Oligopoly dynamics with financial frictions*

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November 15, 2022

Abstract

We develop a baseline model to understand how financial frictions impact industry dynamics. Using state of the art computational tools, we document how key characteristics, such as market size and product differentiation, determine the impact of financing on firms' price and investment strategies, as well as industry dynamics and welfare. We show that investment and prices tend to move in opposite directions, while long run industry concentration generally rises. Strikingly, financing frictions sometimes lead firms to both invest more and charge lower prices. In turn, this behavior often raises consumer surplus and social welfare, even if industry concentration increases.

Keywords: Price and investment competition, financial frictions, industry dynamics

*An earlier draft of this paper was circulated under the title "Dynamic strategic corporate finance: A tug of war with financial frictions". We are grateful to Mike Abito and Juuso Toikka for many helpful discussions and to Rich Kihlstrom and Vojislav Maksimovic for detailed comments on earlier drafts, as well as seminar participants at Guanghua, Microsoft, Ohio State, Wharton, and the World Congress of the Econometric Society. Douglas King and Hugh MacMullan provided invaluable help with setting up computing environments for us on Wharton's HPC Cluster and AWS. Ralf Giering helped us use TAF. Arnav Aggarwal, Alexander Belyakov, and Mehran Ebrahimian provided excellent research assistance. Doraszelski and Gomes gratefully acknowledge financial support from The Wharton School's Dean's Research Fund and the National Science Foundation under award number 2117068.

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1 Introduction

Since, at least, the development of canals and railroads, at the very beginning of the modern era, industries often experience turbulent starts, where firms compete fiercely to achieve dominant positions. Financing is often critical to every firm's likelihood of survival and ultimate success. The recent battles for dominance of new sectors such as rideshare and food delivery offer dramatic examples with key players such as Uber, Lyft, DoorDash and others, requiring multiple capital injections to build large networks and afford the aggressive price strategies that can attract new consumers.

In this paper, we seek to understand how limitations in access to capital markets impact the dynamic strategic interactions between firms and the evolution of oligopolistic industries over time. We develop and analyze a baseline theoretical setting that combines the Budd, Harris & Vickers (1993) model of dynamic competition with one of investment under financial frictions à la Gomes (2001). Our setting is an infinite-horizon dynamic stochastic game where two firms compete for a dominant position through pricing and investment choices, where, crucially, a firm must often rely on (costly) external funds when its profits are insufficient to cover desired investment outlays.

The combination of financial frictions and strategic interactions introduces an important element to the analysis since prices must now be understood and analyzed jointly with firms' other dynamic decisions such as investment, advertising, and R&D. This is because prices and profits directly impact the extent to which (costly) external financing is required to fund investments and, thus, how the industry evolves over time. As a result, the "traditional static-dynamic breakdown" (Doraszelski & Pakes 2007, p. 1892) that underlies much of empirical and theoretical research in industrial organization no longer holds.[1]

Although our model is designed for tractability rather than detailed realism, a

[1] In this regard, our model is also very different from the limited liability effect of debt financing in Brander & Lewis (1986) and from models along the lines of Chevalier & Sharfstein (1996) that treat a low price today as an investment into a high market share tomorrow.

thorough analysis requires the use of state of the art computational tools (Besanko, Doraszelski, Kryukov & Satterthwaite 2010) to systematically detect and numerically evaluate the multiple equilibria inherent in dynamic strategic investment games. Our approach also allows us to fully explore the relevant parameter space governing key industry characteristics such as market size, degree of product differentiation and regulation levels.

We have several new and striking results. First, we show that the impact of financial frictions is often more subtle in the presence of dynamic strategic interactions between firms. Most strikingly, frictions sometimes lead to higher investment as firms seek to strengthen their future competitive position to prevent a rival from catching-up, as decreasing a rival's profits will drive up their future cost of financing. Thus, introducing dynamic strategic interactions between firms challenges the assumption used in virtually every existing empirical study that firms *always* invest less in the presence of financial frictions.

Second, we document that pricing and investment decisions are tightly related in equilibrium, with higher investment often going hand in hand with lower prices, and vice-versa. Since most existing studies typically investigate the impact of financial frictions on pricing or investment decisions in isolation, this explicit connection opens an important avenue for future empirical work.

Finally, we find that, while financial frictions often lead to more asymmetric industry structures, or at least accelerate convergence towards these structures, the opposite is also possible. More importantly, we also show that a combination of higher investment and lower prices can raise consumer surplus and even social welfare, even if long run industry concentration is higher. These insights seem particularly relevant for policymakers in light of recent discussions about rising market power, its origins and its impact on consumers (e.g., Berry, Gaynor & Scott-Morton 2020).

Beyond our main theoretical contributions, we provide novel, motivating evidence that documents how strategic interactions are of auxiliary importance for firm investment, independent of established investment determinants such as cash flow. In addition, we also show that industries—depending on their different characteristics, such as size and product differentiation—differ considerably in their response to the great financial crisis of 2008-09.

Related literature. Many papers study the impact of financial frictions in the context of models à la Hopenhayn (1992), where individual firm actions have no impact on their rivals (some early examples include Cooley & Quadrini (2001), Gomes (2001), Albuquerque & Hopenhayn (2004), and Clementi & Hopenhayn (2006)). There is an even more extensive literature in corporate finance using single-agent models to study the impact of financial frictions on firm investment (for a survey, see Strebulaev & Whited 2011). A common intuition arising from these models is that financial frictions lead to lower investment.

Our framework also expands the existing dynamic industrial organization literature following Ericson & Pakes (1995) that assumes perfect capital markets, where every NPV-positive investment project can be funded in a costless way. First, our framework contributes to this literature by simultaneously studying pricing and investment as integrated decisions rather than abstracting from physical capital investment (Brander & Lewis (1986), Showalter (1995) and Maksimovic (1988)) or having price serve a double role as investment (Chevalier & Sharfstein 1996). Second, all our main results are driven by the combination of strategic interactions and financial frictions.[2,3]

A number of recent papers emphasizes the importance of strategic interactions in different areas of the financial industry, including banks (Egan, Hortacsu & Matvos 2017, Corbae & D'Erasmo 2021, Wang, Whited, Wu & Xiao 2022), shadow banks

[2]Another important early paper is Fershtman & Judd (1987), who study how differences between owners and managers impact product market competition. Although we do not explicitly model the divergent interests of owners and managers, we recognize that financial frictions ultimately arise precisely because of them.

[3]Gilchrist, Schoenle, Sim & Zakrajsek (2017) and Dou, Ji & Wu (2021) contain recent adaptations of the mechanism in Chevalier & Sharfstein (1996). However, this mechanism operates independently of strategic interactions.

(Buchak, Matvos, Piskorski & Seru 2018, Buchak, Matvos, Piskorski & Seru 2000, Jiang 2021), and life insurers (Koijen & Yogo 2016, Koijen & Yogo 2022). However, dynamic decisions regarding investment, advertising, or R&D—and the impact of financial frictions on them—which are the major focus of our paper, play a small role for financial intermediaries.

Closest to our work are Corbae & D'Erasmo (2020), who show that the impact of capital requirements and the transmission of monetary policy changes considerably when banks are imperfectly competitive. Recently, Liu, Mian & Sufi (2022) use a version of the Budd et al. (1993) model to show how a low discount rate, due to either low risk premia or loose monetary policy, can exacerbate industry concentration. However, their model also ignores financial frictions, which create a wedge between monetary policy rates and firm discount rates.

Finally, our paper is also related to the vibrant literature studying the effect of the purported rise in industry concentration on aggregate investment and the rates of innovation and business dynamism (e.g., Akcigit & Ates 2020). This literature also abstracts from financial frictions.

Paper structure. The rest of the paper is organized as follows. Section 2 presents motivating evidence. Section 3 develops our model. Section 4 describes our computational approach and some of the main properties of the equilibria. Section 5 discusses the most important effects of financial frictions on prices, investment, and long run industry concentration. Section 6 discusses some striking welfare implications and Section 7 concludes. We provide all technical details and many additional findings in our Online appendix.

2 Motivating evidence.

While the impact of financing frictions on corporate investment is well documented, there is little evidence on how they affect prices. Similarly, evidence about the impact of strategic interactions on investment is scarce.

In this section, we present novel facts on the importance of strategic interactions and financial frictions for firms. To this end, we use the industry classifications and product similarity data from Hoberg & Phillips (2016), who use text-based analysis of 10-K reports to construct a time-varying measure of similarity for all pairs of firms in Compustat. After excluding financials and utilities, and observations with missing values, we have 29,441 firm-year observations comprising 4,998 distinct firms in 50 distinct industries. Online appendix II provides details on our data and variable construction along with descriptive statistics.

Industry-level facts. To isolate the impact of financing frictions on pricing and investment, we follow Gilchrist et al. (2017) and compare firm behavior during the great financial crisis of 2008-09 with the earlier 2005-06 period. We use gross margin as a measure of pricing power and compute average within-industry changes in investment and margins between 2005-06 and in 2008-09 by averaging across all within-firm changes in an industry.[4]

Figure 1 plots these average changes against industry size, measured by the total industry sales (in $million), and average product differentiation, defined as one minus the average similarity score of Hoberg & Phillips (2016).[5]

Industries differ considerably in their response to the great financial crisis, with the investment decline during the crisis more pronounced in smaller industries as well as those with less product differentiation. The change in margins also exhibits a systematic, but slightly weaker, relationship with industry characteristics. This evidence complements Gilchrist et al. (2017), who use proprietary product-level data to show that financially constrained firms increased prices during the great financial crisis whereas their unconstrained counterparts decreased them.

[4] We exclude all firms without data for any of these four years plus a small group of firms that changes their industry classification over the four years.

[5] Average similarity score is defined as the mean over the closest peer's score per each firm in the industry.

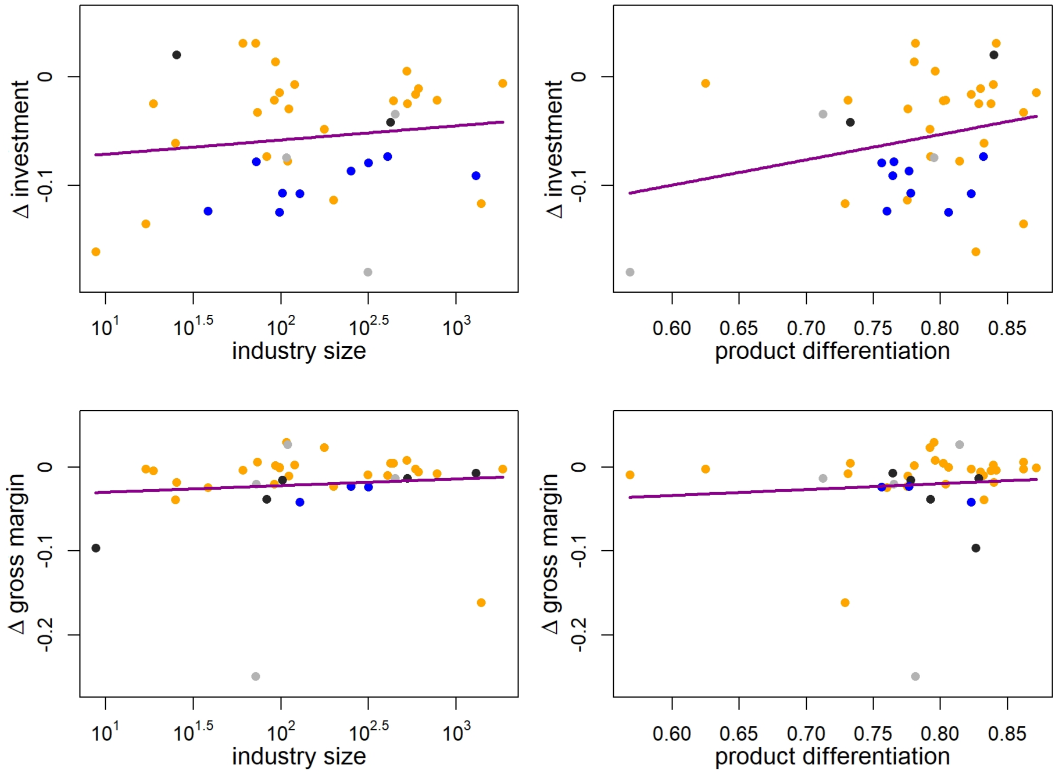

Figure 1: Average change in investment during the great financial crisis in 2008 and 2009 versus 2005 and 2006 by industry (upper panels) and average change in gross margin (lower panels), overlayed by trend line. Blue indicates statistical significance at 1%, dark gray at 5%, light gray at 10%, and yellow no statistical significance at conventional levels.

Firm-level facts. To document the impact of strategic interactions on investment, we consider a standard investment-Q regression, augmented with cash flow and lagged investment terms (Fazzari, Hubbard & Petersen 1988, Eberly, Rebelo & Vincent 2012), and further add (lagged) average investment of a firm's n closest competitors—defined by the firm(s) with the highest Hoberg & Phillips (2016) product similarity score(s).

The upper panel of Table 1 shows that competitor investment appears statistically significant in these regressions, suggesting that strategic considerations matter for firm level investment decisions. Moreover, the coefficients on the standard investment-Q

	baseline	number of closest competitors 1	2	3	4	5
comp. invest. $(t-1)$		0.027^{***}	0.047^{***}	0.057^{***}	0.067^{***}	0.077^{***}
		(0.005)	(0.006)	(0.007)	(0.008)	(0.010)
cash flow (t)	0.075^{***}	0.075^{***}	0.075^{***}	0.075^{***}	0.075^{***}	0.074^{***}
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Tobin's Q $(t-1)$	0.044^{***}	0.044^{***}	0.043^{***}	0.043^{***}	0.043^{***}	0.043^{***}
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
investment $(t-1)$	0.217^{***}	0.215^{***}	0.213^{***}	0.213^{***}	0.213^{***}	0.212^{***}
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)

	baseline	number of randomly drawn firms 1	2	3	4	5
comp. invest. $(t-1)$		-0.004	-0.005	-0.003	-0.004	-0.005
		(0.004)	(0.006)	(0.006)	(0.007)	(0.008)
cash flow (t)	0.075^{***}	0.075^{***}	0.075^{***}	0.075^{***}	0.075^{***}	0.075^{***}
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Tobin's Q $(t-1)$	0.044^{***}	0.044^{***}	0.044^{***}	0.044^{***}	0.044^{***}	0.044^{***}
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
investment $(t-1)$	0.217^{***}	0.217^{***}	0.217^{***}	0.217^{***}	0.217^{***}	0.217^{***}
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)

Table 1: Baseline (upper panel) and placebo analysis (lower panel). Dependent variable is investment. ***, **, and * shows statistical significance at 1%, 5%, and 10%, respectively.

regression variables (cf. column 'baseline') are largely unchanged, suggesting that strategic interactions are a genuinely auxiliary and orthogonal consideration for firms' investment decisions. As the lower panel of Table 1 shows, using a simple *placebo* analysis, this finding is not spurious. The coefficient on competitor investment is not statistically significant when, instead of the lagged average investment of the n closest competitors, we use the lagged average investment of any *randomly* n drawn firms from within the same industry as the focal firm.[6,7]

Collectively, these empirical findings illustrate that: (i) firms' pricing is altered by the presence of financing frictions; and (ii) firms invest strategically in response to their competitors' investment decisions. In the next section, we present a baseline model that builds on both these two key insights and shows how financing frictions and strategic interactions may impact different industries.

3 Model

Our baseline model seeks to capture the impact of financial frictions on the dynamics of an industry and its long-run competitiveness. Absent financial frictions, the setting is similar to Budd et al. (1993). While stylized, their model has the advantage that the industry dynamics it generates are well understood. As such, it is a natural starting point for studying the impact of financial frictions.

We consider a discrete-time, infinite-horizon dynamic stochastic game between two forward-looking firms that compete for a dominant position through their pricing and investment decisions. At any point in time, we let (ω_1, ω_2) denote the underlying quality of the good produced by each firm. While consumer surplus depends both on ω_1 and ω_2, we use the single state variable $\omega = \omega_1 - \omega_2 \in \{-L, -L+1, \ldots, L\}$ to summarize the state of competition in the industry. If $\omega > 0$, firm 1 is the leader and

[6] All regressions use firm, year, industry and industry-year fixed effects.

[7] Online appendix II, shows our findings are also robust to instead using the weighted average of competitors' investments, where the weight is the annual product similarity score.

firm 2 is the follower; if $\omega < 0$, firm 1 is the follower and firm 2 is the leader; and if $\omega = 0$, the firms compete head-to-head. The size of the competitive advantage that the leader enjoys over the follower is defined as $|\omega|$.

By investing, a firm aims to increase its competitive advantage (or decrease its competitive disadvantage), capture market share, and eventually generate profit. The law of motion for the state is

$$\omega' = \max\left\{-L, \min\left\{L, \omega + x_1 - x_2\right\}\right\}, \tag{1}$$

where $x_i \in \{0, 1\}$ is the investment decision of firm $i \in \{1, 2\}$ and we use a prime to distinguish subsequent-period from current-period values. As in a tug of war, investing thus allows a firm to move the state to "its" side.

The state ω captures any potential sources of competitive advantage such as branding, network effects, or learning economies, while investing can accordingly entail spending on marketing or R&D. For concreteness, we think of firms as offering products that differ in how consumers perceive their qualities, (ω_1, ω_2). By investing, a firm aims to increase the quality of its product relative to that of its competitor.

As we discuss in detail below, frictions in accessing capital markets imply that firm i's cost of investing is lower when it can be financed through profits. Financial frictions, therefore, endogenously link the firm's pricing and investment decisions. We begin by detailing the product market and other primitives of the model before moving on to firms' decisions.

Product market. The products offered by the firms are differentiated vertically (as captured by the state ω) and horizontally. The per-period profit of firm 1 is

$$\pi_1(\omega, p_1, p_2) = \frac{M}{1 + \exp\left(\frac{-g(\omega)+\alpha(p_1-p_2)}{\nu}\right)}(p_1 - c), \tag{2}$$

where p_i is the price charged by firm i and where $c \geq 0$ is the marginal cost of production. The (logistic) demand for product 1 is tied to market size, $M > 0$, price sensitivity, $\alpha > 0$, and the degree of horizontal product differentiation,$\nu > 0$. As ν increases, products become more differentiated, softening price competition between the firms. The profit of firm 2 is symmetric to that of firm 1, so that $\pi_2(\omega, p_1, p_2) = \pi_1(-\omega, p_2, p_1)$.

The function $g(\omega)$ maps the state of competition into consumers' quality perceptions and demand. We parameterize

$$g(\omega) = \begin{cases} \frac{\omega}{L} & \text{if} \quad \omega < 0, \\ \tau\frac{\omega}{L} & \text{if} \quad \omega \geq 0, \end{cases} \tag{3}$$

where $\tau \in [0, 1]$ is a handicap parameter. Therefore, a smaller value of τ, imposes a disadvantage on the leader, perhaps because of regulation or antitrust policies.

Online appendix III shows that we can normalize $\alpha = 1$ and $c = 0$ without loss of generality. Hence, in what follows, we restrict attention to the remaining three key demand parameters: market size, M, degree of horizontal product differentiation, ν, and leader handicap, τ.

Investment and financing. To simplify exposition, we focus on the problem of firm 1 from hereon. The expressions for firm 2 are analogous.

Firm 1 can expand its competitive advantage in each period by investing and incurring a cost $F_0 > 0$. Because capital markets are not frictionless, if profits $\pi_1 = \pi_1(\omega, p_1, p_2)$ are not large enough to cover investment outlays, $F_0 x_1$, the firm must raise (costly) external funds in the amount of $F_0 x_1 - \pi_1$. As a result, the net (possibly negative) distribution to the investors, d_1, is

$$d_1 = \begin{cases} \pi_1 - F_0 x_1 & \text{if} \quad \pi_1 - F_0 x_1 \geq 0, \\ (1 + \lambda(x_1, \pi_1))(\pi_1 - F_0 x_1) & \text{if} \quad \pi_1 - F_0 x_1 < 0, \end{cases} \tag{4}$$

where $\lambda(x_1, \pi_1) \geq 0$ the per-dollar cost of raising external funds.

Following Gomes (2001), we allow the per-dollar cost of raising external funds to increase in the amount raised and assume $\lambda(x, \pi) = \zeta \left(F_0 x - \pi\right)^{\kappa-1}$, where $\zeta \geq 0$ governs the severity of the financial frictions and $\kappa \in \mathbb{N}$ is a smoothness parameter.

Equation (4) can then be written more compactly as

$$d_1 = \pi_1 - F_0 x_1 - \zeta \max\{0, F_0 x_1 - \pi_1\}^{\kappa}$$

so that the total cost to firm 1 of investing ($x_1 = 1$) is

$$F(\pi_1) = F_0 + \zeta \max\{0, F_0 - \pi_1\}^{\kappa}$$

and that of not investing ($x_1 = 0$) is zero as long as $\pi_1 \geq 0$.

