Pricing Control and Regulation on Online Service Platforms

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Abstract

Online service platforms enable customers to connect with a large population of independent servers and operate successfully in many sectors, including transportation, lodging, and delivery, among others. We study how prices are chosen and fees are collected on the platform. The platform could assert full control over pricing despite being unaware of the servers' costs (e.g., ride sharing). Or the platform could allow unfettered price competition among the servers (e.g., lodging). This choice influences both the amount of supply available and the overall attractiveness of the platform to consumers. When the platform collects revenue via a commission or a per-unit fee, neither price delegation strategy dominates the other. However, the platform's best payment structure is simple and easy to implement - it is merely the combination of a commission and a per-unit fee (which can be negative, as in a subsidy). Furthermore, this combination enables the delegation of price control to the servers, which may assist in the classification of the servers as contractors rather than employees. A similar approach can be used to maximize profits by fully disintermediated platforms (i.e., no central owner), such as those enabled by blockchain technology.

1 Introduction

Online service platforms establish marketplaces to match independent service providers with potential customers. They have been established in many domains, including ride sharing (e.g., Uber),

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food delivery (e.g., DoorDash), freelance labor (e.g., TaskRabbit), handmade and vintage products (e.g., Etsy), accommodations (e.g., AirBnB), mobile phone applications (e.g., App Store), etcetera.

Pricing is a critical function for a platform's success, both who selects prices and how revenue is generated. On some platforms, servers post their desired fee. We refer to this as *server pricing*. On others, the platform directly selects the price for the servers, which we refer to as *platform pricing*. The platform can collect revenue with a commission on the sales price, or with a per-unit fee, or some other, more elaborate, structure. There is little guidance in the literature to choose among these options.

Although many platforms have only operated with a single pricing structure, most participate in evolving markets and are experiencing a number of potentially significant shocks that could force changes. For example, there is considerable regulatory debate in the gig-economy over the classification of its workers. Are they independent contractors or employees? Most online platforms, such as Uber and Lyft, treat workers as independent contractors, but they are facing sustained regulatory pressure to reclassify their workers, due to the amount of centralized control they exert over them. In California, for instance, a law (AB5) was recently passed that emphasizes a contractor's freedom to do its business without a platform's control and direction. Freedom in setting prices has notably been considered a pre-requisite of this description (Bhuiyan 2020), and some argue that granting price freedom to drivers is a necessary measure for ride-sharing platforms to continue to classify their drivers as contractors (Paul 2016).[1] In the United Kingdom, a recent court ruling granted ride-sharing drivers employee status because of the platform's control over fares and the contractual terms it enforces on the drivers (O'Brien 2021).

Technology presents another potential shock to platform pricing. In particular, distributed ledger technologies, such as blockchain, have enabled new companies to establish disintermediated markets in which there is no centralized agent and therefore no central control over pricing. Examples include Arcade City and Drife in ride-sharing space, Dtravel in house-sharing, and Filecoin and Storj in the market for data storage. A common justification touted across these new platforms is that they take control/power away from central entities, and place it back in the hands of the service providers and platform users. It is not clear whether or how this improves the functioning of these markets.

To address the issue of pricing control and regulation on a platform, we consider a platform

[1] The subsequent passage of a superseding bill in California, in 2020, Proposition 22, temporarily swung the pendulum back towards allowing gig workers to be classified as independent contractors, but less than a year later, the bill was ruled unconstitutional and unenforceable by a California Superior Court Judge (Lyons 2021). Similarly, in Massachusetts, Bill HD2582, which has been called a Proposition 22 "clone", has been met with fierce opposition by drivers (Ongweso 2021).

with the following characteristics: (i) a large number of independent agents, who we refer to as *servers*, offer their services, (ii) servers have private knowledge of their heterogeneous costs, (iii) servers self-schedule their work on a short term basis (i.e., the platform cannot dictate when to work or how much to work), (iv) all participants on the platform choose actions to maximize their objective, correctly anticipating the actions of the other participants, and (v) the consumer choice process is influenced both by the overall attractiveness of the platform as well as the specific price of the server with whom the consumer is matched.

From the first characteristic, it follows that each server is a small portion of the platform, and they know that they individually have no ability to influence the aggregate outcomes on the platform. However, they are sophisticated enough to respond to what happens on the platform in a manner that is best for them (characteristic four).

Server costs' (second characteristic) include both out-of-pocket explicit costs as well as opportunity costs, which can vary considerably (Chen et al. (2019)). For example, a server may have explicit expenses related to the service offered, but also incurs a cost to dedicate time to the platform that could be used for other activities that either yield explicit income (e.g., another paid job) or utility (e.g., leisure). Further, because there is a large number of servers and the relationship with servers is relatively short term, the platform is unable to affordably learn a server's operating cost (i.e., a server's cost is private information to the server). The lack of cost visibility poses a challenge for the platform to make decisions that accommodate servers' heterogeneous preferences, especially given that servers control when and how much they work (third characteristic). For example, Filippas et al. (2021) empirically show that platform pricing harms server participation in a vehicle rental platform due to its inability to fully account for the servers' costs.

The fifth characteristic is distinctive to platforms. Servers provide the explicit task that consumers desire, but consumers are also drawn by the overall performance of the platform. For example, a platform known to have high prices is less likely to be chosen if a customer is aware of more economical alternatives, including potentially forgoing the service altogether. This is true no matter who sets prices. Consequently, a server's demand on a platform comes from two sources. The first is the platform's attractiveness which influences the total demand on the platform. Due to their small size, individual servers have no ability to directly influence the platform's attractiveness. However, the second source of a server's demand depends on the degree of server competition on the platform. The more aggressive server competition is on the platform, the more any one server can influence their own share of total demand. Combining the two effects, platform attractiveness determines the total size of the demand pool and platform competitiveness determines a server's portion of that pool.

For the designer of the platform, the appropriate control and regulation of pricing depends on the tension between letting servers set prices that are suitable given their own costs and the need for an appropriate level of prices across the platform. The limitation of "one size fits all" pricing is that inevitably the platform's price is too high for some servers (those with low costs) and too low for other servers (those with high costs), thereby restricting supply. However, server pricing relies on prices set by individual servers who know they have no power to influence aggregate outcomes. Consequently, competition among servers could lead to prices that are undesirably too high or too low, depending on how easy it is for a server to adjust its share of platform demand. This potential downside of server control over pricing remains even in the fully disintermediated setting where the central firm designing the platform is removed and replaced by a blockchain-based smart contract.

The ideal platform design (i) is responsive to server cost heterogeneity, (ii) properly manages the competition among servers, (iii) is relatively simple to explain and implement, and (iv) delegates pricing control to the servers to enable the classification of servers as contractors (if desired). We show that such a design exists. In fact, it is merely the combination of two simple payment fees. The first is the commonly observed commission per unit based on the price. For example, a server might pay the platform 20% of the selling price per unit. The second is a per-unit fee. Either fee can be negative, which is better described as a subsidy from the platform to the servers. Neither of the two payment fees on its own performs well in all situations, especially when price control is given to the servers (i.e., server pricing). But joining them together allows the firm owning the platform to maximize its profit even when servers choose their own prices. This is achieved because the commission plus per-unit fee enables the platform to fully regulate competition among the servers. For example, when competition is insufficient, the platform subsidizes low-cost servers to deliver more quantity and lower prices. However, when unregulated competition would be destructive, the platform tempers the aggressiveness of the low-cost servers to encourage more supply to participate on the platform. This recipe for the payment structure is sufficiently effective that the platform need not consider more elaborate payments structures.

In sum, through a properly designed and simple fee structure a platform can delegate control of pricing to servers while also receiving the maximum profit possible. To the best of our knowledge, we are the first paper that characterizes whether and how a platform can optimally delegate pricing decisions to a continuum of competing servers with private cost information.

2 Literature Review

This work is related to research on (i) ownership and contracts in supply chains, (ii) price delegation, (iii) platform management, and (iv) the design of decentralized markets through distributed ledger technology.

The structure of a service platform resembles that of a traditional supply chain. There is a single firm (the platform or supplier) that sells a good or service that is distributed through a large number of independent, self-interested, agents (e.g., servers, retailers, distributors).

Pricing control is well studied in supply chains. Identified early on, the double marginalization effect establishes that a retailer chooses a price which is higher than the supplier desires (Spengler (1950)): all else equal, the supplier always prefers the retailer to set a lower price to increase demand. However, lower retail prices also dampen the incentive for retailers to provide costly sales effort that could increase demand, and to stock an ample supply of inventory. Consequently, a supplier may attempt to regulate prices through contractual terms and/or restraints on retail business practice (e.g. Dixit 1983, Rey and Tirole 1986, Deneckere et al. 1996, Padmanabhan and Png 1997, Dana and Spier 2001, Cachon and Lariviere 2005, Song et al. 2008). Direct control of retail prices is legally risky, and so suppliers generally avoid it. But such control is an available option to firms creating platforms.

Asymmetry in cost information complicates the coordination of a supply chain. A number of settings with bilateral relationships have been investigated (e.g. Corbett and De Groote (2000), Ha (2001), Corbett et al. (2004), Mukhopadhyay et al. (2008), Yao et al. (2008), Xie et al. (2014), Ma et al. (2017)) and some with a finite number of competing agents (e.g., Cachon and Zhang (2006)), but none with a large number of small agents.

Throughout the supply chain literature, the competing agents (usually considered to be retailers) always prefer the other agents to raise their prices. One retailer never benefits from a second retailer's price reduction. However, this need not be true on a platform. Because there are a large number of agents on a platform, no single agent has significant influence over the market. Yet, the attractiveness of the platform as a whole is influenced by their collective actions. Consequently, an agent might prefer that all other agents lower their prices so as to attract more demand to the platform. We account for this in our model.

The price delegation literature considers how much control over pricing should a principal assign to an agent. The prototypical setting is a firm deciding the degree of pricing authority to give its salesforce. Initial work debated the value of delegated pricing control (e.g., Weinberg (1975)), and concluded that delegation is better when the agent has better information about demand (e.g., Lal (1986), Joseph (2001), Bhardwaj (2001). Subsequent analysis demonstrates that (due to the revelation principle, Myerson (1981)) there is in fact no inherent advantage of delegation even when the agents have better information as long as the principal is not constrained in the contract that can be offered (Mishra and Prasad (2004, 2005)). A creative principal can allow the agent to choose the price fully knowing that the agent will choose the price that maximizes the principal's objective. This finding somewhat moves the question of who controls prices into the philosophical realm - do agents in fact have pricing control if they choose the prices that the principal knows they will choose, given the principal's payment structure? However, in practice, and to a layperson, the definition of who has price control follows common sense - if the agent directly posts the price, even if the platform designs the agent's compensation, then the agent has control, and if the agent does not explicitly select a price, then the platform has control over pricing. We adopt this straightforward definition of price control.

There has been some empirical work on price delegation. For example, in the auto loan industry, Philipps et al. (2015) finds that giving agents price discretion increases profit relative to centrally chosen prices because the centrally chosen prices are not the best prices given the available information. In our model all agents fully optimize their actions given available data and we do not consider issues related to learning demand.

Our work departs from the price delegation literature in several ways. First, the price delegation literature has not extensively considered the issue of competition among agents. Mishra and Prasad (2005) models the competition between two agents, but each agent works exclusively for a single platform. Hence, there is no notion of a large number of independent agents whose collective actions determines the overall attractiveness of the platform. Second, in the price delegation literature agents select costly effort to increase sales (or to learn about demand, as in Atasu et al. (2021)) and they have homogeneous marginal costs. We do not include costly effort and agents in our model have heterogeneous costs.

There is a growing literature specifically focused on the management of platforms. Some studies only consider fixed prices and focus on the matching process that occurs among the platform participants (Arnosti et al. (2021), Feng et al. (2017), Afèche et al. (2018), Hu and Zhou (2016), Ozkan and Ward (2016), Halaburda et al. (2018)). Others consider revenue maximizing pricing and fee structures given platform control of pricing, i.e. without consideration of server pricing: Riquelme et al. (2015), Gurvich et al. (2016), Cachon et al. (2017), Bai et al. (2018), Taylor (2018), Hu and Zhou (2019), Benjaafar et al. (2021), Bimpikis et al. (2016), Ma et al. (2020), Besbes et al. (2021), Castillo et al. (2017), Hu et al. (2021).

There is some work that considers only server pricing on a platform in which servers always prefer competitors to be less competitive (Allon et al. (2012), Birge et al. (2020), Ke and Zhu (2021)).

Feldman et al. (2019) studies both platform and server prices in a food-delivery platform, but they do not consider competition among servers.

Lobel et al. (2021) evaluates a platform's optimal mix of employees and contractors in a market with uncertain demand but with an exogenous price. Their results demonstrate the value of classifying servers as contractors. Our results address the profit implication if a market legally requires server pricing to continue the use of contractors.

Some work considers platforms that can operate with different types of technologies (e.g., human drivers and autonomous vehicles, Siddiq and Taylor (2019), Lian and van Ryzin (2021)) or platforms in which agents can choose to be a server or a customer (e.g., peer-to-peer sharing in which agents choose to own or rent a vehicle, Benjaafar et al. (2019)). In our platform, the servers operate only with a single technology and agents do not choose to be on the demand or supply side of the market.

Several papers explicitly consider competition among platforms. Liu et al. (2019) find that server retention can be increased by paying discounts along with a commission contract. We do not consider server retention. In Ahmadinejad et al. (2019), ride-sharing platforms compete for drivers and riders. Platforms seek to maximize throughput (number of rides) rather than profit, so the payment structure between drivers and the platform is not considered. Lian et al. (2021) consider a market with multiple platforms that seek to attract a pool of servers, but pricing to consumers is exogenous. Cohen and Zhang (2017) consider pricing both for customers and servers, but do not consider server pricing.

There is work that considers the allocation of decision rights across agents in a market or platform. Hagiu and Wright (2015) and Hagiu and Wright (2018) consider marketing actions other than pricing. In Hagiu and Wright (2019) there is competition across agents, but it does not impact total demand. They use double sided moral hazard and non-contractible actions to explain the price delegation decision, neither of which is required in our analysis.

Growth of distributed ledger technologies is facilitating the operation of completely decentralized platforms, i.e., platforms without even a central firm owning the platform. Aymanns et al. (2020) consider how such a change affects consumer welfare. Others consider various operational controls within blockchain-based decentralized platforms (Benhaim et al. (2021), Chen et al. (2020), Tsoukalas and Falk (2020), Cong et al. (2020), Gan et al. (2021b)). The extant literature, to the best of our knowledge, has not addressed the issue of who should retain pricing control in the presence of market-level and individual-level competition effects. Further, work in this area often highlights the advantages of decentralization but does not consider how the lack of a central firm could reduce value in the system.

3 Model

We model a platform that mediates transactions between customers and a large population of independent servers.

Servers differ in their marginal cost to provide service (e.g., as empirically observed in Filippas et al. (2021)). In particular, there is a unit mass of servers and a server's per-unit-of-demand cost c, is uniformly distributed on the $[0, 1]$ interval.

A server with price p serves $q(p, \overline{p})$ units of demand when the average price paid on the platform is $\overline{p}$ (with the usual constraint that demand cannot be negative),

$$q(p, \overline{p}) = 1 - \beta\overline{p} + \gamma(\overline{p} - p). \tag{1}$$

Each server's demand depends on two factors that separately account for market-level and individual-level competitive effects.

The first factor, $\beta\overline{p}$, reflects the platform's overall attractiveness relative to other (external) options customers may have to fulfill their needs. Hence, we refer to this as the *platform attractiveness* effect and the parameter β measures its strength, with $3/2 < \beta$.[2] For example, if a platform is known to have a low average price compared to the broader market, then this helps to attract demand to the platform. However, if the platform's average price is known to be high, then customers tend to avoid it, leaning towards other options to meet their needs. Consequently, we consider the platform attractiveness effect to operate on time horizons of weeks, months or longer. Furthermore, the platform attractiveness effect is based on the sales-weighted average price on the platform, $\overline{p}$ (formally defined by (3)), because that is the price a customer can expect to actually pay. Consequently, extremely high prices have little impact on demand if customers rarely pay those prices, and low prices with ample demand are given proportionally more importance.

The second factor in a server's demand is the server's own price relative to the average price paid on the platform, $\gamma(\overline{p} - p)$. Once customers seek service from the platform, they naturally gravitate more towards servers with lower prices (or tend to be matched with lower priced servers). We refer

[2] The lower bound ensures the (realistic) situation in which the highest cost server does not participate even if the platform charges no fees.

to this as the *server competition* effect. The parameter $\gamma > 0$ measures the degree to which internal competition can shift the allocation of demand among servers. Large values of γ reflect a market in which consumer demand readily flows to the lowest price options on the platform. However, in practice, consumers of service platforms value other dimensions beyond price, suggesting more moderate values for γ. Put another way, servers provide differentiated products, and due to diverse tastes and needs, price is an important, but not the exclusive, deciding factor when choosing a server.

A server's demand is naturally always decreasing in their own price, p. But servers are of two minds with respect to the average price paid on the platform, $\overline{p}$. From the point of view of platform attractiveness, each server wishes for the average platform price to be low. That will attract many customers to the platform, and, all else equal, a server prefers to participate in a platform with more customers. However, a platform with plenty of demand is of little use to a server with a high price if little demand is matched with the server. In other words, a server's demand increases if their price looks good relative to the average price on the platform. And because of that, the server might prefer that the platform have a higher average price to make the server's own price more enticing.

For a server, the balance of the platform attractiveness and the server competition effects depends on the β and γ parameters. When $\beta < \gamma$, the server's demand is more impacted by competition with other servers than the attractiveness of the platform. In this case, as is typical in competitive settings, each server prefers less competition (i.e., other servers choose high prices). However, when $\gamma < \beta$, a server's demand is influenced more by with overall platform attractiveness than with internal server competition. This means that the server actually prefers for the other servers to act more aggressively (i.e., lower their prices). This is possible because consumers first decide whether to patronize the platform and then choose servers within the platform. If the platform decision looms large, then for a server it can be more important to be part of an attractive platform (ample demand due to a low average price) than to be able to capture share of that demand from other servers, especially if demand is relatively evenly distributed among the servers.

The situation in which the platform attractiveness effect is larger than the server competition effect ($\gamma < \beta$) is distinct to service platforms. For example, it is generally assumed (and empirically observed) that retailers always prefers their competitors to raise their prices. It is fully expected that a retailer's demand only can decrease when other retailers lower their prices. However, the unexpected can occur with a platform because the agents on a platform are small relative to the size of the market. Consequently, consumer decisions are less focused on the attributes of the particular agents/servers and can be governed to some extent by the characteristics of the overall platform.

Figure 1 displays the sequence of actions. The firm owning/designing the platform first establishes who sets prices and how fees are collected from the servers. (We refer to this firm as "the platform".) The platform's fees can be based on the prices charged or the quantities served or a combination of both. One simple fee commonly observed in practice is a *commission fee* in which a server pays ϕp per unit to the platform, where $\phi \leq 1$ is a fixed commission rate and p is the server's price. Typical commission rates range from about 15% in lodging to 25% in ride-sharing. Another simple fee commonly observed in practice is a *per-unit fee* in which a server pays a fixed w per unit to the platform. (In the context of a traditional supply chain, a per-unit fee is usually referred to as a "wholesale price".) When the platform combines commissions with per-unit fees, then one of them can be negative, meaning it is better described as a subsidy rather than a fee.

Next, servers observe the payment terms, their own private cost and their expectation for the average price in the market. Based on that information, the servers decide whether to participate in the market or not. They participate whenever their expected profit exceeds their outside opportunity cost, which we normalize to zero (without loss of generality). In fact, this is their only decision if the platform controls pricing. If the platform allows server pricing, the servers also choose their prices to maximize their individual profit (given, of course, the information they know). Finally, given prices and market entry decisions, demand occurs, revenue is earned, and the platform collects its share of revenue according to its payment terms.

