How Important are Inflation Expectations for the Nominal Yield Curve?*

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Abstract

Less than you think. Relative to the data, standard macro-finance term structure models rely too heavily on the volatility of expected inflation news as a source for variations in nominal yield shocks. In this paper, I develop a nonlinear Bayesian state-space model that accounts for several bond market features, without resorting to an expected inflation channel that counterfactually dominates the variation in nominal yield shocks. The estimation of the model suggests that, for the last two decades, inflation-related risk factors have not played an important role in driving either expected excess bond returns or the term premium component of the nominal yield curve. (JEL C11, C32, C58, E43, E44, G12)

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1 Introduction

Understanding the role of inflation expectations on the nominal yield curve is of key interest for financial and economic decisions, such as risk management, bond pricing, and public policy. Macro-finance term structure models are useful in identifying the economic mechanism behind the substantial variations of nominal yields and their macroeconomic underpinnings. To this end, economists have used long-run risk (e.g., Piazzesi and Schneider 2007; Bansal and Shaliastovich 2013; Song 2016), New Keynesian (e.g., Rudebusch and Swanson 2012; Kung 2015), and habit formation (e.g., Wachter 2006) models. However, as pointed out by Duffee (2016), these models require a counterfactually high volatility of expected inflation news to match the empirical volatility of nominal yield shocks. In the data, the variance of expected inflation news accounts for around 20 percent of the variance of nominal yield shocks, which is strongly at odds with model-implied ratios that, in the case of long-run risk and New Keynesian models, often exceed 100 percent.[1]

In this paper, I develop a nonlinear Bayesian state-space macro-finance endowment model that closely matches the inflation variance ratios proposed by Duffee (2016).[2] The model features time preference shocks, recursive preferences, inflation nonneutrality, multiple stochastic volatility processes, and time-aggregation of consumption. Specifically, I build on the long-run risks setup of Bansal and Shaliastovich (2013) and Schorfheide, Song, and Yaron (2016) in which time variation in expected consumption and inflation and their respective volatilities are key in accounting for bond risk premia dynamics and several other bond market features. The time rate of preference shocks included in the model, on the other hand, primarily affects real rate fluctuations and thus limits the role of expected inflation shocks in driving nominal yield innovations. To assess the empirical validity of these different channels, I estimate the model using a Bayesian MCMC particle filter approach as in Shaliastovich (2015), Schorfheide et al. (2016), and Song (2016). To the best of

[1]While habit formation models fare better with values of around 50 percent, they have problems along other dimensions. For instance, habit models are not able to simultaneously explain the evidence of predictability of bond and currency returns. Wachter (2006) requires a counter-cyclical interest rates to account for bond return predictability, while Verdelhan (2010) relies on a pro-cyclical interest rates to account for the violations of uncovered interest parity in currency markets.

[2]To compute the data-implied inflation variance ratios for long horizons, Duffee (2016) relies on a statistical model of inflation and bond yields. I extend Duffee's analysis by assuming heteroskedasticity in innovations (instead of splitting the estimation in different subperiods) and conclude that the model fit improves significantly and produces ratios of around 30 percent. Nevertheless, the 95 percent of the confidence intervals are around 50 percent, which is still at odds with model-implied values.

my knowledge, this model is the first one that does not rely heavily on the volatility of inflation expectations to match the empirical volatility of yield shocks.

Even though variations in innovations to expected inflation play a limited role in driving yield shocks, the model is able to match the unconditional level, slope, and standard deviation of the nominal interest rates in the data. In addition, it generates sizable variation in bond risk premia, that is, it is able to generate time-varying term premium that mimics the estimates based on reduced-form Gaussian affine term structure models without distorting its macroeconomic fit or requiring a large coefficient of relative risk aversion (roughly 10). Moreover, it qualitatively and quantitatively accounts for the evidence of bond return predictability documented in Cochrane and Piazzesi (2005) and for the failure of the expectation hypothesis, as identified in Campbell and Shiller (1991). Overall, the estimation of the model suggests that inflation expectations are no longer the predominant macroeconomic force behind nominal yield curve variations. While term premium on long-term bonds and expected excess bond returns are positive on average due mostly to inflation-related risk premia, they have been negative for the last decade, driven by real uncertainties.

As in the long-run risks literature, variations in bond risk premia are driven entirely by time-varying conditional volatilities of the predictable components of consumption growth and inflation (e.g., Bansal and Shaliastovich 2013; Doh 2013). My model differs from this literature in adding the time preference shocks channel as an important source of nominal yield shocks. In this model, time preference shocks arise from stochastic changes in agents' discount factor. I show that they are crucial for generating volatile and persistent fluctuations in expected short-term real rates, which in turn allow the model to match the empirical volatility of nominal yield shocks without requiring counterfactually high volatility of inflation expectations innovations. Time preference shocks determine the attitudes of the representative household towards saving and they are, by assumption, orthogonal to the real and nominal economy affecting financial decisions of households. In line with this reasoning, the filtered time preference shocks variable is highly correlated with the adjusted National Financial Conditions Index published by the Federal Reserve of Chicago,[3] which measures risk, liquidity, and leverage in money, debt, and equity markets uncorrelated with

[3]The Adjusted National Financial Conditions Index is provided by the Federal Reserve Bank of Chicago published at https://alfred.stlouisfed.org/series?seid=ANFCI.

economic conditions. Albuquerque, Eichenbaum, Luo, and Rebelo (2016), and Schorfheide et al. (2016), also include time preference shocks in their model specification to account for the weak correlation between stock returns and measurable fundamentals. For instance, the latter show that they are crucial to generate a weak correlation between consumption growth and the risk-free rate as observed in the data. In addition, the former show that they are also able to produce an upward-sloping nominal yield curve. These features are preserved in this model.

While innovations to the time preference shocks are key drivers of yields shocks, they do not play a role in driving fluctuations in bond risk premia.[4] With preferences for early resolution of uncertainty, an increase in real uncertainty lowers bond risk premia, whereas inflation uncertainty raises it, provided that expected inflation shocks negatively affect expected consumption growth. In this setting, nominal shocks are priced, which makes long-term bonds particularly risky since innovations that produce a persistent increase in expected inflation generate persistently low real yields and low expected consumption growth. This is a second complementary mechanism generating a positive slope in the nominal yield curve.

Regarding the other model ingredients, additional time-varying volatilities in the *iid* component of consumption growth and inflation, and time-aggregation of consumption are negligible for the nominal yield curve but are important for tracking the macroeconomic series. Moreover, monthly measurement errors in the process of consumption that average out at the quarterly frequency aid the identification of the persistent parameter of expected consumption growth. In fact, the estimate is remarkably similar with or without considering bond prices in the estimation. Finally, I show that there is enough information in the macroeconomic series, over the sample period 1960 to 2014, to identify a significant negative nonneutral effect of inflation on growth.

This paper is organized as follows. In Section 2, I introduce the model and cast it into a state-space form. Section 3 describes the Metropolis-Hastings sampler used for Bayesian inference, the data set, and prior and posterior distribution of model parameters. Section 4 analyzes the importance of inflation expectations for the nominal yield curve. Finally, Section 5 provides concluding remarks.

[4]This result is based on the assumption of homoskedastic innovations in the time preference shocks.

2 Model specification

In this section, I present the macro-finance model and cast it into a state-space form. In Section 2.1, I describe the preference of the representative household. In Section 2.2, I present the exogenous dynamics of consumption growth and inflation. In Section 2.3, I characterize the model solution and the equilibrium nominal yield curve. Finally, in Section 2.4, I cast the model into a state-space form.

2.1 Preferences

The representative household has Epstein and Zin (1989) recursive preferences and maximizes lifetime utility which is a function of current utility and the certainty equivalent of future utility V^*_{t+1}:

$$V_t = \max_{C_t}[(1-\delta)\lambda_t C_t^{1-\frac{1}{\psi}} + \delta(V^*_{t+1})^{1-\frac{1}{\psi}}]^{\frac{1}{1-\frac{1}{\psi}}} \tag{1}$$

where C_t denotes consumption at time t, ψ is intertemporal elasticity of substitution, δ is the discount factor, and λ_t denotes a time-varying preference parameter. The certainty equivalent of future utility is the guaranteed value of lifetime utility at time $t+1$ given by $V^*_{t+1} = (E_t V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}$, where the parameter γ represents the coefficient of relative risk aversion.

As in Albuquerque et al. (2016) and Schorfheide et al. (2016), I also allow for time preference shocks to the time rate of preferences given by the ratio $x_{\lambda,t} \equiv \lambda_{t+1}/\lambda_t$, which determines how the household trades off current versus future utility. These shocks evolve according to:

$$x_{\lambda,t+1} = \rho_\lambda x_{\lambda,t} + \sigma_\lambda \eta_{\lambda,t+1} \quad \text{with} \quad \eta_{\lambda,t+1} \sim i.i.d.N(0,1) \tag{2}$$

and can be thought of as demand shocks that capture changes in the household attitudes towards savings. The representative household is subject to the following budget constraint: $W_{t+1} = (W_t - C_t)R_{c,t+1}$, where W_t denotes its wealth at time t and $R_{c,t+1}$ is the return on an asset that pays aggregate consumption as dividends.

2.2 Consumption and inflation dynamics

The exogenous dynamics of the logarithm of consumption growth, $\Delta c_{t+1} = log\frac{C_{t+1}}{C_t}$, and the logarithm of the inflation rate, π_{t+1}, are written as follows:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_{c,t} + \sigma_{c,t}\eta_{c,t+1} \\ \pi_{t+1} &= \mu_\pi + x_{\pi,t} + \sigma_{\pi,t}\eta_{\pi,t+1} \end{aligned} \tag{3}$$

where the process of conditional expectations is given by

$$\begin{aligned} x_{c,t+1} &= \rho_{cc}x_{c,t} + \rho_{c\pi}x_{\pi,t} + \sigma_{xc,t}\eta_{xc,t+1} \\ x_{\pi,t+1} &= \qquad\quad\;\; + \rho_{\pi\pi}x_{\pi,t} + \sigma_{x\pi,t}\eta_{x\pi,t+1} \end{aligned}$$

and volatilities evolve according to exponential Gaussian processes

$$\sigma_{i,t} = \varphi_i \sigma exp(h_{i,t}), \quad \text{with} \quad h_{i,t+1} = \rho_{h_i} h_{i,t} + \sigma_{h_i}\omega_{i,t+1}.$$

I normalize $\varphi_c = 1$ and all innovations are distributed according to

$$\eta_{i,t+1}, \quad \omega_{i,t+1}, \quad \epsilon_{i,t+1} \quad \sim i.i.d.N(0,1) \quad \text{for} \quad i = \{c, \pi, xc, x\pi\}.$$

Specification (3) is based on Bansal and Yaron (2004), Piazzesi and Schneider (2007), and Bansal and Shaliastovich (2013), and decomposes real consumption growth and inflation into predictable ($x_{c,t},\ x_{\pi,t}$) and transitory ($\sigma_{c,t}\eta_{c,t+1},\ \sigma_{\pi,t}\eta_{\pi,t+1}$) components. I assume that the predictable components, that is expected consumption growth, x_c, and expected inflation, x_π, follow a bivariate VAR(1) process, with time-varying volatilities given by $\sigma^2_{xc,t}$ and $\sigma^2_{x\pi,t}$, respectively. I allow for expected inflation to directly feed into expected consumption growth via the parameter $\rho_{c\pi}$. This parameter characterizes the nonneutrality of inflation. In contrast to Bansal and Shaliastovich (2013), I consider four separate volatility processes. The long-run stochastic volatilities $\sigma^2_{xc,t}$ and $\sigma^2_{x\pi,t}$ are the drivers of bond risk premia, while the short-run volatilities $\sigma^2_{c,t}$ and $\sigma^2_{\pi,t}$ are negligible for bond prices, but are important for tracking the consumption growth and inflation series. Furthermore, the logarithm of the volatility process is assumed to be normal, which guarantees that

$\sigma_{i,t}$ is positive throughout.

2.3 Model solution

The Euler equation associated with any continuous real return, r_{t+1}, can be written as

$$E_t[exp(m_{t+1} + r_{t+1})] = 1 \tag{4}$$

where the logarithm of the real stochastic discount factor (SDF) implied by the assumed household preferences is

$$m_{t+1} = \theta log\delta + \theta x_{\lambda,t} - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1)r_{c,t+1} \qquad \text{with} \qquad \theta = \frac{1-\gamma}{1-\frac{1}{\psi}}. \tag{5}$$

Note that when $\gamma = \frac{1}{\psi}$ the utility function is of the constant relative risk aversion form and the SDF becomes independent of the log return on the consumption claim denoted by $r_{c,t+1} = log(R_{c,t+1})$. To price nominal payoffs it is useful to specify the nominal discount factor which is equal to the real one minus the inflation rate:

$$m^{\$}_{t+1} = m_{t+1} - \pi_{t+1} \tag{6}$$

To entertain an analytical model solution I use two different approximations. The first, proposed by Campbell and Shiller (1988), involves a log-linear Taylor expansion $r_{c,t+1} \approx \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1}$, where $\kappa_0 = log(1 + exp(\bar{pc})) - \kappa_1 \bar{pc}$ and $\kappa_1 = \dfrac{exp(\bar{pc})}{1 + exp(\bar{pc})}$ are constants determined endogenously by the unconditional mean of the price-consumption ratio, pc_{t+1}, defined as $\bar{pc}$. The second follows Schorfheide et al. (2016) and takes a linear approximation of the volatility process around the unconditional mean of h_i, given by $\sigma^2_{i,t+1} \approx \sigma^2_{i,0} + \nu_i(\sigma^2_{i,t} - \sigma^2_{i,0}) + \sigma_{\omega_i}\omega_{i,t+1}$ for $i = \{c, \pi, xc, x\pi\}$ with $\nu_i = \rho_{h_i}$, $\sigma^2_{i,0} = (\varphi_i \sigma)^2$, and $\sigma_{\omega_i} = 2(\varphi_i\sigma)^2\sigma_{h_i}$.

The first approximation allows me to write innovations to the real SDF as a linear combination in shocks,

$$
\begin{aligned}
m_{t+1} - E_t m_{t+1} = \quad & \underbrace{-\lambda_c \sigma_{c,t} \eta_{c,t+1}}_{\text{short-run consumption risk}} \underbrace{-\lambda_\pi \sigma_{\pi,t} \eta_{\pi,t+1}}_{\text{short-run inflation risk}} \underbrace{-\lambda_\lambda \sigma_{\lambda,t} \eta_{\lambda,t+1}}_{\text{preference risk}} \\
& \underbrace{-\lambda_{xc} \sigma_{xc,t} \eta_{xc,t+1}}_{\text{long-run consumption risk}} \underbrace{-\lambda_{x\pi} \sigma_{x\pi,t} \eta_{x\pi,t+1}}_{\text{long-run inflation risk}} \\
& \underbrace{- \sum_{i \in \{\sigma_c, \sigma_\pi, \sigma_{xc}, \sigma_{x\pi}\}} \lambda_i \sigma_{\omega_i} \omega_{i,t+1}}_{\text{Volatility risk}}
\end{aligned}
\tag{7}
$$

which determine the sources and compensation for risks. The λs denote the equilibrium market prices of risk and are equal to

$$
\lambda_c = \gamma, \quad \lambda_\pi = 0, \quad \lambda_\lambda = \frac{\kappa_1 \rho_\lambda - \theta}{1 - \kappa_1 \rho_\lambda}, \quad \lambda_{xc} = \frac{(\gamma - \frac{1}{\psi})\kappa_1}{1 - \kappa_1 \rho_{cc}}, \quad \lambda_{x\pi} = \rho_{c\pi} \frac{(\gamma - \frac{1}{\psi})\kappa_1^2}{(1 - \kappa_1 \rho_{cc})(1 - \kappa_1 \rho_{\pi\pi})}, \tag{8}
$$

$$
\lambda_{\sigma_c} = \frac{(\gamma - \frac{1}{\psi})(1-\gamma)\kappa_1}{2(1 - \kappa_1 \nu_c)}, \quad \lambda_{\sigma_\pi} = 0, \quad \lambda_{\sigma_{xc}} = \frac{(\gamma - \frac{1}{\psi})(1-\gamma)\kappa_1^3}{2(1 - \kappa_1 \rho_{cc})^2 (1 - \kappa_1 \nu_{xc})}, \quad \lambda_{\sigma_{x\pi}} = \rho_{c\pi}^2 \frac{(\gamma - \frac{1}{\psi})(1-\gamma)\kappa_1^5}{2(1 - \kappa_1 \rho_{cc})^2 (1 - \kappa_1 \rho_{\pi\pi})^2 (1 - \kappa_1 \nu_{x\pi})}.
$$

Note that the market price of short-run consumption risk, λ_c, is equal to the coefficient of relative risk aversion. Since current inflation is independent from the real economy, the prices of risk associated with it ($\lambda_\pi,\ \lambda_{\sigma_\pi}$) are zero. In addition, when agents have preference for early resolution of uncertainty ($\gamma > \frac{1}{\psi}$), the market price of expected consumption growth risk, λ_{xc}, is positive and $\lambda_{x\pi}$ is negative whenever expected inflation is bad news for the real economy ($\rho_{c\pi} < 0$). Furthermore, the market prices of short-, λ_{σ_c}, and long-run, $\lambda_{\sigma_{xc}}$, real uncertainties are negative, as well as the market price of the long-run inflation volatility, $\lambda_{\sigma_{x\pi}}$. The price of the time preference shocks risk, λ_λ, is guaranteed to be positive whenever $\theta \leq 0$ provided that $\rho_\lambda > 0$. Note that when $\gamma = \frac{1}{\psi}$ all prices with the exception of λ_c and λ_λ are zero. Finally, the nominal market prices of risk are all equal to their real counterparts except for $\lambda_\pi^\$$, which now equals one.

I proceed to price the equilibrium nominal bond yields, while Appendix A.2 provides a solution to pc_t and shows an analytical expressions for the equilibrium nominal SDF in terms of the state variables of the economy.

2.3.1 Equilibrium nominal bond yields.

Let $P_{t,n}^\$$ be the price at time t of an n-period zero-coupon nominal bond that pays one unit of the numeraire in n periods. Using the Euler equation, the price can be written recursively as

$$P^{\$}_{t,n} = E_t[M^{\$}_{t+1} P^{\$}_{t+1,n-1}] \tag{9}$$

and can be solved by conjecturing that the logarithm of the nominal bond yields, $y^{\$}_{t,n} = -\frac{1}{n} log P^{\$}_{t,n}$, is linear in the state variables:

$$y^{\$}_{t,n} = \frac{1}{n}((B^{\$}_{0,n} + B^{\$}_{c,n} x_{c,t} + B^{\$}_{\pi,n} x_{\pi,t} + B^{\$}_{\lambda,n} x_{\lambda,t} + B^{\$}_{\sigma_c,n} \sigma^2_{c,t} + B^{\$}_{\sigma_\pi,n} \sigma^2_{\pi,t} + B^{\$}_{\sigma_{xc},n} \sigma^2_{xc,t} + B^{\$}_{\sigma_{x\pi},n} \sigma^2_{x\pi,t}). \tag{10}$$

In Appendix A.4 I provide an analytical expression for the Bs, which measure the sensitivity of bond prices to systemic risks. In order to develop economic intuition in terms of the size and sign of each of these coefficients, Figure 1 shows the equilibrium nominal bond yield loadings (evaluated at the posterior median estimates) as a function of bond maturity n.

[Place Figure 1 about here]

Nominal bonds hedge expected consumption risk. A higher expected consumption growth today decreases bond prices and increases nominal yields ($B^{\$}_{c,n}$>0), due to the agent's motive to smooth consumption intertemporally. In this scenario, the agent wants to borrow to increase today's consumption and since bonds are in zero net supply, aggregate borrowing cannot increase. In equilibrium, returns on bonds must increase to induce the agent to borrow less. Alternatively, with preference for early resolution of uncertainty ($\gamma > \frac{1}{\psi}$), shocks to expected consumption growth are priced. An increase in real volatility increases the agent's uncertainty about future growth and as a result wants to hedge by buying risk-free assets, and in equilibrium, bond yields fall ($B^{\$}_{\sigma_c,n}, B^{\$}_{\sigma_{xc},n} < 0$). Since shocks to expected consumption growth have larger long-term effects, the sensitivity of bond prices to this risk is an order of magnitude larger (in absolute value) than the beta of short-run real volatility risk (see the straight black line in panels (b) and (c) of Figure 1). Furthermore, the longer the bond maturity, the higher the insurance the bond provides against these types of risks and hence the higher its price. This mechanism generates a downward-sloping nominal yield curve. In contrast, the nominal and preference risk channels counteract this mechanism and generate an upward-sloping yield curve.

To understand the nominal channel as described by Bansal and Shaliastovich (2013) and

Eraker, Shaliastovich, and Wang (2016), it is useful to consider the following Fisher-type equation:

$$y_t^{\$,(n)} = y_t^{(n)} + \frac{1}{n} E_t \pi_{t \to t+n} - \frac{1}{2}\frac{1}{n} Var_t \pi_{t \to t+n} + \frac{1}{n} Cov_t(m_{t \to t+n}, \pi_{t \to t+n}),$$

The expected inflation term, $E_t \pi_{t \to t+n}$, directly impacts the valuation of nominal bond prices; nominal yields rise with positive shocks to expected inflation ($B^{\$}_{\pi,n}$>0). In addition, if expected inflation reduces expected consumption growth ($\rho_{c\pi} < 0$), times of high long-term inflation rates are times of low consumption growth. Hence, the covariance term in the above equation is positive and increases with maturity. This term captures the inflation premium in the economy. With preference for early resolution of uncertainty, shocks to expected inflation are also priced, which makes long-term bonds particular risky since shocks that produce a persistent increase in this component generate persistently low real yields and low expected consumption growth. This mechanism is crucial for generating an upward-sloping nominal yield curve.

In contrast to Bansal and Shaliastovich (2013), in this model I include an extra state variable determined by the time preference shocks. As shown in panel (a) of Figure 1, nominal yields are decreasing functions of $x_{\lambda,t}$. When $x_{\lambda,t}$ rises, the representative household values tomorrow's consumption more, relative to the present, and wants to increase savings. In equilibrium, bond prices have to rise ($B^{\$}_{\lambda,n} < 0$) to clear bond markets. This model is also able to generate an upward-sloping nominal yield curve (note that $B^{\$}_{\lambda,m} > B^{\$}_{\lambda,n}$ if $m > n$) via the time preference shocks risk, given that longer maturity bonds have a higher exposure to it. This implies that the model does not need to rely entirely on the expected inflation channel to reproduce this feature of the data. Furthermore, time preference shocks are a promising mechanism to drive volatile and persistent fluctuations in expected short-term real rates which in turn might generate sizable variations in nominal yield shocks without requiring large fluctuations in expected inflation news. I turn to this issue next.