Following Doraszelski & Satterthwaite (2010), we add a privately observed random component $\theta_1 \sim N(0, \sigma^2)$ to the cost of investing $F(\pi_1)$ to avoid mixed strategies. A high realization of θ_1 means that the firm's current investment opportunity is poor. We assume that firm 1 observes θ_1 only after it makes its pricing decision and use $\psi(\theta_1)$ and $\Psi(\theta_1)$ to denote its density and cumulative distribution functions, respectively.[8]

Without loss of generality, we normalize $F_0 = 1$ in what follows. This corresponds to making an appropriate choice of monetary units. We set $\kappa = 3$ to ensure that $F(\pi_1)$ is twice continuously differentiable. This leaves the severity of the financial frictions ζ and the cost volatility σ as the two key cost parameters. In the special case of $\zeta = 0$, financial frictions are absent from our model, as in Budd et al. (1993) and the industrial organization literature more generally.

3.1 Optimal strategies

Given the state ω, we assume that, in each period, firms first decide on prices, $p_i(\omega)$ and then on investments, $x_i(\omega)$. We let $V_i(\omega)$ be the beginning-of-period value function

[8] Because θ_i is privately observed by firm i, firm 1 does not observe θ_2.

of firm i that is determined in the pricing stage and $U_i(\omega)$ be the middle-of-period value function that is determined in the investment stage.

Investment stage. Overloading notation, let $U_1(\omega, \theta_1)$ denote the value function of firm 1 *after* it has observed θ_1. In contrast, $U_1(\omega)$ denotes the value function of firm 1 *before* it has observed θ_1. Similarly, $x_1(\omega, \theta_1) = x_1 \in \{0, 1\}$ denotes the investment decision of firm 1 after it has observed θ_1 and $x_1(\omega) \in [0, 1]$ the probability that firm 1 invests before it has observed θ_1.

The middle-of-period Bellman equation of firm 1 is

$$
\begin{aligned}
U_1(\omega, \theta_1) = \max \Big\{ & - F(\pi_1) - \theta_1 + \beta \left[V_1(\omega^+)(1 - x_2(\omega)) + V_1(\omega) x_2(\omega) \right], \\
& \beta \left[V_1(\omega)(1 - x_2(\omega)) + V_1(\omega^-) x_2(\omega) \right] \Big\},
\end{aligned} \tag{5}
$$

where $\beta \in [0, 1)$ is the discount factor, $x_2(\omega)$ is the probability that firm 2 invests in state ω as seen from the perspective of firm 1, and $\omega^+ = \min\{L, \omega + 1\}$ and $\omega^- = \max\{-L, \omega - 1\}$ are possible successor states to state ω.

The optimal investment decision, $x_1(\omega, \theta_1)$, is characterized by a simple cutoff rule, where $x_1(\omega, \theta_1) = 1$ if

$$
F(\pi_1) + \theta_1 \leq \beta \left[\left(V_1(\omega^+) - V_1(\omega) \right) (1 - x_2(\omega)) + \left(V_1(\omega) - V_1(\omega^-) \right) x_2(\omega) \right]
$$

and $x_1(\omega, \theta_1) = 0$ otherwise. The right-hand side is effectively firm 1's marginal q (as in Hayashi 1982). The implied probability that firm 1 invests in state ω is

$$
x_1(\omega) = \Psi \left(-F(\pi_1) + \beta \left[\left(V_1(\omega^+) - V_1(\omega) \right) (1 - x_2(\omega)) + \left(V_1(\omega) - V_1(\omega^-) \right) x_2(\omega) \right] \right). \tag{6}
$$

Equation (6) captures the layered effects of financial frictions on investment. As usual, the direct impact is to increase the cost of investing $F(\pi_1)$, thus decreasing the investment probability $x_1(\omega)$. As Gomes (2001) points out, increasing $F(\pi_1)$ also has an indirect impact by changing the value function $V_1(\omega)$. The key novelty here is that firm 1 must now account for the impact of financial frictions on firm 2's investment probability $x_2(\omega)$, and vice versa. Strategic interactions thus create a new mechanism for the transmission of financial frictions to investment.

Substituting the optimal investment decision $x_1(\omega, \theta_1)$ into the Bellman equation (5) and integrating both sides with respect to θ_1 yields

$$U_1(\omega) = -F(\pi_1)x_1(\omega) - \int_{-\infty}^{\Psi^{-1}(x_1(\omega))} \theta_1 d\Psi(\theta_1) + \beta \left[V_1(\omega^+)x_1(\omega)(1 - x_2(\omega)) \right.$$
$$\left. +V_1(\omega)(1 - x_1(\omega) - x_2(\omega) + 2x_1(\omega)x_2(\omega)) + V_1(\omega^-)(1 - x_1(\omega))x_2(\omega)\right]. \quad (7)$$

We can show that $\frac{\partial^2 U_1(\omega)}{\partial x_1^2} < 0$ so that equation (6) is necessary and sufficient for a maximum for firm 1. However, there may be multiple solutions to the system of equations given by (6) and its analog for firm 2.

Pricing stage. The beginning-of-period Bellman equation of firm 1 is

$$V_1(\omega) = \max_{p_1} \pi_1(\omega, p_1, p_2(\omega)) + U_1(\omega), \quad (8)$$

where $p_2(\omega)$ is the price that firm 2 charges in state ω and $U_1(\omega)$ is given in equation (7). Recognizing the dependence of $x_1(\omega)$ and $x_2(\omega)$ on $\pi_1(\omega, p_1, p_2(\omega))$ and $\pi_2(\omega, p_1, p_2(\omega))$, the optimal pricing decision $p_1(\omega)$ is characterized by

$$\frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} \left(1 - F'\left(\pi_1(\omega, p_1(\omega), p_2(\omega))\right) x_1(\omega) + \frac{\partial U_1(\omega)}{\partial x_2} \frac{\partial x_2(\omega)}{\partial \pi_1}\right)$$
$$+\frac{\partial \pi_2(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} \frac{\partial U_1(\omega)}{\partial x_2} \frac{\partial x_2(\omega)}{\partial \pi_2} = 0. \quad (9)$$

Equation (9) shows that firm 1 deviates from static Nash pricing to the extent that this allows it to influence firm 2's investment probability $x_2(\omega)$, either through influencing

$\pi_1(\cdot)$ (and thus $x_2(\omega)$ via $\frac{\partial x_2(\omega)}{\partial \pi_1}$) or through influencing $\pi_2(\cdot)$ (via $\frac{\partial x_2(\omega)}{\partial \pi_2}$).

Absent financial frictions, $F'(\pi_1(\cdot)) = \frac{\partial x_2(\omega)}{\partial \pi_j} = 0$ so that equation (9) reduces to the first-order condition $\frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} = 0$ for a static Nash equilibrium (see Online appendix III). As a consequence, pricing decisions can be decoupled from investment decisions.

This "traditional static-dynamic breakdown" (Doraszelski & Pakes 2007, p. 1892) that underlies much of empirical and theoretical research in industrial organization no longer holds in the presence of financial frictions ($\zeta > 0$). Importantly, pricing and investment decisions are linked only because of strategic interactions: in a single-firm model or in a model with a continuum of firms without strategic interactions, the firm would always be at its static optimum.

Finally, if $\zeta > 0$, equation (9) is not sufficient for a maximum for firm 1. As in the investment stage, there may now be multiple solutions to the system of equations given by equation (9) and its analog for firm 2.

3.2 Markov perfect equilibrium

A Markov perfect equilibrium is a solution to the system of equations consisting of the Bellman equations and optimality conditions for firm 1 in equations (6), (7), (8), and (9), and their analogs for firm 2 for all $\omega \in \{-L, \ldots, L\}$. We focus on symmetric equilibria in which the value and policy functions of firm 2 are related to those of firm 1 by

$$V_2(\omega) = V_1(-\omega), \quad U_2(\omega) = U_1(-\omega), \quad p_2(\omega) = p_1(-\omega), \quad x_2(\omega) = x_1(-\omega).$$

In a symmetric equilibrium, it suffices to compute the value and policy functions of firm 1. We detail the resulting system of equations in Online appendix V.

Because equation (9) is necessary but not sufficient, a solution to this system of equations is not necessarily an equilibrium. We thus check that there is no profitable unilateral deviation from a solution, as detailed in Online appendix VII.

Industry dynamics. In equilibrium, the law of motion in equation (1) implies that the expected change in the state ω equals

$$\Delta(\omega) = E[\omega'|\omega] - \omega = \begin{cases} x_1(\omega)(1 - x_2(\omega)) & \text{if} \quad \omega = -L, \\ x_1(\omega) - x_2(\omega) & \text{if} \quad -L < \omega < L, \\ -(1 - x_1(\omega))x_2(\omega) & \text{if} \quad \omega = L. \end{cases} \tag{10}$$

Thus, away from the boundaries, the dynamics of the state are linear in the difference $x_1(\omega) - x_2(\omega)$: the state is expected to increase if firm 1 is more likely to invest than firm 2 and to decrease if firm 1 is less likely to invest than firm 2.[9] The difference in the investment probabilities of the leader and the follower is therefore crucial for the dynamics of the industry and its long-run competitiveness.

Equation (1) formally defines the $(2L + 1) \times (2L + 1)$ state-to-state transition probability matrix P of a Markov chain and, as detailed in Online appendix VIII, we compute its $1 \times (2L + 1)$ limiting distribution $\mu^\infty(\omega)$. In what follows, we summarize the implications of equilibrium behavior for the structure of the industry in the long run using the most likely state of the limiting distribution[10]

$$\widehat{\omega}^\infty = \arg \max_{\omega \in \{0,\ldots,L\}} \mu^\infty(\omega) \tag{11}$$

and the expected size of the competitive advantage

$$\bar{\omega}^\infty = \sum_{\omega=-L}^{L} |\omega| \, \mu^\infty(\omega). \tag{12}$$

[9]The state is expected to increase at the lower boundary $\omega = -L$ and to decrease at the upper boundary $\omega = L$. The boundaries are repulsive in the terminology of Budd et al. (1993) although the degree of repulsion is endogenous in our model.

[10]In a symmetric equilibrium, restricting attention to $\omega \in \{0, \ldots, L\}$ instead of $\omega \in \{-L, \ldots, L\}$ is without loss of generality.

4 Computation and equilibria

In this section, we describe our approach to numerically solving the model and the equilibrium behavior that it generates.

4.1 Parameterization

Table 2 summarizes the parameters and their ranges. We study our baseline model with financial frictions and contrast it to the special case of $\zeta = 0$ without financial frictions. We focus on the four key demand and cost parameters M, ν, τ, and σ. We proceed by specifying grids for these parameters while holding the remaining parameters fixed. We then thoroughly explore this parameter space in our numerical analysis.

	parameter	range	value/grid
state space:			
maximum value of ω	L	$\mathbb{N}$	15
discounting:			
discount factor	β	$[0, 1)$	0.95
product market:			
market size	M	$(0, \infty)$	10^{-2}, $10^{-1.9}$, $10^{-1.8}$, ..., $10^{1.9}$, 10^{2}
price sensitivity	α	$(0, \infty)$	1 (normalization)
degree of horizontal product differentiation	ν	$(0, \infty)$	0.025, 0.075, 0.125,..., 1.925, 1.975
leader handicap	τ	$[0, 1]$	0, 0.05, 0.1, ..., 0.95, 1
marginal cost	c	$[0, \infty)$	0 (normalization)
investment:			
fixed cost	F_0	$(0, \infty)$	1 (normalization)
severity of financial frictions	ζ	$[0, \infty)$	0,1
smoothness of financial frictions	κ	$\mathbb{N}$	3
cost volatility	σ	$(0, \infty)$	0.05, 0.1, 0.15, ..., 0.45, 0.5

Table 2: Parameterization.

To bound the parameter space, we discard "uninteresting" parameterizations for which M and ν are so large that $\pi_1\left(\omega, p_1^N(\omega), p_2^N(\omega)\right) \geq F_0$ for all ω such that financial frictions do not matter.[11] In addition, we discard any parameterization for which there is no meaningful investment absent financial frictions in the sense that $x_1(\omega) < 0.01$ for all ω. Finally, we bound cost volatility $\sigma \leq 2F_0$ to minimize the probability that investment outlays $F_0 + \theta_i$ become negative.

4.2 Homotopy method

We use the homotopy method in Besanko et al. (2010) to thoroughly explore the solution correspondence of our model and systematically search for multiple equilibria. This is extremely complex because it involves a system of $8L + 4$ equations (Bellman equations and optimality conditions) in as many variables (value and policy functions).

We systematically compute slices of the solution correspondence by varying each of the key parameters one at a time while holding the remaining parameters fixed. Each slice must either intersect with all previously computed slices or lead us to an additional solution that, in turn, produces an initial condition to compute an additional slice. We continue this process until all slices match up for all parameter combinations. In this way, we fully explore the solution correspondence over a four-dimensional hypercube in (M, ν, τ, σ)-space to compute as many solutions as possible.

The resulting computational demands are formidable. In total, we deployed more than 82,000 CPUs and spent 8.1 CPU years exploring the solution correspondence for the model with $\zeta = 1$ and a further 7.4 CPU years exploring the solution correspondence of the model with $\zeta = 0$. For a more detailed discussion, see Online appendix VI.

[11] $\left(p_1^N(\omega), p_2^N(\omega)\right)$ denote the prices in a frictionless model where $\zeta = 0$, which also correspond to the static Nash equilibrium.

4.3 Equilibria

We computed 3,800,269 solutions over 91,158 out of 93,041 parameterizations for our baseline model.[12] The number of solutions ranges from 1 to 23,200 across parameterizations. As discussed in Section 3.2, in the presence of financial frictions, a solution to the system of equations $\mathcal{H}(\mathcal{X}, \rho) = 0$ is not necessarily an equilibrium. After checking that there is no profitable unilateral deviation, we retain 2,733,602 equilibria over 88,767 parameterizations.[13] The number of equilibria ranges from 1 to 7,960 across parameterizations.

Figure 2 shows how the average number of equilibria, $\frac{2{,}733{,}602}{88{,}767} = 30.80$, varies across parameter values. To conduct comparative statics and summarize how an outcome of interest, such as the number of equilibria, depends on industry characteristics, we regress the outcome on a constant, dummies for all parameter values of (M, ν, τ, σ) listed in Table 2.[14]

As we can see, the impact of the fundamentals is non-monotonic. The number of equilibria is, in expectation, smallest for an intermediate value of market size M, the lowest value of the degree of horizontal product differentiation ν, the lowest value of the leader handicap τ, and the highest value of cost volatility σ. Conversely, the number of equilibria is largest for an intermediate-to-high market size M, an intermediate-to-low degree of horizontal product differentiation σ, an intermediate-to-high leader handicap τ, and an intermediate-to-low cost volatility σ.

Multiple equilibria: intuition. Multiplicity is rooted in strategic investment behavior. This is easy to see in the special case of $\zeta = 0$, where the static Nash equilib-

[12] The parameterizations for which we have been unable to compute a solution often involve low values of ν. In the limit as $\nu \to 0$, demand becomes discontinuous. The homotopy method exploits the differentiability of the system of equations, as discussed in Section 4.2.

[13] For details, see Online appendix VII.

[14] To facilitate interpretation, we center dummy coefficients for each of the four key parameters around the mean of the outcome of interest. Formally, instead of normalizing the coefficient on one of the dummies for each key parameter to equal zero, we normalize the average of the coefficients on the dummies for each key parameter to equal the average of the outcome of interest.

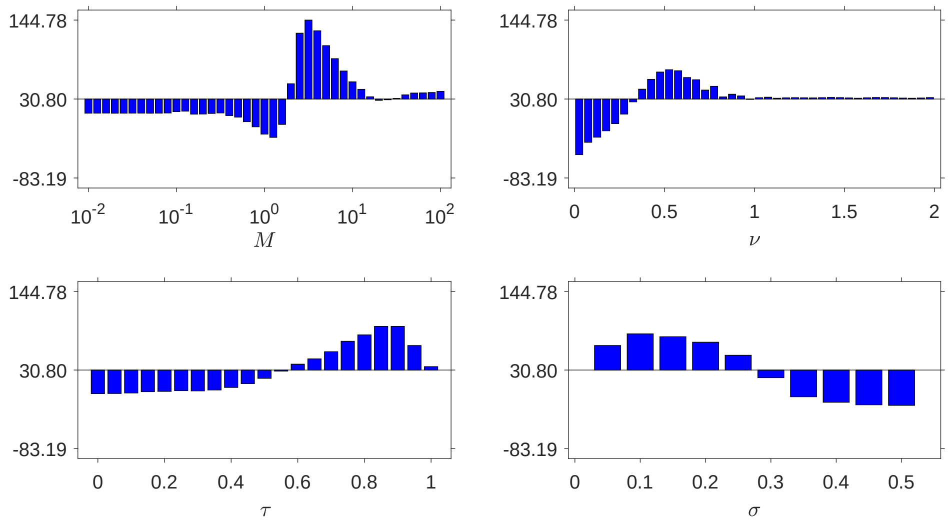

Figure 2: Number of equilibria. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

rium uniquely determines pricing decisions but multiple equilibria remain pervasive.

To build intuition, consider the investment stage in state ω. Holding fixed continuation play, as given by their value functions, the firms play a one-shot game. Equation (6) defines the best reply of firm 1 to firm 2's investment probability $x_2(\omega)$ and implies that

$$\frac{\partial x_1(\omega)}{\partial x_2(\omega)} \propto (V_1(\omega) - V_1(\omega^-)) - (V_1(\omega^+) - V_1(\omega)). \tag{13}$$

Hence, if $V_1(\omega)$ is locally concave, then the best reply of firm 1 is upward-sloping and its investment is a strategic complement to that of firm 2. Conversely, if $V_1(\omega)$ is locally convex, then its investment is a strategic substitute to that of firm 2.

In the limit as $\sigma \to 0$, we revert to mixed strategies and there are five cases to consider in this one-shot game:

1. Neither firm invests for sure, i.e., $x_1(\omega) = 0$ and $x_2(\omega) = 0$.

2. Firm 1 does not invest while firm 2 invests for sure, i.e., $x_1(\omega) = 0$ and $x_2(\omega) = 1$.

3. Firm 1 invests while firm 2 does not invest for sure, i.e., $x_1(\omega) = 1$ and $x_2(\omega) = 0$.

4. Both firms invest for sure, i.e., $x_1(\omega) = 1$ and $x_2(\omega) = 1$.

5. At least one firm mixes between investing and not investing, i.e., $0 \leq x_1(\omega) \leq 1$ and $0 \leq x_2(\omega) \leq 1$, with at least one strict inequality.

Cases 2, 3, and 5 can co-exist if the best replies of both firms are downward-sloping and thus can intersect more than once; this is the case of strategic substitutes. Conversely, cases 1, 4, and 5 can co-exist if the best replies of both firms are upward-sloping and thus can intersect more than once; this is the case of strategic complements.

The combination of $\zeta = 0$ and $\sigma \to 0$ enables us to enumerate all equilibria by checking the $3 \cdot 5^{L-1} \cdot 3$ possible combinations of cases 1 through 5 for states $\omega \in \{0, \ldots, L\}$.[15] The number of possible combinations increases exponentially in L. This suggests that the scope for multiple equilibria is vast once we set $L = 15$.

Equilibrium behavior and industry dynamics. Often, multiplicity does not produce meaningful differences in long run industry structure. But Figure 3 shows that investment behavior and industry dynamics can also be very different for the same parameterization depending on the equilibrium we compute.

The equilibrium in the first row exhibits a pattern of increasing dominance. Firm 1's investment probability $x_1(\omega)$ is large in most states $\omega > 0$, where firm 1 is the leader, and small in most states $\omega < 0$, where firm 1 is the follower. By symmetry, $x_2(\omega) = x_1(-\omega)$, which means the leader invests more than the follower in most states so that $\Delta(\omega) > 0$ if $\omega > 0$ and $\Delta(\omega) < 0$ if $\omega < 0$. As a result, the long-run industry structure is maximally asymmetric, with the limiting distribution $\mu^\infty(\omega)$ tightly concentrated around states $\omega = -L$ and $\omega = L$.

[15] We can rule out cases 2 and 3 in state $\omega = 0$ by symmetry. We can also rule out cases 3 and 4 in state $\omega = L$. Complete derivations for $L = 1$ are available upon request.

