Decentralized or Centralized Control of Online Service Platforms: Who Should Set Prices?

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Abstract

Online service platforms that enable customers to connect with a large population of independent servers have been successfully developed in many sectors, including transportation, lodging, and delivery, among others. We ask a basic, yet fundamentally important, question - who should set the prices on the platform? The platform or the servers? In addition to regulatory implications for the classification of the workers on the platform as either employees or contractors, this choice influences the degree of competition among servers, and in turn determines both the amount of supply available and the overall attractiveness of the platform to consumers. We find that when the platform uses a simple commission contract to earn revenue, the price delegation decision depends on the importance of regulating competition among the large population of servers relative to the value of allowing servers to tailor their prices to their privately known costs. The same tradeoff exists in fully disintermediated platforms, such as those enabled with blockchain technology. However, merely adding appropriate linear quantity discounts or surcharges to the basic commission contract maximizes the platform's revenue and allows all participants to enjoy the benefits of both centralized and decentralized control of prices.

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1 Introduction

Online service platforms establish marketplaces to match independent service providers with potential customers. They have been established in many domains, including ride sharing (e.g.,Uber), food delivery (e.g., DoorDash), freelance labor (e.g., TaskRabbit), handmade and vintage products (e.g., Etsy), accommodations (e.g., AirBnB), mobile phone applications (e.g., App Store), etcetera.

Efficient matching is a key value proposition for any platform. Pricing is also critical for a platform's success. On some platforms, control of pricing is entirely decentralized, with each server posting their desired fee. On others, the platform prefers to maintain centralized pricing control. We seek to understand the conditions under which one pricing control structure is preferred over the other, and to identify pricing mechanisms that can capture the benefits of both approaches.

Although many platforms have only operated with a single pricing structure, most participate in evolving markets and are experiencing a number of potentially significant shocks that could force changes. For example, there is considerable regulatory debate in the gig-economy over the classification of its workers. Are they independent contractors or employees? Most online platforms, such as Uber and Lyft, treat workers as independent contractors, but they are facing sustained regulatory pressure to reclassify their workers, due to the amount of centralized control they exert over them. In California, for instance, a law (AB5) was recently passed that emphasizes a contractor's freedom to do its business without a platform's control and direction. Freedom in setting prices has notably been considered a pre-requisite of this description (Bhuiyan 2020), and some argue that granting price freedom to drivers is a necessary measure for ride-sharing platforms to continue to classify their drivers as contractors (Paul 2016).[1] In the United Kingdom, a recent court ruling granted ride-sharing drivers employee status because of the platform's control over fares and the contractual terms it enforces on the drivers (O'Brien 2021).

Technology presents another potential shock to platform pricing. In particular, distributed ledger technologies, such as blockchain, have enabled new companies to establish disintermediated markets in which there is no centralized control over pricing. Examples include Arcade City and Drife in ride-sharing space, Dtravel in house-sharing, and Filecoin and Storj in the market for data storage. A common justification touted across these new platforms is that they take control/power

[1]The subsequent passage of a superseding bill in California, in 2020, Proposition 22, temporarily swung the pendulum back towards allowing gig-workers to be classified as independent contractors, but less than a year later, the bill was ruled unconstitutional and unenforceable by a California Superior Court Judge (Lyons 2021). Similarly, in Massachusetts, Bill HD2582, which has been called a Proposition 22 "clone", has been met with fierce opposition by drivers (Ongweso 2021).

away from central entities, and place it back in the hands of the service providers and platform users. It is not clear whether or how this improves the functioning of these markets.

To address the issue of pricing control on a platform, we consider a platform with the following characteristics: (i) a large number of independent agents, who we refer to as "servers", offer their services, (ii) servers have private knowledge of their heterogeneous costs, (iii) servers self-schedule their work on a short term basis (i.e., the platform cannot dictate how much they work), and (iv) the consumer choice process is influenced both by the overall attractiveness of the platform as well as the specific price of the server with whom the consumer is matched.

Given that each server is a small portion of the platform, they know that they have essentially no ability to influence the aggregate outcomes on the platform. However, they are sophisticated enough to respond to what happens on the platform in a manner that is best for them.

Each server has their own cost to participate, which is particularly relevant due to the flexible work arrangement offered. This cost includes both out-of-pocket explicit costs as well as opportunity costs, and they can vary considerably (Chen et al. (2019)). For example, in ride sharing a driver must pay for vehicle fuel and maintenance, but also incurs a cost to dedicate time to the platform that could be used for other activities that either yield explicit income (e.g., another paid job) or utility (e.g., leisure). Further, because there is a large number of servers and the relationship with servers is relatively short term, the platform is unable to affordably learn a server's operating cost. The lack of cost visibility poses a challenge for the platform in making decisions that accommodate servers' heterogeneous preferences. For example, Filippas et al. (2021) empirically show that centralized pricing harms server participation in a vehicle rental platform due to its inability to fully account for the servers' costs.

Servers provide the explicit task that consumers desire, but consumers are also influenced by the overall performance of the platform. For example, a platform known to have high prices is less likely to be chosen if a customer is aware of more economical alternatives, including potentially forgoing the service altogether. This is true whether the platform has a single price controlled centrally or if the servers on a platform are capable of posting their own prices. Consequently, a server's demand on a platform comes from two sources. The first is the platform's attractiveness which influences the total demand on the platform, and the second is the degree of server competition that allows a server to regulate their share of the platform's total demand.

We find that when a platform uses a simple commission fee structure, the decision to retain control of pricing or not depends on the tension between letting servers set prices that are suitable given their own costs and the need for an appropriate level of prices across the platform. The limitation of the "one size fits all" approach to centralized pricing is that inevitably the platform's price is too high for some servers (those with low costs) and too low for other servers (those with high costs), thereby restricting supply. However, decentralized pricing relies on prices set by individual servers who know they have no power to influence aggregate outcomes. Consequently, competition among servers could lead to prices which are undesirably too high or too low, depending on how easy it is for a server to adjust its share of platform demand. This potential downside of decentralization remains even in the fully disintermediated setting where the central operator is removed and replaced by a blockchain-based smart contract.

The ideal mechanism for a platform (i) is responsive to server cost heterogeneity and yet also properly manages the competition among servers, and (ii) is relatively simple to explain and implement. We show that such a mechanism exists. Although the platform cannot observe a server's cost, the quantity a server offers is an excellent proxy for that cost - servers with lower costs naturally offer more service. Hence, the platform can use the quantity served to determine payments. In particular, in most cases the platform's ideal mechanism is best described as a quantity discount: the server's fee is linearly decreasing in the quantity the server delivers. This encourages low cost servers to increase their supply offered through lower prices. However, when server competition is intense, the mechanism switches to a quantity surcharge. Here, competition is destructive (from the point of view of the platform), so the platform dampens it by penalizing servers that try to take too much share. In sum, through a properly designed mechanism a platform can combine the advantages of both centralized and decentralized control of pricing.

2 Literature Review

Our work is related to research on (i) ownership and contracts in supply chains, (ii) platform management, and (iii) the design of decentralized markets through distributed ledger technology.

The structure of a service platform resembles that of a traditional supply chain. There is a single firm (the platform or supplier) that sells a good or service that is distributed through a large number of independent, self-interested, agents (e.g., servers, retailers, distributors).

Pricing control is well studied in supply chains. Identified early on, the double marginalization effect establishes that a retailer chooses a price which is higher than the supplier desires (Spengler (1950)). A lower retail price would increase demand, which only benefits the supplier, all else equal. For example, more intense competition among retailers could lower retail margins which would mitigate the negative effects of double marginalization. However, lower retail prices also dampen the incentive for retailers to provide costly services and sales effort that could increase demand, and to stock an ample supply of inventory. Consequently, a supplier may attempt to regulate prices through contractual terms and/or restraints on retail business practice ((e.g. Dixit 1983, Rey and Tirole 1986, Deneckere et al. 1996, Padmanabhan and Png 1997, Dana and Spier 2001, Cachon and Lariviere 2005, Song et al. 2008)). Direct control of retail prices is legally risky, and so suppliers generally avoid it. But such control is an available option to platforms. Asymmetry in cost information complicates the coordination of a supply chain. A number of settings with bilateral relationships have been investigated (e.g. Corbett and De Groote (2000), Ha (2001), Corbett et al. (2004), Mukhopadhyay et al. (2008), Yao et al. (2008), Xie et al. (2014), Ma et al. (2017)). Asymmetric information with competing agents generally focuses on component procurement (e.g., Cachon and Zhang (2006)).

Throughout the supply chain literature, the competing agents (usually considered to be retailers) always prefer the other agents to raise their prices. One retailer never benefits from a second retailer's price reduction. However, this need not be true on a platform. Because there are a large number of agents on a platform, no single agent has significant market control. Yet, the attractiveness of the platform as a whole is influenced by their collective actions. Consequently, an agent might prefer that all other agents lower their prices so as to attract more demand to the platform.

There is a growing literature specifically focused on the management of platforms. Some studies only consider fixed prices and focus on the matching process that occurs among the platform participants (Arnosti et al. (2021),Feng et al. (2017), Afèche et al. (2018), Hu and Zhou (2016), Ozkan and Ward (2016)). Others consider revenue maximizing pricing and fee structures given platform control of pricing, i.e. without consideration of decentralized server pricing: Riquelme et al. (2015), Gurvich et al. (2016), Cachon et al. (2017), Bai et al. (2018), Taylor (2018), Hu and Zhou (2019), Benjaafar et al. (2021), Bimpikis et al. (2016), Besbes et al. (2021), Castilllo et al. (2017).

There is some work that considers only decentralized pricing on a platform in which servers always prefer competitors to be less competitive (Allon et al. (2012), Birge et al. (2020), Ke and Zhu (2021)).

Feldman et al. (2019) study both centralized and decentralized prices in a food-delivery platform, but they do not consider competition among servers.

Lobel et al. (2021) evaluate a platform's optimal mix of employees and contractors in a market with uncertain demand but with an exogenous price. Their results demonstrate the value of classifying servers as contractors. Our results address the profit implication if a market legally requires decentralized pricing to continue the use of contractors.

Some work considers platforms that can operate with different types of technologies (e.g., human drivers and autonomous vehicles, Siddiq and Taylor (2019), Lian and van Ryzin (2021)) or platforms in which agents can choose to be a server or a customer (e.g., peer-to-peer sharing in which agents choose to own or rent a vehicle, Benjaafar et al. (2019)). In our platform, the servers operate only with a single technology and agents do not choose to be on the demand or supply side of the market.

Several papers explicitly consider competition among platforms. Liu et al. (2019) find that server retention can be increased by paying discounts along with a commission contract. We do not consider server retention. Instead, price regulation within the platform is the motivation for quantity based pricing in our model. In Ahmadinejad et al. (2019), ride-sharing platforms compete for drivers and riders. Platforms seek to maximize throughput (number of rides) rather than profit, so the payment structure between between drivers and the platform is not considered. Lian et al. (2021) consider a market with multiple platforms that seek to attract a pool of servers, but pricing to consumers is exogenous. Cohen and Zhang (2017) consider pricing both for customers and servers, but do not consider decentralized server pricing.

There is work that considers the allocation of decision rights across agents in a market or platform. Hagiu and Wright (2015) and Hagiu and Wright (2018) consider marketing actions other than pricing. In Hagiu and Wright (2019) there is competition across agents, but that competition does not impact total demand. Philipps et al. (2015) finds empirical evidence that delegation of pricing to agents can be beneficial because the agents have access to better information, and Atasu et al. (2021) explore analytically how to give sales agents control over pricing when they can choose to exert effort to learn. In our model, agents have private information about their cost but do not engage in costly actions to improve upon what they know.

Growth of distributed ledger technologies is facilitating the operation of completely decentralized platforms, i.e., platforms without even a central agent owning the platform. Aymanns et al. (2020) consider how such a change affects consumer welfare. Others consider various operational controls within blockchain-based decentralized platforms (Benhaim et al. (2021), Chen et al. (2020), Tsoukalas and Falk (2020), Cong et al. (2020), Gan et al. (2021b)). The extant literature, to the best of our knowledge, has not addressed the issue of who should retain pricing control in the presence of market-level and individual-level competition effects. Further, works in this area often highlight the advantages of decentralization but do not consider how the lack of a central agent could reduce value in the system.

3 Model

We model a platform that mediates transactions between customers and a large population of independent servers.

Servers differ in their marginal cost to provide service (e.g., as empirically observed in Filippas et al. (2021)). In particular, there is a unit mass of servers and a server's cost, c, is uniformly distributed on the $(0, 1)$ interval with distribution function $F(c)$.

Each server's demand depends on two factors that separately account for market-level and individual-level competitive effects. The first is the average price paid on the platform, $\overline{p}$. This reflects the platform's overall attractiveness relative to other (external) options customers may have to fulfill their needs. For example, if a platform is known to have a low average price compared to the broader market, then this helps to attract demand to the platform. However, if the platform's average price is known to be high, then customers tend to avoid its use and even make decisions that steer them towards other options to meet their needs. For example, if ride-sharing charges high prices, some customers opt to use public transportation, plan ahead to coordinate with friends for rides, or even incur the costs of car ownership. Consequently, we consider the average price effect to operate on time horizons of weeks, months or longer.

The second factor is the server's own price relative to the average price paid on the platform. Once customers seek service from the platform, they naturally gravitate more towards servers with lower prices.

We let $q(c)$ be the demand server c receives, where

$$q(c) = 1 - \beta\overline{p} + \gamma(\overline{p} - p(c)),$$

and

$$\overline{p} = \frac{\int_0^{\hat{c}} q(c)p(c)\ dc}{\int_0^{\hat{c}} q(c)\ dc}. \tag{1}$$

The parameter $\beta > 0$ measures the sensitivity of a platform's demand to the average price on the platform, $\overline{p}$. It reflects the level of competition the platform faces with other platforms or other service options.

The parameter $\gamma > 0$ measures the degree to which internal competition can shift the allocation of demand among servers. Large values of γ reflect a market in which consumers are primarily concerned with price and mostly seek the lowest price option on the platform. However, in practice, consumers of service platforms value other dimensions beyond price, suggesting less extreme values for γ. For example, in ride sharing, a consumer may prefer a driver/server closer to her location because that leads to a shorter waiting time for pickup. Furthermore, because a driver cannot be in all locations all the time, no driver has a consistent location advantage over the other drivers. Put another way, servers provide differentiated products, and due to diverse tastes and needs, price is not the exclusive deciding factor when choosing a server.

