

Sticky Leverage

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We develop a tractable general equilibrium model that captures the interplay between nominal long-term corporate debt, inflation, and real aggregates. We show that unanticipated inflation changes the real burden of debt and, more significantly, leads to a debt overhang that distorts future investment and production decisions. For these effects to be both large and very persistent it is essential that debt maturity exceeds one period. We also show that interest rate rules can help stabilize our economy.

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The onset of the financial crisis in 2008 triggered the most aggressive monetary policy response in developed countries in at least 30 years. At the same time, financial markets now occupy a much more prominent role in modern macroeconomic theory. Typical models of financial frictions focus on debt and identify leverage as both a source, and an important mechanism of transmission, of economic fluctuations.¹ Surprisingly, the fact that debt contracts are almost always denominated in nominal terms is usually ignored in the literature.^{2,3} Yet, nominal debt creates an obvious link between inflation and the real economy, a potentially important source of monetary nonneutrality even with fully flexible prices.

The goal of this paper is to develop a tractable general equilibrium model that captures the interplay between nominal debt, inflation, and real aggregates, and explore some of its main implications. In our model, as in reality, firms fund themselves by choosing the appropriate mix of nominal defaultable debt and

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¹Some examples include Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), Jermann and Quadrini (2012), Gourio (2013), and Gomes and Schmid (2016).

²Among the very rare exceptions are Dopke and Schneider (2006), Christiano, Motto, and Rostagno (2010), Fernandez-Vilaverde (2010), Bhamra, Fisher and Kuehn (2011), and De Fiore, Teles and Tristani (2011).

³As the end of 2012 U.S. non-financial businesses alone had nearly 12.5 trillion dollars in outstanding credit market debt - about 75 percent of GDP (Federal Reserve Board (FRB 2013)) . Nearly all of these instruments are in the form of nominal liabilities, often issued at fixed rates of interest.

equity securities to issue in every period. Debt is priced fairly by bondholders, who take into account default and inflation risk, but is attractive to issue because of the tax-deductibility of interest payments. Macroeconomic quantities are obtained by aggregating across the optimal decisions of each firm and by ensuring consistency with the consumption, savings and labor choices of representative households.

We have two main results. First, because debt contracts are written in nominal terms, unanticipated changes in inflation, regardless of their source, always have real effects, even if prices and wages are fully flexible. In particular, lower than expected inflation increases the real value of debt, worsens firms' balance sheets, and makes them more likely to default. If defaults and bankruptcies have resource costs, this immediately and adversely impacts output and consumption.

Second, and, more importantly, when debt is long-lived, low inflation *endogenously* creates a debt overhang that persists for many periods - even though debt is freely adjustable in our model. As a result, even surviving firms cut future investment and production plans, as the increased (real) debt lowers the expected rewards to their equity owners. It is this debt overhang phenomenon - emphasized in empirical studies of financial crises but generally missing from standard models with only short term debt - that accounts for most of the effects of changes in inflation on the economy.⁴

After establishing the intuition behind these results in a simple setting with flexible prices, we develop these ideas in the context of a new Keynesian model with sticky prices and a monetary policy rule, linking short-term nominal interest rates to inflation and output. We show that the model produces quantitatively plausible movements in the key macroeconomic quantities. Most notably, with flexible prices, hours are very smooth and investment and consumption tend to respond differently to nominal shocks. By contrast, the model with sticky prices significantly raises the volatility of hours worked and produces more realistic responses to nominal shocks.

The friction we emphasize is probably not suited to understand the response of the economy to all shocks. Nevertheless, we believe an environment with long-term nominal debt contracts offers a perspective of financially driven recessions that is absent from standard models that either abstract from financial frictions or allow only for short term debt. In particular, our setting offers a slightly different insight into the ongoing monetary stimulus around the world. Specifically, in our world standard Taylor rule parameterizations require central banks to try to raise the rate of inflation in response to a debt overhang episode, induced, for example, by a decline in wealth.

One important additional novelty of our paper is that we solve for firms' time-consistent optimal policies for long-term debt when firms can adjust debt freely every period. We present a numerical approach that allows the analysis of model dynamics with perturbation techniques, and as such alleviates the curse of di-

⁴Recent examples include Reinhart and Rogoff (2011) and Mian and Sufi (2014).

dimensionality of fully nonlinear global methods.⁵

While the notion that a debt deflation may have significant macroeconomic consequences goes back at least to Fisher (1933), it has not been incorporated into the modern quantitative macroeconomic literature until quite recently. Our work contributes to this literature by introducing nominal long-term debt in an aggregate business cycle model and studying its role as a nominal transmission channel.

Other macro economic analyses with long-term debt and default include Miao and Wang (2010) and Gomes and Schmid (2016). In both cases the debt is real. In Gomes and Schmid (2016) firms pick their debt at the time of birth and face costly adjustment thereafter. Their focus is on the role of asset prices to capture firm heterogeneity and forecast business cycles. The setting in Miao and Wang (2010) is closer to ours but in their model firms act myopically and fail to take into account that their current leverage choice influences future leverage, and through that, the current value of debt. As a result their approach is not really suitable to fully understand the effects of debt overhang.

The asset pricing implications of allowing for nominal corporate debt in a model driven by productivity and inflation shocks is studied by Kang and Pflueger (2015). Their empirical analysis supports the view that inflation uncertainty raises corporate default rates and bond risk premiums. Their model assumes constant labor and considers only two-period debt.

Fernandez-Villaverde (2010) considers nominal government debt, and Christiano, Motto, and Rostagno (2010) embed nominal entrepreneurial loans into a medium-scale DSGE model. However, both use short-term debt. De Fiore et al. (2011) examine optimal monetary policy when firms have nominal debt with default risk. Eggertson and Krugmann (2012) study leverage and debt overhang with short term debt, but their focus is on real debt and investment plays no role in the analysis. Neither offers a quantitative analysis and both rely on short-term debt. Occhino and Pescatori (2014, 2015) examine debt overhang with one-period debt. Jermann and Yue (2013) study interest rate swaps in conjunction with short-term nominal debt and default.

Some studies on sovereign default have also considered long-term debt in equilibrium models, in particular, Arellano and Ramanarayanan (2012), Aguiar and Amador (2014) and Hatchondo, Martinez and Sosa Padilla (2015). In these studies, debt is real and the problem of a sovereign differs along several dimensions from the problem of a firm in our model. For instance, firms in our model have the ability to issue equity to reduce debt, while sovereigns do not have that choice.

The importance of debt overhang to corporate investment has been studied in the corporate finance literature, but usually in static (real) models that focus solely on optimal firm decisions and where debt overhang arises exogenously. An early example is Myers (1977), and recent dynamic models are provided in

⁵A similar time consistency issue arises in dynamic public policy problems, as studied, for instance, by Klein, Krusell and Rios-Rull (2008).

Hennessy (2004), Moyen (2007), and Chen and Manso (2014).

More broadly, our paper also expands on the growing literature on the macroeconomic effects of financial frictions. This includes Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), Gertler and Karadi (2011) and Jermann and Quadrini (2012).

The next section describes the basic model with flexible prices. Section II examines the key mechanism of regarding the real effects of inflation in a general context. Section III outlines our solution strategy and discusses our quantitative findings for the simple flexible price model. Section IV studies the properties of full model with sticky prices and a monetary policy rule and is followed by a few concluding remarks.

I. Model with Flexible Prices

To isolate the key mechanisms associated with the introduction of long-term nominal debt financing and investment, we first consider a parsimonious model that abstracts from other frictions. Firms own the productive technology and the capital stock in this economy. They are operated by equity holders but partially financed by defaultable debt claims. The firms' optimal choices are distorted by taxes and default costs. Households consume the firms' output and invest any savings in the securities issued by firms. The government plays a minimal role: it collects taxes on corporate income and rebates the revenues to the households in lump-sum fashion. Later we expand this core setting to include other forms of nominal rigidities and a nominal interest rate rule for monetary policy.

A. Firms

We start by describing the behavior of firms and its investors in detail. At any point in time production and investment take place in a continuum of measure one of firms, indexed by j . Some of these firms will default on their debt obligations, in which case they are restructured before resuming operations again. This means that firms remain on-going concerns at all times, so that their measure remains unchanged through time. Although this is not an essential assumption, it greatly enhances tractability to use an environment where all firms make identical choices.⁶

TECHNOLOGY

Each firm produces according to the function:

$$(1) \quad y_t^j = A_t F(k_t^j, n_t^j) = A_t (k)^{\alpha} n^{1-\alpha},$$

⁶Gilchrist, Sim and Zakrajsek (2014) and Gomes and Schmid (2016) present models where the cross-section of firms moves over time with entry and default events.

where A_t is aggregate productivity. Solving for the static labor choice we get the firms' operating profit:

$$(2) \quad R_t k_t^j = \max_{n_t^j} A_t F(k_t^j, n_t^j) - w_t n_t^j$$

where $R_t = \alpha y_t / k_t$ is the implicit equilibrium "rental rate" on capital. Given constant returns to scale, all firms chose identical ratios k^j / n^j , so R_t is identical across firms.

Firm level profits are also subject to additive idiosyncratic shocks, $z_t^j k_t^j$, so that operating profits are equal to:

$$(3) \quad (R_t - z_t^j) k_t^j.$$

We assume that z_t^j is i.i.d. across firms and time, has mean zero, and cumulative distribution $\Phi(z)$ over the interval $[\underline{z}, \bar{z}]$, with $\int_{\underline{z}}^{\bar{z}} \phi(z) dz = \int d\Phi(z)$. We think of these as direct shocks to firms' operating income and not necessarily output. They summarize the overall firm specific component of their business risk. Although they average to zero in the cross section, they can potentially be very large for any individual firm.

Firm level capital accumulation is given by the identity:

$$(4) \quad k_{t+1}^j = (1 - \delta + i_t^j) k_t^j \equiv g(i_t^j) k_t^j$$

where i_t^j denotes the investment to capital ratio.

FINANCING

Firms fund themselves by issuing both equity and defaultable nominal debt. Let B_t^j denote the stock of outstanding defaultable nominal debt at the beginning of period t .

To capture the fact that outstanding debt is of finite maturity, we assume that in every period t a fraction λ of the principal is paid back, while the remaining $(1 - \lambda)$ remains outstanding. This means that the debt has an expected life of $1/\lambda$. In addition to principal amortization, the firm is also required to pay a periodic coupon c per unit of outstanding debt.

