Prospect Theory, Life Insurance, and Annuities

Daniel Gottlieb*

Wharton School, University of Pennsylvania

Abstract

This paper presents a prospect theory-based model of insurance against mortality risk. The model accounts for five main puzzles from life insurance and annuity markets: under-annuitization; insufficient life insurance among the working age; excessive life insurance among the elderly; guarantee clauses; and the simultaneous holding of life insurance and annuities.

Contents

1 Introduction

Twenty-five years ago, while delivering his Nobel Prize banquet speech, Franco Modigliani pointed out that "annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest. It is still ill-understood."[1] In the two and a half decades that followed, not only has this under-annuitization puzzle persisted, but several other puzzles been documented in the study of insurance markets for mortality risk. This paper suggests that prospect theory can simultaneously account for all of these puzzles.

The death of a family member is among the most important financial risks a household faces. About 30 percent of elderly widows live below the poverty line, compared to only eight to nine percent of the married elderly. In addition, over three-quarters of poor widows were not poor before their husband's death.[2] The importance of life insurance and annuities for hedging against mortality risk has been understood since the seminal work of Yaari (1965).[3] Working-aged individuals with dependents should purchase life insurance to financially protect their dependents in the event of an untimely death. Retirees should purchase annuities to protect against outliving their assets. Additionally, because purchasing a life insurance policy is equivalent to selling an annuity, the same person should not buy both at the same time.

The demand for life insurance and annuities observed in practice, however, is strikingly different from what theory predicts. Five puzzles have been extensively documented in the empirical literature. First, as mentioned by Modigliani, few individuals purchase annuities. Second, working-age individuals do not purchase sufficient life insurance. Third, too many retired individuals have life insurance. Fourth, many of those who do purchase annuities hold life insurance policies at the same time. And fifth, most annuity policies have clauses that guarantee a minimum repayment. Overall, it is surprising that individuals treat annuities and life insurance so differently since they are simply mirror images of each other: an initial installment in exchange for payments on the condition either of death (insurance), or of continuing to live (annuities).

This paper presents a dynamic model of the demand for annuities and life insurance based on prospect theory, and suggests that it simultaneously accounts for the apparently diverse behaviors described in these five puzzles. Prospect theory has three main features. First, preferences are defined over gains and losses. Second, preferences have a kink at zero

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[1]Modigliani (1986). Also cited by Brown (2007) and Benartzi, Previtero, and Thaler (2011).

[2]Hurd and Wise (1989).

[3]Davidoff, Brown, and Diamond (2005) generalize Yaari's analysis to environments with incomplete markets.

so that the marginal utility in the domain of gains is smaller than in the domain of losses (loss aversion). And third, individuals are risk averse over gains and risk seeking over losses.[4] In addition, following the original work of Tversky and Kahneman (1981), Redelmeier and Tversky (1992), and Kahneman and Lovallo (1993), most applications of prospect theory assume that lotteries are evaluated in isolation from other risks (narrow framing).[5]

There are different ways of modeling narrow framing in insurance, leading to different reference points. Traditionally, economists have assumed that individuals treat "full insurance" as the reference point. Then, as in other models of first-order risk aversion, prospect theory predicts that people demand "too much insurance" compared to an expected utility maximizer.

More recently, however, some researchers have suggested that individuals view life insurance and annuities as risky investments, which are profitable if the total payment received from the insurance company exceeds the premium. As Brown (2007) puts it: "Without the annuity, the individual has $100,000 for certain. With the annuity, in contrast, there is some positive probability that the individual will receive only a few thousand dollars in income (if he were to die within a few months), some probability that the individual will receive far more than $100,000 (if he lives well past life expectancy), and a full distribution of possibilities in between." Similarly, Kunreuther and Pauly (2012) argue that "[t]here is a tendency to view insurance as a bad investment when you have not collected on the premium you paid the insurer. It is difficult to convince people that the best return on an insurance policy is no return at all."[6] This view of insurance as a risky investment, therefore, suggests that individuals narrowly frame the insurance payments, rather than losses relative to full insurance.

Countless press articles describe the tendency to view life insurance and annuities as risky investments. Consider, for example, the following excerpt from an article recently published by Forbes magazine:

> Every once in a while we get a call on our Financial Helpline from someone whose financial adviser recommended that they invest in a permanent life insurance policy (...). The adviser's pitch can sound compelling. Why purchase temporary term life insurance that you'll likely never use? Isn't that like throwing money away? With permanent life insurance, part of your premiums are invested and

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[4]The other component of prospect theory, which I abstract from in this paper for simplicity, is the idea that people overweight small probability outcomes and underweight large probability outcomes.

[5]Read, Lowenstein, and Rabin (1999) present several examples of narrow framing.

[6]Analogously, in his discussion of Cutler and Zeckhauser (2004), Kunreuther argued that "consumers seem to view insurance as an investment rather than a contingent claim. This view creates a strong preference for low deductibles and rebate schemes so that the insured can *get something back.*"

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some of it can be borrowed tax-free for retirement, or your children's college education, or anything else you'd like and your heirs will get a nice death benefit when you pass away. But is it really always as great as it sounds?

The author then compares a standard permanent life policy (which combines insurance and savings) with a mix of term insurance and a 401(k) plan. While the permanent life policy would pay approximately $600k at retirement, a combination of term insurance and 401(k), which costs exactly the same and provides the same insurance coverage, would pay $980k.[7,8]

Prospect theory's asymmetric evaluation of gains and losses accounts for the difference in how people treat annuities and life insurance. According to prospect theory, consumers are risk averse in the domain of gains and risk seeking in the domain of losses. An annuity involves an initial payment to the insurance company in exchange for a payment stream conditional on being alive. It initially features a considerable chance of loss (if the consumer dies soon), but also the possibility of a gain (if she lives a long time). Because of the kink between the domains of gains and losses domains (loss aversion), consumers under-annuitize. As time passes, surviving consumers collect annuity payments, thereby shifting the support of the distribution to the right. If they live for long enough, the support of the distribution eventually falls entirely in the *gains domain*. Then, the kink lies outside of the support and loss aversion is no longer relevant. However, because consumers are risk averse in the gains domain, they keep under-purchasing annuities.

By contrast, a life insurance policy involves an initial payment to the insurance company in exchange for a repayment to beneficiaries in case of death. Thus, it initially comprises a significant chance of gain (if the consumer dies soon), but also the possibility of a loss (if she lives a long time). The kink between the domains of gains and losses again implies

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[7]Davidson, L. "Should You Use Life Insurance as an Investment? 3 Things to Consider," Forbes, July 2011. For another example, see Richards, C. "Why Life Insurance Is Not an Investment," The New York Times, April 2010. Anagol, Cole, and Sarkar (2011) study the Indian life insurance market and show that whole life insurance policies, which are purchased by a substantial fraction of the population, are dominated by a bundle of term life insurance policies and non-life contingent savings.

[8]The tendency to evaluate insurance in terms of investments dates back to the origins of the life insurance industry. The launch of Tontine insurance policies, which promised payments to consumers who outlived their life insurance policies, was one of the most important marketing innovations in the history of American life insurance. Henry B. Hyde introduced these policies in 1868. He refused to refer to payments as prizes, calling them "investment returns" instead. He acknowledged that "[t]he Tontine principle is precisely the reverse of that upon which Life Assurance is based." By the time of his death, his company had become the largest life insurance company in the world, and the companies selling tontine policies had become the largest financial institutions of the day. The Tontine Coffee House in Manhattan was the first home of what eventually became the New York Stock Exchange (see Hendrick (1907), Jennings and Trout (1982), or McKeever (2009)). Tontine policies were eventually outlawed on the basis of providing gambling rather than insurance products (Baker and Siegelman (2010)). Nevertheless, the idea of including payments in case one does not use the insurance policy persisted. Whole life insurance policies, which pay the face value if policyholders outlive their policy, amount to roughly two-thirds of total policies and one-third of the total face value being issued in the individual life insurance market (ACLI Life Insurers Factbook 2012).

that consumers initially under-purchase life insurance. As time passes, surviving consumers continue to spend on insurance without collecting any payments. Therefore the support of the distribution gradually shifts to the left and, if consumers survive for long enough, it eventually falls entirely in the *losses domain*. At that point, the kink lies outside of the support and no longer affects the decision. Moreover, because consumers are risk seeking in the domain of losses, they buy too much life insurance when they are old. That is, consumers are reluctant to liquidate their life insurance policies after having invested in them for long enough. As a result, consumers underinsure when young and over-insure when old. In fact, I show that they may even hold both annuities and life insurance policies at the same time, if they have previously purchased life insurance and the load is below a certain threshold.

This is not the first paper to suggest a connection between prospect theory and the under-annuitization puzzle. The so-called "hit-by-a-bus concern," according to which individuals may be reluctant to purchase annuities because they fear dying shortly thereafter, can be translated in terms of narrow framing and loss aversion. Brown et al. (2008) present survey results consistent with the idea that individuals frame annuities as an uncertain gamble, wherein one risks losing the invested wealth in case of death shortly after the purchase.[9] Hu and Scott (2007) formally show that prospect theory can explain under-annuitization. Gazzale and Walker (2009) find support for prospect theory as a cause of under-annuitization in a laboratory experiment. This is, however, the first paper to link the under-annuitization puzzle to the other four main puzzles of insurance against mortality risk, and to suggest that these apparently diverse findings can be explained by the same model.[10]

The predictions of the model differ sharply from the predictions of both expected and standard non-expected utility models. The celebrated theorem of Mossin (1968) shows that risk-averse expected-utility consumers will choose full insurance coverage if the load is zero, and they will choose partial insurance coverage if the load is positive. Segal and Spivak (1990) show that Mossin's result also holds for non-expected utility theories satisfying second-order risk aversion. Under first-order risk aversion, consumers will still choose full coverage if the load is zero, but they may also choose full coverage under positive loads.[11] Mossin's result

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[9]Brown and Warshawsky (2004) describe how participants in focus groups conducted by the American Council of Life Insurers consider the purchase of annuities "gambling on their lives." In light of prospect theory, it appears that these individuals do not integrate the annuity payoffs into their wealth but instead evaluate them separately. Brown, Kapteyn, and Mitchell (2011) present experimental evidence that individuals are more likely to delay claiming Social Security benefits when benefits are framed as gains rather than losses, and when they are described in terms of consumption rather than investment. See also Benartzi, Previtero, and Thaler (2011).

[10]Prospect theory has been successfully applied to several other environments – see, e.g., Camerer (2004) and Barberis (2013) for surveys. Marquis and Holmer (1996) study the demand for supplementary health insurance and find that prospect theory fits the data better than expected utility theory.

[11]Karni (1992) and Machina (1995) generalize Mossin's theorem for non-expected utility theories satisfying

also fails to hold in the prospect theory model presented here. However, the departure goes in the opposite direction of standard non-expected utility theories satisfying first-order risk aversion. Because the kink occurs at the point in which there is no insurance, rather than full insurance, *individuals may choose not to purchase insurance even under actuarially fair pricing.*

The remainder of the paper is organized as follows. Section 2 briefly describes empirical evidence from annuities and life insurance markets. Section 3 starts by considering a static insurance setting and contrasting the implications from the insurance-as-investment prospect theory model with those from expected and standard non-expected utility models (Subsection 3.1). It goes on to introduce a general continuous-time framework of mortality risk and the demand for life insurance and annuities (Subsection 3.2). Under expected utility (or standard non-expected utility), consumers generically purchase positive amounts of either life insurance or annuity policies. In the insurance-as-investment prospect theory model, purchasing positive amounts of either life insurance or annuities is no longer a generic property. Section 4 presents the dynamics of life insurance demand, and Section 5 considers the dynamics of annuity demand under the insurance-as-investment prospect theory model. Section 6 shows that, in the presence of positive loads, expected utility consumers never simultaneously demand both life insurance and annuities, whereas prospect theory consumers may. Section 7 then simulates the demands for annuities and life insurance over the life cycle using aggregate data from the United States, and shows that the model provides a good quantitative fit. Section 8 reviews how the predictions of the model relate to the empirical evidence, and Section 9 concludes. All proofs are in the Appendix.

2   Empirical Evidence

In recent decades, the development of the field of information economics brought insurance to the center of economic and financial research. Life insurance and annuity markets became particularly important for several reasons. First, they deal with major and widely-held risks. Second, these markets can potentially shed light on the role of bequests and intergenerational transfers.[12] And third, they appear to be particularly suitable to test standard insurance theory. As Cutler and Zeckhauser (2004) argue:

Mortality risk is a classic case where we expect insurance to perform well. On the

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Fréchet differentiability. Doherty and Eeckhoudt (1995) show that, in the dual theory of Yaari (1987), the solution always entails either full or zero coverage, with full coverage when the load is below a certain threshold.

[12]See, for example, Bernheim (1991).

demand side, the event is obviously infrequent, so administrative costs relative to ultimate payouts are not high. The loss is also well defined, and moral hazard is contained. On the supply side, it is relatively easy to diversify mortality risk across people, since aggregate death rates are generally fairly stable.

Nevertheless, numerous studies have established a striking divergence between predictions of standard insurance theory and empirical observations. The literature documents five puzzles in the demand for life insurance and annuities.

Puzzle 1. Insufficient Annuitization[13]

A major puzzle in annuities research is why so few people buy them. Almost no 401(k) plan even offers annuities as an option, and only 16.6 percent of defined contribution plans do so.[14] In the few cases in which annuities are offered, they are rarely taken. Schaus (2005) examines a sample of retirees for which annuities were offered as part of their 401(k) plan in a sample of 500 medium to large firms and finds that only two to six percent of them accepted. According to a study by Hewitt Associates, only one percent of workers actually purchase annuities.[15] Mitchell, Poterba, Warshawsky, and Brown (1999) show that fees and expenses are not large enough to justify the lack of annuitization. As Davidoff, Brown, and Diamond (2005) argue, "the near absence of voluntary annuitization is puzzling in the face of theoretical results suggesting large benefits to annuitization. (...) It also suggests the importance of behavioral modeling of annuity demand to understand the equilibrium offerings of annuity assets."[16]

Puzzle 2. Insufficient Life Insurance Among the Working-Aged

Several studies find that too few working-age families have life insurance, and many of those that do are still underinsured. A large literature shows that the death of a partner generates large declines in a household's per capita consumption. Bernheim, Forni, Gokhale, and Kotlikoff (2003), for example, argue that two-thirds of poverty among widows and over one-third of poverty among widowers can be attributed to an insufficient purchase of life

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[13]See Brown (2007) and Benartzi, Previtero, and Thaler (2011) for detailed surveys, including discussions of how prospect theory may explain this puzzle.

[14]Based on PSCA's 54th Annual Survey of Profit Sharing and 401(k) Plans.

[15]See Lieber, R., "The Unloved Annuity Gets a Hug From Obama." *The New York Times*, January 29, 2010. Brown (2001) reports that in the AHEAD survey, which consists of households aged 70 and older, less than eight percent of households held privately purchased annuity contracts.

[16]Several explanations have been proposed for the low amount of annuitization, including the presence of pre-annuitized wealth through Social Security and private pension plans (Bernheim, 1991; Dushi and Webb, 2004); the correlation between out-of-pocket health and mortality shocks (Sinclair and Smetters, 2004); the correlation between labor market and stock returns (Chai, Horneff, Maurer, and Mitchell, 2011); and the irreversibility of annuity purchases (Milevsky and Young, 2007). Another leading explanation is the existence of strong bequest motives (Friedman and Warshawsky, 1990, Bernheim, 1991, Lockwood, 2012). Brown (2001) compares annuitization by households with and without children and argues that bequest motives cannot account for under-annuitization.

insurance alone. Roughly two-thirds of secondary earners between ages 22 and 39 would suffer a decline in consumption of more than 20 percent, and nearly one-third would suffer a decline of more than 40 percent if a spouse were to die.[17] Bernheim, Forni, and Kotlikoff (1999) consider individuals between the ages of 52 years old and 61 years old from the Health and Retirement Study and estimate that 30 percent of wives and 11 percent of husbands face living standard reductions of over 20 percent if a spouse were to die. Since life insurance policies are close to actuarially fair (Gottlieb and Smetters, 2011), failure to appropriately insure cannot be attributed to adverse selection or administrative loads.

