity at all time horizons in the coastal region quite well, and especially at high frequency (panel 3, Table 4). The match quality is less good, but tolerable, in the sunbelt region. The model predicts much greater volatility over longer horizons, but underpredicts volatility by one-quarter to one-third in this region. We consistently overpredict construction quantity by at least 25% at each horizon in interior markets.¹⁹

4.3.2. Serial correlation in construction

In stark contrast to the model's complete failure to predict strong persistence in price changes over one-year horizons, it always correctly predicts positive, high frequency serial correlation in construction in all regions, with the match being very good for the interior region. Our estimates are about one-third below what the actual data show for the coastal and sunbelt regions, so complete success for the model cannot be claimed here. We do better at 3-year horizons. Our model estimates correctly mimic the lower level of serial correlation at this longer horizon in all regions. And, our point estimates are very close matches to the data in the coastal and sunbelt regions.

However, the estimates over 5-year horizons do not match the data. As noted above, we are skeptical of the value of creating such differences using only 15 years of data. If we go back and include the 1980s, calculated mean reversion fall by about two-thirds in each region (e.g., from -0.79 to -0.27 in the coastal region; from -0.60 to -0.20 in the sunbelt region; and from -0.72 to -0.24

in the interior region). Thus, it certainly looks as if the short time span over which we have higher quality income data is leading to an upwardly biased level of mean reversion in construction for the model to match. That said, our model estimates still do not match those lower levels of mean reversion.²⁰

5. Conclusion

This paper presents a dynamic linear rational expectations model of housing markets based on cross-city spatial equilibrium conditions. Its aim is to show how well a housing model that focuses on income shocks may approximate certain features of the housing market. The model predicts that housing markets will be largely local, which they are, and that construction persistence is fully compatible with price mean-reversion. The model is also consistent with price changes being predictable.

The model has notable successes and failures at fitting the real data. It generally captures important differences across types of markets, especially coastal ones that have inelastic supply sides to their housing markets. The model also does a decent job of accounting for variation in price changes. An important implicit assumption underlying that conclusion is that the HMDA series more accurately reflects the volatility of local income processes than (say) the BEA's per capita income measure. More in-depth research on this data issue seems warranted given its importance in allowing the model to approximate market price volatility. This conclusion also generally applies to the volatility of quantities as reflected in construction permits.

That said, we still cannot precisely match the very high volatility of three- and five-year price changes observed in the inelastically supplied coastal regions. Thus, it also would be useful for future research to try to pin down whether there is excess volatility in those markets.

The model does tolerably well at accounting for the strong positive serial correlation of construction quantities from one year to the next. It also correctly captures the weakening of this persistence over longer horizons, but fails to match the magnitude of the mean reversion in quantities over longer horizons especially. Some of the failure in matching the magnitude of mean reversion in prices and quantities over longer horizons may be due to data error, but that is not a complete explanation. This is another avenue for fruitful research.

The model fails utterly at explaining the strong, high frequency positive serial correlation of price changes. It does a much better job of accounting for the mean reversion over longer, five-year horizons, especially when one takes into account the likelihood our procedures overstate true mean reversion over this longer time span.

This suggests that housing economists have one very big puzzle to explain, along with some other issues. The major puzzle is the strong persistence in high frequency price changes from one year to the next. This failure must be viewed as stark given that attempt to match moments for a time period that does not include the recent extraordinary boom and bust. Other matters that certainly merit closer scrutiny include the extremely high price change volatility in coast markets over longer time horizons and the inability to match mean reversion in construction over longer horizons. These empirical misses are significant, but it remains true that a dynamic urban model can account for many of the important features of housing markets. We see this model as a starting point for a larger agenda of research on real estate dynamics that starts with

¹⁸ Similar patterns are evident in the other regions.

¹⁹ As was the case for price change volatility, using per capita income from the BEA in lieu of household-level income from HMDA leads us to dramatically underpredict construction volatility. To reiterate, if one believes the BEA series more accurately reflects the true variation of local income processes, then housing markets are far too volatile relative to their fundamentals.