2.3.2 Inflation variance ratios. Following Campbell and Ammer (1993) and Duffee (2016), I decompose an n-period yield into expectations of average ex-ante real rates, future average inflation, and average expected excess returns over the life of the bond:

$$y^{\$}_{t,n} = \frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1} + \frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i} + \frac{1}{n}\sum_{i=1}^{n} E_t rx^{\$}_{r \to t+1, n-i+1}, \tag{11}$$

where the ex-ante real rate is defined as the yield on a one period nominal bond minus expected inflation $r_t = y^{\$}_{t,1} - E_t\pi_{t+1}$. The last term on the right is often described as the bond's nominal term premium, which I write as $tp^{\$}_{t,n}$.

Innovations to the n-maturity yield from $t-1$ to t are equal to the sum of news about these three components:

$$\epsilon^{(n)}_{y^{\$},t} = \epsilon^{(n)}_{r,t} + \epsilon^{(n)}_{\pi,t} + \epsilon^{(n)}_{tp^{\$},t} \tag{12}$$

where the shocks are defined as

$$\epsilon^{(n)}_{y^{\$},t} = E_t y^{\$}_{t,n} - E_{t-1} y^{\$}_{t,n}, \qquad \epsilon^{(n)}_{r,t} = \tfrac{1}{n}\textstyle\sum_{i=1}^{n} E_t r_{t+i-1,1} - \tfrac{1}{n}\sum_{i=1}^{n} E_{t-1} r_{t+i-1,1}.$$

$$\epsilon^{(n)}_{tp^{\$},t} = E_t tp^{\$}_{t,n} - E_{t-1} tp^{\$}_{t,n}, \qquad \epsilon^{(n)}_{\pi,t} = \tfrac{1}{n}\textstyle\sum_{i=1}^{n} E_t \pi_{t+i} - \tfrac{1}{n}\sum_{i=1}^{n} E_{t-1}\pi_{t+i}.$$

In Appendix A.7 I provide an analytical expression for these shocks in terms of model parameters. Duffee (2016) defines the unconditional inflation variance ratio of an n-period nominal bond as the ratio of variance of inflation news to the variance of yield shocks:

$$\text{inflation variance ratio} \equiv \frac{Var(\epsilon^{(n)}_{\pi,t})}{Var(\epsilon^{(n)}_{y^{\$},t})}, \tag{13}$$

and measures the size of expected inflation shocks relative to nominal yield shocks. As shown by Duffee (2016), at quarterly frequency, variances of news about expected inflation account for around 20 percent of variances of yield shocks. This percentage is strongly at odds with standard macro-finance model-implied ratios which often exceed 100 percent. The main problem is that these models do not allow for a large real-rate or excess return channel to generate variations in yield shocks.

This model generates variations in shocks to real rates via four components. To see this it is useful to explicitly write the model-implied expressions for $\epsilon^{(n)}_{r,t}$ as

$$\begin{aligned}\epsilon^{(n)}_{r,t} =& \frac{1}{n}\frac{1}{\psi}\frac{1-\rho^n_{cc}}{1-\rho_{cc}}\sigma_{xc,t-1}\eta_{xc,t} + \frac{1}{n}\frac{\rho_{c\pi}}{1-\rho_{cc}}\frac{1}{\psi}[(1-\rho^{n-1}_{cc}) + \rho_{\pi\pi}(1-\rho^{n-2}_{cc}) + \cdots + \rho_{\pi\pi}^{n-2}(1-\rho_{cc})]\sigma_{x\pi,t-1}\eta_{x\pi,t} \\ &+ \frac{1}{n}\sum_{i\in\{c,\pi,xc,x\pi\}} B^{\$}_{\sigma_i,1}\frac{1-\nu^n_i}{1-\nu_i}\sigma_{\omega_i}\omega_{i,t+1} - \frac{1}{n}\rho_\lambda\frac{1-\rho^n_\lambda}{1-\rho_\lambda}\sigma_\lambda\eta_{\lambda,t}\end{aligned} \tag{14}$$

The first component is related to persistent movements in expected consumption growth. If $\sigma_{xc,t-1}$ is small and the intertemporal elasticity of substitution, ψ, is high, then short-term real rates do not vary much via this component. The second component is associated with the nonneutral effect of expected inflation on expected consumption growth. Bad news for expected inflation translates into bad news for expected consumption growth, which in turn lowers expected future real rates. Hence, variations in nominal yields via this component are mitigated by the negative covariation that it shares with expected inflation shocks, $\epsilon_{\pi,t}^{(n)}$, provided that $\rho_{c\pi} < 0$ (i.e., $cov(\epsilon_{r,t}^{(n)}, \epsilon_{\pi,t}^{(n)}) < 0$). In some models, this negative covariance is able to decrease the volatility of nominal yield shocks below the volatility of inflation news, which generates inflation variance ratios above 100 percent. The third component comes from shocks to the volatility processes and affects shocks to real rates through the precautionary savings channel. However, since σ_{ω_i} are usually small, they cannot contribute much to variations in nominal yield shocks. Finally, the last component is associated with innovations to the time preference shocks. Innovations to x_λ affect the agents' decisions towards saving, which in turn move short-term real rates. If these innovations are volatile and persistent, then they will generate similar movements in real rates. In Section 4.2 I show that this is an important mechanism to match the observed volatility of yield shocks without requiring a counterfactually high volatility of inflation expectations innovations.

2.3.3 Expected excess bond returns. The presence of stochastic volatility gives rise to time variations in bond risk premia. To see this, consider the one-period excess log return on buying an n-period bond at time t and selling it at time $t+1$ as an $n-1$ period bond:

$$rx_{r\to t+1,n}^{\$} = ny_{t,n}^{\$} - (n-1)y_{t+1,n-1}^{\$} - y_{t,1}^{\$}$$

The one-period expected excess return on nominal bonds is determined by the negative of the covariance between the innovations to excess log returns and the innovations to the nominal SDF:

$$\begin{aligned} E_t rx_{r\to t+1,n}^{\$} + \frac{1}{2}V_t rx_{r\to t+1,n}^{\$} &= -cov_t(m_{t+1}^{\$}, rx_{r\to t+1,n}^{\$}) \\ &= \underbrace{-\sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}} B_{i,n-1}^{\$}\lambda_i\sigma_{\omega_i}^2}_{\text{Volatility risk}} \underbrace{-B_{\lambda,n-1}^{\$}\lambda_\lambda\sigma_\lambda^2}_{\text{Preference risk}} \underbrace{-B_{c,n-1}^{\$}\lambda_{xc}\sigma_{xc,t}^2}_{\text{long-run growth risk}} \underbrace{-B_{\pi,n-1}^{\$}\lambda_{x\pi}\sigma_{x\pi,t}^2}_{\text{long-run inflation risk}} \end{aligned} \quad (15)$$

Hence, the one-period conditional excess return on nominal bonds can be attributed to (i) short- and long-run volatility risks, (ii) preference risk, iii) long-run growth risk, and (iv) long-run inflation risk. The bond risk premium is time-varying and driven by compensations for expected consumption and inflation shocks. Note that nominal bond risk premium decreases with real uncertainty ($B_{c,n}^{\$} > 0$

and $\lambda_{xc} > 0$) and increases at times of high inflation uncertainty ($B^{\$}_{\pi,n} > 0$ and $\lambda_{x\pi} < 0$). In addition, long-term bonds also command a constant positive risk premium induced by the time preference shocks risk ($B^{\$}_{\lambda,n-1} < 0$ and $\lambda_{\lambda} > 0$).

2.3.4 Term premium.

Solving $tp^{\$}_{t,n}$ in equation (11) delivers an expression for the n-maturity nominal bond term premium:

$$tp^{\$}_{t,n} = \text{const} + \frac{1}{n}(B^{\$}_{\sigma_{xc},n} - B^{\$}_{\sigma_{xc},1}\frac{1-\nu^n_{xc}}{1-\nu_{xc}})\sigma^2_{xc,t} + \frac{1}{n}(B^{\$}_{\sigma_{x\pi},n} - B^{\$}_{\sigma_{x\pi},1}\frac{1-\nu^n_{x\pi}}{1-\nu_{x\pi}})\sigma^2_{x\pi,t} \tag{16}$$

The term premium measures the additional compensation a risk averse investor needs to be indifferent between purchasing an n-period bond and holding it until maturity or rolling over one-period bonds for a total of n periods. Under the expectation hypothesis the term premium is constant. In this model, the term premium is time-varying due to variations in bond risk premia driven by long-run real and nominal uncertainties. In Section 4.4, I show that the term premium implied by the model is able to account for the evidence of bond return predictability documented in Cochrane and Piazzesi (2005) and for the failure of the expectation hypothesis, as identified in Campbell and Shiller (1991). Furthermore, in Section 4.6, I show that the model is able to generate sizable variations in term premium. Finally, note that from this expression it is straight forward to compute the term premium shocks component, $\epsilon^{(n)}_{tp^{\$},t}$, given by

$$\epsilon^{(n)}_{tp^{\$},t} = \frac{1}{n}\sum_{i\in\{xc,x\pi\}}(B^{\$}_{\sigma_i,n} - B^{\$}_{\sigma_i,1}\frac{1-\nu^n_i}{1-\nu_i})\sigma_{\omega_i}\omega_{i,t+1}. \tag{17}$$

This channel cannot contribute much to the variance of yield shocks, since, as described before, σ_{ω_i} is usually small. Moreover, note that time preference shocks do not contribute to the term premium given the assumption of homoskedastic innovations in agents' rate of preferences.

2.4 State-space representation of the model

The state-space representation of the model consists of two equations. The first is the measurement equation, which links the observed variables with the model-implied ones and adds measurement errors to capture discrepancies that might be present between these two components. In addition, measurement errors account for the stochastic singularity problem that arises when the number of observables is greater than the number of state variables. The second is the state transition equation, which describes the law of motion of the state variables.

2.4.1 Measurement equation. To distinguish observed variables from model-implied ones, I use the superscript o. The observables are consumption growth, Δc^o, inflation, π^o, and zero-coupon nominal bond yields, $y^{\$,o}$.

Measurement equation for consumption growth. To aid the identification of time-varying volatilities, I use the highest frequency available which is monthly (e.g., Drost and Nijman 1993). However, monthly consumption data has nontrivial measurement errors masking the actual dynamics of consumption (e.g., Wilcox 1992). Following Schorfheide et al. (2016), I include monthly measurement errors in the process of consumption that average out at the quarterly frequency and which allow the identification of the persistent component of consumption growth as well as time-varying short- and long-run volatilities.

Let the subscript t represent the monthly time as $t = 3(j-1) + m$, where m indexes the month within quarter j and $m = 1, 2, 3$. Specifically, I assume that:

$$\begin{aligned}\Delta c^o_{3(j-1)+1} &= \Delta c_{3(j-1)+1} + \sigma_\epsilon(\epsilon_{3(j-1)+1} - \epsilon_{3(j-2)+3}) \\ &\quad - \frac{1}{3}\sum_{m=1}^{3}\sigma_\epsilon(\epsilon_{3(j-1)+m} - \epsilon_{3(j-2)+m}) + \sigma^q_\epsilon(\epsilon^q_{(j)} - \epsilon^q_{(j-1)}) \qquad (18) \\ \Delta c^o_{3(j-1)+m} &= \Delta c_{3(j-1)+m} + \sigma_\epsilon(\epsilon_{3(j-1)+m} - \epsilon_{3(j-2)+m-1}) \text{ for } m = 2,3 \text{ and } \epsilon, \epsilon^q \sim N(0,1).\end{aligned}$$

σ_ϵ and σ^q_ϵ denote the standard deviation of monthly and quarterly consumption measurement errors. Note that under this specification, aggregating the monthly consumption series to the quarterly frequency according to

$$\Delta c^{q,o}_j = c^{q,o}_j - c^{q,o}_{j-1} = \sum_{\tau=1}^{5}\frac{3 - |\tau - 3|}{3}\Delta c^o_{3j-\tau+1}$$

averages out the monthly measurement errors; $\Delta c^{q,o}_j = \Delta c^q_j + \sigma^q_\epsilon(\epsilon^q_{(j)} - \epsilon^q_{(j-1)})$. Under this specification, the levels of monthly consumption are constructed by distributing quarterly consumption over the three months of a quarter; this distribution is based on a noisy monthly proxy series. Furthermore, monthly consumption growth rates are proportional to the growth rates of the proxy series and monthly consumption adds up to quarterly consumption. At quarterly frequency, consumption growth is strongly positively autocorrelated, while, at the monthly frequency, it exhibits a significant negative autocorrelation, which provides evidence for a negative moving average component. The assumed measurement error model for consumption is able to reconcile these features.

Measurement equation for inflation. I assume that inflation is measured without any errors and define

$$\pi^o_{3(j-1)+m} = \pi_{3(j-1)+m} \tag{19}$$

because, at the estimation stage, there was no clear way to disentangle the measurement errors from the transitory *iid* shocks.

Measurement equation for Nominal bond yields. I assume that nominal bond yields are contaminated by *iid* measurement errors and define

$$y^{\$,o}_{t,n} = y^{\$}_{t,n} + \sigma_{y_n}\epsilon_{y_n,t+1}, \quad \text{with} \quad \epsilon_{y_n,t+1} \sim N(0,1) \tag{20}$$

where σ_{y_n} denotes the standard deviation of the yields measurement error. Assuming small yield measurement errors is not a strong modeling assumption, since zero-coupon Treasury yields are not directly observed for long maturities and have to be bootstrapped from observed bond prices.

Altogether, the measurement equation of the state-space representation stacks each particular measurement equation and is given by:

$$Y^o_{t+1} = A_{t+1}(D + Zs_{t+1} + Z^v s^v_{t+1}(h_{t+1}) + \Sigma^u u_{t+1}), \quad \text{with} \quad u_{t+1} \sim N(0, I) \tag{21}$$

where the vector of observables Y^o_{t+1} contains consumption growth, Δc^o, inflation, π^o, and nominal bond yields, $y^{\$,o}$. The assumed measurement errors and the model solution provide the link between Y^o_{t+1} and the vectors of state variables s_{t+1} and $s^v_{t+1}(h_{t+1})$. Broadly, s_{t+1} includes the predictable components of consumption growth, $x_{c,t}$, and inflation, $x_{\pi,t}$, as well as the time preference shocks, $x_{\lambda,t}$, innovations to fundamentals, and the measurement errors of consumption. The vector $s^v_{t+1}(h_{t+1})$ is a function of the log volatilities $h_{t+1} = [h_{xc,t+1},\ h_{x\pi,t+1},\ h_{c,t+1},\ h_{\pi,t+1}]'$ and stacks the short-, and long-run real and nominal uncertainties. The vector u_{t+1} consists of bondspecific measurement errors where the diagonal of Σ^u contains the standard deviation of the yields measurement errors, σ_{y_n}. Finally, A_{t+1} is a selection matrix that accounts for changes in the data availability induced by the specification of the measurement error model of consumption growth.

2.4.2 State transition equation. The state transition equation provides the dynamics of the state variables broadly given by the assumed process of the time preference shocks in (2) and by the predictable component of consumption growth and inflation in (3). It can be written as:

$$s_{t+1} = \Phi s_t + v_{t+1}(h_t), \quad \text{and} \quad h_{t+1} = \Psi h_t + \Sigma_h \omega_{t+1} \quad \text{with} \quad \omega_{t+1} \sim N(0, I) \tag{22}$$

where the variance of $v_{t+1}(h_t)$ depends on the log volatilities at time t. In Appendix B, I describe each particular component of the state-space representation in detail. I now proceed to describe the estimation procedure and results.

3 Estimation

In Section 3.1 and 3.2, I present the estimation procedure and the data set used. In Section 3.3, I analyze the prior and posterior distribution of the model parameters, while in Section 3.4, I describe the smoothed latent state estimates.

3.1 Estimation procedure

I use a Metropolis-Hastings sampler for posterior inference. I make inference on two different sets of parameters. One set corresponds to the assumed process for the macroeconomic dynamics denoted by:

$$\Theta_{macro} = \{\Theta_{c\pi}, \Theta_h\} \qquad \text{with}$$

$$\Theta_{c\pi} = \{\mu_c, \sigma, \varphi_{xc}, \rho_{cc}, \rho_{c\pi}, \mu_\pi, \varphi_\pi, \varphi_{x\pi}, \rho_{\pi\pi}, \sigma_\epsilon, \sigma_\epsilon^q\}, \quad \text{and} \quad \Theta_h \quad = \{\rho_{h_c}, \sigma_{h_c}, \rho_{h_\pi}, \sigma_{h_\pi}, \rho_{h_{xc}}, \sigma_{h_{xc}}, \rho_{h_{x\pi}}, \sigma_{h_{x\pi}}\}.$$

The second set of parameters is related to the preference of the representative household:

$$\Theta_{pref} = \{\delta, \psi, \gamma, \rho_\lambda, \sigma_\lambda, \sigma_{y^n}\}$$

I included the yields measurement errors in this second set. I define the union of Θ_{macro} and Θ_{pref} as follows:

$$\Theta = \{\Theta_{macro}, \Theta_{pref}\}.$$

It is important to differentiate these two sets of parameters since I estimate Θ_{macro} with and without considering bond prices in the estimation. If Θ_{macro} does not change much between these two different informational contends, this suggests that there is enough information in the macroeconomic variables to price the nominal yield curve. However, if the difference in parameter estimates is large, the estimation might be achieving a better fit for bond yields while sacrificing the fit of consumption growth and inflation.

To make Bayesian inference about Θ and the latent state vectors, I adopt the econometric approach proposed by Schorfheide et al. (2016). I specify a prior distribution $p(\Theta)$ and update my *a priori* beliefs about the parameter vector Θ, in view of the sample information Y^o. The state of knowledge regarding Θ is summarized by the posterior distribution $p(\Theta|Y)$ and Bayes Theorem provides the formal link:

$$p(\Theta|Y^o) = \frac{p(Y^o|\Theta)p(\Theta)}{p(Y^o)} \tag{23}$$

Given the presence of nonlinearities induced by the volatility states it is not possible to directly evaluate the likelihood function, $p(Y^o|\Theta)$, via the Kalman filter. Instead, I use a sequential Monte Carlo filter to approximate $p(Y^o|\Theta)$. In essence, I represent the distribution of volatilities using a swarm of particles; conditional on these particles, the model has a linear and Gaussian state-space representation that allows me to apply the Kalman filter to equation (21) and (22). Once I am able to draw from the posterior distribution $p(\Theta|Y^o)$, I use a random-walk Metropolis-Hasting algorithm to draw the parameter vector $\{\Theta^{(s)}\}_{s=1}^{N_{sim}}$ as in Fernández-Villaverde and Rubio-Ramírez (2007) and Andrieu, Doucet, and Holenstein (2010). Detailed steps of the algorithm are available in Appendix B.3 and for further reference regarding the implementation of the particle filter see Herbst and Schorfheide (2015).

3.2 Data

The data is standard in the literature and consists of monthly observations of real per-capita consumption growth, inflation, and nominal bond yields with maturities of between one and five years.

The sample period is determined by the availability of the consumption growth series and spans from 1960:M1 to 2014:M12.[5] Specifically, I use the per-capita series of real consumption expenditure on nondurables and services from the National Income and Product Account published by the Bureau of Economic Analysis. I downloaded CPI inflation from the Federal Reserve Bank of St. Louis. I constructed growth rates for consumption and inflation by computing the differences of the log series. Lastly, monthly nominal bond yields were obtained from the CRSP Fama-Bliss zero coupon bond files.

3.3 Prior and posterior distribution

I will first describe my choice of priors and then I proceed to analyze the posterior estimates.

3.3.1 Prior distribution. In general, I attempted to restrict the parameter space to economically plausible values while at the same time be as uninformative as possible. The first columns of Table 1 show the assumed prior distribution, as well as the 5 and 95 percentiles. I will first discuss the priors of the macroeconomic parameters, Θ_{macro}, and then continue with the priors of the preference parameters Θ_{pref}.

The priors for the persistence parameters of the predictable components of consumption growth, ρ_{cc}, and inflation, $\rho_{\pi,\pi}$, as well as the prior for the parameter that measures the non-neutral effect of inflation on growth, $\rho_{c\pi}$, are distributed uniformly over the interval $(-1, 1)$. This range includes magnitudes that imply a zero effect of expected inflation on expected consumption growth, as well as a near *iid* or unit root process for the macroeconomic variables. The parameter φ_π sets the standard deviation of the *iid* component of inflation relative to consumption growth. Here I use a prior that is uniform on the interval $(0, 3)$ and encompasses values that allows π_t to be more or less volatile than Δc_t. A similar logic applies for φ_{xc} and $\varphi_{x\pi}$ with respect to the conditional means of consumption, $x_{c,t}$, and inflation, $x_{\pi,t}$, respectively. The priors for average annualized consumption growth, μ_c, and inflation, μ_π, are normally distributed with a 90 percent interval ranging from ± 10 percent.

Regarding the preferences parameters, Θ_{pref}, my priors are quite agnostic in that they cover values commonly reported in other studies and try not to affect the asset pricing implications of

[5] Monthly consumption growth starts in 1959:M2. However, to have non-missing values for all years I decided to start in 1960:M1.

the model. For instance, I do not restrict the preference of the representative household to have early, $\theta < 0$, or late, $\theta > 0$, resolution of uncertainty: the prior for the intertemporal elasticity of substitution, ψ, and the risk aversion coefficient, γ, cover values below and above one. The prior for the discount factor, δ, covers steady state annualized values for the one period risk free rate between 0.5 and 5 percent. Furthermore, the priors for ρ_λ and φ_λ have the same support as the persistence and standard deviation parameters of the macroeconomic dynamics, where I defined $\sigma_\lambda = \varphi_\lambda \sigma$ in (2). Finally, I fixed the standard deviation of the yields measurement error, σ_{y^n}, at 10 percent of the yields' sample standard deviation, a value consistent with the work of Bekaert, Hodrick, and Marshall (1997).

