The Timing and Location of Entry in Growing Markets: Subgame Perfection at Work*

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Abstract

We develop and analyze a dynamic model in which firms decide when and where to enter a growing market. We do not pre-specify the order of entry, allowing instead for the roles of leader and follower to be determined endogenously. We characterize the subgame perfect equilibria of the dynamic game and show that the times and locations of entry are governed by the threat of preemption. The threat of preemption leads to premature entry, less extreme locations, and the dissipation of rents. The degree of rent dissipation may be far greater than when firms are assumed to enter the market in a pre-specified order, a common assumption in the literature on spatial competition. Because the different assumptions lead to different predictions for the timing and location of entry, we are able to test which one better fits the data in a particular setting. Using data on gas stations, restaurants, and hotels in geographically isolated markets around highway exits and intersections, we find results consistent with subgame perfection for gas stations and three-star hotels.

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1 Introduction

Consider a growing market. At each point in time, a firm must decide whether to enter and, if so, where to position itself within the market. Any gain from being the first entrant generates incentives for firms to preempt each other. In this paper, we account for the threat of preemption by developing a dynamic model of entry around a model of spatial competition. We characterize the subgame perfect equilibria (SPE) and show that the threat of preemption leads to premature entry, less extreme locations, and the dissipation of rents. In short, preemption matters.

Building on Hotelling (1929), the literature on spatial competition attempts to explain an industry's structure by analyzing a static game of location choice followed by price competition. A static game implicitly assumes an unchanging environment, in particular by holding market size fixed. As a result, there is a fixed number of firms in the market. Yet, an industry's structure changes with market size. While the market is small, it supports at most one firm. As the market grows over time, another firm may enter. In other words, the industry starts with a monopoly phase which may be followed by a duopoly phase. Fully understanding when firms choose to enter the market and where they position themselves within it requires a dynamic model.

Adding a time dimension to the model can overturn the principle of maximum differentiation that is central to the literature on spatial competition (see d'Aspremont, Gabszewicz & Thisse (1979), Economides (1984, 1986) and Neven (1985), among others). In a static game of location choice followed by price competition, a firm has an incentive to differentiate itself from its rival to soften price competition. In the extreme, this implies that firms locate at opposite ends of the market. Adding a time dimension to the model can overturn the principle of maximum differentiation because a more central location benefits the first entrant in two ways. First, a more central location increases the incumbent's profits during the monopoly phase. Second, a more central location decreases the potential entrant's profits during the duopoly phase, thereby delaying further entry.[1] The first entrant may therefore choose to position itself at or near the center of the market rather than at its extremes.

Although a small literature considers the timing decision alongside the location decision (Zhou & Vertinsky 2001, Lambertini 2002), it has arguably failed to capture the essence of a fully dynamic entry process because the employed equilibrium concept, by pre-specifying the order of entry, rules out the threat of preemption. At the outset of the game, firms commit, one by one, to a time and location of entry.[2] As pointed out by Fudenberg & Tirole (1985) in their seminal critique of the models of technology adoption in Reinganum (1981*b*, 1981*a*), such a sequential pre-commitment

[1]The observation that a more central location can deter entry has been made before by Neven (1987) in a version of the basic game where location choices are made sequentially rather than simultaneously. Because there is no time dimension in the model, there are no separate monopoly and duopoly phases.

[2]Lambertini (2002) assumes that the follower behaves non-strategically and enters at a fixed time. He shows that the later the follower enters, the closer to the center of the market the leader positions itself. Because there is no fixed cost of entry, the leader always enters immediately.

equilibrium (SEQPE) fails to be subgame perfect. To see this, consider a SEQPE in which firm 1 plans to enter the market at time t_1 and location x_1 and firm 2 at some later time $t_2 > t_1$ and location x_2. We show below that the leader's payoff exceeds the follower's payoff. Firm 2 therefore has an incentive to preempt firm 1 by entering sightly before t_1 at location x_1, thereby securing itself a payoff arbitrarily close to that of firm 1. Because the follower has an incentive to become the leader, the SEQPE is not subgame perfect, and the threat of preemption is operative until payoffs equalize.[3]

In short, if firms pre-commit to a time and location of entry, preemption is ruled out by assumption. While this may be appropriate in some applications, in many others we expect preemption to be critically important for industry structure and dynamics. To account for the threat of preemption, we specify a fully dynamic entry process and characterize the SPE of the dynamic game. In contrast to a SEQPE, the roles of leader and follower are determined endogenously in a SPE. We show that the threat of preemption governs both the times and the locations of entry in a growing market.

By considering the timing decision together with the location decision, we move beyond recent theoretical work that highlights preemption and its implications for the timing decision, such as Smirnov & Wait (2015), who develop a dynamic model with two firms and characterize the SPE, and Shen & Villas-Boas (2010), who develop a dynamic model of a growing market and show that the ability of early entry to deter future competitors' entry leads firms to enter the market at a rate faster than demand is expanding.

In addition to incorporating the time and location dimensions of the entry decision, we establish testable predictions that allow us to distinguish between SEQPE and SPE in the data. Specifically, we show that, conditional on the location of first entry, the rate of market growth does not affect the threshold for market size at the time of first entry under SEQPE, whereas the threshold is a decreasing function of the rate of market growth under SPE. In our empirical application, we study the timing and location decisions of the first entrant for gas stations, restaurants, hotels, and three-star hotels in geographically isolated markets around highway exits and intersections, a nearly ideal setting for testing whether SEQPE or SPE better fits the data: a firm's location has a direct effect on its profits as traffic patterns favor firms in closer proximity to highway exits, whereas firms of the same type that are close to each other may cannibalize sales and incite price competition. Furthermore, highway traffic data provide good measures of market size and the rate of market growth.

We find that gas stations are consistent with SPE, but restaurants and hotels are not. This finding may be explained by institutional details, in particular the fact that restaurants and hotels

[3] Riordan (1992) incorporates asymmetries into the model of Fudenberg & Tirole (1985) and shows that, with two firms, the more efficient firm enters first. Argenziano & Schmidt-Dengler (2013) extend Fudenberg & Tirole (1985) to more than two firms and show that the time of first entry in a duopoly is a lower bound on the time of first entry in any oligopoly and that more firms may delay first entry.

combine spatial differentiation with other dimensions of product differentiation (Mazzeo 2002, Butters & Hubbard 2021). Indeed, when we isolate spatial differentiation by examining three-star hotels, the data are again consistent with SPE. Taken together, we conclude that SPE can be a more natural equilibrium concept than SEQPE for understanding the evolution of growing markets.

We view our approach of testing an implication of subgame perfection as a complement to the structural approaches used to detect preemption and measure its implications. In contrast to the theoretical literature, the empirical literature on preemption is small, with notable contributions by Schmidt-Dengler (2006), Igami & Yang (2016), Zheng (2016), and Fang & Yang (2019, 2022). Schmidt-Dengler (2006) studies the adoption of MRI scanners by hospitals. Following Fudenberg & Tirole (1985) and the ensuing theoretical literature, he defines preemption by contrasting SEQPE and SPE in as similar way we do. One contribution of our paper vis-a-vis Schmidt-Dengler (2006) is that we consider the timing decision together with the location decision and show that the threat of preemption spills over from the former into the latter.

Later papers depart from Fudenberg & Tirole (1985) to define preemption. Igami & Yang (2016) consider the decisions of hamburger chains regarding the number of outlets in a market. Focusing on Markov perfect equilibria (MPE), and thus a subset of SPE, they say that McDonald's has a motive to preempt to the extent that its entry probability in a market is lower in a counterfactual than in the MPE, where the counterfactual aims to force McDonald's rivals to behave as if the number of McDonald's outlets in the market does not matter. Also restricting attention to the timing decision, Fang & Yang (2019, 2022) identify preemption by decomposing a firm's marginal benefit of entry in the equilibrium conditions, where shutting down the relevant term in a counterfactual allows them to measure the implications of preemption. Fang & Yang (2019) use this approach to study the entry decisions of fast casual taco chains, while Fang & Yang (2022) study the decisions of coffee chains regarding the number of outlets in a market. In addition, Zheng (2016) considers the entry decisions of big box retail chains and defines preemption as a one-period deviation from equilibrium; because a retail chain makes decisions simultaneously for all markets, it decides on both the timing and location of entry, similar to our paper.

Our approach of testing an implication of subgame perfection does not allow us to conduct counterfactuals and thus limits what we can learn about the industry being studied. On the other hand, it incorporates the time and location dimensions of the entry decision and may be less dependent on the details of the model and the simplifying assumptions added to make the model computationally tractable.[4]

Our paper is more generally related to recent empirical applications of the Ericson & Pakes (1995) framework, including Arcidiacono, Bayer, Blevins & Ellickson (2016), Collard-Wexler (2013), and Sweeting (2013). While the threat of preemption is of course operative in any MPE, these

[4]For example, Zheng (2016) truncates the time horizon at the end of her sample period and assumes that firms move in alternating periods using a two-stage budgeting process in which they first allocate financial or managerial resources to groups of markets and then, conditional on this budget allocation, make entry decisions.

papers make no attempt to isolate it. Arcidiacono et al. (2016) and Collard-Wexler (2013) consider the entry and exit decisions of retail chains and ready-mix concrete plants, respectively, assuming that geographical markets are independent of one another.[5] Given this assumption, firms (and the analyst) can treat each market separately and only the time dimension of the entry decision matters. Also related, Sweeting (2013) maintains that geographical markets are independent of one another, where in his model each firm decides on the programming format for each of the radio stations it owns. Hence, in a given period, a radio station can be out of the market, or, if it is in the market, in different locations in product space.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the SEQPE and Section 4 the SPE. Section 5 sets up a welfare benchmark by considering an omnipotent social planner that controls the time and location of entry as well as firms' pricing decisions in the product market. Section 6 uses numerical analysis to compare the outcome of the game under SEQPE and SPE and assess their welfare implications. We show that the threat of preemption leads to premature entry, less extreme locations, and the dissipation of rents. The degree of rent dissipation may be far greater under SPE than under SEQPE, and we provide an example where up to 237 times as much surplus is wasted under SPE than under SEQPE. Section 7 contains our empirical application. Section 8 concludes. Appendix A contains details on price competition, Appendices B, C, and D most proofs, and Appendix E details on the data we use in our empirical application.

2 Model

There are two firms $i \in \{1, 2\}$. Firm i decides a time $t_i \geq 0$ and a location $x_i \in [0, 1]$ to enter the market. Firms incur a fixed cost of entry $F > 0$. Time is continuous and the horizon is infinite. Firms discount future cash flows at an interest rate of $r > 0$.

Consumers are uniformly distributed on the unit interval.[6] They have unit demands and incur transportation costs $b > 0$ per unit of distance. A consumer derives a surplus gross of transportation costs and price of $a > 0$ from consumption. Suppose that firm i is located at x_i and charges a price p_i. Then the utility of a consumer located at z from buying from firm i is

$$a - b(z - x_i)^2 - p_i.^7 \tag{1}$$

The total mass of consumers at time t is $m(t)$, where $m' > 0$. We assume market size grows at most exponentially at a rate less than r to ensure that the NPV of future cash flows remains bounded.

[5]This assumption is shared by Schmidt-Dengler (2006), Igami & Yang (2016), and Fang & Yang (2019, 2022).

[6]Tabuchi & Thisse (1995) and Anderson, Goeree & Ramer (1997) consider general distributions.

[7]By contrast, Hotelling (1929) assumes that the consumer's utility is $a - b|z - x_i| - p_i$. As pointed out by d'Aspremont et al. (1979), this raises existence issues when firms compete in prices taking their locations as given. We avoid these issuess by specifying quadratic transportation costs.

Taking their locations as given, firms then compete in prices. The marginal cost of production is $c \geq 0$. We assume $\frac{a-c}{b} > 3$ to ensure the market is fully covered. That is, in equilibrium each consumer prefers buying from either firm 1 or firm 2 over not buying. Let π^M denote a monopolist's instantaneous profits when market size is normalized to unity. Similarly, let π_i^D denote firm i's instantaneous profits when there are two firms in the market and the market size is normalized to unity. In Appendix A, we show that

$$\pi^M(x) = \begin{cases} a - c - b(1-x)^2 & \text{if} \quad x \leq \frac{1}{2}, \\ a - c - bx^2 & \text{if} \quad x > \frac{1}{2}, \end{cases} \tag{2}$$

$$\pi_1^D(x_1, x_2) = \begin{cases} \frac{b(x_2-x_1)}{18}(2 + x_1 + x_2)^2 & \text{if} \quad x_1 \leq x_2, \\ \frac{b(x_1-x_2)}{18}(4 - x_1 - x_2)^2 & \text{if} \quad x_1 > x_2, \end{cases} \tag{3}$$

$$\pi_2^D(x_1, x_2) = \begin{cases} \frac{b(x_2-x_1)}{18}(4 - x_1 - x_2)^2 & \text{if} \quad x_1 \leq x_2, \\ \frac{b(x_1-x_2)}{18}(2 + x_1 + x_2)^2 & \text{if} \quad x_1 > x_2. \end{cases} \tag{4}$$

Note that $\pi^M(x)$ is maximal at $x = \frac{1}{2}$, while $\pi^D(x_1, x_2)$ is maximal at $x_1 = 0$ and $x_2 = 1$ as well as $x_1 = 1$ and $x_2 = 0$. Moreover, $\pi^M(x) \in \left[a - c - b, a - c - \frac{b}{4}\right]$ and $\pi_i^D(x_1, x_2) \in \left[0, \frac{b}{2}\right]$. Consequently, $\frac{a-c}{b} > 3$ implies $\pi^M(x) > \pi_i^D(x_1, x_2)$. Table 1 lists the instantaneous profits $(\pi_1^D(x_1, x_2), \pi_2^D(x_1, x_2))$ for selected combinations of x_1 and x_2.

The existing literature commonly assumes that firms play a static game of location choice followed by price competition. In these games, location choices depend on the number of entrants. The monopolist prefers to be located in the center ($x = \frac{1}{2}$) of the market rather than at its extremes ($x = 0$ or $x = 1$), because $\pi^M(x)$ is maximal at $x = \frac{1}{2}$. That is, the monopolist wants to be where demand is. For a duopolist, on the other hand, we have $\pi_1^D(x, x) = \pi_2^D(x, x) = 0$, as price competition drives profits down to zero if the firms' products are the same. Moreover, $\frac{\partial \pi_1^D}{\partial x_1} < 0$ and $\frac{\partial \pi_2^D}{\partial x_2} > 0$ if $x_1 < x_2$, whereas $\frac{\partial \pi_1^D}{\partial x_1} > 0$ and $\frac{\partial \pi_2^D}{\partial x_2} < 0$ if $x_1 > x_2$. Hence, each firm has an incentive to differentiate its product from that of its rival. In fact, firm i's best reply to firm j's location choice is given by

$$x_i^\circ(x_j) = \begin{cases} 0 & \text{if} \quad x_j > \frac{1}{2}, \\ \{0, 1\} & \text{if} \quad x_j = \frac{1}{2}, \\ 1 & \text{if} \quad x_j < \frac{1}{2}. \end{cases} \tag{5}$$

Consequently, duopolists prefer to locate at opposite ends of the market, and $x_i = 0$ and $x_j = 1$ is the outcome of a subgame perfect Nash equilibrium of the static game of (sequential or simultaneous) location choice followed by price competition. This is of course the principle of maximum differentiation, which stems from the fact that product differentiation alleviates price competition (d'Aspremont et al. 1979).[8]

[8] Economides (1986) considers transportation costs of the form $b|z - x_i|^d$ and shows that maximum differentiation results if $d > 1.67$ but not if $1.26 < d < 1.67$. If $d < 1.26$, there does not exist an equilibrium in the two-stage game of location choice followed by price competition.

Rather than looking at a static game of location choice followed by price competition, we specify a fully dynamic entry process. Suppose firm 1 enters the market before firm 2. Since $t_1 \leq t_2$, we call firm 1 the leader and firm 2 the follower. The NPV of the leader's payoffs is

$$V_1(t_1, t_2, x_1, x_2) = V_1^M(t_1, t_2, x_1, x_2) + V_1^D(t_1, t_2, x_1, x_2) - e^{-rt_1}F, \tag{6}$$

where

$$\begin{aligned} V_1^M(t_1, t_2, x_1, x_2) &= \int_{t_1}^{t_2} e^{-rt}\pi^M(x_1)m(t)dt, \\ V_1^D(t_1, t_2, x_1, x_2) &= \int_{t_2}^{\infty} e^{-rt}\pi_1^D(x_1, x_2)m(t)dt, \end{aligned} \tag{7}$$

and the NPV of the follower's payoffs is

$$V_2(t_2, x_1, x_2) = \int_{t_2}^{\infty} e^{-rt}\pi_2^D(x_1, x_2)m(t)dt - e^{-rt_2}F. \tag{8}$$

Analogous expressions arise if firm 2 enters the market before firm 1.

Before turning to a characterization of the equilibrium, we impose two assumptions on the size of the market.

Assumption 1. *$m(0) = 0$.*

Assumption 2. *There exists a $T < \infty$ such that $\pi_2^D(\frac{1}{2}, 1)m(T) > rF$.*

Assumption 1 stipulates that the market takes off at time zero. Assumption 2 ensures that both firms eventually enter the market. To see this, note that $\pi_2^D(\frac{1}{2}, 1)$ is the minmax instantaneous profit of firm 2, which in turn is equal to the minmax instantaneous profit of firm 1. Hence, Assumption 2 requires that there exists a time T at which the market is so large that the NPV of the worst-case profits $\pi_2^D(\frac{1}{2}, 1)m(T)/r$ covers the fixed cost of entry F.

3 Sequential Pre-commitment Equilibrium

In SEQPE, at the outset of the game each firm must irreversibly commit itself to a time and location at which it enters the market. These decisions are made sequentially. The leader therefore takes the follower's reactions into account when making its own decisions. In deriving the SEQPE, we assume that firm 1 enters before firm 2. Hence, given a SEQPE in which firm 1 is the leader and firm 2 the follower, there is a corresponding SEQPE in which firm 1 is the follower and firm 2 the leader.

The Follower's Problem. The follower solves

$$\max_{t_2 \geq 0, x_2 \in [0,1]} V_2(t_2, x_1, x_2). \tag{9}$$

The derivatives of V_2 with respect to t_2 and x_2 are

$$\frac{\partial V_2}{\partial t_2} = -e^{-rt_2}\pi_2^D(x_1, x_2)m(t_2) + re^{-rt_2}F, \tag{10}$$

$$\frac{\partial V_2}{\partial x_2} = \int_{t_2}^{\infty} e^{-rt}\frac{\partial \pi_2^D(x_1, x_2)}{\partial x_2}m(t)dt. \tag{11}$$

From this it is clear that the follower's timing and location decisions depend solely on its profits from the duopoly phase.

Let $t_2^*(.)$ and $x_2^*(.)$ denote a solution to the follower's problem as a function of x_1.

Proposition 1. *$t_2^*(x_1) = m^{-1}\left(\frac{rF}{\pi_2^D(x_1, x_2^*(x_1))}\right) \in (0, T)$ and $x_2^*(x_1) \in x_2^\circ(x_1)$, as defined in equation* (5)*.*

In what follows, we assume without loss of generality that $x_1 \in [0, \frac{1}{2}]$, which implies $\pi_2^D(x_1, 1) \geq \pi_2^D(x_1, 0)$. That is, $x_2^*(x_1) = 0$ is weakly dominated by $x_2^*(x_1) = 1$. To simplify the exposition, we thus set $x_2^*(x_1) = 1$ and $t_2^*(x_1) = m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)$. Note that $t_2^{*\prime}(x_1) > 0$, because price competition intensifies as the leader moves towards the center of the market, which in turn reduces the follower's instantaneous profits. Hence, the follower has to wait longer until the size of the market has reached a level that allows for profitable entry.

The Leader's Problem. Taking the follower's reactions into account, the leader solves

$$\max_{t_1 \geq 0, x_1 \in [0, \frac{1}{2}]} V_1(t_1, t_2^*(x_1), x_1, 1). \tag{12}$$

The derivatives of V_1 with respect to t_1 and x_1 are

$$\frac{\partial V_1}{\partial t_1} = -e^{-rt_1}\pi^M(x_1)m(t_1) + re^{-rt_1}F, \tag{13}$$

$$\begin{aligned}\frac{\partial V_1}{\partial x_1} = {} & \int_{t_1}^{t_2^*(x_1)} e^{-rt}\pi^{M\prime}(x_1)m(t)dt \\ & + e^{-rt_2^*(x_1)}\pi^M(x_1)m(t_2^*(x_1))t_2^{*\prime}(x_1) \\ & + \int_{t_2^*(x_1)}^{\infty} e^{-rt}\frac{\partial \pi_1^D(x_1, 1)}{\partial x_1}m(t)dt \\ & - e^{-rt_2^*(x_1)}\pi_1^D(x_1, 1)m(t_2^*(x_1))t_2^{*\prime}(x_1).\end{aligned} \tag{14}$$

The leader's timing decision depends on its profits from the monopoly phase. Its location decision is governed by its profits from the monopoly phase and its profits from the duopoly phase. Specifically, the leader's location decision is governed by three considerations. First, by moving towards the center of the market, the leader increases its profits from the monopoly phase since $\pi^{M\prime} > 0$ (first term). Second, the leader decreases its profits from the duopoly phase since $\frac{\partial \pi_1^D}{\partial x_1} < 0$ (third term). Third, by moving towards the center of the market, the leader deters entry by the follower since $t_2^{*\prime} > 0$. This increases the duration of the monopoly phase (second term) and decreases the duration of the duopoly phase (fourth term). The net effect is positive since $\pi^M(x_1) > \pi_1^D(x_1, 1)$.