²⁰ This is the one case in which using the BEA data on income and the longer time series including the 1980s leads to better matches. In this case, the model always predicts at least modest mean reversion in construction over 5-year horizons, and the match quality is quite good for the interior region.

a dynamic spatial equilibrium model. One natural extension is to include interest rate volatility, and we have sketched such an approach in an earlier version of this paper. A second extension is to relax the assumption of perfect rationality for home-buyers, and perhaps builders as well.

Acknowledgments

Glaeser thanks the Taubman Center for State and Local Government at Harvard University, Gyourko thanks the Research Sponsors Program of the Zell/Lurie Real Estate Center at The Wharton School, University of Pennsylvania, and Nathanson thanks the NSF Graduate Research Fellowship Program for financial support. We appreciate the comments of seminar participants at the University of California-Berkeley, the University of Chicago, MIT, the NBER Summer Institute, and the Federal Home Loan Mortgage Corporation on previous versions of the paper. Graham Elliot and James Stock provided helpful guidance. Jiashuo Feng, Andy Moore, and Jon Steinson provided superb research assistance.

Appendix A

A.1. Sequential two-step GMM estimator

Let Z_t^i denote a vector of observed variables that correspond to observation i at period t . This vector may include lagged variables. Denote by ζ the vector of structural parameters that we want to estimate. In our model, the parameter vector ζ corresponds to $(\delta, \theta, \sigma, c_1, c_2)$. Specifically, we use ξ to summarize the vector of wage-related housing demand parameters, (δ, θ, σ) , and γ to denote the vector of housing supply parameters (c_1, c_2) .

We split the vector of moment functions provided by the model into a subvector that depends only on the wage-related structural parameters $\xi, f(Z_t^i; \xi)$, and the remaining subvector of moment functions that depends both on ξ and $\gamma, v(Z_t^i; \xi, \gamma)$. Therefore, using the set of moment functions $f(\cdot)$, we can obtain GMM estimates of ξ that do not depend on the value of $\gamma, \hat{\xi}^{SEQ}$. Using the vector of moment functions $v(\cdot)$ and our estimates of ξ , we then estimate γ in our second step, $\hat{\gamma}^{SEQ}$. These estimates of γ will depend on the values estimated for ξ in the first step.

We estimate $\hat{\xi}^{SEQ}$ by minimizing the objective function:

$$\widehat{Q}_2(\xi) = \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T f(Z_t^i; \xi) \right]' \widehat{W}_{ff} \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T f(Z_t^i; \xi) \right].$$

The weighting matrix \widehat{W}_{ff} is defined as

$$\widehat{W}_{ff} = \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T f(Z_t^i; \hat{\xi}_1) \cdot f(Z_t^i; \hat{\xi}_1)' \right]^{-1},$$

and $\hat{\xi}_1$ minimizes the first stage objective function

$$\widehat{Q}_1(\xi) = \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T f(Z_t^i; \xi) \right]' I \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T f(Z_t^i; \xi) \right],$$

where I denotes the identity matrix. Given that this estimate $\hat{\xi}^{SEQ}$ does not depend on the value of γ , we compute its asymptotic variance as

$$Var(\hat{\xi}^{SEQ}) = \left(\widehat{F}'_{\xi} \widehat{W}_{ff}^{-1} \widehat{F}_{\xi} \right)^{-1},$$

where \widehat{W}_{ff} is defined above and \widehat{F}_{ξ} is

$$\widehat{F}_{\xi} = (NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T \frac{\partial}{\partial \xi} f(Z_t^i; \xi).$$

Using this initial estimate of ξ , we compute an estimate of γ by minimizing the following objective function:

Table A1

Estimated demand and supply parameters: BEA income data, 1980–2003.

	Coastal	Sunbelt	Interior
δ	0.80 (0.11)	0.90 (0.08)	0.73 (0.07)
θ	0.16 (0.13)	-0.01 (0.16)	-0.06 (0.13)
σ_{ϵ}	\$1200 (200)	\$1000 (100)	\$800 (80)
<i>Supply</i>			
c_1	6.08 (1.21)	1.00 (0.09)	2.03 (0.35)
c_2	1.88 (0.40)	0.20 (0.03)	0.48 (0.12)

