The Persistence of the 'Peso Problem' when Policy is Noisy

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The 'peso problem,' the market's belief that a discrete event may occur, has frequently been blamed for the persistence of on-average mistaken forecasts of macroeconomic variables. This paper demonstrates how beliefs that a policy process may have switched can induce apparent ex post biased forecasts of exchange rates even after the switch has occurred. Furthermore, during this 'peso problem' period, exchange rates may appear to contain a speculative bubble component since they will systematically deviate from the levels implied by observing fundamentals ex post; and the conditional variance of exchange rates will exceed the variance implied by observing fundamentals.

Empirical evidence encompassing the recent flexible exchange rate period has rejected the hypothesis that the forward rate is an unbiased predictor of the future spot exchange rate. Furthermore, attempts to relate the prediction error of forward exchange rates to risk characteristics have yet to establish a conclusive connection. As suggested by Cumby and Obstfeld (1984), a contributing factor to the observed bias in forward rates may be a 'peso problem;' that is, expectations by market participants that a discrete event such as a foreign exchange market intervention may occur when the event does not materialize for some time.1

Conventional wisdom concerning the 'peso problem' suggests that the problem disappears after the discrete change in policy occurs. But this paper demonstrates that, under the common assumption that policy and other forcing processes are noisy, the 'peso problem' may appear to persist in plaguing macroeconomic variables even after the discrete change in policy. Although market participants are rational, they require repeated observations to 'learn' the new process. Therefore, during the learning period, empirical phenomena typically associated with the 'peso problem' persist.

In the paper, the effects of this 'learning' period are specifically related to the behavior of the exchange rate although similar results hold for other macroeconomic variables as well. During this period, the 'peso problem' implies three characteristics of the exchange rate behavior that may contribute to

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empirically observed phenomena. First, for a policy switch from 'loose' money to 'tight' money the market expects a depreciating exchange rate as the currency follows an appreciating trend. The result may contribute to periods such as the early 1980s when the dollar exchange rate systematically traded at a forward discount as the dollar followed a basic appreciating path. Second, the exchange rate may appear to be too strong relative to the levels implied by observing the correct set of fundamental forcing variables. Third, the conditional variances of exchange rates exceed those implied by its fundamental variables.

The paper is organized as follows. Section I demonstrates how on-average incorrect forecasts can rationally persist after a policy change. Over time, the market understands the new process and the expected value of forecast errors are zero again. Section II relates the learning mechanism to empirical observations and tests of the exchange rate. Section III reports market efficiency regression results from data generated by the model.

I. Potential Policy Switches and the 'Peso Problem'

A number of works have studied the performance of the forward exchange rate as a predictor of the future spot exchange rate. Evidence strongly refutes the proposition that the forward rate is an unbiased predictor of the future spot rate. The experience of the early 1980s has been particularly damaging to this hypothesis. From roughly 1981 to 1982, the US dollar exchange rate systematically traded at a forward discount while the dollar continued to strengthen. From market survey data, Frankel and Froot (1987) found that expectations concerning the exchange rate were significantly wrong during the period. But in the simple model to be developed below, the market is shown to rationally make on-average mistaken forecasts after a change in a fundamentals process if it does not immediately recognize the change.

To demonstrate how empirical phenomena that resemble a 'peso problem' can result from a policy switch, the analysis below will consider a switch in the process of one of the fundamentals that drive the exchange rate. Although the process changes, the market is not sure whether policy has changed, but learns about this change over time through Bayesian updating. For this purpose, consider a simple determining equation for the exchange rate.

\begin{equation}
\log J_t = \beta'X_t + m_t + \alpha \Delta \log J_t
\end{equation}

where \( s_t = \beta'X_t + m_t + \alpha \Delta s_t \) and \( \Delta s_t = s_t - s_{t-1} \) measures the expected change in the exchange rate.
The analysis will focus upon a switch in the process of the particular fundamental, $m_t$. This variable is an arbitrary forcing process in the exchange rate equation whose coefficient has been set equal to one without loss in generality. The discussion throughout will refer to $m_t$ as the money supply, although this process can be any variable that affects the exchange rate. Indeed, it may even be a variable that is not under the direct control of domestic policy-makers, such as a foreign price or interest rate.

To make the simplest possible case, assume that the money supply process is just a mean level plus noise,

$$m_t = \theta_0 + \epsilon_t,$$

where $\theta_0$ is a constant mean of the process and $\epsilon_t$ is an i.i.d. normally distributed random variable with mean zero and variance $\sigma^2$. However, at a particular point in time, say $t=0$, the market comes to believe that the money supply process has changed. These initial beliefs could come from non-market fundamentals such as the announcement of new elections or statements by government officials or they could arise from fundamentals in an exogenous way. But in either case, the following analysis does not allow this information to affect the subsequent conditional probabilities. Otherwise, a model of how these non-market fundamentals or other relationships affect the exchange rate would be required.

Again, to keep the discussion simple, the new process is assumed to have the same form as the old process except with a different mean, $\theta_1$, and possibly a different variance, $\sigma_1^2$.

$$m_t = \theta_1 + \epsilon_t$$ for $t \geq 0$.

To allow easy discussion of the results below but without loss in generality, it is assumed that $\theta_0 \leq \theta_1$ and that $\theta_1 = 0$. Thus, one can think of the process change as going from 'loose' money to 'tight' money.

According to the market's view of the world, then, at a time 0 they come to believe that money either follows the old process in equation (2) or the new process in equation (3), but market participants are not sure which one. Additionally, two simplifying assumptions are made: (i) once policy changes, it is not expected to shift back, and (ii) the market knows the parameters of the potential new process. The first assumption allows for analytic tractability but does not affect the basic results below. The second assumption implies that, if there is a process switch, equation (3) represents both the actual process and the market's expected process given a switch. The main results below are also not sensitive to this assumption.