A server's demand is naturally always decreasing in their own price, $p(c)$. But servers are of two minds with respect to the average price paid on the platform, $\overline{p}$. From the point of view of platform attractiveness, each server wishes for the average platform price to be low. That will attract many customers to the platform, and, all else equal, a server prefers to participate in a platform with more customers. However, a platform with plenty of demand is of little use to a server with a high price if little demand is matched with the server. In other words, a server's demand increases if their price looks good relative to the average price on the platform. And because of that, the server might prefer that the platform have a higher average price to make the server's own price more enticing. To distinguish between these two components of a server's demand, we refer to the first as the *platform attractiveness* effect (a low average price $\overline{p}$ attracts more demand to the platform) and to the second as the *server competition* effect (a server receives more demand when their price is low relative to the other prices on the platform).

For a server, the balance of the platform attractiveness and the server competition effects depends on the β and γ parameters. When $\beta < \gamma$, the server is more concerned about competition with other servers than the attractiveness of the platform. In this case, as is typical in competitive settings, each server prefers that competing servers choose high prices. However, when $\gamma < \beta$, a server is more concerned with overall platform attractiveness than with internal server competition, meaning that the server actually prefers that the other servers lower their prices. This can occur because consumers first decide whether to patronize the platform and then choose servers within the platform. If the platform decision looms large, then for a server it can be more important to be part of an attractive platform (ample demand due to a low average price) than to be able to capture share of that demand from other servers.

The $\gamma < \beta$ situation is distinct to service platforms. For example, it is generally assumed (and empirically observed) that retailers always prefers their competitors to raise their prices. It is fully expected that a retailer's demand only can decrease when other retailers lower their prices. However, the unexpected can occur in a platform because the agents on a platform are small relative to the size of the market. Consequently, consumer decisions are less focused on the attributes of the particular agents/servers and can be governed to some extent by the characteristics of the overall platform.

Figure 1 displays the sequence of actions. The platform first establishes who sets prices (centralized by the platform or decentralized by the servers) and how fees are collected from the servers. We first consider a commission fee and then more elaborate payment structures. Given those terms, the servers make choices to maximize their profit after observing their own private cost. If the platform controls pricing centrally, the servers only choose whether to participate in the market or not. If the platform chooses decentralized pricing, the servers also choose their price. Servers are capable to service the entire demand assigned to them. Hence, supply is sufficiently elastic to serve the demand, which implies that demand depends exclusively on the offered prices.

Servers recognize that they are small actors in this platform, meaning that the action of a single server has no meaningful impact on market outcomes. Collective actions clearly do influence market outcomes. Consequently, each server anticipates platform performance metrics and bases their decision on those metrics to maximize their earnings. An equilibrium occurs when server expectations are consistent with actual outcomes.

The platform also seeks to maximize its profit. It has the option to take full control over pricing, or it can delegate pricing to the servers. (With some platforms the firm suggests prices to servers while the servers retain control over pricing. In our model there is no role for a firm's price suggestions, so they are not considered.) The platform's fees can be based on the prices charged or the quantities served or a combination of both.

All information is common knowledge with the exception that each server's cost is private information (as already mentioned). For example, the distribution of server cost is commonly known and all prices and quantities are observable.

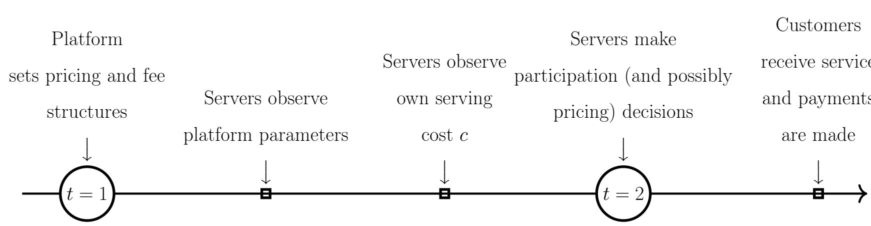


Figure 1: Sequence of events

4 Pricing Policies under Commission Contracts

Two pricing control strategies are considered: *centralized platform pricing*, and *decentralized server pricing.*

With centralized platform pricing, the platform retains control over prices and determines a single price for the platform. This is the typical approach in the ride-sharing industry. Consumers observe a single price for service (within a local market at a moment in time, which is what is modelled here). The only decision for servers is whether to participate in the market or not. The platform can influence that decision, and thus total supply, through the fee structure offered.

With decentralized server pricing, each server selects their own price, which is generally observed in room-sharing or work-for-hire platforms. Now the servers choose whether to participate and their posted price. In addition to the platform's fee structure, this decision is influenced by the competitive dynamics among the servers.

The platform is responsible for designing its fee structure independent of who chooses prices. We begin with a commission contract, which is intuitive, simple, and observed in practice. With a commission contract the servers pay ϕ portion of their revenue to the platform. In ride-sharing, the commission rate is often around 25%. We subsequently consider more elaborate fee structures.

With the commission fee, a server with cost c and price $p(c)$ earns profit

$$\pi(c) = q(c)((1-\phi)p(c) - c). \tag{2}$$

Each server has a zero outside option (without loss of generality), so a server participates only if $\pi(c) \geq 0$. Let $\hat{c}$ be the largest cost among the servers who participate. The platform's profits are

$$\Pi = \phi \int_0^{\hat{c}} q(c)p(c)\, dc = \phi\overline{p} \int_0^{\hat{c}} q(c)\, dc.$$

4.1 Centralized Platform Pricing

With centralized platform pricing all servers are assigned a single price, $p = \overline{p}$ (for a given local market, at a given moment in time). Let $\hat{c}(p, \phi)$ be the highest cost server who participates in the market,

$$\hat{c} = \hat{c}(p, \phi) = (1-\phi)p.$$

All servers with cost $c \le \hat{c}(p, \phi)$ are profitable and enter the market. It follows that the platform's optimization problem can be written as

$$\max_{\overline{p},\phi} \quad \Pi^{\mathcal{P}} = \phi\overline{p}\int_0^{\hat{c}}(1-\beta\overline{p})\,dc$$
$$\text{s.t.} \quad \hat{c} = (1-\phi)\overline{p}.$$

Proposition 1 identifies the platform's optimal decision and the equilibrium market characteristic under centralized platform pricing.

Proposition 1. *With a commission contract and centralized platform pricing, for the platform there exists a unique optimal price and commission rate. The first column in Table 1 summarizes the equilibrium market characteristics under platform pricing.*

	Platform pricing	Server pricing	Optimal mechanism
Platform's profit, Π	$\frac{1}{27\beta^2}$	$\frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$	$\frac{1}{24\beta^2}$
Servers' total profit	$\frac{1}{54\beta^2}$	$\frac{9}{16}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)$	$\frac{1}{48\beta^2}$
Mass of server entry, $\hat{c}$	$\frac{1}{3\beta}$	$\frac{3}{2(2\beta+\gamma)}$	$\frac{1}{2\beta}$
Average market price, $\overline{p}$	$\frac{2}{3\beta}$	$\frac{2}{2\beta+\gamma}$	$\frac{2}{3\beta}$
Server prices, $p(c)$	$\frac{2}{3\beta}$	$\frac{3}{2(2\beta+\gamma)}+c$	$\frac{2}{3\beta}-\frac{1}{6\gamma}+\frac{\beta}{\gamma}c$
Commission rate (ϕ) or fee (f)	$\frac{1}{2}$	$\frac{1}{2}$	$a_0+a_1c+a_2c^2$
Total quantity served, Q	$\frac{1}{9\beta}$	$\frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right)$	$\frac{1}{8\beta}$

Table 1: Equilibrium market characteristics under the three pricing policies.

Centralized pricing gives the platform full control over the platform's attractiveness and it completely eliminates competition among the servers. Thus, the platform cannot rely on server competition to lower the average price in equilibrium. Furthermore, centralized pricing gives limited control over supply: because the platform does not know the servers' costs, the common price is inevitably too low for some servers, who opt to not participate.

4.2 Decentralized Server Pricing

As servers are small actors, they choose prices to maximize their individual profits, taking their expectation for the average market price, $\overline{p}$, as given. An equilibrium occurs when the servers' expectation for the average market price is correct.

Following (2), server c has the following pricing problem:

$$\max_{p(c)} \quad \pi(c) = \big(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\big)((1-\phi)p(c) - c).$$

The server with cost c has a unique optimal price, $p(c)$, to post:

$$p(c) = \frac{1}{2\gamma}\left(1 + (\gamma - \beta)\overline{p} + \frac{c\gamma}{1-\phi}\right). \tag{3}$$

The highest cost server who participates on the platform is

$$\hat{c} = \hat{c}(\phi) = \frac{(1-\phi)(1+(\gamma-\beta)\overline{p})}{\gamma}. \tag{4}$$

Aware of the equilibrium price choices that occur, the platform's optimization problem is

$$\begin{aligned} \max_{\phi} \quad & \Pi^{\mathcal{S}} = \phi\overline{p} \int_0^{\hat{c}(\phi)} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc \\ \text{s.t.} \quad & \text{Eq. } (1), (3), (4). \end{aligned}$$

Proposition 2 identifies the platform's optimal decision under decentralized server pricing.

Proposition 2. *With a commission contract and decentralized server pricing, for any commission rate there exists a unique price equilibrium among the servers. There exists a unique optimal commission rate for the platform. Equilibrium market characteristics of the optimal server pricing policy are included in Table 1, Column 2.*

With decentralized pricing a server with a high cost may post a sufficiently high price to justify participation in the market. All else equal, this expands the total supply offered in the market and generates some revenue for the platform. However, high posted prices also lead to a higher average price on the market, $\overline{p}$, which dampens overall demand. Given that the same commission rate is offered to all servers, the platform is limited in its ability to target the actions of one group of servers over another (e.g., high cost vs. low cost servers).

4.3 Comparing Centralized and Decentralized Pricing

According to Proposition 3 and as illustrated in Figure 2, neither centralized nor decentralized pricing dominates the other in all situations. Server pricing is best when $\gamma = \beta$, generating 12.5%

higher revenue for the platform than centralized/platform pricing. Under those conditions, the platform attractiveness and server competition effects are equally strong, thereby cancelling each other. Consequently the servers operate as local monopolies, unconcerned with the prices set by others in the market. Because the servers do not compete at all, the platform can use the advantage of server pricing (the servers can select a price that is appropriate for them) to yield better results than platform pricing, which is limited to a single price.

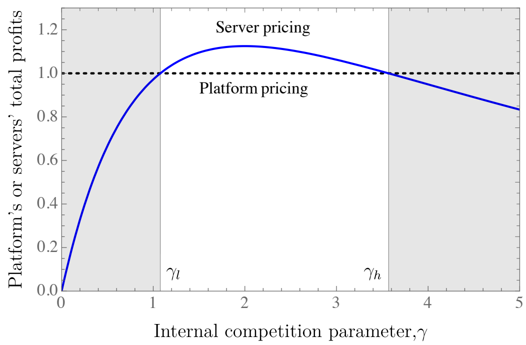


Figure 2: The platform's and servers' total profits under server pricing (solid line), as a fraction of their profits under platform pricing (scaled to 1). Shaded area: Platform pricing is preferred. White area: Server pricing is preferred.

Proposition 3. *The platform's earnings with server pricing are highest when* $\gamma = \beta$*. There exists* $\gamma_l < \beta$ *and* $\beta < \gamma_h$ *such that the platform prefers server pricing when* $\gamma_l < \gamma < \gamma_h$*, and otherwise platform pricing is preferred.*

As γ deviates from β, in either direction, the platform's earnings with server pricing decreases. When $\gamma < \beta$ the platform attractiveness effect dominates, and the average price in the market, $\overline{p}$ is higher than the price the platform would select when it controls pricing. Servers too would prefer they collectively lower their prices, but because all individual servers are powerless to influence the average price, and their ability to steal market share is weak, prices remain high. Alternatively, when $\beta < \gamma$, the server competition effect dominates platform attractiveness. Now, each server aggressively lowers its price to try to capture market share. With all servers trying to take share, the low average price attracts demand to the platform but does not generate ample revenue. In this case the platform would prefer higher prices to generate more revenue, albeit with less demand. In sum, the commission contract is insufficiently capable to lower prices when they are too high ($\gamma < \beta$) or to raise prices when they are too low ($\beta < \gamma$). In extreme situations, when the server competition effect is either too weak $\gamma < \gamma_l$ or too strong $\gamma_h < \gamma$, it is best for the platform to shut off all competition among the servers through centralized control of pricing. The single price in the market does not allow the flexibility for pricing to reflect servers' costs, but in those cases decentralized pricing in unable to prevent extreme pricing. Losing control over the prices can indeed lead to bad outcomes for the platform.

5 Platform's Optimal Contract

Commission contracts are intuitive and observed in practice, but they provide limited flexibility to tailor fees to individual servers. This section evaluates the platform's optimal fee structure.

According to the revelation principle (Myerson (1981)), the search for an optimal mechanism can be restricted to the set of truth-inducing mechanisms. With those mechanisms, the platform announces a menu that maps each possible server cost into a price, quantity, and fee. Servers report a cost (which need not be their true cost) and based on the platform's menu, prices, quantities and fees are determined. The menu is truth inducing if (i) each server prefers to report their cost truthfully (assuming all other servers do so as well) and (ii) a server's earnings from participation in the platform is at least equal to the server's best outside option.

Let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. Let $\pi(c, \tilde{c})$ be a server's earning with cost c that reports costs $\tilde{c}$:

$$\pi(c, \tilde{c}) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))(p(\tilde{c}) - c) - f(\tilde{c}).$$

In any optimal mechanism there exists a $\hat{c}$ such that all servers with costs $c \leq \hat{c}$ participate on the platform whereas all servers with costs $\hat{c} \leq c$ do not. (If a server with cost c did not participate but there were a server with a higher cost that did participate, then the menu could be swapped between the two servers without loss of revenue.) Hence, the platform's optimal mechanism design problem is

$$\begin{aligned} \max_{p(c), f(c), \hat{c}} \quad & \Pi = \int_0^{\hat{c}} f(c)\, dc \\ \text{s.t.} \quad & \pi(c, c) \geq \pi(c, \tilde{c}) \ , \forall c \in (0, \hat{c}), \forall \tilde{c} \in (0, \hat{c}) \\ & \pi(c, c) \geq 0 \ , \forall c \in (0, \hat{c}) \\ & \text{Eq. (1).} \end{aligned}$$

Proposition 4. *The following mechanism maximizes the platform's revenue:* $p(c) = \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{\beta}{\gamma}c$, $f(c) = a_0 + a_1 c + a_2 c^2$, *with* $a_0 = \frac{1}{24}\left(\frac{5}{\beta} - \frac{2}{\gamma}\right), a_1 = \frac{2}{3}\left(\frac{\beta}{\gamma} - 1\right), a_2 = \left(\frac{\beta(\gamma - 2\beta)}{2\gamma}\right)$. *Equilibrium market characteristics of the optimal contract are displayed in Table 1, Column 3.*

Server prices with the optimal mechanism are linearly increasing in the servers' costs. The same applies with the commission contract and decentralized server pricing. But there is an important difference. With decentralized control, the rate at which price increases with cost is independent of market characteristics:

$$\frac{\partial p(c)}{\partial c} = 1.$$

In the optimal mechanism, price is more or less responsive to server costs depending on market parameters,

$$\frac{\partial p(c)}{\partial c} = \frac{\beta}{\gamma}.$$

In particular, when platform attractiveness is more relevant to the servers than internal competition, $\gamma < \beta$, then optimal prices are more sensitive to server costs. In this situation, decentralized control leads to an average price which is too high, so the optimal mechanism skews prices lower for the low cost servers. In contrast, when internal competition dominates, $\beta < \gamma$, the optimal mechanism needs to dampen price competition. It does so through prices that are less responsive to server costs.

