Equilibrium Asset Pricing with Leverage and Default^{*}

João F. Gomes[†] Lukas Schmid[‡]

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ABSTRACT

We develop a general equilibrium model linking the pricing of stocks and corporate bonds to endogenous movements in corporate leverage and aggregate volatility. The model has heterogeneous firms making optimal investment and financing decisions and connects fluctuations in macroeconomic quantities and asset prices to movements in the cross-section of firms. Empirically plausible movements in leverage produce realistic asset return dynamics. Countercyclical leverage drives predictable variation in risk premia, and debt-financed growth generates a high value premium. Endogenous default produces countercyclical aggregate volatility and credit spread movements that are propagated to the real economy through their effects on investment and output.

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 $^{^\}dagger {\rm The}$ Wharton School, University of Pennsylvania. Email: gomesj@wharton.upenn.edu

[‡]The Fuqua School of Business, Duke University. Email: lukas.schmid@duke.edu

We, Joao Gomes and Lukas Schmid, have read and agree with the JF's Submission Guidelines and Policies and the Conflict of Interest Disclosure

Disclosure-statement

- 1. I, Joao Gomes, have research support from the University of Pennsylvania.
- 2. I do not have any paid or unpaid positions with any relevant organization.
- 3. No other party was given the right to review the paper prior to its circulation.

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1 Introduction

It is now well understood that leverage is a major driver of risk exposure and a key contributor to macroeconomic fluctuations. Leverage pushes many corporations to default in downturns often with substantial losses. In turn, expectations of those losses affect the pricing of corporate debt whose issuance aids successful financing of growth options and helps accelerating expansions. Debtfinanced booms and debt-driven busts then contribute to aggregate volatility, and are reflected in asset returns, as the Great Recession of 2008-09 has reminded us. In spite of this, developing a framework suitable to study the joint determination of corporate investment and leverage decisions of firms, macroeconomic fluctuations, and risk premia on stocks and corporate bonds has proved challenging.

This paper is an attempt to fill this gap. It presents a general equilibrium model with heterogeneous firms making optimal investment and financing decisions under uncertainty, and brings together many core insights from asset pricing, capital structure, and macroeconomics. Our model reconciles, in a unified framework, several core stylized facts about asset returns while also addressing many key features in macroeconomic aggregate and firm-level investment and financing variables. Specifically, we show that our model produces a sizable average equity premium and credit spread, together with plausibly low average returns on safe assets. In the time series the model also implies that both price-dividend ratios and credit spreads have substantial predictive power for future stock returns, while the cross-section of stock returns delivers a significant value premium.

In the model, quantitatively realistic asset return dynamics are driven by empirically plausible, endogenous movements in leverage, both in time series and cross-section. In fact, a major contribution of our model is that it delivers an explicit connection between fluctuations in the cross-sectional distribution of firms and the time-series movements in macroeconomic aggregates and financial prices. Indeed, this link is critical, as the mass of firms close to default, and hence the credit spread, becomes a key determinant of aggregate volatility and asset prices.

Endogenous movements in leverage contribute to the amplification and propagation of aggregate consumption risks and volatility. Debt-financed booms and busts amplify aggregate volatility, while accounting for a realistic long-term maturity structure of corporate debt significantly increases the persistence of fluctuations. This amplification raises the volatility of the market price of risk and produces quantitatively realistic risk premia. Importantly, endogenous default also increases the volatility of consumption during recessions, as the mass of firms burdened by excessive leverage and closer to default grows. As a consequence, the equilibrium market price of risk also becomes sharply countercyclical. Endogenous movements in leverage also explain much of our findings about predictability in both time-series and cross-section. Countercyclical leverage drives up risk premia on financial assets in downturns which, in the time series, is naturally reflected in both price-dividend ratios and credit spreads. Cross-sectionally, because investment is, at least partially, debt financed, value firms tend to have higher leverage ratios and these cross-sectional differences in leverage between growth and value firms amplify the dispersion in equity risk, and are a major driver of the value premium.

Some of these mechanisms are also shared by several partial equilibrium models of equity returns, even if leverage is exogenous and there are no financing frictions.¹ In such models however, leverage affects assets' conditional betas only through a direct cash flow effect which is often magnified by correlated, but *exogenous*, movements in discount rates. By contrast, in our general equilibrium setting, the main impact of leverage is felt indirectly though its general equilibrium impact on the stochastic discount factor. This is because movements in leverage are endogenously linked to the dynamics of aggregate consumption. To be sure, in our model, both cash flow and discount rate effects are important and interact with each other. Nevertheless, it is the general equilibrium movements in consumption dynamics and the stochastic discount rate effect that arise as quantitatively more important determinants of asset return dynamics.

Because defaults tend to cluster in downturns, when the market price of risk is high, credit spreads contain a significant and volatile credit risk premium compensating consumers for losses in bad states. Accordingly, credit spreads exhibit significant time-series variation that spills over into the real economy. In expansions, default risk and the market price of risk are low, so that debt-financed investment is cheap, while credit spreads spike up in recessions, due to rises in default rates and the credit risk premium. These endogenous movements in credit prices contribute to amplify the effects of shocks and generate more pronounced business cycle fluctuations. Much like in the data, credit spreads predict business cycles, providing an effective early warning for impending recessions. This is because the risk premium is very informative about the tail of the cross-sectional firm distribution beyond aggregate productivity.²

Finally, our model also informs empirical work linking capital structure and returns by highlighting potential pitfalls associated with the measurement of leverage. Specifically, we show that while there is a close theoretical connection between true market leverage and equity returns, these linkages are much weaker when we construct leverage using only the book value of debt and market values of equity. This finding suggests that using lagged (book) values of debt as a proxy for

¹Carlson, Fisher and Giammarino (2004), Zhang (2005), Livdan, Sapriza, and Zhang (2009), Gomes and Schmid (2010), Ozdagli (2012), Obreja (2013), Kuehn and Schmid (2014).

²Examples of the ability of credit spreads to forecast economic activity include studies by Keim and Stambaugh (1986), Schwert (1989), Stock and Watson (1991), Fama and French (1992), Lettau and Ludvigson (2004), Gilchrist and Zakrajsek (2008), and Mueller (2008).

market leverage, as is generally done in empirical studies, may be at least partially responsible for the failure to find strong link between leverage and returns in the data.

A growing body of work has started to provide an integrated discussion of asset prices, leverage, and aggregate cycles in a modern setting, but our emphasis on risk premia is fairly unique. Existing general equilibrium macro models that explain the cyclical behavior of credit markets and their correlation with macroeconomic aggregates largely abstract from variations in risk premia and asset prices.³ Unlike these classic financial accelerator papers, movements in credit spreads in our paper are mostly due to variations in credit risk premia and do not require large spikes in observed default events. In fact, in our model changes in risk premia drive about two thirds of the credit spread and also account for most of its predictive power.

A parallel literature has sought to link credit risk to the financing decisions of firms and, more recently, to exogenous movements in risk premia and aggregate factors.⁴ Relative to that line of work, we show how embedding a detailed model of credit risk into general equilibrium has important implications for endogenous volatility and risk pricing. ⁵ Closer to our work is Favilukis, Lin, Zhou (2015) who also use a production and investment model with heterogeneous firms to address the impact wage rigidities on the determination of credit spreads. They mostly abstract from other issues such as the patterns in investment and leverage data and the links between credit and equity markets that we emphasize here. Conversely, Begenau and Salomao (2015) study a partial equilibrium model with heterogenous firms that offers a much more detailed analysis of cross-sectional differences in firm financing patterns over the business cycle, while mostly ignoring asset pricing data.

There is also a number of general equilibrium models with production and investment that exploits the role of asset prices and risk premia explicitly. However they all generally ignore the role of credit markets and credit risk.⁶ Relative to these papers, our main contribution is to offer a more detailed general equilibrium model with production and financing and explicitly link the movements in asset prices to endogenous changes in macroeconomic quantities. In this respect, our work is related to general equilibrium models that incorporate firm heterogeneity in order

³Classic examples include Kyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999). More recent contributions are Jermann and Quadrini (2010) and Khan and Thomas (2012).

⁴Building on Leland (1994) recent quantitatively successful contributions include Hackbarth, Miao, and Morellec (2006), Sundaresan and Wang (2010), Chen, Collin Dufresne, and Goldstein (2008), Bhamra, Kuhn, and Strebualev (2010), and Chen (2010). More recently, some papers such as Ai, Kiku, and Li (2013) and Mitra (2014) have developed quantitative models of firm financing based on dynamic contracting in risk-sensitive environments.

 $^{^{5}}$ Miao and Wang (2010) extend our framework to allow for endogenous labor supply, while Gourio (2010) introduces disaster risk in a setting where firms live for two periods to ensure there is no role for firm heterogeneity in equilibrium.

⁶Some examples are Jermann (1998), Tallarini (2000), Lochstoer and Kaltenbrunner (2010), Ai, Croce, Li (2010), Croce (2014), and Kung and Schmid (2015).

to address cross-sectional patterns in stock returns such as the value premium.⁷ In contrast to these contributions, in our model with endogenous financing, leverage emerges as an important determinant of the cross-section of returns.

The rest of the paper is organized as follows. Section 2 describes our general equilibrium model and some of its properties, while Section 3 discusses some of the issues associated with solving it numerically. A detailed discussion of our findings is provided in Section 4, before we conclude.

2 The Model

In this section we describe a general equilibrium model with heterogeneous firms that are financed with both debt and equity. The model is designed to merge many key features of the investment and financing behavior of firms in a modern asset pricing setting.

Firms produce a unique final good that can be used for both consumption and investment. They own, and can add to, their capital stock by taking advantage of stochastic investment opportunities. Debt is used because of its tax benefits and because equity issues are costly. Hence the capital structure reflects and combines the key elements of both modern trade off and pecking order theories. Both debt and equity can be issued stochastically although there are issuance costs. Excessive debt may cause some firms to default. On the other hand, attractive business and credit conditions may also encourage new entrants to join in production.

2.1 Firms

The production sector of the economy is made of a continuum of firms that differ in their productivity, size, and leverage, among other characteristics. In describing the problem of firms we take the stochastic discount factor for the economy as given. We show later how this is determined in general equilibrium by the optimal consumption and savings decisions of households. Nevertheless, it is important to recognize from the outset that firms' discount rates depend on the aggregate state of the economy, denoted s. As we will show, this includes both the current state of the aggregate shocks and the equilibrium cross-sectional distribution of firms.

 $^{^7\}mathrm{Examples}$ along these lines include Gomes, Kogan, and Zhang (2003), Gala (2010) Garleanu, Panageas, and Yu (2010), and Papanikolaou (2010)

2.1.1 Technology

All firms produce the same homogeneous final good that can be used for consumption or investment. The production function denoting the instantaneous flow of output is described by the expression:

$$y_{jt} = \exp(x_t + z_{jt})k_{jt} \tag{1}$$

where k_{jt} denotes the firm's productive capacity and x_t and z_{jt} denote the values of aggregate and firm specific productivity, respectively. The behavior of these follows a first order autoregressive process with normal innovations:

$$x_t = (1 - \rho_x)\bar{x} + \rho_x x_{t-1} + \sigma_x v_{xt}$$
(2)

$$z_{jt} = (1 - \rho_z) + \bar{z}\rho_z z_{j,t-1} + \sigma_z v_{zjt}$$
(3)

where v_{xt} and v_{zjt} are independently and identically distributed shocks drawn from standard normal distributions. We use $N(x_{t+1}|x_t)$ and $N(z_{t+1}|z_t)$ to denote the conditional cumulative c.d.f of these two variables.

A growing literature has emphasized the importance of non-normal or disaster shocks and time variation in volatility (e.g. Bloom (2009), Gourio (2010), Gilchrist et al (2011)). We choose not to include them to illustrate better how a detailed general equilibrium production model can generate endogenously the stochastic consumption volatility that is a key feature of several popular asset pricing models with exogenous consumption (e.g. Drechsler and Yaron (2010)).

2.1.2 Investment Opportunities

Each period firms have the opportunity to increase next period's stock of capital k_{jt+1} . Investment takes place by adopting a new project of discrete size. Each adopted project costs *i* goods per unit of capital, and it scales the stock of capital to $k_{jt+1} = g \times k_{jt}$. In other words, to increase next period's stock of capital by a constant (net) factor of g - 1 the firm must surrender $i \times k_{jt}$ units of current cash flow.

