Poaching Workers in a Supply Chain: Enemy from Within?

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Poaching is the prevalent practice of recruiting workers who are already employed elsewhere. Economy-wide, it represents the primary mode by which workers flow directly from one firm to another. In this paper, we examine the interaction between the flow of goods and the flow of workers (via poaching) between firms linked in a supply chain. We find that the direction of worker poaching between supply chain partners can run counter to classical labor economics results. Specifically, in a supply chain, the *less* productive firm may offer its workers higher wages and poach workers from the *more* productive firm. We also find that worker flows accomplished via poaching impact supply chain management. First, we find that the identity of supply chain bottleneck may depend on whether poaching is available as a means to accomplish worker flows. Next, we find that the benefits of worker flows between supply chain partners in some cases outweigh the costs incurred, so no-poaching agreements would be worse for some supply chains. This net benefit of poaching between members of a supply chain is robust to the presence of outside labor market competitors as long as the competitors do not have high productivity. Thus, poaching workers from supply chain partners can often increase the benefit of operating in manufacturing hubs for all supply chain members.

Key words: worker poaching, passive recruiting, supply chain, capacity management, staffing

1. Introduction

To escape the costs of finding, recruiting, hiring, and training workers, firms often resort to poaching workers. Worker poaching is the practice of actively recruiting workers who are already employed elsewhere. Examples of poaching are prevalent in many industries, including tech, consulting, insurance, finance, law, services, manufacturing, and customer support. In fact, Fallick and Fleischman (2001) found that in the late 1990's, poaching accounted for about 80% of all job-to-job transitions. Only one-fifth of all survey respondents who switched jobs were even searching for a new job. More recently, Jones, et al. (2011) surveyed managers at firms that received stimulus funds through the American Recovery and Reinvestment Act, a legislation designed in part to combat the escalating unemployment crisis during the Great Recession. The authors found that more stimulus funds had actually been spent poaching workers instead of hiring and training the unemployed.

The practice of poaching is becoming even easier as social and professional networks and job search websites develop innovative recruiting solutions. LinkedIn, GlassDoor, CareerBuilder, and Monster all offer services aimed at finding the best candidates to fulfill a firm's needs. They can even provide insight into which workers are most likely to accept an offer and the salary range that would be required to entice workers to leave their current employer. Some even offer a platform to communicate with prospects confidentially. With these novel approaches to recruiting, poaching is likely to become even more widely practiced.

With the prevalence of poaching, it is to be expected among competitors or peers. However, it is also quite prevalent among partners of the same value chain, or supply chain. While poaching from a partner could be perceived as damaging to the business relationship, the economic impact of such a practice is an open question.

We argue in this paper that poaching between members of a supply chain has the potential to actually improve supply chain performance and benefit both suppliers and customers. A simple example of potential benefits of poaching between supply chain partners comes from the shale oil and gas industry, which has become a key supplier for the energy, chemicals, and plastics industries. Shale oil and gas exploration, drilling, and extraction are very labor intense; and the young industry has expanded rapidly, forcing firms to employ very aggressive recruiting strategies. As evidence, the median starting salary of recent graduates from the South Dakota School of Mines and Technology is higher than that of recent graduates from Stanford, Harvard, or any other Ivy League school (PayScale Inc. 2014). To satisfy their skilled labor requirements, shale extraction firms resort to poaching machinists, mechanics, and those with experience in handling chemicals and operating industrial equipment from their customers in plastics and chemicals (Olson et al. 2013). The majority of the workers currently employed in shale oil and gas came from other industries, and many of them were poached. This new industry that has acquired so many workers via poaching is having a large economic impact: BCG estimates that the average household is saving \$425 - \$725 on their energy costs annually (Boston Consulting Group 2013). Shale's customers in energy, plastics, and chemicals are investing hundreds of billions of dollars in capacity expansion to handle the higher availability of their process input (despite their lack of skilled labor, see (Olson et al. 2013)).

Similar examples come from other industries. In pharmaceutical research and development, small pharma firms poach workers from and sell promising drug candidates to large pharmaceutical firms. Many of the European and American survivors of the Ebola virus disease during the 2014 outbreak in West Africa can thank their survival to small pharma firms' experimental drug candidates developed by scientists and engineers, some of whom had been poached from medium and large pharma firms. We also see small tech startups poaching workers from tech giants and then selling developed technologies to the same giants. And firms poach experts from the same firms they contract for consulting.

In other examples, poaching between supply chain members is shunned. Apple, for example, allegedly entered into illegal no-poaching agreements with its main suppliers and customers (including Intel, Foxconn, Best Buy, Comp USA, Mac Zone, and CDW), with its business partners, and even with its competitors (Campbell 2014). Thus, we see some discrepancy: shale oil firms commonly poach from their customers, but Apple allegedly entered into no-poaching agreements with its suppliers and customers. In this paper, we explore the trade-offs involved in poaching from supply chain partners. In doing so, we provide a partial answer to a question appearing in the New York Times, "Is it ever appropriate to agree with a 'partner' company not to poach an employee (Sorkin 2014)?" We go a step further and answer the less intuitive question, "Is it ever appropriate to encourage supply chain partners to poach your workers?" Why is it that poaching provides universal benefit in the shale oil and gas industry but is so damaging to Apple that they enter into illegal no-poaching agreements to escape the costs of poaching?

We first turn to the labor economics literature for insight, where, because of its prevalence, poaching has been widely studied. Firms in poaching environments experience benefits such as: availability of labor (Becker 1962, Katz and Ziderman 1990), labor market risk sharing (Krugman 1991, Rosenthal and Strange 2004), and spillovers (Rosenthal and Strange 2004, Serafinelli 2013, Tambe and Hitt 2014). On the downside, firms must often pay the costs associated with both wage competition and high worker turnover rates. To escape the costs of poaching, firms often require new hires to sign non-compete agreements to minimize spillovers (Marx et al. 2009, Davis 2013), or firms enter into illegal no-poaching agreements (Demirbag et al. 2012).

Throughout the labor economics literature, some intuitive insights are commonly found: (i) firms with higher labor productivity poach from firms with lower labor productivity,

(ii) firms with higher labor productivity offer workers higher wages than firms with lower labor productivity, and (iii) firms often find that the costs of poaching exceed the benefits. In many examples of poaching between supply chain members, however, we observe the exact opposite: the firms with lower labor productivity poach from firms with higher labor productivity, and they offer higher wages. Furthermore, in some cases, all of the supply chain members receive an apparent net benefit from the worker flows accomplished via poaching. Our first contribution in this paper is that we explain why and under what conditions these classical insights from labor economics fail to describe poaching between supply chain members. These counter-intuitive equilibria fall into a class we term Labor-Sharing Equilibria. In Labor-Sharing Equilibria, the supply chain bottleneck firm poaches workers from its supply chain partner. The supply chain partner must replace poached workers and hire additional workers to match the bottleneck's capacity.

We also show the impact of poaching on firm operations. The operations management literature has extensively studied capacity management. Some of this literature examines the interface between labor economics and capacity management (e.g., Milner and Pinker 2001, Gans et al. 2003, Aksin et al. 2007, and references therein). However, the literature generally treats capacity as a decision variable regarding units which can be purchased and sold. Yet, when capacity is driven by employees, capacity must either be trained or poached rather than purchased; and the costs associated with capacity acquisition via these two mechanisms are very different. Furthermore, human resources are decision makers. The existing literature has been silent about the balance of human resource acquisition via training and poaching.

We also add to the operations management literature examining the interaction of flows in a supply chain. The existing literature has studied the flow of goods from supplier to customer and the flow of information and cash from customer to supplier. In this paper, we explore the interaction between the flow of goods and the flow of workers: poaching is the mechanism by which workers flow between firms. When firms linked in a supply chain relationship poach workers from one another, there are consequences related to the flow of goods. For example, a supplier poaching workers from its customer may reduce the amount of the good the customer can purchase. Similarly, a customer poaching workers from its supplier may reduce the supplier's ability to produce. These additional consequences to poaching workers from a supply chain partner help explain the shortfall of the classical intuitive insights from labor economics. They also help explain the impact of poaching on operations.

One of our primary contributions regarding the impact of poaching on operations is related to supply chain capacity management. We find that the identity of the supply chain bottleneck depends on the outcome of equilibrium worker flows. Thus, our first contribution is practical: supply chain capacity management must consider poaching between members of the supply chain when identifying bottleneck resources.

We also show that, in some cases, suppliers or customers *should* allow their workers to be poached by their supply chain partner, and no-poaching agreements can actually hurt supply chain performance and firm profits.

Finally, we show the impact of poaching when supply chain partners operate in manufacturing hubs. Supply chain facility location and industrial agglomeration have been extensively studied in both the operations management and the agglomeration and urbanization economics literatures. The agglomeration and urbanization economics literature has empirically shown aggregate tendencies for firms to co-locate with suppliers and customers (e.g., Klier 1999, Smith and Florida 1994, Ellison and Glaeser 1997, and Krugman 1991). The operations management and urbanization economics literatures have shown that supply chains co-located in manufacturing hubs experience: savings via reduced logistical costs (see Daskin et al. 2005 and references therein), improved supply chain reaction time (e.g., Schonberger 1982 and Harrigan and Venables 2006), and superior environmental sustainability (Sarkis 2001). We show that poaching can actually enhance the value of operating in a manufacturing hub as long as there are no highly productive labor market competitors. However, highly productive labor market competitors can effectively shut down worker flows between supply chain partners.

We now proceed to present the model in Section 2. Then we'll present the solution in Section 3 and insights into how the interaction between the flow of goods and the flow of workers impacts firm operations in Section 4.

2. Model

We begin by providing an overview of the model. In our model, a supplier and a manufacturer interact in both production and labor decisions. Both the manufacturer and the supplier begin with their individual initial pool of workers. Each firm then attempts to poach the other firm's workers while trying to retain its own. After the outcome of the worker poaching and retention decisions is settled, the manufacturer: (i) hires additional workers (if needed) for production, and (ii) orders from the supplier. The supplier then: (i) hires additional workers (if needed) to produce the manufacturer's order, and (ii) produces the good for the manufacturer. Using the supplied good, the manufacturer produces the final good which is then sold in the final goods market. We divide the manufacturer-supplier interaction into three stages: (1) poaching and retention, (2) manufacturer hiring and production, and (3) supplier hiring and production. We next describe each of these stages in more detail.

2.1. Stage 1: Poaching and Retention

In this stage, the manufacturer and supplier start with an exogenous number of workers, denoted by Λ_i, $i = M, S$, where the subscript i refers to either the manufacturer, $i = M$, or the supplier, $i = S$. Workers are heterogeneous in the wage premium that must be offered to entice them to leave their current employer in favor of the supply chain partner. We refer to this heterogeneity as the *employment transfer cost* and we denote it by $\theta \in [0, \infty)$. We denote the number of workers of type $\theta' \leq \theta$ initially employed at firm i by $P_i(\theta)$ for $i = M, S$. In other words, $P_i(\cdot)$ represents the cumulative *population* density of workers initially employed at firm i. We assume that $P_i(\theta)$, $i = M, S$, is continuous for all $\theta > 0$.

Each firm's Stage 1 strategy is the retention wage to offer its own employees, $w_i^R(\theta)$, and the poaching wage to offer employees of the supply chain partner, $w_i^P(\theta)$. We denote each firm's Stage 1 strategy by $W_i(\theta) = \big(w_i^P(\theta), w_i^R(\theta)\big)$ and the set of both firms' Stage 1 strategies by $W(\theta) = (W_M, W_S)$. For notational simplicity, we often suppress the arguments and refer to $W_i(\theta)$ and $W(\theta)$ as W_i and W, respectively.

Each worker, thus, receives two offers: a retention wage offer from their current employer, firm i, and a poaching wage offer from their current employer's supply chain partner, firm j. If workers accept the poaching wage offer, they incur the employment transfer cost, θ. Thus, workers of type θ accept the offer that solves

$$\max\{w_i^R(\theta), w_j^P(\theta) - \theta\}.$$

Worker indifference is resolved in favor of the firm that is *willing* to offer the highest net benefit to the worker. If workers are indifferent and neither firm is willing to offer a higher wage than the other, workers accept the poaching wage offer. Workers also have an outside option (e.g., unemployment) to which they can move without incurring any employment transfer cost. The value of this outside option is equal to an exogenous reservation wage, w, common to all workers. The reservation wage represents the minimum wage that must be paid to keep the workers from preferring this outside option.