Solving equation (1) forward, the current exchange rate depends upon the present value of the expected future path of the forcing variables, including the expected future money supplies:

$$s_t = (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j \left( \beta' \hat{E}_t x_{t-j} + \hat{E}_t m_{t-j} \right),$$

where $\gamma = \lambda/(1 + \lambda)$. Under the assumptions outlined above, the expected money supply for all future periods is just,

$$E_t m_{t+j} = \theta_0 (1 - P_t)$$ for any $j > 0$, $t \geq 0$,

where $P_t$ is the market's assessed probability at time $t$ that the process changed at
time 0. Substituting the expected future money from (5) into (4) gives the following solution to the exchange rate:

\[ s_t = X_t + (1 - \phi_t)\theta_t + (1 - \gamma)\mu_t, \]

where \( X_t = (1 - \gamma) \sum_{j=0}^{\infty} \gamma^j \beta^j E_{t+j} X_{t+j} \). In other words, the exchange rate is affected by a weighted average of the current observation of \( \mu_t \) and the expected future \( \mu_t \).

Clearly, the behavior of the exchange rate depends strongly upon the behavior of the probability of a policy change.

Given the initial beliefs about policy changes at time 0, this conditional probability depends upon subsequent observations of the information set of fundamental variables. To obtain the best forecast of \( \phi_t \), market participants combine their prior beliefs about the probability together with their observations each period to update their posterior probabilities according to Bayes' Rule.

\[ P_{it} = \frac{P_{i,t-1} f(I_t | \theta_i)}{P_{i,t-1} f(I_t | \theta_i) + P_{n,t-1} f(I_t | \theta_n)}, \]

where \( P_{i,t} \) is the conditional probability of 'no change' at \( t = 0 \), \( f(I_t | \theta_i) \) is the probability of observing the information set \( I_t \), given that \( \mu_t \) follows the \( i \)th process. To simplify, assume that the innovations to the \( \kappa \) process are uncorrelated with the \( \epsilon_t \). Then, the information set reduces to the money supply since other fundamentals convey no signal about the noise contained in the observed policy variable. In this case, the ratio of posterior probabilities of each process, the 'posterior odds ratio,' reduces to the following convenient form:

\[ \left( \begin{array}{c} P_{i,t} \\ P_{n,t} \end{array} \right) = \left[ \begin{array}{c} P_{i,t-1} f(\mu_t | \theta_i) \\ P_{n,t-1} f(\mu_t | \theta_n) \end{array} \right] \]

\[ \times \left( \begin{array}{c} (1/\sigma_i) \exp(-(1/2)[\mu_t | \theta_i]'^2) \\ (1/\sigma_n) \exp(-(1/2)[\mu_t - \theta_i]'^2) \end{array} \right). \]

Equation (8) shows that the change from \( t - 1 \) to \( t \) in the relative conditional probabilities depends upon the observation of the current money supply at \( t \). For example, define as \( \bar{\mu} \) the money supply where the probability of being under either policy process is the same; i.e., \( f(\bar{\mu} | \theta_i) = f(\bar{\mu} | \theta_n) \). At this observation of money, the posterior probabilities, \( (P_{i,t} / P_{n,t}) \), equal the prior probabilities, \( (P_{i,t-1} / P_{n,t-1}) \), and hence the conditional probabilities do not change. But observations of money different from \( \bar{\mu} \) convey information causing the probabilities to be revised.

The conditional probability at \( t \) depends in a simple way upon the observation of \( \mu_t \) each period. Figure 1 demonstrates this relationship. Assume first for illustrative purposes that \( \sigma_i = \sigma_n \). Then the probability density functions conditional upon being in each state are identical except for the difference in means. In the figure, this case can be seen by comparing \( f(\bar{\mu} | \theta_i) \) with \( f(\bar{\mu} | \theta_n) \). Here \( \bar{\mu} \), the marginal observation of money where the posterior probability does not change, is just the level of money where the density functions intersect (i.e., \( \theta_i - \theta_n\)). At this point, the ratio of conditional probabilities will be equal to one and, therefore, \( P_{i,t} = P_{n,t} \). However, observations of money greater than \( \bar{\mu} \) will reduce the probability of the old process. The essential analysis remains unchanged when \( \sigma_0 > \sigma_i \). In Figure 1, the market compares the potential new process, \( f(\mu_t | \theta_i, \sigma_i) \), with the old process,
\[ f(m|\theta_0, \sigma_0^2). \] But in this case, more of the probability mass for the new process will be in the tails so that \( \hat{m} \) lies further to the left.

Given that the market's assessed probability of a change in policy is a random variable, the stochastic behavior of the exchange rate depends upon these probabilities. To analyse the behavior of the probabilities, it will prove convenient to linearize the posterior odds ratio by taking the logarithm of equation (8).

\[ \log \left[ \frac{P_{m,t}}{P_{\theta_t}} \right] = \log \left[ \frac{f(m_t|\theta_t)}{f(m_t|\theta_0)} \right] + \log \left[ \frac{f(\theta_t)}{f(\theta_0)} \right]. \]

For analytic simplicity, assume that \( \sigma_0 = \sigma_1 = \sigma \) (although the essential results remain when the variances differ). Then, since the errors are normally distributed,

\[ \log \left[ \frac{f(m_t|\theta_t)}{f(m_t|\theta_0)} \right] = \frac{[m_t - \theta_t]^2}{2\sigma^2}. \]

Then, (9) describes a linear difference equation in the dependent variable, \( \log(P_{m,t}/P_{\theta_t}) \). Given the initial probabilities, \( P_{m,0} \) and \( P_{\theta_0} \), and substituting for the log-likelihood ratio from (10), the solution to this difference equation becomes:

\[ \log(P_{m,t}/P_{\theta_t}) = \log(P_{m,0}/P_{\theta_0}) + \sum_{t=1}^{T} \left[ \theta_t^2 - 2m_t\theta_0 \right] / 2\sigma^2. \]