Servers recognize that they are small actors in this platform, meaning that the action of a single server has no meaningful impact on market outcomes. However, collective actions clearly do influence market outcomes. A price expectations equilibrium occurs when the servers' expectation for the average price is consistent with the actual average price, $\overline{p}$. Similar methods have been used in the literature in the context of static non-atomic games (Aumann 1964, Schmeidler 1973, Ostroy and Zame 1994) and dynamic mean field games (Lasry and Lions 2007, Olszewski and Siegel 2016, Carmona and Wang 2021, Light and Weintraub 2022).

With platform pricing, the platform directly controls prices. Given that the platform cannot distinguish among the servers with low or high costs, with this option the platform selects a single price that applies to all servers. This is the typical approach in the ride-sharing industry. Consumers observe a single price for service within a local market at a moment in time (which is what is modelled here). The only decision for servers is whether to participate in the market or not. The platform can influence that decision, and thus total supply, through the price selected and the fee structure offered.

With server pricing, servers directly select their own price they offer customers on the platform.

This is the typical arrangement observed in room-sharing or work-for-hire platforms.[3] Now the servers choose whether to participate and their posted price. This leads to heterogeneous prices on the platform. In addition to the platform's fee structure, this decision is influenced by the degree of competition among the servers.

All information is common knowledge with the exception that each server's cost is private information (as already mentioned). For example, the distribution of server cost is commonly known and all prices and quantities are observable. All agents maximize their expected profit/earnings (i.e., all are risk neutral) and in equilibrium have valid expectations for future events.

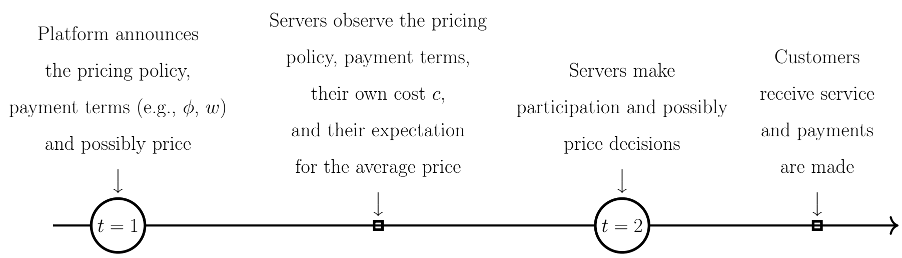

Figure 1: Sequence of events

4 Platform Pricing

When platform pricing is implemented, the platform selects a single price, p, for all servers. Consequently, a unique price expectations equilibrium exists: each server correctly expects p to be the average price on the platform, $p = \overline{p}$.

Platform pricing eliminates the server competition effect - without the ability to select their own price, servers are unable to control the amount of demand they take from other servers. As a result, the platform's demand is equally shared among the servers who participate even though they have heterogeneous costs. Furthermore, the absence of server competition means the platform directly regulates the platform's overall attractiveness to consumers (i.e., total demand) via its selection of the average price, $\overline{p}$.

Given that there is a single price on the platform, for any commission, there exists an equivalent per-unit fee, $w = \phi p$. Hence, all results with the commission payments can be exactly replicated with per-unit fees.

[3] Some platforms attempt a intermediate pricing policy in which prices are suggested to servers while the servers retain control over pricing. This remains server pricing in our model because the platform's suggestion provides no additional information and servers have the skills required to maximize their profit.

A server with cost c_i earns a profit $\pi_i(\overline{p})$,

$$\pi_i(\overline{p}) = q(\overline{p}, \overline{p})((1-\phi)\overline{p} - c_i), \tag{2}$$

where each server's price is the average price, $\overline{p}$. A server's profit is decreasing in cost. So only servers with sufficiently low cost expect to earn a non-negative profit and participate. Let c_h be the largest cost among the servers who participate, i.e., $\pi_h(\overline{p}) = 0$. The platform's expected profit is

$$\Pi^{\mathcal{P}}(\overline{p}, \phi) = \phi \int_0^{c_h} q(\overline{p}, \overline{p})\overline{p}\, dc.$$

Proposition 1 identifies the commission fee that maximizes the firm's profit.

Proposition 1. *With platform pricing and a commission fee, there exists a unique optimal price and commission rate for the platform: $\overline{p} = 2/(3\beta)$ and $\phi = 1/2$. Table 1 summarizes the optimal payment terms and several market metrics with platform pricing.*

As expected, the service competition parameter, γ, does not influence the outcomes because platform pricing eliminates competition among the servers. Servers earn some profit (one third of the total, to be precise) because cost heterogeneity among the servers prevents the platform from earning all of the profit in the system.

	Platform pricing, commission or per-unit fee	Server pricing, commission and per-unit fee	Server pricing, commission only	Server pricing, per-unit fee only
Platform's profit, Π	$\frac{1}{27\beta^2}$	$\frac{1}{24\beta^2}$	$\frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$	$\frac{1}{3}\left(\frac{\gamma}{\beta(2\beta+\gamma)^2}\right)$
Servers' total profit, π	$\frac{1}{54\beta^2}$	$\frac{1}{48\beta^2}$	$\frac{9}{16}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$	$\frac{2}{3}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$
Average market price, $\overline{p}$	$\frac{2}{3\beta}$	$\frac{2}{3\beta}$	$\frac{2}{2\beta+\gamma}$	$\frac{6\beta+\gamma}{3\beta(2\beta+\gamma)}$
Server prices, $p(c)$	$\frac{2}{3\beta}$	$\frac{4\gamma-\beta}{6\gamma\beta} + \frac{\beta c}{\gamma}$	$\frac{3}{2(2\beta+\gamma)} + c$	$\frac{5\beta+\gamma}{3\beta(2\beta+\gamma)} + \frac{c}{2}$
Total quantity served	$\frac{1}{9\beta}$	$\frac{1}{8\beta}$	$\frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right)$	$\frac{\gamma}{(2\beta+\gamma)^2}$

Table 1: Equilibrium market characteristics under the four fee structures (commission and per-use) and pricing policies (platform or server pricing).

5 Server Pricing

With server pricing, servers set their own prices to maximize their profit given their expectation for the average price on the platform, $\overline{p}_e$. Let $p(c,\overline{p}_e)$ be the price of a server with cost c and the expectation $\overline{p}_e$. Let $R(\overline{p}_e,\overline{p})$ be the total revenue on the platform given the price expectation of the servers and the actual average price, $\overline{p}$:

$$R(\overline{p}_e,\overline{p}) = \int_0^1 q(p(c,\overline{p}_e),\overline{p})p(c,\overline{p}_e)dc.$$

Let $Q(\overline{p}_e,\overline{p})$ be the total quantity served:

$$Q(\overline{p}_e,\overline{p}) = \int_0^1 q(p(c,\overline{p}_e),\overline{p})dc.$$

The actual average price on the platform is the ratio of total revenue to total quantity. In a price expectations equilibrium, the actual average price on the platform matches the servers' expected average price, $\overline{p}_e = \overline{p}$:

$$\overline{p} = \frac{R(\overline{p},\overline{p})}{Q(\overline{p},\overline{p})} = \frac{\int_0^1 q(p(c,\overline{p}),\overline{p})p(c,\overline{p})dc}{\int_0^1 q(p(c,\overline{p}),\overline{p})dc}. \tag{3}$$

The existence and uniqueness of a price expectation equilibrium is not assured: expectations are used to select each server's price, and the collection of prices then must yield an aggregate revenue and quantity such that the realized average price matches the original expectation. However, according to Proposition (2), there exists a unique price expectation equilibrium within a broad class of useful payment terms.

Proposition 2. *Consider the class of payment terms in which the servers pay the platform a commission, ϕ, (i.e., a ϕp fee per unit when the price is p) and/or a per-unit fee, w, for each unit of demand served. Within this class of payment terms, there exists a unique price expectations equilibrium when server pricing is implemented.*

We next evaluate actions and performance when the platform implements a commission and per-unit fee together and individually.

5.1 A Commission Plus Per-unit Fee

With server pricing implemented, say the platform charges a commission, ϕ, plus a per-unit fee, w. A server with cost c_i that selects price p and has an average price expectation $\overline{p}$ earns $\pi_i(p, \overline{p})$:

$$\pi_i(p, \overline{p}) = \big(1 - \beta\overline{p} + \gamma(\overline{p} - p)\big)((1-\phi)p - c_i - w). \tag{4}$$

Let $p(c, \overline{p})$ be the optimal price for a server with cost c and price expectation $\overline{p}$:

$$p(c, \overline{p}) = \frac{1}{2}\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma} + \frac{c+w}{1-\phi}\right). \tag{5}$$

Only servers with sufficiently low costs participate. Let c_h be the highest cost server that chooses to participate on the platform,

$$c_h = c_h(\phi, w, \overline{p}) = \frac{(1-\phi)(1-(\beta-\gamma)\overline{p})}{\gamma} - w. \tag{6}$$

The platform's profit is

$$\Pi^{\mathcal{S}}(\phi, w) = (\phi\overline{p} + w)\int_0^{c_h(\phi,w,\overline{p})} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c, \overline{p})))\, dc.$$

With server pricing, because servers have different costs, they post different prices, i.e., there is no single price on the platform. The particular price a server posts, (5), depends on three components. The first is the base amount of demand available to the server, $1 - (\beta - \gamma)\overline{p}$. This impacts all servers equally. The second component is the combination of a server's own cost and the commission. Lower cost servers apply smaller markups than high cost servers:

$$\frac{\partial^2 p(c, \overline{p})}{\partial\phi\partial c} = \frac{1}{2}\frac{1}{(1-\phi)^2} > 0.$$

Prices are least responsive to costs when the platform does not use a commission (i.e., $\phi = 0$) and become more responsive as the platform uses the commission more assertively (either positive or negative ϕ). The third component that influences prices is the per-unit fee. Like the base amount of demand, all servers apply the same markup due to the per-unit fee. Hence, the dispersion of prices on the platform is exclusively regulated with the commission.

The level of prices in equilibrium (i.e., the average price paid) is

$$\overline{p} = \frac{2}{2\beta+\gamma} + \left(\frac{\gamma}{2\beta+\gamma}\right)\left(\frac{w}{1-\phi}\right).$$

The commission influences the level of prices, but the platform cannot use the commission on its own to regulate the level of prices. In contrast, the per-unit fee regulates the price level. Thus, the commission plus per-unit payment structure gives the platform effectively two different tools to adjust prices, one to adjust price dispersion (the commission) and the other to set the overall level (per-unit). Having both tools is useful for the platform. According to Proposition 3, the platform (generally) uses the combination of a commission and a per-unit fee to maximize its profit.

Proposition 3. *With server pricing and a commission plus per-unit fee structure, there exists a unique commission rate and a per-unit fee that maximizes the platform's profit,* $\phi = 1 - \gamma/(2\beta)$, $w = (\gamma - \beta)/(3\beta^2)$. *(See Table 1 for additional details.)*

Although the platform uses a commission and a per-unit fee to maximize its profit, it does not always charge actual fees. When $\gamma < \beta$, server competition is weak leading to high prices. Something is needed to lower prices on the platform. For the platform, that something is a negative per-unit fee, which is better described as a subsidy for each unit served. The subsidy in effect lowers each server's cost, which causes the servers to lower their prices, all by the same amount. The subsidy is obviously costly, but the platform makes up for it with a higher commission. Even with a subsidy, on net the servers pay the platform (otherwise the platform would not earn revenue). But this does not require that all servers on net pay. The platform prefers to subsidize the lowest-cost servers because they serve the most customers and therefore have the largest influence on the average price. When the service competition effect is particularly weak ($\gamma < 2\beta/5$), these servers are allowed to keep all of their revenue and then are, in fact, paid an additional amount. Although the platform loses money on these servers, the lower prices they create makes the platform more attractive and allows the platform to make up for these losses through the fees earned on the higher cost servers.

The platform may also use subsidies at the other extreme of server competition. When $2\beta < \gamma$ service competition is aggressive resulting in prices that are too low. To increase prices, the platform implements a large positive per-unit fee, which raises all prices by the same amount. But all else equal, this means the high-cost servers are less able to pay the fee and earn a profit. To keep enough of them in the market, the platform "charges" a negative commission, i.e., a commission subsidy, which proportionally is more advantageous in absolute terms to high cost servers because they have higher prices. Nevertheless the platform earns something from every server (i.e., there is no server for which the total commission subsidy is greater than the total per-unit payment.)

With intermediate levels of competitiveness, $\beta < \gamma < 2\beta$, the platform continues to use the two different levers of commissions and per-unit, but neither needs to be implemented to the degree that they turn into a subsidy.

As with platform pricing, the platform earns two thirds of total profit and neither the platform's nor the servers' profit depends on the level of server competition, γ. The combination of a properly chosen commission and per-unit fee is able to neutralize the server competition effect (on profits, with the optimal payment terms).

The platform uses the commission and the per-unit fee to counteract each other - when one is increased, the other is decreased. Consequently, there are two special situations in which the combination of a commission and a per-unit fee is not needed. When $\gamma = \beta$, the average price, $\overline{p}$, has no impact on server demand. With no need to regulate the average price, the platform does not need to use a per-unit fee. When $\gamma = 2\beta$, the platform only needs to manage the average price level and the commission is not needed ($\phi = 0$ is optimal).

Although price dispersion exists, the average price paid by customers is the same as what the platform would choose with platform pricing. However, despite the same average price, the total quantity served is greater, because the price dispersion generates more participation.

5.2 Just a Commission

A platform could choose to charge a commission and nothing else. In this case, server earnings, optimal prices and entry decisions follow (4), (5), (6) respectively with $w = 0$. The platform's profit is

$$\Pi^{\mathcal{SC}}(\phi) = \phi\overline{p} \int_0^{c_h(\phi,0,\overline{p})} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c,\overline{p})))dc.$$

The commission on its own cannot regulate the average price level on the platform. It does affect the total revenue (decreasing in ϕ), and, obviously, the platform's share of revenue (increasing in ϕ). Consequently, according to Proposition 4, the platform selects a commission, $\phi = 1/2$, to balance this tension.

Proposition 4. *With server pricing and a commission fee there exists a unique commission rate that maximizes the platform's profit, $\phi = 1/2$. (See Table 1 for additional details.)*

The platform continues to earn two thirds of total profit. However, this is generally two thirds of less-than-optimal profit. When both a commission and per-unit fee are used, the platform characteristics (β and γ) influence how much server prices respond to their costs:

$$\frac{\partial p(c,\overline{p})}{\partial c} = \frac{\beta}{\gamma}.$$

But with just a commission, the platform is unable to regulate how prices adjust to costs as the platform characteristics vary (γ and β):

$$\frac{\partial p(c,\overline{p})}{\partial c} = 1.$$

Consequently, the average price on the platform, $\overline{p}$, can be higher or lower than ideal. The same is true for the total quantity served. Hence, unless the platform operates in the special situation in which platform attractiveness and server competition cancel each other ($\gamma = \beta$), the platform earns less with just the use of a commission.

5.3 Just a Per-unit Fee

Another option for a simple fee structure is to charge only a per-unit fee. In this case the server earnings, optimal prices and entry characteristics follow (4), (5), (6) respectively with $\phi = 0$. Given these, the platform's profit is

$$\Pi^{\mathcal{SU}}(w) = w \int_{0}^{c_h(0,w,\overline{p})} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c, \overline{p})))dc.$$

With per-unit fees the platform is primarily concerned with total demand (i.e., it does not charge a percentage of revenue). A higher fee increases earnings for the platform per unit but also reduces demand. Proposition 5 reports the per-unit fee that best manages this tension.

Proposition 5. *With server pricing and a per-unit fee there exists a unique fee that maximizes the platform's profit,* $w = \frac{1}{3\beta}$*. (See Table 1 for additional details.)*

A per-unit fee reduces the breadth of server participation in the market (reduces c_h) but also raises prices. This combination is helpful for the platform when the service competition effect is moderately strong because dampening excessive server competition is actually beneficial. Consequently, the platform earns the highest profit with per-unit fees when $\gamma = 2\beta$. In that special case, the average market price matches the one achieved with platform pricing or server pricing with also a commission.

The platform's share of total profit is

$$\frac{2 + \gamma/\beta}{3 + \gamma/\beta},$$

which is at least two thirds and is increasing in the level of server competition, γ. However, it is a small consolation to earn a large share when total profit is very low. As with just the commission, server pricing is insufficiently adaptive to the platform characteristics (γ and β):

$$\frac{\partial p(c,)}{\partial c} = \frac{1}{2}.$$

The resulting quantity served is always insufficiently low.

6 Selection of the Pricing Policy and Structure

Which among the pricing policies (platform or server) and structures (commission and/or per-unit) should the platform select? According to Figure 2, if the goal is to maximize the platform's profit, then the answer is simple. The platform is always better off with server pricing and the commission plus per-unit fees ($\Pi^{\mathcal{S}}$).

Platform pricing ($\Pi^{\mathcal{P}}$) is robust to variations in the server competition effect (γ), but so is server pricing with commission plus per-unit fees ($\Pi^{\mathcal{S}}$), and the latter generates 12.5% higher revenue for the platform than platform pricing. Platform pricing addresses the issue of server competition with a blunt instrument, i.e., it eliminates all price dispersion. But this restricts the ability of some higher cost servers to participate on the platform, which is a lost opportunity that can be recovered with server pricing (and a commission plus per-unit fee).

Platform pricing is simple, and it can fare well against the basic versions of server pricing. If the platform constrains itself to only a commission ($\Pi^{\mathcal{SC}}$) or only a per-unit fee ($\Pi^{\mathcal{SU}}$), then it becomes vulnerable with server pricing when the server competition effect is either weak (low γ) or strong (high γ). With either extreme the simple server pricing structures can be arbitrarily bad relative to platform pricing because they are unable to sufficiently regulate the prices on the platform and earn the platform a suitable profit. However, for a considerable range of moderate levels of server competition (intermediate γ), the platform prefers one of the versions of server pricing over platform pricing. In these situations the price dispersion of server pricing allows servers to cater their prices to their costs, and this is desirable for the platform. Corollary 1 formally states these results.

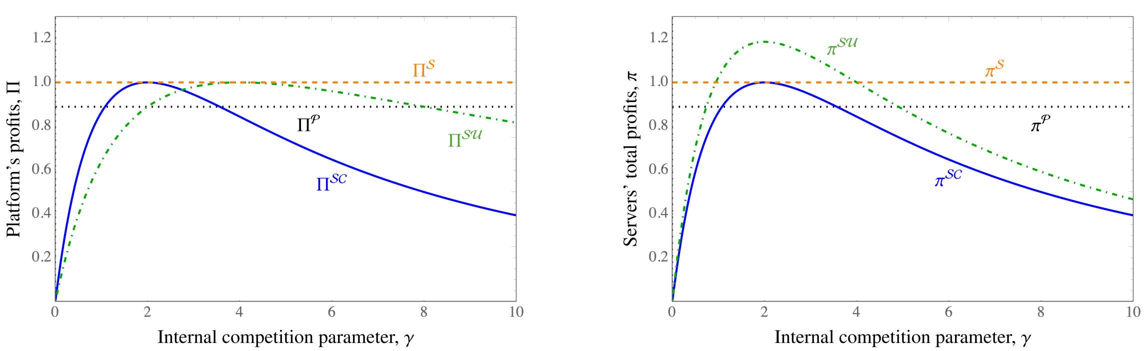

Figure 2: The platform's and servers' profits under platform pricing (dotted line), server pricing with commission (solid line) and server pricing with unit fee (dot-dashed line), as a fraction of their profits under server pricing with commission and per-unit fee (dashed line scaled to 1) for $\beta = 2$.