For convenience our model assumes only one type of debt which is of equal seniority. In practice, corporations are constantly issuing multiple forms of debt instruments which have somewhat different features. What matters for our purpose is that similar debt instruments are generally issued with identical levels of seniority.⁷

⁷For instance, IBM now has over 30 different bonds outstanding that were issued at different times, all with pari-passu clauses. More broadly, senior unsecured bonds currently account for 68 percent of

Letting q_t^j denote the market price of one unit of debt in terms of consumption goods during period t , it follows that the (real) market value of new debt issues during period t is given by:

$$(5) \quad q_t^j (B_{t+1}^j - (1 - \lambda)B_t^j) / P_t = q_t^j (b_{t+1}^j - (1 - \lambda)b_t^j / \mu_t)$$

where $b_t^j = B_t^j / P_{t-1}$, P_t is the overall price level in period t , and we define $\mu_t = P_t / P_{t-1}$ as the economy-wide rate of inflation between period $t - 1$ and t . We will work with the real value of these outstanding liabilities throughout the remainder of the paper.

DIVIDENDS AND EQUITY VALUE

In the absence of new debt issues, (real) distributions to shareholders are equal to:

$$(1 - \tau) \left(R_t - z_t^j \right) k_t^j - ((1 - \tau)c + \lambda) \frac{b_t^j}{\mu_t} - i_t^j k_t^j + \tau \delta k_t^j.$$

where τ is the firm's effective tax rate. The first term captures the firm's operating profits, from which we deduct the required debt repayments and investment expenses and add the tax shields accrued through depreciation expenditures. This expression for equity distributions is consistent with the fact that interest payments are tax deductible.

It follows that the value of the firm to its shareholders, denoted $J(\cdot)$, is the present value of these distributions plus the value of any new debt issues. It is useful to write this value function in two parts, as follows:

$$(6) \quad J \left(k_t^j, b_t^j, z_t^j, \mu_t \right) = \max \left[0, (1 - \tau) \left(R_t - z_t^j \right) k_t^j - ((1 - \tau)c + \lambda) \frac{b_t^j}{\mu_t} + V \left(k_t^j, b_t^j, \mu_t \right) \right]$$

where the continuation value $V(\cdot)$ obeys the following Bellman equation:

$$(7) \quad V \left(k_t^j, b_t^j, \mu_t \right) = \max_{b_{t+1}^j, k_{t+1}^j} \left\{ \begin{array}{l} q_t^j \left(b_{t+1}^j - (1 - \lambda) \frac{b_t^j}{\mu_t} \right) - (i_t^j - \tau \delta) k_t^j \\ + \mathbf{E}_t M_{t,t+1} \int_{\bar{z}}^{\bar{z}} J \left(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j, \mu_{t+1} \right) d\Phi(z_{t+1}) \end{array} \right\}$$

where the conditional expectation \mathbf{E}_t is taken only over the distribution of aggregate shocks. This value function $V(\cdot)$ thus summarizes the effects of the decisions about future investment and financing on equity values.

Several observations about the value of equity (6) will be useful later. First, limited liability implies that equity value, $J(\cdot)$, is bounded and will never fall below zero. This implies that equity holders will default on their credit obligations

corporate debt, with subordinated debt making up only 5 percent. Banks loans and revolving credit facilities account for most of the rest (S&P Ratings Direct (2014)).

whenever their idiosyncratic profit shock z_t^j is above a cutoff level $z_t^* \leq \bar{z}$, defined by the expression:

$$(8) \quad (1 - \tau) \left(R_t - z_t^{j*} \right) k_t^j - ((1 - \tau) c + \lambda) \frac{b_t^j}{\mu_t} + V \left(k_t^j, b_t^j, \mu_t \right) = 0.$$

It is this value z_{t+1}^{j*} that truncates the integral in the continuation value of (7).

Second, the stochastic discount factor $M_{t,t+1}$ is exogenous to the firm and must be determined in equilibrium, in a manner consistent with the behavior of households/investors. Finally, the value function is homogenous of degree one in capital k_t^j and debt b_t^j and so is the default cutoff z_t^{j*} .

DEFAULT AND CREDIT RISK

The firm's creditors buy corporate debt, at price q_t^j , and collect coupon and principal payments, $(c + \lambda) \frac{b_{t+1}^j}{\mu_{t+1}}$, until the firm defaults.

In default, shareholders walk away from the firm, while creditors take over and restructure the firm. Creditors become the sole owners and investors of the firm and are entitled to collect the current after tax operating income $(1 - \tau) \left(R_{t+1} - z_{t+1}^j \right) k_{t+1}^j$. After this restructuring, creditors resume their customary role by selling off the equity portion to new owners while continuing to hold the remaining debt. This means that in addition to the current cash flows, the creditors have a claim that equals the total enterprise, or asset, value, $V \left(k_{t+1}^j, b_{t+1}^j \right) + q_{t+1}^j (1 - \lambda) b_{t+1}^j$.⁸

Restructuring entails a separate loss, in the amount ξk_{t+1}^j , with $\xi \in [0, 1]$.⁹

With these assumptions, the creditors' valuation of their holdings of corporate debt at the end of period t is:¹⁰

$$(9) \quad b_{t+1}^j q_t^j = \mathbb{E}_t M_{t,t+1} \left\{ \begin{array}{l} \Phi(z_{t+1}^{j*}) \left[c + \lambda + (1 - \lambda) q_{t+1}^j \right] \frac{b_{t+1}^j}{\mu_{t+1}} + \int_{z_{t+1}^{j*}}^{\bar{z}} \left[(1 - \tau) \left(R_{t+1} - z_{t+1}^j \right) k_{t+1}^j \right. \\ \left. + V \left(k_{t+1}^j, b_{t+1}^j, \mu_{t+1} \right) + (1 - \lambda) \frac{q_{t+1}^j b_{t+1}^j}{\mu_{t+1}} - \xi k_{t+1}^j \right] d\Phi(z_{t+1}) \end{array} \right\}.$$

The right hand side of this expression can be divided in two parts. The first term reflects the cash flows received if no default takes place, while the integral contains the payments in default, net of the restructuring charges.

It is immediate to establish that this market value of corporate debt is decreas-

⁸This is only one of several equivalent ways of describing the bankruptcy procedures that yields the same payoffs for shareholders and creditors upon default. Equivalently we could assume that they sell debt and continue to run the firm as the new equity holders.

⁹We can think of these costs as including legal fees, but also other efficiency losses and frictions associated with the bankruptcy and restructuring processes. These costs represent a collective loss for bond and equity holders, and may also imply a loss of resources for the economy as a whole.

¹⁰Note that creditors discount the future using the same discount factor as shareholders, $M_{t,t+1}$. This is consistent with our assumption that they belong to the same risk-sharing household.

ing in restructuring losses, ξ , and the default probability, implied by the cutoff z^{j*} . It can also be shown that debt prices are declining in the expected rate of inflation - since equity values increase in μ_{t+1} . Finally, note that q_t^j is homogeneous of degree zero in k_{t+1}^j and b_{t+1}^j .

All together, our assumptions ensure that when the restructuring process is complete a defaulting firm is indistinguishable from a non defaulting firm. All losses take place in the current period and are absorbed by the creditors. Since all idiosyncratic shocks are i.i.d. and there are no adjustment costs, default has no further consequences. As a result, both defaulting and non defaulting firms adopt the same optimal policies and look identical at the beginning of the next period.

B. Households

The general equilibrium is completed with the household sector. This is made of a single representative family that owns all securities and collects all income in the economy, including a rebate on corporate income tax revenues. Preferences are time separable over consumption C and hours worked, N :

$$(10) \quad U = E \left\{ \sum_{t=0}^{\infty} \beta^t [\log C_t + \theta \log (3 - N_t)] \right\}$$

where $\beta \in (0, 1)$ is the rate of inter temporal preference. Assuming log preferences is not crucial but makes it easier to introduce sticky wages later. In addition, this precise specification allows us to normalized the steady state number of hours worked to 1.

As is common in the literature, we find it useful to assume that each member of the family works or invests independently in equities and debt, and all household income is then shared when making consumption and savings decisions.

C. Equilibrium and Aggregation

Given the optimal decisions of firms and households implied by the problems above we can now characterize the dynamic competitive equilibrium in this economy.

We focus on Markov perfect equilibria where the aggregate state vector is $s = (B, K, \mu, A)$, where B and K denote the aggregate levels of debt and capital in the economy. The nature of the problem means that, outside default, this equilibrium is symmetric, in the sense that all firms make identical decisions at all times. The only meaningful cross sectional difference concerns the realization of the shocks z_t^j which induce default for a subgroup of firms with mass $1 - \Phi(z^*)$. Default implies a one time restructuring charge for firms, but these temporary losses have no further impact on the choices concerning future capital and debt. Thus all

firms remain ex-ante identical in all periods so that we can drop all subscripts j for firm specific variables so that in equilibrium $B_t = b_t$ and $K_t = k_t$.

Aggregate output in the economy, Y_t , can be expressed as:

$$(11) \quad Y_t = y_t - [1 - \Phi(z^*)] \xi^r \xi k_t.$$

As discussed above, ξk_t captures the loss that creditors suffer in bankruptcy. Some of these losses may be in the form of legal fees and might be recouped by other members of the representative family. But some may represent a genuine destruction of resources. The relative balance between these two alternatives is governed by the parameter $\xi^r \in [0, 1]$. In the special case where $\xi^r = 0$, default entails no loss of resources at the aggregate level.

The aggregate capital stock, $K_t = k_t$, obeys the law of motion:

$$(12) \quad K_{t+1} = (1 - \delta) K_t + I_t$$

where aggregate investment is $I_t = i_t k_t$.

To complete the description of the economy we require that both goods and labor markets clear. This is accomplished by imposing the aggregate resource constraint:

$$(13) \quad Y_t = C_t + I_t$$

and the labor market consistency condition:

$$(14) \quad N_t = n_t.$$

II. Characterization

To highlight the economic mechanisms at the heart of the model, this section characterizes the firms' leverage choice.