Let t_1^* and x_1^* denote a solution to the leader's problem.

Proposition 2. $t_1^* = m^{-1}\left(\frac{rF}{\pi^M(x_1^*)}\right) \in (0, T)$.

Unfortunately, we cannot say much about the leader's location decision due to the trade-off between the monopoly and the duopoly phases. Zhou & Vertinsky (2001) assume that the market grows linearly without bound and show that the leader chooses either the extreme ($x_1^* = 0$) or the central location ($x_1^* = \frac{1}{2}$) but never an intermediate one ($x_1^* \in (0, \frac{1}{2})$). That is, with a linear market size function, one phase is always much more "important" than the other. By contrast, we consider a general market size function.

SEQPE. The following proposition shows that, in any SEQPE, the payoff to the leader exceeds the payoff to the follower.[9]

Proposition 3. *In any SEQPE,* $V_1(t_1^*, t_2^*(x_1^*), x_1^*, 1) > V_2(t_2^*(x_1^*), x_1^*, 1)$.

Consequently, the follower has an incentive to become the leader, and it is not innocuous to pre-specify the order of entry. To see this, consider a SEQPE with outcome $(t_1^*, t_2^*(x_1^*), x_1^*, 1)$. Since the leader's payoff exceeds the follower's, firm 2 could imitate firm 1 by entering at time $t_1^* - \epsilon$, where $\epsilon > 0$, at location x_1^* and secure itself a payoff of $V_1(t_1^* - \epsilon, t_2^*(x_1^*), x_1^*, 1)$. Given that V_1 is continuous, the resulting payoff for firm 2 is arbitrarily close to firm 1's payoff in the SEQPE and thus in excess of firm 2's payoff in the SEQPE. Because the follower has an incentive to become the leader, the SEQPE is not subgame perfect (Fudenberg & Tirole 1985).

4 Subgame Perfect Equilibrium

In a SEQPE, firms pre-commit to a time and location of entry, with preemption ruled out by assumption. We avoid this assumption here by specifying a fully dynamic entry process and char-

[9] Proposition 2 allows us to replace the leader's problem with $\max_{t_1 \in [0,T], x_1 \in [0, \frac{1}{2}]} V_1(t_1, t_2^*(x_1), x_1, 1)$, where $t_2^*(x_1) = m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)$ by Proposition 1. Since V_1 and $t_2^*(.)$ are continuous functions, a solution exists, and therefore a SEQPE exists. Of course, there may be more than one SEQPE. Note also that $\pi^M(x_1^*) > \pi_2^D(x_1^*, 1)$ implies $t_1^* = m^{-1}\left(\frac{rF}{\pi^M(x_1^*)}\right) < m^{-1}\left(\frac{rF}{\pi_2^D(x_1^*,1)}\right) = t_2^*(x_1^*)$. Hence, our assumption that firm 1 enters before firm 2 is warranted.

acterizing the SPE of the dynamic game. In contrast to a SEQPE, the roles of leader and follower are determined endogenously in a SPE. Hence, the threat of preemption is operative and governs the times and locations of entry in a growing market, as we show below.

To avoid the technical difficulties associated with simultaneous actions in continuous-time games (Simon & Stinchcombe 1989), we follow Gilbert & Harris (1984) and introduce the concept of decision lags. Let $I(t)$ denote the common information on the state of the game at time t. We assume that firm i with decision lag $h_i > 0$ can take an action at time t depending on the information available at time $t - h_i$, as given by $I(t - h_i)$. We further assume that $h_1 < h_2$, so that firm 1 is able to take an action at time t using more recent information than firm 2. We let the decision lags approach zero while maintaining $h_1 < h_2$. In the limit, the delay between information and action is negligible. Both firms observe $I(t)$ at time t, but the action taken by firm 1 is realized first and instantaneously incorporated into the information available to firm 2. Gilbert & Harris (1984) show that the first-mover advantage resulting from decision lags is trivial; the decision lags merely serve to rule out simultaneous entry when it is not optimal for both firms to enter at that time.

The Follower's Problem. Suppose firm 1 has just entered the market at time t_1 in location x_1 and firm 2 has not entered the market yet. That is, firm 1 is the leader and firm 2 the follower. Then firm 2 solves

$$\max_{t_2 \geq t_1, x_2 \in [0,1]} V_2(t_2, x_1, x_2). \tag{15}$$

Note that we now require $t_2 \geq t_1$ as opposed to $t_2 \geq 0$ in Section 3, which reflects the dynamic nature of the entry process: Once time t_1 is reached, there is no going back, and the follower has to choose between entering now or in the future. Let $t_2^*(.)$ and $x_2^*(.)$ denote a solution to the follower's problem as a function of t_1 and x_1.

Proposition 4. *Suppose* $t_1 < \infty$. *Then* $t_2^*(t_1, x_1) = \max\left\{t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1, x_2^*(x_1))}\right)\right\}$, *where* $m^{-1}\left(\frac{rF}{\pi_2^D(x_1, x_2^*(x_1))}\right) \in (0, T)$, *and* $x_2^*(x_1) \in x_2^\circ(x_1)$, *as defined in equation* (5).

In what follows, we assume without loss of generality that $x_1 \in [0, \frac{1}{2}]$, which implies $\pi_2^D(x_1, 1) \geq \pi_2^D(x_1, 0)$. That is, $x_2^*(x_1) = 0$ is weakly dominated by $x_2^*(x_1) = 1$. To simplify the exposition, we thus set $x_2^*(x_1) = 1$ and $t_2^*(t_1, x_1) = \max\left\{t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1, 1)}\right)\right\}$. Note that $t_2^*(t_1, x_1)$ is nondecreasing in both of its arguments.

The Leader's Problem. Suppose firm 1 is about to enter the market at time t_1 and firm 2 has not entered the market yet. Taking the reactions of the lagging firm 2 into account, the leading firm 1 solves

$$\max_{x_1 \in [0, \frac{1}{2}]} V_1(t_1, t_2^*(t_1, x_1), x_1, 1). \tag{16}$$

Let $X_1^*(.)$ denote the set of solutions to the leader's problem as a correspondence of t_1. Since V_1 and $t_2^*(.)$ are continuous functions and $[0, \frac{1}{2}]$ is a compact set, the Theorem of the Maximum implies that $X_1^*(.)$ is nonempty and has a closed graph, and also that $V_1(t_1, t_2^*(t_1, x_1^*(t_1)), x_1^*(t_1), 1)$, where $x_1^*(t_1) \in X_1^*(t_1)$, is a continuous function of t_1.

To assess the properties of $X_1^*(.)$, fix t_1 and split $[0, \frac{1}{2}]$ into two (possibly empty) subsets $\underline{X}_1(t_1) = \left\{x_1 \in [0, \frac{1}{2}] \middle| t_1 \geq m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)\right\}$ and $\overline{X}_1(t_1) = \left\{x_1 \in [0, \frac{1}{2}] \middle| t_1 \leq m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)\right\}$. Hence, if the leader enters at time t_1 at any location $x_1 \in \underline{X}_1(t_1)$, then this triggers immediate entry by the follower, whereas any location $x_1 \in \overline{X}_1(t_1)$ leads to deferred entry. In what follows, we refer to immediate entry by the follower as case 1 and to deferred entry by the follower as case 2.

In Appendix B, we characterize the two subsets $\underline{X}_1(t_1)$ and $\overline{X}_1(t_1)$ and the solution to the leader's problem on each of them. In case 1 (immediate entry by the follower), the leader maximizes instantaneous profits by locating at the extreme of the market. In case 2 (delayed entry by the follower), the leader's location decision is governed by similar considerations to those the leader faces in a SEQPE. In particular, by moving towards the center of the market, the leader increases its profits from the monopoly phase, deceases its profits from the duopoly phase, and increases the duration of the monopoly phase. While the solution to the leader's problem on the set $\overline{X}_1(t_1)$ may not be unique, in Lemma 1 in Appendix B we provide conditions under which the set of solutions $\overline{X}_1^*(t_1)$ shifts to the left — i.e., towards more extreme locations — with t_1.

Because $\pi_2^D(x_1, 1) \leq \pi_2^D(0, 1)$ and $\pi_2^D(x_1, 1) \geq \pi_2^D(\frac{1}{2}, 1)$, the solution to the leader's problem must be given by case 2 (deferred entry by the follower) if $t_1 \in \left[0, m^{-1}\left(\frac{rF}{\pi_2^D(0,1)}\right)\right) = \left[0, m^{-1}\left(\frac{2rF}{b}\right)\right)$ and by case 1 (immediate entry by the follower) if $t_1 \in \left(m^{-1}\left(\frac{rF}{\pi_2^D(\frac{1}{2},1)}\right), \infty\right) = \left(m^{-1}\left(\frac{144rF}{25b}\right), \infty\right)$, as can be seen in Table 1. If $t_1 \in \left[m^{-1}\left(\frac{2rF}{b}\right), m^{-1}\left(\frac{144rF}{25b}\right)\right]$, then the solution may be given by case 1 or case 2 or both. More formally, we therefore have

$$X_1^*(t_1) \subseteq \begin{cases} [0, \frac{1}{2}] & \text{if} \quad t_1 \in \left[0, m^{-1}\left(\frac{2rF}{b}\right)\right), \\ \{0\} \cup [\tilde{x}_1(t_1), \frac{1}{2}] & \text{if} \quad t_1 \in \left[m^{-1}\left(\frac{2rF}{b}\right), m^{-1}\left(\frac{144rF}{25b}\right)\right], \\ \{0\} & \text{if} \quad t_1 \in \left(m^{-1}\left(\frac{144rF}{25b}\right), \infty\right), \end{cases} \tag{17}$$

where $\tilde{x}_1(t_1)$ is the unique solution to $t_1 = m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)$, $\tilde{x}_1' > 0$, $\tilde{x}_1(t_1) = 0$ at $t_1 = m^{-1}\left(\frac{2rF}{b}\right)$, and $\tilde{x}_1(t_1) = \frac{1}{2}$ at $t_1 = m^{-1}\left(\frac{144rF}{25b}\right)$. This is in line with our intuition: If the leader enters early, the follower defers entry, irrespective of the leader's location. Knowing this, the leader faces a trade-off between the monopoly and the duopoly phases, and we are unable to pinpoint its location. By contrast, if the leader enters late, the follower enters immediately, irrespective of the leader's location. Knowing this, the leader chooses the extreme location. Finally, given intermediate times of entry, more extreme locations trigger intermediate entry, whereas more central locations lead to deferred entry by the follower.

While we are in general unable to pinpoint the leader's location choice, we are able to determine how it changes over time. Consider first times of entry in the interval $\left[0, m^{-1}\left(\frac{2rF}{b}\right)\right)$. A straightforward implication of Lemma 1 in Appendix B is that a late first entrant does not choose a less extreme location than an early first entrant. This is the content of the following proposition.

Proposition 5. *Suppose* $t_1, t_1' \in \left[0, m^{-1}\left(\frac{2rF}{b}\right)\right)$ *and* $t_1 < t_1'$. *Then* $\min X_1^*(t_1) \geq \max X_1^*(t_1')$ *with strict inequality if* $\min X_1^*(t_1) \in \left(0, \frac{1}{2}\right)$.

Corollary 1 summarizes the implications of Proposition 5 for the follower's payoff.

Corollary 1. *Suppose* $t_1, t_1' \in \left[0, m^{-1}\left(\frac{2rF}{b}\right)\right)$ *and* $t_1 < t_1'$. *Then* $\max_{x_1 \in X_1^*(t_1)} V_2(t_2^*(t_1, x_1), x_1, 1) \leq \min_{x_1' \in X_1^*(t_1')} V_2(t_2^*(t_1', x_1'), x_1', 1)$ *with strict inequality if* $\min X_1^*(t_1) \in \left(0, \frac{1}{2}\right)$.

Consider next the times of entry in the interval $\left[m^{-1}\left(\frac{2rF}{b}\right), m^{-1}\left(\frac{144rF}{25b}\right)\right]$. Once the leader chooses to locate at the extreme of the market, it continues to do so, which clarifies the relationship between cases 1 and 2. Since we do not use this fact in our characterization of the SPE, we omit the argument.

SPE. Define $L(t_1)$ to be the payoff of the firm that enters first at time t_1, thereby preempting its rival, and define $F(t_1)$ to be the payoff of the firm that enters second and is thus being preempted by its rival at time t_1:

$$\begin{aligned} L(t_1) &= V_1(t_1, t_2^*(t_1, x_1^*(t_1)), x_1^*(t_1), 1), \\ F(t_1) &= V_2(t_2^*(t_1, x_1^*(t_1)), x_1^*(t_1), 1), \end{aligned}$$

where $x_1^*(t_1) \in X_1^*(t_1)$. We sometimes write $L(t_1; x_1^*(t_1))$ and $F(t_1; x_1^*(t_1))$ to show explicitly how payoffs depend on time t_1 and location $x_1^*(t_1)$. Recall that $L(.)$ is a continuous function of t_1. By contrast, $F(.)$ is a correspondence of t_1 since $X_1^*(t_1)$, the set of solutions to the leader's problem at t_1, is not guaranteed to be a singleton. The correspondence $F(.)$ is nonempty and has a closed graph. To simplify the exposition, we write $F(t_1) \geq y$ as shorthand for $\min_{x_1 \in X_1^*(t_1)} F(t_1; x_1) \geq y$, $F(t_1) < y$ for $\max_{x_1 \in X_1^*(t_1)} F(t_1; x_1) < y$, and so on.

Consider the subgame starting at time t_1. There are two classes of subgames, namely the ones where some firm has already entered the market and the ones where no firm has entered the market yet. The first class is easy to deal with because the problem of the remaining firm boils down to the follower's problem that we analyzed above. We therefore focus on the second class in what follows.

To characterize the SPE, we partition the time axis as follows:

- *Region 1:* $\left[0, m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right)\right)$,
- *Region 2:* $\left[m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right), m^{-1}\left(\frac{rF}{a-c-b}\right)\right]$,

- *Region 3:* $\left(m^{-1}\left(\frac{rF}{a-c-b}\right), m^{-1}\left(\frac{2rF}{b}\right)\right)$,
- *Region 4:* $\left[m^{-1}\left(\frac{2rF}{b}\right), m^{-1}\left(\frac{144rF}{25b}\right)\right]$,
- *Region 5:* $\left(m^{-1}\left(\frac{144rF}{25b}\right), \infty\right)$.

Recall that the solution to the leader's problem is given by case 2 (deferred entry by the follower) in regions 1, 2, and 3 and by case 1 (immediate entry by the follower) in region 5. In region 4, the solution may be given by case 1 or case 2 or both. Recall further that $F(.)$ is nondecreasing in regions 1, 2, and 3 by Corollary 1.

In Appendix C, we show that $L(.)$ is increasing in region 1 and decreasing in regions 3, 4, and 5. Hence, $L(.)$ attains a global maximum in region 2. In what follows, we assume that the global maximum of $L(.)$ is unique and that $L(.)$ is increasing to the left of the global maximum and decreasing to the right.

Assumption 3. *$L(.)$ is unimodal.*

A sufficient condition for Assumption 3 to hold is that there exists x_1 such that $x_1 \in X_1^*(t_1)$ for all $t_1 \in \left[m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right), m^{-1}\left(\frac{rF}{a-c-b}\right)\right]$. The reason is that, holding x_1 fixed, $L(t_1) = V_1(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right), x_1, 1)$ is strictly quasiconcave in t_1. Moreover, Assumption 3 always held in the numerical examples we studied in Section 6.

Define $t_1^* = \arg\max_{t_1 \geq 0} L(t_1)$ and further partition region 2 by the global maximum of $L(.)$ as follows:

- *Region 2a:* $\left[m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right), t_1^*\right)$,
- *Region 2b:* $\left[t_1^*, m^{-1}\left(\frac{rF}{a-c-b}\right)\right]$.

Regions 2b, 3, 4, and 5. Working backwards through time, consider the subgame starting at time $t_1 \in [t_1^*, \infty)$. In Appendix C, we show that $L(t_1) \geq F(t_1)$ and

$$L(t_1) > \max\left\{L(t_1'), F(t_1')\right\} \tag{18}$$

for all $t_1' > t_1$. Because the payoff to entering first and becoming the leader at time t_1 is at least as large as the payoff to becoming the follower at time t_1 and larger than the payoff to becoming either the leader or the follower at some later time t_1', it is optimal for a firm to enter at time t_1 and location $x_1 \in X_1^*(t_1)$. Moreover, it is optimal to enter irrespective of the opponent's strategy. Hence, firm 1 enters as it has a shorter decision lag than firm 2. Note that to the extent that $X_1^*(t_1)$ is not a singleton, there may be multiplicity, though this multiplicity has no impact on the outcome of the SPE because, as we show below, the time of first entry is prior to t_1.

Regions 1 and 2a. Consider times of entry in the interval $[0, t_1^*)$. Rent equalization occurs if $F(.)$ cuts $L(.)$ from above at some t_1 to the left of the global maximum of $L(.)$. In what follows, we assume that $F(0) > L(0)$. In essence, this requires that the fixed cost of entry F is sufficiently large.[10] The following proposition shows that $L(t_1^*) > F(t_1^*)$. Note that it does not presume that the global maximum of $L(.)$ is unique.

Proposition 6. *Let* $t_1^* \in \arg\max_{t_1 \geq 0} L(t_1)$*. Then* $L(t_1^*) > F(t_1^*)$*.*

It turns out that there may be more than one SPE. Before stating our main theorem, we set up some notation that allows us to describe the set of SPEs. Define

$$\hat{t}_1 = \sup \left\{ t_1 \in [0, t_1^*] \,|\, F(t_1) \geq L(t_1) \right\}, \tag{19}$$

where $F(t_1) \geq L(t_1)$ is once again shorthand for $\min_{x_1 \in X_1^*(t_1)} F(t_1; x_1) \geq L(t_1)$, and

$$\tilde{T}_1 = \left\{ t_1 \in (\hat{t}_1, t_1^*] \,\middle|\, \max_{x_1 \in X_1^*(t_1)} F(t_1; x_1) \geq L(t_1) \right\}. \tag{20}$$

The sup here accounts for the possibility that $F(.)$ coincides with $L(.)$ up to (but not including) some point. The upper bound t_1^* is necessary because we know that $F(.)$ eventually coincides with $L(.)$.

We know that $\hat{t}_1 \in (0, t_1^*)$ exists because $F(0) > L(0)$ and $F(t_1^*) < L(t_1^*)$ by Proposition 6. By contrast, $\tilde{T}_1 \subseteq (\hat{t}_1, t_1^*)$ may be empty. The following proposition summarizes the properties of $\hat{t}_1$ and $\tilde{T}_1$.

Proposition 7. *(i)* $X_1^*(\hat{t}_1)$ *is a singleton; (ii)* $F(\hat{t}_1) = L(\hat{t}_1)$*; (iii)* $X_1^*(\tilde{t}_1)$ *is not a singleton for any* $\tilde{t}_1 \in \tilde{T}_1$*; (iv)* $\max_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1) = L(\tilde{t}_1)$ *for all* $\tilde{t}_1 \in \tilde{T}_1$*; (v)* $\tilde{T}_1$ *consists of isolated points.*

To illustrate Proposition 7, suppose $\tilde{T}_1 = \varnothing$. Then $F(.)$ cuts $L(.)$ from above at $\hat{t}_1$ and stays below $L(.)$ to the right of $\hat{t}_1$. By contrast, if $\tilde{T}_1 \neq \varnothing$, then $F(.)$ touches $L(.)$ from below at $\tilde{t}_1 \in \tilde{T}_1$ to the right of $\hat{t}_1$. In this sense, $\tilde{T}_1 \neq \varnothing$ is a knife-edge case.

This leads to our main theorem.