Corollary 1. *The platform's profit is 12.5% greater with server pricing and the commission plus* *per-unit fee than with platform pricing. The maximum profit with server pricing is achieved without a per-unit fee when $\gamma = \beta$, and without a commission when $\gamma = 2\beta$. The platform prefers server pricing with a commission over platform pricing if and only if $0.54\beta < \gamma < 1.78\beta$. The platform prefers server pricing with a per-unit fee over platform pricing if and only if $\beta < \gamma < 4\beta$.*

A platform may also have an interest in server welfare because this can influence its ability to recruit servers and could be of interest to regulatory agencies. The right panel of Figure 2 displays the servers' total profit with the considered pricing policies and payment terms. The servers' profit tends to closely mirror the platform's profit. For example, as with the platform's profit, the servers' earnings are invariant to the level of server competition (γ) either with platform pricing (because the single price eliminates server competition) or server pricing with the commission plus per-unit fees. Servers' earnings can be arbitrarily bad with server pricing and either of the basic server pricing payment terms. With just a commission, the servers' earnings peak at the same point as the platform, i.e., when the platform attractiveness effect and server competition effects cancel out so that the average price does not matter ($\gamma = \beta$). That is also the point at which total server earnings peak, but this is achieved with just the per-unit fee. Corollary 2 formalizes these observations.

Corollary 2. *Total server profit is higher with server pricing with a commission and per-unit fee than platform pricing and server pricing with commission only. Servers prefer server pricing with a commission and per-unit fee over server pricing with per-unit fee only if $\gamma < 0.47\beta$ or $\gamma > 2\beta$.*

Consumers are another stakeholder. Without an explicit utility model for consumers, it is not possible to directly measure consumer welfare. However, consumers tend to prefer lower prices and more quantity, all else equal. Based on Corollary 3, server pricing can also be good for consumers.

Corollary 3. *Server pricing with commission plus per-unit fee gives the same average price as platform pricing and a higher number of customers served.*

7 Optimal Mechanism

It is useful to identify the platform's optimal mechanism (i.e., pricing policy and terms) among the set of all possible payment terms the platform could use. According to the revelation principle (Myerson (1981)), the optimal mechanism resides within the set of truth-inducing mechanisms. With those mechanisms, the platform announces a menu that maps each possible server cost into a price and fee. Servers report a cost (which need not be their true cost) and prices and fees are determined based on the announced menu and the resulting demands across the servers. This mechanism is truth inducing if (i) it is optimal for each server to report their cost truthfully (assuming all other servers do so as well) and (ii) a server's earnings from participation in the platform is at least equal to the server's best outside option. The first requirement is referred to as the incentive compatibility constraint and the second is referred to as the individual rationality constraint.

Let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. The combination of $p(c)$ and $f(c)$, is the menu in the platform's mechanism. Let $\pi(c, p(\tilde{c}))$ be a server's earning with cost c that reports costs $\tilde{c}$:

$$\pi(c, p(\tilde{c})) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))(p(\tilde{c}) - c) - f(\tilde{c}).$$

In any optimal mechanism there exists a c_h such that all servers with costs $c \leq c_h$ participate on the platform whereas all servers with costs $c > c_h$ do not. (If a server with cost c did not participate but there were a server with a higher cost that did participate, then the server with cost c would be better off reporting a higher cost.) Hence, the platform's optimal mechanism can be found through the following optimization problem:

$$\begin{aligned}
\max_{p(c), f(c), c_h} \quad & \Pi = \int_0^{c_h} f(c)\, dc \\
\text{s.t.} \quad & \pi(c, p(c)) \geq \pi(c, p(\tilde{c}))\ , \forall c \in (0, c_h), \forall \tilde{c} \in (0, c_h) \\
& \pi(c, p(c)) \geq 0\ , \forall c \in (0, c_h) \\
& \text{Eq. (3).}
\end{aligned}$$

There are several challenges to find the optimal mechanism. First, there is no constraint on the form of the fee structure, $f(c)$. A contract with commission and per-unit fee is one form, but surely not the only form. Second, a server's profit depends on all of the prices in the market through the average price, $\overline{p}$. Hence, the platform's mechanism must have consistent expectations such that given the expected cost reports, the assigned prices are such that the average price leads to the expected quantities and profit. Finally, the relationship between server payoffs and the individual prices are non-linear.

Proposition 6. *The following mechanism maximizes the platform's revenue:* $p(c) = \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{\beta}{\gamma}c$*,* $f(c) = a_0 + a_1 c + a_2 c^2$*, with* $a_0 = \frac{1}{24}\left(\frac{5}{\beta} - \frac{2}{\gamma}\right), a_1 = \frac{2}{3}\left(\frac{\beta}{\gamma} - 1\right), a_2 = \frac{\beta}{2}\left(1 - \frac{2\beta}{\gamma}\right)$.

Proposition 6 identifies the optimal mechanism. Server prices are linearly increasing in the servers' costs, which means that price dispersion is good for the platform. Thus, platform pricing cannot be optimal (because it lacks price dispersion). Payments to the platform are quadratic in server costs, so the optimal mechanism is clearly not a two-part tariff: the heterogeneity in server costs limits the effectiveness of a two-part tariff.

Although the mechanism described in Proposition 6 is optimal, it does not seem practical. With this mechanism servers report their costs, the platform chooses all of the prices, and the servers make payments via a non-linear payment structure. Instead, it would be preferred to identify a mechanism that involves server pricing and linear payments based on observable metrics. Fortunately, according to Proposition 7, such a mechanism exists and it has already been identified.

Proposition 7. *Server pricing with a commission plus per-unit fee payment structure is an optimal mechanism.*

Although the commission and per-unit fees are each alone (generally) insufficient to yield the optimal profit for the platform, together, they work well to stay responsive to the servers' cost heterogeneity and to also regulate the average price. Through appropriately chosen commission and per-unit fees, a platform can achieve the same performance as the optimal mechanism without explicitly controlling the prices or requiring servers to report their costs.

8 Extensions and Robustness

In this section, we discuss several extensions to our model. Section 8.1, explores a market that uses distributed ledger technology (e.g., smart contracts on blockchain platforms) to eliminate the central coordinator (i.e., a platform). Section 8.2 explores a market in which servers have limited capacity. Section 8.3 considers non-uniform distributions for the servers' cost. Section 8.4 allows the platform to maximize throughput rather than profit. In sum, the primary insights from the main model continue to hold in all of these additional circumstances.

8.1 Disintermediated Pricing using Blockchain-Based Smart Contracts

The platform's primary focus is to maximize its own profit, rather than the total value in the system, which also includes the servers' total profits. Consequently, total system value may increase if the platform could be removed and control transferred to the servers. In principle, this may be feasible via smart contracts, enabled by blockchain technology.

Beyond the integrity of transactions, needed to create a viably functioning market, we presume smart contracts can potentially establish a set of observable and enforceable transfers among the servers while also allowing them to have full control over their pricing. Given that, Proposition 8 identifies an optimal disintermediated mechanism that (i) does not include a platform and (ii) maximizes total server profits.

Proposition 8. *Without a central platform earning a profit, the servers' profits can be maximized in equilibrium when a server with cost c selects price* $p(c) = \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{\beta}{2\gamma}c$*, and the server contributes an amount* $f(c)$ *to the system, where* $f(c) = a_0 + a_1 c + a_2 c^2$*, with* $a_0 = \frac{1}{12}\left(\frac{1}{\beta} - \frac{1}{\gamma}\right)$*,* $a_1 = \frac{1}{3}\left(\frac{\beta}{\gamma} - 1\right)$*,* $a_2 = \frac{\beta}{4}\left(1 - \frac{\beta}{\gamma}\right)$*. If* $f(c)$ *is negative, then the server receives a subsidy. This mechanism is budget balancing, i.e., there is a zero net flow of subsidy transfers.*

The mechanism described in Proposition 8 is incentive compatible, i.e., no server wishes to choose a different price conditional that all other servers are following the mechanism and the subsidy transfers are credibly administered. This holds even though some servers select prices that force them to relinquish some of their earnings (when $f(c)$ is positive) and others select prices that allows them to earn a bonus beyond their own earnings (when $f(c)$ is negative).

Similar to the optimal profit-maximizing mechanism, the optimal disintermediated mechanism can be implemented through a commission and per-unit fee structure in which some servers make contributions, others receive contributions, and the net contribution in the system is zero.

The optimal disintermediated contract is simple to characterize, but it may be difficult to implement in practice. First of all, it implies that not all servers will retain all of their earnings. The system of transfers may be viewed as unnatural, or inconsistent with the philosophy behind a disintermediated market. It is also not clear how the specific functional forms for the subsidies would be modified if market conditions change (e.g., shifts in γ or β), and it is not clear that the data collection and computational requirements could be practically satisfied.

A simpler alternative to the optimal mechanism is a market that allows all servers to set their own prices, retain their entire revenue, and distributed ledger technology is merely used to ensure the integrity of all transactions. We refer to this benchmark as "disintermediated server pricing". It is equivalent to the server pricing setting from Section 5, but with commission and fees of

$$\phi = 0, w = 0. \tag{7}$$

Under this setting, the equilibrium is characterized by Equations (3), (5), (6) and (7).

In practice, there are markets in which platforms take no commission, as is displayed in Table 2. However, a direct comparison between centralized and decentralized platforms is difficult, because decentralized platforms rely on alternative monetization mechanisms that may proxy for commission fees, such as stake retention in Initial Coin Offerings (Gan et al. 2021a,b), and a tokenized economy (Cong et al. 2020, Tsoukalas and Falk 2020). Our analysis in this section can thus be interpreted as a "best case scenario" for blockchain-based platforms. We show that even under this best-case assumption, blockchain may not always prevail as the preferred mode of adoption.

Centralized	ϕ		**Decentralized**	ϕ
Uber	25%		Drife	0%
Lyft	20%		Arcade City	0%
AirBnB	15%		Dtravel	7%*
UberEats	22%		Filecoin	0%
Grubhub	25%		Storj.io	0%
Median	20%		Median	0%

Table 2: Typical commission fees per transaction, on centralized vs. decentralized platforms.
*Dtravel commission fee is recycled back into the platform ecosystem.

Proposition 9. *With disintermediated server pricing (i.e., no platform, servers set their prices and retain all earnings), there exists a unique price equilibrium among the servers. There exists* $\gamma_l < \beta$ *and* $\beta < \gamma_h$ *such that the total server profits under disintermediated server pricing is higher than server pricing with a commission and per-unit fee when* $\gamma_l < \gamma < \gamma_h$*, and lower otherwise.*

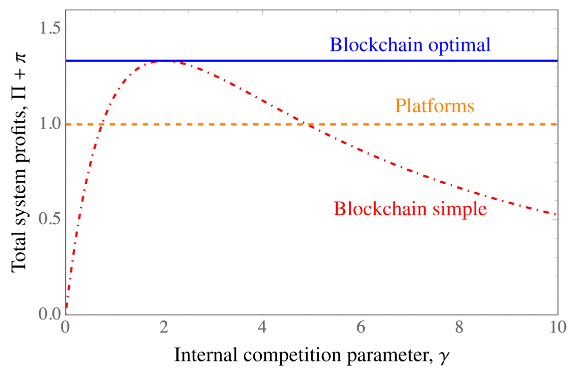

Figure 3: Total system profits with respect to the server competition parameter, γ, for $\beta = 2$ under disintermediated server pricing (dot-dashed line) and disintermediated optimal mechanism (solid line), as a fraction of total profit with the platform's optimal mechanism (server pricing with a commission and per-unit fee) (dashed line scaled to 1).

As displayed in Figure 3, the optimal disintermediated mechanism generates 33.3% more total system profit than the platform's optimal mechanism. This increase is attributed to higher retained earnings by servers, which promotes participation. Under this mechanism, the customers are also better off, with the same average market price and higher quantity served.

Recall, the optimal disintermediated mechanism requires transfers among the servers. When these transfers cannot be implemented, the potential upsides to disintermediation can be large, but this is limited to specific cases. According to Proposition 9, only for intermediate values of server competition, disintermediated server pricing can increase total system profits. In this case, the platform's profit-seeking behavior destroys more value than its price coordination generates. This is not to suggest a platform can or should be be replaced by a disintermediated alternative. A platform may pursue other objectives that are better aligned with servers' objectives (e.g. maximizing throughput) or provide other value added activities that is not captured in the model. In those markets, centralized platforms can exhibit profit-maximizing behavior while maintaining servers' total profits at a desired level (see Appendix, page 59), making it difficult for a disintermediated alternative to provide additional value.

In sum, disintermediation can increase total system value. However, if the optimal disintermediated contract is not practically implementable, a simpler alternative is to merely let the servers price on their own and retain all of their earnings. While this can still increase total system value (by eliminating the distortions associated with the platform's profit-seeking motive), it retains limited control over the level of pricing competition among the servers. Hence, just as in the market regulated by a platform, it is possible that total system value can be reduced via disintermediation, especially if the server competition effect is particularly strong or weak.

8.2 Capacity Constraints

Our primary model presumes that demand constrains the quantity of service provided. However, it is possible that servers may not be able to serve all of the customers that could be allocated to them. This external capacity restriction influences decisions, regardless of who is setting prices or the payment terms. When the platform is in control, it needs to set the platform price sufficiently high to ensure that no demand is left unserved. Similarly, when servers set the price, each individual server wants to extract the maximum profit from the customers and prefers to leave no customer unserved. Proposition 10 characterizes the equilibrium behavior of the optimal mechanism given capacity constraints.

Proposition 10. *In the presence of capacity constraints, the platform's optimal policy is server pricing with a commission and per-unit fee,* $\phi = 1 - \frac{\gamma}{2\beta}$, $w = \left(\frac{4t^2-6t+3}{1-t}\right)\left(\frac{\gamma-\beta}{6\beta^2}\right)$, *where* t *is the capacity of each server.*

The presence of capacity constraints imposes some changes to the platform's optimal contract parameters, but not to the contract structure. The platform's optimal commission is unchanged, but the per-unit fees are now monotone decreasing in t. Because servers have no incentive to cut their prices once their demand matches their capacity, the platform counteracts elevated prices by decreasing server fees across the board.

With either of the simpler payment structures (e.g. only a commission or only a per-unit fee), capacity constraints can be advantageous to the platform. When competition across servers is strong, the platform is better off when servers are moderately limited by capacity. In those cases, capacity constraints dampen the excessive competition between servers, so the platform does not have to try to do so.

8.3 Alternative Cost Distribution Assumptions

The uniform distribution facilitates analytical results and yields an optimal mechanism that can be implemented with a relatively simple structure. For other server cost distributions, prices are non-linear in the servers' costs in the optimal mechanism. Furthermore, the optimal mechanism cannot be implemented with just commission and per-unit fees. Nevertheless, the optimal version of server pricing with commissions plus per-unit fees may perform well.

In this section, we consider a Beta(α_1, α_2) distribution with $\alpha_1 = \alpha_2 = \alpha$, which implies the mean is a constant 0.5 and the density function is symmetric about the mean. We evaluate 400 scenarios using the following parameters: $\gamma \in \{0.1, 0.2, ..., 10\}$, $\beta \in \{2\}$, and $\alpha \in \{2, 3, ..., 5\}$. For each scenario, we compare the optimal non-linear mechanism with the optimal linear mechanism involving server pricing, commission and per-unit fees. We find that the optimal linear mechanism performs very well: it yields on average 99.98% of the optimal profit for the platform and no less than 99.9%.

8.4 Throughput Maximization

Platforms that are in earlier stages of their life-cycle may believe it is more important to grow than to be profitable. Those platforms may prioritize maximizing throughput over profits, which has been considered in other models: e.g. Ahmadinejad et al. (2019), Castro et al. (2020), Yan et al. (2020)). As profit is the product of quantity and margin, maximizing throughput (i.e., quantity) is not a radically different objective. Hence, our main findings continue to hold qualitatively (see Appendix, page 70).

9 Conclusion

Who should control pricing on a service platform and how should fees be collected? Servers know their own costs to participate, so they are best able to tailor their price to their circumstance. A rigid price set by the central platform is likely to exclude some servers who otherwise could contribute. However, servers are small actors. Each knows that they can only respond to the market given to them and this has significant implications for the dynamics on the platform. In particular, there are two effects that need to be regulated. A server can lower their price to take a larger share of the platform's demand. This server competition effect can lead to prices that are destructively too low, limiting revenue potential through less supply (high cost servers drop out) and lower prices. In contrast, there is a platform attractiveness effect - total demand on the platform depends on the average price. Consequently, a server may actually prefer other servers to cut their prices - doing so makes it harder to steal share from them (the typical competition effect) but it also expands the total demand on the platform. Due to each server's limited power, they collectively have little control to balance these effects properly. This creates a value-added opportunity for the platform to regulate pricing.

Commissions and per unit-fees are simple to explain and administer, but both lack precision on their own. This can lead to either prices that are too low due to excessive competition or prices that are too high due to insufficient competition. Consequently, while platform pricing is not ideal in all situations, it performs reasonably well in all cases because when the platform takes full control of pricing it is able to avoid the adverse scenarios of server pricing.

Fortunately, it is possible to combine the robustness of platform pricing with the advantages of decentralization. When servers left on their own are too price aggressive, something is needed to calm them. When independent servers are too timid, something is needed to motivate them to cut their prices. By simply adjusting its commission and per-unit fees to account for server heterogeneity, the platform can realize the maximum profit in all situations. However, in extreme cases this can actually require converting one of the fees into a subsidy.

So the question for a service platform is not so much who should control pricing, but how they do it. For example, server pricing may be necessary to classify servers as contractors rather than employees. Although that classification has implications for who is willing to work on a platform and the costs of their work, it may be possible to give servers full pricing control and yet correct for the potentially negative consequences of doing so. This is a particularly important lesson for platforms that are attempting to use distributed ledger technology (e.g., blockchain) to eliminate any central agent. Such technologies may bring advantages to the market, but without addressing the negative tendencies of server pricing, they face a potentially negative drag on performance.

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Appendix

Proof of Proposition 1. In platform pricing, there exists a single price in the market. The average market price is equivalent to the price set by the platform. Let $\overline{p}$ be the price set and ϕ be the portion of revenue retained. Conditional on participation, a server with cost c_i earns

$$\begin{aligned}\pi_i(\overline{p}) &= q(\overline{p},\overline{p})((1-\phi)\overline{p}-c_i)\\ &= (1-\beta\overline{p})((1-\phi)\overline{p}-c_i).\end{aligned}$$

Server profits are decreasing in cost, c_i, and therefore there exists a threshold cost, c_h, for which server with cost c participates if and only if $c \leq c_h$. The highest cost that participates is

$$c_h = (1-\phi)\overline{p},$$

which leads to zero profit.

Platform's profit maximization problem is

$$\begin{aligned}\max_{\overline{p},\phi} \quad \Pi^{\mathcal{P}}(\overline{p},\phi) \quad &= \phi\overline{p}\int_0^{c_h}(1-\beta\overline{p})\, dc\\ &= \phi\overline{p}\int_0^{(1-\phi)\overline{p}}(1-\overline{p})\, dc\\ &= \phi(1-\phi)\overline{p}^2(1-\beta\overline{p}).\end{aligned}$$

The platform's solution is unique. It's defined by the first order conditions:

$$\begin{aligned}\frac{\partial\Pi^{\mathcal{P}}(\overline{p},\phi)}{\partial\overline{p}} &= \overline{p}(1-\phi)\phi(2-3\beta\overline{p}) = 0,\\ \frac{\partial\Pi^{\mathcal{P}}(\overline{p},\phi)}{\partial\phi} &= \overline{p}^2(1-\beta\overline{p})(1-2\phi) = 0,\end{aligned}$$

giving the solution

$$\begin{aligned}\overline{p} &= \frac{2}{3\beta},\\ \phi &= \frac{1}{2}.\end{aligned}$$

Platform's profit is

$$\Pi^{\mathcal{P}} = \phi(1-\phi)\overline{p}^2(1-\beta\overline{p}) = \frac{1}{27\beta^2}.$$

The highest cost that participates is

$$c_h = (1-\phi)\overline{p} = \frac{1}{3\beta}.$$

Total quantity of customers served in the market is

$$\begin{aligned} Q &= \int_0^{c_h} (1-\beta\overline{p})\, dc \\ &= (1-\phi)\overline{p}(1-\beta\overline{p}) \\ &= \frac{1}{9\beta}. \end{aligned}$$

Servers' total profits is

$$\begin{aligned} \pi^{\mathcal{P}} &= \int_0^{(1-\phi)p} (1-\beta p)((1-\phi)p-c)\, dc \\ &= \frac{1}{2}(1-\beta p)p^2(1-\phi)^2 \\ &= \frac{1}{54\beta^2}. \end{aligned} \tag{8}$$

□

Proof of Proposition 2. Let us consider a broad class of payment structures for which there indeed exists a unique price expectations equilibrium: a server pays the firm a fee consisting of three components: (i) a unit fee, w, per unit served; and (ii) a commission per unit, ϕp, where ϕ is the fixed commission rate and p is the server's price. In total a server with price p and quantity q served pays the firm

$$q\,(w+\phi p)\,.$$

The commission rate does not exceed 100%, i.e., $\phi < 1$. The fixed per-unit fee, w, can be negative, meaning that it is actually a per unit subsidy.