Puzzle 3. Excessive Life Insurance Among the Elderly

Compared to the predictions of standard life cycle models, elderly people hold too much life insurance. Brown (2001) finds that 78 percent of couples aged 70 and older own a life insurance policy. Life insurance is almost 10 times more prevalent than privately purchased annuities in this cohort. In fact, according to the 1993 Life Insurance Ownership Study, individual ownership of life insurance policies is actually *more frequent among those age 65 and older than in any other age cohort*.[18] Moreover, many elderly people *without dependents* have life insurance policies.[19]

Puzzle 4. Annuity Guarantees

Approximately 90 percent of annuities sold in the United States include either a guarantee clause or a refund option.[20] A guarantee clause promises payments for a certain period of time even if the annuitant dies. The majority of annuity contracts feature guarantee clauses, with 10, 15, and 20 years being the most common guarantee periods (they are usually called "term certain annuities"). Refund options reimburse beneficiaries for uncollected payments if the annuitant dies soon after purchase.

Annuities with guarantee clauses or refund options can be seen as a combination of a standard bond and an annuity with a payout date deferred by the guarantee period. Considering this, the prevalence of guarantees is puzzling in two ways. First, several other products offer better payouts at comparable levels of risk than do the bonds implicit in these annuities (Brown, 2007). Second, because the guarantee *de facto* delays the start of annuity, it does not offer protection for the guarantee period. Although the guarantee period may appear to reduce the risk of the annuity when the policy is evaluated in isolation, it does so by delaying insurance against mortality risk, which was presumably the reason for

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[17]Gokhale and Kotlikoff (2002).

[18]Hubener, Maurer, and Rogalla (2013) show that holding life insurance may help protect a surviving spouse from the loss of annuitized income.

[19]See also Cutler and Zeckhauser (2004).

[20]Iqbal, J. and J. E. Montminy. "Recent Research Trumps Immediate Annuities' Biggest Objection," InsuranceNewsNet Magazine, February 2011.

purchasing the annuity.[21]

Puzzle 5. Simultaneous Holding of Life Insurance and Annuities

As Yaari (1965) and Bernheim (1991) point out, purchasing a life insurance policy is equivalent to selling an annuity. Therefore, as long as either of them has a positive load, no one should ever demand both of them at the same time. In practice, however, a substantial number of the families that own annuities also have life insurance policies. According to Brown (2001):

> Of all married households, 50 percent own both a private pension and some form of life insurance. Among widows and widowers, 21 percent own both private pension annuities and life insurance. There are reasons to suspect that private pensions are not strictly voluntary, especially among those aged seventy and up who were likely covered for most of their careers in traditional defined benefit plans. However, even if we restrict ourselves to privately purchased, nonpension annuities, 6.6 percent of married couples own both. Since only 7.7 percent of the sample own such an annuity, however, this means that 86 percent of those married households who have purchased a private, nonpension annuity also own life insurance.

In the presence of positive loads, any portfolio with positive amounts of both life insurance and annuities is first-order stochastically dominated by another portfolio with a lower amount of both. As a result, any decision maker with preferences defined over final wealth cannot simultaneously demand both assets.[22]

3 The General Framework

3.1 Insurance as a Risky Investment

Before introducing a general dynamic model, I will consider a canonical static setting in order to contrast my model with other insurance models. A consumer suffers a financial loss $L > 0$ with probability $p \in (0, 1)$. She is offered insurance policies with a proportional

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[21]Hu and Scott (2007) also propose that prospect theory can account for the existence of guaranteed annuities. However, as Brown (2007) points out, consumers who prefer guarantees in the model of Hu and Scott never purchase annuities. Eckles and Wise (2011) show that, because of loss aversion, prospect theory consumers may over-insure small losses, which could explain the presence of guarantes. In a laboratory experiment, Knoller (2011) finds evidence for the prospect theory rationale for annuity guarantees.

[22]Purchasing life insurance may be desirable for individuals who hold certain deferred annuities and intend to leave part of them as bequests since life insurance benefits are typically income tax-free, whereas annuity payments are not.

loading factor $l \in [0, 1-p)$. That is, each policy costs $p + l$ and pays \$1 in case of loss; insurance is actuarially fair if $l = 0$.

Following a large literature in prospect theory, I assume that individuals evaluate lotteries according to the sum of a "consumption utility function," which satisfies the standard assumptions of expected utility theory, and a "gain-loss utility function."[23] Let the strictly increasing, strictly concave, and differentiable function $U : \mathbb{R}_{++} \to \mathbb{R}$ denote the consumption utility function. Each lottery is also individually evaluated according to the gain-loss utility function

$$\mathcal{V}(X) = \begin{cases} v(X) & \text{if } X \geq 0 \\ -\lambda v(-X) & \text{if } X < 0 \end{cases}, \tag{1}$$

where $v : \mathbb{R}_+ \to \mathbb{R}$ is a weakly concave, twice differentiable function satisfying $v(0) = 0$.[24] A prospect theory decision-maker maximizes the expected sum of consumption utility and gain-loss utility.

Let W denote the consumer's wealth and let I denote the number of insurance policies purchased. Expected consumption utility is

$$pU(W - L + (1 - p - l)I) + (1 - p)U(W - (p + l)I).$$

Buying insurance corresponds to participating in a lottery that pays $[1 - (p + l)]I$ with probability p and $-(p + l)I$ with probability $1 - p$. Therefore, the expected gain-loss utility from insurance is

$$p\mathcal{V}((1 - p - l)I) + (1 - p)\mathcal{V}(-(p + l)I).$$

As argued in the introduction, this paper studies individuals who view insurance as risky investments, which are profitable if the total amount received from the insurance company exceeds the premium. Accordingly, my specification of gains and losses takes the point of zero insurance (status quo) as the reference point: possible outcomes are evaluated against the alternative of not buying any coverage. As a result, individuals in the model treat the outcome of an insurance policy as a gain if the net payment from the policy is positive.[25]

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[23]This "global-plus-local" specification has been widely used in the prospect theory literature. Examples include Barberis and Huang (2001), Barberis, Huang, and Santos (2001), Barberis and Xiong (2012), Koszegi and Rabin (2006, 2007, 2009), Rabin and Weizsacker (2009), and Heidhues and Koszegi (2008, 2010). It is defended by Azevedo and Gottlieb (2012).

[24]Kahneman and Tversky called v a value function and suggested the power utility functional form $v(X) = X^\alpha$, $\alpha \in (0, 1]$. This functional form is concave but it is not differentiable at $X = 0$ when $\alpha \neq 1$, which creates some counterintuitive properties at the kink. The use of the same value function for gains and losses is for notational convenience only; my results immediately generalize to gain-loss utility functions featuring different value functions for gains and losses.

[25]My results remain qualitatively unchanged if we replace the status quo by any other deterministic reference point below the point of full insurance. In fact, when insurance is actuarially fair, the model

Narrow framing and loss aversion introduce a kink at the point of zero insurance. Because of the kink, the consumer prefers not to purchase insurance "more frequently" than if she were maximizing consumption utility only. In fact, she may prefer not to purchase insurance even under actuarially fair prices.[26] Formally, let $\mathcal{I}_0^U$ and $\mathcal{I}_0^{PT}$ denote the sets of wealth levels, losses, loss probabilities, and loads (W, L, p, l) for which zero insurance maximizes expected consumption utility and total utility, respectively. The necessary and sufficient condition for zero insurance to maximize expected consumption utility is

$$p\left(1 - p - l\right) U'\left(W - L\right) - \left(1 - p\right)\left(p + l\right) U'\left(W\right) \leq 0.$$

The necessary local condition for zero insurance to maximize total utility is

$$p\left(1 - p - l\right) U'\left(W - L\right) - \left(1 - p\right)\left(p + l\right) U'\left(W\right) \leq \left\{p\left(1 - p\right)\left(\lambda - 1\right) + l\left[\lambda\left(1 - p\right) + p\right]\right\} v'\left(0\right). \tag{2}$$

Since $\lambda > 1$ and $l \geq 0$, the right-hand side of (2) is strictly positive: Whenever zero insurance maximizes consumption utility, it also maximizes total utility. Thus, when insurance is viewed a risky investment, loss aversion introduces a kink at the point of zero coverage, which makes purchasing insurance less desirable.

Proposition 1. *$\mathcal{I}_0^U \subset \mathcal{I}_0^{PT}$ and the inclusion is strict.*

In most non-expected utility theories, indifference curves are either smooth or have a kink at the point of full insurance. Therefore, as discussed by Segal and Spivak (1990), they predict either just as much or more insurance than expected utility theory does. In contrast, loss-averse decision makers who frame their insurance purchases narrowly may buy no insurance even when prices are actuarially fair. Hence, as with non-expected utility theories featuring first-order risk aversion, Mossin's (1968) theorem no longer holds. However, the result is distorted in the opposite direction: While first-order risk averse individuals may demand full coverage under actuarially unfair prices, prospect theory individuals may not purchase full coverage even when prices are actuarially fair!

Notice that, in each state of the world, gains and losses are determined *relative to not having purchased insurance* (status quo). Hence, when we say that individuals who purchase

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is formally equivalent to one where reference points are set at the expected insurance payments. The predictions of the model are, therefore, qualitative the same as the predictions from Loomes and Sugden's (1986) the theory of disappointment aversion, which sets the (utility) reference point at the expected utility from insurance. On the other hand, my results do not hold if the reference point is set at the point of full insurance (e.g., if the consumer does not frame insurance narrowly).

[26]The online appendix illustrates the differences between the expected utility model, standard non-expected utility models, and the model with narrow framing and loss aversion using the classic Hirshleifer-Yaari diagram.

insurance and suffer a financial loss are in the domain of gains, we mean that, conditional on that loss, they see insurance as a profitable investment. Reciprocally, we say that those who do not suffer a financial loss are in the domain of losses since, in that state of the world, buying insurance is seen as an unprofitable investment. Because the financial loss also decreases consumption utility, the model does *not* imply that individuals would prefer to suffer a financial loss. Hence, gains and losses refer to the outcome of the investment, not to the realized state of the world. In particular, when we apply the theory to mortality risk, an individual who buys life insurance and survives sees the insurance decision as a "poor investment" relative to not having bought insurance, which, of course, does not mean that the individual would rather die earlier.

3.2   Dynamic Model

In order to explore the dynamics of mortality insurance demand, we need to extend the basic framework in two ways: First, we need to consider more than one period. Second, because the marginal utility of consumption is likely to be different if the individual is alive or dead, we must allow consumption utility to be state-dependent.

I consider a continuous-time model of mortality risk. A household consists of a head of the household and one or more heirs. The head of the household lives for a random length $T \in \mathbb{R}_+$ and makes all decisions while alive. For simplicity, I assume that heirs always outlive the household head. Because the head of the household is responsible for making decisions while alive, I refer to her as "the consumer." The functions $U_a : \mathbb{R}_{++} \to \mathbb{R}$ and $U_d : \mathbb{R}_{++} \to \mathbb{R}$ denote the consumption utility in states in which the consumer is alive and dead; $U_d(C)$ can be interpreted as the instantaneous utility of bequeathing C dollars. The consumption utility functions U_a and U_d are strictly increasing, strictly concave, twice continuously differentiable, and satisfy the following Inada condition: $\lim_{C \searrow 0} U_i'(C) = +\infty$, $i = a, d$. Because U_a and U_d satisfy the standard assumptions from expected utility, I will refer to allocations that maximize expected consumption utility as *expected utility maximizing allocations*. A prospect theory decision-maker selects allocations that maximize the expected sum of consumption utility and gain-loss utility (equation 1).

We say that the gain-loss utility is *piecewise linear* when $v(X) = X$. The piecewise linear specification considerably simplifies calculations and is used in several theoretical papers.[27] However, it does not generate the risk-seeking behavior in the domain of losses stressed out by Kahneman and Tversky (1979) and observed in empirical studies. When v is strictly concave, we say that the gain-loss utility is *strictly concave in the gains domain and strictly*

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[27]For example, Barberis, Huang, and Santos, 2001, Barberis and Huang, 2001, and Barberis and Xiong, 2012.

convex in the losses domain. In order to avoid excessively risk-seeking behavior in the losses domain, we will assume that v is "not too concave".[28] For expositional purposes, I focus on gain-loss functions that are either piecewise linear or strictly concave in the domain of gains and strictly convex in the domain of losses. The results can be generalized for weak concavity in the gains domain and weak convexity in the losses domain.

The assumption that lotteries are evaluated in isolation (narrow framing) is an important feature of the model. That is, the decision-maker separately evaluates $\mathcal{V}$ at the outcome of each individual lottery, instead of combining the outcomes from all lotteries. Throughout the paper, life insurance and annuities are treated as different lotteries.[29]

The time of death T is distributed according to an exponential distribution with parameter μ.[30] The probability distribution function of the time of death is $f(T) = \mu \exp(-\mu T)$; the consumer is alive at time t with probability $1 - \int_0^t f(s)\, ds = \exp(-\mu t)$. Expected consumption utility is then

$$\int_0^\infty \exp(-\mu t) \left\{ U_a(C_a(t)) + \mu U_d(C_d(t)) \right\} dt, \tag{3}$$

where $C_a(t)$ denotes consumption at time t (if alive) and $C_d(t)$ denotes the amount of bequests left if the individual dies at t. The consumer earns an income $W_a(t) > 0$ at time t if she is alive. The remaining members of the household have lifetime discounted income $W_d(t) > 0$ if the consumer dies at t.

While alive, the consumer allocates her income between instantaneous life insurance policies, $I(t)$, and instantaneous annuities, $A(t)$. Although it is straightforward to generalize the results for the case of positive proportional loads, I assume that policies are sold at actuarially fair prices to simplify notation.[31] An *instantaneous life insurance policy* costs μdt and pays \$1 if the individual dies in the interval $[t, t+dt]$. Since the probability of death in this interval is μdt, the policy is actuarially fair. Buying an *instantaneous annuity* is equivalent to selling an instantaneous life insurance policy: It pays μdt if the consumer survives the interval $[t, t + dt]$, but costs \$1 if she dies in this interval. Therefore, consumption in each state is $C_a(t) = W_a(t) - \mu[I(t) - A(t)]$ and $C_d(t) = W_d(t) + [I(t) - A(t)]$. Substituting in

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[28]The formal assumption on the concavity of v is presented in Section 4 (Assumption 1).

[29]Because life insurance and annuities are sold as different products and are relevant at very different points in life, narrow framing suggests that individuals would not integrate them into a single gain-loss category.

[30]Although not key to my results, the exponential distribution simplifies the analysis because its memoryless property allows for a parsimonious recursive formulation. In particular, it implies that actuarially fair prices are constant over time.

[31]The simulations in Section 7 incorporate positive loads and mortality probabilities compatible with the ones observed in practice.

the expected consumption utility (3), we obtain

$$\int_0^\infty \exp\left(-\mu t\right) \left\{ U_a\left(W_a(t) - \mu\left[I(t) - A(t)\right]\right) + \mu U_d\left(W_d(t) + \left[I(t) - A(t)\right]\right) \right\} dt. \qquad (4)$$

An expected utility consumer chooses functions A and I to maximize (4).

