

A Positive Theory of Economic Growth and the Distribution of Income*

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First Draft: July 4, 2014

Current Draft: October 10, 2014

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Abstract

This paper is a positive theory of the distribution of income and the growth rate of the economy. It builds on our earlier work, Meltzer and Richard (1981), on the size of government. How does the distribution of income change as an economy grows? To answer this question we build a model of a labor economy in which consumers have diverse productivity. The government imposes a linear income tax which funds equal per capita redistribution. The tax rate is set in a sequence of single issue election in which the median productivity individual is decisive. Economic growth is the result of using a learning by doing technology, so higher taxes discourage labor causing the growth rate of the economy to fall. The distribution of productivity can widen due to increased technological specialization. This causes voters to raise the equilibrium tax rate and reduce growth. The distribution of pre-tax income widens. We estimate the model using data from the U.S., U.K. and France with excellent results.

1 Introduction

How does the distribution of income change as economic growth changes? How does growth change when governments raise tax rates to finance increased redistribution? Economists have discussed these issues for decades, and they have recently become major political issues in developed economies.

*We wish to thank V.V. Chari, Marvin Goodfriend and the seminar participants at the Wharton School for useful comments.

To answer these questions, we analyze a general equilibrium model of a labor economy in which consumers differ in their relative productivity. The wage rate is equal to absolute productivity so a consumer's labor income is the wage rate multiplied by his relative productivity adjusted labor. A linear tax rate on labor income finances government spending for redistribution; the real government budget is balanced. Tax increases lower aggregate hours worked, hence lowering current aggregate income and consumption. Learning-by-doing is the source of economic growth in our model.

Absolute productivity increases with the total amount of productivity adjusted labor expended. Hence increased taxes and redistribution reduces the growth rate and widens the spread in the income distribution. Reducing tax rates and redistribution brings the opposite response. Voters face a trade-off of more current redistribution versus a higher growth rate of wages. As in our earlier work, Meltzer and Richard (1981), the median income voter is decisive in a series of single issue elections to choose the tax rate.

We estimate the model using data for the U.S., the U.K., and France. The data strongly support the model. As well, they support Kuznets (1955) conclusions about the relation of growth to changes in relative incomes.

2 A Selective Literature Survey

The literature on economic growth and redistribution is large and varied. We report on a sample that covers different approaches and reaches very different conclusions.

In a 1955 paper, Simon Kuznets used his extensive knowledge of income data to conclude that economic growth first increases the spread in the income distribution. The reason is that when many unskilled workers enter the labor force, the economy grows, profits rise, and higher incomes increase relatively and absolutely. As workers acquire skills, their productivity and real wages increase relative to profits, so the spread of the income distribution declines. Kuznets' conclusion has remained contentious in part because he did not produce a model showing that his result held in general equilibrium and in part because of the paucity of data he had available.

Many years later, Arthur Okun (1975) discussed the social decision of trading some efficiency or growth for more redistribution achieved by taxing higher incomes. His discussion makes the cost, called the leaky bucket, exceed the amount redistributed. His analysis, like Kuznets, concerns a one-time choice.

Our earlier work Meltzer and Richard (1981) departs from these ideas. In a functioning democratic system, voters make the choice repeatedly not once and for all times. They know their position but are uncertain about their and their children's future. The political choice of redistribution is like economic decisions that optimizing consumers make repeatedly. They vote either to increase current consumption by voting for a higher tax rate or they vote for growth and increased future consumption by lowering tax rates and spending. Although our earlier model is static, it is consistent with voter's decisions. Sometimes they vote for

higher tax rates and spending and sometimes they do the opposite. No society chooses once for all future time.

Much of the recent literature on economic growth focusses on the role of capital as summarized in the influential book by Acemoglu (2009). We think that the emphasis in explaining growth should be on labor productivity not capital. Over the past 200 years real wages have increased 20 fold while the return to capital has remained basically unchanged. Politically, if capital determines growth it is difficult to understand why the taxes on capital income have risen, especially since capital is owned almost exclusively by the upper half of the income distribution. So we choose to ignore capital and focus on labor productivity as the exclusive engine of growth.¹

Treatment of high incomes as rent permits increased taxation to finance redistribution without reducing productive activity. A special use of rents is the claim that most high incomes result from inheritance of wealth produced by an earlier generation and passed on. Evidence does not support this claim both in the U.S. and other developed countries. Kaplan and Rauh, (2013, 46, 48); Becker and Tomes (1979, 1158). Some work suggests that culture and educational attainment of parents has more important influence on later generations than financial inheritance. Becker and Tomes (1979, 1191), Corak (2013, 80)

Another problem with treating high incomes, for example those of the top one percent, as rent is that the population is not fixed but changes. Piketty (2014, 115-16) writes that capital transforms “itself into rents as it accumulates in large enough amounts.” Saez (2013, 24) concludes that “high top tax rates reduce the pre-tax income gap without visible effect on economic growth.” Corak (2013) shows that intergenerational mobility remains large in developed countries like the U.S. and Canada. The main exception in the U.S. is the “least advantaged.” In this paper we address the issue of why some of the least advantaged have stagnant incomes. The least productive choose not to work, so their incomes are all redistribution. Hence they do not acquire productive skills.

Much of the discussion of the top one percent makes no mention of the other 99 percent. Often, the focus is on income from capital, which neglects labor income, about 70% of total income. A very different explanation of the rising share of income earned by the top one percent builds on work on superstars by Rosen (1981). Rosen argued that technological change increases the relative productivity of individuals with exceptional talent in using and developing the new technology. Rosen’s work brings in relative productivity as an explanation of the rising share of the top one percent. Developing new software, like Steve Jobs and others, that create entire markets brings high rewards. Successfully managing a bank or corporation with branches in 50 or 100 countries is an order of magnitude more difficult than managing in a single country or state. Ten years is a long tenure in such jobs, so there is turnover not inheritance of high

¹Piguillem and Schneider (2013) add majority rule voting to the neoclassical growth model. They show that the median voter is decisive assuming that the distribution of capital is strictly proportional to the distribution of relative productivity.

income positions. Highly skilled surgeons adept at operating new technologies should be included.

Major league sport stars in football, baseball, soccer, basketball and hockey no longer perform before audiences restricted to a stadium. Television increased their productivity. Kaplan and Rauh (2013, 42, Figure 3) show the substantial increase in their incomes. Turnover is high; careers at the top are brief. And there is little evidence that the super stars cede their places to their offspring. Rock musicians and entertainment stars often have similar careers with high incomes for short duration. Income of super stars may explain some of the increase in the relative earnings of the top one percent. We doubt that it is a full explanation because the data after 1980 show that the rise in the pre-tax share of the top one percent can be seen in data for the United States, the United Kingdoms, Canada and Sweden but data for France and the Netherlands do not show a similar increase. Roine and Waldenstrom (2006)

The share of pre-tax incomes received by the top one percent includes income from reported capital gains. That makes it more volatile, rising in periods when owners of shares choose to report gains in excess of losses. Also there are substantial differences in the relative shares of different income quintiles when before and after tax and transfers are included. Most economic theory considers consumption, based on permanent not current income, to be a better measure of the economic component of well-being. Table 1 shows data on pre- and post-tax incomes for the United States. The data for 1979 to 2010 are available on the Congressional Budget Office website.²

²<http://www.cbo.gov/publication/44604>

Before Tax (%)				
Year	1979	1989	2007	2010
Lowest 20%	6.2	4.9	4.8	5.1
Middle 20%	15.8	15	13.4	14.2
Highest 20%	44.9	49.3	54.6	51.9
Highest 1%	8.9	12.2	18.7	14.9
After Tax (%)				
Lowest 20%	7.4	5.7	5.6	6.2
Middle 20%	16.5	15.7	14.3	15.4
Highest 20%	42	47.3	51.4	48.1
Highest 1%	7.4	11.8	16.7	12.8

Source: CBO (2014)

Table 1: Selected Income Shares 1979-2010 (2010 Dollars)

The range of data in Table 1 is the range given by CBO. We chose 1989 because it was the end of the Reagan growth years. We chose 2007 because it is the peak year for the income share going to the top one percent. That year is also the peak year for the after tax share of the top one percent.

The table makes clear that it matters considerably whether analysis uses before or after tax income shares. Conceptually, income after tax and transfer is closer to consumption. By 2010 the share of after tax income received by the lowest 20 percent (6.2 percent) is the same as the before tax share received in 1979. Income shares for the lowest and middle 20% fall until 2007, then rise; the share of top one percent and 20 percent rise to 2007, then fall. Most of the rise occurs during the period of relatively high growth in the 1990s. The change is not likely to reflect changes in the return to capital. The data seems more consistent with productivity growth during the boom years.

Of interest in relation to recent discussion, the share of the upper income groups declined from 2007 to 2010. These are years of relatively slow growth combined with increased returns to equity capital and a recovery in many house prices. Again, this suggests that productivity growth is more important than return to capital in explaining income shares.

A main theme of Piketty's (2014 and elsewhere) work is that the tax rates on income and wealth should be raised even though, at some points, he recognizes that the higher rates would lower top incomes but not provide much revenue. Few of the many discussions of his work point out that the choice of tax rate should be an implication of a utility maximizing model, preferably a general

equilibrium model, such as in this paper.