A server's quantity is

$$\begin{aligned} q(p,\overline{p}) &= 1-\beta\overline{p}+\gamma\,(\overline{p}-p) \\ &= 1-(\beta-\gamma)\,\overline{p}-\gamma p \end{aligned}$$

where $\overline{p}$ is the (demand weighted) average price.

Let server costs be uniformly distributed on the interval $[0,1]$. A server with cost c_i earns a profit:

$$\begin{aligned} \pi_i(p,\overline{p}) &= q(p,\overline{p})\,(p-c_i-w-\phi p) \\ &= q(p,\overline{p})\,((1-\phi)\,p-c_i-w) \\ &= (1-\beta\overline{p}+\gamma\,(\overline{p}-p))\,((1-\phi)\,p-c_i-w) \\ &= (1-\phi)\,(1-\beta\overline{p}+\gamma\,(\overline{p}-p))\left(p-\tfrac{c_i+w}{1-\phi}\right). \end{aligned}$$

The server's profit is strictly concave when $\phi < 1$,

$$\begin{aligned}
\frac{\partial \pi_i(p,\overline{p})}{\partial p} &= -\gamma\left(\left(1-\phi\right)p - c_i - w\right) + q(p,\overline{p})\left(1-\phi\right) \\
&= \gamma\left(1-\phi\right)\left(-\left(p - \frac{c_i+w}{1-\phi}\right) + \frac{1}{\gamma}q(p,\overline{p})\right) \\
\frac{\partial^2 \pi_i(p,\overline{p})}{\partial p^2} &= -2\gamma\left(1-\phi\right)
\end{aligned}$$

The server with cost c has optimal price and quantity

$$\begin{aligned}
p^*\left(c,\overline{p}\right) &= \frac{(1-\phi)(1-(\beta-\gamma)\overline{p})+\gamma(c+w)}{2\gamma(1-\phi)} \\
&= \frac{1}{2}\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma} + \frac{c+w}{1-\phi}\right),
\end{aligned}$$

$$\begin{aligned}
q(p^*(c,\overline{p}),\overline{p}) &= 1 - \beta\overline{p} + \gamma\left(\overline{p} - p^*(c,\overline{p})\right) \\
&= \frac{1}{2}\gamma\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma} - \frac{c+w}{1-\phi}\right).
\end{aligned}$$

Define c_0 to be the cost such that $q(p^*(c_0,\overline{p}),\overline{p}) = 0$ for a given expectation for the average price, $\overline{p}$,

$$c_0 = \left(1-\phi\right)\left(\frac{\left(1-\left(\beta-\gamma\right)\overline{p}\right)}{\gamma} - \frac{w}{1-\phi}\right). \tag{9}$$

Begin with the special case $\beta = \gamma$. In this situation the demands and prices of the servers do not depend on the average price. Hence, for whatever prices and quantities are selected, there exists an average price, $\overline{p}$. This is consistent with their expectation and choices, because their choices do not depend on it. Partial participation in the platform requires $0 < q(p^*(0,\overline{p}),\overline{p})$ and $q(p^*(1,\overline{p}),\overline{p}) < 0$, which can be expressed as

$$\frac{1}{\beta} - \frac{1}{1-\phi} < \frac{w}{1-\phi} < \frac{1}{\beta}.$$

Now consider $\gamma \neq \beta$.

$$\begin{aligned}
q(p,\overline{p}) &= 1 - \beta\overline{p} + \gamma\left(\overline{p} - p\right) \\
&= 1 - \left(\beta-\gamma\right)\overline{p} - \gamma p.
\end{aligned}$$

Using (9), define $\overline{p}_e\left(c_0\right)$ as the average price such that $q(p^*(c_0,\overline{p}),\overline{p}) = 0$ when $\overline{p}_e\left(c_0\right)$ is the expectation for the average price,

$$\overline{p}_e\left(c_0\right) = \frac{1}{\beta-\gamma}\left(1 - \gamma\left(\frac{c_0+w}{1-\phi}\right)\right).$$

$\overline{p}_e\left(c_0\right)$ is linear in c_0, increasing if $\beta < \gamma$ and decreasing if $\gamma < \beta$:

$$\frac{d\overline{p}_e\left(c_0\right)}{dc_0} = \frac{\gamma}{\left(\gamma-\beta\right)\left(1-\phi\right)}.$$

In equilibrium, assuming there exists a server with 0 demand, there exists c_0 mass of participating servers in the market. If all servers have non-zero demand, then all of the servers in the market participate. The highest cost server that participates is

$$c_h = \min\left\{c_0, 1\right\}.$$

The total quantity on the platform is

$$\int_0^{\min\{c_0,1\}} q(p^*(c,\overline{p}),\overline{p})dc = \begin{cases} \frac{\gamma}{4(1-\phi)}c_0^2 & ,c_0 < 1, \\ \frac{\gamma(1-2c_0)}{4(\phi-1)} & ,1 < c_0. \end{cases}$$

The total revenue on the platform is

$$\int_0^{\min(c_0,1)} q(p^*(c,\overline{p}),\overline{p})p^*\left(c,\overline{p}\right)dc = \begin{cases} \frac{\gamma}{4(1-\phi)^2}c_0^2\left(\frac{2}{3}c_0+w\right) & ,c_0 < 1, \\ \frac{\gamma\left(3c_0^2+6c_0w-(1+3w)\right)}{12(1-\phi)^2} & ,1 < c_0. \end{cases}$$

Define $\overline{p}_a\left(c_0\right)$ as the actual average price given c_0

$$\overline{p}_a\left(c_0\right) = \frac{\int_0^{\min\{c_0,1\}} q(p^*(c,\overline{p}),\overline{p})p^*\left(c,\overline{p}\right)dc}{\int_0^{\min\{c_0,1\}} q(p^*(c,\overline{p}),\overline{p})dc} = \begin{cases} p_h\left(c_0\right) & ,c_0 < 1, \\ p_l\left(c_0\right) & ,1 < c_0, \end{cases}$$

where

$$p_h\left(c_0\right) = \left(\frac{1}{1-\phi}\right)\left(\frac{2}{3}c_0+w\right),$$

$$p_l\left(c_0\right) = \frac{-3c_0^2-6c_0w+(1+3w)}{3(\phi-1)(2c_0-1)}.$$

$\overline{p}_a\left(c_0\right)$ is continuous, differentiable, increasing, linear for $c_0 < 1$ and strictly concave for $1 < c_0$:

$$\frac{d\overline{p}_a\left(c_0\right)}{dc_0} = \begin{cases} \frac{2}{3(1-\phi)} & ,c_0 < 1, \\ \frac{2}{3(1-\phi)}\left(\frac{1-3c_0+3c_0^2}{(2c_0-1)^2}\right) & ,1 < c_0. \end{cases}$$

Note that

$$p_l\left(1\right) = p_h\left(1\right),$$

which implies that $\overline{p}_a\left(c_0\right)$ is a continuous function.

The average price $\overline{p}$ is a candidate equilibrium if there exists a c_0 such that

$$\overline{p} = \overline{p}_e\left(c_0\right) = \overline{p}_a\left(c_0\right).$$

If $\beta > \gamma$, then $\overline{p}_e\left(c_0\right)$ is strictly decreasing in c_0 while $\overline{p}_a\left(c_0\right)$ is strictly increasing. Thus, there exists a unique c_0 that leads to a candidate equilibrium. If $\beta < \gamma$, then both $\overline{p}_e\left(c_0\right)$ and $\overline{p}_a\left(c_0\right)$ are increasing in c_0. Because for all c_0,

$$\frac{d\overline{p}_a\left(c_0\right)}{dc_0} < \frac{d\overline{p}_e\left(c_0\right)}{dc_0},$$

there exists a unique c_0 for the candidate equilibrium $\overline{p}$.

A candidate interior equilibrium satisfies

$$\overline{p} = \overline{p}_e\left(c_0\right) = \overline{p}_h\left(c_0\right)$$

and that solving for c_0 and $\overline{p}$ yields

$$c_0 = \frac{3\left(\left(1-\phi\right)-\beta w\right)}{2\beta+\gamma} \tag{10}$$

$$\overline{p} \quad = \quad \frac{2\left(1-\phi\right)+\gamma w}{\left(1-\phi\right)\left(2\beta+\gamma\right)}. \tag{11}$$

The stability conditions for the candidate equilibrium to be interior is

$$\begin{gathered} 0 < c_0 < 1, \\ 0 < \overline{p}. \end{gathered} \tag{12}$$

In any equilibrium where the platform earns non-zero profits, a necessary and sufficient condition for the candidate equilibrium to be stable and interior is

$$c_0 < 1 \iff 1 > \frac{3(1-\phi-w\beta)}{2\beta+\gamma}. \tag{13}$$

It is straightforward to see that, if $1 \leq \frac{3(1-\phi-w\beta)}{2\beta+\gamma}$, then by Equation (10), we have $c_0 \geq 1$. That is, all servers participate. We just need to show that, if platform's terms satisfy $1 > \frac{3(1-\phi-w\beta)}{2\beta+\gamma}$, then in any equilibrium where the platform earns non-zero profits, all stability conditions in (12) are satisfied.

For the rest of the proof, assume Equation (13) holds.

In order to prove $c_0 > 0$ is satisfied for any equilibrium with non-zero profits, let us assume for contradiction that $c_0 \leq 0$ and the platform earns non-zero profits. By Equation (10), this implies that

$$c_0 \leq 0 \iff \frac{w}{1-\phi} \geq \frac{1}{\beta}. \tag{14}$$

Since platform earns non-zero profits, there also exists a non-zero mass of servers in the market.

The quantity and the profits of servers are monotone decreasing functions of c. As non-zero mass of servers are in the market, there exists a $c_h \in [0,1]$ such that

$$q(p^*(c,\overline{p}),\overline{p}) > 0, \forall c \in [0,c_h). \tag{15}$$

Since the average price in the market is strictly between the highest price in the market and lowest price in the market (and that the prices are monotone) there exists a server with some cost c_a such that

$$p(c_a,\overline{p}) = \overline{p}, \tag{16}$$

that is the average price is attained by a server with some cost $c_a < c_h$. This server earns non-negative margins:

$$(1-\phi)p(c_a,\overline{p}) - c_a - w \geq 0 \implies p(c_a,\overline{p}) = \overline{p} \geq \frac{c_a + w}{1-\phi}. \tag{17}$$

The quantity served by this server is strictly positive:

$$\begin{aligned}
0 &< q(p^*(c_a,\overline{p}),\overline{p}) \\
&= 1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c_a,\overline{p})) \\
&= 1 - \beta\overline{p} \\
&\leq 1 - \beta\left(\frac{c_a + w}{1-\phi}\right) \\
&\leq 1 - \beta\left(\frac{w}{1-\phi}\right) \\
&\leq 0.
\end{aligned} \tag{18}$$

First line follows from the fact that the server has cost $c_a < c_h$ and Equation (15). Third line follows from Equation (16). Fourth line follows from Equation (17). Fifth line follows from the fact that $c_a \geq 0$. Last line follows from Equation (14).

Equation (18) defines a contradiction (0<0). Hence, our assumption is incorrect. We cannot simultaneously have $c_0 \leq 0$ and the platform earn non-zero profits. If $w/(1-\phi) > 1/\beta$, the platform's fees are too high and no server can add its markup and still retain positive demand. In equilibrium, 0 mass of servers enter and platform earns 0 profits (e.g. there is no market). Since its possible for the platform to earn non-negative profits through other contracts, the platform will never choose a contract that satisfies $w/(1-\phi) > 1/\beta$.

Now, to prove $\overline{p} > 0$ is satisfied for any equilibrium with non-zero profits, let us assume for contradiction that $\overline{p} \leq 0$ and the platform earns non-zero profits. By Equation (11), this implies that

$$\overline{p} \leq 0 \iff -\frac{2}{\gamma} \geq \frac{w}{1-\phi}.$$

As the platform earns non-zero profits, there exists a non-zero mass of servers in the market. By definition, all participating servers serve non-negative demand. Similarly, by definition, the platform and all participating servers earn non-negative profits.

Total value generated is the sum of these two profits (and is non-negative):

$$\begin{aligned}
0 <& \int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})(\phi p^*(c,\overline{p}) + w)\ dc + \int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})((1-\phi)p^*(c,\overline{p}) - c - w)\ dc \\
=& \int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})(p^*(c,\overline{p}) - c)\ dc \\
=& \int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\ dc - \int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})c\ dc \\
\leq& \int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\ dc \\
=& \overline{p} \int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})\ dc \\
\leq& 0.
\end{aligned} \tag{19}$$

The last equality follows from the definition of $\overline{p}$:

$$\overline{p} = \frac{\int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\ dc}{\int_0^{c_0} q(p^*(c,\overline{p}),\overline{p})\ dc}.$$

Last line follows from our knowledge that $q(p^*(c,\overline{p}),\overline{p}) > 0$ for all participating servers and that $\overline{p} \leq 0$.

Equation (19) defines a contradiction. Hence, our original assumption is incorrect. We cannot simultaneously have $\overline{p} \leq 0$ and the platform earn non-zero profits. If $-\frac{2}{\gamma} \geq \frac{w}{1-\phi}$, no profit can be generated in the market. Therefore, the platform will never sets its terms such that $-\frac{2}{\gamma} \geq \frac{w}{1-\phi}$ holds.

Hence, we conclude that for any equilibrium where the platform earns non-zero profits, the equilibrium is interior and satisfies all the stability conditions in (12) if and only if the platform sets its terms such that

$$1 > \frac{3(1-\phi-w\beta)}{2\beta+\gamma}.$$

With server pricing with commission only, we have $w = 0$. Condition (13) is satisfied everywhere if and only if

$$1 > \frac{3(1-\phi)}{2\beta+\gamma}, \forall \phi \in [0,1].$$

The right hand side is monotone decreasing in ϕ and is maximized at $\phi = 0$ (platform doesn't choose $\phi < 0$, otherwise it earns negative profits). Then, we need to impose the condition

$$1 > \frac{3(1-\phi)}{2\beta+\gamma} \geq \frac{3}{2\beta+\gamma}.$$

A sufficient condition is $\beta > 3/2$.

With server pricing with unit fee only, we have $\phi = 0$. Condition (13) is satisfied everywhere if and only if

$$1 > \frac{3w\beta}{2\beta+\gamma}, \forall w > 0.$$

We want this to be true for all w values, that the platform never runs into boundary condition no matter what fee it chooses. Recall that we have $c_0 > 0$ in any profitable equilibrium where platform earns non-zero profits. By Equation (10), this implies

$$c_0 = \frac{3((1-\phi)-\beta w)}{2\beta+\gamma} = \frac{3(1-\beta w)}{2\beta+\gamma} > 0 \implies w < \frac{1}{\beta}.$$

So, in any profitable equilibrium, platform cannot set its wage higher than $1/\beta$. This means that, restricting our attentions to those equilibrium where the platform earn non-zero profits, our condition becomes

$$1 > \frac{3}{2\beta+\gamma}.$$

A sufficient condition is $\beta > 3/2$.

□

Proof of Proposition 3. By Proposition 2 the uniqueness of consistent expectations equilibrium of the average price is guaranteed when platform operates with a commission and/or per-unit fees. Throughout the proofs, due to equivalency of the expected average price, $\overline{p}_e$, and realized average price, $\overline{p}$, we don't make the distinction between the two throughout the proofs.

In server pricing with commission fees, conditional on participation, a server with cost c_i and price p earns

$$\pi_i(p, \overline{p}) = (1 - \beta p + \gamma(\overline{p} - p))((1-\phi)p - c_i - w).$$

Server's profit depends on individual price, p, and also the average market price, $\overline{p}$, which is characterized in the equilibrium. Since each server is small, the price set by an individual server does not influence the average market price.

Server with cost c has the following pricing problem:

$$\max_{p} \big(1-\beta\overline{p}+\gamma(\overline{p}-p)\big)((1-\phi)p-c-w).$$

A server's optimal price is uniquely defined and is characterized by the first order condition. Server with cost c has an optimal price:

$$p^*(c,\overline{p}) = \frac{1}{2\gamma}\left(1+(\gamma-\beta)\overline{p}+\tfrac{(c+w)\gamma}{1-\phi}\right). \tag{20}$$

Server i earns

$$\pi_i(p^*(c_i,\overline{p}),\overline{p}) = \frac{(1-\phi-\overline{p}(1-\phi)(\beta-\gamma)-(c_i+w)\gamma)^2}{4\gamma(1-\phi)}.$$

Server profits are decreasing in cost, c_i, and therefore there exists a threshold cost, c_h, for which server with cost c participates if and only if $c \le c_h$. The highest cost that participates is

$$c_h = \frac{(1-\phi+(\gamma-\beta)(1-\phi)\overline{p})}{\gamma}-w, \tag{21}$$

which leads to zero profit.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. In line with the mean-field approach, the average price that occurs by the server's optimal decisions is consistent with their expectation of the average price. By Equation (3):

$$\begin{aligned}\overline{p} &= \frac{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\,dc}{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})\,dc}\\ &= \frac{\int_0^{c_h}\big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)p^*(c,\overline{p})\,dc}{\int_0^{c_h}\big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)\,dc}\\ &= \frac{2(1+(\gamma-\beta)\overline{p})}{3\gamma}+\frac{w}{3(1-\phi)},\end{aligned}$$

where the last line follows by plugging in expressions for $p^*(c,\overline{p})$ in Equation (20) and c_h in Equation (21).

Solving for $\overline{p}$, we have

$$\overline{p} = \frac{2(1-\phi)+w\gamma}{(2\beta+\gamma)(1-\phi)}. \tag{22}$$

Platform's profit maximization problem is:

$$\begin{aligned}\max_{\phi}\quad \Pi^{\mathcal{S}}(\phi,w) &= (\phi\overline{p}+w)\int_0^{c_h}(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p})))\,dc\\ &= \frac{9\gamma(\beta w+\phi-1)^2(w(\gamma-2\beta(\phi-1))-2(\phi-1)\phi)}{4(\phi-1)^2(2\beta+\gamma)^3}.\end{aligned}$$

.