Because prices are actuarially fair, life insurance and annuities only affect consumption utility through their net amount: $N(t) \equiv I(t) - A(t)$. Moreover, whenever $N(t) \neq 0$, any solution entails either $A(t) > 0$ or $I(t) > 0$ (or both).[32] A net amount N maximizes expected consumption utility (4) if and only if $U_a'\left(W_a(t) - \mu N(t)\right) = U_d'\left(W_d(t) + N(t)\right)$ for almost all t: Since insurance is actuarially fair, an expected utility maximizer equates the marginal utility in both states of the world almost surely (full insurance). Purchasing zero quantities of both life insurance and annuities is optimal if the income paths W_a and W_d satisfy

$$U_a'\left(W_a(t)\right) = U_d'\left(W_d(t)\right) \quad \text{almost surely.} \qquad (5)$$

Let $\mathcal{B}$ denote the space of bounded continuous functions $(W_a, W_d) : \mathbb{R}_+ \to \mathbb{R}_{++}^2$, endowed with the norm of uniform convergence. A property is *generic* if the set of income paths $(W_a, W_d) \in \mathcal{B}$ for which this property is satisfied is a dense and open set. Because U_a and U_d are strictly concave, the marginal utilities are strictly decreasing. Thus, (5) fails generically. As a result, an expected utility maximizer generically purchases a strictly positive quantity of either life insurance or annuity policies:

Proposition 2. *Generically, an expected utility maximizer purchases a strictly positive quantity of actuarially fair life insurance or annuities (i.e., generically, $N^{EU} \neq 0$ almost surely).*

Buying a life insurance path I amounts to participating in a lottery that costs $\mu I(t)\, dt$ for as long as the individual is alive and pays $I(t)$ when she dies. Therefore, the gain-loss utility from life insurance if death occurs at time t is $\mathcal{V}\left(I(t) - \mu \int_0^t I(s)\, ds\right)$. We say that life insurance payments are in the gains domain at time t if the payment received exceeds the total amount spent on life insurance if the individual dies at t: $I(t) \geq \mu \int_0^t I(s)\, ds$. Recall, however, that gains and losses are defined *relative to not buying insurance*. Therefore, when we say that life insurance payments are in the domain of gains at time t, we mean that, conditional on dying at t, the individual sees the life insurance path I as a profitable investment. We do not, of course, mean that such an individual would prefer to die at t than at some other period with a larger gain (since dying at this other period may significantly

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[32]The possibility of holding both life insurance and annuities is an implication of actuarially fair pricing. Section 6 introduces positive loads and shows that expected utility maximizers do not simultaneously hold both life insurance and annuities.

lower her consumption utility).[33]

Similarly, if the person buys $A(t)$ instantaneous annuities at time t, she either survives and receives $\mu A(t)dt$ or dies and loses $A(t)$. Thus, a person who dies at t has collected a total of $\mu \int_0^t A(s)\,ds$ in annuity payments and loses $A(t)$ in annuities that will not be collected; the gain-loss utility is $\mathcal{V}\left(\mu \int_0^t A(s)\,ds - A(t)\right)$. Annuity payments are in the gains domain at time t if the total amount collected in annuities exceed the amount spent on annuities if the person dies at t: $\mu \int_0^t A(s)\,ds \geq A(t)$.

Because the reference point is the point of zero dollars spent on annuities and life insurance, which is constant over time, the consumer is time-consistent. Thus, whether the consumer is evaluating the insurance paths at time 0 or as time unfolds is immaterial for my results. It is natural, therefore, to interpret the paths A and I as (possibly time-varying) annuities and life insurance policies purchased at time $t = 0$.

There are three advantages to interpreting the annuity and life insurance paths as time-varying policies purchased at a fixed period rather than instantaneous policies adjusted at each point in time. First, they are easier to interpret in term of observed policies. A five-year term life insurance, for example, corresponds to the path $I(t) = 1$ if $t \leq 5$ and $I(t) = 0$ if $t > 5$. Second, people do not seem to adjust their life insurance and annuities portfolios on a regular basis. Therefore, requiring them to choose policies continuously might seem unrealistic. And third, it avoids issues related to the possible updates of the reference point.

Let $u_t(N) \equiv U_a(W_a(t) - \mu N) + \mu U_d(W_d(t) + N)$ denote the consumption utility from the net life insurance coverage N at time t. Loss aversion introduces a kink at zero in the total utility function. Suppose, for example, that the gain-loss utility is piecewise linear. A constant amount $\epsilon > 0$ of either life insurance or annuities yields an expected gain-loss utility of $-(\lambda - 1)e^{-1}\epsilon$. Thus, the total utility from the net coverage $N(t) = \epsilon$ is

$$\int_0^\infty \exp(-\mu t)\, u_t(\epsilon)\, dt - (\lambda - 1)e^{-1}|\epsilon|.$$

The total utility function is therefore not Gâteaux differentiable at the status quo point ($N(t) = 0$ for all t). Because of this kink at the status quo, individuals will not purchase either insurance or annuities if the consumption smoothing value is not high enough relative to loss aversion:

Proposition 3. *Suppose $\mathcal{V}$ is piecewise linear and let $\lim_{t \to \infty} e^{-\mu t} \int_0^t u_s'(0)\,ds = 0$. A prospect theory consumer does not purchase any annuities or life insurance policies (i.e., $A = I = 0$ almost surely) if $\sup_{t \in \mathbb{R}_+}\left\{\frac{u_t'(0)}{\mu} - \int_0^t u_s'(0)\,ds\right\} - \inf_{t \in \mathbb{R}_+}\left\{\frac{u_t'(0)}{\mu} - \int_0^t u_s'(0)\,ds\right\} \leq$*

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[33]In fact, preferences over the timing of death are not identified from choices over life insurance and annuities only.

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$\lambda - 1$.

Therefore, purchasing strictly positive quantities of actuarially fair annuity or life insurance policies is *not* a generic property of the prospect theory model. As in the static model, the non-genericity of purchasing either life insurance or annuities is a consequence of the kink at zero introduced by loss aversion. It resembles the loss aversion rationale for the endowment effect and the status quo bias.[34]

3.3 Illustration of Main Results

Next, we will study the dynamics of life insurance and annuity demands separately. However, before presenting the formal analyses, it is helpful to consider an illustrative example in a static setting. Consider an individual who has previously spent an amount $X \geq 0$ on life insurance and faces the probability of death $p \in (0, 1)$ in a *single remaining period*. Life insurance policies are available at actuarially fair prices: Each dollar of coverage costs p.

Let $\mathcal{U}(I)$ denote the expected consumption utility from purchasing an amount I of life insurance coverage. For expositional simplicity, assume that $\mathcal{U}$ is continuously differentiable, strictly concave, and has an interior maximum. Let $\bar{I}$ denote the amount of insurance that maximizes expected consumption utility: $\mathcal{U}'(\bar{I}) = 0$. An insurance coverage I costs pI and repays $(1 - p)I$ in case of death. Therefore, the total net payments from life insurance equal $(1 - p)I - X$ if the individual dies and $-pI - X$ if she survives. The individual maximizes the sum of expected consumption and gain loss utilities:

$$\mathcal{U}(I) + p\mathcal{V}((1 - p)I - X) + (1 - p)\mathcal{V}(-pI - X).$$

The solution depends on whether the net payment in case of death lies in the losses domain $(1 - p)I < X$, in the gains domain $(1 - p)I > X$, or at the kink $(1 - p)I = X$.[35]

In the losses domain, the individual chooses the life insurance coverage that solves the first-order condition:

$$\mathcal{U}'(I) = p(1 - p)\lambda[v'(X + pI) - v'(X - (1 - p)I)] \leq 0,$$

where the inequality follows from the weak concavity of v. Therefore, the solution features $I \geq \bar{I}$. In particular, if the gain-loss utility is piecewise linear, the marginal gain-loss utility is the same whether or not the individual dies (since both are in the losses domain) and

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[34]See, for example, Tversky and Kahneman (1991) and Kahneman, Knetsch, and Thaler (1991).

[35]It is straightforward, but not very informative, to write these regions in terms of primitives rather than in terms of the endogenous variable I.

the individual picks the amount of insurance that maximizes expected consumption utility $I = \bar{I}$. If v is strictly concave, gain-loss utility is convex in the domain of losses, which pushes the individual to purchase more insurance. As a consequence, the individual *over-insures* relative to expected utility: $I > \bar{I}$. The rationale for over-insurance in the domain of losses resembles the anecdotal description of consumers who renew their term insurance policies because they have "invested too much to quit."[36] At the kink, the individual chooses the amount of insurance that would exactly offset the previous expenditure in case of death $\left(I = \frac{X}{1-p}\right)$, which prevents her from entering the losses domain.

In the gains domain, the individual chooses the insurance coverage that solves the following first-order condition:

$$\mathcal{U}'(I) = p(1-p)\left[\lambda v'(pI + X) - v'((1-p)I - X)\right].$$

If the gain-loss utility is piecewise linear, this condition becomes $\mathcal{U}'(I) = p(1-p)(\lambda - 1) > 0$. Therefore, the solution entails *under-insurance* relative to expected consumption utility: $I < \bar{I}$. Under a piecewise linear gain-loss utility function, the marginal gain-loss utility if the individual survives is λ (since it lies in the domain of losses) whereas the marginal gain-loss utility if the individual dies is 1 (gains domain). Therefore, loss aversion ($\lambda > 1$) induces the individual to purchase less insurance in the gains domain.

When v is strictly concave, there are two effects. As before, loss aversion pushes the individual to underinsure. However, convexity of the gain-loss utility in the losses domain may push her to over-insure or under-insure.[37] The individual will prefer to underinsure as long as the gain-loss utility function is not "too convex" so that the loss aversion effect dominates.

Figure 1a presents the amount of insurance that maximizes total utility.[38] When the gain-loss utility function is piecewise linear ($\gamma = 1$), the individual underinsures relative to expected utility if the previous expenditure is small (i.e., in the gains and kink regions) and purchases the coverage that maximizes expected utility if previous expenditure is large (losses region). When it is strictly concave, she underinsures if the previous expenditure is small (gains region) and over-insures if it is large (losses region).

Next, let us examine the purchase of annuities in this static setting. Consider an individual who has previously received $X \geq 0$ in annuity payments and faces the probability

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[36]This rationale is related to the large literature on the escalation of commitment to a failing course of action. See, for example, Staw (1981).

[37]Convexity will push the agent to over-insure if $\left(\frac{1}{2} - p\right)I < X$ and under-insure if the reverse is true.

[38]For the life insurance path in Figure 2a, I assumed $U_d(C) = U_a(C) = C^{\frac{1}{4}}$, $W_A = 10$, $W_D = 0$, and $p = \frac{1}{4}$.
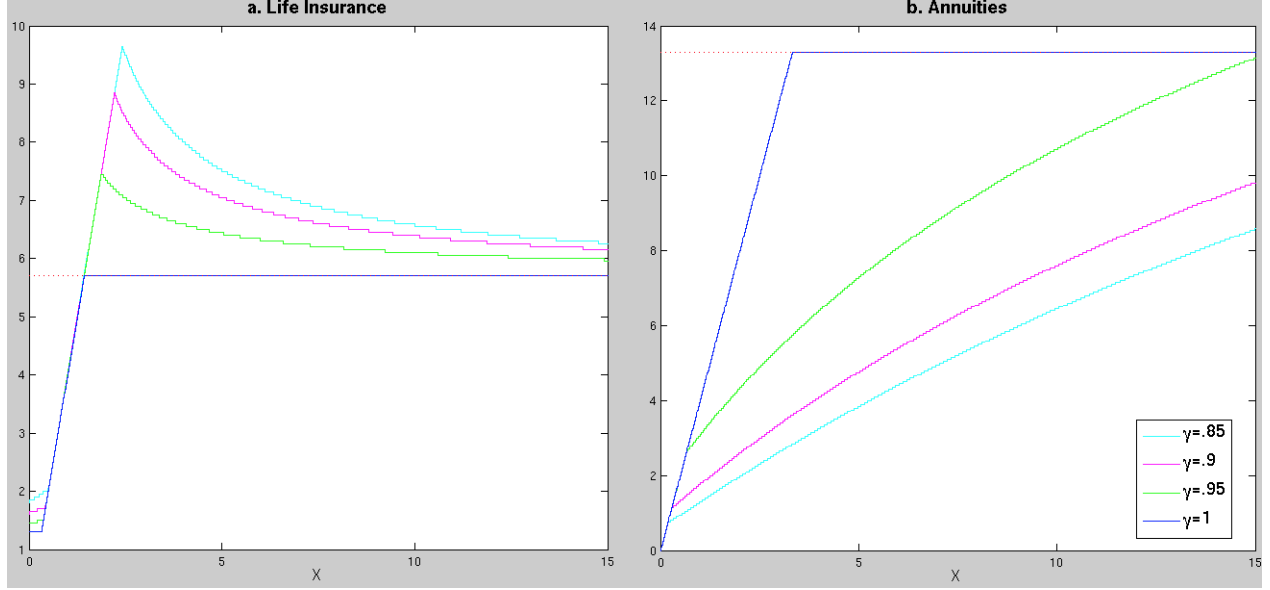
Figure 1: Life insurance coverage and net gains as a function of previous expenditure X. Gain-loss utility: $v(x) = \frac{1}{2}\left[(1+x)^{\gamma} - 1\right]$, $\lambda = 2.25$.

of death $p \in (0,1)$ in a single remaining period. Annuities are available at actuarially fair prices: Each annuity costs $1 - p$ and pays \$1 in case of survival.

With some abuse of notation, let $\mathcal{U}(A)$ denote the individual's expected consumption utility from purchasing A annuities, where $\mathcal{U}$ is continuously differentiable, strictly concave, and has an interior maximum. Let $\bar{A}$ denote the amount of coverage that maximizes expected consumption utility: $\mathcal{U}'(\bar{A}) = 0$. The consumer maximizes total expected utility

$$\mathcal{U}(A) + p\mathcal{V}(X - (1 - p)A) + (1 - p)\mathcal{V}(X + pA).$$

The solution depends on whether the net payment in case of death lies in the gains domain $(X > (1-p)A)$, losses domain $(X < (1-p)A)$, or at the kink. At the kink, the individual buys the amount of annuities that prevents her from entering the losses domain in case of death: $A = \frac{X}{1-p}$.

The first-order condition in the domain of gains is

$$\mathcal{U}'(A) = p(1 - p)\left[v'(X - (1 - p)A) - v'(X + pA)\right] \geq 0,$$

where the inequality follows from the weak concavity of v. Thus, as with life insurance in the domain of losses, the individual chooses an amount of annuities weakly below the one that maximizes expected utility: $A \leq \bar{A}$. If the gain-loss utility is piecewise linear, the individual has the same marginal gain-loss utility if she dies or survives. As a result, she chooses the

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annuity coverage that maximizes expected utility $(A = \bar{A})$. If the gain-loss utility is strictly concave in the domain of gains, the individual under-annuitizes relative to expected utility: $A < \bar{A}$.

In the domain of losses, the first-order condition is

$$\mathcal{U}'(A) = p(1-p)[\lambda v'((1-p)A - X) - v'(X + pA)].$$

When the gain-loss utility is piecewise linear, the term inside brackets becomes $\lambda - 1 > 0$, implying that the consumer under-annuitizes relative to expected utility: $A < \bar{A}$. Thus, loss aversion makes consumers buy less annuities. Strict convexity of the gain-loss utility in the domain of losses introduces an additional effect, which may be either positive or negative.[39] However, as long as the gain-loss utility is "not too convex" in the domain of losses, the loss aversion effect dominates and the individual under-annuitizes relative to expected utility.

Figure 1b presents the annuity coverage that maximizes total utility. When the gain-loss utility is piecewise linear $(\gamma = 1)$, the individual under-annuitizes relative to $\bar{A}$ when the previous expenditure is small (losses and kink regions) and purchases the amount that maximizes expected utility when the previous expenditure is large (gains region). When v is concave $(\gamma < 1)$, the consumer always under-annuitizes.[40] Consistently with the hit-by-a-bus concern discussed in the introduction, loss aversion induces individuals to under-annuitize.

The analysis in this subsection took the previous expenditure X as given. In a dynamic model, the choice of life insurance and annuity coverage in one period affects the expenditure level in all future periods. Purchasing an additional unit of life insurance in one period increases one's total life insurance expenditures by $1-p$ units in all future periods. Similarly, an additional annuity today raises the net annuity expenditure by p units in all future periods. As a result, the optimal decision incorporates the effect of a current increase in coverage in all future gain-loss utility. Sections 4 and 5 consider the dynamic model formally and establish that the optimal solutions are qualitatively similar to the ones from the static model considered here.[41]

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[39]Strict convexity induces the individual to demand more annuities if $\left(\frac{1}{2} - p\right)A > X$.

[40]For Figure 2b, I assumed $U_a(C) = C^{\frac{1}{4}}$, $U_d(C) = 0.2\ C^{\frac{1}{4}}$, $W_A = W_D = 10$, and $p = \frac{1}{4}$. For these parameters, the household's wealth in case of death is not low enough to encourage the individual to ever enter the losses domain. The individual purchases zero annuities if she has not previously done so $(X = 0)$; she purchases enough to avoid entering the losses domain if her previous annuity income is not high enough (zero net gains); and she purchases more if her previous annuity income is high enough (positive net gains).