Long before the Piketty book stimulated renewed interest in income distribution and the choice of tax rate, Becker and Tomes (1979, 1986) developed general equilibrium models of income distribution across family generations. Becker and Tomes (1979) choose a linear tax structure and use revenues for redistribution. They find that “even a progressive tax and public expenditure system may widen the inequality of disposable income.” Becker and Tomes (1986, 533-4) note that some empirical work by Arthur Goldberger found that the widening of inequality does not occur for several generations.

Alesina and Rodrik (1994) use a growth model. As in Meltzer and Richard (1981) voters differ in their endowments, some prefer more, some less, taxation and government spending. The authors show that, in general, voters will not maximize economic growth. Instead, they vote to tax capital to finance redistribution. As in all general equilibrium models, the budget is balanced.

Alesina and Rodrik use the Gini coefficient to measure income inequality. They show empirically that income inequality is negatively related to future economic growth. The reason is that as income inequality rises, voters choose more redistribution, reducing the growth rate.

May increased government spending and taxation increase both growth and redistribution? Of course, it may, but the empirical data in Alesina and Rodrik and elsewhere shows that, in developed economies, the reverse is true. Government spending is mainly for redistribution to augment consumption.

Our contribution to this research builds on the findings in Alesina and Rodrik but incorporates some of the principal ideas offered by Simon Kuznets in his insightful discussions. Kuznets (1979) contains several of his essays. In particular we incorporate technological change Kuznets (1979, 45) as a major source of income growth with substantial effects on income distribution that are not explicitly considered in much of the literature.

In our model, growth of labor productivity and labor income – learning by doing – is a large factor, the largest, in explaining growth of output and living standards. We do not challenge the role of capital or the implication that the return to capital changes very little. The return to labor changes much more. We do not impose our ideas of the desirable extent of income distribution. The workers in our model are voters who choose their preferred tax rate and redistribution. They are aware that an increased tax rate to finance redistribution lowers investment in productivity enhancing investments that add to their future consumption.

Our analysis fits the contours of growth experience. As workers learn, their skills, productivity and incomes increase. They save more, acquire real assets especially housing. They spend to educate their offspring, and they vote for or against redistribution and taxation. In our earlier work, Meltzer and Richard (1981), we showed that rational voters choose a tax rate consistent with the most basic economic theory. They decide whether they want increased taxes to finance more redistribution (consumption today), or lower tax rates to spur investment and future consumption. In the growth model here, the same choice remains central.

3 The Economic Model

We begin by modeling consumers and calculating their lifetime consumption by maximizing their utility. Consumers are endowed with different relative levels of productivity, indexed by n , and one unit of time. A consumer with relative productivity n maximizes his lifetime utility of consumption and leisure:

$$V^n(c^n, \ell^n) = \int_0^\infty e^{-\delta t} [\lambda \ln(c_t^n) + (1 - \lambda) \ln(1 - \ell_t^n)] dt, \quad (1)$$

where $c^n = \{c_t^n\}_0^\infty$ is his consumption stream, $\ell^n = \{\ell_t^n\}_0^\infty$ is his labor stream, and δ is the discount rate. There is a government which levies a linear tax on income at rate τ_t at time t and uses the proceeds for redistribution, equally per capita. The budget constraint for a consumer with productivity n is

$$\ell_t^n = \rho_t w_t + (1 - \tau_t) w_t n \ell_t^n, \quad (2)$$

where $n w_t$ is the wage per unit of labor at time t and $\rho_t w_t$ is amount redistributed at time t . Each individual is a price taker in the labor market, takes the processes $\{\rho_t\}$, $\{w_t\}$, and $\{\tau_t\}$ as given and chooses $\{c_t^n\}$ and $\{\ell_t^n\}$ to maximize utility. The standard Bellman equation for optimal control is³

$$0 = \max \left[\lambda \ln(c_t^n) + (1 - \lambda) \ln(1 - \ell_t^n) - \delta J^n + J_w^n \dot{w}_t + J_\rho^n \dot{\rho}_t + J_\tau^n \dot{\tau}_t \right], \quad (3)$$

where $J^n(w_t, \rho_t, \tau_t)$ is the value function for a consumer with relative productivity n . The standard first-order conditions for equation (3) yield

$$\ell_t^n = \lambda - \frac{\rho_t(1 - \lambda)}{n(1 - \tau_t)}. \quad (4)$$

The maximum fraction of time devoted to working is λ , as can be seen by setting $\rho_t = 0$ in equation (4). Since labor must be positive there is a minimum level of relative productivity, ν_t , below which consumers are *voluntarily* unemployed, living on their redistribution:

$$\nu_t = \frac{\rho_t(1 - \lambda)}{\lambda(1 - \tau_t)}. \quad (5)$$

We call ν_t the voluntary unemployment productivity. Optimal consumption is

$$c_t^n = \rho_t w_t, \text{ for } n < \nu_t. \quad (6a)$$

$$= \lambda(\rho_t + (1 - \tau_t)n)w_t, \text{ for } n \geq \nu_t. \quad (6b)$$

Notice that consumption is increasing and ordered by relative productivity for all choices of ρ_t and τ_t .

³The super dot indicates the time derivative.

Adding stochastic terms to the state equations changes the value function J^n , but does not change the consumers optimal decisions state contingent decisions.

Relative productivity is distributed lognormally, $\ln n \sim N(0, \sigma_t)$, so that the median relative productivity is $m = 1$ and the mean relative productivity is $\bar{n}_t = e^{\frac{1}{2}\sigma_t} > 1$. Hence nw_t is the absolute productivity of a consumer with relative productivity n . Since the median consumer has productivity $m = 1$, w_t is the absolute productivity of the median consumer.

To understand how productivity and economic growth affect the income distribution and redistribution in a mature economy such as the U.S. or western Europe, we need to consider change in relative productivity, σ_t , as well as change in absolute productivity, w_t . An increase in w_t increases the wage earned by all workers, regardless of their level of productivity. An increase in σ_t is meant to capture the effect of technological change with disparate effects, such as the computerization of production the U.S. experienced in the past 40 years. Mean productivity normalized hours worked at time t , Ψ_t , is:

$$\begin{aligned}\Psi_t &= \int_{\nu_t}^{\infty} \frac{n \ell_t^n \exp(-\frac{1}{2}(\frac{\ln n}{\sigma_t})^2)}{n \sigma_t \sqrt{2\pi}} dn \\ &= \int_{\nu_t}^{\infty} \frac{\lambda(n - \nu_t) \exp(-\frac{1}{2}(\frac{\ln n}{\sigma_t})^2)}{n \sigma_t \sqrt{2\pi}} dn \\ &= \lambda \left[\bar{n}_t N\left(-\frac{\ln \nu_t}{\sigma_t} + \sigma_t\right) - \nu_t N\left(-\frac{\ln \nu_t}{\sigma_t}\right) \right].\end{aligned}\quad (7)$$

The government's budget is balanced in that the per capita spending on redistribution, $w_t \rho_t$, equals the tax revenues, $w_t \Psi_t \tau_t$:

$$w_t \tau_t \Psi_t = w_t \rho_t. \quad (8)$$

Everything in this economy is a function of ν_t , σ_t and w_t . It is obvious from equation (7) that Ψ_t is a function of σ_t and ν_t . Substituting equation (5) into equation (4) we find that

$$\ell_t^n = \lambda \left(1 - \frac{\nu_t}{n}\right). \quad (9)$$

Solving equation (8) for ρ_t , substituting the result into equation (5) and then solving for τ_t gives:

$$\tau_t = \frac{\nu_t}{\nu_t + (1 - \lambda)\psi_t} \quad (10)$$

where

$$\psi_t = \Psi_t / \lambda = \bar{n}_t N\left(-\frac{\ln \nu_t}{\sigma_t} + \sigma_t\right) - \nu_t N\left(-\frac{\ln \nu_t}{\sigma_t}\right) \quad (11)$$

is the average fraction of full-time equivalent, productivity-adjusted, units worked. Substituting equation (10) into equation (8) gives:

$$\rho_t = \frac{\lambda \nu_t \psi_t}{\nu_t + (1 - \lambda)\psi_t}. \quad (12)$$

Finally, substituting equations (9), (10), and (12) into equation (6) gives:

$$c_t^n = \lambda w_t \psi_t \frac{\nu_t}{\nu_t + (1 - \lambda) \psi_t}, \text{ for } n < \nu_t. \quad (13a)$$

$$= \lambda w_t \psi_t \frac{\lambda \nu_t + (1 - \lambda) n}{\nu_t + (1 - \lambda) \psi_t}, \text{ for } n \geq \nu_t. \quad (13b)$$

In preparation for determining the size of government we show that selecting ν_t is an equivalent to selecting τ_t . This can be seen by showing that τ_t is a strictly increasing function of ν_t :

$$\frac{d\tau_t}{d\nu_t} = \frac{(1 - \lambda) \bar{n}_t N(-\frac{\ln \nu_t}{\sigma_t} + \sigma_t)}{(\nu_t + (1 - \lambda) \psi_t)^2} > 0. \quad (14)$$

Hence the mapping from ν_t to τ_t is continuous and strictly increasing, so that setting ν_t is equivalent to setting τ_t . Furthermore, increasing (decreasing) ν_t is equivalent to increasing (decreasing) τ_t .