Notes: δ, θ , and σ_{ϵ} are the autocorrelation parameter, moving average parameter and residual variance of an ARMA(1,1) estimated for the component of wages that is not explained by a linear time trend and a metropolitan area-specific constant. c_1 denotes the derivative of expected future housing prices with respect to current investment in housing construction; and c_2 denote the derivative of the physical capital cost of building a home with respect to the stock of houses. The standard errors for the demand parameters are efficient two-step GMM standard errors. The ones for the supply parameters account for error coming from the demand estimates.

$$\widehat{Q}_2(\gamma; \hat{\xi}^{SEQ}) = \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T v(Z_t^i; \gamma, \hat{\xi}^{SEQ}) \right]' \widehat{W}_{vv}(\hat{\xi}^{SEQ}) \times \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T v(Z_t^i; \gamma, \hat{\xi}^{SEQ}) \right],$$

where $\widehat{W}_{vv}(\hat{\xi}^{SEQ})$ is

$$\widehat{W}_{vv}(\hat{\xi}^{SEQ}) = \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T v(Z_t^i; \hat{\gamma}_1, \hat{\xi}^{SEQ}) \cdot v(Z_t^i; \hat{\gamma}_1, \hat{\xi}^{SEQ})' \right]^{-1}$$

and $\hat{\gamma}_1$ minimizes the first stage objective function

$$\widehat{Q}_1(\gamma; \hat{\xi}^{SEQ}) = \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T v(Z_t^i; \gamma, \hat{\xi}^{SEQ}) \right]' I \left[(NT)^{-1} \sum_{i=1}^N \sum_{t=1}^T v(Z_t^i; \gamma, \hat{\xi}^{SEQ}) \right].$$

The correct formula for the asymptotic variance of $\hat{\gamma}^{SEQ}$ must account for the fact that its distribution depends not only on the random vector $\{Z_t^i; \forall i, t\}$ but also on the additional random vector $\hat{\xi}^{SEQ}$. Newey (1984) provides the correct formula for the asymptotic variance of the second step estimator:

Table A2

Volatility and serial correlation in house prices and construction: BEA income data, 1980–2003.

Horizon	Coastal		Sunbelt		Interior	
	Model	Data	Model	Data	Model	Data
<i>Volatility of house price changes (\$)</i>						
1 year	5600	12,650	3400	2600	2300	3800
3 year	8800	32,300	5000	6500	3200	9200
5 year	10,100	44,100	5600	9200	3500	12,600
<i>Serial correlation of house price changes</i>						
1 year	-0.09	0.75	-0.16	0.60	-0.20	0.66
3 year	-0.27	0.09	-0.32	0.21	-0.37	0.17
5 year	-0.36	-0.57	-0.39	-0.24	-0.45	-0.31
<i>Volatility of construction (units)</i>						
1 year	800	2600	2800	5300	700	2100
3 year	1900	6700	6700	14,000	1600	5100
5 year	2600	9800	9500	19,600	2200	6800
<i>Serial correlation of construction</i>						
1 year	0.49	0.75	0.56	0.79	0.44	0.73
3 year	0.12	0.27	0.26	0.37	0.05	0.22
5 year	-0.12	-0.27	-0.04	-0.20	-0.29	-0.24

Notes: The moments computed from the data allows the mean of housing price changes and construction to vary across metropolitan areas. The moments generated from the model use the estimates in Table 1.

$$\text{Var}(\hat{\gamma}^{SEQ}) = \left[\widehat{V}'_{\gamma} \widehat{W}_{vv}^{-1} \widehat{V}_{\gamma} \right]^{-1} + \widehat{V}_{\gamma}^{-1} \widehat{V}_{\xi} \left[\widehat{F}_{\xi} \widehat{W}_{ff}^{-1} \widehat{F}'_{\xi} \right]^{-1} \widehat{V}'_{\xi} \widehat{V}_{\gamma}^{-1'} - \widehat{V}_{\gamma}^{-1} \left[\widehat{V}_{\xi} \widehat{F}_{\xi}^{-1} \widehat{W}_{fv} + \widehat{W}_{vf} \widehat{F}_{\xi}^{-1'} \widehat{V}'_{\xi} \right] \widehat{V}_{\gamma}^{-1'}$$

Following Newey and McFadden (1994), the sequential GMM estimators belong to the more general family of *extremum* estimators. This guarantees that they are consistent, asymptotically normal, and have the asymptotic variance described above.