In this section we show that with long term debt real leverage responds persistently to i.i.d. inflation shocks. That is, nominal leverage is effectively sticky. A debt overhang channel then transmits changes in real leverage to changes in real investment.

Under constant returns to scale, the firm's problem is linearly homogenous in capital and therefore its leverage ratio, defined as $\omega = b/k$, is the single endogenous state variable. As a result, it seems natural to use this measure of leverage ratio in the characterization of our findings.¹¹

Conditionally on not defaulting the value of a firm per unit of capital, $v(\omega) =$

¹¹This definition of leverage is also the most commonly used in the literature and is consistent with the empirical constructs we use to calibrate the model. An alternative but less convenient definition is the so called market leverage ratio, b/V . Regardless of the measure used however, in our model, an inflation shock has no effect on the long run leverage ratio, and a short term transition is produced through a debt overhang effect.

V/k , can be written as:

$$(15) \quad v(\omega) = \max_{\omega', i} \left\{ \begin{array}{l} q \left(\omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) - i + \tau \delta \\ + g(i) EM' \int_{\underline{z}}^{z^{*'}} \left[(1 - \tau) (R' - z') - ((1 - \tau) c + \lambda) \frac{\omega'}{\mu'} + v(\omega') \right] d\Phi(z') \end{array} \right\}$$

where we use primes to denote future values, and the definition $g(i) = (1 - \delta + i)k$. For ease of notation we omit the dependence on the aggregate state variables for the functions $v(\omega)$, $q(\omega')$, as well as for prices M and R .

The market value of the outstanding debt (9) can be expressed as:

$$(16) \quad \omega' q(\omega') = EM' \left\{ \begin{array}{l} \Phi(z^{*'}) \left[c + \lambda \frac{\omega'}{\mu'} + (1 - \lambda) \frac{q'(h(\omega'))\omega'}{\mu'} \right] \\ + (1 - \Phi(z^{*'})) \left[(1 - \tau) R' - \xi + v(\omega') \right] - (1 - \tau) \int_{z^{*'}}^{\bar{z}} z' d\Phi(z) \end{array} \right\}.$$

Explicitly writing next period's debt price $q(h(\omega'))$ as a function of the optimal policy, $\omega' = h(\omega)$, on the right hand side of equation (16), highlights the potential time inconsistency problem faced by the firm. With long lived debt, the price of debt $q(\omega')$ depends on future debt prices and thus on next period's leverage choice ω'' . As no commitment technology is available, time consistency requires that next period's leverage be a function of the current policy choice, so that $\omega'' = h(\omega')$.

Finally, the optimal default cutoff level, z^* , can be expressed as a function of the leverage ratio, as:

$$(17) \quad z^*(\omega) = R - c \frac{\omega}{\mu} - \frac{\lambda}{(1 - \tau)} \frac{\omega}{\mu} + \frac{1}{(1 - \tau)} v(\omega).$$

Differentiating this expression with respect to outstanding leverage ω we get:

$$(18) \quad \frac{\partial z^*(\omega)}{\partial \omega} = - \left(c + \frac{\lambda}{1 - \tau} \right) \frac{1}{\mu} + \frac{1}{(1 - \tau)} \frac{\partial v(\omega)}{\partial \omega} < 0.$$

Intuitively, an increase in outstanding debt increases the required principal and coupon payments, and by reducing the cutoff $z^*(\omega)$, makes default more likely.¹²

A. Debt Overhang and the Impact of Inflation

In this subsection, we characterize firm behavior in response to an exogenous change in the inflation rate, μ . To isolate how a shock to μ is propagated in the model, it is assumed in this section that inflation follows an exogenous i.i.d

¹²The envelope condition implies that:

$$(19) \quad \frac{\partial v(\omega)}{\partial \omega} = -q \frac{1 - \lambda}{\mu} \leq 0.$$

When debt maturity exceeds one period ($\lambda < 1$) an increase in outstanding debt decreases the (expected) future payments to equity holders further encouraging default.

process. For the quantitative analysis later in the paper, the inflation rate is persistent and endogenously driven by real and monetary shocks.

The necessary first order conditions with respect to investment and leverage are given by:¹³,

(20)

$$1 - q(\omega') \omega' = EM' \int_{\underline{z}}^{z^*(\omega')} \left[(1 - \tau)(R' - z') - ((1 - \tau)c + \lambda) \frac{\omega'}{\mu'} + v(\omega') \right] d\Phi(z')$$

and

(21)

$$q(\omega') g(i) + \frac{\partial q(\omega')}{\partial \omega'} \left(\omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) = - (1 - \tau) g(i) EM' \Phi(z^*) \frac{\partial z^*(\omega')}{\partial \omega'}.$$

The first order condition for investment, (20), equates the marginal reduction in equity cash flows today, on the left hand side, to the expected increase in (after tax) dividend and capital gains tomorrow, after netting out any debt payments. This equation captures the debt overhang channel through which the default friction distorts the equilibrium capital allocation.

The optimal condition for leverage, (21), recognizes that the debt price, $q(\omega')$, falls when new debt is issued, $\frac{\partial q(\omega')}{\partial \omega'} < 0$, thus reducing the marginal benefits of more debt today. The marginal costs of new debt, on the right hand side, reflects the impact of new debt on the probability of future default, $\frac{\partial z^*(\omega')}{\partial \omega'}$.

The following proposition shows that the effect from unanticipated inflation depends crucially on the maturity of debt.

PROPOSITION 1: *Consider an economy where optimal choices are described by the optimality conditions (20)-(21), and where: (1) there are no aggregate resource costs associated with bankruptcy, i.e., $\xi^r = 0$; and (2) all realizations of exogenous shocks have been zero for a long time so that at time $t - 1$: $\mu_{t-1} = \bar{\mu}$, $\omega_t = \bar{\omega}$ and all the other variables are at their steady state values. Suppose that at time t the economy experiences a temporary decline in the inflation rate so that $\mu_t < \mu_{t-1}$. Then, if $\lambda = 1$, $\omega_{t+1} = \bar{\omega}_t$ and $i_t = i_{t-1}$.*

Proof. With $\lambda = 1$, the current inflation rate, μ_t , has no direct effect on the choice of $\omega' = \omega_{t+1}$ in (21). Moreover, since μ_t is i.i.d., there is no effect on the expected default cutoff (17) and the equilibrium price of debt, $q(\omega')$. In addition, since $\xi^r = 0$, there are no resource costs and neither aggregate consumption, C , nor the stochastic discount factor, M' , are affected. It follows that there are no indirect general equilibrium effects either, and the optimal choice of leverage, ω_{t+1} is unaffected by the shock. Finally, μ_t does not appear in (20) and therefore has no direct effect on the choice of i . ■

¹³We assume throughout this section that first order conditions are also sufficient. It is straightforward to derive conditions on the distribution for the idiosyncratic shock $\Phi(z)$ to guarantee that this is true when $\lambda = 1$.

Proposition 1 establishes that i.i.d movements in inflation will not be propagated when debt maturity equals one period. As is clear from equation (20) current inflation has no direct impact on the first-order condition for investment. Therefore, without indirect effects through M' and R' , incentives to invest will only change if leverage changes. That is, inflation shocks are transmitted to real investment, if at all, through a debt overhang channel.

When $\lambda < 1$ debt maturity exceeds one period and a decline in μ directly affects the marginal benefit of issuing new debt. Specifically, the unanticipated decline in the rate of inflation μ increases the (real) value of currently outstanding liabilities, $(1 - \lambda)b/\mu$, and everything else equal, raises the marginal benefit from leverage, because $\frac{\partial q(\omega')}{\partial \omega'} < 0$. Intuitively, as $(1 - \lambda)b/\mu$ has increased, an unchanged level of leverage ω' lowers the amount of debt that is issued, $(\omega'g(i) - (1 - \lambda)\frac{\omega}{\mu})$, and thus also reduces the impact on debt prices. As a result equilibrium leverage ω' will typically increase persistently while i declines following even i.i.d. inflation shocks.¹⁴

Persistent responses of real leverage can also be understood as the reflection of the fact that the *marginal* debt price effect in equation (21), $\frac{\partial q(\omega')}{\partial \omega'} (\omega'g(i) - (1 - \lambda)\frac{\omega}{\mu})$, acts like an (endogenous) convex adjustment cost that slows down the response to shocks. Because this marginal price effect is an increasing function of the amount of debt issued, $(\omega'g(i) - (1 - \lambda)\frac{\omega}{\mu})$ it discourages the firm from making rapid changes to its leverage.¹⁵

Figure 1 illustrates this analysis with an example of impulse responses to a one time decrease in the price level - equivalently an i.i.d realization of unexpectedly low inflation. As shown in the third row, the permanent decline in the price level eventually produces an identical decline in *nominal* debt, B and leverage, B/K , so that in the long run there are no real effects to this shock. Initially, however, the drop in nominal debt only partially offsets the inflation shock - nominal debt and leverage are effectively sticky. As a result, we can see in the second row that *real* leverage, ω , and the default rate, $\Phi(z^*)$, both remain elevated for a prolonged period of time while investment also declines persistently.

Finally, note that by relaxing the extreme assumptions in Proposition 1, persistent movements in leverage will occur even when $\lambda = 1$. This will happen when either the underlying inflation movements are persistent or if there are some resource costs associated with default ($\xi^r \neq 0$).¹⁶

¹⁴Although we have no formal proof, it is always true that in our simulations unanticipated low inflation increases ω' and reduces i .

¹⁵Models with collateral constraints, such as Kiyotaki and Moore (1997) or Jermann and Quadrini (2012), require a separate friction on equity or debt issuance to produce real effects from shocks to the collateral constraint. Here, with defaultable long term debt, no additional friction is needed.

¹⁶In our quantitative analysis the latter are typically not very strong.

III. Quantitative Analysis

We now investigate the quantitative importance of the frictions induced by nominal long term debt. To better understand these results we continue to abstract from the effects of any other nominal rigidities and considers only the model with flexible prices. A full quantitative characterization also requires two preliminary steps: a numerical solution strategy and a choice of parameter values. We discuss each of these issues in detail before reporting our main results.

A. Solution Strategy

The competitive equilibrium is characterized by (11)-(21), plus the first-order conditions for consumption and labor supply implied by the household's preferences (10). Later we also add a monetary policy rule guiding the dynamics of inflation.