Assuming the cumulative distribution of investment costs, denoted H(i), is uniform and independent over time, we can write the law of motion for a typical firm's stock of capital as:

$$k_{jt+1} = \begin{cases} k_{jt} & \text{with prob.} & 1 - H(\bar{i}_t) \\ gk_{jt} & \text{with prob.} & H(\bar{i}_t) \end{cases}$$
(4)

Thus, only firms drawing a sufficiently low cost of adopting a new project will choose to increase

their productive capacity. We discuss the determination of the cutoff investment cost, \bar{i}_t , below. Hence our model will produce an endogenous cross-sectional variation in firm size over time as firms optimally take advantage of differing investment opportunities. Finally, we assume maintenance of the existing capital stock entails periodic costs, δk_{jt} , akin to depreciation.

2.1.3 Firm Earnings and Financing

Firms can finance their investments with both debt and equity. We assume that both new debt and new equity issues entail paying proportional issuance costs in the amount of κ_b and κ_e , respectively. These include transaction costs, such as underwriting fees, associated with the issuance of new securities.

Debt takes the form of a consol bond that pays a coupon \tilde{b}_{jt} until random expiration. When an outstanding bond expires, it is retired at market value, and a new bond is (optimally) issued and priced.⁸ Formally, the process for debt expiration and refinancing is captured by the i.i.d. random variable η , which takes the value 1 when outstanding debt expires, and 0 otherwise. We assume that $Prob[\eta = 1] = \zeta$ and $Prob[\eta = 0] = 1 - \zeta$. As a result, expected debt maturity is equal to $\frac{1}{\zeta}$. In what follows we use $\psi(\eta)$ to denote the c.d.f of this distribution.

Given this we can define the after-tax profits to a firm, $\Pi(\cdot)$ as:

$$\Pi(k, \tilde{b}, z, s) = (1 - \tau)(\exp(x + z) - \delta)k - (1 - \tau)\tilde{b}.$$
(5)

where τ denotes the effective tax rate on profits adjusted for taxes on distributions and personal interest income.

2.1.4 Default and Debt Pricing

Limited liability ensures that it is optimal for equity holders to default on their debt obligations whenever the equity value, denoted $V(k, \tilde{b}, z, \eta, s)$, becomes negative. Mathematically, this yields a default cutoff value for the idiosyncratic shock, $\bar{z}(k, \tilde{b}, \eta, s)$, that is defined implicitly by:

$$V(k, \tilde{b}, \bar{z}(k, \tilde{b}, \eta, s), s) = 0 \tag{6}$$

As discussed above, existing bondholders receive a periodic coupon payment as long as the firm does not default or the bond does not expire. Upon expiration the bond must be repurchase at its full market value. On the other hand, if default occurs creditors receive an amount equal to

⁸Our treatment of debt ensures that, at any point in time, each firm only has a single debt security outstanding. Solving the model with multiple types of debt is significantly more costly.

 $\phi(1-\delta+\exp(x+z))k$, where $\phi>0$ is the fraction of the firm's assets (its capital plus current cash flows) that can be recovered.

Given these possibilities, the market value, $B(k, \tilde{b}, z, s)$, of a claim promising to pay a coupon \tilde{b} until (random) expiration, in a firm that is currently in state (k, \tilde{b}, z, s) , obeys the recursion:

$$B(k, \tilde{b}, z, s) = E_{s}M(s, s') \left[\int \int_{\bar{z}(k', \tilde{b}, s', \eta')} [\tilde{b} + B(k', \tilde{b}, z', s')] dN(dz'|z)\psi(d\eta') + \int \int^{\bar{z}(k', \tilde{b}, s', \eta')} \phi(1 - \delta + \exp(x' + z'))k'N(dz'|z)\psi(d\eta') \right]$$
(7)

where we take the investors' stochastic discount factor, M(s, s'), as given for the moment.

Some basic properties of the market value of debt are established in the appendix. In particular, linearity of technology, investment and default costs in k implies that both equity and bond values are also linear in firm size, k. As a result the only endogenous state variable is the ratio $b = \tilde{b}/k$, which can be taken as a measure of firm leverage. It follows that the default threshold obeys $\bar{z}(k, \tilde{b}, \eta, s) = \bar{z}(b, \eta, s)$. Accordingly, we henceforth simplify the notation and work with the normalized equity value function $P(b, z, \eta, s) = V(k, \tilde{b}, z, \eta, s)/k$. Similarly, we will use $Q(b, z, s) = B(k, \tilde{b}, z, s)/k$ to denote the normalized market value of debt.

Equation (7) shows how changes in the recovery rate, ϕ , directly affect the relative price of credit to the firm. Changes in ϕ effectively act as shocks to the supply of credit, changing credit conditions and credit spreads. To better study the effect of these shocks we later consider an expanded version of the model where expected recovery rates ϕ fluctuate stochastically according to the conditional distribution $\Gamma(\phi'|\phi)$.⁹

2.1.5 Equity Value and Optimal Investment

We can now characterize the decisions of equity holders in detail. Conditional on survival, a firm draws an investment cost *i*, and decides whether or not to invest. In the following, $P^0(b, z, \eta, s)$ denotes that of a firm that chooses not to invest at all, and $P^I(b, z, \eta, i, s)$ denotes the equity value of a firm *after* it adjusts its stock of capital. The inaction value, $P^0(\cdot)$ is determined recursively by

⁹Eisfeldt and Rampini (2007) show these types of "liquidity" shocks can be important to explain measured variation in individual firm investment over time, while Jermann and Quadrini (2011) and Khan and Thomas (2013) show how they can be important to explain macroeconomic fluctuations.

the Bellman equation:

$$P^{0}(b, z, \eta, s) = \max_{b'} \left\{ (1 + \chi_{e} \kappa_{e}) \left[\pi(b, z, s) + \eta[(1 - \kappa_{b})Q(b', z, s) - Q(b, z, s)] \right] + E_{s} M(s, s') \int \int_{\bar{z}(b', \eta', s')} P(b', z', \eta', s') N(dz'|z) \psi(d\eta') \right\}$$
(8)

Here $\pi(\cdot) = \Pi(\cdot)/k$ and χ_e is an indicator function that takes the value of 1 when the firm raises new equity. The term $[(1 - \kappa_b)Q(b', z, s) - Q(b, z, s)]$ captures the (optimal) value of net new debt issues net of any issuance costs. As discussed above, new debt issues occur only upon realization of a refinancing shock, i.e. if $\eta = 1$. The final term is the continuation value for equity holders. The integral is truncated to reflect the impact of the option to default on equity values.

The value of equity for a firm that chooses to invest, $P^{I}(\cdot)$, can be similarly expressed as:

$$P^{I}(b, z, \eta, i, s) = \max_{b'} \left\{ (1 + \chi_{e} \kappa_{e}) \left[\pi(b, z, s) - i + \eta [(1 - \kappa_{b})gQ(b', z, s) - Q(b, z, s)] \right] + g E_{s} M(s, s') \int \int_{\bar{z}(b', \eta', s')} P(b', z', \eta', s') N(dz'|z) \psi(d\eta') \right\}$$
(9)

This expression now accounts for both current investment expenditures, i, and the growth in next period's capital stock, g.

Clearly, the firm chooses to invest if and only if $P^{I}(b, z, \eta, i, s) \geq P^{0}(b, z, \eta, s)$. We can get some intuition about the investment decision of each firm by studying the limit case when debt and equity issuances are costless, i.e. $\kappa_{e} = \kappa_{b} = 0$. In this case setting $P^{I}(b, z, \eta, i, s) = P^{0}(b, z, \eta, s)$ yields an optimal investment cutoff:

$$\bar{i}(b,z,\eta,s) = (g-1) \max_{b'} \left\{ \mathcal{E}_{s}M' \int \int_{\bar{z}(b',\eta',s')} P(b',z',\eta',s') N(dz'|z) \psi(d\eta') + \eta Q(b',z,s) \right\}$$
(10)

In this case the term in brackets is exactly Tobin's average q. It equals the expected value of all equity and debt claims on the firm, normalized by the value of the current stock of capital. The optimal investment rule prescribes that a firm should invest if and only if Tobin's q exceeds i/(g-1). For the marginal firm this is exactly 1, so that, at the aggregate level, this economy behaves very much like one with an aggregate investment technology exhibiting convex adjustment costs.

At the beginning of the period, the equity value (per unit of capital) thus obeys:

$$P(b, z, \eta, s) = \max\left\{0, \int \max\{P^{0}(b, z, \eta, s), P^{I}(b, z, \eta, i, s)\}\,dH(i)\right\}$$
(11)

where the first max operator reflects the possibility of default.

This concludes our description of the individual firm decisions. The appendix establishes a number of key properties about the relevant policy functions and Figure 1 illustrates them using the benchmark parameter values discussed below. Most of these properties seem empirically plausible. In particular, note that the investment cutoff $i(\cdot)$ is declining in the existing coupon payment, b, which means high leverage firms are less likely to invest - a classical "debt overhang" result.

Also important, when the discount factor $M(\cdot)$ is constant, the default cut-off $\bar{z}(b, \eta, s)$ becomes linear in x. In this case, changes to aggregate productivity produce symmetric responses in the the default cutoff and default rates over the business cycle.¹⁰ By contrast, allowing for a significant role for risk premia, ties $M(\cdot)$ to x and leads to asymmetric responses to aggregate shocks.

2.2 Aggregation

To characterize the general equilibrium of the model we must aggregate the optimal policies of each individual firm to construct macroeconomic quantities for our economy.

2.2.1 Cross-Sectional Distribution of Firms

We begin by defining $\mu_t = \mu(b, z, \eta, x, \phi)$ as the cross-sectional distribution of firms over leverage, b, idiosyncratic productivity, z, and refinancing shocks, η , at the beginning of period t, when the state of aggregate productivity is x and recovery rates are ϕ .

Our timing is chosen so that that $\mu(\cdot)$ is constructed before any current period decisions take place. As is well known, this cross-sectional distribution will move over time in response to the aggregate state of the economy and will be the main computational obstacle to solving the model.

Given this distribution it is straightforward to define the total mass of firms at the beginning of the current period as:

$$F_t = \int d\mu_t \tag{12}$$

Like μ_t itself, F_t is constructed before individual firms' decisions are made.

Similarly, we can construct the equilibrium default rate in the economy as:

$$D_t = 1 - \frac{\int_{z \ge \bar{z}(b,\eta,s)} d\mu_t}{F_t} \tag{13}$$

Since the default threshold, $\bar{z}(b, \eta, s)$, is decreasing in x this default rate will be countercyclical and, as discussed above, will generally respond asymmetrically to positive and negative shocks in x.

¹⁰Popular examples are Bernanke et al (1997), Gertler and Karadi (2010).

2.2.2 Firm Entry

Entry is required to replace bankrupt firms and ensure a stationary distribution of firms in equilibrium. Accordingly, we assume that every period a mass of potential new entrants arrives in the economy. Potential entrants behave similarly to incumbents but face different initial conditions. Specifically, potential new entrants:

- have no initial level of debt, so that $b_{jt} = 0$; and,
- draw an initial realization of the idiosyncratic shock, z_{jt+1} , from the long-run invariant distribution implied by (3), denoted $N^{\star}(z)$;

All new firms must start with a size equal to $\alpha \bar{k}_t$, where:

$$\bar{k}_t = \frac{\int k_{jt} d\mu_t}{F_t} \tag{14}$$

denotes average firm size at time t and $\alpha < 1$.

Like incumbents, entrants differ in the cost of acquiring this initial stock of capital. For the sake of symmetry and parsimony we assume that the unit cost of their investment opportunities, e, is also drawn from the c.d.f. H(e). As with incumbents, this implies that only firms drawing costs below the cutoff, $\bar{e}(z, s)$, find it optimal to invest and thus enter the market.

2.2.3 Aggregate Investment

Given the optimal behavior of individual firms, gross aggregate investment expenditures are equal to:

$$I_{t} = I(s) = \iint_{0}^{\bar{i}(b,z,\eta,s)} ikdH(i)d\mu + \int_{0}^{\bar{e}(z,s)} e\bar{k}dH(e) - \int_{z \leq \bar{z}(b,\eta,s)} kd\mu + \int \delta k \, d\mu$$
(15)

The first two terms sum the total investment costs of newly adopted projects, incurred by existing firms and by new entrants respectively. We then net out the disinvestment proceeds associated with asset liquidation by defaulting firms. The last term adds the depreciation expenditures of all incumbent firms.