Finally, we determine the mean number of labor units worked per capita, $\bar{\ell}_t$:

$$\bar{\ell}_t = \int_{\nu_t}^{\infty} \frac{\ell_t^n \exp(-\frac{1}{2}(\frac{\ln n}{\sigma_t})^2)}{n \sigma_t \sqrt{2\pi}} dn \quad (15)$$

$$= \int_{\nu_t}^{\infty} \frac{\lambda(1 - \nu_t/n) \exp(-\frac{1}{2}(\frac{\ln n}{\sigma_t})^2)}{n \sigma_t \sqrt{2\pi}} dn \quad (16)$$

$$= \lambda \left[N(-\frac{\ln \nu_t}{\sigma_t}) - \nu_t \bar{n}_t N(-\frac{\ln \nu_t}{\sigma_t} - \sigma_t) \right]. \quad (17)$$

Hence, the fraction of full time labor worked per capita at time t , is $\bar{\ell}_t/\lambda$ and the fraction of full time labor worked per employed person, \bar{L}_t , is

$$\bar{L}_t = \frac{\bar{\ell}_t}{\lambda N(-\frac{\ln \nu_t}{\sigma_t})}. \quad (18)$$

4 The Distribution of Income

We can now show that regardless of how tax rates are determined, the distribution of pre-tax income widens as taxes rise. This widening has nothing to do with technological change or the privileges of the rich. The widening of the distribution of income is the direct consequence of the incentives created by increasing taxes and redistribution. The income of a consumer with relative productivity n at time t is

$$I_t^n = 0 \text{ for } n < \nu_t, \quad (19a)$$

$$= w_t n \ell_t^n \text{ for } n \geq \nu_t. \quad (19b)$$

Substituting equation (9) into equation (19) gives

$$I_t^n = 0 \text{ for } n < \nu_t, \quad (20a)$$

$$= w_t \lambda (n - \nu_t) \text{ for } n \geq \nu_t. \quad (20b)$$

Assuming that he works, the median consumer's income is

$$I_t^m = w_t \lambda (1 - \nu_t). \quad (21)$$

The average income of all consumers, both those who work and those who live on redistribution, is

$$\bar{I}_t = w_t \lambda \psi_t. \quad (22)$$

Higher taxes causes the average income of all consumers (which in equilibrium must equal the average consumption of all consumers) to fall:

$$\frac{\partial \bar{I}_t}{\partial \nu_t} = -w_t \lambda N\left(-\frac{\ln \nu_t}{\sigma_t}\right) < 0, \quad (23)$$

where

$$\frac{\partial \psi_t}{\partial \nu_t} = -N\left(-\frac{\ln \nu_t}{\sigma_t}\right). \quad (24)$$

A commonly used measure of the dispersion of income is the ratio of mean to median income:

$$r_t = \frac{\psi_t}{1 - \nu_t}. \quad (25)$$

Differentiating we get

$$\frac{dr_t}{d\nu_t} = \frac{\bar{n}_t N\left(-\frac{\ln \nu_t}{\sigma_t} + \sigma_t\right) - N\left(-\frac{\ln \nu_t}{\sigma_t}\right)}{(1 - \nu_t)^2} > 0. \quad (26)$$

so that the ratio of mean to median income rises as tax rates increase. In fact all consumers with productivity above (below) median increase (reduce) their income relative to median income as taxes rise:

$$\frac{d(I_t^n / I_t^m)}{d\nu_t} = 0 \text{ for } n < \nu_t, \quad (27a)$$

$$= \frac{n - 1}{(1 - \nu_t)^2} \text{ for } n \geq \nu_t. \quad (27b)$$

Another commonly used measure of the dispersion of income is the fraction earned by the top $k\%$. The upper $k\%$ begins with the consumer with relative productivity

$$n_t^*(k) = \exp(-\sigma_t N^{-1}(k)). \quad (28)$$

For example, the upper 1% begins with relative productivity $n_t^*(0.01) = \exp(-\sigma_t N^{-1}(0.01))$ or $n_t^*(0.01) \approx e^{2.33\sigma_t}$ and the top 10% begins with productivity $n_t^*(0.1) \approx e^{1.28\sigma_t}$.

The total income of the top $k\%$ of consumers is

$$\begin{aligned}\tilde{I}_t(k) &= \lambda w_t \int_{n_t^*(k)}^{\infty} (n - \nu_t) \frac{\exp(-\frac{1}{2}(\frac{\ln n}{\sigma_t})^2)}{\sqrt{2\pi n \sigma_t^2}} dn \\ &= \lambda w_t \left[\bar{n}_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t} + \sigma_t\right) - \nu_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t}\right) \right].\end{aligned}\quad (29)$$

The fraction of income earned by the top $k\%$ is

$$\phi_t(k) = \frac{\tilde{I}_t(k)}{\bar{I}_t} = \frac{\bar{n}_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t} + \sigma_t\right) - \nu_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t}\right)}{\psi_t}.\quad (30)$$

The ratio of the total income of consumers in the top $k\%$ relative to median pre-tax income is

$$\frac{\tilde{I}_t(k)}{I_t^m} = \frac{\bar{n}_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t} + \sigma_t\right) - \nu_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t}\right)}{(1 - \nu_t)}.\quad (31)$$

Differentiating equation (31) with respect to ν_t shows that:

$$\frac{d(\tilde{I}_t(k)/I_t^m)}{d\nu_t} = \frac{\bar{n}_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t} + \sigma_t\right) - N\left(-\frac{\ln n_t^*(k)}{\sigma_t}\right)}{(1 - \nu_t)^2} > 0.\quad (32)$$

Again, "the rich get richer" relative to the median as taxes rise. Again, this is an inevitable consequence of taxation and redistribution.

There is much discussion in the media, and even among academics, of how rising income dispersion is evidence of a more "unequal" society. This is, of course, very misleading because funds collected in taxes are redistributed so that the distribution of consumption actually *narrows* with increased taxes. The welfare implication of increased taxation is a more equal, "fair" society, despite an increase in the dispersion of incomes. In fact all consumers with productivity above (below) median reduce (increase) their consumption relative to median consumption as taxes rise:

$$\frac{d(c_t^n/c_t^m)}{d\nu_t} = \frac{(1 - \lambda)}{(\lambda\nu_t + (1 - \lambda))^2} > 0 \text{ for } n < \nu_t,\quad (33a)$$

$$= \frac{\lambda(1 - \lambda)(1 - n)}{(\lambda\nu_t + (1 - \lambda))^2} \text{ for } n \geq \nu_t.\quad (33b)$$

What about the top $k\%$? The consumption of the top $k\%$ is

$$c_t^*(k) = \lambda w_t \psi_t \frac{\lambda\nu_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t}\right) + (1 - \lambda)\bar{n}_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t} + \sigma_t\right)}{\nu_t + (1 - \lambda)\psi_t}\quad (34)$$

The consumption of the top $k\%$ falls relative to median consumption as taxes increase:

$$\frac{d(c_t^*(k)/c_t^m)}{d\nu_t} = -\frac{\lambda(1 - \lambda)(\bar{n}_t N\left(-\frac{\ln n_t^*(k)}{\sigma_t} + \sigma_t\right) - N\left(-\frac{\ln n_t^*(k)}{\sigma_t}\right))}{(\lambda\nu_t + (1 - \lambda))^2} < 0\quad (35)$$

5 The Median Voter

Until now all consumers have been price takers who have no influence over government tax policy. Romer (1975) and Roberts (1977) shows that if the ordering of individual consumption is independent of the choice of ρ_t and τ_t , the median voter is decisive in a majority rule election to set the tax rate. So the median voter is continuously decisive in elections for $\{\tau_t\}$.

We now turn to analyzing how the median voter would prefer to set tax rates. The choice of tax rates depends on how taxes affect the growth rate of wages. We assume that the growth of wages is due to learning by doing or on the job training. Time spent working contributes to the growth rate of wages. The amount of learning by doing at time t is proportional to ψ_t , the full-time equivalent units of productivity adjusted labor worked at time t ; there is no contribution to learning by doing from those who do not work. We assume that the growth rate of wages (or median productivity) is

$$\frac{\dot{w}_t}{w_t} = g_t \psi_t, \quad (36)$$

where g_t is a technological productivity multiplier which determines how much each full-time equivalent of productivity normalized labor increases wages. In a mature economy, changes to g_t are mainly due to business cycle effects. We assume that

$$\dot{g}_t = \mu_t^g, \quad (37)$$

where μ_t^g is an arbitrary well-behaved function of g_t . Because the consumer's utility function is logarithmic, it will turn out that the exact form of μ_t^g is irrelevant as long as it is independent of ν_t . We assume that the process for σ_t is

$$\dot{\sigma}_t = \mu_t^\sigma \quad (38)$$

Again, as long as μ_t^σ is independent of ν_t , its exact specification is irrelevant.