In the optimal mechanism, the servers pay a fee which is quadratic in their own costs. Consequently, unlike in the simple commission contract, the fee is not a cost share of a server's revenue.

Quantity-based discounts/surcharges

Although it is possible to identify an optimal truth-inducing mechanism, it does not seem practical. A preferable mechanism would not rely on servers reporting their costs, but rather would depend on verifiable and observable performance metrics. Such a mechanism exists.

Consider a quantity-based fee structure. Such structure has been shown to help the efficiency of decentralized supply chains (e.g. Monahan 1984, Weng 1995, Corbett and De Groote 2000), however, their use for price coordination has not been studied in the context of service platforms. To be specific let $q(c)$ be the quantity a server with cost c delivers on the platform and let $f(c)/q(c)$ be the per-unit fee a server pays to the platform when $q(c)$ is the server's quantity. Based on the optimal mechanism, let

$$\frac{f(c)}{q(c)} = q(c)\left(\frac{1}{2\beta} - \frac{1}{\gamma}\right) + \left(\frac{1}{6\beta} + \frac{1}{3\gamma}\right). \tag{5}$$

Suppose the platform announces the fee structure in Equation (5), but otherwise lets the servers choose their prices. Given decentralized pricing, servers choose the prices that maximize their profits knowing that other servers do the same. Thus, server c faces the problem:

$$\max_{p(c)} \quad \pi(c) = q(c)(p(c) - c) - q(c)\left(\frac{f(c)}{q(c)}\right).$$

Given an expected average price $\overline{p}$, the servers' optimal price satisfies

$$p(c) = \frac{2\beta(3c\gamma - 6\gamma\overline{p} - 2) + 6\beta^2\overline{p} + \gamma(6\gamma\overline{p} + 7)}{6\gamma^2}. \tag{6}$$

The highest cost participant is indifferent between participating and not participating:

$$\pi(\hat{c}) = 0. \tag{7}$$

Assuming Equations (1), (6), (7) are uniquely defined, the platform earns:

$$\Pi^{\mathcal{Q}} = \int_0^{\hat{c}} q(c)\left(\frac{f(c)}{q(c)}\right) = \int_0^{\hat{c}} q(c)\left(q(c)\left(\frac{1}{2\beta} - \frac{1}{\gamma}\right) + \left(\frac{1}{6\beta} + \frac{1}{3\gamma}\right)\right)\, dc.$$

Proposition 5. *With the fee structure (5) there exists a unique equilibrium to the servers' decentralized pricing game and this equilibrium replicates all performance metrics of the optimal mechanism, including the platform's profit.*

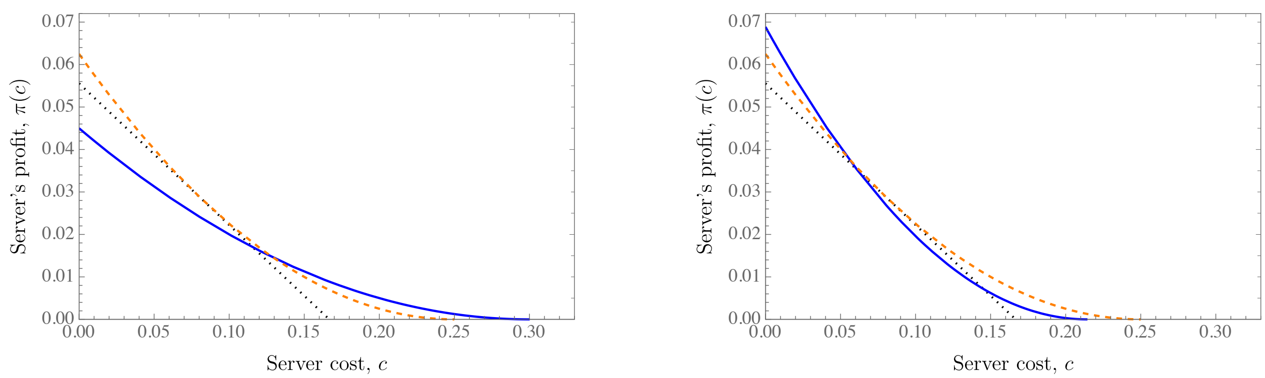
Proposition 5 establishes that the platform can replicate the optimal mechanism with decentralized pricing and a quantity-based fee structure.

When the server competition effect is relatively strong, $2\beta < \gamma$, the platform replicates the optimal mechanism with a quantity surcharge policy. In these markets, competition among the servers is excessive, lowering revenue for the platform. To dampen competition, low cost servers are dissuaded from exploiting their advantage through high quantities. Although not a linear contract, Apple recently announced changes to its App Store that penalizes high volume sellers with a higher commission rate (Higgins and Needleman (2020)).

When the platform attractiveness effect is relatively important, $\gamma < 2\beta$, the platform can replicate the optimal mechanism with a simple quantity discount policy: the per unit fee a server pays the platform is linearly decreasing in the number of units served. Put another way, the more a server serves, the lower the commission rate the server pays. This encourages low cost servers to serve more than they would with a simple fixed commission contract. Doing this lowers the average price paid in the market, which makes the platform more attractive.

Quantity discounts have been observed in ride-sharing platforms. It has been argued that they are used to encourage retention of servers (Kabra et al. (2016), Liu et al. (2019)). Our model does not include a cost to recruit servers, so such a reason does not apply here. Instead, our model suggests that quantity discounts can provide a vital tool for the regulation of a decentralized service platform. In particular, this form of pricing helps the platform to regulate the competition across the different types of servers.

Figures 3a and 3b, illustrate how a server's cost influences their profit. The first figure considers a case in which platform attractiveness is the stronger effect and the second figure displays a case in which server competition is dominant. In both cases, under centralized platform pricing the servers' profits are linearly decreasing in a server's cost, which reflects the fact that every server is given the same price. Servers are more responsive to their costs when they are given control over their prices. This allows higher cost servers to participate on the platform and earn some profit. However, when platform attractiveness is the strong effect (left panel, $\gamma < \beta$), high cost servers are earning too much and low cost servers are not earning enough. Quantity pricing fixes this issue by lowering prices for the high volume servers, which are the low cost servers. In the other situation (right panel, $\beta < \gamma$), the intensity of competition in the market is causing some high price servers to drop out. Now quantity pricing must support the high price servers.



(a) Servers' profits, $\pi(c)$, with respect to server cost, c, for $\gamma = 1$, $\beta = 2$.

(b) Servers profit's, $\pi(c)$, with respect to server cost, c, for $\gamma = 3$, $\beta = 2$.

Figure 3: Servers' profits under the three pricing policies: platform pricing (dotted), server pricing (solid) and quantity discount model (dashed).

Given that quantity pricing allows the platform to better control the actions of independent servers for its own advantage, a natural question is what impact it has on server welfare. Wealth redistribution between the servers is visible in both figures and depends on the relative magnitudes of γ and β: In Figure 3a, for instance, "low-cost" servers, from $c = 0$ until $c = 0.12$, are better off under quantity pricing compared to the other two pricing schemes (the dashed curve is above the two others), while "high-cost" servers from $c = 0.12$ onward are worse off compared to server pricing, and better off compared to platform pricing. Despite this, we can show that quantity pricing leaves servers better off in aggregate. Formally:

Corollary 1. *Total server profit is higher with quantity pricing than both platform or server pricing.*

Corollary 1 suggests that quantity pricing is used primarily to increase the total value generated rather than to shift share of that value towards the platform.

Consumers are another stakeholder. Without an explicit utility model for consumers, it is not possible to directly measure consumer welfare. However, consumers tend to prefer lower prices and more quantity, all else equal. Comparing quantity pricing to platform pricing, the average price consumers pay is the same but there is more supply with quantity pricing. Hence, quantity pricing is also good for consumers relative to platform pricing.

The comparison between quantity pricing and server pricing depends on the regime. When platform attractiveness is more important ($\gamma < \beta$), server pricing leads to higher prices and less quantity served. So on both counts quantity pricing is better for consumers. When server competition is the dominant effect ($\beta < \gamma$), server pricing has lower prices and more quantity in non-extreme situations ($\beta < \gamma < 4\beta$). In these cases, quantity pricing works well for the platform and servers, but potentially at the expense of consumers.

6 Discussion

Our model captures key economic trade-offs in the design of pricing strategies for a platform. The platform wants to offer favorable terms to customers while also attracting a large server supply. The servers are affected by both the platform's overall ability to attract customers and their individual competitiveness. It is up to the platform to find suitable pricing mechanisms that can address both concerns. We find that under a simple commission contract, both platform pricing and server pricing have their own pros and cons. The platform's best strategy is to design a mechanism that capitalizes on server information while restraining extreme competitiveness.

In this section, we discuss several extensions to our model. Section 6.1, explores a market that uses distributed ledger technology (e.g., smart contracts on blockchain platforms) to eliminate the central coordinator (i.e., a platform). Section 6.2 considers non-uniform distributions. Section 6.3 changes the platform's objective from maximizing profit to maximizing throughput. Section 6.4 expands our base model to include heterogeneity in server quality, whereby servers with higher quality can capture greater demand. In sum, the primary insights from the main model continue to hold in those additional circumstances.

6.1 Disintermediated Pricing using Blockchain-Based Smart Contracts

The platform's primary focus is to maximize its own profit, rather than the total value in the system, which also includes the servers' total profits. Consequently, total system value may increase if the platform could be removed and control transferred to the servers. In principle, this may be feasible via smart contracts, enabled by blockchain technology.

Beyond the integrity of transactions needed to create a viably functioning market, we presume smart contracts can potentially establish a set of observable and enforceable transfers among the servers while also allowing them to have full control over their pricing. Given that, Proposition 6 identifies an optimal disintermediated mechanism that (i) does not include a platform and (ii) maximizes total server profits.

Proposition 6. *Without a central platform earning a profit, the servers' profits can be maximized in equilibrium when a server with cost* c *selects price* $p(c) = \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{\beta}{2\gamma}c$*, and the server contributes a subsidy* $f(c)$ *to the system, where* $f(c) = \frac{1}{12}\left(\frac{1}{\beta} - \frac{1}{\gamma}\right) + \frac{1}{3}\left(\frac{\beta}{\gamma} - 1\right)c + \left(\frac{\beta(\gamma-\beta)}{4\gamma}\right)c^2$*. If* $f(c)$ *is negative, then the server receives a subsidy. There is a zero net flow of subsidy transfers.*

The mechanism described in Proposition 6 is incentive compatible, i.e., no server wishes to choose a different price conditional that all other servers are following the mechanism and the subsidy transfers are credibly administered. This holds even though some servers select prices that force them to relinquish some of their earnings (when $f(c)$ is positive) and others select prices that allows them to earn a bonus beyond their own earnings (when $f(c)$ is negative).

Similar to quantity pricing, the optimal disintermediated mechanism can be implemented through a quantity-based fee structure in which some servers make contributions, others receive contributions, and the net contribution in the system is zero.

The optimal disintermediated contract is simple to characterize, but it may be difficult to implement in practice. First of all, it is not possible for all servers to retain all of their earnings. The system of transfers may be viewed as unnatural, or inconsistent with the philosophy behind a disintermediated market. It is also not clear how the specific functional forms for the subsidies would be modified if market conditions change (e.g., shifts in γ or β), and it is not clear that the data collection and computational requirements could be practically satisfied.

A simpler alternative to the optimal mechanism is a market that allows all servers to set their own prices, retain their entire revenue, and distributed ledger technology is merely used to ensure the integrity of all transactions. We refer to this benchmark as "disintermediated server pricing". It is equivalent to the decentralized server pricing setting from Section 4.2, but with commission

$$\phi = 0. \tag{8}$$

Under this setting, the equilibrium is characterized by Equations (1), (3), (4) and (8).

The are markets in which $\phi = 0$, as is displayed in Table 2. However, a direct comparison between centralized and decentralized platforms is difficult, because decentralized platforms rely on alternative monetization mechanisms that may proxy for commission fees, such as stake retention in Initial Coin Offerings (Gan et al. 2021a,b), and a tokenized economy (Cong et al. 2020, Tsoukalas and Falk 2020). Our analysis in this section can thus be interpreted as a "best case scenario" for blockchain-based platforms. We show that even under this best-case assumption, blockchain may not always prevail as the preferred mode of adoption.

Centralized	ϕ	**Decentralized**	ϕ
Uber	25%	Drife	0%
Lyft	20%	Arcade City	0%
AirBnB	15%	Dtravel	7%*
UberEats	22%	Filecoin	0%
Grubhub	25%	Storj.io	0%
Median	20%	Median	0%

Table 2: Typical commission fees per transaction, on centralized vs. decentralized platforms.
*Dtravel commission fee is recycled back into the platform ecosystem.

Proposition 7. *With disintermediated server pricing (i.e., no platform, servers set their prices and retain all earnings), there exists a unique price equilibrium among the servers. There exists* $\gamma_l < \beta$ *and* $\beta < \gamma_h$ *such that the total server profits under disintermediated server pricing is higher than quantity pricing when* $\gamma_l < \gamma < \gamma_h$*, and lower otherwise.*

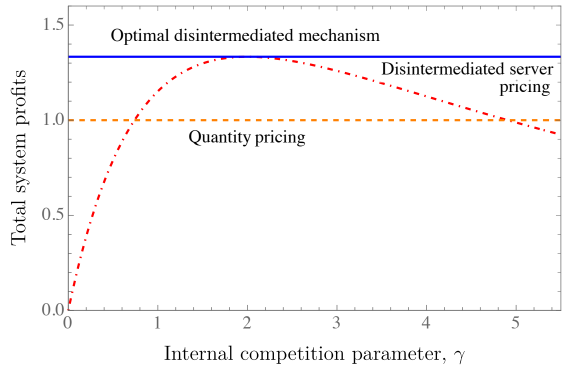


Figure 4: Total system profits with respect to the internal competition parameter, γ, for $\beta = 2$ under , disintermediated server pricing (dot-dashed line) and disintermediated optimal mechanism (solid line), as a fraction of quantity pricing (scaled to 1).