Based on these worker preferences, workers of type θ can be successfully poached by firm i if it is willing to offer its supply chain partner's (firm j's) workers a poaching wage at least as large as

$$\max\left\{w_j^R(\theta), w\right\} + \theta, \tag{1}$$

The term $\max\left\{w_j^R(\theta), w\right\}$, represents the worker's best available option between staying with their current employer and moving to the outside option. In a similar manner, workers of type θ can be successfully retained by firm i if it is willing to pay a retention wage strictly larger than

$$\max\left\{w_j^P(\theta) - \theta, w\right\}. \tag{2}$$

This approach to modeling poaching and retention is analogous to that used by Combes and Duranton (2006). Following Combes and Duranton (2006), we let each worker's employment transfer cost be common knowledge. Thus, firms' poaching and retention wage offers may depend on each worker's employment transfer cost.

We let $\eta_i^P(W)$ and $\eta_i^R(W)$ denote the number of workers poached and retained, respectively, by firm $i = M, S$ under strategies W. Thus, firm $i = M, S$ finishes the first stage with $N_i(W) = \eta_i^P(W) + \eta_i^R(W)$ workers. The state of the system at the conclusion of the first stage is determined by both firms' strategies and is captured by $N(W) = (N_M(W), N_S(W))$. Thus, we often refer to N as the *outcome* of poaching and retention. We let $\theta_i^P(W)$ and $\theta_i^R(W)$ denote the set of workers poached and retained, respectively, by firm i under strategies W. Thus, firm i successfully poaches all of its supply chain partner's workers of type $\theta \in \theta_i^P(W)$ and retains all of its own workers of type $\theta \in \theta_i^R(W)$. For simplicity, we often suppress the arguments of $N(W)$, $\eta_i^P(W)$, $\eta_i^R(W)$, $\theta_i^P(W)$, and $\theta_i^R(W)$ and denote them instead by N, η_i^P, η_i^R, θ_i^P, and θ_i^R, respectively. The profits for firm i under strategies W are given by the equation:

$$\pi_i(W) = V_i(N) - \int_{\theta_i^R} w_i^R(\theta)\,\mathrm{d}P_i(\theta) - \int_{\theta_i^P} w_i^P(\theta)\,\mathrm{d}P_j(\theta). \tag{3}$$

Here, $V_i(N)$ is firm i's continuation value and will be defined in the next two subsections. It captures the value of both firms' hiring and production quantity decisions, and it will depend on the outcome of this stage and on the model primitives (costs of production and hiring and the value obtained from selling the good). The second and third terms are the wages that are be paid to retained and poached workers, respectively, under strategies $w_i^P(\theta)$ and $w_i^R(\theta)$. We will further detail the strategic interaction between the two firms in Section 2.4.

2.2. Stage 2: Manufacturer Hiring and Production

In the second stage of the game, the manufacturer decides: (i) the number of additional workers to hire for production, denoted by η_M^H, and (ii) the quantity, q_M, of the supplied good to order from the supplier. The state of the game at the beginning of Stage 2 is captured by N. The manufacturer's Stage 2 strategy, then, is a pair of functions, $\eta_M^H(N)$ and $q_S(N)$, that give the manufacturer's number of hires and order quantity, respectively, for any state, N, of the game at the beginning of Stage 2. For simplicity, we suppress notation of the strategy's arguments; so we write η_M^H and q_M instead of $\eta_M^H(N)$ and $q_S(N)$, respectively.

Both hiring and production are driven by the manufacturer's potential revenues from selling its product in the final goods market. The manufacturer pays production cost, c_M, for each unit produced; pays the supplier p_S for each unit of the supplied good purchased; and receives p_F for each unit of the manufacturer's product sold in the final goods market. Thus, letting m_M denote the manufacturer's profit margin, we get $m_M = p_F - p_S - c_S$. We assume there is certain unlimited demand for the manufacturer's product in the final goods market. Later in this paper, we relax this assumption and allow for finite and uncertain demand for the manufacturer's product.

Production is limited by capacity, $K_M = k_M\left(N_M + \eta_M^H\right)$, where k_M is the manufacturer's individual worker productivity, and N_M is the number of workers poached and retained by the manufacturer. A capacity-constrained manufacturer may be able to increase sales by increasing capacity. Since we focus mostly on *changes* in capacity, we note that hiring η_M^H workers increases the manufacturer's capacity by $k_M\eta_M^H$. However, hiring is accompanied by a cost: new hires must be found, recruited, hired, trained, and paid. We capture all of these costs in the hiring cost function, $h(\cdot)$. We now describe the main structural assumptions made on the hiring cost function.

Assumption 1. *The hiring cost function satisfies the following properties:*

(a) $h(\cdot)$ *is a strictly convex and continuously differentiable function of the number of workers hired.*

(b) *The initial marginal hiring cost exceeds the workers' reservation wage:* $h'(0) > w$.

(c) *Each firm's labor productivity exceeds the initial marginal hiring cost, i.e.,* $k_i m_i > h'(0)$ *for* $i = M, S$.

The convexity assumption in Assumption 1(a) has been used extensively in the literature (e.g., Sargent 1978) and has been verified empirically (Blatter et al. 2012). Convexity also makes intuitive sense: to increase the number of applicants, firms offer higher wages. Furthermore, when hiring workers from a pool of potential employees, a firm would first choose the workers who best match the firm's requirements and are available at the lowest cost. As the pool of best-matched workers vanishes, firms must work marginally harder to either "train up" the next-best-matched workers or find additional better-matched workers. Before proceeding, we note (and will see in greater detail later) that in our model production is limited because of hiring cost convexity: hiring becomes prohibitively expensive beyond some point. Thus, our model does not apply to industries in which hiring cost convexity is violated.

Assumption 1(b) means that the cost of hiring the first worker is higher than the reservation wage: new hires must be paid at a rate at least as high as the reservation wage (or else they would prefer the outside option), and new hires must be found, recruited, hired, and trained. This assumption implies that firms would prefer retaining workers at the reservation wage over hiring workers during production.

Assumption 1(c) guarantees that both firms are willing to hire during production stages if doing so increases output. Before proceeding, we note that Assumptions 1(b) and 1(c) together imply that the labor productivity, or the financial benefit from a non-idle worker, exceeds the minimum wage required to retain a worker. Mathematically, $k_i m_i > w$.

Now that we have presented structural assumptions on the hiring cost function and have discussed the manufacturer's costs and benefits of production and hiring in the second stage, we mathematically state the manufacturer's problem in the second stage.

$$\begin{aligned} V_M(N) = \max_{q_M, \eta_M} \quad & m_M \min\{q_M, q_S^*(N, q_M)\} - h(\eta_M^H) \\ \text{subject to:} \quad & q_M \geq 0; \quad q_M \leq k_M(N_M + \eta_M^H); \quad \eta_M^H \geq 0 \end{aligned} \tag{4}$$

Here, the firm is maximizing its profits with respect to the order quantity, q_M, and the number of workers to hire for production, η_M^H. Neither of these can be negative, and order quantity is limited by the manufacturer's capacity.

The first term in the objective function represents the net benefit from producing and selling the final good. In the first term, the expression $q_S^*(N, q_M)$ represents the supplier's optimal production quantity in the third stage. The manufacturer cannot produce or sell more than the amount of the supplied good the supplier is willing to provide. The second term reflects the hiring costs incurred from hiring η_M^H additional workers.

2.3. Stage 3: Supplier Hiring and Production

In the final stage of the game, the supplier decides: (i) the number of additional hires, η_S^H, for production, and (ii) production quantity, q_S. By the beginning of Stage 3, the outcome of poaching and retention has been settled, and the manufacturer has placed an order and hired additional workers for production. The supplier-relevant state of the game at the beginning of Stage 3, then, is (N_S, q_M). The supplier's Stage 3 strategy consists of functions $\eta_S^H(N, q_M)$ and $q_S(N, q_M)$ that give the supplier's number of hires and production quantity, respectively, for any state of the system at the beginning of the third stage. Again, for notational simplicity, we use η_S^H and q_S instead of $\eta_S^H(N, q_M)$ and $q_S(N, q_M)$, respectively.

In Stage 3, the supplier faces constraints and trade-offs similar to the manufacturer's. The supplier is subject to its own capacity constraint: $K_S = k_S\left(N_S + \eta_S^H\right)$. And potential production revenues drive higher production quantities and increased capacity. However, increasing capacity by hiring workers becomes increasingly costly. As above, the total cost of hiring η_S^H workers is captured by $h(\eta_S^H)$. The supplier produces its product for the manufacturer, incurring unit production cost c_S, and sells the good to the manufacturer at per-unit price p_S. The supplier's profit margin is thus $m_S = p_S - c_S$. The manufacturer then produces and sells the final good.

Thus, the problem facing the supplier in the third stage is:

$$\begin{aligned} V_S(N, q_M) = \max_{q_S, \eta_S} \quad & p_S \min\{q_S, q_M\} - c_S q_S - h(\eta_S^H) \\ \text{subject to:} \quad & q_S \geq 0; \quad q_S \leq k_S(N_S + \eta_S^H); \quad \eta_S^H \geq 0. \end{aligned} \tag{5}$$

In this problem, the supplier maximizes its profits with respect to production quantity, q_S, and the number of workers to hire for final production, η_S^H. Both q_S and η_S^H are non-negative, and production quantity is limited by supplier capacity.

The supplier's objective function is in many ways similar to the manufacturer's. The first term in the objective function corresponds to profits made from producing and selling the good supplied to the manufacturer. The second term represents the wages and hiring costs associated with the additional η_S^H workers hired.

We see that the game's three stages highlight the trade-offs associated with poaching. The main direct cost of successfully poaching workers is a high wage offer required to entice workers to leave their current employer. An additional possible cost of poaching, however, is a lower capacity for the supply chain partner. Poaching workers may provide benefits, including possible reduction in hiring costs and increased production capacity. Successfully retaining workers may also lead to higher capacity and may reduce production-stage hiring costs. However, retained workers often require high wages to entice them to stay. Furthermore, every retained worker is a worker that is not poached by the supply chain partner; so successful retention may actually prevent the supply chain partner from further increasing its capacity.

2.4. Equilibrium Concept

Before proceeding to the analysis of the model, we present the equilibrium concept and solution approach. To characterize the sub-game perfect Nash equilibrium, we solve for the optimal behavior by analyzing the game going backwards in time. Thus, in the third stage, the supplier's optimal strategy, $\eta_S^{H*}(N, q_M)$ and $q_S^*(N, q_M)$, together give the maximizers of (5) for any given state of the game, (N, q_M), at the beginning of Stage 3. We use the notation η_S^{H*} and q_S^* to refer to the outcomes of optimal behavior starting from a certain point in the game. The arguments of η_S^{H*} and q_S^* are used to indicate the point in the game under consideration. Thus, $\eta_S^{H*}(N)$ and $q_S^*(N)$ are used to express the equilibrium outcome of the supplier's optimal actions in the sub-game beginning in Stage 2 when the state is N. Similarly, η_S^{H*} and q_S^* (without arguments) are used to indicate the equilibrium outcome of the supplier's third-stage optimal strategies with all players behaving optimally in all stages of the game.

In the second stage, the manufacturer correctly anticipates the supplier's optimal Stage 3 hiring and production decisions and strategically chooses $\eta_M^{H*}(N)$ and $q_M^*(N)$ to maximize (4) for any state of the game, N, at the beginning of Stage 2. We use the notation η_M^{H*} and q_M^* (without arguments) to refer to the equilibrium outcome of the manufacturer's optimal second stage strategy with both firms behaving optimally in all stages.

In the first stage, the supplier and the manufacturer simultaneously choose poaching and retention wage offers for each employee type, θ. Each firm's best response is a pair of functions, $w_i^{P*}(\theta)$ and $w_i^{R*}(\theta)$, that maximize (3) given the supply chain partner's poaching and retention wage offers. An equilibrium of the first stage, then, is a set of poaching and retention wage offers, $W^*(\theta) = \left(w_M^{P*}(\theta), w_M^{R*}(\theta), w_S^{P*}(\theta), w_S^{R*}(\theta)\right)$ such that neither firm can unilaterally deviate and strictly improve its payoff. For notational simplicity, we often drop the argument on $W^*(\theta)$ and refer to it simply as W^*.