The reason that we do not need to specify the exact form of μ_t^g or μ_t^σ is the myopic decision making resulting from logarithmic utility. The Bellman equation for the median voter is

$$0 = \max_{\nu_t} \{ \lambda \ln(c_t) + (1 - \lambda) \ln(1 - \ell_t) - \delta J + J_w w_t \psi_t g_t + J_g \mu_t^g + J_\sigma \mu_t^\sigma \}, \quad (39)$$

where we have suppressed the superscript m . We conjecture that

$$J(w_t, g_t) = \frac{\lambda}{\delta} \ln w_t + j(g_t, \sigma_t). \quad (40)$$

Substituting equations (6), (9), and (40) into equation (39) we find

$$0 = \max_{\nu_t} \left\{ \lambda \ln \lambda + \lambda \ln \psi_t - \lambda \ln(\nu_t + (1 - \lambda) \psi_t) + \ln(\lambda \nu_t + (1 - \lambda)) - \delta j + \frac{\lambda \psi_t g_t}{\delta} + j_g \mu_t^g + j_\sigma \mu_t^\sigma \right\}. \quad (41)$$

The derivative of equation (41) with respect to ν_t is:

$$H(\nu_t, g_t, \sigma_t) = \frac{-\lambda N(-\frac{\ln \nu_t}{\sigma_t})}{\psi_t} - \frac{\lambda(1 - (1 - \lambda)N(-\frac{\ln \nu_t}{\sigma_t}))}{\nu_t + (1 - \lambda)\psi_t} + \frac{\lambda}{\lambda\nu_t + (1 - \lambda)} - \frac{\lambda}{\delta} N(-\frac{\ln \nu_t}{\sigma_t}) g_t, \quad (42)$$

The standard conditions for an optimal ν_t are

$$H(\nu_t, g_t, \sigma_t) = 0 \quad (43)$$

and

$$H_\nu(\nu_t, g_t, \sigma_t) < 0. \quad (44)$$

6 Economic Growth

The growth rate of the economy at time t , γ_t , equals the growth rate of aggregate consumption:

$$\begin{aligned} \gamma_t &= \frac{d \ln \bar{c}_t}{dt} \\ &= \frac{\dot{w}_t}{w_t} + \frac{1}{\psi_t} \frac{\partial \psi_t}{\partial \nu_t} \dot{\nu}_t + \frac{1}{\psi_t} \frac{\partial \psi_t}{\partial \sigma_t} \dot{\sigma}_t \\ &= g_t \psi_t - \frac{N_t}{\psi_t} \dot{\nu}_t + \frac{1}{\psi_t} \frac{\partial \psi_t}{\partial \sigma_t} \dot{\sigma}_t, \end{aligned} \quad (45)$$

where $N_t = N(-\frac{\ln \nu_t}{\sigma_t})$,

$$\frac{\partial \psi_t}{\partial \sigma_t} = \sigma_t \bar{n}_t N(-\frac{\ln \nu_t}{\sigma_t} + \sigma) + \nu_t \tilde{n}(-\frac{\ln \nu_t}{\sigma}) > 0, \quad (46)$$

and \tilde{n} is the unit normal probability density function. There are three effects on economic growth captured in equation (45). The first term, $g_t \psi_t$, is the growth rate due to current learning by doing, which is smaller the higher are taxes since $\frac{d\psi_t}{d\nu_t} < 0$. The second term captures the direct reduction in the current growth rate caused by consumers experiencing increasing taxes. Whenever taxes are increasing, so is the level of voluntary unemployment, implying the growth rate of the economy falls. The third term is the effect of technological change on growth. Since the coefficient of $\dot{\sigma}_t$ in equation (45) is positive, an increase in the dispersion of skills causes higher growth.

Increases in σ_t , *ceteris paribus*, causes the government to grow. To see this we need some preliminary calculations. First we need the partial derivative of H with respect to σ_t :

$$H_\sigma = \lambda \left[\frac{N_t}{\psi_t^2} + \frac{(1 - \lambda)(1 - (1 - \lambda)N_t)}{(\nu_t + (1 - \lambda)\psi_t)^2} \right] \frac{\partial \psi_t}{\partial \sigma_t} - \frac{\lambda \tilde{n}(-\frac{\ln \nu_t}{\sigma_t}) \ln \nu_t}{\sigma_t^2} \left[\frac{\nu_t}{\psi_t(\nu_t + (1 - \lambda)\psi_t)} + \frac{g_t}{\delta} \right] > 0. \quad (47)$$

Assuming the median voter works, $\nu_t < 1$ so that $\ln \nu_t < 0$, implying that $H_\sigma > 0$. Taking the total derivative of equation (43) with respect to t , we get

$$\dot{\nu}_t = -\frac{H_g}{H_\nu} \dot{g}_t - \frac{H_\sigma}{H_\nu} \dot{\sigma}_t, \quad (48)$$

where

$$H_g = -\frac{\lambda}{\delta} N\left(-\frac{\ln \nu_t}{\sigma_t}\right) < 0. \quad (49)$$

Because $-\frac{H_\sigma}{H_\nu} > 0$, positive $\dot{\sigma}_t$ causes $\dot{\nu}_t$ to increase, which means taxes rise. Increasing dispersion in relative productivity causes higher tax rates and increased government growth.

Whenever absolute productivity is increasing, $\dot{g}_t > 0$, the economy grows faster. To see this, we substitute equation (48) into equation (45) so we can re-write the growth rate of the economy as

$$\gamma_t = g_t \psi_t + \frac{N_t H_g}{\psi_t H_\nu} \dot{g}_t + \left[\frac{1}{\psi_t} \frac{\partial \psi_t}{\partial \sigma_t} + \frac{N_t H_\sigma}{\psi_t H_\nu} \right] \dot{\sigma}_t. \quad (50)$$

Because $\frac{H_g}{H_\nu} > 0$, increases in g_t causes γ_t to increase so the economy grows faster. The effect of an increase in dispersion, σ_t , on the growth rate of the economy is ambiguous because the bracketed term in equation (50) is of indeterminate sign. The first term, which is the direct effect of σ_t on γ_t , is always positive, but the second term, which is the indirect effect of increasing taxes, is negative.

7 Estimation

We now estimate the model using US data from 1967 - 2011 and 1950-2011, UK data from 1962 - 2011, and French data from 1978 - 2009. The choice of estimation periods reflects the first and last dates when the necessary data are available.⁴ Our data sources are in Table 2.

⁴Our data is available in an Excel spreadsheet at <https://fce.wharton.upenn.edu/profile/972/>. Also available is Matlab code for the calibration.

	France	UK	US
Real GDP per Capita Growth Rate	INSEE	MW	FRED
Productivity Index	FRED	FRED	FRED
Mean Ann. Hours per Engaged Person	FRED	FRED	FRED
Mean & Median Household Income	EuroStat	ONS	CB
Government Expenditures/GDP	EuroStat	UPS	FRED
Income Share of Top 10%	WTID	WTID	WTID

Table 2: Data Sources. INSEE is the French National Institute of Statistics and Economic Studies. MW is MeasuringWorth.com. FRED is the Federal Reserve Economic Data at the Federal Reserve Bank of St. Louis. EuroStat is the economic database of the European Commission. ONS is the UK's Office of National Statistics. CB is the US Census Bureau. UPS is UKPublicSpending.co.uk. WTID is the The World Top Incomes Database.

We estimate two of the unknown model states $\{g_t, \sigma_t\}$, the parameters λ and the number of annual hours equivalent to full time labor, Λ , by minimizing the sum of squared errors in matching four data time series for each country:

1. The growth rate of the economy, γ_t , which is calculated using equation (45) is compared to the Real Growth Rate of Per Capita GDP.
2. Labor participation rate per worker, \bar{L}_t , which is calculated using equation (18) is compared to the Average Annual Hours Worked per Engaged Person.
3. The ratio of mean to median income, r_t , which is calculated using equation (25) is compared to the Ratio of Mean to Median Household Income; OR the fraction of income earned by the top 10%, $\phi_t(0.1)$, calculated using equation (30) is compared to the Income Share of the Top 10% taken from the The World Top Income Database.
4. The tax rate, τ_t , which is calculated using equation (10) is compared to the total government burden which we measure by Total Government Expenditures/GDP.⁵

We set the time discounting factor $\delta = 4\%$.⁶ When reporting the results of each of the estimations we show a graph with four panels, corresponding to the four comparisons of model to data listed above.

Median productivity, w_t , which is the third state variable is computed from the productivity index, P_t . We equate average total output calculated by using

⁵ Measuring the effective tax rate as government expenditure/GDP was suggested by Milton Friedman.

⁶ We do not estimate δ because it is not well identified in the absence of interest rates or other discounting data.

productivity-adjusted labor, equation (22), with average total output calculated using unadjusted labor:

$$\bar{I}_t = w_t \lambda \psi_t = P_t \lambda L_t \quad (51)$$

Solving equation (51) we get

$$w_t = \frac{P_t L_t}{\psi_t}. \quad (52)$$

The estimation is done by a numerical search. The search steps are:

1. Make a starting guess for the states $\{\nu_t, \sigma_t\}$ and λ and Λ .
2. For each t , solve equation (43) for g_t .
3. Compute γ_t, \bar{L}_t, r_t and τ_t .
4. Compute the sum of squared errors.
5. Update the states, λ and Λ using a Nelder-Mead algorithm.
6. Repeat steps (2) - (5) until convergence.

