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A theory of maximalist luxury

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Abstract

The availability of high-quality, low-price counterfeits in many luxury markets threatens the role of luxury goods as a status symbol. If those counterfeits look and feel the same as the authentic counterparts, as many professional authenticators observe, and they are available at a fraction of the price of authentic goods, why would self-interested consumers purchase authentic luxury goods? Then, the future of luxury goods is called into question. In this paper, we propose that the presence of high-quality, low-price counterfeits can, surprisingly, motivate the wealthy consumers to pursue what we term as the "maximalist luxury" strategy. In the presence of these counterfeits, the wealthy can resort to signaling their status by purchasing the maximum number of luxury goods available and put their copious consumption on display, while in the absence of such counterfeits, the wealthy consumers only need to purchase the minimum number of luxury goods to stand out. This new signaling mechanism then highlights the importance of product line decisions by a luxury brand in combating counterfeits and provides a number of managerial insights about how to maintain the role of luxury goods as a status symbol through pricing, adjusting the product line, and limiting its products' functionality.

1 | INTRODUCTION

Luxury goods companies have long waged a battle against counterfeit goods. Today, owing to sales over $460 billion per year (Deloitte, 2018; Mau, 2018) and a dramatic improvement in quality, the battle with counterfeiters is tougher than ever. Today's counterfeits are not the same as yesterday's knockoffs which had inferior quality, but as some industry observers put it, they are nearly identical to the real thing, except for their low prices (Zerbo, 2018). These high-quality, low-price counterfeits go by various names such as "super fakes," "triple-a fakes," or "line-for-lines." Some expert authenticators even lament that it is "borderline impossible to figure out" these counterfeits from the authentic products (Mau, 2018). Combating counterfeits has hopelessly become a game of Whac-a-Mole (Zerbo, 2018), making it clear that luxury brands need more effective strategies. In this paper, we will investigate how the availability of high-quality, low-price counterfeits may change consumer buying behaviors and call for different management strategies for a luxury brand.

Past research has suggested that consumers pay exorbitant prices for luxury goods despite their low-priced equivalents are readily available. This is because luxury consumers desire to stand out and signal their success, sophistication, power, and, of course, wealth, through lavish spending that others cannot afford to mimic (Bagwell & Bernheim, 1996; Veblen, 1899). What a luxury brand needs to do is to focus on branding and charge premium prices to help those customers achieve the coveted separation. However, the availability of high-quality, low-price counterfeits casts some doubts on the validity of this long-held thesis and presents a luxury brand with many new managerial challenges. In a market inundated with such high-quality, low-price counterfeits, many consumers can easily afford to purchase fake luxury goods that look and feel the same as the authentic ones. Then, the ownership of authentic luxury goods can no longer signal lavish spending so as to make the wealthy stand out and deliver to them the symbolic benefits they seek. Indeed, in such markets, it is not clear what would and could motivate self-interested consumers to purchase authentic luxuries at premium prices, and the sales and profitability of luxury brands should then inevitably suffer.

Surprisingly, though, in markets plagued with high-quality, low-price counterfeits, we see consumers engaging in precisely the opposite, maximalist behavior. Not only do some luxury consumers continue to purchase authentic products, but they also pay attention to buying and consuming luxury brands, head to toe. Consumers in these markets manifest themselves as luxury consumers not merely by owning a single Louis Vuitton handbag or a Versace dress, but by putting a luxury lifestyle on display, where they wear Prada earrings, a Prada blazer, a Prada skirt, Prada shoes, and a Prada perfume. Moreover, alongside rampant counterfeits, the sales of luxury goods in countries such as China and India have certainly not suffered, and in fact, they have grown rapidly over the years (Euromonitor International, 2020; Mckinsey & Company, 2019). For instance, according to the 2019 McKinsey China Luxury Report, "China delivered more than half the global growth in luxury spending between 2012-18, and is expected to deliver 65% of the world's additional spending heading into 2025" (Mckinsey & Company, 2019). In this paper, we investigate this apparent puzzle theoretically and propose a new mechanism as to how luxury brands can deliver symbolic benefits despite the presence of high-quality, low-price counterfeits. This new mechanism shows why copious consumption of authentic luxury goods can paradoxically be a rational consumer response to the presence of such counterfeits.

Our theoretical investigation premises on the fact that the distribution channels for authentic luxury goods and those for counterfeits are drastically different. Experts agree, based on preponderance of observations and empirical studies, that the distribution channels for these two types of goods differ in at least four key aspects. First, unlike the supply chain for authentic goods, the one for counterfeits has a limited scale, characterized by fragmented and distributed manufacturing operations, best suited for evading legal detection (Beconcini, 2016; Chow, 2000; Department of Homeland Security, 2020). This difference in supply chain inevitably constrains the size and the assortment of counterfeit vendors. Second, authentic goods are sold in legitimate outlets, that is, a brand's own stores or authorized dealers, whereas counterfeit goods reach consumers mostly "in closed or open-air wholesale markets, small retail stores, side street kiosks, and by unlicensed street vendors and hawkers (Chow, 2000)."[1] In these stores, in an effort to "lower the chances of detection and to minimize legal liability if prosecuted," counterfeiters have to limit their assortment and quantities of products and only focus on fast-moving products. Online vendors follow the same strategy and they are even more scattered than offline vendors.[2] Third, to avoid detection, counterfeiters increasingly use small parcel delivery methods to import and distribute counterfeit products.[3] This implies that, when buying counterfeits online, not only do consumers have to shop across multiple vendors, but also have to purchase in small batches.[4] Finally, aside from the inconvenience for consumers shopping for fakes, there are also other non-pecuniary costs such as possible legal penalties of owning fake goods and lack of after-sale customer service. Because of all these differences, it is a drudgery for counterfeit shoppers to purchase any significant number of fakes. In contrast, a purchaser of authentic goods enjoys one-stop shopping for any number of authentic goods.

Our research leverages the fact that even in an environment of high-quality, low-price counterfeits, a luxury brand has the inimitable advantage over the counterfeiters in setting the length of a product line in the luxury market. Even more importantly, it leverages the fact that a luxury brand has the advantage of the legal, concentrated distribution to sell the entirety of its product line in one place as opposed to illicit, fragmented distributions for competitive counterfeiters. This difference in distribution channels can be readily observed online and offline, for instance in China or India, and affords the luxury consumers the shopping economies of scope. Our theoretical model captures these stylized facts and from them we derive implications related to consumer behavior and suggest normative managerial strategies for a luxury brand. We show that the role of luxury goods as a status symbol can indeed be preserved despite the existence of high-quality counterfeits even at negligible prices.

More specifically, customers who seek to stand out through luxury goods can no longer count on high prices or lavish expenditures alone to do the job, as the prices they pay or the expenditures they incur can no longer be inferred directly due to the existence of nearly identical alternative fakes sold at very low prices. To stand out, they need to leverage the difference in scale economies associated with shopping for authentic versus counterfeit goods by purchasing a portfolio of authentic products or by pursuing what we call the maximalist luxury strategy. This strategy results from the wealthy being motivated by the presence of counterfeits to purchase the maximum number of luxury goods available and actually purchase many more than they would in their absence. In contrast, the wealthy only need to and actually purchase the minimum number of luxury goods to stand out in the absence of high-quality, low-price counterfeits. With this maximalist strategy, a sufficient amount of shopping drudgery, not merely high expenditures, can be imposed on the mimickers who would need to assemble the same portfolio in fakes. This drudgery imposed on mimickers can credibly prevent mimicking. In other words, the signaling mechanism for the wealthy is not only to spend lavishly, but also to purchase and display more items of luxury goods when facing high-quality, low-price counterfeits. Here, spending is necessary, but a copious or maximalist consumption of luxury goods is what makes it difficult for mimickers to assemble the same portfolio of counterfeits. Correspondingly, in this environment, the optimal strategy for a luxury brand is to expand its product line in a way to allow the wealthy to assemble a desired portfolio of products and to accommodate their maximalist luxury strategy.

This new mechanism of achieving symbolic benefits allows us to develop a number of managerial insights about a luxury market populated with high-quality, low-price counterfeits. First, this mechanism suggests that the maximalist luxury consumption can be a rational consumer response to the availability of high-quality, low-price counterfeits. In that light, it is not puzzling or paradoxical that the sales of luxury goods can grow alongside rampant counterfeits. Second, our analysis highlights for the first time the importance of a luxury brand's product line decisions in maintaining the role of luxury goods as a status symbol. We show that the minimum length of product line needs to be long enough to maintain the role of luxury goods as a status symbol. This managerial implication is consistent with, and indeed provides a rationale for, the ongoing trend where nearly three-quarter of luxury brands expand beyond their traditional core as shown in a recent McKinsey survey (Mckinsey & Company, 2012).

To our knowledge, this is the first paper to establish that product line expansion is a necessary and effective weapon to combat high-quality, low-price counterfeits. Our analysis also sheds light on the managerial imperatives for coping with rampant high-quality, low-price counterfeits and warns about the danger of losing a brand's product exclusivity and the possibility of wealthy consumers purchasing counterfeits.

The theoretical discussions on the rationale and workings of the luxury market date back to Veblen's well-cited work “Theory of Leisure Class” (Bagwell & Bernheim, 1996; Veblen, 1899). A significant number of studies in economics, marketing, and psychology subsequently focus on understanding the need for status including the use of counterfeits (Wilcox et al., 2009), signaling via non-conformity (Bellezza et al., 2014; Chaudhuri & Majumdar, 2006) and brand prominence (Han et al., 2010) and how firms can take advantage of this need (Berger et al., 2011; Chaudhuri & Majumdar, 2006; Ferraro et al., 2013; Ordabayeva & Chandon, 2011; Wang & Griskevicius, 2014). A stream of studies in marketing focus on status and vice goods investigating their pricing and competitive strategy (Amaldoss & Jain, 2005a, 2005b, 2015; Jain, 2012; Kuksov & Wang, 2013; Kuksov & Xie, 2012; Pesendorfer, 1995; Rao & Schaefer, 2013; Tereyagoglu & Veeraraghavan, 2012; Yoganarasimhan, 2012) and offering them as limited edition and scarce goods (Balachander & Stock, 2009). An extensive literature in the past also studied what contributes to one's status perception, summarizing factors such as personality traits and physical attractiveness (Anderson et al., 2001), culture (Torelli et al., 2014), financial well-being (Campbell & Henretta, 1980), and social connections (N. Lin, 1999). Among these, perhaps the most relevant to marketers are individuals' revealed product and brand consumption choices. Consumers commonly signal status via possession of status goods such as jewelry, fashion goods, or cars (Bagwell & Bernheim, 1996; Leibenstein, 1950). Ownership of status goods signal wealth, taste, knowledge of quality, and the ability to access high-priced and scarce items.

A rich stream of literature also focuses on product line extension strategies. Joshi et al. (2016) study product line scope and pricing decisions in a horizontally differentiated duopoly and show that only one firm may prefer to expand scope but profits may be higher for both firms, even in the absence of market size expansion. Broader scope permits that firm to effectively price discriminate by raising prices for its core customers. Randall et al. (1998) study if the inclusion of premium or high-quality products in a product line enhances brand equity or if the presence of low-quality products reduces it and show that indeed there is an association between a lower (higher) brand equity and having lower (higher) quality products in the product line. Hamilton and Chernev (2010) study a relevant question, but focusing on the price image of brands. The authors argue that upscale extensions can decrease rather than increase price image, and vice versa for downscale extensions. Others have investigated vertical extensions by studying its impact on profit (Draganska & Jain, 2005, 2006; Hardie et al., 1994) and brand positioning (Horsky & Nelson, 1992) and whether luxury brand extensions may invite consumers who are different than the core, or if their presence can dilute the value of the brand (Bellezza & Keinan, 2014). Our study adds to this stream of research by highlighting the importance of product line decisions in creating symbolic benefits in luxury markets.

In the rest of the paper, we first start by developing our benchmark model in Section 2, we carry out the analysis in Section 3 to identify the new signaling mechanism and present complete analysis of both separating and pooling equilibria. We deliver key managerial insights in Section 4. Then, we extend our benchmark model to investigate the implications of functional benefits for luxury goods in Section 5. Finally, in Section 6, we conclude.

2 | MODEL

Consider a market with a luxury brand (A) and a number of competitive counterfeit retail outlets (C). The luxury brand offers a portfolio of $n \in [1, N]$ products, each priced at p_A. These n products can, for instance, be different apparel, fashion items, accessories. For tractability, we abstract away from the determination of individual prices in a product line such that p_A is taken as the average price. In the conclusion section, we will discuss how relaxing this assumption would allow for additional insights. Consumers in the market are heterogeneous with respect to their wealth, high and low, denoted by $\theta \in \{H, L\}$. H-type consumers have higher wealth (w_H) than that of the L-type consumers (w_L), $w_H > w_L$. Consumers purchase luxury or counterfeit products to signal their social status. In the benchmark model, we will assume that consumers purchase luxury goods only for symbolic benefits (Bagwell & Bernheim, 1996; Han et al., 2010; Veblen, 1899), that is, a luxury good yields utility to its owner only if it helps her to be recognized as an H-type. By assuming away any functional benefits for now, we can isolate consumer purchases purely for the motivation of signaling their type, which means that a consumer will not purchase any authentic or counterfeit goods if doing so cannot generate any symbolic benefit. In Section 5, we extend the utility specification to include functional benefits of goods in addition to symbolic benefits. To model symbolic benefits formally, let a consumer purchase k_A units of authentic luxury goods and k_C units of counterfeit goods in this market. The symbolic utility of the consumer is measured by the probability with which she is recognized as an H-type and is denoted by

$$\mu(\theta = H | k = k_A + k_C).$$

We model the luxury brand-counterfeit competition focusing on two key factors. First, the luxury brand operates in a market such that for each authentic product, consumers can find a high-quality, low-price counterfeit substitute supplied at the competitive price of p_C. We will assume that the price of counterfeit goods is negligible by setting $p_C = 0$. This assumption simplifies the expressions we derive in equilibrium. In a previous version of this paper, we have also shown that this assumption is innocuous as the qualitative insights remain identical when $p_C > 0$.[5] We further assume that consumers buy at most 1 unit of each item so that the total number of goods they buy is always a subset of the product line.

Second, we capture the fact that authentic goods are often sold through authorized stores that provide access to all n products of a brand, while counterfeiters are typically sold via fragmented, small-operation stores of limited product choices, as documented in the introduction section. More specifically, a consumer can typically buy multiple authentic goods in one trip to the brand's store, but to assemble the same set of counterfeits, she is likely to have to visit a number of competing counterfeiters. Formally, we capture this difference by assuming that consumers incur a fixed shopping disutility of $c \geq 0$ when they buy $k_A (\leq n)$ authentic goods. If they buy $k_C (\leq n)$ counterfeit goods, the disutility they incur is bk_C, where b is a positive constant to calibrate the magnitude of this disutility. This specification thus captures the fact that the authorized store has more one-stop shopping advantage over the outlets for counterfeits when consumers purchase a larger portfolio of goods. We further assume that the disutility of shopping for one counterfeit product is lower than the maximum symbolic utility one can obtain, that is, $0 < b < 1$. To avoid trivial outcomes where L-types can always afford as many authentic products as the H-types do for any price, we assume that the L-types' budget is not too large, that is, $0 < w_L < 1 - c$. This assumption also implies that for the L-types, utility associated with signaling type H by expending the entire wealth (w_L) on authentic goods that could have been spent on numeraire goods outweighs the disutility c, that is, $c < 1 - w_L$.[6]

All types can choose to shop from the authorized store, the counterfeiters, or both to assemble up to n products in total. Given the description above, a type θ consumer chooses to purchase k_A authentic and k_C counterfeit products to maximize her utility by solving the following problem:

$$
\begin{aligned}
\max_{k_A,k_C} \mathcal{V}(\theta|\mu) \quad &:= \underbrace{\mu(\theta = H|k = k_A + k_C)}_{\text{symbolic utility}} - \underbrace{(bk_C + cI(k_A > 0))}_{\text{disutility of shopping}} + \underbrace{X}_{\text{utility from numeraire goods}} \\
\text{s. t.} \quad & w_\theta \geq k_A p_A + X \\
& n \geq k_A + k_C \\
& k_A, k_C \in [0, n] \\
& X \geq 0,
\end{aligned} \tag{1}
$$

where $I(\cdot)$ is an indicator variable which equals 1 if a consumer purchases any authentic goods, and 0 otherwise. Here numeraire good consumption indicates any spending that is not on visible goods, such as purchases of groceries, cosmetics, etc.

The timing of the game is as follows:

1. First, the luxury brand determines its product line length n at the authorized store and the price of each product p_A to maximize its revenue from selling to H-types and L-types.
2. The product line and prices are observed by both counterfeiters and consumers. The counterfeiters collectively provide a replica of all authentic products at the competitive market price of $p_C = 0$.
3. Conditional on their wealth and disutility from shopping, consumers decide how many authentic (k_A) and counterfeit items (k_C) to buy, such that $k_A + k_C \leq n$.
4. Consumers purchase and use the products. The public makes an inference about consumer types based on the products they own and display ($k_A + k_C$). Consumers gain symbolic utility μ, which is the probability of being recognized as the H-types.

In this game, given that high-quality, low-cost counterfeits are available, do H-types still purchase authentic luxury goods to signal their status? To answer that question, we start by defining the equilibrium concept.

3 | ANALYSIS

3.1 | Description of equilibrium concept

Recall that there are two types of consumers: H-types and L-types. Let k be the total number of goods purchased by a consumer. We focus on the pure strategy Perfect Bayesian Equilibrium (PBE) profile $(k_A^*(\theta), k_C^*(\theta)) : \{H, L\}^2 \to [0, n]^2$ that satisfies the following three criteria.

First, as the authentic and counterfeit goods are indistinguishable, for any number of products $k \in [0, n]$ that a consumer purchases, there exists a public belief $\mu : [0, n] \mapsto [0, 1]$ corresponding to the probability that a consumer who owns $k(= k_A + k_C)$ items in total is a H-type.

Second, given the public's belief μ, a θ-type consumer maximizes her utility by choosing the number of authentic and counterfeit products to buy, subject to her budget constraint:

$$
\begin{aligned}
\left(k_A^*(\theta), k_C^*(\theta)\right) \quad &\in \arg\max_{k_A,k_C} \mathcal{V}(\theta|\mu) \\
\text{s. t.} \quad & p_A k_A + X \leq w_\theta \\
& 0 \leq k_A + k_C \leq n \\
& X \geq 0,
\end{aligned}
$$

where $\mathcal{V}(\theta|\mu)$ is the consumer's total utility given by (1).

Third, on every equilibrium path, beliefs are consistent with the equilibrium strategies according to Bayes' rule. Specifically, in a separating equilibrium, we have $k_A^*(H) + k_C^*(H) \neq k_A^*(L) + k_C^*(L)$. The belief μ formed upon observing k items satisfies the following conditions:

$$
\mu(\theta = H|k) = \begin{cases} 1 & \text{if } k = k_A^*(H) + k_C^*(H), \\ 0 & \text{if } k = k_A^*(L) + k_C^*(L). \end{cases} \tag{2}
$$

In a pooling equilibrium where $k_A^*(H) + k_C^*(H) = k_A^*(L) + k_C^*(L)$, μ satisfies the following condition:

$$\mu(\theta = H|k = k_A^*(H) + k_C^*(H)) = \rho_H,$$

where $0 < \rho_H < 1$ denotes the population share of H-types in the market.

