Marginal q*

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Abstract

We propose a new method to estimate the marginal value of capital under minimal assumptions. Combining asset prices with fundamentals, our method provides a model-free estimate of marginal q together with a simple correction for measurement error in (average) Tobin's Q. Our measure of marginal q yields plausible and robust estimates of adjustment costs and sensitivities of investment to fundamentals. It explains capital investment better than Tobin's Q, and drives out cash flow. These results raise serious questions about the large body of empirical evidence that relies on Tobin's Q to proxy for marginal q and control for investment opportunities.

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James Tobin's Q theory of investment emphasizes a fundamental connection between financial markets and the real economy: marginal q - i.e. marginal value of capital - is a key driver of business investment, conveniently summarizing all current and future investment opportunities (Hayashi, 1982).

Like any other shadow value, however, the marginal value of capital is not directly observable and empirical researchers have widely resorted to using the "observable" (average) Tobin's Q - i.e. the ratio of market value of capital to its replacement cost - as a proxy.[1] Despite a longstanding consensus that Tobin's Q is an imperfect, and likely misspecified, proxy for marginal q, particularly at the firm level, its use remains predominant in the empirical literature, primarily for lack of reliable and easy-to-compute alternatives.[2]

In this paper, we provide such an alternative. We propose a new, easy to implement, method to estimate marginal q under minimal assumptions regarding the nature of technology, markets, and investor preferences. The key insight rests on the joint measurability of the firm value function, i.e., the firm market value, and its underlying set of firm state variables.

Under simple regularity conditions for the differentiability of the value function, marginal q can be constructed as the elasticity of firm market value with respect to capital, using a very simple two-stage procedure. First, we project the observable market values - i.e. firm value function - onto measurable firm state variables, which include the firm's capital stock. Second, we differentiate the projected market values with respect to the capital stock to obtain an

[1]A Google Scholar search of 'Tobin Q' results in over 5,000 articles referencing Tobin's Q in the last five years. Similarly, Bartlett and Partnoy (2020) found 445 articles referencing Tobin's Q in the recent issues of top three finance journals.

[2]Hayashi (1982), and Abel and Eberly (1994) establish the exact conditions for equivalence between marginal q and (average) Tobin's Q. Some well known examples where these fail to hold include Abel and Eberly (1994, 1997), Gomes (2001), Cooper and Ejarque (2003), Cooper and Haltiwanger (2006) and Abel and Eberly (2011), among others.

estimate of the marginal value of capital - the marginal q.

Unlike previous attempts to estimate marginal q, ours makes an efficient use of market values, and imposes virtually no restrictions on the functional forms of the stochastic discount factor and firm investment technologies, thus making the estimate of capital's shadow value very robust to model mis-specification. Importantly, by constructing marginal q independently from investment technologies, we can easily recover structural parameters of adjustment cost technologies, even in highly nonlinear models, without resorting to simulation-based indirect inference methods.

By using only the variation in market values driven by fundamental state variables, our first-step projection also provides a very simple correction for classical measurement error in Tobin's Q that minimizes concerns induced, for instance, by potential stock market inefficiencies (e.g., Blanchard et al., 1993). We label this estimate Fitted Q. Unlike other popular statistical corrections for classical measurement error in the literature (e.g., Erickson and Whited, 2000, Erickson and Whited, 2012 and Erickson et al., 2014), our fundamental approach produces direct estimates of measurement error-free Tobin's Q rather than just corrected coefficient estimates within standard investment-Q regressions.

While the first-stage Fitted Q estimate corrects for classical measurement error in Tobin's Q, our marginal q estimate further corrects for possible model mis-specification. Thus, by estimating both measures consistently within the same framework we can uniquely identify and quantify the empirical relevance of both classical measurement error and model mis-specification.[3] The empir-

[3]Any correction for classical measurement error can only filter out orthogonal noise from measured (average) Tobin's Q. Since marginal q is driven by the same (or smaller) set of state variables driving (average) Tobin's Q their difference cannot be orthogonal.

ical results indeed show that while accounting for mis-measurement, i.e. using Fitted Q, helps, it is accounting for mis-specification, i.e. using marginal q, that really makes a difference.

Beyond the new methodological contributions, our paper also makes a number of important empirical ones. First, we document that marginal q is statistically different from (average) Tobin's Q or its measurement error-free counterpart, Fitted Q. On average, marginal q is substantially lower (0.81 vs 1.33 vs 2.87), and less volatile (0.49 vs 1.03 vs 5.77) than both Fitted Q and (average) Tobin's Q, respectively.

Second, our empirical estimates of marginal q rather than Tobin's Q, provide much tighter (and more plausible) model-free estimates of upper bounds on capital adjustment costs. Using Tobin's Q as a proxy for marginal q leads to estimates of (maximum) adjustment costs that are 3.5 times larger than the ones we obtain when estimating marginal q directly.

Third, we find that investment is substantially more responsive to changes in marginal q than Tobin's Q or even Fitted Q. In particular, we show that the use of Tobin's Q systematically underestimates the sensitivity of investment to fundamentals between 70% and 80% depending on the exact specification for investment adjustment costs.

Fourth, we revisit the classic investment-q regression using our q measures together with the measurement-error correction in Tobin's Q proposed by Erickson et al. (2014). We find that both Fitted Q and marginal q produce larger and more economically plausible parameter estimates, even in simple OLS specifications with within-group R^2 up to 0.49, that is about 15 times larger than those obtained when just using Tobin's Q.

Finally, we show that using either Fitted Q or marginal q, also drives out the statistical significance of cash flow in investment regressions, further questioning the empirical relevance of cash flow effects and their implications for the existence of financial constraints affecting firms' investment behaviors.

Taken together, these results suggest that our estimates of marginal q offer much stronger empirical support for the seminal - but often questioned - neoclassical theory of investment pioneered by Tobin (1969).

A. Comparison with Literature

Abel and Blanchard (1986) and Gilchrist and Himmelberg (1995) offer the earliest alternative method to estimate marginal q. They use explicit assumptions on functional forms for the marginal revenue product of capital, the marginal adjustment cost, and the stochastic discount factor to construct VAR-based forecasts of the future expected marginal profit of capital.

Abel (1980), Shapiro (1986), and Whited (1992) among others exploit the first-order condition for investment, to replace unobservable marginal q with a parameterized marginal investment cost in the Euler equation, and then estimate it using structural GMM. This approach also requires specific functional form assumptions for the marginal profit of capital and the stochastic discount factor.

More recently, Philippon (2009) proposes an implementation of the Q-theory using only bond rather than equity prices which he finds to work better. Peters and Taylor (2017) construct a new Tobin's q proxy by accounting for intangible capital while Crouzet and Eberly (2021) propose a quantification of the contribution of intangible capital and market power to the gap between average Tobin's Q and marginal q.

Gala et al. (2020) propose to estimate the optimal investment policies as a function of the model's underlying state variables, which is shown to work better empirically than Tobin's Q. By contrast, estimating marginal q directly from the value function provides a superior characterization of a firm's investment behavior and allows to easily recover structural parameters of adjustment cost technologies, even in highly nonlinear models, without resorting to computationally intensive indirect inference methods.

I. A General Model of Investment

To describe and motivate our approach we use a continuous time version of the model in Gala et al. (2020). The framework allows for very general assumptions about the production technology and capital adjustment costs and is flexible enough to subsume the vast majority of investment models in the literature as special cases.

A. Production and Investment Technologies

Consider a firm that uses capital and a vector of costlessly adjustable inputs, such as labor, to produce a nonstorable output. At each point of time, the firm chooses the amounts of costlessly adjustable inputs to maximize the value of its revenue minus expenditures on these inputs.

Let $\Pi(K_{it}, A_{it})$ denote the maximized value of the instantaneous operating profit at time t, where K_{it} is firm i's capital stock at time t and A_{it} is a random variable representing uncertainty in technology, in the prices of costlessly adjustable inputs, and/or in the demand facing the firm. We assume only that $\Pi_K() > 0$ and $\Pi_A() > 0$ and $\Pi_{KK}() \leq 0$.

The random variable A_{it} evolves according to a diffusion process:

$$dA_{it} = \mu_A(A_{it}, \Psi_t)\, dt + \sigma_A(A_{it}, \Psi_t)\, dW_{it}^A \tag{1}$$

where dW_{it}^{A} is standard Wiener process. The vector of aggregate random variables, Ψ_t, summarizes the state of the economy and evolves as

$$d\Psi_t = \mu_\Psi\left(\Psi_t\right)dt + \sigma_\Psi\left(\Psi_t\right)dW_t^\Psi \tag{2}$$

with dW_t^Ψ being a vector of standard Wiener processes independent of dW_{it}^{A}. The general formulation in (1) allows for common systematic variations in shocks to technology, prices of costlessly adjustable inputs, and demand facing the firm.

Capital is acquired by undertaking gross investment at rate I, and the capital stock depreciates at a fixed proportional rate $\delta \geq 0$, so that the capital stock evolves according to

$$dK_{it} = \left(I_{it} - \delta K_{it}\right)dt. \tag{3}$$

When the firm undertakes gross investment, it incurs costs, which reduce operating profits.