Theorem 1. *(i) The following strategy is a SPE: If* $t_1 < \hat{t}_1$*, do not enter; if* $t_1 \geq \hat{t}_1$ *and no firm has entered the market yet, enter at location* $\max X_1^*(t_1)$*. (ii) For any* $\tilde{t}_1 \in \tilde{T}_1$*, the following strategy is a SPE: If* $t_1 < \tilde{t}_1$*, do not enter; if* $t_1 \geq \tilde{t}$ *and no firm has entered the market yet, enter at location* $\min X_1^*(t_1)$*.*

Recall that we focus on subgames where no firm has entered the market yet. To relate Theorem 1 to equations (19) and (20), note that $\min_{x_1 \in X_1^*(t_1)} F(t_1; x_1) = F(t_1; \max X_1^*(t_1))$ and $\max_{x_1 \in X_1^*(t_1)} F(t_1; x_1) = F(t_1; \min X_1^*(t_1))$ because $\frac{\partial \pi_2^D}{\partial x_1} < 0$ if $x_1 \leq x_2$.

[10] This assumption is made purely for analytical convenience. In fact, as long as we are willing to extend the time axis below zero (as we do in Section 6), there always exists a $t_1 < t_1^*$ such that $F(t_1) > L(t_1)$. The reason is that $F(.)$ is positive by Assumption 2, whereas $\lim_{t_1 \to -\infty} L(t_1) = -\infty$.

We prove part (i) of Theorem 1 and relegate the proof of part (ii) to Appendix D. Because we have already dealt with regions 2b, 3, 4, and 5, we focus on regions 1 and 2a in what follows. Working backwards through time, consider the subgame starting at time $t_1 \in [\hat{t}_1, t_1^*]$. If a firm enters first at time t_1 and location $\max X_1^*(t_1)$ according to the prescribed strategy, then it gets $L(t_1)$. If the firm deviates from the prescribed strategy, then its rival enters first and the firm gets $F(t_1; \max X_1^*(t_1))$. We have $\min_{x_1 \in X_1^*(t_1)} F(t_1; x_1) \leq L(t_1)$ for all $t_1 \geq \hat{t}_1$ with strict inequality whenever $t_1 > \hat{t}_1$ by construction (see equation (19)) and $\min_{x_1 \in X_1^*(t_1)} F(t_1; x_1) = F(t_1; \max X_1^*(t_1))$. Hence, the firm has no incentive to deviate from the prescribed strategy. Because it has a shorter decision lag than firm 2, firm 1 enters.

Continuing to work backwards through time, consider the subgame starting at time $t_1 \in [0, \hat{t}_1)$. If a firm does not enter according to the prescribed strategy, then it gets either $L(\hat{t}_1)$ or $F(\hat{t}_1; \max X_1^*(\hat{t}_1))$. If the firm deviates from the prescribed strategy and enters first at time t_1 at location $x_1 \in [0, \frac{1}{2}]$, then it gets at most $L(t_1)$ (and $L(t_1)$ if $x_1 \in X_1^*(t_1)$). We have $\min_{x_1 \in X_1^*(\hat{t}_1)} F(\hat{t}_1; x_1) \geq L(\hat{t}_1)$ by construction and $\min_{x_1 \in X_1^*(\hat{t}_1)} F(\hat{t}_1; x_1) = F(\hat{t}_1; \max X_1^*(\hat{t}_1))$. Moreover, $L(t_1) < L(\hat{t}_1)$ for all $t_1 < \hat{t}_1$ because $L(.)$ is increasing in region 1 and in region 2a by Assumption 3. Taken together, we have

$$L(t_1) < L(\hat{t}_1) \leq F(\hat{t}_1; \max X_1^*(\hat{t}_1)). \tag{21}$$

Hence, the firm has no incentive to deviate from the prescribed strategy.

5 Welfare

The nature of the game being played has stark welfare implications. Consider an omnipotent social planner who controls the time and location of entry as well as firms' pricing decisions in the product market. The planner's goal is to maximize social surplus, consisting of gains from trade net of transportation costs and fixed costs. Let ω^M and ω^D denote instantaneous social surplus under a monopoly and a duopoly, respectively. We have

$$\omega^M(x) = a - c - b\left(x^2 - x + \frac{1}{3}\right), \tag{22}$$

$$\omega^D(x_1, x_2) = \begin{cases} a - c - \frac{b}{3} + \frac{b(x_2 - x_1)}{4}(x_1 + x_2)^2 + bx_2(1 - x_2) & \text{if} \quad x_1 \leq x_2, \\ a - c - \frac{b}{3} + \frac{b(x_1 - x_2)}{4}(x_1 + x_2)^2 + bx_1(1 - x_1) & \text{if} \quad x_1 > x_2. \end{cases} \tag{23}$$

These expressions are independent of prices because demand is inelastic; they are derived in Appendix A. Note that $\omega^M(x)$ is maximal at $x = \frac{1}{2}$ and $\omega^D(x_1, x_2)$ is maximal at $x_1 = \frac{1}{4}$ and $x_2 = \frac{3}{4}$ as well as $x_1 = \frac{3}{4}$ and $x_2 = \frac{1}{4}$. Moreover, $\omega^M(x) \in \left[a - c - \frac{b}{3}, a - c - \frac{b}{12}\right]$ and $\omega^D(x_1, x_2) \in \left[a - c - \frac{b}{3}, a - c - \frac{b}{48}\right]$. Two firms are therefore not necessarily better than one firm.

The NPV of social surplus is

$$W(t_1,t_2,x_1,x_2) = \int_{t_1}^{t_2} e^{-rt}\omega^M(x_1)m(t)dt + \int_{t_2}^{\infty} e^{-rt}\omega^D(x_1,x_2)m(t)dt - e^{-rt_1}F - e^{-rt_2}F. \quad (24)$$

We search for the times and locations of entry that maximize social surplus. That is, we solve the following problem

$$\max_{0\le t_1\le t_2, 0\le x_1\le x_2\le 1, x_1\in[0,\frac{1}{2}]} W(t_1,t_2,x_1,x_2). \quad (25)$$

The planner may decide to have one instead of two firms in the market. In this case, the solution to the above problem is given by $t_1 = m^{-1}\left(\frac{rF}{\omega^M\left(\frac{1}{2}\right)}\right)$ and $x_1 = \frac{1}{2}$ (along with $t_2 = \infty$ and $x_2 \in [\frac{1}{2}, 1]$).

6 Numerical Analysis

We use numerical analysis to contrast the outcomes of the game under SEQPE and SPE and to illustrate their welfare implications. To model a wide range of possible market size functions, we choose the piecewise linear specification

$$m(t) = \begin{cases} 0 & \text{if} \quad t \le 0, \\ \frac{\mu_1}{\tau_1}t & \text{if} \quad 0 < t \le \tau_1, \\ \mu_1 + \frac{1-\mu_1}{\tau_2-\tau_1}(t-\tau_1) & \text{if} \quad \tau_1 < t \le \tau_2, \\ 1 & \text{if} \quad t > \tau_2, \end{cases} \quad (26)$$

where $\tau_1 > 0$ and $\tau_2 > \tau_1$ denote the end of the first and second growth periods, respectively, and $0 < \mu_1 < 1$ is the market size at the end of the first growth period. Note that market size is bounded. Moreover, $m(.)$ is concave, linear, or convex in $t \in [0, \tau_2]$ depending on whether $\frac{\mu_1}{\tau_1} \gtreqless \frac{1-\mu_1}{\tau_2-\tau_1}$.

The structure of the payoffs allows us to normalize some parameters. First, consumers' gross surplus a and firms' marginal cost c appear exclusively in the monopolist's profit function as $a-c$. Hence, without loss of generality we set $c = 0$. Second, if a, b, and F are multiplied by some nonzero constant, then this does not affect the follower's timing decision or the leader's location decision. Hence, $L(.)$ and $F(.)$ are homogenous of degree one in the triple (a, b, F). We therefore normalize $b = 1$. Third, the interest rate r merely determines the time scale and is therefore not of interest by itself. We set $r = 0.05$ in what follows.

We conduct a search over the 15000 parameterizations given by all possible combinations of $a \in \{3, 6, 9\}$, 10 equidistant values of $F \in \left(0, \frac{\pi_2^D(\frac{1}{2},1)}{r}\right)$ (see Assumption 2), $\tau_2 \in \{5, 10, 15, 25, 50\}$, 10 equidistant values of $\mu_1 \in (0, 1)$, and 10 equidistant values of $\tau_1 \in (0, \tau_2)$. That is, we choose τ_1 given τ_2 such that $\tau_1 < \tau_2$. Finally, as discussed in Section 4, we extend the time axis below zero.

To compute $L(.)$ and $F(.)$, we discretize time with a period length of $\Delta = 0.01$.

For each of the parameterizations given above, our computations indicate that $L(.)$ is increasing to the right of t_1^* and decreasing to the left, thus justifying the simplifying assumption made in Theorem 1 in Section 4. Unimodality of $L(.)$ in turn ensures that the SEQPE is unique.[11] Moreover, our computations lead to a unique SPE in which $F(.)$ cuts $L(.)$ from above at $\hat{t}_1$ and stays below $L(.)$ to the right of $\hat{t}_1$ in accordance with part (i) of Theorem 1.

While our 15000 parameterizations do not contain such a case, it is possible that $\tilde{T}_1 \neq \varnothing$ in accordance with part (ii) of Theorem 1. Figure 1 illustrates that there may exist one or more "touching" equilibria besides the "cutting" equilibrium. As can be seen, $F(.)$ cuts $L(.)$ from above at $\hat{t}_1 = -4.52$ and $F(.)$ touches $L(.)$ from below at $\tilde{t}_1 = 15.92$. Note, however, that $\tilde{T}_1 \neq \varnothing$ is a knife-edge case in the sense that, if the fixed cost of entry F is chosen to be slightly larger than in the example, $F(.)$ is below $L(.)$ but does not touch it, whereas $F(.)$ is above $L(.)$ if F is chosen to be slightly smaller.

In what follows, we use superscripts to distinguish between the outcome of the SEQPE, the outcome of the SPE, and the times and locations of entry chosen by the omnipotent social planner, indicated with a W superscript. Our first result summarizes the implications of preemption for the times of entry.

Result 1 (Times of Entry). *We have $t_1^W < t_1^{SEQPE}$ and $t_2^{SEQPE} \leq t_2^{SPE} < t_2^W$. Moreover, in 99.98% of parameterizations we have $t_1^{SPE} < t_1^W$.*

Note that $t_1^{SPE} < t_1^{SEQPE}$ by construction of the SPE. Proposition 5 then implies $x_1^{SEQPE} \leq x_1^{SPE}$ (as stated below in Result 2), and hence $t_2^{SEQPE} \leq t_2^{SPE}$. Table 2 lists the parameterizations with $t_1^{SPE} > t_1^W$, which we describe in greater detail below.

Result 1 says that, from a welfare point of view, the follower enters too early in a SPE as well as in a SEQPE. By contrast, the leader usually enters too early in the SPE but always too late in a SEQPE. In fact, we have $t_1^{SPE} < 0$ in 99.56% of parameterizations. This demonstrates the power of preemption: Since the incentive to preempt persists until payoffs are equalized, the leader is forced to enter too early in a SPE, usually at a time at which there is not even any demand yet.

Our second result concerns the locations of entry. The result is in fact analytic, and we report it here merely for the sake of completeness.

Result 2 (Locations of Entry). *We have $x_1^{SEQPE} \leq x_1^{SPE}$ and $x_2^W \leq x_2^{SEQPE} = x_2^{SPE} = 1$.*

By contrast, the relationship between x_1^W, x_1^{SEQPE}, and x_1^{SPE} is ambiguous. Result 2 says that, since $x_1^{SEQPE} \leq x_1^{SPE}$, a SPE entails less extreme locations and thus less product differentiation than a SEQPE.

Figure 2 presents an example. In the SPE, the first entry occurs at time $t_1^{SPE} = -24.86$ and location $x_1^{SPE} = 0.38$ and the second at time $t_2^{SPE} = 22.93$ and location $x_2^{SPE} = 1$; in the SEQPE,

[11] Recall from Section 3 that, in the SEQPE, the leader enters at time t_1^*.

the first entry occurs at time $t_1^{SEQPE} = 2.53$ and location $x_1^{SEQPE} = 0.37$ and the second at time $t_2^{SEQPE} = 22.89$ and location $x_2^{SEQPE} = 1$. Note that this runs counter to Zhou & Vertinsky's (2001) claim that, in a SEQPE, the first entrant chooses either the extreme ($x_1 = 0$) or the central ($x_1 = \frac{1}{2}$) but never an intermediate ($x_1 \in \left(0, \frac{1}{2}\right)$) location. Of course, our model is more general than theirs because Zhou & Vertinsky (2001) assume that market size grows linearly without bound.

Given that both firms enter too early in a SPE but not in a SEQPE, one expects that the combined payoffs to firms and consumers are lower in a SPE than in a SEQPE. In stating the next result, we use L and F as shorthand for the payoff to the leader and the follower, respectively, and C as shorthand for the payoff to consumers. Expressions for instantaneous consumer surplus under monopoly and duopoly are derived in Appendix A; the NPV of consumer surplus C is defined in the obvious way.

Result 3 (Welfare Comparison)**.** *In 99.89% of parameterizations we have* $\Sigma^{SPE} \equiv L^{SPE} + F^{SPE} + C^{SPE} < L^{SEQPE} + F^{SEQPE} + C^{SEQPE} \equiv \Sigma^{SEQPE}$.

Table 3 lists the parameterizations with $\Sigma^{SPE} > \Sigma^{SEQPE}$. These 16 parameterizations have in common that the market undergoes a long period of sluggish growth followed by a short period of rapid growth as τ_1 is large relative to τ_2 and μ_1 is small. Moreover, gross surplus a is low and the fixed cost of entry F is high. Note that the 3 parameterizations with $t_1^{SPE} > t_1^W$ in Table 2 reappear in the 16 parameterizations with $\Sigma^{SPE} > \Sigma^{SEQPE}$.

The following result quantifies the extent to which rents are dissipated under the two equilibrium concepts. We use W^W as shorthand for $W(t_1^W, t_2^W, x_1^W, x_2^W)$.

Result 4 (Rent Dissipation)**.** *We have*

$$0.21 < \frac{\Sigma^{SPE}}{W^W} < 1, \tag{27}$$

$$0.93 < \frac{\Sigma^{SEQPE}}{W^W} < 1, \tag{28}$$

$$0.95 < \frac{W^W - \Sigma^{SPE}}{W^W - \Sigma^{SEQPE}} < 237. \tag{29}$$

Hence, up to 79% of rents are dissipated by premature entry in a SPE as opposed to at most 7% in a SEQPE. This again demonstrates how powerful the incentive to preempt can be. Comparing rent dissipation under the two equilibrium concepts, Result 4 says that, at least for some parameterizations, 237 times as much surplus is wasted under the SPE than under the SEQPE. Tables 4, 5, and 6 illustrate Result 4. Table 4 shows the parameterizations with the lowest value of $\frac{\Sigma^{SPE}}{W^W}$, Table 5 the ones with the lowest value of $\frac{\Sigma^{SEQPE}}{W^W}$, and Table 6 the ones with the highest value of $\frac{W^W - \Sigma^{SPE}}{W^W - \Sigma^{SEQPE}}$.

7 Empirical Application

The nature of the game being played generates testable predictions. In our empirical application, we focus on the relationship between market size at the time of first entry and the market growth rate. Under a SEQPE, by Proposition 2 the threshold for market size at the time of first entry is

$$m(t_1^{SEQPE}) = \frac{rF}{\pi_1^M(x_1^{SEQPE})}. \tag{30}$$

Hence, conditional on the location of first entry, the market growth rate does not affect the threshold for market size at the time of first entry. This is not the case under SPE, however, where the threat of preemption is operative until rents equalize. Intuitively, as the market growth rate increases, the NPV of the leader's payoff increases more than the NPV of the follower's payoff. Rent equalization therefore requires earlier first entry.

To make this intuition concrete, we specify the exponential market size function $m(t) = e^{\gamma t}$, where $0 < \gamma < r$ is the rate of market growth. In what follows, we index the outcome of the game under SPE by γ and restrict attention to the empirically relevant "cutting" equilibria in part (i) of Theorem 1. Proposition 8 provides a prediction about the relationship between market size at the time of first entry and the rate of market growth under SPE that we use to test whether SEQPE or SPE better fits the data in a particular setting.

Proposition 8. *Suppose* $\frac{a-c}{b} > \frac{7}{2}$ *and* $\gamma' > \gamma$. *If* $x_1^{SPE}(\gamma') = x_1^{SPE}(\gamma)$, *then* $m(t_1^{SPE}(\gamma')) < m(t_1^{SPE}(\gamma))$.

Note that Proposition 8 slightly strengthens our maintained assumption from Section 2 that $\frac{a-c}{b} > 3$. Proposition 8 says that under SPE, conditional on the location of first entry, increasing the rate of market growth decreases the threshold for market size at the time of first entry.

Building on equation (30) and Proposition 8, we test whether SEQPE or SPE better fits the data by regressing market size at the time of first entry $m(t_1)$ on the market growth rate γ while controlling for the location of first entry x_1. This test faces two key challenges. First, as equation (30) makes clear, we have to measure x_1 and how x_1 maps into profits, as well as r and F. This measurement has to be consistent across the markets in our data. Second, the location of first entry is codetermined with market size at the time of first entry. Anything that we cannot measure potentially affects both x_1 on the right-hand side and $m(t_1)$ on the left, thereby creating an endogeneity problem. In our application, we argue that whatever we cannot measure and explicitly control for is either absorbed by fixed effects or can be addressed with instrumental variables.

Although more than two firms may eventually enter a market in our empirical application, we expect the qualitative predictions of our model to remain unchanged. If a second entrant has to account for preemption by another firm under SPE, then this will accelerate entry by the second entrant, which in turn will accelerate entry by the first entrant.

7.1 Data and Estimation

We study the timing and location of first entry by gas stations, restaurants, and hotels in geographically isolated markets around highway exits and intersections. We choose these industries because they vary in the fixed cost of entry and in the extent to which spatial differentiation matters for profitability.

We use the ESRI Data and Maps 2013 data available through ArcGIS at `www.esri.com` to define markets. The data record the latitude and longitude of all highway exits and intersections in the U.S. We visualize the data in panel (a) of Figure 3. To define a market, we group together any road crossings (i.e., highway exits or intersections) that are within 1500m of each other to form a cluster. Each cluster is a market. Panels (b) and (c) illustrate this construction in a close-up of Durham, NC. In panel (c), we draw circles with radius 1500m around all road crossings. If a road crossing lies in the circle drawn around another road crossing, then the two road crossings belong to the same market. In this example, there are four distinct markets, shown with color-coded circles. We define the center of a market as the average latitude and longitude of the road crossings in that market. To avoid urban and sprawling suburban areas without a clear market center, we drop clusters with width or height exceeding 2500m. In urban areas, sequential highway exits and intersections tend to be narrowly spaced and thus tend to form large clusters. By contrast, the elongated clusters along interstate highways typically fall within the limit of 2500m.

We obtain the entry dates and locations of gas stations, restaurants, and hotels in each market from Reference USA's Historical Business Database for 1997–2006. We use each year's database to avoid selection bias, which would be a concern if we used the entry dates and locations for only the current set of firms. We designate as a first entrant a firm that enters a market that had no firms of that type in the previous year. The sample contains 704 markets with a first entry by a gas station, 875 markets with a first entry by a restaurant, 527 markets with a first entry by a hotel, and 534 markets with a first entry by a three-star hotel.[12] Out of these, we have 664 markets with a second entry by a gas station, 774 markets with second entry by a restaurant, 494 markets with a second entry by a hotel, and 383 markets with a second entry by a three-star hotel.

For each firm, we calculate its Euclidian distance to the market center. In our model, the first entrant does not locate farther away from the market center than the second entrant. For those markets in our data with a second entrant, the distance between the market center and the first entrant is more than 10% larger than the distance between the market center and the second entrant in only 6%, 7%, 9%, and 14% of markets for gas stations, restaurants, hotels, and three-star hotels, respectively.[13]

Turning to market size and the rate of market growth, we use highway traffic data that we obtain directly from the Department of Transportation for 1993–2009, the last year the data were

[12] When examining three-star hotels, we ignore hotels of other star levels that may have entered earlier.

[13] We allow ourselves a 10% "buffer" in this exercise due to spatial constraints on entry locations.

available.[14] Using m to index markets and t to index years, we measure market size S_{mt} by annual average daily traffic (AADT) and the rate of market growth γ_{mt} by the annualized growth of AADT between $t-2$ and $t+2$ (i.e., $\gamma_{mt} = \left(1 + \frac{S_{mt+2} - S_{mt-2}}{S_{mt}}\right)^{\frac{1}{4}} - 1$). We provide details on how we allocate AADT to markets in Appendix E.