It is well-known that signaling games suffer from multiplicity of equilibria (Cho & Kreps, 1987; Feltovich & Harbaugh, 2002). The Intuitive Criterion is commonly used to eliminate equilibria which are less intuitive or implausible (Cho & Kreps, 1987). In the analysis that follows, we shall focus on only the equilibrium that satisfies the Intuitive Criterion. For all the equilibria that survive the Intuitive Criterion, we select the equilibria that give the brand the most profit. This profit criterion implies that a profit-driven brand can always find a way to effect the equilibrium that is most beneficial to itself. Among all the profit-maximizing equilibria, we focus on the one that involves the lowest demand for counterfeit goods. If a brand obtains the same profit and the demand for counterfeit goods is the same from offering product lines of different lengths, we focus on the equilibrium with the leanest product line length.

In addition, for our analysis, we invoke two tie-breaking rules. First, if a consumer is indifferent between buying authentic and buying counterfeit goods, she will choose to buy the authentic ones; second, if a consumer is indifferent between buying and not buying, she will buy only if her basket contains authentic goods. Both tie-breaking rules build in the preference and aspiration for the authentic brand and capture the reality of consumers favoring authentic products. Such a preference for authentic goods also implies a bias against consuming counterfeit goods such that when indifferent, consumers will purchase the smallest number of counterfeits possible.[7]

Based on the equilibrium concept described in this section, we can characterize the separating equilibrium in the following proposition. To facilitate our exposition, we define Ω_{SE} as the set of parameters where a separating equilibrium is the most profitable for the luxury brand. A point in Ω_{SE} is denoted by σ, a quadruple variable consisting of four dimensions: ρ_H, c, w_H, and w_L. The definition of Ω_{SE} will be specified shortly later.

Proposition 1. *A separating equilibrium exists where the luxury brand makes a higher profit than in any pooling equilibria if and only if $N \geq \frac{1}{b}$ and $\sigma \in \Omega_{SE}$. In this equilibrium, the optimal product line length (n^*) and the optimal price (p_A^*) are given by*

$$n^* = \begin{cases} \dfrac{1}{b} & \text{if } c > w_L\left(\dfrac{1 - w_H}{w_H}\right) \\ \left(\dfrac{1 - w_L - c + \varepsilon}{b}\right)\left(\dfrac{w_H}{w_H - w_L}\right) & \text{if } c \in \left[1 + \varepsilon - w_L - bN\left(\dfrac{w_H - w_L}{w_H}\right), w_L\left(\dfrac{1 - w_H}{w_H}\right)\right] \\ N & \text{if } c \in \left(0, 1 + \varepsilon - w_L - bN\left(\dfrac{w_H - w_L}{w_H}\right)\right) \end{cases}$$

$$p_A^* = \begin{cases} b(1 - c) & \text{if } c > 1 - w_H \\ bw_H & \text{if } c \in \left(w_L\left(\dfrac{1 - w_H}{w_H}\right), 1 - w_H\right] \\ \dfrac{b(w_H - w_L)}{1 - w_L - c + \varepsilon} & \text{if } c \in \left[1 + \varepsilon - w_L - bN\left(\dfrac{w_H - w_L}{w_H}\right), w_L\left(\dfrac{1 - w_H}{w_H}\right)\right] \\ \dfrac{bw_L}{bN - (1 + \varepsilon - w_L - c)} & \text{if } c \in \left(0, 1 + \varepsilon - w_L - bN\left(\dfrac{w_H - w_L}{w_H}\right)\right). \end{cases}$$

When $N < \frac{1}{b}$ or $\sigma \notin \Omega_{SE}$, a pooling equilibrium exists where the luxury brand makes a higher profit than in any separating equilibria. In this pooling equilibrium, the optimal product line length (n^*) and optimal price (p_A^*) are given by

$$p_A^* = \begin{cases} \max\left\{\dfrac{b(w_H - w_L)}{\rho_H - c - w_L}, \dfrac{bw_L}{b\min\{N, \frac{\rho_H}{b}\} - (\rho_H - c - w_L)}\right\} & \text{if } c < w_L\left(\dfrac{\rho_H}{w_H} - 1\right) \\ \dfrac{bw_L}{w_L + c} & \text{if } w_L\left(\dfrac{\rho_H}{w_H} - 1\right) \le c < w_L + \rho_H, \\ b\left(1 - \dfrac{c}{\rho_H}\right) & \text{if } c \ge w_L + \rho_H \end{cases}$$

$$n^* = \begin{cases} \max\left\{1, \min\left\{N, \dfrac{w_H(\rho_H - c - w_L)}{b(w_H - w_L)}\right\}\right\} & \text{if } c < w_L\left(\dfrac{\rho_H}{w_H} - 1\right) \\ \max\left\{1, \min\left\{N, \dfrac{\rho_H}{b}\right\}\right\} & \text{if } c \ge w_L\left(\dfrac{\rho_H}{w_H} - 1\right) \end{cases}.$$

The proof for this proposition is provided in Appendix A. In the proposition, $\varepsilon > 0$ is an arbitrarily small number we need to pin down precisely the minimum product line length that will break the indifference between mimicking and not for the L-types. Here, Ω_{SE} is defined as $\Omega_{SE} := \{\sigma | \sigma \in \Sigma_1 \cup \Sigma_2 \cup \Sigma_3\}$, where

$$\Sigma_1 \equiv \rho_H \in (0, 1) \cap c \in \left(w_L\left(\frac{1 - w_H}{w_H}\right), 1 - w_L\right)$$

$$\Sigma_2 \equiv \rho_H \in \left\{\left(0, \frac{c}{1 - w_H}\right) \cup (\underline{\rho}, \bar{\rho}\,)\right\} \cap c \in \left(w_L\left(\frac{4w_L}{(w_H + w_L)^2} - 1\right), w_L\left(\frac{1 - w_H}{w_H}\right)\right]$$

$$\Sigma_3 \equiv \rho_H \in \left(0, \frac{c}{1 - w_H}\right) \cap c \in \left(0, w_L\left(\frac{4w_L}{(w_H + w_L)^2} - 1\right)\right]$$

and $\underline{\rho} = \dfrac{(c + w_L)\left(w_L + w_H - \sqrt{(w_L + w_H)^2 - \frac{4w_L^2}{c + w_L}}\right)}{2w_L}$ and $\bar{\rho} = \dfrac{(c + w_L)\left(w_L + w_H + \sqrt{(w_L + w_H)^2 - \frac{4w_L^2}{c + w_L}}\right)}{2w_L}$.[8]

In the case of separating equilibrium, the L-types do not purchase any products ($k_A^*(L) = k_C^*(L) = 0$). The H-types do not purchase any counterfeit products ($k_C^*(H) = 0$) and purchase all available authentic goods as long as $c \ge 1 + \varepsilon - w_L - bN\left(\frac{w_H - w_L}{w_H}\right)$. Otherwise, when $c < 1 + \varepsilon - w_L - bN\left(\frac{w_H - w_L}{w_H}\right)$, they purchase a combination of authentic and counterfeit goods to assemble all available goods. The H-types buying counterfeits is rather expected in this case given the small income difference between the H-types and L-types implied by the condition. In the case of pooling equilibrium, the H-types purchase only authentic goods as long as $c \ge \min\left\{1 + \varepsilon - w_L - bN\left(\frac{w_H - w_L}{w_H}\right), w_L\left(\frac{\rho_H}{w_H} - 1\right)\right\}$. The L-types purchase a combination of authentic and counterfeit goods to match the same total purchase as the H-types.

Proposition 1 allows us to generate a number of insights into managing luxury goods in markets with high-quality, low-cost counterfeits. We will provide these insights as we proceed with our analyses.

3.2 | Counterfeits and impact on the luxury brand

An intriguing question is whether the presence of high-quality, low-price counterfeits can fundamentally change the nature of luxury goods as a status symbol of exclusivity, the way consumers signal their wealth, and the strategy by which the luxury brand manages its product line. To address that question, we proceed to investigate the equilibria in a market where counterfeits are absent. The analysis of this case also applies to the situation where all consumers prefer to buy only authentic goods, possibly due to moral reasons or due to the use of counterfeits being legally penalizable in some countries.[9] The following Lemma summarizes our analysis.

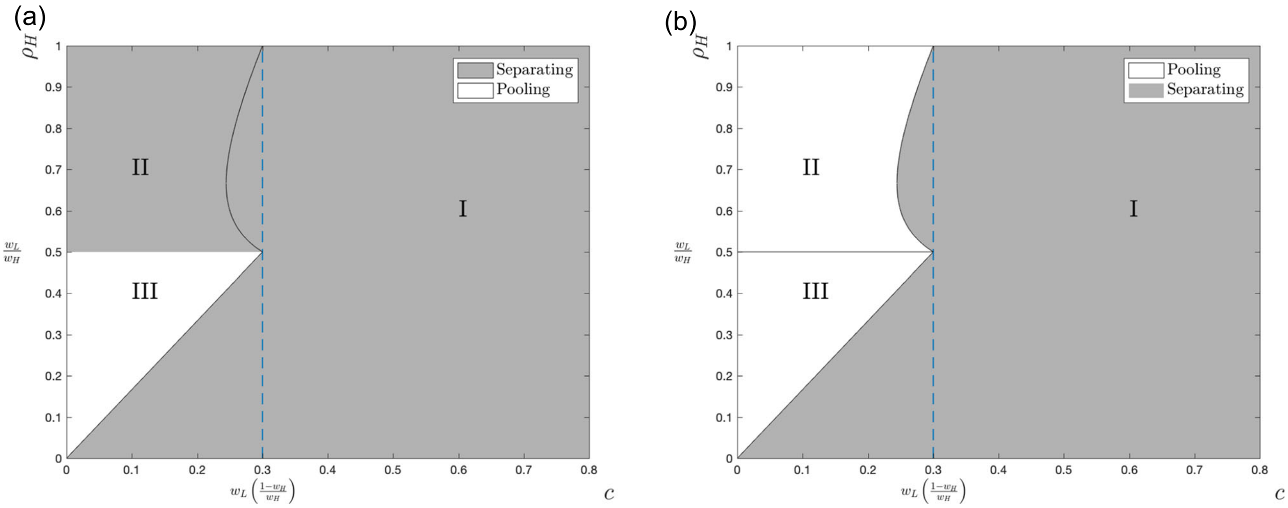

FIGURE 1 Equilibria with and without counterfeits ($w_L = 0.2, w_H = 0.4, b = 0.2, N = 10$). (a) Absent counterfeits (Lemma 1) and (b) with counterfeits (Proposition 1). X-axis and Y-axis illustrate, respectively, the ranges of $c \in (0, 0.8)$ and $\rho_H \in (0, 1)$. The entire parameter space is divided into regions I, II, and III by solid lines. White area indicates pooling equilibria and gray area separating equilibria [Color figure can be viewed at wileyonlinelibrary.com]

Lemma 1 (Absence of Counterfeits). *When high-quality, low-price counterfeits are absent, a separating equilibrium exists where the luxury brand makes a higher profit than in any pooling equilibria if and only if* $c > w_L\left(\frac{1-w_H}{w_H}\right)$ *or* $\rho_H \in \left(0, \frac{c}{1-w_H}\right) \cup \left(\frac{w_L}{w_H}, 1\right)$. *Otherwise, there exists a pooling equilibrium that generates higher profit. In the separating equilibrium, the brand will use the shortest product line length* ($n^* = 1$) *and the* H*-types will purchase* 1 *unit of luxury good at* $p_A^* = \min\{w_H, 1-c\}$ *whereas the* L*-types do not purchase luxury goods. In the pooling equilibrium, we have both the* H*-types and* L*-types purchasing* 1 *unit of authentic good at* $p_A^* = \min\{w_L, \rho_H - c\}$.

The proof for Lemma 1 is in Appendix A and the equilibria are illustrated in Figure 1a. Lemma 1 shows that in the absence of high-quality, low-price counterfeits, the H-types can stand out by outspending the L-types *á* la Veblen in a separating equilibrium. Such a separating equilibrium in regions I and II in Figure 1a always generates higher profit for the luxury brand than any pooling equilibria. In this environment, a brand has an incentive to facilitate H-types' spending, and hence the separation, by taking as high a payment from the H-types as they can afford or are willing to pay. Yet this payment is beyond the L-types' affordability. In region III, the pooling equilibrium can generate more profit for a luxury brand because the brand can benefit from selling the luxury product to all even though it charges a strictly lower price.

In both types of equilibria, the brand's product line decision is inconsequential. This is because in both types of equilibria, the luxury brand is in the position to certify a customer as an H-type with probability 1 in the case of the separating equilibrium and with probability ρ_H in the case of the pooling equilibrium. For that certification, the luxury brand will extract the maximum surplus of $\min\{w_H, 1-c\}$ from the H-types in the case of the separating equilibrium and $\min\{w_L, \rho_H - c\}$ from the L-types in the case of the pooling equilibrium. In either case, regardless of what a brand decides on its product line length, it can always adjust its price to extract the maximum surplus. As such, the shortest product line can always do the job and the consumers in both cases purchase at most one luxury good. By comparing the equilibria in Proposition 1 with those in Lemma 1, both of which are illustrated in Figure 1, we can isolate the impact of the presence of high-quality, low-price counterfeits on the luxury goods market. The next two corollaries summarize our analysis.

Corollary 1 (Maximalist Luxury and Product Line Decision). *The presence of the high-quality, low-price counterfeits makes it harder for the* H*-types to separate from the* L*-types, and thus reduces the region where separating equilibria are more profitable for the luxury brand. To stand out, the* H*-types pursue the maximalist luxury in that they outspend and outnumber the* L*-types in purchase. The brand optimally accommodates maximalist luxury by expanding its product line.*

Figure 1 shows that the region where separating equilibria are more profitable for the luxury brand is reduced from regions I and II to region I due to the presence of counterfeits. Corollary 1 provides three important insights about a market with high-quality, low-price counterfeits. First, since the availability of counterfeits encourages mimicking by the L-types, it may seem to impair the sales of authentic products. However, rather counter-intuitively, the presence of counterfeits need not reduce the number of authentic luxury goods purchased by the H-types. This is because the availability of counterfeits makes it easier or more affordable for the L-types to mimic. To stand out, the H-types need to purchase more authentic goods and thus impose a higher shopping disutility on the L-types if they mimic. In other words, the H-types outspending the L-types is no longer sufficient to stand out, and they must also "outnumber" the L-types. Doing so will impose sufficient shopping disutilities on the L-types so as to eliminate any incentives of their mimicking the H-types. Here, by comparing the purchases by the H-types with and without counterfeits being present, we can see that the H-types are pursuing, in the presence of counterfeits, what we have termed as the "maximalist luxury" strategy. Because of counterfeits, the H-types no longer purchase just one luxury product to stand out. In contrast, they purchase many, while the L-types stay put by buying zero luxury good with or without the presence of counterfeits. Underlying this difference in the number of purchases is a behavioral transformation brought about by the presence of high-quality, low-price counterfeits: the H-types are motivated to purchase the maximum number of luxury goods available in the presence of high-quality, low-price counterfeits, while in their absence, the H-types only need to purchase the minimum number of luxury goods. To the best of our knowledge, our paper is the first to discuss this mechanism of copious or maximalist consumption as a signal for status. This novel mechanism adds to the advertising effect documented in the literature (e.g., Givon et al., 1995) and suggests another avenue through which the presence of counterfeits increases the sales of a luxury brand.

Second, in the presence of high-quality, low-price counterfeits, a brand's product line decision is critical to maintaining the viability of luxury goods as a status symbol. Absent of counterfeits, affordability is the key to signaling status and the length of the product line is immaterial insofar as a luxury brand facilitates consumer signaling. However, when such counterfeits are present, a brand needs to expand its product line to a suitable length to allow the H-types to purchase more and impose a higher disutility on mimicking. Thus, the presence of counterfeits necessitates optimal decisions in product lines. Our analysis thus suggests that it need not be the presence of counterfeits, but rather the mismanagement of a brand's product line that hurts a brand's profitability the most.

Third, our analysis shows that the presence of counterfeits does put a downward pressure on luxury goods pricing when a separating equilibrium occurs. This pressure arises from the fact that a brand needs to motivate H-types to buy a larger portfolio of products to prevent the L-types from mimicking, and the H-types' maximum willingness-to-pay for the symbolic benefits is fixed at $1 - c$. Thus, our analysis suggests that a lower price may be a necessary evil associated with maintaining the viability and profitability of the authentic luxury goods in the presence of high-quality, low-price counterfeits.[10]

Corollary 2. *Ceteris paribus, the presence of counterfeits can lead to an equilibrium switch from a separating equilibrium to a pooling equilibrium, such that the luxury brand becomes better off with counterfeits than without.*

Corollary 2 is illustrated in region II of Figure 1a,b. In region II, with and without counterfeits, both separating and pooling equilibria exist. The switch occurs as the pooling equilibrium is more profitable than the separating equilibrium when counterfeits are present. This is because in a separating equilibrium, the luxury brand sells exclusively to the H-types. In a pooling equilibrium, the luxury brand sells to both types, albeit at a lower price. Given the increase in sales volume, the luxury brand's profit will increase if the price drop is sufficiently small. In region II, an L-type is identified as an H-type with a probability ρ_H and when deviating she will be identified as the H-type with probability 0. Therefore, at a higher ρ_H, she has less an incentive to deviate and the luxury brand can charge a higher price such that in region II the price drop is sufficiently small. However, by selling to both types of consumers, the possession of luxury goods no longer identifies a consumer as the H-type with probability 1 but with probability $\rho_H < 1$. Thus the luxury goods lose some of their luster. Corollary 2 further suggests an interesting insight that the presence of counterfeits is not always detrimental to the luxury brand with regard to profitability, and in fact the luxury brand is better off because of the presence of counterfeits. This is because, in region II of Figure 1a, absent of any counterfeits, the luxury brand's choice is to induce the separating equilibrium where only H-types purchase authentic goods and they exhaust their budget w_H or alternatively to effect the pooling equilibrium where it sells to both H-types and L-types. In the latter case, the luxury brand sells to more customers, but can only claim the revenue of w_L from each customer. For this reason, when ρ_H is larger, the separating equilibrium is more profitable. However, with the presence of counterfeits, the luxury brand's choice is quite different. For instance, in part of region II of Figure 1b, the luxury brand can induce the separating equilibrium where only H-types purchase authentic goods while exhausting their budget w_H and the L-types do not purchase anything, or it can induce the pooling equilibrium where the H-types purchase only authentic goods exhausting their budget w_H and where the L-types purchase a mixture of authentic and counterfeit goods also exhausting their budget w_L. Said differently, the presence of counterfeits gives the luxury brand the ability to sell a high-quality bundle consisting of all authentic goods to the H-types, while the L-types buy a low-quality bundle consisting of a mixture of authentic and counterfeit goods. This ability makes the pooling equilibrium a more profitable choice as the luxury brand can claim the maximum revenue w_H from each H-type and w_L from each L-type. In rest of region II, the luxury brand may not be able to exhaust the budget from the H-types, nevertheless, it still obtains a different revenue from each type of consumers to increase its total revenues from the market. In effect, in the entire region II, the presence of counterfeits enables the luxury brand to implement discrimination at the bundle-price level much in the same way as the availability of a damaged good (Deneckere & Preston McAfee, 1996) enables a firm to do at the product level.[11]

In region III, with and without counterfeits, we always have a pooling equilibrium. This is the region where the cost of shopping for authentic goods (c) is sufficiently small and hence mimicking is easier, and where the share of H-types (ρ_H) is also sufficiently small so that the luxury brand finds it profitable not to distinguish between the two types in selling its products.

4 | PRODUCT LINE STRATEGY

In the previous section, we see that the presence of high-quality, low-price counterfeits will reduce the region where the separating equilibrium is more profitable for the luxury brand but will not eliminate it. Preserving the luxury goods as an exclusive status symbol for the H-types is one of the practitioner's managerial imperatives (Dahlhoff & Zhang, 2020). For that reason, in this section, we shall focus on the separating equilibrium in our derivation for managerial implications. Corollary 1 established the central role of the product line in maintaining a luxury good as an exclusive symbol for the H-types and also the mechanism through which the H-types achieve separation. We see that a luxury brand's decision when combating counterfeits is not limited to pricing but also extends to judicious decisions on the product line. In what follows, we will develop insights into how a luxury brand should adjust its pricing and product line length when (1) counterfeits become more easily accessible and (2) the disutility of shopping for authentic luxury products decreases.