[Place Table 1 about here]

3.3.2 Posterior distribution. The last six columns of Table 1 show the 5, 50, and 95 percentiles of the posterior distribution under two different scenarios. Columns 5 to 7 show the posterior distribution if bond prices are included in the estimation, while columns 8 to 10 only consider the information contained in macro data.[6] It is important to highlight that the posterior intervals are much narrower if bond prices are considered in the estimation, suggesting that bond yields provide additional information that reduces the parameters posterior uncertainty. I will first describe the parameters related to the agent's preference, Θ_{pref}, and then proceed to analyze the parameter estimates in Θ_{macro}.

The coefficient of risk aversion, γ, and the intertemporal elasticity of substitution, ψ, have a posterior median value of around 10 and 1.55, respectively. These estimates are consistent with the parameter values highlighted in the long-run risk literature and imply preferences for early resolution of uncertainty. It is also worth noting that the time preference shocks, $x_{\lambda,t}$, are highly persistent with a posterior median value of 0.95. Furthermore, a value of $\varphi_\lambda \approx 0.22$ suggests that they are considerable less volatile than consumption growth.

[6] To estimate the model that only includes macro data in the estimation, I cast equation 3 into a state-space form and use Metropolis-within-Gibbs sampler for posterior inference iterating over the three conditional distributions. In Appendix C.3 I provide detailed steps of the Metropolis-within-Gibbs sampler algorithm. I also assume a common stochastic volatility process (i.e., $h_{c,t} = h_{xc,t}$ and $h_{\pi,t} = h_{x\pi,t}$). Allowing for four different stochastic volatilities, the 90 percent confidence interval increased considerably for most of the parameters and the log marginal data density decreased from 6055 to 5761. With these evidence, I conclude that there is not enough information in consumption growth and inflation to identify four different stochastic volatilities.

With respect to the parameters associated with the macroeconomic dynamics, Θ_{macro}, I found several features worth noting. The posterior median estimates of $(\rho_{cc},\ \rho_{c\pi},\ \rho_{\pi\pi})$ are remarkably similar with $(0.948,\ -0.146,\ 0.983)$ or without $(0.946,\ -0.209,\ 0.982)$ considering bond prices in the estimation, suggesting that there is enough information in the macro data to identify them. This result is driven by the measurement error model of consumption and the assumed stochastic volatility processes for consumption growth and inflation. Appendix B.4 shows that accounting for measurement errors in consumption reveals a persistent component in consumption growth: In the absence of measurement errors ρ_{cc} drops to -0.20 (See Table B1). Appendix B.4 also shows that adding stochastic volatility to the process of the macro variables improves the model fit considerably (measured by the log marginal data density) and decreases the posterior uncertainty of the parameter estimates (measured by the 90 percent credible interval). Furthermore, there is enough information in the macro data to identify $\rho_{c\pi}$, which is negative and significant as suggested by the 95 percentile in column 10 of Table 1. This negative nonneutral effect of inflation on growth is consistent with Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2013), who used bond yields to identify $\rho_{c\pi}$.

In the model, time-variation in term premia is driven by the stochastic volatilities of expected consumption growth and expected inflation. Once bond prices were included in the estimation, the parameters governing the persistence and standard deviation of the long-run volatility processes increased considerably in order to match the large variations observed in the term premia dynamics -an implicit moment in the likelihood specification. Starting from an agnostic prior, $\sigma_{h_{xc}}$ and $\sigma_{h_{x\pi}}$ increased from an approximate value of 0.01 and 0.1 to 0.20 and 0.22, respectively. Similarly, $\rho_{h_{xc}}$ and $\rho_{h_{x\pi}}$ increased from 0.97 and 0.81 to approximately 0.98. Alternatively, since the high-frequency movements in consumption growth and inflation are negligible for asset prices but important for tracking the macroeconomic data, we should not expect similar changes in the posterior parameter estimates governing these dynamics.[7] Consistent with this idea, the posterior median estimates of $(\rho_{h_c}, \sigma_{h_c})$ and $(\rho_{h_\pi}, \sigma_{h_\pi})$ were quite similar between these two scenarios. Finally, the scale variance parameters associated with consumption growth and inflation decreased in order to counteract

[7] By adding short-run conditional heteroskedasticity by allowing for time variation in $\sigma_{c,t}$ and $\sigma_{\pi,t}$ and shutting down the variation in $\sigma_{xc,t}$ and $\sigma_{x\pi,t}$, the log marginal data density increased from 5870 to 6054, which is almost as high as the value obtained by assuming a common stochastic volatility process (i.e., $h_{c,t} = h_{xc,t}$ and $h_{\pi,t} = h_{x\pi,t}$) equal to 6055. Hence, short-run conditional heteroskedasticity improves considerably the model fit of the assumed process of the macro variables.

the increased variation of the predictable components of consumption growth and inflation: The posterior median of φ_{xc}, $\varphi_{x\pi}$, and φ_{π}, decreased from 0.22, 0.22, and 0.98 to 0.18, 0.08, and 0.79, respectively.

Misspecification Test. The documented difference in the posterior distribution between these two different scenarios raises the concern that there might be a trade off between an improved fit for bond yields and the fit of macroeconomic variables. To assess the extent to which, for instance an increase in $\rho_{h_{xc}}$, leads to a decrease in fit of the joint process of consumption growth and inflation, I re-estimated model (3) conditional on the value of $\rho_{h_{xc}}$ that delivers the highest posterior value under the distribution that includes bond prices in the estimation. Finally, to measure the change in model fit, I computed the log marginal data density of this constraint specification and standardized it using the mean and standard deviation of the log marginal data densities computed from 30 different estimations of the unconstrained version of model (3). I carried out this same procedure for each parameter in θ_{macro}, and in Appendix B.5, I report the estimates in detail. Overall, the drop in the log marginal data densities is small. With the exception of the value for $\rho_{h_{x\pi}}$, all values are inside one standard deviation, suggesting that there is no tension between the parameter estimates obtained with our without including bond prices in the estimation.[8]

3.4 Latent state variables

Now I turn to describe the filtered estimates of the latent state variables. In panels (a) and (b) of Figure 2, I show the 90 percent confidence interval of the smoothed expected consumption growth, $x_{c,t}$, and expected inflation, $x_{\pi,t}$, variables adjusted by their corresponding unconditional mean estimate. I also overlay the observed monthly consumption growth and inflation series. In addition, I include the posterior median values of $x_{c,t}$ and $x_{\pi,t}$ filtered from the model that only considers macro data in the estimation, as well as the National Bureau of Economic Research (NBER) recession dates. The pattern for $x_{c,t}$ is similar to the one reported in Schorfheide et al. (2016): $x_{c,t}$ exhibits declines during each recession, with the largest decline during the recent financial crisis. Notably, this pattern does not change significantly if I do not consider bond prices in the estimation to extract the latent state variable. This last point is also true for $x_{\pi,t}$; the lines follow each other very closely. $x_{\pi,t}$ has its highest point during the Great Inflation period and decreases afterwards.

[8] $\rho_{h_{x\pi}}$ is almost 6 standard deviations away. See Table B2.

Note that recessions during the 1970s and 1980s were accompanied by an increase in the expected inflation component, and that this behavior suddenly reversed during the last three crises, where $x_{\pi,t}$ exhibits a sharp decline.

[Place Figure 2 about here]

In panels (c) and (d) of Figure 2, I plot the volatilities of the long-run growth, h_{xc}, and inflation rate, $h_{x\pi}$, respectively. Both series vary considerably: h_{xc} starts high and shows a decreasing trend with spikes during the crises of 1975 and 1980. It remained at low levels until the late 1990s, after which it started to increase and has stayed at relatively high levels ever since. $h_{x\pi}$ displays different properties. In the 1960s it was at low levels; it jumped around during the 1970s with peaks between 1980 and 1985 and remained at high values until 1990, when it started to decrease with wide fluctuations around 2000. Similarly, in panels (e) and panel (f), I plot the smoothed series for the high frequency components of consumption growth, h_c, and inflation, h_π. The first thing to note is that the magnitude of both series is remarkably smaller. Also of note is that h_c captures the drop in growth volatility that occurred around the 1980s, also known as the Great Moderation period and it shows a decreasing pattern from that date onwards. On the other hand, h_π practically spikes at every recession, but periods of an increase in h_π also occur during expansions.

[Place Figure 3 about here]

Time preference shocks capture the attitudes of the representative household towards savings and they are, by assumption, orthogonal to the real and nominal economy. With this in mind, an interesting candidate to correlate the smoothed demand shocks estimates is the adjusted National Financial Conditions Index (adjusted FCI) published by the Federal Reserve of Chicago. This index measures risk, liquidity, and leverage in money, debt, and equity markets, and isolates a component of financial conditions uncorrelated with economic conditions.[9] Figure 3, shows the 90 percent confidence interval of the smoothed time preference shocks along with the negative value of the adjusted FCI multiplied by a constant to have similar magnitudes.[10] Given these adjustments, a positive value of the adjusted FCI index suggests loose financial markets relative

[9] The Adjusted National Financial Conditions Index is provided by the Federal Reserve Bank of Chicago; the series starts in 1973 and can be downloaded at https://alfred.stlouisfed.org/series?seid=ANFCI

[10] In sum, I multiplied the original adjusted FCI series by -1200.

to an historical average. The correlation between these series is remarkably high, with a value of around 50 percent,[11] suggesting that tighter financial conditions today are associated with a lower willingness of the agents to save for future periods.

I assume that the time preference shocks are orthogonal to consumption growth and to its predictable component. In a general equilibrium framework this will not likely be the case, since innovations to the time preference parameter will affect the household decisions towards consumption, saving, and investment, which in turn will affect production outcomes and the expected path of consumption growth. To address this concern, I computed the 90 percent confidence interval of the correlation between innovations to the time preference shocks, $\sigma_\lambda \eta_{\lambda,t+1}$, and shocks to consumption growth, $\sigma_{c,t}\eta_{c,t+1}$, and between $\sigma_\lambda \eta_{\lambda,t+1}$ and shocks to expected consumption growth, $\sigma_{xc,t}\eta_{xc,t+1}$. These correlations are small, ranging from -0.08 to 0.10 and from 0.09 to 0.2, respectively. These results are consistent with Schorfheide et al. (2016), who also assume uncorrelated shocks between the preference shocks and the cash flow components and conclude that there is no strong evidence against the assumption of orthogonal preference shocks.

Overall, the model estimation fits the bond yields, without sacrificing the fit of the macroeconomic variables. In addition, the latent state variables are directly linked to macroeconomic risk factors or are correlated to measures of risk, liquidity, and leverage in financial markets uncorrelated with economic conditions. In lieu of these findings, I now turn to analyze the importance of inflation expectations for the nominal yield curve.

4 Asset pricing implications

In Section 4.1, I analyze how well the model implied moments match macro and bond yield moments. In Section 4.2, I quantify to what extent the volatility of nominal yield shocks is due to the volatility of expected inflation news. In Section 4.3, I analyze the relative contribution of each risk factor to movements in nominal bond yields. In Section 4.4, I study the bond predictability results implied by the model. Sections 4.5 and 4.6 show the relative contribution of risk factors to movements in expected excess bond returns and term premium, respectively.

[11] This correlation increases to 65 percent if I remove the high frequency components of the series by taking a twelve month moving average.

4.1 Model Implications for macro and bond yield moments

While I am targeting the joint distribution of macroeconomic and bond yield variables, it is informative to document how close the model-implied moments are to those observed in the data. Table 2 shows the percentiles of the posterior predictive distribution for several sample moments,[12] based on parameters drawn from the posterior distribution that considers bond prices (columns 3 to 5), and from that which only considers macro data (columns 6 to 8). For the latter, I use the posterior medians of the preference parameters Θ_{pref}. Finally, in the second column, I show the same moments computed from the observed series. All moments are annualized.

[Place Table 2 about here]

The model does a good job of reproducing the macro moments while at the same time accounting for the level and slope of the nominal yield curve as well as the standard deviation of nominal yields. All of the data moments are well within the 90 percent credible interval. In the results of the estimation that includes bond prices, the model performs well along the lines of macroeconomic moments: the model-implied posterior median values are fairly close to their data estimates. Regarding the bond yield moments, the model-implied posterior medians match the decreasing pattern in the standard deviation of bond yields across maturities and at the same time captures the upward-sloping unconditional mean of the nominal term-structure. It also mimics the difference between the long and the short end (i.e., slope) of the yield curve: in the data and in the model it is around 60 and 50 basis points, respectively. In the results of the estimation that does not include bond prices, the model is still able to match these features of the data and generate credible intervals that contain the sample data moments, as shown in the last three columns of Table 2, although the standard deviation of the nominal yields is lower relative to the estimates obtained by also including bond prices in the estimation.

[12] To this end, I sampled M draws of Θ from its posterior distribution and simulated the model for 660 periods, which corresponds to the number of monthly observations in my estimation sample. This gave me M simulated trajectories of Y^o. For of them, I computed several statistics. In the implementation I set $M = 10,000$.

4.2 Model implications for inflation variance ratios

To compute the data-implied inflation variance ratios for long horizons, Duffee (2016) relies on a statistical model for expected inflation and bond yields. In Section 4.2.1, I extend Duffee's analysis by assuming heteroskedasticity in both inflation news and yield shocks instead of splitting the estimation in different subperiods, and I increase the frequency of the data from quarterly to monthly observations. In Section 4.2.2, I present the model-implied inflation variance ratios.

4.2.1 Inflation variance ratios: Data. Since survey inflation forecasts are focused at relatively short horizons (usually less than one year), I obtained data estimates of the inflation variance ratios from a statistical model. For consistency, and following Stock and Watson (2007), I extracted one- to five-year inflation forecast innovations, $\epsilon_{\pi,t}^{(n)}$, from the process of inflation specified in equation (3). Following Duffee (2016), I also included information from the surveys of market practitioners that are released at a different (quarterly) frequency. As argued by Ang, Bekaert, and Wei (2007), survey-based inflation forecasts are more accurate than model-based ones, which sharpens the inference about $\epsilon_{\pi,t}^{(n)}$. Specifically, I used the CPI forecast from the Survey of Professional Forecasters (SPF) of one to four quarters ahead. The series starts in 1983Q3 .

Similarly, to compute shocks to the n-period yield, $\epsilon_{y^{\$},t}^{(n)}$, I follow Duffee (2016) and assume that bond yields follow martingales

$$y_t^{(n)} = y_{t-1}^{(n)} + \sigma_{y^{(n)},t}\eta_{y^{(n)},t+1} \qquad \text{and} \qquad \eta_{y^{(n)},t+1} \sim i.i.d.N(0,1) \qquad \text{where} \tag{24}$$

$$\sigma_{y^{(n)},t} = \sigma_{y^{(n)}} exp(h_{y,t}), \quad h_{y,t+1} = \rho_{h_y} h_{y,t} + \sigma_{h_y}\omega_{y,t+1}, \quad \text{and} \quad \omega_{y,t+1} \sim i.i.d.N(0,1),$$

where $\epsilon_{y^{\$},t}^{(n)} = \sigma_{y^{(n)},t}\eta_{y^{(n)},t+1}$. For bond yields, the martingale assumption generates lower forecast errors than both, survey based (e.g., Cieslak 2016) and parametrized models (e.g., Duffee 2002). Note that I am assuming a common volatility structure for the bond yields since it produces the best mode fit in terms of the log marginal data density.[13] Finally, given an estimate of $\epsilon_{\pi,t}^{(n)}$ and $\epsilon_{y^{\$},t}^{(n)}$, it is straightforward to compute the inflation variance ratios.[14]

[13] I estimated different versions of the model, and assuming a common volatility structure for bond yields delivers the best model fit.

[14] Regarding the estimation procedure, I cast the statistical model of expected inflation and bond yields (including the quarterly information of CPI forecast from SPF) into a mix-frequency state-space model and used a Metropolis-

[Place Table 3 about here]

Table 3 shows the inflation variance ratios along with the 90 percent confidence interval for maturities of one to five years. For the sake of completeness, I also report the estimates based on the homoskedastic version of the model and, in Appendix C.3, I provide the parameter estimates together with the assumed prior distribution. In the case of homoskedastic innovations, the posterior medians of the unconditional inflation variance ratios for all bond maturities were below 20 percent whereas the 95 percentiles were below 26 percent. These results are by and large consistent with the estimates reported by Duffee (2016), and can be viewed as a corroboration of his findings. Nevertheless, a careful inspection of the innovations suggests that these vary through time and exhibit heteroskedasticity. Therefore, instead of doing subperiod analysis, in the last three columns of Table 3 I report the estimates of the model that allows for stochastic volatility. Allowing for heteroskedasticity in inflation shocks and innovations to bond yields improves the model fit significantly, as evidenced by the log marginal data density reported at the end of the table. It also shifts the distribution of the unconditional inflation variance ratios towards higher values: the posterior medians were around 37 percent for a one-year bond and 28 percent for a five-year bond. However, the 95 percentiles were below 53 percent. These ratios are still strongly at odds with corresponding values from standard macro-finance term structure models.[15]

Robustness. As a robustness check on my findings, I repeated the same exercise for a quarterly version of the statistical model considering different measures of inflation expectations reported by SFP. In Appendix C.4, I report the estimates in detail, while briefly describing the results here.

within-Gibbs sampler for posterior inference, where I iteratively sampled from three conditional posterior distributions. I assume that the survey consensus forecasts are measured with errors where the standard deviation of the measurement errors are free parameters. Similar to the measurement equation for nominal bond yields in Section 2.4, I set the standard deviation of the yield's measurement error at 10 percent of its sample standard deviation. In Appendix C, I provide further details.

[15] The inflation variance ratios are expected to increase under the stochastic volatility case due to a Jensen's term, provided that the correlation between $Var_t(\epsilon^{(n)}_{\pi,t})$ and $Var_t(\epsilon^{(n)}_{y^\$,t})$ is not too high. To see this, note that

$$E\left[\frac{Var_t(\epsilon^{(n)}_{\pi,t})}{Var_t(\epsilon^{(n)}_{y^\$,t})}\right] \approx \frac{E[Var_t(\epsilon^{(n)}_{\pi,t})]}{E[Var_t(\epsilon^{(n)}_{y^\$,t})]} + \frac{E[Var_t(\epsilon^{(n)}_{\pi,t})]}{E[Var_t(\epsilon^{(n)}_{y^\$,t})]^3} Var(Var_t(\epsilon^{(n)}_{y^\$,t})) \left[1 - \rho_{\epsilon^{(n)}_{\pi,t},\epsilon^{(n)}_{y^\$,t}} \sqrt{\frac{Var(Var_t(\epsilon^{(n)}_{\pi,t}))}{Var(Var_t(\epsilon^{(n)}_{y^\$,t}))}} \frac{E[Var_t(\epsilon^{(n)}_{y^\$,t})]}{E[Var_t(\epsilon^{(n)}_{\pi,t})]}\right],$$

where $\rho_{\epsilon^{(n)}_{\pi,t},\epsilon^{(n)}_{y^\$,t}}$ denotes the correlation between the corresponding variance terms. The first term on the right hand side corresponds to the inflation variance ratios under homoskedastic shocks. By assumption, I set $\rho_{\epsilon^{(n)}_{\pi,t},\epsilon^{(n)}_{y^\$,t}}$ equal to zero, and as a result, in my estimation, the inflation variance ratios are expected to increase relative to the homoskedastic case. This exercise can be seen as a best case scenario for the contribution of the Jensen's term to the inflation variance ratios, provided that $\rho_{\epsilon^{(n)}_{\pi,t},\epsilon^{(n)}_{y^\$,t}}$ is nonnegative.

Overall, for all bond maturities considered, expected inflation news does not contribute much to the variation of bond innovations. The posterior medians were around 12 percent. Allowing for stochastic volatility almost doubles the inflation variance ratios relative to the homoskedastic case, improves the model fit; furthermore, the 95 percentile of the inflation variance ratios for all SFP measures considered are below 50 percent.

4.2.2 Inflation variance ratios: Model-implied. In panel (a) of Figure 4, I plot the 90 percent confidence interval of the posterior predictive distribution of the model-implied inflation variance ratios.[16] I also include the posterior median population values implied by the macro-finance model, and those implied by the statistical model reported in Table 3. Note that the data sample moments based on homoskedastic (Data) and heteroskedastic (Data SV) innovations are within the 5 and the 95 percentiles of the posterior predictive distribution for all bond maturities. The macro-finance model produced posterior median population variance ratios of around 12 and 30 percent for short- and long-maturity bonds. Based on this evidence, I conclude that the model is able to generate measures of inflation uncertainty that are close to the data and well below 50 percent. In Appendix C.5, I also show that the model not only matches the inflation variance ratios, but also the standard deviation of inflation and bond yields innovations.

[Place Figure 4 about here]

In panel (b) of Figure 4, I show the same estimates as in panel (a), but based on a model that does not consider time preference shocks.[17] If I turn off the time preference shocks channel, the 90 percent confidence interval of the posterior predictive distribution does not contain the data-implied sample moments based on homoskedastic news and misses two out of five moments based on the heteroskedastic case. Furthermore, the posterior median population values implied by this model are too high, ranging from 51 to around 80 percent. These values are somewhat smaller than the ones implied in the estimation of Bansal and Shaliastovich (2013), which range from 76

[16] Specifically, I sample M draws of Θ from its posterior distribution and simulate the models for 660 periods, which corresponds to the number of monthly observations in my estimation sample. For each one of these simulated trajectories and for each time period t I computed the implied inflation variance ratio and took the average across time for a total of M inflation variance ratios. I set $M = 10,000$.

[17] To this end, I solved the model without time preference shocks, cast it into the state-space form, and estimated it under the same estimation procedure. I made sure that the model estimation matched the same data moments and predictability results as the model that assumes time preference shocks. Results available upon request.

to 113 percent. This result is consistent with Duffee's claim regarding the inability of standard macro-finance models to generate plausible yield shocks without relying heavily on the inflation expectation channel.

Allowing for time preference shocks generates the extra persistent fluctuation in real rates needed to match the empirical volatility of yield shocks without requiring counterfactually high volatility of innovations in inflation expectations. Overall, in the data and in the model, shocks to expected inflation are not the main driver of yield innovations. I now proceed to analyze the relative contribution of each risk factor to movements in the nominal yield curve.