Capital adjustment costs are summarized by the function $\Phi\left(I, K\right)$, which we assume is twice continuously differentiable for $I \neq 0$, with $\Phi_I\left(\cdot\right) \times I \geq 0$ and $\Phi_{II}\left(\cdot\right) \geq 0$. In addition we set $\Phi\left(0, K\right) = 0$ so that adjustment costs are non negative and minimized at $I = 0$. These assumptions are general enough to cover most general non-convex and discontinuous specifications for investment adjustment cost in the literature.[4]

[4]To facilitate exposition our choice of $\Phi(\cdot)$ is still general enough to accommodate adjustment costs in investment growth. However, as we will see below, our method can easily be extended to allow for that, as well as frictions in the adjustment of other variables.

B. Optimal Investment Decisions

Each firm chooses the optimal investment by maximizing the expected present value of operating profit, $\Pi(K, A)$ less total investment cost $\Phi(I, K)$. The value of the firm is thus

$$V(K_{it}, A_{it}, \Psi_t) = \max_{\{I_{t+s}\}} E_t \int_0^\infty \frac{\Lambda_{t+s}}{\Lambda_t} [\Pi(K_{it+s}, A_{it+s}) - \Phi(I_{it+s}, K_{it+s})] \, ds \quad (4)$$

subject to the capital accumulation equation in (3), the firm shock process in (1), the dynamics for the vector of aggregate random variables in (2), and the pricing-kernel dynamics

$$\frac{d\Lambda_t}{\Lambda_t} = -r(\Psi_t) \, dt - \sigma_\Lambda(\Psi_t) \, dW_t^\Psi \quad (5)$$

where r_t denotes the instantaneous riskless rate, and $\sigma_\Lambda(\Psi_t)$ denotes the market prices of risks associated with the vector of aggregate systematic shocks, Ψ_t.[5]

The firm value function $V(K, A, \Psi)$ satisfies the following Hamilton-Jacobi-Bellman (HJB):[6]

$$0 = \max_I \{\Lambda[\Pi(K, A) - \Phi(I, K)] + \mathcal{D}[\Lambda V]\} \quad (6)$$

with $\mathcal{D}[\cdot]$ denoting the infinitesimal generator of the Markov processes A and Ψ, and the process K

$$\mathcal{D}[M(\cdot)] = \mu_A(\cdot) M_A + \frac{\sigma_A^2(\cdot)}{2} M_{AA} + \mu_\Psi(\cdot) M_\Psi + \frac{\sigma_\Psi^2(\cdot)}{2} M_{\Psi\Psi} + (I - \delta K) M_K$$

[5] The vector Ψ_t summarizes the aggregate state of the economy, which potentially includes moments of the cross-sectional firm distribution, aggregate shocks to productivity, wages, relative price of investment goods, and household preferences.

[6] For simplicity of exposition, we have suppressed the firm and time subscripts i and t.

applied to the discounted firm value ΛV, along with the transversality ("no bubble") condition:

$$\lim_{T\to\infty} \mathbf{E}_t \left[\left|\Lambda_{t+T} V_{it+T}\right|\right] = 0.$$

Substituting for $\mathcal{D}[\Lambda V]$ in (6), the optimal investment policy then satisfies

$$I^*(q, K) = \arg\max_I \left[qI - \Phi(I, K)\right] \tag{7}$$

where the marginal value of capital $q \equiv V_K$, by the Fayman-Kac Theorem, is equal to

$$q(K_{it}, A_{it}, \Psi_t) = E_t \int_0^\infty e^{-\delta s} \frac{\Lambda_{t+s}}{\Lambda_t} \left[\Pi_K(K_{it+s}, A_{it+s}) - \Phi_K\left(I^*_{it+s}, K_{it+s}\right)\right] ds. \tag{8}$$

As shown in Abel and Eberly (1994), the marginal q is the present value of the stream of expected marginal profit of capital which consists of two components: Π_K is the marginal operating profit accruing to capital, and $-\Phi_K$ is the reduction in the adjustment cost accruing to the marginal unit of capital.

II. Measuring Marginal q

Marginal q in (8) does not yield an explicit closed-form solution under the general conditions. Hence, we cannot directly test the optimal investment policies in (7), unless we can measure the unobservable marginal q.

A. Projection Method

We propose a new methodology to measure marginal q that rests on the *joint measurability* of firm value function, $V(\cdot)$, and its underlying state variables, $\Omega = \{K_{it}, A_{it}, \Psi_t\}$. Specifically, we can measure marginal q according to its definition as partial derivative of the *observable* value function - i.e. market value of the firm - with respect to its *observable* capital stock, $q \equiv V_K(\Omega)$.[7]

First, we approximate the (scaled) market value of firm i at time t, $Q_{it} \equiv V_{it}/K_{it}$, using a tensor product polynomial in the state variables as

$$v_{it} \equiv \ln Q_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} \sum_{j_\Psi=0}^{n_\Psi} c_{j_k,j_a,j_\Psi} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} \times [\Psi_t]^{j_\Psi} + \epsilon_{it} \tag{9}$$

where $k_{it} \equiv \ln K_{it}$, $a_{it} \equiv \ln A_{it}$, and ϵ_{it} captures measurement error in market values.[8] Given state variables k_{it}, a_{it} and Ψ_t, the coefficients c_{j_k,j_a,j_Ψ} are the subject of the estimation procedure. Then, we estimate the marginal q according to its definition of partial derivative of the value function as

$$\widehat{q}_{it} = \widehat{Q}_{it}\left(1 + \frac{\partial \ln \widehat{Q}_{it}}{\partial \ln K_{it}}\right) = \widehat{Q}_{it}\left(1 + \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} \sum_{j_\Psi=0}^{n_\Psi} \widehat{c}_{j_k,j_a,j_\Psi} \times j_k \times [k_{it}]^{j_k-1} \times [a_{it}]^{j_a} \times [\Psi_t]^{j_\Psi}\right). \tag{10}$$

Notice that, rather than imposing restrictive conditions on the functional forms for the stochastic discount factor and adjustment cost functions, the projection method requires only general regularity conditions for the existence and differentiability of the value function as well as for the measurement of the

[7]The identification of marginal q rests on the ability to identify the *exogenous* state variables, A and Ψ. Therefore, the selection of the relevant state variables for the representation of the value function should always include the *exogenous* state variables implied by the model (or any one-to-one transformation).

[8]Under the null of the model, the value function, V, depends only on the set of state variables Ω. Therefore, we estimate the value function under the standard assumption that firm intrinsic values are observed only with error by the econometrician. The measurement error ϵ_{it} (which can be serially correlated) does not affect firm optimal policies, and as such is orthogonal to the firm intrinsic value.

firm-level state variables.

Given a large panel of firms, our approach is flexible enough so that one can also account for unobserved time-invariant heterogeneity across firms by allowing the constant term $c_{0,0,0}$ in (9) to be firm-specific.

A.1 Measuring the State Variables

To estimate marginal q we need to measure the relevant state variables in Ω. First, we focus on the firm-level (micro) state variables K_{it} and A_{it}. The firm capital stock, K_{it}, is directly observable, but the firm productivity shocks, A_{it}, are not. However, they can be estimated using production function estimation tools from the industrial organization literature (e.g., Olley and Pakes, 1996, Levinsohn and Petrin, 2003, and Ackerberg et al., 2015). The details are provided in Online Appendix B.

Complete knowledge of the aggregate (macro) state variables in Ψ is not required for the purpose of estimating firm level marginal q. We can capture the impact of all unobserved aggregate state variables by allowing for time-specific polynomial coefficients in (9). Specifically, one can fit a separate cross-section of (scaled) firm market values for each year as

$$v_{it} \equiv \ln Q_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} b_{j_k,j_a,t} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \epsilon_{it} \tag{11}$$

where we have suppressed the direct dependence on the aggregate state variables, Ψ, and we have allowed the polynomial coefficients $b_{j_k,j_a,t}$ to vary over time. For ease of exposition and comparison with the existing literature, we focus only on unobserved aggregate variation that affects the value function linearly.[9]

[9]Gala (2012) allows aggregate state variables to enter non-additively the value function

A.2 Polynomial Approximation

A key challenge in implementing this projection method empirically is to determine the order of polynomials in state variables to be included in the estimation. These can be assessed using standard model selection techniques such as Akaike information criterion (AIC). Using step-wise regression analysis, we find that the first and second orders are informative enough to capture main variation in firms' market values and higher order terms are generally not necessary to improve the quality of the approximation.[10]

In Section IV however, we implement a more robust version of our projection method that adds a "step zero" where we employ the least absolute shrinkage and selection operator, *Lasso*, widely used in statistics and machine learning to help optimally select orders of polynomials. We show that this yields very similar estimates to the "reduced" two-step method used in Section III.

Ignoring higher order polynomial terms in k and a, can mechanically introduce endogeneity issues in estimating coefficients of $b_{j_k,j_a,t}$ in (11). In Online Appendix C, we address this concern with a control function approach that uses the investment rate as an instrument (e.g., Olley and Pakes, 1996 and Ackerberg et al., 2015) and show that this delivers similar results to the (easier to implement) OLS regressions reported in the main text.

B. Discussion

The practical appeal of Q-theory lies in the fact that it is possible to summarize all relevant information about firm state variables with a single (relative)

and investigates empirically alternative projection representations of asset prices.