The Internet has certainly enabled easier access to high-quality, low-price counterfeits (Shepard, 2018; Valuiskich, 2018). This easier access means, in the context of our model, a smaller b. How should a luxury brand respond to easing access to counterfeits? The following corollary provides an answer.

Corollary 3 (Accessibility to Counterfeits). *As counterfeits become more easily accessible, or as the disutility from shopping for counterfeits declines ($b \downarrow$), ceteris paribus, the luxury brand should expand its product line ($n^* \uparrow$) until $n^* = N$. The brand should lower its prices ($p_A^* \downarrow$) when $b \geq \frac{w_H(1+\varepsilon-w_L-c)}{N(w_H-w_L)}$ and increase its prices when $b < \frac{w_H(1+\varepsilon-w_L-c)}{N(w_H-w_L)}$ to maintain its profitability.*

Corollary 3 is illustrated in Figure 2. Intuitively, as the counterfeits become more easily accessible, the L-types can avail themselves more of them and hence the H-types must purchase additional authentic goods to set themselves apart. The luxury brand needs to enable the H-types to achieve such a separation by extending its product line. However, to maintain market viability, the brand should also lower its price so that the H-types are willing to purchase a higher number of authentic products for the purpose of signaling. Otherwise, the cost of signaling for the H-types cannot be justified by the symbolic benefits of consumption. This corollary suggests that all else being equal, easier access to counterfeits requires a luxury brand to extend its product line and lower its price (as long as b is not too small) to maintain profitability. This suggests that a good way to support a luxury brand's price is to combat easy access to high-quality, low-price counterfeits vigorously, as many brands do today. Interestingly, as Chinese customers gained increasingly easier access to high-quality counterfeit goods, chiefly through online channels, major luxury brands have cut their prices significantly in China professedly "to shrink the gray market and throttle the thriving arbitrage market for its products."[12]

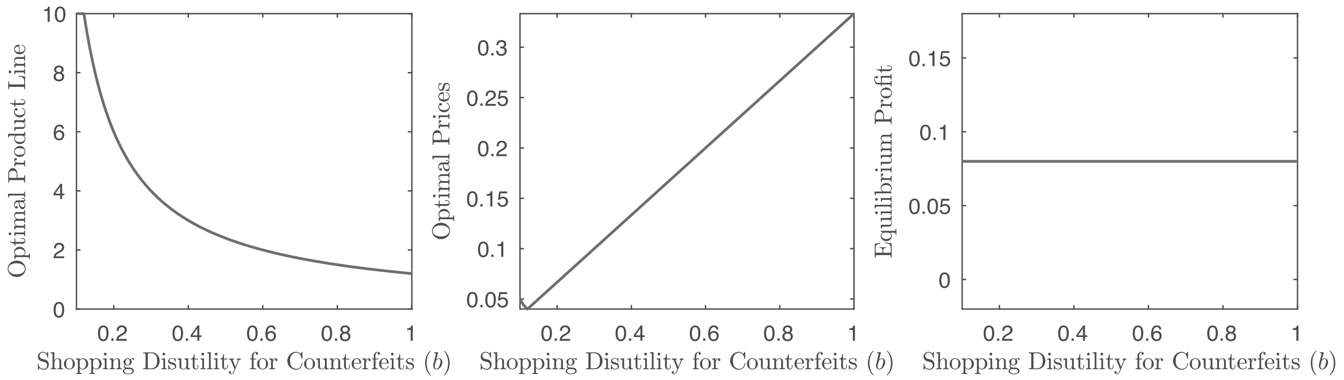

FIGURE 2 Product line (n^*) and price (p_A^*) versus shopping disutility for counterfeits (b) ($w_L = 0.2$, $w_H = 0.4$, $c = 0.2$, $\rho_H = 0.2$, $N = 10$, $1/N = 0.1 \leq b < 1$)

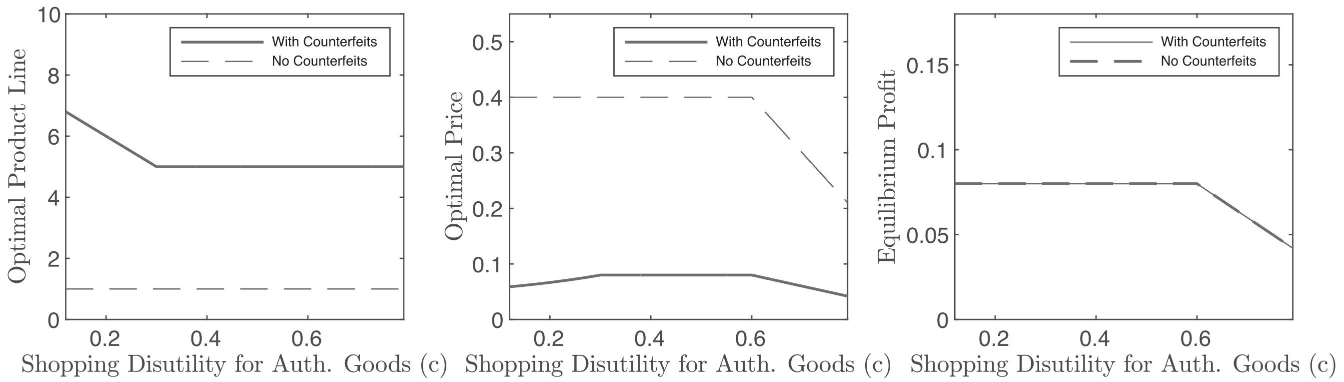

FIGURE 3 Product line (n^*) and price (p_A^*) versus shopping disutility for authentic goods (c) ($w_L = 0.2, w_H = 0.4, b = 0.2, \rho_H = 0.2$, $N = 10$, $\rho_H(1 - w_H) = 0.12 \leq c < 0.8 = 1 - w_L$)

Corollary 4 (Shopping Disutility from Authorized Stores). *As the disutility from shopping for authentic luxury products decreases ($c \downarrow$), ceteris paribus, the luxury brand should*

(i) *increase its price ($p_A^* \uparrow$) when $c > 1 - w_H$ or $c < 1 + \varepsilon - w_L - bN\left(\frac{w_H - w_L}{w_H}\right)$; maintain its current price when $w_L\left(\frac{1 - w_H}{w_H}\right) \leq c \leq 1 - w_H$, and decrease its price ($p_A^* \downarrow$) when $1 + \varepsilon - w_L - bN\left(\frac{w_H - w_L}{w_H}\right) \leq c \leq w_L\left(\frac{1 - w_H}{w_H}\right)$;*

(ii) *maintain its current product line length when $c > w_L\left(\frac{1 - w_H}{w_H}\right)$, and otherwise increase it until $n^* = N$.*

A lower c increases a brand's profitability only when $c > 1 - w_H$.

Figure 3 illustrates how the price, product line length, and profitability change as shopping disutility (c) increases. These three figures illustrate four insights from the corollary. When c is high, decreasing c increases a luxury brand's pricing power. This is because a lower c increases the maximum surplus of the H-types from purchasing authentic goods and hence benefits the brand itself in the form of a higher price. This process stops at $c \leq 1 - w_H$ when the luxury brand has extracted the maximum surplus from the H-types. At that point, the luxury brand maintains its price and product line until the L-types find it tempting to mimic the H-types by purchasing some authentic goods. In response, the luxury brand needs to increase its product line to maintain the separation and also the exclusive consumption of authentic goods by the H-types, both of which necessitate the decrease of its price. However, as c decreases further, the luxury brand's price rises again because this increase will prevent the L-types to mimic the H-types through consuming some authentic goods when the brand has reached the maximum length of its product line. A luxury brand's profit can start decreasing with c. This is because the maximum surplus a luxury brand can extract from the H-types, that is, $1 - c$, is smaller than their budget for a sufficiently large c.

This analysis offers three insights for managing a luxury brand. First, easy access to (i.e., a lower c) and greater economies of scope in purchasing authentic luxury goods is a winning strategy for a brand up to a point. Too low a c, as we have seen above, will encourage mimicking from the L-types. Second, when c is not too low, such a well-managed access to authentic goods, greater economies of scope, and a higher price can result in a new form of exclusivity strategy for luxury brands. In this case, the products are exclusive because of their high prices, but they are sold to consumers with higher convenience. Interestingly, http://JD.com in China seems to have set on that path when they started its new online commerce of luxury goods with white glove deliveries. Third, the product line decision will be a key ingredient to the success of firms following this new strategy. While the change in a brand's product line strategy is weakly monotonic, the change in its pricing strategies can be non-monotonic as c decreases.

5 | EXTENSION: FUNCTIONAL BENEFITS

In our benchmark model, we focus on social benefits as the sole motivation to purchase luxury goods. This simplification allows us to quickly identify the signaling mechanism in the presence of high-quality, low-price counterfeits and explore the implications of counterfeits for a brand's product line choice. In this section, we relax this assumption and show that our findings are qualitatively robust to the introduction of functional benefits, with some new insights.

Intuitively, by adding functional benefits to a luxury product, consumers have additional incentives to purchase the product such that both types of consumers can tolerate more disutility from shopping. Such tolerance can complicate the signaling process as the L-types now have the incentives to purchase the products regardless of whether or not they gain symbolic benefits. More specifically, with functional benefits, a type $\theta \in \{H, L\}$ consumer will purchase k_A authentic and k_C counterfeit items to maximize her utility by solving the following constrained optimization problem:

$$\max_{k_A, k_C} \quad \underbrace{U(k_A, k_C)}_{\text{functional value}} + \underbrace{\mu(\theta = H | k = k_A + k_C)}_{\text{symbolic value}} - \underbrace{(bk_C + cI(k_A > 0))}_{\text{shopping disutility}} + \underbrace{X}_{\text{Utility from Numeraire Goods}}$$

$$\begin{aligned} \text{s. t.} \quad & k_A p_A + X \leq w_\theta \\ & k_A \in [0, n] \\ & k_C \in [0, n] \\ & k_A + k_C \leq n. \end{aligned}$$

The functional value is characterized by the following utility function

$$U(k_A, k_C) = \alpha\sqrt{k_A + k_C},$$

where $\alpha > 0$. Note that this functional form reflects positive but diminishing marginal utility.[13] Our solution concept and the timing of the game remain the same as in the benchmark case. Our analysis is detailed in Appendix A. The following proposition summarizes our results.

Proposition 2. *When consumers care about functional utility, a separating equilibrium exists where the luxury brand makes a higher profit than in any pooling equilibria only if* $N \geq \left(\sqrt{\frac{1}{b}} + \frac{\alpha}{2b}\right)^2$ *and* $\alpha \leq \sqrt{\frac{4bw_H}{w_L}(w_L + c)}$. *Otherwise, the luxury brand either makes a higher profit in a pooling equilibrium than in any separating equilibria or there exists no separating equilibria.*

Proposition 2 and the equilibrium type are illustrated in Figure 4. A quick inspection of Figure 4 suggests a number of new insights. First, as we show in Appendix A, when α approaches zero, the extension converges to our benchmark model (see panel (a) in both Figures 1 and 4). Indeed, adding functional benefits does not qualitatively change the

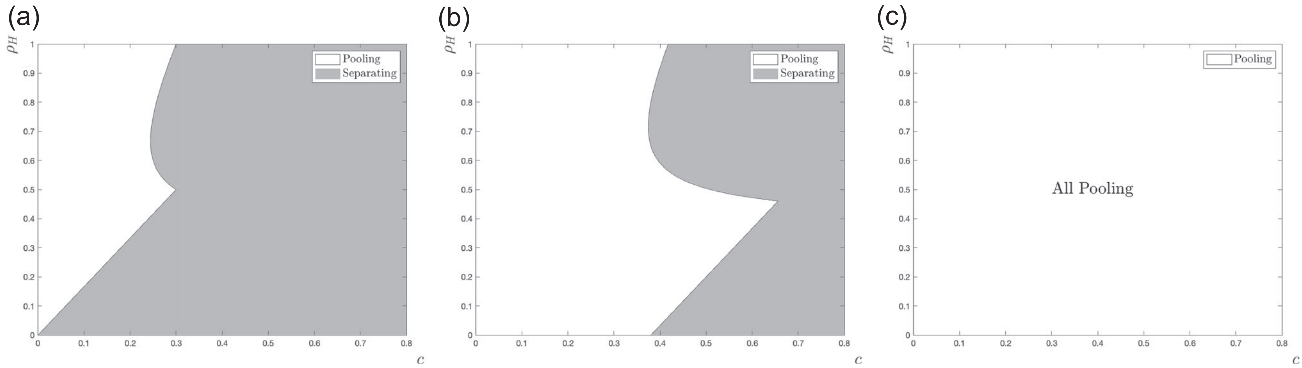

FIGURE 4 Equilibria with functional utility as α varies ($w_L = 0.2$, $w_H = 0.4$, $b = 0.2$, $N = 15$, $c \in (0, 0.8)$, $\rho_H \in (0, 1)$). (a) $\alpha = 0.01 \in (0, \underline{\alpha})$; (b) $\alpha = 0.1 \in \left[\underline{\alpha}, \sqrt{\frac{4bw_H}{w_L}}(w_L + c)\right]$; and (c) $\alpha = 1 \in \left(\sqrt{\frac{4bw_H}{w_L}}(w_L + c), +\infty\right)$

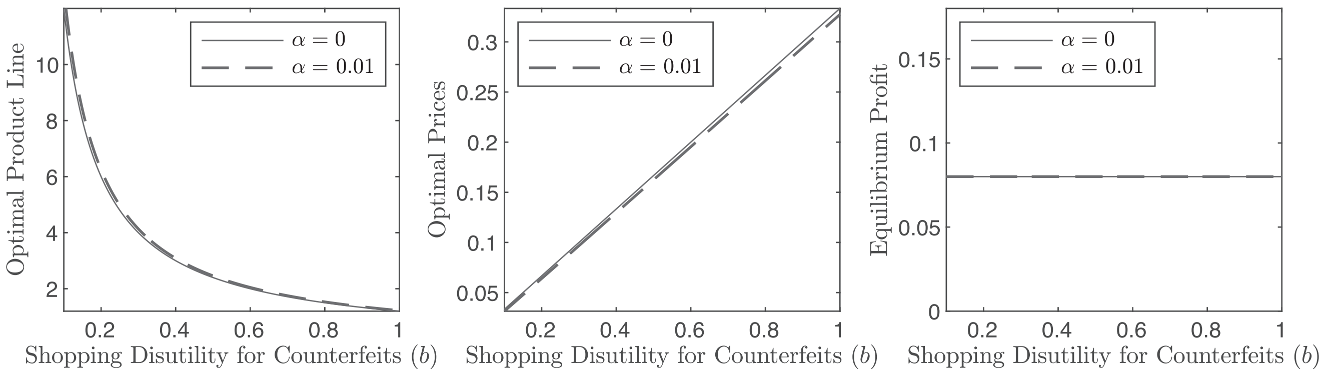

FIGURE 5 Optimal product line and price with functional utility versus disutility of shopping for counterfeits (b) ($w_L = 0.2$, $w_H = 0.4$, $c = 0.2$, $N = 15$, $\rho_H = 0.2$, $b \in [0.1, 1)$)

signaling mechanism identified in our benchmark model as long as $\alpha \leq \sqrt{\frac{4bw_H}{w_L}}(w_L + c)$, and this mechanism still leads to the maximalist luxury.

Second, functional utility gives L-types more incentives to purchase counterfeit goods and to tolerate higher shopping disutility. For that reason, as α increases (from panel (a) to (c) in Figure 4), the region where separating equilibrium is more profitable for the luxury brand shrinks and then disappears. This means that functional utility is inconducive to a luxury brand's exclusivity. In the separating equilibrium, adding functional utility changes consumption patterns from the L-types buying nothing to their possibly buying some counterfeits. Thus, functional utility enhances the viability of the counterfeit market.

Third, as a higher α allows consumers to tolerate higher shopping disutility and enhances the viability of a counterfeit market, the minimum length of product line for a luxury brand to maintain exclusivity is longer with a higher α. Said differently, the luxury brand needs to provide a longer product line when consumers care more about functional utility.

All these insights suggest two important managerial implications. First, promoting the functional utility of a luxury brand can be a slippery slope where a luxury brand can lose its luster as a status symbol. This perhaps explains why luxury brands rarely focus on the functional utility of their products and instead they prefer to promote their brands' exclusivity (Kapferer & Bastien, 2012). Second, it can be extraordinarily tempting for a luxury brand to embrace high sales volumes in exchange for higher profit. However, in doing so, a brand runs the risk of moving away from exclusivity by selling to all types, or it has to provide a long product line.

Furthermore, we can analytically show (see Appendix A) that for sufficiently small α ($\alpha \in (0, \underline{\alpha})$, where $\underline{\alpha}$ is defined in Appendix A), the comparative statics about the product line length, price, and profit with regard to changes in b and c are all similar to the benchmark case, as illustrated in Figures 5 and 6.

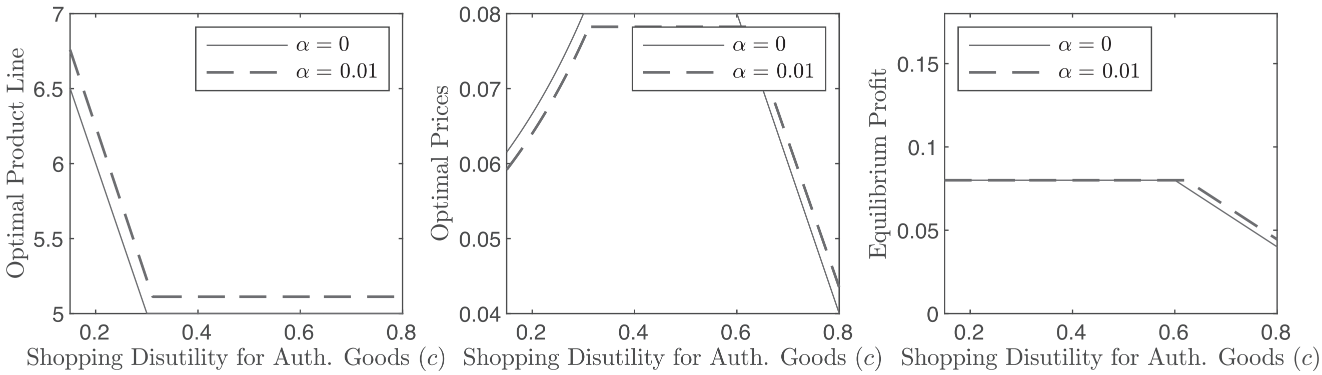

FIGURE 6 Optimal product line and price with functional utility versus disutility of shopping for authentic goods (c) ($w_L = 0.2$, $w_H = 0.4$, $b = 0.2$, $N = 15$, $\rho_H = 0.2$, $c \in [0.15, 0.8)$)

With this extension, our theory closes the loop for the luxury goods: functionality underlies the consumption of high-quality, low-price counterfeits; the consumption of high-quality, low price counterfeits gives rise to counterfeit markets; counterfeit markets create opportunities for mimicking. A brand's pricing and product line decisions can thwart mimicking and deliver status to a select class of consumers or accommodate mimicking and focus on sales volume for its own benefit.