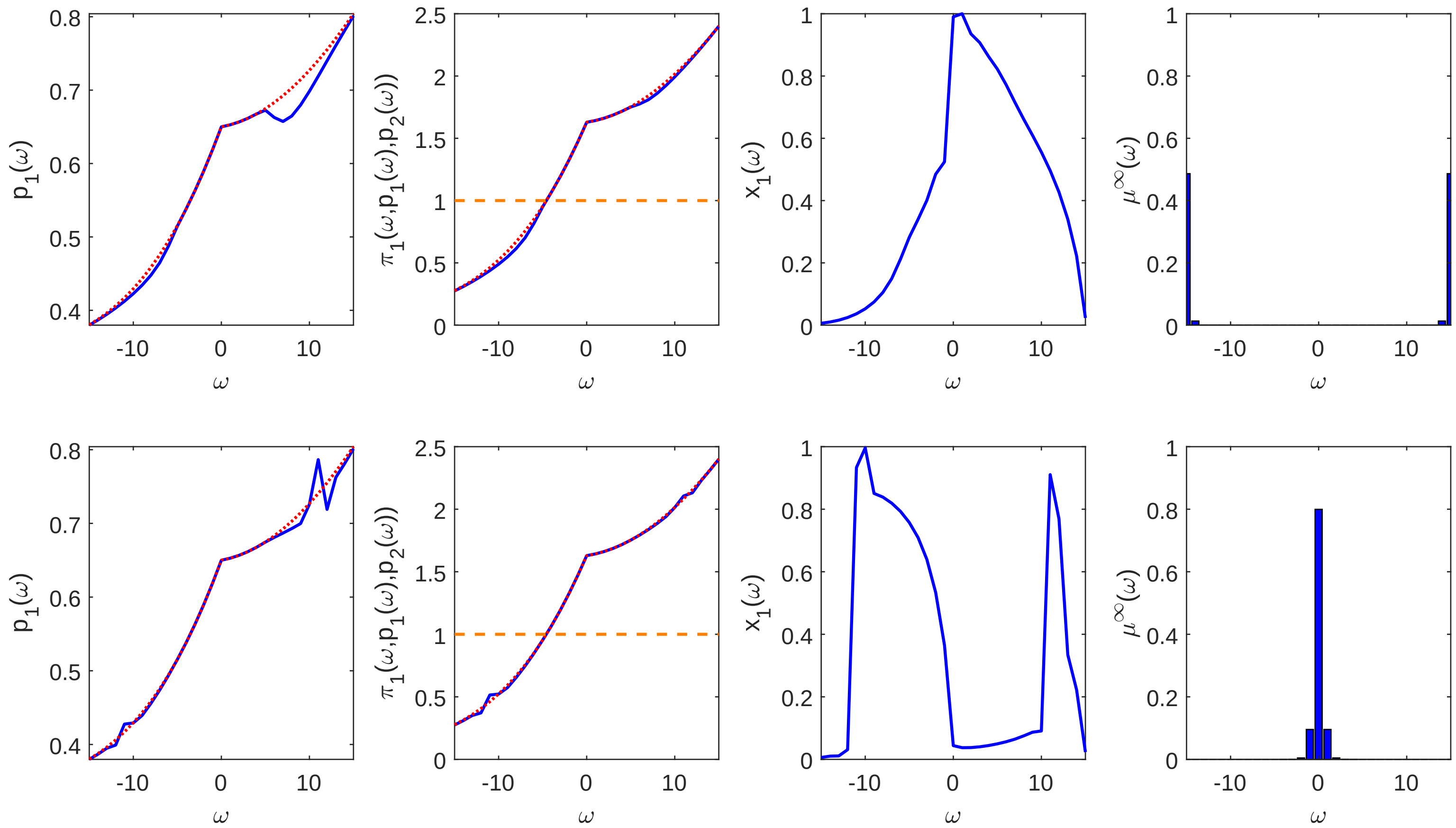

Figure 3: Example of multiple equilibria (first and second row). Pricing decision $p_1(\omega)$ (first column, solid line) overlayed by static Nash equilibrium (dotted line); profit $\pi_1(\omega, p_1(\omega), p_2(\omega))$ (second column, solid line) overlayed by static Nash equilibrium (dotted line) and horizontal line at fixed cost $F_0 = 1$ (dashed lined); investment probability $x_1(\omega)$ (third column); and limiting distribution $\mu^{\infty}(\omega)$ (fourth column). Model with $M = 5.01$, $\nu = 0.33$, $\tau = 0.55$, $\sigma = 0.50$, and $\zeta = 1$.

By contrast, the equilibrium in the second row exhibits catch-up behavior. Firm 1's investment probability $x_1(\omega)$ is large in most states $\omega < 0$, where firm 1 is the follower, and small in most states $\omega > 0$, where firm 1 is the leader. Now the expected change $\Delta(\omega) > 0$ if $-12 < \omega < 0$ and $\Delta(\omega) < 0$ if $0 < \omega < 12$, and the long-run industry structure is minimally asymmetric, with the limiting distribution $\mu^\infty(\omega)$ tightly concentrated around state $\omega = 0$.

Industry concentration. We summarize the long-run industry structure with the expected size of the leader's competitive advantage $\bar{\omega}^\infty$ defined in equation (12). Because the literature offers little guidance regarding equilibrium selection, we view all equilibria that arise for the same primitives as equally likely. Accordingly, we average the value of $\bar{\omega}^\infty$ over all equilibria at a given parameterization.

The long-run industry structure ranges from symmetric, with $\bar{\omega}^\infty \approx 0$, to maximally asymmetric, with $\bar{\omega}^\infty \approx 15$, depending on the parameterization. Figure 4 shows that the long-run industry structure becomes more symmetric in the degree of horizontal product differentiation ν but more asymmetric in the leader handicap τ. Intuitively, a higher ν softens price competition between the firms, while a higher τ imposes a smaller disadvantage on the leader. The long-run industry structure is also more asymmetric for intermediate-to-low market size M, though this relationship is weaker than that of ν and τ. Cost volatility, σ, has no discernible impact on the long-run industry structure.

Average price and investment probability. Online appendix X documents the basic properties of average price and investment policies. We show that average price, $\bar{p}_1 = \frac{1}{2L+1} \sum_{\omega=-L}^{L} p_1(\omega)$, increases in the degree of horizontal product differentiation, ν, and, albeit much less forcefully, in the leader handicap, τ. Market size, M, and cost volatility, σ, have no discernible impact on $\bar{p}_1$.

Average investment probability, $\bar{x}_1 = \frac{1}{2L+1} \sum_{\omega=-L}^{L} x_1(\omega)$, increases in market size, M, while cost volatility, σ, has no discernible impact on $\bar{x}_1$. The impact of product

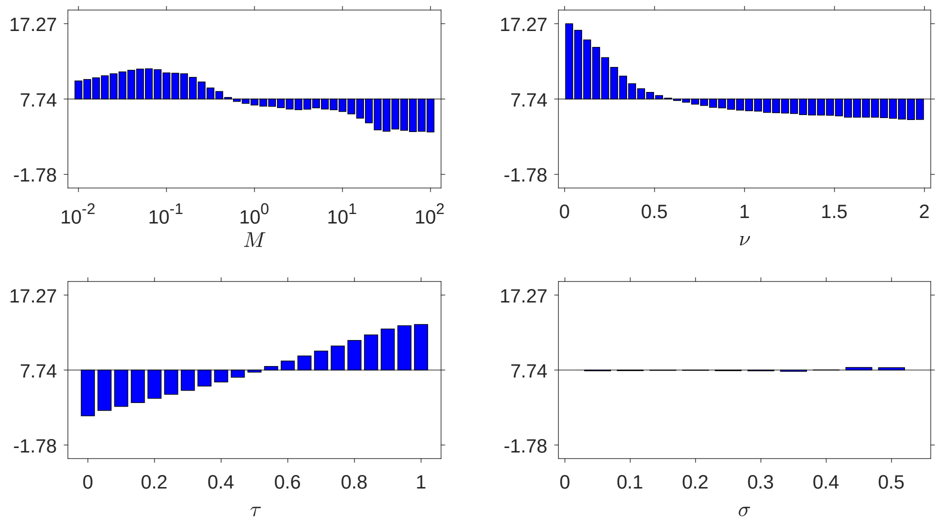

Figure 4: Long-run industry structure as measured by $\bar{\omega}^{\infty}$, averaged over equilibria within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel). Model with $\zeta = 1$.

differentiation, ν, and leader handicap, τ, depends on the firm's competitive position.

5 Impact of financial frictions

Financial frictions can affect price and investment decisions of firms as well as industry dynamics and levels of concentration. To understand these effects, we juxtapose the equilibria that arise in our baseline model with the equilibria from the special case of $\zeta = 0$. For brevity, we henceforth refer to the former as equilibria with financial frictions and to the latter as equilibria without financial frictions and use the superscripts FC and $NOFC$ respectively, to distinguish them.

In line with our choice not to engage in equilibrium selection, we form all possible pairs of equilibria with and without frictions at a given parameterization. This leaves us with 2,864,517,460 pairs over 88,767 parameterizations, with the number of pairs ranging from 1 to 47,019,720 across parameterizations. To account for this wide range, in what follows we average a statistic of interest over all pairs at a given parameterization.

5.1 Impact on price

As discussed in Section 3.1, the combination of financial frictions and strategic interactions impacts equilibrium pricing decisions in complex ways. In our baseline model, firm 1 deviates from static Nash pricing to the extent that this allows it to influence firm 2's investment probability $x_2(\omega)$ through influencing both $\pi_1(\cdot)$ and $\pi_2(\cdot)$.

We summarize the overall impact of financing frictions on prices across all parameterizations by computing the following measures:

$$\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[p_1^{FC}(\omega) > p_1^{NOFC}(\omega) + 0.01||p_1^{NOFC}||_1\right]$$

and

$$\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[p_1^{FC}(\omega) < p_1^{NOFC}(\omega) - 0.01||p_1^{NOFC}||_1\right]$$

where $p_1^{FC}(\omega)$ is firm 1's pricing decision in state ω in the baseline model and $p_1^{NOFC}(\omega)$ is firm 1's pricing decision in state ω in the special case of $\zeta = 0$. The factor $||p_1^{NOFC}||_1 = \sum_{\omega=-L}^{L}\left|p_1^{NOFC}(\omega)\right|$ ensures we focus on economically meaningful differences.

The following result summarizes our findings.

Result 1 *We find that financial frictions meaningfully: (a) decrease average prices in 7.2% of parameterizations; (b) increase average prices in 3.5% of parameterizations.*

Figure 5 depicts these average (normalized) prices with and without financial frictions. Mostly, prices are similar in the baseline model and the special case where $\zeta = 0$, i.e. close to the 45°-line, which confirms that the differences between the two cases are usually relatively small. Interestingly, when the differences are meaningful, we find that financial frictions can increase or decrease prices. Intuitively, by decreasing price a firm can decrease its rival's profit, which in the presence of financial frictions can drive up the rival's cost of investing. Conversely, lower prices can also drive up its own cost of financing/ investing.

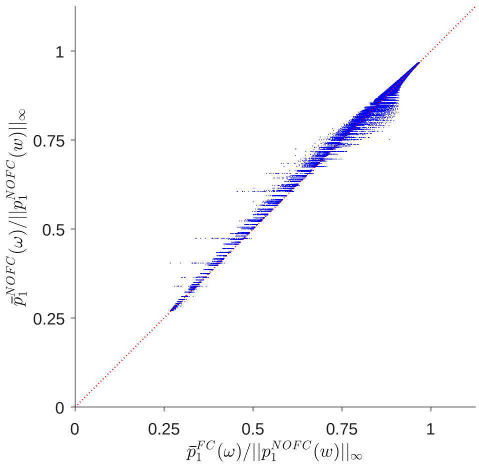

Figure 5: Scatter plot of average price absent financial frictions $\bar{p}_1^{NOFC}$ against average price in the presence of financial frictions $\bar{p}_1^{FC}$. Average price normalized by $||p_1^{NOFC}||_\infty = max_\omega |p_1^{NOFC}(\omega)|$ to improve visibility.

Figure 6 shows that the overall impact of financial frictions on prices depends on industry characteristics. Deviations from static Nash prices tend to occur when the degree of horizontal product differentiation, ν, is low, while the values for leader handicap, τ, and cost volatility, σ, are both large. They are also more common for intermediate values of the market size, M. Conversely, they are very rare for low market size M, a high intermediate-to-low degree of horizontal product differentiation ν, and low value of both τ and σ.[16]

We find that deviations from static Nash pricing, which are optimal in the frictionless world, are slightly more common for leaders than followers. As the leader's profit is always higher than the follower's profit, the leader can more often afford to change prices without incurring additional financing costs to fund investment. Average prices

[16]See Online appendix 7 for the counterpart analysis where financial frictions increase price plus differential analyses for leaders and followers.

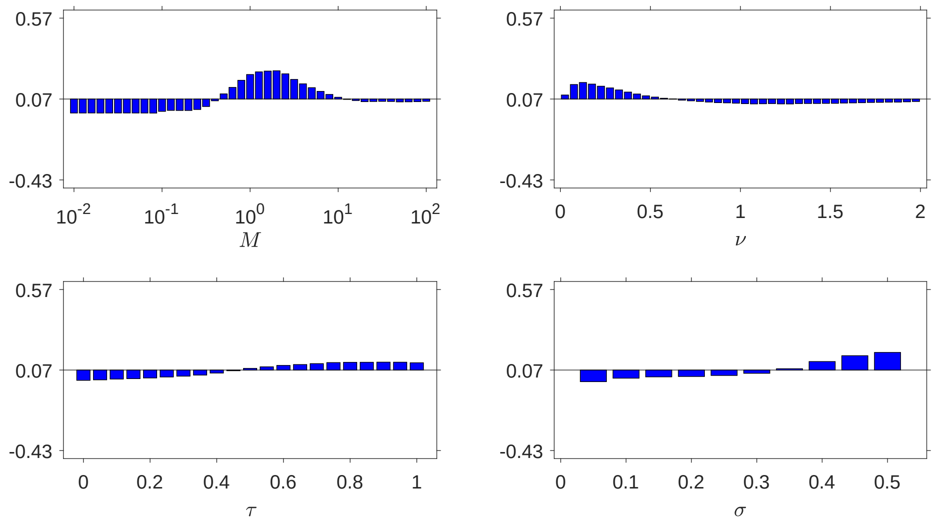

Figure 6: Financial frictions decrease price as measured by $1\left[p_1^{FC}(\omega) < p_1^{NOFC}(\omega) - \frac{0.01}{2L+1}||p_1^{NOFC}||_1 \text{ for some } \omega\right]$, averaged over pairs of equilibria with and without financial frictions within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

charged by industry leaders in the baseline FC case are higher than those in the NOFC case in 4.8% of cases. They are, conversely, lower in 7.9% of parameterizations.[17]

The finding that financial frictions can lead to lower prices is reminiscent of the long-purse or deep-pockets theory of predation in which a cash-rich predator drives a cash-poor prey out of business by reducing the prey's cash flow (Telser 1966, Bolton & Sharfstein 1990). In contrast to these papers, however, the role of predator and prey is not exogenously assigned in our model, and we find that the leader, as well as the follower, may decrease price and thereby their rival's profit.

Profitability. As financial frictions affect the evolution of the industry over time, a price decrease in a given period may be offset by a price increase in a later period.

[17] The comparable numbers for followers are 2.3% and 6.7%, respectively.

We therefore also report the expected net present value of industry-wide profit

$$\Pi^{\infty} = E\left[\sum_{t=0}^{\infty} \beta^t \left(\pi_1(\omega^t, p_1(\omega^t), p_2(\omega^t)) + \pi_2(\omega^t, p_1(\omega^t), p_2(\omega^t))\right) \middle| \omega^0 = 0\right],$$

where the expectation is with respect to the Markov chain defined by equation (10) and the initial state is set to $\omega^0 = 0$. Similar to our price analysis, we now compute

$$1\left[\Pi^{\infty,FC} < \Pi^{\infty,NOFC} - \frac{0.01}{2L+1}\frac{||\pi_1^{NOFC} + \pi_2^{NOFC}||_1}{1-\beta}\right]$$

and

$$1\left[\Pi^{\infty,FC} > \Pi^{\infty,NOFC} + \frac{0.01}{2L+1}\frac{||\pi_1^{NOFC} + \pi_2^{NOFC}||_1}{1-\beta}\right].$$

Again the factor $\frac{||\pi_1^{NOFC}+\pi_2^{NOFC}||_1}{1-\beta}$ ensures we focus on economically meaningful differences. The following result summarizes our findings with respect to the impact of financial frictions on the present value of profits.

Result 2 *We find that financial frictions: (a) decrease the profitability of product market competition in all pairs of equilibria with and without financial frictions at 9.4% of parameterizations; (b) increase the profitability of product market competition in all pairs of equilibria with and without financial frictions at 19.6% of parameterizations.*

Even though financial frictions are slightly more likely to lead to lower average prices, their overall impact is skewed towards increasing expected profitability. In Online appendix X, we document that this increase in the profitability of product market competition is facilitated by low values for the market size parameter, M, degree of horizontal product differentiation ,ν, and leader handicap, τ, as well as high values for cost volatility, σ, although the latter is more muted.

5.2 Impact on investment

Because they directly increase the cost of investing, financial frictions generally lead to lower investment. This is also the core intuition from the single-firm models widely used in corporate finance. As we show below, however, the combination of financial frictions and strategic interactions can lead firms to invest *more*.

As we did with prices, we describe the impact of financial frictions on average investment by computing

$$\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[x_1^{FC}(\omega) > x_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||x_1^{NOFC}||_1\right];$$

and

$$\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[x_1^{FC}(\omega) < x_1^{NOFC}(\omega) - \frac{0.01}{2L+1}||x_1^{NOFC}||_1\right];$$

where $||x_1^{NOFC}||_1 = \sum_{\omega=-L}^{L} \left|x_1^{NOFC}(\omega)\right|$.

The following result summarizes our findings.

Result 3 *We find that financial frictions: (a) decrease average investment in 64.7% of parameterizations; (b) increase average investment in 5.9% of parameterizations.*

Result 3 is striking in that it shows that financial frictions can lead to higher investment. This possibility is completely ruled out in any empirical study on the effects of financial frictions on investment. However, we show that strategic interactions can create a strong incentive to invest since a firm can use this tool to lower its rival's profit in future periods and, thus, drive up its cost of investing. In this sense, a firm can exploit financial frictions to entrench its competitive position.

Not surprisingly, financial frictions are more likely to increase the leader's investment than that of the follower. Figure 7 illustrates this by showing a scatter plot of the average investment probability for both leaders and followers absent financial frictions against those values in the presence of financial fictions. We can see that leader (left panel) under FC out-invests its NOFC counterpart slightly more often than followers

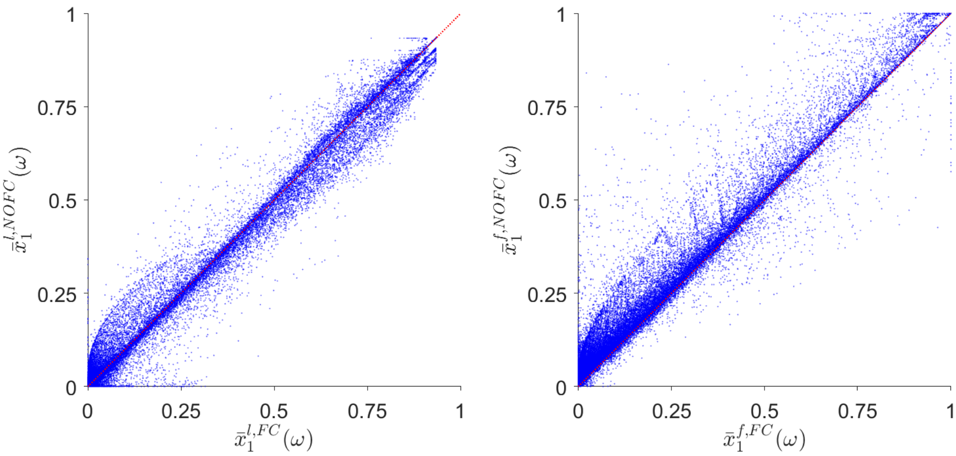

Figure 7: Scatter plot of average investment absent financial frictions $\bar{x}_1^{NOFC}$ against average investment in presence of financial frictions $\bar{x}_1^{FC}$ for leader's average investment (left panel) and follower's average investment (right panel).

(right panel) do, suggesting that leaders use frictions for entrenchment purposes. In turn, this indicates that financial frictions can lead to more long run asymmetry.

Figure 8 shows how different industry characteristics dictate whether average investment is higher in the presence of financial frictions. As we can see, financial frictions lead to higher investment for an intermediate-to-high market size M and very low levels of horizontal product differentiation, ν. This result also becomes more likely when the leader handicap parameter, τ, rises. The cost volatility parameter, σ, has no discernible impact.

Connecting pricing and investment decisions. Comparing Figures 6 and 8 suggests that parameterizations for which average investment is higher in the presence of financial frictions frequently coincide with those for which average prices are lower.

Figure 9 provides a graphical representation of this correlation through a scatter plot of the cases where financial frictions increase average investment against the cases where they decrease average prices (as defined above) together with a regression line. As we can see, there are enough cases to imply a strong positive correlation between

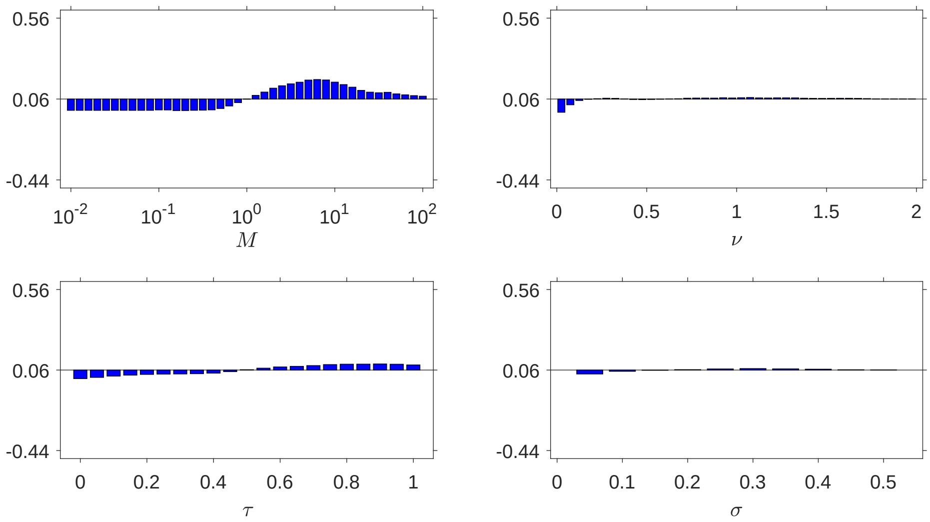

Figure 8: Financial frictions increase investment as measured by $1\left[x_1^{FC}(\omega) > x_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||x_1^{NOFC}||_1 \text{ for some } \omega\right]$, averaged over pairs of equilibria with and without financial frictions within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

lower average prices and higher average investment frequencies in the presence of financial frictions. Online appendix X shows that, conversely, cases where prices tend to be higher with financial frictions are also those where average investment is lower.

