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Too Popular, Too Fast: Optimal Advertising and Entry Timing in Markets with Peer Influence

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Abstract. We study optimal advertising and entry timing decisions for a new product being sold in two-segment markets in which followers are positively influenced by elites, whereas elites are either unaffected or repulsed by product popularity among followers. Key decisions in such markets are not only how much to advertise in each segment over time but also when to enter the follower segment. We develop a continuous-time optimal control model to examine these issues. Analysis yields two sets of two-point boundary value problems where one set has an unknown boundary value condition that satisfies an algebraic equation. A fast solution methodology is proposed. Two main insights emerge. First, the optimal advertising strategy can be U-shaped, that is, decreasing at first to free-ride peer influence but increasing later on to thwart the repulsion influence of overpopularity causing disadoption. Second, in markets where cross-segment repulsion triggers disadoption, advertising is only moderately effective, and entry costs are high, managing both advertising and entry timing can result in significantly higher profits than managing only one of these levers. In markets without disadoption, with high advertising effectiveness or with low entry costs, in contrast, delaying entry may add little value if one already manages advertising optimally. This implies that purveyors of prestige or cool products need not deny followers access to their products in order to protect their profits, and can use advertising to speed up the democratization of consumption profitably.

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1. Introduction

Social influence among consumers can greatly affect the diffusion of new products. It presents marketers with the opportunity to achieve faster diffusion and reduce marketing costs. The intuition is that as the product gains popularity, marketers can free ride on peer influence and cut back on marketing spending. Indeed, diffusion research finds that, in extended Bass models where advertising increases innovative adoption, it is optimal to advertise heavily during product launch and then reduce advertising continuously as the product penetrates the market (Horsky and Simon 1983). Inserting price or competition into the analysis does not alter this basic pattern.[1] In this paper, however, we show that the optimal strategy can be markedly different in markets exhibiting disadoption driven by overpopularity, settings that have received considerable attention recently (Berger and Le Mens 2009, Joshi et al. 2009, Abedi et al. 2014, Smaldino et al. 2017, Yoganarasimhan 2017, Appel et al. 2018, Touboul 2019, Warren et al. 2019).

We examine the marketing policy implications of asymmetric peer effects in a two-segment market, where the first segment is the elite that exercises a positive peer effect on a second segment of followers, whereas the latter has either no influence or a negative influence on the elite. Elite status may correlate with wealth but can stem from other sources of esteem or prestige as well (Hu and Van den Bulte 2014). For instance, in streetwear and lifestyle categories, brand appeal and user status are often based on being perceived to be authentic to a particular subculture rather than based on wealth, and being elite means being strongly associated with that subculture. Examples are surfers for Quicksilver and skaters for Surface (Warren et al. 2019). Similarly, what distinguishes elite or authentic from wannabe Porsche drivers is the ability to handle a sensitive high-performance sports car and not wealth.

Attraction and repulsion dynamics, where adoption by the elite makes the product more attractive to everyone but adoption by the followers repulses the elite to present marketers with notable challenges. For example, Burberry, the upscale apparel company, lost both sales and stature among its traditional upper class customer base after English hooligans and other low-status consumers started wearing the iconic plaid in the late 1990s (Moon 2004). Porsche, the maker of high-performance sports cars like the iconic 911, faced a similar problem when planning the entry of the Cayenne, its first sports utility vehicle. Market research showed that well-heeled potential buyers of SUVs were favorably disposed toward the Porsche brand, but that traditional Porsche sports car enthusiasts were bound to be displeased with the brand becoming associated with soccer moms buying SUVs (Joshi et al. 2009). Diesel Jeans, which long nurtured a very edgy and alternative image appealing to contrarian customers, faced similar problems when it became successful among mainstream customers (Grigorian and Chandon 2004). Many firms enjoying increasing market success struggle to maintain their exclusive or authentic image (Yoganarasimhan 2012, Warren et al. 2019).

Unlike prior literature, we consider the situation where the marketer has the ability to advertise in each of the elite and follower segments and also delay entry into the follower segment. Even when firms cannot strictly prevent followers from buying the product, entry can be delayed effectively by restricting the product's distribution to select channels rarely patronized by followers, as both Burberry and Diesel did initially. Finally, we allow popularity among followers to have two different repulsive effects on the elite: slowing down adoption and triggering disadoption. We use the term advertising to cover a wide variety of marketing efforts. For brands trying to maintain their cool image, marketing communications targeted to the elite segment will often take the form of community events linked to the subculture rather than media advertising suitable for followers (Warren et al. 2019).

Two novel policy insights emerge from our analysis. First, in markets with attraction/repulsion dynamics, the optimal advertising strategy in the elite segment can be nonmonotonic: start high and subsequently decrease but then increase again. This U-shaped pattern results from the desire to engage in prophylactic advertising to the elite as protection against the repulsive effect of growing adoption among followers. U-shaped advertising paths can also be optimal in single-segment markets where overpopularity triggers disadoption. Second, the ability to delay entry into the follower segment will sometimes—but not always—affect the optimal advertising policy and overall profits. Specifically, combining both policy tools can lead to significantly higher profits in markets where cross-segment repulsion triggers disadoption rather merely slower adoption and entry costs are high. In markets without disadoption or with low entry costs, in contrast, delaying entry may add little or no value if one already manages advertising optimally. This implies that purveyors of prestige product need not deny followers access to their products to protect their profits but can use advertising to speed up the democratization of consumption profitably.

Because our model nests those of Horsky and Simon (1983), Kalish et al. (1995), and Joshi et al. (2009), our results also echo several earlier results. For instance, the relative size of each segment's profit pool (segment size multiplied by customer lifetime value (CLV)) greatly affects delayed entry, with a larger profit pool in the elite segment increasing the optimal delay.

Because advertising has dynamic effects, we formulate the model as a two-stage, continuous-time optimal control problem with three decision variables (i.e., advertising in each segment and entry delay into the second segment) and three state variables (i.e., the fraction of adopters in each segment before and after product entry into the follower segment). The solution takes the form of two sets of two-point boundary value problems (TPBVPs) where one set has an unknown boundary value condition that satisfies an algebraic equation. Given the problem's complexity, we provide not only general analytic insights but also a solution methodology that is a hybrid of analytic and numerical analysis.

We proceed as follows: After a brief overview of related work, we set up the model in Section 2, and derive the optimal policy using optimal control methods in Section 3. We present the profit impact analysis in Section 4, further discuss the nature of optimal policies in Section 5, and conclude in Section 6.

1.1. Related Literature

We examine optimal dynamic advertising and entry delay jointly under two market structures: innovator/imitator and attraction/repulsion. The analysis accounts for the costs of entering the second segment and, for markets with attraction/repulsion, the possibility of disadoption by elite customers.

Innovator/imitator dynamics involve a *unidirectional* influence where innovators affect imitators but are unaffected by them (Goldenberg et al. 2002, Muller and Yogev 2006, Van den Bulte and Joshi 2007, Ho et al. 2012, Kim et al. 2016). Several analyses confirm the intuition that focusing initial marketing spending on innovators is optimal (Libai et al. 2005, Lehmann and Esteban-Bravo 2006, Hariharan et al. 2015). Furthermore, when the firm does not advertise but can postpone entering the imitator segment, it can be optimal for the firm to do so if entering the imitator segment requires additional sunk costs, for example, for adapting the product or developing a new sales channel (Kalish et al. 1995). The reason for delaying entry is that, when entry requires additional investment and the firm exhibits finite patience and hence discounts future cash flows, the firm prefers deferring that investment until the installed base in the influential segment is large enough to penetrate the imitator segment and recoup the investment quickly.

Attraction/repulsion dynamics involve *bidirectional* influence between an admired elite and aspiring "wannabe" followers, where adoption by the elite makes the product more attractive to the followers but adoption by those followers makes the product less attractive to the elite (Simmel 1957, Miller et al. 1993, Joshi et al. 2009, Bakshi et al. 2013, Smaldino et al. 2017). Although less studied by marketing scientists, this market structure is especially important for brands associated with status or specific subcultures. Repulsion is a challenge because it cannot only slow down the diffusion among the elite but even lead the elite to disadopt once the product has become too popular with the wannabes, resulting in a chase-and-flight pattern (Simmel 1957; Chung et al. 1987; McCracken 1988, 2008; Berger and Heath 2008; Smaldino et al. 2017; Touboul 2019). Status need not be based on wealth but can also be based on expertise, skill, sophistication, and socially valued traits like "being cool," and may hence result in attraction/repulsion dynamics even for affordable goods like skateboard sneakers or costless innovations like baby names (Berger and Le Mens 2009, Yoganarasimhan 2017, Warren et al. 2019).

We find that a U-shaped pattern of advertising can be optimal even when advertising boosts the tendency to adopt independently but not the susceptibility to peer influence. This stands in contrast to prior research that did not consider popularity-based disadoption (Krishnan and Jain 2006, Fruchter and Van den Bulte 2011, Hariharan et al. 2015). Our analysis also shows that the decision to delay entry should take into account not only repulsion and the cost of entry, as shown earlier by Joshi et al. (2009) and Kalish et al. (1995), respectively, but also (i) the risk of disadoption, (ii) the possibility of targeting advertising toward elites early on to quickly leverage cross-segment influence to recoup the cost of entry into the follower segment more quickly, and (iii) the possibility of targeting advertising toward the elites, both early on and much later the after launch in the follower segment, to withstand the repulsive effect of adoption by followers. Joshi et al. (2009) show that delaying entry in the repelling segment can boost profits, but their analysis does not investigate the optimal dynamic advertising policy either with or without entry delay nor does it investigate scenarios with disadoption. Bakshi et al. (2013), in an analysis allowing for disadoption, show that initial seeding among the elite, similar in spirit to high initial advertising that is eliminated soon after launch, can help the product gain full market penetration in both segments. However, their analysis does not investigate the optimal dynamic advertising policy either with or without entry delay. Table 1 summarizes the literature and our contributions under four topic headings.