6 | CONCLUSIONS

The availability of high-quality, low-price counterfeits is an emerging, unprecedented existential threat to the luxury goods industry and poses many new managerial challenges. If counterfeit goods become so sophisticated in quality that they are virtually indistinguishable from their authentic counterparts, what tactics do consumers use to signal their status through luxury purchases, and how can luxury brands respond to the threat from the high-quality, low-price counterfeits? These questions are perfectly suited for theoretical investigations and we take the first step to address them in this study.

Our paper argues, inspired by some actual encounters with the wealthy, that one such tactic is to assemble and display a portfolio of luxury goods that is harder to imitate via counterfeits. Our thesis relies only on the apparent differences in the distribution channels of these two outlets: one concentrated and legitimate, and the other fragmented and illicit. Because of the difference, it is less convenient to buy a portfolio of fakes because often these goods are not sold at a known store or location, but rather at small shops in the back alleys and shopping for them smacks of illicit activities. Moreover, when consumers have to assemble multiple goods to have that authentic feel and look, the inconvenience of shopping from the counterfeiters can quickly become unbearable. Our analysis shows that this difference in channel structure or shopping disutility can restore the effectiveness in consumer signaling, with accompanying behavioral and managerial insights.

Our first insight is to uncover the mechanism through which luxury goods as a status symbol can be maintained despite the presence of high-quality, low-price counterfeits. When a consumer cannot rely on prices and quality for signaling, we show that a luxury market needs not unravel. In such markets, consumers who are interested in signaling status can rationally assemble a portfolio of products to make it harder for others to mimic them. In other words, in such markets, authentic luxury is manifested not by an LV bag or a Versace dress, which anyone can easily fake, but by a luxury lifestyle on display where a consumer may wear, as we have observed, Prada earrings, a Prada blazer, a Prada skirt, Prada high heels, Prada perfume, etc., and yes, hold a Prada cellphone. This copious display is a mimic-proof signal for status in the environment of high-quality, low-price counterfeits because one can only feasibly assemble them in an authorized store and it is observable by others. Therefore, it is quite reassuring to see that luxury goods can survive and thrive as a status symbol in the face of high-quality, low-price counterfeits.

Our second insight is about the importance of adjusting product line length optimally when facing high-quality, low-price counterfeits. With the presence of counterfeits, consumers may no longer use luxury goods for status signaling, and as a result, a luxury brand can lose its luster as a status symbol. Therefore, it is critical to accommodate the wealthy's desire for an opulent display of luxury goods in that environment to restore the effectiveness of their status signaling to achieve separation from the rest. Our analysis shows that a sufficient length of product line is required to combat the counterfeiters. If a luxury brand has to err on the decision of product line length, it may want to err on the high side.

Our third insight is about the impact of counterfeit goods on the luxury brand. We show that in the presence of high-quality, low-price counterfeits, the likelihood of a pooling equilibrium increases in place of a separating equilibrium. This happens because the existence of counterfeits makes mimicking easier and separation harder. This happens also because a luxury brand can succumb to the temptation of trading its product exclusivity for selling to the masses, all for short-term profitability. Thus, maintaining a brand's exclusivity can become a watershed decision for a luxury brand.

Finally, our analysis shows that indeed there is a trade-off between functionality and symbolic benefits in managing a luxury brand. On the one hand, functional benefits can increase consumer willingness-to-pay for a product and hence it can benefit a brand. On the other hand, higher willingness-to-pay for all consumers in the market will make it more challenging for the wealthy to separate themselves from the rest so that a luxury brand may lose its role as a status symbol. Therefore, it becomes a managerial imperative for a luxury brand to strike a balance between symbolic and functional benefits in its product design and promotions.

As the first theory paper exploring lifestyle luxury where the wealthy buy a portfolio of luxury goods to stand out, this theoretical study sets a new agenda for future empirical research in management strategies for luxury goods. First, do luxury goods consumers purchase a larger portfolio of luxury goods as the high-quality, low-price counterfeits become more available? Anecdotal evidence seems to suggest that this is the case, but more systemic data collection is required to establish that link in reality. Second, do luxury brands offer a longer product line in markets where high-quality, low-price counterfeits are more readily available? Casual observations in emerging markets seem to suggest this. Third, do unit sales of authentic luxury goods increase with the availability of high-quality, low-price counterfeits? Casual observations of growing luxury unit sales in emerging markets suggests the plausibility of that linkage in reality.

On the theory front, our research highlights for the first time the importance of distribution channels, as well as the shopping economies of scope, in maintaining luxury goods as a status symbol. Future research can look further into how different structures of shopping costs may affect luxury brands' performances. As a modeling strategy, we chose to abstract away from the cost associated with expanding a product line. Adding such a cost will clearly reduce the region where a separating equilibrium occurs and increase the region where a pooling equilibrium occurs. In addition, our comparative statics can change somewhat and we leave that investigation to future research. We also focus on the average price in a brand's product line by assuming equal prices for all products. If we were to allow for individual prices, the price distributions, respectively for authentic and counterfeit goods, will both play a role. Future research can verify that depending on how the corresponding prices for authentic and counterfeit goods are correlated, the mechanism we have uncovered in this paper will remain valid.

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ENDNOTES

[1] Y.-C. J. Lin (2011) describes a common way of selling high-quality knock-offs in China where counterfeiters hire people to hang around in busy marketplaces to solicit business, then take interested customers to a nearby apartment where the goods are sold.

[2] Authentic brands typically sell at a single online address but counterfeiters are typically scattered. According to the Department of Homeland Security, counterfeiters "rapidly proliferate" in online environments, setting up multiple accounts in a number of online platforms "in a deliberate effort to complicate enforcement efforts and sell without interruptions" (Department of Homeland Security, 2020) and keep the inventory small. This way, should one account be shut down due to intellectual property infringement complaints, they can continue to sell in another. In 2015, law-enforcement agencies from 27 countries shut down over 37,000 web sites selling counterfeit goods (Europol, 2017; ICE.gov, 2015). Reports show that a known luxury brand sees as many as 5 to 50 domains registered *per day* to sell the brand's counterfeit products (Kemp, 2019).

[3] This high-volume, low-quantity traffic makes monitoring and detection of counterfeits more difficult, especially because goods sold online are shipped in small consignments (Department of Homeland Security, 2020). In 2020, the mail parcel shipments accounted for more than

500 million packages each year and seizures in the small package environment made up 93% of all seizures in 2018 (Department of Homeland Security, 2020). The majority of these shipments carried 10 items or less (OECD, 2016). "Distributing counterfeits across a series of small packages spreads the risk of detection, and lowers the loss from having one or more shipments seized, suggesting that losses to the counterfeiter on an ongoing basis would be within a tolerable range" (OECD, 2016; US Chamber of Commerce, 2016).

[4]There are technical reasons for why it is harder for authorities to detect counterfeits in small parcels than in shipping containers. Section 321 of the Tariff Act of 1930 encourages counterfeiters to favor smaller parcel delivery. "Under Section 321, a foreign good valued at or less than $800 and imported by one person on one day is not subject to the same formal customs entry procedures and rigorous data requirements as higher-value packages entering the United States" (Department of Homeland Security, 2020, p. 4). Cargo containers making entry at a maritime port, on the other hand, "have to legally provide customs officials with information well in advance of arrival" (Department of Homeland Security, 2020, p. 14).

[5]The analysis is available upon request from the authors.

[6]We thank an anonymous referee for suggesting this alternative interpretation.

[7]Different tie-breaking rules can make a difference in the type of equilibria that emerge in some cases. Our tie-breaking rules are not type-dependent and deliver a stable equilibrium.

[8]Note that $\bar{\rho} = \underline{\rho}$ when $c = w_L\left(\frac{4w_L}{(w_H + w_L)^2} - 1\right)$. We also have $\bar{\rho} = 1$ and $\underline{\rho} = \frac{c}{1 - w_H}$ when $c = w_L\left(\frac{1 - w_H}{w_H}\right)$.

[9]Under the French law, for instance, possession of counterfeit items, whether consciously purchased or not, exposes the holder to sanctions such as a fine up to 300,000 Euros and a three-year prison sentence (https://www.europe-consommateurs.eu).

[10]We call this a "necessary evil" as practitioners in the luxury goods industry always abhor any price decrease. See Dahlhoff and Zhang (2020).

[11]We thank an anonymous reviewer for his/her probing question that led to this analysis.

[12]https://www.pricingsolutions.com/pricing-blog/luxury-goods-pricing-dont-envy-their-high-price-tags-and-high-margins.

[13]Our utility function is additive. If we were to specify a multiplicative function, e.g., $(k_A + k_C)^{\alpha}(1 + \mu(\theta|k_A + k_C))$, we would have the symbolic utility interact with the functional utility. In this case, the symbolic utility increases with the functional utility such that more purchases by a consumer not only increase the functional utility but also the symbolic utility. This specification will make it less likely to have a separating equilibrium and also adds considerable complexity to our analysis. However, we can show numerically that our qualitative insights about the maximalist luxury remain intact. We thank an anonymous reviewer for suggesting this robustness check.

[14]While we allow fractional purchase for technical convenience, a luxury brand will always collect the same maximum payment regardless of what fraction of a good is consumed by either type of consumers in this separating equilibrium. Consequently, by the principal of free disposal, fractional consumption between 0 and 1 will never occur as the consumers always prefer more over less for the same payment.

[15]This equilibrium belief is only one of the many possible specifications. However, for any alternative specification, say, the customer is identified as an H-type with probability 1/2 and a L-type with probability 1/2 if they consume more than n^{sep} luxury goods, the equilibrium outcomes that we derived will not be affected. Here is why: in a separating equilibrium, if the L-types do not want to purchase n^{sep} and being identified as the H-types with probability 1, they will surely not want to incur additional cost to be identified as the H-types with a lower probability. In essence, we have the freedom to specify the off-the-equilibrium belief for any messages $k > n^{sep}$, but we chose the simplest and most convenient one that does not have any substantive implications either way.

[16]Note that $\underline{\rho} = \bar{\rho}$ when $w_H = w_L\left(\frac{2}{\sqrt{c + w_L}} - 1\right)$.

[17]Note that with a sufficient large but finite limit (N) of the maximum product line length, p_A cannot be infinitesimally small in a separating equilibrium.

[18]We can also show that, for sufficiently small $\alpha \in (0, \underline{\alpha})$, the comparative statics for the benchmark model and the extended model in corresponding ranges remains the same. To see this, note that p_0 given by Equation (A49) strictly increases in both c and b because LHS of (A49) increases in p_A but decreases in c and b. According to Equation (A48), it is easy to see that n^* strictly decreases in b when c is sufficiently large, and it equals w_H/p_0 thus also strictly decreases in both b and c when c takes intermediate values. When c is sufficiently small, n^* remains constant at N. Hence we can conclude that the optimal product line n^* (weakly) decreases in both b and c.

[19]Strictly speaking, when $p_A = \frac{4b^2 w_H}{\alpha^2}$, we have a Case 3.4 pooling equilibrium since the H-types will purchase the least amount of counterfeits when indifferent. This is also the only price that maximizes a brand's profit in a Case 3.4 pooling equilibrium. When $p_A > \frac{4b^2 w_H}{\alpha^2}$, the brand's profit remains the same at its maximum level but demand for counterfeits increases as p_A increases.

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APPENDIX A

Proof of Lemma 1. First, we verify that when counterfeits are absent, there exists a separating equilibrium such that the H-types purchase only one luxury product and the L-types do not purchase any goods. This strategy profile is incentive compatible as long as $w_L < p_A^* k_A^* \le 1 - c$. In other words, as long as the luxury brand charges a payment that exceeds what the L-types could afford for any goods purchased from the authentic store, the L-types will not be able to mimic, while the high-types still find it worthwhile to signal as long as that payment is less than the symbolic benefit net of shopping disutility.[14] As a result, a luxury brand can provide only 1 product to facilitate the separation. In addition, a brand maximizes its profit by charging the highest price an H-type is willing to pay, that is, $p_A^* = \min\{w_H, 1 - c\}$. The brand obtains a profit of $\rho_H \min\{w_H, 1 - c\}$ in this separating equilibrium.

Next, we derive the conditions for the existence of a pooling equilibrium. Suppose consumers purchase $\hat{k}$ luxury goods in a pooling equilibrium, then it should be both affordable and Incentive Compatible. That is,

$$\hat{k}\, p_A \le w_L < w_H \tag{A1}$$

and

$$\hat{k}\, p_A + c \le \rho_H. \tag{A2}$$

Hence, for p_A that supports a pooling equilibrium, the luxury brand's profit is given by

$$\begin{aligned} \pi \quad &= \hat{k}\, p_A (\rho_H + 1 - \rho_H) = \hat{k}\, p_A \\ \text{s. t. } \hat{k} p_A &\le \min\{w_L, \rho_H - c\}. \end{aligned}$$

In a pooling equilibrium, the maximum surplus a brand can extract from the consumers is equal to $\min\{w_L, \rho_H - c\}$. A brand can facilitate this equilibrium where both the H- and L-types purchase one product with the shortest product line $n^* = 1$ and a price equal to $p_A^* = \min\{w_L, \rho_H - c\}$. There are many beliefs that are compatible with this pooling equilibrium outcome. One such belief is that $\mu(k) = \rho_H$ if $k = 1$, and $\mu(k) = 0$ if $k \in [0, 1)$. We can show that this pooling equilibrium outcome survives the Intuitive Criterion. This is because any off-the-equilibrium-path message $k \in [0, 1)$ is either dominated or undominated (depending on how one modifies the off-the-equilibrium beliefs) for both types of consumers, as they both can afford and are willing to pay for the available luxury goods.

According to our equilibrium selection criterion, the firm will facilitate a separating equilibrium over a pooling equilibrium if and only if the former generates more profit than the latter, that is,

$$\rho_H \min\{w_H, 1 - c\} > \min\{w_L, \rho_H - c\}.$$

The above inequality can be reduced to the following two conditions

$$c > \left(\frac{1 - w_H}{w_H}\right) w_L, \text{or} \tag{A3}$$

$$\rho_H \in \left(0, \frac{c}{1 - w_H}\right) \cup \left(\frac{w_L}{w_H}, 1\right). \tag{A4}$$

□

Proof of Proposition 1. We prove this proposition using backward induction. First, we study the consumer behavior for a given product line n and a price p_A and derive conditions for the luxury market to be viable. Second, we turn to the profit optimization problem of a brand, taking into account both the demand and market viability conditions.

Consumer choice

Given the product line n, price p_A, and the public's belief $\mu(\theta | k_A + k_C)$, a type-$\theta \in \{H, L\}$ consumer chooses k_A, k_C to maximize her utility:

$$\begin{aligned} &\max_{k_A, k_C} \{\mu(\theta | k_A + k_C) - (bk_C + c\mathbf{1}(k_A > 0) + p_A k_A)\} \\ \text{s. t.}\quad & 0 \leq k_A + k_C \leq n \\ & k_A, k_C \in [0, n] \\ & k_A p_A \leq w_\theta. \end{aligned}$$

Let TC_{min}^H and TC_{min}^L denote the minimal total cost of assembling $\hat{k} > 0$ luxury goods for the $H-$ and $L-$types respectively. Formally, we have

$$TC_{min}^H := \min_{k_A \in (0, \hat{k}\,]} \{b\hat{k}, b(\hat{k} - k_A) + c + p_A k_A | w_H \geq p_A k_A\}, \tag{A5}$$

$$TC_{min}^L := \min_{k_A \in (0, \hat{k}\,]} \{b\hat{k}, b(\hat{k} - k_A) + c + p_A k_A | w_L \geq p_A k_A\}. \tag{A6}$$

Note that the only difference between TC_{min}^H and TC_{min}^L is the budget difference between $H-$ and $L-$types in the constraint. Put differently, when their budget is not binding, the H-types and L-types have effectively the same utility maximization problem. Hence, the H- types could separate from the L-types only via buying strictly more items than the L-types based on our tie-breaking rules. It is also straightforward to see that L-types do not purchase any luxury goods in a separating equilibrium. This is because the L-types fail to mimic the H-types and thus do not gain any utility from owning luxury goods in any separating equilibria.

Therefore, on-the-equilibrium-path beliefs in a separating (μ) and pooling equilibrium (μ') are respectively given by

$$\mu = \begin{cases} 1 & \text{if } k_A + k_C = n^{sep} > 0 \\ 0 & \text{if } k_A + k_C = 0 \end{cases}$$

and

$$\mu' = \rho_H \text{ if } k_A + k_C = n^{pool},$$

where n^{sep} and n^{pool} denote respectively the total number of goods H-types purchase in a separating and pooling equilibrium. When a luxury goods market is viable in a pooling equilibrium, it is necessary that $n^{pool} > 0$. The existence of separating and pooling equilibria requires that consumer choices are both individually rational and incentive compatible, translating into the following two conditions. In a separating equilibrium, we must have

$$1 - TC_{min}^{H} \geq 0, 1 - TC_{min}^{L} \leq 0. \tag{A7}$$

In a pooling equilibrium, we must have

$$\rho_H - TC_{min}^{H} \geq 0, \rho_H - TC_{min}^{L} \geq 0. \tag{A8}$$

Next, to simplify the expressions of TC_{min}^{H} and TC_{min}^{L}, we derive the equilibrium conditions in the following three cases (with respect to p_A).

Case 1: $w_L < w_H < \frac{cp_A}{b-p_A}$ $\left(\text{or equivalently,} \frac{bw_H}{w_H+c} < p_A < b\right)$

In this case, $TC_{min}^{H} = TC_{min}^{L} = b\hat{k}, \forall\ \hat{k} \geq 0$. In other words, in this price range, it is the least costly for both the H-$H-$ and L-types to shop only for counterfeit goods to assemble $\hat{k}$ goods in total. According to (A7), a (candidate) separating equilibrium in this case implies that $TC_{min}^{H} = TC_{min}^{L} = 1$, that is, both types of consumers are indifferent between purchasing $n^{sep} = \frac{1}{b}$ counterfeit goods and not purchasing. However, recall that our tie-breaking rules indicate that consumers will choose not to buy when they are indifferent between buying counterfeits and not buying anything. Hence, no separating equilibria exist in this price range.

On the other hand, there exist pooling equilibria where all consumers purchase a total of $k_C^*(H) = k_C^*(L) = n^{pool} < \frac{\rho_H}{b}$ counterfeit goods and zero authentic goods. However, these pooling equilibria cannot be sustained in a viable market because there is no demand for authentic goods.

Case 2: $w_L < \frac{cp_A}{b-p_A} \leq w_H$ $\left(\text{or equivalently,} \frac{bw_L}{w_L+c} < p_A \leq \frac{bw_H}{w_H+c}\right)$

In this case, L-types always prefer to shop only for counterfeits, whereas the H-types' optimal choice depends on $\hat{k}$, the total number of goods they need to assemble. Formally, we have

$$TC_{min}^{H} = \begin{cases} b\hat{k} & \text{if } \hat{k} < \frac{c}{b-p_A} \\ c + p_A\hat{k} & \text{if } \frac{c}{b-p_A} \leq \hat{k} \leq \frac{w_H}{p_A} \\ c + w_H + b\left(\hat{k} - \frac{w_H}{p_A}\right) & \text{if } \hat{k} > \frac{w_H}{p_A} \end{cases}$$

$$TC_{min}^{L} = b\hat{k}, \forall\ \hat{k} \geq 0.$$

Figure A1 illustrates how TC_{min}^{H} and TC_{min}^{L} vary w.r.t. $\hat{k}$ in this case.

