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Differences of Opinion and International Equity Markets

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We develop an international financial market model in which domestic and foreign residents differ in their beliefs about the information content in public signals. We determine how informational advantages of domestic investors in the interpretation of home public signals affect equity markets. We evaluate the ability of our model to generate four international-finance anomalies: (i) the co-movement of returns and capital flows, (ii) home-equity preference, (iii) the dependence of firm returns on home and foreign factors, and (iv) abnormal returns around foreign firm cross-listing in the home market. Their relationships with empirical differences-of-opinion proxies are consistent with the model. (*JEL* F23, F32, G11, G12, G15)

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> In almost all models of economic theory, behavioral differences among consumers are attributed to differences in preferences or in the information they possess. In real life, differences in consumer behavior are often attributed to varying intelligence and ability to process information. Agents reading the same morning newspapers with the same stock price lists will interpret the information differently.
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> —Ariel Rubinstein (1993), cited by Kandel and Pearson (1995)

How do differences in interpreting the same information affect financial markets? This question is at the core of a growing Difference-of-Opinion (hereafter DOO) literature. While differing opinions may exist between agents in any market, international markets provide a natural place to study their effects. Evidence of financial market segmentation across countries has been documented extensively, with different perceptions across those countries often cited as a source. Perhaps people of different countries harbor different opinions that affect their ability to process information relevant to the analysis of economies other than their own. The international financial market may thus be viewed as a natural laboratory for analyzing the effects of differing opinions.

In this paper, therefore, we propose a fully dynamic DOO model based upon international differences in perceptions about economic information, incorporating the equilibrium consumption of utility-maximizing investors. We assume that home investors are better at understanding home signals, and, therefore, are rational about home information. However, investor groups in both countries misinterpret the information about the other country's prospective growth and are, therefore, equally rational or irrational. We call this new feature "foreign sentiment."

The information assumption captures the idea that, while foreigners may see home information as well as home residents do, they do not know how to interpret it. The assumption may be motivated (in an unmodeled way) in at least three different ways, all stemming from the hypothesis that investors start history with prior beliefs that ignore the relationships (i.e., correlations) between the signals and the expected growth rates, and gradually discover them as data comes in. First, it could be that home investors have had a longer time to study the relationship. From this perspective, it may be taking foreigners a longer time to learn how to interpret the home signal and, if so, our model is analyzing a long transitional time period. Second, foreigners may simply have chosen not to become informed about the signal because they have viewed home investment as too risky in the past and consider becoming informed too costly.[1] Third, foreign investors may not be able to learn the same information as home

[1] For example, Van Nieuwerburgh and Veldkamp (2009) show in a noisy-rational-expectations model that, when investors are endowed with a small home information advantage, they choose not to learn what foreigners know.

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residents.[2] The assumption can also be viewed as a natural extension of the Merton (1987) "investor-recognition" hypothesis, to which we add the explicit modeling of information processing.[3] In Section 1, we review the empirical evidence found in the extant literature that supports the DOO hypothesis.

To highlight the effects of DOO, we embed our information processing in a two-country model that is otherwise as stylized and parsimonious as possible. Each country has an output process with an unobserved conditional mean that can be inferred statistically from observable outputs and signals, each generated by subjective Brownian motions. Moreover, markets are complete and all information is public. Investors in each country have access to an identical set of five assets that complete the market: a risk-free asset, two equities with payoffs in the output process in each country, and two futures contracts with payoffs marked to signal innovations. The manner in which information is processed by investors is modeled in Section 2, and the equilibrium is calculated in Section 3.

We then ask to what extent our model can produce international empirical regularities related to asset pricing, portfolio choices, and capital flows. For that purpose, we gather data, described in Section 4.1, to reestimate standard empirical relationships in the literature, allowing comparison with simulated data from our model. After examining the implications of the model and parameter values for some traditional moments of asset prices, capital flows, output, and consumption (Section 4), we evaluate the effects of DOO on four international empirical regularities. Two of these relationships have been found at the country level (Section 5). The first of these empirical regularities is the positive relationship between foreign capital inflows and home stock returns, sometimes called "return-chasing." We show that our model implies this same co-movement when home investors have an informational advantage in interpreting home signals. For the second regularity, we examine "home-equity preference," the observation that home residents weight home assets more heavily than foreign assets in their portfolios. Since home investors ignore foreign signals in our model, they shy away from holding foreign equity because they notice that they time transactions in foreign stocks less accurately than they do in domestic stocks. Therefore, our model generates this relationship as well.

Next we analyze the other two regularities based upon firm-level pricing observations (Section 6). For this purpose, we introduce, in addition to the equity markets of the two countries, the equity security issued by a new firm that operates and is listed in one of the two markets. We first evaluate the finding that the returns of that firm depend upon both home and foreign-market factors. Using our extended model, we show that foreign-sentiment risk indeed generates a home and foreign-factor model in consumption. We also show that

2 Acemoglu, Chernozhukov, and Yildiz (2016) show that when agents are uncertain about the conditional distribution, even a small amount of uncertainty can lead to significant differences in long-run beliefs.

3 On the investor-recognition hypothesis, see also the empirical work of Lehavy and Sloan (2008) and Richardson, Sloan, and You (2012).

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regressing our firm-level excess returns on the home and foreign country stock excess returns implies a somewhat higher beta on the home market than the foreign market, qualitatively consistent with typical empirical findings. We then move to our next pricing observation: stock prices on home firms that cross-list in foreign markets tend to increase around this event and expected returns are lower thereafter, responses often attributed to increased information to foreign investors about the cross-listed firms. To examine this effect in our model, we conjecture that cross-listing by the home firm in the foreign market enables the foreign country's investor to correctly interpret information about the home firm. Because cross-listing aligns perceptions about the information in public signals, the resulting decline in disagreement risk decreases the required return and increases the price, as in the data.

Simulations show that the model can qualitatively generate all four of these regularities with varying degrees of quantitative success. In addition, the model suggests several empirical relationships across countries and firms depending upon their sensitivity to measures of DOO. Using dispersion in professional forecasts of economic activity to examine these relationships, we find evidence consistent with our model.

Overall, our paper provides a significant contribution on at least three fronts. First, we present the first two-country general equilibrium model in which investors rely on public information differently from each other. On the technical side, we modify the model by Dumas, Kurshev, and Uppal (2009) to incorporate multiple trees.[4] Second, to our knowledge, this paper provides the first information-based attempt to explain the above four international empirical regularities simultaneously. Third, we provide new empirical evidence that DOO may affect these regularities.

1. The Difference-of-Opinion Hypothesis

In this section, we argue that the DOO hypothesis is a legitimate contender as an explanation of phenomena in the financial market and that it deserves examination, alongside the alternative hypothesis that says that different people receive different, private information.[5] Indeed, a growing body of direct and indirect empirical evidence suggests that differences of opinion play an important role. We review just a few of the related papers below.

[4] Cochrane, Longstaff, and Santa Clara (2008) and Martin (2013) show that allowing for multiple trees is already far from trivial with homogeneous investors. Osambela (2015) considers a model with two trees, DOO, and funding liquidity constraints, but with deterministic disagreement. To our knowledge, we are the first to solve a multiple-trees model in a setup with DOO and stochastic disagreement, driven by heterogeneous confidence in public signals. Moreover, in Appendix B, we extend the model to correlated trees, still yielding explicit solutions.

[5] Previous contributions to the DOO approach include: Harris and Raviv (1993) and Cecchetti, Lam, and Mark (2000). See Morris (1995) for a discussion of this approach. In both approaches, investors learn about the current state of the economy, knowing exactly how the economy operates, although they do not observe it completely. In yet a third approach, investors are uncertain about the parameters of the model that govern the economy. See Collin-Dufresne, Johannes, and Lochstoer (2016).

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Direct evidence is provided by data on professional forecasters, of the kind we explore in Section 7. Patton and Timmermann (2010) show that differences in individual-forecaster views persist through time. Based on a statistical model of the effect of public versus private information, they conclude that "such differences in opinion cannot be explained by differences in information sets; our results indicate they stem from heterogeneity in priors or models." They also observe that "differences in opinion move countercyclically, with heterogeneity being strongest during recessions where forecasters appear to place greater weight on their prior beliefs" (p. 803). Dovern, Fritsche, and Slacalek (2012) come to a similar conclusion. In standard private information stories based upon Noisy Rational Expectations (NRE) models, forecasters do not disagree more during recessions than during expansions, whereas in our DOO model, disagreement and output growth are found to be correlated, in conformity with that observation.[6]

Indirect evidence comes from the observed relationship between volume of trading and returns in the stock market at the time of public announcements of quarterly earnings. In a sample of more than 60,000 firm announcements, Kandel and Pearson (1995) find that, around a public announcement date, the relationship between volume of trade and returns is clearly different from what it is at other times. They consider many candidate models for the treatment of information by investors, trying to fit them to the observed relationship, and conclude that "it is inconsistent with most existing models in which agents have identical interpretations of the public announcement" (p. 868).[7] Bamber, Barron, and Stober (1999) directly test the relationship between trading volume and a proxy for the measure of differential interpretations that had been suggested by Kandel and Pearson (1995). They find that "around earnings announcements that generate minimal price changes (where many models predict there should be no information-based trading) trading volume increases significantly with Kandel and Pearson's measure of differential interpretations" (p. 370). Furthermore, Jia, Wang, and Xiong (2015) compare the empirical reactions of Chinese and foreign investors to the same public news. Consistent with our model below, they find that Chinese investors react more strongly to earnings forecast revisions published by Chinese analysts, while foreign investors react more strongly to revisions published by foreign analysts. In the international sphere, the dramatic capital flow waves—classified as "surges, stops, flight, and retrenchment"—that Forbes and Warnock (2012) examine make sense if we consider that home and foreign investors hold different and

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[6] Avramov, Kaplanski, and Levy (2016) show that the recommendations of analysts who rely on technical analysis are superior to those relying on fundamental analysis. They both have access to public information, but the former, by construction, use public information only and process it differently.

[7] Similarly, earlier work by Kim and Verrecchia (1991) concluded that differential interpretations had to be present to some degree.

highly fluctuating views about a country's growth prospects, about which there is unlikely to exist much private information.

Further indirect evidence for differences of opinion can also be found in commodity markets. Singleton (2014) observes a positive price drift (which he calls a "boom") in commodities prices—reflecting an additional risk premium—during periods when there is more disagreement among analysts' forecasts. This finding is directly in line with our DOO theory, which says that more disagreement implies more volatility in sentiment risk, potentially increasing risk premia.

Models that rely on private information—such as NRE models—have also been used to explain two of the empirical regularities we study—capital flows and home-equity preference. Early models about these regularities typically assumed that the informational advantage to home residents arises from more precise, privately observed signals. For example, Gehrig (1993) posits that home residents have more certainty about home fundamentals information than do foreigners, thereby reducing the optimal holdings of foreign assets.

Brennan and Cao (1997), in a model in which investors have initially received private signals, show that foreign purchases of home equities are positively correlated with home stock returns because they both react the same way to public signals. In our DOO model, instead of receiving initial private signals, different investors make different assumptions about parameters, but the reaction to public information is very similar.

An important feature introduced by our model is the endogenization of the rate of interest. To our knowledge, all extant implementations of NRE models assume an exogenous and constant rate of interest, so that they are not fully general-equilibrium models. As Loewenstein and Willard (2006) point out, this assumption implies that capital goes in and out of a storage facility that returns the fixed rate of interest so that examining capital flows between investors or between countries provides an incomplete description of capital flows. Albuquerque, Bauer, and Schneider (2007) present an NRE model that delivers persistence of net capital flows, as in the data.[8] The serial dependence of capital flows between investors or between countries when the rate of interest is held fixed is presumably quite different from what it is when the rate of interest is endogenous. Indeed, we demonstrate below that our DOO model can explain only a small degree of capital flow persistence, in part due to the adjustment in the rate of interest.

[8] Dvořák (2003), Albuquerque, Bauer, and Schneider (2007, 2009), and Hau and Rey (2008) all highlight the need to study gross international capital flows in addition to net flows. In order to explain simultaneous foreign purchases and sales, they argue that some U.S. investors must be transacting with other U.S. investors and, therefore, introduce two types of investors within a country. In our paper, by contrast, we consider transactions on only one representative investor per country, summarizing the net securities transactions within that country. In principle, our dynamic equilibrium could also be extended to any number of investors and securities, but this would complicate the model and likely make the DOO channel less transparent. We therefore leave this topic for future research.

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Overall, our endeavor in this paper is not predicated on an assertion that the DOO approach is better than the NRE approach in explaining the international phenomena we study, or that these international phenomena provide a way to discriminate between the two approaches. Nothing at this point positions one approach as the incumbent and the other as the challenger. Indeed, it seems plausible that both differences of opinion and asymmetric information are important forces in the market.

2. The Foreign-Sentiment Risk Model: Information Processing

In this section, we provide a simple framework that captures two features. First, the information in current economic variables and public signals affects forecasts of future variables and hence current prices of financial securities. And, second, investors differ across countries in their beliefs about the informativeness of these currently observed public signals.

The basic features of these differing beliefs and their impact on future expectations can be shown most parsimoniously using a model with two countries that are identical at the initial point in time. Representative investors live in each of the two countries. The countries are completely integrated in that they are open to international trade in securities and in a single, perishable good. Investors in each country are initially endowed with one share of their own output process, itself initialized at the value of 1. The financial market is complete.

2.1 Exogenous outputs and public signals

The output delivered by country $i \in \{A, B\}$ at time t is denoted $\delta_{i,t} dt$. The stochastic process for $\delta_{i,t}$ is

$$\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t} dt + \sigma_\delta dz^\delta_{i,t}, \quad i \in \{A, B\}, \tag{1}$$

where $\left\{z^\delta_i\right\}_{i=A,B}$ are Brownian motions under the effective probability measure, which governs empirical realizations of the process. The conditional expected growth rates $f_{i,t}$ of outputs are also stochastic:

$$df_{i,t} = \zeta\left(\bar{f} - f_{i,t}\right) dt + \sigma_f dz^f_{i,t}, \quad i \in \{A, B\}, \tag{2}$$

where $\zeta > 0$ and $\left\{z^f_i\right\}_{i=A,B}$ are also Brownian motions under the effective probability measure.

Neither the conditional expected growth rates f_i of outputs nor the z shocks are observed by any investor. All investors must estimate, or filter out, the current value of f_i in order to determine the way future conditional mean growth rates affect forward-looking asset prices. They estimate this value by

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observing current outputs and two public signals (s_A, s_B). The signals follow the processes

$$ds_{i,t} = \phi dz^f_{i,t} + \sqrt{1-\phi^2} dz^s_{i,t}, \quad i \in \{A, B\}, \tag{3}$$

where $|\phi| \in [0,1]$ and where $\{z^s_i\}_{i=A,B}$ is a third pair of Brownian motions, under the effective probability measure as well. The term $\phi dz^f_{i,t}$ in the stochastic differential equation for the signals means that the signals are truly informative about expected output growth shocks $dz^f_{i,t}$.

For simplicity of exposition, we describe in the text the model assuming that the six Brownian motions $\left(z^\delta_{A,t}, z^f_{A,t}, z^s_{A,t}, z^\delta_{B,t}, z^f_{B,t}, z^s_{B,t}\right)$ are independent of each other. As we show below, this independence, together with symmetry, provides a significantly simplified version of our model with an intuitive solution. However, for our quantitative analysis below, we allow for cross-country output correlation. Details of the solution with output correlation are provided in Appendix B.

Note that, in these output and signal processes, the parameters are identical across countries for symmetry. Thus, the volatility of the outputs and conditional growth rates, σ_δ and σ_f, the long-run means of the conditional growth rates and their mean reversion parameters, $\bar{f}$ and ζ, and the information in the signal, ϕ, do not depend upon the country.

2.2 Benchmark beliefs: The viewpoint of the "econometrician"

In the information model we develop below, no investor knows the true state of the economy. We introduce, as a benchmark only, the perspective taken by a nonexistent being who interprets data correctly and whom we call "the econometrician." This abstract agent is not a participant in our economy. As in Xiong and Yan (2010), the econometrician observes the same information as do both sets of investors. Like the investors of both countries, the econometrician does not observe the true conditional growth rate of outputs and must filter out this process. The econometrician's measure, therefore, is not the same as the effective probability measure under which we wrote Equations (1), (2), and (3). Rather, as will be true for the country A and B investors, the filtration and measure of the econometrician exclude the true values $\{f_{i,t}\}_{i\in\{A,B\}}$ of the unobserved state variables.

We assume, however, that the econometrician knows the true structure of the economy (Equations (1), (2), and (3)). Accordingly, the econometrician filters the signal process under the hypothesis stated in Equation (3). We formulate the probability measures of home and foreign groups as deviations from the econometrician's probability measure.