This explicit connection between pricing and investment decisions opens up an important avenue for empirical work, which has typically investigated how financial frictions impact these decisions in isolation. For example, the empirical studies in Phillips (1995), Chevalier (1995*b*), Chevalier & Sharfstein (1995, 1996), and Gilchrist et al. (2017) restrict their focus to the impact of financial frictions on price, whereas those in Fazzari et al. (1988), Kovenock & Phillips (1995, 1997), Chevalier (1995*a*), and Kaplan & Zingales (1997) restrict their focus to investment, entry, and exit.

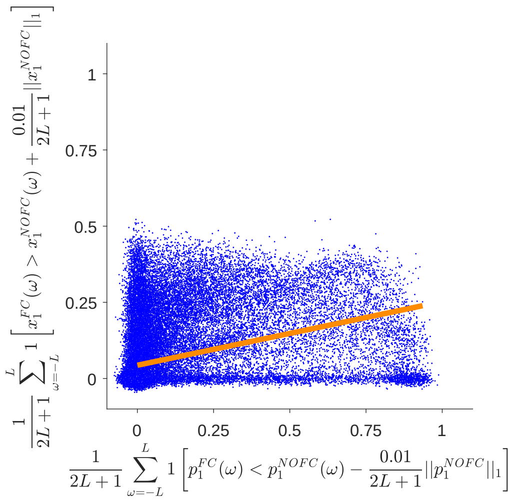

Figure 9: Scatter plot of the parameterizations for which financial frictions increase investment as measured by $\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[x_1^{FC}(\omega) > x_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||x_1^{NOFC}||_1\right]$ against financial frictions decrease price as measured by $\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[p_1^{FC}(\omega) < p_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||p_1^{NOFC}||_1\right]$ overlayed by a linear regression line. Noise added to improve visibility.

5.3 Impact on industry concentration

Figure 3, discussed above, illustrates how strategic pricing and investment behavior can interact in the presence of financial frictions to produce very different long-run outcomes for an industry. In the equilibrium in the first row both firms deviate from static Nash to choose lower prices in some states. This lowers profits and leads to lower follower investment, eventually pushing the industry towards asymmetry. By contrast, in the equilibrium in the second row, firms set higher prices in some states, raising their profits and inducing the follower to invest much more. This behavior, combined with a sharp reduction in the leader's investment, produces a relatively symmetric industry in the long run.

To assess the quantitative significance of these effects, we compute the following measure of meaningful increases in long-run industry concentration under financial frictions

$$1\left[\bar{\omega}^{\infty,FC} > \bar{\omega}^{\infty,NOFC} + 0.01L\right].$$

and meaningful decreases in long-run industry concentration under financial frictions

$$1\left[\bar{\omega}^{\infty,FC} < \bar{\omega}^{\infty,NOFC} - 0.01L\right]$$

The following result summarizes our findings.

Result 4 *We find that financial frictions: (a) increase average long-run industry concentration in 44.6% of parameterizations; (b) decrease average long-run industry concentration in 7.2% of parameterizations.*

Figure 10 shows the scatter plot of the long-run industry structure absent financial frictions, $\bar{\omega}^{\infty,NOFC}$, against the same value in the presence of financial frictions, $\bar{\omega}^{\infty,FC}$. While financial frictions do not always change long-run industry concentration, when they do, they are much more likely to exacerbate asymmetries between firms over time.

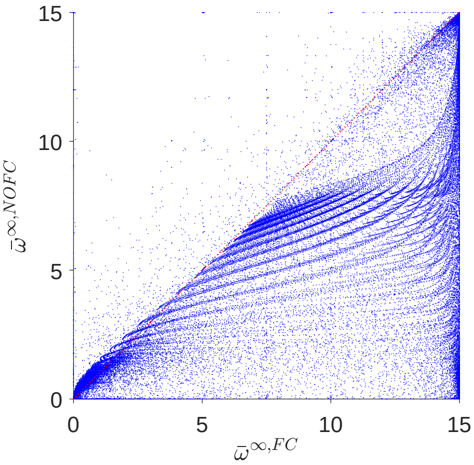

Figure 10: Scatter plot of long-run industry structure absent financial frictions as measured by $\bar{\omega}^{\infty,NOFC}$ against long-run industry structure in presence of financial frictions as measured by $\bar{\omega}^{\infty,FC}$, averaged over pairs of equilibria with and without financial frictions within parameterizations and overlayed by 45° line.

This finding is not surprising, since frictions are more consequential for the followers than leaders, but it accords well with previous theoretical and empirical work by

Cooley & Quadrini (2001), Cabral & Mata (2003), and Angelini & Generale (2008) showing that financial frictions may cause right-skewness in the firm size distribution. More surprising is the fact that financial frictions can also mitigate asymmetries between firms over time, although, as Figure 10 shows, these decreases in industry concentration are less frequent and tend to be less pronounced.

Figure 11 shows how differences in the long-run industry structure, $\bar{\omega}^{\infty,FC} - \bar{\omega}^{\infty,NOFC}$, are impacted by the different parameters of the model. We find that financial frictions exacerbate long-run industry concentration for low values for market size M, degree of horizontal product differentiation, ν, and intermediate-to-high values of the leader handicap parameter,τ. Cost volatility, σ, has no discernible impact on $\bar{\omega}^{\infty,FC} - \bar{\omega}^{\infty,NOFC}$.

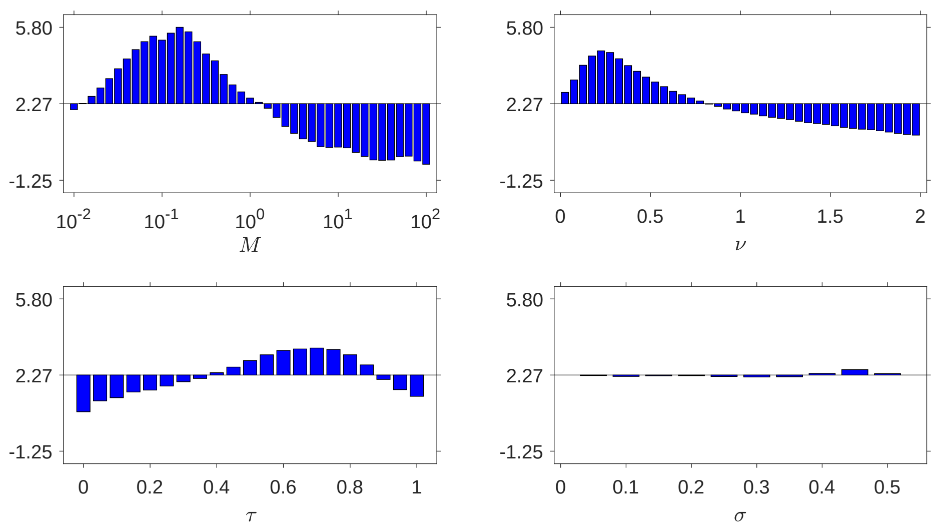

Figure 11: Difference in long-run industry structure as measured by $\bar{\omega}^{\infty,FC} - \bar{\omega}^{\infty,NOFC}$, averaged over pairs of equilibria with and without financial frictions within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

Speed of convergence. Even if financial frictions do not change long-run industry concentration, they may nevertheless impact the speed of convergence to industries' limiting distribution. To investigate this effect, we now define the total variation distance between the transient distribution $\mu^{25}_{\omega^0}(\omega)$ after 25 periods and starting from the initial state ω^0 and the limiting distribution $\mu^{\infty}(\omega)$ as

$$\delta = \frac{\sum_{\omega=-L}^{L} \left|\mu^{25}_{\omega^0}(\omega) - \mu^{\infty}(\omega)\right|}{2}.$$

Thus, a larger value of δ implies slower convergence to the limiting distribution. We then say that financial frictions accelerate convergence to the limiting distribution if $\delta^{FC} < \delta^{NOFC}$. To focus on the most interesting cases, we form all possible pairs of equilibria where both $\bar{\omega}^{\infty,FC} \geq 0.99L$ and $\bar{\omega}^{\infty,NOFC} \geq 0.99L$ so that the industry becomes maximally asymmetric and initialize at $\omega^0 = 0$. Result 5 shows that in long-run asymmetric industries financial frictions are more likely to accelerate than decelerate convergence.

Result 5 *We find that financial frictions: (a) accelerate convergence towards maximally asymmetric industries in 16.5% of parameterizations; (b) decelerate convergence towards maximally asymmetric industries in 3.9% of parameterizations.*

6 Consumer surplus and welfare

Finally, we examine the welfare implications of all our findings above. Social welfare equals the sum of firm values computed above plus the present discounted value of static consumer surplus, denoted CS^{∞}. Computing social welfare is complicated for two reasons. First, static consumer surplus can only be precisely defined for the case where $\tau = 1$. Second, recall that this surplus depends on absolute—not relative—product quality (ω_1, ω_2).

Specifically, when $\tau = 1$, static consumer surplus, denoted $CS(\omega_1, \omega_2)$, is given by

$$CS(\omega_1, \omega_2) = M\frac{\nu}{\alpha} \ln\left(\exp\left(\frac{\frac{\omega_1}{L} - \alpha p_1}{\nu}\right) + \exp\left(\frac{\frac{\omega_2}{L} - \alpha p_2}{\nu}\right)\right) \tag{14}$$

To compute the present value of consumer surplus we must first use a transformation of the law of motion, (1), for the industry state $\omega = \omega_1 - \omega_2$. Our procedure is described in detail in Online appendix IX.

The present value of social welfare for an industry starting from state $(\omega_1, \omega_2) = (0, 0)$ is defined as

$$W^{\infty}(0,0) = CS^{\infty}(0,0) + 2 \times V_1(0) \tag{15}$$

Table 3 summarizes the impact of financial frictions on social welfare by reporting the differences between this value in the baseline model with financial frictions and the special case without frictions ($\kappa = 0$). The table also shows how these differences are correlated with the corresponding differences in firms' average pricing and investment strategies, long run industry concentration, as well as consumer surplus. As before, we focus only on economically meaningful differences in all variables reported.[18]

Recall that, by construction, financial frictions often raise investment costs to firms because $F(\pi_1) \geq F_0$ when $\zeta > 0$. As a result, it is not surprising that they generally lower social welfare, too. However, Table 3 shows that consumer surplus is actually higher under financial frictions in 13% of parameterizations. Notably, these gains are not the result of lower industry concentration since concentration is never reduced when $\tau = 1$. Despite producing more asymmetric industries in the long run, the table shows that households benefit from a combination of higher investment—which raises overall product quality—and lower prices. Of course, these strategies lower overall firm values so that the effect on overall welfare is weakened. Strikingly, however, in about half of the cases, this decrease in firm value is more than compensated for by the increase in consumer surplus such that social welfare is actually higher in the presence of financial frictions.

[18]Specifically, we look for cases where the differences between the baseline and the no-friction model is larger than $|1\%|$ of $\frac{1}{2L+1}||p_1^{NOFC}||_1$, $\frac{1}{2L+1}||x_1^{NOFC}||_1$, L, $|\overline{CS}^{\infty,NOFC}|$, and $|\overline{W}^{\infty,NOFC}|$, respectively.

		social welfare			
		higher	same	lower	total
prices	higher	0%	0%	0%	0%
	same	6%	21%	59%	86%
	lower	1%	2%	11%	14%
investment	higher	5%	5%	2%	12%
	same	1%	10%	4%	15%
	lower	1%	8%	64%	73%
industry concentration	higher	0%	6%	50%	56%
	same	7%	17%	20%	44%
	lower	0%	0%	0%	0%
consumer surplus	higher	7%	3%	3%	13%
	same	0%	19%	11%	30%
	lower	0%	1%	55%	56%
	total	**7%**	**23%**	**70%**	**100%**

Table 3: This table summarizes the overall impact of financial frictions on social welfare. It reports how the difference in social welfare between our baseline model with financial frictions and the special case without frictions ($\zeta = 0$) is correlated with the corresponding differences in firms' average pricing and investment strategies, long run industry concentration, as well as consumer surplus. We define higher (lower) prices, investment, industry concentration, consumer and social welfare to be a difference between the baseline and the no-friction model larger (smaller) than 1% of (the negative) $\frac{1}{2L+1}||p_1^{NOFC}||_1$, $\frac{1}{2L+1}||x_1^{NOFC}||_1$, L, $|\overline{CS}^{\infty,NOFC}|$, and $|\overline{W}^{\infty,NOFC}|$, respectively. Cases within these bounds are declared to have the same outcome.

7 Concluding remarks

We show how limitations in access to capital markets impact the dynamic strategic interactions between firms and the evolution of an industry over time. We have several striking results. First, we show that pricing and investment decisions are tightly related in equilibrium, with higher investment often going hand in hand with lower prices and vice-versa. This connection opens up an important new avenue for empirical work since existing studies have typically investigated the impact of financial frictions on pricing or investment decisions in isolation. Second, we show that the impact of financial frictions is much less clear cut than widely believed and can even lead to higher investment, thus challenging a hypothesis used in countless empirical studies in corporate finance. Finally, we show that, under financing frictions, the combination of higher investment and lower prices can increase consumer surplus and overall welfare even when long run industry concentration is increased. These and other findings confirm that linking the industrial organization and corporate finance literatures is an extremely promising undertaking.

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Online appendix

Not for Publication

I Motivation: Online food delivery industry.

The market for food delivery in the U.S. grew rapidly in recent years to an estimated \$27 billion in 2019. By mid-2020, nearly two-thirds of households had ordered food delivery online at least once. Every major fast food chain is now partnered with at least one major delivery firm such as Grubhub, DoorDash, Postmates, and Uber Eats. Yet, growth opportunities remain vast, given a still low 6% penetration rate of the \$350 billion restaurant market. The size of the opportunity has led firms to rapidly expand their networks of restaurants and cities and use aggressive price discounts to win diners and orders.

Importantly for our paper, this ongoing battle for market dominance was made possible through access to capital markets. As Figure i shows, fierce competition has led to steep losses among the key players and left them without internal funds to finance growth. Total capital injections into the online food delivery industry, especially in the form of venture capital funding and initial public offerings, exceeded \$16 billion by 2020, with DoorDash alone raising new equity eight times between 2014 and 2019. With funding temporarily drying up in 2020, Waitr was forced to scale back expansion plans and lay off employees in many cities while Uber Eats stopped delivering food in South Korea and India. Uber Eats merged with Postmates in the U.S. in mid-2020.

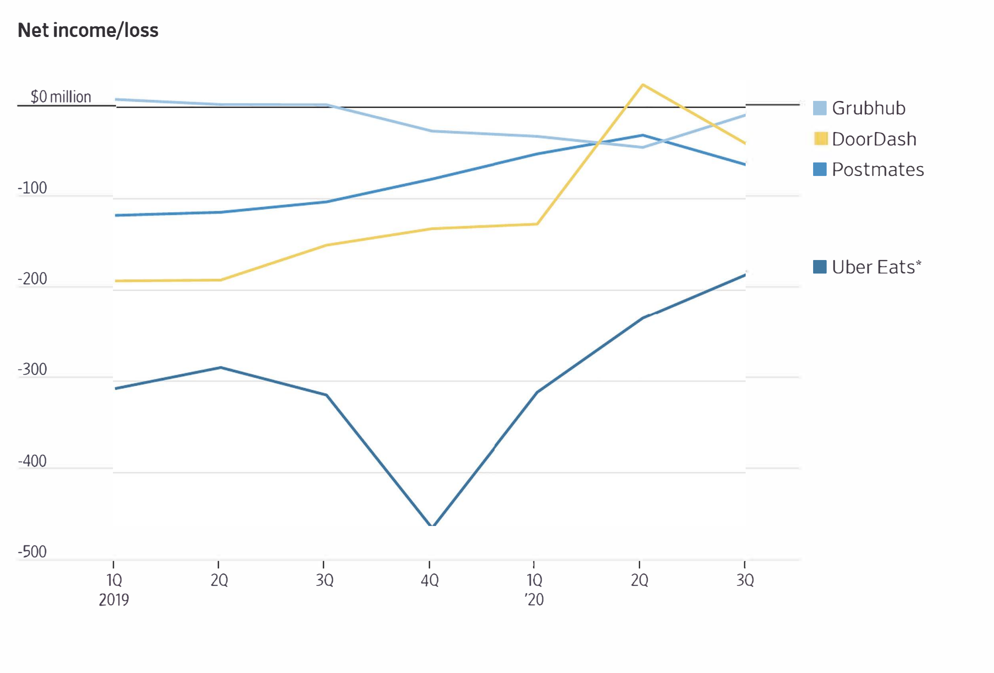

Figure i: Losses of largest online food delivery firms in U.S. Source: Wall Street Journal.

II Data

We follow Gomes (2001) to construct the sample. We merge the 2021 Compustat Industrial Files with the text-based network industry classifications data from Hoberg & Phillips (2016) from 1989 to 2019. The latter is based on the product descriptions in firms' annual 10-K reports and provides us with an annual product similarity score for all pairs of firms in Compustat. The score is normalized to the unit interval, with a higher score indicating greater overlap in product descriptions.

We construct the following variables:

- investment is constructed as capital expenditures (item CAPX), scaled by the beginning-of-period net property, plant, and equipment stock (item PPENT);
- average investment of the n closest competitors of the focal firm as determined by the n highest annual product similarity scores;
- cash flow is the sum of income before extraordinary items and depreciation (items IB and DP), scaled by beginning-of-period net property, plant, and equipment stock (item PPENT);
- Tobin's Q is constructed as total assets (item AT) minus the sum of common equity and deferred taxes on balance sheet (items CEQ and TXDB) plus the product of the fiscal year closing share price and common shares outstanding (item PRCC_F and CSHO), scaled by total assets (item AT);
- gross margin is constructed as sales (item SALE) minus cost of goods sold (item COGS), scaled by sales (item SALE).

We screen the sample as follows. For observations to be included in our sample, we require them to have non-missing data for all variables (contemporaneous or lagged as per the specification used). We exclude financials and utilities (NAICS starting with 21, 22, 23, 52, and 53). Furthermore, we exclude observations where investment of the firm or its competitors exceeds the beginning-of-period net property, plant, and equipment stock (item PPENT); cash flow in absolute value exceeds five times total assets (item AT); Tobin's Q is negative or exceeds ten. Lastly, we winsorize all variables at the 5th and 95th percentiles. We are left with 29,441 firm-year observations, with an average of 981.4 and a minimum (maximum) of 551 (1,373) firms per year. Our sample comprises 4,998 distinct firms, including 1,135 firms with one observation.

	mean	std. dev.	25th pctl.	median	75th pctl.	N
investment	0.21	0.15	0.11	0.17	0.26	29,441
cash flow	0.46	0.44	0.13	0.31	0.63	29,441
Tobin's Q	1.30	0.31	1.05	1.35	1.62	29,441
gross margin	0.67	0.18	0.57	0.69	0.79	29,421
average investment of n closest competitors:						
$n = 1$	0.22	0.16	0.11	0.18	0.28	29,441
$n = 2$	0.22	0.13	0.13	0.19	0.28	29,441
$n = 3$	0.22	0.11	0.14	0.20	0.28	29,441
$n = 4$	0.22	0.11	0.15	0.20	0.28	29,441
$n = 5$	0.22	0.10	0.15	0.21	0.28	29,441

Table i: Descriptive statistics.

Table i provides descriptive statistics for our variables. The bottom part of the table reports the average investment of the n closest competitors of the focal firm, where we vary n from one to five.

In our regressions, we include firm, year, industry, and industry-year fixed effects. We use the fixed industry classifications from Hoberg & Phillips (2016). These are derived from a clustering algorithm that maximizes annual within-industry product similarity scores while targeting 50 distinct industries. On average, an industry comprises 19.6 firms per year.

	baseline	number of closest competitors 1	2	3	4	5
comp. invest. $(t-1)$		0.110***	0.112***	0.098***	0.090***	0.084***
		(0.025)	(0.015)	(0.012)	(0.010)	(0.010)
cash flow (t)	0.075***	0.075***	0.075***	0.075***	0.075***	0.075***
	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)	(0.005)
Tobin's Q $(t-1)$	0.044***	0.044***	0.044***	0.043***	0.043***	0.043***
	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)	(0.003)
investment $(t-1)$	0.217***	0.214***	0.212***	0.211***	0.211***	0.210***
	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)	(0.012)
firm FE	yes	yes	yes	yes	yes	yes
year FE	yes	yes	yes	yes	yes	yes
industry FE	yes	yes	yes	yes	yes	yes
industry-year FE	yes	yes	yes	yes	yes	yes
N	29,441	29,441	29,441	29,441	29,441	29,441

Table ii: Investment regression with weighted average of competitors' investments. Dependent variable is investment (t). ***, **, and * indicates statistical significance at 1%, 5%, and 10%, respectively.

Table ii replaces the unweighted average in the investment regression from Section 2 by a weighted average of competitors' investments, where the weight is the annual product similarity score. We continue to find a positive coefficient that now declines with the number of competitors.