As our focus is on the relationship between market size at the time of first entry and the rate of market growth, a potential concern is that market size and the rate of market growth may be correlated more generally for a variety of reasons. For example, larger markets may experience slower growth due to capacity constraints and congestion of highways. Moreover, Proposition 8 assumes an exponential market size function with a constant rate of market growth, which implies that market size is unrelated to the rate of market growth. To investigate any underlying correlation between market size and the market growth rate, we use a balanced panel of markets in which we observe at least one gas station, restaurant, hotel, or three-star hotel for 1993–2006. The final sample includes 259 first entry events for gas stations, 277 for restaurants, 163 for hotels, and 156 for three-star hotels. In Figure 4, we plot the rate of market growth against the log of market size using a binscatter plot with 20 bins of equal size. Although there is a negative correlation between size and the growth rate for smaller markets, it disappears for larger markets. We therefore restrict the subsequent analyses to markets with AADT exceeding 8000 at the time of first entry.[15] With this restricted sample ($N = 67,890$), we regress the rate of market growth on the log of market size and state and year fixed effects, using two-way clustering on state and year. We do not find a statistically significant relationship between the market growth rate and market size.

We assume that the rate of market growth is not affected by the entry of the firms we study. That is, we assume that highway traffic does not increase simply because of a gas station, restaurant, or hotel, which we consider reasonable. Even with this assumption, we must still account for factors such as entry costs that are likely to affect both the location of first entry and the market's size at that time. Beyond including dummies for the number of highways in the market, the type of the largest road,[16] and state and year fixed effects, we collect data on land prices in 2018 from the American Enterprise Institute and on vegetation in 2001 from the United States Geological Survey LANDFIRE Data Distribution Site. The latter data are typically used to assess fire risk, but we use them as shifters of entry costs, as it costs more to build a new establishment in a dense forest than in a field. Finally, we collect information about the number of other businesses in the market within 500m and 1000m of the market center, which may reflect entry costs at these distances. We provide summary statistics and further details on how we assembled the data in Appendix E.

In addition to these controls, we construct two instruments for the location of first entry to

[14] The Federal Highway Administration switched from raw file format in the earlier years to shapefile format in the later years. As a result, there is no data for 2010.

[15] Our results are robust to restricting the analysis further to markets with AADT in excess of 8000 for all years.

[16] The types are interstate, other freeways and expressways and principal arterials, and other (minor arterial, major collector, minor collector, and local).

address the aforementioned endogeneity concern. Our instruments capture the geographic particularities of a market that may shift the location of first entry. First, we use the log of the diagonal of the bounding box of the market, determined by the maximum difference in the latitudes and longitudes of the road crossings in the market, to measure how geographically spread out the road crossings are. We expect more spread out road crossings to push firms farther out. We interact this instrument with dummies for Michigan, Ohio, Texas, North Carolina, Mississippi, and Georgia, since these states are most represented in the data. This allows the geographic size of the market (i.e., how spread out it is) to have a different sized effect on first entrant's location depending on the state. In the second IV regression, we also use the log distance between the market center and the eventual second entrant, which may also reflect geographic particularities of the market.

To test the relationship between market size at the time of first entry and the rate of market growth, we estimate the model

$$\log S_{mt} = f(\gamma_{mt}) + \alpha \log D_{mt} + X_m \beta + \zeta_{s(m)} + \xi_t + \epsilon_{mt}, \tag{31}$$

where S_{mt} is market size at the time of first entry, $f(\gamma_{mt})$ is a nonparametric function of the rate of market growth at the time of first entry, D_{mt} is the distance between the market center and the first entrant, and X_m is a vector of controls that includes an intercept, dummies for the number of highways, dummies for the type of the largest road, the number of other businesses within 500m and 1000m, and variables for land prices and vegetation (we combine shrub and herb with forest). We also include state fixed effects $\zeta_{s(m)}$, where $s(m)$ is the state in which market m is located, and year fixed effects ξ_t.[17]

We expect $f(\gamma_{mt})$ to be a constant under SEQPE and a decreasing function under SPE. To construct the nonparametric function, we bin the rate of market growth in ranges of 3%. To facilitate estimating equation (31) alternatively by OLS and IV, we replace $f(\gamma_{mt})$ by a linear function of γ_{mt} for our main results in Tables 7–11. We exclude from the estimation those markets that have a negative growth at the time of first entry. Throughout, we proceed separately for gas stations, restaurants, hotels, and three-star hotels.

7.2 Results

We plot the estimated $f(\gamma_{mt})$ in Figure 5 for our four types of firms. For gas stations and three-star hotels, we see a declining relationship between market size at the time of first entry and the rate of market growth, consistent with SPE. However, the error bars are large, especially at higher rates of market growth where we have fewer observations. In contrast, we do not see a clear decline in market size at the time of first entry for restaurants, although there is a decline in point estimate moving from 6-9% market growth to >9% growth. For hotels, the effect is flat until moving from

[17]Our extensive set of controls reduces the useable sample size. For example, states with only one market are fully explained by the state fixed effect.

6-9% market growth to >9% growth, when the point estimate actually increases.

In column (1) of Table 7, we show the OLS estimates for gas stations when replacing the flexible function of growth rate with a linear effect. We find a statistically significant negative coefficient (at 10% with a two-sided test) on γ_{mt}, consistent with SPE. The two IV regressions then follow in columns (2)–(5), with the first stages in columns (2) and (4) and the second stages in columns (3) and (5). Once again, we find a statistically significant negative impact of the rate of market growth (10% and 5%, respectively), with the coefficients not significantly different from the OLS estimates. As expected, in the first stage of the first IV regression, the diagonal of the bounding box of the market has a statistically significant positive impact on the distance between the market center and the first entrant, and this relationship is stronger and weaker for different states. Because we cluster standard errors by state, we report the Kleibergen-Paap first stage F-statistic for the instruments, showing that we do not have weak instruments (Kleibergen 2002). We also report the Hansen J statistic for the test of over-identifying restrictions, which we pass easily. In the first stage of the second IV regression, we find a statistically significant positive impact of the log distance between the market center and the second entrant on the log distance between the market center and the first entrant. This instrument is very strong, leading to a Kleibergen-Paap first stage F-statistic of over 800.

As expected from the Figure 5 results, we do not find a statistically significant relationship between market size at the time of first entry and the rate of market growth for restaurants in Table 8 and hotels in Table 9. This is notable for two reasons. First, using highway exits and intersections to define a market is likely most relevant for gas stations, whereas restaurants and hotels may compete on a wider geographic scale. Second, gas stations have less scope for product differentiation than restaurants and hotels, and thus spatial differentiation is likely more important.

To home in on spatial differentiation and mitigate the impact of other dimensions, we again repeat our analysis for three-star hotels. The OLS and IV estimates in Table 10 show a statistically significant negative impact of the rate of market growth (at 5% for all three specifications), again consistent with SPE.

As an additional check of whether the negative relationship that we find between market size at the time of first entry and the rate of market growth is not spurious, we conduct a placebo test, where we randomly assign the entry date for the first entrant to another year between 1998 and 2006. In the OLS and IV estimates for gas stations in Table 11, there is no longer a statistically significant impact of the rate of market growth. The same is true for the restaurants, hotels, and three-star hotels (not reported due to space constraints).

Our results for market size thresholds resemble those in Bresnahan & Reiss (1990), who consider a static entry game, first with simultaneous and then with sequential moves. They find that the breakeven market size of monopoly and duopoly markets for automobile dealers decreases with positive market growth (as measured by population growth) and increases with negative market growth. This further supports our conclusion that SPE can be a more natural equilibrium concept than SEQPE.

8 Conclusions

In this paper, we contribute to a growing literature in empirical IO by developing and analyzing a dynamic model of entry around a model of spatial competition. In our model, firms decide when and where to enter a growing market. In a SEQPE, firms pre-commit to a time and location of entry, with preemption ruled out by assumption. By contrast, in a SPE the threat of preemption is operative until rents equalize. In analyzing how this affects the timing and location of entry, we show that the threat of preemption leads to premature entry, less extreme locations, and the dissipation of rents.

In our empirical application, we apply this model to study the timing and location of first entry by gas stations, restaurants, and hotels in geographically isolated markets around highway exits and intersections. We show that the nature of the game being played generates testable predictions for the relationship between market size at the time of first entry and the market growth rate. We find that gas stations are consistent with SPE, but restaurants and hotels are not. This finding may be explained by institutional details, in particular the fact that restaurants and hotels combine spatial differentiation with other dimensions of product differentiation. Indeed, when we isolate spatial differentiation by examining three-star hotels, the data are again consistent with SPE. Taken together, we conclude that SPE can be a more natural equilibrium concept than SEQPE for understanding the evolution of growing markets.

A Price Competition

Monopoly. Consider a monopolist located at $x \in [0, \frac{1}{2}]$. Taking its location as given, the monopolist sets its price p to maximize profits. Define $\underline{z}$ and $\overline{z}$, where $\underline{z} \leq \overline{z}$, to be the solutions to $a - b(z-x)^2 - p = 0$. That is,

$$\underline{z} = x - \sqrt{\frac{a-p}{b}}, \quad \overline{z} = x + \sqrt{\frac{a-p}{b}}. \tag{32}$$

Note that $\underline{z}$ and $\overline{z}$ are well-defined for all $p \leq a$. Moreover, we have

$$\underline{z} \geq 0 \Leftrightarrow p \geq a - bx^2, \quad \overline{z} \leq 1 \Leftrightarrow p \geq a - b(1-x)^2 \tag{33}$$

and

$$a - bx^2 \geq a - b(1-x)^2 \Leftrightarrow x \in [0, \frac{1}{2}]. \tag{34}$$

Consumers in the set $[\underline{z}, \overline{z}] \cap [0,1]$ prefer good x over the outside good. Demand is thus given by

$$D(p) = \begin{cases} 0 & \text{if} \quad a \leq p, \\ 2\sqrt{\frac{a-p}{b}} & \text{if} \quad a - bx^2 \leq p < a, \\ x + \sqrt{\frac{a-p}{b}} & \text{if} \quad a - b(1-x)^2 \leq p < a - bx^2, \\ 1 & \text{if} \quad p < a - b(1-x)^2. \end{cases} \tag{35}$$

The first case is obvious, the second case corresponds to $0 \leq \underline{z} \leq \overline{z} \leq 1$, the third to $\underline{z} < 0 \leq \overline{z} \leq 1$, and the fourth to $\underline{z} < 0 < 1 < \overline{z}$. We proceed to calculate profits on a case-by-case basis.

Case 1. Clearly, $\pi^* = 0$.

Case 2. The monopolist solves

$$\max_{p \geq 0} 2\sqrt{\frac{a-p}{b}}(p-c). \tag{36}$$

Solving the FOC yields

$$p^* = c + \frac{2}{3}(a-c) \tag{37}$$

and thus

$$\pi^* = \frac{4}{3}\sqrt{\frac{a-c}{3b}}(a-c). \tag{38}$$

We clearly have $p^* < a$. Moreover,

$$a - bx^2 \leq p^* \Leftrightarrow x \geq \sqrt{\frac{a-c}{3b}}. \tag{39}$$

Case 3. The monopolist solves

$$\max_{p \geq 0} \left(x + \sqrt{\frac{a-p}{b}} \right) (p-c). \tag{40}$$

Solving the FOC yields

$$p^* = c + \frac{2}{3}(a-c) + \frac{2bx}{9}\left(\sqrt{x^2 + \frac{3(a-c)}{b}} - x\right) \tag{41}$$

and thus

$$\pi^* = \frac{2}{3}x(a-c) + \frac{2b}{27}\left(\left(x^2 + \frac{3(a-c)}{b}\right)^{\frac{3}{2}} - x^3\right). \tag{42}$$

We have

$$\begin{aligned} & a - b(1-x)^2 \leq p^* < a - bx^2 \\ \Leftrightarrow \quad & 0 < -7x^2 - 2x\sqrt{x^2 + \frac{3(a-c)}{b}} + \frac{3(a-c)}{b} \leq 9 - 18x. \end{aligned} \tag{43}$$

Case 4. Clearly, $\pi^* = a - c - b(1-x)^2$.

While it is in general not possible to determine which of the four cases is associated with a profit maximum, our intuition suggests that the monopolist chooses to fully cover the market whenever gross surplus is sufficiently high. This is confirmed by the following proposition.

Proposition 9. *Let* $\frac{a-c}{b} > 3$. *Then the monopolist sets price* $p^* = a - b(1-x)^2$ *and makes profits* $\pi^* = a - c - b(1-x)^2$.

Proof. Case 2 is clearly ruled out. Recall that case 3 requires $-7x^2 - 2x\sqrt{x^2 + 3\xi} + 3\xi - 9 + 18x \leq 0$, where $\xi = \frac{a-c}{b}$. Differentiating the LHS of this inequality with respect to ξ yields

$$3\left(1 - \frac{x}{\sqrt{x^2 + 3\xi}}\right) > 0. \tag{44}$$

Moreover, the LHS is zero at $\xi = x^2 - 4x + 3 \leq 3$. This rules out case 3. Recall that case 1 yields profits of zero. Profits in case 4 attain their minimum of $a - c - b$ at $x = 0$. Hence, profits in case 4 exceed those in case 1 whenever $\frac{a-c}{b} > 1$. □

Consider a consumer located at z. His utility is $a - b(z-x)^2 - p^*$. Integrating over all consumers yields instantaneous consumer surplus

$$\sigma^M(x) = \int_0^1 a - b(z-x)^2 - p^* dz = b\left(\frac{2}{3} - x\right). \tag{45}$$

Consider an omnipotent social planner that controls price and thus sets $p = c$ (or any other price for that matter because the price is a transfer from the consumers to the monopolist). Instantaneous social surplus, defined as gains from trade net of transportation costs, is

$$\omega^M(x) = \int_0^1 a - c - b(z-x)^2 dx = a - c - b\left(x^2 - x + \frac{1}{3}\right). \tag{46}$$

Duopoly. There are two firms $i \in \{1, 2\}$. Firm i is located at $x_i \in [0, 1]$. We assume $x_1 \leq x_2$. Taking both locations as given, firm i sets its price p_i to maximize profits. Define $\tilde{z}$ to be the solution to $a - b(z-x_1)^2 - p_1 = a - b(z-x_2)^2 - p_2$. That is,

$$\tilde{z} = \frac{x_1 + x_2}{2} + \frac{p_2 - p_1}{2b(x_2 - x_1)}. \tag{47}$$

For now we assume that $\tilde{z} \in [0,1]$ and that the market is fully covered. Later on we provide conditions such that this is the case in equilibrium.

Consumers in the set $[0, \tilde{z}]$ prefer firm 1 over firm 2, consumers in the set $[\tilde{z}, 1]$ prefer firm 2 over firm 1. Demand is thus given by $D_1(p_1, p_2) = \tilde{z}$ and $D_2(p_1, p_2) = 1 - \tilde{z}$. The profit maximization problems of firms 1 and 2 are $\max_{p_1 \geq 0} D_1(p_1,p_2)(p_1 - c)$ and $\max_{p_2 \geq 0} D_2(p_1,p_2)(p_2 - c)$, respectively. Solving the system of FOCs yields

$$p_1^* = c + \frac{b(x_2 - x_1)}{3}(2 + x_1 + x_2), \tag{48}$$

$$p_2^* = c + \frac{b(x_2 - x_1)}{3}(4 - x_1 - x_2) \tag{49}$$

and thus

$$\pi_1^* = \frac{b(x_2 - x_1)}{18}(2 + x_1 + x_2)^2, \tag{50}$$

$$\pi_2^* = \frac{b(x_2 - x_1)}{18}(4 - x_1 - x_2)^2. \tag{51}$$

It remains to ensure that the prices in equations (48) and (49) constitute an equilibrium.

Proposition 10. *Let* $\frac{a-c}{b} > \frac{5}{4}$*. Then the duopolists set prices* $p_1^* = c + \frac{b(x_2-x_1)}{3}(2+x_1+x_2)$ *and* $p_2^* = c + \frac{b(x_2-x_1)}{3}(4-x_1-x_2)$ *and make profits* $\pi_1^* = \frac{b(x_2-x_1)}{18}(2+x_1+x_2)^2$ *and* $\pi_2^* = \frac{b(x_2-x_1)}{18}(4-x_1-x_2)^2$*.*

Proof. Note that at p_1^* and p_2^* we have $\tilde{z} = \frac{1}{3} + \frac{1}{6}(x_1 + x_2) \in [0,1]$. It remains to check that all consumers strictly prefer one of the inside goods over the outside good.[18] It suffices to consider the consumers located at $z \in \{0, \tilde{z}, 1\}$. At $z = 0$ we have

$$a - bx_1^2 - p_1^* > 0 \Leftrightarrow \frac{a-c}{b} > \frac{2}{3}x_1^2 - \frac{2}{3}x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_2^2, \tag{52}$$

where the RHS of the second inequality has a maximum of 1. At $z = 1$ we have

$$a - b(1-x_2)^2 - p_2^* > 0 \Leftrightarrow \frac{a-c}{b} > \frac{1}{3}x_1^2 - \frac{4}{3}x_1 - \frac{2}{3}x_2 + \frac{2}{3}x_2^2 + 1, \tag{53}$$

where the RHS again has a maximum of 1. Finally, at $z = \tilde{z}$ we have

$$\begin{aligned} & a - b(\tilde{z} - x_1)^2 - p_1^* = a - b(\tilde{z} - x_2)^2 - p_2^* > 0 \\ \Leftrightarrow \quad & \frac{a-c}{b} > \frac{13}{16}x_1^2 - \frac{11}{9}x_1 - \frac{5}{18}x_1x_2 + \frac{7}{9}x_2 + \frac{13}{16}x_2^2 + \frac{1}{9}, \end{aligned} \tag{54}$$

where the RHS has a maximum of $\frac{5}{4}$. □

Consider first a consumer located at $z \leq \tilde{z}$. Her utility is $a - b(z - x_1)^2 - p_1^*$. Next consider a

[18] We use strict inequalities here to rule out so-called kinked equilibria (Salop 1979).

consumer located at $z \geq \tilde{z}$. Her utility is $a - b(z - x_2)^2 - p_2^*$. Integrating over all consumers yields instantaneous consumer surplus

$$\begin{aligned} \sigma^D(x_1, x_2) &= \int_0^{\tilde{z}} a - b(z - x_1)^2 - p_1^* dz + \int_{\tilde{z}}^1 a - b(z - x_2)^2 - p_2^* dz \\ &= a - c - \frac{b}{3} + \frac{b(x_2 - x_1)}{36}(x_1 + x_2)^2 - \frac{b}{9}\left(4x_1^2 - 11x_1 + 2x_2 + 5x_2^2\right). \end{aligned} \tag{55}$$

Consider an omnipotent social planner that controls prices and thus sets $p_1 = p_2 = c$. Consumers to the left of $\frac{x_1+x_2}{2}$ are served by firm 1, consumers to the right by firm 2. Instantaneous social surplus therefore is

$$\begin{aligned} \omega^D(x_1, x_2) &= \int_0^{\frac{x_1+x_2}{2}} a - c - b(z - x_1)^2 dz + \int_{\frac{x_1+x_2}{2}}^1 a - c - b(z - x_2)^2 dz \\ &= a - c - \frac{b}{3} + \frac{b(x_2 - x_1)}{4}(x_1 + x_2)^2 + bx_2(1 - x_2). \end{aligned} \tag{56}$$

B The Leader's Problem

To characterize the two subsets $\underline{X}_1(t_1)$ and $\overline{X}_1(t_1)$, consider the equation $t_1 = m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)$. Recall that $m' > 0$ and $\frac{\partial \pi_2^D}{\partial x_1} < 0$ if $x_1 \leq x_2$. Hence, provided that it exists, the solution $\tilde{x}_1(t_1)$ to this equation is unique. Moreover, we have $\tilde{x}_1' > 0$ and, by construction, $\tilde{x}_1(t_1) = 0$ at $t_1 = m^{-1}\left(\frac{rF}{\pi_2^D(0,1)}\right)$ and $\tilde{x}_1(t_1) = \frac{1}{2}$ at $t_1 = m^{-1}\left(\frac{rF}{\pi_2^D(\frac{1}{2},1)}\right)$. $\underline{X}_1(t_1)$ and $\overline{X}_1(t_1)$ are therefore given by

$$\underline{X}_1(t_1) = \begin{cases} \varnothing & \text{if} \quad t_1 \in \left[0, m^{-1}\left(\frac{rF}{\pi_2^D(0,1)}\right)\right), \\ [0, \tilde{x}_1(t_1)] & \text{if} \quad t_1 \in \left[m^{-1}\left(\frac{rF}{\pi_2^D(0,1)}\right), m^{-1}\left(\frac{rF}{\pi_2^D(\frac{1}{2},1)}\right)\right], \\ [0, \frac{1}{2}] & \text{if} \quad t_1 \in \left(m^{-1}\left(\frac{rF}{\pi_2^D(\frac{1}{2},1)}\right), \infty\right), \end{cases} \tag{57}$$

$$\overline{X}_1(t_1) = \begin{cases} [0, \frac{1}{2}] & \text{if} \quad t_1 \in \left[0, m^{-1}\left(\frac{rF}{\pi_2^D(0,1)}\right)\right), \\ [\tilde{x}_1(t_1), \frac{1}{2}] & \text{if} \quad t_1 \in \left[m^{-1}\left(\frac{rF}{\pi_2^D(0,1)}\right), m^{-1}\left(\frac{rF}{\pi_2^D(\frac{1}{2},1)}\right)\right], \\ \varnothing & \text{if} \quad t_1 \in \left(m^{-1}\left(\frac{rF}{\pi_2^D(\frac{1}{2},1)}\right), \infty\right). \end{cases} \tag{58}$$

We consider the two subsets $\underline{X}_1(t_1)$ and $\overline{X}_1(t_1)$ in turn.