To calculate the econometrician's probability measure, we rewrite the stochastic differential equations in terms of processes that are Brownian motions under his probability measure. For this purpose, we define the four-dimensional process $w^E_t = \left(w^E_{\delta_A,t}, w^E_{\delta_B,t}, w^E_{s_A,t}, w^E_{s_B,t}\right)^\intercal$, where each of the

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elements of w_t^E corresponds to a Brownian component of each of the four observed variables under the probability measure of the econometrician, referred to by the superscript E. Defining $\hat{f}_{i,t}^E$ as the conditional mean of the growth rate of output in country i as estimated by the econometrician, we use filtering theory (see Lipster and Shiryaev 2000, 36, Theorem 12.7) to compute these conditional expected values. For $i \in \{A, B\}$, these expectations are given by

$$d\hat{f}_{i,t}^E = \zeta\left(\bar{f} - \hat{f}_{i,t}^E\right)dt + \frac{\gamma^E}{\sigma_\delta^2}\left(\frac{d\delta_{i,t}}{\delta_{i,t}} - \hat{f}_{i,t}^E dt\right) + \phi\sigma_f ds_{i,t},$$

$$= \zeta\left(\bar{f} - \hat{f}_{i,t}^E\right)dt + \frac{\gamma^E}{\sigma_\delta} dw_{\delta_i,t}^E + \phi\sigma_f dw_{s_i,t}^E, \quad (4)$$

where the number γ^E is the steady-state variance of the econometrician's forecast errors $\hat{f}_A^E - f_A$ and $\hat{f}_B^E - f_B$, these variances being equal to each other by virtue of symmetry:

$$\gamma^E \triangleq \sigma_\delta^2\left(\sqrt{\zeta^2 + \left(1 - \phi^2\right)\frac{\sigma_f^2}{\sigma_\delta^2}} - \zeta\right). \quad (5)$$

This variance would normally be a deterministic function of time. But for simplicity we assume, as did Scheinkman and Xiong (2003) and Dumas, Kurshev, and Uppal (2009), that there has been a sufficiently long period of learning for people of both countries to converge to their long-run level of variance, independent of their prior beliefs.

Equation (4) shows how the econometrician filters out the conditional growth rates based upon observations of outputs and signals. When he sees an increase in the output of country i, he updates his estimate of the conditional mean growth rate by the ratio of its steady-state variance γ^E and the variance of the output σ_δ. When he sees an increase in the signal of country i, he increases his view of $f_{i,t}$ according to $\phi\sigma_f$, the information precision in the signal about this growth rate.

By definition of the conditional expected output growth rates $\hat{f}_{A,t}^E$ and $\hat{f}_{B,t}^E$, we can then write the output growth rates under the econometrician's measure as:

$$\frac{d\delta_{i,t}}{\delta_{i,t}} = \hat{f}_{i,t}^E dt + \sigma_\delta dw_{\delta_i,t}^E, \ i \in \{A, B\}. \quad (6)$$

2.3 The investors' viewpoints

The difference in information processing by the investors of the two countries is implemented as follows. Investors in country A perform their filtering under

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the belief that the signal s_A has the correct conditional correlation with f_A, but they believe incorrectly that the signal s_B has zero correlation with f_B. The "model" they have in mind is

$$ds_{A,t} = \phi dz^f_{A,t} + \sqrt{1-\phi^2} dz^s_{A,t}, \quad ds_{B,t} = dz^s_{B,t}. \tag{7}$$

Notice that investors in country A have the same model of signal s_A as the econometrician (incorporating the true correlation ϕ in Equation (3)) but a different one (incorporating a correlation equal to zero) for the signal s_B. Symmetrically, the "model" that investors of country B have in mind is

$$ds_{A,t} = dz^s_{A,t}, \quad ds_{B,t} = \phi dz^f_{B,t} + \sqrt{1-\phi^2} dz^s_{B,t}. \tag{8}$$

Defining $\hat{f}^i_j$ as the conditional mean of the output growth in country j as estimated by investors in country i, we implement filtering theory one more time to write

$$\begin{aligned} d\hat{f}^i_{i,t} &= \zeta\left(\bar{f} - \hat{f}^i_{i,t}\right)dt + \frac{\gamma^E}{\sigma^2_\delta}\left(\frac{d\delta_{i,t}}{\delta_{i,t}} - \hat{f}^i_{i,t}dt\right) + \phi\sigma_f ds_{i,t}, \\ &= \zeta\left(\bar{f} - \hat{f}^i_{i,t}\right)dt + \frac{\gamma^E}{\sigma_\delta} dw^i_{\delta_i,t} + \phi\sigma_f dw^i_{s_i,t}, \end{aligned} \tag{9}$$

$$\begin{aligned} d\hat{f}^i_{j,t} &= \zeta\left(\bar{f} - \hat{f}^i_{j,t}\right)dt + \frac{\gamma^\times}{\sigma^2_\delta}\left(\frac{d\delta_{j,t}}{\delta_{j,t}} - \hat{f}^i_{j,t}dt\right),\ i \neq j, \\ &= \zeta\left(\bar{f} - \hat{f}^i_{j,t}\right)dt + \frac{\gamma^\times}{\sigma_\delta} dw^i_{\delta_j,t},\ i \neq j, \end{aligned} \tag{10}$$

where the number $\gamma^\times$ is the steady-state variance of the "transnational" forecast errors $\hat{f}^B_A - f_A$ and $\hat{f}^A_B - f_B$, their variances being equal to each other by virtue of symmetry:

$$\gamma^\times = \gamma^E\big|_{\phi=0} = \sigma^2_\delta\left(\sqrt{\zeta^2 + \frac{\sigma^2_f}{\sigma^2_\delta}} - \zeta\right). \tag{11}$$

Note from Equation (5) that γ^E decreases as the information in the signal measured by ϕ^2 rises toward one. Intuitively, the signal s_i allows the econometrician and investors in country i to get a more precise estimate of f_i, thereby reducing the steady-state variance for investors in country i's estimate. By contrast, investors in country $j \neq i$ ignore the information in the signal s_i

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and thereby attribute more of the variability to σ_f. The result is a relationship we use below:

Proposition 1. The steady-state variance of the forecast error of the home-output growth rate made by home investors is lower than the steady-state variance of the same forecast error made by foreign investors; i.e., $\gamma^E < \gamma^\times = \gamma^E\big|_{\phi=0}$.

Since the econometrician's hypothesis about signals is not in line with that of investors in any of the two countries, differences in beliefs are generated. We define the "disagreements" between the econometrician and the investors as

$$\hat{g}^j_{i,t} \triangleq \hat{f}^E_{i,t} - \hat{f}^j_{i,t}; \quad i,j \in \{A,B\}. \tag{12}$$

In principle, $\hat{g}^j_i$ stands for two pairs of disagreements for each country's investor. However, the investors agree with the econometrician about the estimate of the conditional growth rate of their own output. Therefore, $\hat{g}^A_{A,t} \equiv \hat{f}^E_{A,t} - \hat{f}^A_{A,t} = 0$ and $\hat{g}^B_{B,t} = \hat{f}^E_{B,t} - \hat{f}^B_{B,t} = 0$ so that the only disagreements that exist are those between the econometrician and the foreign investor's forecasts of the home output growth rate, $\hat{g}^B_A \equiv \hat{f}^E_{A,t} - \hat{f}^B_{A,t}$ and $\hat{g}^A_B \equiv \hat{f}^E_{B,t} - \hat{f}^A_{B,t}$.[9] Using Equations (9), (10), and (12), we get the dynamics for the disagreements

$$d\hat{g}^j_{i,t} = -\left(\zeta + \frac{\gamma^\times}{\sigma^2_\delta}\right)\hat{g}^j_{i,t}dt + \frac{\gamma^E - \gamma^\times}{\sigma_\delta}dw^E_{\delta_i,t} + \phi\sigma_f dw^E_{s_i,t}; \quad i \neq j; \quad i,j \in \{A,B\}. \tag{13}$$

The econometrician's and investors' filters in Equations (4) and (10), respectively, make clear the drivers for the disagreements in Equation (13). On the one hand, a positive output shock of, say, country A, $dw^E_{\delta_A,t}$, causes the econometrician to increase his estimate $\hat{f}^E_A$ according to γ^E/σ_δ. This same output change induces investors in country B to increase their estimate by $\gamma^\times/\sigma_\delta$. Recall that $\gamma^\times > \gamma^E$ (Proposition 1): because these investors ignore the signal information, they update their estimate by more than the econometrician (and more than investors in country A). Thus, country B investors overadjust their estimate of country A output growth in response to its output shock and the disagreement between these investors and the econometrician declines, $d\hat{g}^B_{A,t} < 0$. Given the definition in Equation (12), this relationship means that country B investors become relatively optimistic about country A output. On the other hand, an increase in the signal $dw^E_{s_A,t}$ induces the econometrician to increase his estimate of the conditional mean $\hat{f}^E_A$ in Equation (4). Since country B investors ignore the signal information, the signal increases the disagreement

[9] When output is correlated, however, both pairs of disagreement move over time and are jointly persistent. See Appendix B for details.

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about country A output, $d\hat{g}^B_{A,t} > 0$. Country B investors become relatively pessimistic about country A. This insight leads to the following proposition.

Proposition 2. Foreign investors underadjust their estimate of home output growth in response to a home signal shock and overadjust their estimate of home output growth when a home output shock occurs. By contrast, home investors properly adjust their estimate of home output growth in response to both home output and home signal shocks.

We now derive the changes in measures between the econometrician and investors. For this purpose, consider also a set of four-dimensional processes for each country that is Brownian under the probability measure of investors in country j; $w^j_t = \left(w^j_{\delta_A,t}, w^j_{\delta_B,t}, w^j_{s_A,t}, w^j_{s_B,t}\right)$, $j \in \{A, B\}$. These processes differ from the econometrician's according to

$$dw^E_{\delta_i,t} = dw^j_{\delta_i,t} - \frac{\hat{g}^j_{i,t}}{\sigma_\delta} dt, \quad dw^E_{s_i,t} = dw^i_{s_i,t}. \tag{14}$$

The probabilities of events will look different from the point of view of the econometrician and the investors in the two countries. Although the signal ds_t is interpreted differently by the econometrician and by the two groups of investors, the signal shock $dw_{s,t}$ is the same under all three of their probability measures (second equation in (14)). The output shock, however, is not observed. Because the signal is interpreted differently, the drift of output is estimated differently by them. Actual output growth is observed, but the output shock $dw_{\delta,t}$ (equal to actual growth minus drift over output volatility) is not viewed the same way under the measures of the econometrician and the two groups (first equation in (14)).

The change in measure between the sets of Brownians perceived by the econometrician, w^E_t, and by the investor in country j, w^j_t, indicates the evolution of their difference in beliefs. Based on Equation (14), we can apply Girsanov's theorem to obtain the changes from the probability measure of the econometrician to those of investors in countries A and B. Doing so implies that the ratios of probability beliefs η_A and η_B of these countries evolve according to:

$$d\eta_{A,t} = -\frac{\hat{g}^A_{B,t}}{\sigma_\delta} \eta_{A,t} dw^E_{\delta_B,t}, \quad d\eta_{B,t} = -\frac{\hat{g}^B_{A,t}}{\sigma_\delta} \eta_{B,t} dw^E_{\delta_A,t}. \tag{15}$$

We give the ratio of beliefs η_i the picturesque name of "country i foreign sentiment" or, more generally, "foreign-sentiment risk." By contrast, the stochastic terms in the disagreement equation (13) are referred to as "disagreement risk." Equation (15) shows the following:

Proposition 3. Except when current disagreements are coincidentally equal to zero, the foreign-sentiment risk of the investors of one country is perfectly correlated with the output shock of the other country (with the sign of the perfect correlation being opposite to the sign of current disagreement).

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Notice that the ratios of probability beliefs are not affected by signal shocks. Also, the evolution of $\eta_{A,t}$ does not depend upon the country A output shock since the econometrician and the country A investors agree about the filter of that process.[10] The change of measure between an investor of a given country and the econometrician is perfectly (positively or negatively) correlated with the output in the other country. Disagreements $\hat{g}^A_{B,t}$ and $\hat{g}^B_{A,t}$ are the drivers of the instantaneous volatilities of the foreign-sentiment variables. For example, $\eta_{A,t}$ depends upon realizations of the output in country B, according to $dw^E_{\delta_B,t}$. The size of this effect depends upon the current disagreement $\hat{g}^A_{B,t}$ between the econometrician and investors in country A. If investors in country A are currently optimistic about country B, then $\hat{g}^A_{B,t}<0$. Since country A investors overadjust their estimate of country B output growth in response to its output realizations, this response will further increase the difference in probabilities and $\eta_{A,t}$ increases.

The Markovian system composed of Equations (4), (6), (13), and (15) completely characterizes the dynamics of eight exogenous state variables that drive the economy, defined by the vector: $Y_t \triangleq \left(\delta_{A,t}, \hat{f}^E_{A,t}, \hat{g}^B_{A,t}, \eta_{B,t}, \delta_{B,t}, \hat{f}^E_{B,t}, \hat{g}^A_{B,t}, \eta_{A,t}\right)^{\mathsf{T}}$. However, since outputs are uncorrelated, the first four components of the vector are only driven by the Brownians on the output and signal of country A, while the last four components of the vector are driven by the corresponding Brownians for country B. Therefore, the state vector can be written as two independent processes: $Y_t = \left\{Y_{A,t}, Y_{B,t}\right\}$ where

$$Y_{i,t} = \left(\delta_{i,t}, \hat{f}^E_{i,t}, \hat{g}^j_{i,t}, \eta_{j,t}\right)^{\mathsf{T}},$$

for $i,j \in \{A,B\}$; $i \neq j$. Although they have equal diffusion matrices, each of these two processes is driven by separate Brownians. In particular,

$$dY_{i,t} = \mu_{i,t}dt + \Omega_{i,t}dw^E_{i,t},$$

where

$$dw^E_{i,t} = \left(dw^E_{\delta_i,t}, dw^E_{s_i,t}\right)^{\mathsf{T}},$$

and

$$\Omega_{i,t} = \begin{bmatrix} \sigma_\delta \delta_{i,t} & 0 \\ \left(\frac{\gamma^E}{\sigma_\delta}\right) & \phi\sigma_f \\ \left(\frac{\gamma^E - \gamma^\times}{\sigma_\delta}\right) & \phi\sigma_f \\ -\eta_{j,t}\left(\frac{\hat{g}^j_{i,t}}{\sigma_\delta}\right) & 0 \end{bmatrix}, \; i \neq j.$$

Thus, the state vector can be evaluated as two independent processes, each governing the evolution of views about each country's output, providing a

[10] This is no longer true when outputs are correlated. See Appendix B for details.

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block diagonal structure exploited in our description of the equilibrium below. This convenient block diagonal structure no longer holds when outputs are correlated, as we describe in Appendix B.

3. The Foreign-Sentiment Risk Model: Equilibrium

We now use the information structure to derive equilibrium pricing relationships. The derivation of equilibrium in a complete market is standard.

The investors in the two countries have identical time-separable utility functions in a common perishable consumption good. For country B investors, the problem can be written as

$$\sup_{c_B} \mathbb{E}_0^E \int_0^\infty \eta_{B,t} e^{-\beta t} \frac{1}{\alpha} c_{B,t}^\alpha dt, \quad \alpha < 1, \tag{16}$$

subject to the lifetime budget constraint:

$$\mathbb{E}_0^E \int_0^\infty \xi_t^E (c_{B,t} - \delta_{B,t}) dt \leq 0, \tag{17}$$

where ξ_t^E is the stochastic discount factor under the econometrician's measure. The optimization problem in Equation (16) is expressed in terms of the expectation of the econometrician, indicated with the superscript E in the expectation operator $\mathbb{E}^E$. We multiply the period utility of B at time t by the ratio of probability beliefs, $\eta_{B,t}$, to get back to the expectation under the measure of B. Country A residents face a symmetric optimization problem.

To solve for the stochastic discount factor, we clear the goods market so that the sum $\delta_{A,t} + \delta_{B,t}$ of country outputs is is equal to "world consumption," $c_{W,t} \triangleq c_{A,t} + c_{B,t}$. Solving this equation for the stochastic discount factor ξ_t^E implies:[11]

$$\xi_t^E(\delta_{A,t}, \delta_{B,t}, \eta_{A,t}, \eta_{B,t}) = e^{-\beta t} \left[\left(\frac{\eta_{A,t}}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_{B,t}}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha} c_{W,t}^{\alpha-1}, \tag{18}$$

where λ_A and λ_B are the Lagrange multipliers of the lifetime budget constraints. The stochastic discount factor relative to the econometrician's measure, ξ_t^E, depends upon world consumption and the ratios of probability measure of both countries. In fact, it is homogeneous of degree 1 in these two variables.

Proposition 4. The stochastic discount factor (18) contains two priced factors: world consumption, $c_{W,t}$, and world (harmonic) average foreign sentiment, defined as

$$\eta_{W,t} \triangleq \left[\left(\frac{\eta_{A,t}}{\lambda^A} \right)^{\frac{1}{1-\alpha}} + \left(\frac{\eta_{B,t}}{\lambda^B} \right)^{\frac{1}{1-\alpha}} \right]^{1-\alpha}. \tag{19}$$

[11] To ease the calculation, we consider only integer levels of risk aversion so that we can later expand the bracket of Equation (18) into powers. For non-integer values of risk aversion, to obtain securities' prices, one would need to apply an inverse Fourier transform.

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The two priced factors are conditionally correlated. As we have seen (Equation (15)), foreign sentiment η_A is perfectly correlated with the output shock of country B, $dw^E_{\delta_B,t}$, with the sign of that correlation depending on the sign of the current disagreement $\hat{g}^A_{B,t}$. Only if and when coincidentally there is full agreement today ($\hat{g}^A_{B,t} = \hat{g}^B_{A,t} = 0$) are these correlations equal to zero. In all cases, the stochastic discount factor in Equation (18) shows that, in the presence of foreign sentiment, equilibrium prices now contain additional risk premia, over and above the classic premium based on world consumption. These premia are related to the risks in individual country output shocks but not related to the risk in signal shocks, since the ratios of probability measures are not impacted by signal shocks.

In equilibrium, each country's share of world consumption is given by a monotonic transformation of the ratio of the two ratios of probability beliefs. Defining country A's share as ω, its equilibrium value is

$$\omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right) = \frac{\left(\frac{\lambda_B}{\lambda A}\frac{\eta_{A,t}}{\eta_{B,t}}\right)^{\frac{1}{1-\alpha}}}{1+\left(\frac{\lambda_B}{\lambda A}\frac{\eta_{A,t}}{\eta_{B,t}}\right)^{\frac{1}{1-\alpha}}}, \tag{20}$$

As in Dumas, Kurshev, and Uppal (2009), individual consumption is linear in world consumption $c_{W,t}$, its slope being ω. Here ω is driven by the ratio of country A beliefs to country B beliefs, $\eta_{A,t}/\eta_{B,t}$.