7.1 The United States

We estimate our model for the United States over two different time periods. During the first period from 1967 to 2011 we use annual observations on real per capita GDP growth rate, average annual hours worked per engaged person, the ratio of mean to median household income, and the burden of government which is total government expenditures/GDP. During the second, longer period we substitute the income share of the top 10% for the ratio of mean to median household income which is not available prior to 1967.

We begin with data from the US from 1967 - 2011. Figure 1 shows a comparison of actual data and model calculations. Obviously the fits of the model to the data are excellent. The r^2 for the fit of actual data to the model are 55.4%, 77.6%, 98.1%, and 86.5%, for growth, hours per employed person, mean to median income, and the tax rate, respectively. The downward trend in hours worked per employed person reflects an international trend as we see below. Mean to median income and government expenditures as a fraction of GDP have both trended upward as a result of increased dispersion in productivity as shown in Figure 2.

Figure 2 shows the optimal estimated states, $\{g_t, \sigma_t, w_t\}$, and ν_t , from 1967 - 2011. The productivity growth multiplier, shown in Panel 1, increased from 1967, reached a peak in 2000, and declined afterward. The dispersion of relative productivity, shown in Panel 2, increased steadily from 1967 through 2000, but has leveled off since then. In contrast to the other state variables, w_t grows throughout the sample, reflecting the continuous growth in US productivity. There has been a steady upward trend in the productivity cutoff for voluntary unemployment.

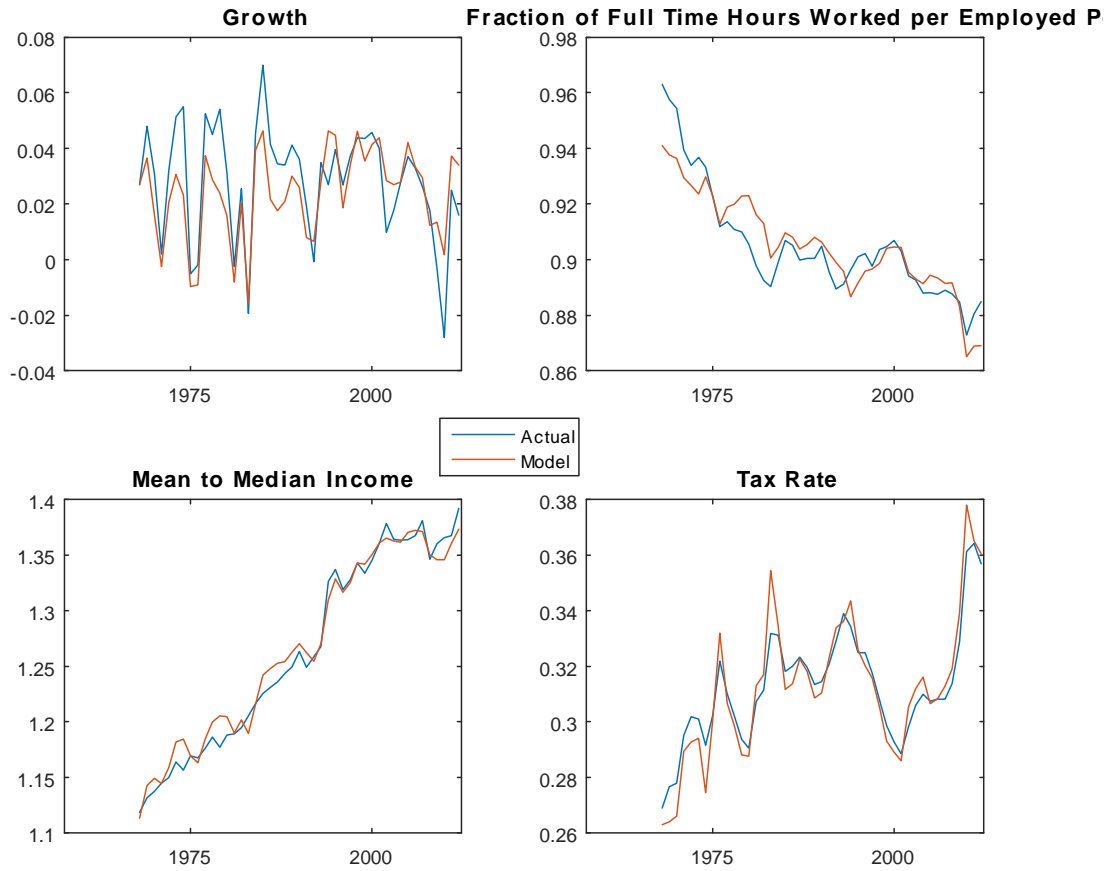


Figure 1: A comparison of actual data and model calculations for the US 1967 - 2011. Panel 1 shows a comparison of the actual rate of real per capita GDP growth to the model's calculated rate of real per capita income growth, γ_t . Panel 2 shows a comparison of the actual average fraction of full time hours worked per engaged person to the model's calculated average fraction of full time worked per employed person, \bar{L}_t . Panel 3 shows a comparison of the actual ratio of mean to median household income to the model's calculated ratio of mean to median personal income, r_t . Panel 4 shows a comparison of the actual ratio of total government expenditures to GDP to the model's linear tax rate, τ_t .

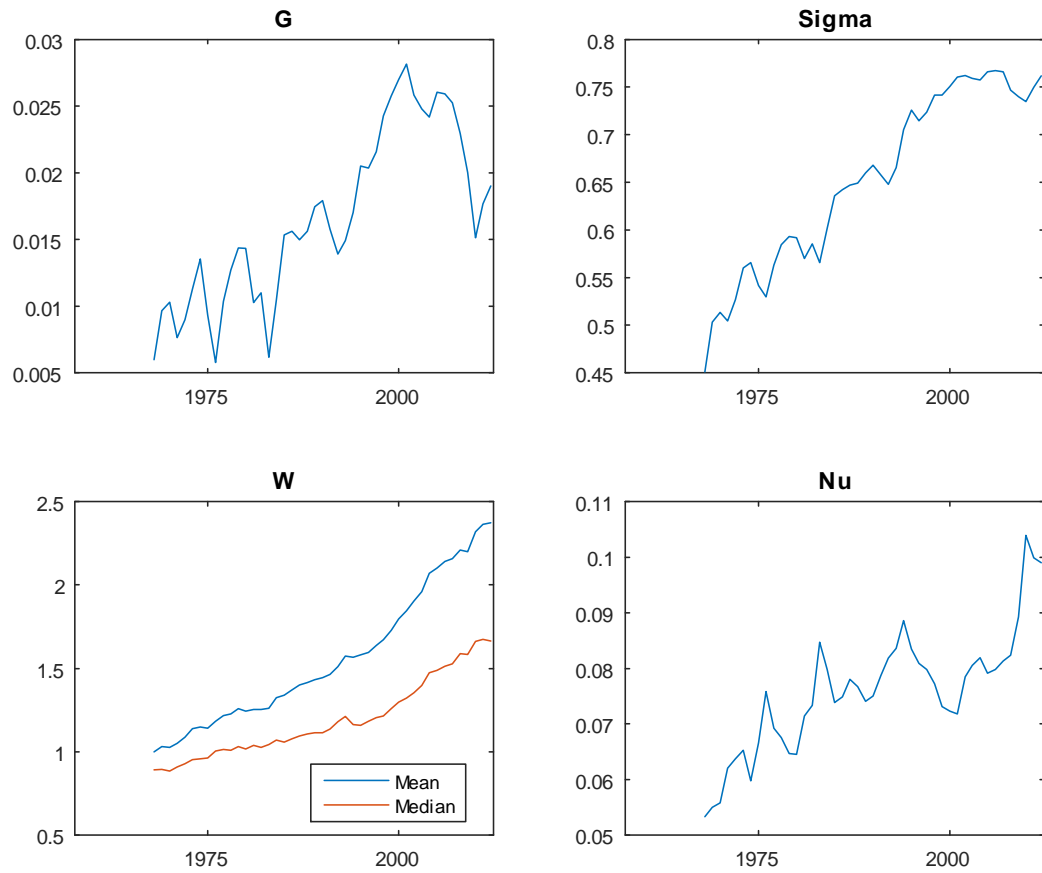


Figure 2: Estimated state variables in the US from 1967 - 2011. The first panel shows the productivity growth multiplier. The second panel show the standard deviation or dispersion of productivity. The third panel shows the absolute productivity index for the median worker; also, for comparison, is the mean productivity index as computed by the BLS. The fourth panel shows the productivity of the last person who voluntarily chooses not to work.

To get a longer time period for estimation we have to substitute the income fraction of the top 10% for the ratio of mean to median household income in all three countries.⁷ Figure 3 reports the estimation in the US from 1950 - 2011. The fit of the model to the data is better than in 1967 - 2011 with r^2 of 88.4%, 88.6%, 77.5%, and 96.6%, respectively. The data trends are similar to 1967 - 2011, except we see that the fraction of income earned by the top 10% did not begin its upward march until the 1980s.

Figure 4 shows the estimated optimal states in the US from 1950-2011. The interesting difference between these charts and Figure 2 is that productivity dispersion did not begin to grow until the 1980s when the share of the top 10% also began to increase.