III Static Nash equilibrium

A static Nash equilibrium $\left(p_1^N(\omega), p_2^N(\omega)\right)$ in state ω is a solution to the system of equations

$$\frac{\partial \pi_1(\omega, p_1^N(\omega), p_2^N(\omega))}{\partial p_1} = 0 \implies \left(\exp\left(\frac{g(\omega) - \alpha(p_1^N(\omega) - p_2^N(\omega))}{\nu}\right) + 1\right)\frac{\nu}{\alpha} - p_1^N(\omega) + c = 0, \quad \text{(i)}$$

$$\frac{\partial \pi_2(\omega, p_1^N(\omega), p_2^N(\omega))}{\partial p_2} = 0 \implies \left(\exp\left(\frac{g(-\omega) + \alpha(p_1^N(\omega) - p_2^N(\omega))}{\nu}\right) + 1\right)\frac{\nu}{\alpha} - p_2^N(\omega) + c = 0. \quad \text{(ii)}$$

The Jacobian of this system is

$$\begin{pmatrix} -\left(\exp\left(\frac{g(\omega)-\alpha(p_1^N(\omega)-p_2^N(\omega))}{\nu}\right)+1\right) & \exp\left(\frac{g(\omega)-\alpha(p_1^N(\omega)-p_2^N(\omega))}{\nu}\right) \\ \exp\left(\frac{g(-\omega)+\alpha(p_1^N(\omega)-p_2^N(\omega))}{\nu}\right) & -\left(\exp\left(\frac{g(-\omega)+\alpha(p_1^N(\omega)-p_2^N(\omega))}{\nu}\right)+1\right) \end{pmatrix}.$$

Standard arguments ensure the existence (Fudenberg & Tirole 1991, Theorem 1.2) and uniqueness (Gale & Nikaido 1965, Theorem 6) of a static Nash equilibrium.

The following proposition implies that we can normalize $\alpha = 1$ and $c = 0$ without loss of generality:

Proposition 1 *Let $(p_1^\circ(\omega), p_2^\circ(\omega))$ be a static Nash equilibrium in state ω for market size $M = 1$, price sensitivity $\alpha = 1$, and marginal cost $c = 0$ and $\pi_i^\circ(\omega, p_1^\circ(\omega), p_2^\circ(\omega))$ the associated profit of firm i. Then $\left(p_1^N(\omega) = \frac{1}{\alpha}p_1^\circ(\omega) + c, p_2^N(\omega) = \frac{1}{\alpha}p_2^\circ(\omega) + c\right)$ is a static Nash equilibrium in state ω for market size $M > 0$, price sensitivity $\alpha > 0$, and marginal cost $c \geq 0$ and the associated profit of firm i is $\pi_i(\omega, p_1^N(\omega), p_2^N(\omega)) = \frac{M}{\alpha}\pi_i^\circ(\omega, p_1^\circ(\omega), p_2^\circ(\omega))$.*

Proof. Plug $\left(p_1^N(\omega), p_2^N(\omega)\right)$ into equations (i) and (ii) that define a static Nash equilibrium in state ω and simplify. Then plug into equations (2) and (**??**). ■

As a corollary, note that since $g(0) = 0$, we have $p_i^\circ(0) = 2\nu$ and $\pi_i^\circ(0, p_1^\circ(0), p_2^\circ(0)) = \nu$ in a static Nash equilibrium in state $\omega = 0$.

IV Properties of the normal distribution

Let $\phi(z)$ and $\Phi(z)$ be the standard normal probability density and cumulative distribution functions, respectively. Then we have that

$$\Psi(\theta_i) = \Phi\left(\frac{\theta_i}{\sigma}\right), \quad \psi(\theta_i) = \frac{1}{\sigma}\phi\left(\frac{\theta_i}{\sigma}\right), \quad \psi'(\theta_i) = -\frac{\theta_i}{\sigma^3}\phi\left(\frac{\theta_i}{\sigma}\right)$$

and

$$\Psi^{-1}(p) = \sigma\Phi^{-1}(p).$$

In addition

$$\Upsilon(\bar{\theta}) = \int_{-\infty}^{\bar{\theta}} \theta_i d\Psi(\theta_i) = -\sigma\phi\left(\frac{\bar{\theta}}{\sigma}\right).$$

IV.1 Comparative statics

Defining

$$A_1 = \beta\left[V_1(\omega^+) - V_1(\omega)\right], \quad B_1 = \beta\left[V_1(\omega) - V_1(\omega^-)\right], \quad Z_1 = -F(\pi_1) + A_1 + (B_1 - A_1)x_2(\omega),$$
$$A_2 = \beta\left[V_2(\omega^-) - V_2(\omega)\right], \quad B_2 = \beta\left[V_2(\omega) - V_2(\omega^+)\right], \quad Z_2 = -F(\pi_2) + A_2 + (B_2 - A_2)x_1(\omega),$$

the system of equations given by equation (6) and its analog for firm 2 is

$$x_1(\omega) = \Psi(Z_1), \tag{iii}$$
$$x_2(\omega) = \Psi(Z_2). \tag{iv}$$

where we use the shorthand $\pi_i = \pi_i(\omega, p_1(\omega), p_2(\omega))$.

First order. To obtain the first-order comparative statics $\frac{\partial x_i(\omega)}{\partial \pi_j}$, we first differentiate equations (iii) and (iv) with respect to π_1 to obtain

$$\frac{\partial x_1(\omega)}{\partial \pi_1} = \psi(Z_1)\left(-F'(\pi_1) + (B_1 - A_1)\frac{\partial x_2(\omega)}{\partial \pi_1}\right), \tag{v}$$

$$\frac{\partial x_2(\omega)}{\partial \pi_1} = \psi(Z_2)\,(B_2 - A_2)\,\frac{\partial x_1(\omega)}{\partial \pi_1}. \tag{vi}$$

Solving yields

$$\frac{\partial x_1(\omega)}{\partial \pi_1} = \frac{1}{Y}\left\{-\psi(Z_1)F'(\pi_1)\right\}, \tag{vii}$$

$$\frac{\partial x_2(\omega)}{\partial \pi_1} = \frac{1}{Y}\left\{-\psi(Z_1)\psi(Z_2)F'(\pi_1)\,(B_2 - A_2)\right\}, \tag{viii}$$

where

$$Y = 1 - \psi(Z_1)\psi(Z_2)\,(B_1 - A_1)\,(B_2 - A_2)\,.$$

Next, we differentiate equations (iii) and (iv) with respect to π_2 to obtain

$$\frac{\partial x_1(\omega)}{\partial \pi_2} = \psi(Z_1)\,(B_1 - A_1)\,\frac{\partial x_2(\omega)}{\partial \pi_2}, \tag{ix}$$

$$\frac{\partial x_2(\omega)}{\partial \pi_2} = \psi(Z_2)\left(-F'(\pi_2) + (B_2 - A_2)\frac{\partial x_1(\omega)}{\partial \pi_2}\right). \tag{x}$$

Solving yields

$$\frac{\partial x_1(\omega)}{\partial \pi_2} = \frac{1}{Y}\left\{-\psi(Z_1)\psi(Z_2)F'(\pi_2)\,(B_1 - A_1)\right\}, \tag{xi}$$

$$\frac{\partial x_2(\omega)}{\partial \pi_2} = \frac{1}{Y}\left\{-\psi(Z_2)F'(\pi_2)\right\}. \tag{xii}$$

It is in general not possible to sign the first-order comparative statics $\frac{\partial x_i(\omega)}{\partial \pi_j}$. Note that absent financial frictions ($\zeta = 0$), $F'(\pi_i) = 0$ and thus $\frac{\partial x_i(\omega)}{\partial \pi_j} = 0$.

Second order. To obtain the second-order comparative statics $\frac{\partial^2 x_i(\omega)}{\partial \pi_j \partial \pi_k}$, we first differentiate equations (v) and (vi) with respect to π_1 and solve to obtain

$$\frac{\partial^2 x_1(\omega)}{\partial \pi_1^2} = \frac{1}{Y}\left\{-W_1 - W_2\psi(Z_1)\left(B_1 - A_1\right)\right\}, \tag{xiii}$$
$$\frac{\partial^2 x_2(\omega)}{\partial \pi_1^2} = \frac{1}{Y}\left\{-W_2 - W_1\psi(Z_2)\left(B_2 - A_2\right)\right\}, \tag{xiv}$$

where

$$W_1 = -\psi'(Z_1)\left(-F'(\pi_1) + (B_1 - A_1)\frac{\partial x_2(\omega)}{\partial \pi_1}\right)^2 + \psi(Z_1)F''(\pi_1),$$
$$W_2 = -\psi'(Z_2)\left((B_2 - A_2)\frac{\partial x_1(\omega)}{\partial \pi_1}\right)^2.$$

Next, we differentiate equations (v) and (vi) with respect to π_2 and solve to obtain

$$\frac{\partial^2 x_1(\omega)}{\partial \pi_1 \partial \pi_2} = \frac{1}{Y}\left\{-W_3 - W_4\psi(Z_1)\left(B_1 - A_1\right)\right\}, \tag{xv}$$
$$\frac{\partial^2 x_2(\omega)}{\partial \pi_1 \partial \pi_2} = \frac{1}{Y}\left\{-W_4 - W_3\psi(Z_2)\left(B_2 - A_2\right)\right\}, \tag{xvi}$$

where

$$W_3 = -\psi'(Z_1)(B_1 - A_1)\frac{\partial x_2(\omega)}{\partial \pi_2}\left(-F'(\pi_1) + (B_1 - A_1)\frac{\partial x_2(\omega)}{\partial \pi_1}\right),$$
$$W_4 = -\psi'(Z_2)\left(-F'(\pi_2) + (B_2 - A_2)\frac{\partial x_1(\omega)}{\partial \pi_2}\right)(B_2 - A_2)\frac{\partial x_1(\omega)}{\partial \pi_1}.$$

It is easy to show that $\frac{\partial^2 x_i(\omega)}{\partial \pi_1 \partial \pi_2} = \frac{\partial^2 x_i(\omega)}{\partial \pi_2 \partial \pi_1}$.

Finally, we differentiate equations (ix) and (x) with respect to π_2 and solve to obtain

$$\frac{\partial^2 x_1(\omega)}{\partial \pi_2^2} = \frac{1}{Y}\left\{-W_5 - W_6\psi(Z_1)\left(B_1 - A_1\right)\right\}, \tag{xvii}$$
$$\frac{\partial^2 x_2(\omega)}{\partial \pi_2^2} = \frac{1}{Y}\left\{-W_6 - W_5\psi(Z_2)\left(B_2 - A_2\right)\right\}, \tag{xviii}$$

where

$$W_5 = -\psi'(Z_1)\left((B_1 - A_1)\frac{\partial x_2(\omega)}{\partial \pi_2}\right)^2,$$
$$W_6 = -\psi'(Z_2)\left(-F'(\pi_2) + (B_2 - A_2)\frac{\partial x_1(\omega)}{\partial \pi_2}\right)^2 + \psi(Z_2)F''(\pi_2).$$

V System of equations for symmetric equilibrium

A symmetric equilibrium is a solution to the system of equations

$$\mathcal{H}(\mathcal{X}) = 0,$$

where

$$\mathcal{X} = (V_1(-L), \ldots, V_1(L), U_1(-L), \ldots, U_1(L), p_1(-L), \ldots, p_1(L), x_1(-L), \ldots, x_1(L))$$

is a vector of $8L + 4$ unknowns and $\mathcal{H}$ is defined by the following $8L + 4$ equations:

$$-V_1(\omega) + \pi_1(\omega, p_1(\omega), p_2(\omega)) + U_1(\omega) = 0, \quad \omega \in \{-L, \ldots, L\}, \tag{xix}$$

$$\begin{aligned} -U_1(\omega) - F\left(\pi_1(\omega, p_1(\omega), p_2(\omega))\right) x_1(\omega) - \int_{-\infty}^{Z_1} \theta_1 d\Psi(\theta_1) + \beta \big[V_1(\omega^+) x_1(\omega)(1 - x_2(\omega)) \\ + V_1(\omega)(1 - x_1(\omega) - x_2(\omega) + 2x_1(\omega)x_2(\omega)) + V_1(\omega^-)(1 - x_1(\omega))x_2(\omega)\big] \\ = 0, \quad \omega \in \{-L, \ldots, L\}, \end{aligned} \tag{xx}$$

$$\begin{aligned} \frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} \left(1 - F'\left(\pi_1(\omega, p_1(\omega), p_2(\omega))\right) x_1(\omega) + \frac{\partial U_1(\omega)}{\partial x_2} \frac{\partial x_2(\omega)}{\partial \pi_1}\right) \\ + \frac{\partial \pi_2(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} \frac{\partial U_1(\omega)}{\partial x_2} \frac{\partial x_2(\omega)}{\partial \pi_2} = 0, \quad \omega \in \{-L, \ldots, L\}, \end{aligned} \tag{xxi}$$

$$-x_1(\omega) + \Psi(Z_1) = 0, \quad \omega \in \{-L, \ldots, L\}, \tag{xxii}$$

where

$$\frac{\partial U_1(\omega)}{\partial x_2} = -B_1 + (B_1 - A_1)x_1(\omega), \tag{xxiii}$$

$\frac{\partial x_2(\omega)}{\partial \pi_j}$ is given in equations (viii) and (xii), and we continue to use the shorthands A_i, B_i, Z_i, and Y defined in IV.1. Throughout it is understood that we use the shorthands

$$V_2(\omega) = V_1(-\omega), \quad U_2(\omega) = U_1(-\omega), \quad p_2(\omega) = p_1(-\omega), \quad x_2(\omega) = x_1(-\omega).$$

Note that we substituted equation (6) into $\int_{-\infty}^{\Psi^{-1}(x_1(\omega))} \theta_1 d\Psi(\theta_1)$ in equation (7) to obtain equation (xx). This substitution avoids numerical issues that arise because our assumption $\theta_i \sim N(0, \sigma^2)$ implies $\lim_{p \to 0+} \Psi^{-1}(p) = -\infty$ and $\lim_{p \to 1-} \Psi^{-1}(p) = \infty$ and because $\Psi^{-1}(-\epsilon)$ and $\Psi^{-1}(1 + \epsilon)$ are undefined for all $\epsilon > 0$.

To simplify the notation, and without loss of generality, we redefine the parameters α and $g(\omega)$ to be $\frac{\alpha}{\nu}$ and $\frac{g(\omega)}{\nu}$. This avoids having to carry along ν. Using this notation, we have

$$\frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} = \frac{M\left(1 + (1 - (p_1(\omega) - c)\alpha\right) \exp(-g(\omega) + \alpha(p_1(\omega) - p_2(\omega))))}{\left(1 + \exp(-g(\omega) + \alpha(p_1(\omega) - p_2(\omega)))\right)^2},$$

$$\frac{\partial \pi_2(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} = \frac{M(p_2(\omega) - c)\alpha \exp(-g(-\omega) - \alpha(p_1(\omega) - p_2(\omega)))}{\left(1 + \exp(-g(-\omega) - \alpha(p_1(\omega) - p_2(\omega)))\right)^2}.$$

To facilitate solving the system of equations and avoid asymptotes, we multiply equation (xxi) by

$$\frac{\left(1 + \exp(-g(\omega) + \alpha(p_1(\omega) - p_2(\omega)))\right)^2}{M\alpha \exp(-g(\omega) + \alpha(p_1(\omega) - p_2(\omega)))}.$$

The Jacobian of the system of equations (xix)–(xxii) is available from the authors upon request.

VI Homotopy method

To understand the method, consider solving a single equation $H(x, \rho) = 0$ in a single variable x that depends on a parameter ρ. When there is more than one value of x that solves $H(x, \rho) = 0$ for a given value of ρ, the mapping $H^{-1}(\rho) = \{x|H(x, \rho) = 0\}$ from parameters into solutions is a correspondence. The homotopy method traces this correspondence by introducing an auxiliary variable s to construct the parametric path $(x(s), \rho(s)) \in H^{-1}(\rho)$. Differentiating $H(x(s), \rho(s)) = 0$ with respect to s yields

$$\frac{\partial H(x(s), \rho(s))}{\partial x} x'(s) + \frac{\partial H(x(s), \rho(s))}{\partial \rho} \rho'(s) = 0. \tag{xxiv}$$

Starting from any known point $(x(s), \rho(s))$ on the path, the basic differential equation (xxiv) prescribes how x and ρ must change to obtain another point on the path. Hence, the homotopy method reduces the task of solving the equation $H(x, \rho) = 0$ to the task of solving the basic differential equation (xxiv) given an initial condition in the form of a known point.

Exploring the solution correspondence of our model is more complex because it involves a system of $8L + 4$ equations $\mathcal{H}(\mathcal{X}, \boldsymbol{\rho}) = 0$ (Bellman equations and optimality conditions) in as many variables $\mathcal{X}$ (value and policy functions). Moreover, $\boldsymbol{\rho}$ is a vector in our model. We therefore compute slices of the solution correspondence by varying each of the key parameters at a time while holding the remaining parameters fixed. We denote a slice of the solution correspondence along, say, market size M by $\mathcal{H}^{-1}(M)$, with the understanding that this slice also depends on the remaining parameters. We analogously construct slices $\mathcal{H}^{-1}(\nu)$, $\mathcal{H}^{-1}(\tau)$, and $\mathcal{H}^{-1}(\sigma)$. A slice may consist of multiple disjoint paths, as illustrated by the slice $\mathcal{H}^{-1}(M)$ in the left panel of Figure ii.[19]

Note that the slice $\mathcal{H}^{-1}(M)$ in the left panel of Figure ii matches up with the slice

[19] The paths in the left panel of Figure ii appear not be disjoint because we display a one-dimensional summary statistic of $\mathcal{X}$ rather than $\mathcal{X}$ itself.

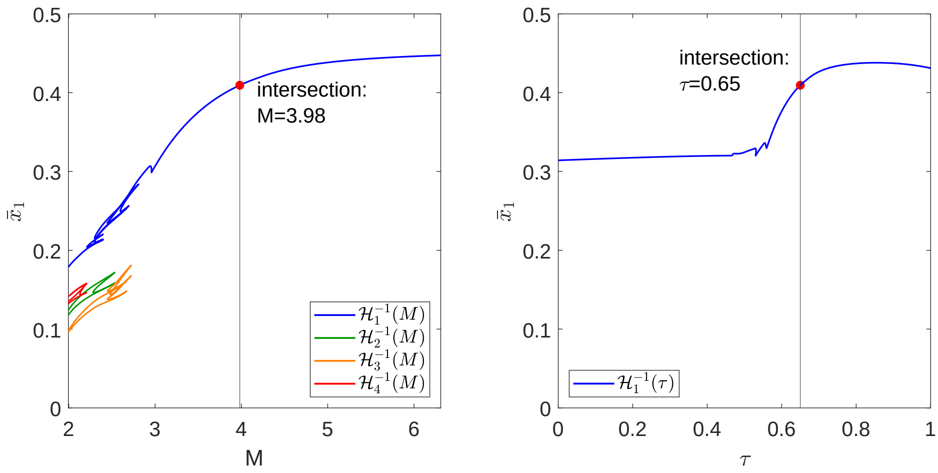

Figure ii: Example of solution correspondence, displayed as average investment probability $\bar{x}_1 = \frac{1}{2L+1}\sum_{\omega=-L}^{L} x_1(\omega)$. Slice $\mathcal{H}^{-1}(M)$ holding fixed $\nu = 0.43$, $\tau = 0.65$, and $\sigma = 0.05$ (left panel) and slice $\mathcal{H}^{-1}(\tau)$ holding fixed $M = 3.98$, $\nu = 0.43$, and $\sigma = 0.05$ (right panel). Model with $\zeta = 1$.

$\mathcal{H}^{-1}(\tau)$ in the right panel, as indicated by the dots. We exploit this by "criss-crossing" the parameter space in an orderly fashion and using solutions on slices $\mathcal{H}^{-1}(M)$ as initial conditions to generate slices $\mathcal{H}^{-1}(\tau)$. Each slice $\mathcal{H}^{-1}(\tau)$ must either intersect with all already computed slices $\mathcal{H}^{-1}(M)$, or lead us to an additional solution that, in turn, gives us an initial condition to compute an additional slice $\mathcal{H}^{-1}(M)$. We continue this process until all slices $\mathcal{H}^{-1}(M)$ and $\mathcal{H}^{-1}(\tau)$ match up. We proceed similarly with the other key parameters ν and σ. In this way, we explore the solution correspondence over a four-dimensional hypercube in (M, ν, τ, σ)-space to compute as many solutions as possible.

While intuitively appealing, this process may occasionally still fail to compute all possible solutions. We refer the reader to Besanko et al. (2010) and Borkovsky, Doraszelski & Kryukov (2010, 2012) for further details on the homotopy method.[20]

[20] We use the automatic differentiation package TAF (Giering, Kaminski & Slawig 2005) to obtain the Jacobian of $\mathcal{H}(\mathcal{X}, \rho)$. Our codes are available upon request.

VII Checking for equilibria

To check that a solution to the system of equations (xix)–(xxii) is an equilibrium, we check that there is no profitable unilateral deviation. Recall that, in the pricing stage, firm 1 anticipates that changing its price changes its investment as well as the investment of firm 2 in the investment stage. We proceed in two steps.