This relationship can be understood intuitively as follows. When the investors of country A have deemed an event more likely to occur than did investors of country B, they have bet on that event through $\eta_{A,t}/\eta_{B,t}$ and, when it occurs, they get to consume more. In this way, ratios of probability beliefs act as endogenous taste shocks in each country.

In the standard case without foreign sentiment, perfect risk sharing would result in consumption growth being perfectly correlated, contra empirical cross-country consumption growth correlations that are typically well below 1, thereby posing a well-known puzzle.[12] With foreign sentiment, however, the presence of the stochastic term $\eta_{A,t}/\eta_{B,t}$ in the sharing rule in Equation (20) means that the cross-country conditional consumption correlation is below one all the time, except in the zero-probability event that disagreements happen to be equal to zero.

In order to obtain the portfolios of the two national groups of investors, we need to derive their total wealth processes. In doing so, we view total wealth as the price of a security with payoffs equal to optimal consumption. The wealth of the representative country i investor is

$$W^i_t\left(\frac{\eta_{A,t}}{\eta_{B,t}}, \delta_{A,t}, \delta_{B,t}, \hat{f}^E_{A,t}, \hat{f}^E_{B,t}, \hat{g}^B_{A,t}, \hat{g}^A_{B,t}\right) = \int_t^\infty \mathbb{E}^E_t\left[\frac{\xi^E_u}{\xi^E_t} c_{i,u}\right] du, \quad i \in \{A, B\}.$$

[12] For example, see the discussions in Backus, Kehoe, and Kydland (1992) and Lewis (1999).

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Since the whole mathematical framework is exponential linear quadratic (as in Cheng and Scaillet 2007), we can obtain the conditional expectation terms and thereby the wealth, pricing functions, and their derivatives, as functions of $\hat{f}^B_{A,t}$, $\hat{f}^B_{B,t}$, $\hat{g}^B_{A,t}$, $\hat{g}^A_{B,t}$, $\delta_{B,t}/\delta_{A,t}$, $\eta_{A,t}/\eta_{B,t}$, and $u-t$. The solutions for these functions are described in Appendix A when outputs are uncorrelated and in Appendix B when they are correlated, taking into account the fact that there are two "trees" in the world with outputs that need to be summed. We obtain the diffusion of wealth from a straightforward application of Itô's lemma. The elements of this diffusion are the equilibrium risk exposures of the investor.

To examine the aggregate equity market implications, we require a set of securities that both completes the market and makes our DOO effects most transparent. To complete the market, we need five securities with nonlinearly dependent payoffs, since we have four linearly independent Brownians that are observable by investors.

Given the aggregate equity market focus of our empirical regularities, stocks that are claims to each country's output are a natural choice to make our effects transparent. Therefore, equities for countries A and B are the first two securities in the menu of assets, with prices denoted $S_{A,t}$ and $S_{B,t}$, respectively. Equities are infinitely long-lived and pay amounts equal to outputs perpetually at every instant. Thus, the stock price of firm i is

$$S_{i,t}\left(\frac{\eta_{A,t}}{\eta_{B,t}},\delta_{A,t},\delta_{B,t},\hat{f}^E_{A,t},\hat{f}^E_{B,t},\hat{g}^B_{A,t},\hat{g}^A_{B,t}\right)=\int_t^\infty \mathbb{E}^E_t\left[\frac{\xi^E_u}{\xi^E_t}\delta_{i,u}\right]du,\quad i\in\{A,B\}. \tag{21}$$

We need three more securities to complete the market. For this purpose, we choose a menu of securities that allows investors to allocate their risk exposures to output shocks exclusively through country equities so that the equity preference is not polluted by indirect allocations via non-equity securities. To that aim, we add three securities with payoffs that are neutral vis-à-vis output shocks. The first one is the instantaneous-maturity riskless bond (in zero net supply), paying the interest rate r_t. The other two are futures contracts (in zero net supply as well). We choose futures contracts that are marked to the fluctuations of the signal shock from each country and designed to be hedges of signal shocks only. Since the market value of a futures contract is always adjusted to be equal to zero, a zero market-value amount of capital flow is induced by transactions in the futures market.

With the available risky menu of the two country stocks and the two futures contracts, investors must replicate the desired exposures. The 1×4 vector $\theta_{i,t}$ represents the numbers of units held by investors in country i of each available financial security:

$$\theta_{i,t}=\begin{bmatrix}\theta^i_{S_A,t} & \theta^i_{S_B,t} & \theta^i_{F_A,t} & \theta^i_{F_B,t}\end{bmatrix},\quad i\in\{A,B\}.$$

Given the closed-form solutions for the wealth of investors and the equity prices, we are able to obtain their diffusions by Itô's lemma. Calling $x_{i,t}$ the

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1×4 diffusion vector of the wealth of Investor i, and Σ_t the 4×4 diffusion matrix of the four risky securities' prices, the vector $\theta_{i,t}$ can be computed directly from a system of linear equations: $x_{i,t} = \theta_{i,t} \cdot \Sigma_t$, $i \in \{A, B\}$.

4. Quantitative Analysis

As argued above, the international financial market is a natural laboratory for analyzing DOO. Therefore, we use our model to examine the implications of DOO for various well-known empirical regularities. In particular, we conduct a simulation under the effective probability measure (as described in Section 2.1). From this measure, we generate 20,000 simulated paths over 50 years with monthly time steps. Using these simulated paths, we then evaluate the ability of our model to replicate patterns similar to the empirical regularities described in the introduction. Although our model is too stylized to replicate these regularities precisely, Section 7 provides evidence using empirical proxies for disagreement that are consistent with its implications. Details of the data and simulation are explained in Appendixes C and E, respectively.

4.1 Data

To provide an empirical reference for our model, we replicate basic findings in the literature. This analysis requires various data series, compiled to match those used in the literature.

First, we follow the literature by measuring capital flows using U.S. net foreign equity purchases data from the Department of Treasury. Given this standard treatment, we refer to capital flows in our model simulations and empirical analysis as "net purchases." For robustness, we analyze three different versions of these net purchases described in detail in Appendix C.3: (i) the raw unscaled net equity purchases; (ii) net purchases scaled by lagged foreign market capitalization as in Albuquerque, Bauer, and Schneider (2007, 2009); and (iii) net purchases scaled by lagged U.S. investment in foreign equities as in Curcuru et al. (2011).[13]

Excess returns are calculated using the corresponding set of stock market returns and the one-month Treasury bill rate, all in U.S. dollars, as detailed in Appendix C.1. To provide baseline results for the international two-factor model commonly found in the literature, we also study a set of non-U.S. companies that are listed in the U.S. market, using the stock returns for these companies described in Appendix C.5. These data are also used to reproduce standard pricing effects around cross-listing events. We focus upon the returns for these firms because they are easily traded across international markets and, hence, are less likely to be affected by forms of market segmentation that are unrelated to DOO such as transactions costs or capital controls.

[13] As suggested by this last study, we also analyze the relationship between active portfolio reallocation and stock returns in Appendix D.

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Table 1
Parameters

Name	Symbol	Value
Parameters for output dynamics		
Long-term average growth rate of output	$\bar{f}$	0.015
Volatility of expected growth rate of output	σ_f	0.03
Volatility of output	σ_δ	0.13
Output correlation	ρ	0, 0.5
Mean reversion parameter	ζ	0.2
Parameters for investors' preferences and beliefs		
Subjective discount rate	β	0.1
Relative risk aversion	$1-\alpha$	3
True correlation between foreign signal and mean foreign growth rate	ϕ	0.95
Perceived correlation between foreign signal and mean foreign growth rate	–	0
Relative Lagrange multipliers of the lifetime budget constraints	λ_B/λ_A	1

This table lists the parameter values used for all the (other) tables and figures in the paper. These values are taken from Dumas, Kurshev, and Uppal (2009).

4.2 Parameters

Table 1 summarizes our specific parameter assumptions. Our calibrated parameters are taken from Dumas, Kurshev, and Uppal (2009). They in turn chose these values based upon Brennan and Xia (2001), who developed a model of learning similar to ours but without differences of opinion and considered a range of parameters for fundamentals and preferences chosen to match key features of U.S. data: consumption, dividends, interest rates, and stock prices. In addition, the parameter ϕ captures the degree to which the home signal conveys valuable information to home residents (ignored by foreign residents) about the conditional home output growth rate. To highlight the potential effects of DOO, this parameter is set at a level of 0.95 in the baseline model. We describe the potential impact of varying degrees of ϕ in our empirical Section 7 below.

To evaluate the effects of foreign-sentiment risk, we calculate some basic unconditional moments for asset prices, capital flows, output, and consumption produced by our model along with their data counterparts. The simulated foreign-sentiment model moments for asset pricing, assuming both uncorrelated output growth as well as a modified version with output growth correlation equal to 0.5, are reported in Panel A of Table 2. For comparison, the panel also provides the simulated model results without sentiment and the moments from the data. As the table shows, our foreign-sentiment model delivers stock-return mean and volatility that are close to the data, though the mean and volatility of the rate of interest are too high.[14]

Panel B of Table 2 also reports consumption and output correlations generated by our model as well as the data. The first two rows give the cross-country consumption growth correlation and cross-country output growth

[14] The difficulty of matching simultaneously the moments of the interest rate and those of the stock market are common in DOO models as shown in Dumas, Kurshev, and Uppal (2009). See also Xiong and Yan (2010).

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Table 2
Moments

Panel A: Asset pricing moments (annualized percentages)

Moment	No foreign sentiment		Foreign sentiment		Data
	$\rho=0$	$\rho=0.5$	$\rho=0$	$\rho=0.5$	
Interest rate mean	7.62	6.35	9.77	9.35	5.43
Interest rate standard deviation	10.79	10.69	7.13	8.03	3.05
Stock return mean	11.58	10.92	11.21	11.43	11.40
Stock return standard deviation	21.79	22.53	15.08	16.75	15.25

Panel B: Macroeconomic moments

Correlations	No foreign sentiment		Foreign sentiment		Data
	$\rho=0$	$\rho=0.5$	$\rho=0$	$\rho=0.5$	
Cross-country consumption	1.00	1.00	0.29	0.30	0.28
Cross-country output	0.00	0.50	0.00	0.50	0.44
Within-country consumption-output	0.67	0.82	0.45	0.61	0.60

Panel C: Capital flow autocorrelation

Net Purchases Measure	Foreign net equity purchases			
	$\rho=0$	$\rho=0.5$	Data mean	Data range
Raw	0.004**	0.066**	0.362**	0.099–0.640
	(<0.001)	(<0.001)	(<0.001)	
Scaled by market cap	−0.008**	−0.01**	0.205**	0.029–0.647
	(<0.001)	(<0.001)	(<0.001)	

This table reports selected moments in the benchmark model with and without foreign sentiment, with and without cross-country correlation ($\rho=0.5$ and $\rho=0$, respectively), and in the data. Panel A provides the means and standard deviations of the interest rate and stock return. The "Data" column reports these statistics using the data described in Appendix C.1. Panel B reports the cross-country and within-country correlations of consumption and output growth. The "Data" column reports the average correlation for countries against the United States, using the data described in Appendix C.2. Panel C gives the autocorrelation in foreign equity purchases, using robust least squares (MM estimation) for the model. The "Data mean" column reports the cross-country mean of the time-series autocorrelations in the U.S. equity net purchases, while "Data range" reports their range, all using the data described in Appendix C.3. p-values for the coefficient being different from zero are in the parentheses. ** indicates significant at the 5% marginal significance level (MSL).

correlation. As could be anticipated from Proposition 3, foreign sentiment lowers the unconditional consumption correlation below one, shown in Panel B. Furthermore, when $\rho=0.5$, the cross-country consumption growth correlation (equal to 0.3) is lower than cross-country output growth correlation, as observed in the averages across countries in our data.[15] Backus, Kehoe, and Kydland (1992) pointed out that complete-markets models have difficulty generating a higher cross-country correlation in output growth than in consumption growth. Clearly, our DOO model provides an important exception due to the incorporation of foreign-sentiment or belief shocks, which are effectively endogenous taste shocks making utility state-dependent. These belief shocks are positively correlated across countries but are also correlated with output shocks, since each output growth provides a signal that is useful for the inference of both home and foreign fundamentals. The combination of these effects generates the

15 While the correlations in the data are calculated at the quarterly frequency, our model is calibrated to generate moments at the monthly frequency to be consistent with capital-flows regressions described in Section 5.

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observed consumption growth correlation, which is little affected by increasing output correlation.

Backus, Kehoe, and Kydland (1992) also noted that the correlation between consumption and output growth within countries is high. In our average of these correlations across countries reported as the "Within-country" correlation, the estimate is 0.67, broadly in line with our simulated foreign-sentiment model-implied correlation of 0.61 when output is correlated.

We next consider the implications for the autocorrelation in capital flows using our base capital flow measure and that measure scaled by market capitalization. Panel C of Table 2 shows that these empirical autocorrelations are positive for both unscaled and scaled capital flows, with the mean and range across countries under "Data mean" and "Data range," respectively. Similarly, Albuquerque, Bauer, and Schneider (2007, Table 3a) estimate the autocorrelation of market capitalization-scaled capital flows for six OECD countries over an earlier period (1977:2 to 2000:3), finding a range from 0.16 to 0.52.

Table 2, Panel C, also reports the autocorrelation from our model for these measures. In our setup, the international capital flows—or "foreign purchases"—are interpreted as the flow demand of home-country (for instance, country B) investors for the foreign (country A) stock valued at current market prices $S_{A,t}\times\left(\theta^B_{S_A,t}-\theta^B_{S_A,t-dt}\right)$, as in balance-of-payments accounting.[16] Scaling by foreign market capitalization as in Albuquerque, Bauer, and Schneider (2007, 2009) produces the change in physical shares of the foreign equity, or $\theta^B_{S_A,t}-\theta^B_{S_A,t-dt}$. As Table 2 shows, this autocorrelation is negligible in our model, regardless of the capital-flow measure and the degree of autocorrelation, showing that the persistence in unscaled capital flows $S_{A,t}\times\left(\theta^B_{S_A,t}-\theta^B_{S_A,t-dt}\right)$ is entirely due to the persistence in the price $S_{A,t}$.[17] Albuquerque, Bauer, and Schneider (2007) are able to generate persistence in capital flows in a model with investor heterogeneity within a country. Here, for tractability, investors are homogeneous within countries, and the focus is on the heterogeneity across countries and on the role of general equilibrium pricing. In our model, the endogenous adjustment of the rate of interest and the lack of persistence of capital flows are part and parcel of the same equilibrium. It would be conceivable to introduce more persistence in the rate of interest and the capital flows by modifying the specification of the processes for the signals.[18] That possibility will be investigated in future research.

16 The empirical literature uses net equity purchases in order to distinguish the securities issued by different countries. Similarly, we focus on flows in equity only. In principle, we could examine flows of all the securities including borrowing and lending at the risk-free rate.

17 One insightful referee pointed out this fact.

18 Specifically, one could let the signal processes have a non-zero drift equal to the conditionally expected growth rates of output. This additional information about the slow-moving expected growth would cause the investors' estimates of that growth to be slower moving.

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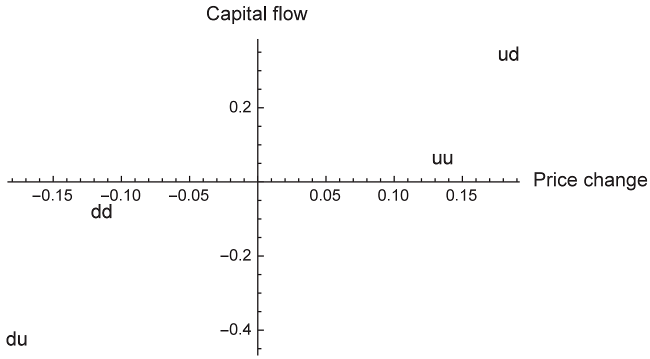

Figure 1
Conditional medians of foreign-equity price change and net foreign purchases by home investors
The figure plots the median foreign-equity price change and capital inflows conditional on the sign of foreign shocks as perceived by the econometrican. The first letter refers to output shock, while the second letter refers to the signal shock. "u" refers to a positive shock, and "d" refers to a negative shock. The conditional medians are calculated across 20,000 paths at $t=50$ years.

5. Portfolios and Capital Flows

5.1 The basic mechanism: Foreign capital inflow and home return co-movement

We now turn to our first international empirical regularity: the co-movement between stock returns and capital flows. As a number of papers have documented, capital flows into countries when the stock market experiences above average returns, a phenomenon sometimes referred to as "return chasing."[19] According to the asymmetric-information model of Brennan and Cao (1997), this phenomenon occurs because foreigners' private signals are less informative about the home country's stock return than are those of the home country's residents. As public information arrives and they see home stock prices going up, less informed foreigners speculate that home investors see in the public signal a confirmation of their private signal, which must have been favorable to home stocks. Accordingly, they buy the home country's stocks. In other words, the reason foreign, uninformed investors buy more of the home asset is that they update their prior beliefs regarding the quality of the home asset after positive public news about that asset.[20]

In our model, the behavioral assumption is different but the logic is similar. All investors fully observe two kinds of information: current output growth and news about the future output growth in the form of the signal. There is a difference in perception about the latter.

[19] See, for example, Brennan and Cao (1997), Bohn and Tesar (1996), Grinblatt and Keloharju (2000), and Brennan et al. (2005).

[20] We are grateful to a referee for emphasizing this interpretation.