In equation (11), the behavior of probabilities depends upon the actual observations of the process. For example, when the money supply this period is large and positive, the market now believes the old process with its higher mean is more likely than the new lower mean process. Specifically, the equation makes clear that when \( m_t \) is large, the posterior probabilities of a policy change will fall at \( t \). Similarly, when the money supply observed today is strongly negative, the market thinks it more likely that policy has changed. Hence, the market's assessed probabilities of a policy change are random variables determined by random observations of the money supply.
Therefore, understanding the expected behavior of exchange rates during this period requires evaluating the expected evolution of the market's beliefs about policy change. Given the market's beliefs about a policy switch at the point in time 0, the expected path is the evolution of probabilities that would arise in a large sample of repeated draws of sequences from the true process. This expected path comes from taking the expected value of log(P_{1,t}/P_{0,0}) conditional on the true process \( \theta \) and the initial probabilities. Hence, taking the expectation of (11) and defining \( \theta_i \) as the true \( \theta \) gives this expected value.

\[
\mu_{0,t} = \log(P_{1,0}/P_{0,0}) + t(\theta_0^2 - 2\theta_0 \theta_1) / 2\sigma^2.
\]

Similarly, the variance of log(P_{1,t}/P_{0,0}) conditional on the true process and the initial probabilities is: \( t\theta_0^2 / 2\sigma^2 \).

Equation (12) clearly shows that the expected value of the 'true' process probability rises over time. To see this result, take first the case where policy has changed, \( \theta = \theta_1 \), so that equation (12) becomes:

\[
\mu_{0,t} = \log(P_{1,0}/P_{0,0}) + t\theta_1^2 / 2\sigma^2.
\]

Then, \( \mu_{0,t+1} > \mu_{0,t} \) and the expected probability of a policy change rises over time. Similarly, when policy has not changed so that \( \theta = \theta_0 \), equation (12) implies,

\[
\mu_{0,t} = \log(P_{1,0}/P_{0,0}) - t\theta_0^2 / 2\sigma^2.
\]

Again, it is clear that \( \mu_{0,t+1} < \mu_{0,t} \).

Thus, even though the probabilities are random variables that may rise or fall according to the realization of \( m \), in any given period, the expected path of the 'true' process probabilities rises and goes to one. That is, taking the limit of (13) as \( t \) goes to infinity shows that \( \mu_{0,t} \) goes to infinity, and similarly from (14), \( \mu_{0,t} \) goes to negative infinity.10 Equations (13) and (14) also show that how quickly the expected probabilities converge depends positively upon \( (\theta_0^2 / \sigma^2) \). This feature indicates that the speed of market learning depends upon the squared signal-to-noise ratio, \( \theta_0 \) being the difference between the two policy means.

Given the analysis of the probability behavior, the effects of this behavior on the exchange rate and its forecast errors can be investigated. Taking the expectation of the exchange rate conditional on \( t - 1 \) information and subtracting the result from the exchange rate given in (6) gives the forecast errors of the exchange rate in terms of each potential process:

\[
\langle 15a \rangle \quad s_t - E_{t-1}s_t = (X_t, - s_{t-1}X_t) + (1 - \gamma)\epsilon_t + \theta_0(P_{1,t-1} - \gamma P_{1,t}),
\]

\( \theta_t = \theta_0 \);  

\[
\langle 15b \rangle \quad s_t - E_{t-1}s_t = (X_t, - s_{t-1}X_t) + (1 - \gamma)\epsilon_t - \theta_0(P_{0,t-1} - \gamma P_{0,t}),
\]

\( \theta_t = \theta_1 \).

Although the expected values of the first two components are zero, the last component depends upon the conditional probabilities. While the market is learning, the behavior of these conditional probabilities depends upon the random observations of the money process, and is not equal to the true values. At time \( t \), the information set that the market uses to both determine the current exchange rate and to make exchange rate forecasts includes the current state of learning as embodied in the posterior probabilities, \( P_{i,t} \). From an initial point in time, then, one can determine the expected value of the path of forecast errors conditional upon the
true process, just as the expected path of probabilities was calculated above in equation \langle 12 \rangle.

Taking expectations of the forecast errors in equation \langle 15b \rangle conditional upon a change in policy and initial probabilities, gives the expected evolution of forecast errors that would result from a large number of repetitions of the sequence \( m \), when policy changes.

\[
\langle 16 \rangle \quad E(s_1 - E_{t-1}s_1 | \theta_1) = -\theta_1 \left[ E(P_{0t-1} | \theta_1) - \gamma E(P_{0t} | \theta_1) \right] < 0.
\]

The inequality follows by recalling both that the discount rate, \( \gamma \), is less than one and that, conditional upon initial probabilities at time 0, \( E(P_{00} | \theta_1) < E(P_{0t-1} | \theta_1) \).

Therefore, if the market does not completely realize that the policy has changed to a 'tighter' money supply process, the market will on-average expect a weaker exchange rate than subsequently occurs. Similarly, when the policy process does not switch, the expected evolution of forecast errors is on-average positive.

These on-average systematic errors reflect the interaction of two forecasts. At \( t - 1 \), the market overestimates the exchange rate at time \( t \) by \( \theta_0 P_{t-1} \). However, at time \( t \) they still mistakenly believe that policy may not have changed as measured by \( P_{0t} \). They therefore anticipate a higher money supply in the future, causing a weaker exchange rate today by \( \gamma \theta_0 P_{t-1} \). Since on average the probability of 'no-change', \( P_{0t} \), declines over time, the effect at time \( t - 1 \) dominates the effect at time \( t \) in expected value.