The solution of the model is significantly complicated by the presence of the derivative $\frac{\partial q(\omega')}{\partial \omega'}$ in (21). To understand the role of this derivative let us differentiate the debt price function (16) to obtain:

$$(22) \quad q(\omega') + \omega' \frac{\partial q(\omega')}{\partial \omega'} = EM' \left\{ \begin{array}{l} \Phi(z^*(\omega'))(c + \lambda) \frac{1}{\mu'} + (1 - \Phi(z^*)) v_\omega(\omega') \\ + (1 - \lambda) \frac{q(h(\omega'))}{\mu'} + (1 - \lambda) \frac{\omega'}{\mu'} \frac{\partial q}{\partial \omega'}(h(\omega')) \cdot h_\omega(\omega') \\ + \frac{\partial z^*}{\partial \omega}(\omega') \phi(z^*(\omega')) \left(\begin{array}{l} -(1 - \tau)(R' - z^*(\omega')) \\ + (c + \lambda) \frac{\omega'}{\mu'} + \xi - v(\omega') \end{array} \right) \end{array} \right\}.$$

Hence, the derivative of the debt price, $\frac{\partial q(\omega')}{\partial \omega'}$, is linked to the marginal impact of the current leverage choice, ω' , on the future choice, ω'' , here captured by the derivative of the policy function $h_\omega(\omega')$. The presence of this term complicates the solution by standard local approximation methods because $h_\omega(\omega)$ is unknown and must be computed together with the policy function itself, $h(\omega)$. Essentially, there is one additional variable to solve for, namely $h_\omega(\omega)$, without an additional equation.

Our strategy is to differentiate both the derivative of the debt price function, equation (22), and the first-order condition for leverage, (21). The two resulting equations now include second-order derivatives for the debt price and the policy function, $\frac{\partial^2 q(\omega')}{\partial \omega'^2}$ and $h_{\omega\omega}(\omega)$. Again we need a boundary condition, but this will now be imposed on a higher order term. Specifically, we assume that the derivative of the policy function exhibits constant elasticity in ω , but is otherwise unrestricted, so that:

$$\ln h_\omega = A_1(s) + h_1 \ln \omega$$

where $A_1(s)$ is allowed to be any arbitrary function of the state vector, s . This implies a restriction that can be used as an additional equation to characterize

local dynamics:

$$(23) \quad h_{\omega\omega} = \frac{h_{\omega}}{\omega} h_1.$$

since h_1 is a constant that can be determined from the deterministic steady state. This approach does not constrain the first-order dynamics of h_{ω} , and yields a system of equations that can be fully characterized using first-order perturbation methods.¹⁷

Solving for the deterministic steady state is also more involved than for standard models, because the presence of $h_{\omega}(\omega)$ leaves the system of nonlinear equations that characterize the deterministic steady state short by one equation. To address this problem, we instead compute the deterministic steady state using value function iteration over a grid for ω . Computing time is relatively short because the model is deterministic and there is a univariate grid. Also, this global solution only needs to produce the steady state value for ω , and not all the derivatives of the policy function. This is because the nonlinear system of equations for the deterministic steady state is only short one equation. Effectively, knowing the steady state value for ω provides us with the missing equation. Our appendix provides additional details.

As an alternative, Miao and Wang (2010) solve a model with real long-term debt by taking the extreme approach of setting $h_{\omega}(\omega') = 0$, thus ignoring any of these effects. Effectively, in their solution, firms act myopically not realizing that their current leverage choice influences future leverage and through that the current value of debt.

The presence of derivatives of unknown functions that characterizes the solution of our model is also a feature of time-consistent solutions for problems of dynamic public policy, as studied, for instance, by Klein, Krusell and Rios-Rull (2008). In their model, the government anticipates how future policy will depend on current policy via the state of the economy. Our solution method shares some of the features of the approach described in their paper.

Our solution approach allows us to overcome the particular challenges implied by time-consistent firm behavior with long-term debt without all the limitations of fully nonlinear global methods. Indeed, once the deterministic steady state for the firms' problem is found, our approach can take advantage of the scalability of perturbation methods to easily handle additional state variables, such as a nominal interest included in a monetary policy rule.

B. Parameter Choices

The model is calibrated at quarterly frequency. While most parameters are chosen to match steady state targets, some are pinned down by model simulations.

¹⁷Assuming instead that $h_{\omega\omega}$ equals its constant steady state value, produces slightly different local dynamics.

With a few exceptions noted below the empirical moments are computed over the 1955.I–2012.IV period. All macroeconomic data comes from the Federal Reserve Bank of St. Louis web site (FRED). Table 1 summarizes our baseline parameter choices.

TECHNOLOGY AND PREFERENCES

Our choices for the capital share α , depreciation rate δ , and the subjective discount factor β all correspond to fairly common values in the literature. Together they determine the capital-output and investment-output ratios, as well as the average rate of return on capital in our economy. As long as they remain in a plausible range, the quantitative properties of the model are not very sensitive to these parameter values.

PRODUCTIVITY SHOCKS

We assume the following general AR(1) representation for the stationary component of aggregate productivity:

$$\ln A_t = \rho_a \ln A_{t-1} + \sigma_a \varepsilon_t^a.$$

We construct series for Solow residuals using data on GDP, hours, capital stock and the GDP deflator. This yields estimates of $\rho_a = 0.97$ and $\sigma_a = 0.007$.

To summarize the behavior of idiosyncratic profit shocks we adopt a general quadratic approximation to its pdf:

$$(24) \quad \phi(z) = \eta_1 + \eta_2 z + \eta_3 z^2.$$

Our earlier assumptions about the distribution's mean imply that $\eta_2 = 0$. Imposing symmetry on $\phi(z)$ with $\bar{z} = -\underline{z} = 1$ ensures that there is only one free parameter, η_1 . Its value is selected to target the unconditional volatility of the leverage ratio, a key variable in our model. While η_1 is closely linked to the volatility of leverage, for the range of values for η_1 for which the model can be solved, the model cannot produce enough volatility in leverage to fully match the empirical calibration target.

Together with the average leverage ratio, ω , and expected debt life, $1/\lambda$, the value for η_1 is a crucial determinant of the impact and persistence of shocks on firm leverage and investment. This is because the choice of η_1 governs the mass of firms accumulated around the default of $\phi(z^*)$. If this mass is sizable, shocks can have large impacts on the default probability, $\Phi(z^*)$, and on bond prices, $q(\omega)$.

LEVERAGE PARAMETERS

The parameter λ determines average debt maturity. This is an important parameter for determining the propagation of shocks. We choose a value of $\lambda = 0.05$

per quarter, implying an average expected maturity of 5 years, similar to initial maturities of industrial and commercial loans, and significantly shorter than those for corporate bonds. Given the importance of this parameter, we prefer to focus on the results when debt maturity is conservatively calibrated. We also document how results change when average debt maturities change. The periodic coupon rate, c is much less important. We set it to $c = \exp(\mu)/\beta - 1$, so that the price of default free debt is equal to one.

Given c and λ , the expressions for bond prices (16) and optimal investment (20) link the default loss parameter, ξ , and the tax wedge, τ , with the steady-state levels of the default rate $\Phi(z^*)$ and the leverage ratio, ω . Default rates are chosen to closely match Moody's average default rate of 0.26 percent per quarter (Moody's Investor Service (2014)). The leverage ratio is constructed using data from the FRB (2013) and defined as the ratio of credit market instruments to real assets (at current cost) plus cash and cash equivalent holdings for the U.S. non-financial business sector. This equals 42 percent for both entire sample and the sub period since 1971. Recent values however are much higher, with corporate debt ratios averaging almost 52 percent between 2005 and 2009.

With this data we estimate values of $\xi = 0.29$ and $\tau = 0.4$. The chosen default cost parameters implies average steady-state recovery rates at default of about 30 percent. Higher default cost implies that firms are less likely to default in equilibrium so that overall expected default costs remain unchanged. Finally we assume all restructuring charges involve a deadweight resource loss so that $\xi^r = 1$.¹⁸

INFLATION DYNAMICS

In the basic model with flexible prices the dynamics of inflation is exogenous and determined by the monetary authority. Hence, in this section we will directly model the dynamics of inflation by assuming it follows an AR(1) process that matches both the quarterly volatility and persistence of this variable. For the period between 1955.I – 2012.IV these are estimated to be 0.004 and 0.85, respectively.

SUMMARY

Table 1 summarizes our parameter choices for the benchmark calibration. The model is quite parsimonious and requires only 10 structural parameters, in addition to the stochastic process for the shocks. Table 2 shows the implications of these choices for the first and second (unconditional) moments of a number of key variables.

¹⁸The (statutory) wedge implied by corporate income rates and the tax treatment of individual interest and equity income during this period is about 25 percent. As a result we should think of τ as capturing other relative benefits of using debt rather than equity (e.g. issuance costs).

The second panel in Table 2 shows that our quantitative model shares many of the properties of other variations of the stochastic growth model. All the main aggregates, have plausible volatilities, except for labor, as is typical in standard stochastic growth model. The empirical standard deviation for default rates is available only at an annual frequency, and is equal to 1.4 percent (Moody's Investor Service (2014)). The model counterpart is quite close at 1.6 percent.

TABLE 1—Calibration

Parameter	Description	Value
Preferences and Technology		
β	Subjective Discount Factor	0.99
γ	Risk Aversion	1.00
θ	Elasticity of Labor	0.63
α	Capital Share	0.36
δ	Depreciation Rate	0.025
Leverage and Default		
λ	Debt Amortization Rate	0.05
ξ^r	Fraction of Resource Cost	1.00
ξ	Default Loss	0.29
τ	Tax Wedge	0.40
η_1	Distribution Parameter	0.6815
Shocks		
ρ_a	Persistence Technology Shock	0.97
σ_a	Volatility Technology Shock	0.007
ρ_μ	Persistence Inflation	0.85
$\sigma_{\epsilon,\mu}$	Volatility Inflation Shocks	0.004

C. Findings

We now investigate the quantitative importance of the frictions induced by nominal long-term debt. We are interested in determining how the model responds to movements in the inflation rate and how much endogenous propagation can plausibly be generated by the combination of our sticky leverage and debt overhang mechanisms.