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Figure 1 shows the basic mechanism graphically using our simulated data for our basic net equity purchases measure: $S_{A,t} \times \left(\theta^B_{S_A,t} - \theta^B_{S_A,t-dt}\right)$, where as above we treat B as the "home" and A as the "foreign" country that B invests in.[21] This figure plots the median net purchases and price changes conditional on the sign of foreign shocks. To calculate these medians, we categorize 20,000 simulated outcomes into four different groups depending upon the signs of the country A shocks; that is, whether the output shock is positive "u" or negative "d" and the signal shock is positive "u" or negative "d." The labels in the figure then plot the median for the specific subsample, where the first letter refers to output shock, while the second letter refers to the signal shock. For example, "ud" is the median for all simulations when the output shock is positive $\left(dw^E_{\delta_A,t} > 0\right)$ and the signal shock is negative $\left(dw^E_{s_A,t} < 0\right)$.

Then, to see the relationship between net purchases and price changes, suppose a positive output shock occurs abroad (first label "u"). All investors attempt to buy, driving the price up; that is, $S_{A,t} - S_{A,t-dt} > 0$. Half of the time this positive output shock is accompanied by a negative foreign signal (second label "d")—a fact that home investors ignore, which leads to disagreement. In that case, foreign investors are willing to sell and home investors are able to execute their buys, increasing their net foreign purchases for the "ud" pair. The combination of the four label pairs clearly shows a positive relationship between net purchases and price movements, consistent with the empirical finding.

Figure 1 should really be plotted in three dimensions reflecting three conditioning shocks: the foreign signal shock, the foreign output shock as viewed by home investors, and the foreign output shock as viewed by foreign investors. All three shocks simultaneously cause the price change and the capital flow, because both groups participate in the demand and the supply of securities. In Figure 1, we have conditioned on the output shock under the econometrician's measure. This is an appropriate shortcut because the output shocks as estimated by the two groups and the econometrician remain quite correlated with each other. We have verified that a plot like Figure 1, but conditioned on the output shock as viewed by one group or the other, looks very similar.

We further illustrate this relationship by running on simulated data a standard regression found in the literature. Beginning with Brennan and Cao (1997), a number of papers have regressed net purchases of foreign equities by U.S. investors on the foreign-market return.[22] To reproduce the relationship implied by this regression, we consider two types of regression. This first type follows the relationship in Figure 1 with raw net equity purchases. We use our

21 A similar relationship holds when plotting the scaled capital flows, $\theta^B_{S_A,t} - \theta^B_{S_A,t-dt}$.

22 See, for example, Bohn and Tesar (1996) and, more recently, Curcuru et al. (2011). Albuquerque, Bauer, and Schneider (2007) consider the correlation between net foreign purchases and foreign market returns, finding a positive relationship.

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Table 3
Country-level regularities

Panel A: Capital flow and stock return

	Net purchases, raw			Net purchases, scaled by market cap		
	$\rho=0$	$\rho=0.5$	Data	$\rho=0$	$\rho=0.5$	Data
Brennan-Cao coef	0.575**	0.734**	0.387**	0.541**	0.952**	0.224**
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.038)
Adjusted R^2	0.016	0.015	0.085	0.001	0.002	0.035

Panel B: Home-equity preference

	No foreign sentiment		Foreign sentiment		Data	
	$\rho=0$	$\rho=0.5$	$\rho=0$	$\rho=0.5$	Mean	Range
Home equity share of wealth	0.500	0.500	0.596	0.566	0.745	0.651−0.803

Panel C: Holdings of financial assets and trading volume

	No foreign sentiment		Foreign sentiment	
	$\rho=0$	$\rho=0.5$	$\rho=0$	$\rho=0.5$
Number of home shares held	0.500	0.500	0.635	0.637
Number of foreign shares held	0.500	0.500	0.366	0.363
Number of home futures held	0.000	0.000	−0.021	−0.040
Number of foreign futures held	0.000	0.000	0.021	0.040
Trading volume of stocks	0.000	0.000	0.073	0.063
Trading volume of futures	0.000	0.000	0.027	0.028

This table reports country-level regularities in the model and in the data. Panel A reports coefficients, p-values of the hypothesis that the coefficients differ from zero (in parentheses), and R^2 from regressions of net purchases of foreign equities on foreign returns in the data and in the model using robust least squares (MM estimation). ** indicates significant at the 5% MSL. Data measures of net purchases are described in Appendix C.3. Panel B shows the value of home stock held as a proportion of wealth, for the model simulations and for the mean and range over time in the data described in Appendix C.4. Panel C gives the median value generated by the model for home investor asset holdings and their changes including the number of home and foreign equity shares, the number of home and foreign futures held, and the absolute value of changes in his holdings of each asset class, as measures of the stock and futures trading volumes.

simulations to regress foreign purchases by country B investors for the foreign stock valued at current market prices, $S_{A,t} \times \left(\theta^B_{S_A,t} - \theta^B_{S_A,t-dt}\right)$, on the returns of the foreign country given by the change in the stock price, $S_{A,t} - S_{A,t-dt}$. The columns under the heading "Net purchases, raw" in Panel A of Table 3 show significant regression coefficients in our simulation of 0.575 and 0.734 from our model without and with output growth correlation (labeled $\rho=0$ and $\rho=0.5$, respectively).[23] By way of comparison, the column labeled "Data" provides the coefficient for a pooled regression in our data set. The coefficient increases with correlated output, and the pooled-data estimate is lower than in the model. But all are within the range across countries reported in Brennan and Cao (1997) of 0.12 to 5.13.

[23] Without foreign sentiment, the capital flows would obviously be equal to zero, as in that case investors are identical and hence there is no incentive for financial trade. Accordingly, we only present results for our model with foreign sentiment.

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For our second type of regression, we use net purchases scaled by market capitalization. Following Albuquerque, Bauer, and Schneider (2007), this measure provides a particularly useful interpretation of net purchases as the change in foreign shares, $\theta^B_{S_A,t}-\theta^B_{S_A,t-dt}$. The Brennan-Cao coefficients obtained by regressing these measures on the foreign return or, in our model, $(S_{A,t}-S_{A,t-dt})/S_{A,t-dt}$, are reported in Table 3, Panel A, in the columns under the heading "Net purchases, scaled by market cap."[24] Similar to the results with unscaled net purchases, the Brennan-Cao coefficients increase with correlated output growth from 0.541 to 0.952. Intuitively, when home output is high at the same time foreign output is high (which, with the positive correlation, happens more often than not), foreign investors, who ignore the home signal, ascribe a higher probability that the home conditional mean is high. Therefore, the foreign investors buy more home stocks, while home residents, using the home signal, are willing to sell. As a result, an increase in home stock price is associated with even more foreign buying of the home stock than when output is uncorrelated.

As these results show, foreign sentiment generates a positive co-movement between capital flows and returns. By contrast, there would be no capital flows in the absence of differences in perceptions of the news content in signals, as parameterized by ϕ. Thus, the tendency to find a positive Brennan-Cao coefficient depends upon how much the investors differ in their interpretation of the foreign signal, a relationship to which we return in Section 7.

5.2 Home-equity preference

Home-equity preference is the observation that home investors tilt their portfolios toward home equity beyond the level suggested by standard theory.[25] Explanations proposed for this preference include nontradable goods or leisure, incomplete markets, and asymmetric information. We next show that our foreign-sentiment risk model also generates home-equity preference even though in our model all goods are tradable, markets are fully complete, and all information is public.

The mechanism by which equity preference is generated in the model is one reflecting intertemporal hedging, and it stems from the capital-flow mechanism that we outlined in the previous section. Sitting at time t, an investor knows that at time $t+dt$ and thereafter he will revise his portfolio. He faces reinvestment risk, which is precisely the risk that intertemporal hedging is meant to alleviate, to the extent possible. Looking at time $t+dt$ and thereafter, this investor is aware of the correlation between net purchases and price movement that we described

24 Results using our third net purchases measure scaling by foreign investment were similar and, therefore, we omit them for parsimony.

25 An extensive literature has documented this regularity. For a few examples of studies spanning several decades, see Grubel (1968), French and Poterba (1991), and more recently Ahearne, Griever, and Warnock (2004). Lewis (1999, 2011) and Coeurdacier and Rey (2013) synthesize potential explanations.

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in the previous subsection. He knows that, when it comes to foreign equity, he will be able to buy on the occasion of a high output shock that causes the price to be high and he will be able to sell on the occasion of a low output shock that causes the price to be low, whereas there is no such tendency on the home equity market. For that reason, even though rates of return in equilibrium are symmetric between the two types of equity, he knows that, his timing being inferior in the foreign market, he will not be able to earn as much holding and revising his portfolio of foreign equity than he does doing the same with home equity. As a hedge against this, he holds less of foreign than he does of domestic equity, while still holding both for the sake of diversification. This is true as an average across states of nature (or across simulation paths).

We use our simulated model to evaluate portfolios held by an investor. Panels B and C of Table 3 show the median of these holdings. The columns labeled "No foreign sentiment" show that the median share invested abroad is equal to 50% without foreign-sentiment risk. By contrast, the simulations including foreign-sentiment risk (columns "Foreign sentiment") show a clear bias toward greater home equity holdings. In Panel C, the median across paths is approximately 64% for the number of home shares held, and in Panel B, the proportion of wealth held is between 57% and 60%. The table also shows that holdings of futures are small compared with equity holdings and that the volume of trading in futures is less than half the volume of trading in stocks.

6. Pricing Issues and Firm-Level Returns

6.1 Two-factor consumption CAPM

In integrated markets, risk factors are common to all securities and all securities are priced with these same factors. In our model, with no market segmentation and with all tradable goods, equilibrium prices, such as Equation (21), are functions of seven state variables, each of which is driven by four Brownian motions—that is, two in each country when outputs are uncorrelated. Moreover, Proposition 4 and the stochastic discount factor in Equation (18) reveal that ultimately only two factors are priced: world consumption and world average foreign sentiment. The next proposition says that our model is consistent with a two-factor consumption CAPM.

Proposition 5. The following consumption-CAPM holds:

$$\widehat{\mu}^{E}_{S_i} - r_t = (1-\alpha) Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{dc_{W,t}}{c_{W,t}}\right) - Cov\left(\frac{dS_{i,t}}{S_{i,t}}, \frac{d\eta_{W,t}}{\eta_{W,t}}\right), \quad i \in \{A, B\}, \tag{22}$$

where $c_{W,t} = c_{A,t} + c_{B,t}$ is world consumption and $\eta_{W,t}$ (defined in Equation (19)) is a measure of world average foreign-sentiment risk, with dynamics given by

$$\frac{d\eta_{W,t}}{\eta_{W,t}} = \omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)\frac{d\eta_{A,t}}{\eta_{A,t}} + \left[1-\omega\left(\frac{\eta_{A,t}}{\eta_{B,t}}\right)\right]\frac{d\eta_{B,t}}{\eta_{B,t}}.$$

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Proof. The market price of risk is obtained by applying Itô's lemma to the stochastic discount factor, and identifying its diffusion vector. The CAPM risk premia are derived from the market price of risk. ■

As in the standard consumption-based CAPM, a security risk premium is positively correlated with the covariance of its return with the world consumption growth. In our model, the risk premium is also decreasing in the covariance of the security's return with the world average foreign sentiment. The conditionally expected excess returns on the left-hand side of the CAPM relationship correspond to the way in which the econometrician would collect and process data on returns.

In empirical studies, a home risk factor often appears to be priced, in addition to a world, or foreign, factor. Our interpretation of that fact is based on Proposition 3 above, which says that the foreign-sentiment risk of the investors in one country is conditionally perfectly correlated with the output shock in the other country (with the sign of the perfect correlation being opposite to the sign of current disagreement). In CAPM Equation (22), the apparent pricing of home output risk, over and above world consumption risk, derives from its correlation with world average foreign-sentiment risk. The latter is the true unobserved risk factor.

6.2 Pricing puzzle: Factor model

Much of the empirical literature on international stock returns focuses not upon a consumption-based CAPM, but rather on the factor structure of the returns. In this literature, international firm returns appear to depend upon home factors as well as foreign or world factors, an observation often interpreted as evidence for market segmentation or nontradable risks.[26] Moreover, the beta on the home market is typically higher than that on the foreign market.

To evaluate this relationship in our model, we now introduce a new firm C in country A with an output process similar to that of A and B. The dynamics of firm C's output and conditional expected output growth rate are

$$\frac{d\delta_{C,t}}{\delta_{C,t}} = f_{C,t}dt + \sigma_\delta dz^\delta_{C,t},$$

$$df_{C,t} = -\zeta\left(f_{C,t} - \bar{f}\right)dt + \sigma_f dz^f_{C,t},$$

where $z^\delta_{C,t}$ and $dz^f_{C,t}$ are independent Brownian motions under the effective probability measure. Similarly to A and B, investors also observe a signal

[26] For studies finding home and foreign factors in returns and the related debate on the number of factors, see Agmon (1973), Lessard (1976), Heston and Rouwenhorst (1994), Cavaglia, Brightman, and Aked (2000), Cavaglia and Moroz (2002), Brooks and Del Negro (2005, 2006), and Bekaert, Hodrick, and Zhang (2009). For evidence that the factors are related to market segmentation, see Bekaert and Harvey (1995), among others.

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Table 4
Firm-level regularities

Panel A: Factor model

	No foreign sentiment	Foreign sentiment	Percentage difference	Data
Home excess return	0.473**	0.515**	8.858	0.861**
	(<0.001)	(<0.001)		(<0.001)
Foreign excess return	0.473**	0.485**	2.452	0.241**
	(<0.001)	(<0.001)		(0.011)
Adjusted R^2	0.837	0.927	10.780	0.003

Panel B: Impact of cross-listing

	Price change		Return change	
	Foreign sentiment	Data	Foreign sentiment	Data
Cross-listing firm	4.487	3.418	−0.142	−0.194

This table reports firm-level regularities in the model and in the data. Panel A reports coefficients, p-values of the hypothesis that the coefficient differs from zero (in parentheses), and R^2 from regressing the excess equity return of a firm on that of its home market and the foreign market. The first two columns give the statistics from the simulated model without and with foreign sentiment, respectively, while the third column calculates their percentage differences. The column labeled "Data" contains the coefficient estimates for the pooled regression in Equation (23) in the text using excess returns constructed from firm return data described in Appendix C.5 and market return and Treasury bill data described in Appendix C.1. ** indicates significant at the 5% MSL. Panel B reports the firm's mean percentage change in price and the mean change in return due to cross-listing in the model under the "Foreign sentiment" columns. Under "Data," the panel reports results from the pooled regression in Equation (24) in the text where the "Price change" is the annualized abnormal return for the 14 weeks up until cross-listing and where the "Return change" is the change in mean fitted estimates of $\left(b^{POST} - b^{PRE}\right) R_{mt}$. Data for returns and cross-listing dates are in Appendix C.5, excluding Canadian firms.

$s_{C,t}$ that is correlated with the output growth rate $f_{C,t}$. Further assumptions and notational details are spelled out in Appendix F. Since firm C is listed in country A, investors of country A process properly the signal that is correlated with the output growth of firm C, while investors of country B ignore it.

With this expanded model, we examine the regression relationship of individual firm returns on home and foreign markets using our simulated data for the uncorrelated output model.[27] Specifically, we regress individual firm C's stock excess return, $(dS_C + \delta_C dt)/S_C - r_t dt$, on the corresponding country A stock excess return $(dS_A + \delta_A dt)/S_A - r_t dt$, and country B stock excess return, $(dS_B + \delta_B dt)/S_A - r_t dt$.[28] Panel A of Table 4 shows the results of the two-factor regressions run on our simulated data under "No foreign sentiment" and "Foreign sentiment." Without foreign sentiment, the symmetry assumed in our model implies foreign and home factors have the same betas. However, the difference of interpretation of signals captured by ϕ in our model generates foreign-sentiment risk. When this risk is present, the beta on the foreign factor

27 The significantly greater computational burden of adding another firm prohibits simulations for the correlated outputs case.

28 The return on the stock market index of country A would normally include firm C. However, in our regression of $(dS_C + \delta_c dt)/S_C - rdt$ on the country A index excess return, we exclude firm C in the index for the stock market of country A, as this would bias the beta against that index.

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is smaller than the beta on the home factor, which is in accordance with the empirical evidence in the literature. A similar pattern can be seen in the column labeled "Data," reporting the estimates of the pooled regression:

$$R_{it} = \beta_0 + \beta_{Home} R_{mt}^{Home} + \beta_{US} R_{mt}^{US} + u_t, \qquad (23)$$

where R_{it} is the excess return of firm i and R_{mt}^{Home} and R_{mt}^{US} are the excess returns of firm i's home market and the U.S. market, respectively, at time t. In Section 7 we go on to show empirically that the tendency for home and foreign betas to differ is greater for firm returns that are more sensitive to variations in the dispersion of forecasters' views.

6.3 "Abnormal" cross-listing returns

Cross-listing events present another feature of international security-return behavior, often associated with differing economic perceptions across countries. At the time foreign firms list in home markets, the returns on equity shares of the cross-listing firm become abnormally high relative to the market, generally between 1.5% and 7%. On the other hand, the cost of capital is lowered. For example, typical estimates for the cost of capital drop after cross-listing are between 0.22% and 1.3%.[29] An oft-cited explanation for these responses is that cross-listing the foreign firm in the home market improves home-investor information processing about its future behavior. Ahearne, Griever, and Warnock (2004) show that U.S. investors are more willing to invest in firms from countries that list on U.S. exchanges, noting that cross-listing provides information from these firms that is easier to interpret.[30]

To evaluate this pricing behavior with our model using the motivation from the literature, we return to the three-firms extended model above. We now compare the equilibrium in which firm C is listed in country A to the equilibrium in which it is listed both in country A and in country B. To capture the idea that cross-listing provides home investors with increased ability to process information about the foreign firm, we postulate that, if firm C is cross-listed, investors in country B know how to correctly interpret the public information about firm C. Therefore, under cross-listing, country B investors now recognize that the signal process conveys information about firm C. This assumption implies that the dependence of returns on forecast disagreements will change because firms cross-list, a relationship we examine in Section 7.