If these forecast errors could be viewed *ex post*, this behavior might lead an observer to incorrectly conclude that the market were behaving irrationally. However, such an observer would be assuming that the private sector had full knowledge of the fundamental processes that drive the exchange rate. But in the example above, the market is uncertain about the policy process and only learns the true process over time. Therefore, conditional on initial prior beliefs and the subsequent observations of the process, they form their forecasts rationally.\(^{11}\)

Consider the effects of this persistent 'peso problem' upon a simple market efficiency diagnostic. For the sake of argument, suppose that there were no risk premium so that the forward exchange rate identified the expected future spot exchange rate. Furthermore, assume that the sample period begins when the policy switch occurs, a point in time where one might split the sample period of the data in the hopes of avoiding contamination by the policy change.

But, if the market is learning during this period, the sample average would continue to be non-zero on average. Substituting from equation \langle 16 \rangle above, the expected value of the sample average following a policy switch is:

\[
(1/T) E \left\{ \sum_{t=1}^{T} (s_t - E_{t-1}s_t) | \theta_1 \right\} = -\left( \frac{\theta_0}{T} \right) E \left\{ \sum_{t=1}^{T} (P_{0t-1} - \gamma P_{0t}) | \theta_1 \right\} < 0.
\]

Since on average the probability of the old process declines over time, the expected value of the forecast error sample mean is negative. However, as the expected values of the 'wrong' process probabilities decline over time, the expected value of the forecast errors conditional on a policy change diminish in absolute value. As \( T \) becomes large, the expected values of \( P_{0t} \) and \( P_{0t-1} \) become close to zero so that the expected forecast errors conditional on learning go to zero. As more observations are added to the sample, the expected value of the sample mean goes to zero.
It should be noted that the role played by the conditional probabilities upon the behavior of the exchange rate and its forecast errors is not particular to foreign exchange. For example, studies of the behavior of real interest rates might incorrectly conclude that \textit{ex ante} rates were higher than in actuality if, say, the market did not yet believe that inflation had fallen to a new lower level.\footnote{12}

\section*{II. Implications for Exchange Rate Behavior}

The persistence of uncertainty about the current policy regime has implications for other empirical regularities as well. This section illustrates how this 'peso problem' can help explain the following phenomenon:

1. The currency trades at a forward discount while the exchange rate follows an appreciating trend.
2. The exchange rate deviates from the level implied by observing the correct set of fundamentals variables \textit{ex post}.
3. The conditional variances of exchange rates exceed the conditional variance implied by observing fundamentals \textit{ex post}.\footnote{13}

\subsection*{II.A. A Persistent Forward Discount for an Appreciating Currency}

A motivating example used in the first part of this paper was the behavior of the US dollar forward rate prediction error during the early 1980s. From 1981 to 1982, the dollar exchange rate traded at a forward discount against major currencies. However, during this period, the dollar followed a basic appreciating trend.

To demonstrate how the market’s learning about policy could contribute to this observation, suppose that policy switched from a looser money supply, $\theta_0$, to a tighter money supply, $\theta_1$. Furthermore, assume that the forward rate equals the expected future spot exchange rate so that the forward discount is the expected rate of depreciation. Leading equation (6) forward one period, taking conditional expectations, and subtracting the current exchange rate gives the following form of the forward discount.

\begin{equation}
E_{t+1} s - s_t = (X_{t+1} - X_t) - (1 - \gamma) \epsilon_t^f + (1 - \gamma) P_{0, t} \theta_0.
\end{equation}

Clearly, the actual forward premium depends upon both the expected change in the fundamental processes as well as the probability of a process switch. The $X$ and $\epsilon$ contribute noise to this series. To focus on the process switch, suppose that the unconditional expected change in other fundamentals is zero. Then, the expected value of the sample mean of the forward discount while the market is learning about policy is positive.

\begin{equation}
(1/T) E \left\{ \sum_{t=1}^{T} (E_{t+1} s - s_t) \right\} = (1 - \gamma)(\theta_0/T) E \left\{ \sum_{t=1}^{T} P_{0, t} \theta_0 \right\} > 0.
\end{equation}

Intuitively, to the extent that the market thinks policy may not have changed, it thinks that a larger component of today’s money supply is transitory noise. Therefore, it anticipates a depreciation on average.

On the other hand, the \textit{actual} exchange rate appreciates on average. Leading equation (6) and subtracting from $s_t$ gives the actual change in the exchange rate...
from \( t \) to \( t + 1 \). Taking the expected value of the sample mean of this exchange rate change gives,

\[
(1/T)E\left\{ \sum_{t=1}^{T} (s_{t+1} - s_t) \right\} = -(\gamma \theta_0/T)E\left\{ \sum_{t=1}^{T} (P_{0,t} - P_{0,t+1})|\theta_1 \right\} < 0.
\]

As the market comes to recognize that policy has switched to a tighter monetary policy, the exchange rate follows a general appreciating trend. Thus, learning of this sort could potentially contribute to the foreign exchange experience of the early 1980s.

**II.B. A 'Pseudo-Bubble'**

Another implication of this policy process uncertainty is that the level of the exchange rate deviates from the level implied by observing its 'fundamentals' *ex post*, behavior that some observers associate with speculative bubbles. This implication is also noteworthy since a frequently-used test procedure for detecting explosive bubbles terms requires that the non-explosive 'fundamentals' component of the exchange rate be equal to the exchange rate implied by observing fundamentals *ex post*. Under the null hypothesis of 'no explosive bubbles components,' the actual exchange rate and the exchange rate implied by observed fundamentals should be the same, implying cross-equation constraints between the two variables. According to the test, these constraints will not be rejected under the null hypothesis. Since this procedure has been discussed elsewhere, here it will simply be shown that the level of the exchange rate implied by observing the true process *ex post* will no longer equal the actual exchange rate during learning. Therefore, cross-equation constraints based upon this equality will be rejected, even though by construction the exchange rate contains no explosive bubble terms.