7.2 United Kingdom

We estimate our model for the United Kingdom from 1962 - 2011.⁸ We use annual observations on real per capita GDP growth rate, average annual hours worked per engaged person, the income share of the top 10%, and the burden of government which is total government expenditures/GDP. Figure 5 shows a comparison of actual data and model calculations. The fits of the model to the data are very good, but not as good as the US. The r^2 are , 67.4%, 71.7%, 86.3%, and 35.4%, respectively. As in the US, the income share of the top 10% began trending upward in the 1980s, although the shares are lower in the UK than the US.

Figure 6 shows the estimated optimal states in the UK from 1962 - 2011. Notice in Panel 2 that, as in the US, relative productivity dispersion began to increase in the 1980s coincident with the increase of the income share of the top 10%.

7.3 France

We estimate our model for France from 1978 - 2009.⁹ We use annual observations on real per capita GDP growth rate, average annual hours worked per engaged person, the income share of the top 10%, and the burden of government which is total government expenditures/GDP. Figure 7 shows a comparison of actual data and model calculations. The fits of the model to the data are good,

⁷We think that the household income data is likely to be better measured because it is based on a census bureau survey of the US population. In contrast, the income fraction of the top 10% is taken from income tax returns. Changes in tax laws and non-compliance means the tax data is not necessarily a consistent sample from year to year.

⁸Mean and median household income data in the UK are available only after 1977. Furthermore, following the European Union standard, the data are adjusted for changing household composition, including the number of people in the household and number of unemployed people. We do not use these data because the adjustments may bias our estimates.

Prior to 1962, the income share of the top 10% in the UK is spotty.

⁹We begin in 1978 because prior to then we cannot find data on total French government expenditures. We end in 2009 because that is the last year in which the income share of the top 10% is reported. Mean and median household income data are only available from 1995 onward.

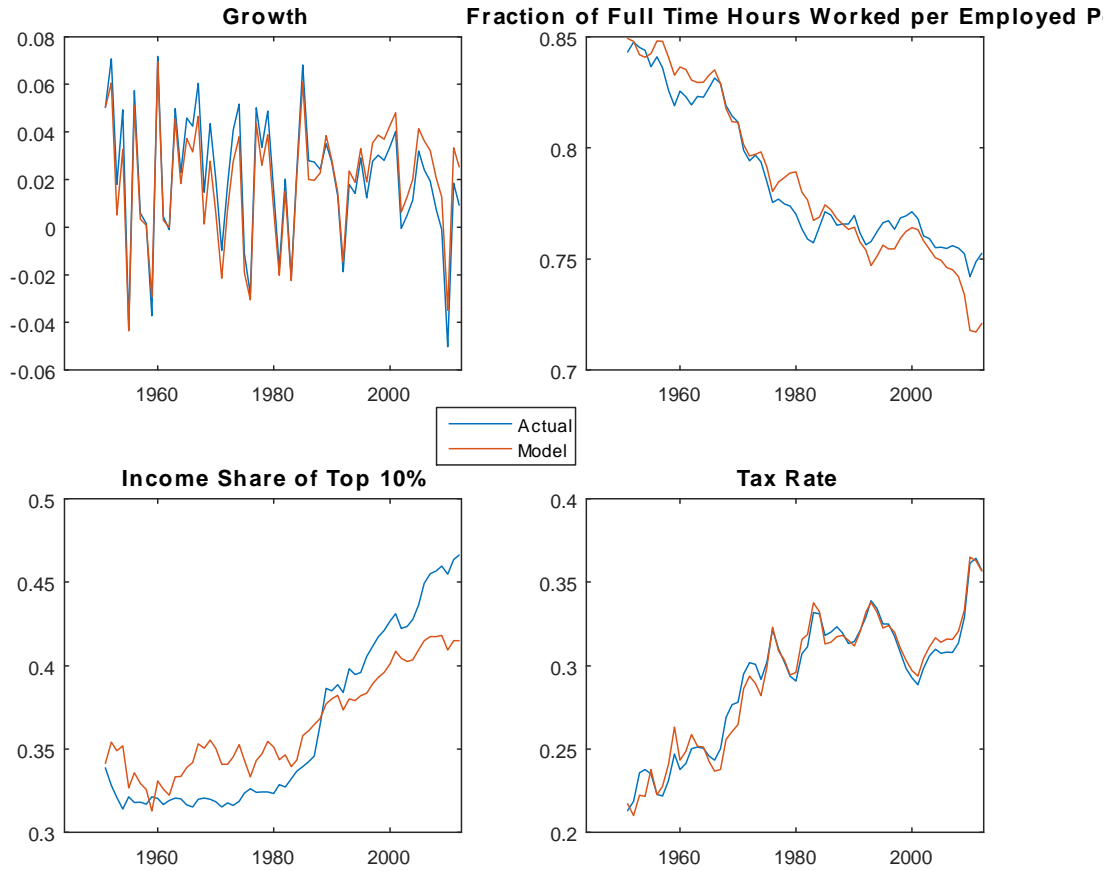


Figure 3: A comparison of actual data and model calculations for the US 1950 - 2011. Panel 1 shows a comparison of the actual rate of real per capita GDP growth to the model's calculated rate of real per capita income growth, γ_t . Panel 2 shows a comparison of the actual average fraction of full time hours worked per engaged person to the model's calculated average fraction of full time worked per employed person, \bar{L}_t . Panel 3 shows a comparison of the actual share of income earned by the top 10% to the model's calculated fraction of total income earned by the top 10%, $\phi_t(0.1)$. Panel 4 shows a comparison of the actual ratio of total government expenditures to GDP to the model's linear tax rate, τ_t .

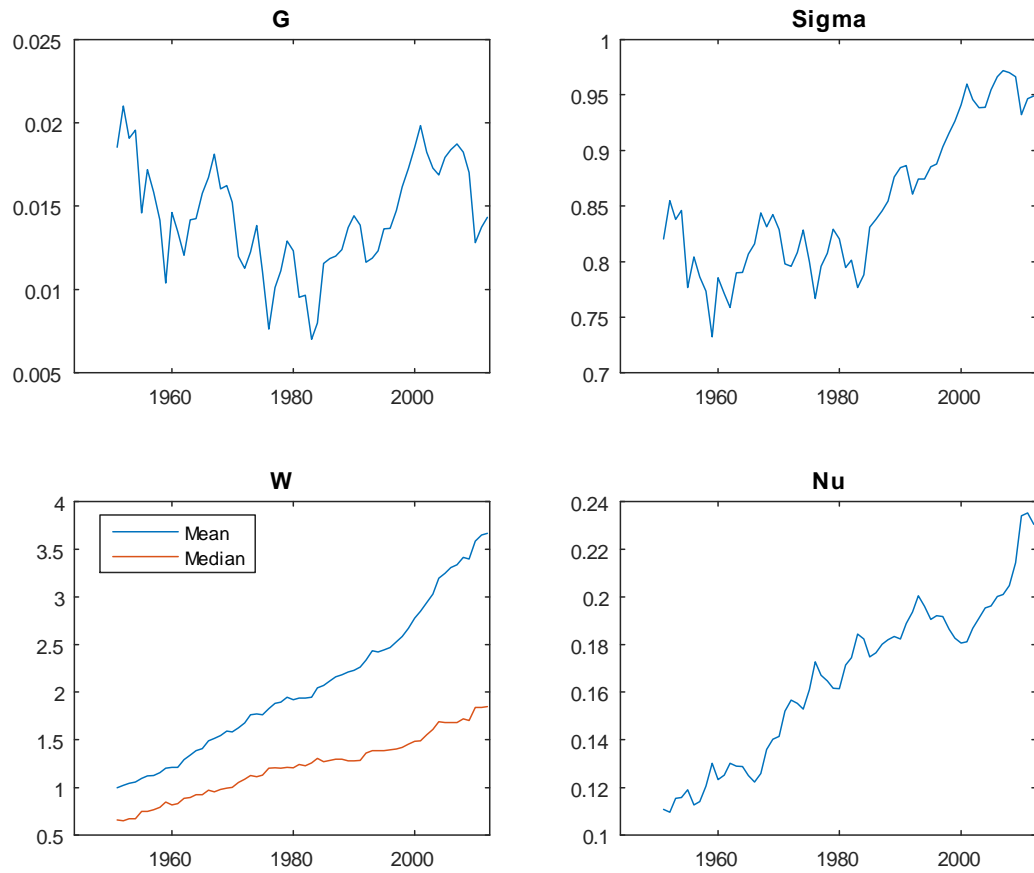


Figure 4: Estimated state variables in the US from 1950 - 2011. The first panel shows the productivity growth multiplier. The second panel show the standard deviation or dispersion of productivity. The third panel shows the absolute productivity index for the median worker; also, for comparison, the mean productivity index as computed by the BLS. The fourth panel shows the productivity of the last person who voluntarily chooses not to work.