[10] To facilitate interpretation, we report only natural polynomials in the paper, but the Online Appendix shows our estimation results are robust to using orthogonal terms instead.

market price.[11] Unfortunately, it is well known that the required knife-edge homogeneity assumptions for this result are at odds with micro data, and the identification and measurement of marginal q with (average) Tobin's Q offers a poor fit to the data at the firm level (e.g. Gala et al., 2020).

Here, we propose to use the same exact asset price information to instead directly estimate marginal q but under minimal assumptions on technology and preferences that are much less restrictive than those required by previous methods.[12]

In addition, since we estimate marginal q independently from adjustment cost technologies we can more easily recover the structural parameters of adjustment cost technologies, even in highly nonlinear and heterogeneous models, and without resorting to computationally intensive simulation-based estimation methods.

Finally and importantly, since we only use the variation in market values driven by fundamental state variables in constructing marginal q, we can also minimize any concerns about classical measurement error induced, for instance, by potential stock market inefficiencies (Blanchard et al., 1993).

III. Empirical Implementation

We now describe the data used in the empirical analysis and additional issues concerning the projection representation of marginal q. We then use the projection measure of marginal q to estimate capital adjustment costs and investigate the shape of the investment policy function.

[11]Formally Abel and Eberly (1994) show that marginal q is proportional to (average) Tobin's Q: $q = \rho \frac{V}{K}$ if $\Pi(\cdot)$ and $\Phi(\cdot)$ are homogeneous of degree ρ in both I and K.

[12]For example the VAR-based methods used in Abel and Blanchard (1986), and Gilchrist and Himmelberg (1995) or the Euler-equation approaches in Abel (1980), Shapiro (1986), and Whited (1992).

A. **Data**

Our data come from the combined annual COMPUSTAT files. To facilitate comparison with much of the literature, our sample is made of an unbalanced panel of firms for the years 1973 to 2019, that includes only manufacturing firms (SIC 2000-3999).

We keep only firm-years that have non-missing information required to construct the primary variables of interest, namely: investment, I, firm size, K, (average) Tobin's Q, and sales revenues, Y. These variables are constructed as follows. Firm size, or the capital stock, is defined as the gross book value of property, plant and equipment (item *ppegt*). Investment is defined as capital expenditures in property, plant and equipment (item *capx*). Sales are measured by net sales revenues (item *sale*). These last two variables are scaled by the beginning-of-year capital stock. Finally, Tobin's Q is measured by the end-of-year market value of capital, defined as market value of outstanding equity ($prcc\_f \times csho$) plus the book value of debt ($dltt + dlc$) net of current assets (act), scaled by gross property, plant and equipment.[13]

The sample is filtered to exclude observations where (average) Tobin's Q and sales are either zero or negative. Furthermore, we require the gross capital stock to be greater than 5 million in dollar adjusted to 1982, as is standard in literature (e.g., Erickson and Whited, 2012 and Peters and Taylor, 2017). To ensure that the measure of investment captures the purchase of property, plant and equipment, we eliminate any firm-year observation in which a firm made an acquisition. All regression variables are trimmed at the top and bottom 0.5% of their distributions to reduce the influence of any outliers, which are

[13]Erickson and Whited (2006) show that using a perpetual inventory algorithm to estimate the replacement cost of capital and/or a recursive algorithm to estimate the market value of debt barely improves the measurement quality of the various proxies for Q.

common in accounting ratios. Finally, we require a firm to have at least two-year observations in the sample. This procedure yields a base sample of 47,141 firm-year observations. More details about the sample construction is in Online Appendix A. Table 1 reports summary statistics including mean, standard deviation and main percentiles for the variables of interest.

Table 1: **Summary Statistics**

This table reports summary statistics for the primary variables of interest from Compustat over the period 1973-2019. Investment rate, I/K, is defined as capital expenditures in property, plant and equipment scaled by the beginning-of-year capital stock. The capital stock, K, is defined as gross property, plant and equipment. Firm size, $\ln(K)$, is the natural logarithm of the beginning-of-year capital stock. The sales-to-capital ratio, $\ln(Y/K)$, is computed as the natural logarithm of end-of-year sales scaled by the beginning-of-year capital stock. Tobin's Q is defined as the market value of capital (market value of equity plus debt net of current assets) scaled by gross property, plant and equipment.

	Obs.	Mean	Std. Dev.	25th	50th	75th
I/K	$47,141$	0.155	0.185	0.057	0.102	0.178
$\ln(K)$	$47,141$	4.638	2.000	3.051	4.290	5.954
$\ln(Y/K)$	$47,141$	0.777	0.818	0.317	0.835	1.297
Q	$47,141$	2.872	5.767	0.420	1.004	2.694

With the constructed sample, we now describe our main findings. We first examine the variation of market values and investment rates across portfolios sorted by firm size, K, and profitability shock, A. We then proceed to estimate marginal q, and use it for the estimation of adjustment costs and investment-q sensitivity. Lastly, we revisit the investment-q regressions by using our measure of marginal q.

B. Market Values and Investment by Size and Profitability

To gain some insights about the role of size and profitability shock in spanning the true underlying state space for market values and investment rates, we sort all firms into 25 portfolios double-sorted on the empirical distribution of profitability shock conditional on firm size. Specifically, each firm is allocated annually first across five firm size quintiles, and then, within each size quintile, to five productivity quintiles. Table 2 reports the equally-weighted average market values and investment rates across the resulting 25 conditionally double-sorted portfolios.

Across most firm size quintiles the pattern in average market values and investment rates shows a monotonic increasing relation with productivity. These relations are statistically and economically significant across these portfolios. This table confirms that there is substantial variation in market values and investment rates as functions of the underlying state variables. We now turn to estimate marginal q using the projection method.

C. First Step: Empirical Value Function

We now turn to formally estimate firms' value function as a function of the firm-level state variables in (9).

Table 3 reports the empirical estimates for various specifications of the value function polynomial regression:

$$v_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} c_{j_k,j_a} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \delta_i + \eta_t + \epsilon_{it} \tag{12}$$

As discussed above, all estimates use year- and firm-fixed effects to account

Table 2: **Market Value and Investment by State-Variable Portfolios**

This table reports equal-weighted averages of market values and investment rates for portfolios based on conditional sorts on firm size, K, and profitability shock, A. The sample period is 1973 to 2019.

Panel A: Market Value, V($ billion)

		Profitability Shock (A)				
		Q1	2	3	4	Q5
Firm Size (K)	Q1	0.042	0.058	0.081	0.128	0.168
	2	0.057	0.086	0.125	0.194	0.390
	3	0.092	0.166	0.239	0.368	0.626
	4	0.318	0.649	0.955	1.224	1.108
	Q5	3.570	4.899	6.655	7.193	10.918

Panel B: Investment Rate, I/K

		Profitability Shock (A)				
		Q1	2	3	4	Q5
Firm Size (K)	Q1	0.045	0.118	0.202	0.351	0.723
	2	0.038	0.091	0.152	0.235	0.428
	3	0.035	0.073	0.106	0.154	0.303
	4	0.030	0.061	0.086	0.124	0.227
	Q5	0.031	0.062	0.090	0.136	0.190

for potential aggregate shocks and unobserved firm heterogeneity, including variations in firms' input prices and depreciation rates.

We find that first and second order terms are all strongly statistically significant. The complete second order polynomial in k and a, which also includes the interaction term, explains up to 65% (including fixed effects) of the total variation in log (scaled) market values.[14] Based on the Akaike information criteria, we then choose the complete second order polynomial in firm size

[14] We omit higher order terms because they are mostly insignificant and do not improve the overall quality of the approximation.

Table 3: **Empirical Value Function**

This table reports estimates from the empirical value function:

$$v_{it} = \sum_{j_k=0}^{n_k} \sum_{j_a=0}^{n_a} c_{j_k,j_a} \times [k_{it}]^{j_k} \times [a_{it}]^{j_a} + \delta_i + \eta_t + \epsilon_{it}$$

where the left-hand-side is the log market value scaled by capital, $\ln(V/K)$, k is firm size, $\ln K$, a is profitability shock, $\ln A$, δ_i is a firm fixed effect, and η_t is a year fixed effect. Robust standard errors are clustered by firm and reported in parenthesis. $\overline{R}^2$ denotes adjusted R-square and AIC is the Akaike Information Criterion. qQ-test is a Wald test of the equivalence between marginal q and average Q as described in (13). P-values are reported. The sample period is 1973 to 2019.

	(1)	(2)	(3)
$\ln K$	-0.339	-0.618	-0.511
	(0.017)	(0.034)	(0.047)
$\ln A$	0.971	0.310	0.524
	(0.030)	(0.107)	(0.128)
$(\ln K)^2$		0.031	0.027
		(0.003)	(0.004)
$(\ln A)^2$		0.292	0.289
		(0.045)	(0.045)
$\ln A \times \ln K$			-0.048
			(0.014)
$\overline{R}^2$	0.646	0.649	0.649
AIC	$113,894$	$113,434$	$113,407$
qQ-test	0.000	0.000	0.000

and productivity shock (column 3) as the best parsimonious state variable representation of market values empirically.[15]

As discussed above, Section IV uses a *Lasso* approach to help optimally select orders of polynomials to obtain very similar results. Finally, in the

[15]Even if the quadratic and interaction terms do not increase substantially the overall fit of the value function, they are statistically significant and might be still important to explain variation in investment through marginal q.