[41]The model from this subsection can also be interpreted as representing consumers who readjust their policies annually without taking into account the effect of their current decisions on future gains and losses, as in the myopic loss aversion theory of Benartzi and Thaler (1995). The only difference between our approaches is that, while Benartzi and Thaler argue that investors typically evaluate their portfolios annually and, therefore, assume that reference points for stock returns are reset yearly, I assume that insurance purchases are evaluated relative to the status quo of not having purchased insurance and, therefore, are kept constant.

4 Demand for Life Insurance

This section studies the dynamics of life insurance demand. In order to focus on the dynamic effects introduced by prospect theory, I assume that income is constant. I will, therefore, omit the time subscript from the consumption utility function: $u(N) \equiv U_a(W_a - \mu N) + \mu U_d(W_d + N)$. Moreover, in order to focus on the demand for life insurance, I assume that the marginal utility of consumption when alive is greater than when dead in the absence of any coverage: $U_a'(W_a) > U_d'(W_d)$. This assumption ensures that consumers do not purchase annuities.

Let $\bar{I}$ denote the amount of insurance purchased by an expected utility maximizer:[42]

$$U_a'(w - \mu \bar{I}) = U_d'(\bar{I}).$$

I will refer to $\bar{I}$ as the "efficient" amount of insurance.[43] Notice that the efficient amount $\bar{I}$ is constant over time: I have abstracted from the life cycle component of life insurance demand by assuming a constant income in order to focus on the dynamic effects introduced by prospect theory. Therefore, any departure from the constant insurance policy $\bar{I}$ will be caused by prospect theory only. Section 7 simulates the dynamic life insurance demand under more realistic life cycle assumptions.

A prospect theory consumer chooses I to maximize

$$\int_0^\infty \exp(-\mu t) \left\{ u(I(t)) + \mu \mathcal{V}\left(I(t) - \mu \int_0^t I(s)\,ds \right) \right\} dt.$$

Let $G_I(t) \equiv I(t) - \mu \int_0^t I(s)\,ds$ denote the "net gain" from life insurance payments if the individual dies in period t. Life insurance payments are in the gains domain if $G_I(t) > 0$, in the losses domain if $G_I(t) < 0$, and at the kink if $G_I(t) = 0$.

Let I^{PL} and G_I^{PL} denote the optimal life insurance and net gain paths in the benchmark case of a piecewise linear gain-loss utility. The following proposition establishes the main properties of the optimal life insurance path:

Proposition 4. $I^{PL}(t)$ *is continuous. Moreover, there exist* $t_1^{PL} > 0$ *and* $t_2^{PL} > t_1^{PL}$ *such that:*

    1. $I^{PL}(t) < \bar{I}$ *is decreasing and* $G_I^{PL}(t) > 0$ *when* $t < t_1^{PL}$;

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[42]For brevity, I will omit the qualifier "for almost all t" in the remainder of the paper.

[43]Referring to $\bar{I}$ as efficient implicitly takes expected utility as a normative benchmark and treats the gain-loss utility as a "mistake." As with most behavioral welfare analyses, this is not an uncontroversial assumption, although it seems to be the case in standard judgements about whether individuals buy appropriate amounts of life insurance and annuities.
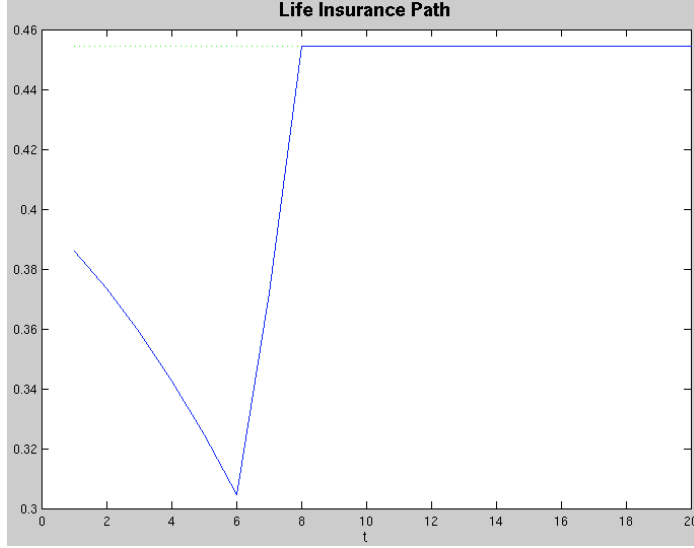
Figure 2: Life Insurance with a piecewise linear gain-loss utility. $U_A(C) = 2\ln C$, $U_D(C) = \ln(C)$, $W_a = 1$, $W_d = 0$, $\lambda = 2.25$, and $\mu = 0.2$.

2. $I^{PL}(t) < \bar{I}$ is increasing and $G_I^{PL}(t) = 0$ when $t_1^{PL} \leq t \leq t_2^{PL}$; and

3. $I^{PL}(t) = \bar{I}$ and $G_I^{PL}(t) < 0$ when $t > t_2^{PL}$.

The proof is presented in the appendix along with the expression for the life insurance path. Here, I discuss the intuition behind it. Time is partitioned in three intervals: the gains domain $[0, t_1)$, the kink (t_1, t_2), and the losses domain $(t_2, +\infty)$. The individual is underinsured in the gains domain and at the kink, and she is efficiently insured in the losses domain (see Figure 2).

Purchasing an instantaneous life insurance policy has two effects: (i) it spreads consumption from the state in which the individual is alive to the state in which she is not (*consumption utility effect*); and (ii) since each policy pays $1 if the consumer dies but adds μdt to the stock of insurance expenditure if she survives, each policy raises the gain in the current period while decreasing it in all future periods (*gain-loss utility effect*).

In the losses domain $(t > t_2)$, the marginal gain-loss utility λ is constant. Since policies are actuarially fair, the law of iterated expectations implies that shifting payments intertemporally does not affect the gain-loss utility, and the second effect vanishes. More precisely, the individual's gain-loss utility for $t > t_2$ is $\int_{t_2}^{+\infty} \exp(-\mu t)\,\mu \left[\lambda\left(I(t) - \mu \int_0^t I(s)\,ds\right)\right] dt$. Applying integration by parts, this expression simplifies to $-\mu\lambda \exp(-\mu t_2) \int_0^{t_2} I(t)\,dt$, which is not a function of $I(t)$ for $t > t_2$. Thus, the *gain-loss utility cancels out*: The consumer maximizes expected consumption utility only, which yields the efficient insurance level in

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this last interval.[44]

In the gains domain $(t < t_1)$, the marginal gain-loss utility is 1. Since the individual may eventually reach the losses domain (if she lives past t_2), where the marginal gain-loss utility is $\lambda > 1$, she underinsures. Moreover, the probability of reaching the losses domain is increasing, inducing her to further reduce the amount of life insurance over time. At $t = t_1$, the individual reaches the kink. At this point, she chooses exactly the amount needed to avoid entering the losses domain. In order to do so, the amount of insurance needs to grow at rate μ.[45]

Next, consider the model with a strictly concave v so that the gain-loss utility is strictly concave in the gains domain and strictly convex in the losses domain. I will make two assumptions that rule out uninteresting cases. First, in order to ensure that consumers will not spend their entire wealth on life insurance in the losses domain, I assume that v is strictly concave, but not "too concave":

Assumption 1. $\frac{u''(I)}{\mu} < \lambda v''(X) < 0$ for all $X, I \in \mathbb{R}$.

Second, to rule out the case where consumers never purchase insurance, I assume that the discounted marginal consumption utility exceeds the expected marginal gain-loss utility at the point of zero coverage:[46]

Assumption 2. $\frac{u'(0)}{\mu} > v'(0)(\lambda - 1)e^{-1}$.

Let I^C denote the optimal life insurance path when v is strictly concave. Strict convexity of the gain-loss utility in the losses domain induces the consumer to demand more insurance than in the piecewise linear case (efficient level). Since the insurance path eventually falls in the losses domain, consumers over-insure if they live for long enough. The following proposition states this result formally:

Proposition 5. *Suppose Assumptions 1 and 2 hold. There exists $t^* > 0$ such that $I^C(t) > \bar{I}$ for all $t > t^*$.*

Therefore, the life insurance path follows the pattern described in Puzzles 2 and 3. Because of loss aversion, consumers initially buy too little life insurance. Moreover, because consumers are risk seeking in the domain of losses, they eventually over-insure.

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[44]If insurance were actuarially unfair, the law of iterated expectations would cancel all terms except for the positive insurance loads. Then, loss aversion induce lower insurance purchases due to the losses from insurance loads. Section 7 presents simulations using positive insurance loads.

[45]Notice that the comparative statics in the losses domain allows us to distinguish between the myopic model in which the individual does not take the effect of current insurance purchases on future reference points (Subsection 3.3) and the model in which she takes this effect into account. In the myopic model, insurance is increasing over time in all regions (see Figure 1 for $\gamma = 1$). When she takes into account the effect of current purchases on future reference points, coverage is decreasing in the losses domain.

[46]By the Inada condition, Assumption 2 is always satisfied if the dependent's wealth W_d is low enough.

5   Demand for Annuities

This section considers the dynamics of annuity demand. As in Section 4, we will consider constant incomes in order to focus on the dynamic effects introduced by prospect theory. We also assume that there is no role for life insurance and an expected utility individual would demand a strictly positive amount of annuities: $U_a'(W_a) > U_d'(W_d)$. At time t, household consumption equals $W_a + \mu A(t)$ if the individual survives and $W_d - A(t)$ if she dies. Let $\bar{A}$ denote the amount of annuities purchased by an expected utility maximizer: $U_a'\left(W_a + \mu \bar{A}\right) = U_d'\left(W_d - \bar{A}\right)$. I will refer to $\bar{A}$ as the *efficient* amount of annuities.

A prospect theory consumer chooses A to maximize

$$\int_0^\infty \exp\left(-\mu t\right) \left\{ U_a\left(W_a + \mu A(t)\right) + \mu \left[U_d\left(W_d - A(t)\right) + \mathcal{V}\left(\mu \int_0^t A(s)\, ds - A(t)\right) \right] \right\} dt.$$

Let $G_A(t) \equiv \mu \int_0^t A(s)\, ds - A(t)$ denote the gain from annuities if the individual dies at t. Annuity payments are: in the gains domain if $G_A(t) > 0$, in the losses domain if $G_A(t) < 0$, and at the kink if $G_A(t) = 0$.

Let A^{PL} and G_A^{PL} denote the optimal annuity and net gain paths in the benchmark case of a piecewise linear gain-loss utility. The following proposition establishes the main properties of the dynamic annuity demand in this benchmark case:

Proposition 6. $A^{PL}(t)$ *is continuous. Moreover, there exist* $t_1^{PL} > 0$ *and* $t_2^{PL} > t_1^{PL}$ *such that:*

1. $A^{PL}(t) < \bar{A}$ is decreasing and $G_A^{PL}(t) < 0$ when $t \in [0, t_1^{PL})$,

2. $A^{PL}(t) < \bar{A}$ is increasing and $G_A^{PL}(t) = 0$ when $t \in (t_1^{PL}, t_2^{PL})$, and

3. $A^{PL}(t) = \bar{A}$ and $G_A^{PL}(t) > 0$ when $t > t_2^{PL}$.

The proof of Proposition 6 is analogous to the one from Proposition 4. The main difference is that, with annuities, the solution starts in the losses domain $(t < t_1)$, reaches the kink $(t_1 < t < t_2)$, and then eventually lies in the gains domain $(t > t_2)$. The consumer under-annuitizes in the losses domain and at the kink, and buys the efficient amount of annuities in the gains domain.

As in the case of life insurance, an annuity has two effects. It spreads consumption from the state in which the individual dies to the one in which she survives ("consumption utility effect"); and it reduces the gain-loss utility by \$1 if the individual dies but increases the stock of annuity payments in all future periods by μdt if she survives ("gain-loss utility effect"). In
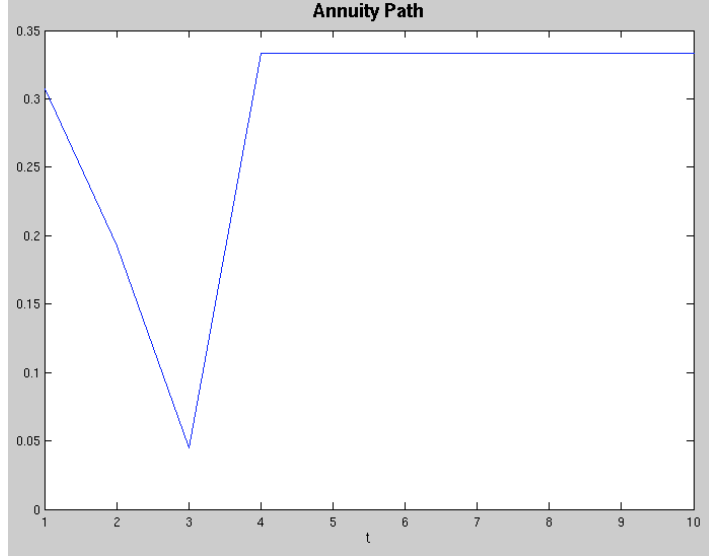
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Figure 3: Annuity path with a piecewise linear gain-loss utility. $U_A(C) = U_D(C) = \ln(C)$, $W_a = 0$, $W_d = 1$, $\lambda = 2.25$, and $\mu = 2$.

the domain of gains, the marginal gain loss utility is constant (equal to 1). Therefore, intertemporally shifting payments does not affect the gain-loss utility. Formally, the gain-loss utility in the gains domain is $\int_{t_2}^{+\infty} \exp(-\mu t)\,\mu\left(\mu \int_0^t A(s)\,ds - A(t)\right)dt$. Applying integration by parts, this term can be simplified to $\mu \exp(-\mu t_2)\int_0^{t_2} A(t)\,dt$, which is not a function of $A(t)$ for $t > t_2$. Thus, the gain-loss utility effect vanishes and the consumer maximizes consumption utility.

In the domain of losses $(t < t_1)$, current marginal gain-loss utility (λ) is higher that expected future marginal gain-loss utility (since there is a positive probability of reaching the gains domain, where it equals $1 < \lambda$). Therefore, the prospect of loosing money on an uncollected annuity is particularly salient in the domain of losses, leading to under-annuitization. Moreover, the chances of reaching the gains domain increases over time, further reducing the amount of annuitization. At $t = t_1$, the consumer reaches the kink, where she chooses exactly the amount needed to avoid entering the domain of losses. In order to do so, annuitization has to grow at rate μ.

As described in Section 2 (Puzzle 4), the vast majority of annuities have guarantee clauses or refund options, which effectively reduce the coverage during the guarantee period. Consistently with the hit-by-a-bus argument, Proposition 7 shows that prospect theory consumers initially purchase small amounts of instantaneous annuities, which limits exposure in the domain of losses. Thus, the model predicts an annuity path with a reduced initial coverage (see Figure 3).

There is an important dynamic difference between annuities and life insurance when v

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is strictly concave. Seen as an investment, life insurance is "profitable" if the consumer does not live for too long: payments start in the gains domain and eventually move into the losses domain. Because the gain-loss utility is convex in the domain of losses, individuals over-insure if they live for long enough. As an investment, an annuity is "profitable" if the consumer lives for long enough: payments start in the losses domain and eventually reach the domain of gains. Because the gain-loss utility is concave in the gains domain, the consumer under-annuitizes even if she lives for long enough. The next proposition states this result formally:

Proposition 7. *Let v be strictly concave. There exist $\bar{t} \geq 0$ such that $A^C(t) < \bar{A}$ for all $t > \bar{t}$.*

Propositions 5 and 7 show how the asymmetry between gains and losses in prospect theory can account for the difference in how people treat annuities and life insurance. Life insurance starts in the gains domain but eventually moves into the losses domain, where people are risk seeking. Therefore, prospect theory predicts that individuals eventually over-purchase life insurance (if they live for long enough). On the other hand, annuity payments start in the losses domain and eventually reach the gains domain. Because individuals are risk averse in the gains domain, they keep under-purchasing annuities, no matter how long they live.[47]

6   Simultaneous Holding of Life Insurance and Annuities

As described previously, a substantial fraction of families that own voluntary annuities *also* own life insurance policies (Puzzle 5). This is puzzling because any portfolio with positive amounts of life insurance and annuities is first-order stochastically dominated by another portfolio with a lower amount of both when either of them is not actuarially fair.