To see this result, first calculate the 'fundamentals' value of the exchange rate that would arise if the market knew the true process. Using the actual process in equation (6) gives:

\[
s_{t}^f = X_t + \theta_0 + (1 - \gamma)\theta_1,
\]

Therefore, subtracting (21) from (6) gives the deviation of the actual exchange rate from its 'fundamentals' level,

\[
s_t - s_t^f = (-1)^{t+1} \gamma \theta_0 (1 - P_{0,t}) \neq 0,
\]

As long as the market doubts the 'true' policy process and therefore assigns it a probability less than one, this doubt will drive a wedge between the exchange rate and the level implied by observing fundamentals *ex post*. To illustrate, suppose that there has not been a change in policy to the tighter money supply as the market expects. Then the exchange rate will be kept relatively strong by this anticipation. In this case, the pseudo-'bubble' term is negative,

\[
s_t - s_t^f = -\gamma \theta_1 P_{0,t} < 0.
\]

Thus, even in a period of loose money supply, the exchange rate may be kept temporarily stronger than implied by observed fundamentals if the market believes that a new tighter policy is in force. Hence, observers may incorrectly claim that the exchange rate contains a speculative bubble. Since this wedge depends upon the
random probabilities, the deviations will also be random variables. As the probabilities converge over time, the size of the wedge vanishes.

II.C. 'Excessive' Volatility

The variances of the forecast errors further characterize the behavior of exchange rates during this learning period. For purposes of comparison, consider the variance of exchange rates if the true policy process were known by the market. In this case, the respective probabilities of each state will be zero or one. From equations (15), the forecast errors in this case would only involve \( x \) and \( \epsilon \). Therefore, if the true policy process were known, then the variance of exchange rate forecast errors would only depend upon the variance of these fundamentals. Since the focus of the discussion is on the switching policy process, the following equations will omit the variance of the \( x \)s.\(^{16}\) Defining \( a = (1 - \gamma) \), the variance of forecast errors due to the variance of the money supply can be written:

\[
E_{r-1}[D^*(s_i)^2 | \theta_j] = a^2 \sigma_j^2.
\]

However, when the market is uncertain about which policy process is being followed, the conditional variances of the forecast errors will be affected by the unanticipated movements in the conditional probabilities. Taking the conditional variances of equations (15) gives,

\[
E_{r-1}[D^*(s_i)^2 | \theta_j] = a^2 \sigma_j^2 + b V_{r-1}(\theta_j) + (-1)^{j-1} 2ab \text{Cov}_{r-1}(\epsilon_i, \theta_j), \quad j \neq i
\]

where \( b = \gamma \theta_j \) and \( V_{r-1} \) and \( \text{Cov}_{r-1} \) are the variance and covariance operators, respectively on \( t - 1 \) information. As shown above, the conditional variances can be decomposed into three parts. The first term on the right-hand side of equation (25) is the variance arising from the true underlying fundamental process as in equation (24). The second term is the variance arising from unanticipated disturbances in the 'wrong' conditional probabilities. Clearly, this term is positive and diminishes in expected value over time as the probabilities decline. The third term captures the interaction between the disturbance to the true process and the unexpected change in the probabilities. The following result establishes that this interaction contributes unambiguously to a higher conditional variance in both processes. Its proof is given in the appendix.

\[
\text{Cov}_{r-1}(\epsilon_i, \theta_j) > 0, \quad \text{Cov}_{r-1}(\epsilon_i, \theta_{j'}) < 0.
\]

Intuitively, a positive disturbance to \( m \) will cause a positive forecast error in the exchange rate. To the extent that the market thinks it now more likely that the process is 'loose money,' \( \theta_j \), rather than 'tight money,' \( \theta_{j'} \), there will be a greater negative movement in \( P_{i'} \). Hence, the interactions of the disturbances with the probabilities exacerbates the variance of exchange rates.

This result indicates that if the market is unclear about the direction of policy, rational forecasts of exchange rates will not only appear to be on-average wrong but will also experience greater variance than that implied by observing fundamentals \textit{ex post}. Interestingly, the behavior of these conditional variances is time-varying. Hence, after a policy switch, forward rate prediction errors would exhibit conditional heteroscedasticity with respect to stationary processes such as...
the $x_i$. This heteroscedasticity will disappear over time as the variance of the conditional probabilities vanishes in expected value.

III. Market Efficiency Regressions Using Data During 'Learning'

In order to illustrate how a persistent peso problem after a policy regime change might affect regression results, data were constructed from the model above. Even though the process in this model is too simple to describe an actual fundamental process in the exchange rate, this exercise gives some insights into potential problems in practice. In this example, the money process was assumed to move from a $N(0.5, 2)$ to a $N(0, 3)$ distribution. Repeating the process fifty times generated fifty possible sequences of exchange rate forecasts according to different realizations from the process. From these series, each of two sets of regressions was performed fifty times. The first set of regressions is a market efficiency test relating the forward exchange rate premium to the actual change in the exchange rate. Studies that have conducted this regression are summarized in Levich (1985a). If there were no risk premium, the forward rate in this model would be equal to the expected future spot exchange rate. So regressing the exchange rate change on the forward premium would translate into the following regression:

$$s_{t+1} - s_t = \delta_0 + \delta_1(E_t s_{t+1} - s_t) + \epsilon_{t+1},$$

where $\delta_i$ are regression coefficients and $\epsilon_t$ is a disturbance term. Thus, a typical market efficiency test is to run this regression and test: $\delta_0 = 0, \delta_1 = 1$.