Using our simulated data, we calculate average "abnormal returns" due to firm C cross-listing from country A to country B, as described above. Although these results simply compare the price with and without foreign-sentiment risk

[29] On abnormal returns and the lower cost of returns, see Hail and Leuz (2009), Sarkissian and Schill (2009), and Gozzi, Levine, and Schmukler (2008), for a few examples. Karolyi (2006) provides a survey.

[30] For example, Coffee (1999, 2002) argues that the cross-listing stock prices increase because these firms commit to abide by the stricter reporting standards, thereby reducing investor uncertainty.

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in firm C, they demonstrate the effects of removing informational uncertainty across countries, as suggested by the literature.

As a comparison to data, we use the firm-level return data to reestimate a standard pooled regression conditioning on cross-listing:[31]

$$R_{it} = b^{PRE} R_{mt} + b^{POST} R_{mt} I_t + \epsilon_t, \quad (24)$$

where $R_{mt} \equiv \left[1, R_{mt}^{Home}, R_{mt}^{US}\right]$, b^{PRE} and b^{POST} are the conformable factor loadings, and I_t is a dummy variable equal to one if the period is after cross-listing and equal to zero otherwise.

The results are displayed in Panel B of Table 4. Without foreign sentiment, the abnormal returns would obviously be equal to zero. With foreign sentiment, the model generates a mean percentage price increase due to cross-listing equal to 4.487% for the firm that cross-lists. This level is within the range in the literature noted above and close to our estimated price response of 3.418%. Our model also generates a lower cost of capital, as in the literature. In the model, the mean reduction in the cost of capital is equal to 0.142%, somewhat lower than the range of values found empirically, but similar to our estimate of 0.194%. Overall, the improvement in the processing of information about a firm's growth prospects due to cross-listing affects its price quite clearly but affects its cost of capital much less, consistent with the empirical evidence mentioned above.

7. Empirical Regularities and Difference of Opinion Proxies

We have shown that DOO can potentially help explain several well-known international finance anomalies both qualitatively and quantitatively. Therefore, additional information might be found by looking at empirical evidence about DOO. Although testing our model is precluded by its highly stylized nature, we can examine evidence by noting that countries presumably face varying degrees of DOO about the information content in news, captured in our model by ϕ. For some countries and firms, opinions about the information content in news may be rather unanimous, while for others there may be more disagreement. Our model then suggests we should find greater evidence of financial anomalies for countries and firms that are more sensitive to differences in information processing.

Therefore, in this section, we empirically investigate some relationships predicted by our model using proxies for these differences. The evidence will only be suggestive, as we are not in position to control for variables other than the degree of DOO. We begin by describing these measures and relationships before turning to the results.

[31] See, for example, Foerster and Karolyi (1999) and Sarkissian and Schill (2009). The latter paper provides a breakdown into pre-listing and post-listing periods, as in our analysis.

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7.1 Data and Model Relationships

To proxy for DOO about future economic activity, we use data based upon the forecasts of professional forecasters. We conjecture that these forecasters likely have common information so that disagreements about macroeconomic predictions result from differences of opinion.[32] In principle, we would like to have measures of differences in forecasts for many countries. Unfortunately, a sufficiently long history of the dispersion of forecasts in non-U.S. countries is unavailable. We, therefore, focus upon professional forecasts of the U.S. economy as well as some indirect measures that may include forecast dispersion of global variables. Moreoever, the forecasters are individuals and institutions from various countries, thereby providing a range of international views.

In the empirical analysis, we study three sets of variables detailed in Appendix C.6. The first set is the difference between the 75th and 25th percentiles of forecasts for U.S. GDP and investment growth, both residential and nonresidential, from the Survey of Professional Forecasters (SPF). A second proxy for differing opinions is the variance of the SPF individual forecasts for the four-quarter-ahead GDP growth, a variable analyzed by Bansal and Shaliastovich (2010) and Shaliastovich (2015). The third set of variables includes the "sentiment risk indices" proposed by Baker and Wurgler (2006) for the cross-section of U.S. data, and Baker, Wurgler, and Yuan (2012) for global data. Unlike our other measures, these variables need not directly relate to disagreements, but we include them since they have been associated with sentiment risk in the literature and provide some measure of global views. Although these various measures likely capture different effects arising from disagreements, we collectively call these measures "DOO proxies" for simplicity.

Using these proxies, we can then ask whether relationships suggested from the model are borne out by the data. Accordingly, we examine relationships based upon three of the regularities in the model.[33] The first relationship is that countries with net equity purchases that are more sensitive to DOO have a higher Brennan-Cao coefficient. That is, as noted in Section 5, countries with greater differences in views about the information content of news arising from higher ϕ generate greater co-movement between capital inflows and returns. The second suggested empirical relationship is that firms with returns that are more sensitive to differing opinions have a greater difference between their home and foreign betas. We showed in Section 6.2 that differences in processing the foreign signal generated a wedge between home and foreign betas, so, conversely, firm returns that face no disagreements should have no such wedge. The third empirical relationship arises from our model conjecture in Section 6.3 that cross-listing changes the way in which the investors in the newly listed market

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[32] We are grateful to Geert Bekaert for emphasizing this point to us.

[33] We have insufficient annual observations to econometrically analyze the remaining regularity, "home-equity preference."

view the cross-listed company, implying a shift in information processing. If so, the effects on firm returns due to differing views about public information should change after cross-listing. We next examine empirical evidence of these three relationships.

7.2 Empirical Evidence

To consider the first empirical relationship, we ask how net equity purchases would respond differently across countries depending upon their sensitivity to DOO proxies. For this purpose, we first establish whether, for each country i, proxies help explain net purchases with the following time-series regressions:

$$NP_t^i = a_0^i + a_1^i D_t + u_t^i, \tag{25}$$

and

$$NP_t^i = a_0^i + a_1^i D_t + a_2^i R_{mt}^i + e_t^i,$$

where NP_t^i are U.S. net purchases of equity of country i, D_t are the DOO proxies, and R_{mt}^i is the market return of country i.

Studying the effects of Brennan-Cao coefficients across countries requires a cross-section of countries. Therefore, for this analysis we focus on net purchases scaled by U.S. holdings of foreign securities because this scaling provides a wider cross-section of 42 countries, while market capitalization scaling limits the number of countries to 13. Using these net purchases scaled by total foreign securities, the initial regressions verify that the coefficients on DOO measures, a_1^i, are generally significantly different from zero (not reported for parsimony).

For each DOO proxy, we then sort countries into three groups based upon the absolute value of their country-specific a_1 coefficients. The absolute value captures the possibility that the net equity purchases of some countries may be positively related to the DOO proxy, while for others the net purchases may be negatively related, but any deviation from zero generates exposure. We next run a pooled regression for each of the sorted groups of their net purchases on their respective market returns. Panel A of Table 5 reports the coefficients on the market returns, that is, their Brennan-Cao coefficients, where each column provides the sort on a different DOO proxy and each row is a different group based upon the initial regression (25) alone for parsimony.[34] Notably, the average response of net purchases to equity market return increases for countries with greater sensitivity to DOO proxies. For example, using GDP forecasts, the Brennan-Cao coefficient for the "Low $|\hat{a}_1|$" country group is only 0.005, while that of the "High $|\hat{a}_1|$" country group is 0.096. A similar pattern can be seen for almost all the other proxies. Thus, consistent with the first

[34] We also examined the robustness of these results along several dimensions including different time periods, using a smaller subsample of countries also studied by Curcuru et al. (2011), and using lagged returns as regressors, finding a similar pattern in all these cases.

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Table 5
Regularities and DOO Proxies

Panel A: Brennan-Cao coefficients by groups sorted on DOO sensitivity

	GDP	Resid invest	Nonresid invest	GDP var	US sent	Global sent
Group 1 (low $\lvert a_1 \rvert$)	0.005**	0.019**	0.003**	0.008**	0.007**	0.005**
	(0.002)	(0.004)	(0.001)	(0.002)	(0.003)	(0.001)
Group 2 (medium $\lvert a_1 \rvert$)	0.021**	0.020**	0.036**	0.020**	0.018**	0.014**
	(0.004)	(0.005)	(0.006)	(0.004)	(0.004)	(0.003)
Group 3 (high $\lvert a_1 \rvert$)	0.096**	0.087**	0.097**	0.106**	0.084**	0.140**
	(0.015)	(0.015)	(0.016)	(0.015)	(0.014)	(0.017)

Panel B: Cross-sectional regression of $|\beta_{Home} - \beta_{US}|$ on DOO sensitivity

	GDP	Resid invest	Nonresid invest	GDP var	US sent	Global sent
d_1	0.178**	0.113**	0.378**	0.107**	1.312**	0.291**
	(0.001)	(0.001)	(0.002)	(0.001)	(0.014)	(0.009)

Panel C: Changes in sensitivity to DOO across cross-listing events

	GDP	Resid invest	Nonresid invest	GDP var	US sent	Global sent
Before: b_D^{PRE}	−0.063	−0.041	−0.028	−0.078	−0.007	0.075**
	(0.140)	(0.163)	(0.161)	(0.132)	(0.139)	(0.030)
Change: $b_D^{POST} - b_D^{PRE}$	0.824**	0.410*	0.968**	0.763**	−0.142	−0.083**
	(0.289)	(0.220)	(0.293)	(0.265)	(0.233)	(0.041)

Panel A reports coefficients from pooled Brennan-Cao regressions where country groups are sorted by low, medium, and high absolute values of a_1, from initial time-series regressions by country given by Equation (25) in the text using net purchases of equity scaled by total foreign investment as described in Appendix C.3 and the data for DOO proxies described in Appendix C.6. Panel B reports estimates from cross-sectional regressions of $\left|\beta^i_{Home} - \beta^i_{US}\right| = d_0 + d_1 \left|\beta^i_D\right| + e_t$, where β^i_{Home} and β^i_{US} are estimates from firm-level time-series regressions given in Equation (26) in the text using excess returns constructed from the firm return data described in Appendix C.5 and the market return and Treasury bill data described in Appendix C.1. Panel C reports coefficients on DOO proxies in the pooled time-series regression given in Equation (27) in the text using the same data as in Panel B along with the cross-listing dates described in Appendix C.5. In all panels, the DOO proxies as described in Appendix C.6 are defined as: "GDP," "Resid invest," and "Nonresid invest" are the differences between the 75th and 25th percentiles of forecasts for U.S. GDP, nonresidential investment, and residential investment, respectively; "GDP var" is the variance of GDP forecasts; and "U.S. sent" and "Global sent" are the U.S. and Global Sentiment Indices, respectively. Standard errors are in parentheses. ** (*) indicates significant at the 5% (10%) MSL.

relationship suggested by our model, the countries with net purchases that are more sensitive to opinion dispersion are also the countries with net purchases that are more sensitive to domestic returns.

We next consider the second empirical relationship suggested by our model. Since the presence of differing opinions in our symmetric two-country model generates a wedge between home and foreign betas, conversely, firms with more sensitivity to forecast dispersion should have more difference between these two betas. To evaluate this possibility, we run initial time-series CAPM regressions of the excess returns R_{it} for each of our non-U.S. firms on the excess returns of their home and foreign (U.S.) markets, R^{Home}_{mt} and R^{US}_{mt}, respectively, and on the DOO proxies as given by:

$$R_{it} = \beta^i_0 + \beta^i_{Home} R^{Home}_{mt} + \beta^i_{US} R^{US}_{mt} + \beta^i_D D_t + u^i_t. \qquad (26)$$

We generally find that the coefficient β^i_D in these regressions are significantly different from zero. To further ask whether firms with returns that are more sensitive to DOO are more likely to have a greater wedge between home

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and foreign betas, we could potentially study the relationship between these beta differences, $\beta^i_{Home} - \beta^i_{US}$, and their coefficient, β^i_D. This evidence cannot directly be interpreted in light of our model for two reasons, however. First, countries are symmetric in our model although they are not in the data, leading to deviations between estimates of β^i_{Home} and β^i_{US} due to asymmetric exposures of firms to each market. Second, due to the wide range of asymmetries across our sample of firms, there is a corresponding range of deviations between β^i_{Home} and β^i_{US}, such that some firms have higher betas against the foreign market than home. For both of these reasons, we ask the more modest question of whether firms with greater exposure to DOO have a greater absolute difference between home and foreign betas. To address this question, we regress the absolute value of the difference between home and U.S. estimates $\left|\beta^i_{Home} - \beta^i_{US}\right|$ on the exposure to DOO as captured by $\left|\beta^i_D\right|$.[35] Panel B of Table 5 reports the results of this regression, showing that the coefficient is indeed positive across all measures. Thus, as suggested by our symmetric model, firms with higher sensitivity to DOO also have higher deviations between home and foreign betas.

The third relationship coming from our framework pertains to the effects of disagreements on firm returns and prices after cross-listing. To estimate this change, we follow the literature by running a pooled regression of the foreign company excess returns on the "Foreign" (U.S.) market and the "Home" market and interacting these variables with a dummy after the cross-listing event. In other words, we run regressions of the form:

$$R_{it} = b^{PRE} R_{mt} + b^{POST} R_{mt} I_t + b^{PRE}_D D_t + b^{POST}_D D_t I_t + \epsilon_t, \tag{27}$$

where, as in Equation (24), R_{mt} is the three-dimensional vector of a constant and home and foreign market returns, b^{PRE} and b^{POST} are the factor loadings, and I_t is a dummy variable equal to one if the period is after cross-listing and equal to zero otherwise. In these regressions, the empirical relationship suggested by our framework would mean a change in sensitivity to DOO measures, that is: $b^{POST}_D - b^{PRE}_D \neq 0$.

Table 5, Panel C, reports the estimates for the coefficients on the DOO variables alone, subsuming the others for parsimony.[36] Given in the row labeled "Before: b^{PRE}_D," the coefficient estimates on D_t prior to cross-listing are generally insignificantly different from zero. Consistent with the suggested model relationship, the row labeled "Change: $b^{POST}_D - b^{PRE}_D$" indeed shows a significant change in this coefficient after cross-listing for most of the DOO variables. Overall, therefore, the evidence suggests that the market returns of

[35] Our larger data set of more than 500 firms allows this cross-sectional regression, while the shorter data set of 42 countries precluded this possibility for capital flows above. Note that the standard errors from these cross-sectional regressions should be viewed with caution because they ignore the sampling error in the first-stage firm-level time-series regressions.

[36] The estimated factor loadings reflect standard findings in the literature, such as an increase in betas on the U.S. market and little or no change on the beta from the firm's home market.

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foreign firms that cross-list in the United States are more sensitive to difference of opinion about U.S. variables, consistent with our model.

8. Conclusions

By allowing international investors to differ in their interpretation of home and foreign public information, we showed that four regularities in international finance can be at least partially explained: (i) the co-movement of returns and international capital flows, (ii) home-equity preference, (iii) the dependence of firm returns on home and foreign factors, and (iv) abnormal returns around foreign firm cross-listing in the home market. We also analyzed how differences in forecasts relate to the regularities across countries and firms, finding evidence consistent with our model.

Overall, our model clearly demonstrates the effects of differing opinion across countries. According to that viewpoint, if someone asked: "What is a foreigner?" our answer would be: "A foreigner is one who interprets news about home firms less correctly than home residents do." Because of this behavioral phenomenon, risk and risk premia are created, over and beyond the risk of the fundamentals, by the risk that the opinions of investors living in different countries will in the future diverge from one another.

Appendix A Transform Analysis

In order to obtain the prices of financial securities as well as the country wealth processes (needed for constructing the portfolios), we need to compute the expected values of the product of the change of measure with the payoffs. From the equations for the equilibrium stochastic discount factor, and the expressions obtained for the stock prices and wealths, it is clear that we need the joint conditional distribution of $\left(\eta_{A,u}, \eta_{B,u}, \delta_{A,u}, \delta_{B,u}\right)^{\intercal}$ at some future date u given the current state $\left(\eta_A, \eta_B, \delta_A, \delta_B, \hat{f}_A^E, \hat{f}_B^E, \hat{g}_A^B, \hat{g}_B^A\right)^{\intercal}$ at current time t. We can derive a moment function or Fourier transform that allows us to obtain the required expressions,

$$\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{1+\frac{\delta_{j,u}}{\delta_{i,u}}}{1+\frac{\delta_j}{\delta_i}}\right)^{\psi}\right] \text{ for } i\in\{A,B\} \text{ and} i\neq j.$$

In a "one-tree" version of our economy, Dumas, Kurshev, and Uppal (2009), following Yan (2008), show that, by assuming that risk aversion $1-\alpha$ is a positive integer (which can be true only when investors have risk aversion greater than or equal to 1) and using the binomial theorem, the moment function of outputs and sentiment is enough for obtaining prices and portfolios. While this property is useful in our setup, our problem is further complicated by the fact that the stochastic discount factor (see Equation (18)), in our model, contains a power of the sum of two outputs: $\left(\delta_{A,t}+\delta_{B,t}\right)^{\alpha-1}$. To see why this complicates our problem, note that, since investors are risk averse, $\alpha-1<0$, the binomial theorem cannot be used to expand that term. Clearly, obtaining exact solutions for the stock prices and portfolio choice is more challenging in our "two-trees" setup.

The moment-generating function in Proposition 6 contains precisely these types of elements, and can be used to obtain stock prices and wealths.