Case 1: Immediate Entry by the Follower. On the set $\underline{X}_1(t_1)$, we have $t_2^*(t_1, x_1) = t_1$, which is independent of x_1, and the leader's problem simplifies to

$$\max_{x_1 \in \underline{X}_1(t_1)} \int_{t_1}^{\infty} e^{-rt}\pi_1^D(x_1, 1)m(t)dt - e^{-rt_1}F. \tag{59}$$

As long as $t_1 < \infty$, this is equivalent to maximizing instantaneous profits. The solution to the leader's problem on the set $\underline{X}_1(t_1)$ is thus $x_1^\circ(1) = 0$.

Case 2: Deferred Entry by the Follower. On the set $\overline{X}_1(t_1)$, we have $t_2^*(t_1, x_1) = m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)$, which is independent of t_1, and the leader's problem simplifies to

$$\max_{x_1 \in \overline{X}_1(t_1)} \quad \int_{t_1}^{m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)} e^{-rt}\pi^M(x_1)m(t)dt + \int_{m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)}^{\infty} e^{-rt}\pi_1^D(x_1,1)m(t)dt - e^{-rt_1}F. \tag{60}$$

The derivative with respect to x_1 is

$$\begin{aligned}\frac{\partial(.)}{\partial x_1} &= \int_{t_1}^{m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)} e^{-rt}\pi^{M\prime}(x_1)m(t)dt \\ &\quad -e^{-rm^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)}\pi^M(x_1)\frac{(rF)^2}{\pi_2^D(x_1,1)^3}\frac{1}{m'\left(m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)\right)}\frac{\partial \pi_2^D(x_1,1)}{\partial x_1} \\ &\quad + \int_{m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)}^{\infty} e^{-rt}\frac{\partial \pi_1^D(x_1,1)}{\partial x_1}m(t)dt \\ &\quad +e^{-rm^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)}\pi_1^D(x_1,1)\frac{(rF)^2}{\pi_2^D(x_1,1)^3}\frac{1}{m'\left(m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right)\right)}\frac{\partial \pi_2^D(x_1,1)}{\partial x_1}.\end{aligned} \tag{61}$$

The leader's location decision is governed by three considerations. First, by moving towards the center of the market, the leader increases its profits from the monopoly phase since $\pi^{M\prime} > 0$ (first term). Second, the leader decreases its profits from the duopoly phase since $\frac{\partial \pi_1^D}{\partial x_1} < 0$ (third term). Third, by moving towards the center of the market, the leader deters entry by the follower. This increases the duration of the monopoly phase (second term) and decreases the duration of the duopoly phase (fourth term) since $\frac{\partial \pi_2^D}{\partial x_1} < 0$. The net effect is positive since $\pi^M(x_1) > \pi_1^D(x_1, 1)$.

Let $\overline{X}_1^*(t_1)$ denote the set of solutions to the leader's problem on the set $\overline{X}_1(t_1)$. While the solution to the leader's problem may not be unique, we can determine how $\overline{X}_1^*(t_1)$ shifts with t_1. The following lemma provides conditions under which $\overline{X}_1^*(t_1)$ shifts to the left.

Lemma 1. *Suppose $t_1 < t_1'$, $x_1 \in \overline{X}_1^*(t_1)$, and $x_1 < x_1'$. If $x_1 \in \overline{X}_1(t_1')$, then $x_1' \notin \overline{X}_1^*(t_1')$. Moreover, if $x_1 \in \text{int}\overline{X}_1(t_1')$, then $x_1 \notin \overline{X}_1^*(t_1')$.*

Proof. The cross-partial derivative of V_1 with respect to x_1 and t_1 is

$$\frac{\partial^2 V_1}{\partial x_1 \partial t_1} = -e^{-rt_1}\pi^{M\prime}(x_1)m(t_1) \leq 0 \tag{62}$$

with strict inequality whenever $t_1 > 0$.

To see that $x_1' \notin \overline{X}_1^*(t_1')$ note that

$$\begin{aligned}
& V_1\left(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1',1)}\right), x_1', 1\right) - V_1\left(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right), x_1, 1\right) \\
= & \int_{x_1}^{x_1'} \frac{\partial V_1\left(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(s,1)}\right), s, 1\right)}{\partial x_1} ds \\
< & \int_{x_1}^{x_1'} \frac{\partial V_1\left(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(s,1)}\right), s, 1\right)}{\partial x_1} ds \\
= & V_1\left(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1',1)}\right), x_1', 1\right) - V_1\left(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right), x_1, 1\right) \\
\leq & \ 0,
\end{aligned} \tag{63}$$

where the first inequality follows since the cross-partial derivative of V_1 with respect to x_1 and t_1 is negative whenever $t_1 > 0$ and the second inequality follows because $x_1 \in \overline{X}_1^*(t_1)$ (i.e., x_1 is optimal at t_1) and $x_1 < x_1'$ implies $x_1' \in \overline{X}_1(t_1)$ (i.e., x_1' is feasible at t_1). Since $x_1 \in \overline{X}_1(t_1')$ (i.e., x_1 is feasible at t_1') and $V_1\left(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1',1)}\right), x_1', 1\right) < V_1\left(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right), x_1, 1\right)$, x_1' cannot be a solution to the leader's problem at t_1'. This proves the first part of the claim.

To prove the second part of the claim, note that if $x_1 \in \text{int}\overline{X}_1(t_1') \subseteq \text{int}\overline{X}_1(t_1)$, then we have

$$0 = \frac{\partial V_1\left(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right), x_1, 1\right)}{\partial x_1} > \frac{\partial V_1\left(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1,1)}\right), x_1, 1\right)}{\partial x_1}. \tag{64}$$

Hence, x_1 cannot be a solution to the leader's problem at t_1'. □

C SPE

Region 5. Consider the subgame starting at time $t_1 \in \left(m^{-1}\left(\frac{144rF}{25b}\right), \infty\right)$. We have $L(t_1) = V_1(t_1, t_1, 0, 1) = V_2(t_1, 0, 1) = F(t_1)$. Moreover, we have $L' = F' < 0$ and, in fact, $\lim_{t_1\to\infty} L(t_1) = \lim_{t_1\to\infty} F(t_1) = 0$ because of discounting for all $t_1' > t_1$. This implies

$$L(t_1) > L(t_1') = F(t_1'). \tag{65}$$

Hence, it is optimal for a firm to enter at time t_1 and location $x_1 \in X_1^*(t_1) = \{0\}$. Moreover, it is optimal to enter irrespective of the opponent's strategy.

Region 4. Consider the subgame starting at time $t_1 \in \left[m^{-1}\left(\frac{2rF}{b}\right), m^{-1}\left(\frac{144rF}{25b}\right)\right]$. Note that

$$L(t_1) \geq F(t_1) \tag{66}$$

for all t_1. The reason is that the leader is always free to locate at the extreme of the market, which then causes the follower to enter immediately and leads to equal payoffs for both firms. More formally, we have $L(t_1; x_1) \geq L(t_1; 0) = F(t_1; 0) \geq F(t_1; x_1)$ for all $x_1 \in X_1^*(t_1)$ and all t_1. Moreover, as Proposition 11 below shows, we have

$$L(t_1) > L(t_1') \tag{67}$$

for all $t_1' > t_1$. Hence, we have

$$L(t_1) > \max\left\{L(t_1'), F(t_1')\right\} \tag{68}$$

for all $t_1' > t_1$, and it is optimal for a firm to enter at time t_1 and location $x_1 \in X_1^*(t_1)$ irrespective of the opponent's strategy.

Note that, to the extent that $X_1^*(t_1)$ is not a singleton, there may be multiplicity, but this multiplicity has no impact on the outcome of the SPE because, as we will show, the time of first entry is prior to t_1.

Proposition 11. *Suppose $t_1, t_1' \in \left[m^{-1}\left(\frac{2rF}{b}\right), m^{-1}\left(\frac{144rF}{25b}\right)\right]$ and $t_1 < t_1'$. Then $L(t_1) > L(t_1')$.*

Proof. Note that

$$L(t_1) = \max\left\{V_1(t_1, t_1, 0, 1), \max_{x_1 \in \overline{X}_1(t_1)} V_1(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1, 1)}\right), x_1, 1)\right\}. \tag{69}$$

We show that both arguments of the maximum operator are decreasing.

Consider the first argument. Differentiating V_1 with respect to t_1 yields

$$-e^{-rt_1}\pi_1^D(0,1)m(t_1) + re^{-rt_1}F = -e^{-rt_1}\left(\pi_1^D(0,1)m(t_1) - rF\right) \leq 0 \tag{70}$$

whenever $t_1 \geq m^{-1}\left(\frac{rF}{\pi_1^D(0,1)}\right) = m^{-1}\left(\frac{2rF}{b}\right)$ with strict inequality whenever $t_1 > m^{-1}\left(\frac{rF}{\pi_1^D(0,1)}\right) = m^{-1}\left(\frac{2rF}{b}\right)$.

Consider the second argument. Suppose to the contrary that the second argument is nondecreasing. Then there exists $x_1 \in \overline{X}_1^*(t_1)$ and $x_1' \in \overline{X}_1^*(t_1')$ such that

$$V_1(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1, 1)}\right), x_1, 1) - V_1(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1', 1)}\right), x_1', 1) \leq 0. \tag{71}$$

Since $\overline{X}_1(t_1') \subseteq \overline{X}_1(t_1)$, this implies

$$V_1(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1', 1)}\right), x_1', 1) - V_1(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1', 1)}\right), x_1', 1) \leq 0. \tag{72}$$

Using A to denote the LHS of the above inequality, we have

$$A \equiv \int_{t_1}^{t_1'} e^{-rt}\pi^M(x_1')m(t)dt - e^{-rt_1}F + e^{-rt_1'}F. \tag{73}$$

To establish a contradiction, it remains to show that $A > 0$. Clearly, $A = 0$ for $t_1' = t_1$. Differentiating A with respect to t_1' yields

$$e^{-rt_1'}\pi^M(x_1')m(t_1') - re^{-rt_1'}F = e^{-rt_1'}\left(\pi^M(x_1')m(t_1') - rF\right) > 0 \tag{74}$$

whenever $t_1' > m^{-1}\left(\frac{rF}{\pi^M(x_1')}\right) \in \left[m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right), m^{-1}\left(\frac{rF}{a-c-b}\right)\right]$. Our assumption that $t_1' \in \left[m^{-1}\left(\frac{2rF}{b}\right), m^{-1}\left(\frac{144rF}{25b}\right)\right]$ ensures this. Thus $A > 0$ whenever $t_1' > t_1$. □

Region 3. Consider the subgame starting at time $t_1 \in \left(m^{-1}\left(\frac{rF}{a-c-b}\right), m^{-1}\left(\frac{2rF}{b}\right)\right)$. Corollary 1 says that

$$F(t_1) \leq F(t_1') \tag{75}$$

for all $t_1' > t_1$. Moreover, Proposition 12 below shows that

$$L(t_1) > L(t_1') \tag{76}$$

for all $t_1' > t_1$. From equation (66) we also know that

$$L\left(m^{-1}\left(\frac{2rF}{b}\right)\right) \geq F\left(m^{-1}\left(\frac{2rF}{b}\right)\right). \tag{77}$$

It follows that

$$L(t_1) \geq F(t_1) \tag{78}$$

for all t_1. (In fact, the inequality is strict.) Hence, we again have

$$L(t_1) > \max\left\{L(t_1'), F(t_1')\right\} \tag{79}$$

for all $t_1' > t_1$, and it is optimal for a firm to enter at time t_1 and location $x_1 \in X_1^*(t_1)$ irrespective of the opponent's strategy. The comment on multiplicity made at the end of our discussion of region 4 again applies.

Proposition 12. *Suppose* $t_1, t_1' \in \left(m^{-1}\left(\frac{rF}{a-c-b}\right), m^{-1}\left(\frac{2rF}{b}\right)\right)$ *and* $t_1 < t_1'$. *Then* $L(t_1) > L(t_1')$.

Proof. Similar to the proof of Proposition 11. □

Region 2b. Consider the subgame starting at time $t_1 \in \left[t_1^*, m^{-1}\left(\frac{rF}{a-c-b}\right)\right]$. We have $L(t_1) > L(t_1')$ by Assumption 3 and, by Corollary 1, $F(t_1) \leq F(t_1')$ for all $t_1' > t_1$. Consequently, we can repeat the argument for region 3 given above to establish that it is optimal for a firm to enter at time t_1 and location $x_1 \in X_1^*(t_1)$ irrespective of the opponent's strategy. The comment on multiplicity made at the end of our discussion of region 4 again applies.

Region 1. While $L(.)$ is decreasing in regions 3, 4, and 5, $L(.)$ is increasing in region 1.

Proposition 13. *Suppose* $t_1, t_1' \in \left[0, m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right)\right)$ *and* $t_1 < t_1'$. *Then* $L(t_1) < L(t_1')$.

Proof. Note that

$$L(t_1) = \max_{x_1 \in \overline{X}_1(t_1)} V_1(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1, 1)}\right), x_1, 1). \tag{80}$$

Suppose to the contrary that this is nonincreasing. Then there exists $x_1 \in \overline{X}_1^*(t_1)$ and $x_1' \in \overline{X}_1^*(t_1')$ such that

$$V_1(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1, 1)}\right), x_1, 1) - V_1(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1', 1)}\right), x_1', 1) \geq 0. \tag{81}$$

Since $\overline{X}_1(t_1) = \overline{X}_1(t_1') = \left[0, \frac{1}{2}\right]$, this implies

$$V_1(t_1, m^{-1}\left(\frac{rF}{\pi_2^D(x_1, 1)}\right), x_1, 1) - V_1(t_1', m^{-1}\left(\frac{rF}{\pi_2^D(x_1, 1)}\right), x_1, 1) \geq 0. \tag{82}$$

Using A to denote the LHS of the above inequality, we have

$$A \equiv \int_{t_1}^{t_1'} e^{-rt}\pi^M(x_1)m(t)dt - e^{-rt_1}F + e^{-rt_1'}F. \tag{83}$$

To establish a contradiction, it remains to show that $A < 0$. Clearly, $A = 0$ for $t_1' = t_1$. Differentiating A with respect to t_1' yields

$$e^{-rt_1'}\pi^M(x_1)m(t_1') - re^{-rt_1'}F = e^{-rt_1'}\left(\pi^M(x_1)m(t_1') - rF\right) < 0 \tag{84}$$

whenever $t_1' < m^{-1}\left(\frac{rF}{\pi^M(x_1)}\right) \in \left[m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right), m^{-1}\left(\frac{rF}{a-c-b}\right)\right]$. Our assumption that $t_1' \in \left[0, m^{-1}\left(\frac{rF}{a-c-\frac{b}{4}}\right)\right)$ ensures this. Thus $A < 0$ whenever $t_1' > t_1$. □

D Additional Proofs

Proof of Proposition 1. Fix x_1. We start by showing that $t_2^*(x_1) < \infty$. Note that $\lim_{t_2\to\infty} V_2(t_2, x_1, x_2) = 0$. Suppose now that the follower enters at time T in location $x_2^\circ(x_1)$. This guarantees it instantaneous profits of at least $\pi_2^D(x_1, x_2^\circ(x_1))$ from time T on. Note that $\pi_2^D(x_1, x_2^\circ(x_1)) \geq \pi_2^D(\frac{1}{2}, 1)$. The NPV of the follower's payoffs is thus at least

$$\int_T^\infty e^{-rt}\pi_2^D(\frac{1}{2}, 1)m(t)dt - e^{-rT}F \geq e^{-rT}\left(\frac{\pi_2^D(\frac{1}{2}, 1)m(T)}{r} - F\right) > 0, \tag{85}$$

where the last inequality is due to Assumption 2. Hence, the follower enters in finite time. This in turn implies that any solution to its problem entails maximizing its instantaneous profits. Thus $x_2^*(x_1) \in x_2^\circ(x_1)$.

The second derivative of V_2 with respect to t_2 is

$$\frac{\partial^2 V_2}{\partial t_2^2} = re^{-rt_2}\pi_2^D(x_1, x_2)m(t_2) - e^{-rt_2}\pi_2^D(x_1, x_2)m'(t_2) - r^2e^{-rt_2}F. \tag{86}$$

Note that

$$\frac{\partial V_2}{\partial t_2} = 0 \Rightarrow \frac{\partial^2 V_2}{\partial t_2^2} = -e^{-rt_2}\pi_2^D(x_1, x_2)m'(t_2) < 0 \tag{87}$$

for all $x_2 \in x_2^\circ(x_1)$. Hence, V_2 is strictly quasiconcave in t_2 for all $x_2 \in x_2^\circ(x_1)$, which ensures that $t_2^*(x_1)$ is uniquely pinned down. Moreover, at $t_2 = 0$, equation (10) reduces to

$$rF > 0 \tag{88}$$

because of Assumption 1. Evaluating equation (10) at $t_2 = T$ yields

$$-e^{-rT}\pi_2^D(x_1, x_2)m(T) + re^{-rT}F \leq -e^{-rT}\left(\pi_2^D(\frac{1}{2}, 1)m(T) - rF\right) < 0 \tag{89}$$

for all $x_2 \in x_2^\circ(x_1)$, where the first inequality follows because $\pi_2^D(\frac{1}{2}, 1)$ is the minmax instantaneous profit of firm 2 and the second inequality follows because of Assumption 2. It follows that equation (10) has a zero in the interval $(0,T)$ for all $x_2 \in x_2^\circ(x_1)$. This zero in turn determines $t_2^*(x_1)$. □

Proof of Proposition 2. The second derivative of V_1 with respect to t_1 is

$$\frac{\partial^2 V_1}{\partial t_1^2} = re^{-rt_1}\pi^M(x_1)m(t_1) - e^{-rt_1}\pi^M(x_1)m'(t_1) - r^2e^{-rt_1}F. \tag{90}$$

Note that

$$\frac{\partial V_1}{\partial t_1} = 0 \Rightarrow \frac{\partial^2 V_1}{\partial t_1^2} = -e^{-rt_1}\pi^M(x_1)m'(t_1) < 0 \tag{91}$$

for all x_1. Hence, V_1 is strictly quasiconcave in t_1 for all x_1, which ensures that t_1^* is uniquely pinned down. Moreover, at $t_1 = 0$, equation (13) reduces to

$$rF > 0 \tag{92}$$

because of Assumption 1. Evaluating equation (13) at $t_1 = T$ yields

$$-e^{-rT}\pi^M(x_1)m(T) + re^{-rT}F < -e^{-rT}\left(\pi_2^D(\frac{1}{2},1)m(T) - rF\right) < 0, \tag{93}$$

where the first inequality follows because $\pi^M(x) > \pi_i^D(x_1,x_2)$ and the second inequality follows because of Assumption 2. It follows that equation (13) has a zero in the interval $(0,T)$ for all x_1. This zero in turn determines t_1^*. □

Proof of Proposition 3. The leader is always free to choose time $t_2^*(0) - \epsilon$ and location 0, where ϵ is small but positive. Hence,

$$V_1(t_1^*, t_2^*(x_1^*), x_1^*, 1) \geq A + B, \tag{94}$$

where

$$A = \int_{t_2^*(0)-\epsilon}^{t_2^*(0)} e^{-rt}\pi^M(0)m(t)dt - e^{-r(t_2^*(0)-\epsilon)}F + e^{-rt_2^*(0)}F, \tag{95}$$

$$B = \int_{t_2^*(0)}^{\infty} e^{-rt}\pi_1^D(0,1)m(t)dt - e^{-rt_2^*(0)}F. \tag{96}$$

Since $\pi_1^D(0,1) = \pi_2^D(0,1)$, the leader makes the same profits as the follower during the duopoly phase. Hence, $B = V_2(t_2^*(0),0,1) \geq V_2(t_2^*(x_1^*),x_1^*,1)$, where the inequality follows from applying the envelope theorem to the follower's problem in equation (9) and the fact that $\frac{\partial \pi_2^D}{\partial x_1} < 0$ if $x_1 \leq x_2$. It remains to show that $A > 0$ for some $\epsilon > 0$. Clearly, $A = 0$ if $\epsilon = 0$. Differentiating A with respect to ϵ and evaluating the result at $\epsilon = 0$ yields

$$e^{-rt_2^*(0)}\pi^M(0)m(t_2^*(0)) - re^{-rt_2^*(0)}F > e^{-rt_2^*(0)}\left(\pi_2^D(0,1)m(t_2^*(0)) - rF\right) = 0, \tag{97}$$

where the inequality follows because $\pi^M(x) > \pi_i^D(x_1,x_2)$ and the equality follows because of Proposition 1. The claim follows. □

Proof of Proposition 4. Similar to the proof of Proposition 1. □

Proof of Proposition 6. Similar to the proof of Proposition 3. □

Proof of Proposition 7. (i/ii) Note that $\max_{x_1 \in X_1^*(\hat{t}_1)} F(\hat{t}_1; x_1) \geq \min_{x_1 \in X_1^*(\hat{t}_1)} F(\hat{t}_1; x_1) \geq L(\hat{t}_1)$. If $X_1^*(\hat{t}_1)$ is not a singleton, then the first inequality is strict; if $F(\hat{t}_1) > L(\hat{t}_1)$, then the last inequality is strict. Consider $t_1 = \hat{t}_1 + \epsilon$, where ϵ is small but positive. Since $F(.)$ is nondecreasing, we have $F(t_1) > L(\hat{t}_1)$. Using the fact that $L(.)$ is continuous, this implies $F(t_1) \geq L(t_1)$ provided that ϵ is sufficiently small. Hence, $\hat{t}_1$ cannot be the sup of the set in question.