4.3 Model implications for bond yield dynamics

The estimation of the model matches the yields well. The mean absolute pricing errors range from 4.3 to 6.3 basis points for the different bond yields considered. These values are comparable with the pricing errors reported in the literature on macro-finance term structure models (e.g., Bikbov and Chernov 2010; Doh 2013). To highlight the importance of each state variable, in Figure 5, I show the contribution of expected consumption growth, $x_{c,t}$, expected inflation $x_{\pi,t}$, time preference shocks, $x_{\lambda,t}$, and long-run real, $\sigma^2_{xc,t}$, and nominal, $\sigma^2_{x\pi,t}$, uncertainties to the 5-Year bond yield. I do not explicitly show the contribution of the short-run real, $\sigma^2_{c,t}$, and nominal, $\sigma^2_{\pi,t}$, uncertainties because their share is too small (less than 0.5 percent) to distinguish them clearly in the graph.

[Place Figure 5 about here]

The unconditional mean of the 5-Year bond yield is 5.97 percent; however, the yield exhibits substantial variations induced by movements in the state variables. The constant $\frac{1}{n}B^{\$}_{0,n}$ sets the level at roughly 5.93 percent, and from there on the yield goes up or down depending on the relative contribution of each risk factor. In the 1970s, the decade of inflation in the United States, the 5-Year bond yield was primarily above 5.93 percent due to a rise in inflation expectations (recall that $B^{\$}_{\pi} > 0$). In the 1980s, inflation expectations began to decline (see Figure 2) as a result of the program of disinflation under Chairman Volcker; however, nominal uncertainty, $\sigma^2_{x\pi,t}$, was at an historic high level which moved the long-term yields ($B^{\$}_{\sigma_{xc},n} > 0$ for high n) in the opposite direction. During that period, time preference shocks were big and negative (see Figure 3), probably related to tighter financial conditions, which increased the nominal yield even more. The 1990s and

2000s were periods of relative stability in prices and, as such, $x_{\pi,t}$ and $\sigma^2_{x\pi,t}$ took a secondary role in explaining movements in bond prices. The 2000s were characterized by high real volatility (see Figure 2) which increased the uncertainty about future growth. To hedge this risk, the demand for risk-free assets increased, and in equilibrium, pushed the yield below 5.93 percent. $\sigma^2_{xc,t}$ was the dominant risk factor during the 2000s. A similar reasoning applies to the remaining bond maturities. Now I proceed to analyze the model implications for bond risk premia.

4.4 Model implications for bond return predictability

The model is also able to match the bond predictability results documented in Campbell and Shiller (1991) and Cochrane and Piazzesi (2005). Table 4 shows that the model-implied distribution of the slope coefficients in the term spread regression of Campbell and Shiller (1991) are in line with the estimates based on observed data. The posterior median estimates are negative, decline with maturity, and are remarkably close to the actual point estimates. Furthermore, I ran the same regressions as in Cochrane and Piazzesi (2005) and concluded that the model-simulated yields also exhibit excess bond return predictability. The posterior median coefficients were able to match the sign, pattern, and magnitude of the point estimates based on observed bond yields. The same holds for the R^2, although the posterior median estimates are somewhat larger than the ones based on observed yields.

[Place Table 4 about here]

In Section 3.3, I document that in order to match the large variations observed in the term premium dynamics, the parameters governing the standard deviation of the volatility process increase considerably. To assess the asset pricing implications of these parameter differences, the last three columns of Table 4 replicate the same set of regressions considering only information contained in the macroeconomic variables.[18] Regarding the Campbell-Shiller regressions slope, the model is unable to replicate the sign nor the increasing pattern (in absolute value) observed in the data estimates. In fact, the posterior median estimates are close to one, which is exactly the value predicted by the expectation hypothesis. However, the model is still able to generate some excess

[18]To this end, I use the posterior medians of the preference parameters Θ_{pref} along with the posterior distribution of Θ_{macro} obtained by only considering macro data in the estimation.

return predictability as evidenced by the R^2, though the level is considerably smaller than the one estimated with bond prices. The inclusion of asset prices in the estimation alleviates these problems by generating the extra variability needed in the stochastic volatilities of expected consumption growth and expected inflation to match the term premium dynamics.

4.5 Model implications for expected excess returns

In the model, movements in the one-period expected excess nominal bond returns reflect compensation for innovations to expected consumption growth and to expected inflations driven by their stochastic volatilities. Figure 6 depicts the posterior median contribution of preference risk, long-run real and inflation risk for a one-period expected excess return on a 5-Year bond.[19] The total annualized unconditional expected excess return is positive at around 0.062 percent. However, the conditional return varies substantially in the time period considered, ranging from -0.23 percent to 0.43 percent. In the 1980s to early 1990s the major source of risk premium for holding long maturity bonds was long-run inflation risk, which accounted for approximately 71 percent of the expected excess bond returns. However, inflation-related risk premia has been practically zero since 1995. In the 1960s and 2000s the risk premium was negative and driven mostly by long-run real risk. In a stable inflation environment, interest variation comes mostly from changes in the real rates which makes long-term bonds safer assets for long-term investors. Short term bonds have higher exposure to the reinvestment risk caused by changes in the short-term real rates, and as such, they should bear a bond risk premia. As explained in the model section, bonds also command a positive risk premium driven by the preference shocks. This source of risk is constant across time and its contribution to the expected excess return is around 0.065 percent.

[Place Figure 6 about here]

4.6 Model implications for bond term premium

As in Bansal and Shaliastovich (2013), this model generates variability in term premia through time-varying quantity of risk driven by the long-run real and inflation volatilities. Conversely, reduced

[19] I do not explicitly show the contribution of volatility risk since its share is too small (less than 0.55 percent) to distinguish it clearly in the graph.

form Gaussian affine term structure models (ATSM) produce time-variation in term premia via time-varying prices of risk (e.g., Kim and Wright 2005; Bauer, Rudebusch, and Wu 2012; Adrian, Crump, and Moench 2013). Central banks around the world rely on reduced form models because of their tractability and reliability. It is still an empirical open question whether time-varying quantity of risk is able to mimic the term premia dynamics based on reduced form models.

In panel (a) of Figure 7, I start by plotting the one- to five-years estimated posterior median term premiums. As expected, the level and variation of the term premium increases with the maturity of the bond. During most of the 1960s the term premium component was practically zero; it increased throughout the 1970s, peaked in the early 1980s, and then began a declining trend for the rest of the time period. According to my estimates, the term premium became negative since the onset of the financial crisis which might be traced back to a flight to quality effect or to the large-scale asset purchases by the Federal Reserve. In this environment, negative term premium makes sense. We should expect a positive term premium in periods with unstable inflation and relative constant real rates such as during the 1970s and 1980s, since short-term bonds have a lower exposure to the former and a higher loading on the latter (see Figure 1). In this environment, it is safer to roll over short-term bonds that adapt faster to inflation news, than to buy long term bonds.

[Place Figure 7 about here]

In panel (b), I plot the 5-Year term premium from the model along with the estimates of Adrian et al. 2013 (ACM) and Kim and Wright 2005 (KW) for the same bond maturity. These last two models belong to ATSM class and are published by the Federal Reserve Banks of New York and St. Louis, respectively.[20] Panel (b) shows that the model is able to account for the level and variability of the term premium and even matches the estimates of reduced form ATSM. In fact, the correlation between the long-run risk model-implied term premium and the estimates of ACM and KW is about 80 and 98 percent, respectively. The correlation between ACM and KW is 69 percent. Note that the ACM estimate has been considerably higher since the last crisis, which implies a lower future path of short-term interest rates, relative to KW or my estimates. Alternatively, the term premium based on the long-run risk model is considerably higher during the 1980s.

[20]The term premium based on Adrian et al. 2013 can be found here: https://www.newyorkfed.org/research/data_indicators/term_premia.html, while the the term premium based on Kim and Wright 2005 can be found here: https://fred.stlouisfed.org/series/THREEFYTP5

An advantage of using a macro-finance model is that I am able trace back the movements in the term premium to changes in macroeconomic risks, while ACM and KW have to rely on correlations. With this in mind, panel (c) of Figure 7, depicts the contribution of long-run real, $\sigma^2_{xc,t}$, and nominal uncertainties, $\sigma^2_{x\pi,t}$, to the 5-Year nominal term premium. Without stochastic volatilities, the term premium is constant at around 1 percent. From the 1970s to the late 1990s, long-run inflation uncertainty increased the term premium above that threshold, while real uncertainties decreased it to below 1 percent during the 1960s and even to negative values in the late 2000s. Finally, in panel (d) I show the decomposition of the 5-Year yield into the term premium component and the expected path of short-term Treasury yields over the next five years. On average, the term premia is positive and accounts for 6 percentage of the 1-Year bond yield; it goes up to 18 percent for the 5-Year bond yield. It is instructional to note that the term premium for all bond maturities considered strongly co-moves with observed yields, which can be seen as a graphical and informal way to corroborate the failure of the expectations hypothesis presented in Table 4.

Overall, recursive preference with time-varying quantity of risk is able to generate plausible term premia dynamics without compromising the fit of the macroeconomic variables. The term premium is driven by real and nominal uncertainties and the key driver is time dependent. For the last two decades, however, nominal uncertainties have not played an important role in driving bond risk premia dynamics, while real uncertainties have played the predominant role.

5 Conclusion

I developed a nonlinear Bayesian state-space model in which time preference shocks, inflation non-neutrality, multiple stochastic volatility processes, and time-aggregation of consumption generate bond yield dynamics largely consistent with the data without sacrificing the fit of the macroeconomic variables. The model accounts for bond return predictability, and mimics the term premia estimates based on reduced form Gaussian affine term structure models without the need to rely on a big inflation expectations channel. The model-implied inflation variance ratios are consistent with the corresponding values based on monthly U.S. data. I find that inflation-related risk factors have not played an important role in term premia dynamics and expected excess bond returns for the last two decades: The main driver comes from the real side of the economy.

Appendix

A Economic Model

Appendix A is organized as follows. In Section A.1, I introduce the model set-up. In Section A.2, I present the solution to the price-consumption ratio. In Section A.3, I write the equilibrium real and nominal stochastic discount factor (SDF). In Section A.4, I solve the equilibrium nominal bond yield loadings. In sections A.5 and A.6, I derive the equilibrium expected excess bond returns and the term premium dynamics. Finally, in Section A.7, I solve the model-implied inflation variance ratios.

A.1 Model set-up

In this section, I provide a solution for the economic model. The assumed stochastic process for the logarithm of consumption growth, $\Delta c_{t+1} = log\frac{C_{t+1}}{C_t}$, and the logarithm of the inflation rate, π_{t+1} are:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + x_{c,t} + \sigma_{c,t}\eta_{c,t+1} \\ \pi_{t+1} &= \mu_\pi + x_{\pi,t} + \sigma_{\pi,t}\eta_{\pi,t+1} \end{aligned} \tag{A.1}$$

where the process for the predictable components and the stochastic volatilities are given by:

$$\begin{aligned} x_{c,t+1} &= \rho_{cc}x_{c,t} + \rho_{c\pi}x_{\pi,t} + \sigma_{xc,t}\eta_{xc,t+1} \\ x_{\pi,t+1} &= \qquad\qquad + \rho_{\pi\pi}x_{\pi,t} + \sigma_{x\pi,t}\eta_{x\pi,t+1} \end{aligned} \tag{A.2}$$

and

$$\sigma_{i,t} = \varphi_i \sigma exp(h_{i,t}), \quad \text{with} \quad h_{i,t+1} = \rho_{h_i}h_{i,t} + \sigma_{h_i}\omega_{i,t+1}$$

All innovations are distributed according to

$$\eta_{i,t+1}, \quad \omega_{i,t+1}, \quad \epsilon_{i,t+1} \quad \sim i.i.d.N(0,1) \quad \text{for} \quad i = \{c, \pi, xc, x\pi\}$$

Given the assumed preference for the representative agent, the logarithm of the real stochastic discount factor (SDF) is given by

$$m_{t+1} = \theta log\delta + \theta x_{\lambda,t} - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{c,t+1} \tag{A.3}$$

with θ defined as

$$\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$$

and where the time preference shocks evolve as:

$$x_{\lambda,t+1} = \rho_\lambda x_{\lambda,t} + \sigma_\lambda \eta_{\lambda,t+1} \quad \text{with} \quad \eta_{\lambda,t+1} \sim i.i.d.N(0,1) \tag{A.4}$$

To entertain an analytical model solution I rely on two different approximations. The first, proposed by Campbell and Shiller (1988), involves a log-linear Taylor expansion to $r_{c,t+1}$:

$$r_{c,t+1} = \kappa_0 + \kappa_1 pc_{t+1} - pc_t + \Delta c_{t+1} \tag{A.5}$$

where pc_{t+1} is the log price-to-consumption ratio, and κ_0 and κ_1 are constants determined endogenously by the unconditional mean of the price-consumption ratio, $\bar{pc}$, in the economy and are given by

$$\kappa_1 = \frac{exp(\bar{pc})}{1 + exp(\bar{pc})} \quad \text{and} \quad \kappa_0 = log(1 + exp(\bar{pc})) - \kappa_1 \bar{pc}$$

The second approximation, considers a linear approximation of the volatility process around the unconditional mean of h as in Schorfheide et al. (2016):

$$\begin{aligned}
\sigma_{i,t+1}^2 &\approx (\varphi_i\sigma)^2 + 2(\varphi_i\sigma)^2 h_{i,t+1} \\
&= (\varphi_i\sigma)^2 + 2(\varphi_i\sigma)^2 \rho_{h_i} h_{i,t} + 2(\varphi_i\sigma)^2 \sigma_{h_i}\omega_{i,t+1} \\
&\approx (\varphi_i\sigma)^2 + \rho_{h_i}(\sigma_{i,t}^2 - (\varphi_i\sigma)^2) + 2(\varphi_i\sigma)^2 \sigma_{h_i}\omega_{i,t+1} \\
&= \sigma_{i,0}^2 + \nu_i(\sigma_{i,t}^2 - \sigma_{i,0}^2) + \sigma_{\omega_i}\omega_{i,t+1} \quad \text{for} \quad i = \{c, \pi, xc, x\pi\}
\end{aligned} \tag{A.6}$$

where I defined

$$\nu_i = \rho_{h_i}, \quad \sigma_{i,0}^2 = (\varphi_i\sigma)^2, \quad \text{and} \quad \sigma_{\omega_i} = 2(\varphi_i\sigma)^2\sigma_{h_i}$$

The Euler equation is

$$E_t[exp(m_{t+1} + r_{c,t+1})] = 1 \tag{A.7}$$

which can be used to price any asset in the economy.

A.2 Solution for the price-consumption ratio

In equilibrium, the price-consumption ratio is linear in the state variables:

$$pc_t = A_0 + A_c x_{c,t} + A_\pi x_{\pi,t} + A_\lambda x_{\lambda,t} + A_{\sigma_c}\sigma^2_{x,t} + A_{\sigma_\pi}\sigma^2_{\pi,t} + A_{\sigma_{xc}}\sigma^2_{xc,t} + A_{\sigma_{x\pi}}\sigma^2_{x\pi,t} \tag{A.8}$$

To solve for the coefficients As I use the log-linear approximation for $r_{c,t+1}$ in (A.5) and substitute it along with the SDF in (A.3) in the Euler equation in (A.7). Using the method of undetermined coefficient it follows that:

$$A_c = \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho_{cc}}, \quad A_\pi = \kappa_1\rho_{c\pi}\frac{1-\frac{1}{\psi}}{(1-\kappa_1\rho_{cc})(1-\kappa_1\rho_{\pi\pi})}, \quad A_\lambda = \frac{\rho_\lambda}{1-\kappa_1\rho_\lambda} \tag{A.9}$$

$$A_{\sigma_c} = \frac{(1-\gamma)(1-\frac{1}{\psi})}{2(1-\kappa_1\nu_c)}, \quad A_{\sigma_\pi} = 0, \quad A_{\sigma_{xc}} = \theta\frac{(\kappa_1 A_c)^2}{2(1-\nu_{xc})}, \quad A_{\sigma_{x\pi}} = \theta\frac{(\kappa_1 A_\pi)^2}{2(1-\nu_{x\pi})}$$

and the constant is given by

$$A_0 = (1-\kappa_1)^{-1}\{log\delta + (1-\frac{1}{\psi})\mu_c + \kappa_0 + \frac{1}{2}\theta(1+\kappa_1 A_\lambda)^2\sigma^2_\lambda + \kappa_1[\sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}}(1-\nu_i)A_i\sigma^2_{0,i} + \frac{1}{2}\kappa_1\theta A_i^2\sigma^2_{\omega_i}]\} \tag{A.10}$$

To complete the derivation of the equilibrium price-consumption ratio I need to solve for steady state value of pc_t. To this end, I need to solve the following system of equations:

$$\bar{pc} = A_0(\kappa_1,\kappa_0) + \sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}} A_i(\kappa_1)\sigma^2_{i,0}$$
$$\kappa_1 = \frac{exp(\bar{pc})}{1+exp(\bar{pc})}$$
$$\kappa_0 = log(1+exp(\bar{pc})) - \kappa_1\bar{pc}$$

which can be done numerically. Having solved these constants, I now proceed to derive an expression for the SDF which we will use to price the term structure of interest rates.

A.3 Expression for the real and nominal SDF

Evaluating $r_{c,t+1}$ with the solution that I just derived for pc_t in (A.9) and (A.10) and using the assumed process for consumption growth I can rewrite the real SDF in terms of the state variables and the underlying shocks (risk). In particular, I have:

$$\begin{aligned} m_{t+1} = m_0 + m_c x_{c,t} + m_\pi x_{\pi,t} + m_\lambda x_{\lambda,t} + m_{\sigma_c}\sigma^2_{x,t} + m_{\sigma_\pi}\sigma^2_{\pi,t} + m_{\sigma_{xc}}\sigma^2_{xc,t} + m_{\sigma_{x\pi}}\sigma^2_{x\pi,t} \\ - \lambda_c\sigma_{c,t}\eta_{c,t+1} - \lambda_\pi\sigma_{\pi,t}\eta_{\pi,t+1} - \lambda_\lambda\sigma_{\lambda,t}\eta_{\lambda,t+1} - \lambda_{xc}\sigma_{xc,t}\eta_{xc,t+1} - \lambda_{x\pi}\sigma_{x\pi,t}\eta_{x\pi,t+1} \\ - \sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}} \lambda_i\sigma_{\omega_i}\omega_{i,t+1} \end{aligned} \tag{A.11}$$

Where the λ's represent the market price of each source of risk. The discount factor parameters and market price of risks are equal to:

$$m_c = -\tfrac{1}{\psi}, \quad m_\pi = 0, \quad m_\lambda = \rho_\lambda, \quad m_{\sigma_c} = (1-\theta)(1-\kappa_1\nu_c)A_{\sigma_c}, \quad m_{\sigma_\pi} = 0 \tag{A.12}$$

$$m_{\sigma_{xc}} = (1-\theta)(1-\kappa_1\nu_{xc})A_{\sigma_{xc}}, \quad m_{\sigma_{x\pi}} = (1-\theta)(1-\kappa_1\nu_{x\pi})A_{\sigma_{x\pi}}$$

the constant is given by

$$m_0 = log\delta - \frac{1}{\psi}\mu_c + \frac{1}{2}(1-\theta)\theta(1+\kappa_1 A_\lambda)^2\sigma^2_\lambda + \frac{1}{2}(1-\theta)\theta\kappa_1^2[\sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}} A_i^2\sigma^2_{\omega_i}] \tag{A.13}$$

and the market prices of risk are

$$\lambda_c = \gamma, \quad \lambda_\pi = 0, \quad \lambda_\lambda = \frac{\kappa_1\rho_\lambda - \theta}{1-\kappa_1\rho_\lambda}, \quad \lambda_{xc} = (1-\theta)\kappa_1 A_c, \quad \lambda_{x\pi} = (1-\theta)\kappa_1 A_\pi \tag{A.14}$$

$$\lambda_{\sigma_c} = (1-\theta)\kappa_1 A_{\sigma_c}, \quad \lambda_{\sigma_\pi} = 0, \quad \lambda_{\sigma_{xc}} = (1-\theta)\kappa_1 A_{\sigma_{xc}}, \quad \lambda_{\sigma_{x\pi}} = (1-\theta)\kappa_1 A_{\sigma_{x\pi}}$$

To price nominal payoffs is useful to specify the nominal discount factor which is equal to the real one minus the inflation rate:

$$m^\$_{t+1} = m_{t+1} - \pi_{t+1} \tag{A.15}$$

I can derive a similar expression for $m^\$_{t+1}$ as in equation (A.11), where the nominal discount factor parameters and the nominal market price of risks are equal to the real counterparts with the exception of $m_0^\$$, $m_\pi^\$$ and $\lambda_\pi^\$$, which are equal to:

$$m_0^\$ = m_0 - \mu_p, \quad m_\pi^\$ = m_\pi - 1 \quad \text{and} \quad \lambda_\pi^\$ = \lambda_\pi + 1$$

For convenience, it is useful to express the nominal version of equation (A.11) in terms of a time t expected component and innovations to the nominal SDF:

$$\begin{aligned} m_{t+1}^\$ - E_t m_{t+1}^\$ = & \underbrace{-\lambda_c^\$ \sigma_{c,t}\eta_{c,t+1}}_{\text{short-run consumption risk<0}} \underbrace{-\lambda_\pi^\$ \sigma_{\pi,t}\eta_{\pi,t+1}}_{\text{short-run inflation risk>0}} \underbrace{-\lambda_\lambda^\$ \sigma_{\lambda,t}\eta_{\lambda,t+1}}_{\text{preference risk <0}} \\ & \underbrace{-\lambda_{xc}^\$ \sigma_{xc,t}\eta_{xc,t+1}}_{\text{long-run consumption risk<0}} \underbrace{-\lambda_{x\pi}^\$ \sigma_{x\pi,t}\eta_{x\pi,t+1}}_{\text{long-run inflation risk>0}} \\ & \underbrace{- \sum_{i \in \{\sigma_c, \sigma_\pi, \sigma_{xc}, \sigma_{x\pi}\}} \lambda_i^\$ \sigma_{\omega_i} \omega_{i,t+1}}_{\text{Volatility risk>0}} \end{aligned} \tag{A.16}$$

Using the nominal discount factor in equation (A.15), I can solve the equilibrium nominal yields in the economy.