2. Model Formulation

We consider a monopolist launching a new product at time $t = 0$ in a market consisting of two segments: the elite in segment 1 and the followers in segment 2. Segments are allowed to differ in both size and profit rate. Because our focus is on cross-segment influence patterns rather than segment size, we normalize the size of each segment to one. The firm can control the advertising in each segment separately at any point in time between the time of launch and the end of the planning horizon T. It may also choose to delay entry into segment 2.

Let $x(t)$ and $y(t)$ be the fractions of customers in segments 1 and 2, respectively, who use the product at time t. Let τ be the entry time into the second segment. We assume a Bass like model for the diffusion dynamics. Thus, until entry into the second segment (i.e., $t < \tau$) the diffusion dynamics is given by

$$\frac{dx(t)}{dt} = \left(p_1 + \rho_1\sqrt{u_1(t)} + q_{11}x(t)\right)(1 - x(t)),\ x(0) = 0. \quad (1)$$

Here p_1 and q_{11} are the standard Bass model parameters and $u_1(t)$ is the advertising level. The advertising effect, $\rho_1\sqrt{u_1(t)}$ is independent of penetration and boosts the tendency to adopt independently of peers, p_1. The model is identical to that of Horsky and Simon (1983) except for using $\sqrt{u_1(t)}$ instead of $\ln(u_1(t))$. Our specification is directly transposable into a model with a linear advertising control in the diffusion equation and a quadratic cost of advertising in the objective function, an alternative specification often used in the optimization literature (Sethi et al. 2008).

Following entry of the product into the second segment, that is, for $t \in [\tau, T]$, the diffusion dynamics for the two segments are as follows:

$$\begin{aligned}\frac{dx(t)}{dt} &= \left(p_1 + \rho_1\sqrt{u_1(t)} + q_{11}x(t) + q_{12}y(t)\right)(1 - x(t)) - kx(t)y(t)\\ \frac{dy(t)}{dt} &= \left(p_2 + \rho_2\sqrt{u_2(t)} + q_{21}x(t) + q_{22}y(t)\right)(1 - y(t)),\ y(\tau) = 0.\end{aligned} \quad (2)$$

The specification distinguishes between two repulsion effects on the elite segment because of popularity among followers: slowdown in adoption (when $q_{12} < 0$) and disadoption by those who have already adopted (when $k > 0$).[2] The attraction/repulsion dynamics is that followers are attracted to elites, whereas the latter are repulsed by followers' adoptions. Consequently, the model accommodates disadoption because of repulsion in the elite segment (via parameter k) but not in the follower segment. The disadoption is proportional to the

Table 1. Related Literature and Contributions of the Present Study

Topic	Literature	Contribution
Optimal delay of entry, aka simultaneous versus sequential entry	• Analyses of sequential entry do not consider optimal advertising (Kalish et al. 1995, Lehmann and Weinberg 2000, Elberse and Eliashberg 2003, Joshi et al. 2009). • Simultaneous launch accelerates future revenue and is best if the follower segment has a large revenue potential, the cost of entry of entry is low, or the discount factor is small.	• Optimal delay adds only a small improvement over advertising optimally when there is no repulsion-based disadoption, advertising is highly effective, or entry costs are low.
Optimal advertising in multiple segments	• Only few papers investigate optimal dynamic advertising in segmented markets, and they do so without considering peer effects or delayed entry (Chiu et al. 2018, Villena and Contreras 2019).	• A new methodology to analyze the optimal combination of dynamic advertising and entry delay. • Combining both levers can lead to markedly higher profits in markets with cross-segment repulsion, moderate advertising effectiveness, and high entry costs.
Optimal dynamic advertising with peer influence	• When advertising increases innovative adoption, advertising should be high early on and then subside as the product penetrates the market and positive peer influence develops (Horsky and Simon 1983). • In markets with influentials and followers, advertising should be targeted toward influentials (Libai et al. 2005, Lehmann and Esteban-Bravo 2006, Hariharan et al. 2015).	• In markets with popularity-based disadoption, the optimal advertising path in the elite segment can be U-shaped: Firms may benefit from prophylactic advertising later on to counter the repulsive effect of adoption by followers. • Prophylactic advertising resulting in a U-shape can also be optimal in single-segment markets with disadoption.
Attraction/repulsion dynamics	• Attraction/repulsion dynamics has long been recognized as an important phenomenon (Simmel 1957, Miller et al. 1993) and interest has been especially keen recently (Berger and Heath 2008, Berger and Le Mens 2009, Joshi et al. 2009, Bakshi et al. 2013, Smaldino et al. 2017, Yoganarasimhan 2017, Appel et al. 2018, Touboul 2019, Warren et al. 2019). • Joshi et al. (2009) consider optimal entry delay to reduce the negative impact of repulsion by followers on the profits from the elite segment.	• Using both entry delay and advertising is more profitable than using either in isolation when repulsion triggers disadoption, advertising is not highly effective, and entry costs are high. • Otherwise, delaying entry offers little benefit if one already optimizes advertising. • Consequently, purveyors of prestige or cool product can use advertising to speed up the democratization of consumption profitably without denying anyone access to their products.

fraction of adopters in each segment because xy represents the likelihood of interaction between elite and follower adopters. When both fractions are large, xy is high, leading to repulsive effects, but when either one is small, there is little disadoption because of repulsion. Although we focus on settings where q_{12} is nil or negative, it will be positive if the contagion from followers to elite customers boosts adoption. As for the remaining parameters, we assume $p_1,\ p_2 > 0$ and $q_{11},\ q_{21},\ q_{22} \geq 0$. Consistent with prior theory on attraction/repulsion, fashion cycles, and chase-and-flight dynamics (Simmel 1957), and with the notion that elite disadopters are dissatisfied with the product's popularity among followers rather than the product itself, we assume that disadopters do not generate negative word-of-mouth and can readopt the product.

The firm's objective function is the net present value (NPV) of its profit stream over the time horizon $[0, T]$. This is maximized with respect to the two advertising controls (u_1, u_2) and the entry time τ into the follower segment:

$$\max_{u_1,\, u_2,\, \tau} \left\{ \int_0^{\tau} (m_1 \dot{x} - u_1) e^{-rt} dt + \int_{\tau}^{T} (m_1 \dot{x} + m_2 \dot{y} - u_1 - u_2) e^{-rt} dt - M e^{-r\tau} + s(x(T), y(T)) e^{-rT} \right\}, \tag{3}$$

subject to state dynamics described by Equations (1) and (2).[3]

Parameter $r > 0$ is the discount rate, m_1 and m_2 are the profit pools (number of customers multiplied by

CLV without disadoption) for the two segments, and $M > 0$ is the one-time sunk cost to launch the product into segment 2. Finally, $s(\cdot)$ is the salvage value at time T. We specify it as $s(x(T), y(T)) = q_1^T x(T) + q_2^T y(T)$ where parameters q_1^T and q_2^T are nonnegative.

Our model formulation applies to consumables and durables for which a large part of the profit is realized after the initial purchase. First, consider how x enters the objective function. The literature tends to use $m\dot{x}$ for durable goods and $mx(t)$ for repeat-purchase goods or leased durables (Sethi et al. 2008). However, these expressions are interchangeable given a proper scaling of the profit pool term m. Specifically, $\int_0^\tau e^{-rt} m\dot{x}dt + \int_\tau^T e^{-rt} m\dot{x}dt + s(x(T))e^{-rT}$ transforms after integration-by-parts into $\int_0^\tau e^{-rt} rmxdt + \int_\tau^T e^{-rt} rmxdt$ $+ S(x(T))e^{-rT}$, where $S(x(T)) \equiv mx(T) + s(x(T))$. Thus, the analysis is unaffected by the change of specification.

More important is the interpretation of m_1 and m_2 as profit pools (Gadiesh and Gilbert 1998) rather than mere segment sizes. Specifically, $m_i = N_i \phi_i$ where N_i is the number of customers in the segment and ϕ_i is the CLV of such a customer in dollars. For a durable purchased only once and without any subsequent profit streams, ϕ_i is the gross margin on the transaction price, obtained at acquisition and reimbursed at disadoption. For repeat-purchased goods and for durables that are leased or generate a large fraction of their profit after the initial purchase, ϕ_i is the CLV at the time of acquisition, excluding the cost of acquisition, that is, the present value of the gross margin on the perpetuity of future revenue streams, and valued at the time of acquisition (not yet discounted back to $t = 0$). For consumables and durables, the profits of which are generated predominantly through cash flows after the initial adoption until the customer churns, like cars,[4] the company does not return money to the disadopting customers. Rather, Equations (2) and (3) are consistent with the firm booking the full CLV ϕ_i at the time of acquisition and, if disadoption occurs at a later date, deducting it again. Because of discounting, customers who disadopt still add value to the firm.

Analysis of the dynamic optimization problem in (3) involves forming Hamiltonians H^1 and H^2 as follows (Kamien and Schwartz 1991, p. 247):

$$\begin{aligned} H^1 &= (m_1 + \lambda_1)\dot{x} - u_1, \ t \in [0, \tau], \\ H^2 &= (m_2 + \lambda_2)\dot{x} - u_1 + (m_2 + \mu)\dot{y} - u_2, \ t \in [\tau, T], \end{aligned} \tag{4}$$

where $\lambda_1(t)$ and $\lambda_2(t)$ are the current multipliers of x in time intervals $t \in [0, \tau]$ and $t \in [\tau, T]$, respectively, and $\mu(t)$ the current multiplier of y in the time interval $t \in [\tau, T]$. The multipliers represent shadow prices associated with a unit change in the fraction of users in each respective segment, that is, the net benefit (or loss) to the firm of having one additional unit of users in each segment at time t than currently exists.