This section shows that, while an expected utility consumer will never simultaneously hold life insurance and annuity policies, a prospect theory consumer may. It builds on the

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[47]There are two effects from prospect theory on the demand for annuities in the gains domain. Loss aversion makes people prefer to buy less annuities (as in Proposition 6), while convexity in the losses domain makes people prefer to buy more annuities. As long as the gain-loss utility is "not too convex" in the losses domain (relative to the loss aversion coefficient λ), people will always prefer to under-purchase annuities. In fact, as I show in Section 7, prospect theory consumers prefer to buy zero annuities in all periods under the usual coefficients from prospect theory and typical distributions of income and mortality. Similarly to life insurance, the comparative statics in the gains domain allows us to distinguish between the myopic model in which the individual does not take the effect of current purchases on future reference points (Subsection 3.3) and the model in which she takes this effect into account. In the myopic model, annuity purchases are increasing over time in all regions (see Figure 1 for $\gamma = 1$). When she takes into account the effect of current purchases on future reference points, coverage is decreasing in the gains domain.

intuition that consumers with strictly convex gain-loss utility functions will be reluctant to terminate their life insurance policies after having spent enough on life insurance. Then, when the value of annuities is sufficiently high, they may purchase annuities but still keep some life insurance. In particular, individuals who have purchased a large amount of life insurance before retiring may choose to keep their life insurance policies after retirement while, at the same time, purchasing annuities.

The model modifies the annuities framework from Section 5 in two ways. First, we need to allow prices to be actuarially unfair to make the simultaneous holding of annuities and life insurance non-trivial. I will assume that annuities and life insurance policies both have a proportional loading factor $l \in (0, \mu)$. Second, I will assume that the individual has already spent an amount $X_I > 0$ on life insurance, but, as in Section 5, life insurance no longer has value in terms of consumption utility: $U_a'(W_a) > U_d'(W_d)$. It is therefore natural to interpret this setting as the post-retirement continuation problem in a life cycle model. The following proposition establishes that the consumer will simultaneously hold annuities and life insurance as long as the loading factor is not too large and the consumer is sufficiently loss averse:

Proposition 8. *Let v be strictly concave with a bounded second derivative v''. There exist $\bar{\lambda}(X_I, v) > 0$ and $\bar{l}(X_I, v) > 0$ such that whenever $\lambda > \bar{\lambda}(X_I, v)$ and $l < \bar{l}(X_I, v)$, the solution entails $A(t) > 0$ for almost all t and $I(t) > 0$ in a set with positive measure.*

Simultaneously purchasing life insurance and annuities reduces consumption uniformly by the insurance loads. For "small" insurance loads, the effect on consumption utility is of lower order relative to the effect on gain-loss utility. Because the consumer has previously spent an amount $X_I > 0$, life insurance payments are in the losses domain for "small" amounts of insurance I. Thus, purchasing an infinitesimal amount of life insurance increases gain-loss utility by an amount proportional to $-\lambda v''(\mu X_I) > 0$. Purchasing more annuities in the domain of gains affects the gain-loss utility by an amount proportional to $v''\left(\mu \int_0^t A(s)\, ds - A(t)\right) < 0$. When λ is large enough, the gain from the life insurance gain-loss utility exceeds the loss from the annuity gain-loss utility. As a result, the consumer simultaneously holds both.[48]

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[48]Because of the dynamic asymmetry between life insurance and annuities, the opposite pattern is never optimal: prospect theory consumers who have previously purchased a large amount of annuities and currently need life insurance do not find it optimal to simultaneously purchase both.

7 Simulations

I now use actual data to simulate the demand for life insurance and annuities using an augmented version of the model studied previously. Recall that Sections 4 and 5 abstracted away from life cycle aspects by considering constant incomes, a memoryless mortality distribution, and no savings. Thus, any deviation from the constant policies that maximize expected utility was unequivocally due to the prospect theory component of preferences. In this section, I introduce these features in the model and show that it generates a dynamic pattern of life insurance and annuity demands that approximates the empirical demands quite closely.

Life Insurance Marketing Research Association (LIMRA), a large trade association representing major life insurers, regularly surveys the U.S. population in order to study trends in life insurance ownership and coverage. Table 1 presents the mean ownership and coverage by age from their last survey, conducted in 2010. In order to contrast the predictions of the model with the life insurance coverage data, I consider a 6-period model, each of them corresponding to an age bracket. In each period $t \in \{1, ..., 6\}$, the head of the household ("the consumer") allocates an income w_t between risk-free assets $s_t \geq 0$, life insurance $I_t \geq 0$, and annuities $A_t \geq 0$. Inflation and the interest rate on risk-free assets are set to zero. At the end of each period, the consumer dies with probability p_t. In case of death, her dependents receive the household's assets s_t, life insurance payments I_t, and a lifetime income from their own labor. In case of survival, the consumer receives income w_t, the household's assets s_t, and annuity payments A_t.

For the income path w_t, I use head of the household mean income data from the 2011 Current Population Survey of the U.S. Census Bureau. For the dependent's income, I use the same path as the head of the household's income w_t with a lag of 30 years (i.e., I assume that the average age between the head of the household and her dependents is 30 years, and that dependents have the same expected income as the household head). Mortality rates are calculated based on the Social Security Administration's 2007 Period Life Table. I assume

Ages	Percent Covered	Mean Coverage
18-24	36	75,600
25-34	54	202,400
35-44	63	231,400
45-55	66	166,700
55-64	67	165,000
65 and older	54	92,300

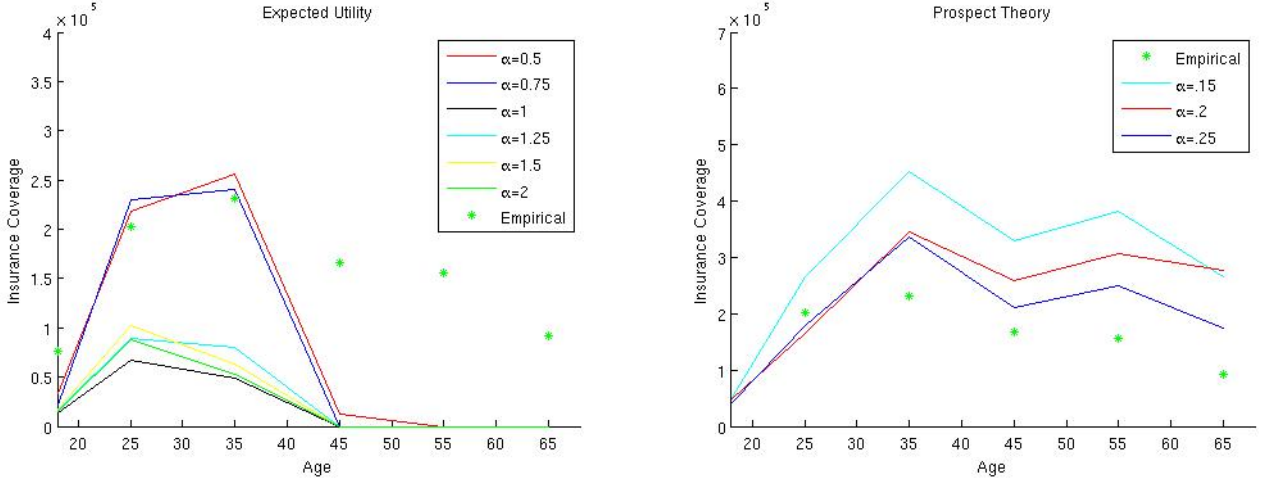Table 1: Life Insurance ownership and coverage by age. Source: LIMRA, 2011.

Figure 4: Life insurance demand under Expected Utility (for $\theta = 1$) and Prospect Theory (for $\theta = .9$) under different parameters α.

a 2% loading factor which, according to Hubener, Maurer, and Rogalla (2013), corresponds to the explicit expense loadings reported by industry leaders. I assume that only individuals who have been married have a bequest motive and use data from the 2008 Survey of Income and Program Participation of the U.S. Census Bureau on the proportion of individuals who have been married.

In order to remain as close as possible to the Macroeconomic literature, I assume a state-dependent CRRA consumption utility specification:

$$u_D(x) = \begin{cases} \frac{x^{1-\theta}-1}{1-\theta} \text{ for } 0 < \theta \neq 1 \\ \ln x \text{ for } \theta = 1 \end{cases}, \quad u_A(x) = \alpha \times u_D(x),$$

where $\alpha > 0$ measures the importance of consumption relative to bequests and θ is the coefficient of relative risk aversion (which equals the inverse of the elasticity of intertemporal substitution). The literature suggests that, for expected utility, θ should lie between 1 and 2, with $\theta = 1$ being the most common assumption. Accordingly, I will take $\theta = 1$ for the expected utility calculations in the main text. In the appendix, I show that the results remain essentially unchanged for other parameter values.

Following Tversky and Kahneman (1992), the loss aversion parameter equals $\lambda = 2.25$. For the gain-loss utility, I use a CARA value function: $v(x) = 1 - \exp(x)$. This is the most common functional form that satisfies our assumptions.[49] Combining a logarithmic consumption utility with a gain-loss utility function generates excessive risk aversion for

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[49]As noted previously, the commonly used CRRA value function is has an infinite slope at zero, which generates excessive loss aversion near zero.
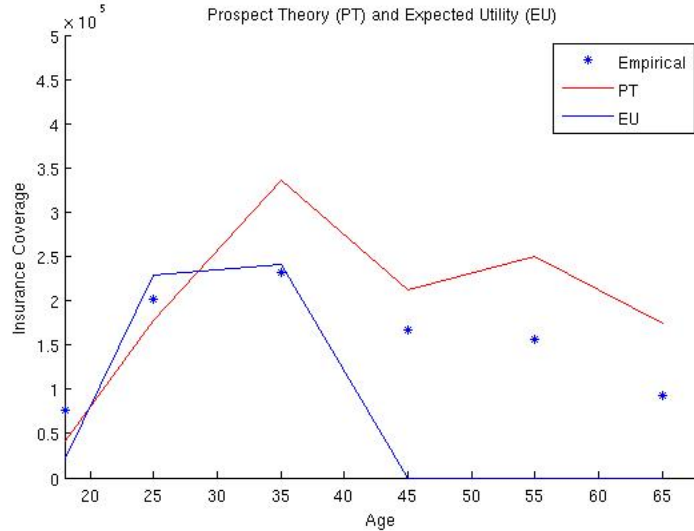
Figure 5: Life insurance demand under Prospect Theory ($\theta = .9$, $\alpha = .25$) and Expected Utility ($\theta = 1$, $\alpha = .75$).

most lotteries. In order to select an appropriate parameter value for θ, I consider a lottery in which one either gains or loses half of one's income with equal probabilities. I then calculate the coefficient under which someone with the average income has the same risk premium over such lottery as in the original formulation of prospect theory with the parameters estimated by Tversky and Kahneman (1992). The resulting coefficient is $\theta \approx 0.9$.

The only free parameter in the model is the importance of consumption relative to bequests α. Interestingly, for any α under which prospect theory consumers buy positive amounts of life insurance, the demand for annuities is zero. This is consistent with the fact that the vast majority of the U.S. population does not voluntarily purchase annuities.

Figure 4 shows the life insurance paths for the range of parameters α that fit the data better. The graph on the left depicts the life insurance demand under expected utility. Consistently with the arguments from the literature, expected utility consumers should be buying significantly more coverage when young and should not be covered by life insurance after age 55. Therefore, our basic model with expected utility generates Puzzles 2 and 3 discussed previously. In the Appendix, I present the same results for other inverse elasticities of intertemporal substitution θ and show that the results remain qualitatively unaffected. The graph on the right shows the life insurance demand when gain-loss utility is also taken into account. The prospect theory model fits the data much better than the expected utility specification. Consistently with the data, simulated demand is increasing until the 35-44 age bracket and is positive in all brackets. Figure 5 contrasts the life insurance demands for the parameters α that better fit the data in each case.

8 The Puzzles Revisited

This section more directly relates the predictions of the model with the puzzles noted in Section 2.

Puzzle 1. Insufficient Annuitization. Prospect theory consumers are reluctant to purchase annuities (Proposition 3). Those who do buy annuities, choose an insufficient amount in the losses domain (Proposition 6). Moreover, when the value function v is concave, they also choose an inefficiently low level of coverage in the gains domain (Proposition 7). Therefore, the model generates under-annuitization. In fact, the simulations from Section 7 generate zero annuitization even with actuarially fair policies.

Puzzle 2. Insufficient Life Insurance Among the Working-Aged. As in the case of annuities, prospect theory consumers are reluctant to purchase life insurance (Proposition 3). Furthermore, the ones who buy insurance initially choose an insufficient amount of coverage (Proposition 5). As Figure 5 shows, the prospect theory model predicts substantially less life insurance purchases than expected utility at earlier ages.

Puzzle 3. Excessive Life Insurance Among the Elderly. Life insurance payments eventually lie in the domain of losses. When the value function v is concave, consumers hold "too much" life insurance later in life (Proposition 5). Thus, the model predicts that individuals will be initially reluctant to purchase life insurance but will later be unwilling to get out of it. As Figure 5 shows, prospect theory generates considerably more life insurance than expected utility at later ages.

Puzzle 4. Guaranteed Annuities. Annuities with guarantee clauses or refund options have a reduced coverage during an initial period. Because of loss aversion, prospect theory consumers demand annuity policies with this feature (Proposition 6).

Puzzle 5. Simultaneous Holding of Life Insurance and Annuities. Since buying a life insurance policy is equivalent to selling an annuity, consumers should not demand both of them at the same time. In contrast, the majority of families that own annuities also have life insurance policies. Because prospect theory consumers who have purchased life insurance are reluctant to get out of it, they demand both life insurance and annuities as long as they are sufficiently loss averse and loads are not too large (Proposition 8).

9 Conclusion

This paper studies an insurance model based on prospect theory. The predictions of the model differ from those of expected and standard non-expected utility models. In particular, decision-makers may prefer to buy zero coverage even when policies are actuarially fair. I

apply the model to a dynamic setting of mortality risk insurance, and show that it can shed light on the five main empirical puzzles from life insurance and annuity markets.

There are several possible directions for future research. It would be interesting to test whether differences in loss aversion can account for these puzzles empirically. While Brown, Kling, Mullainathan, and Wrobel (2008), Gazzale and Walker (2009), and Knoller (2011) present evidence for under-annuitization and the presence of annuity guarantees, the empirical study of the other puzzles remains to be done. Additionally, this paper abstracted from probability weighting for simplicity. Nevertheless, probability weighting may exacerbate the effects obtained here since mortality risk usually involves small probabilities, which are overweighted in prospect theory.[50]

The model can also be applied more broadly. In particular, several recent attempts to introduce new insurance products have consistently encountered remarkably low demands. Karl Case, Robert Shiller, and Allan Weiss, for example, attempted to introduce home equity insurance, which protects homeowners against declines in the prices of their homes. Because residential equity is the single largest component of household wealth for most households and tends to be extremely under-diversified, and because index policies are largely immune from adverse selection and moral hazard concerns, they should be highly valuable. Very few households, however, are willing to pay for them.[51]

A similar pattern is observed in development economics. Cole et al. (2013) document an attempt to introduce a new rainfall insurance for Indian farmers. Although rainfall was widely cited as the most significant source of risk, only 5-10 percent of households accepted to purchase rainfall insurance. Stein and Tobacman (2011) present results from an experiment in which individuals from Gujarat, India are offered rainfall insurance and savings products. They find that individuals prefer either pure savings or pure insurance to mixes between the two. This result is consistent with the model presented here, where loss aversion introduces a kink at zero for both products. Bryan (2010) studies the low adoption of crop insurance in India and limited liability credit insurance in Kenya.

In this paper, I have focused on the demand side, taking available insurance policies as exogenous. In practice, however, policies are determined by insurance firms who are largely aware of consumer biases. Another avenue for future research is to study the supply of insurance policies when consumers behave according to prospect theory. One attempt

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[50]Introducing probability weighting generates dynamic inconsistency, which complicates the analysis. However, it has been shown to generate interesting dynamic effects in other contexts – c.f., Barberis, Huang, and Thaler (2006), Barberis (2012), or Ebert and Strack (2012).