A.4 Solution for equilibrium nominal bond yields

In this section I solve the yields of nominal zero-coupon bonds of different maturities. Let $P_{t,n}^\$$ be the time-t price of an n-period zero-coupon nominal bond that pays one unit of numeraire in n periods. Using the Euler equation I can write the price in logs recursively as

$$p_{t,n}^\$ = logE_t exp(m_{t+1}^\$ + p_{t+1,n-1}^\$) \tag{A.17}$$

where I define $p_{t,n}^\$ = logP_{t,n}^\$$. Conjecture that $p_{t,n}^\$$ is a linear function of the state variables

$$p_{t,n}^\$ = -(B_{0,n}^\$ + B_{c,n}^\$ x_{c,t} + B_{\pi,n}^\$ x_{\pi,t} + B_{\lambda,n}^\$ x_{\lambda,t} + B_{\sigma_c,n}^\$ \sigma_{x,t}^2 + B_{\sigma_\pi,n}^\$ \sigma_{\pi,t}^2 + B_{\sigma_{xc},n}^\$ \sigma_{xc,t}^2 + B_{\sigma_{x\pi},n}^\$ \sigma_{x\pi,t}^2) \tag{A.18}$$

To solve the Bs I again use the method of undetermined coefficients and it follows that:

$$\begin{aligned}
B^{\$}_{c,n} &= B^{\$}_{c,n-1}\rho_{cc} - m^{\$}_{c} \\
B^{\$}_{\pi,n} &= B^{\$}_{c,n-1}\rho_{c\pi} + B^{\$}_{\pi,n-1}\rho_{\pi\pi} - m^{\$}_{\pi} \\
B^{\$}_{\lambda,n} &= B^{\$}_{\lambda,n-1}\rho_{\lambda} - m^{\$}_{\lambda} \\
B^{\$}_{\sigma_c,n} &= B^{\$}_{\sigma_c,n-1}\nu_c - \frac{1}{2}(\lambda^{\$}_c)^2 - m^{\$}_{\sigma_c} \\
B^{\$}_{\sigma_\pi,n} &= B^{\$}_{\sigma_\pi,n-1}\nu_\pi - \frac{1}{2}(\lambda^{\$}_\pi)^2 - m^{\$}_{\sigma_\pi} \\
B^{\$}_{\sigma_{xc},n} &= B^{\$}_{\sigma_{xc},n-1}\nu_{xc} - \frac{1}{2}(\lambda^{\$}_c + B^{\$}_{c,n-1})^2 - m^{\$}_{\sigma_{xc}} \\
B^{\$}_{\sigma_{x\pi},n} &= B^{\$}_{\sigma_{x\pi},n-1}\nu_{x\pi} - \frac{1}{2}(\lambda^{\$}_\pi + B^{\$}_{\pi,n-1})^2 - m^{\$}_{\sigma_{x\pi}} \\
B^{\$}_{0,n} &= B^{\$}_{0,n-1} - \frac{1}{2}(\lambda_\lambda + B^{\$}_{\lambda,n-1})^2\sigma^2_\lambda - \sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}} [(B^{\$}_{i,n-1})\sigma^2_{i,0}(1-\nu_i) + \frac{1}{2}(\lambda_i + B^{\$}_{i,n-1}))^2\sigma^2_{\omega_i}] - m^{\$}_0
\end{aligned} \tag{A.19}$$

with initial conditions $B^{\$}_{c,n} = B^{\$}_{\pi,n} = B^{\$}_{\lambda,n} = B^{\$}_{\sigma_c,n} = B^{\$}_{\sigma_\pi,n} = B^{\$}_{\sigma_{xc},n} = B^{\$}_{\sigma_{x\pi},n} = B^{\$}_{0,n} = 0$. Given the price of a zero-coupon bond, bond yields are defined $y^{\$}_{t,n} = -\frac{1}{n} lnP^{\$}_{t,n}$. Hence,

$$y^{\$}_{t,n} = \frac{1}{n}(B^{\$}_{0,n} + \underbrace{B^{\$}_{c,n}}_{>0} x_{c,t} + \underbrace{B^{\$}_{\pi,n}}_{>0} x_{\pi,t} + \underbrace{B^{\$}_{\lambda,n}}_{<0} x_{\lambda,t} + \underbrace{B^{\$}_{\sigma_c,n}}_{<0}\sigma^2_{c,t} + \underbrace{B^{\$}_{\sigma_\pi,n}}_{<0}\sigma^2_{\pi,t} + \underbrace{B^{\$}_{\sigma_{xc},n}}_{<0}\sigma^2_{xc,t} + \underbrace{B^{\$}_{\sigma_{x\pi},n}}_{>0 \text{ for high } n}\sigma^2_{x\pi,t})$$

A.5 Expected excess bond returns

Define the excess log return on buying an n period bond at time t and selling it at time $t+1$ as an $n-1$ period bond:

$$rx^{\$}_{r\to t+1,n} = ny^{\$}_{t,n} - (n-1)y^{\$}_{t+1,n-1} - y^{\$}_{t,1}$$

As we did for the nominal SDF, the excess log return can be expressed in terms of a time t expected component and return innovations:

$$rx^{\$}_{r\to t+1,n} - E_t rx^{\$}_{r\to t+1,n} = -B^{\$}_{c,n-1}\sigma_{xc,t}\eta_{xc,t+1} - B^{\$}_{\pi,n-1}\sigma_{x\pi,t}\eta_{x\pi,t+1} - B^{\$}_{\lambda,n-1}\sigma_\lambda\eta_{\lambda,t+1} - \sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}} B^{\$}_{i,n-1}\sigma_{\omega_i}\omega_{i,t+1}$$

In the model, one-period expected excess return on nominal bonds is determined by the negative covariation between the innovations to excess log returns and the innovations to the nominal SDF in equation (A.16):

$$\begin{aligned}
E_t rx^{\$}_{r\to t+1,n} + \frac{1}{2}V_t rx^{\$}_{r\to t+1,n} &= -cov_t(m_{t+1}, rx^{\$}_{r\to t+1,n}) \\
&= \underbrace{-\sum_{i\in\{\sigma_c,\sigma_\pi,\sigma_{xc},\sigma_{x\pi}\}} B^{\$}_{i,n-1}\lambda_i\sigma^2_{\omega_i}}_{\text{Volatility risk}} \underbrace{-B^{\$}_{\lambda,n-1}\lambda_\lambda\sigma^2_\lambda}_{\text{Preference risk}} \underbrace{-B^{\$}_{c,n-1}\lambda_{xc}\sigma^2_{xc,t}}_{\text{long-run growth risk}} \underbrace{-B^{\$}_{\pi,n-1}\lambda_{x\pi}\sigma^2_{x\pi,t}}_{\text{long-run inflation risk}}
\end{aligned} \tag{A.20}$$

A.6 Term Premium

I can decompose the n-period yield into expectations of average expected future short rates and average excess returns over the life of the bond:

$$y^{\$}_{t,n} = \frac{1}{n}\sum_{i=1}^{n} E_t y^{\$}_{t+i-1,1} + \frac{1}{n}\sum_{i=1}^{n} E_t rx^{\$}_{r\to t+1,n-i+1} \tag{A.21}$$

The last term on the right is often described as the bond's nominal term premium, and we will denote it as $tp^{\$}_{t,n}$. In this model, the term premia is time-varying due to variations in bond risk premia driven by short and long-run real and inflation volatilities. Specifically,

$$\begin{aligned} tp^{\$}_{t,n} &= y^{\$}_{t,n} - \frac{1}{n}\sum_{i=1}^{n} E_t y^{\$}_{t+i-1,1} \\ &= \frac{1}{n}B^{\$}_{0,n} - B^{\$}_{0,1} - \frac{1}{n}\sum_{i\in\{c,\pi,xc,x\pi\}} B^{\$}_{\sigma_i,1}(n-\nu_i\frac{1-\nu_i^n}{1-\nu_i})\sigma^2_{i,0} + \frac{1}{n}\sum_{i\in\{xc,x\pi\}}(B^{\$}_{\sigma_i,n} - B^{\$}_{\sigma_i,1}\frac{1-\nu_i^n}{1-\nu_i})\sigma^2_{i,t} \end{aligned} \tag{A.22}$$

A.7 Inflation variance ratios: Model-implied

Equation (A.21) decomposes an n-period yield into expectations of average expected future short rates and a term premium component. If I define the ex-ante real rate as the yield of a one-period nominal bond minus expected inflation,

$$r_t = y^{\$}_{t,1} - E_t\pi_{t+1}$$

I can further decompose the bond yield as

$$y^{\$}_{t,n} = \frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1} + \frac{1}{n}\sum_{i=1}^{n} E_t\pi_{t+i} + tp^{\$}_{t,n} \tag{A.23}$$

From this last equation it is clear that innovations in the n- maturity yield from $t-1$ to t are equal to the sum of news about ex-ante real rates, expected average inflation and term premium shocks:

$$\epsilon^{(n)}_{y^{\$},t} = \epsilon^{(n)}_{r,t} + \eta^{(n)}_{\pi,t} + \epsilon^{(n)}_{tp^{\$},t} \tag{A.24}$$

with

$$\epsilon^{(n)}_{y^{\$},t} = E_t y^{\$}_{t,n} - E_{t-1}y^{\$}_{t,n}, \qquad \epsilon^{(n)}_{r,t} = \tfrac{1}{n}\textstyle\sum_{i=1}^{n} E_t r_{t+i-1,1} - \tfrac{1}{n}\sum_{i=1}^{n} E_{t-1}r_{t+i-1,1}.$$

$$\epsilon^{(n)}_{tp^{\$},t} = E_t tp^{\$}_{t,n} - E_{t-1} tp^{\$}_{t,n}, \qquad \epsilon^{(n)}_{\pi,t} = \tfrac{1}{n}\textstyle\sum_{i=1}^{n} E_t \pi_{t+i} - \tfrac{1}{n}\sum_{i=1}^{n} E_{t-1}\pi_{t+i}.$$

Given the equilibrium solution of bond yields and the assumed process for the inflation rate I am able to derive analytical expressions for each one of this shocks. In particular, ex-ante real rates are given by

$$\begin{aligned}\frac{1}{n}\sum_{i=1}^{n} E_t r_{t+i-1,1} = {} & \frac{1}{n}(B^{\$}_{0,1} - \mu_\pi + \sum_{i\in\{c,\pi,xc,x\pi\}} B^{\$}_{\sigma_i,1}(n - \nu_i \frac{1-\nu_i^n}{1-\nu_i})\sigma^2_{i,0}) + \frac{1}{n}B^{\$}_{c,1}\frac{1-\rho^n_{cc}}{1-\rho_{cc}}x_{c,t} + \frac{1}{n}B^{\$}_{\lambda,1}\frac{1-\rho^n_{\lambda}}{1-\rho_{\lambda}}x_{\lambda,t} \\ & + \frac{1}{n}\frac{\rho_{c\pi}}{1-\rho_{cc}}B^{\$}_{c,1}[(1-\rho^{n-1}_{cc}) + \rho_{\pi\pi}(1-\rho^{n-2}_{cc}) + \rho^2_{\pi\pi}(1-\rho^{n-3}_{cc}) + \cdots + \rho^{n-2}_{\pi\pi}(1-\rho_{cc})]x_{\pi,t} \\ & + \frac{1}{n}\sum_{i\in\{c,\pi,xc,x\pi\}} B^{\$}_{\sigma_i,1}\frac{1-\nu_i^n}{1-\nu_i}\sigma^2_{i,t}\end{aligned}$$

Thus, we can write the innovations to this component as

$$\begin{aligned}\epsilon^{(n)}_{r,t} = {} & \frac{1}{n}B^{\$}_{c,1}\frac{1-\rho^n_{cc}}{1-\rho_{cc}}\sigma_{xc,t-1}\eta_{xc,t} + \frac{1}{n}B^{\$}_{\lambda,1}\frac{1-\rho^n_{\lambda}}{1-\rho_{\lambda}}\sigma_{\lambda}\eta_{\lambda,t} \\ & + \frac{1}{n}\frac{\rho_{c\pi}}{1-\rho_{cc}}B^{\$}_{c,1}[(1-\rho^{n-1}_{cc}) + \rho_{\pi\pi}(1-\rho^{n-2}_{cc}) + \rho^2_{\pi\pi}(1-\rho^{n-3}_{cc}) + \cdots + \rho^{n-2}_{\pi\pi}(1-\rho_{cc})]\sigma_{x\pi,t-1}\eta_{x\pi,t} \\ & + \frac{1}{n}\sum_{i\in\{c,\pi,xc,x\pi\}} B^{\$}_{\sigma_i,1}\frac{1-\nu_i^n}{1-\nu_i}\sigma_{\omega_i}\omega_{i,t+1}\end{aligned} \tag{A.25}$$

Similarly, expected average inflation is equal to

$$\frac{1}{n}\sum_{i=1}^{n} E_t \pi_{t+i} = \mu_\pi + \frac{1}{n}B^{\$}_{\pi,1}\frac{1-\rho^n_{\pi\pi}}{1-\rho_{\pi\pi}}x_{\pi,t}$$

with shocks given by

$$\epsilon^{(n)}_{\pi,t} = \frac{1}{n}B^{\$}_{\pi,1}\frac{1-\rho^n_{\pi\pi}}{1-\rho_{\pi\pi}}\sigma_{x\pi,t-1}\eta_{x\pi,t} \tag{A.26}$$

Furthermore, equation (A.22) provides an expression for the term premia. It follows that shocks to this component are

$$\epsilon^{(n)}_{tp^{\$},t} = \frac{1}{n}\sum_{i\in\{xc,x\pi\}}(B^{\$}_{\sigma_i,n} - B^{\$}_{\sigma_i,1}\frac{1-\nu_i^n}{1-\nu_i})\sigma_{\omega_i}\omega_{i,t+1} \tag{A.27}$$

It is straightforward to show that the sum of $\epsilon^{(n)}_{r,t}$, $\epsilon^{(n)}_{\pi,t}$ and $\epsilon^{(n)}_{tp^{\$},t}$ (given by equations (A.25), (A.26) and (A.27)) is equal to innovations in the n- maturity yield:

$$\epsilon^{(n)}_{y^{\$},t} = B^{\$}_{c,n}\sigma_{xc,t-1}\eta_{xc,t} + B^{\$}_{\pi,n}\sigma_{x\pi,t-1}\eta_{x\pi,t} + B^{\$}_{\lambda,n}\sigma_{\lambda}\eta_{\lambda,t} + \sum_{i\in\{c,\pi,xc,x\pi\}} B^{\$}_{\sigma_i,n}\sigma_{\omega_i}\omega_{i,t+1} \tag{A.28}$$

Finally, the model-implied inflation variance ratios are equal to the variance of (A.26) divided by the variance of (A.28).

B State-space representation of the macro-finance model

In this section I describe a state-space representation of the model and its estimation procedure. In Section B.1, I derive the measurement equation, while in Section B.2, I show the state transition equation. In Section B.3, I describe the algorithm for posterior inference.

B.1 Measurement equation

The goal of this section is to derive the coefficients of the measurement equation described in Section 2.4.1 given by:

$$Y^o_{t+1} = A_{t+1}(D + Zs_{t+1} + Z^v s^v_{t+1}(h_{t+1}) + \Sigma^u u_{t+1}), \quad \text{with} \quad u_{t+1} \sim N(0, I) \tag{B.1}$$

To have an expression of the observed variables in terms of model parameters and measurement errors, recall the assumed process for the macroeconomic variables and the equilibrium solutions for bond yields presented in section A, together with the measurement-error structure described in Section 2.4.1. In particular suppose that $t+1$ is the last month of quarter m and write:

$$\begin{aligned}
\Delta c^o_{t+1} &= \mu_c + x_{c,t} + \sigma_{c,t}\eta_{c,t+1} + \sigma_\epsilon(\epsilon_{t+1} - \epsilon_t) \\
\Delta c^o_t &= \mu_c + x_{c,t-1} + \sigma_{c,t-1}\eta_{c,t} + \sigma_\epsilon(\epsilon_t - \epsilon_{t-1}) \\
\Delta c^o_{t-1} &= \mu_c + x_{c,t-2} + \sigma_{c,t-2}\eta_{c,t-1} - \frac{1}{3}\sigma_\epsilon(\epsilon_{t+1} + \epsilon_t) + \frac{2}{3}\sigma_\epsilon\epsilon_{t-1} - \frac{2}{3}\sigma_\epsilon\epsilon_{t-2} + \frac{1}{3}\sigma_\epsilon(\epsilon_{t-3} + \epsilon_{t-4}) + \sigma^q_\epsilon(\epsilon^q_{t+1} + \epsilon^q_{t-2}) \\
\pi^o_{t+1} &= \mu_\pi + x_{\pi,t} + \sigma_{\pi,t}\eta_{\pi,t+1} \\
y^{\$,o}_{t+1,n} &= \frac{1}{n}((B^\$_{0,n} + B^\$_{c,n}x_{c,t+1} + B^\$_{\pi,n}x_{\pi,t+1} + B^\$_{\lambda,n}x_{\lambda,t+1} + B^\$_{\sigma_c,n}\sigma^2_{c,t+1} + B^\$_{\sigma_\pi,n}\sigma^2_{\pi,t+1} + B^\$_{\sigma_{xc},n}\sigma^2_{xc,t+1} + B^\$_{\sigma_{x\pi},n}\sigma^2_{x\pi,t+1}) + \sigma_{y_n}\epsilon_{y_n,t+1}
\end{aligned}$$

where to simplify, the notation $y^{\$,o}_{t+1,n}$ denotes the one- to five-year zero-coupon bonds. Given these expressions, it is just a matter of ordering all the pieces adequately. The state vector s_{t+1} and s^v_{t+1} are given by:

$$\begin{aligned}
s_{t+1} = [&x_{c,t+1},\ x_{c,t},\ x_{c,t-1},\ x_{c,t-2}\ \sigma_{c,t}\eta_{c,t+1},\ \sigma_{c,t-1}\eta_{c,t},\ \sigma_{c,t-2}\eta_{c,t-1},\ \sigma_\epsilon\epsilon_{t+1},\ \sigma_\epsilon\epsilon_t,\ \sigma_\epsilon\epsilon_{t-1},\ \sigma_\epsilon\epsilon_{t-2},\ \sigma_\epsilon\epsilon_{t-3},\ \sigma_\epsilon\epsilon_{t-4}, \\
&\sigma^q_\epsilon\epsilon^q_{t+1},\ \sigma^q_\epsilon\epsilon^q_t,\ \sigma^q_\epsilon\epsilon^q_{t-1},\ \sigma^q_\epsilon\epsilon^q_{t-2},\ x_{\pi,t+1},\ x_{\pi,t},\ \sigma_{\pi,t}\eta_{\pi,t+1},\ x_{\lambda,t+1},\ x_{\lambda,t}]',
\end{aligned}$$

and

$$s^v_{t+1} = [\sigma^2_{xc,t+1},\ \sigma^2_{x\pi,t+1},\ \sigma^2_{c,t+1},\ \sigma^2_{\pi,t+1}]'.$$

Given these state vectors, I write the matrices D, Z, Z^v and Σ^u as follows:

$$
Z=\begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0\\
\frac{1}{n}B^{\$}_{c,n} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{n}B^{\$}_{\pi,n} & 0 & \frac{1}{n}B^{\$}_{\lambda,n} & 0
\end{bmatrix}
$$

$$
D=\begin{bmatrix}\mu_c\\ \mu_c\\ \mu_c\\ \mu_\pi\\ \frac{1}{n}B^{\$}_{0,n}\end{bmatrix},\quad
Z^v=\begin{bmatrix}
0 & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
0 & 0 & 0 & 0\\
\frac{1}{n}B^{\$}_{\sigma_{xc},n} & \frac{1}{n}B^{\$}_{\sigma_{x\pi},n} & \frac{1}{n}B^{\$}_{\sigma_c,n} & \frac{1}{n}B^{\$}_{\sigma_\pi,n}
\end{bmatrix},\quad
\Sigma^u=\begin{bmatrix}
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \sigma_{y_n}
\end{bmatrix}
$$

Finally, the vector of observables Y^o_{t+1} and the selection matrix A_{t+1} can be written as:

- If t+1 is the last month of the quarter:

$$
Y^o_{t+1}=\begin{bmatrix}\Delta c^o_{t+1}\\ \Delta c^o_t\\ \Delta c^o_{t-1}\\ \pi^o_{t+1}\\ y^{\$}_{t+1,n}\end{bmatrix},\qquad
A_{t+1}=\begin{bmatrix}
1 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0\\
0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 1
\end{bmatrix}'
$$

- If t+1 is not the last month of the quarter:

$$
Y^o_{t+1}=\begin{bmatrix}\pi^o_{t+1}\\ y^{\$}_{t+1,n}\end{bmatrix},\qquad
A_{t+1}=\begin{bmatrix}
0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 1
\end{bmatrix}.
$$

B.2 State transition equation

The goal of this section is to derive the coefficients of the measurement equation described in Section 2.4.2 given by:

$$
s_{t+1}=\Phi s_t+v_{t+1}(h_t) \tag{B.2}
$$

and

$$
h_{t+1}=\Psi h_t+\Sigma_h\omega_{t+1},\quad h_{t+1}=[h_{xc,t+1},\ h_{x\pi,t+1},\ h_{c,t+1},\ h_{\pi,t+1}]'\quad\text{with}\quad \omega_{t+1}\sim N(0,I). \tag{B.3}
$$

The state variables evolve according to

$$
\begin{aligned}
x_{c,t+1} &= \rho_{cc}x_{c,t}+\rho_{c\pi}x_{\pi,t}+\sigma_{xc,t}\eta_{xc,t+1}\\
x_{\pi,t+1} &= \qquad\qquad +\rho_{\pi\pi}x_{\pi,t}+\sigma_{x\pi,t}\eta_{x\pi,t+1}\\
x_{\lambda,t+1} &= \rho_\lambda x_{\lambda,t}+\sigma_\lambda\eta_{\lambda,t+1}
\end{aligned}
$$

Hence, Φ and $v_{t+1}(h_t)$ just make sure that these dynamics are preserved with zeros and ones in the adequate places:

$$
\Phi = \begin{bmatrix}
\rho_{cc} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{c\pi} & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\pi\pi} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\lambda} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}, \quad v_{t+1}(h_t) = \begin{bmatrix}
\sigma_{xc,t}\eta_{xc,t+1} \\ 0 \\ 0 \\ 0 \\ \sigma_{c,t}\eta_{c,t+1} \\ 0 \\ 0 \\ \sigma_{\epsilon}\epsilon_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma^q_{\epsilon}\epsilon^q_{t+1} \\ 0 \\ 0 \\ 0 \\ \sigma_{x\pi,t}\eta_{x\pi,t+1} \\ 0 \\ \sigma_{\pi,t}\eta_{\pi,t+1} \\ 0 \\ \sigma_{\lambda}\eta_{\lambda,t+1}
\end{bmatrix}
$$

Finally, Ψ, Σ_h and ω_{t+1} can be written as

$$
\Psi = \begin{bmatrix}
\rho_{h_{xc}} & 0 & 0 & 0 \\
0 & \rho_{h_{x\pi}} & 0 & 0 \\
0 & 0 & \rho_{h_c} & 0 \\
0 & 0 & 0 & \rho_{h_\pi}
\end{bmatrix}, \quad \Psi = \begin{bmatrix}
\sigma_{h_{xc}} & 0 & 0 & 0 \\
0 & \sigma_{h_{x\pi}} & 0 & 0 \\
0 & 0 & \sigma_{h_c} & 0 \\
0 & 0 & 0 & \sigma_{h_\pi}
\end{bmatrix}, \quad \omega_{t+1} = \begin{bmatrix}
\omega_{xc,t+1} \\ \omega_{x\pi,t+1} \\ \omega_{c,t+1} \\ \omega_{\pi,t+1}
\end{bmatrix}
$$

with

$$
\sigma_{i,t} = \varphi_i \sigma exp(h_{i,t}), \quad \text{for} \quad i = \{c, \pi, xc, x\pi\}
$$

and I normalized $\varphi_c = 1$.