A.2. Moment conditions

A.2.1. Estimation of housing demand parameters

The vectorial moment condition

$$\mathbb{E}[f(\widehat{W}^i; (\delta, \theta, \sigma))] = 0$$

is based on the following vector of moment functions:

$$f(\widehat{W}^i; (\delta, \theta, \sigma)) = \begin{cases} \tau_t^i \\ \tau_t^i \widehat{W}_{t-s}^i & \forall s \geq 3, \\ (\tau_t^i)^2 - (2\theta^2 - 2\theta + 2)\sigma_{\epsilon}^2 \\ \tau_t^i \tau_{t-1}^i - (-\theta^2 + 2\theta - 1)\sigma_{\epsilon}^2 \\ \tau_t^i \tau_{t-2}^i - (-\theta)\sigma_{\epsilon}^2 \end{cases}$$

with

$$\tau_t^i = \Delta \widehat{W}_t^i - \delta \Delta \widehat{W}_{t-1}^i - (1 - \delta)w_1^i = \epsilon_t^i + (\theta - 1)\epsilon_{t-1}^i - \theta\epsilon_{t-2}^i,$$

and $\Delta \widehat{W}_t^i = \widehat{W}_t^i - \widehat{W}_{t-1}^i$. Intuitively, one can think of the random variable τ_t^i as close to (but not exactly) a double-difference of the productivity measure \widehat{W} . The moment function $f(\widehat{W}^i; (\delta, \theta, \sigma))$ is based on the expectation, variance, and serial correlation of this double difference, as well as its covariance with lagged values of the productivity measure \widehat{W} .

A.2.2. Estimation of housing supply parameters

The vectorial moment condition

$$\mathbb{E}[v(\widehat{W}^i; (\delta, \theta, \sigma))] = 0$$

is based on the following vector of moment functions:

$$v(H^i, N^i, I^i; (c_1, c_2)) = \begin{cases} v_t^i \\ v_t^i N_{t-s}^i \\ \kappa_t^i & \forall s \geq 1 \\ \kappa_t^i N_{t-s}^i & \forall s \geq 0, \\ (v_t^i)^2 - \left[\frac{(\bar{\phi} + \hat{\theta})^2}{(\bar{\phi} - \delta)^2} + \hat{\theta}^2 \right] \hat{\sigma}_{\epsilon}^2 \\ (\kappa_t^i)^2 - \left[\frac{(1+r)^2 (\delta + \hat{\theta})^2}{(c_1)^2 (\bar{\phi} - \delta)^2} \right] \sigma_{\epsilon}^2 \end{cases}$$

with

$$v_t^i = ((H_t^i - \widehat{H}_t^i) - \delta(H_{t-1}^i - \widehat{H}_{t-1}^i) + \frac{\alpha(1+r)}{1+r-\phi} ((N_t^i - \widehat{N}_t^i) - \delta(N_{t-1}^i - \widehat{N}_{t-1}^i))),$$

$$\kappa_t^i = ((I_t^i - \widehat{I}_t^i) - \delta(I_{t-1}^i - \widehat{I}_{t-1}^i) + (1 - \phi) ((N_t^i - \widehat{N}_t^i) - \delta(N_{t-1}^i - \widehat{N}_{t-1}^i))).$$

Intuitively, one can think of the random variables v and κ as functions of the differences between the current values of the observable variables (H, I, N) and their steady state values, $(\widehat{H}, \widehat{I}, \widehat{N})$. The moment function $v(H^i, N^i, I^i; (c_1, c_2))$ is based on the expectation and variance of v and κ , as well as their covariances with lagged values of the number of households, N .

A.3. Stochastic processes predicted by the model

If shocks are known as they occur, then our model implies the following ARMA(2,3) process for housing prices

$$\Delta H_t^i = a_0^i + a_1 \Delta H_{t-1}^i + a_2 \Delta H_{t-2}^i + b_0 \epsilon_t^i + b_1 \epsilon_{t-1}^i + b_2 \epsilon_{t-2}^i + b_3 \epsilon_{t-3}^i,$$

where a_0^i denotes a metropolitan area effect, and the parameter vector $(a_1, a_2, b_0, b_1, b_2, b_3)$ is restricted in the following way:

$$a_1 = \phi + \delta,$$

$$a_2 = -\phi\delta,$$

$$b_0 = \frac{\bar{\phi} + \theta}{\bar{\phi} - \delta},$$

$$b_1 = \frac{\delta + r(\delta + \theta) - \theta(\delta + \phi) - \bar{\phi}(1 + \delta + \phi)}{\bar{\phi} - \delta},$$

$$b_2 = \frac{\phi\bar{\phi} - \theta(1 + r + \phi(\bar{\phi} - 1)) + \delta(\bar{\phi} - 1 - r + \theta + \theta\phi)}{\bar{\phi} - \delta},$$

$$b_3 = \phi\theta.$$

The model also predicts an ARMA(2,1) process for the construction of new houses:

$$I_t^i = d_0^i + d_1 I_{t-1}^i + d_2 I_{t-2}^i + e_0 \epsilon_{t-1}^i + e_1 \epsilon_{t-2}^i,$$

where d_0^i denotes a metropolitan area effect and the parameter vector (d_1, d_2, e_1, e_2) is restricted in the following way:

$$d_1 = \phi + \delta,$$

$$d_2 = -\phi\delta,$$

$$e_0 = \frac{(1+r)(\delta + \theta)}{c_1(\bar{\phi} - \delta)},$$

$$e_1 = -\frac{(1+r)(\delta + \theta)}{c_1(\bar{\phi} - \delta)}.$$

Appendix B. Supplementary material

Supplementary data associated with this article can be found, in the online version, at <http://dx.doi.org/10.1016/j.jue.2014.02.003>.

References

Alonso, W., 1964. Location and Land Use. Harvard University Press, Cambridge, MA.

Arellano, M., Bond, S., 1991. Some tests of specification for panel data: Monte Carlo evidence and an application to employment equations. *Review of Economic Studies* 58 (2), 277–297.

Case, K.E., Shiller, R., 1989. The efficiency of the market for single family homes. *American Economic Review* 79 (1), 125–137.

Cutler, D., Poterba, J., Summers, L., 1991. Speculative dynamics. *Review of Economic Studies* 58 (3), 529–546.

Glaeser, E., Gyourko, J., 2005. Urban decline and durable housing. *Journal of Political Economy* 113 (2), 345–375.

Glaeser, E., Gyourko, J., Sacks, R., 2005. Why have house prices gone up? *American Economic Review* 95 (2), 329–333.

Glaeser, E., Gyourko, J., Saiz, A., 2008. Housing supply and housing bubbles. *Journal of Urban Economics* 64 (2), 198–217.

Gyourko, J., Saiz, A., 2006. Construction costs and the supply of housing structure. *Journal of Regional Science* 46 (4), 661–680.

Gyourko, J., Mayer, C., Sinai, T., 2013. Superstar cities. *American Economic Journal: Economic Policy* 5 (4), 167–199.

Hansen, L., 1982. Large sample properties of generalized method of moments estimators. *Econometrica* 50, 1029–1054.

Newey, W., 1984. A method of moments interpretation of sequential estimators. *Economic Letters* 14, 201–206.

Newey, W., McFadden, D., 1994. Large sample estimation and hypothesis testing. In: Engle, R.F., McFadden, D.L. (Eds.), *Handbook of Econometrics*, vol. IV. North-Holland Publishing Co, Amsterdam.

Polinsky, A., Ellwood, D., 1979. An empirical reconciliation of micro and grouped estimates of the demand for housing. *Review of Economics and Statistics* 61 (2), 199–205.

Roback, J., 1982. Wages, rents, and the quality of life. *Journal of Political Economy* 90 (4), 1257–1278.

Rosen, S., 1979. Wage-based indexes of urban quality of life. In: Mieszkowski, P., Straszheim, M. (Eds.), *Current Issues in Urban Economics*. Johns Hopkins University Press, Baltimore.

Saiz, A., 2003. Room in the kitchen for the melting pot: immigration and rents. *Review of Economics and Statistics* 85 (3), 502–521.

Topel, R., Rosen, S., 1988. Housing investment in the United States. *Journal of Political Economy* 96 (4), 718–740.

Van Nieuwerburgh, S., Weill, P., 2010. Why has house price dispersion gone up? *Review of Economic Studies* 77 (4), 1567–1606.