Online Appendix we show that these results are robust to the industry-year portfolio level regressions as well as the inclusion of interactive fixed effects between firm and year as suggested by Bai (2009).

D. Second Step: Marginal q

Given the estimates of the (scaled) value function in (12), we can then compute the Fitted Q and marginal q. The Fitted Q is computed from the fitted values of the specification in (12), and it provides a measurement error-free measure of (average) Tobin's Q, maximally correlated with the fundamental state variables. The firm's marginal q is computed according to its definition as a partial derivative of the value function with respect to its capital stock as in (10).

Table 4 reports summary statistics for the empirical distributions of estimated marginal q, Fitted Q, and observed Tobin's Q. Tobin's Q is on average higher and more volatile than both Fitted Q and marginal q. Fitted Q is on average higher and more volatile than marginal q.

Table 4: **Distribution of Marginal** q

This table reports summary statistics for (average) Tobin's Q, and the estimated marginal q and Fitted Q. Fitted Q is computed as the fitted value of the value function approximation in (12). Marginal q is computed according to its definition as partial derivative of the value function approximation with respect to the capital stock. The sample period is 1973 to 2019.

	Obs.	Mean	Std. Dev.	25th	50th	75th
Tobin's Q	$47,141$	2.872	5.767	0.420	1.004	2.694
Fitted Q	$47,141$	1.325	1.032	0.624	1.038	1.689
Marginal q	$47,141$	0.811	0.492	0.462	0.685	1.016

Figure 1 plots the empirical distributions of logarithm of the ratios of Tobin's Q to marginal q (top panel), and Fitted Q to marginal q (bottom panel). Marginal q can take on values substantially higher than observed Tobin's Q. However, marginal q is always lower than Fitted Q, which is consistent with concavity of the value function (Hayashi, 1982).

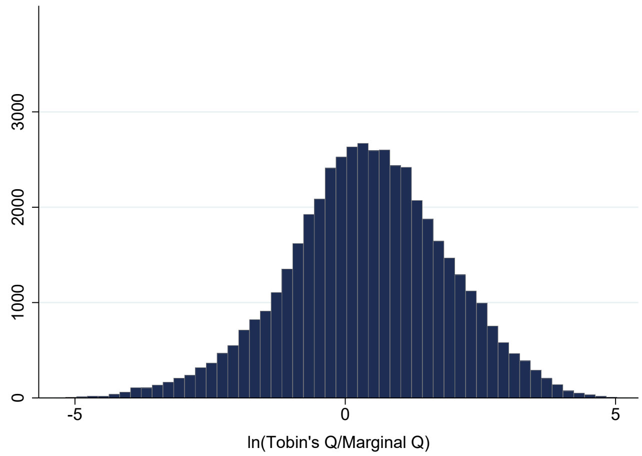
To understand the differences among these alternative Q measures and quantify the contribution of marginal q in explaining variation in both Tobin's Q and Fitted Q, we perform a series of regressions of (average) Tobin's Q and Fitted Q on marginal q. Figure 2 reports the change in R-squares when including various covariates. In Panel A, it shows that firm fixed effects explain about 60% of total variation in Tobin's Q while marginal q does not explain much of the total variation in Tobin's Q. On the other hand, in the regression of Fitted Q, the R-square increases from 70% to 98% after adding marginal q as a covariate.

D.1 Equivalence Between Marginal and Average Q

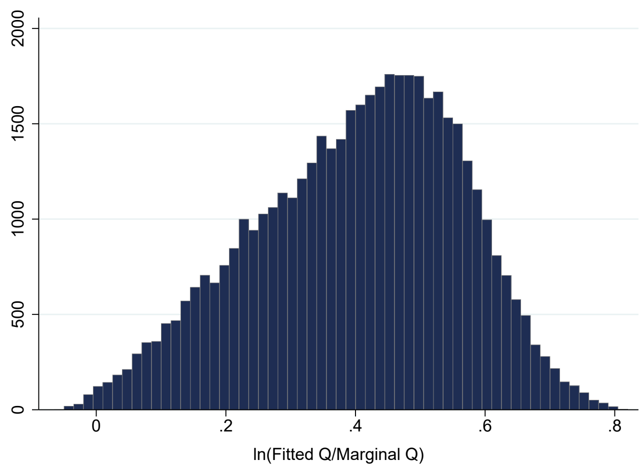
For each state variable representation of the value function in terms of polynomials in k and a, we can test directly the equivalence between marginal q and average Q. Testing such an equivalence requires that $\partial\widehat{v}_{it}/\partial k_{it} = 0$, or equivalently that all coefficients corresponding to terms involving k are jointly equal to zero:

$$c_{j_k,j_a} = 0 \quad \text{for } j_k = 1, ..., n_k; \text{ and } \forall j_a. \tag{13}$$

The null hypothesis in (13) corresponds to a test of linear restrictions on the coefficient estimates. Such an hypothesis can be tested using a Wald statistic ("qQ-test"), which is distributed as χ^2_r with degrees of freedom r equal to the number of restrictions.



(a) Distribution of *Tobin* Q/q

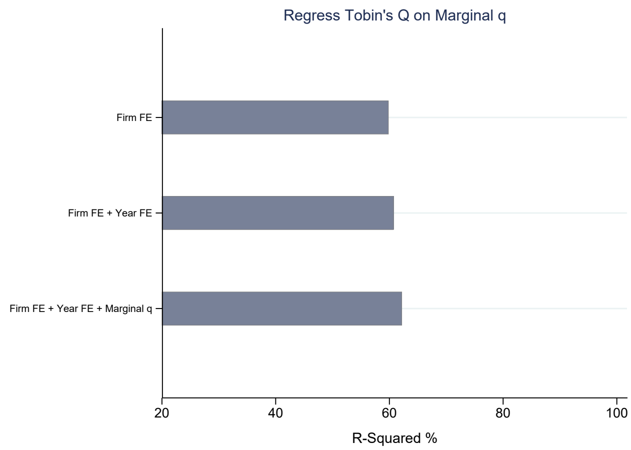


(b) Distribution of *Fitted* Q/q

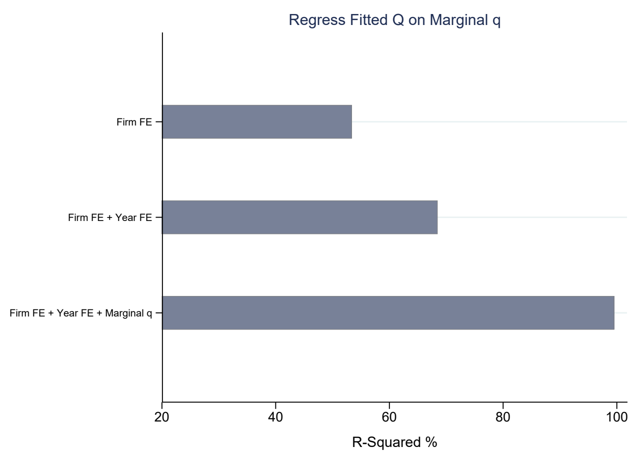
This figure plots the empirical distributions of the natural logarithm of ratios of (average) Tobin's Q to marginal q (top panel) and Fitted Q to marginal q (bottom panel). The sample period is 1973 to 2019.

Figure 1: **Empirical Distribution of Q Wedges**

The last row of Table 3 reports the p-values corresponding to the "qQ-test" for each polynomial specification. In all cases, we can *strongly* reject the null hypothesis that marginal q is equal to (average) Tobin's Q.



(a) Tobin's Q on marginal q



(b) Fitted Q on marginal q

This figure plots the change in R-squares by regressing (average) Tobin's Q (Panel A) and Fitted Q (Panel B) on Marginal q. The y-axis denotes regressors included in the regression and the x-axis is the R-square (percentage) for each regression. The sample period is 1973 to 2019.

Figure 2: **R-Squares of Regressing Q on Marginal q**

E. Estimating Adjustment Costs

Under the optimal investment policy, the maximand in (7) - i.e. $qI - \Phi(I, K)$ - is nonnegative. As such, the estimate of marginal q provides an upper bound on the total capital adjustment costs paid by the firm as a share of capital expenditure, $\Phi(\cdot)/I$.

Therefore, one can use the empirical distribution of marginal q to draw inference about the upper bound on the total capital adjustment costs as a share of investment. As shown in Table 4, marginal q is on average much lower and less volatile than both Fitted Q and Tobin's Q. As such, the total adjustment costs (including the purchase price) never exceed on average 81 percent of the cost of investment for the average firm in the sample. In contrast, under Hayashi (1982)'s assumptions of homogeneity and perfect competition, one would have estimated on average an upper bound of 287 percent of the cost of investment based on the implied equivalence between observed (average) Tobin's Q and marginal q.

Therefore, these estimates of marginal q provide much tighter (and plausible) bounds on the total adjustment costs as a share of investment, *regardless* of the specific assumptions concerning the investment technology.