B.3 Posterior inference

For posterior inference I use a Bayesian MCMC particle filter to the state-space representation described by equations B.1, B.2 and B.3, which is linear conditional on the log volatilities h_t. I briefly summarize the main steps below; for further details, refer to Algorithm 13 in Herbst and Schorfheide (2015).

1. I initialize the Markov Chain at some parameter values Θ^0.
2. Given Θ^k
   (a) **Initialization.** At time $t = 0$, I draw M particles $\{h_0^j\}_{j=1}^M$ from the unconditional distribution of equation B.3. Conditional on each particle h_0^j, I generate s_0^j from the

unconditional distribution of equation B.2. I set each particle weight to $\pi_0^j = \frac{1}{M}$, $j = 1, \ldots, M$.

(b) **Recursion.** For $t = 1, \ldots, T$:

i. **Forecasting s_t.** I propagate each particle h_{t-1}^j using the law of motion in equation B.3 to get h_t^j. Given s_{t-1}^j and (h_{t-1}^j, h_t^j) I run one iteration of the Kalman filter using the state-space system given by equations B.1 and B.2, which is conditionally linear. This step delivers the distribution of s_t^j, denoted by $p(s_t|y_t^o, s_{t-1}^j, h_{t-1}^j, h_t^j)$, which is normal with mean $s_{t|t}^j$ and variance $P_{t|t}^j$. Hence,

$$s_t^j \sim N(s_{t|t}^j, P_{t|t}^j).$$

ii. **Forecasting y_t.** I compute the incremental weights $\tilde{\omega}_t^j$ according to

$$\tilde{\pi}_t^j = \pi_{t-1}^j \times p(y_t^o|\tilde{y}_{t|t-1}^j, F_{t|t-1}^j)$$

where the Kalman filter step in i delivers $\tilde{y}_{t|t-1}^j$ and $F_{t|t-1}^j$. The likelihood $p(y_t^o|\tilde{y}_{t|t-1}^j, F_{t|t-1}^j)$ is conditionally Gaussian and follows from the measurement equation B.1.

iii. **Updating.** I define the normalized weights by

$$\pi_t^j = \frac{\tilde{\pi}_t^j}{\sum_{i=1}^M \tilde{\pi}_t^i}$$

and resample the particles h_t^j and states s_t^j using multinomial resampling with weights π_t^j.

(c) **Likelihood Approximation.** The approximation of the log-likelihood function is given by

$$ln\hat{p}(y_t^o|Y_{1:t-1}^o) = ln\hat{p}(y_{t-1}^o|Y_{1:t-2}^o) + ln(\sum_{i=1}^M \tilde{\pi}_t^j)$$

(d) **Metropolis-Hastings algorithm** Once I am able to approximate the likelihood function, I use a standard random walk Metropolis-Hastings algorithm to obtain a new parameter draw Θ^{k+1}. See Algorithm 18 in Herbst and Schorfheide (2015) for further details regarding this step.

(e) I repeat steps (a) to (b) N_{sim} times.

In the implementation, I used 10,000 particles ($M = 10,000$), generated $50,000$ draws ($N_{sim} = 50,000$) and set the burn in period at $25,000$. I targeted an acceptance rate of approximately 30 percent. In addition, I checked that the results do not changed if I increase the number of particles.

B.4 Role of measurement error model of consumption and stochastic volatility

In Section 3.3.2 of the main text, I documented that the posterior median estimates of $\{\rho_{cc}, \rho_{c\pi}, \rho_{\pi\pi}\}$ are remarkably similar with or without including bond prices in the estimation. The result is mostly driven by the assumed measurement error model for consumption. To see this, in Table B1,I show the

5, 50, 95 percentiles of the posterior distribution of the persistent parameters under three different specifications of the consumption growth and the inflation process. In (a) I assume homoskedastic innovations and that consumption growth is measured without any errors, (i.e., $c_t^o = c_t$). In (b) I add the measurement error model of consumption, while in (c), I assume the measurement error model of consumption and consider short- and long-run stochastic volatility. In the last specification, I impose a common stochastic volatility process (i.e., $h_{c,t} = h_{xc,t}$ and $h_{\pi,t} = h_{x\pi,t}$). Furthermore, in the last column of the table, I report the log marginal data densities to provide formal support in terms of model fit for each specification. In all cases I only consider macro data in the estimation.

[Place Table B1 about here]

Without the measurement error model, the posterior median ρ_{cc} is equal to -0.20 in order to match the monthly negative autocorrelation observed in the data of -0.17. Nevertheless, this version of the model cannot reconcile the monthly negative autocorrelation with a positive autocorrelation observed at a quarterly or annual frequency. Accounting for measurement errors in consumption that average out every quarter alleviates this problem. Under this specification, the posterior median of ρ_{cc} jumps to 0.90, and the fit of the model measured by the log marginal data density improves significantly from 5855 to 5870. At the same time, the persistence parameter associated with expected inflation, $\rho_{\pi\pi}$, remains largely unchanged at a value of 0.91, while the parameter that measures the effect of expected inflation on expected growth, $\rho_{c\pi}$, increases from -0.13 to -0.02. Adding stochastic volatility to the process of the macro variables improves the model fit considerably, the posterior uncertainty decreases (tighter credible intervals) and the persistence parameters ρ_{cc} and $\rho_{\pi\pi}$ jump to 0.94 and 0.98, respectively.[21] $\rho_{c\pi}$ remained at -0.02. In terms of model fit, adding stochastic volatility increases the log marginal data density from 5870 to 6055.[22] Finally, it is important to highlight that under the three different specification, $\rho_{c\pi}$, is negative and significant as suggested by the 90 percent credible interval.

[21]The intuition for why the ρ_{cc} and $\rho_{\pi\pi}$ increases goes as follows. The presence of stochastic volatility allows for sharp fluctuations in Δc_{t+1}^o and π_{t+1}^o by incurring in similar movements in their conditional variance without the need of large temporary shocks by increasing σ and φ_π and reducing the estimates of the persistent parameters.

[22]In specification (Me. Sv.) I impose a common stochastic volatility process (i.e.$h_{c,t} = h_{xc,t}$ and $h_{\pi,t} = h_{x\pi,t}$). If I allow for four different stochastic volatility processes, the confidence interval for most of the parameters increased considerably and the log marginal data density decreases to 5761. With these evidence, I conclude that there is not enough information in consumption growth and inflation to identify four different stochastic volatilities

B.5 Misspecification test

To assess the extent to which the documented change in the parameter estimates documented in Section 3.3.2 leads to a decrease in fit of the macroeconomic variables, I re-estimated model (3) conditional on different parameter values and re-computed the marginal data densities. In particular, I took the set of parameters that deliver the highest posterior value under the model that includes and excludes bond prices in the estimation and fixed one parameter at a the time. From these estimations, I obtain constraint log marginal data densities and I standardize those values using the mean and standard deviation of the log marginal data densities computed from 30 different estimations of the unconstrained version of the model. The results are summarized in Table B2, where I report the values only for the parameters that are most relevant for matching the nominal yield curve distribution. As expected, all standardized marginal data densities related to the high posterior macro parameters (Macro data) are positive or less than one and a half standard deviations away from the unconstrained model. The key finding here is that if I move to the high posterior parameters associated with the information contained in bond prices, the drop in the log marginal data densities is small. With the exception of $\rho_{h_{x\pi}}$, all values are inside one standard deviation, indicating that there is essentially no or a small tension between the parameter estimates obtained with our without bond yield data.

[Place Table B2 about here]

C Inflation variance ratios: Data

In this section I provide further details on how to compute the inflation variance ratios from the data. In Section C.1, I show how to compute news to expected average inflation and to nominal bond yields, which are the main ingredients for the inflation variance ratios derived from the data. In Section C.2, I write the state-space representation of the model, while in Section C.3, I present the estimation procedure and the prior and posterior distribution of the model parameters. Finally, in Section C.4, I carry out some robustnesses checks based on a quarterly version of the model.

C.1 News to expected average inflation and to nominal bond yields

From equation (11) in Section 2.3.2, I showed that innovations to the n- maturity yield from $t-1$ to t are equal to the sum of news about ex-ante real rates, expected average inflation and term premium shocks:

$$\epsilon^{(n)}_{y^{\$},t} = \epsilon^{(n)}_{r,t} + \epsilon^{(n)}_{\pi,t} + \epsilon^{(n)}_{tp^{\$},t} \tag{C.1}$$

Next I provide further details on how to obtained an estimate of $\epsilon^{(n)}_{\pi,t}$. For consistency, I extracted one- to five-year inflation forecasts from the same model assumed in the macro-finance model:

$$\begin{aligned} \pi_{t+1} &= \mu_{\pi} + x_t + \sigma_{\pi,t}\eta_{\pi,t} \\ x_t &= \rho x_{t-1} + \sigma_{x,t}\eta_{x,t} \end{aligned} \tag{C.2}$$

with

$$\eta_{\pi,t}, \quad \eta_{x,t} \quad \sim i.i.d.N(0,1)$$

Given this process for inflation, time t inflation expectations for j months ahead is given by

$$E_t\pi_{t+j} = \mu_{\pi} + \rho^{j-1}x_t$$

Hence the expected inflation innovation from t to $t+1$ in period $t+j$ can be written as

$$E_{t+1} - E_t\pi_{t+j} = \rho^{j-1}\sigma_{x,t}\eta_{x,t+1}$$

From here, I can express the innovations to expected average inflation as

$$\epsilon^{(n)}_{\pi,t} = E_{t+1} - E_t\frac{1}{n}\sum_{i=1}^{m}\pi_{t+j} = \frac{1}{n}\frac{1-\rho^n}{1-\rho}\sigma_{x,t}\eta_{x,t,t+1} \tag{C.3}$$

Alternatively, to compute shocks to the n-period yield, $\epsilon^{(n)}_{y^{\$},t}$, I assume that bond yields follow martingales

$$y^{(n)}_t = y^{(n)}_{t-1} + \sigma_{y^{(n)},t}\eta_{y^{(n)},t+1} \qquad \text{with} \qquad \eta_{y^{(n)},t+1} \sim i.i.d.N(0,1) \tag{C.4}$$

where

$$\epsilon^{(n)}_{y^{\$},t} = \sigma_{y^{(n)},t}\eta_{y^{(n)},t,t+1} \tag{C.5}$$

Finally, the process for the stochastic volatilities is also the same as in the baseline model

$$\sigma_{i,t} = \sigma_i exp(h_{i,t}), \qquad \text{with} \quad h_{i,t+1} = \rho_{h_i} h_{i,t} + \sigma_{h_i}\omega_{i,t+1} \text{ and } \omega_{i,t+1} \sim i.i.d.N(0,1) \text{ for } i = \{\pi, x, y\}$$

assuming a common volatility structure for the bond yields.

C.2 State-space representation

The state-space representation is given by the measurement equation:

$$Y^o_{t+1} = A_{t+1}(D + Z\alpha_{t+1} + H\epsilon_{t+1}) \quad \text{with} \quad \epsilon_{t+1} \sim N(0, I) \tag{C.6}$$

and the state transition equation is

$$\alpha_{t+1} = T\alpha_t + \eta_{t+1}(h_t) \quad \text{and} \quad h_{t+1} = \Psi h_t + \Sigma_h \omega_{t+1} \quad \text{with} \quad \omega_{t+1} \sim N(0, I) \tag{C.7}$$

with $h_{t+1} = [h_{\pi,t+1},\ h_{x,t+1},\ h_{y,t+1}]'$.

Following Duffee (2016), I include information from the surveys of market practitioners that are released in a different (quarterly) frequency to sharpen inference. In particular, I consider the CPI forecasts from the Survey of Professional Forecasters of one to four quarters ahead. To this end, I assume that the forecasts are published at the second month of the quarter, which is consistent with the release date of the Federal Reserve Bank of Philadelphia. To gain some insight into how to include this information, let t be the second month of the quarter. Then the market practitioners make an inflation forecast for say, one quarter ahead, starting from period $t + 1$, which I label as $\tilde{\pi}^{o,1}_{t+1}$. Using the process for inflation in equation C.2 the following relation follows:

$$\tilde{\pi}^{o,1}_{t+1} = E_t[\pi_{t+2} + \pi_{t+3} + \pi_{t+4}] = 3\mu_p + \tilde{\rho}_1 x_t$$

where to simplify the notation I defined $\tilde{\rho}_1 = \rho + \rho^2 + \rho^3$. The same applies for the inflation forecasts of two $\tilde{\pi}^{o,2}_{t+1}$, three $\tilde{\pi}^{o,3}_{t+1}$ and four $\tilde{\pi}^{o,4}_{t+1}$ quarters ahead. Following this notation, the observed variables in terms of the state variables are given by

$$\begin{aligned}
\pi^o_{t+1} &= \mu + x_t + \sigma_{\pi,t}\eta_{\pi,t+1} \\
\tilde{\pi}^{o,1}_{t+1} &= 3\mu + \tilde{\rho}_1 x_t + \sigma^1_\epsilon \epsilon^1_{t+1} \\
\tilde{\pi}^{o,2}_{t+1} &= 3\mu + \tilde{\rho}_2 x_t + \sigma^2_\epsilon \epsilon^2_{t+1} \\
\tilde{\pi}^{o,3}_{t+1} &= 3\mu + \tilde{\rho}_3 x_t + \sigma^3_\epsilon \epsilon^3_{t+1} \\
\tilde{\pi}^{o,4}_{t+1} &= 3\mu + \tilde{\rho}_4 x_t + \sigma^4_\epsilon \epsilon^4_{t+1} \\
y^{\$,o}_{t+1,n} &= y^{\$}_{t+1,n} + \sigma_{\epsilon,y_n}\epsilon_{y_n,t+1}
\end{aligned}$$

where to simplify the notation $y^{\$,o}_{t+1,n}$ denotes the one- to five-year zero-coupon bonds and I assumed that the survey consensus forecasts are measured with errors. The standard deviation of the measurement errors are free parameters given by σ^i_ϵ. To be consistent with the macro-finance model, I set the standard deviation of the yield's measurement error, σ_{ϵ,y_n}, at 10 percent of their sample standard deviation, similar to the assumed value for the macro-finance model. Furthermore, I defined

$$\tilde{\rho}_2 = \rho^4 + \rho^5 + \rho^6, \quad \tilde{\rho}_3 = \rho^7 + \rho^8 + \rho^9, \qquad \text{and} \quad \tilde{\rho}_4 = \rho^{10} + \rho^{11} + \rho^{12}.$$

The state vector can be written as

$$\alpha_{t+1} = \begin{bmatrix} x_{t+1} & x_t & \sigma_{\pi,t}\eta_{\pi,t+1} & \sigma^1_\epsilon\epsilon^1_{t+1} & \sigma^2_\epsilon\epsilon^2_{t+1} & \sigma^3_\epsilon\epsilon^3_{t+1} & \sigma^4_\epsilon\epsilon^4_{t+1} & y_{t+1,n} & y_{t,n} \end{bmatrix}'.$$

Hence, the D, Z and H can be written as

$$D = \begin{bmatrix} \mu \\ 3\mu \\ 3\mu \\ 3\mu \\ 3\mu \\ 0 \end{bmatrix}, \quad Z = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{\rho}_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{\rho}_2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \tilde{\rho}_3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & \tilde{\rho}_4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{\epsilon,y_n} \end{bmatrix}$$

Finally, the vector of observables Y^o_{t+1} and the selection matrix A_{t+1} can be written as:

- If t+1 is the last month of the quarter:

$$Y^o_{t+1} = \begin{bmatrix} \pi^o_{t+1} \\ \tilde{\pi}^{o,1}_{t+1} \\ \tilde{\pi}^{o,2}_{t+1} \\ \tilde{\pi}^{o,3}_{t+1} \\ \tilde{\pi}^{o,4}_{t+1} \\ y^{\$,o}_{t+1,n} \end{bmatrix}, \qquad A_{t+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}'$$

- If t+1 is not the last month of the quarter:

$$Y^o_{t+1} = \begin{bmatrix} \pi^o_{t+1} \\ y^{\$}_{t+1,n} \end{bmatrix}, \qquad A_{t+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Regarding the coefficients associated with the state transition equation, I write T and $\eta_{t+1}(h_t)$ as follows

$$T = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad \eta_{t+1}(h_t) = \begin{bmatrix} \sigma_{x,t}\eta_{x,t} \\ 0 \\ \sigma_{\pi,t}\eta_{\pi,t+1} \\ \sigma^1_\epsilon \epsilon^1_{t+1} \\ \sigma^2_\epsilon \epsilon^2_{t+1} \\ \sigma^3_\epsilon \epsilon^3_{t+1} \\ \sigma^4_\epsilon \epsilon^4_{t+1} \\ \sigma_{y^{(n)},t}\eta_{y^{(n)},t+1} \\ 0 \end{bmatrix}$$

Finally, the matrices associated with the log volatility process are:

$$\Psi = \begin{bmatrix} \rho_{h_\pi} & 0 & 0 \\ 0 & \rho_{h_x} & 0 \\ 0 & 0 & \rho_{h_y} \end{bmatrix}, \qquad \Psi = \begin{bmatrix} \sigma_{h_\pi} & 0 & 0 \\ 0 & \sigma_{h_x} & 0 \\ 0 & 0 & \sigma_{h_y} \end{bmatrix}, \qquad \omega_{t+1} = \begin{bmatrix} \omega_{\pi,t+1} \\ \omega_{x,t+1} \\ \omega_{y,t+1} \end{bmatrix}$$

with

$$\sigma_{i,t} = \sigma_i exp(h_{i,t}), \quad \text{for} \quad i = \{\pi, x, y^{(n)}\}.$$

C.3 Posterior inference

I use a Metropolis-within-Gibbs sampler for posterior inference. To this end, it is useful to sample from the conditional distributions for a subset of parameters and latent variables conditional on all the remaining ones. In particular, I iterate over three conditional distributions. I summarize the parameters and latent volatilities accordingly:

$$\Theta_\pi = \{\mu,\ \sigma_\pi,\ \rho,\ \sigma_x,\ \sigma_{y^{(12)}},\ \sigma_{y^{(24)}},\ \sigma_{y^{(36)}},\ \sigma_{y^{(48)}},\ \sigma_{y^{(60)}},\ \sigma^1_\epsilon,\ \sigma^2_\epsilon,\ \sigma^3_\epsilon,\ \sigma^4_\epsilon,\ \}, \tag{C.8}$$

$$\Theta_h = \{\rho_{h_\pi},\ \sigma_{h_\pi},\ \rho_{h_x},\ \sigma_{h_x},\ \rho_{h_y},\ \sigma_{h_y}\}, \quad H^{1:T} = \{h^{1:T}_\pi, h^{1:T}_x, h^{1:T}_y\}$$

I use Markov chain Monte Carlo (MCMC) algorithm that iterates over the three conditional distributions to generate a sequence of draws

$$\{\Theta_\pi^{(s)}, (H^{1:T})^{(s)}, \Theta_h^{(s)}\}_{s=1}^{N_{sim}}$$

from the posterior distribution defined by Bayes' Theorem as:

$$p(\Theta_\pi, \Theta_h, H^{1:T}|Y^o) \propto p(Y^o|\Theta_\pi, \Theta_h, H^{1:T})p(H^{1:T}|\Theta_\pi, \Theta_h)p(\Theta_\pi, \Theta_h)$$

The main steps of the Metropolis-within-Gibbs sampler are as follows:

1. I initialize the sample starting from$\{(H^{1:T})^{(0)}, \Theta_h^{(0)}\}$.
2. I use a random walk Metropolis-Hastings step to draw from the posterior of Θ_π conditional on $\{Y^o, (H^{1:T})^{(s-1)}, \Theta_h^{(s-1)}\}$. For the covariance matrix of the proposal distribution, I follow Schorfheide (2005) and use the negative inverse of the posterior density Hessian evaluated at the mode. I target an acceptance rate of approximately 30 percent by adjusting the scaling factor of the proposal distribution accordingly.
3. I sample the stochastic volatilities using the procedure of Kim, Shephard, and Chib (1998) conditioning on $\{Y^o, \Theta_\pi^{(s)}, \Theta_h^{(s-1)}\}$.
4. I draw Θ_h using a standard Gibbs sampling approach conditional on $\{(H^{1:T})^{(s)}\}$. I retain draws that imply roots inside the unit circle.

I generated 50,000 draws and I set the burn in period at 25,000. Table C1 shows the prior and posterior distribution.

[Place Table C1 about here]

C.4 Robustness

As a robustness check, I repeated the same exercise for a quarterly version of the homoskedastic and heteroskedastic model considering three different measures of inflation expectations reported by SFP. Table C2 shows the estimated inflation variance ratios.[23] In panel (a) I replicate the same exercise as in Table 3, but using quarterly frequency instead of using mix-frequency data. In panel (b), I use the GDP deflator (PGDP) as the measure of the inflation rate and I use the survey

[23]Parameter estimates are available upon request.

consensus forecasts of current-quarter inflation (nowcasts) rather than actual observed inflation as my measure of π^o. In panel (c), I repeated the same exercise as in panel (b) with CPI inflation instead of PGDP. Overall, expected inflation news does not contribute much to the variation of bond innovations (below 30 percent) for all bond maturities considered. Allowing for stochastic volatility almost doubles the inflation variance ratios relative to the homoskedastic case, and increases the model fit; furthermore, all inflation variance ratios are below 50 percent.