The current multipliers λ_1, λ_2, and μ must satisfy the following conditions:

$$\frac{d\lambda_1}{dt} = r\lambda_1 - (m_1 + \lambda_1)\Big[q_{11}(1 - x) - \Big(p_1 + \rho_1\sqrt{u_1} + q_{11}x\Big)\Big], \tag{5}$$

$$\begin{aligned} \frac{d\lambda_2}{dt} = r\lambda_2 - (m_1 + \lambda_2)\Big[&q_{11}(1 - x) \\ &- \Big(p_1 + \rho_1\sqrt{u_1} + q_{11}x + (k + q_{12}y)\Big)\Big] \\ &- (m_2 + \mu)q_{21}(1 - y), \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{d\mu}{dt} = r\mu &- (m_1 + \lambda_2)[q_{12}(1 - x) - kx] \\ &- (m_2 + \mu)\Big[q_{22}(1 - y) - \Big(p_2 + \rho_2\sqrt{u_2} + q_{21}x + q_{22}y\Big)\Big], \end{aligned} \tag{7}$$

$$\lambda_1(\tau) = \lambda_2(\tau), \ \lambda_2(T) = q_1^T, \ \mu(T) = q_2^T. \tag{8}$$

We characterize the optimal advertising and entry policies in Section 3, and use these results to present insights in the remaining sections.

3. Optimal Policies

The required conditions for the optimal advertising and entry time in the dynamic optimization problem (3) are presented in Theorem 1. All proofs are in Appendix A.

Theorem 1. *The optimal advertising in the two segments, u_1 and u_2, and the optimal entry time τ, must satisfy Equations (5)–(8) and the following:*

$$\begin{aligned} u_1 &= \begin{cases} \frac{1}{4}\Big(\rho_1(m_1 + \lambda_1)(1 - x)\Big)^2, & t \in [0, \tau], \\ \frac{1}{4}\Big(\rho_1(m_1 + \lambda_2)(1 - x)\Big)^2, & t \in [\tau, T]. \end{cases} \\ u_2 &= \frac{1}{4}\Big(\rho_2(m_2 + \mu)(1 - y)\Big)^2, \ t \in [\tau, T], \end{aligned} \tag{9}$$

and, letting $L(\tau) \equiv -\frac{1}{4}(m_2 + \mu(\tau))\Big[\rho_2^2(m_2 + \mu(\tau)) + 4(p_2 + q_{21}x(\tau))\Big] + rM$,

$$\tau = \begin{cases} 0 & \text{if } L(\tau) < 0 \text{ in } (0, T), \\ L^{-1}(0) & \text{if } L(\tau) = 0 \text{ in } (0, T), \\ T & \text{if } L(\tau) > 0 \text{ in } (0, T). \end{cases} \tag{10}$$

It can be seen from the results in Theorem 1 that the $(1 - x)$ and $(1 - y)$ terms are a force toward advertising declining over time if market penetration evolves monotonically. Also observe that there is still the possibility of advertising at the terminal time T because $u_1(T) = \frac{1}{4}\Big(\rho_1\Big(m_1 + q_1^T\Big)(1 - x(T))\Big)^2$ can be strictly larger than zero as long as the elite segment is not fully penetrated, that is, $x(T) < 1$.

From Theorem 1, the condition for sequential entry (launching the product into the second segment after

Table 2. Optimal Delay and Profit vs. Peer Influence

Peer influence	q_{12}	τ^*	Π
Negative	−0.1	3.35	0.76
Zero	0	0.75	0.82
Positive	0.4	0	0.99

a delay $\tau > 0$) to be optimal is derived from the condition that $H^1(\tau)$ minus the marginal cost to enter, rM, should equal $H^2(\tau)$. At the time of launch into the second segment, the valuation of x in the one-segment and two-segment regimes must coincide, that is, $\lambda_1(\tau) = \lambda_2(\tau)$. If the condition is not satisfied inside the time horizon, the firm will face a corner solution and it would be optimal to either introduce the product simultaneously into the two segments at $t = 0$, or to never introduce it into the second segment.

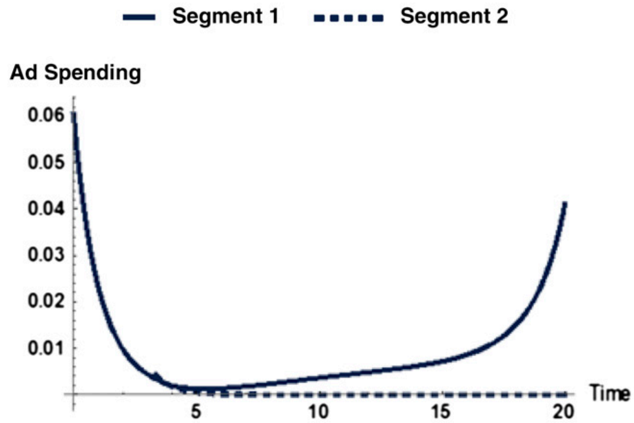
3.1. Optimal Delay

Continuing the discussion about the timing of entry, some conclusions can be drawn from Theorem 1. Because $m_2 + \mu(t) \geq 0$, $p_2 > 0$, and $q_{21} \geq 0$, the follower segment should be entered simultaneously if M is zero. In contrast, it is optimal to not enter the follower segment at all if M is very large, and entering sequentially can be optimal only if M is nonzero yet not excessively large.

Table 2 illustrates how the optimal entry delay decision is impacted by the influence from followers on elite customers, as implied by Theorem 1. It does so by reporting the optimal delay for three values of q_{12} while fixing other diffusion parameters to $p_1 = 0.03$, $p_2 = 0.001$, and $q_{11} = q_{21} = q_{22} = 0.4$; the profit pools $m_1 = m_2 = 1$; and $\rho_1 = \rho_2 = 1$, $M = 0.4$, $k = 0.2$, and $q_1^T = q_2^T = 0$. In this and subsequent example and analyses, we follow Peres and Van den Bulte (2014, p. 90) and select parameter values with the objective of conveying model-based insights while acknowledging that insights from models and experiments may be more valuable when parameters or manipulations are roughly consistent with quantities reported in prior empirical work when such reports exist.[5] Appendix B provides further information on the algorithm developed for these numerical calculations. Table 2 illustrates how higher values of q_{12} results in shorter optimal delays and higher profits.

The online appendix analyzes how the delay is affected by the relative size of the two profit pools. A larger profit pool in the elite segment delays the introduction in the follower segment, whereas a larger pool in the follower segment brings it forward. Also, numerical analysis suggests that the relation between τ and with m_1/m_2 becomes increasingly S-shaped as advertising effectiveness increases. The higher the advertising effectiveness, the closer the optimal entry timing decision comes to "now or never."

Figure 1. (Color online) Example of Optimal Advertising over Time ($q_{12} = -0.1$, $k = 0.2$)



3.2. U-Shaped Advertising in the Elite Segment

Next, we state Corollary 1 to Theorem 1, which is that a U-shaped advertising path can be optimal, but only if $k > 0$, that is, only if disadoption occurs.

Corollary 1. *From Theorem 1, for a sufficiently small discount rate r, if and only if there is a time $\bar{t} \in [\tau, T]$ such that, for all $t \in [\bar{t}, T]$,*

$$ky - q_{11}(1-x)^2 - q_{21}\frac{m_2 + \mu}{m_1 + \lambda_2}(1-y)(1-x) > 0, \qquad (11)$$

then the optimal advertising u_1 in segment 1 is U-shaped.

Figure 1 plots the optimal advertising in both segments over time for the case in Section 3.1 (Table 2) with $q_{12} = -0.1$. The optimal advertising path is U-shaped in the elite segment and monotonically decreasing in the follower segment. Also, the great bulk of advertising is allocated to segment 1, because it boosts growth in segment directly, boosts growth in segment 2 indirectly, and protects growth in segment 1 from repulsion from segment 2.

Corollary 1 states that the optimal advertising level need not decline monotonically over time when $k > 0$ and may be U-shaped instead. The next section characterizes in greater detail how marketing support should vary over time in markets with attraction/repulsion dynamics.

3.3. How Strength of Repulsion-Based Disadoption Affects Advertising

Figure 2 illustrates how the strength of repulsion k affects the split of advertising spend aggregated over time between segments and between before versus after entry in segment 2. It does so by means of a numerical analysis. We use the same parameters as in Figure 1 but now vary k from 0 to 0.3.