The expression for aggregate investment (15) integrates, in a parsimonious way, elements of rising marginal adjustment costs and partial irreversibility, both of which are important to generate quantitatively interesting behavior in asset prices. Because optimal investment cutoffs are increasing in productivity, the marginal cost of (aggregate) investment rises in good times, much like it would in a simple aggregate model with standard convex adjustment costs (e.g. Jermann (1998)). And since bankruptcy is costly, investment becomes in effect only partially reversible, thus adding endogenous counter-cyclical variation to consumption growth in general equilibrium. This endogenously increases the market price of risk during recessions and exacerbates underlying variations in equilibrium asset prices.

In addition, linearity of the aggregate production technology and investment expenditures ensures that our economy will grow endogenously over time at a stochastic rate that is linked to average aggregate productivity x_t . Faced with these aggregate shocks, our economy will exhibit persistent variation over time in the growth rates of output and consumption among others, providing a natural laboratory to investigate the effects of shocks to long run growth rates in a general equilibrium context with endogenous quantities and prices.¹¹

2.2.4 Other Aggregate Quantities

All other aggregate quantities can be define in straightforward fashion. In particular, aggregate output is given by:

$$Y(s) = \int \exp(x+z)k \, d\mu \tag{16}$$

while the losses associated with bankruptcy are given by:

$$\Phi(s) = \int_{z \le \bar{z}(b,\eta,s)} (1-\phi)(1+\exp(x+z))k \, d\mu$$
(17)

Similarly, we can also construct the aggregate market value of corporate equity and debt respectively with the expressions:

$$V(s) = \int P(b, z, \eta, s) k \, d\mu \tag{18}$$

and

$$B(s) = \int Q(b, z, s) k \, d\mu \tag{19}$$

These definitions for the aggregate quantities make it clear that the aggregate state of our economy s is the triplet (x, ϕ, μ) . All aggregate quantities and prices depend on the average state of productivity, financial conditions as well as the cross-sectional variation in firm productivities and leverage.

¹¹If the arrival of investment projects to new and old firms $H(\cdot)$ is time varying, the model easily accommodates the type of investment specific technological shocks that have been emphasized recently in the literature (e.g.Papanikolaou (2011) and Kogan and Papanikolaou (2013))

2.3 Households

To close our general equilibrium model we now describe the behavior and constraints faced by the households/investors. We assume that the economy is populated by a competitive representative agent household, that derives utility from the consumption flow of the single consumption good, C_t . This representative household maximizes the discounted value of future utility flows, defined through the Epstein-Zin (1991) and Weil (1990) recursive function:

$$U_t = \{(1-\beta)u(C_t)^{1-1/\sigma} + \beta E_t [U_{t+1}^{1-\gamma}]^{1/\kappa}\}^{1/(1-1/\sigma)}.$$
(20)

The parameter $\beta \in (0, 1)$ is the household's subjective discount factor and $\gamma > 0$ is the coefficient of relative risk aversion. The parameter $\sigma \ge 0$ denotes the elasticity of intertemporal substitution and $\kappa = (1 - \gamma)/(1 - 1/\sigma)$.

The household invests in shares of each existing firm as well as a riskless bond in zero net supply that earns a period rate of interest r_t . We also assume that there are no constraints on short sales or borrowing and that households receive the proceeds of corporate income taxes as a lump-sum rebate equal to:

$$T(s) = \tau \int \exp(x+z)k \, d\mu \tag{21}$$

Given these assumptions the equilibrium stochastic discount factor that must be used to compare cash flows across two adjacent periods is defined by the expression:

$$M_{t,t+1} = \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma} R_{W,t+1}^{1-1/\kappa}\right]^{\kappa}.$$
(22)

where

$$R_{W,t+1} = \frac{W_{t+1}}{W_t - C_t}.$$
(23)

is the return on total household wealth, including bonds and tax proceeds.

As is well known, the absence of arbitrage implies that all gross asset returns in this economy will satisfy:

$$E_t[M_{t+1}R_{i,t+1}] = 1, (24)$$

for all assets i, including the equity and bond investments in the firms described above, as well as total household wealth. Accordingly, we can construct wealth as the value of a claim to household's future consumption.

2.4 General Equilibrium

Optimal investor behavior determines the equilibrium stochastic discount factor, M(s, s'), given household wealth. Earlier we described optimal firm behavior given the stochastic discount factor and showed how it determines aggregate investment and output as well as household wealth. Ensuring consistency between these two pieces of the economy requires that aggregate consumption by households is equal to aggregate production, net of investment and deadweight losses.

Formally our competitive equilibrium can then be constructed by imposing the additional consistency condition:

$$C_t = C(s) = Y(s) - I(s) - \chi \Phi(s)$$
 (25)

This ensures that the stochastic discount factor used by each firm corresponds to that implied by optimal household behavior. The parameter χ determines to what extent bankruptcy costs are deadweight losses. While this will be our baseline assumption, we can entertain the possibility that these costs are rebated to households, for example in the form of legal and accounting costs, by setting $\chi = 0.^{12}$

Finally we also need to specify a law of motion for the cross-sectional measure of firms over time. Given optimal firm policies this measure satisfies the following relation:

$$\mu(b', z', \eta', x', \phi') = Prob(b_{t+1} < b', z_{t+1} < z', \eta_{t+1} < \eta', x_{t+1} < x', \phi_{t+1} < \phi')$$

$$= \left[\int_{z' \ge \bar{z}(b,\eta',s')} N(z'|z)\phi(\eta')\Omega_{b(b,z,\eta,i,x,\phi) < b'} d\mu(b, z, \eta, x, \phi) + N^{\star}(z')\Omega_{b(0,\bar{z},i,x,\phi) < b'} \right] N(x'|x)\Gamma(\phi'|\phi)$$
(26)

where Ω is an indicator function that takes the value of 1 if the optimal policy function $b(b, z, \eta, i, x, \phi) < b'$ and 0 otherwise. $N(\cdot), N^{\star}(\cdot)$ and $\Gamma(\cdot)$ are the cumulative distributions defined earlier.

The terms outside brackets in equation (26) capture the exogenous evolution in the aggregate states. The first term inside the brackets sums all the surviving firms which choose an optimal leverage below b' across all current states next period. The second term adds the mass of all entering firms that also choose optimal leverage below b'. Recall that new firms arrive at the current period with b = 0.

Figure 2 shows how the cross-sectional distribution changes after a long sequence of positive or negative shocks to aggregate productivity, x. Each panel depicts the equilibrium marginal distribution over debt commitments, $\mu(b, \cdot, \cdot, \cdot, \cdot)$. This reflects both the effects of truncation by exit, refinancing and investment from incumbent firms, and the lumpy additions from new entrants.

 $^{^{12}}$ We follow the convention of considering that bankruptcy costs are deadweight losses but in a general equilibrium model this is a somewhat debatable choice, since some of these costs might be in the form of legal and accounting fees that accrue to other types of firms in the economy.

In expansions, most firms find it optimal to choose higher levels of debt upon refinancing so as to aid funding the exercise of valuable growth options. Similarly, new entrants will be relatively highly levered, and these two effects combine to thin out the left tail of the distribution. During contractions however, many firms will find themselves burdened with excessive debt and optimally choose to default. At the same time, less attractive growth opportunities lead to lower levels of new debt issues upon refinancing and to a larger concentration of low (book) debt firms. Taken together, these effects will reduce the mass of firms at the right tail of the distribution. As we will show later, the predictive power of credit spreads comes from their ability to summarize the tail behavior of the cross-sectional distribution, $\mu(\cdot)$.

3 Computation and Calibration

This section offers a brief description of our approach to solve the model in section 2 and the choice of parameter values. As discussed above, the main obstacle to the computation of the competitive equilibrium is the fact that the cross-sectional measure of firms $\mu(\cdot)$ changes over time. In spite of this, and the level of detail in capturing firm behavior, our model remains relatively parsimonious and relies on relatively few independent parameters.

3.1 Computation

Computing the competitive equilibrium requires the following three basic steps:

- Given an initial stochastic discount factor M(s, s') solve the problem of each individual firms and determine the equilibrium level of entry and default
- Aggregate individual firm decisions and use the consistency condition (25) to compute aggregate consumption and wealth
- Ensure that the implied aggregate quantities are consistent with the initial process for M(s, s').

Convergence of this procedure delivers the equilibrium values for all individual and aggregate quantities in the model. Our online appendix (Gomes and Schmid (2017)) describes this procedure in more detail, along with a number of robustness checks.

3.2 Parameter Choices

In the benchmark model there are no credit market shocks and the recovery rate, ϕ , does not move over time. This requires us to specify the value of fifteen parameters: three for preferences, seven for technology, and another five to capture institutional or policy features. Table 1 summarizes our choices.

The preference parameters are β , γ and σ . They are chosen to ensure that the model matches the key properties of the risk free rate and the aggregate equity premium in the economy. Several studies have already shown how to combine time non-separable preferences and persistent shocks to aggregate growth to produce these results. More recent papers have expanded this analysis to general equilibrium models with all equity firms. Our parameter values are quite similar to several papers in this literature.¹³

For the technology parameters we set the maintenance cost of capital δ to 2% per quarter, a value consistent with standard estimates of capital depreciation rates. Our choice of α is set to be consistent with the relative size of new entrants, reported by Davis and Haltiwanger (1992). The size of growth options, g, as well as the support of the uniform distribution from which i is drawn, are chosen to be consistent with the evidence on the lumpy nature of firm-level investment. In particular, we set the support to match the empirical frequency of investment spikes of about 0.06 per quarter (Davis and Haltiwanger (1992)), and g to match the empirical investment rate.

The volatility and persistence of the aggregate productivity process are set to $\rho_x = 0.96$ and $\sigma_x = 0.012$, largely in line with other macro studies and ensuring that we match the volatility and persistence of output growth in the data. The parameters for idiosyncratic shocks can be chosen to match a number of different moments of the cross-sectional distribution of firms. We choose $\rho_z = 0.9$ in order to obtain a realistic cross-sectional dispersion in investment rates, and pick $\sigma_z = 0.15$ to match the empirical three-year default rate.

Finally, we need to specify a number of choices to capture the institutional environment. First, we assume that leverage, b, remains constant, alternatively that coupons, \tilde{b} , scale linearly with capital, within refinancing cycles. This assumption is useful but not essential, as we show when investment and refinancing cycles exactly coincide. The marginal corporate tax rate, τ is set to 20% to reflect the effect of of individual taxes on distributions and interest on the effective marginal tax rate. We choose the recovery parameter, ϕ to generate average recoveries on defaulted bonds around 45% of face value (Warner (1977)). Formally, we set the value of ϕ so that in default:

$$\frac{\phi(1-\delta+\exp(x+z))}{Q(s_0,b,z)} = 0.45$$
(27)

where $Q(s_0, b, z)$ is the value of debt (relative to k) initially raised by the firm on average.

Next, we determine equity and bond issuance costs, κ_e and κ_b . Although they are commonly

 $^{^{13}}$ Early examples include Bansal and Yaron (2004) in models with exogenous quantities and Lochstoer and Kaltenbrunner (2009), Croce (2010) and Kung and Schmid (2015) in settings with production.

used in the literature, we show below that these costs only marginally improve our quantitative results and are not really essential. We set κ_e to match the empirical frequency of equity issuances. The small bond issuance cost κ_b helps matching the dispersion in credit spreads but, again, given the stochastic nature of bond refinancing, they are not really important. The bond refinancing probability ζ effectively pins down the average maturity of corporate bonds, which we take to be about eight years, as reported in Chen, Xu, and Yang (2016). In this case refinancing cycles last roughly twice as long as investment cycles. Below, we also consider a version of the model where debt maturity is halved so that these cycles are effectively aligned.

For the version with credit market specific shocks we need two extra parameters. We assume that recovery rates in bankruptcy fluctuate over time, as a result of exogenous shocks to liquidation values. To accomplish this we calibrate the recovery rate using data provided from Moody's 2013 Annual Default Study.¹⁴ This study contains annual data on creditor recoveries on all bonds between 1982 and 2013. We obtain an annual standard deviation of 0.1 and with an annual persistence of 0.6, and fit an AR(1) process to match these facts.

4 Findings

We begin describing our quantitative findings by summarizing the basic implications of our model for means and dynamics of major aggregate quantities and asset prices. Next, we examine the role of leverage in shaping macroeconomic risks and risk premia. In particular, we show how time series and cross-sectional variation in leverage leads to predictable patterns in expected stock returns over time and across firms. Finally we discuss how the evolution of the cross-sectional distribution of firms over time becomes a determinant of economic cycles. In particular we illustrate how movements in the cross-section amplify aggregate fluctuations and how credit spreads emerge as an indicator and predictor informative about these movements.