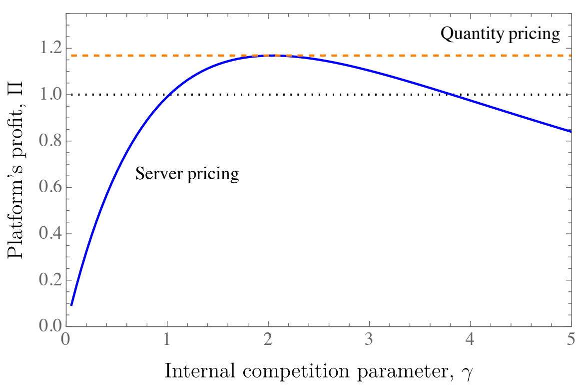
As displayed in Figure 4, the optimal disintermediated mechanism maximizes total system value. Compared to quantity pricing, it increases total profit generated in the system by 33.3%. This increase is attributed to higher retained earnings by servers, which promotes participation. Under this mechanism, the customers are also better off, with the same average market price and higher quantity served than quantity pricing.

When the optimal disintermediated mechanism cannot be implemented, the potential upsides to disintermediation can be large, but this is limited to specific cases. According to Proposition 7, only for intermediate values of server competition, disintermediated server pricing can increase total system profits. In this case, the platform's profit seeking behavior destroys more value than its price coordination generates. This is not to suggest a platform can or should be be replaced by a disintermediated alternative. A platform may pursue other objectives that are better aligned with servers' objectives (e.g. maximizing throughput) or provide other value added that is not captured in the model. In those markets, centralized platforms can exhibit profit-maximizing behavior while maintaining servers' total profits at a desired level (see Appendix, page 45), making it difficult for a disintermediated alternative to provide excess value.

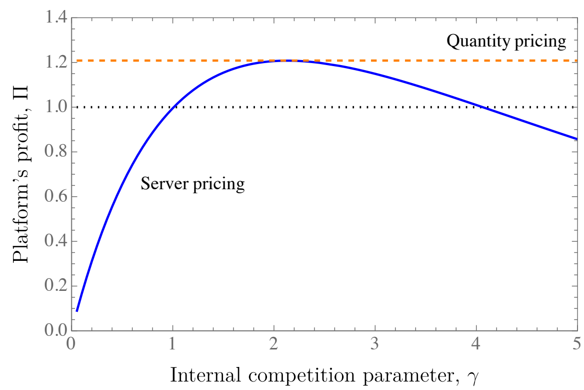
In sum, disintermediation via an optimal contract can clearly increase total system value. However, if that contract is not implementable, a simpler alternative is to merely let the servers price on their own and retain all of their earnings. While this can increase total system value (by eliminating the distortions associated with the platform's profit seeking motive), it retains limited control over the level of pricing competition among the servers. Hence, just as in the market regulated by a platform, it is possible that total system value can be reduced via disintermediation, especially if the server competition effect is particularly strong or weak.

6.2 Variation in Server Costs

The uniform distribution facilitates analytical results and results in an optimal quantity pricing mechanism that is linear in the quantity a server delivers. The optimal mechanism can be evaluated numerically for other distributions for the servers' costs. We consider $F \sim \text{Beta}(\alpha_1, \alpha_2)$ with $\alpha_1 = \alpha_2 = \alpha$, which implies the mean is a constant 0.5 and the density function is symmetric about the mean. For each pricing strategy, we evaluate 250 scenarios using the following parameters: $\gamma \in \{0.1, 0.2, ..., 5\}$, $\beta \in \{2\}$, and $\alpha \in \{2, ..., 5\}$. We numerically evaluate for each scenario the optimal mechanism, which is non-linear in a server's quantity, and the optimal linear mechanism. We find the linear mechanism performs very well: the linear quantity pricing mechanism's platform profit yields on average 99.9% of the optimal profit and no less than 99%. Figures 5a and 5b, display the platform's profit for $\alpha = 2$ and $\alpha = 5$ respectively. These correspond to two cases where the variance of costs in the market is relatively high and relatively low. In either case, as with the uniform distribution, server pricing is preferred only for intermediate values of server competition.



(a) Platform's relative profit under server pricing for $\alpha = 2$, $\beta = 2$.



(b) Platform's profit under server pricing for $\alpha = 5$, $\beta = 2$.

Figure 5: The platform's relative profit for $\alpha = 2$, $\beta = 2$ (left) and $\alpha = 5$, $\beta = 2$ (right). Legend: server pricing (solid line) and optimal mechanism (dashed line), as a fraction of platform pricing (scaled to 1).

6.3 Throughput Maximization

Platforms that are in earlier stages of their life-cycle may believe it is more important to grow than to be profitable. Those platforms may prioritize maximizing throughput over profits, which has been considered in other models: e.g. Ahmadinejad et al. (2019),Castro et al. (2020),Yan et al. (2020)). As profit is the product of quantity and margin, maximizing throughput (i.e., quantity) is not a radically different objective. Hence, the negative consequences of extreme server competition continue to hold. Consequently, our main findings continue to hold qualitatively (see Appendix, page 48).

6.4 Correlation Between Server Cost and Customer Value

In our base model, server costs are not correlated with customer value, that is, servers provide homogeneous service, regardless of their cost heterogeneity. This is applicable to markets like ride-sharing where servers are heterogeneous in their preferences for non value-adding factors like time-of-day or location. In other markets, like home rental and freelance labor, servers also exhibit heterogeneity in costly value-added activities that influence service quality. In those markets, the service may not be homogeneous: servers with high costs can provide higher customer value, and therefore attract more demand, which could have non-trivial implications. To address this, we consider an extension in which the demand intercept term is modified to scale linearly with server costs. A server with cost c faces the demand function:

$$q(c) = 1 + \alpha c - \beta \overline{p} + \gamma(\overline{p} - p(c)).$$

The other characteristics of the setting follow decentralized server pricing in Section 4.2. The full analysis is provided in the Appendix.

Figure 7 displays the platform's profit for different levels of $\alpha \in \{0, 0.3, 0.5\}$.

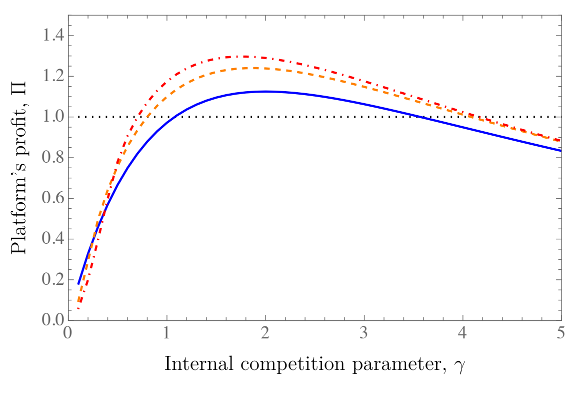


Figure 6: Platform's profit under server pricing for $\beta = 2$.

Figure 7: Platform's profits under server pricing for $\alpha = 0$ (solid), $\alpha = 0.3$ (dashed) and $\alpha = 0.5$ (dot-dashed), as a fraction of platform pricing (scaled to 1).

Allowing service quality to increase with costs makes the platform want to include more "high-cost" servers. However, a one-size-fits-all, centralized pricing approach, does not allow the platform to incentivize their participation. On the other hand, a decentralize pricing approach, whereby servers set their own prices, gives more flexibility to high-costs servers to enter. Consequently, as the sensitivity α increases, so does the region in which server pricing is preferred. This effect is directly observable in the figure, as long as market competition.

In sum, the benefits of decentralized pricing are stronger in markets like home rental or freelance labor, where cost is a more direct proxy for service quality.

7 Conclusion

Who should control pricing on a service platform? Servers know their own costs to participate, so they are best able to tailor their price to their circumstance. A rigid price set by the central platform is likely to exclude some servers who otherwise could contribute. However, servers are small actors. Each knows that they can only respond to the market given to them and this has significant implications for the dynamics on the platform. In particular, there are two effects that need to be regulated. A server can lower their price to take a larger share of the platform's demand. This server competition effect can lead to prices that are destructively too low, limiting revenue potential through less supply (high cost servers drop out) and lower prices. In contrast, there is a platform attractiveness effect - total demand on the platform depends on the average price. Consequently, a server may actually prefer other servers to cut their prices - doing so makes it harder to steal share from them (the typical competition effect) but it also expands the total demand on the platform. Due to each server's limited power, they collectively have little control to balance these effects properly. This creates a value-added opportunity for the platform to regulate pricing.

A commission contract is simple to explain and administer, but it lacks precision. Consequently, while centralized platform pricing is not ideal in all situations, it performs reasonably well in all cases, and can prevent some extreme adverse scenarios of decentralized server pricing.

Fortunately, it is possible to combine the robustness of centralized platform pricing with the advantages of decentralization. When servers left on their own are too price aggressive, something is needed to calm them. When independent servers are too timid, something is needed to motivate them to cut their prices. Although the platform cannot observe a server's cost, the platform can observe the quantity a server offers. As low cost servers work more on the platform, this suggests the platform can target servers through the quantity they deliver. In fact, through simple quantity pricing, we show that the platform can achieve the maximum profit in all situations. As a bonus, the servers are also better off, and frequently consumers as well.

So the question for a service platform is not so much who should control price, but how they do it. For example, decentralized server pricing may be necessary to classify servers as contractors rather than employees. Although that classification has implications for who is willing to work on a platform and the costs of their work, it may be possible to give servers full pricing control and yet correct for the potentially negative consequences of doing so. This is a particularly important lesson for platforms that are attempting to use distributed ledger technology (e.g., blockchain) to eliminate any central agent. Such technologies may bring advantages to the market, but without addressing the negative tendencies of decentralized server pricing, they face a potentially negative drag on performance.

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Appendix

Proof of Proposition 1. In centralized platform pricing, let p be the price set by the platform and ϕ be the portion of revenue retained. Conditional on participation, a server with cost c earns

$$\begin{aligned}\pi(c) &= q(c)((1-\phi)p - c) \\ &= (1-\beta p)((1-\phi)p - c).\end{aligned}$$

Server profits are decreasing in cost, c, and therefore there exists a threshold cost, $\hat{c}$, for which server with cost c participates if and only if $c < \hat{c}$. The highest cost that participates is

$$\hat{c} = (1-\phi)p,$$

which leads to zero profit.

Platform's profit maximization problem is

$$\begin{aligned}\max_{p,\phi} \quad \Pi &= \phi p \int_0^{\hat{c}} (1-\beta p)\, dc \\ &= \phi p \int_0^{(1-\phi)p} (1-p)\, dc \\ &= \phi(1-\phi)p^2(1-\beta p).\end{aligned}$$

The platform's solution is unique. It's defined by the first order conditions:

$$\begin{aligned}\frac{\partial \Pi}{\partial p} &= p(1-\phi)\phi(2-3\beta p) = 0, \\ \frac{\partial \Pi}{\partial \phi} &= p^2(1-\beta p)(1-2\phi) = 0,\end{aligned}$$

giving the solution

$$\begin{aligned}p &= \frac{2}{3\beta}, \\ \phi &= \frac{1}{2}.\end{aligned}$$

Platform's profit is

$$\Pi = \phi(1-\phi)p^2(1-\beta p) = \frac{1}{27\beta^2}.$$

The highest cost that participates and the average market price are

$$\begin{aligned}\hat{c} &= (1-\phi)p = \frac{1}{3\beta}, \\ \bar{p} &= p = \frac{2}{3\beta}.\end{aligned}$$

Total quantity of customers served in the market is

$$Q = \int_0^{\hat{c}} (1 - \beta p)\, dc$$
$$= (1-\phi)p(1-\beta p) = \frac{1}{9\beta}.$$

Servers' total profit is

$$\int_0^{\hat{c}} \pi(c) = \int_0^{(1-\phi)p} (1-\beta p)((1-\phi)p - c)\, dc$$
$$= \frac{1}{2}(1-\beta p)p^2(1-\phi)^2 = \frac{1}{54\beta^2}. \tag{9}$$

□

Proof of Proposition 2. In decentralized server pricing, conditional on participation, a server with cost c earns

$$\pi(c) = (1 - \beta p + \gamma(\overline{p} - p(c)))((1-\phi)p(c) - c).$$

Server's profit depends on individual price, $p(c)$, and also the average market price, $\overline{p}$, which is characterized in the equilibrium. Since each server is small, the price set by an individual server does not influence the average market price.

Server c has the following pricing problem:

$$\max_{p(c)} \quad \pi(c) = \big(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\big)((1-\phi)p(c) - c).$$

A server's optimal price is uniquely defined and is characterized by the first order condition

$$\frac{\partial \pi(c)}{\partial p(c)} = 1 - \phi - \overline{p}(1-\phi)(\beta - \gamma) + c\gamma - 2\gamma(1-\phi)p(c) = 0,$$

giving the solution

$$p(c) = \frac{1}{2\gamma}\left(1 + (\gamma - \beta)\overline{p} + \tfrac{c\gamma}{1-\phi}\right). \tag{10}$$

Server c earns

$$\pi(c) = \frac{(1 - \phi - \overline{p}(1-\phi)(\beta-\gamma) - c\gamma)^2}{4\gamma(1-\phi)}.$$

Server profits are decreasing in cost, c, and therefore there exists a threshold cost, $\hat{c}$, for which server with cost c participates if and only if $c < \hat{c}$. The highest cost that participates is

$$\hat{c} = \frac{(1-\phi)(1 + (\gamma - \beta)\overline{p})}{\gamma}, \tag{11}$$

which leads to zero profit.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. Due to cyclical nature of the definitions, the weighted average of all prices itself depends on the average market price. By Equation (1):

$$\begin{aligned}
\overline{p} &= \frac{\int_0^{\hat{c}} q(c)p(c)\, dc}{\int_0^{\hat{c}} q(c)\, dc} \\
&= \frac{\int_0^{\hat{c}} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\big)p(c)\, dc}{\int_0^{\hat{c}} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\big)\, dc} \\
&= \frac{2(1 + (\gamma - \beta)\overline{p})}{3\gamma},
\end{aligned}$$

where the last line follows by plugging in expressions for $p(c)$ in Equation (10) and $\hat{c}$ in Equation (11).