[Place Table C2 about here]

C.5 Inflation variance ratios

Table C3 reports the model-implied unconditional standard deviation of monthly shocks to both expected inflation and to bond yields as well as the inflation variance ratios. I report values implied by the macro-finance model and the "data" assuming homoskedastic (No sv) and heteroskedastic (sv) innovations. I report the posterior median values of the population estimates. The macro-finance model produces population variance ratios of around 12 percent at short maturities and less than 30 percent at long maturities. Hence, the model is able to generate measures of inflation uncertainty that are close to the data and well below a half. It is important to highlight that the model not only matches the inflation variance ratios, but also the standard deviation of the innovations in inflation and bond yields.

[Place Table C3 about here]

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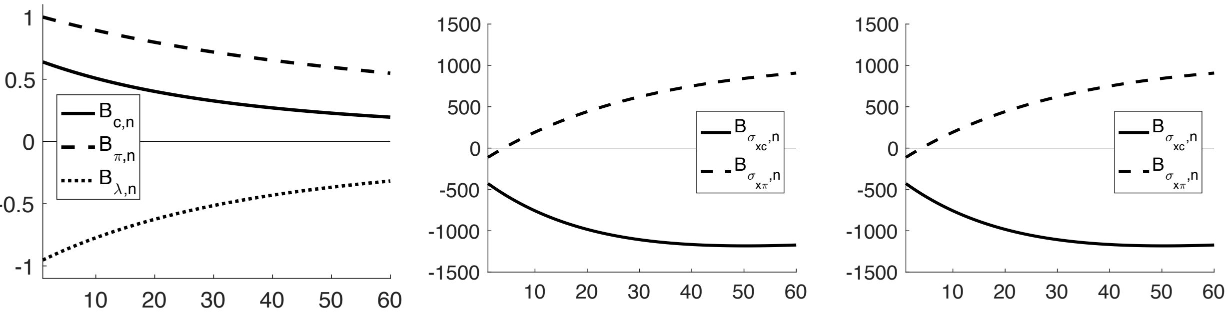


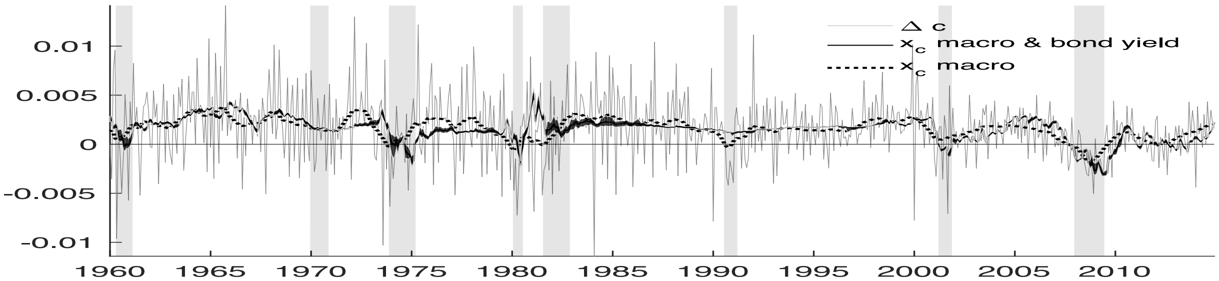
(a) $B^{\$}_{c,n}$, $B^{\$}_{\pi,n}$, and $B^{\$}_{\lambda,n}$

(b) $B^{\$}_{\sigma_{xc},n}$ and $B^{\$}_{\sigma_{x\pi},n}$

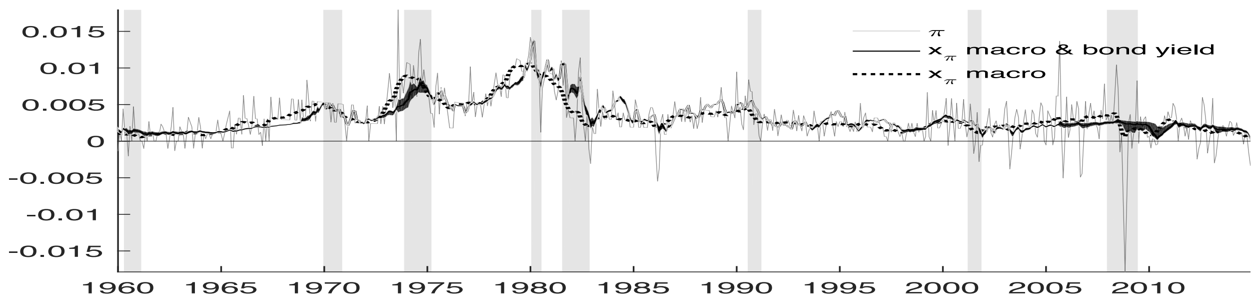
(c) $B^{\$}_{\sigma_{c},n}$ and $B^{\$}_{\sigma_{\pi},n}$

Figure 1.

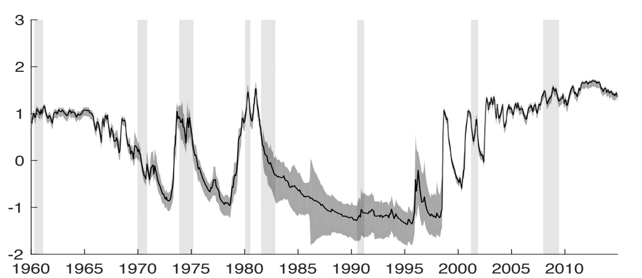
Equilibrium nominal bond yield loadings. This figure shows model implied nominal bond yield loadings evaluated at posterior median values reported in Table 1. Panel (a) shows the loadings with respect to expected consumption growth, $x_{c,t}$, expected inflation, $x_{\pi,t}$ and time preference shocks, $x_{\lambda,t}$, respectively. Panel (b) shows the loadings with respect to long-run real, $\sigma^2_{xc,t}$, and nominal, $\sigma^2_{x\pi,t}$, uncertainties. Panel (c) shows the loadings with respect to short-run real, $\sigma^2_{c,t}$ and nominal, $\sigma^2_{\pi,t}$, uncertainties. Maturity is in months.



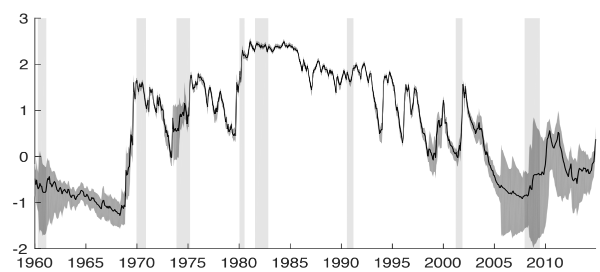
(a) Long-Run Growth x_c



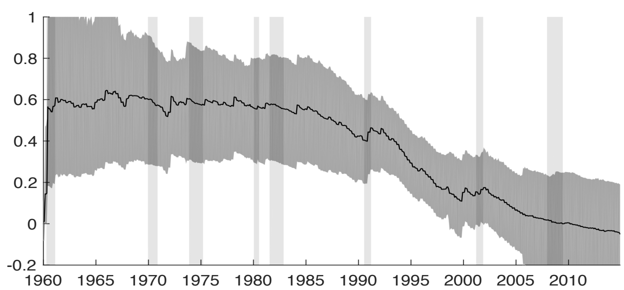
(b) Long-Run Inflation x_π



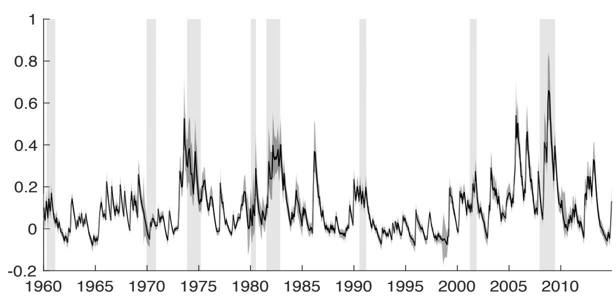
(c) Long-Run Growth Vol h_{xc}



(d) Long-Run Inflation Vol $h_{x\pi}$



(e) Short-Run Growth Vol h_c



(f) Short-Run Inflation Vol h_π

Figure 2.
Macroeconomic latent state variables. This figure shows the smoothed mean and volatility states filtered from the macro-finance model. In panels (a) and (b), I overlay the smoothed states x_c and x_π obtained without bond prices (dashed line) along with the observed series of consumption growth and inflation, respectively. Dark gray shaded areas correspond to the 90 percent credible intervals. Light shaded bars represent recessions as defined by the National Bureau of Economic Research. The estimation sample is from 1960:M1 to 2014M12.

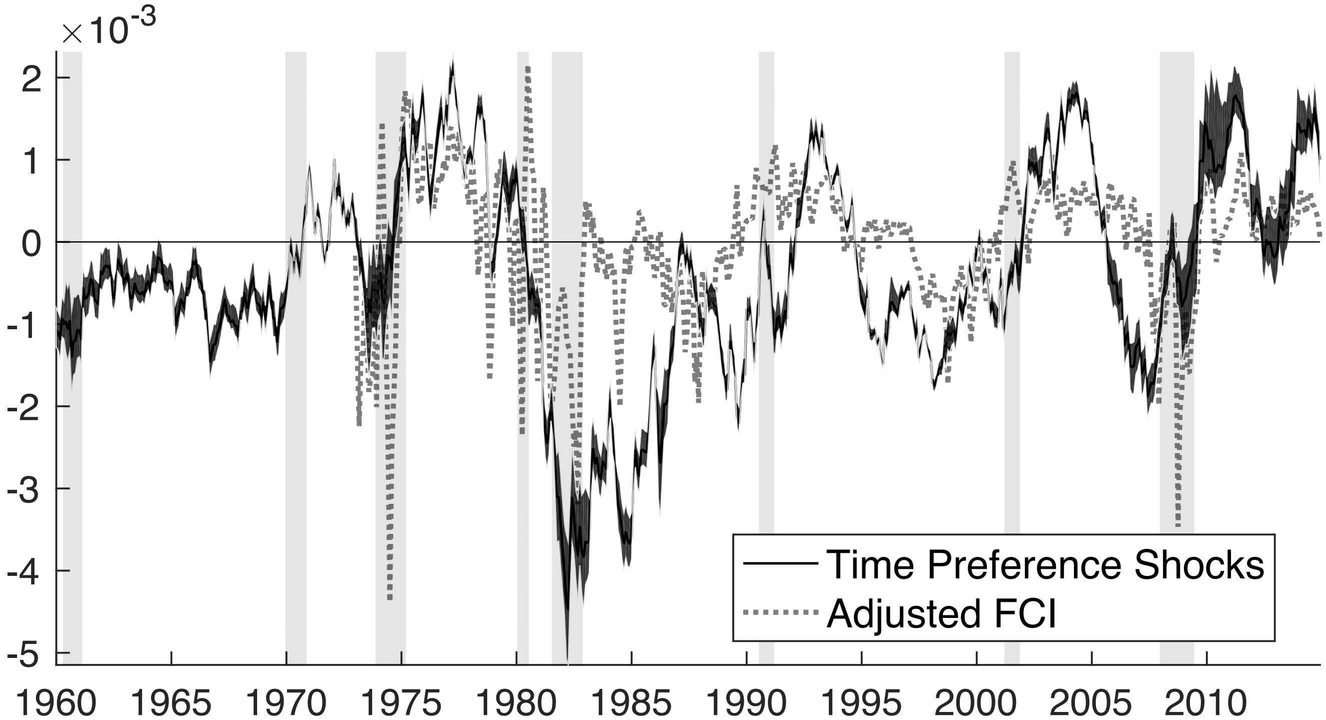
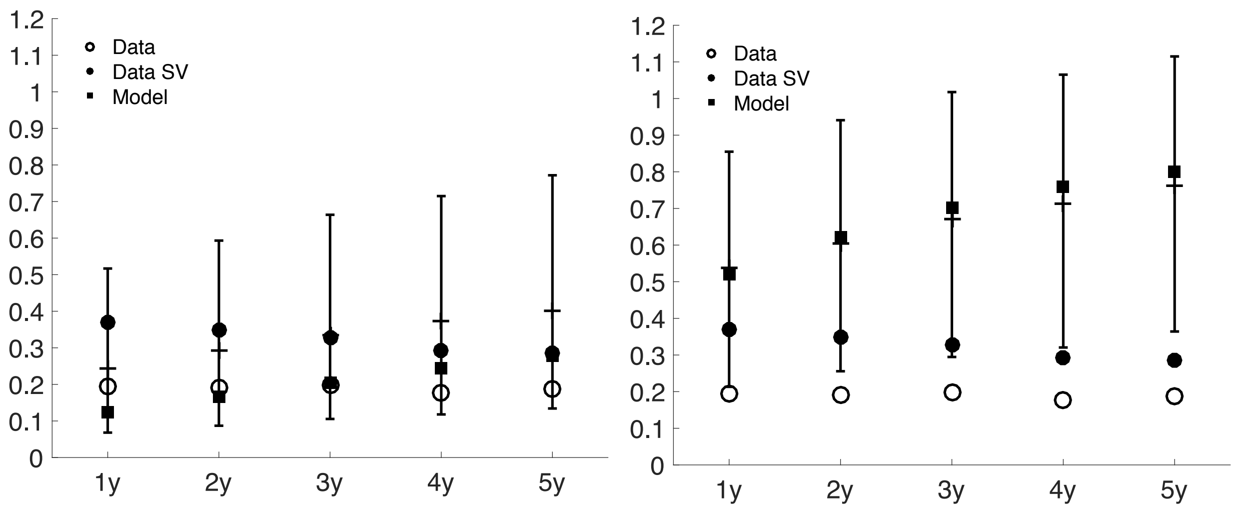


Figure 3.
Time preference shocks and financial condition Index. This figure shows the latent time preference shocks state variable filtered from the macro-finance model along with the adjusted National Financial Conditions Index published by the Federal Reserve Bank of Chicago. Dark gray shaded areas correspond to 90 percent credible interval. Light shaded bars represent recessions as defined by the National Bureau of Economic Research. The estimation sample is from 1960:M1 to 2014M12. The adjusted National Financial Conditions Index starts in 1973 and is published at https://alfred.stlouisfed.org/series?seid=ANFCI.



(a) With Time Preference Shocks

(b) Without Time Preference Shocks

Figure 4.

Inflation variance ratios: Data and model. This figure shows the inflation variance ratios implied by different assumed models. The black and white circles denote the posterior median population values from the statistical model under the assumption of homoskedastic (Data) and heteroskedastic (Data SV) shocks, respectively. The black squares represent the posterior median value of the macro-finance model with (panel (a)) and without (panel (b)) time preference shocks. I also include the 90 percent confidence interval of the macro-finance models to account for finite sample uncertainty.

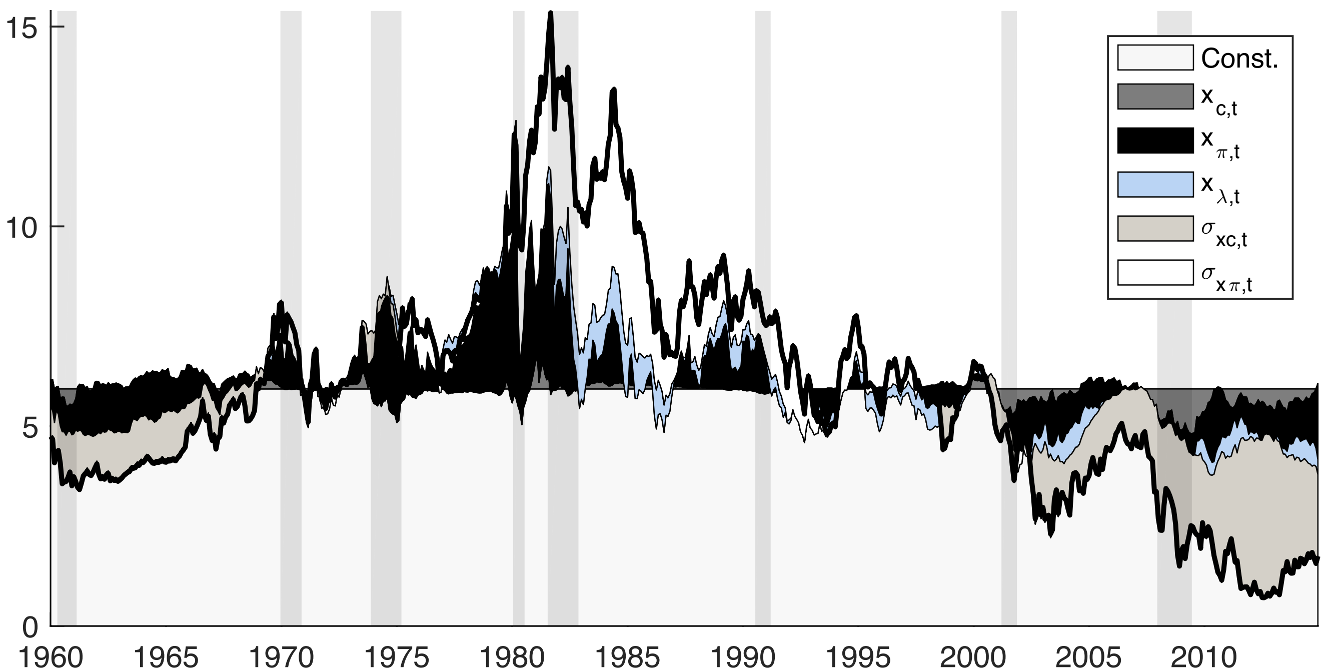


Figure 5.
Decomposition of the 5-Year bond. This figure shows the model-implied nominal bond yields. I show the posterior median contribution of each state variable to the 5-Year bond yield. Light shaded bars represent recessions as defined by the National Bureau of Economic Research. The annualized mean absolute pricing errors $E(|u_{y,t}||Y^o)$ in basis points are equal to 5.66 (for the 1-Year bond, 1y), 5.97 (2y), 5.71 (3y), 4.26 (4y) and 6.26 (5y). The estimation sample is from 1960:M1 to 2014M12.

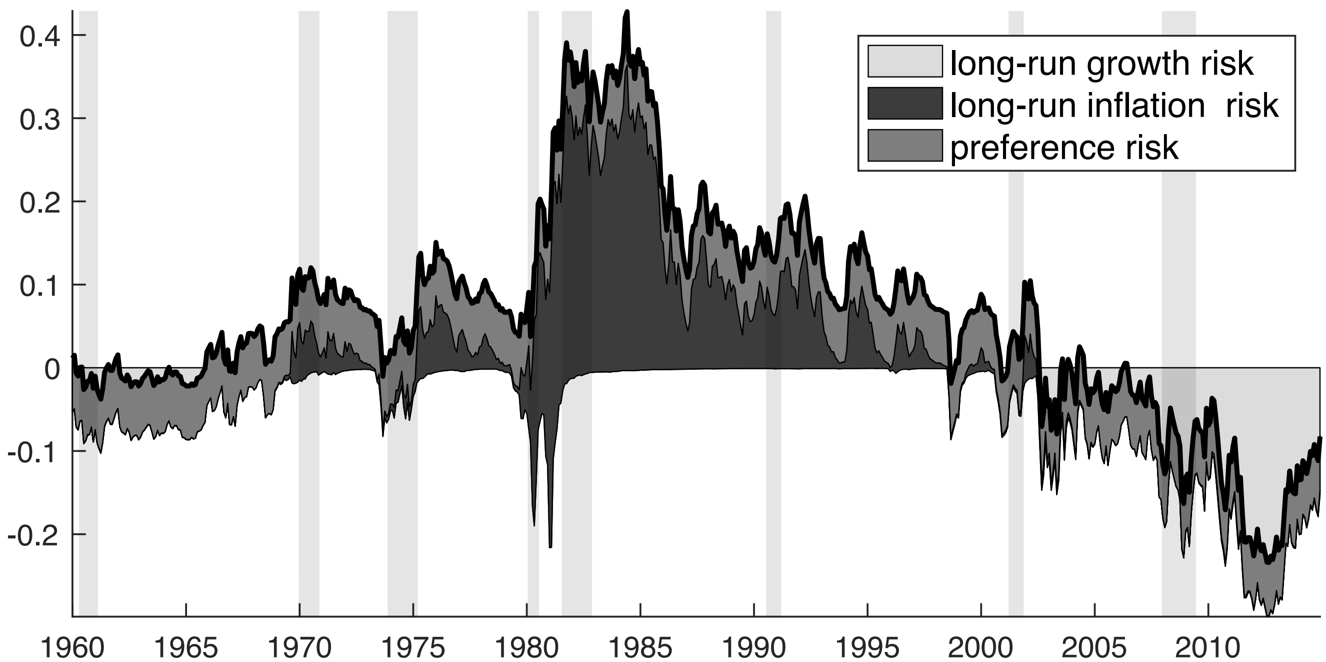
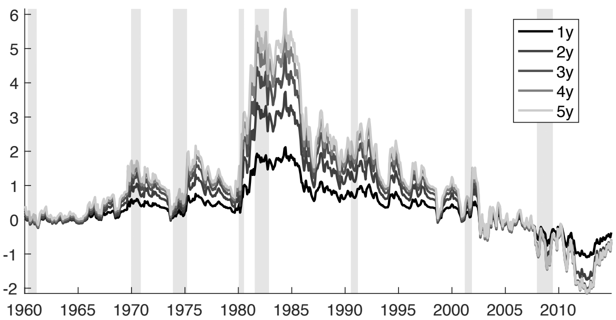
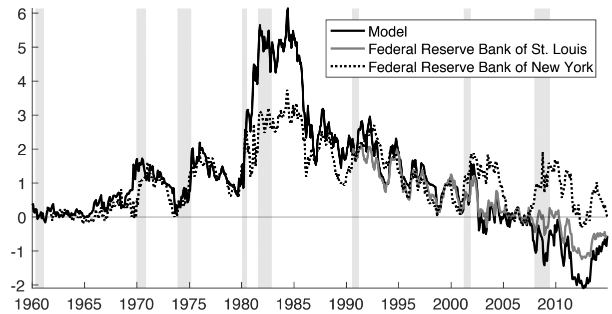


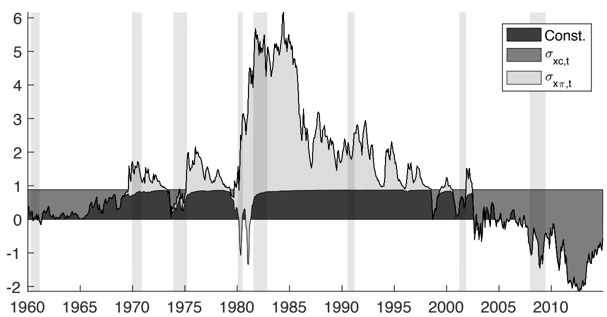
Figure 6.
Decomposition of one period expected excess bond returns for the 5-Year bond. This figure shows the contribution of long-run growth risk, long-run inflation risk, and preference risk to the one-period expected excess bond returns for the 5-Year bond. Light shaded bars represent recessions as defined by the National Bureau of Economic Research. The estimation sample is from 1960:M1 to 2014M12.