Most of our quantitative results are based on simulations. To construct the statistics reported below we solve the benchmark model and alternative specifications by numerical dynamic programming as detailed in Section 3. We then simulate the implied equilibrium policies at quarterly frequency to construct 1000 independent panels of 64 years each and report averages across all simulations. Unless otherwise noted we always report the relevant empirical moments for the sample period between 1951 and 2014.

¹⁴We thank Hui Chen and Carla Nunes for providing us with the detailed data.

4.1 Leverage and Macroeconomic Risks

Table 2 reports basic macroeconomic and financial moments from the benchmark model, as well as two informative, alternative specifications. In particular, we report results from a version in which we restrict firms to using only equity financing, as well as the specification in which we allow for stochastic movements in debt recovery rates.

We start by noting that the benchmark model is quantitatively consistent with salient features of US business cycles, as captured by the volatilities of consumption, output, and investment. Similarly, the share of investment (and hence consumption) is plausible and close to the actual data.

Moreover, both the level of the risk free rate and the equity premium are very close to those observed in the data, and this match does not require the very large movements in the risk free rate often associated with habit preferences. Essentially, this is because the persistent stochastic variation in growth rates generated by our model increases the household's precautionary savings thereby lowering equilibrium interest rates.

While Bansal and Yaron (2004) have shown that accounting for long run movements in consumption and dividends, combined with preferences for an early resolution of uncertainty, delivers realistic risk premia in an endowment economy setting, this has proved harder to implement in general equilibrium production economies (Kaltenbrunner and Lochstoer (2010), Campanale, Castro and Clementi (2009), Croce (2010)). This is because in a production economy, general equilibrium restrictions often tie dividends very closely to consumption, while empirically, dividends are much more volatile than consumption. In our setup, however, financial leverage (endogenously) breaks the tight link between dividends and consumption and renders dividends an order of magnitude more volatile. This allows us to generate a more realistic amount of stock market volatility and is crucial in matching the aggregate equity premium.¹⁵

Table 2 also shows that our baseline calibration implies realistic levels of corporate leverage ratios, default rates and credit spreads (these statistics are based on the average properties of the cross-sectional distribution of firms). Plausibly, low equilibrium leverage ratios reflect realistic pricing of corporate debt. As in recent work by Bhamra et al (2008) and Chen (2011), our success in matching credit market data relies on the the fact that default occurs in periods of very high marginal utility, thereby significantly increasing the effective cost of default and the required compensation to bondholders. In our model, however, cash flow and discount rates are jointly endogenously determined.

¹⁵Although we do not report these numbers here, the model also generates a slow moving pattern in leverage (Lemmon, Roberts and Zender (2008)) and the long run movements in aggregate dividends observed in the data (Bansal and Yaron (2004)).

Tying macroeconomic fluctuations to variation in default rates is then the key component of the large credit spreads and the reason we can match the data along this dimension. Given our default rate, risk neutral valuation implies a credit spread of a few basis points. Instead, our calibration generates a credit spread of 110 basis points, much closer to its empirical value. Unlike other popular macro models with credit markets, it is this credit *risk premium*, induced by the (endogenous) covariance between default rates and the market price of risk, and not default rates that account for the large credit spreads in our model.¹⁶

Finally, Table 2 documents that our benchmark heterogeneous firm economy also generates realistic cross-sectional dispersion in equity risk premia. More specifically, the model generates a spread between the returns on portfolios of the highest and the lowest quintile of book-to-market sorted companies, that is, a value premium $E[r^v - r^g]$, in line with the empirical evidence. A quantitatively realistic value premium is broadly consistent with the literature modeling links between irreversible investment and asset returns (see e.g. Gomes et al (2003), Zhang (2005), Garleanu et al (2012)), much as we do. On the other hand, our approach differs from these models in that growth option exercise is linked to capital structure through debt financing. We explore the asset pricing implications of this added element of realism below.

4.1.1 The Role of Leverage and Credit Risk

By examining a variety of alternative model specifications we can understand in more detail the role of leverage in shaping aggregate risk and risk premia in our economy.

Risk Premia Comparing the benchmark model with the two alternative specifications in Table 2 provides us with some first insights into the role of leverage for aggregate fluctuations and risk premia.

First, and foremost, the all equity version of our economy reported in column 3 does not generate enough volatility in equity returns and macro quantities and fails to match the observed equity premium. This model also produces an unrealistically high equilibrium risk free rate. Intuitively, this is because smoother aggregate cycles weaken households' savings motives. Compared with this all equity economy, the unconditional volatility of consumption is about 35% higher in our baseline, levered, economy.

Similarly, the value premium implied by the all equity model, while still positive, is also substantially smaller. Intuitively, in our model, growth options are partially debt financed, so that value

 $^{^{16}}$ Our decomposition is also consistent with Elton and Gruber (2001) who estimate that about two thirds of the credit spreads are due to the credit risk premium.

firms in the cross-section also exhibit higher leverage ratios.¹⁷ The cross-sectional differences in leverage between growth and value firms thus amplify the dispersion in equity risk. In the context of our model, this dispersion accounts for a substantial fraction of the observed value premium. Thus, while a positive value premium is consistent with models with irreversibilities in investment and operating leverage (Gomes et al (2003), Zhang (2005), Carlson et al (2006), Garleanu et al (2012)), our findings can be seen instead as ascribing an important role to financial leverage.

Table 2 also examines the case of a full model with credit shocks, in the form of stochastic variation in the recovery parameter ϕ . We see that most unconditional moments remain very close to the baseline specification. In particular, we also obtain realistic default rates and credit spreads. Essentially, this is due to the fact that credit shocks are estimated to be smaller, and less persistent than productivity shocks, and, in contrast to them, do not affect aggregate output on impact. Nevertheless, the importance of leverage as a determinant of cross-sectional patterns in stock returns, means that we obtain a slightly larger value premium in the version with credit shocks.

Leverage and Risk Premia Inspection of Table 3 further explores the role of leverage in generating risk premia in our model. First, we take the all-equity model as a benchmark, and simply lever it up exogenously using the average leverage ratio from our benchmark model. This type of Modigliani-Miller argument is a common way to incorporate leverage in the asset pricing literature and produce levered equity returns. Column three shows that while both levered returns and their volatility are significantly higher than in the all-equity economy, they are still well below the values in the benchmark model. Exogenous "levering" of returns misses the effects of leverage on the dynamics of risks and significantly underestimates the critical role of long term debt in amplifying aggregate risk.

Table 3 also shows that, at the cross-sectional level, levering up returns on growth and value portfolios using the average leverage ratio for these firms doubles the value premium relative to the all equity model. Nevertheless, the baseline model, with endogenous leverage adds two forces that interact to further amplify the value premium. First, the dispersion of leverage across growth and value portfolios is countercyclical. Second, in the absence of Modigliani-Miller, firms' investment and financing choices also impact aggregate consumption, so that episodes of widening cross-sectional dispersion endogenously coincide with an elevated market price of risk.

The last column of Table 3 studies a version of the model where we modify the default decision so that firms will default on their debt obligations whenever internal cash flows fall short of their

 $^{^{17}}$ The average leverage ratio in the baseline model is about 0.55 for value firms but only around 0.2 for growth firms. Ozdagli's (2012) reports similar estimates in the CRSP data.

debt obligations - a so called involuntary or "liquidity" default. Formally, we modify the default cut-off (6) to:

$$\exp(x + \bar{z}) - \delta = b \tag{28}$$

Now firms are prevented from covering liquidity shortfalls by contracting additional capital in equity and debt markets. As the table shows, this form of default has a slightly dampening effect on aggregate volatility. Intuitively, firms anticipate they will default earlier and more often, so they will take slightly less debt. Notably, however although default rates in this case are substantially higher, we see only a modest increase in average credit spreads. This is because default boundaries now tied to profitability not equity values, are far less sensitive to cyclical movements in the aggregate economy.

4.1.2 Alternative Parameter Choices

Table 4 examines the sensitivity of some of the main moments to a few parameters. The table shows that when shocks to the stochastic growth rate are not sufficiently persistent, so that ρ_x is lower, the model fails to generate enough volatility in equity returns and thus cannot match the observed equity premium. In this case, the model also produces an unrealistically high equilibrium risk free rate. Similarly, lowering the persistence, ρ_x , reduces default rates and credit spreads. Clearly, firms find it easier to avoid default when temporary bad times are expected to be shorter. In turn, equilibrium leverage ratios are higher than in the baseline case. Persistence has profound effects on the asset pricing implications of our model but does not significantly alter the level or volatility of the main macro quantities, at least not at these relatively short horizons.

Removing either equity issuance or debt issuance costs provides firms with additional financial flexibility to cover cash shortfalls by issuing new equity or to refinance more cheaply. This tends to dampen aggregate volatility slightly and produce a corresponding reduction in aggregate risk premia. Eliminating issuance costs also produces a small increase in average leverage. Similarly, eliminating default costs χ slightly dampens aggregate volatility, and reduces risk premia, but again not substantially so.

Finally, we also consider a specification in which firm-level financing and investment cycles are effectively aligned, in that firms refinance with the same frequency as they exercise growth options. Relative to the benchmark parameterization, this requires halving the implied average debt maturity by setting $\zeta = 0.06$, so that now both coupons \tilde{b} and leverage b are effectively constant within cycles. Giving firms more frequent access to debt markets leads to higher investment and dampens aggregate volatility. It comes with larger debt issuances and higher leverage, so that the there is only a very modest reduction in equity and credit risk premiums.

4.1.3 Cyclical Patterns

Corporate Policies Risk premia on both stocks and corporate bonds reflects firms' performance and policies across economic cycles, induced by variation in aggregate productivity growth rates and credit conditions. Table 5 documents the cyclical behavior of several key corporate investment and financing variables by reporting their cross-correlations with GDP growth.

For the baseline model without credit shocks, we see that persistence in productivity shocks produces a strongly pro-cyclical behavior in both aggregate investment and net entry as new firms enter the market and build up productive capacity in anticipation of higher future profits. As in the data, our firms are more likely to issue equity during good times in the model, although the correlation with economic activity is modest. This is because equity issues take place to both fund investment in good times and also to recapitalize the firm in bad times. Debt issues are procyclical for the same reason and the fact that refinancing is more attractive in expansions when default probabilities fall.

Not surprisingly, since market values of firms and price-dividend ratios are both strongly procyclical, the model naturally generates a realistic countercyclical pattern in market leverage.¹⁸ Elevated leverage in downturns contributes to strongly countercyclical default rates and credit spreads, since default becomes less attractive when profits are high. This endogenous comovement underlies the substantial credit risk premium embedded in the pricing of corporate bonds. Importantly, the cyclicality in market leverage is masked when we use its empirical approximation, quasi-market leverage (QML), which replaces the unobservable market value of debt with its book value (Strebulaev (2006)). For the baseline model, the correlation between output growth and the model generated QML is only -0.16, which is almost exactly the value in the data.¹⁹ Thus, relying on book values of debt obscures important information incorporated in debt valuations.

Although all variables exhibit the correct cyclical behavior in the baseline model, the implied correlations are sometimes higher than in the data. Without financing shocks, and a single source of aggregate uncertainty innovations in output growth are just too closely tied to those in aggregate productivity. In the model with credit shocks the cyclical patterns in firms' financing and investment policies become much more realistic. These shocks immediately impact both leverage and spreads while spreading to GDP growth more slowly, through changes in the optimal firm policies.

Aggregate Risk and the Amplification of Macro Shocks Figure 3 looks at the impact of fluctuations in credit markets on the key quantities driving aggregate risk. It shows the response to

¹⁸The model is thus consistent with evidence that leverage is procyclical at refinancing points, while countercyclical in the cross-section, see e.g. Bhamra, Kuehn, and Strebulaev (2010), Danis, Rettl, and Whited (2014)

¹⁹In the model the book value of debt is the market value of debt at time of issuance.

an exogenous technology shock in our benchmark economy with levered firms and compares it to the response in the all equity model. We can see that leverage introduces a powerful amplification mechanism in the model with output and investment growth responding by between 35% to 50% more in the baseline model. This is because in that model positive productivity shocks reduce the probability of default and thus lower the effective cost of new debt. This raises ex-ante firm value and encourages firm entry and investment spending in incumbents.²⁰ These amplifications results are only a little stronger than those in Bernanke et al (1999). This is expected since both models are calibrated to similar investment to output ratios and average credit spreads. Importantly however, our transmission mechanism relies largely on movements in credit risk premium, instead of actual default rates, to produce realistic variations in credit spreads and the cost of capital.