Figure 2. (Color online) How the Strength of Repulsion-Based Disadoption (k) Affects Optimal Advertising

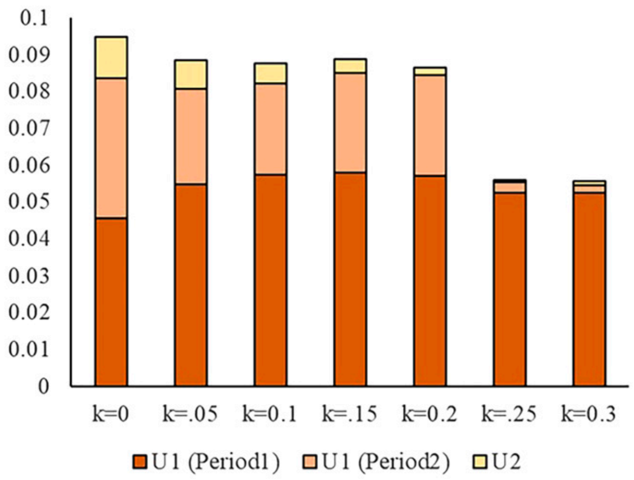


Figure 2 shows several patterns. First, overall ad spend tends to decrease as k increases, but there is a very notable decrease around $k = 0.25$. This coincides with a dramatic increase of the entry delay τ from 2–3 to about 15. Second, advertising in period 1, that is, before entry into segment 2, is relatively stable. It increases slightly as k increases from 0 to 0.1 and remains constant as k increases to 0.2. This slight increase is to boost the installed base in segment 1 before it gets exposed to repulsion from segment 2. Further increases of k do not trigger even more pre-entry advertising because the dramatic increase in entry delay provides protection. The third pattern in Figure 2 is how the advertising in period 2, that is, after entry into segment 2, varies with k. It starts fairly high and decreases as k increases from 0 to 0.10 as the company accommodates the mild repulsion reducing the effective demand in segment 1 by reducing its ad spend. However, as k increases beyond 0.10, postentry advertising increases again as the company starts fighting repulsion with prophylactic advertising. Finally, once k reaches 0.25, it is more profitable to dramatically delay the entry into segment 2 rather than to fight repulsion with prophylactic advertising. Overall, Figure 2 illustrates how the ability to delay entry into the repulsive segment can dramatically affect the optimal level of advertising spending.

4. Profit Impact Analysis

In this section, we use the solution concepts and the algorithm presented in Appendix B to investigate the relative profitability of four archetypical strategies using different combinations of dynamic advertising and delay. These strategies are as follows:

1. No Delay and No Advertising ($u_1 = u_2 = 0, \tau = 0$).
2. No Delay and Optimal Advertising ($u_1 \geq 0, u_2 \geq 0, \tau = 0$).
3. Optimal Delay and No Advertising ($u_1 = u_2 = 0, \tau \geq 0$):
4. Optimal Delay and Optimal Advertising ($u_1 \geq 0, u_2 \geq 0, \tau \geq 0$).

Strategy 1 involves no optimization at all. Strategy 2 involves simultaneous entry in both segments. Strategy 3 involves delaying entry into the second segment but no advertising in either segment. Strategy 4 involves optimal dynamic advertising and optimal entry delay, the solution of which is characterized in Theorem 1.

Setting aside the relative size of the profit pools addressed in Section 3.1, we expect the benefits of advertising and delay to vary as a function of three market characteristics. The first is the effectiveness of advertising (parameter $\rho \geq 0$), because optimizing advertising should affect profits more when advertising affects the diffusion process greatly. The second is the presence of negative influence from segment 2 on segment 1 operating through negative adoption contagion when $q_{12} < 0$ and repulsion-based disadoption when $k > 0$. As advertising in segment 1 can counter cross-segment repulsion, it expectedly has a greater profit impact when such repulsion is at work. The third is the cost of entry in segment 2, $M \geq 0$. Having the opportunity to delay entry into the second segment until the word-of-mouth in segment 1 is strong enough to withstand repulsion is likely to have a greater profit impact when the cost of entry in the second segment is large or when the repulsion from segment 2 onto segment 1 is strong.

Thus, we manipulate four parameters: the effectiveness of advertising, $\rho = \rho_1 = \rho_2 = \{1.0, 0.2\}$; the cross-segment contagion parameter $q_{12} = \{0.4, 0, -0.1\}$; the presence of repulsion-based disadoption in segment 1, $k = \{0, 0.2\}$; and the cost of entry in segment 2, $M = \{0, 0.4\}$. This yields 24 scenarios. As to the remaining parameters, we set symmetric profit pools $m_1 = m_2 = 1$ consistent with Joshi et al. (2009) and with the fact that sports cars like the 911 account for about 40% of Porsche sales but generate higher unit margins than its SUVs. The discount rate is $r = 0.1$ which is realistic if time is in years (Peres and Van den Bulte 2014). The diffusion parameters are $p_1 = 0.03$, $p_2 = 0.001$, and $q_{21} = q_{22} = 0.4$, broadly consistent with prior estimates at an annual scale (Van den Bulte and Joshi 2007). The planning horizon is $T = 20$ years. For each scenario, we calculate the NPV of strategies 1, 2, 3, and 4. Given the long horizon and the 10% discount rate, residual values are ignored.

Before assessing the four strategies across the 24 scenarios, we confirmed that in a market with a single segment where advertising boosts the tendency to adopt independently of peer influence and where disadoption is precluded, the optimal advertising strategy is to launch hard and cut advertising once contagion kicks in. Because our model nests that of Horsky and Simon (1983) except for the square root versus natural log transformation of advertising, it is not surprising that we replicate their key insight when $m_2 = 0$.

As a second preliminary analysis, we investigated the optimal advertising policy in the asymmetric influence model proposed by Van den Bulte and Joshi (2007), where segment 1 influences segment 2 but not vice versa ($q_{21} > 0$; $q_{12} = k = 0$). The optimal strategy here is still to launch hard and decrease advertising monotonically over time in both segments, but there is marked increase in the advertising spending in segment 1 when popularity in that segment has a positive influence on segment 2 ($q_{21} = 0.4$ rather than $q_{21} = 0$). This increase occurs both through spending more at the time of launch and through reaching zero spending later. This increase in advertising in segment 1 is paired with a reduction in spending in segment 2. In short, our analysis confirms the intuition that the presence of asymmetric influence leads to increased spending in the elite or influential segment, and decreased spending in the follower or imitator segment.

4.1. Markets Without Disadoption ($k = 0$)

Table 3 presents the NPV of the four strategies for the 12 scenarios where $k = 0$. The table reports the NPV of strategy 1 (zero advertising and delay) as a benchmark and the improvements over it of the other strategies. Strategy 2 (optimizing advertising only) boosts the NPV by 21% on average, whereas strategy 3 (optimizing delay only) boosts the NPV by 4%. There is no value in delaying entry unless there is a cost of entry (Kalish et al. 1995). Strategy 4 (optimizing both) boosts the NPV by 23%.

For the scenarios studied, strategy 4 always improves notably over the strategy 3, that is, the use of delayed entry analyzed earlier by Kalish et al. (1995) and Joshi et al. (2009). Strategy 4 notably improves on strategy 2 only in scenarios 11 and 12: Using a combination of entry delay and advertising is notably better than using merely advertising only when the cost of entry is sizable and advertising effectiveness is moderate. Overall, optimizing advertising tends to be more profitable than optimizing entry delay, and optimizing both rather than merely advertising is important only when the entry cost is sizable and advertising is moderately effective.

4.2. Markets with Disadoption ($k > 0$)

The profit impact analysis just reported allowed adoption in segment 2 to slow adoption in segment 1 ($q_{12} < 0$) but not to trigger disadoption ($k = 0$). When cross-segment repulsion is so strong that uptake in segment 2 induces disadoption in segment 1, one would expect the benefits of dynamic advertising and entry delay to be larger than those documented in the previous section. We therefore repeat the analysis of the same four strategies, but now for the 12 scenarios with $k = 0.2$ instead of $k = 0$.

Comparing the profits obtained in strategy 1 in Table 4 with those in Table 3 shows, unsurprisingly, that disadoption lowers NPV. Of greater interest is that the NPV improvements of the active strategies are notably larger in markets with cross-segment repulsion. The average impact of strategy 2 (optimizing advertising) increases from 21% in Table 3 to 30% in Table 4, that of strategy 3 (delaying entry) increases from 4% to 16%, and that of strategy 4 (both advertising and entry timing) increases from 23% to 36%. Also notable is that, when repulsion triggers disadoption ($k > 0$), delay can boost profitability even in the absence of entry cost (strategy 3 in scenarios 3 and 9). This does not occur when $k = 0$ (Table 3) (Kalish et al. 1995).

Contrasting the combination strategy with the advertising-only and the delay-only strategies leads to the same insights as in Table 3. Strategy 4 always improves notably on the strategy 3 and improves notably on strategy 2 only when the cost of entry is positive and advertising is moderately rather than highly effective.

The pattern across all scenarios in Tables 3 and 4 shows that only using prophylactic marketing communications to the cool or elite segment is more profitable than only denying the uncool or nonelite access to the product in 21 of the 24 scenarios. The three exceptions occur when advertising is only moderately effective, entry costs are notable, and popularity in the second segment slows adoption or triggers disadoption in the first segment. In those three cases, using both advertising and delay rather than only delay allows the company to make its product available to all customers somewhat faster and boost its profitability by 5%–7% while doing so.

5. Single-Segment Markets with Popularity-Based Disadoption

Corollary 1 highlighted the critical role of popularity-based disadoption in the optimality of monotonically decreasing advertising in a two-segment market. We now investigate whether and how disadoption can result in U-shaped advertising paths in single-segment markets as well. This simplification not only provides some sharper results but is also of substantive interest because marketers may need to address popularity-based disadoption also in homogenous markets or in heterogeneous markets where they cannot effectively differentiate between the two segments.