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Proposition 6. The moment-generating function needed for solving stock prices and wealths is

$$\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{1+\frac{\delta_{j,u}}{\delta_{i,u}}}{1+\frac{\delta_j}{\delta_i}}\right)^{\psi}\right]=H\left(\hat{g}_i^B,t,u,\chi_i\right)\times H\left(\hat{g}_j^A,t,u,\chi_j\right)$$

$$\times J\left(\hat{f}_i^E,\hat{g}_i^j,t,u,\varepsilon_i,\chi_j\right)\times G\left(\frac{\delta_j}{\delta_i},\hat{f}_i^E,\hat{f}_j^E,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j,\psi\right),$$

with

$$H(\hat{g}_t,t,u,\chi)=e^{B_1(t,u,\chi)+\hat{g}_t^2B_4(t,u,\chi)},$$

$$J\left(\hat{f},\hat{g},t,u,\varepsilon,\chi\right)=e^{\varepsilon\left[K_2(t,u)+\hat{f}K_3(t,u)+\hat{g}B_3(t,u,\chi)\right]+\varepsilon\left[K_1(t,u)+B_2(t,u,\chi)\right]},$$

and

$$G\left(\frac{\delta_j}{\delta_i},\hat{f}_i^E,\hat{f}_j^E,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\chi_i,\chi_j,\psi\right)=\int_{-\infty}^{+\infty}\left(1+\frac{\delta_j}{\delta_i}e^{\frac{y-\mu_y\left(\hat{f}_i^E,\hat{f}_j^E,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\chi_i,\chi_j\right)}{\sigma_y\left(t,u,\chi_i,\chi_j\right)}}\right)^{\psi}n(y)dy,$$

where

$$\mu_y\left(\hat{f}_i^E,\hat{f}_j^E,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\chi_i,\chi_j\right)=\left(\hat{f}_j^E-\hat{f}_i^E\right)K_3(t,u)+\hat{g}_j^iB_3(t,u,\chi_i)-\hat{g}_i^jB_3\left(t,u,\chi_j\right)$$
$$-2\varepsilon_i\left[K_1(t,u)+B_2\left(t,u,\chi_j\right)\right],$$

$$\sigma_y\left(t,u,\chi_i,\chi_j\right)=\sqrt{2K_1(t,u)+B_2(t,u,\chi_i)+B_2\left(t,u,\chi_j\right)},$$

and $n(\cdot)$ is a univariate standard normal density function. The functions K_1, K_2, K_3, B_1, B_2, B_3, and B_4 are given explicitly in the proof.

Proof. We want to compute $\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{1+\frac{\delta_{j,u}}{\delta_{i,u}}}{1+\frac{\delta_j}{\delta_i}}\right)^{\psi}\right]$. Consider first obtaining a similar moment function:

$$Z\left(\delta_i,\delta_j,\hat{f}_i^E,\hat{f}_j^E,\eta_i,\eta_j,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right)=\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{\delta_{j,u}}{\delta_j}\right)^{\varepsilon_j}\right].$$

This function satisfies the following PDE:

$$0\equiv\mathcal{L}Z\left(\delta_i,\delta_j,\hat{f}_i^E,\hat{f}_j^E,\eta_i,\eta_j,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right)$$
$$+\frac{\partial Z}{\partial t}\left(\delta_i,\delta_j,\hat{f}_i^E,\hat{f}_j^E,\eta_i,\eta_j,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right).$$

with the initial condition $Z\left(\delta_i,\delta_j,\hat{f}_i^E,\hat{f}_j^E,\eta_i,\eta_j,\hat{g}_i^j,\hat{g}_j^i,t,t,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right)=\delta_i^{\varepsilon_i}\delta_j^{\varepsilon_j}\eta_i^{\chi_i}\eta_j^{\chi_j}$, and where $\mathcal{L}$ is the differential generator of $\left(\delta_i,\delta_j,\hat{f}_i^E,\hat{f}_j^E,\eta_i,\eta_j,\hat{g}_i^j,\hat{g}_j^i\right)$ under the probability measure of the econometrician.

Because the system of state variables $\left(\delta_i,\delta_j,\hat{f}_i^E,\hat{f}_j^E,\eta_i,\eta_j,\hat{g}_i^j,\hat{g}_j^i\right)$ is in the exponential-linear-quadratic class, we can obtain the solution of this PDE from the solution of a simpler system of ODEs

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(see Cheng and Scaillet 2007). Moreover, because of the block-diagonal structure of the diffusion matrix of the state variables (as shown in Section 2.3), we obtain the solution in closed-form, as shown in the remainder of this proof.

We have

$$\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{\delta_{j,u}}{\delta_j}\right)^{\varepsilon_j}\right]=\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\delta_{j,u}}{\delta_j}\right)^{\varepsilon_j}\right] \times\mathbb{E}_t^E\left[\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\right] =Q\left(\hat{f}_j^E,\hat{g}_j^i,t,u,\varepsilon_j,\chi_i\right)\times Q\left(\hat{f}_i^E,\hat{g}_i^j,t,u,\varepsilon_i,\chi_j\right).$$

Therefore, we can split the required moment function into the product of two separate ones, one for each subset of independent state variables. Since the dynamics of the state variables in each of these groups are similar to those in Dumas, Kurshev, and Uppal (2009), and since the object is also the same as in that paper, we obtain a very similar result:

$$Q\left(\hat{f},\hat{g},t,u,\varepsilon,\chi\right)=H_{\hat{f}}\left(\hat{f},t,u,\varepsilon\right)\times H_{\hat{g}}(\hat{g},t,u,\varepsilon,\chi),$$

with

$$H_{\hat{f}}\left(\hat{f},t,u,\varepsilon\right)=e^{\varepsilon^2K_1(t,u)+\varepsilon\left[K_2(t,u)+\hat{f}K_3(t,u)\right]},$$

$$H_{\hat{g}}(\hat{g},t,u,\varepsilon,\chi)=e^{B_1(t,u,\chi)+\varepsilon^2B_2(t,u,\chi)+\varepsilon\hat{g}B_3(t,u,\chi)+\hat{g}^2B_4(t,u,\chi)},$$

and

$$K_1(t,u)=\left(\frac{\gamma^E}{\zeta}+\frac{1}{2}\sigma_\delta^2+\frac{1}{2\zeta^2}\left[\left(\frac{\gamma^E}{\sigma_\delta}\right)^2+\left(\phi\sigma_f\right)^2\right]\right)(u-t)+\frac{1-e^{-2\zeta(u-t)}}{4\zeta^3}\left[\left(\frac{\gamma^E}{\sigma_\delta}\right)^2+\left(\phi\sigma_f\right)^2\right] -\frac{1-e^{-\zeta(u-t)}}{\zeta}\left(\frac{\gamma^E}{\zeta}+\frac{1}{\zeta^2}\left[\left(\frac{\gamma^E}{\sigma_\delta}\right)^2+\left(\phi\sigma_f\right)^2\right]\right),$$

$$K_2(t,u)=\left(\overline{f}-\frac{1}{2}\sigma_\delta^2\right)(u-t)-\frac{1}{\zeta}\overline{f}\left[1-e^{-\zeta(u-t)}\right],$$

$$K_3(t,u)=\frac{1}{\zeta}\left[1-e^{-\zeta(u-t)}\right],$$

$$B_1(t,u,\chi)=\frac{a}{2}\int_t^u B_4(t,\tau,\chi)d\tau,$$

$$B_2(t,u,\chi)=\int_t^u B_3(t,u,\chi)\left[m+ne^{-\zeta(\tau-t)}+\frac{a}{4}B_3(t,u,\chi)\right]d\tau,$$

$$B_3(t,u,\chi)=\frac{\sum_{i=1}^5\vartheta_i(\chi)e^{-\upsilon_i(\chi)(u-t)}}{q(\chi)+b(\chi)+[q(\chi)-b(\chi)]e^{-2q(\chi)(u-t)}},$$

$$B_4(t,u,\chi)=\frac{c(\chi)\left(1-e^{-2q(\chi)(u-t)}\right)}{q(\chi)+b(\chi)+[q(\chi)-b(\chi)]e^{-2q(\chi)(u-t)}},$$

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where

$$a=2\left[\left(\frac{\gamma^E-\gamma^\times}{\sigma_\delta}\right)^2+\left(\phi\sigma_f\right)^2\right], \qquad l(\chi)=\chi\,\frac{\gamma^E}{\zeta}\,\frac{1}{\sigma_\delta^2},$$
$$b(\chi)=\zeta+\frac{\gamma^\times}{\sigma_\delta^2}+\chi\left(\frac{\gamma^E-\gamma^\times}{\sigma_\delta^2}\right), \qquad m=\gamma^E-\gamma^\times+\frac{\gamma^E}{\zeta}\left(\frac{\gamma^E-\gamma^\times}{\sigma_\delta^2}\right)+\frac{1}{\zeta}\left(\phi\sigma_f\right)^2,$$
$$c(\chi)=\frac{1}{2}\chi(\chi-1)\frac{1}{\sigma_\delta^2}, \qquad n=-\frac{\gamma^E}{\zeta}\left(\frac{\gamma^E-\gamma^\times}{\sigma_\delta^2}\right)-\frac{1}{\zeta}\left(\phi\sigma_f\right)^2,$$
$$k(\chi)=-\chi\left[1+\frac{\gamma^E}{\zeta}\,\frac{1}{\sigma_\delta^2}\right], \qquad q(\chi)=\sqrt{b(\chi)^2-ac(\chi)},$$

and

$$\upsilon_1=0, \qquad \vartheta_1(\chi)=\frac{2c(\chi)m+k(\chi)[b(\chi)+q(\chi)]}{q(\chi)},$$
$$\upsilon_2(\chi)=2q(\chi), \qquad \vartheta_2(\chi)=\frac{2c(\chi)m+k(\chi)[b(\chi)-q(\chi)]}{q},$$
$$\upsilon_3=\zeta, \qquad \vartheta_3(\chi)=\frac{2c(\chi)n+l(\chi)[b(\chi)+q(\chi)]}{q(\chi)-\zeta},$$
$$\upsilon_4(\chi)=2q(\chi)+\zeta, \qquad \vartheta_4(\chi)=\frac{2c(\chi)n+l(\chi)[b(\chi)-q(\chi)]}{q(\chi)+\zeta},$$
$$\upsilon_5(\chi)=q(\chi), \qquad \vartheta_5(\chi)=-[\vartheta_1(\chi)+\vartheta_2(\chi)+\vartheta_3(\chi)+\vartheta_4(\chi)].$$

Using this solution we can write

$$\begin{aligned}\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{\delta_{j,u}}{\delta_j}\right)^{\varepsilon_j}\right]&=\mathbb{E}_t^E\left[e^{\chi_j\ln\frac{\eta_{j,u}}{\eta_j}+\chi_i\ln\frac{\eta_{i,u}}{\eta_i}+\varepsilon_i\ln\frac{\delta_{i,u}}{\delta_i}+\varepsilon_j\ln\frac{\delta_{j,u}}{\delta_{j,t}}}\right]\\
&=e^{B_1(t,u,\chi_i)+\left(\hat{g}_j^i\right)^2B_4(t,u,\chi_i)}\\
&\quad\times e^{B_1\left(t,u,\chi_j\right)+\left(\hat{g}_i^j\right)^2B_4\left(t,u,\chi_j\right)}\times\Phi,\end{aligned}$$

where

$$\begin{aligned}\Phi=&\,e^{\varepsilon_i\left[K_2(t,u)+\hat{f}_i^EK_3(t,u)+\hat{g}_i^jB_3\left(t,u,\chi_j\right)\right]+\varepsilon_i^2\left[K_1(t,u)+B_2\left(t,u,\chi_j\right)\right]}\\
&\times e^{\varepsilon_j\left[K_2(t,u)+\hat{f}_j^EK_3(t,u)+\hat{g}_j^iB_3(t,u,\chi_i)\right]+\varepsilon_j^2[K_1(t,u)+B_2(t,u,\chi_i)]}\end{aligned}$$

corresponds to the moment function of a bivariate normal distribution:

$$\begin{pmatrix}\ln\frac{\delta_{i,u}}{\delta_i}\\ \ln\frac{\delta_{j,u}}{\delta_j}\end{pmatrix}\sim N\left(\mu_\delta\left(\hat{f}_i^E,\hat{f}_j^E,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\chi_i,\chi_j\right),\Sigma_\delta\left(t,u,\chi_i,\chi_j\right)\right),$$

with

$$\mu_\delta\left(\hat{f}_i^E,\hat{f}_j^E,\hat{g}_i^j,\hat{g}_j^i,t,u,\varepsilon_i,\chi_i,\chi_j\right)=\begin{bmatrix}K_2(t,u)+\hat{f}_i^EK_3(t,u)+\widehat{g}_i^jB_3\left(t,u,\chi_j\right)\\ K_2(t,u)+\hat{f}_j^EK_3(t,u)+\widehat{g}_j^iB_3(t,u,\chi_i)\end{bmatrix},$$

$$\Sigma_\delta\left(t,u,\chi_i,\chi_j\right)=\begin{bmatrix}2\left[K_1(t,u)+B_2\left(t,u,\chi_j\right)\right] & 0\\ 0 & 2[K_1(t,u)+B_2(t,u,\chi_i)]\end{bmatrix}$$

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From properties of normal distributions, it follows that Φ also corresponds to this alternative bivariate normal distribution:

$$\begin{pmatrix} \ln \frac{\delta_{i,u}}{\delta_i} \\ \ln \frac{\delta_{j,u}}{\delta} - \ln \frac{\delta_{i,u}}{\delta_i} \end{pmatrix} \sim N\left(\widetilde{\mu}_\delta\left(\hat{f}_i^E, \hat{f}_j^E, \hat{g}_i^j, \hat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j\right), \widetilde{\Sigma}_\delta\left(t, u, \chi_i, \chi_j\right)\right),$$

where

$$\widetilde{\mu}_\delta\left(\hat{f}_i^E, \hat{f}_j^E, \hat{g}_i^j, \hat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j\right) = \begin{bmatrix} K_2(t,u) + \hat{f}_i^E K_3(t,u) + \widehat{g}_i^j B_3\left(t,u,\chi_j\right) \\ \left(\hat{f}_j^E - \hat{f}_i^E\right) K_3(t,u) + \hat{g}_j^i B_3(t,u,\chi_i) - \hat{g}_i^j B_3\left(t,u,\chi_j\right) \end{bmatrix},$$

$$\widetilde{\Sigma}_\delta\left(t,u,\chi_i,\chi_j\right) = \begin{bmatrix} 2\left[K_1(t,u) + B_2\left(t,u,\chi_j\right)\right] & -2\left[K_1(t,u) + B_2\left(t,u,\chi_j\right)\right] \\ -2\left[K_1(t,u) + B_2\left(t,u,\chi_j\right)\right] & 4K_1(t,u) + 2\left[B_2(t,u,\chi_i) + B_2\left(t,u,\chi_j\right)\right] \end{bmatrix}.$$

Therefore, we obtain that the moment function given in Proposition 6 is equivalent to

$$\begin{aligned} &\mathbb{E}_t^E\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{\delta_{j,u}}{\delta_{i,u}} \Big/ \frac{\delta_j}{\delta_i}\right)^{v}\right] \\ &= \mathbb{E}_t^E\left[e^{\chi_j \ln \frac{\eta_{j,u}}{\eta_j} + \chi_i \ln \frac{\eta_{i,u}}{\eta_i} + \varepsilon_i \ln \frac{\delta_{i,u}}{\delta_i} + v \ln\left(\frac{\delta_{j,u}}{\delta_j} - \ln \frac{\delta_{i,u}}{\delta_i}\right)}\right] \\ &= H\left(\hat{g}_i^B, t, u, \chi_i\right) \times H\left(\widehat{g}_j^A, t, u, \chi_j\right) \times J\left(\hat{f}_i^E, \hat{g}_i^j, t, u, \varepsilon_i, \chi_j\right) \\ &\quad \times e^{v\left[\mu_y\left(\hat{f}_i^E, \hat{f}_j^E, \hat{g}_i^j, \hat{g}_j^i, t, u, \varepsilon_i, \chi_i, \chi_j\right)\right] + v^2 \sigma_y^2\left(t, u, \chi_i, \chi_j\right)}, \end{aligned}$$

where the functions H, J, μ_y, and σ_y^2 are given in the proposition.
In order to use this moment function to compute the required expectation, we integrate over the log output ratio conditional normal distribution, which is implicit in this moment function. ■

Appendix B Introducing Output Correlation

In this section we show how to solve our model when outputs are correlated across countries. To introduce output correlation symmetrically, we posit that the stochastic process for $\delta_{i,t}$ $(i \in \{A,B\})$ is

$$\frac{d\delta_{i,t}}{\delta_{i,t}} = f_{i,t}dt + \sigma_\delta\left(\sqrt{\frac{1+\sqrt{1-\rho^2}}{2}}dz_{i,t}^\delta + \sqrt{\frac{1-\sqrt{1-\rho^2}}{2}}dz_{j,t}^\delta\right),\ i \neq j;\ i,j \in \{A,B\},$$

where z_i^δ are independent Brownian motions under the effective probability measure, which governs empirical realizations of the process. The dynamics of the conditional growth rates $f_{i,t}$ of outputs and of the signals under the true measure remain exactly the same as in Equations (2) and (3), respectively.