Figure 4 examines the response of the aggregate economy to a credit market shock, that takes the form of a positive innovation in recovery rates. Such an easing of credit conditions facilitates debt financed investment spending. In the absence of an immediate effect on output, the ensuing investment boom, albeit considerably smaller and less persistent than that following productivity shocks, comes at the expense of consumption which falls on impact. This behavior is reminiscent of that following investment-specific technology shocks, documented in Papanikolaou (2011) and Kogan and Papanikoloau (2014), and can have important implications for the cross-section of returns.

Asymmetric Cycles and Volatility Countercyclical movements in leverage also have notable implications for the dynamics of risks themselves. In particular, as Figure 5 shows, the economy responds more sensitively to adverse shocks than to positive innovations in our levered benchmark economy. This happens in part because, as discussed above, the default rate D_t itself, is asymmetric and in part because investment is only somewhat reversible since it is costly to convert the capital goods lost in default into consumption goods. These asymmetric responses are important since they effectively generate endogenously conditional movements in risk that render aggregate volatility countercyclical in our levered economy. As a result, investors will require compensation for these countercyclical movements in risk. We now turn to discuss this issue.

4.2 Leverage and Predictability

We have seen that our model both produces cross-sectional variation in leverage across firms and predicts quite realistic countercyclical movements in leverage over time. We now show how such variation leads to predictable patterns in expected stock returns both over time and across firms.

²⁰Because we abstract from variations in labor supply these results are probably a lower bound on the amount of endogenous propagation that this mechanism can generate.

4.2.1 Time Series Return Predictability

Table 6 documents evidence regarding time variation in risk premia by showing how popular indicators of equity and credit market conditions, such as valuation ratios and credit spreads, are informative about long-horizon stock market excess returns over the risk free rate. We see that, as in the data, high price-dividend ratios predict lower expected stock returns going forward, while credit spreads forecast high average stock returns.

In the model leverage is an equally good predictor of future equity returns but this is not true in data where predictive power of empirical measures of leverage for future excess returns is weaker than that of standard predictors such valuations ratios. Our model suggests that, as argued in Strebulaev (2006), this finding is due to the lack of a good empirical measure of leverage. Panel C shows that if we instead use quasi-market leverage (QML) return predictability drops considerably and become statistically significant only at longer horizons.

Some of the patterns regarding predictability in returns are reminiscent of the literature studying the links between growth options and returns (see e.g. Gomes et al (2003), Gala (2010), Garleanu, et al (2012)). Our work however is distinct in that it links movements in quantities and risk premia to movements in credit markets. Tables 7, 8 and 9 offer an in depth examination of the role of leverage in generating the patterns in time-varying expected stock returns documented above.

The easiest way to do this is by looking at the implications of an all equity model. In this case valuation ratios are still able to predict future equity returns, but the effects are substantially muted. Given the response of the all-equity model in Figure 5, this is not surprising. Without default, business cycles are only slightly asymmetric, reflecting some non-convexities in investment, but significantly less so than in the benchmark model. The all equity model therefore features significantly less countercyclical risk dynamics, which is reflected in the predictability results.

Table 7 also shows that exogenously "levering" equity returns by assuming firms have a constant leverage ratio, even if this ratio is the same as in the baseline model, has virtually no impact on the predictability regressions. Another useful thought-experiment is to "lever up" the returns from an all equity model, by using an exogenous leverage process that has both the same mean *and* the same cyclical correlation of leverage in the baseline model. This exogenously countercyclical leverage enhances return predictability since the more realistic leverage dynamics amplifies movements in conditional consumption betas. Our results suggest that these more realistic countercyclical leverage dynamics account for a significant fraction of the movements in expected stock returns.

Table 8 shows that aligning firms' financing and investment cycles (by setting $\zeta = 0.06$) and shortening average corporate debt maturity slightly exacerbates return predictability in that firms finance larger investment projects in expansions when debt is relatively cheap, thereby reinforcing cyclical movements in leverage. On the other hand, perhaps not surprisingly, lowering persistence and risk aversion substantially reduces predictability, as overall risk premiums fall.

Finally, allowing for credit shocks has only a relatively minor impact on the link between pricedividend ratios and excess returns or, as shown in Table 9, in the ability of credit spreads to forecast future stock returns.

4.2.2 Cross-Sectional Predictability

All moments documented so far have been obtained by aggregating across the cross-sectional distribution of firms in our model, $\mu(\cdot)$. However, our model produces a very rich cross-sectional heterogeneity at the firm level that can be explored in a number of ways. We now explore the connections between individual firm characteristics and risk premia, which are useful to understand how movements in the cross-sectional distribution of firms feed back into aggregate cycles and risks. We first examine the link between the cross-sectional distribution in leverage and credit spreads, and then turn to the cross-section of stock returns.

The Cross Section of Firms Table 10 reports a first set of results. Panel A documents properties of firm-level investment and financing policies along with their cross-sectional dispersion. The benchmark model is generally consistent with basic facts about corporate policies. Notably, the model reproduces the lumpy nature of firms' investment and financing behavior quite accurately. Firms' investment comes in rare but sizable spikes, as does equity issuance. Assuming liquidity default produces only a slight reduction in the amount of cross-sectional dispersion in our model, while adding credit shocks tends to widen the cross-sectional dispersion. This is especially apparent in investment rates, equity issuance, and leverage. Intuitively, this is because firms with investment opportunities are disproportionately affected by changes in financing conditions, and use debt and equity to finance investment expenditures.

Panel B shows that the cross-sectional dispersion of firm characteristics also exhibits some distinct cyclical patterns. While the cross-sectional dispersion of market leverage and credit spreads widens in downturns, differences in firms' investment opportunities are mildly exacerbated in expansions. As shown in Table 5 above, debt issuance is procyclical since firms are less likely to pay the costs associated with debt restructuring in downturns. Thus, firms are less likely to refinance and tend to be further away from their optimal capital structure in recessions. This widening in the dispersion in leverage and credit spreads during recessions means that the tail of the distribution of firms plays an important role in determining the economy-wide response to shocks. The Cross-Section of Leverage and Credit Spreads Panel A in Table 11 compares the model's predictions to core conditional moments of the cross-sectional distribution of firm level leverage identified in the empirical capital structure literature. These are based on empirical regressions relating relating corporate leverage to several financial indicators (e.g. Rajan and Zingales (1995)). The table shows that our model reproduces the observed negative relationships between leverage and both profitability and Tobin's Q. With persistent shocks, both highly profitable and high Q firms have large investment opportunities going forward. Because equity issuances are costly, such firms will borrow more prudently to avoid floatation costs. The table also shows the model produces a strong positive relation between firm size and leverage.

These firm characteristics are also reflected in the pricing of corporate bonds, as documented in panel B. We include asset volatility to control for the impact of idiosyncratic risk on credit spreads. Again, the table shows that our rationalizes the observed empirical patterns quite well. Notably, both market-to-book and size are economically important determinants of spreads, the latter (unsurprisingly) with a negative sign. To the extent that the market-to-book is interpreted as a proxy for growth opportunities, these will lead to higher credit spreads.

Leverage and the Cross-Section of Stock Returns We now examine how the links between firm characteristics and leverage are reflected in the cross-section of stock returns. Table 12 documents the equity returns associated with various quintile portfolios that can be obtained by sorting stocks (uni and bi-variately) according to their leverage (market and quasi-market) and book-to-market equity.

Panel A reports the results for our benchmark model without credit shocks. The univariate sorts confirm that, in line with standard intuition, our model predicts that leverage increases risks to shareholders and this is reflected in higher expected stock returns. On the other hand, sorting by quasi-market leverage substantially weakens that link, as the second univariate sort demonstrates. This is because quasi-market leverage refers to the market value of lagged debt, which is decoupled from its current market value reflected in contemporaenous stock returns. At the same time, the final univariate sort shows that expected returns are monotonically increasing in book-to-market equity, giving rise to a value premium in our model. These univariate sorts then make clear the importance of cross-sectional differences in leverage to the value premium. This is because firms with higher book-to-market equity also tend to have higher leverage ratios, as growth options arising from the opportunity to adopt new projects vary persistently with firm and aggregate level shocks. To disentangle these forces, the panels on the right consider double sorts on the leverage measures and book-to-market. These sorts show that there remains sizable spread in book-to-market sorted portfolios even when controlling for leverage. This reflects the role of investment irreversibility in the model, also well documented in previous work (see Gomes, Kogan, Zhang (2003) or Zhang (2005)).

Panel B documents that leverage and value spreads widen in the model specification with credit shocks, although the effect is not very significant. As we have discussed earlier, with long-term debt, firms with higher leverage are less likely to adopt new investment projects - a debt overhang effect. As a result these firms' investment policies are less affected by sudden shifts in financing conditions. By contrast, firms with low leverage and ample growth options are more willing to take advantage of favorable financing conditions to fund investment expenditures, making their valuations more sensitive to movements in the recovery rate, ϕ . These firms then have a positive β with respect to credit risk and serve as a hedge against such shocks thereby earning lower expected stock returns.²¹ The effects of credit shocks on the cross section of stock returns are thus qualitatively similar to those of investment-specific technology shocks (eg, Papanikolaou (2011), Kogan and Papanikolaou (2014)).

Table 13 provides evidence regarding our sorts in the data. Empirically, as documented previously in the literature (see Gomes and Schmid (2010)) and confirmed here, the direct link between leverage and stock returns is somewhat weak. However, our model, by virtue of Table 12, provides a plausible rationale in that again the widespread use of quasi market leverage in empirical work may account for this. Because the book value is only a lagged value of market debt it clearly blurs the relationship between leverage measures and stock returns. This effect is likely exacerbated with long term debt.

4.2.3 Business Cycle Predictability

Several empirical studies have shown that credit spreads are a useful predictor of macroeconomic cycles.²² In this section we show that the same is true in our general equilibrium model. We then explore its structure to shed some new light on the economic mechanism underlying the predictive power of credit spreads.

Table 14 shows the results of regressing one year ahead (log) output and investment growth on the average (value-weighted) credit spread at time t. It shows that elevated credit spreads forecast future declines in aggregate output and investment in ways that are both statistically and economically meaningful. Moreover, the estimated coefficients are similar in magnitude to those found in the data. The table shows that this predictability survives even after we control for the current state of aggregate productivity, x_t . Recall that in the baseline model, without credit shocks,

 $^{^{21}}$ Our model (plausibly) predicts a negative price of credit risk. This is because the low persistence of credit shocks implies that the effect on the stochastic discount factor of realized consumption growth dominates.

²²E.g. Gilchrist et al (2008), Lettau and Ludvigson (2004) and Mueller (2008).

aggregate variables such as output and investment, are driven by movements in the two-dimensional aggregate state $s = (x, \mu(\cdot))$. It follows that variation credit spreads are capturing, at least partially, the dynamics of firm heterogeneity. In particular, credit spreads should contain information about conditions for firms in the tail of the distribution which are burdened by elevated leverage, and thus more susceptible to changes in aggregate conditions.

The model can also be used to decompose the credit spread in two components, a risk neutral spread, that captures expectations of default losses over a given horizon, and the credit risk premium, which accounts for the covariation of these losses with agents' marginal utility. Table 14 shows that in our model the risk neutral credit spread component does not significantly forecast future movements in either output or investment, confirming the crucial role played by risk premia in our model.

Panel C recalculates these predictability regressions for the augmented version of our model with credit shocks. Now the aggregate state becomes $s = (x, \phi, \mu)$ and credit spreads must now capture variation in both the distribution of firms and exogenous credit market conditions. As a result, credit spreads become even stronger predictor of future economic activity, especially future investment. Finally, Panel D in Table 14 shows that in a model with liquidity default, credit spreads contain somewhat less information about future economic activity. This is due to the fact that default boundaries are now tied to profitability and not equity values, and thus become less sensitive to cyclical movements in the aggregate economy.

Table 15 sheds further light on the nature of the information content of credits spreads for future economic activity, by providing some sensitivity of the predictive regressions with respect to some key parameters. All else the same, a lower debt maturity (higher refinancing frequency) limits risk premia, thus reducing credit spreads' predictive power, although the quantitative effects are somewhat modest. Lowering aggregate persistence not only lowers macroeconomic volatility, and predictable movements in output and investment, but, with recursive preferences, also (credit) risk premia, so that the predictive power of both credit spreads and risk-neutral spreads is generally dampened. Lowering risk aversion mainly works through substantially reducing risk premia, so that the predictive power of credit spreads now approaches that of risk-neutral spreads.