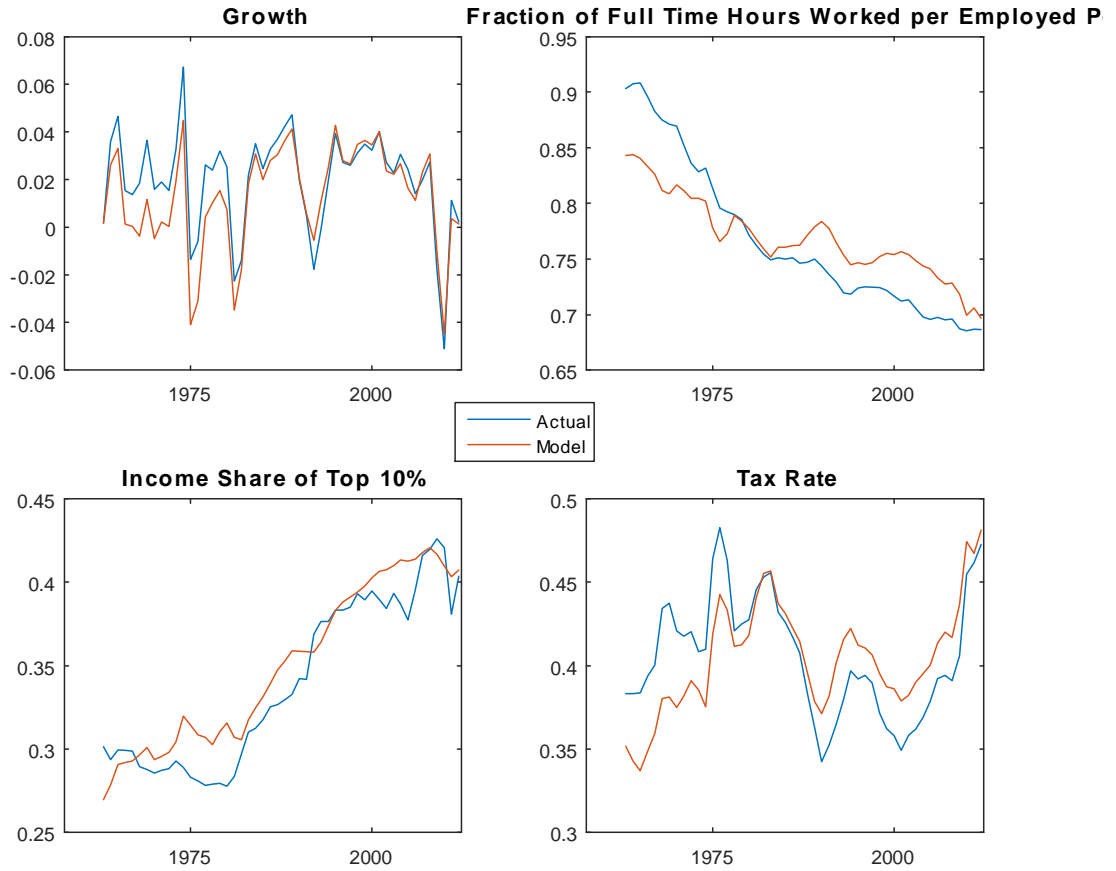


Figure 5: A comparison of actual data and model calculations for the UK 1962 - 2011. Panel 1 shows a comparison of the actual rate of real per capita GDP growth to the model's calculated rate of real per capita income growth, γ_t . Panel 2 shows a comparison of the actual average fraction of full time hours worked per engaged person to the model's calculated average fraction of full time worked per employed person, \bar{L}_t . Panel 3 shows a comparison of the actual share of income earned by the top 10% to the model's calculated fraction of total income earned by the top 10%, $\phi_t(0.1)$. Panel 4 shows a comparison of the actual ratio of total government expenditures to GDP to the model's linear tax rate, τ_t .

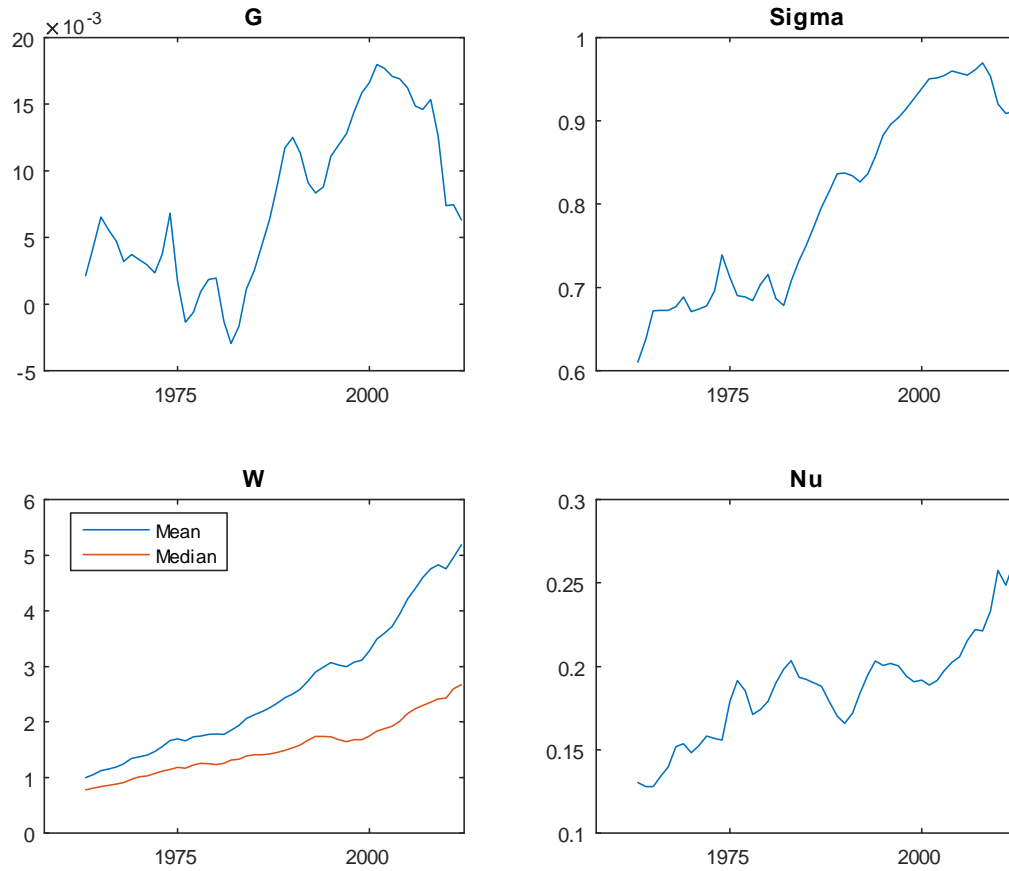


Figure 6: Estimated state variables in the UK from 1962 - 2011. The first panel shows the productivity growth multiplier. The second panel show the standard deviation or dispersion of productivity. The third panel shows the absolute productivity index for the median worker; also, for comparison, the mean productivity index as computed by the BLS. The fourth panel shows the productivity of the last person who voluntarily chooses not to work.

but not as good as the US or UK. The r^2 are 60.2%, 80.2%, -70.7%, and 79.5%, respectively. The fit of the model to the actual income share of the top 10% is poor; since 1995 the model requires a larger share for the top 10% than the tax data shows.

Figure 8 shows the estimated optimal states in France from 1978 - 2009. Notice in Panel 2 that, as in the US and UK, relative productivity dispersion began to increase in the 1980s coincident with the increase of the income share of the top 10%.

7.4 Technological Specialization and the Dispersion of Productivity

In all three countries, an important cause for the change in the distribution of productivity is technological specialization. New technologies result in divergent growth in productivity, which increase σ_t . Increased returns to specialization cause the distribution of relative productivity to widen. Evidently, as shown in Panel 2 of Figures 2, 4, 6, and 8, there has been a significant widening in the dispersion of relative productivity, σ_t , in the U.S., UK and France, respectively. This dispersion has been attributed to the growth of computer technology.¹⁰ Those who are able to lever their skills through technology have become relatively more productive in comparison with the median worker. This technological change has increased the growth rate of the economy and the dispersion of pre-tax income.

7.5 Statistics

We compute the model parameters by minimizing the sum of squared errors which is equivalent to maximizing the likelihood. Hence we can find the standard errors of the model parameters using the outer product of gradients estimator. The two unknown parameters in each country are the number of annual hours comprising full time work, Λ , and the maximum fraction of time devoted to work, λ .

¹⁰See Gordon (2002). More recently Gordon and Mokyr have joined in a lively debate over whether continued technological change will fuel future productivity growth Aeppel (2014).