Solving for $\overline{p}$, we have

$$\overline{p} = \frac{2}{2\beta + \gamma}. \tag{12}$$

Platform's profit maximization problem is:

$$\begin{aligned}
\max_{\phi} \quad \Pi &= \phi\overline{p} \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc \\
&= \frac{9\gamma(1-\phi)\phi}{2(2\beta+\gamma)^3}.
\end{aligned}$$

.

The platform's solution is unique. It's defined by the first order conditions:

$$\frac{\partial \Pi}{\partial \phi} = \frac{9\gamma(1-2\phi)}{2(2\beta+\gamma)^3} = 0,$$

giving the solution

$$\phi = \frac{1}{2}.$$

Platform's profit is

$$\Pi = \frac{9\gamma(1-\phi)\phi}{2(2\beta+\gamma)^3} = \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right).$$

The highest cost that participates and the average market price are

$$\begin{aligned}
\hat{c} &= \frac{3}{2(2\beta+\gamma)}, \\
\overline{p} &= \frac{2}{2\beta+\gamma}.
\end{aligned}$$

Total quantity of customers served in the market is

$$
\begin{aligned}
Q &= \int_0^{\hat{c}} q(c)\, dc \\
&= \int_0^{\hat{c}} \left(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\right)\, dc \\
&= \frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right).
\end{aligned}
$$

Servers' total profit is

$$
\begin{aligned}
\int_0^{\hat{c}} \pi(c) &= \int_0^{\hat{c}} \frac{(1-\phi-\overline{p}(1-\phi)(\beta-\gamma)-c\gamma)^2}{4\gamma(1-\phi)}\, dc \\
&= \frac{9\gamma(1-\phi)^2}{4(2\beta+\gamma)^3} = \frac{9}{16}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right).
\end{aligned} \tag{13}
$$

□

Proof of Proposition 3. The ratio of platform's profits under server pricing and platform pricing is:

$$
\frac{\Pi^{\mathcal{S}}}{\Pi^{\mathcal{P}}}(\gamma,\beta) = \frac{\frac{9}{8}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)}{\frac{1}{27\beta^2}} = \frac{243}{8}\left(\frac{\beta^2\gamma}{(2\beta+\gamma)^3}\right).
$$

The ratio of prices is increasing in γ for $\gamma < \beta$ and decreasing for $\gamma > \beta$. Therefore, the ratio of prices are quasi-concave in γ and is maximized at $\gamma = \beta$, where it takes a value of 9/8. At two extremes, where γ approaches 0 or infinity, the ratio converges to 0. By intermediate value theorem applied to both sides, there exists some $0 < \gamma_l < \beta$ and $\gamma_h > \beta$ such that

$$
\frac{\Pi^{\mathcal{S}}}{\Pi^{\mathcal{P}}}(\gamma_l,\beta) = 1,
$$

$$
\frac{\Pi^{\mathcal{S}}}{\Pi^{\mathcal{P}}}(\gamma_h,\beta) = 1.
$$

By quasi-concavity, we can further conclude that

$$
\frac{\Pi^{\mathcal{S}}}{\Pi^{\mathcal{P}}}(\gamma,\beta) > 1 \iff \gamma_l < \gamma < \gamma_h.
$$

□

Proof of Proposition 4. In the optimal mechanism, let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. Let $\pi(c, \tilde{c})$ be a server's earning with cost c by reporting cost $\tilde{c}$:

$$\pi(c, \tilde{c}) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))(p(\tilde{c}) - c) - f(\tilde{c}).$$

Let

$$u(c, \tilde{c}) = (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - c).$$

Then, server's net earnings is

$$\pi(c, \tilde{c}) = u(c, \tilde{c}) - f(\tilde{c}).$$

Notice that marginal utility from higher $p(\tilde{c})$ is increasing with cost c. Specifically,

$$\frac{\partial}{\partial c}\left(\frac{\partial \pi(c, \tilde{c})}{\partial p(\tilde{c})}\right) = \frac{\partial}{\partial c}\left(\gamma(c + \overline{p} - 2p(\tilde{c})) - \beta\overline{p} + 1\right) = \gamma > 0. \tag{14}$$

The Individual Rationality (IR) and the Incentive Compatibility (IC) constraints are:

$$\begin{aligned}
&\pi(c, \tilde{c}) = u(c, \tilde{c}) - f(\tilde{c}) \geq 0,\\
&\pi(c, c) \geq \pi(c, \tilde{c}),
\end{aligned}$$

for all $c \in \mathcal{C}$, $\tilde{c} \in \mathcal{C}$, where $\mathcal{C}$ is the set of server costs that participate in equilibrium.

By IC constraints,

$$\begin{aligned}
\pi(c, c) \geq \pi(c, \tilde{c}) &= (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - c) - f(\tilde{c})\\
&> (1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))\,(p(\tilde{c}) - \tilde{c}) - f(\tilde{c}) = \pi(\tilde{c}, \tilde{c})
\end{aligned}$$

for all $\tilde{c} > c$. Therefore, server earnings are strictly decreasing in cost. This implies there will exists a cost $\hat{c}$ such that a server with cost c participates if and only if $c \leq \hat{c}$. Otherwise, platform can uniformly increase the fee, $f(\tilde{c})$, for all participating servers, increasing its profit.

By IR constraints, we can alternatively formulate a server's earnings as

$$\pi(c, \tilde{c}) = \pi(\tilde{c}, \tilde{c}) - u(\tilde{c}, \tilde{c}) + u(c, \tilde{c}).$$

The pair of inequalities IC constraints imposes for servers with costs c and $\tilde{c}$ are:

$$\begin{aligned}
\pi(c, c) &\geq \pi(c, \tilde{c}) = \pi(\tilde{c}, \tilde{c}) - u(\tilde{c}, \tilde{c}) + u(c, \tilde{c}),\\
\pi(\tilde{c}, \tilde{c}) &\geq \pi(\tilde{c}, c) = \pi(c, c) - u(c, c) + u(\tilde{c}, c).
\end{aligned}$$

These inequalities can be combined:

$$
\begin{aligned}
& u(\tilde{c}, c) - u(c, c) \leq \pi(\tilde{c}, \tilde{c}) - \pi(c, c) \leq u(\tilde{c}, \tilde{c}) - u(c, \tilde{c}) \\
\iff & \int_{c}^{\tilde{c}} \frac{\partial u(c_k, c)}{\partial c_k} \, dc_k \leq \pi(\tilde{c}, \tilde{c}) - \pi(c, c) \leq \int_{c}^{\tilde{c}} \frac{\partial u(c_k, \tilde{c})}{\partial c_k} \, dc_k,
\end{aligned} \tag{15}
$$

where $\frac{\partial u(c_k, \tilde{c})}{\partial c_k}$ is the partial derivative of u with respect to its first argument evaluated at the point $(c_k, \tilde{c})$.

This implies

$$
\begin{aligned}
0 &\leq \int_{c}^{\tilde{c}} \frac{\partial u(c_k, \tilde{c})}{\partial c_k} - \int_{c}^{\tilde{c}} \frac{\partial u(c_k, c)}{\partial c_k} \\
&= \int_{c}^{\tilde{c}} \int_{p(c)}^{p(\tilde{c})} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} \, dp(c_z) \, dc_k.
\end{aligned} \tag{16}
$$

By Equation (14), the second derivative of u is non-negative. Therefore the expression above implies $p(\tilde{c}) \geq p(c)$ for all $\tilde{c} > c$. If platform sets $p(\tilde{c}) = p(c)$, then

$$
\begin{aligned}
u(c, \tilde{c}) &= u(c, c), \\
u(\tilde{c}, c) &= u(\tilde{c}, \tilde{c}),
\end{aligned}
$$

which then implies

$$
\pi(\tilde{c}, \tilde{c}) - \pi(c, c) = u(\tilde{c}, \tilde{c}) - u(c, c) \iff f(c) = f(\tilde{c}).
$$

By fixing one end point and letting the other converge towards it in Equation (15), we can also infer that $u(c_k, c_k)$ is continuous and its derivative satisfies:

$$
\frac{\partial u(c, c)}{\partial c} = \frac{d\pi(c, c)}{dc},
$$

where $\frac{d\pi(c,c)}{dc}$ is the total derivative of π with respect to c evaluated at (c, c) and $\frac{\partial u(c,c)}{\partial c}$ is the partial derivative of u with respect to its first argument evaluated at the point (c, c). Then,

$$
\pi(c, c) + \int_{c}^{\tilde{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k = \pi(\tilde{c}, \tilde{c}).
$$

Setting $\tilde{c} = \hat{c}$, the equation simplifies to

$$
\pi(c, c) = -\int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k,
$$

which is equivalent to

$$f(c) = u(c,c) + \int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k. \tag{17}$$

Notice that the monotonicity of $p(\tilde{c})$ and Equation (17) are necessary conditions implied by IC. We can also show that they are sufficient conditions. With that aim, let us assume Equation (17) holds. If $\tilde{c} > c$, we have

$$\begin{aligned}
\pi(c,\tilde{c}) &= u(c,\tilde{c}) - f(\tilde{c}) \\
&= u(c,\tilde{c}) - u(\tilde{c},\tilde{c}) - \int_{\tilde{c}}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
&= u(c,\tilde{c}) - u(\tilde{c},\tilde{c}) - \int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k + \int_{c}^{\tilde{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
&= -\int_{c}^{\tilde{c}} \frac{\partial u(c_k, \tilde{c})}{\partial c_k} \, dc_k - \int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k + \int_{c}^{\tilde{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
&= -\int_{c}^{\tilde{c}} \left(\frac{\partial u(c_k, \tilde{c})}{\partial c_k} - \frac{\partial u(c_k, c_k)}{\partial c_k} \right) \, dc_k - \int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
&= -\int_{c}^{\tilde{c}} \int_{p(c_k)}^{p(\tilde{c})} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} \, dp(c_z) \, dc_k + \pi(c,c) \leq \pi(c,c),
\end{aligned}$$

where the inequality follows from Equation (16). If $\tilde{c} < c$, we have

$$\begin{aligned}
\pi(c,\tilde{c}) &= u(c,\tilde{c}) - f(\tilde{c}) \\
&= u(c,\tilde{c}) - u(\tilde{c},\tilde{c}) - \int_{\tilde{c}}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
&= u(c,\tilde{c}) - u(\tilde{c},\tilde{c}) - \int_{\tilde{c}}^{c} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k - \int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
&= \int_{\tilde{c}}^{c} \frac{\partial u(c_k, \tilde{c})}{\partial c_k} \, dc_k - \int_{\tilde{c}}^{c} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k - \int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
&= -\int_{\tilde{c}}^{c} \left(\frac{\partial u(c_k, c_k)}{\partial c_k} - \frac{\partial u(c_k, \tilde{c})}{\partial c_k} \right) \, dc_k - \int_{c}^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k} \, dc_k \\
- &= \int_{\tilde{c}}^{c} \int_{p(\tilde{c})}^{p(c_k)} \frac{\partial^2 u(c_k, c_z)}{\partial c_k \partial p(c_z)} \, dp(c_z) \, dc_k + \pi(c,c) \leq \pi(c,c),
\end{aligned}$$

where the inequality follows from Equation (16). Hence, the conditions above are sufficient for IC.

The platform's problem is to choose $p(c)$, $f(c)$ and $\hat{c}$ to maximize total profits subject to IR and IC constraints:

$$
\begin{aligned}
\max_{p(c),f(c),\hat{c}} \quad & \int_0^{\hat{c}} f(c)\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0,\hat{c}) \\
& p'(c) \geq 0, \forall c \in (0,\hat{c}) \\
& f(c) = u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k,c_k)}{\partial c_k}\, dc_k \\
= \max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} \left(u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k,c_k)}{\partial c_k}\, dc_k \right) dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0,\hat{c}) \\
& p'(c) \geq 0, \forall c \in (0,\hat{c}).
\end{aligned}
$$

Notice that $u(c,c)$ is equivalent to the welfare generated by server i. Therefore the platform's optimal mechanism will not be welfare-maximizing. By integration by parts,

$$
\begin{aligned}
\int_0^{\hat{c}} \left(\int_c^{\hat{c}} \frac{\partial u(c_k,c_k)}{\partial c_k}\, dc_k \right) dc &= \left[\left(\int_c^{\hat{c}} \frac{\partial u(c_k,c_k)}{\partial c_k}\, dc_k \right) c \right]_0^{\hat{c}} - \int_0^{\hat{c}} \left(-\frac{\partial u(c,c)}{\partial c} \right) c\, dc \\
&= \int_0^{\hat{c}} \frac{\partial u(c,c)}{\partial c} c\, dc.
\end{aligned} \tag{18}
$$

The platform's problem converts to

$$
\begin{aligned}
\max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} \left(u(c,c) + \frac{\partial u(c,c)}{\partial c} c \right) dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0,\hat{c}) \\
& p'(c) \geq 0, \forall c \in (0,\hat{c}) \\
= \max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} \Big((1-\beta\overline{p}+\gamma(\overline{p}-p(c)))(p(c)-c) - (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))c \Big)\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0,\hat{c}) \\
& p'(c) \geq 0, \forall c \in (0,\hat{c}) \\
= \max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} (1-\beta\overline{p}+\gamma(\overline{p}-p(c)))(p(c)-2c)\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0,\hat{c}) \\
& p'(c) \geq 0, \forall c \in (0,\hat{c}).
\end{aligned}
$$

Let us focus on the problem where the first constraint is relaxed. Let

$$
p(c) = \frac{1-\beta\overline{p}+\gamma\overline{p}+2c\gamma}{2\gamma} + \delta(c),
$$

where $\delta(c)$ is any deviation from the price that maximizes the profits from each individual server. Let us fix $\overline{p}$ and $\hat{c}$ to a number and re-write platform's problem as a function of $\delta(c)$. After we plug in $p(c)$, the problem simplifies to:

$$\begin{aligned} \max_{\delta(c)} \quad & \int_0^{\hat{c}} \frac{(-2c\gamma + (\gamma-\beta)\overline{p}+1)^2}{4\gamma}\, dc - \gamma \int_0^{\hat{c}} \delta(c)^2\, dc \\ \text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\, dc = \tfrac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \tfrac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\ & \delta'(c) \geq -1. \end{aligned}$$

Notice that since $\overline{p}$ and $\hat{c}$ are fixed, the optimal solution is equivalent to the solution of the following problem:

$$\begin{aligned} \max_{\delta(c)} \quad & -\gamma \int_0^{\hat{c}} \delta(c)^2\, dc \\ \text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\, dc = \frac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\ & \delta'(c) \geq -1 \\ = -\gamma \min_{\delta(c)} \quad & \int_0^{\hat{c}} \delta(c)^2\, dc \\ \text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\, dc = \frac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\ & \delta'(c) \geq -1. \end{aligned}$$