Table 3. Relative Profitability of Strategies ($k = 0$)

					% Improvement in NPV				
No.	ρ	M	q_{12}	NPV of strategy 1	2 vs. 1	3 vs. 1	4 vs. 1	4 vs. 2	4 vs. 3
1	1	0	0.4	1.2422	23.50%	0.00%	23.50%	0.00%	23.50%
2	1	0	0	1.1274	29.1	0	29.1	0	29.1
3	1	0	−0.1	1.0564	34.7	0	34.7	0	34.7
4	1	0.4	0.4	0.8422	34.6	2.1	34.6	0	31.8
5	1	0.4	0	0.7274	45.1	7.3	45.2	0.1	35.3
6	1	0.4	−0.1	0.6564	55.8	15.2	56.9	0.7	36.2
7	0.2	0	0.4	1.2422	6.9	0	6.9	0	6.9
8	0.2	0	0	1.1274	7.1	0	7.1	0	7.1
9	0.2	0	−0.1	1.0564	8.4	0	8.4	0	8.4
10	0.2	0.4	0.4	0.8422	10.1	2.1	10.1	0	7.8
11	0.2	0.4	0	0.7274	11.0	7.3	14.9	3.5	7.1
12	0.2	0.4	−0.1	0.6564	13.6	15.2	23.3	8.5	7.0
Average					21.4	4.1	22.6	1.1	17.6

5.1. Main Analysis

We simplify the main model to the following single-segment optimization problem:[6]

$$\max_{u_1} \Pi = \int_0^T (m_1\dot{x} - u_1)e^{-rt}dt + q_1^T(x(T))e^{-rT} \qquad (12)$$

$$\text{s.t. } \frac{dx(t)}{dt} = \left(p + \rho_1\sqrt{u_1(t)} + q_{11}x(t)\right)(1 - x(t)) - kx(t), \; x(0) = 0.$$

This simplification generates two insights. The first is that disadoption ($k > 0$) prevents the product from reaching full market penetration without advertising. For instance, when $\rho_1 = q_{11} = 0$, then $x(t \to \infty) = \frac{p}{p+k} < 1$. When $\rho_1 = 0$, $q_{11} > k$ and $p \to 0$, then $x(t \to \infty) = 1 - k/q_{11} < 1$. That the penetration ceiling is decreasing with k is difficult to discern from the full model and explains why nonzero marketing spending can be optimal a long time after the product launch, in contrast to settings traditionally studied where $k = 0$.

The second insight is stated in Theorem 2, derived after applying optimal control methods.

Theorem 2. *In the case of the one-segment model in (12), and for a sufficiently small discount rate, the following holds:*

(a) *If there is no disadoption (i.e., $k = 0$), then the optimal advertising policy is monotonically decreasing over time.*

(b) *If there is disadoption (i.e., $k > 0$) and if and only if there exists a $\bar{t}$ such that for all $t \geq \bar{t}$,*

$$\Delta(t) = -q_{11}(1 - x(t))^2 + k > 0 \qquad (13)$$

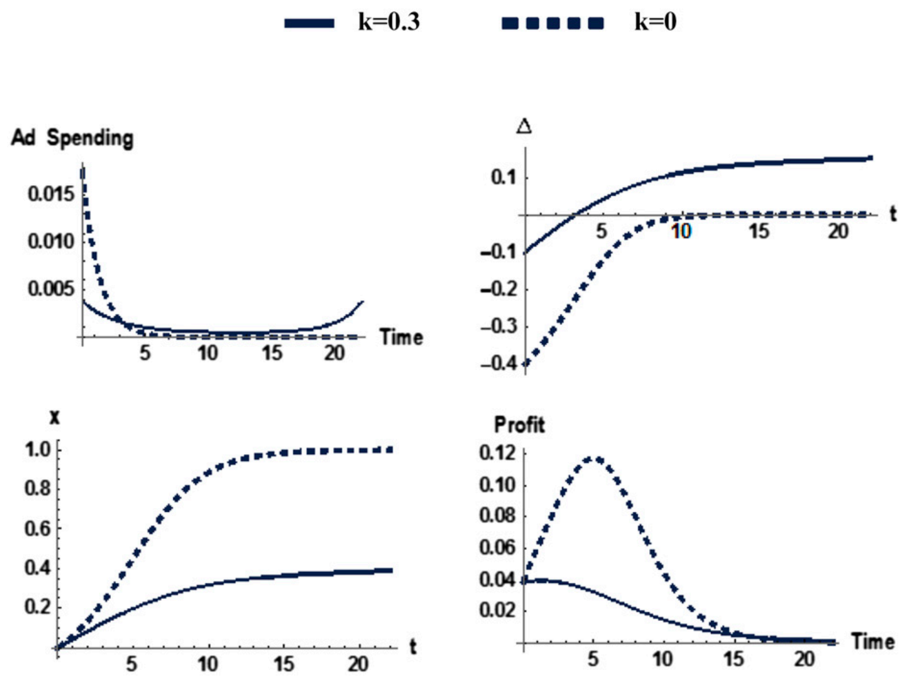
then the optimal advertising policy is U-shaped.

The result in part (a) is as in Horsky and Simon (1983). Part (b) stands in contrast to this traditional result: a U-shaped advertising policy can be optimal in an unsegmented market provided that popularity-

Table 4. Relative Profitability of Strategies ($k = 0.2$)

					% Improvement in NPV				
No.	ρ	M	q_{12}	NPV of strategy 1	2 vs. 1	3 vs. 1	4 vs. 1	4 vs. 2	4 vs. 3
1	1	0	0.4	1.1261	23.00%	0.00%	23.00%	0.00%	23.00%
2	1	0	0	0.9339	30	0	30	0	30.0
3	1	0	−0.1	0.7378	53	5.2	53	0	45.4
4	1	0.4	0.4	0.7261	35.7	2.9	35.7	0	31.9
5	1	0.4	0	0.5339	52.5	14.7	53.8	0.9	34.1
6	1	0.4	−0.1	0.3378	115.8	72.1	125.5	4.5	31.0
7	0.2	0	0.4	1.1261	6.5	0	6.6	0.1	6.6
8	0.2	0	0	0.9339	6.4	0	6.4	0	6.4
9	0.2	0	−0.1	0.7378	13.3	5.2	13.7	0.4	8.1
10	0.2	0.4	0.4	0.7261	10.1	2.9	10.2	0.1	7.1
11	0.2	0.4	0	0.5339	11.3	14.7	21.3	9.0	5.8
12	0.2	0.4	−0.1	0.3378	29	72.1	80.4	39.8	4.8
Average					30.3	15.8	36.4	4.6	17.6

Figure 3. (Color online) Illustration of Theorem 2



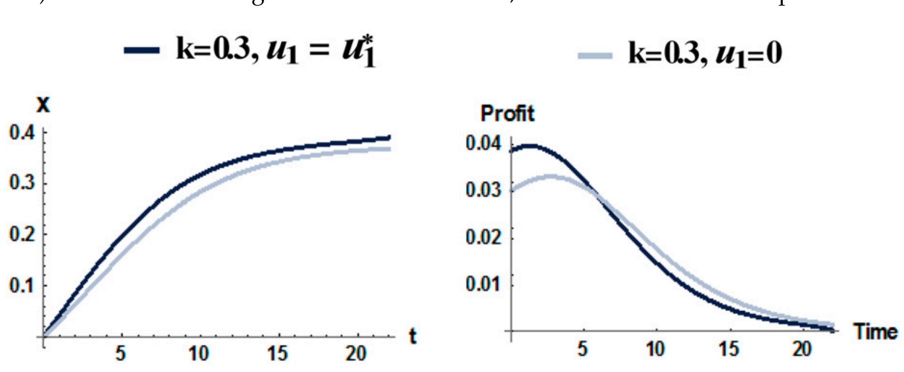
based disadoption occurs. This last insight can also be derived from our two-segment model because the dynamics of (12) is nested in (2), and part (b) is nested in Corollary 1 by setting $y = 1$.

Figure 3 illustrates Theorem 2 for the set of parameters $\rho_1 = 0.2$, $q_{11} = 0.4$, $p_1 = 0.03$, $m_1 = 1$, $r = 0.1$, and $T = 22$. The full lines pertain to the case where $k = 0.3$, a scenario that satisfies the condition involving Δ in part (b) of Theorem 2 and that results in a U-shaped advertising path, with advertising decreasing until $t = 15$ and increasing again thereafter. The dashed lines pertain to the case $k = 0$ where the condition involving Δ is not satisfied, which results in monotonically decreasing advertising. Note how disadoption ($k > 0$) shifts the marketing ceiling downward in the bottom left panel.

Next, we investigate the impact of advertising on the evolution of the user base in the presence of disadoption. Figure 4 is a continuation of the analysis using the parameters for the full lines in Figure 3 ($k = 0.3$) and shows how optimal advertising affects the user base and (undiscounted) profits. The left panel shows how the size of the user base evolves over time: It grows quickly at first but stalls below 40% of the full market potential. Advertising generates a modest increase in the user base, throughout the planning window. As shown in Figure 3, the optimal advertising path is U-shaped. The timing of the marked uptick in advertising spending around $t > 15$ coincides with the stalling of the growth of the user base stemming from disadoption, which worsens as the size of the user base increases.

Three additional points are worth noting. First, the profit impact of advertising is concentrated early on, when the product has a small user base and hence

Figure 4. (Color online) Effect of Advertising on User Base and Profit, in the Presence of Disadoption



does not benefit from peer influence. This is consistent with the standard "launch hard and cut back later once you can free-ride peer influence" strategy. Second, the marked uptick in advertising to stem disadoption from over-popularity once $t > 15$ is not wasted, and the profits in the last five periods are only slightly lower than those without advertising. The reason that the profits after $t \approx 7$ are lower with advertising than without advertising is that the advertising in the first seven period builds the user base faster and hence accelerates the onset of disadoption. Third, considering the user base and profits graphs jointly, the NPV impact of advertising stems more from a leftward shift rather than an upward shift in the user base and profit curves, suggesting that it stems from diffusion acceleration and protection against over-popularity rather than demand expansion (Libai et al. 2013, Appel et al. 2018).