E.1 **Marginal q under Smooth Adjustment Costs**

With smooth adjustment costs, the optimality condition for investment requires the marginal cost of investment equal to marginal q. Therefore, the distribution of marginal q corresponds exactly to the distribution of marginal adjustment costs.

Under smooth adjustment costs, the average of the firm marginal adjustment cost of investment (including the purchase price) is only about 0.81 for each additional dollar of investment. This estimate is about 39% smaller than the estimate under the Fitted Q. Thus, if we were to use the Fitted Q as a measurement error-free estimate of marginal q under the assumption of homogeneity, we would have over-estimated the firm marginal adjustment costs. This is even more pronounced, if we were to use Tobin's Q. In this case, we would have estimated the average marginal cost of investment at about 2.87, which is about 2.5 times higher than under marginal q.

F. Investment-q Sensitivity

In this section, we estimate the implied shape of the investment policy function by providing structural estimates of investment-q sensitivities and capital adjustment costs under alternative measures of Tobin's Q, including the new measure of marginal q.

In line with the existing literature, we focus on a generalized adjustment cost function, $\Phi(\cdot)$, that is homogeneous of degree one in investment and capital. Specifically, we use the following polynomial specification for the adjustment cost function:

$$\frac{\Phi\left(I_{it}, K_{it}; \delta_i, \eta_t\right)}{K_{it}} = \left(\delta_i + \eta_t\right)\left(\frac{I_{it}}{K_{it}}\right) + \sum_{m=2}^{M} \frac{\gamma_m}{m}\left(\frac{I_{it}}{K_{it}}\right)^m \tag{14}$$

where the variables δ_i and η_t allow for firm- and year-specific elements to the investment price. For example, the price of capital may systematically vary across firms due to tax considerations such as the value of investment tax credits and depreciation allowances. While this function is not restricted to be globally convex, we verify that the estimates imply convexity.

This functional form yields the following expression for the optimal investment policy in (7) and can be estimated as

$$q_{it} = \sum_{m=2}^{M} \gamma_m \left(\frac{I_{it}}{K_{it}}\right)^{m-1} + \delta_i + \eta_t + \varepsilon_{it} \tag{15}$$

where the error term ε captures measurement or estimation error in the alternative measures of Q. Empirically, we use various measures of Q discussed in the paper as the dependent variable in (15).

F.1 Quadratic Adjustment Costs

Table 5 reports the estimates of the adjustment cost parameters γ_m in (15) obtained with standard Tobin's Q, Fitted Q, and the projection measure of marginal q. We concentrate first on the linear-quadratic adjustment costs specification. Columns (7) of Table 5 shows that the positive and significant coefficient on investment when using marginal q in (15). It implies an adjustment cost parameter of only about 1.3. This is much smaller than the value implied by the use of (average) Tobin's Q in column (1), 4.53, and Fitted Q in column (4), 2.67.

Following Abel and Eberly (2002), one could obtain an estimate of the quadratic adjustment costs as a share of investment expenditure by multiplying $\gamma/2$ with 0.16, the sample mean for the investment rate.[16] The magnitude of quadratic adjustment costs as a share of investment expenditure implied by the coefficient estimate under marginal q is about 10 percent of investment expenditure.[17] As a comparison, these numbers imply that the estimated quadratic adjustment costs under the Fitted Q and the observed Tobin's Q specifications are about two times and 3.5 times as large as them under the

[16] The total amount of quadratic adjustment costs is given by $\frac{\gamma}{2}\left(\frac{I}{K}\right)^2 K$. Therefore, the ratio of total adjustment costs to investment I is $\frac{\gamma}{2}\frac{I}{K}$.

[17] Given the quadratic adjustment cost specification, we can also compare these estimates with previous studies. For instance, Gilchrist and Himmelberg (1995) find estimates of $\gamma = 20$ when using (average) Tobin's Q and $\gamma = 5.46$ when using the VAR-based measure of marginal q (i.e. Fundamental Q). These estimates, which correspond ($\simeq \frac{\gamma}{2}\times\overline{I/K} = \frac{\gamma}{2}\times 0.16$) to 160 percent and 44 percent, respectively, are still much higher than the ones reported here.

Table 5: **Investment-q Sensitivity**

This table reports estimates from the following regression:

$$Y_{it} = \sum_{m=2}^{M} \gamma_m \left(\frac{I_{it}}{K_{it}}\right)^{m-1} + \delta_i + \eta_t + \varepsilon_{it}$$

where the left-hand side variable Y_{it} is either marginal q, Fitted Q, or (average) Tobin's Q, and the right-hand side variables include the investment rate (I/K), firm fixed effects δ_i, and year fixed effects η_t. Specifications (1)-(3) report estimates using (average) Tobin's Q as the dependent variable. Specifications (4)-(6) report estimates using Fitted Q as the dependent variable. Specifications (7)-(9) report estimates using marginal q as the dependent variable. Robust standard errors are clustered by firm and reported in parenthesis. $\overline{R}^2$ denotes adjusted R-square. The sample period is 1973 to 2019.

	Tobin's Q			Fitted Q			Marginal q		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
γ_2	4.527	7.539	9.315	2.671	4.173	4.997	1.299	2.221	2.858
	(0.253)	(0.472)	(0.802)	(0.052)	(0.082)	(0.127)	(0.024)	(0.036)	(0.055)
γ_3		−2.707	−6.638		−1.351	−3.174		−0.829	−2.238
		(0.423)	(1.729)		(0.082)	(0.300)		(0.036)	(0.128)
γ_4			1.856			0.861			0.665
			(0.839)			(0.156)			(0.066)
$\overline{R}^2$	0.571	0.572	0.573	0.802	0.816	0.817	0.827	0.849	0.853

marginal q specification.

These estimates also help us infer the sensitivity of investment to fundamentals.[18] We show that the inferences about investment sensitivity to fundamentals are substantially different when using alternative measures of Q. For example, the sensitivity of investment as measured by marginal q is more than three times that estimated in conventional Tobin's Q regressions. Accounting

[18]The investment sensitivity to fundamentals given the quadratic adjustment costs is defined as

$$\partial\left(\frac{I_{it}}{K_{it}}\right)/\partial q_{it} = 1/\gamma_2.$$

for measurement error in Tobin's Q - i.e. using Fitted Q - increases the investment sensitivity relative to observed Tobin's Q. But it is still less than half of the sensitivity estimated from marginal q.

Importantly, investment is substantially more correlated with marginal q than with the measures of Tobin's Q including the measurement error-free Fitted Q. While accounting for mismeasurement - i.e. using Fitted Q - helps to some extent, accounting for misspecification - i.e. using marginal q - substantially improves both the correlation with investment and the structural estimates of investment sensitivity to fundamentals.

F.2 **Polynomial Adjustment Costs**

The evidence above rests on the strong assumption of a linear relationship between investment and marginal q. However previous work has shown that the relationship between firms' investment and their Tobin's Q can be highly nonlinear (Barnett and Sakellaris, 1999; Abel and Eberly, 2002). This suggests that a higher order specification of adjustment costs should be considered when examining the empirical relationship between investment and Q measures.

Table 5 also reports the adjustment cost parameters for higher-order specifications. The empirical results suggest to include only up to quartic terms ($M = 4$). At all instances of cubic terms ($M = 3$), for example, we estimated this polynomial adjustment cost function to be convex for the range of investment rates observed in the sample.

Given the estimates in Table 5, one can obtain the structural estimate of the convex adjustment costs as a share of investment expenditure.[19] In Figure

[19]For the cubic case ($M = 3$), the total amount of convex adjustment costs is given by $\frac{\gamma_2}{2}\left(\frac{I}{K}\right)^2 K + \frac{\gamma_3}{3}\left(\frac{I}{K}\right)^3 K$. Therefore, the ratio of total adjustment costs to investment I is $\frac{\gamma_2}{2}\frac{I}{K} + \frac{\gamma_3}{3}\left(\frac{I}{K}\right)^2$. Similarly, for the quartic case ($M = 4$), the ratio of total adjustment costs

3, we plot the ratio of adjustment costs to investment expenditures by varying investment rates (I/K) from 0 to 1. Panel A demonstrates the magnitudes of adjustment costs as a share of investment expenditure implied by coefficient estimates in the cubic case. This ratio increases from 0% to 287% as the investment rate rises from 0 to 1 when inferred from the estimates of (average) Tobin's Q. It implies that, for instance, the estimated convex adjustment costs are about 287% of investment expenditure when $I/K = 1$. The convex adjustment cost ratios implied by the estimates from Fitted Q specification is lower than these from (average) Tobin's Q. It is between 0% and 164% as the investment rate increases from 0 to 1. In contrast, the coefficients from the marginal q specification yield the tightest and most plausible estimates for the convex adjustment costs, varying between between 0% and 83%. Focusing on the mean investment rate $I/K = 0.16$, these numbers imply the estimated convex adjustment costs under Tobin's Q and Fitted Q specifications are about 3.4 times and 1.8 times as large as the estimate under the marginal q specification.