Let us solve the relaxed problem without the inequality constraint. We can use calculus of variations to solve this problem. The Lagrangian is

$$L = \int_0^{\hat{c}} \delta(c)^2\, dc + \lambda\left(\tfrac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \tfrac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) - \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\, dc\right).$$

The Lagrange-Euler equation is

$$\frac{\partial}{\partial\delta(c)}\Big(\delta(c)^2 - \lambda\left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\Big) = 0,\ \forall c \in (0, \hat{c}),$$

which implies

$$\delta(c) = \frac{\gamma\lambda\overline{p} - 2c\gamma\lambda}{2(\gamma\lambda - 1)},\ \forall c.$$

We plug this into the constraint to find λ. We get two roots:

$$\lambda = \frac{1}{\gamma} \mp \frac{\sqrt{\frac{\hat{c}^2}{3} - \hat{c}\overline{p} + \overline{p}^2}}{\beta\overline{p} - 1}.$$

We plug in the λ to find the optimal $\delta(c)$ and $p(c)$. If $\lambda = \frac{1}{\gamma} - \frac{\sqrt{\frac{4\hat{c}^2}{3} - 2\hat{c}\overline{p} + \overline{p}^2}}{\beta\overline{p}-1}$, we have

$$p(c) = \frac{6c(\beta\overline{p}-1) + \overline{p}\left(-\sqrt{3}(\beta - 2\gamma)\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} - 3\beta\overline{p} + 3\right) + \sqrt{3}\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}}{2\sqrt{3}\gamma\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}}.$$

Since price is decreasing in costs, the solution is not incentive compatible. If $\lambda = \frac{1}{\gamma} + \frac{\sqrt{\frac{4\hat{c}^2}{3} - 2\hat{c}\overline{p} + \overline{p}^2}}{\beta\overline{p} - 1}$, we get

$$p(c) = \frac{c(6 - 6\beta\overline{p}) + \overline{p}\left(-\sqrt{3}(\beta - 2\gamma)\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} + 3\beta\overline{p} - 3\right) + \sqrt{3}\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}}{2\sqrt{3}\gamma\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}}.$$

The price is increasing in c, so the solution is feasible. The marginal server serves non-negative demand:

$$q(\hat{c}) = -\frac{\left(\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} - 2\sqrt{3}\hat{c} + \sqrt{3}\overline{p}\right)(\beta\overline{p} - 1)}{2\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}} \geq 0,$$

which implies $\hat{c} < 3\overline{p}/4$. We now plug the optimal prices in platform's objective. Platform's problem becomes:

$$\begin{aligned} \max_{\overline{p},\hat{c}} \quad & \tfrac{1}{6}\hat{c}\left(\sqrt{3}\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} - 3\hat{c} + 3\overline{p}\right)(1 - \beta\overline{p}) \\ \text{s.t.} \quad & \hat{c} \leq 3\overline{p}/4. \end{aligned}$$

In the optimal mechanism, the highest cost that participates and the average market price are:

$$\overline{p} = \frac{2}{3\beta},$$
$$\hat{c} = \frac{1}{2\beta}.$$

The equilibrium characteristics are as follows:

$$\begin{aligned} p(c) &= \frac{c(6 - 6\beta\overline{p}) + \overline{p}\left(-\sqrt{3}(\beta - 2\gamma)\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} + 3\beta\overline{p} - 3\right) + \sqrt{3}\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}}{2\sqrt{3}\gamma\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2}} \\ &= \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{c\beta}{\gamma}, \\ f(c) &= u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k \\ &= c^2\left(\frac{\beta(\gamma - 2\beta)}{2\gamma}\right) + \frac{2c}{3}\left(\frac{\beta}{\gamma} - 1\right) + \frac{1}{24}\left(\frac{5}{\beta} - \frac{2}{\gamma}\right). \end{aligned}$$

The platform earns

$$\begin{aligned} \Pi &= \frac{1}{6}\hat{c}\left(\sqrt{3}\sqrt{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} - 3\hat{c} + 3\overline{p}\right)(1 - \beta\overline{p}) \\ &= \frac{1}{24\beta^2}. \end{aligned}$$

Total quantity of customers served in the market is

$$Q = \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc = \frac{1}{8\beta}.$$

Servers' total profit is

$$\int_0^{\hat{c}} \pi(c) = \int_0^{\hat{c}} ((1 - \beta\overline{p} + \gamma(\overline{p} - p(\tilde{c})))(p(\tilde{c}) - c) - f(\tilde{c}))\, dc$$
$$= \frac{1}{48\beta}.$$

□

Proof of Proposition 5. The platform's fee under quantity pricing is

$$f(c)/q(c) = q(c)\left(\frac{1}{2\beta} - \frac{1}{\gamma}\right) + \left(\frac{1}{6\beta} + \frac{1}{3\gamma}\right).$$

The platform announces the fee structure above, but otherwise lets the servers choose their prices. Servers choose the quantities that maximize their profits knowing that other servers do the same. Thus, server c faces the problem:

$$\max_{p(c)} \quad \pi(c) = q(c)(p(c) - c) - q(c)\left(f(c)/q(c)\right).$$

Given an expected average price $\overline{p}$, the servers' optimal quantity choice satisfies

$$p(c) = \frac{2\beta(3c\gamma - 6\gamma\overline{p} - 2) + 6\beta^2\overline{p} + \gamma(6\gamma\overline{p} + 7)}{6\gamma^2}. \tag{19}$$

Server c earns

$$\pi(c) = \frac{(2\beta(2 - 3c\gamma + 3\overline{p}(\gamma - \beta)) - \gamma)^2}{72\beta\gamma^2}.$$

Server profits are decreasing in cost, c, and therefore there exists a threshold cost, $\hat{c}$, for which server with cost c participates if and only if $c < \hat{c}$. The highest cost that participates is

$$\hat{c} = \frac{2}{3\gamma} + \overline{p} - \frac{\beta\overline{p}}{\gamma} - \frac{1}{6\beta}, \tag{20}$$

which leads to zero profit.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. Due to cyclical nature of the definitions, the weighted average of all prices itself depends on the average market price. By Equation (1),

$$\begin{aligned}
\overline{p} &= \frac{\int_0^{\hat{c}} q(c)p(c)\, dc}{\int_0^{\hat{c}} q(c)\, dc} \\
&= \frac{\int_0^{\hat{c}} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\big)p(c)\, dc}{\int_0^{\hat{c}} \big(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\big)\, dc} \\
&= \frac{6\beta^2\overline{p} - \beta(15\gamma\overline{p} + 4) + \gamma(9\gamma\overline{p} + 10)}{9\gamma^2},
\end{aligned}$$

where the last line follows by plugging in the expressions for $p(c)$ in Equation (19) and $\hat{c}$ in Equation (20).

Solving for $\overline{p}$ gives:

$$\overline{p} = \frac{2}{3\beta},$$

which implies

$$\begin{aligned}
\hat{c} &= \frac{2}{3\gamma} + \overline{p} - \frac{\beta\overline{p}}{\gamma} - \frac{1}{6\beta} \\
&= \frac{1}{2\beta}.
\end{aligned}$$

Plugging the relevant expressions into platform's objective, the platform's optimal profit is

$$\Pi = \frac{1}{24\beta^2}.$$

Notice that the platform's profit under this policy is equivalent to the optimal mechanism. We further find that

$$\begin{aligned}
p(c) &= \frac{2\beta(3c\gamma - 6\gamma\overline{p} - 2) + 6\beta^2\overline{p} + \gamma(6\gamma\overline{p} + 7)}{6\gamma^2} \\
&= \frac{2}{3\beta} - \frac{1}{6\gamma} + \frac{c\beta}{\gamma}.
\end{aligned}$$

Total quantity of customers served in the market is

$$\begin{aligned}
Q &= \int_0^{\hat{c}} \left(1 - \beta\overline{p} + \gamma(\overline{p} - p(c))\right)\, dc \\
&= \frac{1}{8\beta}.
\end{aligned}$$

Servers' total profit is

$$\begin{aligned}
\int_0^{\hat{c}} \pi(c) &= \int_0^{\hat{c}} \frac{(2\beta(2 - 3c\gamma + 3\overline{p}(\gamma - \beta)) - \gamma)^2}{72\beta\gamma^2}\, dc \\
&= \frac{1}{48\beta}.
\end{aligned} \tag{21}$$

That is, the equilibrium market characteristics under quantity pricing is equivalent to the optimal mechanism.

Proof of Corollary 1. By Equations, (9), (13), (21), we have

$$\int_0^{\hat{c}} \pi^{\mathcal{Q}}(c) = \frac{1}{48\beta} \geq \frac{1}{54\beta} = \int_0^{\hat{c}} \pi^{\mathcal{P}}(c)$$

and

$$\int_0^{\hat{c}} \pi^{\mathcal{Q}}(c) = \frac{1}{48\beta} \geq \frac{9}{16}\left(\frac{\gamma}{(2\beta+\gamma)^2}\right) = \int_0^{\hat{c}} \pi^{\mathcal{S}}(c).$$

Quantity pricing, which maximizes platform profits, also maximizes servers' total profits.

Proof of Proposition 6. Let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. By Proposition 4, the monotonicy of prices, $p(c)$, and Equation (17) are necessary and sufficient conditions for servers' IC constraints. Then, the equilibrium fees charged to server c is characterized as

$$f(c) = u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k.$$

The total system profits generated is

$$\int_0^{\hat{c}} q(c)(p(c) - c)\, dc.$$

The optimal truth-inducing contract that maximizes total system profits is characterized through the following problem:

$$\begin{aligned}
\max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - c)\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0, \hat{c}) \\
& p'(c) \geq 0, \forall c \in (0, \hat{c}) \\
& f(c) = u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k, \forall c \in (0, \hat{c}).
\end{aligned}$$

Let

$$p(c) = \frac{1 - \beta\overline{p} + \gamma\overline{p} + c\gamma}{2\gamma} + \delta(c),$$

where $\delta(c)$ is the deviation of the optimal prices from the price that maximizes individual server profits. We fix $\overline{p}$ and $\hat{c}$ to a number and re-write platform's problem as a function of $\delta(c)$. After we plug in the values of $p(c)$ and simplify the expressions the problem becomes:

$$\begin{aligned}
\max_{\delta(c)} \quad & \int_0^{\hat{c}} \frac{(-c\gamma+(\gamma-\beta)\overline{p}+1)^2}{4\gamma}\,dc - \gamma\int_0^{\hat{c}} \delta(c)^2\,dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\,dc = \frac{\hat{c}(1-\beta\overline{p})^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& \delta'(c) \geq -\tfrac{1}{2}.
\end{aligned}$$

Notice that since $\overline{p}$ and $\hat{c}$ are fixed, the optimal solution is equivalent to the solution of the following problem:

$$\begin{aligned}
\max_{\delta(c)} \quad & -\gamma\int_0^{\hat{c}} \delta(c)^2\,dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\,dc = \frac{\hat{c}(1-\beta\overline{p})^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& \delta'(c) \geq -\tfrac{1}{2} \\
= -\gamma\min_{\delta(c)} \quad & \int_0^{\hat{c}} \delta(c)^2\,dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\,dc = \frac{\hat{c}(1-\beta\overline{p})^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& \delta'(c) \geq -\tfrac{1}{2}.
\end{aligned}$$

Let us solve the relaxed problem without the inequality constraint. We will see later that the constraint is not binding for the relaxed problem. We can use calculus of variations to solve this problem. The Lagrangian is

$$L = \int_0^{\hat{c}} \delta(c)^2\,dc + \lambda\left(\frac{\hat{c}(1-\beta\overline{p})^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2\right) - \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\,dc\right).$$

The Lagrange-Euler equation is

$$\frac{\partial}{\partial\delta(c)}\Big(\delta(c)^2\,dc - \lambda\left(\gamma\delta(c)^2 + c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\Big) = 0,\ \forall c \in (0,\hat{c}).$$

This gives

$$\delta(c) = \frac{\gamma\lambda\overline{p} - c\gamma\lambda}{2(\gamma\lambda-1)},\ \forall c \in (0,\hat{c}).$$

We plug this into the constraint to find λ. We get two roots

$$\lambda = \frac{1}{\gamma} \mp \frac{\sqrt{\frac{\hat{c}^2}{3} - \hat{c}\overline{p} + \overline{p}^2}}{\beta\overline{p}-1}.$$

We can plug in the λ to find the optimal $\delta(c)$ and therefore the optimal $p(c)$.

If $\lambda = \frac{1}{\gamma} - \frac{\sqrt{\frac{\hat{c}^2}{3} - \hat{c}\overline{p} + \overline{p}^2}}{\beta\overline{p}-1}$, we get

$$p(c) = \frac{3c(1-\beta\overline{p}) + \overline{p}\left(-\sqrt{3}(\beta-2\gamma)\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2} - 3\beta\overline{p} + 3\right) + \sqrt{3}\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2}}{2\sqrt{3}\gamma\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2}}.$$

Notice that this solution is not feasible, since the price is decreasing in cost, c. Let us look at the other solution. If $\lambda = \frac{1}{\gamma} + \frac{\sqrt{\frac{\hat{c}^2}{3} - \hat{c}\overline{p} + \overline{p}^2}}{\beta\overline{p} - 1}$, we get

$$p(c) = \frac{3c(\beta\overline{p}-1) + \overline{p}\left(-\sqrt{3}(\beta-2\gamma)\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2} - 3\beta\overline{p} + 3\right) + \sqrt{3}\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2}}{2\sqrt{3}\gamma\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2}}.$$

The price is increasing in c, so the solution is feasible. As $p(c)$ is a linear function, we can conclude that in the optimal mechanism prices will be linear. The marginal server serves non-negative demand:

$$q(\hat{c}) = -\frac{\left(\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2} + \left(-\sqrt{3}\right)\hat{c} + \sqrt{3}\overline{p}\right)(\beta\overline{p}-1)}{2\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2}} \geq 0,$$

which implies $\hat{c} < 3\overline{p}/2$. We now plug the optimal prices in the objective. The problem becomes:

$$\begin{aligned} \max_{\overline{p},\hat{c}} \quad & \tfrac{1}{12}\hat{c}\left(-2\sqrt{3}\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2} + 3\hat{c} - 6\overline{p}\right)(\beta\overline{p}-1) \\ \text{s.t.} \quad & \hat{c} < 3\overline{p}/2. \end{aligned}$$

Solving for maximizers, we get

$$\overline{p} = \frac{2}{3\beta},$$
$$\hat{c} = \frac{1}{\beta}.$$

Total value generated under optimal mechanism is

$$\Pi = \frac{1}{12}\hat{c}\left(-2\sqrt{3}\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2} + 3\hat{c} - 6\overline{p}\right)(\beta\overline{p}-1) = \frac{1}{12\beta^2}.$$

□

Proof of Proposition 7. The equilibrium is defined similar to server pricing with $\phi = 0$:

$$\begin{aligned} p(c) &= \frac{1}{2\gamma}\left(1 + (\gamma-\beta)\overline{p} + \frac{c\gamma}{1-\phi}\right) \\ &= \frac{3}{2(2\beta+\gamma)} + \frac{1}{2}c. \end{aligned}$$

Server c earns

$$\pi(c) = \frac{(1 - \overline{p}(\beta-\gamma) - c\gamma)^2}{4\gamma}.$$

Highest cost that participates in the market is

$$\hat{c} = \frac{(1-\phi)(1 + (\gamma-\beta)\overline{p})}{\gamma} = \frac{3}{2\beta+\gamma},$$

which leads to zero profit.