The set of actions played along the equilibrium path, then, is a set $\left(W^*, \eta_M^{H*}, q_M^*, \eta_S^{H*}, q_S^*\right)$ such that neither firm can unilaterally deviate in any decision at any stage of the game to strictly improve its payoff.

3. Analysis

In this section, we analyze the game formulated above. We start by analyzing the manufacturer's and supplier's problems in Stages 2 and 3.

3.1. Stages 2 and 3: Production

Recall that in the second stage, the manufacturer decides the amount of the supplied good to order and the number of additional workers to hire for production. The manufacturer strategically makes its hiring and production decisions with correct anticipation of the supplier's optimal response. Further recall that the outcome of the firms' poaching and retention decisions has been settled by this point in the game. In the third stage of the game, the supplier observes the manufacturer's order quantity and the outcome of poaching and retention decisions. Using this information, the supplier decides how much to produce and how many additional workers to hire for production.

By Assumption 1(c), the firms' labor productivities, $k_i m_i$, are greater than the initial marginal hiring cost, $h'(0)$. Thus, the manufacturer and supplier are willing to hire additional workers in Stages 2 and 3 if doing so leads to increased production. The following Lemma captures the outcome of firms' optimal Stage 2 and 3 decisions for any outcome, N, of the poaching and retention stage.

Lemma 1. *Given that the manufacturer and supplier begin Stage 2 with N_M and N_S workers, respectively, the outcome of optimal strategies in Stages 2 and 3 is:*

$$\begin{aligned} \eta_i^{H*}(N) &= \left[\min\left\{\bar{\eta}_i^H, \frac{k_j\,\left(N_j + \bar{\eta}_j^H\right)}{k_i} - N_i\right\}\right]^+ \\ q_i^*(N) &= \min\left\{k_S\,\left(N_S + \bar{\eta}_S^H\right), k_M\,\left(N_M + \bar{\eta}_M^H\right)\right\}. \end{aligned} \tag{6}$$

where $i = M, S$ and firm j is firm i's supply chain partner. In these equations, $\bar{\eta}_i^H$ is firm i's maximum optimal hiring level *and is given by the solution to $h'(\bar{\eta}_i^H) = k_i m_i$.*

Each firm optimally hires workers until it hits the minimum of its *internal* or *external* limitations. If additional hiring becomes prohibitively expensive, the firm is limiting its workforce based on *internal* economic considerations. As noted in Lemma 1, the point at which hiring becomes prohibitively expensive is the *maximum optimal hiring level.* The maximum optimal hiring level is the point at which a firm's labor productivity matches the marginal hiring cost. When a firm's production is limited by internal economic considerations, the firm's optimal production quantity, q_i^*, corresponds to its capacity at its maximum optimal hiring level: $k_i \left(N_i + \bar{\eta}_i^H\right)$. If the firm's supply chain partner first reaches its maximum optimal hiring level, the firm ceases to hire additional workers because of *external* limitations: hiring and producing beyond the supply chain partner's maximum optimal capacity incur costs without providing any additional revenue. When limited by external considerations, the firm hires $\left[\frac{k_j}{k_i}\left(N_j + \bar{\eta}_j^H\right) - N_i\right]^+$ additional workers and perfectly satisfies its supply chain partner's maximum optimal production quantity, $k_j \left(N_j + \bar{\eta}_j^H\right)$.

In Lemma 1, we see that in the case that both firms optimally hire workers in the production stages, the two firms have matched production capacities. With matched capacities, we extend the traditional definition of the supply chain bottleneck to be the firm that ceases to hire because it faces its *internal* hiring limitations. The supply chain throughput corresponds to the maximum optimal production capacity of the bottleneck firm.

We also see an interesting insight into policies such as the American Recovery and Reinvestment Act (ARRA) that are designed to increase hiring of the unemployed: only the supply chain bottleneck firm needs additional incentive to hire extra workers. Our model aligns with intuition and recommends that lowering marginal hiring cost is among the easier ways for government to provide such incentive (temporarily lowering payroll taxes for newly-hired employees would one such example). Once the supply chain bottleneck increases its maximum optimal hiring level, all other members of the supply chain optimally hire additional workers to match the bottleneck's capacity whether they receive government incentive or not. This effect is more pronounced if the bottleneck's individual worker productivity is high relative to worker productivities found at the other firms in the supply chain. This combination of being the supply chain bottleneck and having high worker productivity is most likely to occur in industries with low profit margin.

We can also state the profits for each firm when both are behaving optimally in the second and third stages by substituting optimal outcomes from (6) into (3).

$$\pi_i(N) = m_i\, q^*(N) - h\left[\eta_i^{H*}(N)\right] - \int_{\theta_i^R} w_i^R(\theta)\,\mathrm{d}P_i(\theta) - \int_{\theta_i^P} w_i^P(\theta)\,\mathrm{d}P_j(\theta). \tag{7}$$

3.2. Stage 1: Poaching and Retention

We now turn to solving for optimal behavior in the poaching and retention stage. Based on the optimal firm hiring and production outcomes in (6), we see that there are two basic equilibrium outcomes in the production stages: one in which the supply chain throughput is limited by the supplier's maximum optimal hiring level in the third stage and another in which the throughput is limited by the manufacturer's maximum optimal hiring level in the second stage. We show in the Appendix (see the proof of Theorem 1) that these equilibrium outcomes are mutually exclusive. That is, existence of an equilibrium in which the manufacturer is the supply chain bottleneck precludes the existence of equilibria in which the supplier is the bottleneck for the same set of model primitives. For illustrative purposes, we focus throughout this section on equilibria in which the supplier is the supply chain bottleneck. We show in the Appendix that the results of this section are general, so symmetric results hold for equilibria in which the manufacturer is the supply chain bottleneck.

We begin by considering the maximum wage each firm would be willing to pay for an additional poached or retained worker. We then use these maximum wage offers to construct equilibrium poaching and retention wage strategies. Finally, we use these equilibrium poaching and retention wage strategies to discover the equilibrium outcome (i.e., the magnitude and direction of worker flows and the types of workers poached and retained by each firm).

With the supplier as the bottleneck, the supply chain throughput corresponds to the supplier's capacity at its maximum optimal hiring level. The supplier hires at its maximum optimal hiring level, $\bar{\eta}_S^H$, and the manufacturer hires the number of workers to match the supplier's production capacity (if needed). From (6), we see the mathematical representation of this intuition: $q^* = k_S\,(N_S + \bar{\eta}_S^H)$, $\eta_S^* = \bar{\eta}_S^H$, and $\eta_M^* = \left(\frac{k_S\,(N_S + \bar{\eta}_S^H)}{k_M} - N_M\right)^+$. Substituting these expressions into (7), we find that the supplier's marginal benefit from poaching or retaining an additional worker of type θ is $k_S\, m_S$, and the marginal cost is

$w_S^z(\theta)$, where $z \in \{P, R\}$ for poaching or retention, respectively. Thus, the highest wage the supplier would be willing to pay for an additional poached or retained worker is:

$$w_{S,\max} = k_S\, m_S. \tag{8}$$

For the manufacturer, the marginal benefit of poaching or retaining an additional worker is complicated by the fact that higher poaching and retention may reduce the number of workers available to the supply chain bottleneck (the supplier). Thus, while poaching and retaining workers presents a benefit by reducing the total hiring costs, it may result in reduction of the supply chain's bottleneck capacity in Stages 2 and 3. Anticipating optimal Stage 2 and 3 decisions, we find that the marginal benefit to the manufacturer from poaching or retaining an additional worker who would otherwise be employed at the supplier is:

$$\left(1 + \frac{k_S}{k_M}\right) h'\left(\frac{k_S\,(N_S + \bar{\eta}_S^H)}{k_M} - N_M\right) - k_S\, m_M$$

if $k_S\,(N_S + \bar{\eta}_S^H) > k_M N_M$. Otherwise, the manufacturer has an excess of workers, a case to which we return at the end of this section. For now, we consider equilibria in which the manufacturer must hire workers to match the supplier's maximum optimal capacity, i.e., $k_S\,(N_S + \bar{\eta}_S^H) > k_M N_M$. The marginal benefit of retaining a worker who would not otherwise be poached by the supplier is:

$$h'\left(\frac{k_S\,(N_S + \bar{\eta}_S^H)}{k_M} - N_M\right),$$

again, if $k_S\,(N_S + \bar{\eta}_S^H) > k_M N_M$. The marginal cost of poaching or retaining a worker is $w_M^z(\theta)$. Thus, we get that the equilibrium maximum wage that the manufacturer is willing to offer a worker who would otherwise be employed by the supplier is:

$$w_{M,\max} = \left(1 + \frac{k_S}{k_M}\right) h'\left(\frac{k_S\,(N_S + \bar{\eta}_S^H)}{k_M} - N_M\right) - k_S\, m_M. \tag{9}$$

The first term in (9) represents the *marginal cost of double hiring*: when the manufacturer poaches an additional worker from the supplier or when the manufacturer retains an additional worker that could have been poached by the supplier, it saves the marginal cost of hiring both a replacement for that worker and additional workers to match the supplier's higher capacity from the additional worker. The second term represents the opportunity cost the manufacturer incurs by not allowing the supplier to poach or retain that additional worker. Had the manufacturer allowed the supplier to poach or retain an additional worker, the supplier's throughput would have increased by k_S. These additional k_S units are worth $k_S m_M$ to the manufacturer. Thus, the maximum wage represents the savings in hiring cost minus the opportunity cost of poaching and retention.

The following lemma shows optimal equilibrium poaching wage offers. We let $w_{i,\max}^{(eq)}$ represent the maximum wage firm i is willing to offer to poach or retain an additional worker in equilibrium.

LEMMA 2 **(Optimal Poaching and Retention Wage Offers)**. *In equilibria in which both firms hire a strictly positive number of workers in the production stages, firm $i \in \{M,S\}$ offers workers employed by its supply chain partner (firm j) poaching wages equal to:*

$$w_i^{P*}(\theta) = \min\left\{w_{i,\max}^{(eq)}, \max\left\{w_{j,\max}^{(eq)}, w\right\} + \theta\right\}. \tag{10}$$

Firm i offers its own workers retention wages equal to:

$$w_i^{R*}(\theta) = \max\left\{w, \min\left\{w_{j,\max}^{(eq)} - \theta, w_{i,\max}^{(eq)}\right\}\right\}. \tag{11}$$

We recall that to successfully poach a worker, a firm must be willing to offer a wage that is at least as large as the sum of the worker's transfer cost and the worker's best alternative. The worker's best alternative is the maximum of its current employer's retention wage offer and the reservation wage, w. In an attempt to prevent poaching, the worker's employer would be willing to offer a retention wage up to its equilibrium maximum wage offer. Thus, a worker initially employed at firm $j \in \{M,S\}$ who is successfully poached would have a best alternative of $\max\{w_{j,\max}, w\}$. To successfully poach a worker, then, a firm must be willing to offer the worker a wage at least as high as $\max\{w_{j,\max}, w\} + \theta$. A firm attempting to poach workers, however, would not be willing to offer a poaching wage that exceeds the marginal benefit of an additional worker. Similarly, the current employer's supply chain partner (firm j), would be willing to offer a poaching wage no larger than $w_{j,\max}$. Thus, to successfully retain a worker, the current employer must be willing to offer a wage higher than $w_{j,\max} - \theta$. It is of interest to note that, under this model's assumptions, only one firm successfully poaches workers from the other in equilibrium, and the identity of the firm that poaches workers is the firm with the highest $w_{\max}^{(eq)}$.

Now that we have determined the equilibrium poaching and retention wage offers, we proceed to analyze the equilibrium outcome of the first stage by determining the magnitude of worker flows. For convenience, we define the *equilibrium poaching threshold* to be:

$$\theta_S^T = w_{S,\max}^{(eq)} - \max\left\{w, w_{M,\max}^{(eq)}\right\}, \tag{12}$$

where the subscript S reminds us that the supplier is the bottleneck. We quickly note that (12) will be used to characterize the equilibrium outcome.