A second set of regressions was based upon studies such as Cumby and Obstfeld (1981). Under rational expectations and no risk premium, past forward rate prediction errors should not contain information useful for predicting future prediction errors. Therefore, an autoregression of the prediction error should result in coefficients that are all zero. Using the current model constructed from rational expectations and no risk premium, the regression becomes,

$$s_{t+1} - E_t s_{t+1} = \delta_0' + \delta_1'(s_t - E_t s_t) + \delta_2'(s_{t-1} - E_{t-2} s_{t-1}) + \epsilon_{t+1}'.$$

Both of these regressions were carried out fifty times as summarized in Table 1. The table reports the number of times each of the hypotheses was rejected. Since the regressions were conducted fifty times, one would expect to reject the hypotheses at 10 per cent confidence levels roughly five times and at 5 per cent confidence levels about two or three times. However, for the regressions using a data set of fifty observations, the hypothesis that $\delta_0 = 0$ is rejected thirty-six times. Furthermore, the hypothesis that $\delta_1 = 1$ is nearly always rejected. In fact, the hypothesis that $\delta_1 = 0$ and positive is only rejected in two cases out of fifty. In some cases, $\delta_1$ is significantly negative. In addition, the point estimates of $\delta_1$ are often negative. For example, when the number of observations is thirty, $\delta_1$ is negative twenty-five times out of the fifty regressions, despite the fact that its theoretical value is positive. Thus, although the severity of the problem cannot be concluded from this particular example, these regressions support the notion that a persistent peso problem after a tightening of monetary policy can cause a downward bias in the prediction of the forward rate.

The high incidence of negative $\delta_1$ is particularly interesting in light of the results obtained in Fama (1984) and confirmed by Hodrick and Srivastava (1986). For a number of currencies against the US dollar during the floating rate period, they...
TABLE I. Market efficiency regressions using simulated data.

<table>
<thead>
<tr>
<th>Equation:</th>
<th>$s_{t+1} - s_t = \delta_0 + \delta_1 (E_s s_{t+1} - s_t) + \epsilon_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean:</td>
<td>$-0.28 (-0.73)$ \hspace{1cm} $-0.01 (0.01)$</td>
</tr>
<tr>
<td>Range:</td>
<td>$-1.45/-0.01 (-1.97/-0.01)$ \hspace{1cm} $-0.61/0.49 (-0.74/0.42)$</td>
</tr>
<tr>
<td>No. of rejections:</td>
<td>$H_0: \delta_0 = 0$ \hspace{1cm} $H_0: \delta_1 = 1$ \hspace{1cm} $H_0: \delta_2 = 0$</td>
</tr>
<tr>
<td>at 5 per cent</td>
<td>11 (36) \hspace{1cm} 47 (46) \hspace{1cm} 1 - /2 + (1 - /2 +)</td>
</tr>
<tr>
<td>at 10 per cent</td>
<td>20 (38) \hspace{1cm} 48 (48) \hspace{1cm} 5 - /2 + (1 - /3 +)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation:</th>
<th>$s_{t+1} - E_s s_{t+1} = \delta_0 + \delta_1 (s_t - E_s s_{t-1}) + \delta_2 (s_{t-1} - E_s s_{t-1}) + \epsilon_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean:</td>
<td>$-0.15 (-0.08)$ \hspace{1cm} $-0.02 (-0.01)$ \hspace{1cm} $-0.10 (-0.06)$</td>
</tr>
<tr>
<td>Range:</td>
<td>$-0.80/0.22 (-0.57/0.31)$ \hspace{1cm} $-0.48/0.27 (-0.50/0.25)$ \hspace{1cm} $-0.39/0.19 (-0.38/0.24)$</td>
</tr>
<tr>
<td>No. of rejections that $\delta = 0$:</td>
<td>at 5 per cent \hspace{1cm} 7 (7) \hspace{1cm} 10 (1) \hspace{1cm} 1 (4)</td>
</tr>
<tr>
<td>at 10 per cent</td>
<td>12 (10) \hspace{1cm} 11 (4) \hspace{1cm} 5 (7)</td>
</tr>
</tbody>
</table>

Note: The reported figures are for a sample of 30 (50) observations. The +/ - distinction in the $\delta_1$ column indicates the sign of the coefficient when significantly different than zero.

regress the actual change in the exchange rate on the forward premium. They find that this coefficient is negative. Fama (1984) demonstrates how the negative coefficient can imply that the variance in the risk premium exceeds the variance in the expected change in the exchange rate.

The prevalence of negative $\delta_1$ in Table 1 suggests another factor that may help explain the negative coefficients in the regressions in Fama (1984) and Hodrick and Srivastava (1986). Although the model was constructed without any risk premium, the market systematically expects a depreciation while on average the exchange rate is appreciating. Therefore, the expected change in the exchange rate and the actual change are negatively correlated during 'learning.'

Table 1 also reports the results of regressing exchange rate forecast errors on its own lagged values as in equation (28). The hypothesis that the coefficients $\delta_1 = 0$ are rejected more often than the confidence levels would suggest. In particular at the 10 per cent level, the hypothesis that $\delta_1$ is zero is rejected ten times with thirty observations and twelve times with fifty observations. With thirty observations, a zero coefficient is also rejected relatively frequently for $\delta_1$, although less frequently for $\delta_2$. Thus, the autocorrelation appears to die out rather quickly. However, this pattern between $\delta_1$ and $\delta_2$ is reversed when the sample is expanded to fifty observations. In summary, the results in Table 1 suggest that even after a policy switch empirical estimation may be affected by the market learning about the policy.

IV. Concluding Remarks

This paper has provided an example of how a policy process switch can cause persistence in empirical phenomena that resemble a 'peso problem' in macroeconomic variables. In contrast to the conventional view that a 'peso problem' disappears immediately after the discrete policy change, the problem can...
continue to contaminate empirical studies during a learning period after the change. The properties of this problem for flexible exchange rates and other prices also have implications for how they appear to respond to fundamentals. Even if the correct set of fundamentals were known and there were no speculative bubble component, these prices would deviate from the levels implied by observing fundamentals. Furthermore, as long as the 'peso problem' persists, the variance of exchange rates will exceed the variance implied by their fundamentals.