Finally, we present the feedback solution of the optimization problem in (12), expressing the optimal advertising as a function of the state variable (size of the user base) rather than time. Compared with open-loop solutions that depend on time, feedback policies are robust as they do not require revision if the change in the size of user base during a time interval deviates from what was expected.[7]

Theorem 3. *Consider the optimization problem* (12). *There exists a unique local optimal time-invariant feedback solution for* u_1^* *of the following form:*

$$u_1^*(x, \Phi(x)) = \frac{1}{4}\left[\rho_1(m_1 + \Phi(x))(1 - x)\right]^2,$$

where the function $\Phi(x)$ *is the unique solution of the following backward differential equation:*

$$\Phi'(x)\left[\left(p + \frac{1}{2}[\rho_1^2(m_1 + \Phi(x))(1 - x)] + q_{11}x\right)(1 - x) - kx\right]$$
$$= r\Phi(x) - (m_1 + \Phi(x))[q_{11}(1 - x) - (p + \frac{1}{2}[\rho_1^2(m_1 + \Phi(x))$$
$$(1 - x)] + q_{11}x + k)], \Phi(x(T)) = q_1^T.$$

Although Theorem 3 guarantees the existence of the feedback solution only locally around the steady state, the solution exists globally by the following argument (Hartl 1987). The solution of the optimal control problem is unique and the state $x(t)$ evolves monotonically. The latter implies a one-to-one relationship between the size of the user base $x(t)$ and time t. If $x(t) = g$ then $x^{-1}(g) = t$. Therefore, one can write the costate (optimal advertising) not only as a function of time but also as a function of the state (user base).

In short, Theorem 3 enables one to express the current optimal advertising level as a function of the current size of the user base, rather than time. Figure 5 illustrates Theorem 3 for the same set of parameters used in Figures 3 and 4, with $k = 0.3$. The left graph shows the feedback solution of the optimal advertising path as a function of the size of user base, and the right graph shows the open-loop solution of optimal advertising path as a function of time. The feedback solution shows more dramatically how prophylactic advertising ramps up once the user base reaches 38%.

5.2. Dynamic vs. Time-Invariant Optimal Advertising

To evaluate the benefit of the optimal solution in Theorem 2, we compare it against a policy where advertising is optimized but under constraint of not varying over time. The model is

$$\max_u \ \Pi = \int_0^T (m\dot{x} - u)e^{-rt}dt + q^T(x(T))e^{-rT}$$
$$\text{s.t. } \frac{dx}{dt} = \left(p + \rho\sqrt{u} + qx\right)(1 - x) - kx, \ x(0) = 0.$$

Subscripts and arguments are omitted for clarity. Let us simplify as advertising is constant and set $q^T = 0$ anticipating a longer horizon. This gives $\max_u \int_0^T m\dot{x}e^{-rt}dt - \frac{u}{r}(1 - e^{-rT})$. Let us define two constants, $A = p + \rho\sqrt{u}$ and $B = A + k - q$. The former is clearly positive, whereas the latter may be negative. Thus, $\dot{x} = A - Bx - qx^2$, $x(0) = 0$, which implies that $\int_0^x \frac{dx}{qx^2 + Bx - A} = -t$. To evaluate the left side of this, note that $B^2 + 4Aq > 0$ because both A and q are positive.

Figure 5. (Color online) Feedback and Open-Loop Solutions for the Scenario in Figures 3 and 4 ($k = 0.3$)

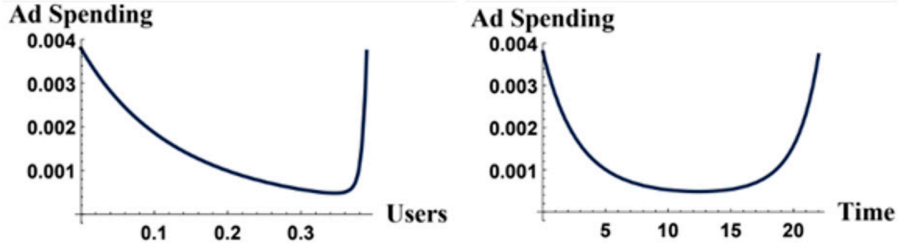


Table 5. NPV of Optimal Constant vs. Dynamic Advertising as a function of k

k value	Constant ad NPV	Dynamic ad NPV	% Gain
0.00	0.559	0.573	2.50%
0.05	0.49	0.5	2.04%
0.10	0.425	0.433	1.88%
0.15	0.365	0.37	1.36%
0.20	0.311	0.315	1.29%
0.25	0.263	0.265	0.76%
0.30	0.221	0.222	0.45%

Let us define a term $v = \sqrt{B^2 + 4Aq}$. Then, the solution is

$$x = \frac{2A(1-e^{-vt})}{(v+B)+(v-B)e^{-vt}} \Rightarrow \frac{dx}{dt} = \frac{4Av^2e^{-vt}}{(v+B+(v-B)e^{-vt})^2}.$$

Thus, the problem becomes to maximize the following profit function with respect to u,

$$\Pi = 4mAv^2 \int_0^T \frac{e^{-vt}}{[(v+B)+(v-B)e^{-vt}]^2} e^{-rt}dt - \frac{u}{r}(1-e^{-rT}).$$

We compare the relative performance of the two strategies numerically, for k ranging from 0 to 0.3, and using the same values as before for the remaining parameters: $\rho = 0.2$, $q_{11} = 0.4$, $p = 0.03$, $m = 1$, $r = 0.1$, and $T = 22$.

With $k = 0$, the optimal advertising decreases monotonically over time, and with $k = 0.3$, it is U-shaped. For $k = 0.3$, the analysis of the time-invariant optimal strategy gives $A = 0.03 + 0.2\sqrt{u}$, $B = -0.07 + 0.2\sqrt{u}$, and $v = \sqrt{.04u + .097 + .292\sqrt{u}}$.

Table 5 shows that allowing the optimal advertising to vary over time produces a boost in NPV of less than 3%. Perhaps surprisingly, the gains from allowing advertising to vary over time are larger the lower k is. The reason is that a flat level of advertising better approximates a U-shaped advertising path (optimal when k is moderate or high) than a monotonically decreasing advertising path (optimal when k is very low).

6. Conclusion

We investigated the optimal dynamic advertising and entry delay decisions for a new product in markets with asymmetric peer influence, either one-way innovator/imitator influence or two-way attraction/repulsion influence. The latter case occurs when the elite segment positively influences the follower segment to adopt while itself being negatively influenced by popularity in the follower segment. This repulsive peer effect can cause slower adoption and even disadoption among the elite.

Our model specifications and optimization analyses nest those by Horsky and Simon (1983), Kalish et al. (1995), and Joshi et al. (2009), by considering both optimal dynamic advertising and optimal delay and by considering both influential/imitator and attraction/repulsion as cross-segment influences. Doing so required solving a complex problem of two sets of TPBVPs, where one set has an unknown boundary value condition that satisfies an algebraic equation. The solution methodology provides analytic solutions the properties of which are investigated and illustrated using numerical analysis.

Two main novel policy insights emerge from our analysis. The first is that, in markets with repulsion-based disadoption, the optimal advertising strategy for the elite segment can be U-shaped. The firm may want to advertise heavily at first and decrease advertising spending subsequently, but increase spending again later to fight the repulsive effect of the growing popularity among followers.[8] Whereas high levels of advertising are profitable early on to accelerate diffusion when one cannot yet leverage and free-ride positive peer influence among elite customers, high levels of advertising can also be profitable much later on as prophylactic advertising provides protection against negative peer influence once has become popular among followers. U-shaped advertising paths can be optimal also in single-segment markets, provided that they exhibit popularity-based disadoption.

The second main insight is that using advertising and delayed entry in combination can, but need not, lead to markedly higher profits than using either exclusively. The combination can be especially profitable in markets where cross-segment repulsion triggers disadoption, advertising is only moderately effective, and entry costs are high. In markets without disadoption, with highly effective advertising, or with low entry costs, in contrast, delaying entry may add little value if one already manages advertising optimally. The relative size of the profit pool in each segment is another important contingency. A greater profit pool in the elite segment delays the introduction in the follower segment, whereas a greater profit pool in the follower segment brings it forward.

Investigating the effectiveness of policy tools other than delayed entry (or no entry at all) into the second segment has become more important recently as companies put greater weight on considerations of inclusion and equity, as opposed to only profit. Marketers of prestige product are often concerned that broadening their customer base beyond the elite may harm their profitability. Consequently, some choose to never reach out to nonelite customers or do so after considerable delay. Such exclusionary policies studied by Kalish et al. (1995) and Joshi et al. (2009) may not be as acceptable today as they once were to many consumers, employees, and other stakeholders. We provide insights into how quickly firms can start catering beyond the elite segments, and how advertising can be used to speed up this democratization of consumption profitably.

Our analysis, enabled by a novel solution approach, provides policy guidance to marketing decision makers operating in markets exhibiting attraction/repulsion and the resulting chase-and-flight dynamics. These types of phenomena have gained considerable interest recently (Berger and Heath 2008, Berger and Le Mens 2009, Joshi et al. 2009, Abedi et al. 2014, Smaldino et al. 2017, Yoganarasimhan 2017, Appel et al. 2018, Touboul 2019, Warren et al. 2019). Firms may consider additional policy tools to manage peer influence across segments beyond the two we studied. One is initial seeding to leverage the influence of opinion leaders and counter repulsion (Lehmann and Esteban-Bravo 2006, Bakshi et al. 2013). Another approach is to use pricing to manage the uptake in each segment. The effectiveness of such policies will depend on whether segments have different reservation prices and price sensitivities, and on whether the firm can charge different prices to different segments. Third, companies often attempt to achieve segment decoupling by marketing distinct product designs to different segments (Han et al. 2010), thereby weakening the repulsive effect. The novel solution rules and results about optimal advertising and entry delay presented here will, we hope, motivate and facilitate research on the effectiveness of other marketing policies used to manage cross-segment attraction/repulsion and the resulting chase-and-flight dynamics.