We now show how the number of workers poached and retained by each firm depends on the poaching threshold by applying the rules for successful worker poaching and retention from (1) and (2) to the optimal poaching and retention wage offers from Lemma 2. The resulting equilibrium worker flows are given by:

$$\begin{aligned}
\eta_S^{P\,(eq)}\left(\theta_S^T\right) &= P_M\left[(\theta_S^T)^+\right] \\
\eta_S^{R\,(eq)}\left(\theta_S^T\right) &= \bar{P}_S\left[(-\theta_S^T)^+\right] \\
\eta_M^{P\,(eq)}\left(\theta_S^T\right) &= P_S\left[(-\theta_S^T)^+\right] \\
\eta_M^{R\,(eq)}\left(\theta_S^T\right) &= \bar{P}_M\left[(\theta_S^T)^+\right].
\end{aligned} \tag{13}$$

From these equations, we see that the bottleneck supplier poaches from the manufacturer if and only if $\theta_S^T > 0$. Similarly, the manufacturer poaches from the bottleneck supplier if and only if $\theta_S^T < 0$. In the special case that $\theta_S^T = 0$, neither firm poaches from the other. We also see the practical meaning of the poaching threshold: $\theta_S^T > 0$ represents the highest worker type poached by the supplier if the supplier is the poacher, and $-\theta_S^T > 0$ captures the highest worker type poached by the manufacturer if the manufacturer is the poacher. Thus, the resulting number of workers employed at each firm after poaching and retention is:

$$\begin{aligned}
N_S^{(eq)}\left(\theta_S^T\right) &= \begin{cases} \Lambda_S + P_M(\theta_S^T), & \text{if } \theta_S^T \geq 0 \\ \Lambda_S - P_S(-\theta_S^T), & \text{o.w.} \end{cases} \\
N_M^{(eq)}\left(\theta_S^T\right) &= \begin{cases} \Lambda_M - P_M(\theta_S^T), & \text{if } \theta_S^T \geq 0 \\ \Lambda_M + P_S(-\theta_S^T), & \text{o.w.} \end{cases}
\end{aligned} \tag{14}$$

We substitute the results from (14) into (9) and substitute the resulting equilibrium maximum wage offers into the equation for the equilibrium poaching threshold given in

(12). For convenience, we define the following function describing the number of workers the manufacturer must hire in Stage 2 to match the supplier's maximum optimal capacity:

$$\mu_M(\theta) = \begin{cases} \frac{k_S}{k_M}\left[\Lambda_S + \bar{\eta}_S^H + P_M(\theta)\right] - \Lambda_M + P_M(\theta) & \text{,if } \theta \geq 0 \\ \frac{k_S}{k_M}\left[\Lambda_S + \bar{\eta}_S^H - P_M(-\theta)\right] - \Lambda_M - P_M(-\theta) & \text{,o.w.} \end{cases} \tag{15}$$

The resulting equation, shown in the following Lemma, characterizes the equilibrium poaching threshold for equilibria in which the supplier is the supply chain bottleneck. The equation characterizes all equilibria of primary interest throughout the remainder of the paper. With the equilibrium poaching threshold, the game's complete equilibrium outcome can quickly be obtained by substitution into the appropriate equations.

Lemma 3 **(Equilibrium Poaching Threshold)**. *In equilibria in which the supplier is the bottleneck and the manufacturer optimally hires workers in the second stage, the equilibrium poaching threshold satisfies:*

$$\theta_S^T = k_S m_S - \max\left\{w, \left(1 + \frac{k_S}{k_M}\right) h'\left(\mu_M\left(\theta_S^T\right)\right) - k_S m_M\right\}. \tag{16}$$

As mentioned earlier, many of the equations we have presented in the Stage 1 analysis only apply to equilibria in which the supplier is the bottleneck and the manufacturer must hire a strictly positive number of workers in the second stage to match the supplier's maximum optimal capacity. However, if $k_M \Lambda_M \geq k_S\left(\Lambda_S + \bar{\eta}_S^H\right)$, then it may be optimal for the manufacturer to refrain from hiring workers in Stage 2. To characterize the equilibria in which the supplier is the bottleneck and the manufacturer finds it optimal to refrain from hiring in Stage 2, we first consider the unique root of (15) (if it exists), which we denote θ_0. θ_0 represents the worker type such that if all of the manufacturer's workers of type $\theta \leq \theta_0$ are poached by the supplier, the final optimal production capacities would be matched without any need for the manufacturer to hire additional workers in Stage 2. The equilibrium outcome can be divided into three cases, depending on the existence and value of θ_0. In the first case, either θ_0 does not exist because $\mu_M(\theta) < 0$ for all θ or θ_0 exists and satisfies $\theta_0 > k_S m_S - w$. In this case, the manufacturer has no need to hire workers in Stage 2. In fact, before poaching and retention wages are offered, it first lays off workers (who would not be poached by the supplier) until its starting workforce is reduced to

$$\frac{k_S}{k_M}\left(\Lambda_S + \bar{\eta}_S^H\right) + \left(1 + \frac{k_S}{k_M}\right) P_M\left(k_S m_S - w\right).$$

Then, in equilibrium, it allows the supplier to poach the most it can afford, $P_M(k_S m_S - w)$, and retains all workers remaining. In the second case, $\theta_0 < k_S m_S - \max\left\{w, \left(1+\frac{k_S}{k_M}\right)h'(0) - k_S m_M\right\}$. Here, the manufacturer allows so many workers to be poached by the supplier that it must hire workers in Stage 2 to match the supplier's maximum optimal capacity. Thus, Lemma 3 applies to this case. In the final case, $k_S m_S - \max\left\{w, \left(1+\frac{k_S}{k_M}\right)h'(0) - k_S m_M\right\} \le \theta_0 \le k_S m_S - w$. Here, the manufacturer is willing to allow the supplier to poach workers of type $\theta \le \theta_0$ but is unwilling to allow the supplier to poach more workers since the manufacturer would then have to pay the marginal cost of double hiring.

For the remainder of the paper, we focus on equilibria in which the conditions of Lemma 3 are satisfied. In other words, the equilibria we consider see the manufacturer optimally hiring a strictly positive number of workers in the second stage. We do this because we are interested in the situation that could arise that one firm finds it optimal to share workers with its supply chain partner. The equilibria in which the conditions of Lemma 3 are not satisfied are trivial: the manufacturer *wants* to reduce its workforce. In the first and third cases described in the preceding paragraph, all workers poached by the supplier would provide no value if they were retained by the manufacturer. We presented these equilibria for completeness; but we turn our attention to the non-trivial case in which the manufacturer optimally hires workers in the second stage.

4. Results and Discussion

Now that we have completed analysis of the core model, we turn to development and discussion of the model's results.

We see in (13) that the sign of θ_S^T determines the direction of worker flows between the supply chain firms. Corresponding to the different signs of θ_S^T (i.e., the different directions of worker flows), then, we get two different equilibrium types.

Definition 1 (Labor-Sharing Equilibrium). A Labor-Sharing Equilibrium (LSE) is an equilibrium in which one firm is both limiting supply chain throughput and poaching from its supply chain partner even though the supply chain partner must hire additional workers to replace the workers poached.

In Labor-Sharing Equilibria in which the supplier is the supply chain bottleneck, $\theta_S^T > 0$, and the supplier poaches $P_M\left(\theta_S^T\right)$ workers from the manufacturer. In the special case that $P_M\left(\theta_S^T\right)=0$, no workers flow from the manufacturer to the supplier since no affordable workers are available. However, we still refer to this as a Labor-Sharing Equilibrium since the manufacturer would be willing to share workers with the supplier if it currently employed workers of type $\theta \leq \theta_S^T$.

Now we turn to the second equilibrium type.

Definition 2 (Labor-Hoarding Equilibrium). A Labor-Hoarding Equilibrium (LHE) is an equilibrium in which a firm poaches workers from the supply chain bottleneck.

Labor-Hoarding Equilibria in which the supplier limits throughput have $\theta_S^T < 0$. In such equilibria, the manufacturer poaches all workers of type $\theta \leq -\theta_S^T$ from the supplier. We call these Labor-Hoarding Equilibria because the manufacturer is retaining its own workers (and even poaching low-θ workers from the supplier) even though the manufacturer is not the supply chain bottleneck. Such hoarding of workers may reduce overall supply chain throughput.

Close inspection of Definition 1 reveals a counter-intuitive idea: in Labor-Sharing Equilibria, the supply chain bottleneck poaches from its supply chain partner. Intuitively, we associate the bottleneck with the less productive firm and the poacher with the more productive firm. To illustrate the counter-intuitive nature of these equilibria, we consider the extreme special case that firms' initial capacities are matched ($k_S\Lambda_S = k_M\Lambda_M$), the supplier has lower individual worker productivity ($k_S < k_M$), and the supplier has a lower labor productivity ($k_S m_S < k_M m_M$). It can be shown that in this special case, the supplier is the supply chain bottleneck. However, Labor-Sharing Equilibria may still exist in which such a supplier that is less productive in every way poaches from its supply chain partner. This phenomenon is very counter-intuitive from the perspective of labor economics. In labor economics, the firm with higher labor productivity poaches from firms with lower labor productivity. Thus, even the existence of Labor-Sharing Equilibria is of interest. We will see later in the paper that these Labor-Sharing Equilibria also have important implications for operating in a manufacturing hub and entry of a highly productive labor market competitor. We first focus on results related to the existence of these counter-intuitive Labor-Sharing Equilibria and then we proceed to the operational implications of these equilibria.

Theorem 1 **(Necessary Conditions for Existence of an LSE)**.
A Labor-Sharing Equilibrium (LSE) in which the supplier limits supply chain throughput exists if and only if there exists a $\theta_S^T > 0$ satisfying all of the following conditions:

1. $\mu_M\left(\theta_S^T\right) > 0$
2. $\theta_S^T = k_S m_S - \max\left\{w, \left[\left(1+\frac{k_S}{k_M}\right) h'\left(\mu_M\left(\theta_S^T\right)\right) - k_S m_M\right]\right\}$
3. $k_S\left[\Lambda_S + \bar{\eta}_S^H + P_M\left(\theta_S^T\right)\right] < k_M\left[\Lambda_M + \bar{\eta}_M^H - P_M\left(\theta_S^T\right)\right]$.

Furthermore, if there exists such a θ_S^T, the game has a unique equilibrium outcome with θ_S^T as the equilibrium poaching threshold.

Corollary 1. *A Labor-Hoarding Equilibrium (LHE) in which the supplier is the supply chain bottleneck exists if and only if there exists a $\theta_S^T < 0$ that satisfies both of the following conditions:*

1. $\theta_S^T = k_S m_S - \left[\left(1+\frac{k_S}{k_M}\right) h'\left(\mu_M\left(\theta_S^T\right)\right) - k_S m_M\right]$
2. $k_S\left[\Lambda_S + \bar{\eta}_S^H - P_S\left(-\theta_S^T\right)\right] < k_M\left[\Lambda_M + \bar{\eta}_M^H + P_S\left(-\theta_S^T\right)\right]$.

Existence of such a θ_S^T implies that: (i) the game has a unique equilibrium outcome corresponding to this value of θ_S^T, and (ii) $k_M m_M > k_S m_S$.

For $\theta_S^T > 0$, the supplier is poaching workers from the manufacturer in equilibrium since it is willing to offer higher wages. The first condition in Theorem 1 guarantees that the manufacturer must hire a positive number of workers in the second stage to match the supplier's maximum optimal production capacity. The second condition in Theorem 1 is the equation characterizing the equilibrium poaching threshold. The final condition is necessary for the supplier to be the firm limiting supply chain throughput at θ_S^T. The conditions of Corollary 1 carry similar meaning, except there is no requirement that both firms hire a strictly positive number of workers.

The uniqueness results in Theorem 1 and Corollary 1 are important. They imply that a given set of model primitives cannot lead to both a Labor-Sharing Equilibrium and a Labor-Hoarding Equilibrium. Furthermore, they imply that if a Labor-Sharing Equilibrium or a Labor-Hoarding Equilibrium exists in which the supplier is the supply chain bottleneck, then there are no equilibria in which the manufacturer is the supply chain bottleneck.

Based on the necessary conditions in Theorem 1, we develop sufficient conditions for the existence of a Labor-Sharing Equilibrium in the special case that the manufacturer's initial capacity, $k_M\Lambda_M$, is less than the supplier's no-poaching maximum optimal capacity, $k_S\left(\Lambda_S + \bar{\eta}_S^H\right)$. For convenience, we let

$$Q_S = k_S m_S - \left[\left(1+\frac{k_S}{k_M}\right) h'\left(\frac{k_S\left(\Lambda_S + \bar{\eta}_S^H\right) - k_M\Lambda_M}{k_M}\right) - k_S m_M\right]. \tag{17}$$

THEOREM 2 **(Sufficient Conditions for Existence of an LSE)**.
In the case that $k_S\left(\Lambda_S+\bar{\eta}_S^H\right)>k_M\Lambda_M$*, the following conditions are sufficient for the existence of a Labor-Sharing Equilibrium (LSE):*

1. $Q_S>0$
2. $k_S\left[\Lambda_S+P_M\left(Q_S\right)+\bar{\eta}_S^H\right]<k_M\left[\Lambda_M-P_M\left(Q_S\right)+\bar{\eta}_M^H\right]$.