While these results were obtained under a number of simplifying assumptions, they indicate that empirical work should exercise caution in interpreting results from macroeconomic data in periods following anticipated or actual policy changes. During such periods, measures of future forecasts may appear to be biased \textit{ex post} although \textit{ex ante} market participants are rationally using all available information to learn about the policy change. Also, even if speculative bubbles do not exist, tests based upon, for example, the deviation of the exchange rate from its fundamental level are likely to find what may appear to be a bubble. Finally, during such periods, the variances of macroeconomic variables will tend to exceed those implied by observing fundamentals \textit{ex post}.

\section*{Appendix}

\textit{Convergence of the Probabilities}

Define first the following terms:

\[ F(x, \theta_i) \equiv \sigma \exp \left[ -(1/2) \left( \frac{x - \theta_i}{\sigma} \right)^2 \right] \]

\[ H(x) \equiv \frac{F(x, \theta_0)}{F(x, \theta_0) + F(x, \theta_1)} \]

\[ K = \frac{P_{t-1|t-1}}{P_{t-1}} \]

\[ \hat{m} = \theta_0/2 \]

Then, the probability of either process \( \theta_i \) can be written:

\[ P_{t,i} = \frac{P_{t-1|t-1} F(x, \theta_i)}{P_{t-1|t-1} F(x, \theta_0) + P_{t-1|t-1} F(x, \theta_1)} \]

Consider an initial probability of the new process, \( P_{1,t-1} \). Combining over a common denominator, the innovation in the probability becomes:

\[ P_{1,t} - P_{1,t-1} = (P_{0,t-1} P_{1,t-1}) \left[ \frac{F(x, \theta_1) - F(x, \theta_0)}{P_{0,t-1} F(x, \theta_0) + P_{1,t-1} F(x, \theta_1)} \right] \]

\[ = P_{0,t-1} \left[ \frac{1 - H(x)}{1 + KH(x)} \right] \]

Then the expected value of a change in \( P_{1,t} \) given that the true process is \( \theta_1 \) and \( P_{1,t-1} \) is:

\[ E\{ P_{1,t} - P_{1,t-1} | \theta_1, P_{1,t-1} \} = P_{0,t-1} \sigma \sqrt{2\pi} \int_{-x}^{x} Z(x, \theta_1) dx \]

where \( Z(x, \theta_0) \equiv (x - \theta_0)[F(x, \theta_1) - F(x, \theta_0)]/(1 + KH(x)) \).

This can be divided into positive and negative components by splitting the integral at \( \hat{m} \).

\[ E\{ P_{1,t} - P_{1,t-1} | \theta_1, P_{1,t-1} \} = P_{0,t-1} \sigma \sqrt{2\pi} \left[ \int_{-x}^{x} Z(x, \theta_1) dx + \int_{-x}^{x} Z(x) dx \right] \]
Persistence of the 'Peso Problem' when Policy is Noisy

where the first term is negative and the second is positive. Next, note that for every \( x \) on \((-\infty, \hat{m})\) there corresponds a \( z = 2\hat{m} - x \) on \( (\hat{m}, \infty) \) such that:

(i) \( F(x, \theta_1) - F(x, \theta_0) = -[F(z, \theta_0) - F(x, \theta_1)] \),

(ii) \( H(x) < H(y) \).

These two facts together imply that \( \langle A2 \rangle \), the expected change in the true probability is positive. Since \( \langle A2 \rangle \) is positive for any positive prior probability, it is immediate that as the expected probability is iterated forward, the expected change is always positive. Therefore, the expected probability \( E(P_{t+1}\mid \theta_0) \) converges to the upper bound of unity.

To determine what happens to the variance of the probability distribution as the mean converges, note that the variance of the innovation in \( \langle A1 \rangle \) conditional on the expected path of the probabilities can be written,

\[
\langle A3 \rangle \quad \text{Var}\{P_{t,i} - P_{t-1,i}\mid \theta_1, E(P_{t,i-1})\} = E(P_{0,i-1})\sigma^2 \sqrt{2\pi} \int_{-\infty}^{\infty} \frac{1 - H(x)}{1 + E(P_{0,i-1})/P_{0,i-1}H(x)} F(x, \theta_1)dx.
\]

Taking the limit of \( \langle A3 \rangle \) as \( t \) goes to infinity, the variance goes to zero.

**Proof of Equation \( \langle 26 \rangle \)**

\[
\langle 26 \rangle \quad \text{Cov}_{t,i}(\varepsilon^i, P_{0,i}) > 0.
\]

In terms of the notation above, the covariance of the disturbance when the true process is the old, \( \theta_0 \), is given by,

\[
\text{Cov}_{t,i}(\varepsilon^i, P_{0,i}) = P_{t,i-1} \sigma^2 \sqrt{2\pi} \int_{-\infty}^{\infty} Z(x, \theta_0)dx,
\]

where \( Z(x, \theta_0) = (x - \theta_0)[F(x, \theta_0) - F(x, \theta_1)]/[1 + (KH(x))^{-1}] \).