VII.1 Local deviations

First, we examine local deviations. Without imposing $\frac{\partial U_1(\omega)}{\partial x_1} = 0$ from the envelope theorem, equation (9) reads

$$\frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} \left(1 - F'\left(\pi_1(\omega, p_1(\omega), p_2(\omega))\right) x_1(\omega) + \frac{\partial U_1(\omega)}{\partial x_1}\frac{\partial x_1(\omega)}{\partial \pi_1} + \frac{\partial U_1(\omega)}{\partial x_2}\frac{\partial x_2(\omega)}{\partial \pi_1}\right) + \frac{\partial \pi_2(\omega, p_1(\omega), p_2(\omega))}{\partial p_1}\left(\frac{\partial U_1(\omega)}{\partial x_1}\frac{\partial x_1(\omega)}{\partial \pi_2} + \frac{\partial U_1(\omega)}{\partial x_2}\frac{\partial x_2(\omega)}{\partial \pi_2}\right) = 0, \tag{xxv}$$

where $\frac{\partial U_1(\omega)}{\partial x_2}$ is given in equation (xxiii), $\frac{\partial x_1(\omega)}{\partial \pi_j}$ in equations (vii) and (viii), and we continue to use the shorthands A_i, B_i, Z_i, and Y defined in IV.1 (with $\pi_i = \pi_i(\omega, p_1(\omega), p_2(\omega))$). Direct calculation using Leibniz's rule yields

$$\frac{\partial U_1(\omega)}{\partial x_1} = -F(\pi_1(\omega, p_1(\omega), p_2(\omega))) - \Psi^{-1}(x_1(\omega)) + A_1 + (B_1 - A_1)x_2(\omega).$$

Restricting attention to local deviations, we examine the derivative of the left-hand side equation (xxv) with respect to $p_1(\omega)$:

$$\frac{\partial^2 \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1^2}\left(1 - F'(\pi_1(\omega, p_1(\omega), p_2(\omega)))x_1(\omega) + \frac{\partial U_1(\omega)}{\partial x_1}\frac{\partial x_1(\omega)}{\partial \pi_1} + \frac{\partial U_1(\omega)}{\partial x_2}\frac{\partial x_2(\omega)}{\partial \pi_1}\right)$$
$$+ \frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1}\Bigg[-F''(\pi_1(\omega, p_1(\omega), p_2(\omega)))\frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1}x_1(\omega)$$
$$-F'\left(\pi_1(\omega, p_1(\omega), p_2(\omega))\right)\left(\frac{\partial x_1(\omega)}{\partial \pi_1}\frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1} + \frac{\partial x_1(\omega)}{\partial \pi_2}\frac{\partial \pi_2(\omega, p_1(\omega), p_2(\omega))}{\partial p_1}\right)$$
$$-F'\left(\pi_1(\omega, p_1(\omega), p_2(\omega))\right)\frac{\partial \pi_1(\omega, p_1(\omega), p_2(\omega))}{\partial p_1}\frac{\partial x_1(\omega)}{\partial \pi_1}$$

$$
\begin{aligned}
&+\frac{\partial^2 U_1(\omega)}{\partial x_1^2}\left(\frac{\partial x_1(\omega)}{\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial x_1(\omega)}{\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\frac{\partial x_1(\omega)}{\partial \pi_1}\\
&+\frac{\partial^2 U_1(\omega)}{\partial x_1 \partial x_2}\left(\frac{\partial x_2(\omega)}{\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial x_2(\omega)}{\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\frac{\partial x_1(\omega)}{\partial \pi_1}\\
&+\frac{\partial U_1(\omega)}{\partial x_1}\left(\frac{\partial^2 x_1(\omega)}{\partial \pi_1^2}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial^2 x_1(\omega)}{\partial \pi_1\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\\
&+\frac{\partial^2 U_1(\omega)}{\partial x_2 \partial x_1}\left(\frac{\partial x_1(\omega)}{\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial x_1(\omega)}{\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\frac{\partial x_2(\omega)}{\partial \pi_1}\\
&+\frac{\partial U_1(\omega)}{\partial x_2}\left(\frac{\partial^2 x_2(\omega)}{\partial \pi_1^2}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial^2 x_2(\omega)}{\partial \pi_1\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\Bigg]\\
&+\frac{\partial^2 \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1^2}\left(\frac{\partial U_1(\omega)}{\partial x_1}\frac{\partial x_1(\omega)}{\partial \pi_2}+\frac{\partial U_1(\omega)}{\partial x_2}\frac{\partial x_2(\omega)}{\partial \pi_2}\right)\\
&+\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\Bigg[-F'\left(\pi_1(\omega,p_1(\omega),p_2(\omega))\right)\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\frac{\partial x_1(\omega)}{\partial \pi_2}\\
&+\frac{\partial^2 U_1(\omega)}{\partial x_1^2}\left(\frac{\partial x_1(\omega)}{\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial x_1(\omega)}{\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\frac{\partial x_1(\omega)}{\partial \pi_2}\\
&+\frac{\partial^2 U_1(\omega)}{\partial x_1 \partial x_2}\left(\frac{\partial x_2(\omega)}{\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial x_2(\omega)}{\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\frac{\partial x_1(\omega)}{\partial \pi_2}\\
&+\frac{\partial U_1(\omega)}{\partial x_1}\left(\frac{\partial^2 x_1(\omega)}{\partial \pi_2^2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial^2 x_1(\omega)}{\partial \pi_2\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\\
&+\frac{\partial^2 U_1(\omega)}{\partial x_2 \partial x_1}\left(\frac{\partial x_1(\omega)}{\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial x_1(\omega)}{\partial \pi_2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\frac{\partial x_2(\omega)}{\partial \pi_2}\\
&+\frac{\partial U_1(\omega)}{\partial x_2}\left(\frac{\partial^2 x_2(\omega)}{\partial \pi_2^2}\frac{\partial \pi_2(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}+\frac{\partial^2 x_2(\omega)}{\partial \pi_2\partial \pi_1}\frac{\partial \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1}\right)\Bigg], \quad \text{(xxvi)}
\end{aligned}
$$

where $\frac{\partial^2 x_1(\omega)}{\partial \pi_j \partial \pi_k}$ is given in equations (xiii)–(xviii), and

$$
\begin{aligned}
\frac{\partial^2 U_1(\omega)}{\partial x_1^2} &= -\frac{1}{\psi\left(\Psi^{-1}(x_1(\omega))\right)},\\
\frac{\partial^2 U_1(\omega)}{\partial x_1 \partial x_2} &= \frac{\partial^2 U_1(\omega)}{\partial x_2 \partial x_1} = B_1 - A_1,\\
\frac{\partial^2 \pi_1(\omega,p_1(\omega),p_2(\omega))}{\partial p_1^2} &= -\frac{1}{\left(1+\exp(-g(\omega)+\alpha(p_1(\omega)-p_2(\omega)))\right)^3}\\
&\quad \cdot \{M\alpha \exp(-g(\omega)+\alpha(p_1(\omega)-p_2(\omega)))\\
&\quad \cdot (2(1+\exp(-g(\omega)+\alpha(p_1(\omega)-p_2(\omega)))) + (p_1(\omega)-c)\alpha(1-\exp(-g(\omega)+\alpha(p_1(\omega)-p_2(\omega)))))\},
\end{aligned}
$$

$$\frac{\partial^2 \pi_2(\omega, p_1(\omega), p_2(\omega))}{\partial p_1^2} = -\frac{1}{(1+\exp(-g(-\omega)-\alpha(p_1(\omega)-p_2(\omega))))^3}$$
$$\cdot \left\{ M\alpha^2 \exp(-g(-\omega)-\alpha(p_1(\omega)-p_2(\omega)))(p_2(\omega)-c)\left(1-\exp(-g(-\omega)-\alpha(p_1(\omega)-p_2(\omega)))\right)\right\}.$$

Firm 1 has a profitable unilateral local deviation in state ω if the derivative in equation (xxvi) evaluated at the candidate solution is positive. In this case, the candidate solution is not an equilibrium.

VII.2 Global deviations

Second, we examine global deviations by solving the saddle point problem

$$\max_{p_1} \min_{x_1,x_2} \pi_1(\omega, p_1, p_2(\omega)) - F(\pi_1(\omega, p_1, p_2(\omega)))x_1 - \int_{-\infty}^{\Psi^{-1}(x_1)} \theta_1 d\Psi(\theta_1)$$
$$+\beta \left[V_1(\omega^+)x_1(1-x_2) + V_1(\omega)(1-x_1-x_2+2x_1x_2) + V_1(\omega^-)(1-x_1)x_2\right] \quad \text{(xxvii)}$$

subject to equation (6) and its analog for firm 2 (with $x_1(\omega)$ and $x_2(\omega)$ replaced by x_1 and x_2). In the spirit of simple penal codes (Abreu 1988), we assume that after deviating in the pricing stage firm 1 faces the worst possible continuation in the investment stage. This allows the model to generate the widest set of possible equilibrium behaviors.

Accounting for numerical precision, we say that firm 1 has a profitable unilateral global deviation in state ω if the value of the saddle point problem in equation (xxvii) is larger than the value of the objective function evaluated at the candidate solution and if p_1, x_1, and x_2 in the saddle point problem are sufficiently different from the candidate solution. In this case, the candidate solution is not an equilibrium.

To solve the saddle point problem, we nest the inner minimization problem given p_1 into the outer maximization problem over p_1. Starting with the inner minimization problem given p_1, we substitute the analog of equation (6) for firm 2 into equation (6) and aim to obtain all solutions to the resulting univariate equation in x_1 by a combination of a grid search and a derivative-free bisection algorithm. We select the solution that is associated with the worst possible continuation for firm 1. Turning to the outer maximization problem over p_1, we use a derivative-free golden section search algorithm.

Note that the existence of a profitable unilateral local deviation does not imply the existence of a profitable unilateral global deviation. This is because in equation (xxvi) we perturb the continuation in the investment stage around the candidate solution via the second-order comparative statics $\frac{\partial^2 x_i(\omega)}{\partial \pi_j \partial \pi_j}$ whereas in the saddle point problem in equation (xxvii) we condition on the worst possible continuation in the investment stage.

VIII Limiting distribution

Let P denote the $(2L+1) \times (2L+1)$ state-to-state transition probability matrix constructed in equation (**??**) with typical element $P_{\omega,\omega'}$. The assumption $\theta_i \sim N(0, \sigma^2)$ ensures $x_i(\omega) \in (0,1)$ and thus $P_{\omega,\omega-1} > 0$, $P_{\omega,\omega} < 1$, and $P_{\omega,\omega+1} > 0$. It follows that the entire state space is one closed communicating class. The $1 \times (2L+1)$ limiting distribution μ^∞ is a solution to the system of linear equations

$$\mu^\infty P = \mu^\infty \Longleftrightarrow \mu^\infty (P - I) = 0,$$

where I is the $(2L+1) \times (2L+1)$ identity matrix and 0 is a $1 \times (2L+1)$ vector of zeros. Because the system of linear equations is homogenous, if μ^∞ is a solution, then so is $\alpha\mu^\infty$ for any $\alpha \in \mathbb{R}$. We are therefore free to fix the scale of μ^∞ (or normalize any solution after obtaining it).

We develop a recursive formula for computing μ^∞. To reduce the number of unknowns and equations, we exploit that P is symmetric in the sense that $P_{-\omega,-\omega'} = P_{\omega,\omega'}$ for all $\omega, \omega' \in \{0, 1, \ldots, L\}$. We thus have

$$\begin{pmatrix} \mu^\infty(0) & \mu^\infty(1) & \ldots & \mu^\infty(\omega) & \ldots & \mu^\infty(L-1) & \mu^\infty(L) \end{pmatrix}$$
$$\cdot \begin{pmatrix} P_{0,0}-1 & P_{0,1} & 0 & 0 & 0 & \cdots & 0 \\ 2P_{1,0} & P_{1,1}-1 & P_{1,2} & 0 & 0 & \cdots & 0 \\ 0 & P_{2,1} & P_{2,2}-1 & P_{2,3} & 0 & \cdots & 0 \\ 0 & \cdots & \ddots & \ddots & \ddots & \cdots & 0 \\ 0 & \cdots & P_{\omega,\omega-1} & P_{\omega,\omega}-1 & P_{\omega,\omega+1} & \cdots & 0 \\ 0 & \cdots & \ddots & \ddots & \ddots & \cdots & 0 \\ 0 & \cdots & 0 & P_{L-2,L-3} & P_{L-2,L-2}-1 & P_{L-2,L-1} & 0 \\ 0 & \cdots & 0 & 0 & P_{L-1,L-2} & P_{L-1,L-1}-1 & P_{L-1,L} \\ 0 & \cdots & 0 & 0 & 0 & P_{L,L-1} & P_{L,L}-1 \end{pmatrix} = 0,$$

where the multiplication of $P_{1,0}$ by 2 in the second row and first column is the necessary adjustment for the dropped equations. Using that each row of P sums to 1, this can be rewritten as

$$\begin{pmatrix} \mu^\infty(0) & \mu^\infty(1) & \dots & \mu^\infty(\omega) & \dots & \mu^\infty(L-1) & \mu^\infty(L) \end{pmatrix}$$
$$\cdot \begin{pmatrix} -2P_{0,1} & P_{0,1} & 0 & 0 & 0 & \cdots & 0 \\ 2P_{1,0} & -(P_{1,0}+P_{1,2}) & P_{1,2} & 0 & 0 & \cdots & 0 \\ 0 & P_{2,1} & -(P_{2,1}+P_{2,3}) & P_{2,3} & 0 & \cdots & 0 \\ 0 & \cdots & \ddots & \ddots & \ddots & \cdots & 0 \\ 0 & \cdots & P_{\omega,\omega-1} & -(P_{\omega,\omega-1}+P_{\omega,\omega+1}) & P_{\omega,\omega+1} & \cdots & 0 \\ 0 & \cdots & \ddots & \ddots & \ddots & \cdots & 0 \\ 0 & \cdots & 0 & P_{L-2,L-3} & -(P_{L-2,L-3}+P_{L-2,L-1}) & P_{L-2,L-1} & 0 \\ 0 & \cdots & 0 & 0 & P_{L-1,L-2} & -(P_{L-1,L-2}+P_{L-1,L}) & P_{L-1,L} \\ 0 & \cdots & 0 & 0 & 0 & P_{L,L-1} & -P_{L,L-1} \end{pmatrix} = 0,$$

where the multiplication of $P_{0,1}$ by 2 in the first row and first column is the necessary adjustment for the dropped equations, or

$$-2P_{0,1}\mu^\infty(0) + 2P_{1,0}\mu^\infty(1) = 0,$$
$$P_{0,1}\mu^\infty(0) - (P_{1,0} + P_{1,2})\mu^\infty(1) + P_{2,1}\mu^\infty(2) = 0,$$
$$P_{1,2}\mu^\infty(1) - (P_{2,1} + P_{2,3})\mu^\infty(2) + P_{3,2}\mu^\infty(3) = 0,$$
$$\vdots$$
$$P_{\omega-1,\omega}\mu^\infty(\omega-1) - (P_{\omega,\omega-1} + P_{\omega,\omega+1})\mu^\infty(\omega) + P_{\omega+1,\omega}\mu^\infty(\omega+1) = 0,$$
$$\vdots$$
$$P_{L-3,L-2}\mu^\infty(L-3) - (P_{L-2,L-3} + P_{L-2,L-1})\mu^\infty(L-2) + P_{L-1,L-2}\mu^\infty(L-1) = 0,$$
$$P_{L-2,L-1}\mu^\infty(L-2) - (P_{L-1,L-2} + P_{L-1,L})\mu^\infty(L-1) + P_{L,L-1}\mu^\infty(L) = 0,$$
$$P_{L-1,L}\mu^\infty(L-1) - P_{L,L-1}\mu^\infty(L) = 0.$$

Fixing $\mu^\infty(0)$, the first equation yields

$$\mu^\infty(1) = \frac{P_{0,1}}{P_{1,0}}\mu^\infty(0).$$

Plugging this into the second equation yields

$$\mu^{\infty}(2) = \frac{P_{1,2}P_{0,1}}{P_{2,1}P_{1,0}}\mu^{\infty}(0) = \frac{P_{1,2}}{P_{2,1}}\mu^{\infty}(1).$$

Plugging this into the third equation yields

$$\mu^{\infty}(3) = \frac{P_{2,3}P_{1,2}P_{0,1}}{P_{3,2}P_{2,1}P_{1,0}}\mu^{\infty}(0) = \frac{P_{2,3}}{P_{3,2}}\mu^{\infty}(2).$$

Continuing in this way yields the recursion

$$\mu^{\infty}(\omega) = \frac{\prod_{i=1}^{\omega} P_{i-1,i}}{\prod_{i=1}^{\omega} P_{i,i-1}}\mu^{\infty}(0) = \frac{P_{\omega-1,\omega}}{P_{\omega,\omega-1}}\mu^{\infty}(\omega-1), \quad \omega \in \{1, 2, \ldots, L\}.$$

To account for numerical precision, we construct

$$P_{\omega-1,\omega} = x_1(\omega-1)(1-x_2(\omega-1)), \quad P_{\omega,\omega-1} = (1-x_1(\omega))x_2(\omega), \quad \omega \in \{1, 2, \ldots, L\},$$

by first evaluating the right-hand side of equation (6) and its analog for firm 2 using symbolic math with infinite precision arithmetic. We then execute the recursion using symbolic math to prevent over- and underflows.

IX Consumer Surplus

Let $(\omega_1, \omega_2) \in \mathbb{Z}^2$ be the underlying state. For $\tau = 1$ (no leader handicap), the formula for consumer surplus is

$$\begin{aligned}
CS(\omega_1, \omega_2, p_1, p_2) &= M\frac{\nu}{\alpha} \ln\left(\exp\left(\frac{\frac{\omega_1}{L} - \alpha p_1}{\nu}\right) + \exp\left(\frac{\frac{\omega_2}{L} - \alpha p_2}{\nu}\right)\right) \\
&= M\frac{\nu}{\alpha}\left(\frac{\frac{\omega_1}{L} - \alpha p_1}{\nu} + \ln\left(1 + \exp\left(\frac{-\frac{\omega_1 - \omega_2}{L} + \alpha(p_1 - p_2)}{\nu}\right)\right)\right) \\
&= M\frac{\nu}{\alpha}\left(\frac{\frac{\omega_1}{L} - \alpha p_1}{\nu} + \ln\left(1 + \exp\left(\frac{-\frac{\omega}{L} + \alpha(p_1 - p_2)}{\nu}\right)\right)\right) = CS(\omega_1, \omega, p_1, p_2),
\end{aligned} \tag{xxviii}$$

where $\omega = \omega_1 - \omega_2 \in \mathbb{Z}$ is the state that matters for firms' decisions, i.e., we have $x_i(\omega)$ and $p_i(\omega)$ for all $i \in \{1, 2\}$. We can therefore regard the underlying state space either as (ω_1, ω_2) or as (ω_1, ω).