Following the same argument as in Case 1, we can rule out the possibility of $TC_{min}^{H} = TC_{min}^{L} = 1$ in any separating equilibrium. Hence according to condition (A7), a separating equilibrium may exist in this case only if the following two conditions hold:

$$1 \geq \frac{bc}{b-p_A}, \tag{A9}$$

$$\frac{1}{b} \leq n^{sep} \leq \frac{\min\{1-c, w_H\}}{p_A} + \frac{\max\{0, 1-c-w_H\}}{b}. \tag{A10}$$

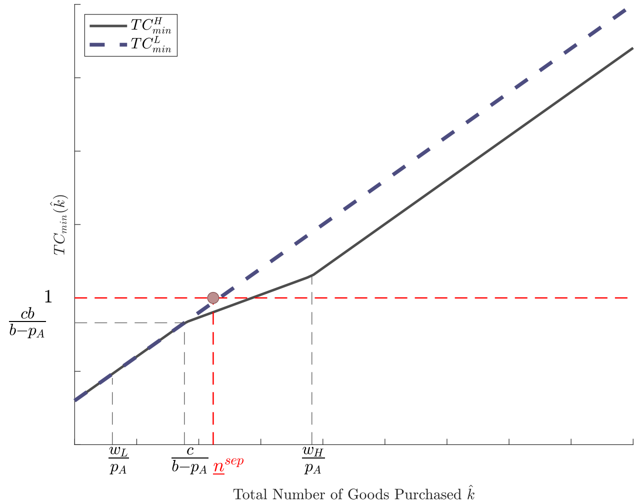


FIGURE A1 Case 2 $\left(w_L < \frac{cp_A}{b-p_A} \leq w_H\right)$ [Color figure can be viewed at wileyonlinelibrary.com]

For the solution set to be nonempty, we need the following condition to be satisfied:

$$p_A \leq b \min\left\{1-c, \frac{w_H}{w_H+c}\right\}. \tag{A11}$$

Next we show that conditions (A10) and (A11) are sufficient to support a separating equilibrium. First, the lower bound on n^{sep} given by (A10) implies that $TC^L_{min} \leq 1$, that is, the L-types prefer not to mimic the H-types (even when indifferent based on our tie-breaking rules) and thus $k^*_A(L) = k^*_C(L) = 0$. Second, given (A10) and (A11), by signaling their type, the H-types will need to consume n^{sep} goods in total where their basket contains at least some authentic goods, that is, $k^*_A(H) > 0$. Hence, based on our tie-breaking rules, even when indifferent between buying or not buying, the H-types will choose to buy. Therefore, the strategy profile where the H-types buy n^{sep} goods in total (subject to the conditions (A10) and (A11)) and the L-types do not buy anything constitutes a separating equilibrium. There are many beliefs that could support such a separating equilibrium outcome. One such belief is to infer that a consumer is a H-type if the consumer purchases at least n^{sep} goods in total, and otherwise an L-type. That is, $\mu(k) = 0$ if $k < n^{sep}$ and $\mu(k) = 1$ if $k \geq n^{sep}$.[15]

Lastly, we prove that the only separating equilibrium outcome that survives Intuitive Criterion is the Riley outcome where the H-types assemble the least amount of goods necessary to separate from the L-types. Let $\underline{n}^{sep}$ denote this number. This implies that L-types' IC constraint must be satisfied at $k = \underline{n}^{sep}$. That is,

$$1 - TC^L_{min}(\underline{n}^{sep}) \leq 0. \tag{A12}$$

We can show that any separating equilibria where the H-types purchase $\hat{k} = k_A(H) + k_C(H) > \underline{n}^{sep}$ fails the Intuitive Criterion. Suppose there exists such a separating equilibrium where the H-types purchase in total $\hat{k} > \underline{n}^{sep}$ goods and they obtain a symbolic utility of 1 in equilibrium, whereas the L-types obtain zero symbolic utility and thus their best response is not to purchase anything, that is, $k^*_A(L) = k^*_C(L) = 0$. Given that $TC^\theta_{min}(k)$ strictly increases in k for all admissible k, there must exist an off-the-equilibrium-path message $k' \in [\underline{n}^{sep}, \hat{k})$ such that

$$1 - TC^H_{min}(k') > 1 - TC^H_{min}(\hat{k}), \tag{A13}$$

and given inequality (A12), we also have

$$1 - TC^L_{min}(k') < 0. \tag{A14}$$

The right hand sides of inequalities (A13) and (A14) are the equilibrium payoffs for the H-types and L-types, respectively; whereas the left hand sides are the maximum payoffs that the H-types and L-types could obtain by consuming k' products in total given that the public holds rational beliefs, in this case $\mu(\theta = H|k') = 1$. Therefore, k' is dominated for the L-types and not for the H-types. Once we restrict the public's beliefs to $\mu(\theta = H|k') = 1$, the maximum payoff that the H-types obtain by consuming k' products is larger than their equilibrium payoff. Hence this separating equilibrium fails the Intuitive Criterion.

Therefore, we focus on the separating equilibrium where the H-types assemble the least amount of goods, in this case, $\underline{n}^{\,sep} = \frac{1}{b}$. In this equilibrium, H-types' consumption is given by

$$k_A^*(H) = \begin{cases} \frac{1}{b} & \text{if } p_A \le \min\{bw_H, b(1-c)\} \\ \frac{w_H}{p_A} & \text{if } bw_H < p_A \le b\left(\frac{w_H}{w_H + c}\right) \end{cases}, \quad k_C^*(H) = \begin{cases} 0 & \text{if } p_A \le \min\{bw_H, b(1-c)\} \\ \frac{1}{b} - \frac{w_H}{p_A} & \text{if } bw_H < p_A \le b\left(\frac{w_H}{w_H + c}\right) \end{cases}.$$

Note that if $w_H > 1 - c$, we have $bw_H > b\left(\frac{w_H}{w_H + c}\right)$, and hence the H-types consumption will reduce to $k_A^*(H) = \frac{1}{b}$ and $k_C^*(H) = 0$.

On the other hand, there may exist pooling equilibria where the L-types purchase only counterfeits while the H-types purchase only authentic goods or authentic plus counterfeit goods in Case 2. However, in a pooling equilibrium, the luxury brand can extract a surplus of no more than $\min\{w_H, \rho_H - c\}$ from each H-type according to (A8) and zero surplus from the L-types, which is no more than the maximum surplus from a separating equilibrium. Moreover, even when the brand obtains equal profits, demand for counterfeits is higher in a pooling equilibrium (where L-types would buy some counterfeits) than in a separating equilibrium (where L-types do not buy any counterfeits). Hence we rule out these pooling equilibria based on our equilibrium selection criterion.

Case 3: $\frac{cp_A}{b - p_A} \le w_L < w_H$ $\left(\text{or equivalently,} p_A \le \frac{bw_L}{w_L + c}\right)$

In this case, both the L-types and H-types' optimal portfolio depends on $\hat{k}$, the total number of goods they need to assemble. Formally, we have

$$TC_{min}^H = \begin{cases} b\hat{k} & \text{if } \hat{k} < \frac{c}{b - p_A} \\ c + p_A\hat{k} & \text{if } \frac{c}{b - p_A} \le \hat{k} \le \frac{w_H}{p_A} \\ c + w_H + b\left(\hat{k} - \frac{w_H}{p_A}\right) & \text{if } \hat{k} > \frac{w_H}{p_A} \end{cases}$$

and

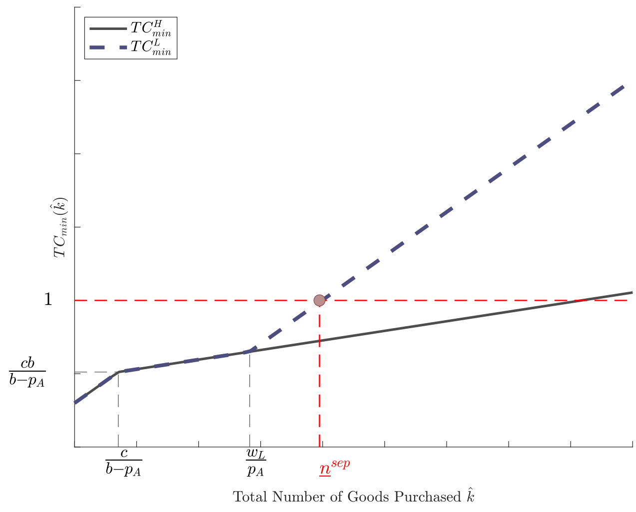
$$TC_{min}^L = \begin{cases} b\hat{k} & \text{if } \hat{k} < \frac{c}{b - p_A} \\ c + p_A\hat{k} & \text{if } \frac{c}{b - p_A} \le \hat{k} \le \frac{w_L}{p_A}. \\ c + w_L + b\left(\hat{k} - \frac{w_L}{p_A}\right) & \text{if } \hat{k} > \frac{w_L}{p_A} \end{cases}$$

Figure A2 illustrates how TC_{min}^H and TC_{min}^L vary w.r.t. $\hat{k}$ in Case 3.

Again, we can rule out the possibility of $TC_{min}^H = TC_{min}^L = 1$ in any separating equilibrium. This is because under those cases, either both types of consumers are indifferent between buying counterfeit goods and buying nothing, or they are indifferent between buying authentic goods and buying nothing. In the former case, they both will choose not to buy anything, while in the latter case, they will buy equal number of authentic goods. Hence according to condition (A7), separating equilibrium may exist in this case only if the following two conditions hold:

$$1 > \left(\frac{w_L}{p_A}\right)p_A + c = w_L + c, \tag{A15}$$

FIGURE A2 Case 3 $\left(\frac{cp_A}{b-p_A} \le w_L < w_H\right)$ [Color figure can be viewed at wileyonlinelibrary.com]



$$\frac{1-c-w_L}{b}+\frac{w_L}{p_A}<n^{sep}\le\frac{\min\{1-c,w_H\}}{p_A}+\frac{\max\{0,1-c-w_H\}}{b}. \tag{A16}$$

Since condition (A15) holds by assumption, for the solution set to be nonempty for condition (A16), we simply need $p_A < b$ which also clearly holds in case 3. Note that the lower bound of n^{sep} does not hold at equality in condition (A16). This is because, based on our tie-breaking rules, the L-types will mimic the H-types by cross-shopping from both authentic and counterfeit stores when indifferent between assembling $\hat{k}=\frac{1-c-w_L}{b}+\frac{w_L}{p_A}$ goods and not buying anything. Hence in the separating equilibrium that satisfies Intuitive Criterion (i.e., where the H-types assemble the least amount of goods), we have

$$\underline{n}^{\,sep}=\frac{1-c-w_L+\varepsilon}{b}+\frac{w_L}{p_A}, \tag{A17}$$

where $\varepsilon>0$ is arbitrarily small. In this equilibrium, H-types' consumption is given by

$$k_A^*(H)=\begin{cases}\frac{1-c-w_L+\varepsilon}{b}+\frac{w_L}{p_A} & \text{if } p_A\le\min\left\{b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right),\frac{bw_L}{w_L+c}\right\}\\ \frac{w_H}{p_A} & \text{if } b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right)<p_A\le\frac{bw_L}{w_L+c}\end{cases},$$

$$k_C^*(H)=\begin{cases}0 & \text{if } p_A\le\min\left\{b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right),\frac{bw_L}{w_L+c}\right\}\\ \frac{1-c-w_L+\varepsilon}{b}+\frac{w_L-w_H}{p_A} & \text{if } b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right)<p_A\le\frac{bw_L}{w_L+c}\end{cases}.$$

Note that if $w_H\ge\frac{w_L}{w_L+c}$, we have $b\left(\frac{w_H-w_L}{1-c-w_L}\right)\ge\frac{bw_L}{w_L+c}$, and thus the H-types' consumption reduces to $k_A^*(H)=\frac{1-c-w_L+\varepsilon}{b}+\frac{w_L}{p_A}$ and $k_C^*(H)=0$.

On the other hand, there may exist pooling equilibria where both the L-types and H-types purchase at least *some* authentic goods in Case 3. According to (A8), we must have

$$n^{pool}\le\begin{cases}\frac{\rho_H-c-w_L}{b}+\frac{w_L}{p_A} & \text{if } w_L<\rho_H-c\\ \frac{\rho_H-c}{p_A} & \text{if } w_L\ge\rho_H-c.\end{cases} \tag{A18}$$

These pooling equilibrium outcomes can be supported by the belief that $\mu'(k)=\rho_H$ if $k=n^{pool}$ and $\mu'(k)=0$ if $k\ne n^{pool}$. Note that for any given p_A, the upper bound of n^{pool} defined by (A18) is strictly less than the lower bound of n^{sep} defined by

(A17) since $\rho_H < 1$. Therefore, these pooling equilibrium outcomes will survive the Intuitive Criterion as long as the product line is set to $n = n^{pool}$. In a Case 3-pooling equilibrium, the H-types and L-types' consumption are given respectively by

$$k_A^*(H) = \begin{cases} \min\left\{\frac{w_H}{p_A}, n^{pool}\right\} & \text{if } p_A \le \min\left\{\frac{bw_L}{w_L+c}, b - \frac{c}{n^{pool}}\right\} \\ 0 & \text{if } b - \frac{c}{n^{pool}} < p_A \le \frac{bw_L}{w_L+c} \end{cases},$$

$$k_C^*(H) = \begin{cases} \max\left\{n^{pool} - \frac{w_H}{p_A}, 0\right\} & \text{if } p_A \le \min\left\{\frac{bw_L}{w_L+c}, b - \frac{c}{n^{pool}}\right\} \\ n^{pool} & \text{if } b - \frac{c}{n^{pool}} < p_A \le \frac{bw_L}{w_L+c} \end{cases}.$$

and

$$k_A^*(L) = \begin{cases} \min\left\{\frac{w_L}{p_A}, n^{pool}\right\} & \text{if } p_A \le \min\left\{\frac{bw_L}{w_L+c}, b - \frac{c}{n^{pool}}\right\} \\ 0 & \text{if } b - \frac{c}{n^{pool}} < p_A \le \frac{bw_L}{w_L+c} \end{cases},$$

$$k_C^*(L) = \begin{cases} \max\left\{n^{pool} - \frac{w_L}{p_A}, 0\right\} & \text{if } p_A \le \min\left\{\frac{bw_L}{w_L+c}, b - \frac{c}{n^{pool}}\right\} \\ n^{pool} & \text{if } b - \frac{c}{n^{pool}} < p_A \le \frac{bw_L}{w_L+c} \end{cases}.$$

Note that pooling equilibria in a viable market (where consumers purchase at least some authentic goods) require that $p_A \le b - \frac{c}{n^{pool}}$ $\left(\text{or equivalently } n^{pool} \ge \frac{c}{b - p_A}\right)$.

Firm's optimization problem

Given the consumer behavior discussed above, a luxury brand chooses its price p_A and product line length n to maximize its total profit. We first derive firm's optimal decisions both in the pooling and separating equilibria. Then we compare their respective profit and select the equilibrium outcome that generates the highest profit. Note that $\underline{n}^{\,sep} \ge \frac{1}{b}$ in any of the above 3 cases. Therefore, if the limit of product line length N is below $\frac{1}{b}$, there does not exist any separating equilibrium. In the analysis that follows, we compare the profit of a separating versus pooling equilibrium when $N \ge \frac{1}{b}$.

Recall that in a separating equilibrium, the L-types do not purchase any goods as they obtain zero symbolic benefits, which is the only motivation for consumption in our benchmark model. Thus in a separating equilibrium, the firm solves the following profit-maximizing problem:

$$\max_{p_A} \pi = \rho_H k_A^*(H) p_A$$

$$\text{s. t.} \quad k_A^*(H) = \begin{cases} \begin{cases} \frac{1-c-w_L+\varepsilon}{b} + \frac{w_L}{p_A} & \text{if } p_A \le b\left(\frac{w_H - w_L}{1-c-w_L+\varepsilon}\right) \\ \frac{w_H}{p_A} & \text{if } b\left(\frac{w_H - w_L}{1-c-w_L+\varepsilon}\right) < p_A \le \frac{bw_H}{w_H+c} \end{cases} & \text{if } w_H \in \left(w_L, \frac{w_L}{w_L+c}\right], \\ \begin{cases} \frac{1-c-w_L+\varepsilon}{b} + \frac{w_L}{p_A} & \text{if } p_A \le \frac{bw_L}{w_L+c} \\ \frac{1}{b} & \text{if } \frac{bw_L}{w_L+c} < p_A \le bw_H \\ \frac{w_H}{p_A} & \text{if } bw_H < p_A \le b\left(\frac{w_H}{w_H+c}\right) \end{cases} & \text{if } w_H \in \left(\frac{w_L}{w_L+c}, 1-c\right], \\ \begin{cases} \frac{1-c-w_L+\varepsilon}{b} + \frac{w_L}{p_A} & \text{if } p_A \le \frac{bw_L}{w_L+c} \\ \frac{1}{b} & \text{if } \frac{bw_L}{w_L+c} < p_A \le b(1-c) \end{cases} & \text{if } w_H > 1-c. \end{cases}$$

Recall that if the firm obtains equal profit with multiple possible product line lengths, we focus on the leanest product line. We examine the firm's profit-maximization problem in the following four cases.

(1) If $w_H \in \left(w_L, \frac{bNw_L}{bN-(1+\varepsilon-w_L-c)}\right)$, there exists no separating equilibrium when $\underline{n}^{sep} = \frac{w_L}{p_A} + \frac{1-w_L-c+\varepsilon}{b} > N$, that is, when $p_A < \frac{bw_L}{bN-(1-w_L-c+\varepsilon)}$. Also note that given the range of w_H, we have $\frac{bw_L}{bN-(1-w_L-c+\varepsilon)} > b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right)$. Therefore, the relevant price range in this case is $p_A \in \left[\frac{bw_L}{bN-(1-w_L-c+\varepsilon)}, \frac{bw_H}{w_H+c}\right]$. The brand's profit remains constant at $\pi = \rho_H w_H$ in this price range. However, the higher the price, the more counterfeit goods H-types need to assemble to signal. Based on our selection criteria, we choose the profit-maximizing equilibrium with the lowest demand for counterfeits, that is, when price is at its lowest, $p_A^* = \frac{bw_L}{bN-(1-w_L-c+\varepsilon)}$.

(2) If $w_H \in \left[\frac{bNw_L}{bN-(1+\varepsilon-w_L-c)}, \frac{w_L}{w_L+c}\right]$, profit first strictly increases in price when $p_A < b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right)$ then remains constant at $\pi = \rho_H w_H$ when $p_A \in \left[b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right), \frac{bw_H}{w_H+c}\right]$. Note that, however, when p_A^* exceeds $b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right)$, the H-types cannot afford all $\underline{n}^{sep} < N$ authentic goods and hence would start purchasing counterfeits to signal their status. Based on our selection criteria, we choose the profit-maximizing equilibrium with the lowest demand for counterfeits, that is, $p_A^* = b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right)$ such that the H-types are (just) able to afford all authentic goods.

(3) If $w_H \in \left(\frac{w_L}{w_L+c}, 1-c\right]$, profit first strictly increases in price when $p_A < bw_H$ then remains constant at $\pi = \rho_H w_H$ when $p_A \in \left[bw_H, \frac{bw_H}{w_H+c}\right]$. In addition, when p_A^* exceeds bw_H, the H-types cannot afford all $\underline{n}^{sep} = \frac{1}{b}$ authentic goods and hence would start purchasing counterfeits to signal their status. Therefore, based on our selection criteria, we choose the equilibrium with $p_A^* = bw_H$.

(4) If $w_H > 1-c$, profit always strictly increases in price and thus the optimal price is given by the upper bound on price, that is, $p_A^* = 1-c$.