(iii) Suppose that $X_1^*(\tilde{t}_1)$ is a singleton. Then we have $\min_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1) = \max_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1) \geq L(\tilde{t}_1)$. It follows that $\hat{t}_1 \geq \tilde{t}_1$, a contradiction.

(iv) Suppose that $\max_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1) > L(\tilde{t}_1)$ and consider $t_1 = \tilde{t}_1 + \epsilon$, where ϵ is small but positive. Since $F(.)$ is nondecreasing, we have $F(t_1) > L(\tilde{t}_1)$. Using the fact that $L(.)$ is continuous, this implies $F(t_1) \geq L(t_1)$ provided that ϵ is sufficiently small. Hence, $\hat{t}_1 > \tilde{t}_1$, a contradiction.

(v) Suppose not. Then there exists $\bar{\epsilon} > 0$ such that $\tilde{t}_1 + \epsilon \in \tilde{T}_1$ for all $0 \leq \epsilon < \bar{\epsilon}$. Using properties (iii) and (iv), we have

$$\max_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1) = L(\tilde{t}_1) > \min_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1), \tag{98}$$

$$\max_{x_1 \in X_1^*(\tilde{t}_1+\epsilon)} F(\tilde{t}_1 + \epsilon; x_1) = L(\tilde{t}_1 + \epsilon) > \min_{x_1 \in X_1^*(\tilde{t}_1+\epsilon)} F(\tilde{t}_1 + \epsilon; x_1). \tag{99}$$

Since $F(.)$ is nondecreasing, this implies

$$L(\tilde{t}_1) = \max_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1) \leq \min_{x_1 \in X_1^*(\tilde{t}_1+\epsilon)} F(\tilde{t}_1 + \epsilon; x_1) < L(\tilde{t}_1 + \epsilon) \tag{100}$$

for all $0 < \epsilon < \bar{\epsilon}$. Taking the limit as $\epsilon \to 0$ and using the fact that $L(.)$ is continuous yields

$$\max_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1) = \min_{x_1 \in X_1^*(\tilde{t}_1)} F(\tilde{t}_1; x_1), \tag{101}$$

a contradiction, since $X_1^*(\tilde{t}_1)$ is not a singleton. □

Proof of part (ii) of Theorem 1. We again focus on regions 1 and 2a in what follows. Working backwards through time, consider the subgame starting at time $t_1 \in [\tilde{t}_1, t_1^*]$. If a firm enters first at time t_1 and location $\min X_1^*(t_1)$ according to the prescribed strategy, then it gets $L(t_1)$. If the firm deviates from the prescribed strategy, then its rival enters first and the firm gets $F(t_1; \min X_1^*(t_1))$. We claim that $L(t_1) \geq F(t_1; \min X_1^*(t_1))$, so that the firm has no incentive to deviate from the prescribed strategy.

To see this, recall that $\max_{x_1 \in X_1^*(t_1)} F(t_1; x_1) = F(t_1; \min X_1^*(t_1))$. Suppose first that $t_1 \in \tilde{T}_1$. Then $\max_{x_1 \in X_1^*(t_1)} F(t_1; x_1) = L(t_1)$ by part (iv) of Proposition 7 and the claim follows. Next suppose that $t_1 \notin \tilde{T}_1$. Then $\max_{x_1 \in X_1^*(t_1)} F(t_1; x_1) < L(t_1)$ by construction (see equation (20)) and the claim follows.

Continuing to work backwards through time, consider the subgame starting at time $t_1 \in [0, \tilde{t}_1)$. If a firm does not enter according to the prescribed strategy, then it gets $L(\tilde{t}_1) = F(\tilde{t}_1; \min X_1^*(\tilde{t}_1))$, where the equality follows from part (iv) of Proposition 7 and $\max_{x_1 \in X_1^*(t_1)} F(t_1; x_1) = F(t_1; \min X_1^*(t_1))$. If the firm deviates from the prescribed strategy and enters first at time t_1 at location $x_1 \in [0, \frac{1}{2}]$, then it gets at most $L(t_1)$ (and $L(t_1)$ if $x_1 \in X_1^*(t_1)$). We have $L(t_1) < L(\tilde{t}_1)$

for all $t_1 < \tilde{t}_1$ because $L(.)$ is increasing in region 1 and in region 2a by Assumption 3. Taken together, we have

$$L(t_1) < L(\tilde{t}_1) = F(\tilde{t}_1; \min X_1^*(\tilde{t}_1)). \tag{102}$$

Hence, the firm has no incentive to deviate from the prescribed strategy. □

Proof of Proposition 8. The "cutting" equilibria in part (i) of Theorem 1 entail rent equalization by part (ii) of Proposition 7. In line with our focus on the threshold for market size at the time of first entry, we change variables from entry times $\left(t_1^{SPE}(\gamma), t_2^{SPE}(\gamma)\right)$ to market sizes $\left(m_1^{SPE}(\gamma) = m(t_1^{SPE}(\gamma)), m_2^{SPE}(\gamma) = m(t_2^{SPE}(\gamma))\right)$. We establish below that $\frac{\partial m_1^{SPE}(\gamma)}{\partial \gamma} < 0$, conditional on entry locations $\left(x_1^{SPE}(\gamma), x_2^{SPE}(\gamma) = 1\right)$. To simplify the notation in what follows we omit superscripts and indices.

Because there is deferred entry by the follower in regions 1, 2, and 3, we have $m_1 < m_2$ and

$$H_2(m_1, m_2, \gamma) = m_2 - \frac{rF}{\pi_2^D(x_1, 1)} = 0. \tag{103}$$

Using equations (6), (7), and (8), we write the condition for rent equalization as

$$\begin{aligned} H_1(m_1, m_2, \gamma) = &\left(\frac{\pi^M(x_1)}{r-\gamma} m_1 - F\right) e^{-rm^{-1}(m_1)} \\ &- \left(\frac{\pi^M(x_1) - \pi_1^D(x_1, 1) + \pi_2^D(x_1, 1)}{r-\gamma} m_2 - F\right) e^{-rm^{-1}(m_2)} = 0, \end{aligned} \tag{104}$$

where $t = m^{-1}(m) = \frac{1}{\gamma} \ln m$ is the inverse of $m = m(t) = e^{\gamma t}$. Plugging in yields

$$H(m_1, \gamma) = \left(\frac{\pi^M}{r-\gamma} m_1 - F\right) m_1^{-\frac{r}{\gamma}} - \left(\frac{\pi^M - \pi_1^D + \pi_2^D}{r-\gamma} \frac{rF}{\pi_2^D} - F\right) \left(\frac{rF}{\pi_2^D}\right)^{-\frac{r}{\gamma}} = 0, \tag{105}$$

where we have suppressed the dependency of instantaneous profits on entry locations to simplify the notation. Equation (105) determines how m_1 changes with γ.

We begin by establishing that, given γ, equation (105) has a unique solution in m_1 that is attained in the interval $\left[\frac{(r-\gamma)F}{\pi^M}, \frac{rF}{\pi^M}\right]$. Because $\pi^M > \pi_1^D$, $\pi_2^D > 0$, and $0 < \gamma < r$, we have

$$\frac{\pi^M - \pi_1^D + \pi_2^D}{r-\gamma} \frac{rF}{\pi_2^D} - F = \left(\left(\frac{\pi^M - \pi_1^D}{\pi_2^D} + 1\right) \frac{r}{r-\gamma} - 1\right) F > 0. \tag{106}$$

It follows that at a solution to equation (105), it must be that

$$\frac{\pi^M}{r-\gamma} m_1 - F > 0 \Longleftrightarrow m_1 > \frac{(r-\gamma)F}{\pi^M}. \tag{107}$$

Turning from the lower bound to the upper bound, we have

$$\frac{\partial H}{\partial m_1} = -\pi^M \frac{1}{\gamma} m_1^{-\frac{r}{\gamma}} + F \frac{r}{\gamma} m_1^{-\frac{r}{\gamma}-1} = m_1^{-\frac{r}{\gamma}-1} \frac{1}{\gamma} \left(-\pi^M m_1 + rF\right). \tag{108}$$

Hence, $H(m_1, \gamma)$ attains its maximum at $\widehat{m}_1 = \frac{rF}{\pi^M}$ and is increasing (decreasing) to the left (right) of $\widehat{m}_1$. Moreover, $\widehat{m}_1 < \frac{rF}{\pi_2^D}$ because $\pi^M > \pi_2^D$.

We have $\lim_{m_1 \to 0+} H(m_1, \gamma) = -\infty$. We also have $H\left(\frac{rF}{\pi_2^D}, \gamma\right) \geq 0$, with strict inequality as long as $\pi_1^D > \pi_2^D$, and therefore $H(\widehat{m}_1, \gamma) > 0$. It follows that equation (105) has a unique solution that is attained in the interval $\left[\frac{(r-\gamma)F}{\pi^M}, \frac{rF}{\pi^M}\right]$.

Next we characterize how the solution in m_1 to equation (105) changes with γ. We obtain $\frac{\partial m_1}{\partial \gamma}$ from the implicity function theorem as

$$\frac{\partial m_1}{\partial \gamma} = -\frac{\partial H/\partial \gamma}{\partial H/\partial m_1}. \tag{109}$$

Hence, showing that $\frac{\partial m_1}{\partial \gamma} < 0$ amounts to showing that $\frac{\partial H}{\partial m_1}$ and $\frac{\partial H}{\partial \gamma}$ have the same sign.

We have $\frac{\partial H}{\partial m_1}$ in equation (108). Because the solution to equation (105) is attained in the interval $\left[\frac{(r-\gamma)F}{\pi^M}, \frac{rF}{\pi^M}\right]$, it follows that $\frac{\partial H}{\partial m_1} \geq 0$, with strict inequality as long as $m_1 < \frac{rF}{\pi^M}$.

Turning from $\frac{\partial H}{\partial m_1}$ to $\frac{\partial H}{\partial \gamma}$, we have

$$\begin{aligned} \frac{\partial H}{\partial \gamma} &= \frac{\pi^M}{(r-\gamma)^2} m_1^{-\frac{r}{\gamma}+1} + \left(\frac{\pi^M}{r-\gamma} m_1 - F\right) \frac{r}{\gamma^2} m_1^{-\frac{r}{\gamma}} \ln m_1 \\ -\frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} \frac{rF}{\pi_2^D} \left(\frac{rF}{\pi_2^D}\right)^{-\frac{r}{\gamma}} &- \left(\frac{\pi^M - \pi_1^D + \pi_2^D}{r-\gamma} \frac{rF}{\pi_2^D} - F\right) \frac{r}{\gamma^2} \left(\frac{rF}{\pi_2^D}\right)^{-\frac{r}{\gamma}} \ln\left(\frac{rF}{\pi_2^D}\right). \end{aligned} \tag{110}$$

Using equation (105) to rewrite equation (110) yields

$$\begin{aligned} \frac{\partial H}{\partial \gamma} &= \frac{\pi^M}{(r-\gamma)^2} m_1^{-\frac{r}{\gamma}+1} - \frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} m_2^{-\frac{r}{\gamma}+1} \\ &+ \left(\frac{\pi^M - \pi_1^D + \pi_2^D}{r-\gamma} m_2 - F\right) \frac{r}{\gamma^2} m_2^{-\frac{r}{\gamma}} \ln\left(\frac{m_1}{m_2}\right), \end{aligned} \tag{111}$$

where $m_2 = \frac{rF}{\pi_2^D}$. Our goal is to provide a sufficient condition for $\frac{\partial H}{\partial \gamma} > 0$. To this end, we bound the above expression for $\frac{\partial H}{\partial \gamma}$ from below. We have

$$\begin{aligned} \frac{\partial H}{\partial \gamma} &= m_2^{-\frac{r}{\gamma}+1} \left(\frac{\pi^M}{(r-\gamma)^2} \left(\frac{m_1}{m_2}\right)^{-\frac{r}{\gamma}+1} - \frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} \right. \\ &\left. + \left(\frac{\pi^M - \pi_1^D + \pi_2^D}{r-\gamma} - \frac{F}{m_2}\right) \frac{r}{\gamma^2} \ln\left(\frac{m_1}{m_2}\right) \right) \end{aligned} \tag{112}$$

$$\begin{aligned} &\geq m_2^{-\frac{r}{\gamma}+1} \left(\frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} \left(\frac{m_1}{m_2}\right)^{-\frac{r}{\gamma}+1} - \frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} \right. \\ &\left. + \left(\frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} - \frac{F}{(r-\gamma)m_2}\right) \frac{r(r-\gamma)}{\gamma^2} \ln\left(\frac{m_1}{m_2}\right) \right) \end{aligned} \tag{113}$$

$$= m_2^{-\frac{r}{\gamma}+1} \frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} \Bigg(\left(\frac{m_1}{m_2}\right)^{-\frac{r}{\gamma}+1} - 1$$
$$+ \left(1 - \frac{F(r-\gamma)}{(\pi^M - \pi_1^D + \pi_2^D)m_2}\right) \frac{r(r-\gamma)}{\gamma^2} \ln\left(\frac{m_1}{m_2}\right) \Bigg), \tag{114}$$

where the inequality follows from $\pi_1^D \geq \pi_2^D$. Recall that $\ln\left(\frac{m_1}{m_2}\right) < 0$. Hence, to further bound $\frac{\partial H}{\partial \gamma}$ from below, we have to replace $1 - \frac{F(r-\gamma)}{(\pi^M - \pi_1^D + \pi_2^D)m_2}$ by something larger or, equivalently, $\frac{F(r-\gamma)}{(\pi^M - \pi_1^D + \pi_2^D)m_2}$ by something smaller. We know that $m_1 \leq \frac{rF}{\pi^M}$ or, equivalently, that $F(r-\gamma) \geq \frac{\pi^M m_1 (r-\gamma)}{r}$. Hence,

$$\frac{\partial H}{\partial \gamma} \geq m_2^{-\frac{r}{\gamma}+1} \frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} \Bigg(\left(\frac{m_1}{m_2}\right)^{-\frac{r}{\gamma}+1} - 1$$
$$+ \left(1 - \frac{\pi^M}{\pi^M - \pi_1^D + \pi_2^D} \frac{m_1}{m_2} \frac{r-\gamma}{r}\right) \frac{r(r-\gamma)}{\gamma^2} \ln\left(\frac{m_1}{m_2}\right) \Bigg). \tag{115}$$

To further bound $\frac{\partial H}{\partial \gamma}$ from below, we again have to replace $1 - \frac{\pi^M}{\pi^M - \pi_1^D + \pi_2^D} \frac{m_1}{m_2} \frac{r-\gamma}{r}$ by something larger or, equivalently, $\frac{\pi^M}{\pi^M - \pi_1^D + \pi_2^D}$ by something smaller. Because $\pi_1^D \geq \pi_2^D$, we have that $\frac{\pi^M}{\pi^M - \pi_1^D + \pi_2^D} \geq 1$. Hence,

$$\frac{\partial H}{\partial \gamma} \geq m_2^{-\frac{r}{\gamma}+1} \frac{\pi^M - \pi_1^D + \pi_2^D}{(r-\gamma)^2} \Bigg(\left(\frac{m_1}{m_2}\right)^{-\frac{r}{\gamma}+1} - 1$$
$$+ \left(1 - \frac{m_1}{m_2} \frac{r-\gamma}{r}\right) \frac{r(r-\gamma)}{\gamma^2} \ln\left(\frac{m_1}{m_2}\right) \Bigg). \tag{116}$$

The term in brackets is a function of $0 < \frac{\gamma}{r} < 1$ and $0 < \frac{m_1}{m_2} < 1$. It is easily checked numerically that the term in brackets is positive if $\frac{m_1}{m_2} < \frac{1}{5}$ irrespective of the value of $\frac{\gamma}{r}$ or if $\frac{m_1}{m_2} < \frac{1}{4}$ and $\frac{\gamma}{r} < \frac{17}{20}$. Hence, $\frac{\partial H}{\partial \gamma} > 0$ if one of these two conditions holds.

To complete the proof, we provide a sufficient condition for $\frac{m_1}{m_2} < \frac{1}{5}$. Because $m_1 \leq \frac{rF}{\pi^M(x_1)}$ from our earlier analysis of equation (105) and $m_2 = \frac{rF}{\pi_2^D(x_1,1)}$, where we make explicit the dependency of instantaneous profits on entry locations, we have

$$\frac{m_1}{m_2} \leq \frac{\pi_2^D(x_1,1)}{\pi^M(x_1)} = \frac{\frac{b(1-x_1)}{18}(4-x_1-1)^2}{a-c-b(1-x_1)^2} = \frac{(1-x_1)(3-x_1)^2}{18\left(\frac{a-c}{b} - (1-x_1)^2\right)}$$
$$\leq \frac{1}{2\left(\frac{a-c}{b} - 1\right)}, \tag{117}$$

where the last inequality uses $x_1 \in [0, \frac{1}{2}]$. Hence, a sufficient condition for $\frac{m_1}{m_2} < \frac{1}{5}$ is that $\frac{a-c}{b} > \frac{7}{2}$. □

E Data Details

Annual average daily traffic (AADT) is the number of vehicles that pass through a given highway segment during a day, averaged over a calendar year. The segments vary in length and are not aligned with exits and intersections. To allocate AADT to markets, we consider all segments that are within 1000m of any road crossing in the market. Along a highway, we first average the traffic volumes of multiple segments, with each segment's length as weight. This gives the average traffic volume for that highway in the market. We then sum the average traffic volumes of all highways in the market.

For land prices, we use the landpriceindex field from the 2018 LANDDATA.MSA file available from the American Enterprise Institute at `https://www.aei.org/housing/land-price-indicators/`. The land price index normalizes land prices in different MSAs relative to an average value of one. We use the land price for the MSA that is closest to a market in terms of Euclidean distance.

Vegetation data are from the United States Geological Survey LANDFIRE Data Distribution Site at `https://landfire.gov/viewer/viewer.html`. We use vegetation data for 2001, the middle of our sample period. Figure 6 shows these data for the U.S. and for a close-up. The map visualization tool has both a detailed legend and provides the ability to click anywhere on the map to get detailed vegetation data. Measurement tools for distance are also included. Two undergraduate research assistants visually assessed the proportion of each vegetation type within 500m and 1000m of the market center. The correlations in the vegetation measures are between 0.56 and 0.80 and we use the average for each market. We use these proportions to calculate the area (in square kilometers) occupied by each vegetation type. Table 12 provides summary statistics.

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Figure 1: Example of a "touching" equilibrium. Payoffs (left panel), times of entry (middle panel), and locations of entry (right panel). A dark (bright) line designates the first (second) entrant. Parameters are $a = 3$, $b = 1$, $c = 0$, $F = 3.47$, $r = 0.05$, $\tau_1 = 22.73$, $\mu_1 = 0.09$, and $\tau_2 = 25$.