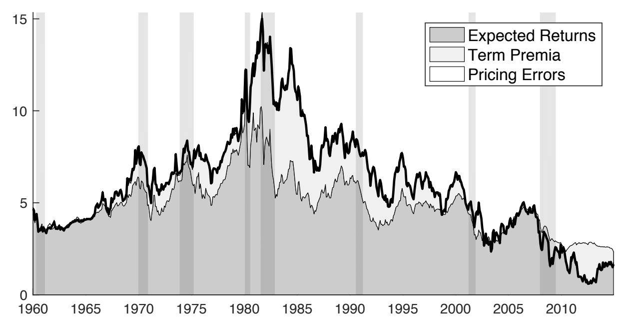
(a) Term premium by bond maturity



(b) 5-Year term premium: Different models



(c) Decomposition of the 5-Year Term Premium



(d) 5-Year bond: Expected rates and term premium

Figure 7.
Model-implied term premium. This figure shows the model-implied bond term premium. Panel (a) shows the posterior median term premium for different bond maturities. Panel (b) shows the posterior median model-implied term premium for the 5-Year bond together with the term premium estimated by Kim and Wright (2005) (gray line, Federal Reserve Bank of St. Louis) and Adrian et al. (2013) (black dashed line, Federal Reserve Bank of New York) for the same bond maturity. Panel (c) decomposes the posterior median of the 5-Year bond term premium into its different sources of risk. Panel (d) decomposes the 5-Year bond into expected average short-term rates and the term premium component. Light shaded bars represent recessions as defined by the National Bureau of Economic Research. The estimation sample is from 1960:M1 to 2014M12.

	Prior Distribution			Macro data and bond prices			Macro data		
	Distr.	5%	95%	5%	50%	95%	5%	50%	95%
				Household Preferences					
δ	B	0.99679	0.99997	0.99916	0.99939	0.99946	-	-	-
ψ	G	0.52277	3.69669	1.53386	1.55565	1.57737	-	-	-
γ	G	2.73680	15.49909	9.63995	9.85970	10.08695	-	-	-
				Time preference shocks					
ρ_λ	U	-0.90013	0.90010	0.95096	0.95142	0.95276	-	-	-
φ_λ	U	0.14972	2.85078	0.22062	0.22581	0.22872	-	-	-
				Consumption growth					
μ_c	N	-0.00667	0.00984	0.00160	0.00164	0.00167	0.00011	0.00133	0.00203
σ_c	IG	0.00079	0.00612	0.00149	0.00151	0.00153	0.00124	0.00162	0.00207
φ_{xc}	U	0.15065	2.85003	0.18402	0.18774	0.19145	0.14766	0.22489	0.34413
ρ_{cc}	U	-0.89950	0.90028	0.94698	0.94815	0.94995	0.89076	0.94603	0.97630
$\rho_{c\pi}$	U	-0.89978	0.90014	-0.01512	-0.01467	-0.01382	-0.04398	-0.02090	-0.00209
				Inflation					
μ_π	N	-0.00512	0.01132	0.00308	0.00314	0.00322	0.00122	0.00284	0.00403
φ_π	IG	0.14966	2.84986	0.78484	0.79180	0.80489	0.75878	0.98835	1.26624
$\varphi_{x\pi}$	IG	0.15010	2.85092	0.07602	0.07799	0.08162	0.16288	0.22976	0.33315
$\rho_{\pi\pi}$	U	-0.89968	0.89992	0.98304	0.98347	0.98389	0.96405	0.98199	0.99485
				Long-run volatility					
$\rho_{h_{xc}}$	N^T	0.81766	0.98222	0.97952	0.98222	0.98881	0.85926	0.97316	0.99478
$\sigma_{h_{xc}}$	IG	0.00334	0.19515	0.20044	0.20186	0.20352	0.00349	0.01367	0.12274
$\rho_{h_{x\pi}}$	N^T	0.81761	0.98231	0.98437	0.98700	0.98979	0.70757	0.81366	0.89535
$\sigma_{h_{x\pi}}$	IG	0.00333	0.19458	0.21894	0.22100	0.22226	0.04830	0.09609	0.16833
				Short-run Volatility					
ρ_{h_c}	N^T	0.81768	0.98224	0.99649	0.99864	0.99956	0.85926	0.97316	0.99478
σ_{h_c}	IG	0.00334	0.19426	0.01079	0.01425	0.01768	0.00349	0.01367	0.12274
ρ_{h_π}	N^T	0.81781	0.98229	0.79965	0.81893	0.84364	0.70757	0.81366	0.89535
σ_{h_π}	IG	0.00334	0.19567	0.10019	0.10196	0.10771	0.04830	0.09609	0.16833
				Consumption measurement errors					
σ_ϵ	IG	0.00079	0.00613	0.00138	0.00139	0.00140	0.00129	0.00155	0.00175
σ_ϵ^q	IG	0.00067	0.03877	0.00126	0.00131	0.00133	0.00119	0.00145	0.00164

Table 1.
Prior and posterior model estimates. This table reports the 5 and 95 percentiles of the prior distribution of the model parameter along with the 5, 50, and 95 percentiles of their posterior distribution. I present the posterior distribution with (Macro data and bond prices) and without (Macro data) bond prices included in the estimation. The assumed prior distributions are the following: B - beta , G - gamma , IG - inverse gamma, N -normal , N^T - truncated normal outside of the $(-1, 1)$ interval and U - uniform. I assume that the measurement errors of consumption average out every quarter. I assume a common stochastic volatility process (i.e., $h_{c,t} = h_{xc,t}$ and $h_{\pi,t} = h_{x\pi,t}$) in the estimation with only considering macro data. The estimation sample is from 1960:M1 to 2014M12.

		Parameters Estimates are based on					
		Macro data & and bond prices			Macro data		
		Macroeconomic Moments					
Δc	Data	5%	50%	95%	5%	50%	95%
Mean	1.95	0.05	1.96	3.82	0.11	1.68	3.05
Std. Dev.	1.31	1.04	2.23	6.54	0.85	1.37	2.41
AC(1)	0.52	0.40	0.67	0.84	0.37	0.64	0.85
π							
Mean	3.84	1.09	3.77	6.46	0.39	3.26	5.75
Std. Dev.	2.76	0.68	1.96	8.44	1.45	2.14	3.57
AC(1)	0.73	0.36	0.74	0.90	0.52	0.74	0.89
Corr($\Delta c, \pi$)	-0.04	-0.65	-0.18	0.22	-0.68	-0.30	0.15
		Bond Yield Moments					
Mean	Data	5%	50%	95%	5%	50%	95%
1y	5.31	0.95	5.00	7.34	2.04	4.45	6.60
2y	5.51	0.60	5.16	7.72	2.55	4.80	6.84
3y	5.70	0.41	5.30	8.06	2.86	5.06	7.06
4y	5.86	0.43	5.41	8.35	3.11	5.24	7.24
5y	5.97	0.54	5.50	8.52	3.28	5.37	7.40
Std. Dev.							
1y	3.18	1.38	3.06	16.77	1.48	2.04	3.19
2y	3.13	1.17	3.02	22.02	1.18	1.70	2.90
3y	3.04	1.03	3.00	25.40	0.96	1.46	2.70
4y	2.96	0.93	2.95	27.25	0.80	1.27	2.54
5y	2.88	0.84	2.87	27.86	0.67	1.13	2.41

Table 2.
Moments of macroeconomic variables and bond yields: Data and model. This table reports descriptive statistics for consumption growth, inflation, and nominal yields. I included the sample data moments and the 5, 50, and 95 percentiles of the model-implied moments based on 10,000 parameter draws of the posterior distribution with and without bond prices in the estimation. I simulated the series for 660 periods, which equals the number of periods in the sample. To compute the bond yield moments with only considering macroeconomic data, I set the preference parameters δ, ψ, γ, ρ_λ and φ_λ to the posterior median estimates from Table 1. The estimation sample is from 1960M1 to 2014M12.

n	Homoskedastic 5%	50%	95%	Heteroskedastic 5%	50%	95%
1y	0.148	0.193	0.245	0.272	0.371	0.526
2y	0.148	0.193	0.255	0.264	0.347	0.480
3y	0.152	0.198	0.253	0.244	0.329	0.453
4y	0.137	0.177	0.226	0.216	0.293	0.407
5y	0.138	0.177	0.239	0.212	0.287	0.385
$lnp(Y^o)$		18410			19075	

Table 3.
Unconditional inflation variance ratios: Data. This table reports the population values of inflation variance ratios obtained from the statistical model of inflation and bond yields. I include the 5, 50, and 95 percentiles based on 10,000 parameter draws of the posterior distribution. The statistical model assumes that inflation is the sum of a constant and an AR(1) component. Shocks to bond yields are computed assuming they are martingales. The table reports inflation variance ratio under the assumption of homoskedastic and heteroskedastic innovations. The estimation sample is from 1983M7 to 2014M12.

		Parameters Estimates are based on					
		Macro data and bond prices			Macro data		
n	Data	5%	50%	95%	5%	50%	95%
Campbell-Shiller Regression: Slope							
2y	-0.67	-2.96	-0.78	1.09	0.32	1.00	1.83
3y	-1.07	-4.04	-1.02	1.23	0.40	1.12	2.05
4y	-1.47	-5.22	-1.19	1.32	0.43	1.15	2.12
5y	-1.50	-6.46	-1.26	1.42	0.45	1.16	2.15
Cochrane-Piazzesi Regression: Slope							
2y	0.45	0.35	0.44	0.56	0.50	0.60	0.71
3y	0.86	0.77	0.85	0.92	0.86	0.94	1.03
4y	1.25	1.16	1.20	1.23	1.08	1.16	1.23
5y	1.44	1.36	1.51	1.66	1.15	1.30	1.46
Cochrane-Piazzesi Regression: R^2							
2y	0.21	0.04	0.38	0.76	0.00	0.03	0.10
3y	0.23	0.05	0.37	0.69	0.00	0.03	0.10
4y	0.26	0.06	0.34	0.63	0.00	0.03	0.10
5y	0.24	0.06	0.31	0.58	0.00	0.03	0.10

Table 4.
Bond risk premia: Data and model. This table reports bond return predictability evidence. I included the 5, 50, and 95 percentiles of the model-implied estimates based on two different regressions as well as the estimates obtained from observed bond yields. The model-implied bond yield series is based on 10,000 parameter draws of the posterior distribution with and without considering bond prices in the estimation. I simulated the series for 660 periods, which equals the number of periods in the sample. To compute the model-implied bond yields based only on macro data, I set the preference parameters δ, ψ, γ, ρ_λ and φ_λ to the posterior median estimates from Table 1. For the Campbell-Shiller regression I report the slope, β_n, from the following regression: $y_{t+12,n-12} - y_{t,n} = \alpha_n + \beta_n \frac{12}{n-12}(y_{t,n} - y_{t,12}) + \epsilon_{t+12} \quad \text{for} \quad n \in \{24, 36, 48, 60\}$. For the Cochrane-Piazzesi regression I report the slope, b_n, and R^2 from the following regression: $rx^{\$}_{r\to t+12,n} = a_n + b_n \hat{rx}_t + \epsilon_{t+1,n}$ where $\hat{rx}_t$ is the fitted value from $\frac{1}{4}\sum_{n=2y}^{5y} rx^{\$}_{r\to t+1,n} = \gamma_0 + \gamma_1 f^{\$}_{t,1y} + \gamma_2 f^{\$}_{t,2y} + \gamma_3 f^{\$}_{t,3y} + \gamma_4 f^{\$}_{t,4y} + \gamma_5 f^{\$}_{t,5y} + \epsilon_{t+1}$. $rx^{\$}_{r\to t+1,n}$ denotes the excess log return on buying an n period bond at time t and selling it at time $t+1$as an $n-1$ period bond defined by $rx^{\$}_{r\to t+1,n} = ny^{\$}_{t,n} - (n-1)y^{\$}_{t+1,n-1} - y^{\$}_{t,1} \quad \text{for} \quad n \in \{24, 36, 48, 60\}$. $f^{\$}$ denotes the corresponding forward rates. The estimation sample is from 1960M1 to 2014M12.

Macro data													
	Posterior Distribution												
	ρ_{cc}				$\rho_{c\pi}$				$\rho_{\pi\pi}$				$lnp(Y^o)$
	5%	50%	95%	Width	5%	50%	95%	Width	5%	50%	95%	Width	
(a) No Me.	-0.286	-0.206	-0.130	0.156	-0.223	-0.128	-0.0392=	0.184	0.849	0.912	0.963	0.1139	5855
(b) Me.	0.828	0.892	0.941	0.112	-0.041	-0.019	-0.0008	0.040	0.873	0.9363	0.972	0.098	5870
(c) Me. Sv.	0.890	0.946	0.976	0.085	-0.044	-0.021	-0.002	0.041	0.964	0.9820	0.994	0.030	6055

Table B1.
Posterior estimates of the persistence parameters. This table reports the 5, 50, and 95 percentiles of the posterior distribution for the persistent parameters ρ_{cc}, $\rho_{c\pi}$ and $\rho_{\pi\pi}$ for the monthly version of the model. I also include the 90% confidence width and the log marginal data density. I consider three different specifications of the process of consumption growth and inflation. No Me., consumption growth is measured without any errors, (i.e., $c_t^o = c_t$), and homoskedastic innovations; Me. assumes the measurement error model of consumption; Me. Sv., assumes the measurement error model of consumption and short- and long-run stochastic volatility imposing a common stochastic volatility process (i.e.$h_{c,t} = h_{xc,t}$ and $h_{\pi,t} = h_{x\pi,t}$). The estimation sample is from 1960:M1 to 2014M12.

Time Period: 1960 to 2014				
Fixed Param	Macro data and bond prices	Std. MDD	Macro data	Std. MDD
Persistence parameters				
ρ_{cc}	0.9504	-0.6905	0.9001	-0.1828
$\rho_{c\pi}$	-0.0139	1.8419	-0.0152	2.7896
$\rho_{\pi\pi}$	0.9833	-0.8736	0.9702	-1.4128
Scale volatility parameters				
σ_c	0.0015	-0.2544	0.0012	0.4817
φ_{xc}	0.1858	0.5237	0.2250	0.9632
φ_{π}	0.7817	1.1512	1.2921	0.5140
$\varphi_{x\pi}$	0.0785	0.1980	0.2974	0.0722
Long-run Volatility				
$\rho_{h_{xc}}$	0.9853	-0.9074	0.8292	1.1632
$\sigma_{h_{xc}}$	0.2028	1.1607	0.2509	1.2429
$\rho_{h_{x\pi}}$	0.9873	-5.8628	0.7099	1.2272
$\sigma_{h_{x\pi}}$	0.2218	4.3508	0.1482	1.8991
No Constrains:		$lnp(Y)$: 6055	Std. Dev. $lnp(Y)$: 6.05	

Table B2.
Standardized marginal data densities for macroeconomic model. This table reports standardized log marginal data densities for the process of consumption growth and inflation conditional on different parameter values. To standardize the values, I used the mean and standard deviation of the log marginal data density from the unconstrained version of the model based on 30 different values computed from the same number of unconstrained estimations. The estimation sample is from 1960:M1 to 2014M12.

	Prior Distribution			Homoskedastic			Heteroskedastic		
	Distr.	5%	95%	5%	50%	95%	5%	50%	95%
				Inflation					
μ	N	-5.12e-03	1.13e-02	5.75e-03	6.90e-03	8.36e-03	5.91e-03	6.91e-03	8.06e-03
σ_π	IG	7.94e-04	6.10e-03	2.42e-03	2.56e-03	2.71e-03	1.72e-03	1.83e-03	1.95e-03
ρ	U	-0.900	0.900	0.995	0.996	0.998	0.995	0.996	0.997
σ_x	IG	7.94e-04	6.14e-03	1.29e-04	1.44e-04	1.65e-04	1.16e-04	1.31e-04	1.52e-04
				Bond yields					
$\sigma_{y^{12}}$	IG	7.94e-04	6.09e-03	3.01e-04	3.22e-04	3.43e-04	1.96e-04	2.12e-04	2.28e-04
$\sigma_{y^{24}}$	IG	7.96e-04	6.12e-03	2.96e-04	3.16e-04	3.36e-04	2.01e-04	2.14e-04	2.30e-04
$\sigma_{y^{36}}$	IG	7.94e-04	6.11e-03	2.86e-04	3.03e-04	3.25e-04	2.01e-04	2.16e-04	2.32e-04
$\sigma_{y^{48}}$	IG	7.96e-04	6.11e-03	2.95e-04	3.15e-04	3.37e-04	2.09e-04	2.23e-04	2.42e-04
$\sigma_{y^{60}}$	IG	7.94e-04	6.09e-03	2.84e-04	3.01e-04	3.24e-04	2.06e-04	2.23e-04	2.38e-04
				Stochastic volatilities					
$\rho_{h_{xc}}$	N^T	0.523	0.982	-	-	-	0.697	0.953	0.990
$\sigma_{h_{xc}}$	IG	0.010	0.584	-	-	-	0.002	0.005	0.031
$\rho_{h_{x\pi}}$	N^T	0.523	0.982	-	-	-	0.812	0.894	0.947
$\sigma_{h_{x\pi}}$	IG	0.010	0.584	-	-	-	0.031	0.056	0.102
$\rho_{h_{x\pi}}$	N^T	0.523	0.982	-	-	-	0.498	0.603	0.707
$\sigma_{h_{x\pi}}$	IG	0.010	0.584	-	-	-	0.237	0.310	0.393
				Measurement errors SPF					
σ_ϵ^1	IG	7.93e-04	6.11e-03	6.73e-04	7.59e-04	8.43e-04	6.74e-04	7.48e-04	8.26e-04
σ_ϵ^2	IG	7.94e-04	6.11e-03	2.72e-04	3.16e-04	3.65e-04	2.70e-04	3.16e-04	3.67e-04
σ_ϵ^3	IG	7.94e-04	6.12e-03	2.00e-04	2.39e-04	2.87e-04	2.04e-04	2.43e-04	2.84e-04
σ_ϵ^4	IG	7.93e-04	6.11e-03	3.42e-04	3.88e-04	4.49e-04	3.46e-04	3.85e-04	4.34e-04
$\ln p(Y^o)$					18410.781			19075.976	

Table C1.
Prior and posterior estimates from the statistical model of inflation and bond yields. This table reports the 5 and 95 percentiles of the prior distribution of the model parameter models along with the 5, 50, and 95 percentiles of their posterior distribution. I present the posterior distribution with (Homoskedastic) and without (Heteroskedastic) stochastic volatility in the innovations of inflation, expected inflation and bond yields. The assumed prior distributions are the following: IG - inverse gamma, N -normal , NT - truncated normal outside of the (-1,1) interval and U - uniform. The estimation sample is from 1983M7 to 2014M12.

Quarterly Frequency						
	Homoskedastic			Heteroskedastic		
	5%	50%	95%	5%	50%	95%
(a)	π - SPF for observed quarter CPI Time period 1981Q3-2014Q4					
1y	0.075	0.103	0.142	0.164	0.236	0.327
2y	0.082	0.108	0.145	0.164	0.230	0.333
3y	0.091	0.119	0.163	0.167	0.225	0.325
4y	0.087	0.121	0.168	0.145	0.203	0.292
5y	0.084	0.120	0.167	0.136	0.193	0.274
ln$p(Y^o)$		7088.304			7422.010	
(b)	π - PGDP Time period 1968Q1-2014Q4					
1y	0.095	0.124	0.162	0.188	0.274	0.394
2y	0.102	0.134	0.174	0.208	0.292	0.420
3y	0.102	0.139	0.182	0.198	0.285	0.402
4y	0.101	0.137	0.181	0.165	0.242	0.352
5y	0.097	0.131	0.174	0.141	0.211	0.307
ln$p(Y^o)$		9499.774			10115.738	
(c)	π - SPF for current quarter CPI Time period 1981Q3-2014Q4					
1y	0.078	0.104	0.137	0.163	0.240	0.342
2y	0.080	0.110	0.145	0.161	0.235	0.341
3y	0.090	0.120	0.158	0.157	0.226	0.328
4y	0.087	0.122	0.163	0.151	0.213	0.299
5y	0.087	0.118	0.158	0.139	0.201	0.288
ln$p(Y^o)$		7197.171			7558.242	

Table C2.
Unconditional inflation variance ratios. This table reports the population values of inflation variance ratios for three different measures of inflation forecasts. The inflation variance ratio is computed as the ratio of the variance of shocks to expected inflation to the variance of yield shocks. I include the 5, 50, and 95 percentiles of the moments based on 10,000 parameter draws of the posterior distribution. The model assumes that inflation is the sum of a constant and an AR(1) component. Shocks to the yields are computed assuming they are martingales. The table reports inflation variance ratio under the assumption of homoskedastic and heteroskedastic innovations.

	Std. Dev. of Inflation News			Std. Dev. of Yield News			Inflation Variance Ratios		
	Data		Model	Data		Model	Data		Model
	No sv	sv		No sv	sv		No sv	sv	
1y	0.169	0.155	0.129	0.386	0.254	0.366	0.193	0.371	0.125
2y	0.166	0.152	0.118	0.379	0.257	0.289	0.193	0.347	0.165
3y	0.162	0.148	0.107	0.364	0.259	0.236	0.198	0.329	0.206
4y	0.159	0.145	0.098	0.378	0.268	0.198	0.177	0.293	0.245
5y	0.156	0.142	0.090	0.362	0.267	0.170	0.186	0.287	0.280

Table C3.
Estimates of inflation variance ratios: Data and model. This table reports the posterior median population values of the standard deviation of expected inflation innovations and bond yield news as well as the inflation variance ratios. I include the estimates based on the homoskedastic (No sv) and heteroskedastic (sv) version of the statistical model of inflation and bond yields (Data). I also show the model-implied estimates of the macro-finance model (Model). The median is based on 10,000 parameter draws of the posterior distribution for each particular model.