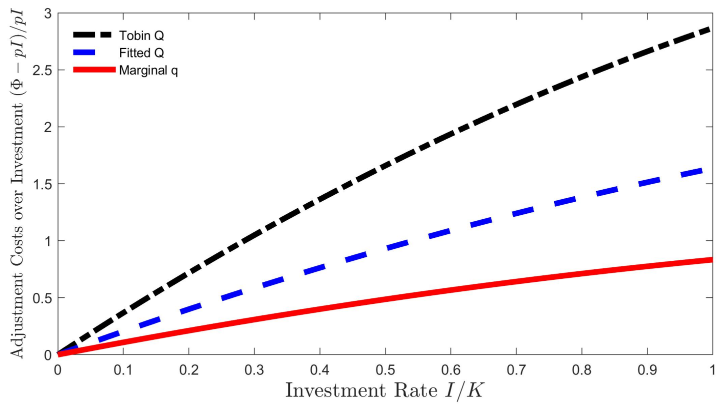
Panel B of Figure 3 demonstrates the magnitudes of adjustment costs implied by coefficients in the quartic case. The ratios of adjustment costs to investment expenditure are quite similar to Panel A when varying the investment rate, indicating that the estimates of adjustment costs are robust to inclusion of higher-order polynomials (e.g., the fourth-order polynomial).

For the cubic polynomials, we can also obtain a closed-form solution for the sensitivity of investment to fundamentals implied by these estimates.[20] For an

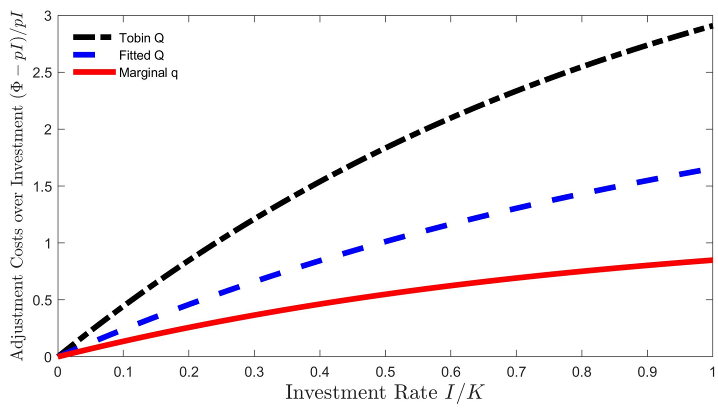
to investment I is $\frac{\gamma_2}{2}\frac{I}{K} + \frac{\gamma_3}{3}\left(\frac{I}{K}\right)^2 + \frac{\gamma_4}{4}\left(\frac{I}{K}\right)^3$.

[20]The investment sensitivity to fundamentals given the cubic adjustment costs is defined as

$$\partial\left(\frac{I_{it}}{K_{it}}\right)/\partial q_{it} = 1/\sqrt{\gamma_2^2 + 4\gamma_3 q_{it} - 4\gamma_3(\delta_i + \eta_t)}.$$



(a) Polynomial Adjustment Costs ($M = 3$)



(b) Polynomial Adjustment Costs ($M = 4$)

This figure plots the ratio of adjustment costs over investment expenditures - i.e. $(\Phi - pI)/pI$ - as a function of investment rates (I/K). Each panel displays the adjustment costs ratio implied by marginal q (red solid line), Fitted Q (blue long-dashed line), and (average) Tobin's Q (black dashed line). Panel A displays adjustment costs ratios under the third-degree polynomial adjustment costs ($M = 3$), while Panel B displays adjustment costs ratios under the fourth-degree polynomial adjustment costs ($M = 4$).

Figure 3: **Ratio of Adjustment Costs to Investment**

average firm, these are estimated to be 0.14 for Tobin's Q, 0.28 for Fitted Q, and 0.67 for marginal q, when computed at the sample mean level of Q equal to 0.81 (the mean value of marginal q in Table 4).

Consistently with the empirical evidence based on quadratic adjustment costs, investment is substantially more correlated with marginal q than with the observed Tobin's Q or the measurement error-free Fitted Q. Based on these empirical evidence, one can conclude that accounting for misspecification - i.e. using marginal q - increases substantially the correlation with investment, the investment sensitivity to fundamentals, and yields plausible estimates of capital adjustment costs.

G. Investment-q Regressions

In this section, we consider the classic corporate investment-q regression augmented with cash flow following Fazzari et al. (1988).

As a benchmark, Panel A of Table 6 reports the regression results of capital investment rates on Q (without cash flow) using the different measures. In column (1), we report the OLS regression result, which largely replicates regression results of previous studies. It generates a positively significant coefficient on Tobin's Q, though the magnitude of coefficient ahead of Tobin's Q is quite small. Following Erickson et al. (2014), columns (2) and (3) adopt the fourth- and fifth-order cumulant estimators to correct for classical measurement error in Tobin's Q regressions. Consistent with Erickson et al. (2014), the coefficient on Tobin's Q is now much larger, suggesting attenuation bias in the OLS estimate. Both estimates generate higher within-group R-square than does OLS.

Columns (4) and (5) report results using Fitted Q and marginal q instead. As argued, our Fitted Q is a measurement-error free estimate of Tobin's Q and the projection measure of marginal q better captures firms' investment opportunities. Notably, both measures yield much larger coefficients than the

Table 6: **Regression of Investment on Q and Cash Flow**

This table reports regression results of investment on cash flow and Q measures, either marginal q, Fitted Q or (average) Tobin's Q from

$$I/K = \alpha \cdot C/K + \beta \cdot Q + \delta_i + \eta_t + \epsilon_{it}$$

following Fazzari et al. (1988), where the left-hand side variable is the investment rate (I/K), and the right-hand side variables include the cash flow rate (C/K) defined as the firm's cash flow level scaled by its capital stock, firm fixed effects δ_i, and year fixed effects η_t. Panel A reports regression results of investment on Q without including cash flow. Panel B reports regression results of investment on Q and cash flow. Columns (1), (4), and (5) report the OLS regression results. Columns (2) and (3) report results using the fourth and fifth order cumulant estimators following Erickson et al. (2014). Fixed effects are controlled by a within transformation for all variables when using the cumulant estimator. ρ^2 is the within-group R^2 from a hypothetical regression of investment on true Q, and τ^2 is the within-group R^2 from a hypothetical regression of our constructed Q measures on true Q. $\overline{R}^2$ denotes adjusted R-square. Sargan Test is the test of the model overidentifying restrictions. The firm-level clustered standard errors are reported in parenthesis.

Panel A: Investment Rate on Q

	OLS	*Fourth*	*Fifth*	*OLS*	*OLS*
	(1)	(2)	(3)	(4)	(5)
Tobin's Q	0.007	0.028	0.037		
	(0.000)	(0.003)	(0.002)		
Fitted Q				0.167	
				(0.003)	
Marginal q					0.376
					(0.005)
Firm FE	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y
ρ^2	0.033	0.112	0.147	0.445	0.488
		(0.014)	(0.014)		
$\overline{R}^2$	0.329			0.615	0.645
τ^2		0.212	0.161		
		(0.027)	(0.016)		
Sargan Test		4.237	17.005		
p-value		0.120	0.004		

Panel B: Investment Rate on Q and Cash Flow

	OLS	*Fourth*	*Fifth*	*OLS*	*OLS*
	(1)	(2)	(3)	(4)	(5)
Tobin's Q	0.007	0.030	0.038		
	(0.000)	(0.003)	(0.002)		
Fitted Q				0.167	
				(0.003)	
Marginal q					0.376
					(0.005)
C/K	0.015***	0.012	0.007	0.003	0.000
	(0.005)	(0.006)	(0.007)	(0.003)	(0.003)
Firm FE	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y
ρ^2	0.035	0.121	0.152	0.445	0.488
		(0.015)	(0.015)		
$\overline{R}^2$	0.330			0.615	0.645
τ^2		0.197	0.156		
		(0.025)	(0.016)		
Sargan Test		1.915	16.279		
p-value		0.384	0.006		

cumulant estimates and the marginal q regression generates the highest within-group R-square (about 49%) among all specifications. This confirms that marginal q is superior to both (average) Tobin's Q and Fitted Q in explaining investment. Fitted Q still provides sizable correction for measurement errors in Tobin's Q and also greatly improves performance relative to Tobin's Q. Overall, marginal q almost doubles the adjusted R-square compared to Tobin's Q.

Panel B of Table 6 extends the regressions in Panel A by including cash flow. Coefficient estimates of Q measures are largely consistent with these in Panel A. For the cash flow variable, column (1) produces a positively significant coefficient. In contrast, columns (2) and (3) deliver smaller coefficients on cash flow compared to their OLS counterparts, and it becomes insignificant for the fifth-order cumulant estimation. These results are largely consistent with Erickson et al. (2014). Columns (4) and (5) report results using Fitted Q and marginal q instead. In both specifications, the estimates of cash flow coefficient are statistically insignificant, with the cash flow coefficient shrinking to zero when using marginal q.

IV. Robustness

In this section, we discuss a number of important robustness checks for our marginal q estimates. The results of most of these tests are summarized in Table 7 with further details included in Online Appendices B and C.

First, our baseline second order polynomial approximation of firm values may raise potential endogeneity concerns since any missing higher-order polynomials could likely be correlated with the terms included in the projection and bias our coefficient estimates. We address this concern using the control function approach developed by Olley and Pakes (1996) and Ackerberg et al. (2015) in a two-stage GMM procedure. This procedure is described in detail in Online Appendix B where we also show that the bias-corrected parameter estimates have similar magnitude and statistical significance with those reported in Table 3.