The platform's solution is unique. It's defined by the first order conditions:

$$\frac{\partial \Pi^{\mathcal{S}}(\phi, w)}{\partial \phi} - \frac{9\gamma(\beta w + \phi - 1)\left(\beta w^2(\beta(-\phi) + \beta + \gamma) + \beta w(\phi - 2)(\phi - 1) + (\phi - 1)^2(2\phi - 1)\right)}{2(\phi - 1)^3(2\beta + \gamma)^3} = 0,$$
$$\frac{\partial \Pi^{\mathcal{S}}(\phi, w)}{\partial w} = \frac{9\gamma(\beta w + \phi - 1)\left(-6\beta\phi^2 + \phi\left(8\beta + \gamma - 6\beta^2 w\right) + (2\beta + \gamma)(3\beta w - 1)\right)}{4(1 - \phi)^2(2\beta + \gamma)^3} = 0,$$

giving the solution

$$\phi = 1 - \frac{\gamma}{2\beta}, w = \frac{\gamma - \beta}{3\beta^2}.$$

Platform's profit is

$$\Pi^{\mathcal{S}} = \frac{9\gamma(\beta w + \phi - 1)^2(w(\gamma - 2\beta(\phi - 1)) - 2(\phi - 1)\phi)}{4(\phi - 1)^2(2\beta + \gamma)^3} = \frac{1}{24\beta^2}.$$

The highest cost that participates and the average market price are

$$c_h = \frac{1}{2\beta},$$
$$\overline{p} = \frac{2}{3\beta}.$$

Total quantity of customers served in the market is

$$\begin{aligned} Q &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc \\ &= \int_0^{c_h} \left(1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p}))\right)\, dc \\ &= \frac{1}{8\beta}. \end{aligned}$$

Servers' total profits ard

$$\begin{aligned} \pi^{\mathcal{SC}} &= \int_0^{c_h} \frac{(1 - \phi - \overline{p}(1 - \phi)(\beta - \gamma) - (c_i + w)\gamma)^2}{4\gamma(1 - \phi)}\, dc \\ &= \frac{9\gamma(\beta w + \phi - 1)^3}{4(\phi - 1)(2\beta + \gamma)^3} \\ &= \frac{1}{48\beta^2}. \end{aligned} \tag{23}$$

$\square$

Proof of Proposition 4. In server pricing with commission fees, conditional on participation, a server with cost c_i earns

$$\pi_i(p(c_i), \overline{p}) = (1 - \beta p + \gamma(\overline{p} - p(c_i)))((1-\phi)p(c_i) - c_i).$$

Server's profit depends on individual price, $p(c_i)$, and also the average market price, $\overline{p}$, which is characterized in the equilibrium. Since each server is small, the price set by an individual server does not influence the average market price.

Server with cost c has the following pricing problem:

$$\max_p \big(1 - \beta\overline{p} + \gamma(\overline{p} - p)\big)((1-\phi)p - c).$$

A server's optimal price is uniquely defined and is characterized by the first order condition, giving the solution

$$p^*(c, \overline{p}) = \frac{1}{2\gamma}\left(1 + (\gamma - \beta)\overline{p} + \tfrac{c\gamma}{1-\phi}\right). \tag{24}$$

Server i earns

$$\pi_i(p^*(c_i), \overline{p}) = \frac{(1 - \phi - \overline{p}(1-\phi)(\beta - \gamma) - c_i\gamma)^2}{4\gamma(1-\phi)}.$$

Server profits are decreasing in cost, c_i, and therefore there exists a threshold cost, c_h, for which server with cost c participates if and only if $c \leq c_h$. The highest cost that participates is

$$c_h = \frac{(1-\phi)(1 + (\gamma - \beta)\overline{p})}{\gamma}, \tag{25}$$

which leads to zero profit.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. In line with the mean-field approach, the average price that occurs by the server's optimal decisions is consistent with their expectation of the average price. By Equation (3):

$$\begin{aligned}
\overline{p} &= \frac{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})p^*(c, \overline{p})\, dc}{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc} \\
&= \frac{\int_0^{c_h} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p}))\big)p^*(c, \overline{p})\, dc}{\int_0^{c_h} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p}))\big)\, dc} \\
&= \frac{2(1 + (\gamma - \beta)\overline{p})}{3\gamma},
\end{aligned}$$

where the last line follows by plugging in expressions for $p^*(c, \overline{p})$ in Equation (24) and c_h in Equation (25).

Solving for $\overline{p}$, we have

$$\overline{p} = \frac{2}{2\beta + \gamma}. \tag{26}$$

Platform's profit maximization problem is:

$$\begin{aligned}\max_{\phi} \quad \Pi^{\mathcal{SC}}(\phi, 0) &= \phi\overline{p}\int_0^{c_h}(1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p})))\, dc \\ &= \frac{9\gamma(1-\phi)\phi}{2(2\beta+\gamma)^3}.\end{aligned}$$

.

The platform's solution is unique. It's defined by the first order conditions:

$$\frac{\partial \Pi^{\mathcal{SC}}(\phi, 0)}{\partial \phi} = \frac{9\gamma(1-2\phi)}{2(2\beta+\gamma)^3} = 0,$$

giving the solution

$$\phi = \frac{1}{2}.$$

Platform's profit is

$$\Pi^{\mathcal{SC}} = \frac{9\gamma(1-\phi)\phi}{2(2\beta+\gamma)^3} = \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right).$$

The highest cost that participates and the average market price are

$$\begin{aligned}c_h &= \frac{3}{2(2\beta+\gamma)}, \\ \overline{p} &= \frac{2}{2\beta+\gamma}.\end{aligned}$$

Total quantity of customers served in the market is

$$\begin{aligned}Q &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc \\ &= \int_0^{c_h} (1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p})))\, dc \\ &= \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right).\end{aligned}$$

Servers' total profits is

$$\begin{aligned}\pi^{\mathcal{SC}} &= \int_0^{c_h} \frac{(1 - \phi - \overline{p}(1-\phi)(\beta-\gamma) - c\gamma)^2}{4\gamma(1-\phi)}\, dc \\ &= \frac{9\gamma(1-\phi)^2}{4(2\beta+\gamma)^3} \qquad (27) \\ &= \frac{9}{16}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right).\end{aligned}$$

□

Proof of Proposition 5. In server pricing with per-unit fees, conditional on participation, a server with cost c_i earns

$$\pi_i(p(c_i), \overline{p}) = (1 - \beta p + \gamma(\overline{p} - p(c_i)))(p(c_i) - c_i - w).$$

Server's profit depends on individual price, $p(c_i)$, and also the average market price, $\overline{p}$, which is characterized in the equilibrium. Since each server is small, the price set by an individual server does not influence the average market price.

Server with cost c has the following pricing problem:

$$\max_p \big(1 - \beta\overline{p} + \gamma(\overline{p} - p)\big)(p - c - w).$$

A server's optimal price is uniquely defined and is characterized by the first order condition, giving the solution

$$p^*(c, \overline{p}) = \frac{1}{2\gamma}\left(1 + (\gamma - \beta)\overline{p} + (c + w)\gamma\right). \tag{28}$$

Server i earns

$$\pi_i(p^*(c_i), \overline{p}) = \frac{(1 - \overline{p}(\beta - \gamma) - (c_i + w)\gamma)^2}{4\gamma}.$$

Server profits are decreasing in cost, c_i, and therefore there exists a threshold cost, c_h, for which server with cost c participates if and only if $c \leq c_h$. The highest cost that participates is

$$c_h = \frac{(1 + (\gamma - \beta)\overline{p})}{\gamma} - w, \tag{29}$$

which leads to zero profit.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. In line with the mean-field approach, the average price that occurs by the server's optimal decisions is consistent with their expectation of the average price. By Equation (3):

$$\begin{aligned}
\overline{p} &= \frac{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p}) p^*(c, \overline{p})\, dc}{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc} \\
&= \frac{\int_0^{c_h} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p}))\big) p^*(c, \overline{p})\, dc}{\int_0^{c_h} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p}))\big)\, dc} \\
&= \frac{2(1 + (\gamma - \beta)\overline{p}) + \gamma w}{3\gamma},
\end{aligned}$$

where the last line follows by plugging in expressions for $p^*(c, \overline{p})$ in Equation (28) and c_h in Equation (29).

Solving for $\overline{p}$, we have

$$\overline{p} = \frac{2 + \gamma w}{2\beta + \gamma}. \tag{30}$$

Platform's profit maximization problem is:

$$\begin{aligned}\max_{w} \quad \Pi^{\mathcal{SU}}(0, w) &= w \int_{0}^{c_h} (1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p})))\ dc \\ &= \frac{9\gamma w(1 - \beta w)^2}{4(2\beta + \gamma)^2}.\end{aligned}$$

.

The platform's problem is quasi-concave. The solution is unique. It's defined by the first order conditions:

$$\frac{\partial \Pi^{\mathcal{SU}}(0, w)}{\partial \phi} = \tfrac{9\gamma(1-\beta\phi)(1-3\beta\phi)}{4(2\beta+\gamma)^2} = 0,$$

giving the solution

$$\phi = \frac{2}{3\beta}.$$

Platform's profit is

$$\Pi^{\mathcal{SU}} = \frac{9\gamma w(1 - \beta w)^2}{4(2\beta + \gamma)^2} = \left(\frac{\gamma}{3\beta(2\beta + \gamma)^2}\right).$$

The highest cost that participates and the average market price are

$$\begin{aligned}c_h &= \frac{2}{(2\beta + \gamma)}, \\ \overline{p} &= \frac{6\beta + \gamma}{3\beta(2\beta + \gamma)}.\end{aligned}$$

Total quantity of customers served in the market is

$$\begin{aligned}Q &= \int_{0}^{c_h} q(p^*(c, \overline{p}), \overline{p})\ dc \\ &= \int_{0}^{c_h} (1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c, \overline{p})))\ \ dc \\ &= \left(\frac{\gamma}{(2\beta + \gamma)^2}\right).\end{aligned}$$

Servers' total profits is

$$\begin{aligned}\pi^{\mathcal{SU}} &= \int_{0}^{c_h} \frac{(1 - \overline{p}(\beta - \gamma) - (c + w)\gamma)^2}{4\gamma}\ dc \\ &= \frac{9\gamma(1 - \beta w)^3}{4(2\beta + \gamma)^3} \qquad (31) \\ &= \frac{2}{3}\left(\frac{\gamma}{(2\beta + \gamma)^3}\right).\end{aligned}$$

$\square$

Proof of Corollary 1. The platform's profits under server pricing with per-unit fee is maximized at $\gamma = 2\beta$ and is equal to $1/(24\beta^2)$.

The ratio of platform's profits under server pricing with per-unit fee and platform pricing, evaluated for the parameters *gamma*, *beta* is:

$$\frac{\Pi^{\mathcal{SU}}}{\Pi^{\mathcal{P}}}(\gamma,\beta) = \frac{\dfrac{1}{3}\left(\dfrac{\gamma}{\beta(2\beta+\gamma)^2}\right)}{\dfrac{1}{27\beta^2}} = 9\left(\frac{\beta^2\gamma}{\beta(2\beta+\gamma)^2}\right).$$

The platform prefers server pricing with unit fee over platform pricing if

$$\frac{\Pi^{\mathcal{SU}}}{\Pi^{\mathcal{P}}}(\gamma,\beta) > 1 \iff \beta < \gamma < 4\beta.$$

The platform's profits under server pricing with per-unit fee is maximized at $\gamma = \beta$ and is equal to $1/(24\beta^2)$.

The platform prefers server pricing with commission over platform pricing if

$$\frac{\Pi^{\mathcal{SC}}}{\Pi^{\mathcal{P}}}(\gamma,\beta) > 1 \iff 0.539\beta < \gamma < 1.785\beta.$$

$\square$

Proof of Corollary 2. The ratio of servers' total profits under server pricing with two components and platform pricing is:

$$\frac{\pi^{\mathcal{S}}}{\pi^{\mathcal{P}}}(\gamma,\beta) = \frac{\frac{1}{48\beta^2}}{\frac{1}{54\beta^2}} = \frac{9}{8} > 1.$$

Servers always collectively prefer server pricing with both components over platform pricing.

The ratio of servers' total profits under server pricing with two components and server pricing with commission is:

$$\frac{\pi^{\mathcal{S}}}{\pi^{\mathcal{SC}}}(\gamma,\beta) = \frac{\frac{1}{48\beta^2}}{\frac{9\gamma}{16(2\beta+\gamma)^3}} = \frac{(2\beta+\gamma)^3}{27\gamma\beta^2} \geq 1.$$

Servers always collectively prefer server pricing with both components over server pricing with commission.

The ratio of servers' total profits under server pricing with two components and server pricing with per-unit fee is:

$$\frac{\pi^{\mathcal{S}}}{\pi^{\mathcal{SU}}}(\gamma,\beta) = \frac{\frac{1}{48\beta^2}}{\frac{2\gamma}{3(2\beta+\gamma)^3}} = \frac{(2\beta+\gamma)^3}{32\gamma\beta^2}.$$

Servers collectively prefer server pricing with both components over server pricing with unit fee if

$$\frac{\pi^{\mathcal{S}}}{\pi^{\mathcal{SU}}}(\gamma,\beta) > 1 \iff 0.472\beta < \gamma < 2\beta.$$

$\square$

Proof of Corollary 3. Both platform pricing and server pricing with two components yield the same average price, $\overline{p} = 2/(3\beta)$. The ratio of quantity served under server pricing with two components and platform pricing is:

$$\frac{Q^{\mathcal{S}}}{Q^{\mathcal{P}}}(\gamma, \beta) = \frac{\frac{1}{8\beta}}{\frac{1}{9\beta}} = \frac{9}{8} > 1.$$

□

Proof of Proposition 6. Let us assume that under the platform's optimal mechansim, there exists a unique average price, $\overline{p}$, that satisfies consistent expectations equilibrium. In the optimal mechanism, let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. Let $\pi(c, p(\tilde{c}))$ be a server's earning with cost c by reporting cost $\tilde{c}$:

$$\pi(c, p(\tilde{c})) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))(p(\tilde{c}) - c) - f(\tilde{c}).$$

Let

$$u(c, p(\tilde{c})) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - c).$$

Then, server's net earnings is

$$\pi(c, p(\tilde{c})) = u(c, p(\tilde{c})) - f(\tilde{c}).$$

Notice that marginal utility from higher $p(\tilde{c})$ is increasing with cost c. Specifically,

$$\frac{\partial^2 \pi(c, p(\tilde{c}))}{\partial c \partial p(\tilde{c})} = \frac{\partial}{\partial c}\left(\frac{\partial \pi(c, p(\tilde{c}))}{\partial p(\tilde{c})}\right) = \frac{\partial}{\partial c}\left(\gamma(c + \overline{p} - 2p(\tilde{c})) - \beta\overline{p} + 1\right) = \gamma > 0. \tag{32}$$

The Individual Rationality (IR) and the Incentive Compatibility (IC) constraints are:

$$\pi(c, p(c)) \geq 0,$$
$$\pi(c, p(c)) \geq \pi(c, p(\tilde{c})),$$

for all $c \in \mathcal{C}$, $\tilde{c} \in \mathcal{C}$, where $\mathcal{C}$ is the set of server costs that participate in equilibrium.

By IC constraints,

$$\begin{aligned}\pi(c, p(c)) \geq \pi(c, p(\tilde{c})) &= (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - c) - f(\tilde{c}) \\ &> (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - \tilde{c}) - f(\tilde{c}) = \pi(\tilde{c}, p(\tilde{c}))\end{aligned}$$

for all $\tilde{c} > c$. Therefore, server earnings are strictly decreasing in cost. This implies there will exists a cost c_h such that a server with cost c participates if and only if $c \leq c_h$. Furthermore,, the server with cost c_h will earn 0 profits under the optimal mechanism. Otherwise, platform can uniformly increase the fee, $f(\tilde{c})$, for all participating servers, increasing its profit.

By IR constraints, we can alternatively formulate a server's earnings as

$$\pi(c, p(\tilde{c})) = \pi(\tilde{c}, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) + u(c, p(\tilde{c})).$$

The pair of inequalities IC constraints imposes for servers with costs c and $\tilde{c}$ are:

$$\begin{aligned}
\pi(c, p(c)) &\geq \pi(c, p(\tilde{c})) = \pi(\tilde{c}, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) + u(c, p(\tilde{c})), \\
\pi(\tilde{c}, p(\tilde{c})) &\geq \pi(\tilde{c}, p(c)) = \pi(c, p(c)) - u(c, p(c)) + u(\tilde{c}, p(c)).
\end{aligned}$$

These inequalities can be combined:

$$\begin{aligned}
& u(\tilde{c}, p(c)) - u(c, p(c)) \leq \pi(\tilde{c}, p(\tilde{c})) - \pi(c, p(c)) \leq u(\tilde{c}, p(\tilde{c})) - u(c, p(\tilde{c})) \\
\iff & \int_c^{\tilde{c}} \frac{\partial u(c_k, p(c))}{\partial c_k}\, dc_k \leq \pi(\tilde{c}, p(\tilde{c})) - \pi(c, p(c)) \leq \int_c^{\tilde{c}} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k,
\end{aligned} \tag{33}$$

where $\frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}$ is the partial derivative of u with respect to its first argument evaluated at the point $(c_k, p(\tilde{c}))$.

Ignoring the middle term, Equation (33) implies

$$\begin{aligned}
0 &\leq \int_c^{\tilde{c}} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k - \int_c^{\tilde{c}} \frac{\partial u(c_k, p(c))}{\partial c_k}\, dc_k \\
&= \int_c^{\tilde{c}} \int_{p(c)}^{p(\tilde{c})} \frac{\partial^2 u(c_k, p(c_z))}{\partial c_k \partial p(c_z)}\, dp(c_z)\, dc_k.
\end{aligned} \tag{34}$$

By Equation (32), the cross-partial derivative of u is non-negative everywhere. Given $\tilde{c} \geq c$, the expression above implies $p(\tilde{c}) \geq p(c)$ for all $\tilde{c} > c$. In an incentive compatible scheme, a lower cost server cannot have a higher price than a higher cost server. If platform sets $p(\tilde{c}) = p(c)$, then

$$\begin{aligned}
u(c, p(\tilde{c})) &= u(c, p(c)), \\
u(\tilde{c}, p(c)) &= u(\tilde{c}, p(\tilde{c})),
\end{aligned}$$

which then implies

$$\pi(\tilde{c}, p(\tilde{c})) - \pi(c, p(c)) = u(\tilde{c}, p(\tilde{c})) - u(c, p(c)) \iff f(c) = f(\tilde{c}).$$

By fixing one end point (i.e., c or $\tilde{c}$) and letting the other converge towards it in Equation (33), we can also infer that $\pi(c_k, p(c_k))$ is continuous with respect to the Lebesgue measure, and is differentiable almost everywhere. The derivative is

$$\frac{d\pi(c, p(c))}{dc} = \frac{\partial u(c, p(c))}{\partial c},$$

where $\frac{d\pi(c,p(c))}{dc}$ is the total derivative of π with respect to c evaluated at $(c, p(c))$ and $\frac{\partial u(c,p(c))}{\partial c}$ is the partial derivative of u with respect to its first argument evaluated at the point $(c, p(c))$. Then,

$$\pi(c, p(c)) + \int_{c}^{\tilde{c}} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k = \pi(\tilde{c}, p(\tilde{c})).$$

Setting $\tilde{c} = c_h$, the equation simplifies to

$$\pi(c, p(c)) = -\int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k,$$

which is equivalent to

$$f(c) = u(c, p(c)) + \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k. \tag{35}$$

Equation (35) ensures that each server's pay-off is non-negative:

$$\pi(c, p(c)) = u(c, p(c)) - f(c) \quad = -\int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k > 0,$$

since $\dfrac{\partial u(c_k, p(c_k))}{\partial c_k} < 0$ for all c_k.