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Appendix A. Proofs

A.1. Proof of Theorem 1
The necessary conditions are as follows.

For the time interval $t \in [0, \tau]$,

$$\frac{\partial H^1}{\partial u_1} = 0 \Rightarrow u_1 = \frac{1}{4}\Big(\rho_1 (m_1 + \lambda_1)(1 - x)\Big)^2. \tag{A.1}$$

$$\frac{d\lambda_1}{dt} = r\lambda_1 - \frac{\partial H^1}{\partial x} = r\lambda_1 - (m_1 + \lambda_1)\Big[q_{11}(1 - x) - \Big(p_1 + \rho_1\sqrt{u_1} + q_{11}x\Big)\Big]. \tag{A.2}$$

For the time interval $t \in [\tau, T]$,

$$\frac{\partial H^2}{\partial u_1} = 0 \Rightarrow u_1 = \frac{1}{4}\Big(\rho_1 (m_1 + \lambda_2)(1 - x)\Big)^2. \tag{A.3}$$

$$\frac{\partial H^2}{\partial u_2} = 0 \Rightarrow u_2 = \frac{1}{4}\Big(\rho_2 (m_2 + \mu)(1 - y)\Big)^2. \tag{A.4}$$

$$\frac{d\lambda_2}{dt} = r\lambda_2 - \frac{\partial H^2}{\partial x} \Rightarrow$$

$$\frac{d\lambda_2}{dt} = r\lambda_2 - (m_1 + \lambda_2)\big[q_{11}(1 - x) - \Big(p_1 + \rho_1\sqrt{u_1} + q_{11}x + (k + q_{12}y)\Big)\big] - (m_2 + \mu)q_{21}(1 - y). \tag{A.5}$$

$$\frac{d\mu}{dt} = r\mu - \frac{\partial H^2}{\partial y} \Rightarrow$$

$$\frac{d\mu}{dt} = r\mu - (m_1 + \lambda_2)\big[q_{12}(1 - x) - kx\big] - (m_2 + \mu)\Big[q_{22}(1 - y) - \Big(p_2 + \rho_2\sqrt{u_2} + q_{21}x + q_{22}y\Big)\Big]. \tag{A.6}$$

Finally,

$$\lambda_2(T) = q_1^T \text{ and } \mu(T) = q_2^T. \tag{A.7}$$

Thus, necessary conditions for entry into segment 2 are

$$\lambda_1(\tau) = \lambda_2(\tau), \text{ and} \tag{A.8}$$

$$H^1(\tau) + rM - H^2(\tau) \begin{cases} < 0 & \text{if } \tau = 0, \\ = 0 & \text{if } \tau \in (0, T), \\ > 0 & \text{if } \tau = T. \end{cases} \tag{A.9}$$

Where,

$$H^1 = (m_1 + \lambda_1)dx/dt - u_1, \; t \in [0, \tau],$$
$$H^2 = (m_2 + \lambda_2)dx/dt - u_1 + (m_2 + \mu)dy/dt - u_2, \; t \in [\tau, T]. \tag{A.10}$$

Taking into account that $y(\tau) = 0$ and (A.7), we obtain from (A.9) that

$$-\frac{1}{4}(m_2 + \mu(\tau))\Big[\rho_2^2(m_2 + \mu(\tau)) + 4\big(p_2 + q_{21}x(\tau)\big)\Big] + rM \begin{cases} < 0 & \text{if } \tau = 0, \\ = 0 & \text{if } \tau \in (0, T), \\ > 0 & \text{if } \tau = T. \end{cases} \tag{A.11}$$

Let, $L(\tau) \equiv -\frac{1}{4}(m_2 + \mu(\tau))\Big[\rho_2^2(m_2 + \mu(\tau)) + 4\big(p_2 + q_{21}x(\tau)\big)\Big] + rM$. Then,

$$\tau = \begin{cases} 0 & \textit{if } L(\tau) < 0, \textit{in } (0, T) \\ t \textit{ in } (0, T) & \textit{if } L(\tau) = 0, \\ T & \textit{if } L(\tau) > 0, \textit{ in } (0, T). \end{cases} \tag{A.12}$$

$L(\tau)$ is continuous, and the necessary and sufficient conditions for optimality of τ are that $L(\tau) = 0$, $L(\tau^-) > 0$ and $L(\tau^+) < 0$. If multiple points satisfy, we pick the one with highest profit

A.2. Proof of Corollary 1
Consider the derivative of u_1,

$$\frac{du_1}{dt} = 2\rho_1\sqrt{u_1}\,\Big\{r\lambda_2(1 - x) + (m_1 + \lambda_2)(ky - q_{11}(1 - x)^2) - q_{21}(m_2 + \mu)(1 - y)(1 - x)\Big\}.$$

First set $r = 0$. To show that u_1 is non-monotonic it suffices to show that $\exists(\bar{t} > 0)$ such that $\frac{du_1}{dt}(\bar{t}) > 0$. Define $\Delta(t) \equiv ky - q_{11}(1 - x)^2 - q_{21}\frac{m_2 + \mu}{m_1 + \lambda_2}(1 - y)(1 - x)$. Then,

$$\frac{du_1}{dt} = 2\rho_1\sqrt{u_1}\,(m_1 + \lambda_2)\Delta, \tag{A.13}$$

and a necessary and sufficient condition that $\frac{du_1}{dt}(\bar{t}) > 0$ is that $\Delta(\bar{t}) > 0$, or,

$$\frac{du_1}{dt} > 0 \Longleftrightarrow \Delta(t) > 0. \tag{A.14}$$

If $\forall\, t \geq \bar{t}$, $\Delta(t) > 0$, then u_1 will be U-shaped. By continuity, this property holds in a neighborhood of $r = 0$, $r > 0$ completing the proof.

A.3. Proof of Theorem 2

Consider the profit maximization problem in the text ((12a) and (12b)):

$$\max_{u_1} \ \Pi = \int_0^T (m_1\dot{x} - u_1)e^{-rt}dt + q_1^T(x(T))e^{-rT}$$
$$\text{s.t.} \ \ \frac{dx(t)}{dt} = \left(p + \rho_1\sqrt{u_1(t)} + q_{11}x(t)\right)(1 - x(t)) - kx(t), \ x(0) = 0.$$

Part (a): We form the Hamiltonian, $\boldsymbol{H} = (\boldsymbol{m}_1 + \boldsymbol{\lambda}_1)\dot{\boldsymbol{x}} - \boldsymbol{u}_1$, and obtain the following necessary conditions:

$$\frac{d\lambda_1}{dt} = r\lambda_1 - (m_1 + \lambda_1)[q_{11}(1 - x) - \left(p + \rho_1\sqrt{u_1} + q_{11}x\right)], \quad \lambda_1(T) = q_1^T \tag{A.15}$$

$$u_1 = \frac{1}{4}[\rho_1(m_1 + \lambda_1)(1 - x)]^2. \tag{A.16}$$

Consider the sign of the derivative of u_1. From (A.16) we obtain

$$\dot{u_1} = 2\sqrt{u_1}\rho_1\{\dot{\lambda_1}(1 - x) - \dot{x}(m_1 + \lambda_1)\}. \tag{A.17}$$

Plugging $\dot{x}$ and $\dot{\lambda_1}$ from (12) and (A.15) into (A.17) we obtain,

$$\dot{u_1} = [\rho_1^2(m_1 + \lambda_1)(1 - x)][r\lambda_1(1 - x) - (m_1 + \lambda_1)q_{11}(1 - x)^2]. \tag{A.18}$$

If $r = 0$, the sign of $\dot{u_1}$ from (A.18) is negative, and from continuity there is a neighborhood of $r = 0$ in which $r > 0$, in which $\dot{u_1}$ remains negative $\forall\, t$, completing the proof.
Part (b): We form the Hamiltonian, $H = (m_1 + \lambda_1)\dot{x} - u_1$, and apply the techniques of dynamic optimization to get the following necessary conditions:

$$\frac{d\lambda_1}{dt} = r\lambda_1 - (m_1 + \lambda_1)\left[q_{11}(1 - x) - \left(p + \rho_1\sqrt{u_1} + q_{11}x + k\right)\right], \quad \lambda_1(T) = q_1^T \tag{A.19}$$

$$u_1 = \frac{1}{4}[\rho_1(m_1 + \lambda_1)(1 - x)]^2. \tag{A.20}$$

Because (A.20) is exactly (A.16), to prove the proposition, we consider the derivative of u_1 in (A.17). Plugging $\dot{x}$ and $\dot{\lambda_1}$ from (12) and (A.19) into (A.17), we obtain

$$\dot{u_1} = \left[\rho_1^2(m_1 + \lambda_1)(1 - x)\right][r\lambda_1(1 - x) - (m_1 + \lambda_1)q_{11}(1 - x)^2 - k]. \tag{A.21}$$

Let us first consider $r = 0$. Given $0 < k < q_{11}$, considering (A.21), we have

$$\dot{u_1}(0) < 0. \tag{A.22}$$

Let $\Delta(t) = -q_{11}(1 - x(t))^2 + k$. To show u_1 is nonmonotonic, it suffices to show that there is a point $\bar{t} > 0$ such that $\dot{u_1}(\bar{t}) > 0$. Because $\dot{u_1} = \rho_1^2(m_1 + \lambda_1(t))^2(1 - x)\Delta(t)$, a necessary and sufficient condition is that $\Delta(\bar{t}) > 0$. If

$$\forall\, t \geq \bar{t}, \Delta(t) > 0, \tag{A.23}$$

then u_1 will have a U-shape. From continuity, this property remains true for a neighborhood of $r = 0$ where $r > 0$. This completes the proof.