In a second robustness check to address potential endogeneity concerns due to missing higher-order polynomials, we use orthogonalized polynomials in the state variables, $k = \ln K$ and $a = \ln A$ to estimate the value function (12). Table 7, shows that estimated marginal q based on these orthogonalized state variables is nearly perfectly correlated with that constructed from our baseline estimates in Table 3. Online Appendix C reports the estimated coefficients using the orthogonalized state variables which are also consistent with our main results.

Table 7: **Comparison of Marginal qs Estimated with Various Methods**

This table reports the correlation and distance between marginal qs derived from the OLS regression in Table 3 (marginal q_{OLS}), the regression with orthogonalized $\ln A$ and $\ln K$ in Table A4 (marginal q_{Orthg}), the regression with interactive fixed effects in Table A5 (marginal q_{IntFE}), and the post-Lasso OLS estimation in Table A6 (marginal q_{Lasso}). Panel A reports the correlations between marginal qs. Panel B reports the distance between marginal qs, computed as the average ratio of the absolute difference between the row measure and the column measure divided by the mean value of the column measure. For example, row (2) column (1) of Panel B indicates that the distance between Marginal q_{Orthg} and Marginal q_{OLS} is equal to 0.0065, which is computed as the average of $abs(\text{Marginal } q_{Orthg} - \text{Marginal } q_{OLS})/(\text{Mean of Marginal } q_{OLS} = 0.811)$. $abs(\cdot)$ denotes the absolute value.

Panel A: Correlation

	Marginal q_{OLS}	Marginal q_{Orthg}	Marginal q_{IntFE}	Marginal q_{Lasso}
Marginal q_{OLS}	1.0000			
Marginal q_{Orthg}	0.9999	1.0000		
Marginal q_{IntFE}	0.9901	0.9902	1.0000	
Marginal q_{Lasso}	0.9490	0.9488	0.9065	1.0000

Panel B: Distance

	Marginal q_{OLS}	Marginal q_{Orthg}	Marginal q_{IntFE}	Marginal q_{Lasso}
Marginal q_{OLS}	0.0000			
Marginal q_{Orthg}	0.0065	0.0000		
Marginal q_{IntFE}	0.1193	0.1183	0.0000	
Marginal q_{Lasso}	0.1126	0.1114	0.1522	0.0000

Next, to mitigate potential endogeneity concerns due to omitted state variables, we follow Bai (2009) and re-estimate Eqn.(12) allowing for interactive fixed-effects, which can be correlated with the regressors.[21] Again, Table 7 shows that the resulting estimate of marginal q is highly correlated (0.99) with

[21] The interactive fixed-effects model (Bai, 2009) generalizes the standard fixed-effects model by allowing for firm-specific effects of unobservable time-varying variables.

our baseline marginal q. The regression coefficient estimates are collected in Online Appendix C.

Finally, to address any lingering concerns about our approach to polynomial selection, we use the least absolute shrinkage and selection operator (Lasso) widely used in statistics and machine learning. Lasso is a regression analysis for the regularization of data models and feature selection (Tibshirani, 1996). Specifically, we investigate a list of candidate covariates with a series of polynomials in $k = \ln K$ and $a = \ln A$ up to their tenth order (a total of 120 variables). The Lasso regression solves the following minimization problem

$$\beta^*(lasso) = \arg\min_{\beta} |\mathbf{y} - \sum_{j=1}^{p=120} \mathbf{x}_j \beta_j|^2 + \lambda \sum_{j=1}^{p=120} |\beta_j|$$

where $\lambda \sum_{j=1}^{p=120} |\beta_j|$ is the penalty term. Picking the non-negative regularization parameter, λ, is a key step in the Lasso regression and significantly impacts the prediction performance of the fitted model. Here we use the adaptive Lasso algorithm developed by (Zou, 2006) to perform the regression, in which the adaptive weights are used for penalizing different coefficients in the penalty term. These weights are data-dependent and rely on an initial estimator to calculate penalty loadings.

In Online Appendix C we show that this Lasso analysis again yields very similar specification to our baseline regression results in Table 3, by selecting both the first- and second-order polynomials in $k = \ln K$ and $a = \ln A$, together with only a few additional higher-order terms. Further, post-estimation OLS based on Lasso's model selection outcomes also produces very similar magnitudes for the estimated coefficients.

More importantly, the marginal q constructed from the post-Lasso OLS estimation is still very highly correlated with our baseline estimates, with a correlation coefficient 0.949. On average, the absolute differences between these two measures is only 11.26% of the mean value of marginal q.

V. Conclusions

We propose a novel and practical way to measure firm's marginal q using asset prices under very minimal assumptions concerning technology and preferences. We show that this measure differs substantially from average Tobin's Q, has a much higher correlation with investment and yields more plausible estimates of capital adjustment costs as well as responses of investment to fundamentals.

Our approach can be easily extended to include additional state variables and accommodate complex investment models with labor market frictions as in Hall (2004), Merz and Yashiv (2007), Belo et al. (2014), and Michaels et al. (2019) and financial frictions as in Bond and Meghir (1994), Hennessy et al. (2007), Bustamante (2016), Bolton et al. (2011), Hugonnier et al. (2015), and Bazdresch et al. (2018). In such instances, the projection method can be used to measure not only marginal q, but also to measure the marginal value of labor, cash/debt and intangible assets.

Unlike existing methodologies, this measure of marginal q is independent of assumptions on capital adjustment costs. This independence can be exploited to validate empirically alternative class of adjustment costs including those depending on the growth rate of investment as in Eberly et al. (2012) and also allows for a fully nonparametric kernel density estimation of marginal q, which seems a new direction of promising research.

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For Online Publication

Online Appendix for "Marginal q"

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Appendix A: Data Construction

We follow previous literature (e.g., Peters and Taylor, 2017 and Bazdresch et al., 2018) to construct the sample used in the paper. The sample is from the 2021 Compustat Industrial Files and run from 1973 to 2019. We screen the sample as follows. We restrict to manufacturing firms (SIC code 2000 to 3999). We also exclude observations with negative or zero sales or (average) Tobin's Q, or with less than $ 5 million in gross capital stock, which we deflate by the GDP deflator index with a base year of 1982. Besides, we also exclude any firm-year observation where a firm made an acquisition in a given year, to avoid the contamination from the effects of mergers and acquisitions. Specifically, we delete all firm-year observations if (i) the firm was recorded to have one or more M&A deals in a year in the SDC Mergers and Acquisition Database, or (ii) an acquisition (item AQC) exceeds 15% of total assets (item AT), or (iii) the absolute difference between $CAPX$ and $CAPXV$ over $PPEGT$ exceeds 0.5 and at the same time it observes a substantial increase (greater than 20%) of the absolute growth rate of $PPEGT$. Moreover, we require that the variables needed to construct our regression variables not be missing, and trim regression variables over the entire panel at 0.5% level to remove outliers. Lastly, we require a firm to have at least two-year observations. We are left with 47,141 firm-year observations, with between 660 and 1,286 firms per year.

In our regressions, the capital stock of a firm is defined as Compustat item $PPEGT$. Investment rate is constructed as item $CAPX$ scaled by the gross beginning-of-period capital stock $PPEGT$. Cash flow is the sum of items IB and DP scaled by $PPEGT$. Tobin's Q is constructed as $DLTT$ plus DLC plus $PRCC\_F$ times $CSHO$ minus AC, then scaled by $PPEGT$. In estimating the profitability shocks, firms' revenues is defined as the real net sales $SALE$.

Appendix B: Estimating Profitability Shocks

We follow Olley and Pakes (1996)'s methodology with the correction proposed by Ackerberg et al. (2015) to estimate profitability shocks. We assume that each firm has a Cobb-Douglas revenue function $F(Z,K,N) = ZK^{\alpha_K}N^{\alpha_N}$, where Z denotes the productivity shock, K is physical capital, N is the variable factor(s), and W is the price of the variable factor(s). The equations that follow are based on one variable factor for expositional purposes but extend easily to multiple variable factors. Maximization of operating profit, $\Pi(Z,K,N) = F(Z,K,N) - WN$, over the flexible factor, N, leads to a reduced form profit function, $\Pi(K,A) = AK^{\theta}$, where $A = (1-\alpha_N)\left[Z(\alpha_N/W)^{\alpha_N}\right]^{\frac{1}{(1-\alpha_N)}}$ includes shocks to productivity as well as variations in factor prices and in demand. The exponent on capital θ is $\alpha_K/(1-\alpha_N)$. Similarly, the revenue function evaluated at the optimal flexible factor takes the reduced form $F(K,A) = \frac{A}{(1-\alpha_N)}K^{\theta}$.

The coefficient on K measuring the degree of returns-to-scale in capital (θ) in both the revenue and profit functions is the same. Moreover, the properties of the shocks to revenue and profits are the same up to a factor of proportionality. Thus, the estimation strategy is to estimate θ from a regression of the log revenue on the log capital stock:

$$\pi_{it} = a_{it} + \theta k_{it} + \varepsilon_{it}$$

where $\pi_{it} = \ln(F(K,A))$, $a_{it} = \ln(A_{it})$, $k_{it} = \ln(K_{it})$, and ε_{it} is noise. However, running an OLS potentially introduces bias in estimating θ due to the classic endogeneity problem: k_{it} can be correlated with a_{it} since the econometric unobservable a_{it} might be observed or partially observed by the firm prior to choosing k_{it}.