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Applying filtering under these assumptions, the conditional mean of the growth rate of output in country i as estimated by the econometrician, $\hat{f}^E_{i,t}$, has dynamics

$$d\hat{f}^E_{i,t}=\zeta\left(\bar{f}-\hat{f}^E_{i,t}\right)dt+\frac{\gamma^E-\rho\widetilde{\gamma}^E}{\sigma_\delta\left(1-\rho^2\right)}dw^E_{\delta_i,t}+\frac{\widetilde{\gamma}^E-\rho\gamma^E}{\sigma_\delta\left(1-\rho^2\right)}dw^E_{\delta_j,t}+\phi\sigma_f dw^E_{s_i,t},$$

where $\widetilde{\gamma}^E$ is the steady-state covariance of $\hat{f}^E_A-f_A$ and $\hat{f}^E_B-f_B$, while γ^E is the steady-state variance of $\hat{f}^E_A-f_A$ and it is also the steady-state variance of $\hat{f}^E_B-f_B$, these variances being equal to each other by virtue of symmetry:[37]

$$\widetilde{\gamma}^E\triangleq\sigma^2_\delta\rho\left(\frac{1}{|\rho|}\sqrt{\frac{\zeta^2\left(1+\rho^2\right)+\left(1-\phi^2\right)\frac{\sigma^2_f}{\sigma^2_\delta}-\sqrt{\left(1-\rho^2\right)\left(\zeta^4\left(1-\rho^2\right)+2\zeta^2\left(1-\phi^2\right)\frac{\sigma^2_f}{\sigma^2_\delta}+\left(1-\phi^2\right)^2\frac{\sigma^4_f}{\sigma^4_\delta}\right)}}{2}}-\zeta\right),$$

$$\gamma^E\triangleq\sigma^2_\delta\left(\sqrt{\left(1-\rho^2\right)\left(\zeta^2\left(1-\rho^2\right)+\left(1-\phi^2\right)\frac{\sigma^2_f}{\sigma^2_\delta}-\frac{\left(\widetilde{\gamma}^E\right)^2}{\sigma^4_\delta}-2\zeta\rho\frac{\widetilde{\gamma}^E}{\sigma^2_\delta}\right)}-\zeta\left(1-\rho^2\right)\right)+\rho\widetilde{\gamma}^E.$$

We can then write the dynamics of the output processes under the measure of the econometrician:

$$\frac{d\delta_{i,t}}{\delta_{i,t}}=\hat{f}^E_{i,t}dt+\sigma_\delta\left(\sqrt{\frac{1+\sqrt{1-\rho^2}}{2}}dw^E_{\delta_i,t}+\sqrt{\frac{1-\sqrt{1-\rho^2}}{2}}dw^E_{\delta_j,t}\right);\quad i\neq j;\quad i,j\in\{A,B\}.$$

The investors' interpretation of the signals is exactly the same as the one described in Section 2.3. Defining $\hat{f}^i_j$ as the conditional mean of the output growth in country j as estimated by investors in country i, we obtain

$$\begin{aligned}d\hat{f}^i_{i,t}&=\zeta\left(\bar{f}-\hat{f}^i_{i,t}\right)dt+\frac{\gamma^X-\rho\widetilde{\gamma}}{\sigma^2_\delta\left(1-\rho^2\right)}\left(\frac{d\delta_{i,t}}{\delta_{i,t}}-\hat{f}^i_{i,t}dt\right)+\frac{\widetilde{\gamma}-\rho\gamma^X}{\sigma^2_\delta\left(1-\rho^2\right)}\left(\frac{d\delta_{j,t}}{\delta_{j,t}}-\hat{f}^i_{j,t}dt\right)+\phi\sigma_f ds_{i,t},\\&=\zeta\left(\bar{f}-\hat{f}^i_{i,t}\right)dt+\frac{\gamma^X-\rho\widetilde{\gamma}}{\sigma_\delta\left(1-\rho^2\right)}dw^i_{\delta_i,t}+\frac{\widetilde{\gamma}-\rho\gamma^X}{\sigma_\delta\left(1-\rho^2\right)}dw^i_{\delta_j,t}+\phi\sigma_f ds_{i,t},\\d\hat{f}^i_{j,t}&=\zeta\left(\bar{f}-\hat{f}^i_{j,t}\right)dt+\frac{\widetilde{\gamma}-\rho\gamma^Y}{\sigma^2_\delta\left(1-\rho^2\right)}\left(\frac{d\delta_{i,t}}{\delta_{i,t}}-\hat{f}^i_{i,t}dt\right)+\frac{\gamma^Y-\rho\widetilde{\gamma}}{\sigma^2_\delta\left(1-\rho^2\right)}\left(\frac{d\delta_{j,t}}{\delta_{j,t}}-\hat{f}^i_{j,t}dt\right),\\&=\zeta\left(\bar{f}-\hat{f}^i_{j,t}\right)dt+\frac{\widetilde{\gamma}-\rho\gamma^Y}{\sigma_\delta\left(1-\rho^2\right)}dw^i_{\delta_i,t}+\frac{\gamma^Y-\rho\widetilde{\gamma}}{\sigma_\delta\left(1-\rho^2\right)}dw^i_{\delta_j,t},\end{aligned}$$

[37] When $\rho=0$, $\widetilde{\gamma}^E=0$ and γ^E collapses to the expression in Equation (5).

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where γ^X is the steady-state variance of $\hat{f}^i_{i,t} - f_i$, γ^Y is the steady-state variance of $\hat{f}^i_{j,t} - f_j$, and $\widetilde{\gamma}$ is their covariance:[38]

$$\widetilde{\gamma} \triangleq \rho\sigma_\delta^2 \left(\sqrt{\frac{\left\{ \zeta^2\left(2\left(1+\rho^2\right)\left(1-\phi^2\right)+\phi^4\right)+\left(2-3\phi^2+\phi^4\right)\frac{\sigma_f^2}{\sigma_\delta^2} -2\left(1-\phi^2\right)\sqrt{\left(1-\rho^2\right)\left(\zeta^4\left(1-\rho^2\right)+\zeta^2\left(2-\phi^2\right)\frac{\sigma_f^2}{\sigma_\delta^2}+\left(1-\phi^2\right)\frac{\sigma_f^4}{\sigma_\delta^4}\right)} \right\}}{\phi^4+4\rho^2\left(1-\phi^2\right)}} - \zeta \right),$$

$$\gamma^X \triangleq \sigma_\delta^2 \left(\sqrt{\left(1-\rho^2\right)\left(\zeta^2\left(1-\rho^2\right)+\left(1-\phi^2\right)\frac{\sigma_f^2}{\sigma_\delta^2}-\frac{\widetilde{\gamma}^2}{\sigma_\delta^4}-2\zeta\rho\frac{\widetilde{\gamma}}{\sigma_\delta^2}\right)} - \zeta\left(1-\rho^2\right) \right) + \rho\widetilde{\gamma},$$

$$\gamma^Y = \gamma^X\big|_{\phi=0} = \sigma_\delta^2 \left(\sqrt{\left(1-\rho^2\right)\left(\zeta^2\left(1-\rho^2\right)+\frac{\sigma_f^2}{\sigma_\delta^2}-\frac{\widetilde{\gamma}^2}{\sigma_\delta^4}-2\zeta\rho\frac{\widetilde{\gamma}}{\sigma_\delta^2}\right)} - \zeta\left(1-\rho^2\right) \right) + \rho\widetilde{\gamma}.$$

As in the text, the "disagreements" between the econometrician and the investors are defined as:

$$\hat{g}_i^j \equiv \hat{f}^E_{i,t} - \hat{f}^j_{i,t};\ i,j \in \{A,B\}.$$

With output correlation, investors disagree with the econometrician about the estimate of the conditional growth rate of their own output, because the behavioral bias in their estimate of the foreign output growth rate forecast spills over into the home output growth rate forecast. Therefore, $\hat{g}_i^j$ stands for two pairs of disagreements for each country's investor, with dynamics

$$d\hat{g}^i_{i,t} = -\left[\frac{\sqrt{1+\sqrt{1-\rho^2}}\left(\widetilde{\gamma}-\rho\gamma^X\right)-\sqrt{1-\sqrt{1-\rho^2}}\left(\gamma^X-\rho\widetilde{\gamma}\right)}{\sqrt{2}\sigma_\delta^2\left(1-\rho^2\right)^{\frac{3}{2}}}\hat{g}^i_{j,t} + \left(\zeta+\frac{\sqrt{1+\sqrt{1-\rho^2}}\left(\gamma^X-\rho\widetilde{\gamma}\right)-\sqrt{1-\sqrt{1-\rho^2}}\left(\widetilde{\gamma}-\rho\gamma^X\right)}{\sqrt{2}\sigma_\delta^2\left(1-\rho^2\right)^{\frac{3}{2}}}\right)\hat{g}^i_{i,t} \right]dt + \frac{\left(\widetilde{\gamma}^E-\widetilde{\gamma}\right)-\rho\left(\gamma^E-\gamma^X\right)}{\sigma_\delta\left(1-\rho^2\right)}dw^E_{\delta_j,t} + \frac{\left(\gamma^E-\gamma^X\right)-\rho\left(\widetilde{\gamma}^E-\widetilde{\gamma}\right)}{\sigma_\delta\left(1-\rho^2\right)}dw^E_{\delta_i,t},$$

38 When $\rho=0$, $\widetilde{\gamma}=0$, $\gamma^X=\gamma^E$ (given in Equation (5)), and $\gamma^Y=\gamma^\times$ (given in Equation (11)).

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$$d\hat{g}^i_{j,t}=-\left[\frac{\sqrt{1+\sqrt{1-\rho^2}}\left(\widetilde{\gamma}-\rho\gamma^Y\right)-\sqrt{1-\sqrt{1-\rho^2}}\left(\gamma^Y-\rho\widetilde{\gamma}\right)}{\sqrt{2}\sigma_\delta^2\left(1-\rho^2\right)^{\frac{3}{2}}}\hat{g}^i_{i,t}\right.$$

$$\left.+\left(\zeta+\frac{\sqrt{1+\sqrt{1-\rho^2}}\left(\gamma^Y-\rho\widetilde{\gamma}\right)-\sqrt{1-\sqrt{1-\rho^2}}\left(\widetilde{\gamma}-\rho\gamma^Y\right)}{\sqrt{2}\sigma_\delta^2\left(1-\rho^2\right)^{\frac{3}{2}}}\right)\hat{g}^i_{j,t}\right]dt$$

$$+\frac{\left(\widetilde{\gamma}^E-\widetilde{\gamma}\right)-\rho\left(\gamma^E-\gamma^Y\right)}{\sigma_\delta\left(1-\rho^2\right)}dw^E_{\delta_j,t}+\frac{\left(\gamma^E-\gamma^Y\right)-\rho\left(\widetilde{\gamma}^E-\widetilde{\gamma}\right)}{\sigma_\delta\left(1-\rho^2\right)}dw^E_{\delta_i,t}+\phi\sigma_f dw^E_{s_i,t},$$

where $i\neq j;\ \ i,j\in\{A,B\}$.

The changes from the probability measure of the econometrician to those of investors in countries A and B are then

$$\frac{d\eta_{A,t}}{\eta_{A,t}}=-\frac{\sqrt{1+\sqrt{1-\rho^2}}\,\hat{g}^A_{A,t}-\sqrt{1-\sqrt{1-\rho^2}}\,\hat{g}^A_{B,t}}{\sigma_\delta\sqrt{2\left(1-\rho^2\right)}}dw^E_{\delta_A,t}$$

$$-\frac{\sqrt{1+\sqrt{1-\rho^2}}\,\hat{g}^A_{B,t}-\sqrt{1-\sqrt{1-\rho^2}}\,\hat{g}^A_{A,t}}{\sigma_\delta\sqrt{2\left(1-\rho^2\right)}}dw^E_{\delta_B,t},$$

$$\frac{d\eta_{B,t}}{\eta_{B,t}}=-\frac{\sqrt{1+\sqrt{1-\rho^2}}\,\hat{g}^B_{A,t}-\sqrt{1-\sqrt{1-\rho^2}}\,\hat{g}^B_{B,t}}{\sigma_\delta\sqrt{2\left(1-\rho^2\right)}}dw^E_{\delta_A,t}$$

$$-\frac{\sqrt{1+\sqrt{1-\rho^2}}\,\hat{g}^B_{B,t}-\sqrt{1-\sqrt{1-\rho^2}}\,\hat{g}^B_{A,t}}{\sigma_\delta\sqrt{2\left(1-\rho^2\right)}}dw^E_{\delta_B,t}.$$

In order to obtain the prices of financial securities as well as the country wealth processes (needed for constructing the portfolios), we need the joint conditional distribution of $\left(\eta_{A,u},\eta_{B,u},\delta_{A,u},\delta_{B,u}\right)^\intercal$ at some future date u given the current state $(\eta_A,\eta_B,\delta_A,\delta_B,\hat{f}^E_A,\hat{f}^E_B,\hat{g}^B_A,\hat{g}^B_B,\hat{g}^A_A,\hat{g}^A_B)^\intercal$ at current time t. As shown in Appendix A (see in particular the proof of Proposition 6), we can obtain the prices of financial securities as well as the country wealth processes from the following moment function:

$$Z\left(\delta_i,\delta_j,\hat{f}^E_i,\hat{f}^E_j,\eta_i,\eta_j,\hat{g}^j_i,\hat{g}^j_j,\hat{g}^i_i,\hat{g}^i_j,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right)$$

$$=\mathbb{E}^E_t\left[\left(\frac{\eta_{i,u}}{\eta_i}\right)^{\chi_i}\left(\frac{\eta_{j,u}}{\eta_j}\right)^{\chi_j}\left(\frac{\delta_{i,u}}{\delta_i}\right)^{\varepsilon_i}\left(\frac{\delta_{j,u}}{\delta_j}\right)^{\varepsilon_j}\right].$$

This function satisfies the following PDE:

$$0\equiv\mathcal{L}Z\left(\delta_i,\delta_j,\hat{f}^E_i,\hat{f}^E_j,\eta_i,\eta_j,\hat{g}^j_i,\hat{g}^j_j,\hat{g}^i_i,\hat{g}^i_j,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right)$$

$$+\frac{\partial Z}{\partial t}\left(\delta_i,\delta_j,\hat{f}^E_i,\hat{f}^E_j,\eta_i,\eta_j,\hat{g}^j_i,\hat{g}^j_j,\hat{g}^i_i,\hat{g}^i_j,t,u,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right).$$

with the initial condition $Z\left(\delta_i,\delta_j,\hat{f}^E_i,\hat{f}^E_j,\eta_i,\eta_j,\hat{g}^j_i,\hat{g}^j_j,\hat{g}^i_i,\hat{g}^i_j t,t,\varepsilon_i,\varepsilon_j,\chi_i,\chi_j\right)=\delta_i^{\varepsilon_i}\delta_j^{\varepsilon_j}\eta_i^{\chi_i}\eta_j^{\chi_j}$, and where $\mathcal{L}$ is the differential generator of $\left(\delta_i,\delta_j,\hat{f}^E_i,\hat{f}^E_j,\eta_i,\eta_j,\hat{g}^j_i,\hat{g}^j_j,\hat{g}^i_i,\hat{g}^i_j\right)$ under the probability measure of the econometrician.

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Because the system of state variables $\left(\delta_i, \delta_j, \hat{f}_i^E, \hat{f}_j^E, \eta_i, \eta_j, \hat{g}_i^j, \hat{g}_j^j, \hat{g}_i^i, \hat{g}_j^i\right)$ is in the exponential linear quadratic class, we can obtain the solution of this PDE from the solution of a simpler system of ODEs (see Cheng and Scaillet 2007). With output correlations, however, the diffusion matrix of the state variables does not have a block diagonal structure, and the solution of the system of ODEs, while in principle available in closed form, becomes complicated and is more quickly obtained numerically. Our solution is still explicit, and it is exact up to the numerical solution of a Riccati system of ODEs, which can be obtained at arbitrarily high precision in Mathematica.

Appendix C Data Description

This appendix describes the data employed in the text, in order of appearance, as well as some robustness checks.

C.1 Equity Market Returns and U.S. Interest Rate

The risk-free interest rate is the one-month Treasury rate from Datastream. The market equity returns for the United States and 42 other countries are constructed from Morgan Stanley Capital International Total Return Indices, all measured in U.S. dollars. These 42 countries are identical to the full sample of potential countries noted in Curcuru et al. (2011). Specifically, they are: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, Columbia, Czech Republic, Denmark. Finland, France. Germany, Great Britain, Greece, Hong Kong, Hungary, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Malaysia, Mexico, the Netherlands, Norway, Pakistan, Peru, Philippines, Poland, Portugal, Russia, Singapore, South Africa, Spain, Sweden, Switzerland, Taiwan, Thailand, and Turkey. To be consistent with that study, these data are analyzed over the period 1980 to 2008.

C.2 Consumption and Output

The consumption and output data are taken from the IMF International Financial Statistics from 1960 to 2014. Output is Real Gross Domestic Product, while consumption is Household Consumption Expenditure deflated by the GDP Deflator. Both measures are converted into growth rates, although adjusting by HP filters as in Backus, Kehoe, and Kydland (1992) gave similar results.

C.3 Net Equity Purchases

Following the literature, capital flow data are measured with U.S. net purchases of foreign equity reported by the U.S. Treasury International Capital (TIC) database. We analyze three different monthly measures: (i) raw net purchases, (ii) net equity purchases scaled by beginning of period foreign market capitalization as in Albuquerque, Bauer, and Schneider (2007, 2009), and (iii) net equity purchases scaled by beginning of period holdings of foreign securities by U.S. investors following Curcuru et al. (2011). For analysis with set (ii), we use the six non-U.S. countries considered by Albuquerque, Bauer, and Schneider (2007) along with another seven countries for which we have a full set of net equity purchases and foreign market capitalization over the full period. Scaling the data in this way provides capital flows from the following thirteen countries: Austria, Australia, Canada, Switzerland, Germany, Denmark, France, Great Britain, Italy, Japan, the Netherlands, Singapore, and South Africa. For the set (iii) results, we use the 42 countries in Curcuru et al. (2011) listed in Section C.1. We analyze all these data over the same period of 1980 to 2008 in order to be consistent with the latter study. Moreover, as a robustness check, much of the general results in the paper were also estimated using the smaller subset of 20 countries reported in the tables of Curcuru et al. (2011). These estimates generally yielded similar results to those using our full set of countries.