The first condition guarantees that the supplier is the equilibrium poacher given that the supplier is the bottleneck. The second condition is sufficient for the supplier to be the supply chain bottleneck in the case that $Q_S>0$. We note that these sufficient conditions are expressed purely in terms of exogenous parameters, so we do not need the exact equilibrium to know that a Labor-Sharing Equilibrium exists in which the supplier is the supply chain bottleneck. We note that if we know *a priori* that the supplier is the equilibrium supply chain bottleneck, then the first condition of Theorem 2 is alone sufficient for existence of a Labor-Sharing Equilibrium.

As we have already pointed out, these Labor-Sharing Equilibria often demonstrate counter-intuitive properties when viewed through the lens of traditional labor economics. Thus, these sufficient conditions are important for recognizing situations in which we could expect to see counter-intuitive poaching phenomena. These equilibria are also important, however, when viewed through the lens of operations management. We show throughout the remainder of this paper that Labor-Sharing Equilibria carry important implications for supply chain throughput, firm profits, and the labor costs and benefits of operating in a manufacturing hub. The necessary conditions of Labor-Hoarding Equilibria in Corollary 1 are no less important. We will show that the traditional way of thinking about supply chain bottlenecks can be significantly impacted under Labor-Hoarding Equilibria.

4.1. Model Comparison

In this section, we compare the core model of the paper with two variants to determine the effect of the supply chain relationship and to determine the effect of poaching. We refer to the paper's core model as *Model WG* since it considers both the flow of workers (W) and the flow of goods (G). We compare Model WG first with a variant that removes the supply chain ties. In other words, this variant has no flow of goods between the firms. We refer to this variant as *Model W* since workers flow between the firms but no goods flow between them. Then we compare Model WG to the variant that removes the possibility of worker flows between firms. We refer to this variant as Model G since only goods can flow between firms. We begin with the comparison of Models WG and W.

Model Comparison I: Effect of the Supply Chain

In this section, we examine the effect of the supply chain by comparing the solution from the Models WG and W. In Model W, the manufacturer and supplier still have the opportunity to poach one another's workers, but the supplier no longer provides any of the supplied good to the manufacturer. For convenience, we let firm H denote the firm with the highest labor productivity, $k_i m_i$, and firm L denote the firm with the lowest labor productivity. In Stages 2 and 3 of Model W, both firms hire their maximum optimal hiring level and produce at their maximum possible optimal capacities. Optimal poaching and retention wage offers in the first stage lead to the following worker flows:

$$\begin{aligned}
\eta_H^R &= \Lambda_H, \\
\eta_H^P &= P_L\left(k_H m_H - k_L m_L\right), \\
\eta_L^R &= \Lambda_L - P_L\left(k_H m_H - k_L m_L\right), \\
\eta_L^P &= 0.
\end{aligned}$$

We see that the poaching firm has the highest labor productivity and offers the highest wages. This result provides a check on the validity of our model: when the model contains only the labor economics component, we get the labor economics results. Comparing Models WG and W highlights the importance of the supply chain relationship on the existence of equilibria which contradict the "normal" labor economics results.

We also see the poaching-related costs of operating in a manufacturing hub with strong labor market competitors but without supply chain members. If a firm with low labor productivity operates in a manufacturing hub without any supply chain partners, its profits suffer from both lower production output and higher retention costs.

Model Comparison II: Effect of Poaching

Now we turn to the second model variant (Model G) in which we examine the performance of the supply chain in the case that poaching is not available as a means to accomplish worker flows between supply chain firms. We get the following proposition comparing supply chain throughput in Models WG and G.

PROPOSITION 1. (a) *If a Labor-Sharing Equilibrium is the outcome in Model WG, the supply chain throughput is* higher *in Model WG than in Model G. Furthermore, the same firm is limiting throughput in both models.*

(b) *If a Labor-Hoarding Equilibrium is the outcome in Model WG and the same firm is the bottleneck in both Models WG and G, then supply chain throughput is* lower *in Model WG than in Model G.*

(c) *Consider the case that* $k_S\left(\Lambda_S+\bar{\eta}_S^H\right)>k_M\left(\Lambda_M+\bar{\eta}_M^H\right)$ *so that the manufacturer is the bottleneck in Model G. Let*

$$\bar{\theta}_S=\sup\left\{\theta<0:\,(k_S+k_M)\;P_S\left(-\theta\right)>k_S\,\left(\Lambda_S+\bar{\eta}_S^H\right)-k_M\left(\Lambda_M+\bar{\eta}_M^H\right)\right\}.$$

Then, the supplier is the supply chain bottleneck in a Labor-Hoarding Equilibrium of Model WG if $k_S\bar{\eta}_S^H<k_M\left(\Lambda_S+\Lambda_M+\bar{\eta}_M^H\right)$ *and*

$$\bar{\theta}_S>k_Sm_S+k_Sm_M-\left(1+\frac{k_S}{k_M}\right)h'\left(\frac{k_S}{k_M}\left[\Lambda_S+\bar{\eta}_S^H-P_S\left(-\bar{\theta}_S\right)\right]-\Lambda_M-P_S\left(-\bar{\theta}_S\right)\right).$$

This proposition captures some important insights provided by our model into the interaction between the flow of goods and the flow of workers accomplished via poaching. First, we see that the flow of goods between supplier and manufacturer can be either higher or lower when poaching is available as a means to accomplish worker flows. Furthermore, we see very clearly that the type of equilibrium (LSE vs. LHE) in Model WG can inform us about how the availability of poaching affects the flow of goods. We see that in Labor-Sharing Equilibria, the worker flows accomplished via poaching allow increased capacity at the supply chain's bottleneck.

In Proposition 1(c), we see a very important result: worker flows between supply chain partners can change the identity of the supply chain bottleneck. In other words, there exist Labor-Hoarding Equilibria in Model WG in which a firm poaches so many workers from its supply chain partner that its supply chain partner *becomes* the bottleneck. In our model, the identity of the supply chain bottleneck is endogenous: it is dictated by equilibrium poaching, retention, and hiring. Thus, when identifying supply chain bottleneck resources, worker flows accomplished via poaching must be considered.

The important results we see in Proposition 1 highlight the impact of the flow of workers on the throughput of goods in the supply chain. Now we turn to examine how the interaction between worker and material flows affects firms' profits. We find that the following equations characterize the difference between firm profits in Models WG and G. We focus on the case that Model WG leads to a Labor-Sharing Equilibrium since it allows the possibility that both firms' profits are higher in Model WG than in Model G. We let (without loss of generality) the supplier be the supply chain bottleneck in Model WG.

$$
\begin{aligned}
\Delta\pi_{S,\mathrm{LSE}} &= \int_0^{\theta_S^T} P_M(\theta)\,\mathrm{d}\theta - \int_0^{\max\left\{w, w_{M,max}\right\}-w} P_S(\theta)\,\mathrm{d}\theta \\
\Delta\pi_{M,\mathrm{LSE}} &\geq \left(1+\frac{k_S}{k_M}\right) P_M\left(\theta_S^T\right) h'\left[\frac{k_S\left(\Lambda_S+\bar{\eta}_S^H\right)}{k_M} - \Lambda_M + \left(1+\frac{k_S}{k_M}\right) P_M\left(\theta_S^T\right)\right] \\
&\quad - h\left[\frac{k_S\left(\Lambda_S+\bar{\eta}_S^H\right)}{k_M} - \Lambda_M + \left(1+\frac{k_S}{k_M}\right) P_M\left(\theta_S^T\right)\right] \\
&\quad + h\left[\frac{k_S\left(\Lambda_S+\bar{\eta}_S^H\right)}{k_M} - \Lambda_M\right] - \int_{\theta_S^T}^{m_S k_S - w} P_M(\theta)\,\mathrm{d}\theta
\end{aligned}
\tag{18}
$$

The expression for $\Delta\pi_{M,\mathrm{LSE}}$ is satisfied with equality if $w_{M,\max} \geq w$. Although this expression for $\Delta\pi_{M,\mathrm{LSE}}$ is quite complex, we can see that convexity of the hiring cost function, $h(\cdot)$, immediately implies that the first three terms sum to a value greater than zero. We can also see that the sum of the first three terms is higher for larger values of the Model WG poaching threshold, θ_S^T; and the absolute value of the final term is lower for higher values of θ_S^T. Thus, the manufacturer benefits the most from poaching when Model WG gives higher poaching thresholds. This is also when the most workers are poached from the manufacturer. There are two forces driving this counter-intuitive result. First, for higher poaching thresholds, supply chain throughput would be increased to a greater extent since more workers would be poached. Second, higher poaching thresholds correspond to a higher manufacturer profit margin and/or lower marginal cost of hiring the workers needed to match the supplier's capacity. We see, then, that if the poaching threshold from Model WG is high enough in a Labor-Sharing Equilibrium, both firms' profits are actually *higher* with poaching. Under these circumstances, non-poaching agreements would actually hurt both firms. The benefit from higher throughput made possible by supply chain worker flows exceeds the cost of the retention and poaching wage competition. We can also see here the poaching-related labor benefits of manufacturing hubs: in manufacturing hubs, between-firm worker flows are easily accomplished since employees don't have to relocate to switch employers. This can improve efficiency of worker flows. In manufacturing hubs, then, the interaction between the flow of goods and the flow of workers can benefit all supply chain members.

We see in (18) the reason for the poaching success in the shale oil and gas industry. With the shale extraction firms poaching skilled labor, the downstream customers have a relatively low marginal cost of double hiring. Furthermore, the customers experience a large production payoff for allowing their workers to be poached. Thus, their maximum equilibrium wages are low. This leads to a high poaching threshold. Both shale oil and its customers benefit because poaching is available as a means to accomplish worker flows. However, if Apple and members of its value chain were to allow poaching of developers and managers, the marginal hiring costs would be high. Furthermore, the firms would receive little benefit from allowing workers to be poached by value chain partners. Thus, the maximum equilibrium wages are high and the poaching threshold is low. Apple and its partners would thus be worse off without no-poaching agreements.

4.2. Comparative Statics

We now proceed to present comparative statics on the poaching threshold, θ_i^T. With the comparative statics results, we show how the worker flows depend on the model primitives. If the poaching threshold increases in a Labor-Sharing Equilibrium, a weakly larger number of workers flow to the bottleneck firm from its supply chain partner. However, increasing the poaching threshold in a Labor-Hoarding Equilibrium leads to a weakly smaller number of the bottleneck firm's workers poached by its supply chain partner. We let firm $i \in \{M, S\}$ be the supply chain bottleneck, and we let firm j be firm i's supply chain partner.

PROPOSITION 2 **(Comparative Statics)**. *In equilibria in which firm $i \in \{M, S\}$ is the supply chain bottleneck (with supply chain partner j), an increase in k_j (firm j's individual worker productivity) or p_F (price of the final good) causes a weak increase in the poaching threshold, θ_i^T. Increasing production costs c_M or c_S results in a weakly decreasing poaching threshold, θ_i^T.*

With workers who are individually more productive, firm j is able to achieve higher production from retained and poached workers and higher production from hired workers. Furthermore, the number of workers firm j must hire to match firm i's production capacity is reduced. Thus, firm j is more prone to allow its workers to be poached. In a manufacturing hub, then, we see that labor flows can distribute the benefits of enhanced worker productivity even without knowledge spillover effects: the supply chain bottleneck retains and poaches weakly more workers, experiences weakly higher production capacity and supply chain throughput, and earns weakly higher profits when firm j's workers become more productive.

The supply chain partners also share the benefits when the price of the final good increases. If the manufacturer is the supply chain bottleneck, an increase in p_F implies that the manufacturer can afford to offer a higher poaching wage. Thus, throughput increases. If the supplier is the bottleneck, then the manufacturer becomes more willing to allow its workers to be poached so that it can capitalize on higher throughput.

Changes in production costs are also shared. Increasing production costs, c_S or c_M, decrease the firm margins. Labor sharing becomes less beneficial since the payoff for sharing workers is reduced. Furthermore, with increased production costs, the bottleneck cannot afford wages as high and thus cannot afford to poach as many workers.