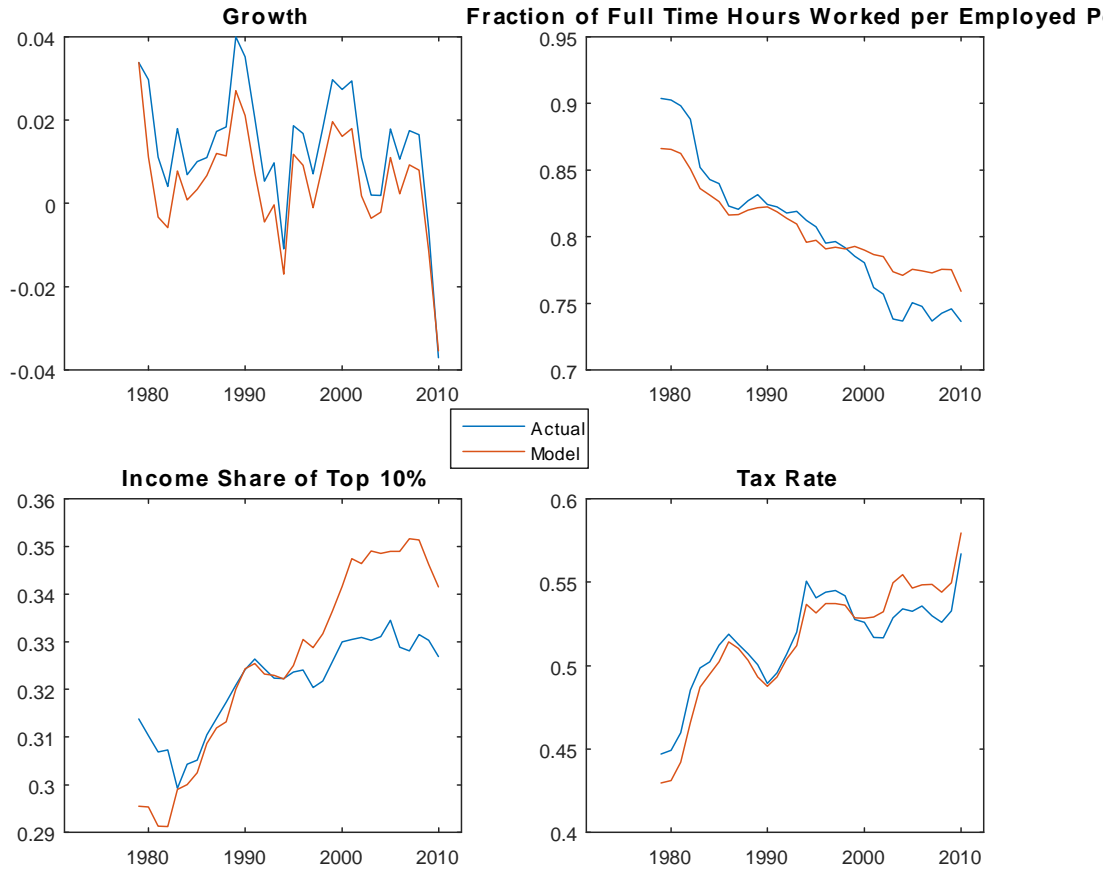


Figure 7: A comparison of actual data and model calculations for France 1978 - 2009. Panel 1 shows a comparison of the actual rate of real per capita GDP growth to the model's calculated rate of real per capita income growth, γ_t . Panel 2 shows a comparison of the actual average fraction of full time hours worked per engaged person to the model's calculated average fraction of full time worked per employed person, \bar{L}_t . Panel 3 shows a comparison of the actual share of income earned by the top 10% to the model's calculated fraction of total income earned by the top 10%, $\phi_t(0.1)$. Panel 4 shows a comparison of the actual ratio of total government expenditures to GDP to the model's linear tax rate, τ_t .

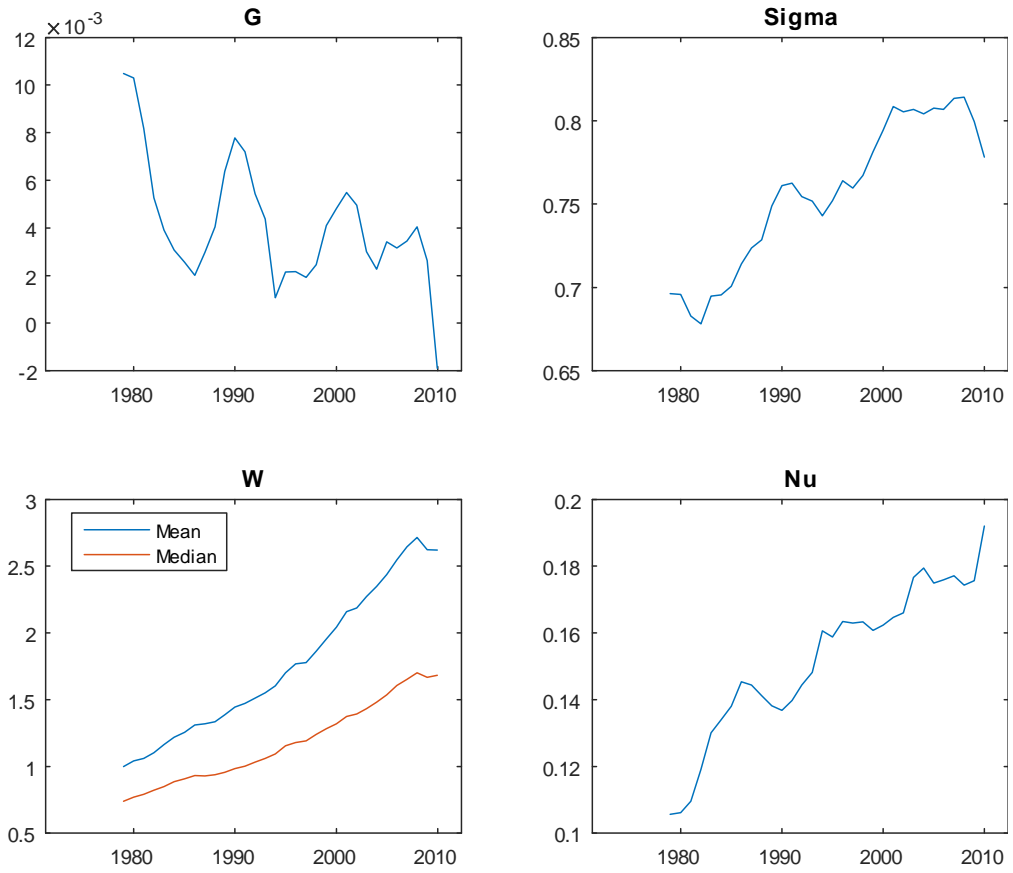


Figure 8: Estimated state variables in France from 1978 - 2009. The first panel shows the productivity growth multiplier. The second panel show the standard deviation or dispersion of productivity. The third panel shows the absolute productivity index for the median worker; also, for comparison, the mean productivity index as computed by the BLS. The fourth panel shows the productivity of the last person who voluntarily chooses not to work.

	λ	Λ
US 1967 - 2011	0.86 (97)	1925 (187)
US 1950 - 2011	0.69 (21)	2264 (55)
UK 1962 - 2011	0.78 (16)	2404 (25)
FR 1978 - 2009	0.88 (64)	1999 (52)

Table 3: Estimated model parameters and their T-statistics. The estimates are asymptotically consistent.

Our estimated parameters are shown in Table 3. The estimated full time annual hours worked ($\lambda\Lambda$) ranges from 1562 in the US from 1950 - 2011 to 1875 in the UK from 1962 - 2011.

8 Conclusion

Our contribution to the large and very diverse literature on growth and income distribution takes the form of a general equilibrium model of growth in labor income and consumption. The tax rate, as measured by the total government burden, and the amount spent on redistribution are endogenous variables. In developed, democratic countries voters chose the tax rate in single issue elections. The budget is balanced, so spending and tax collections are equal. By assumption, all spending is for redistribution.

The model extends our earlier work on a static economy, Meltzer and Richard (1981), to a growing economy. Consumers are endowed with different initial levels of productivity. Output and labor income change, as does productivity and, with it, the distribution of income among income groups. In our model, labor productivity changes as workers learn more productive skills on the job and as technology changes. This changes relative and absolute incomes and the spread between the top and the bottom (or other aspects) of the income distribution.

Our model analyzes consumption over time. Consumption is an endogenous variable that depends, *inter alia*, on taxation. Voters choose the tax rate in periodic elections. Sometimes they choose to increase current consumption by increasing tax rates and redistribution. Since higher tax rates reduce investment in learning by doing, the growth rate falls. Voters can vote to increase growth by subsequently voting to reduce tax rates to increase future consumption. The spread between top and bottom of the income distribution declines. Estimation of the model shows good correspondence to the historical data for the tax rate, average hours worked per employed person, the distribution of pre-tax income, and the growth rate of the economy which means the model captures the main facts about redistribution and economic growth.

The model answers the puzzling result emphasized by Piketty (2014). As did Karl Marx, Piketty concludes that because the return on capital repeatedly exceeds the growth rate of developed economies and does not change much over time, developed economies will face ever-increasing capital stocks. Since returns to capital go mainly to the highest income groups, the distribution of income widens over time and will continue to do so. Another possibility, of course, is that capital owners either consume or donate to charity the capital output in excess of the economic growth rate, so that capital does not accumulate faster than the economy grows. The puzzle for Piketty's conjecture is why there is no evidence anywhere that the capital stock has approached saturation. That fact opens the way for an alternative explanation of the relative constancy of the return to capital. Unlike Piketty who bases his conclusion on a comparison of the before tax income of the top 1 or 0.1 percent to before redistribution to the lowest income groups, we compare incomes available for consumption by the different income classes. Piketty's choice greatly overstates what has happened in developed countries. Our measure is more closely related to income after tax and after redistribution, hence to consumption. In our model, labor is the source of income. Unlike the return to capital, the return to labor has increased considerably over time. It is subject to cyclical and other changes in relative share. And it changes with productivity growth, thereby increasing at times the relative shares of those in the working classes while reducing their relative share in periods of low growth, and therefore consumption. As Kuznets conjectured, we must look to changes in labor income to explain changes in the spread between high and low income shares. Data for the three countries we study support our model and the Kuznets' conjecture.

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