RESPONSE TO INFLATION CHANGES

Proposition 1 shows that under very general conditions, even pure i.i.d movements in inflation can induce prolonged movements in corporate leverage and

TABLE 2—**Aggregate Moments.** THE REPORTED FIRST MOMENTS ARE COMPUTED AS AVERAGES IN THE DATA, AND STEADY STATE VALUES IN THE MODEL. SECOND MOMENTS ARE STANDARD DEVIATIONS BASED ON HP1600-FILTERED DATA AND MODEL SERIES. STANDARD ERRORS ARE NEWEY-WEST CORRECTED.

	Model	Data	Standard Error
First Moments			
Investment/Output, I/Y	0.24	0.20	0.0071
Leverage	0.42	0.42	0.0081
Default Rate, $1 - \Phi(z^*)$	0.0022	0.0026	0.0015 ^a
Second Moments			
σ_Y	1.44	1.65	0.0013
σ_I/σ_Y	3.46	4.23	0.1824
σ_C/σ_Y	0.37	0.52	0.0277
σ_N/σ_Y	0.38	1.07	0.1422
σ_ω	0.67	1.7	0.0015
σ_μ	0.5	0.5	0.0005

^a standard error of the annual default rate

investment. Figure 2 shows how more realistic changes in the inflation rate are reflected in the general equilibrium response of the key macroeconomic aggregates.

Following a one standard deviation negative shock to inflation, the default rate increases as the real value of outstanding corporate liabilities increases. This increase in the default rate immediately produces output losses since restructuring costs are not rebated to households and represent real deadweight losses. For a 0.4 percent innovation in inflation, investment and output decline at impact by 2.5 percent and 0.5 percent, respectively.

Inflation shocks have particularly strong and persistent effects on investment. Strong impact effects on investment are not particularly surprising since other models of financial frictions often produce large distortions in the investment Euler equation (for instance, Jermann and Quadrini (2012)). Because our household/investors have log preferences, risk premia are very small and the source of the financial friction is tied to the *expected* loss upon default - which depends on the default loss parameter, ξ .

The more striking result, a priori, is the large persistence in the investment response, which mirrors that in default rates and leverage, and is driven by the “sticky leverage” and debt overhang effects discussed above. As Proposition 1 implies, leverage - here we report its market value, $q\omega'$ - rises and remains elevated for a long time even though inflation quickly returns to its long run mean, leading to a prolonged, and significant, contraction in investment spending.

Figure 2 captures one weakness in this flexible price model. Initially, at least, changes in inflation mainly induce intratemporal reallocation between consump-

tion and investment expenditures. While the increase in the required rental rate on capital works to stifle investment, it leads to a short term boom in consumption. Eventually, the lower capital stock leads to lower output, and consumption begins to decline. Labor initially mirrors the behavior of consumption, as households seek to smooth their leisure decisions as well, but, over time, reduced capital contributes to lowering the marginal product of labor. To address this issue we will consider a version of the model with sticky prices in the next section.¹⁹

Finally, it is worth restating that, with flexible prices, the exact source of inflation is irrelevant. The responses of the key variables will be similar regardless of whether the inflation movements are exogenous or induced by some monetary policy rule. All that matters for these dynamics is the actual behavior of the inflation rate.

VARIANCE DECOMPOSITION

Table 3 shows the contribution of inflation shocks to the total variance of key macro and financial aggregates in the flexible price model as well as the sensitivity of this measure with respect to some key assumptions.²⁰

Regardless of our calibration, inflation shocks are always the dominant source of variations in leverage and default rates. In addition, in our benchmark model inflation is also responsible for 44 percent of the variance of investment and 23 percent of the variance of output. Clearly, even in this flexible price model, inflation nonneutralities can be quantitatively important.

In the baseline case, leverage matches the data for the period since 1955. When our economy is calibrated to an average leverage ratio of only 32 percent, inflation still accounts for about a quarter of the movements in investment. If, instead, the leverage ratio matches the value observed in the period between 2005-2009, which is 52 percent, inflation shocks account for two thirds of the investment variance and nearly half of the variance of output.

Table 3 also shows that with one-period debt, real quantities are almost unaffected by movements in the inflation rate, and we essentially recover monetarily neutrality. There are two reasons for this. First, for a given inflation process, percentage gains and losses on bonds produced by inflation are quite small with short maturities. Second, the debt overhang channel is entirely absent with one-period debt.

IV. Model with Nominal Rigidities

The flexible price model is very useful to illustrate the key role of “stickiness” of nominal liabilities and the persistent effect of inflation movements on economic

¹⁹Another possible alternative is to instead use GHH preferences. In this case consumption and labor do not move on impact and will decline afterwards.

²⁰Since movements in TFP are the only other source of fluctuations here, the fraction of the variance coming from TFP shocks is 1 minus the number reported in the table.

TABLE 3—Variance Decomposition (due to inflation)

	Output	Investment	Consumption	Hours	Leverage	Default
Benchmark	0.23	0.44	0.16	0.12	0.88	0.99
Shorter Maturity, $\lambda = 0.06$	0.22	0.42	0.14	0.10	0.86	0.99
One-period Debt, $\lambda = 1.00$	0.00	0.00	0.00	0.00	0.98	0.99
High Leverage (0.52)	0.43	0.70	0.58	0.46	0.93	0.98
Low Leverage (0.32)	0.10	0.22	0.04	0.03	0.64	0.99

aggregates induced by the overhang of long-term debt. However, the model fails to match the volatility of aggregate hours. More significantly, as shown in Figure 2, the model suffers from a basic “co-movement” problem in that debt overhang leads to impact to an intratemporal reallocation of resources towards consumption.

In addition, the flexible price model isolates the determination of inflation, thus failing to allow for the joint endogeneity of aggregate prices and quantities. To address these concerns we now consider an expanded version of model that allows for nominal price rigidities. Adding this standard form of money non-neutrality both improves the model’s ability to fit the data and allows us to better appreciate the relative importance of long-term nominal debt to macroeconomic fluctuations.

To accomplish this we modify the model by introducing a new economic agent: monopolistically-competitive retailers. While our firms remain perfectly competitive they now produce (intermediate) goods, that are sold to retailers, before being finally passed to the final consumers. We now also explicitly model the behavior of the monetary authority.

A. Retailers

There is a continuum of retailers, indexed by $r \in [0, 1]$. Retailers buy goods from the intermediates producers at real (or deflated) price/cost P_{mt} , package them, and sell them to households at nominal price P_{rt} . Each retailer acts as a monopolist. The final good, used for consumption and investment, is a CES aggregator of the retailers output, denoted Y_{rt} :

$$(25) \quad Y_t = \left[\int_0^1 (Y_{rt})^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}$$

where ϵ is linked to the elasticity of substitution across the different varieties of goods sold by retailers. It follows that each retailer faces the demand curve:

$$(26) \quad Y_{rt} = Y_t \left[\frac{P_{rt}}{P_t} \right]^{-\epsilon}.$$

Following Calvo (1983) and Christiano, Eichenbaum and Evans (2005), we assume each retailer can change their price optimally in period t with probability $1 - \psi$. A retailer then chooses P_t^* optimally to solve:

$$(27) \quad \max_{P_t^*} \mathbb{E}_t \sum_{i=0}^{\infty} \psi^i M_{t,t+i} \left[\frac{P_t^*}{P_{t+i}} - P_{m,t+i} \right] Y_{r,t+i}.$$

Note that this implies that the optimal price, P_t^* , will be identical across the retailers setting prices at time t .

The optimal price setting behavior implies the following law of motion for the aggregate price level (Calvo (1983)):

$$(28) \quad P_t = \left[\int_0^1 P_{ft}^{1-\epsilon} \right]^{1/(1-\epsilon)} = \left[(1 - \psi)(P_t^*)^{1-\epsilon} + \psi [P_{t-1}]^{1-\epsilon} \right]^{1/(1-\epsilon)}$$

This formulation requires us to specify only two new parameter values, the elasticity of substitution, ϵ , and the probability of price fixing, ψ .

B. Monetary Authority

In accordance with the literature we assume the monetary authority uses the Taylor rule:

$$(29) \quad \ln r_t = v_0 + \rho_r \ln r_{t-1} + (1 - \rho_r) [v_\mu \ln \mu_t + v_y \ln (Y_t/Y_{t-1})] + \zeta_t,$$

where r_t is the nominal (gross) one period interest rate, which obeys the Euler equation:

$$(30) \quad r_t = \frac{1}{\mathbb{E}_t M_{t,t+1} / \mu_{t+1}},$$

and ζ_t is an exogenous monetary policy shock.

C. Quantitative Analysis

To study the properties of this sticky price model we need to specify the values for a number of additional parameters. Regarding the monetary policy rule, the responses to inflation and output growth are set to $v_y = 0.2$ and $v_\mu = 1.5$ while the smoothing parameter is estimated to be $\rho_r = 0.5$. These values are consistent with estimates from the literature.²¹ The volatility and persistence of the monetary policy shock, ζ_t , are set so that the model matches the quarterly volatility and persistence of the inflation rate.

²¹See Clarida, Gali, and Gertler (2000).

The elasticity of substitution between goods sold by retailers, ϵ , is set to 5, close to the value in Gertler and Karadi (2011), and the estimates in Primiceri, Schaumburg, and Tambalotti (2006). Regarding price stickiness, we follow Christiano, Eichenbaum and Evans (2005) and assume $\psi = 0.6$ so that retailers re-optimize prizes once every 2.5 quarters, on average.²²

Table 4 compares the predictions of the flexible and sticky price versions of the model to the unconditional moments of key macro variables. Notably, the version of the model with sticky prices produces much higher volatility in work hours, matching their volatility relative to output over the business cycle.²³

TABLE 4—**Aggregate Moments.** THE REPORTED FIRST MOMENTS ARE COMPUTED AS AVERAGES IN THE DATA, AND STEADY STATE VALUES IN THE MODEL. SECOND MOMENTS ARE STANDARD DEVIATIONS BASED ON HP1600-FILTERED DATA AND MODEL SERIES. STANDARD ERRORS ARE NEWEY-WEST CORRECTED.