4.3. Labor Market Competition

Thus far in our analysis, we have considered a single supplier and a single manufacturer. Thus, the firms have not had to compete with other firms for workers, and the workers have not had specific employment options outside the supply chain. Now we examine the impact of additional labor market competition on our results. Specifically, we assume that a labor market competitor, firm C, is also attempting to poach and retain workers. This labor market competitor does not participate in the supply chain and only serves as a potential source of workers to poach or a potential poacher. We let $w_{C,\max} > w$ be the exogenous maximum wage that the labor market competitor is willing to pay in the poaching and retention stage. Without loss of generality, we let firm i represent the firm (either the supplier or manufacturer) limiting supply chain throughput, and firm j represents firm i's supply chain partner.

Our first result relates to the presence of a labor market competitor that is more productive than the supply chain bottleneck.

PROPOSITION 3. *The presence of a labor market competitor makes supply chain worker flows impossible if:*

1. $w_{C,\max} > k_i m_i$,

2. *there exists a* $w^C_{j,\max}$ *satisfying*

$$w^C_{j,\max} = h'\left[\frac{k_i\left[\Lambda_i - P_i\left(w_{C,\max} - k_i m_i\right) + \bar{\eta}^H_i\right]}{k_j} - \Lambda_j + P_j\left(w_{C,\max} - w^C_{j,\max}\right)\right]$$

such that $w_{C,\max} > w^C_{j,\max}$, $w_{C,\max} > \left(1 + \frac{k_i}{k_j}\right) w^C_{j,\max} - k_i m_j$, *and* $k_i\left[\Lambda_i - P_i\left(w_{C,\max} - k_i m_i\right) + \bar{\eta}^H_i\right] < k_j\left[\Lambda_j - P_j\left(w_{C,\max} - w^C_{j,\max}\right) + \bar{\eta}^H_j\right]$.

Supply chain throughput is reduced if such a labor market competitor is present.

The conditions in Proposition 3 guarantee: (i) the labor market competitor is willing to offer wages higher than any wage offers from the manufacturer or supplier, and (ii) firm i is indeed limiting supply chain throughput in equilibrium.

In these equilibria, both supply chain partners must pay high wages to retain workers, and the poaching by the labor market competitor reduces supply chain throughput. Thus, we see that the labor costs of being in a manufacturing hub with highly productive labor market competitors have the potential to be far more substantial than simple costs of wage competition. Highly productive labor market competition can hurt supply chain throughput and eliminate worker flows between supply chain partners. This insight also gives us some additional information regarding entry of a new labor market competitor. A highly productive entrant can shut down supply chain labor sharing. In this case, poaching would no longer serve as a means to increase supply chain bottleneck capacity.

We find, however, that if $k_i m_i > w_{C,\max}$, then Labor-Sharing Equilibria may continue to exist under conditions similar to those in the original model. Thus, the existence of Labor-Sharing Equilibria and the subsequent results are robust to the presence of labor market competition as long as the competitors are not more productive than the supply chain bottleneck. However, labor market competition often leads to lower profits as the firms must offer higher wages to ward off the competitor's poaching attempts.

4.4. Model Robustness

We performed a number of tests on the model to ensure robustness. We briefly mention robustness with respect to demand uncertainty. In our test, the manufacturer faces demand uncertainty in a Newsvendor setting. We found that the results of the model continued with minor discrepancy. The primary difference between our model and the model extension with demand uncertainty comes as a result of decreasing returns to production associated with the uncertain demand. Decreasing returns to production imply that the manufacturer's marginal payoff for hiring workers decreases, so if the manufacturer poaches and retains more workers, its maximum optimal hiring level decreases. Because of this, the supplier may be less willing to allow its workers to be poached since the supplier's value of each additional worker poached by the manufacturer is weakly decreasing. Despite the differences, however, Labor-Sharing Equilibria continue to exist with uncertain demand.

5. Conclusion

In this paper, we've highlighted interesting results and insights stemming from the interaction between the flow of goods and the flow of workers via poaching in a supply chain. We have shown that it may be optimal to allow a less productive firm to poach workers from members of its supply chain so that the supply chain throughput can be enhanced. In fact, the more productive the workers employed by the supply chain partners and the greater the partners' profit margin, the greater the tendency for labor sharing. These insights run counter to classical intuition from labor economics. Furthermore, labor sharing via poaching actually provides a net benefit to all supply chain firms in the case that the marginal revenues from increased throughput exceed both the marginal costs of hiring workers and the higher wages that must be paid to poach and retain workers.

This insight provides an explanation for the differences in the response to poaching observed between the shale industry and Apple. The customers of the shale oil and gas industry get high payoffs from higher supplier poaching. And the marginal cost of "double hiring" is relatively low for the skilled laborers poached by the shale oil extraction firms. Thus, worker flows made available via poaching are able to increase throughput and the resulting profits for all members of the supply chain. However, with Apple and its partners, the marginal benefit of allowing an additional engineer or developer to be poached is low; and the marginal hiring cost is high for these workers. Thus, poaching and retention wages would be very high without no-poaching agreements. Since the higher wages and hiring costs exceed the benefits of allowing engineers and developers to be poached, it becomes desirable for Apple and its business partners to avoid poaching altogether by entering into no-poaching agreements.

One of our primary results relates to the idea that poaching has the potential to impact our views of supply chain capacity management. Traditionally, we think of the bottleneck as the resource with the lowest capacity. In our model, the bottleneck firm is the supply chain partner that is unwilling to further increase its capacity by hiring additional workers. We have shown that the availability of poaching can change the identity of the bottleneck firm. Thus, consideration of worker flows accomplished via poaching can be critical understanding supply chain bottleneck capacity management.

We also find that the presence of a highly productive labor market competitor shuts down the possibility of labor sharing, decreases throughput, and decreases profits. However, labor sharing between supply chain partners is robust to labor market competition if the competitor has lower labor productivity than the supply chain bottleneck. The robustness of labor sharing between members of a supply chain even in the presence of outside labor competition highlights the poaching-induced labor benefits of operating in a manufacturing hub. Close proximity to other members of a supply chain may help provide for efficient worker flows without excessive poaching/retention wages.

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Appendix. Proofs

We omit the proof of Proposition 3 since it is so similar to the proofs of Lemma 3 and Proposition 1.

Proof of Lemma 1: This proof follows by solving the supplier's problem in (5) and using the supplier's optimum strategy to solve the manufacturer's problem in (4). We use the two optimum strategies together to determine the equilibrium outcome in the sub-game beginning at the start of Stage 2 in state N.

We begin by noting that the supplier has no incentive in (5) to have $q_S > q_M$. Such production levels incur the production cost but don't add to revenue. Thus, an equivalent optimization problem is:

$$V_S(N, q_M) = \max_{q_S, \eta_S} \quad m_S q_S - h(\eta_S^H)$$

$$\text{subject to:} \quad q_S \geq 0; \quad q_S \leq k_S(N_S + \eta_S^H); \quad q_S \leq q_M; \quad \eta_S^H \geq 0.$$

The Lagrangian for this problem (with all λ representing the multipliers for the inequality constraints) is:

$$\mathcal{L} = m_S q_S - h(\eta_S^H) + \lambda_q q_S + \lambda_K \left[k_S(N_S + \eta_S^H) - q_S\right] + \lambda_M(q_M - q_S) + \lambda_\eta \eta_S^H$$

For first-order conditions, then, we get:

$$[q_S]: \quad 0 = m_S + \lambda_q^* - \lambda_K^* - \lambda_M^*$$

$$[\eta_S^H]: \quad 0 = \lambda_\eta^* + \lambda_K^* k_S - h'\left(\eta_S^{H\,*}(N, q_M)\right)$$

Since $m_S > 0$, $\lambda_q^* = 0$; so it's optimal for the supplier to increase capacity until its capacity constraint is binding, until the manufacturer's order constraint is binding, or until both are binding. If the manufacturer's order constraint is binding but the capacity constraint is not, $q_S^*(N, q_M) = q_M$ and $\eta_S^{H\,*} = 0$: if the supplier can completely fulfill the manufacturer's order without being capacity-constrained, it will perfectly satisfy the order and not hire any additional workers. If the capacity constraint is binding but the manufacturer's order constraint is not, the supplier hires up to the point that $h'(\eta_S^H) = k_S m_S$. This level of hiring represents the maximum optimal hiring level, $\bar{\eta}_S^H$. The supplier produces at the level corresponding to its capacity, $q_S^*(N, q_M) = k_S(N_S + \bar{\eta}_S^H)$. In the final case, both the capacity constraint and the manufacturer's order constraint are binding. In other words, the supplier hires enough workers so that its capacity is equal to the manufacturer's order quantity. Thus, in this case, the supplier must hire $\eta_S^{H\,*}(N, q_M) = \frac{q_M}{k_S} - N_S$ workers to satisfy the manufacturer's order, and the order quantity corresponds to the manufacturer's order, $q_S^*(N, q_M) = q_M$. Thus, we get that the supplier's optimal strategy is:

$$q_S^*(N, q_M) = \min\left\{q_M, k_S\left(N_S + \bar{\eta}_S^H\right)\right\}$$

$$\eta_S^{H\,*}(N, q_M) = \left[\min\left\{\bar{\eta}_S^H, \frac{q_M}{k_S} - N_S\right\}\right]^+$$

Now we turn to the manufacturer's Stage 2 optimum strategy. We begin by substituting the supplier's optimal Stage 3 strategy into (4) to get:

$$V_M(N) = \max_{q_M, \eta_M} \quad m_M \min\left\{q_M, k_S\left(N_S + \bar{\eta}_S^H\right)\right\} - h(\eta_M^H)$$
$$\text{subject to:} \;\; q_M \geq 0; \quad q_M \leq k_M(N_M + \eta_M^H); \quad \eta_M^H \geq 0.$$

Since no benefit is provided by ordering more than $k_S\left(N_S + \bar{\eta}_S^H\right)$, we can re-write the problem again as:

$$V_M(N) = \max_{q_M, \eta_M} \quad m_M q_M - h(\eta_M^H)$$
$$\text{subject to:} \;\; q_M \geq 0; \quad q_M \leq k_M(N_M + \eta_M^H); \quad q_M \leq k_S\left(N_S + \bar{\eta}_S^H\right); \quad \eta_M^H \geq 0.$$

This problem is identical to the supplier's except that the supplier's production is bounded above by q_M, whereas the manufacturer's is bounded above by $k_S\left(N_S + \bar{\eta}_S^H\right)$. Thus, using the solution to the supplier's problem, we get that the manufacturer's optimal strategy is:

$$q_M^*(N) = \min\left\{k_M\left(N_M + \bar{\eta}_M^H\right), k_S\left(N_S + \bar{\eta}_S^H\right)\right\}$$
$$\eta_M^{H*}(N) = \left[\min\left\{\bar{\eta}_M^H, \frac{k_S\left(N_S + \bar{\eta}_S^H\right)}{k_M} - N_M\right\}\right]^+$$

Combining the optimum manufacturer and supplier strategies in Stages 2 and 3, we get (6) as the equilibrium outcome for the sub-game beginning in Stage 2 in state N.

□

Proof of Lemma 2: Consider firm $i \in \{M, S\}$ with supply chain partner j. We look for equilibrium wage offers; so each firm's wage offers must be best responses to the supply chain partner's wage offers. We assume firm j offers poaching and retention wages according to (10) and (11), respectively. We show that firm i's best response is to offer poaching and retention wages according to (10) and (11), respectively. To do this, we note that firm i would not be willing to pay more than $w_{i,\max}^{(eq)}$ (the marginal benefit of an additional worker) in equilibrium to poach or retain a worker. We further recall that the equations for successful poaching and retention are given in (1) and (2), respectively.