5 Conclusion

This paper studies a general equilibrium model with heterogeneous firms making optimal investment and financing decisions under uncertainty, and brings together many core insights from asset pricing, capital structure, and macroeconomics. The model reconciles, in a unified framework, many stylized facts about asset returns with several key features in both macroeconomic aggregate and firm-level investment and financing variables. Specifically, we show that our model produces a sizable average equity premium and credit spread, together with plausibly low average returns on safe assets. It also implies that, in the time series, both price-dividend ratios and credit spreads have substantial predictive power for future stock returns, while generating a significant value premium in the crosssection of stock returns. A major contribution of our model is that it delivers an explicit connection between fluctuations in the cross-sectional distribution of firms and the time-series movements in macroeconomic aggregates and financial prices.

In the model, quantitatively realistic asset return dynamics are driven by empirically plausible, endogenous movements in leverage, both in time series and cross-section. Endogenous movements in leverage and risk premia contribute to the amplification and propagation of aggregate consumption risks and volatility. This raises the volatility of the market price of risk and produces quantitatively realistic risk premia. Importantly, endogenous default also increases the volatility of consumption during recessions, as the mass of firms burdened by excessive leverage and closer to default grows. As a consequence, the equilibrium market price of risk also becomes sharply countercyclical. Countercyclical leverage drives up risk premia on financial assets in downturns which, in the time series, is naturally reflected in both price-dividend ratios and credit spreads. As a consequence, expected returns on stocks and bonds are higher in recessions, raising the cost of capital and lowering investment and output growth. Cross-sectionally, because investment is, at least partially, debt financed, value firms tend to have higher leverage ratios and these cross-sectional differences in leverage between growth and value firms amplify the dispersion in equity risk, and are a major driver of the value premium.

Endogenous movements in credit markets thus allow our model to match the observed conditional and unconditional movements in both financial prices and macroeconomic quantities in a parsimonious setting.

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A Appendix: Properties of the Firm's Problem

This appendix establishes some of the basic properties of main value functions used to describe the problem of equity and bondholders.

A.1 Homogeneity

We begin by establishing linear homogeneity of equity and bond values in k and \tilde{b} . To ease notation, we consider a version of the firm's problem without aggregate shocks. This avoids any dependence on x and ϕ , as well as the aggregate state, s. The stochastic discount factor also simplifies to M(s, s') = M.

A.1.1 Liquidity Default

In this case the proof involves the following in several steps

1. Homogeneity of the default threshold:

$$(\exp(\bar{z}) - \delta)k - \tilde{b} = 0 \implies \bar{z} = \bar{z}(k, \tilde{b}) = \bar{z}(\lambda k, \lambda \tilde{b})$$

Hence

$$\bar{z}(k,\tilde{b}) = \bar{z}(1,b) = \bar{z}(b)$$

2. Conjecture homogeneity of the optimal policies

$$\begin{aligned} k'(\lambda k,\lambda \tilde{b},z,\eta,i) &= k'(k,\tilde{b},z,\eta,i) \\ b'(\lambda k,\lambda \tilde{b},z,\eta,i) &= b'(k,\tilde{b},z,\eta,i) \end{aligned}$$

3. Homogeneity of bond prices in k and \tilde{b} :

$$\begin{split} B(\lambda k, \lambda \tilde{b}, z) &= M \int \left[\int_{\bar{z}(\tilde{b}/k')} [\lambda \tilde{b} + B(\lambda k', \lambda \tilde{b}, z')] dN(z'|z) \right. \\ &+ \int^{\bar{z}(\tilde{b}/k')} \phi(1 - \delta + \exp(z')) \lambda k' dN(z'|z) \right] \psi(d\eta') \\ &= \lambda B(k, \tilde{b}, z) \end{split}$$

4. Homogeneity of the equity issuance threshold, denoted \bar{z}_e :

$$\Pi(\lambda k, \lambda \tilde{b}, \bar{z}_e) - i\lambda k + \eta[(1 - \kappa_b)B(\lambda k', \lambda \tilde{b}', \bar{z}_e) - B(\lambda k, \lambda \tilde{b}, \bar{z}_e)] = \lambda[\Pi(k, \tilde{b}, \bar{z}_e) - ik + \eta[(1 - \kappa_b)B(k', \tilde{b}', \bar{z}_e) - B(k, \tilde{b}, \bar{z}_e)]] = \Pi(k, \tilde{b}, \bar{z}_e) - ik + \eta[(1 - \kappa_b)B(k', \tilde{b}', \bar{z}_e) - B(k, \tilde{b}, \bar{z}_e)] = 0$$

This implies that

$$\bar{z}_e(\lambda k, \lambda \tilde{b}, \eta, i) = \bar{z}_e(k, \tilde{b}, \eta, i) = \bar{z}_e(b, \eta, i)$$

and thus $\chi_e = \chi_e(b, b', z, \eta, i).$

- 5. Homogeneity of the present discounted equity values $V^0(k, \tilde{b}, z, \eta), V^I(k, \tilde{b}, z, \eta, i)$ and $V(k, \tilde{b}, z, \eta)$ follows from the fact that periodic equity payoffs are linear homogeneous (Stokey (1996)).
- 6. Homogeneity of the optimal investment cutoff

$$V^{I}(k,\tilde{b},z,\eta,\bar{i}) = V^{0}(k,\tilde{b},z,\eta) \implies V^{I}(\lambda k,\lambda\tilde{b},z,\eta,\bar{i}) = V^{0}(\lambda k,\lambda\tilde{b},z,\eta)$$

so that

$$\bar{i}(\lambda k,\lambda \tilde{b},z,\eta)=\bar{i}(k,\tilde{b},z,\eta)=\bar{i}(b,z,\eta)$$

- 7. Finally, verify the conjectures:
 - (a) Capital

$$\begin{aligned} k'(\lambda k, \lambda \tilde{b}, z, \eta, i) &= \left[(1 - H(\bar{i}(\lambda k, \lambda \tilde{b}, z, \eta))) + gH(\bar{i}(\lambda k, \lambda \tilde{b}, z, \eta)) \right] \lambda k \\ &= \lambda k'(k, \tilde{b}, z, \eta, i) \end{aligned}$$

(b) Optimal coupon

$$b'(\lambda k, \lambda \tilde{b}, z, \eta, i) = (1 - \zeta)\lambda b + \zeta \arg \max_{b'} V(\lambda k, \lambda \tilde{b}, z, \eta)$$
$$= \lambda \left[(1 - \zeta)b + \zeta \arg \max_{b'} V(k, \tilde{b}, z, \eta) \right]$$
$$= \lambda b'(k, \tilde{b}, z, \eta, i)$$

A.1.2 Optimal Default

Now the default threshold is defined implicitly by:

$$V(k,\tilde{b},\bar{z},\eta) = 0$$

The proof is nearly identical but now starts with the conjecture that the value function $V(k, \tilde{b}, \bar{z}, \eta)$ is linear homogeneous in k and \tilde{b} .

1. Homogeneity of the default threshold is established by noting that

$$V(\lambda k, \lambda \tilde{b}, \bar{z}, \eta) = \lambda V(k, \tilde{b}, \bar{z}, \eta) = 0$$

- 2. It then follows from the analysis above that $B(k, \tilde{b}, z)$ is linear homogeneous in k and \tilde{b} .
- 3. Replicate the remaining steps above to verify the conecture that $V(k, \tilde{b}, \bar{z}, \eta)$ is also linear homogeneous in k and \tilde{b} .

A.2 Other Properties

We can now establish additional properties of the optimal policies choices. We assume the the economy is at its deterministic steady-state so the cross section distribution of firms, μ , is constant over time.

Lemma 1. The market value of debt, $Q(\cdot)$, is increasing in x, z and ϕ

Proof Monotonicity in x follows from the facts that $\overline{z}(\cdot)$ is decreasing (see below) and the recovery payment increasing in x. Monotonicity in z follows from the persistence in (3) and the fact that the recovery payment also increasing in z'. Monotonicity in ϕ follows from the fact that the recovery payment increasing in ϕ . \Box

Lemma 2. The normalized equity value $P(\cdot)$, is increasing in z and x, and declining in leverage b;

Proof This follows directly from the fact that equity cash flows, $\pi(\cdot)$, net of investment spending, *i*, are increasing in *z* and *x* while declining in *b*. \Box

Monotonicity of the value function ensures the existence of a (unique) default threshold. In addition to these properties, we can show that limited liability which endows equity with an exit option and increases the value of uncertainty, implies that $P(\cdot)$ will be convex in $\exp(z)$. **Lemma 3.** The default cutoff, $\bar{z}(\cdot)$, is increasing in the coupon b and declining in x

Proof This follows from the fact that $P(\cdot)$ is declining in b and increasing in $x.\Box$

Lemma 4. The investment cutoff, $\overline{i}(\cdot)$, is increasing in z and x and (weakly) decreasing in the existing coupon payment b. The optimal debt policy $\overline{b}(\cdot)$ is increasing in z.

Proof The monotonicity of the policy functions in z and x follows directly from Lemmas 1 and 2. In addition, higher b might trigger the equity issuance indicator $\chi_e > 0$ to switch from 0 to 1, reducing $\bar{i}(\cdot)$ implied by (10). \Box

Parameter	Description	Model
A. Preferences		
β	Subjective discount factor	0.994
ψ	Elasticity of intertemporal substitution	2
γ	Risk aversion	10
B. Technology		
g	Size of growth options	1.14
α	Relative size of entrants	0.2
δ	Maintenance investment rate	0.02
$ ho_x$	Persistence of aggregate shock	0.96
σ_x	Volatility of aggregate shock	0.012
$ ho_z$	Persistence of idiosyncratic shock	0.90
σ_{z}	Volatility of idiosyncratic shock	0.16
χ	Resource cost of default	1
C. Institutions		
au	Effective corporate tax rate	0.2
ϕ	Bankruptcy cost	0.4
κ_e	Equity issuance cost	0.025
κ_b	Bond issuance cost	0.004
ζ	Bond refinancing probability	0.03

Table	1:	Quarterly	Calibi	ration
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This table reports the basic parameter choices for our model and their associated empirical targets. These choices are discussed in detail in subsection 3.2. The model is calibrated at quarterly frequency.

	Data	Benchmark	All Equity	Credit Shocks
A. Macro Moments				
$\sigma[\Delta c]$	1.68	1.73	1.28	1.79
$rac{\sigma[\Delta c]}{\sigma[\Delta y]}$	0.70	0.69	0.67	0.70
$\frac{\sigma[\Delta \tilde{i}]}{\sigma[\Delta u]}$	4.59	4.09	3.88	4.22
$\frac{I}{Y}$	0.19	0.21	0.20	0.22
B. Asset Pricing Moments				
$E[r^f]$	1.69	1.27	1.95	1.16
$\sigma[r^f]$	2.21	1.36	1.13	1.28
$E[r^e - r^f]$	4.29	4.21	1.67	4.12
$\sigma[r^e]$	17.79	12.76	4.88	13.74
C. Cross Sectional Moments				
3-Year Default rate	2.93%	2.88%	0.00%	3.05%
Credit spread	0.95%	1.10%	0.00%	1.18%
Market leverage	0.35	0.34	0.00	0.33
$E[r^v - r^g]$	5.37	4.06	1.08	4.37

Table 2: Aggregate Moments

This table reports unconditional sample moments generated from the simulated data of our benchmark model. The model's moments come from averages across 1000 simulations of 64 years each. The empirical sample comes from the BEA and CRSP. In the table Δw denotes the log difference in the variable W. The return on equity, r^e refers to the value weighted aggregate stock market return, while the risk free rate r^f denotes the return on a one year government bond. The credit spread is the spread between AAA-rated and BAA-rated bonds. The spread $E[r^v - r^g]$ captures the return difference between the highest and lowest quintiles of book-to-market portfolios. All returns are adjusted for annual CPI inflation. The default rate is from Moody's. The parameter values used in the benchmark simulation are reported in Table 1. Firms in the all equity model have no leverage. Finally, the credit shocks model assumes the recovery rate on debt is stochastic and follows the Markov process in the text. All data are annualized.