The average market price is

$$\overline{p} = \frac{2}{2\beta + \gamma},$$

equivalent to server pricing.

The total system profits is

$$\int_0^{\hat{c}} \pi(c) = \int_0^{\hat{c}} q(c)(p(c) - c)\, dc$$
$$= \frac{9}{4}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right).$$

The ratio of total system profits under disintermediated server pricing and quantity pricing is:

$$\frac{W^{\mathcal{D}}}{W^{\mathcal{Q}}}(\gamma, \beta) = \frac{\frac{9}{4}\left(\frac{\gamma}{(2\beta+\gamma)^3}\right)}{\frac{3}{48\beta^2}} = \frac{36\beta^2\gamma}{(2\beta+\gamma)^3}.$$

The ratio of prices is increasing in γ for $\gamma < \beta$ and decreasing for $\gamma > \beta$. Therefore, the ratio of prices are quasi-concave in γ and is maximized at $\gamma = \beta$, where it takes a value of 4/3. At two extremes, where γ approaches 0 or infinity, the ratio converges to 0. Then, by intermediate value theorem, there exists some $0 < \gamma_l < \beta$ and $\gamma_h > \beta$ such that

$$\frac{W^{\mathcal{D}}}{W^{\mathcal{Q}}}(\gamma_l, \beta) = 1,$$
$$\frac{W^{\mathcal{D}}}{W^{\mathcal{Q}}}(\gamma_h, \beta) = 1.$$

By quasi-concavity, we can further conclude that:

$$\frac{W^{\mathcal{D}}}{W^{\mathcal{Q}}} > 1 \iff \gamma_l < \gamma < \gamma_h.$$

□

Extension of Platform's Optimal Contract with a Total Server Profits Target. Let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. By Proposition 4, the monotonicy of prices, $p(c)$, and Equation (17) are necessary and sufficient conditions for servers' IC constraints. Then, the equilibrium fees charged to server c is characterized as

$$f(c) = u(c, c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k.$$

The total system profits generated is

$$\int_0^{\hat{c}} q(c)(p(c) - c)\, dc.$$

Platform's optimal truth-inducing contract that maximizes its profit subject to the constraint servers earn a total profit of K is defined by:

$$
\begin{aligned}
\max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} f(c)\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0, \hat{c}) \\
& p'(c) \geq 0, \forall c \in (0, \hat{c}) \\
& f(c) = u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k, \forall c \in (0, \hat{c}) \\
& \int_0^{\hat{c}} q(c)(p(c) - c)\, dc - \int_0^{\hat{c}} f(c)\, dc = K \\
= \max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - 2c)\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0, \hat{c}) \\
& p'(c) \geq 0, \forall c \in (0, \hat{c}) \\
& f(c) = u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k, \forall c \in (0, \hat{c}) \\
& \int_0^{\hat{c}} q(c)(p(c) - c)\, dc - \int_0^{\hat{c}} f(c)\, dc = K,
\end{aligned}
$$

where the equivalency of two models follow from Equation (18).

Let

$$
p(c) = \frac{1 - \beta\overline{p} + \gamma\overline{p} + 2c\gamma}{2\gamma} + \delta(c),
$$

where $\delta(c)$ is the deviation of the optimal prices from the price that maximizes individual server profits. We fix $\overline{p}$ and $\hat{c}$ to a number and re-write platform's problem as a function of $\delta(c)$. After we plug in the values of $p(c)$ and simplify the expressions the problem becomes:

$$
\begin{aligned}
\max_{\delta(c)} \quad & \int_0^{\hat{c}} \frac{(-2c\gamma + (\gamma - \beta)\overline{p} + 1)^2}{4\gamma}\, dc - \gamma \int_0^{\hat{c}} \delta(c)^2\, dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\, dc = \frac{\hat{c}(\beta\overline{p} - 1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& \int_0^{\hat{c}} c\gamma\delta(c)\, dc = \frac{1}{12}\hat{c}^2(-4\gamma\hat{c} + 3\overline{p}(\gamma - \beta) + 3) \\
& \delta'(c) \geq -1.
\end{aligned}
$$

Notice that since $\overline{p}$ and $\hat{c}$ are fixed, the optimal solution is equivalent to the solution of the following problem:

$$\begin{aligned}
&\max_{\delta(c)} && -\gamma \int_0^{\hat{c}} \delta(c)^2 \, dc \\
&\text{s.t.} && \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) \, dc = \frac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& && \int_0^{\hat{c}} c\gamma\delta(c) \, dc = \frac{1}{12}\hat{c}^2(-4\gamma\hat{c} + 3\overline{p}(\gamma-\beta) + 3) \\
& && \delta'(c) \geq -\tfrac{1}{2} \\
= -\gamma &\min_{\delta(c)} && \int_0^{\hat{c}} \delta(c)^2 \, dc \\
&\text{s.t.} && \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) \, dc = \frac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& && \int_0^{\hat{c}} c\gamma\delta(c) \, dc = \frac{1}{12}\hat{c}^2(-4\gamma\hat{c} + 3\overline{p}(\gamma-\beta) + 3) \\
& && \delta'(c) \geq -\tfrac{1}{2}.
\end{aligned}$$

Let us solve the relaxed problem without the inequality constraint. We will see later that the constraint is not binding for the relaxed problem. We can use calculus of variations to solve this problem. The Lagrangian is

$$\begin{aligned}
L = \; & \int_0^{\hat{c}} \delta(c)^2 \, dc + \lambda \left(\frac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) - \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) \, dc \right) \\
& + \tau \left(\frac{1}{12}\hat{c}^2(-4\gamma\hat{c} + 3\overline{p}(\gamma-\beta) + 3) - \int_0^{\hat{c}} c\gamma\delta(c) \right)
\end{aligned}$$

The Lagrange-Euler equation is

$$\frac{\partial}{\partial\delta(c)}\Big(\delta(c)^2 \, dc - \lambda\left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) - \tau c\gamma\delta(c)\Big) = 0, \;\forall c \in (0, \hat{c}).$$

This gives

$$\delta(c) = \frac{-2c\lambda - c\tau + \lambda\overline{p}}{2(\lambda-1)}, \;\forall c \in (0, \hat{c}).$$

We plug this into the constraint to find λ. We get two roots

$$\begin{aligned}
\lambda^1 &= 1 - \frac{\gamma\hat{c}^2\overline{p}}{\sqrt{\hat{c}^4(\beta\overline{p}-1)^2 - 24\hat{c}^2K(\beta\overline{p}-1) - 48K^2}}, \\
\tau^1 &= \frac{3\overline{p}\left(\hat{c}^2(1-\beta\overline{p}) + \sqrt{\hat{c}^4(\beta\overline{p}-1)^2 - 24\hat{c}^2K(\beta\overline{p}-1) - 48K^2} - 4K\right)}{2\hat{c}\sqrt{\hat{c}^4(\beta\overline{p}-1)^2 - 24\hat{c}^2K(\beta\overline{p}-1) - 48K^2}} - 2,
\end{aligned}$$

and

$$\begin{aligned}
\lambda^2 &= \frac{\gamma\hat{c}^2\overline{p}}{\sqrt{\hat{c}^4(\beta\overline{p}-1)^2 - 24\hat{c}^2K(\beta\overline{p}-1) - 48K^2}} + 1, \\
\tau^2 &= \frac{3\overline{p}\left(\hat{c}^2(\beta\overline{p}-1) + \sqrt{\hat{c}^4(\beta\overline{p}-1)^2 - 24\hat{c}^2K(\beta\overline{p}-1) - 48K^2} + 4K\right)}{2\hat{c}\sqrt{\hat{c}^4(\beta\overline{p}-1)^2 - 24\hat{c}^2K(\beta\overline{p}-1) - 48K^2}} - 2.
\end{aligned}$$

We can plug in the λ and τ values to find the optimal $\delta(c)$ and therefore the optimal $p(c)$.

Using the first solution, we get

$$p(c) = \frac{2\hat{c}\left(\hat{c}^2(\beta(-\overline{p})+2\gamma\overline{p}+1)+\sqrt{\hat{c}^4(\beta\overline{p}-1)^2-24\hat{c}^2K(\beta\overline{p}-1)-48K^2}\right)-3c\left(\hat{c}^2(\beta\overline{p}-1)+\sqrt{\hat{c}^4(\beta\overline{p}-1)^2-24\hat{c}^2K(\beta\overline{p}-1)-48K^2}+4K\right)}{4\gamma\hat{c}^3}.$$

Notice that this solution is not feasible, since the price is decreasing in cost, c. Let us look at the other solution. We get

$$p(c) = \frac{3c\left(\beta\hat{c}^2(-\overline{p})+\hat{c}^2+\sqrt{\hat{c}^4(\beta\overline{p}-1)^2-24\hat{c}^2K(\beta\overline{p}-1)-48K^2}-4K\right)+2\hat{c}^3(\beta(-\overline{p})+2\gamma\overline{p}+1)-2\hat{c}\sqrt{\hat{c}^4(\beta\overline{p}-1)^2-24\hat{c}^2K(\beta\overline{p}-1)-48K^2}}{4\gamma\hat{c}^3}.$$

The price is increasing in c as long as $\hat{c} \geq \sqrt{2}\sqrt{-\frac{K}{\beta\overline{p}-1}}$. We will later see that this inequality is never binding. As $p(c)$ is a linear function, we can conclude that in the optimal mechanism prices will be linear. The marginal server serves non-negative demand:

$$q(\hat{c}) = -\frac{\left(\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2} + \left(-\sqrt{3}\right)\hat{c} + \sqrt{3}\overline{p}\right)(\beta\overline{p} - 1)}{2\sqrt{\hat{c}^2 - 3\hat{c}\overline{p} + 3\overline{p}^2}} \geq 0,$$

which implies $\hat{c} \leq 2\sqrt{-\frac{K}{\beta\overline{p}-1}}$. We now plug the optimal prices in the objective. The problem becomes:

$$\begin{aligned}
\max_{\overline{p},\hat{c}} \quad & \frac{1}{8}\left(\frac{\overline{p}\left(\sqrt{\hat{c}^4(\beta\overline{p}-1)^2-24\hat{c}^2K(\beta\overline{p}-1)-48K^2}+12K\right)}{\hat{c}} + \hat{c}\overline{p}(1-\beta\overline{p}) - 16K\right) \\
\text{s.t.} \quad & \sqrt{2}\sqrt{-\frac{K}{\beta\overline{p}-1}} \leq \hat{c} \leq 2\sqrt{-\frac{K}{\beta\overline{p}-1}}.
\end{aligned}$$

Solving for maximizers, we get

$$\overline{p} = \frac{2}{3\beta},$$

$$\hat{c} = 2\sqrt{3}\sqrt{K}.$$

Platform's total profit under optimal mechanism is

$$\Pi = \frac{1}{8}\left(\frac{\overline{p}\left(\sqrt{\hat{c}^4(\beta\overline{p}-1)^2 - 24\hat{c}^2K(\beta\overline{p}-1) - 48K^2} + 12K\right)}{\hat{c}} + \hat{c}\overline{p}(1-\beta\overline{p}) - 16K\right) = \frac{\sqrt{K}}{\sqrt{3}\beta} - 2K.$$

□

Extension with Throughput-Maximization. Under platform pricing, the platform's quantity-maximization problem is characterized as

$$\begin{aligned}
\max_{p,\phi} \quad Q \quad &= \int_0^{\hat{c}} (1-\beta p)\, dc \\
&= (1-\phi)p(1-\beta p).
\end{aligned}$$

The platform's solution is unique:

$$p(c) = \frac{1}{2\beta},$$
$$\phi = 0.$$

Total quantity of customers served in the market is

$$Q = (1-\phi)p(1-\beta p) = \frac{1}{4\beta}.$$

Under server pricing, servers' optimal pricing and entry decisions are defined by Equations (10) and (11). Average market price is defined by Equation (12). The platform's quantity-maximization problem is characterized as

$$\begin{aligned} \max_{\phi} \quad Q &= \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc \\ &= \frac{9\gamma(1-\phi)}{4(2\beta+\gamma)^2}. \end{aligned}$$

The platform's solution is unique:

$$\phi = 0.$$

Total quantity of customers served in the market is

$$Q = \frac{9\gamma(1-\phi)}{4(2\beta+\gamma)^2} = \frac{9\gamma}{4(2\beta+\gamma)^2}.$$

In the quantity maximizing optimal contract, let $p(c)$ be the price the platform assigns to server c and $f(c)$ be the fee collected. By Proposition 4, the monotonicy of prices, $p(c)$, and Equation (17) are necessary and sufficient conditions for servers' IC constraints. Then, the equilibrium fees charged to server c is characterized as

$$f(c) = u(c,c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k.$$

The total quantity served in the market is

$$\int_0^{\hat{c}} q(c)\, dc = \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc.$$

The optimal truth-inducing contract that maximizes total quantity served subject to non-negative profit constraint is characterized through the following problem:

$$
\begin{aligned}
\max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0, \hat{c}) \\
& p'(c) \geq 0, \forall c \in (0, \hat{c}) \\
& f(c) = u(c, c) + \int_c^{\hat{c}} \frac{\partial u(c_k, c_k)}{\partial c_k}\, dc_k, \forall c \in (0, \hat{c}) \\
& \int_0^{\hat{c}} f(c)\, dc \geq 0 \\
= \max_{p(c),\hat{c}} \quad & \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))\, dc \\
\text{s.t.} \quad & \pi(c) \geq 0, \forall c \in (0, \hat{c}) \\
& p'(c) \geq 0, \forall c \in (0, \hat{c}) \\
& \int_0^{\hat{c}} (1 - \beta\overline{p} + \gamma(\overline{p} - p(c)))(p(c) - 2c)\, dc \geq 0,
\end{aligned}
$$

where the equivalence of the last constraints follow from Equation (18). Plugging in

$$
p(c) = \frac{1 - \beta\overline{p} + \gamma\overline{p} + 2c\gamma}{2\gamma} + \delta(c)
$$

and fixing $\overline{p}$ and $\hat{c}$ to a number, the problem simplifies to

$$
\begin{aligned}
\max_{\delta(c)} \quad & \int_0^{\hat{c}} \frac{1}{2}(-2c\gamma + \overline{p}(\gamma - \beta) + 1)\, dc - \gamma \int_0^{\hat{c}} \delta(c)\, dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right)\, dc = \tfrac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \tfrac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& \delta'(c) \geq -1 \\
& \frac{(2\gamma\hat{c} + \beta\overline{p} - \gamma\overline{p} - 1)^3 - (\overline{p}(\beta - \gamma) - 1)^3}{24\gamma^2} \geq \int_0^{\hat{c}} \gamma\delta(c)^2\, dc.
\end{aligned}
$$