To sign this integral, it is useful to break it up into different intervals. So, rewriting gives the following form:

\[
\text{Cov}_{t,i}(\varepsilon^i, P_{0,i}) = P_{t,i-1} \sigma^2 \sqrt{2\pi} \left[ \int_{-\infty}^{\theta_0} Z(x, \theta_0)dx + \int_{\theta_0}^{2\theta_0 - \hat{m}} Z(x, \theta_0)dx \right] + \int_{2\theta_0 - \hat{m}}^{\infty} Z(x, \theta_0)dx.
\]

For \( x \) on \((-\infty, \hat{m})\), \((\theta_0, 2\theta_0 - \hat{m})\), and \((2\theta_0 - \hat{m}, \infty)\), inspecting the components of \( Z(x, \theta_0) \) verifies that \( Z(x, \theta_0) > 0 \). However, for \( x \) on \((\hat{m}, \theta_0)\), \( Z(x, \theta_0) < 0 \). Therefore, a sufficient condition for the integral to be positive is:

\[
\left| \int_{\theta_0}^{2\theta_0 - \hat{m}} Z(x, \theta_0)dx \right| > \left| \int_{-\infty}^{\theta_0} Z(x, \theta_0)dx \right|.
\]

But this follows immediately since for every \( x \) on \((\hat{m}, \theta_0)\) there corresponds a \( y = 2\theta_0 - x \) on \((\theta_0, 2\theta_0 - \hat{m})\) such that:

(i) \( |y - \theta_0| = |x - \theta_0| \)

(ii) \( |F(x, \theta_0) - F(x, \theta_1)| < |F(y, \theta_0) - F(y, \theta_1)| \)

(iii) \( 1 + KH(x) > 1 + KH(y) \).

To show the result for \( \text{Cov}_{t,i}(\varepsilon^i, P_{0,i}) \) requires redefining \( Z(x, \theta_0) \) in terms of \( \varepsilon^i = (x - \theta_i) \). By following the same steps as above, the integral can be shown positive.
Notes

1. The phenomenon has been called the 'peso problem' because it was initially associated with the recurring expectation of a devaluation in the Mexican peso market. The earliest formulation appears to be Rogoff (1979). Before the devaluation occurred, the forward and futures markets persistently underpredicted the value of the peso. See also discussions in Lizondo (1983), Krasker (1980), and Borensztein (1987).

2. See, for example, Hsieh (1984), Cumby and Obstfeld (1981), and Hansen and Hodrick (1980), among many others. The literature is surveyed by Levich (1985a). This forward prediction error may arise from a risk premium as described in Hodrick (1981). However, as Cumby (1986) and Borensztein (1987) have pointed out, the risk premium implied by the forward market during much of the early 1980s was against the dollar although during this period market analysts claimed there was a 'flight into dollars.'

3. In a recent study of the period January 1980 to June 1985, Levich (1985b) finds the mean forward rate prediction error for the US dollar versus Swiss franc exchange rate from January 1980 to June 1985 to be a statistically significant -1.4 per cent on a monthly basis. For the German deutsche mark, British pound, and Japanese yen, the size of the average prediction error is also significantly negative but slightly smaller in absolute value.

4. The basic model to be developed below is due to the 'asset market approach to the exchange rate' as in Mussa (1982) and Frenkel and Mussa (1985). In a simple monetary model, \( \pi \) represents the interest semi-elasticity of money demand.

5. These initial beliefs are the market's priors used in the Bayesian updating.

6. Similar assumptions also appear in related settings such as in deterministic theoretical models like Flood and Garber (1982) and Obstfeld (1984) where speculative runs on the central bank force the abandonment of the fixed exchange rate regime. Thereafter, the exchange rate floats indefinitely with no return to fixed exchange rates.

7. In a related literature on convergence of rational expectations models through Bayesian learning, the parameters of the model are uncertain. For an early work, see Taylor (1975); more recent examples include Bray and Savin (1986). As described in Lewis (1987), results similar to those in the text also obtain in this framework.

8. However, these disturbances are likely to be correlated in reality. If the innovations in the two sets of processes are correlated, observations of the \( x_i \) processes will provide more information about the 'true' policy state. In such a case, the market will use its information about the joint probability distribution of \( m \) and \( x \) in forming its probability of policy change.

9. If the variances of the two distributions are sufficiently different, there may be more than one such money supply level. Since this definition is only employed as an expositional device, the discussion in the text will proceed as though there were only one \( m \).

10. Technically, this result only establishes that the expected value of the log ratio of probabilities converge. But, the appendix demonstrates that, with a little more algebra, the levels of the probabilities converge to their true values in mean-squared error.

11. Their forecasts are optimal in the sense that they minimize the mean squared errors based upon their priors at each point in time.


13. For example, Huang (1981) and West (1987) reject variance bounds tests of the exchange rate in the simple monetary model. However, West finds that the tests are not rejected when allowing for structural shocks that may arise from money demand or purchasing power parity.

14. Of course, the monetary policy regime switches that occurred in 1979 and 1982 may to some extent be considered endogenous rather than exogenous as described in the analysis here. Allowing for endogenous switches would be interesting but requires a different framework.

15. Empirical studies that identify a speculative bubble to be the deviation of an asset price from its fundamental level include West (1984) for the stock market, Meese (1986) for the foreign exchange market, and Flood and Garber (1980) for the German hyperinflation. Hamilton and Whiteman (1985) generalize the point made by Flood and Garber (1980) that, using this fundamental specification, one cannot distinguish between a speculative bubble and a switch in a fundamental process; a point also emphasized by Flood and Hodrick (1986) and Obstfeld (1985).

16. Since the \( x_i \) have been assumed uncorrelated with \( m \), the variances of the \( x_i \) will not contribute to the characteristics of the 'peso problem' anyway.

17. See Lewis (1987) for an empirical investigation into the potential effects upon exchange rate forecast errors from market learning.
18. Fama (1984) finds that evidence for the negative coefficient is strongest during the subsample from May 1978 to December 1982, the period that includes two years of systematically incorrect prediction in the forward market.

19. The result comes from the negative covariance between the expected change in the exchange rate and the premium. Fama (1984) finds this covariance puzzling, but Hodrick and Srivastava (1986) demonstrate that such a relationship is perfectly plausible on economic grounds.

References