A.4. Proof of Theorem 3

Consider the optimal solution of Theorem 2:

$$u_1 = \frac{1}{4}\left[\rho_1(m_1 + \lambda_1)(1 - x)\right]^2 \tag{A.24}$$

and let

$$\lambda_1 = \Phi(x).$$

Now the theorem follows immediately by substituting (A.24) and its derivative with respect to t into (A.19) and (A.20) and (12b) and considering the relation $\dot{\lambda_1} = \Phi'(x)\dot{x}$ and the existence and uniqueness theorem of differential equations (Boyce and Diprima 1986).

Appendix B. Algorithm to apply Theorem 1

A numerical approach is required to solve the set of coupled first-order nonlinear ordinary differential equations with boundary conditions, or TPBVPs, occurring in Theorem 1. Solving the problem and characterizing the optimal marketing policies in specific settings presents some challenges because the system of equations is divided into two stages, namely, before and after the entry into segment 2, and there are more variables in the second stage than the first. Thus, the problem can be represented as

Stage 1 for $t \in [0, \tau)$: Solve $\{\frac{dx}{dt} = f_1(x, \lambda, \boldsymbol{p}), \frac{d\lambda}{dt} = g_1(x, \lambda, \boldsymbol{p})\}$

Stage 2 for $t \in [\tau, T]$:

$$\text{Solve} \left\{\frac{dx}{dt} = f_2(x, \lambda, y, \mu, \boldsymbol{p}), \ \frac{d\lambda}{dt} = g_2(x, \lambda, y, \mu, \boldsymbol{p}), \ \frac{dy}{dt} = h_2(x, \lambda, y, \mu, \boldsymbol{p}), \ \frac{d\mu}{dt} = r_2(x, \lambda, y, \mu, \boldsymbol{p})\right\}$$

where, $\boldsymbol{p} = \{q_{11}, q_{12}, q_{21}, q_{22}, p_1, p_2, \rho_1, \rho_2, k, m_1, m_2, r, M\}$ is the set of parameters.

One can solve this system of equations using the Runga-Kutta approach implemented in the NDSolve solver in *Mathematica* (Method→"*ExplicitRungaKutta*"). However, that solution algorithm is cumbersome and slow because of the numerical integration challenge posed by widely separated boundary conditions, in our case at times $t = 0$, $t = \tau$ and $t = T$, with complex dynamics in the interval. We thus propose an alternative algorithm that produces the same solution in a fraction of the time. It does so using the following approach:[9]

- First solving the stage 1 system of equations over a short time interval using a slow but stable solver (Method →

"*ImplicitRungaKutta*"), falsely applying the terminal boundary condition at the end of the short interval (i.e., falsely assuming $\tau = T$).

• Next, solving the Stage 2 system of equations using the middle of the interval as initial estimates for a shooting method application (Method → {"*Shooting*", "*StartingInitialConditions*" → estimated starting points}) to stretch the solution interval by an amount $\Delta\tau$.

• Iterating till the solution converges.

Differential equations with initial or terminal value conditions are called boundary value problems (BVPs). These can be numerically solved using different techniques, including the Runga-Kutta method and the Shooting method.

In optimal control problems, the solution often takes the form of a two-point boundary valued problem, that is, two differential equations with the state equation having a known initial value and the co-state equation having a known terminal value. Such problems are called a TPBVP.

The shooting method for solving a BVP with a terminal condition is to make guesses about the initial value, run the differential equation forward, and check what value of the initial guess most reduces the discrepancy with the terminal condition. (Essentially, one can write the discrepancy as a function of the initial variable and numerically find the root.) The shooting method has the advantage of being fast, but it is not as robust as finite difference methods.

In our case, there are four coupled differential equations with initial and terminal boundary conditions and separated by a long time horizon. We used the following approach to stabilize the solution given the large time horizon and complex dynamics in the interval. First, solving the system of equations over a short time interval with the terminal boundary condition (falsely) applied at the end of this interval, and then using the solution at the middle of the interval as initial estimates to stretch the solution interval by an increment.

Figure B.1 illustrates this process for an initial time interval of two (setting $\lambda(2) = 0$) and thereafter an extended time interval of four. The red points are the midpoint values of the first solution. The green points are the midpoint values of the second solution where the red points were used as initial conditions on the shooting method for the second solution. This procedure is iteratively continued until the end.

Endnotes

[1] When advertising boosts not only innovative adoption but also imitation, the optimal advertising policy follows an inverted-U pattern (Dockner and Jørgensen 1988). However, advertising affecting imitation is less supported by the data than advertising affecting only the tendency to adopt independently of peers (Ruiz-Conde et al. 2014). Still less appealing is the assumption that changes rather than levels of advertising boost both types of adoption, as this leads to an odd optimal policy of launching with a miniscule ad budget and increasing spending over time (Fruchter and Van den Bulte 2011).

[2] When $u = 0$ and $k = 0$, imposing $p_1 + q_{12} \geq 0$ guarantees that $\frac{dx}{dt} \geq 0, \; \forall\, t$. In numerical examples where we do not impose these restrictions, we check that $p_1 + q_{11}x + q_{12}y \geq 0$ throughout the time window considered.

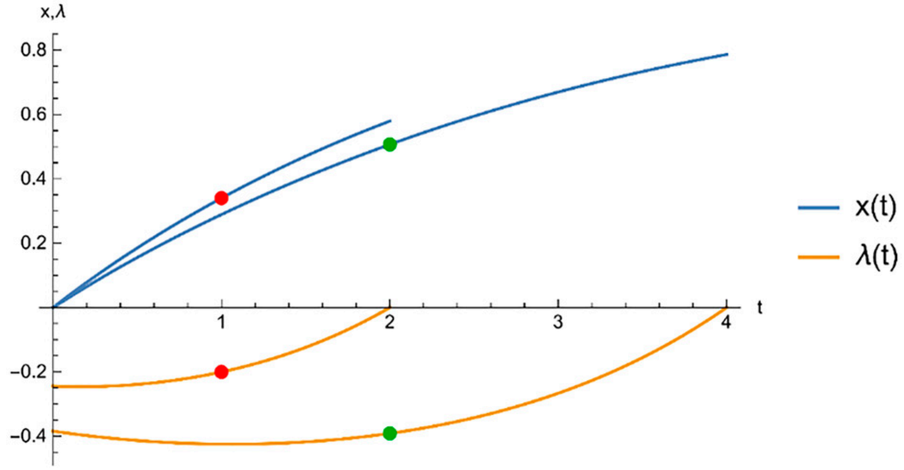
[3] Here and later, we omit the time argument on the state, co-state, and control variables when no confusion arises.

[4] After-sales revenue streams make up more than 50% of the entire revenue streams generated from selling a car (Gadiesh and Gilbert 1998, Ealey and Troyana-Bermudez 2000). The reason is that many car manufacturers including BMW, Lexus, Acura, Toyota, Ford, and GM participate extensively in downstream activities like insurance, refinancing, communication and emergency services, maintenance and repair services, and so on (Ealey and Troyana-Bermudez 2000). The share of vehicle sales in the total profit stream (as opposed to revenue stream) is expected to shrink from about 40% in 2015 to 22%–29% by 2030 (Bernhart et al. 2016).

[5] Except for $q_{12} < 0$, the values of the diffusion coefficients are consistent with estimates reported by Lilien et al. (2000) and Van den Bulte and Joshi (2007). Because ky is the effective churn rate and the average value of y is less than $1/2$, setting $k = 0.2$ puts the effective churn rate in the range of the estimates reported by Libai et al (2009). The profit pools are set to equal each other, consistent with the segment sizes estimated by Joshi et al. (2009), and are arbitrarily set to unity without loss of generality. The advertising effectiveness parameters are purposely set somewhat larger than the 0.5–0.8 estimates reported by Naik et al. (2008, p. 133) using a related model. To provide insight, we set M higher than 0 but lower than $m_2 = 1$. Note from Theorem 1 that M affects only the entry timing decision and not the advertising policy. Finally, the salvage values q^T (unrelated to the q diffusion coefficients) can be ignored when r and T are sufficiently large.

[6] The dynamics of this single-segment model stem from the same tension as in the main model: popularity can cause disadoption due

Figure B.1. (Color online) Iterative Approach to Solving TPBVP



to the loss of exclusivity appeal. Appel et al. (2018) present an agent-based model with the same property. The diffusion path among the elite in our two-segment model reduces to the diffusion path in our one-segment model if $y = 1$ throughout or if y has the same dynamics as x up to a scalar multiple.

[7] A *feedback* strategy depends on both time and the current state of the system. Thus, a feedback solution is of the form $u^*(t) = \vartheta(t,\ x(t))$. Becausece our paper deals with a finite time horizon, and therefore one cannot assume stationary (i.e., time invariant) values function. In our model, the value function is separable into time dependent and state dependent, a term that leads to the optimal advertising being stationary. The value function is $V(t, x)) = e^{-rt}\Phi(x)$ and does depend explicitly on time. Regarding the methodology, dynamic programming via Hamilton-Jacobi-Bellman (HJB) equation and optimal control will generate the same solution (Kamien and Schwartz 1991, p. 260).

[8] In contrast to prior research (Krishnan and Jain 2006, Fruchter and Van den Bulte 2011, Hariharan et al. 2015), we find that such a nonmonotonic pattern can be optimal even without suboptimally high initial spending and even when advertising boosts only the tendency to adopt independently (p) rather than both the tendency to adopt independently and the susceptibility to peer influence (q).

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