We first focus on firm i's optimal poaching wages. For all of firm j's workers for whom $w_{i,\max}^{(eq)} > \max\left\{w_j^{R*}(\theta), w\right\} + \theta$, it's optimal for firm i to offer a poaching wage equal to $w_i^\theta = \max\left\{w_j^{R*}(\theta), w\right\} + \theta$, which is the minimum wage that must be offered to successfully poach a worker of type θ from firm j. With this wage offer, all of firm j's workers for whom $w_{i,\max}^{(eq)} > \max\left\{w_j^{R*}(\theta), w\right\} + \theta$ are successfully poached by firm i. For all of firm j's workers for which $w_{i,\max}^{(eq)} \leq \max\left\{w_j^{R*}(\theta), w\right\} + \theta$, it's optimal for firm i to offer a poaching wage equal to $w_i^\theta = w_{i,\max}^{(eq)}$. This amount represents the maximum firm i would be willing to pay. However, since these workers are not successfully poached, firm i does not actually have to pay this amount. Substituting in $w_j^{R*}(\theta)$, we see that all of firm j's workers of type $\theta \leq w_{i,\max}^{(eq)} - \max\left\{w, w_{j,\max}^{(eq)}\right\}$ are successfully poached by firm i, and they must be offered $\max\left\{w, w_{j,\max}^{(eq)}\right\} + \theta$ to entice them to leave. The remaining workers employed at firm j are not affordable by firm i and are not poached. Thus, it is optimal to offer them $w_{i,\max}^{(eq)}$. Thus, we find that $w_i^{P*}(\theta) = \min\left\{w_{i,\max}^{(eq)}, \max\left\{w, w_{j,\max}^{(eq)}\right\} + \theta\right\}$.

Now we turn to firm i's optimal retention wages. The marginal benefit of retaining a worker depends on whether the worker could be poached by the supply chain partner. If the worker could not be poached, then the marginal benefit of retaining the worker depends only on either firm i's labor productivity or its marginal hiring cost (depending on whether firm i is the bottleneck or not, respectively), which are both greater than the reservation wage, w. Since the worker could not be poached by the supply chain partner, however, firm i needs only offer w to successfully retain the worker. Given firm j's poaching wage strategy, this holds for all workers of type $\theta \geq \max\left\{w_{j,\max}^{(eq)}, w\right\} - w$.

If the worker could be poached, then firm i must account for the fact that an additional worker retained translates into one less worker firm j could poach. This effect is captured in $w_{i,\max}^{(eq)}$. Thus, for workers of type $\theta \leq \max\left\{w_{j,\max}^{(eq)}, w\right\} - w_{i,\max}^{(eq)}$, firm i would have to offer more than $\max\left\{w_{j,\max}^{(eq)}, w\right\}$ to successfully retain them. However, if $w_{j,\max}^{(eq)} > w_{i,\max}^{(eq)}$, then firm i cannot afford to offer enough to retain the workers; so it optimally offers the most it can afford, $w_{i,\max}^{(eq)}$, to retain them. For worker types $\max\left\{w_{j,\max}^{(eq)}, w\right\} - w_{i,\max}^{(eq)} < \theta < \max\left\{w_{j,\max}^{(eq)}, w\right\} - w$, firm i can retain them with an offer of $\max\left\{w, w_{j,\max}^{(eq)} - \theta\right\}$. Since, for these workers, $w_{i,\max}^{(eq)} > \max\left\{w, w_{j,\max}^{(eq)} - \theta\right\}$, firm i optimally offers $\max\left\{w, w_{j,\max}^{(eq)} - \theta\right\}$ and successfully retains them. All of the cases for optimal retention, then, can be written by $w_i^{R*}(\theta) = \max\left\{w, \min\left\{w_{j,\max}^{(eq)} - \theta, w_{i,\max}^{(eq)}\right\}\right\}$.

Thus, the poaching and retention wages in Lemma 2 are best-responses to firm j's strategy when firm j uses these strategies. Thus, these are equilibrium poaching and retention wage offers.

□

Proof of Lemma 3: In equilibria in which the supplier is the supply chain bottleneck and the manufacturer optimally hires workers in the second stage, the manufacturer's equilibrium maximum optimal wage is given by (9) with (14) substituted in for N_S and N_M. Substituting the resulting expressions into (12) gives the result.

□

Proof of Theorem 1: Here, we prove the general version of Theorem 1 that allows for either firm to be limiting supply chain throughput.

We first assume that an LSE exists in which the firm i is the bottleneck and firm j is firm i's supply chain partner. By the definition of an LSE, the manufacturer must hire a strictly positive number of workers to match, so $\mu_j(\theta_i^T) > 0$. This also implies satisfaction of the second condition by Proposition 3. The fact that firm i is the bottleneck in equilibrium implies that firm i's maximum optimal production capacity at poaching threshold θ_i^T is less than firm j's maximum optimal production capacity at the same poaching threshold. Thus, the third condition is satisfied.

Now we assume that there exists a $\theta_i^T > 0$ that satisfies the three conditions of Theorem 1. Existence of such a poaching threshold implies that $\mu_M(\theta_S^T) > 0$, so the manufacturer must hire a strictly positive number of workers to match the supplier's maximum optimal production capacity. Satisfaction of the second condition implies that the θ_i^T corresponds to an equilibrium poaching threshold. And the third condition implies that firm i is the supply chain bottleneck. Thus, this value of θ_i^T corresponds to an LSE since the equilibrium outcome it fits the definition.

Now we turn to the more involved portion of the proof: showing that if there exists a $\theta_i^T > 0$ that satisfies all of the conditions of Theorem 1, the game has a unique equilibrium outcome. We begin by assuming existence of a $\theta_i^T > 0$ that satisfies all three conditions of Theorem 1. We first show that existence of such a θ_i^T precludes existence of any other equilibrium outcome with firm i as the supply chain bottleneck. To do this, we focus on the second condition of Theorem 1. The left-hand side is continuous and strictly increasing in θ_i^T. The right-hand side is weakly decreasing in θ_i^T since $\mu_j(\cdot)$ is weakly increasing $h'(\cdot)$ is strictly increasing. Thus, since the left-hand side is strictly increasing and the right-hand side is weakly decreasing in θ_i^T, there can exist at most one solution to the second condition. Thus, there are no other equilibria in which firm i is the bottleneck.

Now we show that there are no equilibria in which firm j is the supply chain bottleneck. We first show that there are no equilibrium $\theta_j^T \geq 0$. Existence of such an equilibrium θ_j^T would imply that $k_j\left[\Lambda_j + \bar{\eta}_j^H + P_i\left(\theta_j^T\right)\right] \leq k_i\left[\Lambda_i + \bar{\eta}_i^H - P_i\left(\theta_j^T\right)\right]$, which implies that $k_j\left(\Lambda_j + \bar{\eta}_j^H\right) \leq k_i\left(\Lambda_i + \bar{\eta}_i^H\right)$ since $P(\cdot) \geq 0$. However, since there exists a $\theta_i^T > 0$ satisfying the third condition of Theorem 1, we have for this θ_i^T that $k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(\theta_i^T\right)\right] < k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(\theta_i^T\right)\right]$, so that $k_i\left(\Lambda_i + \bar{\eta}_i^H\right) < k_j\left(\Lambda_j + \bar{\eta}_j^H\right)$, a contradiction. Thus, there are no equilibria in which firm j is the bottleneck and $\theta_j^T \geq 0$.

Finally, we show that no equilibria exist with $\theta_j^T < 0$ given that there exists a $\theta_i^T > 0$ that satisfies the conditions of Theorem 1. To prove this by contradiction, we assume that there exists an equilibrium $\theta_j^T < 0$ that satisfies:

1. $\theta_j^T = k_j m_j + k_j m_i - \left(1 + \frac{k_j}{k_i}\right) h'\left[\frac{k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(-\theta_j^T\right)\right] - k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(-\theta_j^T\right)\right]}{k_i} + \bar{\eta}_i^H\right]$
2. $k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(-\theta_j^T\right)\right] \leq k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(-\theta_j^T\right)\right]$.

Consider the set of all $\theta_j \leq 0$ that satisfy the second condition. That is, let $\Theta_j = \left\{\theta_j < 0 : k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(-\theta_j\right)\right] \leq k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(-\theta_j\right)\right]\right\}$. If the set Θ_j is empty, then the proof is done: there are no $\theta_j \leq 0$ that satisfy the second condition, so there are no equilibria with $\theta_j^T < 0$. So let's assume that the set Θ_j is non-empty. Let $\bar{\theta}_j = \sup \Theta_j$. Since $P_j(\cdot)$ is a positive weakly increasing function, all $\bar{\theta}_j$ satisfy $k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(-\bar{\theta}_j\right)\right] = k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(-\bar{\theta}_j\right)\right]$. We see, then, that our definition of Θ_j and the third condition of Theorem 1 imply that $0 < \theta_i^T < -\theta_j$ for any $\theta_j \in \Theta_j$. Specifically, these imply that $0 < \theta_i^T < -\bar{\theta}_j$. We substitute our definition of $\bar{\theta}_j$ into the right-hand side of the first equilibrium condition and get:

$$
\begin{aligned}
& k_j m_j + k_j m_i - \left(1 + \frac{k_j}{k_i}\right) h'\left[\frac{k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(-\bar{\theta}_j\right)\right] - k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(-\bar{\theta}_j\right)\right]}{k_i} + \bar{\eta}_i^H\right] \\
&= k_j m_j + k_j m_i - \left(1 + \frac{k_j}{k_i}\right) h'\left(\bar{\eta}_i^H\right) \\
&= k_j m_j + k_j m_i - \left(1 + \frac{k_j}{k_i}\right) k_i m_i \\
&= k_j m_j + k_j m_i - k_i m_i - k_j m_i \\
&= k_j m_j - k_i m_i,
\end{aligned}
$$

where the second line follows because $\bar{\theta}_j$ satisfies $k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(-\bar{\theta}_j\right)\right] = k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(-\bar{\theta}_j\right)\right]$ and the third line follows because of the definition of $\bar{\eta}_i^H$. Since the right-hand side of the first equilibrium condition above is weakly decreasing in θ_j^T and the left-hand side is strictly increasing, we get that any $\theta_j^T < 0$ that satisfies both equilibrium conditions above also satisfies $k_j m_j - k_i m_i \leq \theta_j^T \leq \bar{\theta}_j$. This, in turn, implies that it also satisfies $k_i m_i - k_j m_j \geq -\theta_j^T \geq -\bar{\theta}_j$. Now we turn back to examining the $\theta_i^T > 0$ satisfying the equilibrium conditions of Theorem 1. The third equilibrium condition of Theorem 1 implies that the argument of $h'(\cdot)$

in the second equilibrium condition is less than $\bar{\eta}_j^H$. Thus, the strict convexity of $h(\cdot)$ from Assumption 1(a) implies that:

$$h'\left[\frac{k_i\left[\Lambda_i+\bar{\eta}_i^H+P_j\left(\theta_i^T\right)\right]-k_j\left[\Lambda_j+\bar{\eta}_j^H-P_j\left(\theta_i^T\right)\right]}{k_j}+\bar{\eta}_j^H\right]<h'\left(\bar{\eta}_j^H\right)=k_jm_j.$$

Thus, we get that:

$$\begin{aligned}\left(1+\frac{k_i}{k_j}\right)h'\left[\frac{k_i\left[\Lambda_i+\bar{\eta}_i^H+P_j\left(\theta_i^T\right)\right]-k_j\left[\Lambda_j+\bar{\eta}_j^H-P_j\left(\theta_i^T\right)\right]}{k_j}+\bar{\eta}_j^H\right]-k_im_j &< \left(1+\frac{k_i}{k_j}\right)k_jm_j-k_im_j\\ &=k_jm_j.\end{aligned}$$

By Assumptions 1(b) and 1(c), we also have that $k_jm_j>w$. Thus, we get that:

$$\max\left\{w,\left(1+\frac{k_i}{k_j}\right)h'\left[\frac{k_i\left[\Lambda_i+\bar{\eta}_i^H+P_j\left(\theta_i^T\right)\right]-k_j\left[\Lambda_j+\bar{\eta}_j^H-P_j\left(\theta_i^T\right)\right]}{k_j}+\bar{\eta}_j^H\right]-k_im_j\right\}<k_jm_j.$$

Finally, we get that the right-hand side of the second condition of Theorem 1 satisfies:

$$k_im_i-\max\left\{w,\left(1+\frac{k_i}{k_j}\right)h'\left[\frac{k_i\left[\Lambda_i+\bar{\eta}_i^H+P_j\left(\theta_i^T\right)\right]-k_j\left[\Lambda_j+\bar{\eta}_j^H-P_j\left(\theta_i^T\right)\right]}{k_j}+\bar{\eta}_j^H\right]-k_im_j\right\}>k_im_i-k_jm_j.$$

Thus, we get that the right-hand side of the first equilibrium condition of Theorem 1 is always greater than $k_im_i-k_jm_j$ for all θ_i^T that satisfy the third equilibrium condition. Since the right-hand side of the second condition of Theorem 1 is weakly decreasing in θ_i^T and the left-hand side is strictly increasing in θ_i^T, we get that any $\theta_i^T>0$ that satisfies all equilibrium conditions of Theorem 1 also satisfies $\theta_i^T>k_im_i-k_jm_j$. However, from our analysis of the $\theta_j^T<0$ that satisfies the equilibrium conditions above for a Labor-Hoarding Equilibrium with firm j as the bottleneck, we found that $k_im_i-k_jm_j\geq-\theta_j^T\geq-\bar{\theta}_j$. We also found that any $\theta_i^T>0$ that satisfies the second equilibrium condition of Theorem 1 also satisfies $0<\theta_i^T<-\bar{\theta}_j$. Combining these inequalities, we get that:

$$k_im_i-k_jm_j\geq-\theta_j^T\geq-\bar{\theta}_j>\theta_i^T>k_im_i-k_jm_j,$$

a contradiction. Thus, existence of a $\theta_i^T>0$ that satisfies the equilibrium conditions of Theorem 1 precludes existence of a $\theta_j^T<0$ that satisfies the equilibrium conditions above for a Labor-Hoarding Equilibrium with firm j as the bottleneck.