	Flexible Prices	Sticky Prices	Data	Standard Error
First Moments				
Investment/Output, I/Y	0.24	0.20	0.20	0.0071
Leverage, ω	0.42	0.42	0.42	0.0081
Default Rate, $1 - \Phi(z^*)$	0.0022	0.0026	0.0026	0.0015 ^b
Second Moments				
σ_Y	1.44	1.45	1.65	0.0013
σ_I/σ_Y	3.46	4.98	4.23	0.1824
σ_C/σ_Y	0.37	0.41	0.52	0.0277
σ_N/σ_Y	0.38	1.05	1.07	0.1244
σ_ω	0.67	0.69	1.7	0.0015
σ_μ	0.5	0.5	0.5	0.0005

^b standard error of the annual default rate

Figures 3 and 4 document the response of the key variables in the model to monetary and total factor productivity shocks. In both figures the blue lines show the baseline responses in the model with long-term debt, while the green lines depict the case where debt maturity is just one period.

It is apparent that the responses are always larger in the model with long-term debt. The differences are particularly significant for monetary policy shocks. Figure 3 shows that even though inflation dynamics are very similar, the model with long-term debt produces a much larger drop in investment, hours worked and output. As before, these general equilibrium responses are largely due to the

²²Our choice is also close to the micro evidence from Bills and Klenow (2004).

²³We also recalibrate the distribution parameter for the idiosyncratic shocks, η_1 , to 0.695. This preserves the volatility of leverage across model versions.

long-lasting movements in corporate leverage and the effects of debt overhang on investment.

The figure also shows that the sticky price model does not suffer from a “co-movement” problem: now both consumption and investment drop following a monetary tightening. This is because sticky prices lead to a much larger immediate drop in output in response to this shock.

It is well known that the effects of technology shocks can be amplified by adding financing frictions to corporate investment. Figure 4 shows that these effects can be further amplified (and propagated) when these frictions take the form of long-term nominal debt. While the differences between the long-term and short-term debt cases are smaller than for the monetary shocks, the responses of the main macro quantities exhibit more persistence when debt maturity is longer.

Notably, with long-term debt, the model can generate a realistic drop in hours worked. This is not the case when debt matures after one period. To obtain an initial drop in hours worked in this case we would need to allow for a larger (and less plausible) degree of price rigidity, ψ .

VARIANCE DECOMPOSITION

Table 5 re-examines the contribution of monetary shocks to the total variance of key macro and financial aggregates with sticky prices. The model with sticky prices exhibits even stronger monetary non neutralities. Now even when debt lasts for only one period ($\lambda = 1$), nominal shocks account for about 34 percent of the total variance of GDP and nearly 47 percent of that in investment.

Interestingly, the table also shows that the effects of two nominal frictions are almost additive. The contribution of nominal shocks to the variance of GDP in the sticky price model with long-term debt (38 percent) is essentially equal to the sum of the contributions of sticky prices (24 percent, for $\lambda = 1$) plus that of sticky long-term debt (23 percent, in the flexible price model).

For investment and consumption, the two nominal frictions have typically also about the same impact. Sticky prices are more important for the variation of hours, but corporate leverage and default are driven almost exclusively by debt.

A COLLAPSE IN WEALTH

As a final experiment, we examine the case of a large decline in the value of the stock of capital in the economy. This experiment can be viewed as capturing some aspects of the contraction seen since 2007/08, with sharply declining real estate values. Formally, this is implemented by an unexpected decrease in the value of the capital stock k of 5 percent, through a one-time increase in the depreciation rate, δ . Our implementation is similar to that in Gertler and Karadi (2011) and Gourio (2013). Unlike these authors, however, our model does not require a persistent shock to generate a prolonged decline in investment after the initial period.

TABLE 5—Variance Decompositions (due to inflation)

Panel A: Flexible Price Model						
	Output	Investment	Consumption	Hours	Leverage	Default
Benchmark	0.23	0.44	0.16	0.12	0.88	0.99
Shorter Maturity, $\lambda = 0.06$	0.22	0.42	0.14	0.10	0.86	0.99
One-period Debt, $\lambda = 1.00$	0.00	0.00	0.00	0.00	0.98	0.99
High Leverage (0.52)	0.43	0.70	0.58	0.46	0.93	0.98
Low Leverage (0.32)	0.10	0.22	0.04	0.03	0.64	0.99

Panel B: Sticky Price Model						
	Output	Investment	Consumption	Hours	Leverage	Default
Benchmark	0.38	0.73	0.06	0.90	0.92	0.99
Shorter Maturity, $\lambda = 0.06$	0.37	0.72	0.06	0.90	0.90	0.98
One-period Debt, $\lambda = 1.00$	0.24	0.47	0.02	0.87	0.94	1.00
High Leverage (0.52)	0.44	0.82	0.13	0.92	0.94	0.99
Low Leverage (0.32)	0.35	0.66	0.05	0.89	0.90	0.98

Figure 5 examines the effects of this shock in the neoclassical (flexible price) model with long-term debt, where inflation does not move at all, and in the full New Keynesian model where the monetary authority induces an endogenous response of inflation. On impact, the destruction of the capital stock lowers both firm and equity values. This leads to an immediate spike in corporate defaults, and an increase in leverage ratios. Without a monetary response inflation remains constant (green lines), and the debt overhang leads to long-lasting real declines in investment, consumption and output. It is particularly striking that despite the large destruction of capital, investment is below average for several periods after the shock.

In the New Keynesian model (blue lines) however, monetary policy responds with a sustained increase in inflation which reduces the burden of outstanding liabilities and default rates. This in turn mitigates the magnitude of the decline in the macroeconomic aggregates and the economy recovers a lot faster. With a 5 percent reduction in the capital stock, the model's implied inflation rises about 1 percent above its steady state level over the first year after the shock.

Although only suggestive, this experiment is quite consistent with a policy prescription in the immediate aftermath of the crisis and summarized by Rogoff (Bloomberg, May 19, 2009):

“I’m advocating 6 percent inflation for at least a couple of years. It would ameliorate the debt bomb and help us work through the deleveraging process.”

V. Conclusion

In this paper we have presented a general equilibrium model with nominal long-term debt that can help us better understand the monetary non-neutralities associated with Irving Fisher's (1933) debt deflation. The model also sheds some light on the slowdown following the 2008 financial crisis and possible monetary policy responses. Our model is capable of generating very large and persistent movements in output and investment, even without price rigidities. Adding standard price rigidities helps the model producing more volatile labor and more realistic co-movements between investment and consumption.

Almost unavoidably, our attempt to write a parsimonious and tractable model leaves out many important features. In particular, we ignore nominal debt contracts other than those held by firms, even though household debt is roughly equal in magnitude and subject to similarly large restructuring costs.

Our analysis also abstracts from the role of movements in credit risk premia and the behavior of asset prices in general. In addition, while convenient, the assumption of constant returns to scale, which nearly eliminates firm heterogeneity and renders the model so tractable, also limits our ability to study firm behavior more comprehensively.

We leave the explorations of these and other simplifying assumptions for future work. Nevertheless we believe none is essential to the main ideas in the paper.

REFERENCES

- Aguar, Mark and Manuel Amador. 2014. "Take the Short Route: How to Repay and Restructure Sovereign Debt with Multiple Maturities." Unpublished.
- Allen, Franklin, Elena Carletti and Douglas Gale. 2013. "Money, Financial Stability and Efficiency." *Journal of Economic Theory*.
- Arellano and Ramanarayanan. 2012. "Default and the Maturity Structure in Sovereign Bonds." *Journal of Political Economy*.
- Bhamra, Harjoat, Adlai Fisher and Lars-Alexander Kuehn. 2011. "Monetary Policy and Corporate Default." *Journal of Monetary Economics*.
- Bernanke, Ben, and Mark Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review*.
- Bernanke, Ben, Mark Gertler, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework", in *Handbook of Macroeconomics*, Michael Woodford and John Taylor, ed. North Holland.
- Bils, Mark and Peter Klenow. 2004. "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy*.
- Calvo, Guillermo. 1983. "Staggered Prices in a Utility Maximizing Framework." *Journal of Monetary Economics*.

- Carlstrom, Charles and Timothy Fuerst. 1997. "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis." *American Economic Review*.
- Chen, Hui and Gustavo Manso. 2014. "Macroeconomic Risk and Debt Overhang." Unpublished.
- Chen, Hui, Michael Michaux and Nikolai Roussanov. 2016. "Houses as ATMs? Mortgage Refinancing and Macroeconomic Uncertainty." Unpublished.
- Christiano, Lawrence, Martin Eichenbaum, and Charles Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*.
- Christiano, Lawrence, Roberto Motto, and Massimo Rostagno. 2010. "Financial Factors in Economic Fluctuations." European Central Bank Working Paper 1192.
- Clarida, Richard, Jordi Gali, and Mark Gertler, 2000. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *Quarterly Journal of Economics*.
- Cochrane, John, 2011, "Determinacy and Identification with Taylor Rules". *Journal of Political Economy*, Vol. 119, No. 3, pp. 565-615.
- Cooley, Thomas, Ramon Marimon and Vincenzo Quadrini. 2004. "Aggregate Consequences of Aggregate Contract Enforceability." *Journal of Political Economy*.
- De Fiore, Fiorella, Pedro Teles, and Oreste Tristani. 2011. "Monetary Policy and the Financing of Firms." *American Economic Journal: Macroeconomics*.
- Doepke, Matthias and Martin Schneider. 2006. "Inflation and the Redistribution of Nominal Wealth." *Journal of Political Economy*.
- Fernandez-Vilaverde, Jesus, 2010. "Fiscal Policy in a Model With Financial Frictions." *American Economic Review: Papers and Proceedings*.
- Eggertsson, Gauti and Paul Krugman. 2012. "Debt, Deleveraging, and the Liquidity Trap." *Quarterly Journal of Economics*.
- Federal Reserve Economic Data. 1952-2012. <https://research.stlouisfed.org/fred2> (accessed May 4, 2016). Federal Reserve Bank of St. Louis.
- Federal Reserve Board. 2013. "Financial Accounts of the United States: Nonfinancial Business. <https://www.federalreserve.gov/releases/z1/> (accessed March 7, 2013).
- Fisher, Irving. 1933. "The Debt-Deflation Theory of Great Depressions." *Econometrica*.
- Gertler, Mark, Simon Gilchrist, and Fabio Natalucci. 2007. "External Constraints on Monetary Policy and the Financial Accelerator." *Journal of Money, Credit and Banking*.
- Gertler, Mark and Karadi, Peter. 2011. "A Model of Unconventional Monetary Policy." *Journal of Monetary Economics*.