Our analysis so far indicates that monotonicity of $p(\tilde{c})$ and Equation (35) are necessary conditions implied by IC. We can also show that they are sufficient conditions. With that aim, let us assume Equation (35) holds. If $\tilde{c} > c$, we have

$$\begin{aligned}
\pi(c, p(\tilde{c})) &= u(c, p(\tilde{c})) - f(\tilde{c}) \\
&= u(c, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) - \int_{\tilde{c}}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= u(c, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) - \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k + \int_{c}^{\tilde{c}} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= -\int_{c}^{\tilde{c}} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k - \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k + \int_{c}^{\tilde{c}} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= -\int_{c}^{\tilde{c}} \left(\frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k} - \frac{\partial u(c_k, p(c_k))}{\partial c_k} \right) dc_k - \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= -\int_{c}^{\tilde{c}} \int_{p(c_k)}^{p(\tilde{c})} \frac{\partial^2 u(c_k, p(c_z))}{\partial c_k \partial p(c_z)}\, dp(c_z)\, dc_k + \pi(c, p(c)) \leq \pi(c, p(c)),
\end{aligned}$$

where the inequality follows from Equation (34). If $\tilde{c} < c$, we have

$$
\begin{aligned}
\pi(c, p(\tilde{c})) &= u(c, p(\tilde{c})) - f(\tilde{c}) \\
&= u(c, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) - \int_{\tilde{c}}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= u(c, p(\tilde{c})) - u(\tilde{c}, p(\tilde{c})) - \int_{\tilde{c}}^{c} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k - \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= \int_{\tilde{c}}^{c} \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k}\, dc_k - \int_{\tilde{c}}^{c} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k - \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= -\int_{\tilde{c}}^{c} \left(\frac{\partial u(c_k, p(c_k))}{\partial c_k} - \frac{\partial u(c_k, p(\tilde{c}))}{\partial c_k} \right) dc_k - \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \\
&= -\int_{\tilde{c}}^{c} \int_{p(\tilde{c})}^{p(c_k)} \frac{\partial^2 u(c_k, p(c_z))}{\partial c_k \partial p(c_z)}\, dp(c_z)\, dc_k + \pi(c, p(c)) \leq \pi(c, p(c)),
\end{aligned}
$$

where the inequality follows from Equation (34). Hence, the conditions above are sufficient for IC.

The platform's problem is to choose $p(c)$, $f(c)$ and c_h to maximize total profits subject to IR and IC constraints, and the natural restriction that the equilibrium demand of a server needs to be non-negative:

$$
\begin{aligned}
&\max_{p(c), f(c), c_h} && \int_0^{c_h} f(c)\, dc \\
&\text{s.t.} && \pi(c, p(c)) \geq \pi(c, p(\tilde{c})), \forall c \in (0, c_h), \forall \tilde{c} \in (0, c_h) \\
& && \pi(c, p(c)) \geq 0, \forall c \in (0, c_h) \\
& && \text{Eq. (3)} \\
= &\max_{p(c), f(c), c_h} && \int_0^{c_h} f(c)\, dc \\
&\text{s.t.} && p'(c) \geq 0, \forall c \in (0, c_h) \\
& && f(c) = u(c, p(c)) + \int_c^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k, \forall c \in (0, c_h) \\
& && 1 - \beta \overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in (0, c_h) \\
& && \text{Eq. (3)} \\
= &\max_{p(c), c_h} && \int_0^{c_h} \left(u(c, p(c)) + \int_c^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\, dc_k \right) dc \\
&\text{s.t.} && p'(c) \geq 0, \forall c \in (0, c_h) \\
& && 1 - \beta \overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in (0, c_h) \\
& && \text{Eq. (3).}
\end{aligned}
$$

By integration by parts,

$$\int_0^{c_h}\left(\int_c^{c_h}\frac{\partial u(c_k,p(c_k))}{\partial c_k}\,dc_k\right)dc=\left[\left(\int_c^{c_h}\frac{\partial u(c_k,p(c_k))}{\partial c_k}\,dc_k\right)c\right]_0^{c_h}-\int_0^{c_h}\left(-\frac{\partial u(c,p(c))}{\partial c}\right)c\,dc$$
$$=\int_0^{c_h}\frac{\partial u(c,p(c))}{\partial c}c\,dc. \tag{36}$$

The platform's problem converts to

$$\begin{aligned}
&\max_{p(c),c_h} && \int_0^{c_h}\left(u(c,p(c))+\frac{\partial u(c,p(c))}{\partial c}c\right)\,dc\\
&\text{s.t.} && p'(c)\geq 0,\forall c\in(0,c_h)\\
& && 1-\beta\overline{p}+\gamma(\overline{p}-p(c))\geq 0,\forall c\in(0,c_h)\\
& && \text{Eq. (3)}\\
=&\max_{p(c),c_h} && \int_0^{c_h}\Big((1-\beta\overline{p}+\gamma(\overline{p}-p(c)))(p(c)-c)-(1-\beta\overline{p}+\gamma(\overline{p}-p(c)))c\Big)\,dc\\
&\text{s.t.} && p'(c)\geq 0,\forall c\in(0,c_h)\\
& && 1-\beta\overline{p}+\gamma(\overline{p}-p(c))\geq 0,\forall c\in(0,c_h)\\
& && \text{Eq. (3)}\\
=&\max_{p(c),c_h} && \int_0^{c_h}(1-\beta\overline{p}+\gamma(\overline{p}-p(c)))(p(c)-2c)\,dc\\
&\text{s.t.} && p'(c)\geq 0,\forall c\in(0,c_h)\\
& && 1-\beta\overline{p}+\gamma(\overline{p}-p(c))\geq 0,\forall c\in(0,c_h)\\
& && \text{Eq. (3)}.
\end{aligned}$$

Let us re-formulate this as a problem of quantities. Let $q(c)$ be the demand platform attains to server with cost c. By Equation (1), we can find a one-to-one equivalence between price and quantity:

$$p(c)=\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}.$$

The average price in Equation (3) can also be formulated as function of quantities:

$$\begin{aligned}
\overline{p}&=\frac{\int_0^{c_h}\frac{q(c)(1+(\gamma-\beta)\overline{p}-q(c))}{\gamma}\,dc}{\int_0^{c_h}q(c)\,dc}\\
&=\frac{\int_0^{c_h}\frac{q(c)(1-q(c))}{\gamma}\,dc}{\int_0^{c_h}q(c)\,dc}+\overline{p}\left(1-\frac{\beta}{\gamma}\right),
\end{aligned}$$

which simplifies to

$$\overline{p}=\frac{\int_0^{c_h}q(c)\,(1-q(c))\;dc}{\beta\int_0^{c_h}q(c)\,dc}. \tag{37}$$

Platform's optimal quantity-choice problem is

$$\begin{aligned}
&\max_{q(c),c_h} \int\limits_0^{c_h} q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-2c\right)\,dc \\
&\text{s.t. } q'(c)\le 0, \forall c\in(0,c_h) \\
&\qquad q(c)\ge 0, \forall c\in(0,c_h) \\
&\qquad \text{Eq. (37).}
\end{aligned} \tag{38}$$

We can further simplify the objective function. Notice that Equation (37) implies:

$$\overline{p}\int_0^{c_h} q(c)\,dc = \frac{1}{\beta}\int_0^{c_h} q(c)\left(1-q(c)\right)\,dc.$$

Using this relationship, the platform's objective function can be reformulated as

$$\begin{aligned}
\Pi &= \int\limits_0^{c_h} q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-2c\right)\,dc \\
&= \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\gamma}-2c\right)\,dc + \left(\frac{\gamma-\beta}{\gamma}\right)\overline{p}\int\limits_0^{c_h} q(c)\,dc \\
&= \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\gamma}-2c\right)\,dc + \left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta}\int_0^{c_h} q(c)\left(1-q(c)\right)\,dc \\
&= \int\limits_0^{c_h}\left(q(c)\left(\frac{1-q(c)}{\gamma}-2c\right)+\left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta}q(c)\left(1-q(c)\right)\right)\,dc \\
&= \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc.
\end{aligned} \tag{39}$$

The problem terms no longer depend on the average price and we can drop Equation (37) from the constraints. Problem is re-formulated:

$$\begin{aligned}
&\max_{q(c),c_h} \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc \\
&\text{s.t. } q'(c)\le 0, \forall c\in(0,c_h) \\
&\qquad q(c)\ge 0, \forall c\in(0,c_h).
\end{aligned}$$

Holding c_h constant and relaxing the first constraint, we can decompose the problem into individual sub-problems for all servers:

$$\begin{aligned}
&\max_{q(c)} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc \\
&\text{s.t. } q(c)\ge 0.
\end{aligned}$$

This is the maximization of a simple quadratic function with a linear constraint. The optimal quantity is:

$$q^*(c) = \max\left\{0, \frac{1}{2} - \beta c\right\}.$$

The objective values of the sub-problems are always strictly positive for all servers with positive quantities. Therefore, platform is always better off hiring more server as long as there is a server in the market that can generate non-negative demand:

$$c_h = \min\{1, \max\{c : q^*(c) \geq 0\}\} \implies c_h = \min\left\{1, \frac{1}{2\beta}\right\}.$$

This solution also satisfies:

$$q^{*\prime}(c) = \max\{-\beta, 0\} \leq 0.$$

Therefore, the solution our relaxed problem is also optimal for the platform's optimal mechanism.

If $\beta \geq \frac{1}{2}$, then only a subset of servers participate in the equilibrium. Instead, if $\beta < \frac{1}{2}$, platform hires all the servers in the market. The average market price is:

$$\begin{aligned}\overline{p} &= \frac{\int_0^{c_h} q^*(c)\,(1 - q^*(c))\; dc}{\beta \int_0^{c_h} q^*(c)\, dc} \\ &= \begin{cases} \frac{2}{3\beta} & \text{, if } \beta \geq \frac{1}{2}, \\ \frac{3-4\beta^2}{6\beta-6\beta^2} & \text{, if } \beta < \frac{1}{2}. \end{cases}\end{aligned}$$

Other equilibrium characteristics are as follows:

$$\begin{aligned}p^*(c) &= \frac{1 + (\gamma - \beta)\overline{p} - q^*(c)}{\gamma} \\ &= \begin{cases} \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{c\beta}{\gamma} & \text{, if } \beta \geq \frac{1}{2}, \\ \frac{\beta^2(-4\beta+4\gamma+3)-3\gamma}{6(\beta-1)\beta\gamma} + \frac{c\beta}{\gamma} & \text{, if } \beta < \frac{1}{2}, \end{cases} \\ f^*(c) &= u(c, p^*(c)) + \int\limits_c^{c_h} \frac{\partial u(c_k, p^*(c_k))}{\partial c_k}\; dc_k \\ &= \begin{cases} c^2\left(\frac{\beta(\gamma-2\beta)}{2\gamma}\right) + \frac{2c}{3}\left(\frac{\beta}{\gamma} - 1\right) + \frac{1}{24}\left(\frac{5}{\beta} - \frac{2}{\gamma}\right) & \text{, if } \beta \geq \frac{1}{2}, \\ \frac{\beta c^2}{2}\left(1 - \frac{2\beta}{\gamma}\right) + \frac{c}{6\gamma}\left(\frac{(4\beta^2-3)(\beta-\gamma)}{1-\beta}\right) + \frac{1}{12\beta\gamma}\left(\frac{\beta(\beta(\beta(6\gamma-4)-8\gamma+3)+6\gamma)-3\gamma}{\beta-1}\right) & \text{, if } \beta < \frac{1}{2}. \end{cases}\end{aligned}$$

The platform earns

$$\begin{aligned}\Pi &= \int\limits_0^{c_h} q^*(c)\left(\frac{1 - q^*(c)}{\beta} - 2c\right)\; dc \\ &= \begin{cases} \frac{1}{24\beta^2} & \text{, if } \beta \geq \frac{1}{2}, \\ \frac{1}{12}\left(4\beta + \frac{3}{\beta} - 6\right) & \text{, if } \beta < \frac{1}{2}. \end{cases}\end{aligned}$$

By Proposition 3, the optimal mechanism is equivalent to server pricing with commission and per-unit fee, for which Proposition 2 guarantees the uniqueness of consistent expectations average price equilibrium.

Total quantity of customers served in the market is

$$\begin{aligned} Q &= \int_0^{c_h} q^*(c)\, dc \\ &= \begin{cases} \frac{1}{8\beta} & \text{, if } \beta \geq \frac{1}{2}, \\ \frac{1-\beta}{2} & \text{, if } \beta < \frac{1}{2}. \end{cases} \end{aligned}$$

Servers' total profits is

$$\begin{aligned} \pi &= \int_0^{c_h} ((1 - \beta\overline{p} + \gamma(\overline{p} - p^*(c)))(p^*(c) - c) - f^*(c))\, dc \\ &= \begin{cases} \dfrac{1}{48\beta^2} & \text{, if } \beta \geq \frac{1}{2}, \\ \frac{1}{4} - \frac{\beta}{3} & \text{, if } \beta < \frac{1}{2}. \end{cases} \end{aligned}$$

□

Proof of Proposition 7. Notice that platform's profits under server pricing with a commission and per-unit fee, $\Pi^{\mathcal{S}} = 1/(24\beta^2)$ is equivalent to its profits under the optimal mechanism, $\pi = 1/(24\beta^2)$. Hence, platform can replicate its optimal performance through server pricing.

□

Proof of Proposition 8. Let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee charged to server c to participate in the market. Unlike the centralized mechanism, the fee collected is not necessarily retained by the platform. Instead, $f(c)$ functions as a re-allocation lever that re-distributes wealth among servers. Since additional money cannot be injected into the system, total fee charged should be equal to 0:

$$\int_0^{c_h} f(c)\, dc = 0.$$

As a consequence of this $f(c)$ can take either negative or positive values, meaning some servers may retain more money than they generate, and others retain less money than they generate.

By Proposition 6, the monotonicy of prices, $p(c)$, and Equation (35) are necessary and sufficient conditions for servers' IR and IC constraints. Then, the equilibrium fees charged to server c is characterized as

$$f(c) = u(c, p(c)) + \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k} \, dc_k.$$

The total system profits generated is

$$\int_{0}^{c_h} q(p(c), \overline{p})(p(c) - c) \, dc.$$

The optimal truth-inducing contract that maximizes total system profits is characterized through the following problem:

$$\begin{aligned}
\max_{p(c), c_h} \quad & \int_{0}^{c_h} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - c) \, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in (0, c_h) \\
& f(c) = u(c, p(c)) + \int_{c}^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k} \, dc_k, \forall c \in (0, c_h) \\
& \int_{0}^{c_h} f(c) \, dc = 0 \\
& 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)) \geq 0, \forall c \in (0, c_h) \\
& \text{Eq. (3).}
\end{aligned}$$

Let us relax the second and third constraints, and similar to the proof of Proposition 6, characterize the platform's problem as a function of quantities.

$$\begin{aligned}
\max_{q(c), c_h} \int_{0}^{c_h} & q(c) \left(\frac{1 + (\gamma - \beta)\overline{p} - q(c)}{\gamma} - c \right) \, dc \\
\text{s.t.} \;\; & q'(c) \leq 0, \forall c \in (0, c_h) \\
& q(c) \geq 0, \forall c \in (0, c_h) \\
& \text{Eq. (37).}
\end{aligned}$$

We can further simplify the objective function. Notice that Equation (37) implies:

$$\overline{p} \int_{0}^{c_h} q(c) \, dc = \frac{1}{\beta} \int_{0}^{c_h} q(c) \left(1 - q(c)\right) \, dc.$$

Using the relationship, the platform's objective function can be reformulated as

$$
\begin{aligned}
\Pi &= \int\limits_0^{c_h} q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-c\right)\,dc \\
&= \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\gamma}-c\right)\,dc + \left(\frac{\gamma-\beta}{\gamma}\right)\overline{p}\int\limits_0^{c_h} q(c)\,dc \\
&= \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\gamma}-c\right)\,dc + \left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta}\int_0^{c_h} q(c)\left(1-q(c)\right)\,dc \\
&= \int\limits_0^{c_h} \left(q(c)\left(\frac{1-q(c)}{\gamma}-c\right) + \left(\frac{\gamma-\beta}{\gamma}\right)\frac{1}{\beta}q(c)\left(1-q(c)\right)\right)\,dc \\
&= \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\beta}-c\right)\,dc.
\end{aligned}
$$

We can drop Equation (37) from the constraints. Problem is re-formulated:

$$
\begin{aligned}
&\max_{q(c),c_h} \int\limits_0^{c_h} q(c)\left(\frac{1-q(c)}{\beta}-c\right)\,dc \\
&\text{s.t. } q'(c) \le 0, \forall c \in (0, c_h) \\
&\quad q(c) \ge 0, \forall c \in (0, c_h).
\end{aligned}
$$

Holding c_h constant and relaxing the first constraint, we can decompose the problem into individual sub-problems for all servers:

$$
\begin{aligned}
&\max_{q(c),c_h} q(c)\left(\frac{1-q(c)}{\beta}-c\right)\,dc \\
&\text{s.t. } q(c) \ge 0, \forall c \in (0, c_h).
\end{aligned}
$$

This is the maximization of a simple quadratic function with a linear constraint. The optimal quantity is:

$$
q^*(c) = \max\left\{0, \frac{1}{2}\left(1-\beta c\right)\right\}.
$$

The objective values of the sub-problems are always strictly positive for all servers with positive quantities. Therefore, platform is always better off hiring more server as long as the server has non-negative demand:

$$
q^*(c_h) = 0 \implies c_h = \frac{1}{\beta}.
$$

Assuming an interior equilibrium ($c_h < 1$), the average price is

$$
\begin{aligned}
\overline{p} &= \frac{\int_0^{c_h} q^*(c)\left(1-q^*(c)\right)\,dc}{\beta\int_0^{c_h} q^*(c)\,dc} \\
&= \frac{2}{3\beta}.
\end{aligned}
$$

The optimal price for server with cost c is

$$p^*(c) = \frac{1 + (\gamma - \beta)\overline{p} - q^*(c)}{\gamma}$$
$$= \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{\beta}{2\gamma}c.$$

The subsidy a server with cost c contributes to the system is

$$f^*(c) = u(c, p^*(c)) + \int_c^{c_h} \frac{\partial u(c_k, p^*(c_k))}{\partial c_k}\, dc_k$$
$$= \frac{1}{12}\left(\frac{1}{\beta} - \frac{1}{\gamma}\right) + \frac{1}{3}\left(\frac{\beta}{\gamma} - 1\right)c + \left(\frac{\beta(\gamma - \beta)}{4\gamma}\right)c^2.$$

This solution also satisfies:

$$q^{*\prime}(c) = \max\left\{-\frac{1}{2}\beta, 0\right\} \leq 0$$

and

$$\int_0^{c_h} f^*(c)\, dc = 0.$$

Therefore, the solution our relaxed problem is also feasible for the optimal mechanism.

Total value generated under optimal mechanism is

$$\Pi + \pi = \int_0^{c_h} q^*(c)\left(\frac{1 - q^*(c)}{\beta} - c\right)\, dc$$
$$= \frac{1}{12\beta^2}.$$

With server pricing, let the platform sets its terms,

$$\phi = 1 - \frac{\gamma}{\beta},\ w = \frac{2(\gamma - \beta)}{3\beta^2},$$

and assume servers expect an average price of

$$\overline{p} = \frac{2}{3\beta}.$$

A server with cost c has a profit-maximizing problem of

$$\max_p (1 - \beta\overline{p} + \gamma(\overline{p} - p))((1 - \phi)p - c - w),$$

giving an optimal price of

$$p^*(c, \overline{p}) = \frac{1}{2}\left(\frac{1 - (\beta - \gamma)\overline{p}}{\gamma} + \frac{c + w}{1 - \phi}\right)$$
$$= \frac{4\gamma - \beta + 3\beta^2 c}{4\beta\gamma}.$$

Let c_h be the highest cost server that can participate with non-negative demand:

$$q(p^*(c_h, \overline{p}), \overline{p}) = 0 \implies c_h = \frac{1}{\beta}.$$

The realized average price is consistent with expectation

$$\begin{aligned}\overline{p} &= \frac{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p}) p^*(c, \overline{p})\, dc}{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc} \\ &= \frac{2}{3\beta},\end{aligned}$$

consistent with expectation.

The platform's profit is

$$\begin{aligned}\Pi &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})(\phi p^*(c, \overline{p}) + w)\, dc \\ &= 0.\end{aligned}$$

The servers' total profits is

$$\begin{aligned}\pi &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})((1-\phi) p^*(c, \overline{p}) - c - w)\, dc \\ &= \frac{1}{12\beta^2}.\end{aligned}$$

Total value generated is

$$\Pi + \pi = \frac{1}{12\beta^2},$$

same as the optimal mechanism.