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C.4 Home Equity Preference

Home equity preference in the data is calculated as the ratio of the value of U.S. foreign equity investment divided by U.S. wealth from 2001 to 2006. U.S. foreign equity investment is from the IMF Coordinated Portfolio Investment Survey, while U.S. wealth is measured as the market value of the Datastream U.S. Equity Market index plus Net Investment Abroad from the Bureau of Economic Analysis's International Economics Accounts.

C.5 Firm-level Equity Returns and Cross-listing Dates

In order to provide returns with sufficient liquidity as well as time-series data, we chose the set of foreign companies traded in the United States by 2004 on the NYSE or NASDAQ, a filter that provides 576 potential firms.[39] Weekly returns were calculated beginning in 1970 or the earliest availability from the Datastream International Total Return Indices in U.S. dollars for each of these firms as well as for their home markets. To provide a standard benchmark using the DOO measures described later, we conducted our firm-level analysis from 1980 to the beginning of 2010, when the U.S. Sentiment Index data ends. For individual time-series regressions, all available data are used. For pooled regressions, periods with missing observations are dropped.

For the analysis based upon pre-listing and post-cross-listing, we dropped firms with fewer than 52 observations before cross-listing, resulting in 311 firms for that analysis. The constituent companies and their cross-listing dates were obtained from the Citibank website cross-checked against the Bank of New York website for the ADRs and from Doidge, Karolyi, and Stulz (2004) for the Canadian firms.

C.6 Difference of Opinion Measures

In Section 7, we consider the relationship between some regularities and various DOO measures used in the literature. The first two sets are monthly variables calculated from the Survey of Professional Forecasters (SPF) database from January 1980 to January 2010. The other set of measures are the Sentiment Risk Indices based upon the first principal components of a number of variables considered to capture U.S. investor sentiment by Baker and Wurgler (2006) and global investor sentiment by Baker, Wurgler, and Yuan (2012). For each of these variables, we use the data over the available time period, although their frequencies vary. The U.S. sentiment indices are monthly and available through 2010. However, the Global Sentiment Index is annual and ends in 2005. For this proxy, therefore, we generate a monthly series using the annual sentiment data following the steps in Yu (2013).

Appendix D Active Portfolio Reallocation Measures

In addition to robustness checks based upon the data sets described earlier, we considered robustness of our model to other measures of capital flows. In particular, while the positive coefficients found by regressing U.S. net equity purchases on foreign returns have been called "return chasing," Curcuru et al. (2011) note that this positive relationship may simply reflect a passive increase in the value of foreign assets when their returns increase. They address this possibility by isolating the change in the portfolio allocation in foreign equities that result from active changes in share holdings, an approach also taken by Hau and Rey (2008). When they regress these active portfolio reallocations on returns, they find on average much lower coefficients that are generally insignificantly different

[39] Specifically, these foreign stocks are traded either as American Depositary Receipts or are directly listed on the exchanges, as in the case of Canadian companies.

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from zero. To investigate this possibility, we calculate the "active portfolio reallocations" specified in Curcuru et al. (2011) as[40]

$$\text{Active Portfolio Reallocations} = \frac{S_{A,t} \times \theta^{B}_{S_A,t}}{S_{A,t} \times \theta^{B}_{S_A,t} + S_{B,t} \times \theta^{B}_{S_B,t}} - \frac{S_{A,t} \times \theta^{B}_{S_A,t-dt}}{S_{A,t} \times \theta^{B}_{S_A,t-dt} + S_{B,t} \times \theta^{B}_{S_B,t-dt}}.$$

Thus, the active reallocation is the change in the value of foreign portfolio shares owned by country B investors, due to changes in equity holdings and not in prices.

To determine whether our model captures the relationship found by Curcuru et al. (2011), we simulate this variable in the uncorrelated and correlated output growth versions of our model. We find that the coefficient on Active Portfolio Reallocations is closer to zero than the corresponding coefficient on foreign returns alone, for both correlated and uncorrelated output models. This finding reflects a more muted relationship between active portfolio reallocation and returns, than for total foreign purchases, consistent with the Curcuru et al. (2011) results. Moreover, the coefficients on Active Portfolio Reallocations are not significantly different from zero as in Curcuru et al. (2011). Overall, therefore, our highly stylized model is remarkably consistent with two co-occurring findings in the literature about capital flows—a positive Brennan-Cao coefficient and little to no response in Active Portfolio Reallocations.

Appendix E Non-stationarity and Simulation

E.1 Stationarity

Existing studies on the survival of irrational traders ask whether excessively pessimistic or optimistic agents survive in the long run in an economy in which one population of agents knows the true probability distribution. These studies include Kogan et al. (2006), Yan (2008), and Dumas, Kurshev, and Uppal (2009). These studies conclude that, although "irrational" traders do not survive in the long run, they disappear very slowly, in terms of consumption shares. In our model, both types of investors are symmetrically ignoring the foreign signals. It is not obvious ex ante, therefore, whether the equilibrium we have obtained is stationary or not.

To throw some light on the issue, we obtain, as in Dumas, Kurshev, and Uppal (2009), the probability distribution of the future country A's consumption share (ω_u) under the effective probability measure, by Fourier inversion of the characteristic function of the ratio of foreign sentiments η_A/η_B. Figure 2 plots the probability density function of ω_u for different initial values ω_t. The top panel displays the case in which country A's consumption share is currently smaller ($\omega_t = 0.25$), and the bottom panel displays the case in which both countries' consumption shares are currently the same ($\omega_t = 0.5$). We see from this figure that, independently of the current relative consumption shares, as time passes, the probability distribution expands to the edges, exhibiting non-stationarity of the consumption shares. But it does so rather slowly.

E.2 Details of the simulation analysis

Using monthly time steps, we perform an exact simulation of the state variables $\left(\delta_A, \delta_B, \hat{f}^E_A, \hat{f}^E_B, \hat{g}^B_A, \hat{g}^A_B\right)$ for which exact transition probability distributions are known. For variables (η_A, η_B), we rely on a simple Euler-Maruyama discretization. We sometimes get outlier paths. Prompted by a referee, we have verified that these are legitimate observations and not the result of simulation errors. To confirm that, we have gradually reduced the time step of the simulation (down from one year to 1/32 of a year) and observed no change in the histograms of any of the state variables. We have also found no systematic increase or decrease in the min or

40 This paper only considers the share of equities as a proportion of foreign equities. In our two-country model, we use the same concept to consider foreign equities as a proportion of all equities, in this case home and abroad.

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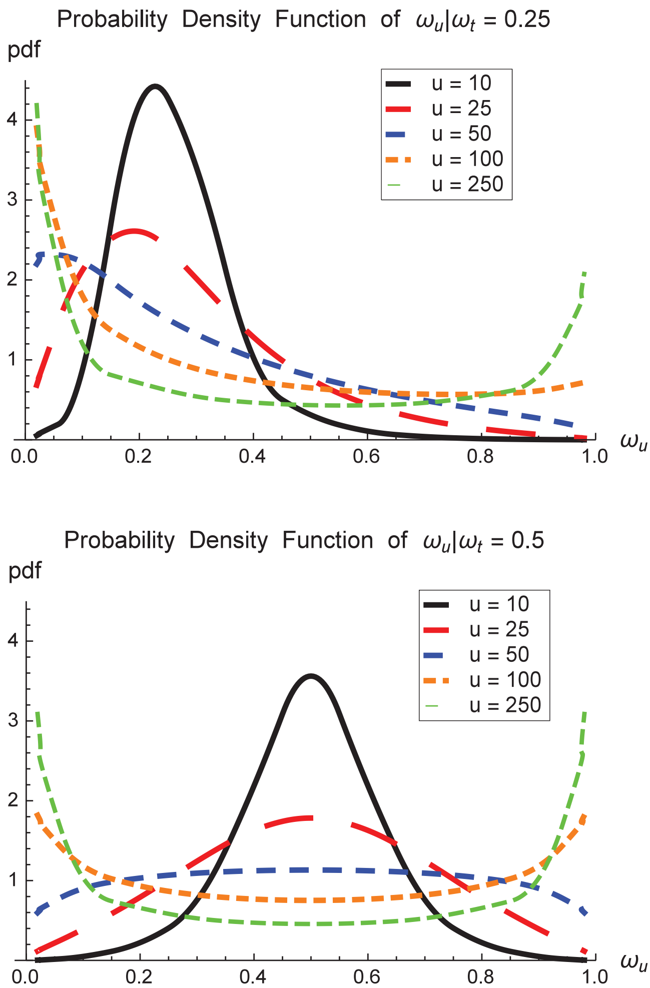

Figure 2
Non-stationarity of the equilibrium
Top panel: probability density function of the consumption share for various horizons (in years) for an intial share equal to $\omega_t = 0.25$. Bottom panel: the same for an initial share $\omega_t = 0.5$.

the max of any of them. The referee provided the rationale for these outliers being actually part of the solution: "Consider the process for disagreement described in Equation (13). The public shock decreases disagreement because $\gamma^{\times} > \gamma^{E}$. But a high $\gamma^{\times}$ also increases mean reversion in disagreement accelerating the decrease in disagreement. Therefore, periods of high disagreement that are followed by output news that everyone follows may look like outliers."

Although these paths occur very rarely, statistics calculated across paths that involve portfolio choices are sensitive to them. For that reason, we resort to robust statistics whenever portfolio choices are involved. This choice is also made, for example, by Beeler and Campbell (2012).

While the simulation is conducted under the effective probability measure (as in Section 2.1), the comparison of simulated data with empirical results remains a delicate exercise because of the

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non-stationarity of our model. As a way of alleviating the problem created by the non-stationarity of our model, we take several precautions. First, to obtain moments that are as little dependent on initial conditions as possible, or to approach unconditional moments, we run the simulations over as long a span of time as we can afford. We chose $T=50$ years (600 monthly observations).[41] Second, we draw many paths, namely 20,000 paths. Third, we split these 20,000 paths into five subsamples, and use different sets of initial values for the state variables in each subsample. Fourth, we compute moments across paths at the terminal points of the simulation. Moments calculated across paths are valid representations of the behavior of the model, whether or not the model is stationary.

We verify that the frequency distribution of the consumption share obtained from the simulation for the 50-year horizon closely matches the theoretical probability distribution shown in Figure 2.

We are careful to impose symmetry between the two markets A and B.[42] And, when we introduce the third firm C, we run the simulation in such a way that A and B remain symmetric in their joint relationship with C. In this way, we have a clean no-foreign-sentiment benchmark when we examine the empirical regularities.

Appendix F A Cross-Section

As indicated in the text, let there be one firm listed in country B, which we call firm B, and let there be two firms listed in country A, which we call firm A and firm C. Under the effective probability measure, the outputs of firms A and B are as they were before, but we now introduce the output of the new firm C as

$$\frac{d\delta_{C,t}}{\delta_{C,t}} = f_{C,t}dt + \sigma_\delta dz^\delta_{C,t},$$

where $z^\delta_{C,t}$ is an independent Brownian motion under the effective probability measure, which governs empirical realizations of the process.

The conditional expected growth rate $f_{C,t}$ of output is also stochastic:

$$df_{C,t} = \zeta\left(\bar{f} - f_{C,t}\right)dt + \sigma_f dz^f_{C,t}.$$

As before, all investors must estimate, or filter out, the current value of $f_{i,t}$ and its future behavior. They do so by observing the current cash flows and the three public signals (s_A, s_B, s_C) The signal correlated with $dz^f_{C,t}$ evolves according to

$$ds_{C,t} = \phi dz^f_{C,t} + \sqrt{1-\phi^2}dz^s_{C,t}, \tag{A.1}$$

where z^s_C is a Brownian motion under the effective probability measure as well. All the Brownian motions are independent from one another.

Investors in country A (where firms A and C are listed) perform their filtering under the belief that the signals s_A and s_C have the correct correlation with f_A and f_C; but they believe incorrectly that the signal s_B has zero correlation with f_B, which means that they ignore the information about the firms listed in the other country. The “model” they have in mind for this signal, in addition to those for the signal processes of A and B in Equation (7), posits that the dynamics of the signal process of C are those given in Equation (A.1).

Investors in country B (where firm B is listed) perform their filtering under the belief that the signal s_B has the correct correlation with f_B; but they believe incorrectly that the signals s_A and s_C have zero correlation with f_A and f_C, which means that they ignore the information about the

41 Although we run the model over 50 years, the economic agents in our simulation have an infinite horizon, not a 50-year horizon.

42 For more precision in the means and medians, we have also used the technique of antithetic variates.

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firms listed in the other country. The "model" they have in mind, in addition to those for the signal processes of A and B in Equation (8), posits that

$$ds_{C,t} = dz^{s}_{C,t}.$$

By following exactly the same steps as in Section 2, we can show that the vector of exogenous state variables under the reference measure of the econometrician is given by the Markovian system composed of Equations (4), (6), (13), and (15), to which we now add the following three analogous equations for firm C's output, expected conditional growth rate, and disagreement, respectively:

$$\frac{d\delta_{C,t}}{\delta_{C,t}} = \hat{f}^{E}_{C,t}dt + \sigma_{\delta} dw^{E}_{\delta_C,t},$$

$$d\hat{f}^{E}_{C,t} = \zeta\left(\bar{f} - \hat{f}^{E}_{C,t}\right)dt + \frac{\gamma^{E}}{\sigma_{\delta}} dw^{E}_{\delta_C,t} + \phi\sigma_f dw^{E}_{s_C,t},$$

$$d\hat{g}^{B}_{C,t} = -\left(\zeta + \frac{\gamma^{\times}}{\sigma^2_{\delta}}\right)\hat{g}^{B}_{C,t}dt + \frac{\gamma^{\times} - \gamma^{E}}{\sigma_{\delta}} dw^{E}_{\delta_C,t} + \phi\sigma_f dw^{E}_{s_C,t},$$

and the new change from the measure of investor B to that of the econometrician $\widetilde{\eta}_{B,t}$:

$$\frac{d\widetilde{\eta}_{B,t}}{\widetilde{\eta}_{B,t}} = -\frac{1}{\sigma_{\delta}}\left(\hat{g}^{B}_{A,t}dw^{E}_{\delta_A,t} + \hat{g}^{B}_{C,t}dw^{E}_{\delta_C,t}\right).$$

Therefore, we get an extended vector of ten exogenous state variables that drive the economy: $\widetilde{Y}_t \equiv \left(\delta_{A,t}, \hat{f}^{E}_{A,t}, \hat{g}^{B}_{A,t}, \eta_{B,t}, \delta_{B,t}, \hat{f}^{E}_{B,t}, \hat{g}^{A}_{B,t}, \eta_{A,t}, \delta_{c,t}, \hat{f}^{E}_{c,t}, \hat{g}^{B}_{C,t}, \eta_{C,t}\right)^{\mathsf{T}}$ where η_C is defined as:

$$\frac{d\eta_{C,t}}{\eta_{C,t}} = -\frac{1}{\sigma_{\delta}}\hat{g}^{B}_{C,t}dw^{E}_{\delta_C,t},$$

so that $\widetilde{\eta}_{B,t} = \eta_{B,t} \times \eta_{C,t}$. The structure is very similar to that of our two-firm model of Section 2. But there is now a third system $Y_{C,t}$ where

$$Y_{C,t} = \left(\delta_{C,t}, \hat{f}^{E}_{C,t}, \hat{g}^{B}_{C,t}, \eta_{C,t}\right)^{T}.$$

Therefore, this system can be written as

$$dY_{C,t} = \mu_{C,t}dt + \Omega_{C,t}dw^{E}_{C,t},$$

where

$$dw^{E}_{C,t} = \left(dw^{E}_{\delta_C,t}, dw^{E}_{s_C,t}\right)^{\mathsf{T}},$$

and

$$\Omega_{C,t} = \begin{bmatrix} \sigma_{\delta}\delta_{i,t} & 0 \\ \left(\frac{\gamma^{E}}{\sigma_{\delta}}\right) & \phi\sigma_f \\ \left(\frac{\gamma^{E} - \gamma^{\times}}{\sigma_{\delta}}\right) & \phi\sigma_f \\ -\eta_{C,t}\left(\frac{\hat{g}^{B}_{C,t}}{\sigma_{\delta}}\right) & 0 \end{bmatrix}.$$

Thus, the state vector can be evaluated as three independent sets of processes. For instance, the full vector of twelve state variables $\widetilde{Y}_t$ can be written as

$$d\widetilde{Y}_t = \widetilde{\mu}_t dt + \widetilde{\Omega}_t d\overrightarrow{w}^{E}_{i,t},$$

where

$$d\overrightarrow{w}^{E}_{i,t} = \left(dw^{E}_{\delta_A,t}, dw^{E}_{\delta_B,t}, dw^{E}_{\delta_C,t}, dw^{E}_{s_A,t}, dw^{E}_{s_B,t}, dw^{E}_{s_C,t}\right),$$

and

$$\widetilde{\Omega}_t = \begin{bmatrix} \Omega_{A,t} & 0 & 0 \\ 0 & \Omega_{B,t} & 0 \\ 0 & 0 & \Omega_{C,t} \end{bmatrix},$$

where $\widetilde{\Omega}_t$ is still block diagonal, a property we exploit again in seeking the equilibrium in this extended model.

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In this setting we now have six different Brownian motions. Hence we need seven linearly independent securities in order to complete the market. Since we intend to replicate a regression of a firm stock excess return on a home country stock excess return and a foreign country stock excess return, we choose our menu of securities accordingly. In particular, we consider three stocks: (i) a stock S_C, which is a claim on the output of firm C (δ_C); (ii) a stock $S_{A,C}$, which is a claim on the output of firm A, taken to represent the equity market of country A (δ_A); and (iii) a stock S_B, which is a claim on the foreign output δ_B. As in Section 3, we complete the market with a locally riskless, instantaneously maturing bond deposit and three zero-net supply futures contracts whose prices are marked to the fluctuations of the three signal shocks, and are designed to hedge these shocks only.

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