Thus, we have ruled out equilibria with $\theta_j^T<0$ that satisfy the first Labor-Hoarding Equilibrium condition. However, we must also rule out equilibria that may have $\theta_j^T<0$ with a discontinuous jump in the right-hand side of the first Labor-Hoarding Equilibrium condition at θ_j^T. Such an equilibrium would require that $\mu_i\left(-\theta_j^T\right)=0$ and

$$\theta_j^T>k_jm_j+k_jm_i-\left(1+\frac{k_j}{k_i}\right)h'(0)=k_jm_j-h'(0)+\frac{k_j}{k_i}\left[k_im_i-h'(0)\right]>0,$$

where the final inequality follows because of Assumption 1(c). Thus, equilibria in which there exists a discontinuity at the equilibrium poaching threshold cannot exist for $\theta_j^T\leq 0$. Thus, we have now ruled out existence of any equilibria with $\theta_j^T<0$.

Thus, we have shown that existence of a $\theta_i^T>0$ that satisfies the conditions of Theorem 1 precludes existence of any other equilibrium θ_i^T, any equilibrium $\theta_j^T\geq 0$, or any equilibrium $\theta_j^T<0$. Since any equilibrium outcome must fit into one of these bins, it follows that any $\theta_i^T>0$ that satisfies the three conditions of Theorem 1 represents the unique equilibrium outcome.

□

Proof of Corollary 1: We begin by assuming existence of a Labor-Hoarding Equilibrium in which firm i is the bottleneck. By the definition, firm j is the poacher and thus offers a higher wage. This implies that at this equilibrium, $\theta_i^T < 0$. As we showed in the proof of Theorem 1, there cannot exist any $\theta_i^T < 0$ in which (16) is not satisfied with equality in equilibrium. Since $k_i m_i > w$ by Assumption 1(b) and (c), and firm j's equilibrium maximum wage offer is higher than firm i's (which is equal to $k_i m_i$), we get that firm j's maximum wage offer is greater than w. Thus, we get that (16) reduces to the first condition and must be satisfied with equality. Since firm i is the bottleneck, its maximum optimal capacity at the equilibrium poaching threshold must be lower. Thus, the second condition must be satisfied.

Now we assume that there exists a $\theta_i^T < 0$ that satisfies the two conditions of Corollary 1. Since $\theta_i^T < 0$, firm j is willing to offer higher wages at this poaching threshold and is thus the poacher. Also, since the second condition is satisfied, firm i is the supply chain bottleneck at this poaching threshold. Finally, since the first condition is satisfied, this poaching threshold satisfies the equilibrium requirement given that firm i is the supply chain bottleneck at this poaching threshold. Thus, this poaching threshold corresponds to an equilibrium in which firm i is the bottleneck but firm j is poaching workers from firm i. In other words, it corresponds to a Labor-Hoarding Equilibrium.

For uniqueness, we begin by following the proof of Theorem 1 and notice since all equilibria with $\theta_i^T < 0$ must satisfy (16) with equality, the left-hand side of (16) is strictly increasing, and the right-hand side of (16) is weakly decreasing and continuous for all $\theta \geq \theta_i^T$ satisfying the first condition, it must be the only equilibrium with firm i as the bottleneck. Furthermore, as we showed in the proof of Theorem 1, existence of a LHE firm firm i as the bottleneck rules out existence of any LSE with firm j as the bottleneck. Finally, we showed in the proof of Theorem 1 that in any LHE with firm j as the bottleneck, $k_i m_i > k_j m_j$. Since a symmetric result holds in any LHE in which firm i is the bottleneck, it must be that $k_j m_j > k_i m_i$, so no LHE can exist with firm j is the bottleneck since it would imply that $k_i m_i > k_j m_j > k_i m_i$, a contradiction. Thus, we get a similar uniqueness result and $k_j m_j > k_i m_i$.

□

Proof of Theorem 2: We begin by assuming that the two conditions of Theorem 2 are satisfied. Since $k_i\left(\Lambda_i + \bar{\eta}_i^H\right) > k_j \Lambda_j$, $\mu_j(\theta) > 0$. Thus, the right-hand side of (16) is well-defined, continuous, and weakly decreasing for all $\theta \geq 0$. Since $Q_i > 0$ and $k_i m_i > w$, the right-hand side of (16) is positive for $\theta_i^T = 0$. Thus, since the left-hand side is continuous and strictly increasing from zero and the right-hand side is positive for $\theta_i^T = 0$, continuous, and weakly decreasing from Q_i, we get that the fixed point of (16) exists and is strictly greater than zero and weakly less than Q_i. Thus, since $P_j(\cdot)$ is weakly increasing, $P_j\left(Q_i\right) > P_j\left(\theta_i^T\right)$. This implies, then, that $k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(\theta_i^T\right)\right] < k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(\theta_i^T\right)\right]$. Thus, there exists a $\theta_i^T > 0$ that satisfies all conditions of Theorem 1 and thus corresponds to a Labor-Sharing Equilibrium.

□

Proof of Proposition 1: (a) WLOG, let firm i be the supply chain bottleneck. In an LSE, then, the equilibrium poaching threshold, θ_i^T, satisfies $k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(\theta_i^T\right)\right] < k_j\left[\Lambda_j + \bar{\eta}_j^H - P_j\left(\theta_i^T\right)\right]$. The throughput in Model WG is $k_i\left[\Lambda_i + \bar{\eta}_i^H + P_j\left(\theta_i^T\right)\right]$. Since $P_j(\cdot) \geq 0$, the previous inequality implies that $k_i\left(\Lambda_i + \bar{\eta}_i^H\right) <$

$k_j\left(\Lambda_j+\bar{\eta}_j^H\right)$, which implies that firm i is the bottleneck firm in Model G and supply chain throughput is $k_i\left(\Lambda_i+\bar{\eta}_i^H\right)$. Thus, throughput is weakly higher in Model WG.

(b) WLOG, let firm i be the supply chain bottleneck in both Models WG and G. Throughput in Model WG in an LHE is given by $k_i\left[\Lambda_i+\bar{\eta}_i^H-P_i\left(-\theta_i^T\right)\right]$, and throughput in Model G is given by $k_i\left(\Lambda_i+\bar{\eta}_i^H\right)$. Thus, throughput is lower in Model WG than in Model G.

(c) WLOG, let firm j be the bottleneck in Model G. Thus, $k_j\left(\Lambda_j+\bar{\eta}_j^H\right)<k_i\left(\Lambda_i+\bar{\eta}_i^H\right)$. We let

$$\Theta_i=\left\{\theta<0:\left(k_i+k_j\right) P_i(-\theta)>k_i\left(\Lambda_i+\bar{\eta}_i^H\right)-k_j\left(\Lambda_j+\bar{\eta}_j^H\right)\right\}$$

and $\bar{\theta}_i=\sup \Theta_i$. Here, the set Θ_i represents the set of all poaching thresholds that would see firm i as the bottleneck in Model WG. If $k_i \bar{\eta}_i^H<k_j\left(\Lambda_i+\Lambda_j+\bar{\eta}_j^H\right)$, then the set Θ_i is non-empty since $P_i(\cdot) \leq \Lambda_i$. We also see that, based on the definition of $\bar{\theta}_i$,

$$\frac{k_i}{k_j}\left[\Lambda_i+\bar{\eta}_i^H-P_i\left(-\bar{\theta}_i\right)\right]-\Lambda_j-P_i\left(-\bar{\theta}_i\right)>0$$

since it is either equal to $\bar{\eta}_j^H$ or $k_i\left(\Lambda_i+\bar{\eta}_i^H\right)-k_j \Lambda_j$, both of which are strictly positive. Thus, if

$$\bar{\theta}_i>k_i m_i+k_i m_j-\left(1+\frac{k_i}{k_j}\right) h^{\prime}\left(\frac{k_i}{k_j}\left[\Lambda_i+\bar{\eta}_i^H-P_i\left(-\bar{\theta}_i\right)\right]-\Lambda_j-P_i\left(-\bar{\theta}_i\right)\right),$$

then the right-hand side of (16) is less than the left-hand side at $\bar{\theta}_i$. Furthermore, if there exists a worker type, θ_0 such that

$$\frac{k_i}{k_j}\left[\Lambda_i+\bar{\eta}_i^H-P_i\left(-\theta_0\right)\right]-\Lambda_j-P_i\left(-\theta_j\right)=0,$$

then we know (from the proof of Theorem 1) that the right-hand side (16) is greater than zero at θ_0. Furthermore, since $P_i(\cdot)$ is weakly increasing, $\theta_0<\bar{\theta}_i$. We also see that the right-hand side of (16) is continuous for all possible poaching thresholds between θ_0 and $\bar{\theta}_i$. If no θ_0 exists, then the right-hand side of (16) is continuous for all possible poaching thresholds no greater than $\bar{\theta}_i$. Thus, since the right-hand side of (16) is less than the left-hand side at $\bar{\theta}_i$ and is continuous and weakly decreasing, it intersects the left-hand side for some $\theta_i^T<\bar{\theta}_i$. Thus, the conditions of Corollary 1 are satisfied, and the equilibrium poaching threshold corresponds to a Labor-Hoarding Equilibrium with firm i as the bottleneck.

□

Proof of Proposition 2: We begin with equilibrium poaching thresholds in which $\mu_j\left(\theta_i^T\right)=0$ so that (16) is satisfied with equality. Then we turn to the case in which $\mu_j\left(\theta_i^T\right)=0$.

In equilibria in which (16) is satisfied with equality, we show the impact on the poaching threshold by showing the impact on the right-hand side of (16). If a change increases the right-hand side for all θ such that $\mu_j(\theta)>0$, then it weakly increases the equilibrium poaching threshold since the new equilibrium still represents the intersection between the right- and left-hand sides of (16) (for infinitesimal changes). An increase in k_j strictly decreases $\mu_j(\theta)$ for all θ, causing a strict decrease of $h^{\prime}(\cdot)$ for all θ. Thus, the change weakly increases the right-hand side of (16) for all θ and thus weakly increases the poaching threshold. It's possible, however, than an increase in k_j could force θ_i^T to increase to the point that $\mu_j(\theta_i^T)=0$. The transition between regimes occur in a continuous manner at any point at which $\mu_j\left(\theta_i^T\right)=0$ and

$$\theta_i^T=k_i m_i-\max \left\{w,\left(1+\frac{k_i}{k_j}\right) h^{\prime}(0)-k_i m_j\right\}.$$

We define $\theta_0 = \{\theta : \mu_j(\theta) = 0\}$. Using (15), we find that $P_j(\theta_0)$ is strictly increasing in k_j; so we find that the equilibrium poaching threshold is always weakly increasing accompanying an increase in k_j.

Now we turn to the comparative statics on p_F. An increase in p_F strictly (weakly) increases the right-hand side of (16) if the manufacturer (supplier) is the bottleneck. Thus, in equilibria with (16) holding with equality, increasing p_F at least weakly increases the poaching threshold. In equilibria that don't see (16) holding with equality, increasing p_F has no effect until equilibria are reached in which (16) holds again with equality. Thus, increasing p_F causes a weak increase in the poaching threshold. The proof showing the impact of a change in production costs is essentially identical to the proof for the impact of a change in p_F, so we omit them.

$\square$