Since $\overline{p}$ and $\hat{c}$ are fixed, the optimal solution is equivalent to the solution of the following problem:

$$\begin{aligned}
\max_{\delta(c)} \quad & -\gamma \int_0^{\hat{c}} \delta(c)\, dc \\
\text{s.t.} \quad & \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) dc = \frac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& \delta'(c) \geq -1 \\
& \frac{(2\gamma\hat{c} + \beta\overline{p} - \gamma\overline{p} - 1)^3 - (\overline{p}(\beta-\gamma)-1)^3}{24\gamma^2} \geq \int_0^{\hat{c}} \gamma\delta(c)^2\, dc \\
= -\gamma \min_{\delta(c)} \quad & \int_0^{\hat{c}} \delta(c)\, dc \\
\text{s.t.} \quad & \gamma \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) dc = \tfrac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \tfrac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) \\
& \delta'(c) \geq -1 \\
& \frac{(2\gamma\hat{c} + \beta\overline{p} - \gamma\overline{p} - 1)^3 - (\overline{p}(\beta-\gamma)-1)^3}{24\gamma^2} \geq \int_0^{\hat{c}} \gamma\delta(c)^2\, dc.
\end{aligned}$$

Let us solve the relaxed problem without the first inequality constraint. Later, we will see that this constraint is not binding. In the equilibrium, the budget constraint is binding, as otherwise, the platform can distribute all of its profits among participating servers. We can use calculus of variations to solve this problem. The Lagrangian is

$$L = \int_0^{\hat{c}} \delta(c)\, dc + \lambda \left(\frac{\hat{c}(\beta\overline{p}-1)^2}{4\gamma} - \frac{1}{12}\gamma\hat{c}\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right) - \int_0^{\hat{c}} \left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) dc \right)$$
$$+\tau \left(\frac{(2\gamma\hat{c} + \beta\overline{p} - \gamma\overline{p} - 1)^3 - (\overline{p}(\beta-\gamma)-1)^3}{24\gamma^2} - \int_0^{\hat{c}} \gamma\delta(c)^2\, dc \right).$$

The Lagrange-Euler equation is

$$\frac{\partial}{\partial\delta(c)} \Big(\delta(c) - \lambda\left(\gamma\delta(c)^2 + 2c\gamma\delta(c) - \gamma\overline{p}\delta(c)\right) - \tau\gamma\delta(c)^2 \Big) = 0,\ \forall c \in (0, \hat{c}),$$

which implies

$$\delta(c) = \frac{-2c\gamma\lambda + \gamma\lambda\overline{p} + 1}{2\gamma(\lambda+\tau)},\ \forall c \in (0, \hat{c}).$$

We plug this into the two equality constraints to find:

$$\lambda = \frac{6\hat{c} - 6\overline{p}}{4\gamma\hat{c}^2 - 6\gamma\hat{c}\overline{p} + 3\gamma\overline{p}^2} + \frac{1}{\beta\overline{p}-1},$$
$$\tau = -\frac{6(\hat{c}-\overline{p})}{\gamma\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right)}.$$

This implies

$$p(c) = \overline{p} - \frac{3(\overline{p}-2c)(\overline{p}-\hat{c})(\beta\overline{p}-1)}{\gamma\left(4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2\right)}.$$

The price is increasing in c, so the solution is feasible. The marginal server serves non-negative demand:

$$q(\hat{c}) = \frac{\hat{c}(2\hat{c} - 3\overline{p})(\beta\overline{p} - 1)}{4\hat{c}^2 - 6\hat{c}\overline{p} + 3\overline{p}^2} \geq 0,$$

which implies $\hat{c} < 3\overline{p}/2$. We now plug the optimal prices in platform's objective. Platform's problem becomes:

$$\begin{aligned} \max_{\overline{p},\hat{c}} \quad & \tfrac{\hat{c}^3(1-\beta\overline{p})}{4\hat{c}^2-6\hat{c}\overline{p}+3\overline{p}^2} \\ \text{s.t.} \quad & \hat{c} \leq 3\overline{p}/2. \end{aligned}$$

In the optimal mechanism, the highest cost that participates and the average market price are:

$$\overline{p} = \frac{1}{2\beta},$$
$$\hat{c} = \frac{3}{4\beta}.$$

Total quantity of customers served in the market is

$$Q = \frac{9}{32\beta},$$

$$f(c) = \frac{(\gamma - 2\beta)(4\beta c - 3)(4\beta c - 1)}{32\beta\gamma}.$$

Based on the optimal mechanism, let

$$\frac{f(c)}{q(c)} = q(c)\left(\frac{1}{2\beta} - \frac{1}{\gamma}\right) + \left(\frac{1}{2\gamma} - \frac{1}{4\beta}\right).$$

The platform announces the fee structure above, but otherwise lets the servers choose their prices. Servers choose the quantities that maximize their profits knowing that other servers do the same. Thus, server c faces the problem:

$$\max_{p(c)} \quad \pi(c) = q(c)(p(c) - c) - q(c)\left(\frac{f(c)}{q(c)}\right).$$

Given an expected average price $\overline{p}$, the servers' optimal quantity choice satisfies

$$p(c) = \frac{-2\beta + 4\beta c\gamma + 3\gamma + 4\beta^2\overline{p} - 8\beta\gamma\overline{p} + 4\gamma^2\overline{p}}{4\gamma^2}. \tag{22}$$

Server c earns

$$\pi(c) = \frac{(2\beta(-2c\gamma + 2\overline{p}(\gamma - \beta) + 1) + \gamma)^2}{32\beta\gamma^2}.$$

Server profits are decreasing in cost, c, and therefore there exists a threshold cost, $\hat{c}$, for which server with cost c participates if and only if $c < \hat{c}$. The highest cost that participates is

$$\hat{c} = \frac{2\beta + \gamma - 4\beta^2\overline{p} + 4\beta\gamma\overline{p}}{4\beta\gamma}, \tag{23}$$

which leads to zero profit.

The average market price is defined in the equilibrium as a weighted average of all prices set in the market. Due to cyclical nature of the definitions, the weighted average of all prices itself depends on the average market price. By Equation (1):

$$\begin{aligned}\overline{p} &= \frac{\int_0^{\hat{c}} q(c)p(c)\, dc}{\int_0^{\hat{c}} q(c)\, dc} \\ &= \frac{\int_0^{\hat{c}} \big(1-\beta\overline{p}+\gamma(\overline{p}-p(c))\big)p(c)\, dc}{\int_0^{\hat{c}} \big(1-\beta\overline{p}+\gamma(\overline{p}-p(c))\big)\, dc} \\ &= \frac{4\beta^2\overline{p}-2\beta(5\gamma\overline{p}+1)+\gamma(6\gamma\overline{p}+5)}{6\gamma^2},\end{aligned}$$

where the last line follows by plugging in the expressions for $p(c)$ in Equation (22) and $\hat{c}$ in Equation (23).

Solving for $\overline{p}$ gives:

$$\overline{p} = \frac{1}{2\beta},$$

which implies

$$\begin{aligned}\hat{c} &= \frac{2\beta+\gamma-4\beta^2\overline{p}+4\beta\gamma\overline{p}}{4\beta\gamma} \\ &= \frac{3}{4\beta}.\end{aligned}$$

Plugging the relevant expressions into platform's objective, the total quantity served is

$$Q = \frac{9}{32\beta}.$$

Notice that the total quantity under this policy is equivalent to the optimal mechanism.

Extension with Correlation Between Server Cost and Customer Value. Consider a setting defined by the demand function:

$$q(c) = 1+\alpha c-\beta\overline{p}+\gamma(\overline{p}-p(c)).$$

First, let us consider platform pricing, where all servers are assigned a single price, $p=\overline{p}$ by the platform. The highest cost server who participates in the market is

$$\hat{c}(p,\phi) = (1-\phi)p. \tag{24}$$

All servers with cost $c \leq \hat{c}(p, \phi)$ are profitable and enter the market. It follows that the platform's optimization problem can be written as

$$\max_{\overline{p},\phi} \quad \Pi^{\mathcal{P}} = \phi \overline{p} \int\limits_{0}^{\hat{c}(\overline{p},\phi)} (1 + \alpha c - \beta \overline{p})\, dc$$
$$\text{s.t.} \quad \text{Eq. (24).}$$

The platform's solution is uniquely defined by the first order conditions. The optimal price and commission are

$$p(c) = \frac{\alpha - 4\beta}{3\beta(\alpha - 2\beta)},$$
$$\phi = \frac{\alpha - 2\beta}{\alpha - 4\beta}.$$

Platform's total profit is

$$\Pi = \frac{2}{54\beta^2 - 27\alpha\beta}.$$

Now let us consider server pricing, where each server chooses the price that maximize individual profits. Server c has the following pricing problem:

$$\max_{p(c)} \quad \pi(c) = \big(1 + \alpha c - \beta \overline{p} + \gamma(\overline{p} - p(c))\big)((1 - \phi)p(c) - c).$$

The server with cost c has a unique optimal price, $p(c)$, to post:

$$p(c) = \frac{1}{2}\left(\frac{1 + \alpha c - \beta \overline{p}}{\gamma} + \frac{c}{1 - \phi} + \overline{p}\right). \tag{25}$$

Server c earns

$$\pi(c) = \frac{(c(\gamma - \alpha(1 - \phi)) - (1 - \phi)(1 - \overline{p}(\beta - \gamma)))^2}{4\gamma(1 - \phi)}.$$

Depending on the size of ϕ, the server profits can be either increasing or decreasing in costs. Formally, server profits is increasing in costs if $\frac{\alpha-\gamma}{\alpha} > \phi \geq 0$, and decreasing if $1 \geq \phi > \frac{\alpha-\gamma}{\alpha}$. Let $\hat{c}_l$ be the lowest cost that participates in the first case and $\hat{c}_h$ be the highest cost that participates in the second case. In either case, the threshold server cost, at which a server is indifferent between participating or not participating earns zero profits. Threshold server cost satisfies

$$\pi(\hat{c}_l(\phi)) = 0$$

in the first case, and

$$\pi(\hat{c}_l(\phi)) = 0$$

in the second case. This means that, if $\frac{\alpha-\gamma}{\alpha} > \phi \geq 0$, a server with cost c participates if and only if

$$c \geq \hat{c}_l(\phi) = \tfrac{(1-\phi)(1-\overline{p}(\beta-\gamma))}{\gamma-\alpha(1-\phi)}. \tag{26}$$

If $1 \geq \phi > \frac{\alpha-\gamma}{\alpha}$, a server with cost c participates if and only if

$$c \leq \hat{c}_h(\phi) = \tfrac{(1-\phi)(1-\overline{p}(\beta-\gamma))}{\gamma-\alpha(1-\phi)}. \tag{27}$$

The average market price is defined by:

$$\overline{p} = \begin{cases} \dfrac{\int_0^{\hat{c}} q(c)p(c)\,dc}{\int_0^{\hat{c}} q(c)\,dc} & , 1 \geq \phi > \frac{\alpha-\gamma}{\alpha}, \\[2ex] \dfrac{\int_{\hat{c}}^{1} q(c)p(c)\,dc}{\int_{\hat{c}}^{1} q(c)\,dc} & , \frac{\alpha-\gamma}{\alpha} > \phi \geq 0. \end{cases}$$

Solving for $\overline{p}$ gives:

$$\overline{p} = \begin{cases} \dfrac{\alpha\phi - \alpha + 2\gamma}{\alpha\beta\phi - \alpha\beta + 2\alpha\gamma\phi - 2\alpha\gamma + 2\beta\gamma + \gamma^2} & , 1 \geq \phi > \frac{\alpha-\gamma}{\alpha}, \\[2ex] \dfrac{\alpha^2\phi^2 - 2\alpha^2\phi + \alpha^2 + \alpha\phi^2 - 2\alpha\phi + \alpha - \gamma^2 + 2\gamma\phi - 2\gamma}{(\phi-1)\left(\alpha\beta\phi - \alpha\beta + 2\alpha\gamma\phi - 2\alpha\gamma + 2\beta\gamma + \gamma^2\right)} & , \frac{\alpha-\gamma}{\alpha} > \phi \geq 0. \end{cases} \tag{28}$$

Aware of the equilibrium price choices that occur, the platform's optimization problem is

$$\begin{aligned} \max_{\phi} \quad & \Pi^{\mathcal{S}} = \begin{cases} \phi\overline{p} \int_0^{\hat{c}_h(\phi)} (1 + \alpha c - \beta\overline{p}_h + \gamma(\overline{p}_h - p(c)))\,dc & , 1 \geq \phi \geq \frac{\alpha-\gamma}{\alpha}, \\ \phi\overline{p} \int_{\hat{c}_l(\phi)}^{1} (1 + \alpha c - \beta\overline{p}_l + \gamma(\overline{p}_l - p(c)))\,dc & , \frac{\alpha-\gamma}{\alpha} \geq \phi \geq 0 \end{cases} \\ \text{s.t.} \quad & \text{Eq. } (25), (26), (27), (28). \end{aligned}$$

We solve for the platform's problem numerically. Since we solve for relatively small α values, we never observe cases with $\phi < \frac{\alpha-\gamma}{\alpha}$ in equilibrium. This effectively makes our findings comparable with previous findings, where only the servers with sufficiently low costs participate. So, for the cases we consider, the platform's solution is mathematically equivalent to

$$\begin{aligned} \max_{\phi} \quad & \Pi^{\mathcal{S}} = \phi\overline{p} \int_0^{\hat{c}_h(\phi)} (1 + \alpha c - \beta\overline{p}_h + \gamma(\overline{p}_h - p(c)))\,dc \\ \text{s.t.} \quad & \text{Eq. } (25), (26), (27), (28). \end{aligned}$$

□