- Gilchrist, Simon, Jae W. Sim and Egon Zakrajsek. 2014. "Uncertainty, Financial Frictions, and Investment Dynamics." NBER Working Paper 20038.
- Gomes, Joao and Lukas Schmid, 2016. "Equilibrium Asset Pricing with Leverage and Default." Unpublished..
- Gourio, Francois, 2013 "Credit Risk and Disaster Risk." American Economic Journal: Macroeconomics.
- Hatchondo, Juan Carlos, Leonardo Martinez and César Sosa Padilla. 2015. "Debt Dilution and Sovereign Default Risk." Center for Applied Economics and Policy Research 2015-092.
- Hennessy, Christopher. 2004. "Tobin's Q, Debt Overhang, and Investment." Journal of Finance.
- Jermann, Urban, and Vincenzo Quadrini, 2012. "Macroeconomic Effects of Financial Shocks." American Economic Review.
- Jermann, Urban, and Vivian Yue. 2013. "Interest Rate Swaps and Corporate Default.", Unpublished.
- Kang, Johnny and Carolin Pflueger. 2015. "Inflation Risk in Corporate Bonds." Journal of Finance.
- Kiyotaki, Nobuhiro and John Moore. 1997. "Credit Cycles." Journal of Political Economy.
- Klein, Paul, Per Krusell and Jose Victor Rios-Rull. 2008. "Time-Consistent Public Policy." Review of Economic Studies.
- Mian, Atif and Amir Sufi. 2014. "What Explains the 2007-2009 Drop in Employment?" Econometrica
- Miao, Jianjun and Pengfei Wang. 2010. "Credit Risk and Business Cycles." Boston University Working Papers 2010- 033.
- Moody's Investor Service. 2014 "Annual Default Study: Corporate Default and Recovery Rates 1920-2013: Exhibit 30"
- Moyen, Nathalie. 2007. "How Big is the Debt Overhang Problem." Journal of Economic Dynamics and Control.
- Myers, Stuart, 1977. "Determinants of Corporate Borrowing." Journal of Financial Economics.
- Occhino, Filippo and Andrea Pescatori. 2014. "Leverage, Investment, and Optimal Monetary Policy." The B.E. Journal of Macroeconomics, Contributions.
- Occhino, Filippo and Andrea Pescatori. 2015. "Debt Overhang in a Business Cycle Model." European Economic Review.
- Piazzesi, Monika and Martin Schneider. 2012. "Inflation and the Price of Real Assets." Unpublished.
- Primiceri, Giorgio, Ernst Schaumburg, and Andrea Tambalotti. 2006. "Intertemporal Disturbances." NBER Working paper 12243.

Reinhart, Carmen and Kenneth Rogoff. 2011. "From Financial Crash to Debt Crisis." *American Economic Review*.

Bloomberg, 2009. "U.S. Needs More Inflation to Speed Recovery, Say Mankiw, Rogoff." Bloomberg, May 9. <http://www.bloomberg.com>.

Standard and Poor's Capital IQ. 2014. Ratings Direct, McGraw Hill.

VI. Appendix: Solution Method

The dynamics of the model are characterized using a first-order perturbation approximation around the deterministic steady state.

To compute the deterministic steady state, R is initially taken as given. Three equations, (15), (16), and (17), are used to solve for the steady state level of leverage by value function iteration. The first-order condition for investment is then used to find an updated value for R , and this is repeated until convergence.

The full model's steady state is then obtained by combining the steady state value for leverage with the equilibrium conditions (11)-(21), the first-order conditions for consumption and labor supply implied by the household's preferences (10), (22) and the derivatives of (22) and (21) with respect to ω .

To study the model's dynamics, this system of equations is augmented with one equation determining the behavior of the second derivative of the policy function, $h_{\omega\omega} = (h_{\omega}/\omega) h_1$, which is based on the assumption that the first derivative of the policy function exhibits constant elasticity in ω . The coefficient h_1 is set equal to $h_{\omega\omega} (\omega/h_{\omega})$ evaluated at the deterministic steady state.

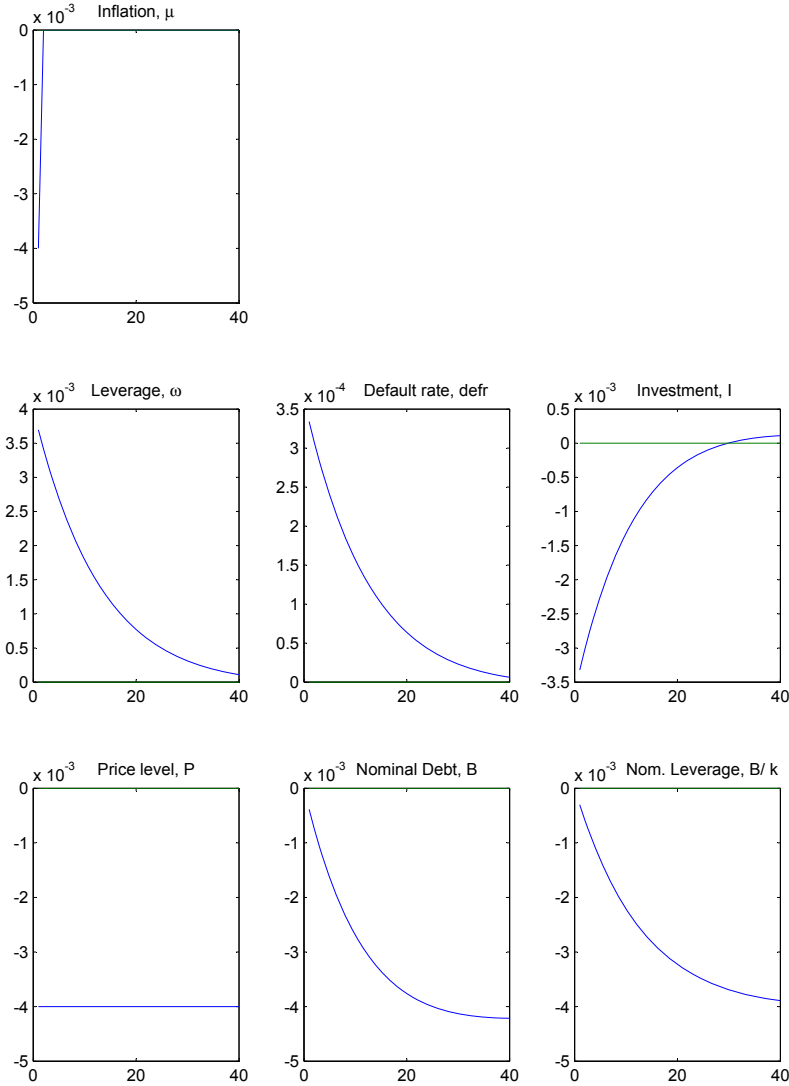


FIGURE 1. A DECLINE IN THE PRICE LEVEL. THIS FIGURE SHOWS THE EFFECT OF AN UNANTICIPATED ONE TIME DECLINE IN THE PRICE LEVEL, ON THE KEY VARIABLES OF THE MODEL.

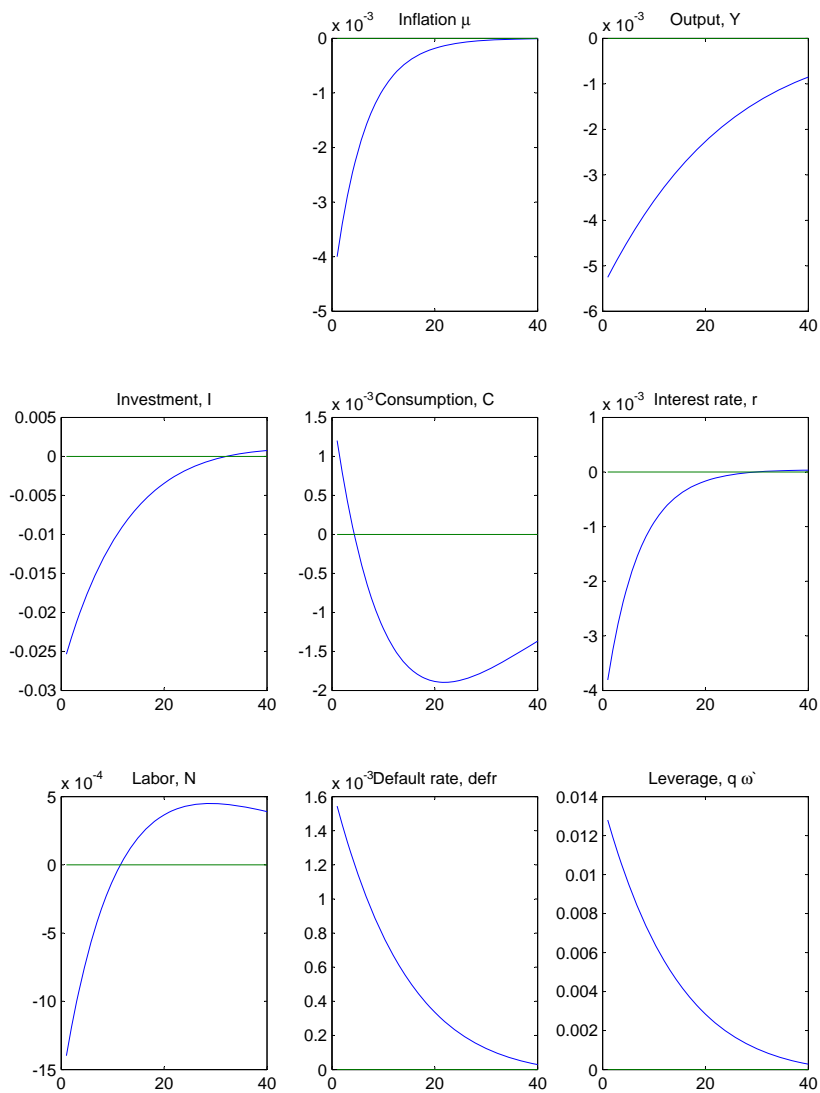


FIGURE 2. AN INFLATION SHOCK. THIS FIGURE SHOWS THE EFFECT OF A UNANTICIPATED ONE STANDARD DEVIATION DECLINE IN THE INFLATION RATE, μ_t , ON THE KEY VARIABLES OF THE MODEL.