[51]See Shiller (2008), who documents the attempt to commercialize these policies and also mentions that prospect theory may help understand why it failed. Consistently with the model presented here, Kunreuther and Pauly (2012) argue that the tendency to view insurance as an investment may explain the widespread reluctance to purchase flood and earthquake insurance.

in this direction is pursued by Gottlieb and Smetters (2011), who study the supply of life insurance policies in a market where consumers frame their risks narrowly.

Finally, while the model seems to match the evidence for mortality risk insurance and provides implications consistent with other insurance markets, there are some markets that do not fit the predictions of the model. For example, although the model predicts that individuals will be initially reluctant to purchase insurance, there are several markets in which consumers seem to be "too eager" to purchase insurance. Examples include small durable goods insurance (such as cell phones or DVD players), identity theft insurance, and flight insurance.[52] An analysis of which behavioral theories are more relevant to which markets seems to be a particularly fruitful avenue for future research.

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[52]See, for example, Cutler and Zeckhauser (2004) or Kunreuther and Pauly (2005). "Excessive" insurance is consistent with a number of other models, including Koszegi and Rabin (2007) and, more broadly, the class of non-expected utility theories displaying first-order risk aversion. One possible cause for this divergence may be the effect of framing in different insurance markets, which could shift the reference point.

Appendix

Other Simulations

Figure 4 presented the life insurance demands for expected utility under a unit elasticity of substitution ($\theta = 1$) for different parameters of importance of consumption relative to bequests α. Figure 6 depicts the demands under different inverse elasticities of substitutions θ. When consumption is significantly more important than bequests (α is "large enough"), expected utility predicts zero insurance for individuals over 45. When consumption is not very important relative to bequests (α is "small enough"), the model predicts that young individuals would buy much more insurance than what we observe in practice.
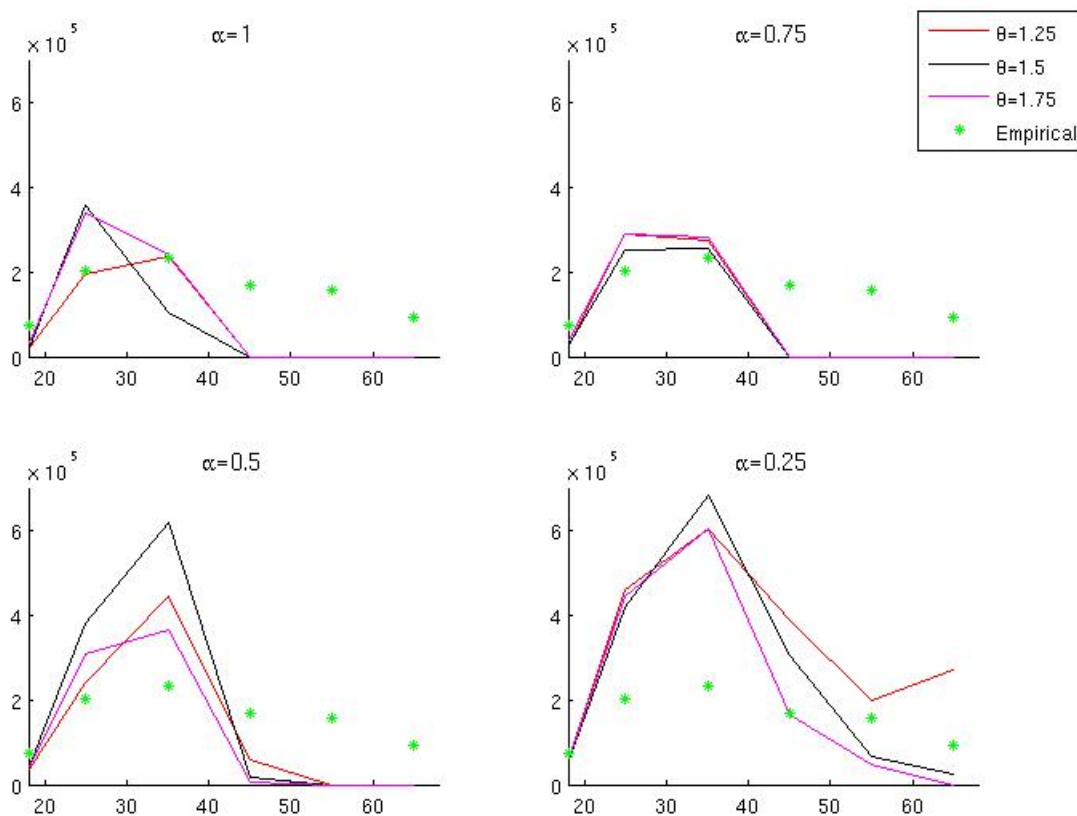

Figure 6: Life insurance demand under Expected Utility.

Figure 7 depicts the demand for life insurance in the prospect theory model for different parameters of importance of consumption relative to bequests α. When consumption is relatively unimportant (α is "small"), the model predicts too much insurance relative to the observed data. When it is sufficiently important (α is "sufficiently large"), the individual buys less insurance but insurance is increasing after a certain age. The best fit is attained for parameters around $\alpha = 0.25$.
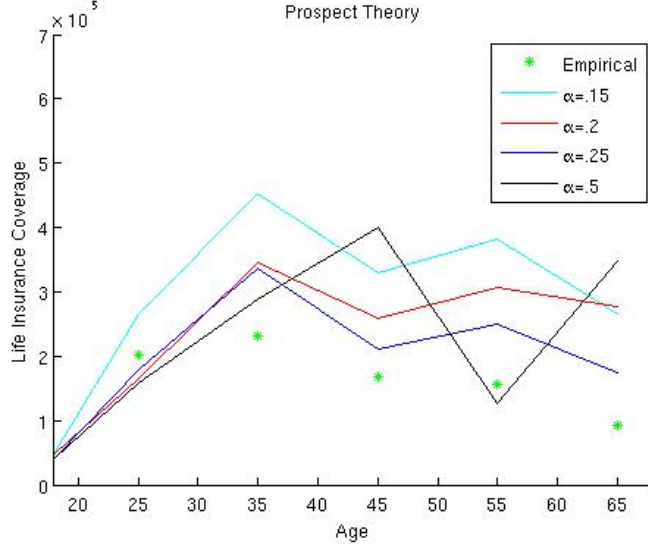
Figure 7: Life insurance demand under Prospect Theory.

Proof of Proposition 3

First, we reformulate the consumer's problem in order to write it as an optimal control program. Note that the piecewise linear gain-loss utility function can be written as $\mathcal{V}(x) = \min\{x, \lambda x\}$. Therefore, the expected gain-loss utility from life insurance equals

$$\int_0^\infty e^{-\mu t} \left[\min\left\{ I(t) - \mu \int_0^t I(s)\, ds,\ \lambda \left(I(t) - \mu \int_0^t I(s)\, ds \right) \right\} \right] dt$$

$$= \int_0^\infty e^{-\mu t} \left[I(t) - \mu \int_0^t I(s)\, ds + \min\left\{ 0,\ (\lambda - 1) \left(I(t) - \mu \int_0^t I(s)\, ds \right) \right\} \right] dt.$$

Integration by parts establishes that $\int_0^\infty e^{-\mu t} \left[I(t) - \mu \int_0^t I(s)\, ds \right] dt = 0$. Thus, the expected gain-loss utility from life insurance is equal to $\int_0^\infty e^{-\mu t} \min\left\{ 0,\ (\lambda - 1) \left(I(t) - \mu \int_0^t I(s)\, ds \right) \right\} dt$. Analogously, the expected gain-loss utility from annuities equals

$$\int_0^\infty e^{-\mu t} \min\left\{ 0,\ (\lambda - 1) \left(\mu \int_0^t A(s)\, ds - A(t) \right) \right\} dt.$$

Let $X_I(t) \equiv \mu \int_0^t I(s)\, ds$ and $X_A(t) \equiv \mu \int_0^t I(s)\, ds$ denote the total expenditure in life insurance and annuities up to time t, and introduce the following auxiliary variables:

$$L_I(t) \equiv \min\{0, I(t) - X_I(t)\}, \tag{6}$$

$$L_A(t) \equiv \min\{0, X_A(t) - A(t)\}. \tag{7}$$

In the remainder of the proof, I will omit the time argument to simplify notation. The consumer's

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program is equivalent to:

$$\max_{I,L_I,X_I,A,L_A,X_A} \int_0^\infty e^{-\mu t} \left[u_t (I - A) + \mu (\lambda - 1) (L_I + L_A) \right] dt$$

subject to (6), (7), $\dot{X}_I = \mu I$, $\dot{X}_A = \mu A$, $X_I (0) = X_A (0) = 0$.

Note also that (6) can be written as $L_I \leq 0$ and $L_I \leq I - X_I$ with one of them holding with equality. Similarly, (7) can be written as $L_A \leq 0$ and $L_A \leq X_A - A$ with one of them holding with equality.

Consider the following program

$$\max_{I,L_I,X_I,A,L_A,X_A} \int_0^\infty e^{-\mu t} \left[u_t (I - A) + \mu (\lambda - 1) (L_I + L_A) \right] dt \tag{8}$$

subject to $L_I \leq 0$, $L_I \leq I - X_I$, $L_A \leq 0$, $L_A \leq X_A - A$, $\dot{X}_I = \mu I$, $\dot{X}_A = \mu A$, and $X_I (0) = X_A (0) = 0$.

Because the objective function is strictly increasing in L_I, we cannot have a solution to this program where neither $L_I \leq 0$ nor $L_I \leq I - X_I$ bind. Analogously, we cannot have a solution where neither $L_A \leq 0$, nor $L_A \leq X_A - A$ bind. Therefore, the solution to program (8) coincides with the solution to the consumer's problem.

The Mangasarian sufficient conditions for a solution to (8) are:

$$e^{-\mu t} u'_t (I - A) + \gamma_I \mu + \dot{\gamma}_I \leq 0 \ \text{ with } = \text{ if } I > 0, \tag{9}$$

$$-e^{-\mu t} u'_t (I - A) + \gamma_A \mu + \dot{\gamma}_A \leq 0 \ \text{ with } = \text{ if } A > 0, \tag{10}$$

$$e^{-\mu t} \mu (\lambda - 1) \geq \dot{\gamma}_I \ \text{ with } = \text{ if } L_I < 0, \tag{11}$$

$$e^{-\mu t} \mu (\lambda - 1) \geq -\dot{\gamma}_A \ \text{ with } = \text{ if } L_A < 0, \tag{12}$$

$$\min \{ \dot{\gamma}_I; \ I - X_I - L_I \} = \min \{ -\dot{\gamma}_A; \ X_A - A - L_A \} = 0, \tag{13}$$

and transversality conditions $\lim_{t \to \infty} \gamma_I \geq 0$, $\lim_{t \to \infty} \gamma_A \geq 0$, and $\lim_{t \to \infty} \gamma_I X_I = \lim_{t \to \infty} \gamma_A X_A = 0$.

Let $I = A = X_I = X_A = L_I = L_A = 0$. Note that conditions (11)-(13) become:

$$e^{-\mu t} \mu (\lambda - 1) \geq \dot{\gamma}_I \geq 0, \tag{14}$$

$$e^{-\mu t} \mu (\lambda - 1) \geq -\dot{\gamma}_A \geq 0. \tag{15}$$

We proceed by verifying that there exist co-state variables γ_I and γ_A that satisfy the sufficient conditions above. Take

$$\gamma_I (t) = e^{-\mu t} \left[-Z - \int_0^t u'_s (0) \, ds \right], \ \ \gamma_A (t) = e^{-\mu t} \left[Z + \int_0^t u'_s (0) \, ds \right],$$

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for a real number Z satisfying

$$\lambda - 1 + \inf_{t \in \mathbb{R}} \left\{ \frac{u_t'(0)}{\mu} - \int_0^t u_s'(0) \, ds \right\} \geq Z \geq \sup_{t \in \mathbb{R}} \left\{ \frac{u_t'(0)}{\mu} - \int_0^t u_s'(0) \, ds \right\}. \tag{16}$$

Such a Z exists because, by assumption,

$$\lambda - 1 \geq \sup_{t \in \mathbb{R}} \left\{ \frac{u_t'(0)}{\mu} - \int_0^t u_s'(0) \, ds \right\} - \inf_{t \in \mathbb{R}} \left\{ \frac{u_t'(0)}{\mu} - \int_0^t u_s'(0) \, ds \right\}.$$

By construction, γ_I and γ_A satisfy (9) and (10) with equality. Moreover, they satisfy the transversality conditions since $\lim_{t \to \infty} e^{-\mu t} \int_0^t u_s'(0) \, ds = 0$ and $\gamma_I X_I = \gamma_A X_A = 0$ for all t. The only conditions that remain to be verified are (14) and (15).

Differentiating γ_I and substituting in (14), we obtain:

$$\lambda - 1 \geq -\frac{u_t'(0)}{\mu} + \int_0^t u_s'(0) \, ds + Z \geq 0.$$

Moreover, differentiating γ_A and substituting in (15) yields exactly the same condition. Adding $\frac{u_t'(0)}{\mu} - \int_0^t u_s'(0) \, ds$ to both sides yields

$$\lambda - 1 + \frac{u_t'(0)}{\mu} - \int_0^t u_s'(0) \, ds \geq Z \geq \frac{u_t'(0)}{\mu} - \int_0^t u_s'(0) \, ds.$$

From the definition of Z (equation 16), this inequality is satisfied for all t. We have therefore verified that conditions (14) and (15) both hold, concluding the proof.

Proof of Proposition 4

Following the same approach as in the proof of Proposition 3, the program can be written as

$$\max_{I,L,X} \int_0^\infty e^{-\mu t} \left[u(I) + \mu (\lambda - 1) L \right] dt \tag{17}$$

subject to $L \leq 0$, $L \leq I - X$, $\dot{X} = \mu I$, and $X(0) = 0$.

The Mangasarian sufficient conditions for a solution of this optimal control problem are:[53]

$$u'(I) + e^{\mu t} (\dot{\gamma} + \mu \gamma) \leq 0 \ \text{ with } = \text{ if } I > 0, \tag{I}$$

$$\mu (\lambda - 1) \geq e^{\mu t} \dot{\gamma} \ \text{ with } = \text{ if } L < 0, \tag{L}$$

$$\min \{ I - X - L, \dot{\gamma} \} = 0 \tag{CS}$$

$$\lim_{t \to \infty} \gamma = 0, \quad \lim_{t \to \infty} \gamma X = 0. \tag{Tr}$$

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[53] CS stands for complementary slackness and Tr stands for transversality.

Let t_1 and $t_2 > t_1$ be a solution to the system:

$$\frac{e^{\mu(t_1-t_2)}\bar{I}}{\mu} = \int_0^{t_1} u'^{-1}\left(e^{\mu(t-t_1)}u'\left(e^{\mu(t_1-t_2)}\bar{I}\right)\right)dt, \quad \text{and} \tag{18}$$

$$\frac{u'\left(e^{\mu(t_1-t_2)}\bar{I}\right)}{\mu} + \int_{t_1}^{t_2} u'\left(e^{\mu(s-t_2)}\bar{I}\right)ds = \lambda - 1. \tag{19}$$

We will establish that such a solution exists later. Take the following candidate for a solution of the program:

$$I(t) = \begin{cases} u'^{-1}\left(e^{\mu(t-t_1)}u'\left(e^{\mu(t_1-t_2)}\bar{I}\right)\right) & \text{if } t < t_1 \\ e^{\mu(t-t_2)}\bar{I} & \text{if } t_1 \le t \le t_2 \\ \bar{I} & \text{if } t > t_2 \end{cases},$$

$$X(t) = \begin{cases} \mu\int_0^t u'^{-1}\left(e^{\mu(t-t_1)}u'\left(e^{\mu(t_1-t_2)}\bar{I}\right)\right) & \text{if } t < t_1 \\ e^{\mu(t-t_2)}\bar{I} & \text{if } t_1 \le t \le t_2 \\ [1+\mu(t-t_2)]\bar{I} & \text{if } t > t_2 \end{cases},$$

$$L(t) = \begin{cases} 0 & \text{if } t \le t_2 \\ -\mu(t-t_2)\bar{I} & \text{if } t > t_2 \end{cases}, \quad \text{and}$$

$$\gamma(t) = \begin{cases} -e^{-\mu t_1}\frac{u'\left(e^{\mu(t_1-t_2)}\bar{I}\right)}{\mu} & \text{if } t < t_1 \\ -e^{-\mu t}\left[\frac{u'\left(e^{\mu(t_1-t_2)}\bar{I}\right)}{\mu} + \int_{t_1}^t u'\left(e^{\mu(s-t_2)}\bar{I}\right)ds\right] & \text{if } t_1 \le t \le t_2 \\ -e^{-\mu t}(\lambda-1) & \text{if } t > t_2 \end{cases}.$$

Note that I is a continuous function. For notational simplicity, let $\bar{\gamma} \equiv -\frac{e^{-\mu t_1}u'\left(e^{\mu(t_1-t_2)}\bar{I}\right)}{\mu} < 0$.