Hence we can derive the optimal price and product line length as follows:

$$p_A^* = \begin{cases} b(1-c) & \text{if } w_H > 1-c \\ bw_H & \text{if } w_H \in \left(\frac{w_L}{w_L+c}, 1-c\right] \\ b\left(\frac{w_H-w_L}{1-c-w_L+\varepsilon}\right) & \text{if } w_H \in \left[\frac{bNw_L}{bN-(1+\varepsilon-w_L-c)}, \frac{w_L}{w_L+c}\right], \\ \frac{bw_L}{bN-(1+\varepsilon-w_L-c)} & \text{if } w_H \in \left(w_L, \frac{bNw_L}{bN-(1+\varepsilon-w_L-c)}\right) \end{cases} \tag{A19}$$

and

$$n^* = \begin{cases} \frac{1}{b} & \text{if } w_H > \frac{w_L}{w_L+c} \\ \min\left\{N, \left(\frac{1-w_L-c+\varepsilon}{b}\right)\left(\frac{w_H}{w_H-w_L}\right)\right\} & \text{if } w_H \le \frac{w_L}{w_L+c} \end{cases}. \tag{A20}$$

At the optimal price p_A^*. The H-types' consumption in this separating equilibrium is thus given by

$$k_A^*(H) = \begin{cases} \frac{1}{b} & \text{if } w_H > \frac{w_L}{w_L+c} \\ \left(\frac{1-w_L-c+\varepsilon}{b}\right)\left(\frac{w_H}{w_H-w_L}\right) & \text{if } w_H \in \left[\frac{bNw_L}{bN-(1+\varepsilon-w_L-c)}, \frac{w_L}{w_L+c}\right] \\ \frac{w_H\left(bN-(1+\varepsilon-w_L-c)\right)}{bw_L} & \text{if } w_H \in \left(w_L, \frac{bNw_L}{bN-(1+\varepsilon-w_L-c)}\right) \end{cases} \tag{A21}$$

$$k_C^*(H) = \begin{cases} 0 & \text{if } w_H \geq \frac{bNw_L}{bN - (1+\varepsilon - w_L - c)} \\ N - \frac{w_H\left(bN - (1+\varepsilon - w_L - c)\right)}{bw_L} & \text{if } w_H \in \left(w_L, \frac{bNw_L}{bN-(1+\varepsilon-w_L-c)}\right) \end{cases}. \tag{A22}$$

The luxury brand obtains a profit of $\pi^{sep*} = \rho_H \min\{w_H, 1 - c\}$.

In a (Case 3) pooling equilibrium, both the H-types and L-types purchase some authentic goods. Hence the firm solves the following profit-maximizing problem:

$$\begin{aligned} \max_{p_A} \pi \quad &= \rho_H k_A^*(H) p_A + (1-\rho_H) k_A^*(L) p_A \\ \text{s. t.} \quad k_A^*(H) &= \begin{cases} \min\left\{\frac{w_H}{p_A}, n^{pool}\right\} & \text{if } p_A \leq \min\left\{b - \frac{c}{n^{pool}}, \frac{bw_L}{w_L + c}\right\}, \\ 0 & \text{if o/w} \end{cases} \\ k_A^*(L) \quad &= \begin{cases} \min\left\{\frac{w_L}{p_A}, n^{pool}\right\} & \text{if } p_A \leq \min\left\{b - \frac{c}{n^{pool}}, \frac{bw_L}{w_L + c}\right\}, \\ 0 & \text{if o/w} \end{cases} \\ n^{pool} \quad &\leq \begin{cases} \frac{\rho_H - c - w_L}{b} + \frac{w_L}{p_A} & \text{if } w_L < \rho_H - c \\ \frac{\rho_H - c}{p_A} & \text{if } w_L \geq \rho_H - c \end{cases}. \end{aligned}$$

Note that $n^{pool} p_A \leq \rho_H - c$ holds for any admissible price p_A in a pooling equilibrium. Hence we consider the following three cases separately.

(1) $w_H > w_L \geq \rho_H - c$

This means the maximum surplus a brand can obtain from a pooling equilibrium is $\pi^* = \rho_H - c$. To achieve the maximum surplus, the L-types' IC constraint is binding, that is, $n^{pool} = \frac{\rho_H - c}{p_A}$. A viable market implies that $p_A \leq b\left(1 - \frac{c}{\rho_H}\right) < \frac{bw_L}{w_L + c}$. Note that in this case neither H-types nor L-types purchase any counterfeit goods. Hence we focus on the profit-maximizing equilibrium with the leanest product line which is accompanied with the highest feasible price, that is, $p_A^* = b\left(1 - \frac{c}{\rho_H}\right)$ and $n^{pool} = \frac{\rho_H - c}{p_A^*} = \frac{\rho_H}{b}$.

(2) $w_H \geq \rho_H\left(\frac{w_L}{w_L + c}\right)$ and $w_L < \rho_H - c$.

To extract the maximum surplus, L-types' IC constraint must be binding, that is, $n^{pool} = \frac{\rho_H - c - w_L}{b} + \frac{w_L}{p_A}$. In addition, we have $w_L < n^{pool} p_A \leq \rho_H\left(\frac{w_L}{w_L + c}\right) \leq w_H$. We can rewrite the profit function as follows:

$$\pi = \rho_H\left(p_A\left(\frac{\rho_H - c - w_L}{b}\right) + w_L\right) + \rho_L w_L.$$

Clearly the brand's profit strictly increases in price. Therefore in the profit-maximizing pooling equilibrium, we have $p_A^* = \min\{\frac{bw_L}{w_L + c}, b - \frac{c}{n^{pool}}\} = \frac{bw_L}{w_L + c}$ and $n^{pool} = \frac{\rho_H}{b}$.

(3) $w_H < \rho_H\left(\frac{w_L}{w_L + c}\right)$ and $w_L < \rho_H - c$

To extract the maximum surplus, L-types' IC constraint must be binding, that is, $n^{pool} = \frac{\rho_H - c - w_L}{b} + \frac{w_L}{p_A}$. We can rewrite the profit function as follows:

$$\pi = \rho_H \min\left\{p_A\left(\frac{\rho_H - c - w_L}{b}\right) + w_L, w_H\right\} + \rho_L w_L.$$

The brand's profit strictly increases in price when $p_A \in \left(0, \frac{b(w_H - w_L)}{\rho_H - c - w_L}\right)$ then remains constant at $\pi = \rho_H w_H + (1 - \rho_H) w_L$ when $p_A \in \left[\frac{b(w_H - w_L)}{\rho_H - c - w_L}, \frac{bw_L}{w_L + c}\right]$. However, the higher the price, the more counterfeit both the H- and the L-types would purchase in equilibrium. Therefore we select the profit-maximizing equilibrium with the lowest demand for counterfeits, that is, $p_A^* = \max\left\{\frac{b(w_H - w_L)}{\rho_H - c - w_L}, \frac{bw_L}{bN - (\rho_H - c - w_L)}\right\}$ and $n^{pool} = \min\left\{N, \frac{w_H(\rho_H - c - w_L)}{b(w_H - w_L)}\right\}$. Hence the optimal price and product line are given respectively by

$$p_A^* = \begin{cases} \max\left\{\frac{b(w_H - w_L)}{\rho_H - c - w_L}, \frac{bw_L}{bN - (\rho_H - c - w_L)}\right\} & \text{if } w_L < \rho_H - c, w_H < \rho_H\left(\frac{w_L}{w_L + c}\right) \\ \frac{bw_L}{w_L + c} & \text{if } w_L < \rho_H - c, w_H \geq \rho_H\left(\frac{w_L}{w_L + c}\right), \\ b\left(1 - \frac{c}{\rho_H}\right) & \text{if } w_L \geq \rho_H - c \end{cases} \tag{A23}$$

$$n^* = \begin{cases} \max\left\{1, \min\left\{N, \frac{w_H(\rho_H - c - w_L)}{b(w_H - w_L)}\right\}\right\} & \text{if } w_H < \rho_H\left(\frac{w_L}{w_L + c}\right) \\ \max\left\{1, \frac{\rho_H}{b}\right\} & \text{if } w_H \geq \rho_H\left(\frac{w_L}{w_L + c}\right) \end{cases}. \tag{A24}$$

At the optimal price p_A^* and product line length n^*, the H-types and L-types' equilibrium consumption are given, respectively, by

$$k_A^*(H) = \begin{cases} \frac{\rho_H}{b} & \text{if } w_H \geq \rho_H\left(\frac{w_L}{w_L + c}\right) \\ \frac{w_H(\rho_H - c - w_L)}{b(w_H - w_L)} & \text{if } w_H < \rho_H\left(\frac{w_L}{w_L + c}\right), N \geq \frac{w_H(\rho_H - c - w_L)}{b(w_H - w_L)} \\ \frac{bNw_H - w_H(\rho_H - c - w_L)}{bw_L} & \text{if } w_H < \rho_H\left(\frac{w_L}{w_L + c}\right), N < \frac{w_H(\rho_H - c - w_L)}{b(w_H - w_L)} \end{cases} \tag{A25}$$

$$k_C^*(H) = \begin{cases} 0 & \text{if o/w} \\ \frac{w_H(\rho_H - c - w_L) - bN(w_H - w_L)}{bw_L} & \text{if } w_H < \rho_H\left(\frac{w_L}{w_L + c}\right), N < \frac{w_H(\rho_H - c - w_L)}{b(w_H - w_L)} \end{cases} \tag{A26}$$

and

$$k_A^*(L) = \begin{cases} \frac{w_L + c}{b} & \text{if } w_L < \rho_H - c \\ \frac{\rho_H}{b} & \text{if } w_L \geq \rho_H - c \end{cases}, \quad k_C^*(L) = \begin{cases} n^* - \frac{w_L + c}{b} > 0 & \text{if } w_L < \rho_H - c \\ 0 & \text{if } w_L \geq \rho_H - c \end{cases}. \tag{A27}$$

In a pooling equilibrium, the luxury brand obtains a maximum profit of

$$\pi^{*pool} = \begin{cases} \rho_H - c & \text{if } w_L \geq \rho_H - c \\ \rho_H\left(\frac{\rho_H w_L}{w_L + c}\right) + (1 - \rho_H) w_L & \text{if } w_L < \rho_H - c, w_H \geq \rho_H\left(\frac{w_L}{w_L + c}\right). \\ \rho_H w_H + (1 - \rho_H) w_L & \text{if } w_L < \rho_H - c, w_H < \rho_H\left(\frac{w_L}{w_L + c}\right) \end{cases}$$

Finally, the firm obtains more profit from facilitating a separating equilibrium than a pooling equilibrium if and only if $\pi^{sep*} > \pi^{pool*}$. That is, the parameters satisfy the following conditions:

$$\rho_H \in \begin{cases} (0,1) & \text{if } c \in \left(w_L\left(\frac{1-w_H}{w_H}\right), 1-w_L\right) \\ \left(0, \frac{c}{1-w_H}\right) \cup (\underline{\rho}, \bar{\rho}), & \text{if } c \in \left(w_L\left(\frac{4w_L}{(w_H+w_L)^2}-1\right), w_L\left(\frac{1-w_H}{w_H}\right)\right]. \\ \left(0, \frac{c}{1-w_H}\right) & \text{if } c \in \left(0, w_L\left(\frac{4w_L}{(w_H+w_L)^2}-1\right)\right] \end{cases}$$

Specifically, $\underline{\rho} = \frac{(c+w_L)\left(w_L+w_H-\sqrt{(w_L+w_H)^2-\frac{4w_L^2}{c+w_L}}\right)}{2w_L}$ and $\bar{\rho} = \frac{(c+w_L)\left(w_L+w_H+\sqrt{(w_L+w_H)^2-\frac{4w_L^2}{c+w_L}}\right)}{2w_L}$.[16] □

Proof of Proposition 2. In the benchmark model, we have assumed away the functional benefits of luxury consumption. As a result, the L-types purchase nothing in a separating equilibrium. In this extended model, since luxury goods provide both functional and symbolic benefits, the L-types' consumption in a separating equilibrium is determined solely by the functional benefits of a good because they obtain zero symbolic benefits. Consumers obtain a (net) marginal functional utility of MU_c from consuming a counterfeit good and MU_A from an authentic good, where

$$\begin{aligned} MU_C &= \frac{\alpha}{2\sqrt{k_A+k_C}} - b, \\ MU_A &= \frac{\alpha}{2\sqrt{k_A+k_C}} - p_A. \end{aligned}$$

The market for authentic luxury goods is viable only if $p_A < b$, otherwise, all consumers would prefer counterfeit to authentic goods because in that case counterfeit goods are not only visibly identical to the authentic goods, but provide higher net marginal functional utility than the authentic goods ($MU_C > MU_A$, $\forall\, k_A > 0$). To maximize their utility, consumers have four consumption options: (i) shop only for counterfeit goods ($k_A = 0, k_C > 0$), (ii) shop only for authentic goods ($k_A > 0, k_C = 0$), (iii) cross-shop from both the authentic outlet and counterfeiters ($k_A > 0, k_C > 0$), or (iv) do not purchase any goods ($k_A = k_C = 0$). Let V_C, V_A, V_M, V_0 denote respectively the functional utility (net of shopping dis-utility and price) of a consumer when choosing (i), (ii), (iii) and (iv). Given a type-θ consumer's budget constraint, we have

$$\begin{aligned} &V_C = \alpha\sqrt{k_C} - bk_C \\ &V_A = \alpha\sqrt{k_A} - p_A k_A - c \\ &V_M = \alpha\sqrt{\left(\frac{w_\theta}{p_A}\right) + k_C} - bk_C - p_A k_A - c \\ &V_0 = 0. \end{aligned}$$

Next we derive and compare the maximum net functional utility a consumer can obtain when choosing (i), (ii), and (iii).

$$V_C^* \equiv \max_{k_C>0}\{V_C\} = \frac{\alpha^2}{4b},$$

$$V_A^* \equiv \max_{0<k_A\le\frac{w_\theta}{p_A}}\{V_A\} = \begin{cases} \frac{\alpha^2}{4p_A} - c & \text{if } \frac{w_\theta}{p_A} > \left(\frac{\alpha}{2p_A}\right)^2 \\ \alpha\sqrt{\frac{w_\theta}{p_A}} - w_\theta - c & \text{if } \frac{w_\theta}{p_A} \le \left(\frac{\alpha}{2p_A}\right)^2 \end{cases},$$

$$V_M^* \equiv \max_{k_C>0,0<k_A\le\frac{w_\theta}{p_A}}\{V_M\} = \frac{\alpha^2}{4b} + b\left(\frac{w_\theta}{p_A}\right) - w_\theta - c.$$

Note that when consumers enjoy functional utility, purchasing nothing is a strictly dominated strategy ($V_0 < V_C^*$). Thus, we can derive the maximum net functional utility that a type-θ consumer can obtain, denoted by V_{max}^{θ}, as follows.

If $p_A \in \left(\frac{bw_\theta}{w_\theta + c}, b\right)$, we have

$$V_{max}^{\theta} = V_C^*.$$

If $p_A \in \left(\frac{b(w_\theta - c)}{w_\theta}, \frac{bw_\theta}{w_\theta + c}\right]$, we have

$$V_{max}^{\theta} = \begin{cases} V_C^* & \text{if } \alpha^2 \in \left(0, 4b^2\left(\sqrt{\frac{w_\theta}{p_A}} - \sqrt{\frac{w_\theta}{p_A} - \frac{c + w_\theta}{b}}\right)^2\right] \\ V_A^* & \text{if } \alpha^2 \in \left(4b^2\left(\sqrt{\frac{w_\theta}{p_A}} - \sqrt{\frac{w_\theta}{p_A} - \frac{c + w_\theta}{b}}\right)^2, \frac{4b^2 w_\theta}{p_A}\right] \\ V_M^* & \text{if } \alpha^2 \in \left(\frac{4b^2 w_\theta}{p_A}, \infty\right) \end{cases}.$$

If $p_A \in \left(0, \frac{b(w_\theta - c)}{w_\theta}\right]$, we have

$$V_{max}^{\theta} = \begin{cases} V_C^* & \text{if } \alpha^2 \in \left(0, \frac{4bcp_A}{b - p_A}\right) \cup \left[4p_A w_\theta, \min\left\{\frac{4b^2 c}{b - p_A}, 4b^2\left(\sqrt{\frac{w_\theta}{p_A}} - \sqrt{\frac{w_\theta}{p_A} - \frac{c + w_\theta}{b}}\right)^2\right\}\right] \\ V_A^* & \text{if } \alpha^2 \in \left(\frac{4bcp_A}{b - p_A}, 4p_A w_\theta\right) \cup \left(\min\left\{\frac{4b^2 c}{b - p_A}, 4b^2\left(\sqrt{\frac{w_\theta}{p_A}} - \sqrt{\frac{w_\theta}{p_A} - \frac{c + w_\theta}{b}}\right)^2\right\}, \frac{4b^2 w_\theta}{p_A}\right] \\ V_M^* & \text{if } \alpha^2 \in \left(\frac{4b^2 w_\theta}{p_A}, \infty\right) \end{cases}.$$

The existence of separating and pooling equilibria requires that consumer choices are both individually rational and incentive compatible, translating into the following two conditions. Suppose the equilibrium consumption of the H-types is n^{sep} in a separating equilibrium and n^{pool} in a pooling equilibrium. In a separating equilibrum, we must have

$$1 + U(n^{sep}) - TC_{min}^{H}(n^{sep}) \geq V_{max}^{H} \text{ and } 1 + U(n^{sep}) - TC_{min}^{L}(n^{sep}) \leq V_{max}^{L}, \tag{A28}$$

where $U(k) = \alpha\sqrt{k}$ denotes the functional utility of assembling k goods. In a pooling equilibrium, we must have

$$\rho_H + U(n^{pool}) - TC_{min}^{H}(n^{pool}) \geq V_{max}^{H}, \text{ and } \rho_H + U(n^{pool}) - TC_{min}^{L}(n^{pool}) \geq V_{max}^{L}. \tag{A29}$$

Next, to simplify the expressions of TC_{min}^H and TC_{min}^L, we derive the equilibrium conditions in the following three cases.

Case 1: $w_L < w_H < \frac{cp_A}{b - p_A}$ $\left(\text{or equivalently}, \frac{bw_H}{w_H + c} < p_A < b\right)$

In this case, $TC_{min}^H = TC_{min}^L = b\hat{k}, \forall\, \hat{k} \geq 0$. In other words, in this price range, it is the least costly for both the $H-$ and $L-$ types to shop only for counterfeit goods to assemble any number of goods in total. Therefore, we also have $V_{max}^H = V_{max}^L = V_C^*$. According to (A28), a (candidate) separating equilibrium in this case implies that the H-types need to purchase exactly $\hat{k}$ goods that solves

$$b\hat{k} = 1 + U(\hat{k}) - V_C^*.$$

In this case, both types of consumers are indifferent between purchasing $\hat{k}$ counterfeit goods and purchasing $\frac{\alpha^2}{4b^2}$ counterfeit goods. However, our tie-breaking rules imply a bias against consuming counterfeit goods such that when indifferent, both types of consumers will purchase the smallest (and thus equal) number of counterfeits possible. Hence no separating equilibria exist in this price range.

On the other hand, there exist pooling equilibria where both the H-types and L-types purchase a total of $\left(\frac{\alpha}{2b}\right)^2 \le n^{pool} \le \left(\frac{\alpha}{2b} + \sqrt{\frac{\rho_H}{b}}\right)^2$ counterfeit goods and zero authentic goods. However, these pooling equilibria cannot be sustained in a viable market because there is no demand for authentic goods.