Consistent with Roy's identity

$$\begin{aligned}
\frac{\partial CS(\omega_1, \omega, p_1, p_2)}{\partial p_1} &= M\frac{\nu}{\alpha}\left(-\frac{\alpha}{\nu} + \frac{1}{1 + \exp\left(\frac{-\frac{\omega}{L} + \alpha(p_1 - p_2)}{\nu}\right)} \frac{\alpha}{\nu} \exp\left(\frac{-\frac{\omega}{L} + \alpha(p_1 - p_2)}{\nu}\right)\right) \\
&= -M\frac{1}{1 + \exp\left(\frac{-\frac{\omega}{L} + \alpha(p_1 - p_2)}{\nu}\right)},
\end{aligned} \tag{xxix}$$

$$\begin{aligned}
\frac{\partial CS(\omega_1, \omega, p_1, p_2)}{\partial p_2} &= M\frac{\nu}{\alpha}\left(\frac{1}{1 + \exp\left(\frac{-\frac{\omega}{L} + \alpha(p_1 - p_2)}{\nu}\right)} \left(-\frac{\alpha}{\nu}\right) \exp\left(\frac{-\frac{\omega}{L} + \alpha(p_1 - p_2)}{\nu}\right)\right) \\
&= -M\frac{1}{1 + \exp\left(\frac{\frac{\omega}{L} - \alpha(p_1 - p_2)}{\nu}\right)},
\end{aligned} \tag{xxx}$$

Recall that law of motion for the state in the paper defines the state-to-state transition probabilities at an interior state $\omega \in \{-L + 1, \ldots, L - 1\}$ as

$$\Pr(\omega - 1 | \omega, x_1(\omega,), x_2(\omega,)) = (1 - x_1(\omega))x_2(\omega), \tag{xxxi}$$

$$\Pr(\omega + 1 | \omega, x_1(\omega), x_2(\omega)) = x_1(\omega)(1 - x_2(\omega)), \tag{xxxii}$$

$$\Pr(\omega|\omega, x_1(\omega), x_2(\omega)) = 1 - \Pr(\omega - 1|\omega, x_1(\omega), x_2(\omega)) - \Pr(\omega + 1|\omega, x_1(\omega), x_2(\omega)) \tag{xxxiii}$$

and at a boundary state $\omega \in \{-L, L\}$ as

$$\Pr(-L + 1| - L, x_1(-L), x_2(-L)) = x_1(-L)(1 - x_2(-L)), \tag{xxxiv}$$

$$\Pr(-L| - L, x_1(-L), x_2(-L)) = 1 - \Pr(-L + 1| - L, x_1(-L), x_2(-L)), \tag{xxxv}$$

$$\Pr(L - 1|L, x_1(L), x_2(L)) = (1 - x_1(L))x_2(L), \tag{xxxvi}$$

$$\Pr(L|L, x_1(L), x_2(L)) = 1 - \Pr(L - 1|L, x_1(L), x_2(L)). \tag{xxxvii}$$

It follows that the law of motion on the underlying state space (ω_1, ω_2) if $\omega = \omega_1 - \omega_2 \in \{-L + 1, \dots, L - 1\}$, obeys

$$\Pr(\omega_1 + 1, \omega_2 + 1|\omega_1, \omega_2) = x_1(\omega)x_2(\omega), \tag{xxxviii}$$

$$\Pr(\omega_1 + 1, \omega_2|\omega_1, \omega_2) = x_1(\omega)(1 - x_2(\omega)), \tag{xxxix}$$

$$\Pr(\omega_1, \omega_2 + 1|\omega_1, \omega_2) = (1 - x_1(\omega))x_2(\omega), \tag{xl}$$

$$\Pr(\omega_1, \omega_2|\omega_1, \omega_2) = (1 - x_1(\omega))(1 - x_2(\omega)); \tag{xli}$$

if $\omega = \omega_1 - \omega_2 = L$, then

$$\Pr(\omega_1 + 1, \omega_2 + 1|\omega_1, \omega_2) = x_1(\omega), \tag{xlii}$$

$$\Pr(\omega_1 + 1, \omega_2|\omega_1, \omega_2) = 0, \tag{xliii}$$

$$\Pr(\omega_1, \omega_2 + 1|\omega_1, \omega_2) = (1 - x_1(\omega))x_2(\omega), \tag{xliv}$$

$$\Pr(\omega_1, \omega_2|\omega_1, \omega_2) = (1 - x_1(\omega))(1 - x_2(\omega)); \tag{xlv}$$

and if $\omega = \omega_1 - \omega_2 = -L$, then

$$\Pr(\omega_1 + 1, \omega_2 + 1|\omega_1, \omega_2) = x_2(\omega), \tag{xlvi}$$

$$\Pr(\omega_1+1,\omega_2|\omega_1,\omega_2) = x_1(\omega)(1-x_2(\omega)), \tag{xlvii}$$

$$\Pr(\omega_1,\omega_2+1|\omega_1,\omega_2) = 0, \tag{xlviii}$$

$$\Pr(\omega_1,\omega_2|\omega_1,\omega_2) = (1-x_1(\omega))(1-x_2(\omega)). \tag{xlix}$$

Note that this specification ensures that $\omega' = \omega_1' - \omega_2' \in \{-L,\ldots,L\}$.

The above law of motion, of course, lends itself to simulation. Starting from $(\omega_1,\omega_2) = (0,0)$, say, we simulate for T periods, each period incrementing (ω_1,ω_2) as required. However, we can also compute the expected NPV of consumer surplus $CS^\infty(0,0)$ starting from $(\omega_1,\omega_2) = (0,0)$ recursively. To do so, we impose that an upper boundary K on ω_i instead of a finite time horizon T. Note that it takes at least K periods to get from $\omega_i = 0$ to $\omega_i = K$. Hence, by the time the boundary is reached, discounting weighs heavily. Using the shorthand $CS(\omega_1,\omega_2)$ for the per-period consumer surplus in equation (xxviii), recursively compute

$$CS^\infty(K,K) = \frac{1}{1-\beta}CS(K,K), \tag{l}$$

$$CS^\infty(K,K-1) = CS(K,K-1)$$
$$+\beta\Big[(1-(1-x_1(1))x_2(1))CS^\infty(K,K-1) + (1-x_1(1))x_2(1)CS^\infty(K,K)\Big], \tag{li}$$

$$CS^\infty(K,K-2) = CS(K,K-2)$$
$$+\beta\Big[(1-(1-x_1(2))x_2(2))CS^\infty(K,K-2) + (1-x_1(2))x_2(2)CS^\infty(K,K-1)\Big], \tag{lii}$$

$$\vdots$$

$$CS^\infty(K,K-L) = CS(K,K-L)$$
$$+\beta\Big[(1-(1-x_1(L))x_2(L))CS^\infty(K,K-L) + (1-x_1(L))x_2(L)CS^\infty(K,K-L+1)\Big], \tag{liii}$$

$$CS^\infty(K-1,K) = CS^\infty(K,K-1), \tag{liv}$$

$$CS^\infty(K-2,K) = CS^\infty(K,K-2), \tag{lv}$$

$$\vdots$$

$$CS^{\infty}(K-L,K) = CS^{\infty}(K,K-L), \tag{lvi}$$

$$\begin{aligned}CS^{\infty}(K-1,K-1) &= CS(K-1,K-1)\\ +\beta\Big[(1-x_1(0))(1-x_2(0))CS^{\infty}(K-1,K-1) &+ x_1(0)(1-x_2(0))CS^{\infty}(K,K-1)\\ +(1-x_1(0))x_2(0)CS^{\infty}(K-1,K) &+ x_1(0)x_2(0)CS^{\infty}(K,K)\Big],\end{aligned} \tag{lvii}$$

$$\begin{aligned}CS^{\infty}(K-1,K-2) &= CS(K-1,K-2)\\ +\beta\Big[(1-x_1(1))(1-x_2(1))CS^{\infty}(K-1,K-2) &+ x_1(1)(1-x_2(1))CS^{\infty}(K,K-2)\\ +(1-x_1(1))x_2(1)CS^{\infty}(K-1,K-1) &+ x_1(1)x_2(1)CS^{\infty}(K,K-1)\Big],\end{aligned} \tag{lviii}$$

$$\begin{aligned}CS^{\infty}(K-1,K-3) &= CS(K-1,K-3)\\ +\beta\Big[(1-x_1(2))(1-x_2(2))CS^{\infty}(K-1,K-3) &+ x_1(2)(1-x_2(2))CS^{\infty}(K,K-3)\\ +(1-x_1(2))x_2(2)CS^{\infty}(K-1,K-2) &+ x_1(2)x_2(2)CS^{\infty}(K,K-2)\Big],\end{aligned} \tag{lix}$$

$$\vdots$$

$$\begin{aligned}CS^{\infty}(K-1,K-L-1) &= CS(K-1,K-L-1)\\ +\beta\Big[(1-x_1(L))(1-x_2(L))&CS^{\infty}(K-1,K-L-1)\\ +(1-x_1(L))x_2(L)CS^{\infty}(K-1,K-L) &+ x_1(L)CS^{\infty}(K,K-L)\Big],\end{aligned} \tag{lx}$$

$$CS^{\infty}(K-2,K-1) = CS^{\infty}(K-1,K-2), \tag{lxi}$$

$$CS^{\infty}(K-3,K-1) = CS^{\infty}(K-1,K-3), \tag{lxii}$$

$$\vdots$$

$$CS^{\infty}(K-L-1,K-1) = CS^{\infty}(K-1,K-L-1), \tag{lxiii}$$

$$\vdots$$

$$\begin{aligned}CS^{\infty}(0,0) &= CS(0,0)\\ +\beta\Big[(1-x_1(0))(1-x_2(0))CS^{\infty}(0,0) &+ x_1(0)(1-x_2(0))CS^{\infty}(1,0)\\ +(1-x_1(0))x_2(0)CS^{\infty}(0,1) &+ x_1(0)x_2(0)CS^{\infty}(1,1)\Big].\end{aligned} \tag{lxiv}$$

Notice that each equation can be easily solved for the term $CS^{\infty}(\omega_1,\omega_2)$ on the left-hand side. The recursion amounts to computing $(K+1)(2L+1)$ numbers. Notice that the state-to-state transitions if either $\omega_1 = K$ or $\omega_2 = K$ are made up to respect the imposed upper boundary K on ω_i. The remaining state-to-state transitions follow from the law of motion on (ω_1, ω_2).

X Additional results.

The figures in this appendix supplement the main text as follows:

- Figures iii, iv, v, vi, vii, viii, and ix pertain to the paragraph titled "Average price and investment probability" in Section 4.3;
- Figures x, xi, and xii pertain to Section 5.1 titled "Impact on price";
- Figures xiii and xiv pertain to the paragraph titled "Profitability" in Section 5.1;
- Figures xv, xvi, and xvii pertain to Section 5.2 titled "Impact on investment";
- Figures xviii and xix pertain to Section 5.3 titled "Impact on industry concentration";
- Figure xx pertains to Section 6 titled "Consumer surplus and welfare ".

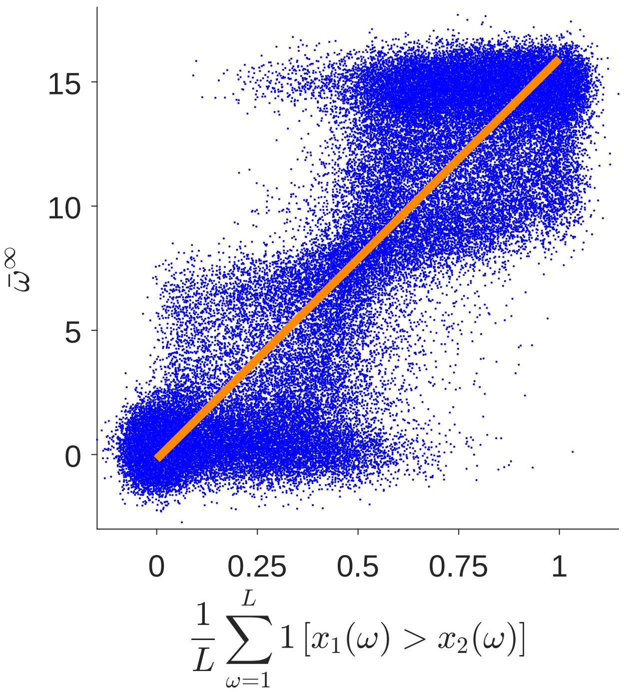

Figure iii: Scatter plot of long-run industry structure as measured by $\bar{\omega}^{\infty}$ against fraction of states where leader invests more than follower as measured by $\frac{1}{L}\sum_{\omega=1}^{L} 1\left[x_1(\omega) > x_2(\omega)\right]$, averaged over equilibria within parameterizations and overlayed by trend line. Noise added to improve visibility. Model with $\zeta = 1$.

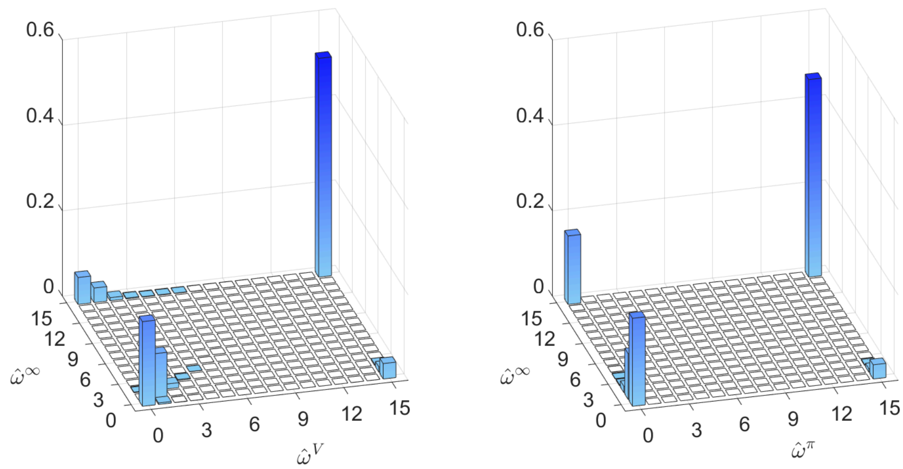

Figure iv: Joint distribution over most likely long-run industry structure $\widehat{\omega}^{\infty}$ and state $\widehat{\omega}^{V}$ where joint payoff is largest (left panel) respectively state $\widehat{\omega}^{\pi}$ where joint profit in static Nash equilibrium is largest (right panel). Model with $\zeta = 1$.

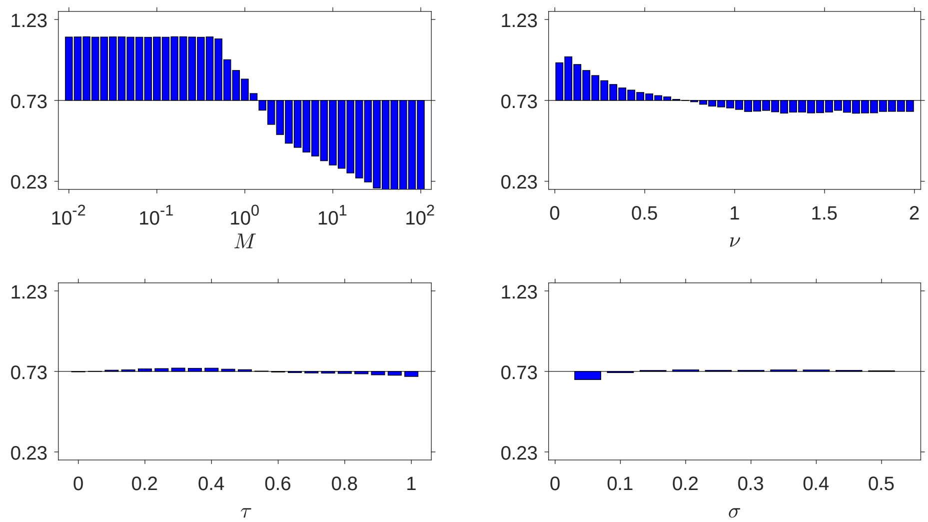

Figure v: Fraction of states where financial frictions matter as measured by $\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[\pi_1(\omega, p_1(\omega), p_2(\omega)) < F_0\right]$, averaged over equilibria within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel). Model with $\zeta = 1$.

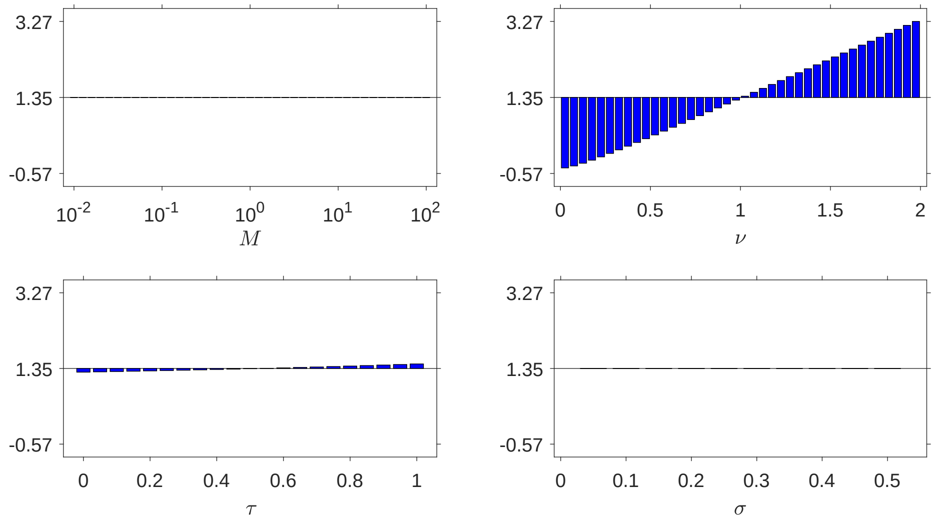

Figure vi: Average price $\bar{p}_1$, averaged over equilibria within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel). Model with $\zeta = 1$.

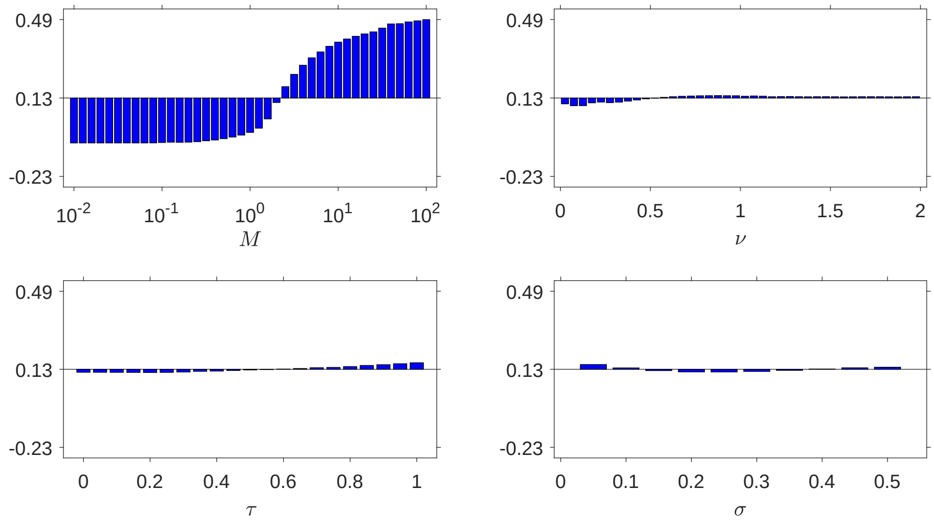

Figure vii: Average investment probability $\bar{x}_1$, averaged over equilibria within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel). Model with $\zeta = 1$.

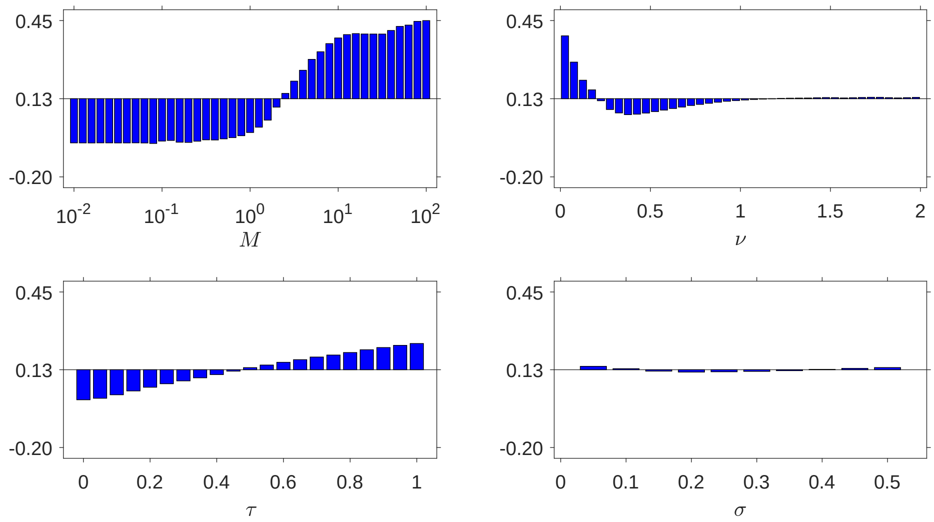

Figure viii: Leader's average investment probability $\bar{x}_1^l$, averaged over equilibria within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel). Model with $\zeta = 1$.

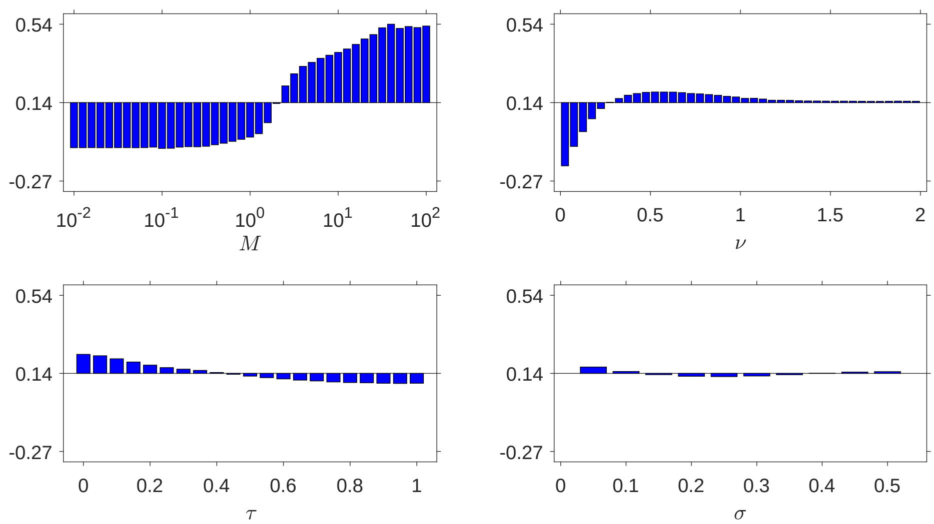

Figure ix: Follower's average investment probability $\bar{x}_1^f$, averaged over equilibria within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel). Model with $\zeta = 1$.