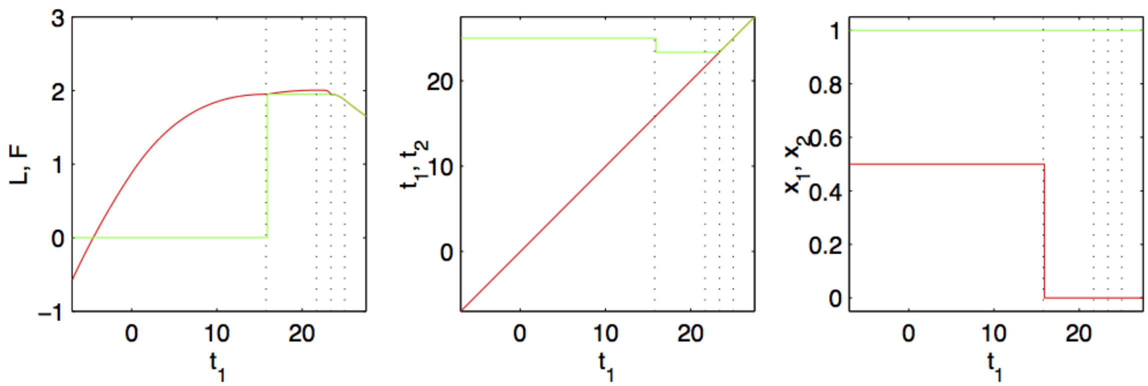


Figure 2: Example of an intermediate location. Payoffs (left panel), times of entry (middle panel), and locations of entry (right panel). A dark (bright) line designates the first (second) entrant. Parameters are $a = 3$, $b = 1$, $c = 0$, $F = 1.58$, $r = 0.05$, $\tau_1 = 22.73$, $\mu_1 = 0.27$, and $\tau_2 = 25$.

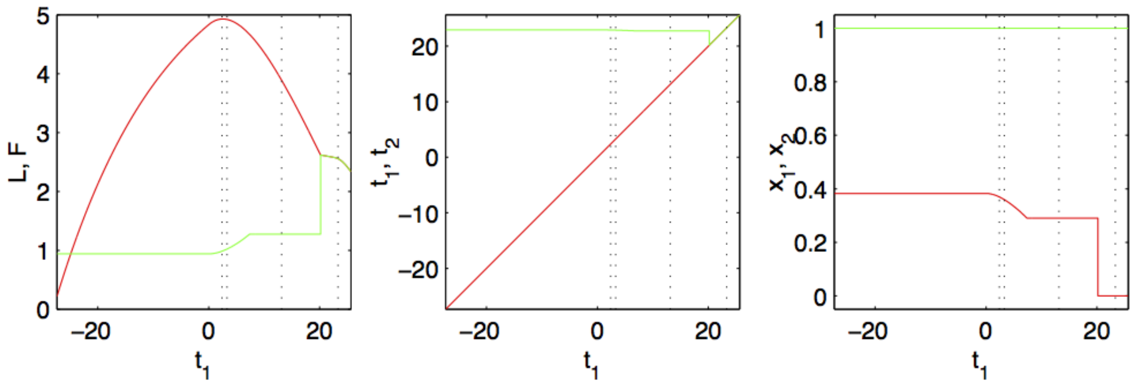
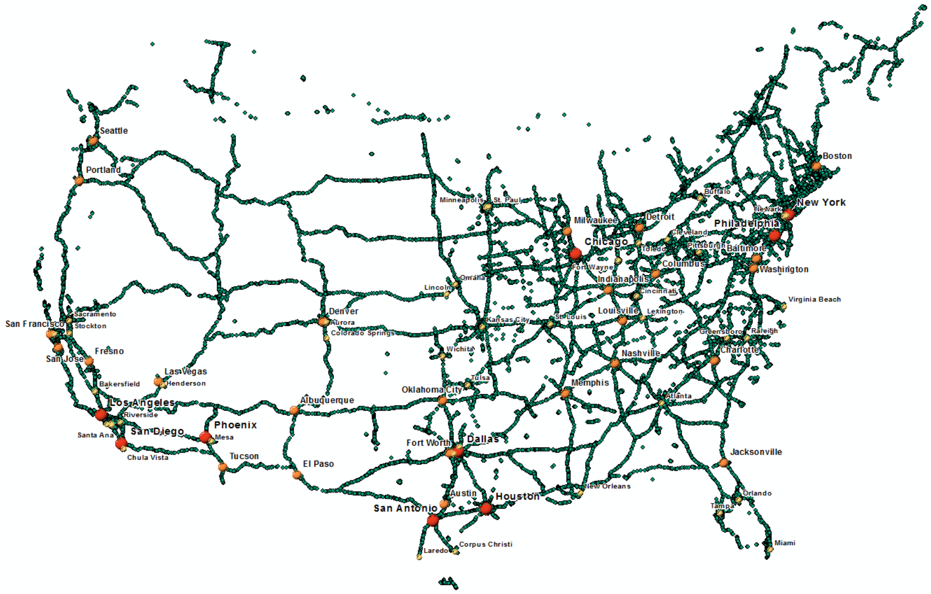
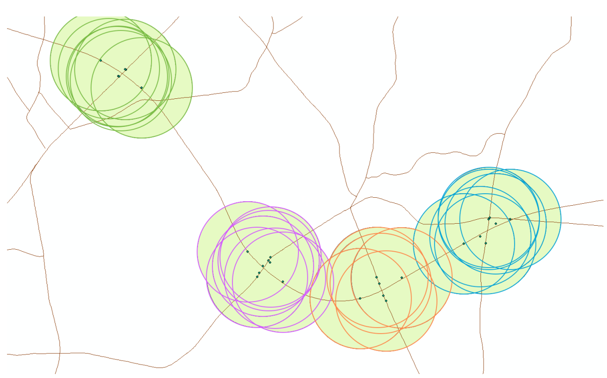


Figure 3: Market definition.



(a) Highway exits and intersections

(b) Close-up of Durham, NC: exits and intersections



(c) Close-up of Durham, NC: four markets

Figure 4: Rate of market growth and market size.

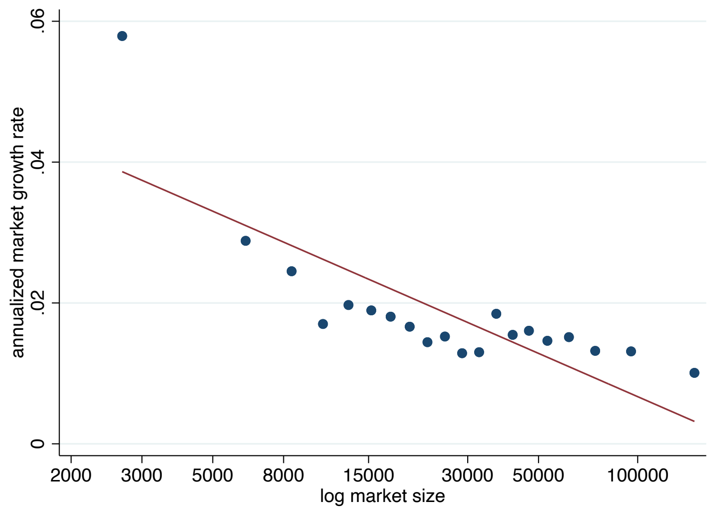
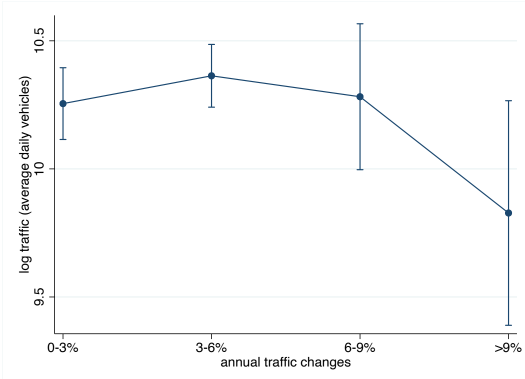
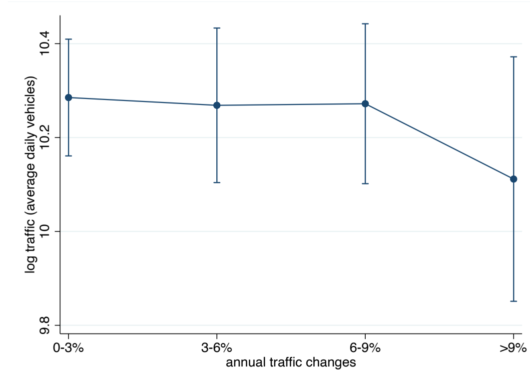


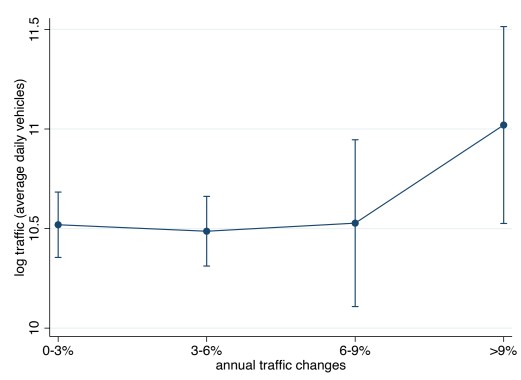
Figure 5: Market size threshold and rate of market growth.



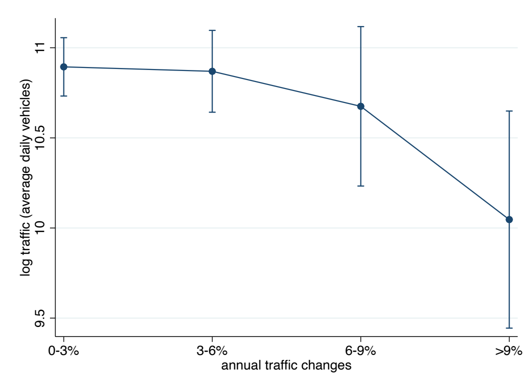
(a) Gas stations ($N = 272$)



(b) Restaurants ($N = 286$)

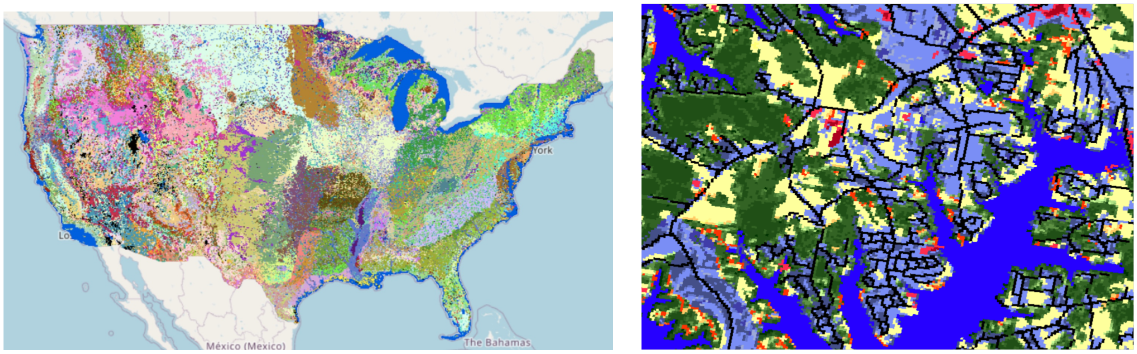


(c) Hotels ($N = 196$)



(d) Three-star hotels ($N = 174$)

Figure 6: 2001 vegetation data. Blue = water, purple/red/black = developed, yellow = field and grass, green = forest, brown=shrub and herb



(a) U.S. (b) Close-up

	$x_2 = \frac{1}{2}$	$x_2 = 1$
$x_1 = 0$	$(\frac{25b}{144}, \frac{49b}{144})$	$(\frac{b}{2}, \frac{b}{2})$
$x_1 = \frac{1}{2}$	$(0, 0)$	$(\frac{49b}{144}, \frac{25b}{144})$

Table 1: Instantaneous profits $(\pi_1^D(x_1, x_2), \pi_2^D(x_1, x_2))$.

#	a	F	τ_1	μ_1	τ_2		t_1	t_2	x_1	L	F	C
1	3	3.16	9.09	0.09	10	SPE	5.43	9.32	0.00	4.22	4.22	23.92
						W	5.41	∞	0.50			
2	3	3.16	13.64	0.09	15	SPE	8.27	13.97	0.00	3.32	3.32	18.90
						W	8.12	∞	0.50			
3	3	3.16	22.73	0.09	25	SPE	14.15	23.29	0.00	2.05	2.05	11.81
						W	13.53	∞	0.50			

Table 2: Parameterizations with $t_1^{SPE} > t_1^{W}$. $x_2^{SPE} = 1$ omitted.

#	a	F	τ_1	μ_1	τ_2		t_1	t_2	x_1	L	F	C	Σ
1	3	2.53	9.09	0.09	10	SPE	3.30	9.25	0.00	4.62	4.62	23.99	33.23
						SEQPE	6.31	9.25	0.00	4.69	4.62	23.91	33.22
2	3	2.53	13.64	0.09	15	SPE	5.16	13.88	0.00	3.63	3.63	18.99	26.25
						SEQPE	9.47	13.88	0.00	3.72	3.63	18.89	26.24
3	3	2.53	22.73	0.09	25	SPE	9.23	23.13	0.00	2.25	2.24	11.92	16.41
						SEQPE	15.78	23.13	0.00	2.34	2.24	11.80	16.39
4	3	2.84	4.55	0.09	5	SPE	2.17	4.64	0.00	5.63	5.63	30.30	41.55
						SEQPE	3.55	4.64	0.00	5.66	5.63	30.25	41.54
5	3	2.84	8.18	0.09	10	SPE	3.34	8.57	0.00	4.50	4.50	24.54	33.54
						SEQPE	6.39	8.57	0.00	4.58	4.50	24.45	33.54
6	3	2.84	9.09	0.09	10	SPE	4.44	9.28	0.00	4.42	4.42	23.96	32.80
						SEQPE	7.10	9.28	0.00	4.48	4.42	23.88	32.78
7	3	2.84	12.27	0.09	15	SPE	5.21	12.85	0.00	3.57	3.57	19.64	26.78
						SEQPE	9.59	12.85	0.00	3.67	3.57	19.53	26.78
8	3	2.84	13.64	0.09	15	SPE	6.82	13.93	0.00	3.47	3.47	18.95	25.90
						SEQPE	10.65	13.93	0.00	3.54	3.47	18.85	25.86
9	3	2.84	22.73	0.09	25	SPE	11.86	23.21	0.00	2.15	2.15	11.86	16.16
						SEQPE	17.76	23.21	0.00	2.22	2.15	11.75	16.12
10	3	3.16	4.09	0.09	5	SPE	1.95	4.32	0.00	5.42	5.42	30.64	41.49
						SEQPE	3.55	4.32	0.00	5.47	5.42	30.59	41.48
11	3	3.16	4.55	0.09	5	SPE	2.68	4.66	0.00	5.38	5.38	30.28	41.03
						SEQPE	3.95	4.66	0.00	5.40	5.38	30.23	41.01
12	3	3.16	8.18	0.09	10	SPE	4.02	8.63	0.00	4.30	4.30	24.50	33.10
						SEQPE	7.10	8.63	0.00	4.38	4.30	24.41	33.08
13	3	3.16	9.09	0.09	10	SPE	5.43	9.32	0.00	4.22	4.22	23.92	32.37
						SEQPE	7.89	9.32	0.00	4.27	4.22	23.84	32.33
14	3	3.16	12.27	0.09	15	SPE	6.21	12.95	0.00	3.40	3.40	19.60	26.41
						SEQPE	10.65	12.95	0.00	3.50	3.40	19.48	26.39
15	3	3.16	13.64	0.09	15	SPE	8.27	13.97	0.00	3.32	3.32	18.90	25.54
						SEQPE	11.84	13.97	0.00	3.37	3.32	18.81	25.50
16	3	3.16	22.73	0.09	25	SPE	14.15	23.29	0.00	2.05	2.05	11.81	15.91
						SEQPE	19.73	23.29	0.00	2.10	2.05	11.71	15.86

Table 3: Parameterizations with $\Sigma^{SPE} > \Sigma^{SEQPE}$. $x_2^{SPE} = 1$ and $x_2^{SEQPE} = 1$ omitted.

#	a	F	τ_1	μ_1	τ_2		t_1	t_2	x_1	L	F	C	Σ, W	$\frac{\Sigma^{SPE}}{W^W}$
1	9	3.16	4.55	0.55	50	SPE	-66.78	40.91	0.50	0.07	0.03	23.07	23.17	0.21
						W	0.15	∞	0.50				109.35	
2	9	3.16	4.55	0.45	50	SPE	-64.96	42.42	0.50	0.05	0.03	21.42	21.50	0.22
						W	0.18	∞	0.50				99.95	
3	9	3.16	4.55	0.64	50	SPE	-68.24	38.64	0.50	0.06	0.03	25.68	25.77	0.22
						W	0.13	∞	0.50				118.76	
4	6	3.16	4.55	0.55	50	SPE	-58.48	40.91	0.50	0.05	0.03	15.45	15.53	0.22
						W	0.22	∞	0.50				71.51	
5	6	3.16	4.55	0.45	50	SPE	-56.66	42.42	0.50	0.05	0.03	14.33	14.41	0.22
						W	0.27	∞	0.50				65.27	

Table 4: Examples of rent dissipation: lowest values of $\frac{\Sigma^{SPE}}{W^W}$. $x_2^{SPE} = 1$ omitted.

#	a	F	τ_1	μ_1	τ_2		t_1	t_2	x_1	L	F	C	Σ, W	$\frac{\Sigma^{SEQPE}}{W^W}$
1	3	3.16	22.73	0.09	25	SEQPE	19.73	23.29	0.00	2.10	2.05	11.71	15.86	0.93
						W	13.53	∞	0.50				16.97	
2	3	3.16	13.64	0.18	15	SEQPE	5.92	13.86	0.00	3.82	3.33	19.23	26.37	0.94
						W	4.06	∞	0.50				28.16	
3	3	3.16	12.27	0.09	15	SEQPE	10.65	12.95	0.00	3.50	3.40	19.48	26.39	0.94
						W	7.31	∞	0.50				28.16	
4	3	3.16	8.18	0.27	10	SEQPE	2.37	8.29	0.00	5.19	4.33	25.13	34.66	0.94
						W	1.62	∞	0.50				36.96	
5	3	3.16	9.09	0.27	10	SEQPE	2.63	9.14	0.00	5.14	4.24	24.47	33.86	0.94
						W	1.80	∞	0.50				36.09	

Table 5: Examples of rent dissipation: lowest values of $\frac{\Sigma^{SEQPE}}{W^W}$. $x_2^{SEQPE} = 1$ omitted.

#	a	F	τ_1	μ_1	τ_2		t_1	t_2	x_1	L	F	C	Σ, W	$\frac{W^W-\Sigma^{SPE}}{W^W-\Sigma^{SEQPE}}$
1	9	3.16	9.09	0.55	50	SPE	-64.07	41.82	0.50	0.04	0.03	21.94	22.01	215.40
						SEQPE	0.30	41.82	0.50	74.62	0.03	21.94	96.59	
						W	0.30	∞	0.50				96.93	
2	9	3.16	4.55	0.36	50	SPE	-62.83	43.51	0.50	0.04	0.03	20.25	20.32	219.75
						SEQPE	0.23	43.51	0.50	69.95	0.03	20.25	90.23	
						W	0.22	∞	0.50				90.55	
3	9	3.16	4.55	0.64	50	SPE	-68.24	38.64	0.50	0.06	0.03	25.68	25.77	227.83
						SEQPE	0.13	38.64	0.50	92.63	0.03	25.68	118.35	
						W	0.13	∞	0.50				118.76	
4	9	3.16	4.55	0.45	50	SPE	-64.96	42.42	0.50	0.05	0.03	21.42	21.50	232.49
						SEQPE	0.18	42.42	0.50	78.17	0.03	21.41	99.61	
						W	0.18	∞	0.50				99.95	
5	9	3.16	4.55	0.55	50	SPE	-66.78	40.91	0.50	0.07	0.03	23.07	23.17	236.69
						SEQPE	0.15	40.91	0.50	85.89	0.03	23.07	108.99	
						W	0.15	∞	0.50				109.35	

Table 6: Examples of rent dissipation: highest values of $\frac{W^W-\Sigma^{SPE}}{W^W-\Sigma^{SEQPE}}$. $x_2^{SPE}=1$ and $x_2^{SEQPE}=1$ omitted.