□

Proof of Proposition 9. The equilibrium is defined similar to server pricing with $\phi = 0$, $w = 0$. Server i chooses a price

$$\begin{aligned}p^*(c_i) &= \frac{1}{2\gamma}\left(1 + (\gamma - \beta)\overline{p} + \frac{c_i \gamma}{1-\phi}\right) \\ &= \frac{3}{2(2\beta + \gamma)} + \frac{1}{2} c_i\end{aligned}$$

and earns

$$\pi_i(p^*(c_i), \overline{p}) = \frac{(1 - \overline{p}(\beta - \gamma) - c_i\gamma)^2}{4\gamma}.$$

Highest cost that participates in the market is

$$c_h = \frac{1 + (\gamma - \beta)\overline{p}}{\gamma}.$$

The average market price is

$$\overline{p} = \frac{2}{2\beta + \gamma},$$

equivalent to server pricing $\phi = 0$, $w = 0$.

The total system profits is

$$\begin{aligned}\Pi^{\mathcal{DS}} + \pi^{\mathcal{DS}} &= \int_0^{c_h} q(p^*(c_i), \overline{p})(p^*(c_i) - c_i)\, dc_i \\ &= \frac{9}{4}\left(\frac{\gamma}{(2\beta + \gamma)^3}\right).\end{aligned}$$

The ratio of total system profits under disintermediated server pricing and server pricing with commission and per-unit fee is:

$$\frac{\Pi^{\mathcal{DS}} + \pi^{\mathcal{DS}}}{\Pi^{\mathcal{S}} + \pi^{\mathcal{S}}}(\gamma, \beta) = \frac{\frac{9}{4}\left(\frac{\gamma}{(2\beta + \gamma)^3}\right)}{\frac{3}{48\beta^2}} = \frac{36\beta^2\gamma}{(2\beta + \gamma)^3}.$$

The ratio of prices is increasing in γ for $\gamma < \beta$ and decreasing for $\gamma > \beta$. Therefore, the ratio of prices are quasi-concave in γ and is maximized at $\gamma = \beta$, where it takes a value of 4/3. At two extremes, where γ approaches 0 or infinity, the ratio converges to 0. Then, by intermediate value theorem, there exists some $0 < \gamma_l < \beta$ and $\gamma_h > \beta$ such that

$$\begin{aligned}\frac{\Pi^{\mathcal{DS}} + \pi^{\mathcal{DS}}}{\Pi^{\mathcal{S}} + \pi^{\mathcal{S}}}(\gamma_l, \beta) = 1, \\ \frac{\Pi^{\mathcal{DS}} + \pi^{\mathcal{DS}}}{\Pi^{\mathcal{S}} + \pi^{\mathcal{S}}}(\gamma_h, \beta) = 1.\end{aligned}$$

By quasi-concavity, we can further conclude that:

$$\frac{\Pi^{\mathcal{DC}} + \pi^{\mathcal{DC}}}{\Pi^{\mathcal{S}} + \pi^{\mathcal{S}}}(\gamma, \beta) > 1 \iff \gamma_l < \gamma < \gamma_h.$$

□

Extension of Platform's Optimal Contract with a Total Server Profits Target. Following the proof of Proposition 6, the platform's optimal mechanism design problem is defined by Equation

(38), with the added constraint that the total profits earned by servers is equal to or exceeds some k:

$$\begin{aligned}
\int_0^{c_h} \pi(c,p(c))\,dc &= \int_0^{c_h} u(c,p(c))\,dc - \int_0^{c_h} f(c)\,dc \\
&= \int_0^{c_h} q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-c\right)\,dc - \int_0^{c_h} q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-2c\right)\,dc \\
&= \int_0^{c_h} q(c)c\,dc \geq k.
\end{aligned}$$

The platform's problem is

$$\begin{aligned}
\max_{q(c),c_h} \int_0^{c_h} & q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-2c\right)\,dc \\
\text{s.t. } & q'(c) \leq 0, \forall c \in (0,c_h) \\
& q(c) \geq 0, \forall c \in (0,c_h) \\
& \int_0^{c_h} q(c)c\,dc \geq k \\
& \text{Eq. (37).}
\end{aligned}$$

The third constraint is not binding for $k \leq \frac{1}{48\beta^2}$, as that's the total profit servers earn under the optimal mechanism. Similarly, the maximum profits servers can earn in the system is $\frac{1}{12\beta^2}$. Therefore, our problem is feasible and bounded by profit constraint if and only if $k \in \left[\frac{1}{24\beta^2}, \frac{1}{12\beta^2}\right]$. In that case, the platform's problem is

$$\begin{aligned}
\max_{q(c),c_h} \int_0^{c_h} & q(c)\left(\frac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma}-2c\right)\,dc \\
\text{s.t. } & q'(c) \leq 0, \forall c \in (0,c_h) \\
& q(c) \geq 0, \forall c \in (0,c_h) \\
& \int_0^{c_h} q(c)c\,dc = k \\
& \text{Eq. (37).}
\end{aligned}$$

Following the same steps as Equation (39), we can transform the objective function such that it doesn't depend on $\overline{p}$:

$$\max_{q(c),c_h} \int_0^{c_h} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc$$

$$\text{s.t. } q'(c) \leq 0, \forall c \in (0, c_h)$$

$$q(c) \geq 0, \forall c \in (0, c_h)$$

$$\int_0^{c_h} q(c)c\,dc = k.$$

Let us fix c_h, relax the first two constraints. We can use calculus of variations to solve this problem. The Lagrangian is

$$L = \int_0^{c_h} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc + \lambda\left(\int_0^{c_h} q(c)c\,dc - k\right).$$

The Euler-Lagrange equation is

$$\frac{\partial}{\partial q(c)}\left(q(c)\left(\frac{1-q(c)}{\beta}-2c\right)+\lambda q(c)c\right) = 0,\ \forall c \in (0, c_h).$$

This gives

$$q^*(c) = \frac{1}{2} - \left(1-\frac{\lambda}{2}\right)\beta c\ \forall c \in (0, c_h). \tag{40}$$

We plug this into the constraint to find λ. We get a single solution

$$\int_0^{c_h} q^*(c)c\,dc = k \iff \lambda = \frac{4\beta c_h^3 - 3c_h^2 + 12k}{2\beta c_h^3}.$$

This gives an optimal objective of

$$\int_0^{c_h} q^*(c)\left(\frac{1-q^*(c)}{\beta}-2c\right)\,dc = \frac{c_h^4 + 24c_h^2 k - 48k^2}{16\beta c_h^3} - 2k.$$

To show that this maximizer is a global maximizer, let us consider any feasible deviation $z(c)$ from the optimal solution. In order to remain feasible, the deviation, $z(c)$, needs to satisfy:

$$\int_0^{c_h} z(c)c\,dc = 0.$$

The new objective is:

$$\int_0^{c_h} (q^*(c)+z(c))\left(\frac{1-(q^*(c)+z(c))}{\beta}-2c\right)\,dc = \frac{c_h^4+24c_h^2k-48k^2}{16\beta c_h^3}-2k$$

$$-\frac{\left(c_h^2(3-4\beta c_h)-12k\right)}{2\beta c_h^3}\int_0^{c_h} z(c)c\,dc-\frac{1}{\beta}\int_0^{c_h} z(c)^2\,dc$$

$$=\frac{c_h^4+24c_h^2k-48k^2}{16\beta c_h^3}-2k-\frac{1}{\beta}\int_0^{c_h} z(c)^2\,dc$$

$$\leq \frac{c_h^4+24c_h^2k-48k^2}{16\beta c_h^3}-2k.$$

This means, any feasible deviation from the optimal $q^*(c)$ defined above makes the objective worse off. Hence, Equation (40) defines the optimal solution to the relaxed quantity choice problem. Notice that the resulting objective function is monotone increasing in c_h and it satisfies the first inequality. However, for sufficiently large c_h values, some of the quantities can be negative. More specifically, that occurs if $c_h > 2\sqrt{3}\sqrt{k}$. We will now show that, in the non-relaxed problem, the optimal quantity will be equal to Equation (40) for those with non-negative $q(c)$ and 0 otherwise. As a result platform's profit will be

$$\frac{\sqrt{k}}{\sqrt{3}\beta}-2k.$$

First, recall that a server's profit under relaxed problem is strictly increasing in c_h. This means that, as long as the highest cost participant has positive demand, platform is better off hiring more servers. There cannot be an equilibrium where the highest cost participate servers strictly positive demand. What is not clear is whether it is indeed optimal for platform to set quantities of 0 for servers with cost higher than $2\sqrt{3}\sqrt{k}$. To see this is true, let us look at what happens if the platform deviates from that solution by some function $z(c)$ such that:

$$\int_0^1 z(c)c\,dc = 0,$$

$$z(c)\geq 0,\quad \forall c > 2\sqrt{3}\sqrt{k}.$$

The new objective is:

$$\int_0^1 (q(c)+z(c))\left(\frac{1-(q(c)+z(c))}{\beta}-2c\right)\,dc = \int_0^{2\sqrt{3}\sqrt{k}} (q(c)+z(c))\left(\frac{1-(q(c)+z(c))}{\beta}-2c\right)\,dc$$
$$+\int_{2\sqrt{3}\sqrt{k}}^1 z(c)\left(\frac{1-z(c)}{\beta}-2c\right)\,dc$$
$$=\frac{\sqrt{k}}{\sqrt{3}\beta}-2k-\int_{2\sqrt{3}\sqrt{k}}^1\left(\frac{2\beta c-1}{\beta}\right)z(c)\,dc-\frac{1}{\beta}\int_{2\sqrt{3}\sqrt{k}}^1 z(c)^2\,dc$$
$$-\frac{\left(c_h^2(4\beta c_h-3)+12k\right)}{2\beta c_h^3}\int_0^{2\sqrt{3}\sqrt{k}} z(c)c\,dc-\frac{1}{\beta}\int_0^{2\sqrt{3}\sqrt{k}} z(c)^2\,dc$$
$$=\frac{\sqrt{k}}{\sqrt{3}\beta}-2k+\int_{2\sqrt{3}\sqrt{k}}^1\left(\frac{\left(6-\frac{\sqrt{3}c}{\sqrt{k}}\right)}{6\beta}\right)z(c)\,dc-\frac{1}{\beta}\int_0^1 z(c)^2\,dc$$
$$\le\frac{\sqrt{k}}{\sqrt{3}\beta}-2k.$$

Any deviations from the solution we found makes the platform worse off. The platform's profit under this mechanism is

$$\Pi=\frac{\sqrt{k}}{\sqrt{3}\beta}-2k.$$

The platform can earn non-negative profits for all feasible values of $k\in\left[0,\frac{1}{12\beta^2}\right]$.

$\square$

Proof of Proposition 10. Let us assume that servers have a capacity of t customers.

Under platform pricing, let us continue from the proof of proposition 1. The highest number of customers a server serves is

$$q(\overline{p},\overline{p})=1-\beta\overline{p}=\frac{1}{3}.$$

The capacity is binding if and only if $t\le\frac{1}{3}$. In this case, if platform sets a price $\overline{p}$, the profit of a server with cost c is

$$\pi_i(\overline{p})=t((1-\phi)\overline{p}-c_i).$$

The highest cost that participates is

$$\pi_h(\overline{p})=0\implies c_h=(1-\phi)\overline{p}.$$

The platform's profits is:

$$\begin{aligned}\Pi^{\mathcal{P}}(\overline{p},\phi) &= \int_0^{c_h} t((1-\phi)\overline{p})\ dc \\ &= t\phi\overline{p}(1-\phi)\overline{p}.\end{aligned} \tag{41}$$

It's monotone increasing in p. So, the platform never chooses a price below the amount that sets its demand to t. It also cannot set its price above that, since that means that the capacity is not binding, in which case our optimal solution from the unconstrained problem hold. So, the optimal price in a capacity constrained setting is the one that sets the demand of each server exactly equal to their capacity:

$$q(p^*(c,\overline{p}),\overline{p}) = 1-\beta\overline{p}+\gamma(\overline{p}-\overline{p}) = t \implies \overline{p} = \frac{1-t}{\beta}.$$

For a given p, the platform's optimal commission is given by the FOC:

$$\frac{\partial}{\partial\phi}(t\phi p(1-\phi)p) = tp(1-2\phi)p = 0 \implies \phi = \frac{1}{2}.$$

Platform's optimal profit is

$$\Pi^{\mathcal{P}} = \frac{t(1-t)^2}{4\beta^2}.$$

Under server pricing with commission fees, let us continue from the proof of proposition 4. Since the equilibrium demand is monotone decreasing in cost, the highest number of customers a server serves is

$$q(p^*(0,\overline{p}),\overline{p}) = 1-\beta p^*(0,\overline{p})+\gamma(\overline{p}-p^*(0,\overline{p})) = \frac{3\gamma}{4\beta+2\gamma}.$$

The capacity is binding if and only if $t \le \frac{3\gamma}{4\beta+2\gamma}$. In this case, the servers need to incorporate the capacity constraint in their pricing decision. A server's profit conditional on participation is

$$\pi_i(p(c_i),\overline{p}) = \min\{(1-\beta\overline{p}+\gamma(\overline{p}-p(c_i)), t\}((1-\phi)p(c_i)-c_i).$$

Server c sets the price that maximizes their own profits:

$$\max_p \min\{(1-\beta\overline{p}+\gamma(\overline{p}-p), t\}((1-\phi)p-c).$$

Without the capacity constraints, server's problem is concave in its decision variable. The optimal price is unique:

$$p^*(c,\overline{p}) = \frac{1}{2}\left(-\frac{c}{\phi-1}+\frac{1}{\gamma}-\frac{\beta\overline{p}}{\gamma}+\overline{p}\right).$$

This is equivalent to a quantity of

$$q(p^*(c,\overline{p}),\overline{p}) = \frac{1}{2}\left(\frac{c\gamma}{\phi-1}+\overline{p}(\gamma-\beta)+1\right).$$

If a server is capacity constrained ($t < q(p^*(c,\overline{p}),\overline{p})$), then a server's profit is increasing in own price. So, a server never sets a price lower than the amount that sets its demand equal to the capacity. Hence, the optimal price of a server that is constrained by capacity is:

$$p^*(c,\overline{p}) = \frac{-\beta\overline{p} + \gamma\overline{p} - t + 1}{\gamma}.$$

The quantity served is decreasing in costs. The lowest cost server that is not capacity constrained has exactly a demand of t units.

$$q(p^*(\tilde{c},\overline{p}),\overline{p}) = t \implies \tilde{c} = \frac{(\phi-1)(\overline{p}(\beta-\gamma)+2t-1)}{\gamma}.$$

Servers' equilibrium prices are

$$p^*(c,\overline{p}) = \begin{cases} \frac{-\beta\overline{p}+\gamma\overline{p}-t+1}{\gamma}, & c \le \tilde{c}, \\ \frac{1}{2}\left(-\frac{c}{\phi-1} + \frac{1}{\gamma} - \frac{\beta\overline{p}}{\gamma} + \overline{p}\right), & \tilde{c} < c \le c_h. \end{cases}$$

Servers with cost higher than c_h cannot profitably participate.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. In line with the mean-field approach, the average price that occurs by the server's optimal decisions is consistent with their expectation of the average price. By Equation (3):

$$\begin{aligned}
\overline{p} &= \frac{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\,dc}{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})\,dc} \\
&= \frac{\int_0^{c_h}\big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)p^*(c,\overline{p})\,dc}{\int_0^{c_h}\big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)\,dc} \\
&= \frac{\int_0^{\tilde{c}} tp^*(c,\overline{p})\,dc + \int_{\tilde{c}}^{c_h}\big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)p^*(c,\overline{p})\,dc}{\int_0^{\tilde{c}} t\,dc + \int_{\tilde{c}}^{c_h}\big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)\,dc} \\
&= \frac{3(\overline{p}(\gamma-\beta)+1)^2 + 6t(\overline{p}(\beta-\gamma)-1) + 4t^2}{3\gamma(\overline{p}(\gamma-\beta)-t+1)}.
\end{aligned}$$

There is a single feasible equilibrium average price

$$\overline{p} = \frac{\sqrt{-12\beta^2t^2 + 12\beta\gamma t^2 + 9\gamma^2(t-1)^2} + 6\beta(t-1) - 3\gamma(t-1)}{6\beta(\beta-\gamma)}. \tag{42}$$

Platform's profit maximization problem is:

$$\begin{aligned}
\max_{\phi} \quad \Pi^{\mathcal{SC}}(\phi,0) &= \phi\overline{p}\int_0^{c_h}(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p})))\,dc \\
&= \frac{t\left(-t\sqrt{-12\beta^2t^2+12\beta\gamma t^2+9\gamma^2(t-1)^2}+\sqrt{-12\beta^2t^2+12\beta\gamma t^2+9\gamma^2(t-1)^2}+2\beta t^2+3\gamma(t-1)^2\right)}{6\beta^2\gamma}(1-\phi)\phi.
\end{aligned}$$

.

The platform's solution is unique. It's defined by the first order conditions and is equal to

$$\phi = \frac{1}{2}.$$

Platform's profit is

$$\Pi^{\mathcal{SC}}(\phi, 0) = \frac{t\left(-t\sqrt{-12\beta^2 t^2 + 12\beta\gamma t^2 + 9\gamma^2(t-1)^2} + \sqrt{-12\beta^2 t^2 + 12\beta\gamma t^2 + 9\gamma^2(t-1)^2} + 2\beta t^2 + 3\gamma(t-1)^2\right)}{24\beta^2\gamma}.$$

Under server pricing with per-unit fees, let us continue from the proof of proposition 5. Since the equilibrium demand is monotone decreasing in cost, the highest number of customers a server serves is

$$q(p^*(0,\overline{p}),\overline{p}) = 1 - \beta p^*(0,\overline{p}) + \gamma(\overline{p} - p^*(0,\overline{p})) = \frac{\gamma}{2\beta+\gamma}.$$

The capacity is binding if and only if $t \leq \frac{\gamma}{2\beta+\gamma}$. In this case, the servers need to incorporate the capacity constraint in their pricing decision. A server's profit conditional on participation is

$$\pi_i(p(c_i),\overline{p}) = \min\{(1 - \beta\overline{p} + \gamma(\overline{p} - p(c_i)), t\}(p(c_i) - c_i - w).$$

Server c set the price that maximizes their own profits:

$$\max_p \min\{(1 - \beta\overline{p} + \gamma(\overline{p} - p), t\}(p - c - w).$$

Without the capacity constraints, server's problem is concave in its decision variable. The optimal price is unique:

$$p^*(c,\overline{p}) = \frac{\gamma(c + \overline{p} + w) + \beta(-\overline{p}) + 1}{2\gamma}.$$

This is equivalent to a quantity of

$$q(p^*(c,\overline{p}),\overline{p}) = \frac{1}{2}(\gamma(-(c+w)) + \overline{p}(\gamma - \beta) + 1).$$

If a server is capacity constrained ($t < q(p(c,\overline{p}),\overline{p})$), then a server's profit is increasing in own price. So, a server never sets a price lower than the amount that sets its demand equal to the capacity. Hence, the optimal price of a server that is constrained by capacity is:

$$p^*(c,\overline{p}) = \frac{-\beta\overline{p} + \gamma\overline{p} - t + 1}{\gamma}.$$

The quantity served is decreasing in costs. The lowest cost server that is not capacity constrained has exactly a demand of t units.

$$q(p^*(\tilde{c},\overline{p}),\overline{p}) = t \implies \tilde{c} = \frac{-\beta\overline{p} + \gamma\overline{p} - 2t - \gamma w + 1}{\gamma}.$$

So, servers' equilibrium prices are

$$p^*(c,\overline{p}) = \begin{cases} \frac{-\beta\overline{p}+\gamma\overline{p}-t+1}{\gamma}, & c \le \tilde{c}, \\ \frac{\gamma(c+\overline{p}+w)+\beta(-\overline{p})+1}{2\gamma}, & \tilde{c} < c \le c_h. \end{cases}$$

Servers with cost higher than c_h cannot profitably participate.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. In line with the mean-field approach, the average price that occurs by the server's optimal decisions is consistent with their expectation of the average price. By Equation (3):

$$\begin{aligned}
\overline{p} &= \frac{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})p^*(c,\overline{p})\, dc}{\int_0^{c_h} q(p^*(c,\overline{p}),\overline{p})\, dc} \\
&= \frac{\int_0^{c_h} \big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)p^*(c,\overline{p})\, dc}{\int_0^{c_h} \big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)\, dc} \\
&= \frac{\int_0^{\tilde{c}} tp^*(c,\overline{p})\, dc + \int_{\tilde{c}}^{c_h} \big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)p^*(c,\overline{p})\, dc}{\int_0^{\tilde{c}} t\, dc + \int_{\tilde{c}}^{c_h} \big(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p}))\big)\, dc} \\
&= -\frac{3t(2\overline{p}(\beta-\gamma)+\gamma w-2)+3(\overline{p}(\beta-\gamma)-1)(\overline{p}(\beta-\gamma)+\gamma w-1)+4t^2}{3\gamma(\overline{p}(\beta-\gamma)+t+\gamma w-1)}.
\end{aligned}$$

There is a single feasible equilibrium average price

$$\overline{p} = -\frac{\sqrt{-12\beta^2t^2+12\beta\gamma t^2+9\gamma^2(t+\beta w-1)^2}-3\gamma(t-1)+3\beta(2t+\gamma w-2)}{6\beta(\beta-\gamma)}.$$

The capacity is binding, $\tilde{c} > 0$, if and only if platform sets its wage such that $w \le \frac{3\gamma-4\beta t-2\gamma t}{3\beta\gamma}$. The platform never sets $w > \frac{3\gamma-4\beta t-2\gamma t}{3\beta\gamma}$. That's because, as long as $t \le \frac{\gamma}{2\beta+\gamma}$, the platform's doesn't achieve the optimal at a point where capacity is not binding. Setting $w > \frac{3\gamma-4\beta t-2\gamma t}{3\beta\gamma}$ leads to a non-capacity binding equilibrium, which is strictly worse than the optimal.