Case 2: $w_L < \frac{cp_A}{b - p_A} \le w_H$ $\left(\text{or equivalently}, \frac{bw_L}{w_L + c} < p_A \le \frac{bw_H}{w_H + c}\right)$

In this case, L-types always prefer to shop only for counterfeits, whereas the H-types' optimal choice depends on $\hat{k}$, the total number of goods they need to assemble. Formally, we have

$$TC^H_{min} = \begin{cases} b\hat{k} & \text{if } \hat{k} < \frac{c}{b - p_A} \\ c + p_A\hat{k} & \text{if } \frac{c}{b - p_A} \le \hat{k} \le \frac{w_H}{p_A} \\ c + w_H + b\left(\hat{k} - \frac{w_H}{p_A}\right) & \text{if } \hat{k} > \frac{w_H}{p_A} \end{cases}$$

$$TC^L_{min} = b\hat{k}, \forall\ \hat{k} \ge 0.$$

Here, we always have $V^L_{max} = V^*_C$. Therefore, whether in a pooling or a separating equilibrium, the brand only sells to the H-types. In a pooling equilibrium where the H-types buy at least some authentic goods, the L-types will buy more than $\frac{\alpha^2}{4b^2}$ counterfeit goods to achieve some symbolic utility, whereas the L-types will only buy exactly $\frac{\alpha^2}{4b^2}$ counterfeit goods in a separating equilibrium b/c they gain zero symbolic utility. Hence we focus on only separating equilibria in this case based on our selection criterion.

To further simplify the expressions of V^H_{max} in this case, we consider the following 3 subcases as α varies.

(i) Case 2.1 ($V^H_{max} = V^*_C$)

Figure A3 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

According to (A28), a separating equilibrium may exist in this case only if the following two conditions hold:

$$TC^H_{min}(n^{sep}) \le \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b} \tag{A30}$$

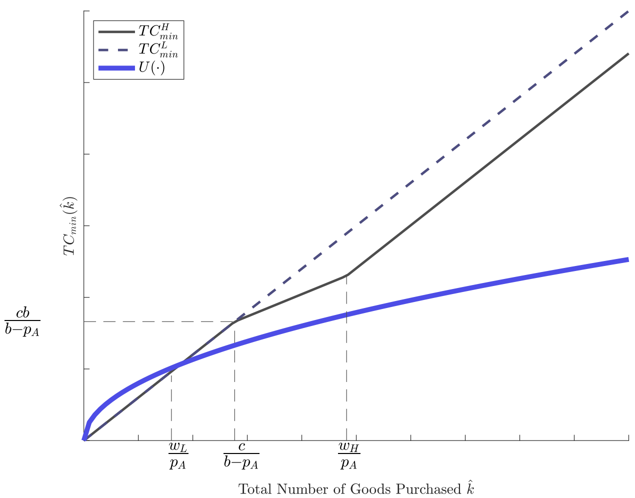


FIGURE A3 Case 2.1 $\left(V^H_{max} = V^L_{max} = V^*_C = \frac{\alpha^2}{4b}\right)$ [Color figure can be viewed at wileyonlinelibrary.com]

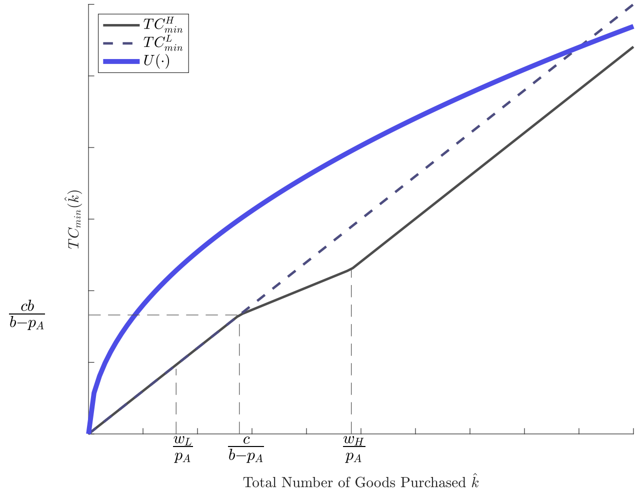


FIGURE A4 Case 2.2 ($V^H_{max} = V^*_A$, $V^L_{max} = V^*_C$) [Color figure can be viewed at wileyonlinelibrary.com]

$$TC^L_{min}(n^{sep}) \geq \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b} \tag{A31}$$

Condition (A31) implies that

$$n^{sep} \geq \left(\frac{\alpha}{2b} + \sqrt{\frac{1}{b}}\right)^2 \equiv \underline{n}^{\,sep}. \tag{A32}$$

Since $TC^H_{min}(\hat{k}) \leq TC^L_{min}(\hat{k})$ for any $\hat{k} > 0$, we know that condition (A30) is satisfied at $n^{sep} = \underline{n}^{\,sep}$. Hence we focus on the Riley outcome where the H-types consume the least amount of goods necessary to separate. Following the same logic in Case 1, we rule out the possibility of $TC^H_{min} = TC^L_{min} = bn^{sep}$ in any separating equilibrium based on our tie-breaking rules. Therefore, in a separating equilibrium, we must also have $\underline{n}^{\,sep} \geq \frac{c}{b - p_A}$, or equivalently, $p_A \leq b - \frac{c}{\left(\frac{\alpha}{2b} + \sqrt{\frac{1}{b}}\right)^2}$.

(ii) Case 2.2 ($V^H_{max} = V^*_A$)

Figure A4 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

According to (A28), a separating equilibrium may exist in this case only if condition (A31) as well as the following condition hold:

$$TC^H_{min}(n^{sep}) = w_H + c + b\left(n^{sep} - \frac{w_H}{p_A}\right) \leq \alpha\sqrt{n^{sep}} + 1 - \alpha\sqrt{\frac{w_H}{p_A}} + w_H + c \tag{A33}$$

We can re-write inequality (A33) as $n^{sep} \in \mathcal{S} \equiv \left[\frac{\alpha - \sqrt{\alpha^2 - 4b\left(\alpha\sqrt{\frac{w_H}{p_A}} - 1 - \frac{bw_H}{p_A}\right)}}{2b}, \frac{\alpha + \sqrt{\alpha^2 - 4b\left(\alpha\sqrt{\frac{w_H}{p_A}} - 1 - \frac{bw_H}{p_A}\right)}}{2b}\right]$. It is easy to see that $\underline{n}^{\,sep} = \left(\frac{\alpha}{2b} + \sqrt{\frac{1}{b}}\right)^2 \in \mathcal{S}$. Hence condition (A33) is satisfied at $n^{sep} = \underline{n}^{sep}$.

(iii) Case 2.3 ($V^H_{max} = V^*_M$)

Figure A5 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

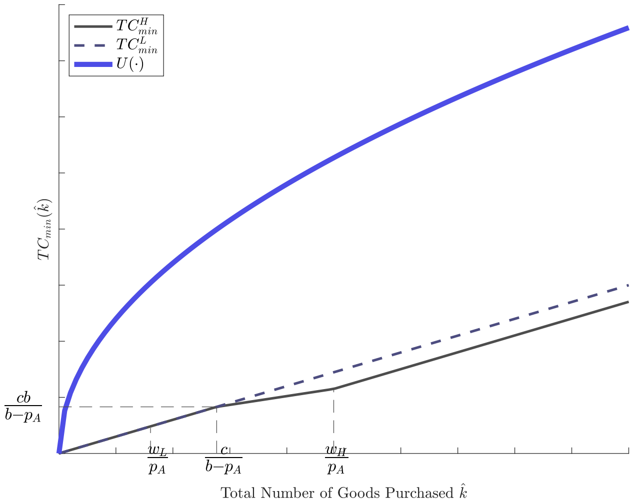


FIGURE A5 Case 2.3 $\left(V^H_{max} = V^*_M = \frac{\alpha^2}{4b} - w_H - c + \frac{w_H}{p_A}\right)$ [Color figure can be viewed at wileyonlinelibrary.com]

According to (A28), a separating equilibrium may exist in this case only if condition (A31) as well as the following condition hold:

$$TC^H_{min}(n^{sep}) = w_H + c + b\left(n^{sep} - \frac{w_H}{p_A}\right) \leq \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b} + w_H + c - \frac{bw_H}{p_A}, \tag{A34}$$

which reduces to $bn^{sep} \leq \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b}$. Combining conditions (A31) and (A34), we have only one solution that satisfies both conditions, that is, $n^{sep} = \underline{n}^{sep} = \left(\frac{\alpha}{2b} + \sqrt{\frac{1}{b}}\right)^2$. In this separating equilibrium, the L-types are indifferent between buying $\underline{n}^{sep}$ and buying $\frac{\alpha^2}{4b^2}(<\underline{n}^{sep})$ counterfeit goods, whereas the H-types are also indifferent but their basket contains some authentic goods. Hence based on our tie-breaking rules, the H-types will signal their status by purchasing a total of $\underline{n}^{sep}$ goods, authentic and counterfeits combined, whereas the L-types will choose not to mimic by purchasing only $\frac{\alpha^2}{4b^2}$ counterfeit goods.

Case 3: $\frac{cp_A}{b-p_A} \leq w_L < w_H$ $\left(\text{or equivalently, } p_A \leq \frac{bw_L}{w_L + c}\right)$

In this case, both the L-types and H-types' optimal portfolio depends on $\hat{k}$, the total number of goods they need to assemble. Formally, we have

$$TC^H_{min} = \begin{cases} b\hat{k} & \text{if } \hat{k} < \frac{c}{b-p_A} \\ c + p_A\hat{k} & \text{if } \frac{c}{b-p_A} \leq \hat{k} \leq \frac{w_H}{p_A} \\ c + w_H + b\left(\hat{k} - \frac{w_H}{p_A}\right) & \text{if } \hat{k} > \frac{w_H}{p_A} \end{cases}$$

and

$$TC^L_{min} = \begin{cases} b\hat{k} & \text{if } \hat{k} < \frac{c}{b-p_A} \\ c + p_A\hat{k} & \text{if } \frac{c}{b-p_A} \leq \hat{k} \leq \frac{w_L}{p_A}. \\ c + w_L + b\left(\hat{k} - \frac{w_L}{p_A}\right) & \text{if } \hat{k} > \frac{w_L}{p_A} \end{cases}$$

To further simplify the expressions of V^H_{max} and V^L_{max} in this case, we consider the following 5 subcases as α varies.

(i) Case 3.1 $(V^L_{max} = V^H_{max} = V^*_C)$

FIGURE A6 Case 3.1 $\left(V^H_{max} = V^L_{max} = V^*_C = \frac{\alpha^2}{4b}\right)$ [Color figure can be viewed at wileyonlinelibrary.com]

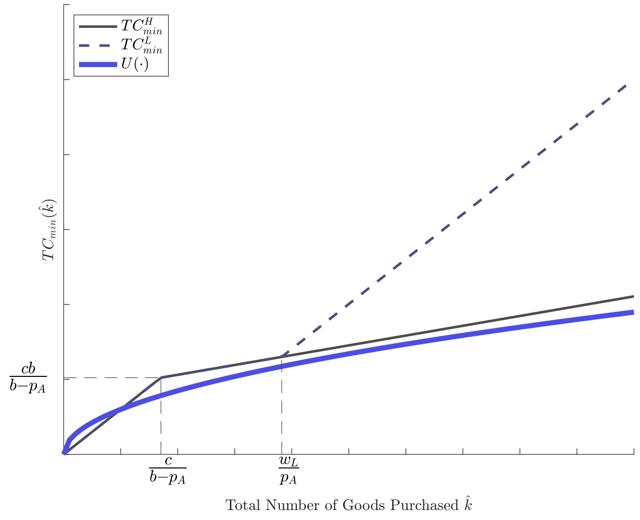


Figure A6 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

Similar to Case 2.1, a separating equilibrium may exist in this case only if conditions (A30) and (A31) hold. Based on our tie-breaking rules, we can rule out the possibility of $TC^H_{min} = TC^L_{min}$ in any separating equilibrium. Therefore, our tie-breaking rule and condition (A31) imply that

$$n^{sep} \geq \underline{n}^{sep} = \left(\frac{\alpha + \sqrt{4b\left(1 + \varepsilon - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2, \text{ and} \tag{A35}$$

$$\underline{n}^{sep} > \frac{w_L}{p_A}. \tag{A36}$$

It is straightforward to see that inequality (A36) is satisfied given that $w_L < 1 - c$. Since $TC^H_{min}(\hat{k}) \leq TC^L_{min}(\hat{k})$ for any $\hat{k} > 0$, we know that condition (A30) is satisfied at $n^{sep} = \underline{n}^{sep}$ given by (A35). Hence we focus on the Riley outcome where the H-types consume the least amount of goods necessary to separate, that is, $k_A(H) + k_C(H) = \underline{n}^{sep}$.

On the other hand, there exist pooling equilibria where both the H-types and L-types purchase a total of n^{pool} luxury goods, where

$$\left(\frac{\alpha}{2b}\right)^2 \leq n^{pool} \leq \begin{cases} \left(\frac{\alpha + \sqrt{4b\left(\rho_H - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2 & \text{if } p_A < \frac{\alpha\sqrt{w_L}}{w_L - (\rho_H - c) - \frac{\alpha^2}{4b}} \text{ or } w_L \leq \rho_H - c + \frac{\alpha^2}{4b} \\ \left(\frac{\alpha + \sqrt{\alpha^2 + 4p_A\left(\frac{\alpha^2}{4b} + \rho_H - c\right)}}{2p_A}\right)^2 & \text{if } p_A \geq \frac{\alpha\sqrt{w_L}}{w_L - (\rho_H - c) - \frac{\alpha^2}{4b}} \text{ and } w_L > \rho_H - c + \frac{\alpha^2}{4b} \end{cases}.$$

Note that among those pooling equilibria, only those with $n^{pool} \geq \frac{c}{b - p_A}$ can be sustained in a viable market (where there are at least some demand for authentic goods).

(ii) Case 3.2 ($V^L_{max} = V^*_C$; $V^H_{max} = V^*_A$)

Figure A7 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

According to (A28), a separating equilibrium may exist in this case only if the following two conditions hold:

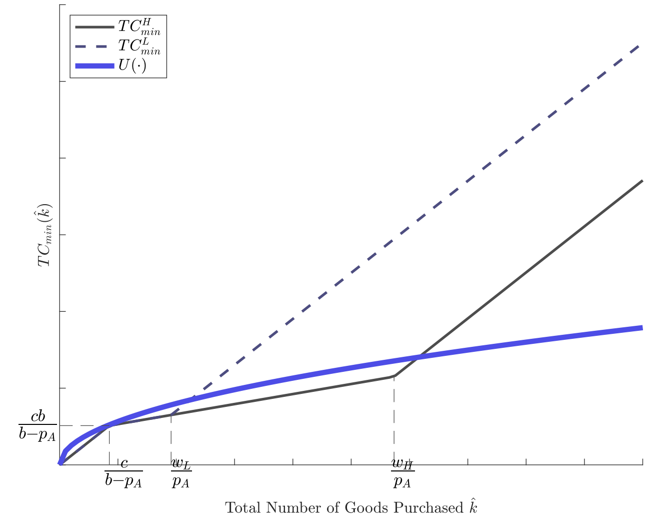


FIGURE A7 Case 3.2 ($V^L_{max} = V^*_C$, $V^H_{max} = V^*_A$) [Color figure can be viewed at wileyonlinelibrary.com]

$$w_H + c + b\left(n^{sep} - \frac{w_H}{p_A}\right) \leq \alpha\sqrt{n^{sep}} + 1 - \alpha\sqrt{\frac{w_H}{p_A}} + w_H + c, \tag{A37}$$

$$w_L + c + b\left(n^{sep} - \frac{w_L}{p_A}\right) \geq \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b}. \tag{A38}$$

We can rewrite condition (A37) as

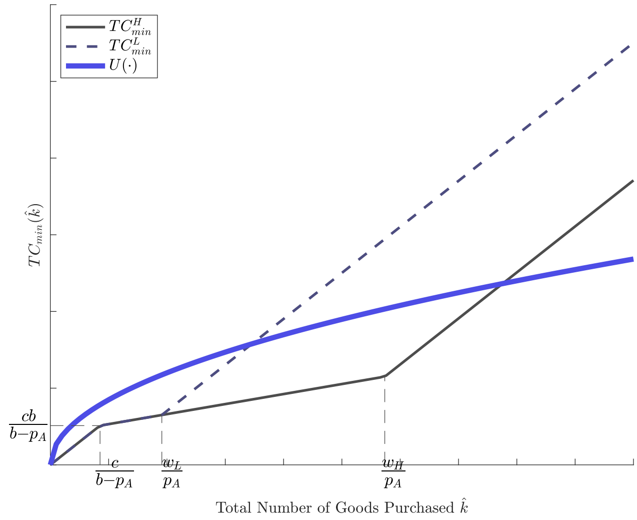
$$n^{sep} \in \mathcal{S} \equiv \left[\left(\frac{\alpha - \sqrt{\alpha^2 + 4b\left(\frac{bw_H}{p_A} + 1 - \alpha\sqrt{\frac{w_H}{p_A}}\right)}}{2b}\right)^2, \left(\frac{\alpha + \sqrt{\alpha^2 + 4b\left(1 + \frac{bw_H}{p_A} - \alpha\sqrt{\frac{w_H}{p_A}}\right)}}{2b}\right)^2\right].$$

Similar to Case 3.1, our tie-breaking rule and condition (A38) imply that $n^{sep} \geq \left(\frac{\alpha + \sqrt{4b\left(1 + \varepsilon - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2$. It is easy to see that $\left(\frac{\alpha + \sqrt{4b\left(1 + \varepsilon - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2 \in \mathcal{S}$. Therefore, the Riley outcome in this case is $n^{sep} = \underline{n}^{sep} = \left(\frac{\alpha + \sqrt{4b\left(1 + \varepsilon - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2$.

On the other hand, the pooling equilibrium outcome is the same as in Case 3.2. That is, both the H-types and the L-types purchase a total of n^{pool} goods, where

$$\left(\frac{\alpha}{2b}\right)^2 \leq n^{pool} \leq \begin{cases} \left(\frac{\alpha + \sqrt{4b\left(\rho_H - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2 & \text{if } p_A < \frac{\alpha\sqrt{w_L}}{w_L - (\rho_H - c) - \frac{\alpha^2}{4b}} \text{ or } w_L \leq \rho_H - c + \frac{\alpha^2}{4b} \\ \left(\frac{\alpha + \sqrt{\alpha^2 + 4p_A\left(\frac{\alpha^2}{4b} + \rho_H - c\right)}}{2p_A}\right)^2 & \text{if } p_A \geq \frac{\alpha\sqrt{w_L}}{w_L - (\rho_H - c) - \frac{\alpha^2}{4b}} \text{ and } w_L > \rho_H - c + \frac{\alpha^2}{4b} \end{cases}.$$

FIGURE A8 Case 3.3 ($V^L_{max} = V^*_A$, $V^H_{max} = V^*_A$) [Color figure can be viewed at wileyonlinelibrary.com]



(iii) Case 3.3 ($V^L_{max} = V^H_{max} = V^*_A$)

Figure A8 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

According to (A28), a separating equilibrium may exist in this case only if the following condition two conditions hold:

$$w_H + c + b\left(n^{sep} - \frac{w_H}{p_A}\right) \le \alpha\sqrt{n^{sep}} + 1 - \alpha\sqrt{\frac{w_H}{p_A}} + w_H + c, \tag{A39}$$

$$w_L + c + b\left(n^{sep} - \frac{w_L}{p_A}\right) \ge \alpha\sqrt{n^{sep}} + 1 - \alpha\sqrt{\frac{w_L}{p_A}} + w_L + c. \tag{A40}$$

Our tie-breaking rule and the above two conditions imply that

$$n^{sep} \in \left[\left(\frac{\alpha - \sqrt{\alpha^2 + 4b\left(\frac{bw_H}{p_A} + 1 - \alpha\sqrt{\frac{w_H}{p_A}}\right)}}{2b}\right)^2, \left(\frac{\alpha + \sqrt{\alpha^2 + 4b\left(1 + \frac{bw_H}{p_A} - \alpha\sqrt{\frac{w_H}{p_A}}\right)}}{2b}\right)^2\right], \tag{A41}$$

$$n^{sep} \ge \left(\frac{\alpha + \sqrt{\alpha^2 + 4b\left(\frac{bw_L}{p_A} + 1 + \varepsilon - \alpha\sqrt{\frac{w_L}{p_A}}\right)}}{2b}\right)^2. \tag{A42}$$

It is easy to see that the Riley outcome in this case is $\underline{n}^{sep} = \left(\frac{\alpha + \sqrt{\alpha^2 + 4b\left(\frac{bw_L}{p_A} + 1 + \varepsilon - \alpha\sqrt{\frac{w_L}{p_A}}\right)}}{2b}\right)^2$.