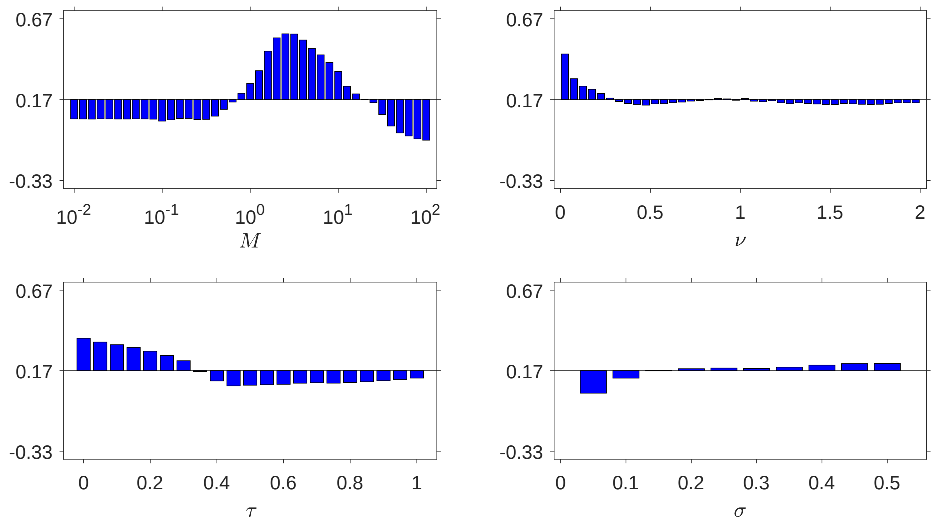

Figure x: Financial frictions increase price as measured by $1\left[p_1^{FC}(\omega) > p_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||p_1^{NOFC}||_1 \text{ for some } \omega\right]$, averaged over pairs of equilibria with and without financial frictions. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

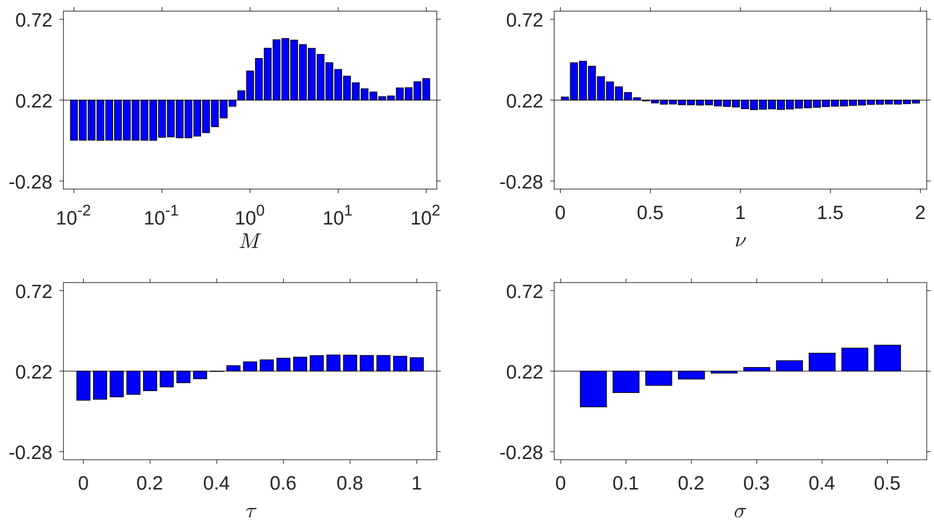

Figure xi: Financial frictions decrease leader's price as measured by $1\left[p_1^{FC}(\omega) < p_1^{NOFC}(\omega) - \frac{0.01}{2L+1}||p_1^{NOFC}||_1 \text{ for some } \omega > 0\right]$, averaged over pairs of equilibria with and without financial frictions. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

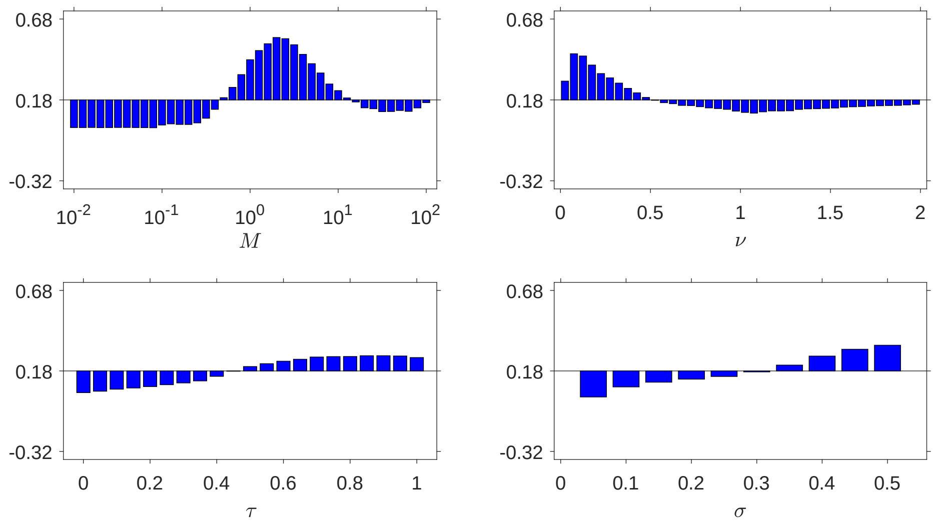

Figure xii: Financial frictions decrease follower's price as measured by $1\left[p_1^{FC}(\omega) < p_1^{NOFC}(\omega) - \frac{0.01}{2L+1}||p_1^{NOFC}||_1 \text{ for some } \omega < 0\right]$, averaged over pairs of equilibria with and without financial frictions. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

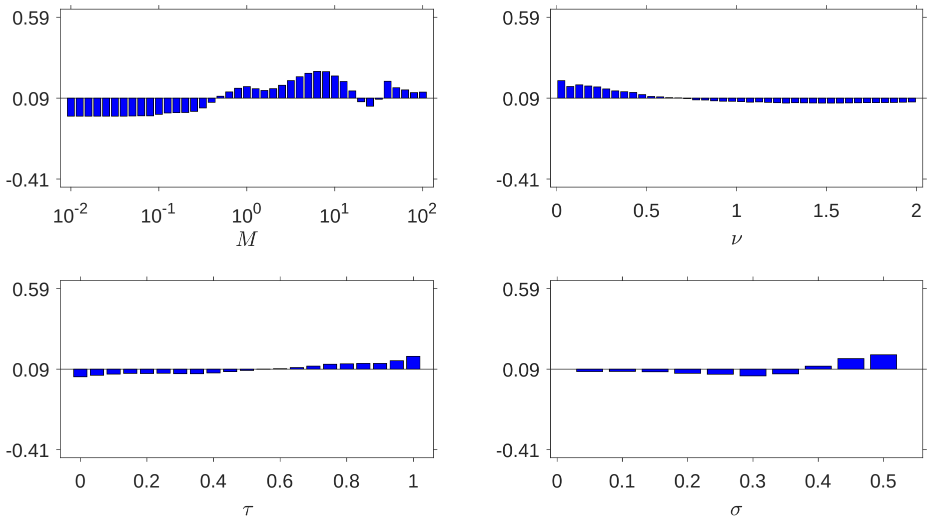

Figure xiii: Financial frictions decrease profitability of product market competition as measured by $1\left[\Pi^{\infty,FC} < \Pi^{\infty,NOFC} - \frac{0.01}{2L+1}\frac{||\pi_1^{NOFC}+\pi_2^{NOFC}||_1}{1-\beta}\right]$, averaged over pairs of equilibria with and without financial frictions. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

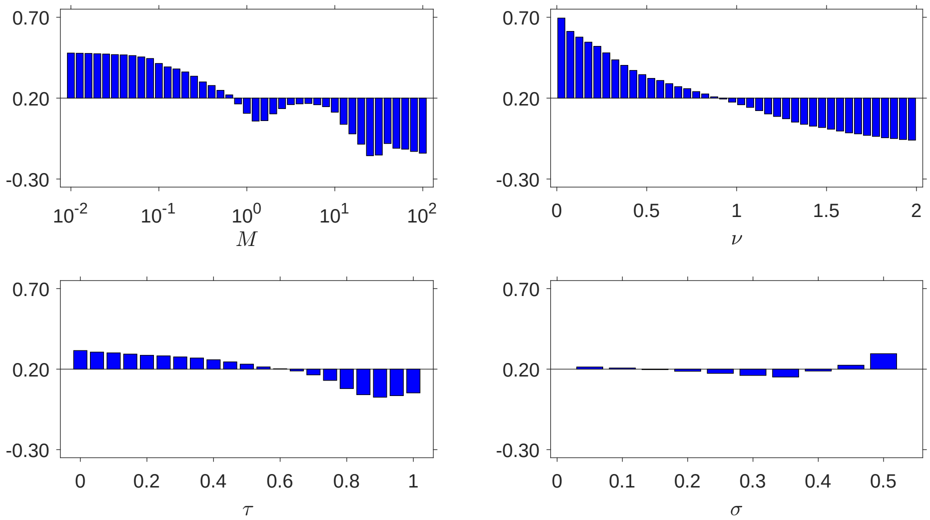

Figure xiv: Financial frictions increase profitability of product market competition as measured by $1\left[\Pi^{\infty,FC} > \Pi^{\infty,NOFC} + \frac{0.01}{2L+1}\frac{||\pi_1^{NOFC}+\pi_2^{NOFC}||_1}{1-\beta}\right]$, averaged over pairs of equilibria with and without financial frictions. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

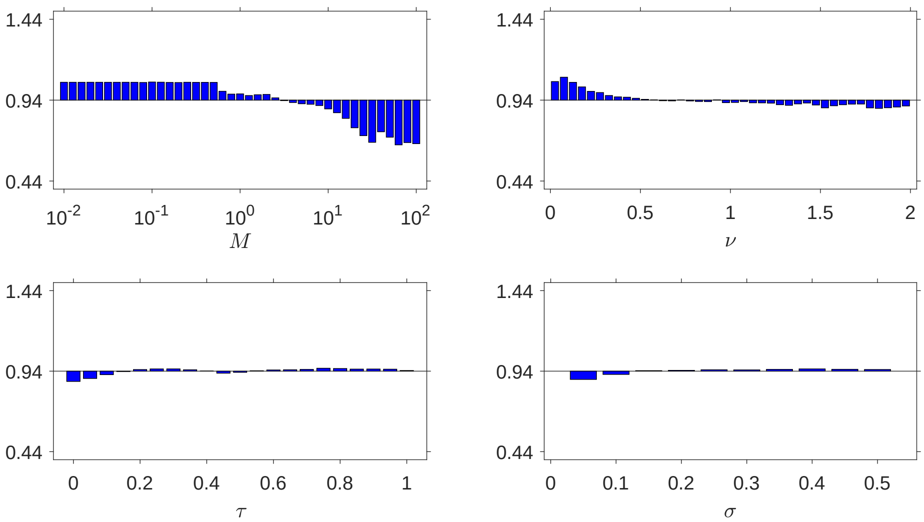

Figure xv: Financial frictions decrease investment as measured by $1\left[x_1^{FC}(\omega) < x_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||x_1^{NOFC}||_1 \text{ for some } \omega\right]$, averaged over pairs of equilibria with and without financial frictions within parameterizations. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

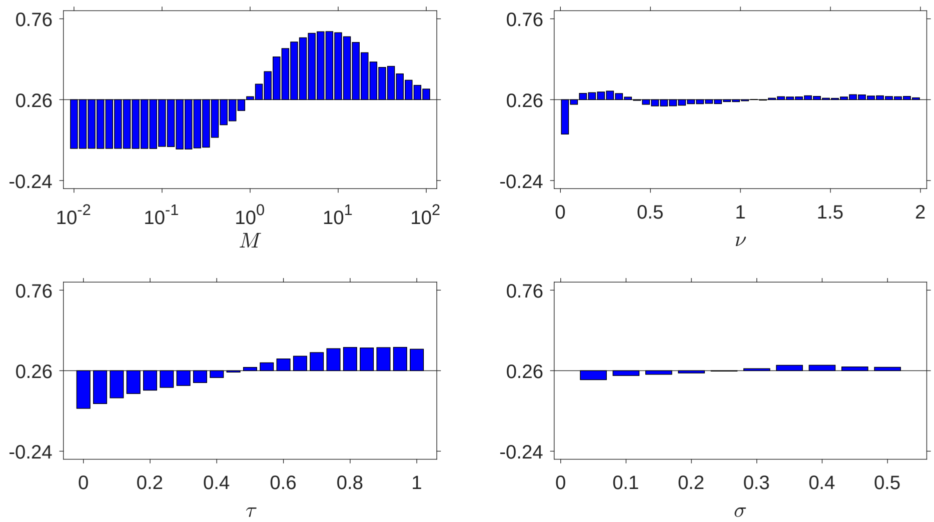

Figure xvi: Financial frictions increase leader's investment as measured by $1\left[x_1^{FC}(\omega) > x_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||x_1^{NOFC}||_1 \text{ for some } \omega > 0\right]$, averaged over pairs of equilibria with and without financial frictions. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

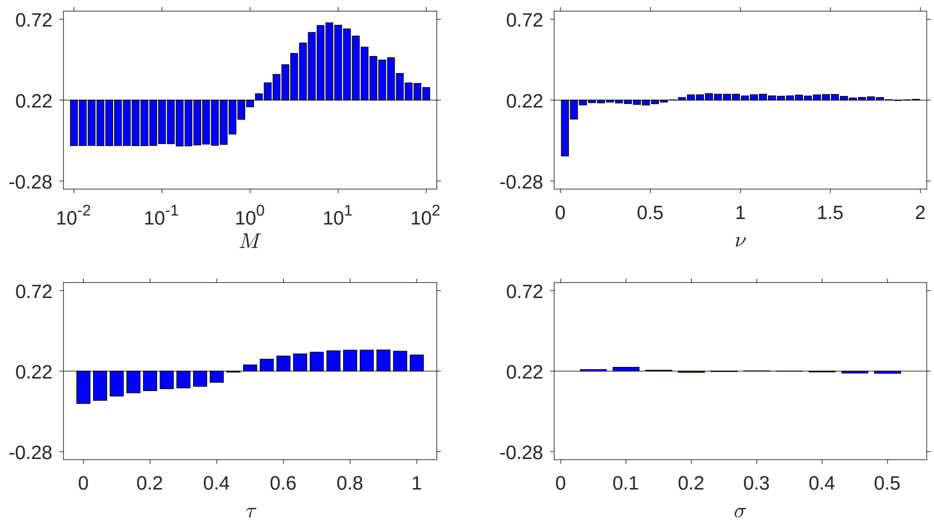

Figure xvii: Financial frictions increase follower's investment as measured by $1\left[x_1^{FC}(\omega) > x_1^{NOFC}(\omega) + \frac{0.01}{2L+1}||x_1^{NOFC}||_1 \text{ for some } \omega < 0\right]$, averaged over pairs of equilibria with and without financial frictions. Comparative statics with respect to market size M (upper left panel), degree of horizontal product differentiation ν (upper right panel), leader handicap τ (lower left panel), and cost volatility σ (lower right panel).

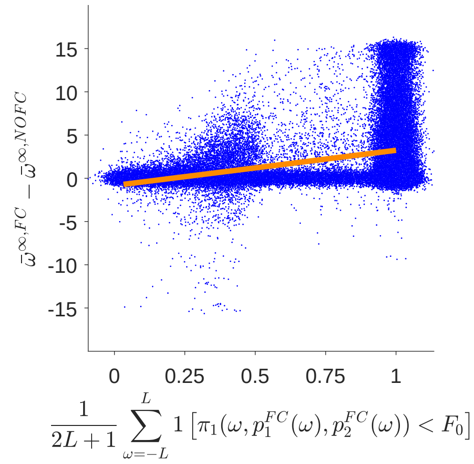

Figure xviii: Scatter plot of difference in long-run industry structure as measured by $\bar{\omega}^{\infty,FC} - \bar{\omega}^{\infty,NOFC}$ against fraction of states where financial frictions matter as measured by $\frac{1}{2L+1}\sum_{\omega=-L}^{L} 1\left[\pi_1\left(\omega, p_1^{FC}(\omega), p_2^{FC}(\omega)\right) < F_0\right]$, averaged over pairs of equilibria with and without financial frictions within parameterizations and overlayed by trend line. Noise added to improve visibility.

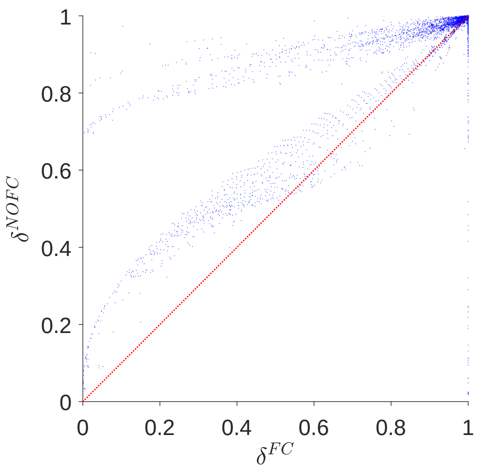

Figure xix: Scatter plot of speed of convergence absent financial frictions as measured by δ^{NOFC} against speed of convergence in presence of financial frictions as measured by δ^{FC}, averaged over pairs of equilibria with financial frictions and $\bar{\omega}^{\infty,FC} \geq 0.99L$ and equilibria without financial frictions and $\bar{\omega}^{\infty,NOFC} \geq 0.99L$ within parameterizations and overlayed by 45° line.

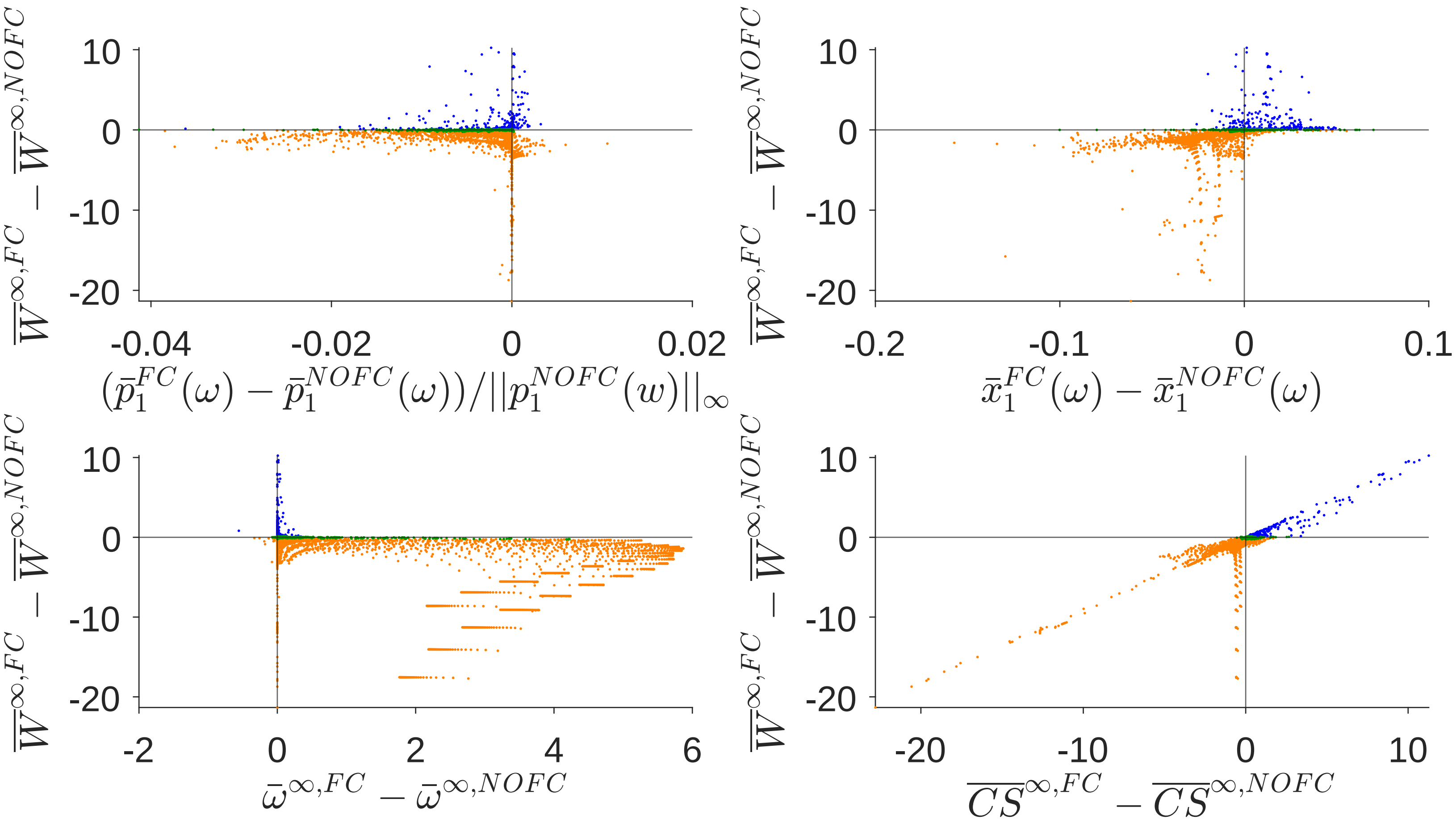

Figure xx: Scatter plot of the social welfare difference between our baseline model with financial frictions and the special case without frictions ($\kappa = 0$) against the difference in firms' price (row 1, column 1) and investment strategies (1,2), industry concentration (2,1), and consumer welfare (2,2), averaged over pairs of equilibria with financial frictions and equilibria without financial frictions within parameterizations. For comparison across industries, we normalize the consumer surplus and welfare measures by market size M. Average price normalized by $||p_1^{NOFC}||_\infty = max_\omega |p_1^{NOFC}(\omega)|$ to improve visibility. Blue (orange) dots indicate cases with higher (lower) social welfare, defined to be a difference between the baseline and the no-friction model larger (smaller) than 1% of (the negative) $|\overline{W}^{\infty,NOFC}|$. Green dots indicate cases within these bounds.