Variable	OLS	IV 1		IV 2	
	Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size
rate of market growth	-1.6603*	-1.8313	-1.3356*	0.1492	-1.6646**
	(0.8865)	(1.2771)	(0.7758)	(0.2395)	(0.8390)
log distance to center (m)	-0.0955**		0.0499		-0.0974**
	(0.0437)		(0.1143)		(0.0390)
land price index	0.0453	0.2399	0.0104	0.0854	0.0458
	(0.2972)	(0.2761)	(0.3073)	(0.0941)	(0.2809)
largest road interstate highway	0.7539**	0.7124	0.5697*	0.0967	0.7564**
	(0.3378)	(0.4598)	(0.3176)	(0.1052)	(0.3163)
largest road freeway/principle arterial	0.3684**	-0.4512*	0.4465***	-0.0068	0.3674**
	(0.1725)	(0.2270)	(0.1514)	(0.0272)	(0.1662)
three-road intersection	0.0630	0.2587**	0.0215	-0.0248	0.0636
	(0.0987)	(0.1120)	(0.0932)	(0.0173)	(0.0934)
four-road intersection	0.2153	0.0374	0.1933	-0.0126	0.2156
	(0.1454)	(0.1350)	(0.1373)	(0.0406)	(0.1370)
five-road intersection	0.3685**	0.3060**	0.2988*	0.0083	0.3695**
	(0.1585)	(0.1177)	(0.1664)	(0.0580)	(0.1504)
log diagonal of bounding box (m)		0.3572*		-0.0347	
		(0.1891)		(0.0554)	
MI X log diagonal of bounding box (m)		-0.4749**		-0.1163**	
		(0.1972)		(0.0498)	
OH X log diagonal of bounding box (m)		0.8688***		0.0381	
		(0.2539)		(0.0763)	
TX X log diagonal of bounding box (m)		-0.3850*		-0.0112	
		(0.1972)		(0.0527)	
NC X log diagonal of bounding box (m)		-0.1556		0.0079	
		(0.1981)		(0.0578)	
MS X log diagonal of bounding box (m)		0.6081**		-0.0844	
		(0.2970)		(0.0698)	
GA X log diagonal of bounding box (m)		1.2789***		-0.0038	
		(0.2362)		(0.0599)	
log distance of second entrant to center (m)				1.0114***	
				(0.0211)	
number of firms within 500m	0.0021	0.0010	0.0018	-0.0003	0.0021
	(0.0029)	(0.0030)	(0.0027)	(0.0006)	(0.0028)
number of firms within 1000m	0.0016	-0.0023	0.0020	0.0003	0.0015
	(0.0015)	(0.0016)	(0.0015)	(0.0004)	(0.0014)
forest coverage 500m circle (sq. km)	-0.9128	1.6131	-1.2056*	-0.5134	-0.9089
	(0.6151)	(1.0717)	(0.6918)	(0.3561)	(0.5812)
forest coverage 1000m circle (sq. km)	0.0516	-0.1946	0.0967	0.1713	0.0510
	(0.1890)	(0.3185)	(0.2002)	(0.1017)	(0.1793)
field/grass coverage 500m circle (sq. km)	-0.7284	-0.5975	-0.6763	-0.1480	-0.7291
	(0.5904)	(1.0525)	(0.6224)	(0.2212)	(0.5576)
field/grass coverage 1000m circle (sq. km)	-0.1001	0.1163	-0.1093	0.0724	-0.0999
	(0.2114)	(0.2831)	(0.2148)	(0.0778)	(0.2000)
water coverage 500m circle (sq. km)	-0.3888	0.5812	-0.5444	0.3373	-0.3867
	(1.4231)	(1.9522)	(1.3645)	(0.6550)	(1.3530)
water coverage 1000m circle (sq. km)	0.2305	-0.0371	0.2508	-0.0617	0.2302
	(0.4461)	(0.7047)	(0.4293)	(0.1537)	(0.4236)
R-squared	0.240	0.234	0.206	0.967	0.240
N	272	272	268	272	268
Kleibergen-Paap rk Wald F statistic		73.53		1263.99	
Hansen J statistic		4.781		5.65	

Table 7: Regression results for gas stations. $p < 0.10$ (*), $p < 0.05$ (**), $p < 0.01$ (***). Standard errors clustered by state.

	OLS	IV 1		IV 2	
Variable	Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size
rate of market growth	-1.0371	0.8568	-1.1947	-0.0800	-1.0389
	(1.1491)	(1.8118)	(1.1623)	(0.2842)	(1.0881)
log distance to center (m)	-0.0340		0.0621		-0.0329
	(0.0458)		(0.1975)		(0.0444)
land price index	-0.1003	-0.3250	-0.0684	0.0264	-0.1000
	(0.2375)	(0.3199)	(0.2302)	(0.0387)	(0.2260)
largest road interstate highway	0.9302***	0.6281**	0.8141*	0.0530	0.9289***
	(0.3216)	(0.2858)	(0.4315)	(0.0404)	(0.3065)
largest road freeway/principle arterial	0.5511***	-0.0709	0.5525***	-0.0153	0.5511***
	(0.1154)	(0.1815)	(0.1081)	(0.0212)	(0.1096)
three-road intersection	0.1684*	-0.1442	0.1757*	0.0079	0.1685*
	(0.0965)	(0.1336)	(0.1018)	(0.0218)	(0.0916)
four-road intersection	0.2850***	0.1235	0.2634***	0.0122	0.2847***
	(0.1006)	(0.1956)	(0.0911)	(0.0305)	(0.0956)
five-road intersection	0.3064**	-0.0163	0.2927**	-0.0184	0.3062***
	(0.1222)	(0.1967)	(0.1210)	(0.0277)	(0.1157)
log diagonal of bounding box (m)		0.2568*		-0.0419	
		(0.1270)		(0.0300)	
MI X log diagonal of bounding box (m)		0.1266		-0.0763*	
		(0.2125)		(0.0399)	
OH X log diagonal of bounding box (m)		0.0025		-0.0392	
		(0.2548)		(0.0507)	
TX X log diagonal of bounding box (m)		0.1253		0.0215	
		(0.1346)		(0.0191)	
NC X log diagonal of bounding box (m)		0.3621**		-0.0259	
		(0.1538)		(0.0356)	
MS X log diagonal of bounding box (m)		2.0985***		0.1302	
		(0.4324)		(0.0881)	
GA X log diagonal of bounding box (m)		0.0530		-0.0625*	
		(0.1933)		(0.0311)	
log distance of second entrant to center (m)				1.0007***	
				(0.0152)	
number of firms within 500m	-0.0061	-0.0558***	0.0001	-0.0022	-0.0060
	(0.0145)	(0.0158)	(0.0192)	(0.0026)	(0.0141)
number of firms within 1000m	0.0056*	0.0004	0.0055*	0.0005	0.0056*
	(0.0032)	(0.0031)	(0.0031)	(0.0008)	(0.0031)
forest coverage 500m circle (sq. km)	-0.0036	0.9972	-0.1194	0.2059	-0.0049
	(0.6884)	(0.8599)	(0.7752)	(0.1800)	(0.6561)
forest coverage 1000m circle (sq. km)	-0.1141	-0.2372	-0.0872	-0.0290	-0.1138
	(0.1612)	(0.1963)	(0.1730)	(0.0592)	(0.1540)
field/grass coverage 500m circle (sq. km)	-0.4654	3.1862**	-0.7397	0.1585	-0.4686
	(0.6112)	(1.2555)	(0.8535)	(0.2329)	(0.5823)
field/grass coverage 1000m circle (sq. km)	-0.0444	-0.8408**	0.0262	-0.0261	-0.0436
	(0.1806)	(0.3237)	(0.2357)	(0.0601)	(0.1734)
water coverage 500m circle (sq. km)	-3.3385**	-1.7108	-3.1573**	0.2815	-3.3364**
	(1.5710)	(2.0903)	(1.5294)	(0.5546)	(1.4935)
water coverage 1000m circle (sq. km)	0.5949*	-0.2055	0.6143*	-0.0512	0.5951*
	(0.3442)	(0.4178)	(0.3197)	(0.1299)	(0.3272)
R-squared	0.180	0.268	0.168	0.978	0.180
N	286	286	281	286	281
Kleibergen-Paap rk Wald F statistic		29.49		1922.07	
Hansen J statistic		5.51		6.25	

Table 8: Regression results for restaurants. $p < 0.10$ (*), $p < 0.05$ (**), $p < 0.01$ (***). Standard errors clustered by state.

	OLS	IV 1		IV 2	
Variable	Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size
rate of market growth	-2.2372	1.8441	-2.3031	-0.0291	-2.2280
	(1.5468)	(1.3285)	(1.6976)	(0.2154)	(1.4466)
log distance to center (m)	0.0074		0.0393		0.0029
	(0.0609)		(0.3297)		(0.0585)
land price index	0.1558	-0.6666***	0.1768	0.0436	0.1528
	(0.1796)	(0.1911)	(0.2584)	(0.0308)	(0.1665)
largest road interstate highway	1.0368***	0.4164	1.0077***	0.1768***	1.0409***
	(0.2306)	(0.3461)	(0.3685)	(0.0497)	(0.2153)
largest road freeway/principle arterial	0.5175**	-0.1277	0.5230**	0.0464	0.5168**
	(0.2317)	(0.2562)	(0.2235)	(0.0351)	(0.2170)
three-road intersection	0.3450**	-0.0200	0.3438***	0.0208	0.3452***
	(0.1278)	(0.1440)	(0.1208)	(0.0256)	(0.1191)
four-road intersection	0.3840**	0.4291***	0.3696	0.0270	0.3860**
	(0.1891)	(0.1311)	(0.2524)	(0.0273)	(0.1759)
five-road intersection	0.6594***	0.0440	0.6517***	-0.0094	0.6604***
	(0.1550)	(0.1620)	(0.1764)	(0.0369)	(0.1451)
log diagonal of bounding box (m)		0.3396**		-0.0309	
		(0.1426)		(0.0200)	
MI X log diagonal of bounding box (m)		-0.1871		0.0161	
		(0.1713)		(0.0252)	
OH X log diagonal of bounding box (m)		0.2824		0.0462	
		(0.2102)		(0.0443)	
TX X log diagonal of bounding box (m)		-0.0973		0.0503***	
		(0.1359)		(0.0155)	
NC X log diagonal of bounding box (m)		0.2269		0.0213	
		(0.1397)		(0.0311)	
MS X log diagonal of bounding box (m)		0.7116***		0.0297	
		(0.1701)		(0.0482)	
GA X log diagonal of bounding box (m)		0.2984		0.0908***	
		(0.2442)		(0.0298)	
log distance of second entrant to center (m)				0.9726***	
				(0.0168)	
number of firms within 500m	-0.0013	0.0013	-0.0013	0.0000	-0.0013
	(0.0029)	(0.0021)	(0.0026)	(0.0003)	(0.0027)
number of firms within 1000m	0.0004	-0.0004	0.0004	0.0000	0.0004
	(0.0016)	(0.0008)	(0.0014)	(0.0001)	(0.0014)
forest coverage 500m circle (sq. km)	0.0651	2.5703***	-0.0345	0.1007	0.0792
	(0.8269)	(0.8542)	(1.4368)	(0.1450)	(0.7664)
forest coverage 1000m circle (sq. km)	-0.1543	-0.4567**	-0.1356	-0.0150	-0.1570
	(0.1738)	(0.2078)	(0.2453)	(0.0400)	(0.1609)
field/grass coverage 500m circle (sq. km)	1.4208*	0.0364	1.4066**	-0.2373	1.4227**
	(0.7551)	(1.2515)	(0.7094)	(0.2729)	(0.7044)
field/grass coverage 1000m circle (sq. km)	-0.3877	-0.0361	-0.3841*	0.0824	-0.3882*
	(0.2520)	(0.3267)	(0.2293)	(0.0601)	(0.2347)
water coverage 500m circle (sq. km)	0.6240	-0.1184	0.6038	0.1287	0.6269
	(1.8309)	(2.5191)	(1.7525)	(0.6004)	(1.7059)
water coverage 1000m circle (sq. km)	-0.1016	0.1842	-0.1069	0.0992	-0.1008
	(0.3919)	(0.4435)	(0.3904)	(0.1117)	(0.3657)
R-squared	0.241	0.403	0.240	0.984	0.241
N	196	196	193	196	193
Kleibergen-Paap rk Wald F statistic		19.27		2272.74	
Hansen J statistic		6.75		9.03	

Table 9: Regression results for hotels. $p < 0.10$ (*), $p < 0.05$ (**), $p < 0.01$ (***). Standard errors clustered by state.

Variable	OLS	IV 1		IV 2	
	Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size
rate of market growth	-1.7374**	-1.3857**	-1.8771**	-0.0371	-1.8299**
	(0.7858)	(0.5758)	(0.7677)	(0.2831)	(0.7162)
log distance to center (m)	-0.1208		-0.2287		-0.1923*
	(0.0934)		(0.2229)		(0.0998)
land price index	0.5909*	-0.2779	0.5623*	0.3756*	0.5719*
	(0.3304)	(0.4562)	(0.3252)	(0.2096)	(0.3082)
largest road interstate highway	0.4549	-0.2047	0.5492	0.1431	0.5174
	(0.5095)	(0.5067)	(0.5704)	(0.1391)	(0.4784)
largest road freeway/principle arterial	0.2947	-0.5132	0.2476	0.0049	0.2635
	(0.5134)	(0.3683)	(0.4219)	(0.1260)	(0.4632)
three-road intersection	0.0310	0.0172	0.0406	0.0019	0.0374
	(0.1806)	(0.1163)	(0.1709)	(0.0627)	(0.1678)
four-road intersection	0.0614	0.0989	0.0903	-0.0378	0.0805
	(0.1950)	(0.2141)	(0.1624)	(0.0643)	(0.1760)
five-road intersection	0.1514	0.1710	0.2016	0.0781	0.1847
	(0.2476)	(0.1570)	(0.2446)	(0.0688)	(0.2273)
log diagonal of bounding box (m)		0.5476***		-0.0688	
		(0.1293)		(0.0660)	
MI X log diagonal of bounding box (m)		0.0816		0.1419***	
		(0.1516)		(0.0393)	
OH X log diagonal of bounding box (m)		-0.1412		-0.0570	
		(0.1099)		(0.0776)	
TX X log diagonal of bounding box (m)		-0.0753		0.0435	
		(0.1340)		(0.0459)	
NC X log diagonal of bounding box (m)		0.2388		0.0165	
		(0.1789)		(0.0615)	
MS X log diagonal of bounding box (m)		0.9509**		0.2212*	
		(0.3599)		(0.1279)	
GA X log diagonal of bounding box (m)		-0.0713		0.1081***	
		(0.0910)		(0.0378)	
log distance of second entrant to center (m)				0.9243***	
				(0.0665)	
number of firms within 500m	-0.0018	-0.0021	-0.0020	-0.0005	-0.0019
	(0.0017)	(0.0032)	(0.0018)	(0.0007)	(0.0016)
number of firms within 1000m	0.0006	0.0001	0.0006	0.0001	0.0006
	(0.0004)	(0.0008)	(0.0004)	(0.0002)	(0.0004)
forest coverage 500m circle (sq. km)	1.5246	-0.8346	1.4507	0.1327	1.4756
	(1.0800)	(1.5022)	(0.9896)	(0.7187)	(0.9845)
forest coverage 1000m circle (sq. km)	-0.4308	0.3594	-0.4080	-0.0068	-0.4157
	(0.3207)	(0.3647)	(0.2900)	(0.1467)	(0.2936)
field/grass coverage 500m circle (sq. km)	3.5179*	-0.1188	3.5774**	-0.6064	3.5573**
	(1.8696)	(1.5523)	(1.7000)	(0.4293)	(1.7118)
field/grass coverage 1000m circle (sq. km)	-1.1475**	-0.0687	-1.1731***	0.0999	-1.1645***
	(0.4580)	(0.3692)	(0.4150)	(0.1173)	(0.4152)
water coverage 500m circle (sq. km)	-2.0660	-0.2568	-2.1651	-1.3025	-2.1317
	(1.5716)	(1.9443)	(1.4869)	(1.4209)	(1.4351)
water coverage 1000m circle (sq. km)	-0.0997	-0.2357	-0.1176	0.4347*	-0.1116
	(0.4878)	(0.3369)	(0.4312)	(0.2484)	(0.4472)
R-squared	0.202	0.527	0.192	0.919	0.197
N	174	174	170	174	170
Kleibergen-Paap rk Wald F statistic		7.84		110.13	
Hansen J statistic		7.39		7.413	

Table 10: Regression results for three-star hotels. $p < 0.10$ (*), $p < 0.05$ (**), $p < 0.01$ (***). Standard errors clustered by state.

	OLS	IV 1		IV 2	
Variable	Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size	1st Stage: Log Distance	2nd Stage: Log Market Size
rate of market growth	-1.0922	2.5708**	-0.7747	0.0661	-1.0681
	(1.0928)	(1.1749)	(1.1386)	(0.2461)	(1.0344)
log distance to center (m)	-0.0197		-0.1538		-0.0299
	(0.0446)		(0.1795)		(0.0411)
land price index	0.0623	-0.1152	0.0421	-0.0275	0.0608
	(0.1109)	(0.1371)	(0.1048)	(0.0212)	(0.1047)
largest road interstate highway	0.5830**	0.4469	0.7407***	0.1005	0.5950**
	(0.2538)	(0.3095)	(0.2504)	(0.0775)	(0.2382)
largest road freeway/principle arterial	0.3770**	-0.1139	0.3621**	-0.0363	0.3759***
	(0.1522)	(0.1776)	(0.1564)	(0.0365)	(0.1441)
three-road intersection	0.1153	0.2178	0.1509	-0.0176	0.1180
	(0.1088)	(0.1342)	(0.1233)	(0.0164)	(0.1019)
four-road intersection	0.2055**	-0.0236	0.2130**	-0.0435	0.2061**
	(0.0982)	(0.1783)	(0.0937)	(0.0280)	(0.0925)
five-road intersection	0.2992**	0.4604***	0.3783**	-0.0604	0.3052**
	(0.1470)	(0.1571)	(0.1900)	(0.0409)	(0.1389)
log diagonal of bounding box (m)		0.4856***		-0.0599**	
		(0.1464)		(0.0270)	
MI X log diagonal of bounding box (m)		-0.9095***		-0.0708**	
		(0.1777)		(0.0324)	
OH X log diagonal of bounding box (m)		0.4411*		0.0955*	
		(0.2242)		(0.0548)	
TX X log diagonal of bounding box (m)		-0.2355*		-0.0298	
		(0.1317)		(0.0286)	
NC X log diagonal of bounding box (m)		0.3345*		0.0859***	
		(0.1953)		(0.0288)	
MS X log diagonal of bounding box (m)		0.2133		-0.0258	
		(0.3791)		(0.0818)	
GA X log diagonal of bounding box (m)		-0.3786		0.1557**	
		(0.2869)		(0.0612)	
log distance of second entrant to center (m)				1.0308***	
				(0.0140)	
number of firms within 500m	0.0025	-0.0042	0.0022	-0.0010	0.0025
	(0.0027)	(0.0050)	(0.0027)	(0.0007)	(0.0025)
number of firms within 1000m	0.0032**	-0.0008	0.0030***	0.0004	0.0031***
	(0.0012)	(0.0019)	(0.0011)	(0.0002)	(0.0011)
forest coverage 500m circle (sq. km)	-0.4546	-0.1981	-0.4912	-0.2718	-0.4574
	(0.6940)	(0.9196)	(0.7210)	(0.1806)	(0.6599)
forest coverage 1000m circle (sq. km)	0.0509	0.0195	0.0607	0.0784	0.0516
	(0.2401)	(0.2247)	(0.2390)	(0.0644)	(0.2275)
field/grass coverage 500m circle (sq. km)	-0.6436	0.5433	-0.6050	-0.2055	-0.6407
	(0.9205)	(0.9000)	(0.9390)	(0.1903)	(0.8736)
field/grass coverage 1000m circle (sq. km)	0.0463	-0.1223	0.0356	0.0368	0.0455
	(0.2875)	(0.2475)	(0.2891)	(0.0630)	(0.2728)
water coverage 500m circle (sq. km)	-0.4513	2.9466*	-0.0333	0.5285	-0.4196
	(1.2044)	(1.5148)	(1.2653)	(0.5778)	(1.1498)
water coverage 1000m circle (sq. km)	0.2991	-0.9328*	0.1924	-0.1935	0.2910
	(0.3169)	(0.5326)	(0.3390)	(0.1487)	(0.3039)
R-squared	0.177	0.227	0.150	0.972	0.177
N	279	279	278	279	278
Kleibergen-Paap rk Wald F statistic		42.40		1627.38	
Hansen J statistic		4.76		5.54	

Table 11: Placebo results for gas stations. $p < 0.10$ (*), $p < 0.05$ (**), $p < 0.01$ (***). Standard errors clustered by state.

	Mean	Std. dev.
	500m	
water (sq. km)	0.019	0.037
developed (sq. km)	0.398	0.158
field and grass (sq. km)	0.17	0.139
forest (sq. km)	0.143	0.122
shrub and herb (sq. km)	0.052	0.091
	1000m	
water (sq. km)	0.125	0.166
developed (sq. km)	1.245	0.605
field (sq. km)	0.811	0.592
forest (sq. km)	0.714	0.535
shrub and herb (sq. km)	0.229	0.372
N	1632	

Table 12: 2001 vegetation data summary statistics.