Platform's profit maximization problem is:

$$\begin{aligned}
\max_{\phi} \quad \Pi^{\mathcal{SU}}(0,w) \quad &= w\int_0^{c_h}(1-\beta\overline{p}+\gamma(\overline{p}-p^*(c,\overline{p})))\, dc = \frac{tw\Big(\sqrt{-12\beta^2t^2+12\beta\gamma t^2+9\gamma^2(t+\beta w-1)^2}-3\gamma(t+\beta w-1)\Big)}{6\beta\gamma} \\
\text{s.t.} \quad w &\le \frac{3\gamma-4\beta t-2\gamma t}{3\beta\gamma}.
\end{aligned}$$

The objective function is quasi-concave in w. The platform's solution is unique. It's defined by the first order conditions and is equal to

$$w = \frac{4\beta^2t^2-4\beta\gamma t^2-3\gamma^2(t-1)^2}{6\beta\gamma^2(t-1)}.$$

Platform's profit is

$$\Pi^{\mathcal{SU}} = \frac{-4\beta^2 t^3 + 4\beta\gamma t^3 + 3\gamma^2 (t-1)^2 t}{12\beta^2\gamma^2}.$$

Under the optimal mechanism, let us continue from the proof of proposition 6. Since the equilibrium demand is monotone decreasing in cost, the highest number of customers a server serves is $q^*(0) = 1/2$.

The capacity is binding if and only if $t \leq \frac{1}{2}$. If the capacity is binding, the platform's optimal decisions will change.

Our analysis for the optimal mechanism in Proposition 6 hold true for the capacity constrained model up until Equation (9). The capacity constraints add an additional constraint to the platform's optimization problem

Problem is re-formulated:

$$\begin{aligned}
\max_{q(c), c_h} & \int_0^{c_h} q(c) \left(\frac{1 - q(c)}{\beta} - 2c \right) \, dc \\
\text{s.t. } & q'(c) \leq 0, \forall c \in (0, c_h) \\
& q(c) \geq 0, \forall c \in (0, c_h) \\
& q(c) \leq t, \forall c \in (0, c_h).
\end{aligned}$$

Holding c_h constant and relaxing the first constraint, we can decompose the problem into individual sub-problems for all servers:

$$\begin{aligned}
\max_{q(c), c_h} & \; q(c) \left(\frac{1 - q(c)}{\beta} - 2c \right) \, dc \\
\text{s.t. } & q(c) \geq 0, \forall c \in (0, c_h) \\
& q(c) \leq t, \forall c \in (0, c_h).
\end{aligned}$$

This is the maximization of a simple quadratic function with two linear constraints. The optimal quantity is:

$$q^*(c) = \min \left\{ t, \max \left\{ 0, \frac{1}{2} - \beta c \right\} \right\}.$$

The objective values of the sub-problems are always strictly positive for all servers with positive quantities. Therefore, platform is always better off hiring more server as long as the server has non-negative demand:

$$q^*(c_h) = 0 \implies c_h = \frac{1}{2\beta}.$$

This solution also satisfies:

$$q^{*\prime}(c) = \max\{-\beta, 0\} \leq 0.$$

Therefore, the solution our relaxed problem is also optimal for the platform's optimal mechanism.

Assuming an interior equilibrium ($c_h < 1$), the average market price is:

$$\begin{aligned}\overline{p} &= \frac{\int_0^{c_h} q^*(c)\left(1-q^*(c)\right)\, dc}{\beta\int_0^{c_h} q^*(c)\, dc}\\ &= \frac{4t^2-6t+3}{3\beta-3\beta t}.\end{aligned}$$

The platform earns

$$\begin{aligned}\Pi\ &= \int_0^{c_h} q^*(c)\left(\frac{1-q^*(c)}{\beta}-2c\right)\, dc\\ &= \frac{t\left(4t^2-6t+3\right)}{12\beta^2}.\end{aligned}$$

With server pricing, let the platform sets its terms,

$$\phi = 1-\frac{\gamma}{2\beta},\ w = \frac{\left(4t^2-6t+3\right)(\beta-\gamma)}{6\beta^2(t-1)},$$

and assume the servers expect an average price of

$$\overline{p} = \frac{4t^2-6t+3}{3\beta(1-t)}.$$

Without capacity restrictions, a server with cost c has a profit-maximizing problem of

$$\max_p (1-\beta\overline{p}+\gamma(\overline{p}-p))((1-\phi)p-c-w),$$

giving an optimal price of

$$\begin{aligned}p^*(c,\overline{p}) &= \frac{1}{2}\left(\frac{1-(\beta-\gamma)\overline{p}}{\gamma}+\frac{c+w}{1-\phi}\right)\\ &= \frac{3\beta-6\beta^2c+6\beta^2ct-6\gamma+8\beta t^2-8\gamma t^2-9\beta t+12\gamma t}{6\beta\gamma(t-1)}.\end{aligned} \tag{43}$$

Let c_t be the highest cost server that is bounded by capacity:

$$q(p^*(c_t,\overline{p}),\overline{p}) = t \implies c_t = \frac{1-2t}{2\beta}.$$

All servers with cost less than c_t set a price that makes its demand exactly equal to t,

$$p^*(c_t,\overline{p}) = \frac{1-(\beta-\gamma)\overline{p}-t}{\gamma} = \frac{-3\gamma+\beta t^2-4\gamma t^2+6\gamma t}{3\beta\gamma t-3\beta\gamma},$$

others will set the price as defined in Equation (43).

Let c_h be the highest cost server that can participate with non-negative demand:

$$q(p^*(c_h,\overline{p}),\overline{p}) = 0 \implies c_h = \frac{1}{2\beta}.$$

The realized average price is consistent with expectation:

$$\begin{aligned}\overline{p} &= \frac{\int_0^{c_t} tp^*(c_t, \overline{p})\ dc + \int_{c_t}^{c_h} q(p^*(c, \overline{p}), \overline{p}) p^*(c, \overline{p})\ dc}{\int_0^{c_t} t\ dc + \int_{c_t}^{c_h} q(p^*(c, \overline{p}), \overline{p})\ dc} \\ &= \frac{4t^2 - 6t + 3}{3\beta(1-t)}.\end{aligned}$$

The platform earns

$$\begin{aligned}\Pi &= \int_0^{c_t} t(\phi p^*(c_t, \overline{p}) + w)\ dc + \int_{c_t}^{c_h} q(p^*(c, \overline{p}), \overline{p})(\phi p^*(c, \overline{p}) + w)\ dc \\ &= \frac{t\left(4t^2 - 6t + 3\right)}{12\beta^2},\end{aligned}$$

same as the optimal mechanism.

$\square$

Extension with Throughput-Maximization. In the quantity maximizing optimal contract, let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. By Proposition 6, the monotonicity of prices, $p(c)$, and Equation (35) are necessary and sufficient conditions for servers' IC constraints. The equilibrium fees charged to server c is characterized as

$$f(c) = u(c, p(c)) + \int_c^{c_h} \frac{\partial u(c_k, p(c_k))}{\partial c_k}\ dc_k.$$

The total quantity served in the market is

$$\int_0^{c_h} q(c)\ dc = \int_0^{c_h} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\ dc.$$

The optimal truth-inducing contract that maximizes total quantity served subject to non-negative profit constraint is characterized through the following problem:

$$
\begin{aligned}
\max_{p(c),c_h} \quad & \int_0^{c_h} (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in (0,c_h) \\
& 1-\beta\overline{p}+\gamma(\overline{p}-p(c)) \geq 0, \forall c \in (0,c_h) \\
& f(c) = u(c,p(c)) + \int_c^{c_h} \frac{\partial u(c_k,p(c_k))}{\partial c_k}\, dc_k, \forall c \in (0,c_h) \\
& \int_0^{c_h} f(c)\, dc \geq 0 \\
& \text{Eq. (3)} \\
= \max_{p(c),c_h} \quad & \int_0^{c_h} (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))\, dc \\
\text{s.t.} \quad & p'(c) \geq 0, \forall c \in (0,c_h) \\
& 1-\beta\overline{p}+\gamma(\overline{p}-p(c)) \geq 0, \forall c \in (0,c_h) \\
& \int_0^{c_h} (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))(p(c)-2c)\, dc \geq 0 \\
& \text{Eq. (3)},
\end{aligned}
$$

where the equivalence of the fourth constraints follow from Equation (36).

We reformulate the problem as a function of quantities:

$$
\begin{aligned}
\max_{q(c),c_h} \quad & \int_0^{c_h} q(c)\, dc \\
\text{s.t.} \quad & q'(c) \leq 0, \forall c \in (0,c_h) \\
& q(c) \geq 0, \forall c \in (0,c_h) \\
& \int_0^{c_h} q(c) \left(\tfrac{1+(\gamma-\beta)\overline{p}-q(c)}{\gamma} - 2c \right) dc \geq 0 \\
& \text{Eq. (37)}.
\end{aligned}
$$

Following the same steps as Equation (39), we transform the objective function such that it doesn't depend on $\overline{p}$:

$$
\begin{aligned}
\max_{q(c),c_h} \quad & \int_0^{c_h} q(c)\, dc \\
\text{s.t.} \quad & q'(c) \leq 0, \forall c \in (0,c_h) \\
& q(c) \geq 0, \forall c \in (0,c_h) \\
& \int_0^{c_h} q(c) \left(\frac{1-q(c)}{\beta} - 2c \right) dc \geq 0.
\end{aligned}
$$

Let us look at a complimentary problem, where the platform maximizes its earning subject to a fixed cap on total quantity served:

$$\begin{aligned}
\max_{p(c),c_h} \quad & \int_0^{c_h} q(c)\left(\frac{1-q(c)}{\beta}-2c\right)\,dc \\
\text{s.t.} \quad & q'(c)\le 0, \forall c\in(0,c_h) \\
& q(c)\ge 0, \forall c\in(0,c_h) \\
& \int_0^{c_h} q(c)\,dc = k,
\end{aligned}$$

where k is some non-negative number. Let us relax the first two constraints and solve the problem by calculus of variation. We will solve the problem sequentially. First, let us look at the quantity choice game. The Lagrangian is

$$L = q(c)\left(\frac{1-q(c)}{\beta}-2c\right)+\lambda q(c).$$

The Euler-Lagrange equation is

$$\frac{\partial}{\partial q(c)}\left(q(c)\left(\frac{1-q(c)}{\beta}-2c\right)+\lambda q(c)\right)=0,\ \forall c\in(0,c_h).$$

This gives

$$q^*(c)=\frac{\beta-2\beta c\lambda+\lambda}{2\lambda},\ \forall c\in(0,c_h). \tag{44}$$

We plug this into the constraint to find λ. We get a single solution

$$\int_0^{c_h} q^*(c)\,dc = k \iff \lambda=\frac{\beta c_h}{\beta c_h^2-c_h+2k}.$$

This gives an optimal objective of

$$\int_0^{c_h} q^*(c)\left(\frac{1-q^*(c)}{\beta}-2c\right)\,dc=\frac{\beta c_h^3}{12}+\frac{k(c_h-k)}{\beta c_h}-c_hk.$$

To show that this maximizer is a global maximizer, let us consider any feasible deviation, $z(c)$, from the optimal solution we found above. For feasibility, the deviation needs to satisfy

$$\int_0^{c_h} z(c)\,dc=0.$$

The new objective is:

$$\int_0^{c_h} (q^*(c) + z(c)) \left(\frac{1 - (q^*(c) + z(c))}{\beta} - 2c \right) \, dc = \frac{\beta c_h^3}{12} + \frac{k(c_h - k)}{\beta c_h} - c_h k - \left(c_h - \frac{c_h - 2k}{\beta c_h} \right) \int_0^{c_h} z(c) \, dc$$
$$- \frac{1}{\beta} \int_0^{c_h} z(c)^2 \, dc$$
$$= \frac{\beta c_h^3}{12} + \frac{k(c_h - k)}{\beta c_h} - c_h k - \frac{1}{\beta} \int_0^{c_h} z(c)^2 \, dc$$
$$\leq \frac{\beta c_h^3}{12} + \frac{k(c_h - k)}{\beta c_h} - c_h k.$$

This means, any feasible deviation from the optimal $q^*(c)$ defined in Equation (44) makes the objective worse off. Hence, Equation (44) defines the optimal solution to the relaxed quantity choice problem. Notice that the resulting objective function is monotone increasing in c_h and it satisfies the first inequality. However, for sufficiently large c_h values, some of the quantities can be negative. More specifically, that occurs if $c_h > \sqrt{2}\sqrt{\frac{k}{\beta}}$. We will now show that, in the non-relaxed problem, the optimal quantity will be equal to Equation (44) for those with non-negative $q(c)$ and 0 otherwise. As a result platform's profit will be

$$\frac{k}{\beta} - \frac{4\sqrt{2}k^{3/2}}{3\sqrt{\beta}}.$$

First, recall that a server's profit under relaxed problem is strictly increasing in c_h. This means that, as long as the highest cost participant has positive demand, platform is better off hiring more servers. There cannot be an equilibrium where the highest cost participate servers strictly positive demand. What is not clear is whether it is indeed optimal for platform to set quantities of 0 for servers with cost higher than $\sqrt{2}\sqrt{\frac{k}{\beta}}$. To see this is true, let us look at what happens if the platform deviates from that solution by some function $z(c)$ such that:

$$\int_0^1 z(c) = 0,$$

$$z(c) \geq 0, \quad \forall c > \sqrt{2}\sqrt{\frac{k}{\beta}}.$$

The new objective is:

$$\int_0^1 (q^*(c)+z(c))\left(\frac{1-(q^*(c)+z(c))}{\beta}-2c\right)\,dc = \int_0^{\sqrt{2}\sqrt{\frac{k}{\beta}}} (q(c)+z(c))\left(\frac{1-(q(c)+z(c))}{\beta}-2c\right)\,dc$$

$$+\int_{\sqrt{2}\sqrt{\frac{k}{\beta}}}^1 (z(c))\left(\frac{1-z(c)}{\beta}-2c\right)\,dc$$

$$=\frac{k}{\beta}-\frac{4\sqrt{2}k^{3/2}}{3\sqrt{\beta}}-\int_{\sqrt{2}\sqrt{\frac{k}{\beta}}}^1 \left(\frac{2\beta c-1}{\beta}\right)z(c)\,dc-\frac{1}{\beta}\int_{\sqrt{2}\sqrt{\frac{k}{\beta}}}^1 z(c)^2\,dc$$

$$-\frac{\left(2\sqrt{2}\sqrt{\beta}\sqrt{k}-1\right)}{\beta}\int_0^{\sqrt{2}\sqrt{\frac{k}{\beta}}} z(c)\,dc-\frac{1}{\beta}\int_0^{\sqrt{2}\sqrt{\frac{k}{\beta}}} z(c)^2\,dc$$

$$=\frac{k}{\beta}-\frac{4\sqrt{2}k^{3/2}}{3\sqrt{\beta}}-\int_{\sqrt{2}\sqrt{\frac{k}{\beta}}}^1 \left(c-\frac{\sqrt{2}\sqrt{k}}{\sqrt{\beta}}\right)z(c)\,dc-\frac{1}{\beta}\int_0^1 z(c)^2\,dc$$

$$\leq\frac{k}{\beta}-\frac{4\sqrt{2}k^{3/2}}{3\sqrt{\beta}}.$$

That is, any feasible deviation from the optimal quantity allocation makes the platform worse off. The highest profit platform can earn while ensuring total demand is equal to some k is:

$$\int_0^{c_h} q^*(c)\left(\frac{1-q^*(c)}{\beta}-2c\right)\,dc = k\left(\frac{1}{\beta}-\frac{4}{3}\sqrt{2}\sqrt{\frac{k}{\beta}}\right).$$

The profit of the firm is non-negative if and only if

$$k\leq\frac{9}{32\beta}.$$

Hence, the highest total demand the platform can achieve without earning negative profits is $k=\frac{9}{32\beta}$.

In the optimal mechanism, the highest cost that participates and the average market price are:

$$\overline{p}=\frac{1}{2\beta},$$
$$c_h=\frac{3}{4\beta}.$$

Total quantity of customers served in the market is

$$Q=\frac{9}{32\beta},$$

The fee platform charges to a server is

$$f^*(c) = \frac{(\gamma - 2\beta)(4\beta c - 3)(4\beta c - 1)}{32\beta\gamma}.$$

With server pricing, let the platform sets its terms,

$$\phi = 1 - \frac{\gamma}{2\beta},\ w = \frac{\gamma - 2\beta}{4\beta^2},$$

and assume servers expect an average price of

$$\overline{p} = \frac{1}{2\beta}.$$

A server with cost c has a profit-maximizing problem of

$$\max_p (1 - \beta\overline{p} + \gamma(\overline{p} - p))((1 - \phi)p - c - w),$$

giving an optimal price of

$$\begin{aligned} p^*(c, \overline{p}) &= \frac{1}{2}\left(\frac{1 - (\beta - \gamma)\overline{p}}{\gamma} + \frac{c + w}{1 - \phi}\right) \\ &= \frac{2\gamma - \beta + 4\beta^2 c}{4\beta\gamma}. \end{aligned}$$

Let c_h be the highest cost server that can participate with non-negative demand:

$$q(p^*(c_h, \overline{p}), \overline{p}) = 0 \implies c_h = \frac{3}{4\beta}.$$

The realized average price is consistent with expectation

$$\begin{aligned} \overline{p} &= \frac{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p}) p^*(c, \overline{p})\, dc}{\int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc} \\ &= \frac{1}{2\beta}, \end{aligned}$$

consistent with expectation.

The platform's profit is

$$\begin{aligned} \Pi &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})(\phi p^*(c, \overline{p}) + w)\, dc \\ &= 0. \end{aligned}$$

Total quantity served in the market is

$$\begin{aligned} Q &= \int_0^{c_h} q(p^*(c, \overline{p}), \overline{p})\, dc \\ &= \frac{9}{32\beta}, \end{aligned}$$

same as the optimal mechanism.

□