	Data	Benchmark	Exog. Lev	Liq. Def.
A. Macro Moments				
$\sigma[\Delta c]$	1.68	1.73	1.27	1.55
$rac{\sigma[\Delta c]}{\sigma[\Delta y]}$	0.70	0.69	0.67	0.65
$\frac{\sigma[\Delta i]}{\sigma[\Delta y]}$	4.59	4.09	3.91	4.03
$\frac{I}{Y}$	0.19	0.21	0.20	0.20
B Asset Pricing Moments				
$E[r^f]$	1.69	1 27	1 92	1 54
$\sigma[r^f]$	2.21	1.36	1.11	1.20
$E[r^e - r^f]$	4.29	4.21	2.43	3.64
$\sigma[r^e]$	17.79	12.76	7.48	10.19
C. Cross Sectional Moments				
3-Vear Default rate	2 03%	288%	0.00%	4 16%
Gradit grad	2.9370	2.0070	0.0070	4.1070
Credit spread	0.95%	1.10%	0.00%	1.81%
Market leverage	0.35	0.34	0.34	0.32
$E[r^v - r^g]$	5.37	4.06	2.18	3.23
Market leverage $E[r^v - r^g]$	$0.35 \\ 5.37$	$\begin{array}{c} 0.34\\ 4.06\end{array}$	$0.34 \\ 2.18$	$\begin{array}{c} 0.32\\ 3.23\end{array}$

 Table 3: Aggregate Moments: Role of Leverage

This table reports unconditional sample moments generated from the simulated data of our benchmark model. The model's moments come from averages across 1000 simulations of 64 years each. The empirical sample comes from the BEA and CRSP. In the table Δw denotes the log difference in the variable W. The return on equity, r^e refers to the value weighted aggregate stock market return, while the risk free rate r^f denotes the return on a one year government bond. The credit spread is the spread between AAA-rated and BAA-rated bonds. The spread $E[r^v - r^g]$ captures the return difference between the highest and lowest quintiles of book-to-market portfolios. All returns are adjusted for annual CPI inflation. The default rate is from Moody's. The parameter values used in the benchmark simulation are reported in Table 1. Results for the exogenous leverage model are constructed by simply "levering up" returns in the all equity model with the average market leverage ratio in the benchmark model. The liquidity default model modifies the default decision of the firm to $\exp(x + \bar{z}(b, x)) - \delta < b$. All data are annualized.

	Data	Benchmark	$\rho_x = 0.9$	$\kappa_e = 0$	$\kappa_b = 0$	$\chi = 0$	$\zeta = 0.06$
A. Macro Moments							
$\sigma[\Delta c]$	1.68	1.73	1.54	1.64	1.73	1.70	1.65
$rac{\sigma[\Delta c]}{\sigma[\Delta u]}$	0.7	0.69	0.68	0.62	0.66	0.65	0.64
$\frac{\sigma[\Delta i]}{\sigma[\Delta u]}$	4.59	4.09	3.85	3.96	4.04	4.01	4.12
$\frac{I}{Y}$	0.19	0.21	0.19	0.21	0.21	0.21	0.23
B. Asset Pricing Moments							
$E[r^f]$	1.69	1.27	3.46	1.33	1.31	1.41	1.47
$\sigma[r^f]$	2.21	1.36	1.13	1.19	1.27	1.23	1.29
$E[r^e - r^f]$	4.29	4.21	1.42	3.48	4.25	4.12	4.15
$\sigma[r^e]$	17.79	12.76	5.93	11.92	12.78	12.25	12.43
C. Cross Sectional Moments							
3-Year Default rate	2.93%	2.88%	1.34%	2.18%	2.72%	2.76%	2.97%
Credit spread	0.95%	1.10%	0.35%	0.76%	1.02%	0.84%	1.02%
Market leverage	0.35	0.34	0.44	0.40	0.34	0.35	0.38
$E[r^v - r^g]$	5.37	4.06	1.24	2.76	4.03	4.16	4.14

 Table 4: Aggregate Moments: Robustness

This table reports unconditional sample moments generated from the simulated data of some key variables of our model under different parameter specifications. We report averages across 1000 simulations of 64 years. All data are annualized. The empirical sample comes from the BEA and CRSP. In the table Δw denotes the log difference in the variable W. The return on equity, r^e refers to the value weighted aggregate stock market return, while the risk free rate r^f denotes the return on a one year government bond. The credit spread is the spread between AAA-rated and BAA-rated bonds. The spread $E[r^v - r^g]$ captures the return difference between the highest and lowest quintiles of book-to-market portfolios. All returns are adjusted for annual CPI inflation. The default rate is from Moody's. The return on equity refers to the value weighted aggregate stock market return. The parameter values used in the benchmark simulation are reported in Table 1. Data counterparts come from the BEA and CRSP.

Correlation with Δy	Data	Benchmark	All Equity	Credit Shock
Investment growth, Δi	0.81	0.71	0.67	0.55
Net entry	0.44	0.81	0.82	0.66
Market leverage	-0.11	-0.38	0.00	-0.32
Price-Dividend ratio, PD	0.42	0.72	0.71	0.64
Debt issuance	0.33	0.37	0.00	0.29
Equity issuance	0.10	0.23	0.28	0.14
Default rate, D	-0.33	-0.65	0.00	-0.43
Credit spread, CS	-0.36	-0.68	0.00	-0.52

Table 5: Business Cycle Properties

This table reports the correlation of key macro and financial variables with changes in log GDP, Δy in the data and in our benchmark model. The model's moments come from averages across 1000 simulations of 64 years each. The empirical sample comes from the BEA, CRSP, Moody's and the Board of Governors of the Federal Reserve. The parameter values used in the benchmark simulation are reported in Table 1. Firms in the all equity model have no leverage. Finally, the credit shocks model assumes the recovery rate on debt is stochastic and follows the Markov process in the text. All data are annualized.

Predictor	Horizon (in years)					
		1	2	3	4	5
A. $\log PD$	Dat	a				
	β_n	-0.132	-0.231	-0.292	-0.340	-0.430
	-	(-3.21)	(-2.96)	(-2.94)	(-3.03)	(-3.18)
	\mathbb{R}^2	0.090	0.157	0.193	0.214	0.254
	Ben	chmark N	Model			
	β_n	-0.082	-0.161	-0.236	-0.308	-0.378
		(-2.42)	(-2.76)	(-3.02)	(-3.23)	(-3.48)
	\mathbb{R}^2	0.066	0.126	0.180	0.234	0.273
B. CS	Dat	a				
	β_n	3.293	1.758	1.326	2.102	2.755
		(4.30)	(2.97)	(2.49)	(4.04)	(5.22)
	\mathbf{R}^2	0.039	0.018	0.017	0.034	0.048
	Ben	chmark N	Model			
	β_n	0.909	1.751	2.612	3.385	4.180
	,	(2.34)	(2.61)	(2.80)	(3.05)	(3.14)
	\mathbf{R}^2	0.031	0.062	0.089	0.115	0.139
C. QML	Dat	a				
·	β_n	0.653	0.531	0.638	0.825	1.074
	, 10	(1.18)	(1.08)	(1.53)	(2.21)	(3.14)
	\mathbf{R}^2	0.032	0.047	0.081	0.174	0.293
	Ben	chmark N	Model			
	β_n	0.237	0.455	0.686	0.892	1.106
	1- 10	(1.69)	(1.91)	(2.07)	(2.18)	(2.26)
	\mathbf{R}^2	0.034	0.067	0.096	0.125	(1.15)
D. ML	Ben	chmark N	Model			
	β_n	0.484	0.938	1.367	1.768	2.146
	<i> - 11</i>	(2.44)	(2.96)	(3.22)	(3.56)	(3.67)
	\mathbf{R}^2	0.071	0.131	0.186	0.239	0.283

Table 6: Return Predictability: Benchmark

This table reports excess stock return, $r_{t,t+n}^e - r_{t,t+n}^f$ forecasts for horizons, n, between one and five years in both the data and in our benchmark model. Simulated moments cathe from averages across 1000 simulations of 64 years each. The empirical sample comes from the CRSP. The parameter values used in the benchmark simulation are reported in Table 1. We report excess stock return forecasts of the form : $r_{t,t+n}^e - r_{t,t+n}^f = \alpha_n + \beta_n X_t + \epsilon_{t+1}$, with X_t the log price-dividend ratio (panel A), the value-weighted credit spread (panel B), the value-weighted quasi market leverage ratio (panel C), and the value weighted market leverage ratio (panel D). T-statistics are reported in parentheses. All standard errors are corrected with Newey-West.

	Horizon (in years)				
	1	2	3	4	5
A. Data					
β_n	-0.132	-0.231	-0.292	-0.340	-0.430
R^2	0.090	0.157	0.193	0.214	0.254
B. Benchmark Model					
β_n	-0.062	-0.121	-0.177	-0.226	-0.285
\mathbb{R}^2	0.036	0.072	0.105	0.128	0.154
C. All Equity β_n \mathbf{R}^2	-0.015 0.015	-0.029 0.027	-0.041 0.037	-0.052 0.046	-0.062 0.052
D. Exogenously Constant Leverage β_n \mathbf{R}^2	-0.017 0.011	-0.035 0.018	-0.053 0.023	-0.064 0.029	-0.078 0.033
E. Exogenously Countercyclical Leverage $\beta_n \ {\rm R}^2$	-0.052 0.039	-0.099 0.064	-0.148 0.091	-0.191 0.107	-0.215 0.129
F. Credit Shocks β_n \mathbf{R}^2	-0.065 0.043	-0.121 0.078	-0.173 0.107	-0.216 0.125	-0.264 0.149

Table 7: Stock Return Predictability with PD ratios

This table reports excess stock return, $r_{t,t+n}^e - r_{t,t+n}^f$ forecasts for horizons, n, between one and five years in both the data and in our benchmark model. We report population estimates. The empirical sample comes from the CRSP. The parameter values used in the benchmark simulation are reported in Table 1. Firms in the all equity model (Panel B) have no leverage. In panel D equity returns are levered exogenously by a constant market leverage ratio of 0.34. Panel E refers to a specification with exogenous market leverage with a mean of 0.34 and correlation with log changes in GDP, Δy , of -0.38. Finally, the credit shocks model assumes the recovery rate on debt is stochastic and follows the Markov process in the text. Excess stock return forecasts using the log-price-dividend ratio are of the form: $r_{t,t+n}^e - r_{t,t+n}^f = \alpha_n + \beta_n \log(P_t/D_t) + \epsilon_{t+1}$.

	Horizon (in years)					
	1	2	3	4	5	
A. Data	-0.132	-0 231	-0 292	-0.340	-0.430	
\mathbf{B}^{2}	0.090	0.157	0.193	0.214	0.254	
B. Benchmark Model β_n \mathbf{R}^2	-0.062 0.036	-0.121 0.072	-0.177 0.105	-0.226 0.128	-0.285 0.154	
C. $\zeta = 0.06$ β_n \mathbf{R}^2	-0.068 0.041	-0.132 0.081	-0.194 0.113	-0.252 0.145	-0.307 0.172	
D. $\rho_x = 0.9$ β_n \mathbf{R}^2	-0.021 0.009	-0.038 0.016	-0.056 0.023	-0.072 0.028	-0.086 0.032	
E. $\gamma = 2$ β_n \mathbf{R}^2	-0.009 0.002	-0.017 0.003	-0.024 0.005	-0.031 0.006	-0.036 0.007	

Table 8: Stock Return Predictability with PD ratios - Sensitivity

This table reports excess stock return, $r_{t,t+n}^e - r_{t,t+n}^f$ forecasts for horizons, n, between one and five years in both the data and in our benchmark model. We report population estimates. The empirical sample comes from the CRSP. The parameter values used in the benchmark simulation are reported in Table 1. Sensitivity is provided with respect to the refinancing probability (panel C), aggregate persistence (panel D), and risk aversion (panel E). Excess stock return forecasts using the log-price-dividend ratio are of the form: $r_{t,t+n}^e - r_{t,t+n}^f = \alpha_n + \beta_n \log(P_t/D_t) + \epsilon_{t+1}$.