On the other hand, there exist pooling equilibria where both the H-types and L-types purchase a total of n^{pool} luxury goods, where $\frac{w_L}{p_A} \le n^{pool} \le \left(\frac{\alpha + \sqrt{4b\left(\rho_H - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2$.

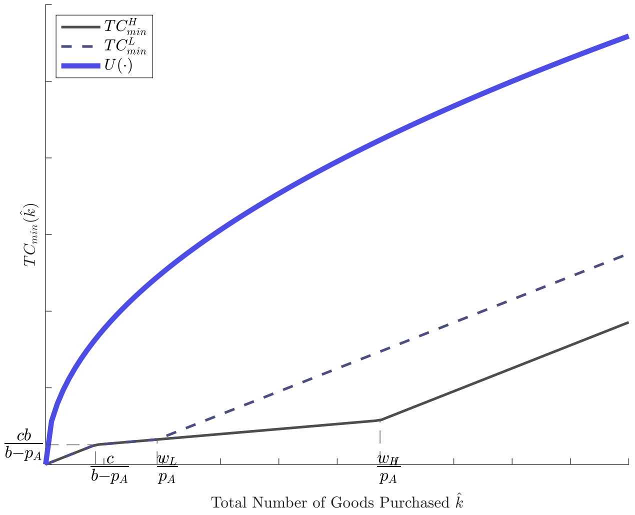


FIGURE A9 Case 3.4 ($V^L_{max} = V^*_M$, $V^H_{max} = V^*_A$) [Color figure can be viewed at wileyonlinelibrary.com]

(iv) Case 3.4 ($V^L_{max} = V^*_M$; $V^H_{max} = V^*_A$)

Figure A9 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

According to (A28), a separating equilibrium may exist in this case only if the following condition two conditions hold:

$$w_H + c + b\left(n^{sep} - \frac{w_H}{p_A}\right) \leq \alpha\sqrt{n^{sep}} + 1 - \alpha\sqrt{\frac{w_H}{p_A}} + w_H + c, \tag{A43}$$

$$w_L + c + b\left(n^{sep} - \frac{w_L}{p_A}\right) \geq \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b} + w_L + c - \frac{bw_L}{p_A}. \tag{A44}$$

Our tie-breaking rule and condition (A44) imply that $n^{sep} \geq \left(\frac{\alpha}{2b} + \sqrt{\frac{1+\varepsilon}{b}}\right)^2$. Let $n \in [\underline{n_0}, \overline{n_0}]$ denote the solution set to condition (A43). It is easy to see that $\overline{n_0} > \left(\frac{\alpha}{2b} + \sqrt{\frac{1+\varepsilon}{b}}\right)^2 > \underline{n_0}$. Therefore, the Riley outcome in this case is $\underline{n}^{sep} = \left(\frac{\alpha}{2b} + \sqrt{\frac{1+\varepsilon}{b}}\right)^2$.

On the other hand, there exist pooling equilibria where both the H-types and L-types purchase a total of n^{pool} luxury goods, where $\frac{w_L}{p_A} \leq n^{pool} \leq \left(\frac{\alpha + \sqrt{4b\left(\rho_H - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2$.

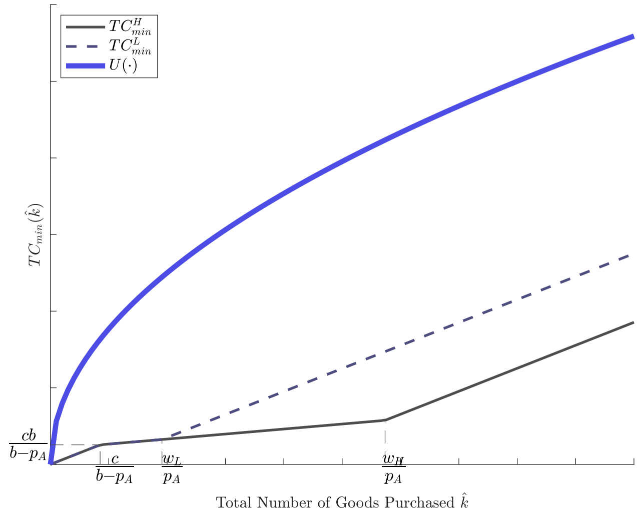
(v) Case 3.5 $\left(V^L_{max} = V^*_M, V^H_{max} = V^*_M\text{, or equivalently, } p_A \in \left(\frac{4b^2 w_H}{\alpha^2}, \frac{bw_L}{w_L + c}\right]\right)$

Figure A10 illustrates how TC^H_{min}, TC^L_{min}, and $U(\cdot)$ vary w.r.t. $\hat{k}$ in this case.

According to (A28), a separating equilibrium may exist in this case only if the following condition two conditions hold:

$$TC^H_{min}(n^{sep}) = w_H + c + b\left(n^{sep} - \frac{w_H}{p_A}\right) \leq \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b} + w_H + c - \frac{bw_H}{p_A}, \tag{A45}$$

FIGURE A10 Case 3.5 ($V_{max}^{H} = V_M^*, V_{max}^{L} = V_M^*$) [Color figure can be viewed at wileyonlinelibrary.com]



$$TC_{min}^{L}(n^{sep}) = w_L + c + b\left(n^{sep} - \frac{w_L}{p_A}\right) \geq \alpha\sqrt{n^{sep}} + 1 - \frac{\alpha^2}{4b} + w_L + c - \frac{bw_L}{p_A}. \tag{A46}$$

Re-arranging the inequalities, we can derive that only one solution that satisfies both conditions, that is, $n^{sep} = \underline{n}^{sep} = \left(\frac{\alpha}{2b} + \sqrt{\frac{1}{b}}\right)^2$. In this separating equilibrium, both the H-types and the L-types are indifferent between buying $\underline{n}^{sep}$ goods, authentic and counterfeit combined, and buying $\frac{\alpha^2}{4b^2}(<\underline{n}^{sep})$ counterfeit goods. Based on our tie-breaking rules, the L-types will mimic the H-types when indifferent because their basket could include authentic goods. In other words, there is no separating equilibrium in this case.

On the other hand, there exist pooling equilibria where both the H-types and L-types purchase a total of $\left(\frac{\alpha}{2b}\right)^2 \leq n^{pool} \leq \left(\frac{\alpha}{2b} + \sqrt{\frac{\rho_H}{b}}\right)^2$ luxury goods, including both authentic and counterfeit goods.

Firm's optimization problem

Given the consumer behavior discussed above, a luxury brand chooses its price p_A and product line length n to maximize its total profit. Note that $\underline{n}^{sep} \geq \left(\sqrt{\frac{1}{b}} + \frac{\alpha}{2b}\right)^2$ in any of the above three cases. Therefore, if the limit of product line length N is below $\left(\sqrt{\frac{1}{b}} + \frac{\alpha}{2b}\right)^2$, there does not exist any separating equilibrium. We proceed to show that, first, as α converges to 0, the equilibrium outcomes converge to our results displayed in the benchmark model; Second, when α is intermediate $\left(\underline{\alpha} \leq \alpha \leq \sqrt{\frac{4bw_H}{w_L}(w_L + c)}\right)$, the parameter range that supports a profit-maximizing separating equilibrium shrinks compared to that in the benchmark model. Finally, when α is sufficiently large $\left(\alpha > \sqrt{\frac{4bw_H}{w_L}(w_L + c)}\right)$, the profit-maximizing equilibrium is always a pooling equilibrium.

- When α is sufficiently small ($\alpha < \underline{\alpha}$)

Here $\underline{\alpha} \equiv \min\left\{\frac{4bc\,\underline{p}_A}{b - \underline{p}_A}, 4b^2\left(\sqrt{\frac{w_H}{\underline{p}_A}} - \sqrt{\frac{w_H}{\underline{p}_A} - \frac{c + w_H}{b}}\right)^2\right\}$ *where* $\underline{p}_A = \frac{bw_L}{bN - (1 + \varepsilon - w_L - c)}$ *is the minimum level of* p_A *that could possibly support a separating equilibrium given* N.[17] *In this case,* $V_{max}^{H} = V_C^*$, *that is, only Case 2.1 and Case 3.1 are admissible. Recall that a separating equilibrium in Case 2.1 requires that* $\underline{n}^{sep} = \left(\sqrt{\frac{1}{b}} + \frac{\alpha}{2b}\right)^2$

and $p_A \in \left(\frac{bw_L}{w_L+c}, \min\left\{b - \frac{c}{\left(\sqrt{\frac{1}{b}}+\frac{\alpha}{2b}\right)^2}, \frac{bw_H}{w_H+c}\right\}\right]$; *A separating equilibrium in Case 3.1 requires that* $\underline{n}^{sep} = \left(\frac{\alpha + \sqrt{4b\left(1+\varepsilon - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2$ *and* $p_A \le \frac{bw_L}{w_L+c}$. *We can thus derive the optimal price* (p_A^*) *and product line length* (n^*) *that supports a separating equilibrium as follows:*

$$p_A^* = \begin{cases} b - \frac{c}{\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2} & \text{if } c > b\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2 - w_H \\ \frac{w_H}{\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2} & \text{if } c \in \left(b\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2\left(\frac{w_L}{w_H}\right) - w_L, b\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2 - w_H\right] \\ p_0 & \text{if } c \in \left[\frac{bNw_L}{w_H} - \frac{(2b\sqrt{N}-\alpha)^2}{4b} + 1 + \varepsilon - w_L, b\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2\left(\frac{w_L}{w_H}\right) - w_L\right] \\ \frac{bw_L}{\frac{(2b\sqrt{N}-\alpha)^2}{4b} - (1+\varepsilon - w_L - c)} & \text{if } c < \frac{bNw_L}{w_H} - \frac{(2b\sqrt{N}-\alpha)^2}{4b} + 1 + \varepsilon - w_L \end{cases} \tag{A47}$$

and

$$n^* = \begin{cases} \left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2 & \text{if } c > b\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2\left(\frac{w_L}{w_H}\right) - w_L \\ \frac{w_H}{p_0} & \text{if } c \in \left[\frac{bNw_L}{w_H} - \frac{(2b\sqrt{N}-\alpha)^2}{4b} + 1 + \varepsilon - w_L, b\left(\frac{\alpha}{2b}+\sqrt{\frac{1}{b}}\right)^2\left(\frac{w_L}{w_H}\right) - w_L\right], \\ N & \text{if } c < \frac{bNw_L}{w_H} - \frac{(2b\sqrt{N}-\alpha)^2}{4b} + 1 + \varepsilon - w_L \end{cases} \tag{A48}$$

where $p_A = p_0$ solves the following equation

$$p_A\left(\frac{\alpha + \sqrt{4b\left(1+\varepsilon - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2 = w_H. \tag{A49}$$

We can show that there exists a unique solution $p_A = p_0 \in \left(0, \frac{bw_L}{w_L+c}\right)$ that satisfies equation (A49). This is because the left-hand-side (LHS) of equation (A49) monotonically increases in p_A and we also have $LHS < RHS$ when $p_A = 0$ and $LHS > RHS$ when $p_A = \frac{bw_L}{w_L+c}$. In a separating equilibrium, the authentic brand obtains a maximum profit of

$$\pi^{sep*} = \rho_H \min\left\{b\left(\sqrt{\frac{1}{b}}+\frac{\alpha}{2b}\right)^2 - c, w_H\right\} \tag{A50}$$

by selling only to the H-types. Also note that, as α approaches zero, the optimal price and product line given by (A47) and (A48) (as well as the profit) converge to those in the benchmark model characterized by (A19) and (A20).

On the other hand, the brand may obtain higher profit from facilitating a (Case 3.1) pooling equilibrium where the authentic brand sells to both the H-types and L-types. Recall that in the candidate pooling equilibrium, both types of consumers purchase a total number of n^{pool} goods where

$$\left(\frac{\alpha}{2b}\right)^2 \le n^{pool} \le \begin{cases} \left(\dfrac{\alpha+\sqrt{4b\left(\rho_H - w_L - c + b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2 & \text{if } p_A < \dfrac{\alpha\sqrt{w_L}}{w_L-(\rho_H-c)-\frac{\alpha^2}{4b}} \text{ or } w_L \le \rho_H - c + \dfrac{\alpha^2}{4b} \\ \left(\dfrac{\alpha+\sqrt{\alpha^2+4p_A\left(\frac{\alpha^2}{4b}+\rho_H-c\right)}}{2p_A}\right)^2 & \text{if } p_A \ge \dfrac{\alpha\sqrt{w_L}}{w_L-(\rho_H-c)-\frac{\alpha^2}{4b}} \text{ and } w_L > \rho_H - c + \dfrac{\alpha^2}{4b} \end{cases}$$

$$\text{s. t.}\quad n^{pool} \ge \frac{c}{b-p_A}$$

$$p_A \le \frac{bw_L}{w_L+c}.$$

We can derive the optimal price (p_A^*) and product line length (n^*) that supports a pooling equilibrium as follows:

$$p_A^* = \begin{cases} \max\left\{\tilde{p}_0, \dfrac{bNw_L}{\frac{(2b\sqrt{N}-\alpha)^2}{4b}-(\rho_H-w_L-c)}\right\} & \text{if } w_L < \rho_H - c + \frac{\alpha^2}{4b}, w_H < n_1\left(\frac{w_L}{w_L+c}\right) \\ \dfrac{bw_L}{w_L+c} & \text{if } w_L < \rho_H - c + \frac{\alpha^2}{4b}, w_H \ge n_1\left(\frac{w_L}{w_L+c}\right), \\ p_1 & \text{if } w_L \ge \rho_H - c + \frac{\alpha^2}{4b} \end{cases} \tag{A51}$$

$$n^* = \begin{cases} \max\{1, n_1\} & \text{if } w_H \ge n_1\left(\frac{bw_L}{w_L+c}\right) \\ \max\{1, \min\{N, n_2\}\} & \text{if } w_H < n_1\left(\frac{bw_L}{w_L+c}\right) \end{cases}, \tag{A52}$$

where $p_A = \hat{p}_0$ solves the following equation:

$$p_A\left(\frac{\alpha+\sqrt{4b\left(\rho_H-w_L-c+b\left(\frac{w_L}{p_A}\right)\right)}}{2b}\right)^2 = w_H$$

and $p_A = p_1$ solves the following equation:

$$\frac{c}{b-p_A} = \left(\frac{\alpha+\sqrt{\alpha^2+4p_A\left(\frac{\alpha^2}{4b}+\rho_H-c\right)}}{2p_A}\right)^2.$$

We also define $n_1 := \left(\dfrac{\alpha+\sqrt{\alpha^2+4p_1\left(\frac{\alpha^2}{4b}+\rho_H-c\right)}}{2p_1}\right)^2$ and $n_2 := \left(\dfrac{\alpha+\sqrt{4b\left(\rho_H-w_L-c+b\left(\frac{w_L}{\tilde{p}_0}\right)\right)}}{2b}\right)^2$.

In a pooling equilibrium, the luxury brand obtains a maximum profit of

$$\pi^{*pool} = \begin{cases} \rho_H - c + \frac{\alpha^2}{4b} & \text{if } w_L > \rho_H - c + \frac{\alpha^2}{4b} \\ \rho_H n_1\left(\frac{bw_L}{w_L+c}\right) + (1-\rho_H)w_L & \text{if } w_L \le \rho_H - c + \frac{\alpha^2}{4b}, w_H \ge n_1\left(\frac{bw_L}{w_L+c}\right). \\ \rho_H w_H + (1-\rho_H)w_L & \text{if } w_L \le \rho_H - c + \frac{\alpha^2}{4b}, w_H < n_1\left(\frac{bw_L}{w_L+c}\right) \end{cases}$$

The firm obtains more profit from facilitating a separating equilibrium than a pooling equilibrium if and only if $\pi^{sep*} > \pi^{pool*}$. Again, note that as α approaches zero, the optimal price and product line given by (A51) and (A52) as well as the maximum profit in a pooling equilibrium converge back to those in the benchmark model characterized by (A23) and (A24). Hence the parameter range that supports a separating equilibrium should converge to that in the benchmark as well.[18]

- When α is intermediate $\left(\underline{\alpha} \leq \alpha \leq \sqrt{\frac{4bw_H}{w_L}(w_L + c)}\right)$

In this case, we have Cases 2.1, 2.2, 3.1, and 3.2 that are admissible. Note that in those cases, $V_{max}^L = V_C^*$ and thus the L-types do not purchase any authentic goods in a separating equilibrium, that is, $k_A^*(L) = 0$. On the other hand, as α increases, the H-types need to assemble a sufficient amount of products by cross-shopping from both authentic and counterfeit store to prevent the L-types from mimicking. That is, $k_A^*(H) > 0$, $k_C^*(H) > 0$. It is also easy to see that as α increases, the region of parameter range that supports a separating equilibrium will shrink for two reasons. First, the gap between the product line length that supports a separating and a pooling equilibrium ($n^{sep} - n^{pool}$) increases with α in all those cases. Hence for a given N, as α increases, it is more likely that only pooling equilibrium will survive. Second, even when N is sufficiently large that both pooling and separating equilibria exist, the maximum surplus a brand can extract from both the H-types and the L-types in a pooling equilibrium also increases with α while there is an upper limit to that surplus a brand can extract from the H-types in a separating equilibrium, that is w_H. Hence as α increases, the profit-maximizing equilibrium with the leanest product line will more likely be a pooling equilibrium. It will be worthwhile for the brand to facilitate a separating equilibrium by maintaining a long enough product line only when w_H and ρ_H is sufficiently large as α increases.

- When α is sufficiently large $\left(\alpha > \sqrt{\frac{4bw_H}{w_L}(w_L + c)}\right)$

In this case, we have Cases 2.3, 3.3, 3.4, and 3.5 that are admissible. Recall that in Case 3.5 $\left(p_A \in \left(\frac{4b^2 w_H}{\alpha^2}, \frac{bw_L}{w_L + c}\right]\right)$, the luxury brand always obtains a maximum profit of $\pi = \rho_H w_H + (1 - \rho_H) w_L$ by maintaining a pooling equilibrium. First, this is clearly the maximum profit a brand could obtain from any equilibrium outcome. Second, we know from our earlier discussions that the shortest n possible to facilitate any separating equilibrium is at least $\left(\frac{\alpha^2}{4b^2} + \sqrt{\frac{1}{b}}\right)^2$. Thus, by facilitating a (Case 3.5) pooling equilibrium with $n^* = \frac{\alpha^2}{4b^2}$, a brand obtains the maximum profit with the leanest product line. To minimize the demand for counterfeits, the brand sets the price at its lowest[19] $p_A^* = \frac{4b^2 w_H}{\alpha^2}$ so that in this pooling equilibrium, the H-types buy all available authentic goods and the L-types buy a combination of authentic and counterfeit goods to match the same number of H-types purchase. Formally, we have

$$k_A^*(H) = \frac{\alpha^2}{4b^2}, \quad k_C^*(H) = 0, \tag{A53}$$

$$k_A^*(L) = \left(\frac{w_L}{w_H}\right)\frac{\alpha}{4b^2}, k_C^*(L) = \frac{\alpha^2}{4b}\left(1 - \frac{w_L}{w_H}\right). \tag{A54}$$

In this case, even without symbolic utility, the brand can extract the maximum surplus from both types simply because the sufficiently large functional utility itself motivates both types of consumers to purchase as many luxury goods as are available. □