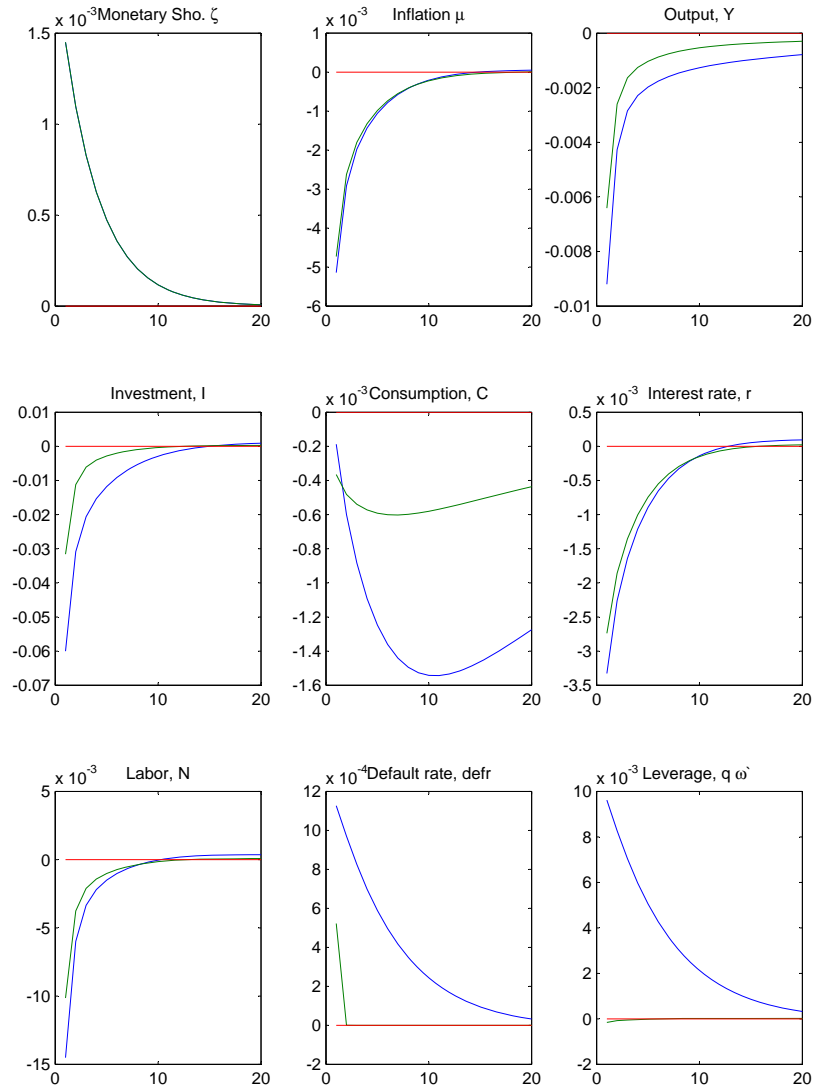


FIGURE 3. A MONETARY SHOCK. THIS FIGURE SHOWS THE EFFECT OF AN EXOGENOUS SHOCK TO THE MONETARY POLICY RULE ON THE KEY VARIABLES OF THE MODEL, WITH LONG-TERM DEBT, $\lambda = 0.05$ (BLUE LINE) AND SHORT-TERM DEBT, $\lambda = 1$ (GREEN LINE).

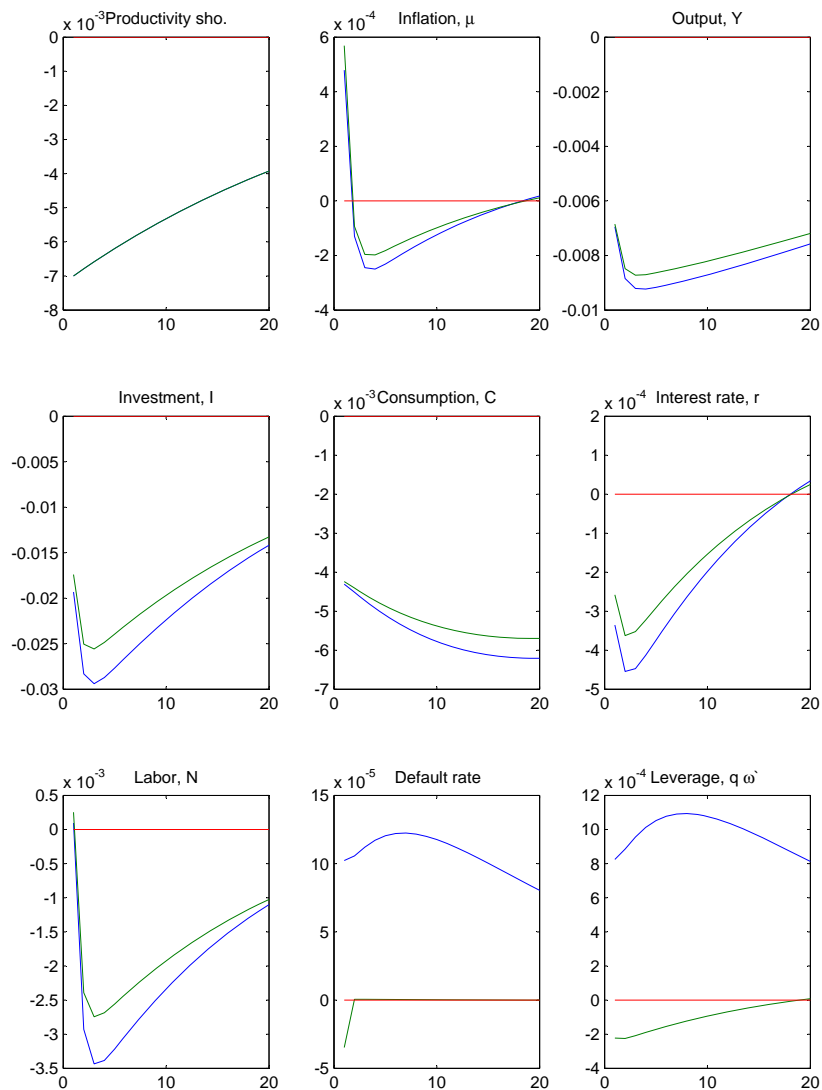


FIGURE 4. A NEGATIVE PRODUCTIVITY SHOCK. THIS FIGURE SHOWS THE EFFECT OF A NEGATIVE PRODUCTIVITY SHOCK ON THE KEY VARIABLES OF THE MODEL, WITH LONG-TERM DEBT, $\lambda = 0.05$ (BLUE LINE) AND SHORT-TERM DEBT, $\lambda = 1$ (GREEN LINE).

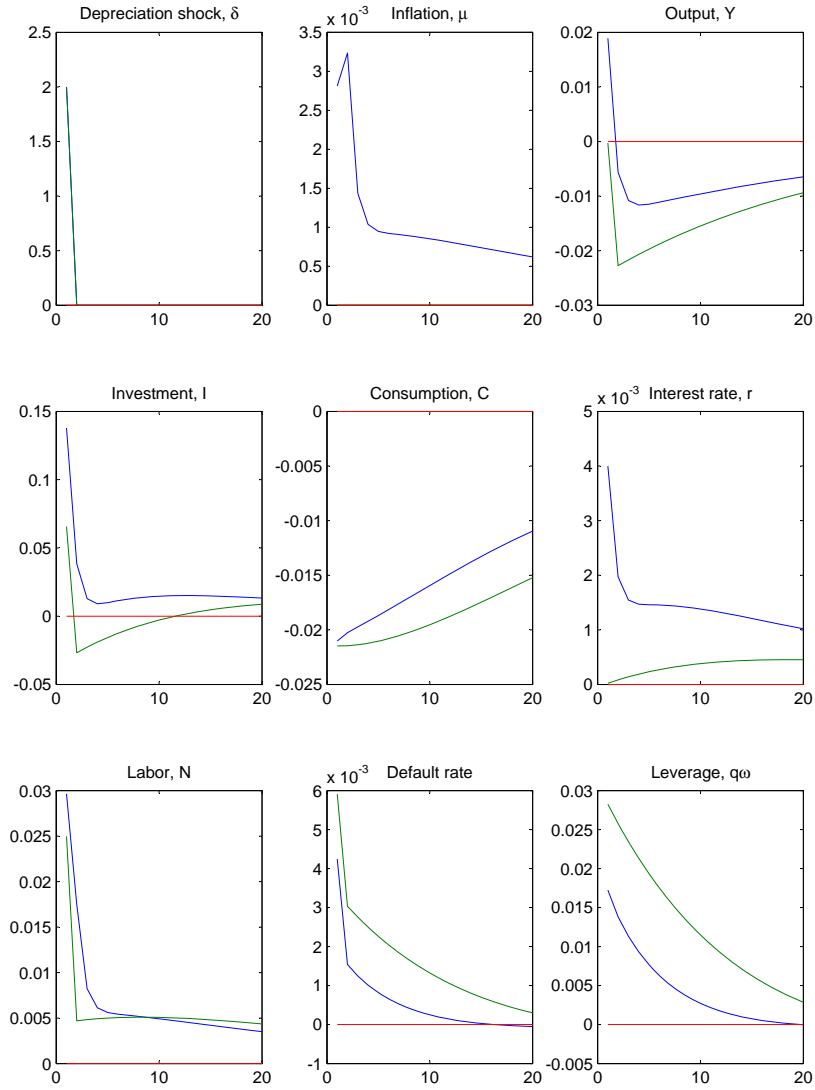


FIGURE 5. A WEALTH SHOCK. THIS FIGURE SHOWS THE EFFECT OF A ONE-TIME EXOGENOUS DESTRUCTION OF THE CAPITAL STOCK, k , BY 5 PERCENT THROUGH AN INCREASE OF THE DEPRECIATION RATE, δ . IT COMPARES THE MODEL'S RESPONSES IN THE NEOCLASSICAL MODEL WHEN INFLATION IS UNCHANGED (GREEN LINE) AND IN THE NEW KEYNESIAN MODEL WHEN IT ADJUSTS ENDOGENOUSLY ACCORDING TO THE MONETARY POLICY RULE (BLUE LINE).