	Horizon (in years)				
	1	2	3	4	5
A. Data β_n \mathbf{R}^2	$3.293 \\ 0.039$	$1.758 \\ 0.018$	$1.326 \\ 0.017$	$2.102 \\ 0.034$	$2.755 \\ 0.048$
B. Benchmark Model β_n \mathbf{R}^2	$0.722 \\ 0.021$	$1.351 \\ 0.035$	$1.938 \\ 0.049$	$2.554 \\ 0.073$	$3.106 \\ 0.124$
C. Credit Shocks β_n \mathbf{R}^2	$0.796 \\ 0.028$	$\begin{array}{c} 1.481 \\ 0.046 \end{array}$	$2.094 \\ 0.066$	$2.592 \\ 0.087$	$\begin{array}{c} 2.981 \\ 0.091 \end{array}$

Table 9: Stock Return Predictability with Credit Spreads

This table reports excess stock return, $r_{t,t+n}^e - r_{t,t+n}^f$ forecasts for horizons, n, between one and five years in both the data and in our benchmark model. We report population estimates. The empirical sample comes from the CRSP. The parameter values used in the benchmark simulation are reported in Table 1. Panel C reports the results for a model where the recovery rate on assets upon default is stochastic and follows the Markov process in the text. Excess stock return forecasts using the value-weighted credit spread are of the form: $r_{t,t+n}^e - r_{t,t+n}^f = \alpha_n + \beta_n \operatorname{CS}_t + \epsilon_{t+1}$ (panel B).

Table 10:	Cross-sectional	Moments
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Data	Benchmark	All Equity	Liquid. Default	Credit Shocks
0.03	0.03	0.02	0.03	0.03
0.06	0.06	0.05	0.05	0.07
0.08	0.11	0.22	0.00	0.12
0.19	0.27	0.00	0.20	0.29
0.08	0.12	0.07	0.09	0.14
0.41%	0.49%	0.00%	0.35%	0.52%
-0.21	-0.69	0.00	-0.55	-0.64
0.17	0.22	0.10	0.14	0.21
-0.84	-0.82	0.00	-0.67	-0.81
	Data 0.03 0.06 0.08 0.19 0.08 0.41% -0.21 0.17 -0.84	Data Benchmark 0.03 0.03 0.06 0.06 0.08 0.11 0.19 0.27 0.08 0.12 0.41% 0.49% -0.21 -0.69 0.17 0.22 -0.84 -0.82	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

This table reports average cross-sectional moments of firm characteristics across model specifications. The parameter values used in the benchmark simulation are reported in Table 1. The liquidity default model modifies the default decision of the firm to $\exp(x + \bar{z}(b, x)) - \delta < b$. Finally, the credit shocks model assumes the recovery rate on debt is stochastic and follows the Markov process in the text. The data are from the quarterly CRSP-Compustat file covering the years 1984 to 2014, except for the investment frequency that comes from Davis and Haltiwanger (1992). Credit sprads (CDS rates) are based on CMA data. All moments are reported at quarterly frequency.

	Data	Benchmark	Credit Shocks
Panel A: Leverage Regressions			
Size,	0.009	0.023	0.019
	(10.65)	(3.62)	(3.25)
Market-to-book, Q	-0.076	-0.102	-0.092
	(-40.42)	(-2.46)	(-2.13)
Profitability	-0.286	-0.393	-0.413
	(-15.42)	(-2.42)	(-2.81)
Panel B: Credit Spread Regressions			
Leverage	0.090	0.098	0.107
	(8.68)	(3.73)	(3.64)
Asset volatility	0.064	0.062	0.079
	(7.73)	(2.86)	(2.92)
Market-to-book	0.006	0.015	0.013
	(4.85)	(2.15)	(2.20)
Size	-0.002	-0.004	-0.004
	(-2.18)	(-2.16)	(-2.27)
Profitability	-0.071	-0.023	-0.027
	(-2.28)	(-2.34)	(-2.28)

Table 11: Cross-sectional Regressions

This table reports pooled panel regressions of (panel A) market leverage and (panel (B) credit spreads on firm characteristics in the data and on simulated data from the model. In the data, credit spreads (CDS rates) and asset volatility are based on CMA data. Leverage, market-to-book, size, and profitability come from CRSP-Computat. The parameter values used in the benchmark simulation are reported in Table 1. The credit shocks model assumes the recovery rate on debt is stochastic and follows the Markov process in the text. The empirical sample covers quarterly data of on average 320 firms between 2004 and 2011. *t*-statistics are clustered at the firm level and reported in parentheses.

Table 12: Leverage and Stock Ret	turns in the Model	
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Univ	variate S	Sorts		Double Sorts										
ML	QML	BM		ML QI					ΛL					
				Panel A: Benchmark										
						BM						BM		
			ML	Low				High	QML	Low				High
2.97	3.67	2.38	Low	2.16	2.75	3.72	4.83	5.66	Low	1.85	2.94	3.76	4.91	6.17
3.85	3.83	2 .00 3 .40	Tou	2.32	$\frac{2.19}{3.09}$	3.91	4.94	5.83	1011	2.19	3.08	4.20	5.05	6.22
4.39	4.22	4.29		2.46	3.40	4.21	5.05	5.96		2.38	3.44	4.27	5.14	6.38
4.98	4.86	5.24		2.55	3.57	4.54	5.32	6.29		2.52	3.59	4.41	5.52	6.55
5.57	5.18	6.45	High	2.76	3.68	4.73	5.70	6.75	High	2.61	3.87	4.73	5.72	6.84
					Panel	B: Cre	dit She	ocks						
						BM						BM		
			ML	Low		DI		High	QML	Low		DI		High
2.73	3.43	2.05	Low	1.88	2.56	3.60	4.73	5.59	Low	1.67	2.84	3.63	4.79	6.13
3.63	3.64	3.04		2.06	2.91	3.76	4.83	5.74		1.94	2.90	4.03	5.02	6.24
4.12	4.02	4.12		2.24	3.28	4.13	4.92	5.93		2.14	3.24	4.17	5.11	6.36
4.87	4.69	5.17		2.37	3.46	4.45	5.25	6.26		2.36	3.40	4.32	5.54	6.51
5.46	5.03	6.43	High	2.65	3.61	4.72	5.62	6.71	High	2.57	3.61	4.66	5.65	6.73

This table reports average realized annual returns of: (a) univariate portfolio sorts based on the market leverage ratio (ML), the quasi-market leverage ratio (QML), and book-to-market (BM) equity (left panel); and (b) unconditional double sorts based on market leverage and book-to-market equity, and on quasi-market leverage and book-to-market equity (right panel), both in the benchmark model (panel A), and in the credit shocks model that assumes the recovery rate on debt is stochastic and follows the Markov process in the text (panel B). Statistics are obtained by averaging the results from simulating the model economy 1000 times over 64 years. Each portfolio corresponds to a quintile. All returns are value weighted

Univari	ate Sorts	Double Sorts									
QML	BM										
			BM								
		QML	Low				High				
6.99	6.34	Low	4.76	8.02	6.79	6.49	8.68				
6.48	8.48		8.08	9.09	7.39	10.26	9.31				
8.38	7.55		6.42	5.27	8.93	9.56	6.88				
9.01	9.43		7.42	10.85	11.00	10.03	10.2				
10.35	10.71	High	9.50	11.84	8.54	13.37	13.44				

Table 13: Leverage and Stock Returns in the Data

This table reports average realized annual excess returns in the data: (a) univariate portfolio sorts based on the quasi-market leverage ratio (QML), and book-to-market (BM) equity (left panel); and (b) unconditional double sorts based on quasi-market leverage and book-to-market equity. The data comes from Data are from CRSP/Compustat merged dataset for all firms excluding financials and utilities covering the period since Jan 1977 and December 2015. Each portfolio corresponds to a quintile. All returns are value weighted

		$\Delta y_{t,t+4}$			$\Delta i_{t,t+4}$	
Panel A: Data						
CS_t	-3.89 (-2.82)			-9.71 (-2.39)		
Panel B: Benchmark						
CS_t x_t CS_t^{RN}	-2.33 (-2.58)	$\begin{array}{c} -1.09 \\ (-2.44) \\ 2.68 \\ (3.62) \end{array}$	-0.43 (-1.32)	-6.61 (-3.36)	$\begin{array}{c} -2.82 \\ (-2.35) \\ 4.35 \\ (3.43) \end{array}$	-1.54 (-1.41)
Panel C: Credit Shocks						
CS_t x_t CS_t^{RN}	-2.48 (-3.12)	$\begin{array}{c} -1.32 \\ (-2.63) \\ 2.34 \\ (2.61) \end{array}$	-0.39 (-1.15)	-7.70 (-3.75)	$\begin{array}{c} -4.12 \\ (-2.69) \\ 3.63 \\ (2.74) \end{array}$	-1.85 (-1.28)
Panel D: Liquidity Default						
CS_t x_t CS_t^{RN}	-1.17 (-2.34)	$\begin{array}{c} -0.76 \\ (-1.51) \\ 2.75 \\ (3.83) \end{array}$	-0.27 (-0.83)	-4.23 (-2.45)	$\begin{array}{c} -2.26 \\ (-1.74) \\ 4.53 \\ (3.26) \end{array}$	-1.19 (-1.12)

Table 14: Business Cycle Forecasting

This table reports forecasting regressions for output and investment growth in a number of model specifications and the data. We regress 4-period ahead log growth in output and investment, respectively: $\Delta y_{t,t+4} = \log Y_{t+4}/\log Y_t$ and $\Delta i_{t,t+4} = \log I_{t+4}/\log I_t$ on the value weighted aggregate credit spread CS_t at time t, and additional control variables. The risk-neutral credit spread CS_t^{RN} is the spread on bonds valued using the risk-neutral measure. Tstatistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 64 years. Standard errors are corrected using Newey-West.

		$\Delta y_{t,t+4}$			$\Delta i_{t,t+4}$	
Panel A: Data						
CS_t	-3.89 (-2.82)			-9.71 (-2.39)		
Panel B: $\zeta=0.06$						
CS_t	-1.91 (-2.55)	-0.96 (-2.34)		-5.29 (-3.01)	-2.14 (-2.23)	
x_t		(3.26)			3.96 (2.94)	
CS_t^{RN}		(0.20)	-0.53 (-1.18)		(2.01)	-1.44 (-1.21)
Panel C: $\rho_x = 0.9$						
CS_t	-1.19 (-2.21)	-0.72 (-1.84)		-3.54 (-2.63)	-1.08 (-2.19)	
x_t		2.14			2.71	
CS_t^{RN}		(3.05)	-0.32 (-1.14)		(3.23)	-1.13 (-1.27)
Panel D: $\gamma = 2$						
CS_t	-0.74 (-1.79)	-0.51 (-1.58)		-2.67 (-3.42)	-1.26 (-2.18)	
x_t		2.42			3.51	
CS_t^{RN}		(3.47)	-0.41 (-1.26)		(3.34)	-1.17 (-1.39)

Table 15: Business Cycle Forecasting: Sensitivity

This table reports forecasting regressions for output and investment growth in a number of model specifications and the data. We regress 4-period ahead log growth in output and investment, respectively: $\Delta y_{t,t+4} = \log Y_{t+4}/\log Y_t$ and $\Delta i_{t,t+4} = \log I_{t+4}/\log I_t$ on the value weighted aggregate credit spread CS_t at time t, and additional control variables. The risk-neutral credit spread CS_t^{RN} is spread on bonds valued using the risk-neutral measure. T-statistics are reported in parentheses below. Statistics for the model are obtained by averaging the results from simulating the economy 1000 times over 64 years. Standard errors are corrected using Newey-West.



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Figure 1: Optimal Policy Functions. This figure plots the policy functions for investment, $i(b, z, s, \eta)$, for $\eta = 1$ and $\eta = 0$, and default, $\bar{z}(b, s, 0)$, respectively for our baseline calibration at the mean productivity (solid line). The top panels shows the impact of a one standard deviation increase (dashed) and decrease (dotted) in aggregate productivity, x while the bottom panels show the effects of aone standard deviation increase (dashed) and decrease (dotted) in z.



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Figure 2: Cross-Sectional Distribution of Firms. This figure depicts the equilibrium cross-sectional distribution of firms, $\mu(s, b, z)$ for our baseline model. The top panel shows the impact of a one standard deviation increase in aggregate productivity, x on $\mu(\cdot)$ while the bottom panel shows the effects of a one-standard deviation decrease in x.



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Figure 3: Business Cycle Amplification. This figure shows the response of output, consumption and investment growth to a one standard deviation positive innovation in aggregate technology in both our baseline levered economy (solid) and an alternative scenario where all investment is financed with equity alone (dashed).



Figure 4: A Credit Supply Shock. This figure shows the response of output, investment and consumption growth to an improvement in credit market conditions induced by to a one standard deviation positive innovation in the recovery rate on assets ϕ in our augmented economy with credit shocks.



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Figure 5: Booms and Busts. This figure shows the responses of output, investment growth, and consumption growth to a one standard deviation positive and negative innovation in aggregate technology in our baseline levered economy (solid) and in an alternative scenario where all investment is financed with equity alone (dashed).