Olley and Pakes (1996) propose a control function approach to tackle this endogeneity problem, which is later improved by Levinsohn and Petrin (2003) and Ackerberg et al. (2015). Typically the primitives of the model are assumed to satisfy the following assumptions:

ASSUMPTION 1—First-order Markov: $p(a_{it}|I_{it-1}) = p(a_{it}|a_{it-1})$ where I_{it-1} is the firm's information set at $t-1$.

ASSUMPTION 2—Timing of input choices: Capital choice of firms is dynamic and evolves according to $k_{it} = K(k_{it-1}, i_{it-1})$ where investment i_{it-1} is chosen before period t.

ASSUMPTION 3—Strict monotonicity: $i_{it} = f(a_{it}, k_{it})$ is strictly increasing in a_{it}.

The estimation can be implemented through a two-stage GMM. Given *ASSUMPTION* 3, we can invert investment $a_{it} = f^{-1}(k_{it}, i_{it})$, and substitute into the production function:

$$\pi_{it} = \theta k_{it} + f^{-1}(k_{it}, i_{it}) + \varepsilon_{it} = \Phi_t(k_{it}, i_{it}) + \varepsilon_{it}.$$

Then the first stage moment condition is

$$E[\varepsilon_{it}|I_{it}] = E[\pi_{it} - \Phi_t(k_{it}, i_{it})|I_{it}] = 0.$$

After obtaining a nonparametric estimate $\widehat{\Phi_t}(k_{it}, i_{it})$ of $\Phi_t(k_{it}, i_{it})$. We estimate θ in the second stage using the following second stage conditional moment:

$$E[\pi_{it} - \theta k_{it} - g(\Phi_{t-1}(k_{it-1}, i_{it-1}) - \theta k_{it-1})|I_{it-1}] = 0,$$

which comes from the Markov property of a_{it} such that $a_{it} = E[a_{it-1}|I_{it-1}] +$

$\zeta_{it} = E(a_{it}|a_{it-1}) + \zeta_{it}$ with ζ_{it} being the unexpected innovation in a_{it}.

To implement the method in Stata, we follow Asker et al. (2019) and Baqaee and Farhi (2020) and use the *prodest* Stata package. Specifically,

- outcome variable is log sales,
- "state" variable is log capital stock, i.e., log PPEGT in the Compustat,
- "proxy" variable is log investment, which is an instrument for productivity,
- "control" variables include SIC 3-digit and SIC 4-digit sales shares.

We do not include labor in our production function estimation because of a lot of missing values in Compustat. We hence set "free" variable equal to a constant zero. Following Baqaee and Farhi (2020), we use 3-year rolling windows and the estimate of θ for year t is based on data in years $t-1$, t, and $t+1$. So in the raw data we extend the sample to years 1972 and 2020. The estimation procedure has two stages. The first stage is a projection stage. We regress log sales on the 3rd degree polynomial of all state, proxy, and control variables in order to remove the measurement errors. The second stage is the estimation stage using the predicted log sales as the dependent variable.

After obtaining unbiased estimates of θ, we recover the productivity shock from $\hat{a}_{it} = \hat{\Phi}_{it} - \hat{\theta}k_{it}$. Table A1 tabulates summary statistics for the estimates of θ across years. Figure A1 exhibits the distribution of estimated a_{it} in the sample. The mean of $\hat{a}_{it}$ is equal to 1.376 and the standard deviation is 0.335.

Appendix C: Additional Tables and Figures

Table A1: Estimates of Productivity Shocks θ

This table reports summary statistics for the estimates of productivity shocks θ over the period 1973-2019. Detailed estimation procedures are provided in Online Appendix B.

	Obs.	Mean	Std. Dev.	25th	50th	75th
θ	47	0.848	0.015	0.836	0.844	0.864

Table A2: Empirical Value Function - Control Function Approach

This table replicates Table 3, except we follow Ackerberg et al. (2015) to use the control function approach to correct the potential endogeniety issue after omitting higher order polynomial terms in $\ln K$ and $\ln A$. Fixed effects are controlled by a within transformation for all variables. Other details are the same as Table 3.

	(1)	(2)	(3)
$\ln K$	-0.373	-0.628	-0.551
	(0.000)	(0.000)	(0.006)
$\ln A$	0.981	0.299	0.429
	(0.000)	(0.000)	(0.007)
$(\ln K)^2$		0.028	0.026
		(0.002)	(0.002)
$(\ln A)^2$		0.298	0.320
		(0.001)	(0.007)
$\ln A \times \ln K$			-0.015
			(0.014)
Firm FE	Y	Y	Y
Year FE	Y	Y	Y
N	$47,141$	$47,141$	$47,141$

Table A3: Empirical Value Function - Portfolio-level Estimation

This table replicates Table 3, except we estimate the empirical value function at the portfolio level. Each observation is a 4-digit SIC industry by year, in which we take an average of the firm-level variables. Standard errors are clustered at the indudstry level. We include 4-digit SIC codes and year fixed effects in the regression. Other details are the same as Table 3.

	(1)	(2)	(3)
$\ln K$	-0.118	-0.290	-0.308
	(0.021)	(0.088)	(0.129)
$\ln A$	1.575	0.568	0.526
	(0.086)	(0.310)	(0.379)
$(\ln K)^2$		0.017	0.017
		(0.009)	(0.009)
$(\ln A)^2$		0.445	0.445
		(0.130)	(0.130)
$\ln A \times \ln K$			0.010
			(0.043)
Industry FE	Y	Y	Y
Year FE	Y	Y	Y
$\overline{R}^2$	0.512	0.515	0.515
N	$8,902$	$8,902$	$8,902$

Table A4: Empirical Value Function - Orthogonal $\ln K$ and $\ln A$

This table replicates Table 3, except we orthogonalize $\ln K$ and $\ln A$ as $\widehat{\ln K}$ and $\widehat{\ln A}$ and then project $\ln(V/K)$ on the polynomial of the orthogonalized variables. Other details are the same as Table 3.

	(1)	(2)	(3)
$\widehat{\ln K}$	-0.343	-0.623	-0.581
	(0.017)	(0.034)	(0.036)
$\widehat{\ln A}$	0.971	1.114	1.327
	(0.030)	(0.035)	(0.071)
$(\widehat{\ln K})^2$		0.031	0.027
		(0.003)	(0.004)
$(\widehat{\ln A})^2$		0.289	0.289
		(0.045)	(0.045)
$\widehat{\ln A} \times \widehat{\ln K}$			-0.049
			(0.014)
Firm FE	Y	Y	Y
Year FE	Y	Y	Y
$\overline{R}^2$	0.646	0.649	0.649
N	$47,141$	$47,141$	$47,141$

Table A5: Empirical Value Function - Interactive Fixed Effects

This table replicates Table 3, except we allow the interactive fixed effects between firms and years in the regression following Bai (2009). Other details are the same as Table 3.

	(1)	(2)	(3)
$\ln K$	-0.260	-0.552	-0.484
	(0.029)	(0.066)	(0.077)
$\ln A$	0.600	0.233	0.351
	(0.036)	(0.129)	(0.159)
$(\ln K)^2$		0.029	0.027
		(0.006)	(0.006)
$(\ln A)^2$		0.164	0.168
		(0.053)	(0.054)
$\ln A \times \ln K$			-0.029
			(0.017)
Firm FE	Y	Y	Y
Year FE	Y	Y	Y
N	$47,141$	$47,141$	$47,141$

Table A6: Empirical Value Function - Lasso Analysis

This table reports the regression results for the Lasso analysis and the post-estimation OLS. Candidate covariates in the model include 10th-order polynomials in $\ln K$ and $\ln A$. Column (1) reports the Lasso regression results by using the adaptive Lasso algorithm proposed by Zou (2006). Column (2) reports the OLS regression results after selecting variables from the Lasso regression in column (1). Fixed effects are indicated at the bottom of the table. Other details are the same as Table 3.

	(1)	(2)
	Lasso	Post-est OLS
$\ln A$	0.251	0.258
$(\ln A)^2$	0.171	0.126
$(\ln A)^{10}$	0.000	-0.000
$\ln K$	-0.799	-0.920
$(\ln K)^2$	0.057	0.069
$(\ln K)^6$	-0.000	-0.000
$(\ln A)^{10} \times \ln K$	0.000	0.000
$(\ln A)^3 \times \ln K$	0.007	0.013
$(\ln A)^7 \times (\ln K)^2$	-0.000	-0.000
Firm FE	Y	Y
Year FE	Y	Y
$\overline{R}^2$	.	0.651
N	$47,141$	$47,141$

Figure A1: Distribution of Estimated Productivity Shocks

This figure plots the distributions of the estimated productivity shock ($\ln(A)$) following Olley and Pakes (1996) and Ackerberg et al. (2015). Detailed procedure to estimate the productivity shock is provided in Online Appendix B.