We need to verify that (i) the previous conditions are satisfied; (ii) $\dot{X} = \mu I$; and (iii) the state X and the co-state γ are continuous. We will proceed by a series of lemmata:

Lemma 1. *Condition (I) is satisfied.*

Proof. We will establish that $u'(I) + e^{\mu t}(\dot{\gamma}+\mu\gamma) = 0$ holds for all t. For $t < t_1$, we have

$$u'(I) + e^{\mu t}(\dot{\gamma}+\mu\gamma) = -\mu e^{\mu t}\bar{\gamma} + e^{\mu t}(\mu\bar{\gamma}) = 0.$$

Differentiating γ at $t \in [t_1, t_2]$, yields

$$\dot{\gamma} = -\mu\gamma - e^{-\mu t}u'\left(e^{\mu(t-t_2)}\bar{I}\right) \therefore u'\left(e^{\mu(t-t_2)}\bar{I}\right) + e^{\mu t}(\dot{\gamma}+\mu\gamma) = 0.$$

For $t > t_2$, we have

$$\underbrace{u'(\bar{I})}_{0} + e^{\mu t}(\dot{\gamma}+\mu\gamma) = e^{\mu t}\left(\mu(\lambda-1)e^{-\mu t} - \mu(\lambda-1)e^{-\mu t}\right) = 0.$$

Hence, the condition (I) is satisfied. $\qquad\square$

Lemma 2. *Condition (L) is satisfied.*

Proof. The condition holds with strict inequality at $t < t_1$ (since $\dot{\gamma} = 0$) and with equality at $t > t_2$. For $t \in [t_1, t_2]$, we have $\gamma = e^{\mu(t_1 - t)}\bar{\gamma} - e^{-\mu t} \int_{t_1}^{t} u' \left(e^{\mu(s - t_2)}\bar{I} \right) ds$. Differentiating with respect to time, gives

$$\dot{\gamma} = e^{-\mu t} \left[u' \left(e^{-\mu \Delta}\bar{I} \right) - u' \left(e^{\mu(t - t_2)}\bar{I} \right) \right] + \mu e^{-\mu t} \int_{t_1}^{t} u' \left(e^{\mu(s - t_2)}\bar{I} \right) ds.$$

Substituting in the condition that needs to be verified, $\mu(\lambda - 1) \geq e^{\mu t}\dot{\gamma}$, we obtain:

$$\mu(\lambda - 1) \geq u' \left(e^{\mu(t_1 - t_2)}\bar{I} \right) - u' \left(e^{\mu(t - t_2)}\bar{I} \right) + \mu \int_{t_1}^{t} u' \left(e^{\mu(s - t_2)}\bar{I} \right) ds.$$

The left-hand side of this condition is constant in t, while the right-hand side is increasing since its derivative is $-u'' \left(e^{\mu(t - t_2)}\bar{I} \right) \mu e^{\mu(t - t_2)}\bar{I} + \mu u' \left(e^{\mu(t - t_2)}\bar{I} \right) > 0$. Thus, condition (L) is satisfied if and only if it is satisfied for $t = t_2$. Evaluating it at t_2, yields:

$$\mu(\lambda - 1) \geq u' \left(e^{\mu(t_1 - t_2)}\bar{I} \right) - \underbrace{u'(\bar{I})}_{0} + \mu \int_{t_1}^{t_2} u' \left(e^{\mu(s - t_2)}\bar{I} \right) ds,$$

which is satisfied (with equality) by (19). $\qquad\square$

Lemma 3. *Condition (CS) is satisfied.*

Proof. We need to verify that $\min\{I - X - L, \dot{\gamma}\} = 0$. For $t < t_1$, we have $\dot{\gamma} = 0$, and $L = 0$. Therefore, we need to show that $I - X = u'^{-1} \left(-\mu e^{\mu t}\bar{\gamma} \right) - \mu \int_{0}^{t} u'^{-1} \left(-\mu e^{\mu s}\bar{\gamma} \right) ds \geq 0$. Since this expression is decreasing in t (recall that $\bar{\gamma} < 0$), it suffices to verify that $\lim_{t \nearrow t_1} [I(t) - X(t)] \geq 0$, which is true because both I and X are continuous at t_1 and $I(t_1) = X(t_1)$.

For $t \in [t_1, t_2]$, we have $I - X - L = 0$. Therefore, we need to show that $\dot{\gamma} \geq 0$. Differentiating γ gives

$$\dot{\gamma} = e^{-\mu t} \left[u' \left(e^{-\mu \Delta}\bar{I} \right) - u' \left(e^{\mu(t - t_2)}\bar{I} \right) \right] + \mu e^{-\mu t} \int_{t_1}^{t} u' \left(e^{\mu(s - t_2)}\bar{I} \right) ds.$$

Note that the second term is positive. The first is also positive since

$$e^{\mu t_1} \leq e^{\mu t} \implies e^{\mu(t_1 - t_2)}\bar{I} \leq e^{\mu(t - t_2)}\bar{I} \implies u' \left(e^{-\mu \Delta}\bar{I} \right) \geq u' \left(e^{\mu(t - t_2)}\bar{I} \right), \tag{20}$$

where the first inequality used the fact that $t \geq t_1$. Thus $\dot{\gamma} \geq 0$.

For $t > t_2$, we have $I - X - L = 0$ and $\dot{\gamma} = \mu(\lambda - 1) e^{-\mu t} > 0$, so that $\min\{I - X - L, \dot{\gamma}\} = 0$. $\qquad\square$

It is immediate to verify that the transversality condition (Tr) holds. It also follows directly from inspection that $\dot{X} = \mu I$. Therefore, all that remains to be checked is that the state and the

co-state variables X and γ are continuous. Notice that, by construction, X is continuous at all $t \neq t_1$ and γ is continuous at all $t \neq t_2$. Moreover, by condition (18), X is continuous at t_1 and, by condition (19), γ is continuous at t_2. This establishes that our candidate indeed solves the program.

Before we conclude, let us establish that t_1 and $t_2 > t_1$ implicitly defined as a solution to the nonlinear system of equations (18)-(19) exist. It is simpler to work with the system written in terms of t_1 and $\Delta \equiv t_2 - t_1$. Then, we need to show that the following system:

$$\frac{e^{-\mu\Delta}\bar{I}}{\mu} = \int_0^{t_1} u'^{-1}\left(e^{\mu(t-t_1)}u'\left(e^{-\mu\Delta}\bar{I}\right)\right)dt, \quad \text{and}$$

$$\frac{u'\left(e^{-\mu\Delta}\bar{I}\right)}{\mu} + \int_{t_1}^{t_1+\Delta} u'\left(e^{\mu(s-t_1-\Delta)}\bar{I}\right)ds = \lambda - 1,$$

has a strictly positive solution $(t_1^*, \Delta^*) \in \mathbb{R}_{++}^2$. As the claim below establishes, this is actually a block-diagonal system. Let

$$M\left(x, y\right) \equiv \frac{u'\left(e^{-\mu y}\bar{I}\right)}{\mu} + \int_x^{x+y} u'\left(e^{\mu(s-x-y)}\bar{I}\right)ds - \left(\lambda - 1\right).$$

Claim 1. $M\left(x_0, y\right) = M\left(x_1, y\right)$ for all $x_0, x_1 \in \mathbb{R}$.

Proof. Differentiating M with respect to x, yields

$$\frac{\partial M}{\partial x}\left(x, y\right) = -u'\left(e^{-\mu y}\bar{I}\right) - \int_x^{x+y} u''\left(e^{\mu(s-x-y)}\bar{I}\right)\mu e^{\mu(s-x-y)}\bar{I}ds,$$

where I have used the fact that $u'\left(\bar{I}\right) = 0$. Note, however, that

$$\frac{d}{ds}\left[u'\left(e^{\mu(s-x-y)}\bar{I}\right)\right] = \mu e^{\mu(s-x-y)}\bar{I}u''\left(e^{\mu(s-x-y)}\bar{I}\right).$$

Substituting in the derivative of M and applying the Fundamental Theorem of Calculus, yields

$$\frac{\partial M}{\partial x}\left(x, y\right) = -u'\left(e^{-\mu y}\bar{I}\right) - \underbrace{u'\left(\bar{I}\right)}_{0} + u'\left(e^{-\mu y}\bar{I}\right) = 0.$$

$\square$

Therefore, M is constant in x. In particular, equation (19) can be rewritten as

$$M\left(0, \Delta\right) = \frac{u'\left(e^{-\mu\Delta}\bar{I}\right)}{\mu} + \int_0^{\Delta} u'\left(e^{\mu(s-\Delta)}\bar{I}\right)ds - \left(\lambda - 1\right) = 0.$$

Note that $M\left(0, \cdot\right)$ is a continuous function, $M\left(0, 0\right) = -\left(\lambda - 1\right)$, and $\lim_{\Delta \to \infty} M\left(0, \Delta\right) = +\infty$. Therefore, there exists $\Delta^* > 0$ such that $M\left(0, \Delta^*\right) = 0$.

Let $L(t, \Delta) \equiv \frac{e^{-\mu\Delta\bar{I}}}{\mu} - \int_0^{t_1} u'^{-1}\left(e^{\mu(t-t_1)}u'\left(e^{-\mu\Delta}\bar{I}\right)\right) dt$. We need to show that there exists t_1^* such that $L(t_1^*, \Delta^*) = 0$. Note that $L(\cdot, \Delta^*)$ is a continuous function, $L(0, \Delta^*) = \frac{e^{-\mu\Delta^*\bar{I}}}{\mu} > 0$, and $\lim_{t_1 \to \infty} L(t_1, \Delta) = -\infty$. Therefore, there exists such a $t_1^* > 0$. It thus follows that there exist $t_1^* > 0$ and $t_2^* \equiv t_1^* + \Delta > t_1^*$ that solve the system (18)-(19).

Proof of Proposition 5

The proof will follow a sequence of lemmata. To reduce notation, let us renormalize the value function: $w(x) \equiv \begin{cases} \mu v(x) & \text{if } x \geq 0 \\ \lambda\mu v(x) & \text{if } x < 0 \end{cases}$, and let $X(t) \equiv \mu \int_0^t I(s) ds$ denote the total amount spent on insurance up to time t. Our first lemma establishes that $I(t) = 0$ for almost all t is not optimal:

Lemma 4. *Let I be an optimal insurance path. Then, the set $\{t \in \mathbb{R} : I(t) > 0\}$ has positive measure.*

Proof. If $I = 0$ for almost all t, the consumer's total utility equals $\int_0^\infty e^{-\mu t} u(0) dt = \frac{u(0)}{\mu}$. Suppose the consumer buys a constant amount $I(t) = \epsilon$ for all t. Then, her total utility becomes

$$\frac{u(\epsilon)}{\mu} + \int_0^\infty e^{-\mu t} \mu \mathcal{V}(\epsilon(1 - \mu t)) dt.$$

Under this constant profile, she is in the losses domain if $t > \frac{1}{\mu}$ and in the gains domain if $t < \frac{1}{\mu}$. Substituting the value functions in each of these domains, we obtain:

$$\frac{u(\epsilon)}{\mu} + \int_0^{\frac{1}{\mu}} e^{-\mu t} \mu v(\epsilon(1 - \mu t)) dt - \lambda \int_{\frac{1}{\mu}}^\infty e^{-\mu t} \mu v(\epsilon(\mu t - 1)) dt.$$

It suffices to show that the derivative of the total utility with respect to ϵ evaluated at $\epsilon = 0$ is strictly positive. Computing that derivative, yields

$$\frac{u'(0)}{\mu} + v'(0)\left[\int_0^{\frac{1}{\mu}} (1 - \mu t) e^{-\mu t} \mu dt + \lambda \int_{\frac{1}{\mu}}^\infty (1 - \mu t) e^{-\mu t} \mu dt\right],$$

which, after solving the integrals, becomes $\frac{u'(0)}{\mu} - v'(0)(\lambda - 1) e^{-1}$. Thus, deviating to $I(t) = \epsilon$ increases the consumer's total utility for ϵ small enough if $\frac{u'(0)}{\mu} > v'(0)(\lambda - 1) e^{-1}$. $\square$

The next lemma obtains necessary conditions associated with the optimal insurance path.

Lemma 5. *Let I be an optimal insurance path. Then,*

$$\dot{I} = \mu \frac{u'(I) + w''(I - \mu X) I}{u''(I) + w''(I - \mu X)}. \tag{21}$$

if $I > \mu X$,

$$\dot{I} = \mu \frac{u'(I) - w''(\mu X - I) I}{u''(I) - w''(\mu X - I)} \tag{22}$$

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if $I < \mu X$, and $\dot{I} = \mu I$ if $I = \mu X$.

Equations (21) and (22) are the Euler-Lagrange equations associated with the optimal insurance path in the gains and losses domains, respectively.

Proof. The optimal insurance path solves

$$\max_{\dot{X},X} \int_0^\infty \exp(-\mu t) \left[u\left(\dot{X}\right) + \mu \mathcal{V}\left(\dot{X} - \mu X\right) \right] dt.$$

The proof follows by applying a standard calculus of variations.

Let us consider the losses domain first. Let X be an optimal path, and let $[a,b]$ be an interval in the losses domain: $\dot{X}(t) < \mu X(t)$ for all $t \in [a,b]$. Consider the following perturbed path: $\tilde{X}_\epsilon \equiv X + \epsilon h$, for a smooth function $h : \mathbb{R}_+ \to \mathbb{R}_+$ such that $h(t) = 0$ for all $t \in [0,a] \bigcup [b,+\infty)$. The objective function evaluated at the perturbed path equals

$$W_\epsilon := \int_0^\infty \exp(-\mu t) \left[u\left(\dot{X} + \epsilon \dot{h}\right) - w\left(\mu(X + \epsilon h) - \dot{X} - \epsilon \dot{h}\right) \right] dt.$$

Taking the derivative with respect to ϵ and evaluating at $\epsilon = 0$ yields

$$\left| \frac{\partial W_\epsilon}{\partial \epsilon} \right|_{\epsilon=0} = \int_a^b \exp(-\mu t) \left\{ \left[u'\left(\dot{X}\right) + w'\left(\mu X - \dot{X}\right) \right] \dot{h} - w'\left(\mu X - \dot{X}\right) \mu h \right\} dt.$$

Applying integration by parts (and using the fact that $h(a) = h(b) = 0$), we can rewrite this condition as

$$\left| \frac{\partial W_\epsilon}{\partial \epsilon} \right|_{\epsilon=0} = -\int_a^b \exp(-\mu t) \left\{ u''\left(\dot{X}\right) \ddot{X} + w''\left(\mu X - \dot{X}\right) \left(\mu \dot{X} - \ddot{X}\right) - \mu u'\left(\dot{X}\right) \right\} h \, dt.$$

The Fundamental Lemma of Calculus of Variations then yields:

$$u''(I)\dot{I} + w''(\mu X - I)\left(\mu I - \dot{I}\right) - \mu u'(I) = 0.$$

Rearranging this condition establishes the Euler-Lagrange condition:

$$\dot{I} = \mu \frac{u'(I) - w''(\mu X - I)I}{u''(I) - w''(\mu X - I)}.$$

The condition for the gains domain is analogous. The kink is defined as points for which $I = \mu X$. Thus, any interval of points at the kink must satisfy $\dot{I} = \mu I$, which establishes the last condition. $\square$

Lemma 6. *There exists t_0 such that $\mu X > I$ for all $t > t_0$.*

Proof. Since X is increasing ($\dot{X} = I \geq 0$), there are two possibilities: either $\lim_{t \to \infty} X(t) = X^*$ for some $X^* \in \mathbb{R}$, or $\lim_{t \to \infty} X(t) = +\infty$.

First, suppose $\lim_{t\to\infty} X(t) = X^*$ for some $X^* \in \mathbb{R}$. Then, it must be the case that $\lim_{t\to\infty} I(t) = 0$. Since $I(t) > 0$ in a set of non-zero measure (by the previous lemma), it follows that $X^* > 0$. Thus, we must be in the losses domain for t large enough:

$$\lim_{t\to\infty} I - \mu X = -\mu X^* < 0.$$

Now, suppose that $\lim_{t\to\infty} X(t) = +\infty$. Then, since $I \leq \frac{W}{\mu}$ for all t, we must have

$$\lim_{t\to\infty} I - \mu X = -\infty < 0.$$

$\square$

Lemma 7. $\lim_{t\to\infty} X(t) = +\infty$.

Proof. As in Lemma 6, note that because X is increasing, it must be the case that either $\lim_{t\to\infty} X(t) = +\infty$, or $\lim_{t\to\infty} X(t) = X^*$ for some $X^* \in \mathbb{R}$. Suppose, in order to obtain a contradiction, that $\lim_{t\to\infty} X(t) = X^* < \infty$. Then, it must be the case that $\lim_{t\to\infty} I(t) = 0$. Moreover, by Lemma 6, there exists $\bar{t}$ such that we are in the losses domain whenever $t > \bar{t}$. Then, it must satisfy the Euler-Lagrange condition on equation (22):

$$\dot{I} = \mu \frac{u'(I) - w''(\mu X - I)I}{u''(I) - w''(\mu X - I)} \quad \text{for } t > \bar{t}.$$

However, note that

$$\lim_{t\to\infty} \frac{u'(I) - w''(\mu X - I)I}{u''(I) - w''(\mu X - I)} = \frac{u'(0)}{u''(0) - w''(\mu X^*)} > 0,$$

which is strictly positive since $u'(0) > 0$ and, by Assumption 1, $u''(0) < w''(\mu X^*)$. Thus, $\lim_{t\to\infty} \dot{I}(t) > 0$, which contradicts $\lim_{t\to\infty} I(t) = 0$. $\square$

We are now ready to establish the main result:

Lemma 8. *There exists $\bar{t}$ such that $I > \bar{I}$ for all $t > \bar{t}$.*

Proof. From Lemma 6, we can without loss of generality suppose that $\bar{t}$ is in the losses domain. In order to obtain a contradiction, let $\hat{t} > \bar{t}$ be such that $I(\hat{t}) \leq \bar{I}$ (that is, $u'(I(\hat{t})) \geq 0$). Then, equation (22) yields

$$\dot{I}(\hat{t}) = \mu \frac{\overbrace{u'(I(\hat{t})) - w''(\mu X(\hat{t}) - I(\hat{t}))I(\hat{t})}^{+}}{\underbrace{u''(I(\hat{t})) - w''(\mu X(\hat{t}) - I(\hat{t}))}_{-}} < 0,$$

so that $I(t) \leq \bar{I}$ is strictly decreasing for all $t > \hat{t}$.

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Since I is bounded below ($I(t) \geq 0$) and decreasing, it converges: There exists $I^* \geq 0$ such that $\lim_{t\to\infty} I(t) = I^*$. Thus, it must be the case that $\lim_{t\to\infty} \dot{I} = 0$, and, by equation (22),

$$\lim_{t\to\infty} \frac{u'(I) - w''(\mu X - I)I}{u''(I) - w''(\mu X - I)} = 0. \tag{23}$$

From Assumption 1, it follows that for all $\tilde{I}, \tilde{X} \in \mathbb{R}$,

$$u''\left(\tilde{I}\right) < u''\left(\tilde{I}\right) - w''\left(\tilde{X}\right) < 0.$$

Moreover, $u''(I)$ is bounded below in the interval $I \in [0, \bar{I}]$.[54] Thus, the denominator in (23) is bounded and we must have

$$\lim_{t\to\infty} \left[u'(I) - w''(\mu X - I)I\right] = 0.$$

Since $\lim_{t\to\infty} u'(I) = u'(I^*)$, it then follows that

$$\lim_{t\to\infty} w''(\mu X - I)I = u'(I^*). \tag{24}$$

There are two possible cases: $I^* = 0$, and $I^* > 0$.

First, consider $I^* = 0$. Then, because w'' is bounded (Assumption 1), it follows that

$$u'(I^*) = \lim_{t\to\infty} w''(\mu X - I)I = 0,$$

implying that $I^* = \bar{I} > 0$, which contradicts the assumption that $I^* = 0$.

Next, suppose $I^* > 0$. Then, we must have $u'(I^*) = I^* \times \lim_{t\to\infty} w''(\mu X - I)$. Since $w''\left(\tilde{X}\right) < 0$ for all $\tilde{X}$, it follows that $\lim_{t\to\infty} w''(\mu X - I) \leq 0$. Thus, $u'(I^*) \leq 0 \therefore I^* \geq \bar{I}$, which contradicts the fact that $I(t)$ is strictly decreasing and $I(t) < \bar{I}$ for all $t > \hat{t}$. $\square$

Proof of Proposition 6

The proof follows the same exact steps as Proposition 4 and is, therefore, omitted.

Proof of Proposition 7

To simplify notation, let $\tilde{u}(A) \equiv U_a(W_a + \mu A) + \mu U_d(W_d - A)$ denote the instantaneous consumption utility from an instantaneous annuity purchase A, and let $X(t) \equiv \mu \int_0^t A(s)ds$ denote the total amount spent on annuities up to time t (so that $\dot{X} = A$).

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[54]Note that $u''(I) = \mu^2 U_a''(W_a - \mu I) + \mu U_d''(W_d + I)$. Since $\bar{I} < \frac{W_a}{\mu}$ (interior efficient coverage), and $U_d''(W_d) > -\infty$ (because $W_d > 0$ and U is twice continuously differentiable), it follows that $u''(I) > -\infty$ for all I.

The optimal annuity path solves:

$$\max_{X,\dot{X}} \int_0^\infty e^{-\mu t} \left[\tilde{u}\left(\dot{X}\right) + \mu \mathcal{V}\left(\mu X - \dot{X}\right) \right] dt.$$

The result is immediate if the solution is at the boundary: $\dot{X} = 0$ for (almost) all t. Therefore, suppose the optimal annuity path is interior $(A(t) > 0)$ in some time interval. Proceeding as in the proof of Proposition 5, the necessary Euler-Lagrange condition for an optimal annuity path in the gains domain $(\mu X > A)$ is

$$\dot{A} = \mu \frac{\tilde{u}'(A) + \mu v''(\mu X - A) A}{\tilde{u}''(A) + \mu v''(\mu X - A)},$$

for all t where A is interior. We will use the following lemma:

Lemma 9. *There exists t_0 such that $\mu X > A$ for all $t > t_0$.*

Proof. Since X is increasing $(\dot{X} = A \geq 0)$, there are two possibilities: either $\lim_{t\to\infty} X(t) = X^*$ for some $X^* \in \mathbb{R}$, or $\lim_{t\to\infty} X(t) = +\infty$.

First, suppose $\lim_{t\to\infty} X(t) = X^*$ for some $X^* > 0$ (it has to be positive since $X(t) > 0$ for some $t - A(t) > 0$ in a set with positive measure – and X is non-decreasing). Then, it must be the case that $\lim_{t\to\infty} A(t) = 0 < \mu X^*$, which establishes that we must be in the gains domain for t large enough.

Next, suppose that $\lim_{t\to\infty} X(t) = +\infty$. Then, since $A \leq W_d$ for all t, we must have $\lim_{t\to\infty} \mu X - A = \infty > 0$. $\square$

We are now ready to establish the main result:

Proof of the Proposition. Let t_0 be as defined by the lemma and take $t \geq t_0$. Suppose, in order to obtain a contradiction, that $A(t) \geq \bar{A}$ (so that $\tilde{u}'(A) < 0$). Then, by the Euler-Lagrange equation,

$$\dot{A} = \mu \frac{\tilde{u}'(A) + \mu v''(\mu X - A) A}{\tilde{u}''(A) + \mu v''(\mu X - A)} > 0,$$

which gives $A(t) > \bar{A}$ for all $t > t_0$. Since A is bounded (by W_d), it must be the case that A converges:

$$\lim_{t\to\infty} A = A^* > \bar{A}.$$

Thus, we must have $\dot{A} \to 0$. Since $\tilde{u}''(A) + \mu v''(\mu X - A)$ is a bounded function of A (follows from the continuity of U_a and U_d, and the assumption that W_d and W_a are both strictly positive), we have

$$\lim_{t\to\infty} \tilde{u}'(A) + \mu v''(\mu X - A) A = 0.$$

But this requires

$$\tilde{u}'(A^*) = - \lim_{t\to\infty} \mu v''(\mu X - A) A \geq 0,$$

where the inequality follows from $v''(x) < 0$ for all x. But this contradicts $A(t) > \bar{A}$ for all t.

Proof of Proposition 8

An instantaneous annuity policy pays $(\mu - l)\,dt$ in case of survival and costs \$1 in case of death; an instantaneous life insurance policy costs $(\mu + l)\,dt$ and pays \$1 in case of death. Consumption in each state equals $C_a(t) = W_a(t) + \mu\left[A(t) - I(t)\right] - l\left[A(t) + I(t)\right]$, and $C_d(t) = W_d(t) - \left[A(t) - I(t)\right]$. Notice that any path in which the set $\{t : I(t) > 0,\ A(t) > 0\}$ has positive measure is first-order stochastically dominated – reducing both $A(t)$ and $I(t)$ leaves $C_d(t)$ unchaged but raises $C_a(t)$ by $2l$.

For notational simplicity, assume that the load equals zero $l = 0$; the result for positive loads will follow from the continuity of the total utility function in l. Let A and I denote a solution to the maximization of expected total utility. In order to obtain a contradiction, suppose that $I(t) = 0$ for almost all t.

By the Lemma 9, there exists t_2^C such that $\mu \int_0^t A(s)ds > A(t)$ for all $t > t_2^C$ (i.e., $t > t_2^C$ lies in the gains domain). Fix $\bar{t} > t_2^C$. Because $\mu \int_0^t A(s)\,ds$ is a strictly increasing function of t, there exists $\bar{\delta} > 0$ such that $\mu \int_0^{\bar{t}} A(s)ds > A(t) + \delta$ for all $\delta \in (0, \bar{\delta})$. Consider a uniform increase in $A(t)$ and $I(t)$ for all $t > \bar{t}$ by an amount $\delta \in (0, \bar{\delta})$. Consumption in each state remains unchanged. The gain-loss utility conditional on dying before time $\bar{t}$ also remains unchanged.

The gain-loss utility from life insurance conditional on dying at $t \geq \bar{t}$ is $\mathcal{V}\left(\delta - \mu\left[X_I + \delta(t - \bar{t})\right]\right)$. Note that all subsequent life insurance payments remain in the losses domain whenever $\delta < \frac{\mu X_I}{1 + \bar{t}\mu}$. Therefore, for δ small enough, this expression equals $-\lambda v\left(\mu X + \delta\left[\mu(t - \bar{t}) - 1\right]\right)$.

The gain-loss utility from annuities conditional on dying at $t \geq \bar{t}$ is

$$\mathcal{V}\left(\mu \int_0^t A(s)\,ds + \mu(t - \bar{t})\delta - A(t) - \delta\right) = v\left(\mu \int_0^t A(s)\,ds - A(t) + \delta\left[\mu(t - \bar{t}) - 1\right]\right),$$

where I used the fact that, by construction, t lies in the gains domain. Therefore, the change in expected total utility equals:

$$\int_{\bar{t}}^\infty \exp(-\mu t)\,\mu \left\{ \begin{array}{c} \left[v\left(\mu \int_0^t A(s)\,ds - A(t) + \left[\mu(t - \bar{t}) - 1\right]\delta\right) - v\left(\mu \int_0^t A(s)\,ds - A(t)\right)\right] \\ -\lambda\left[v\left(\mu X_I + \left[\mu(t - \bar{t}) - 1\right]\delta\right) - v\left(\mu X_I\right)\right] \end{array} \right\} dt.$$

Applying a second-order Taylor expansion, we obtain that this term is positive if the following condition holds:

$$\int_{\bar{t}}^\infty \exp(-\mu t)\,\mu\left[\mu(t - \bar{t}) - 1\right]^2 \left\{ v''\left(\mu \int_0^t A(s)\,ds - A(t)\right) - \lambda v''\left(\mu X_I\right) \right\} > 0.$$

Because $\exp(-\mu t)\,\mu\left[\mu(t - \bar{t}) - 1\right]^2 > 0$ for all $t > \bar{t}$, a sufficient condition is

$$v''\left(\mu \int_0^t A(s)\,ds - A(t)\right) > \lambda v''\left(\mu X_I\right).$$

Let $\bar{v} = \sup\{|v''(x)|\}$ (which exists because v'' is bounded). Setting $\lambda(X_I, v) \equiv \frac{\bar{v}}{v''(\mu X_I)}$ concludes the proof.

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Hirshleifer-Yaari Diagram (for online appendix)

For comparative purposes, it is helpful to represent preferences in the classic Hirshleifer-Yaari diagram. Let $C_L = W - L + (1 - p - l) I$ and $C_{NL} = W - (p + l) I$ denote consumption in the states of loss and no loss. Total utility can be written in terms of state-dependent consumption and the point of zero insurance $(W - L, W)$ as

$$pU(C_L) + (1 - p) U(C_{NL}) + pV(C_L - (W - L)) + (1 - p) V(C_{NL} - W).$$

Thus, under consumption utility only (expected utility), indifference curves are smooth, with slope equal to $-\frac{p}{1-p} \frac{U'(C_L)}{U'(C_{NL})}$. In particular, the marginal rate of substitution at certainty (45 degree line) equals the odds ratio: $-\frac{p}{1-p}$.
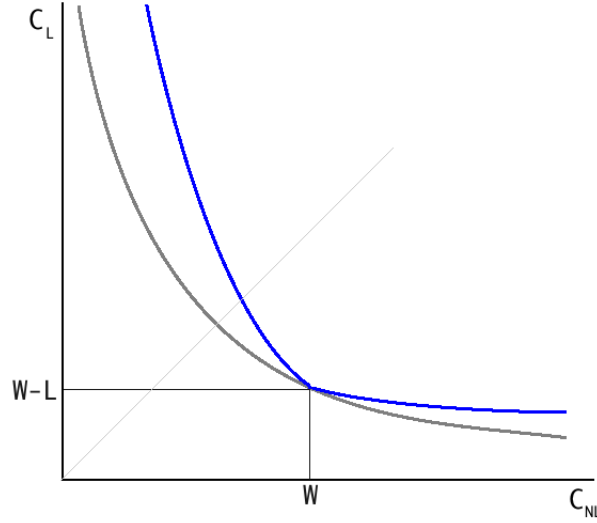

Figure 8: Hirshleifer-Yaari diagram. Indifference curves from consumption utility (smooth curve) and total utility (kink at zero insurance).

Because of reference dependence, the same consumption profile is ranked differently if it is the consumer's original endowment or if it is obtained by acquiring insurance. Thus, an individual's preferences over consumption profiles depends on the point of zero insurance. Indifference curves in the model with narrow framing and loss aversion have slope

$$\frac{dC_{NL}}{dC_L} = -\frac{p}{1-p} \frac{U'(C_L) + V'(C_L - (W - L))}{U'(C_{NL}) + V'(C_{NL} - W)}.$$

The slope converges to $-\frac{p}{1-p} \frac{U'(W-L)+v'(0)}{U'(W)+\lambda v'(0)}$ as $I \searrow 0$, and $-\frac{p}{1-p} \frac{U'(W-L)+\lambda v'(0)}{U'(W)+v'(0)}$ as $I \nearrow 0$. Thus, loss aversion introduces a kink at the point of *zero insurance*.

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While indifference curves from standard non-expected utility models with first-order risk aversion also have a kink, the kink is usually at the point of full insurance (45 degree line).[55] Therefore, they predict that consumers will over-insure relative to expected utility. In particular, they may fully insure even if prices are not actuarially fair. Here, because the kink occurs at the point of zero coverage, consumers are reluctant to buy any coverage.

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[55]This class of models also includes prospect theory without narrow framing.