
David Besanko† Ulrich Doraszelski‡ Yaroslav Kryukov§

October 15, 2013

Abstract

To detect the presence of predatory pricing, antitrust authorities routinely ask whether a firm sacrifices current profit in exchange for the expectation of higher future profit following the exit of its rival. Because predatory pricing is an inherently dynamic phenomenon, we show in this paper how to construct sacrifice tests for predatory pricing in a modern industry-dynamics framework along the lines of Ericson & Pakes (1995). In particular, we adapt the definitions of predation due to Ordover & Willig (1981) and Cabral & Riordan (1997) to this setting and construct the corresponding sacrifice tests.

*Most computations have been done on the Wharton School Grid and we are indebted to Hugh MacMullan for technical support.
†Kellogg School of Management, Northwestern University, Evanston, IL 60208, d-besanko@kellogg.northwestern.edu.
‡Wharton School, University of Pennsylvania, Philadelphia, PA 19104, doraszelski@wharton.upenn.edu.
§Tepper School of Business, Carnegie Mellon University, Pittsburgh, PA 15213, kryukov@cmu.edu.
1 Introduction

To detect the presence of predatory pricing, antitrust authorities routinely ask whether a
firm sacrifices current profit in exchange for the expectation of higher future profit following
the exit of its rival. Because predatory pricing is an inherently dynamic phenomenon, we
show in this paper how to construct sacrifice tests for predatory pricing in a modern industry-
dynamics framework that endogenizes competitive advantage and industry structure. Due
to its presence in a number of high-profile predatory pricing cases, we focus on learning-by-
doing.

At the core of predatory pricing is the trade-off between lower profit in the short run
due to aggressive pricing and higher profit in the long run due to reduced competition. De-
termining what constitutes an illegitimate profit sacrifice—and thus predatory pricing—is
especially difficult when firms face other intertemporal trade-offs such as learning-by-doing,
network effects, or switching costs that can give rise to aggressive pricing with subsequent
recoupment. As Farrell & Katz (2005) point out, “[d]istinguishing competition from pre-
dation is even harder in network markets than in others. With intertemporal increasing
returns, there may innocently be intense initial competition as firms fight to make initial
sales and benefit from the increasing returns” (p. 204). Yet, allegations of predation (or, in
an international context, dumping) sometimes arise in settings where learning-by-doing is a
key feature of the industrial landscape. Examples include the “semiconductor wars” between
the U.S. and Japan during the 1970s and 1980s (Flamm 1993, Flamm 1996, Dick 1991), the
allegations by U.S. color television producers against Japanese producers during the 1960s
and 1970s that are at the core of the Matsushita Electric Corp. predatory pricing case
(Yamamura & Vandenberg 1986), and most recently the debate about Chinese solar panels.

In these and many other industries, a firm has an incentive to price aggressively because
its marginal cost of production decreases with its cumulative experience.\(^1\) While this makes
it difficult to disentangle predatory pricing from mere competition for efficiency on a learn-
ing curve, being able to do so is crucial when predation is alleged. In practice, antitrust
authorities find a price predatory if there is evidence of an illegitimate profit sacrifice. This,
in turn, requires a notion of what constitutes an illegitimate profit sacrifice in the first place.

In this paper, we show how the definitions of predation due to Ordover & Willig (1981)
and Cabral & Riordan (1997) can be used to determine what constitutes an illegitimate profit
sacrifice. In contrast to antitrust authorities, the economics literature focuses more directly
on the impact that a price cut has on reshaping the structure of an industry. According to
the definitions of predation due to Ordover & Willig (1981) and Cabral & Riordan (1997), a
price is predatory if it had not been worth charging absent its impact on the probability that

\(^1\)See footnote 2 in Besanko, Doraszelski, Kryukov & Satterthwaite (2010) for references to the empirical
literature on learning-by-doing.
the rival exits the industry. While the idea that predatory pricing can be usefully defined
by a “but-for” scenario has greatly influenced economists’ thinking, to our knowledge it has
rarely been formalized outside simple models such as the one in Cabral & Riordan (1997).2
In this paper, we show how to adapt the definitions of predation due to Ordover & Willig
(1981) and Cabral & Riordan (1997) to an industry-dynamics framework along the lines of
Ericson & Pakes (1995). We then show how to construct sacrifice tests from these definitions.
The economic definitions of predation in the extant literature therefore amount to particular
ways of disentangling an illegitimate profit sacrifice stemming from predatory pricing from
a legitimate effort to increase cost efficiency through aggressive pricing.

To construct sacrifice tests in a dynamic pricing model similar to the models of learning-
by-doing in Cabral & Riordan (1994) and Besanko et al. (2010), we build on Besanko,
Doraszelski & Kryukov (2013) and decompose the equilibrium pricing condition. The insight
in that paper is that the price set by a firm reflects two goals besides short-run profit. First,
by pricing aggressively, the firm may move further down its learning curve and improve its
competitive position in the future, giving rise to an advantage-building motive. Second, the
firm may prevent its rival from moving further down its learning curve and becoming a more
formidable competitor, giving rise to an advantage-denying motive.

To isolate the probability of rival exit—the linchpin of the definitions of predation due
to Ordover & Willig (1981) and Cabral & Riordan (1997)—we go beyond Besanko et al.
(2013) and decompose the equilibrium pricing condition with even more granularity. One
component of the advantage-building motive is the advantage-building/exit motive. This is
the marginal benefit to the firm from the increase in the probability of rival exit that results
if the firm moves further down its learning curve. The advantage-denying/exit motive is
analogously the marginal benefit from preventing the decrease in the probability of rival
exit that results if the rival moves further down its learning curve. Other terms in the
decomposed equilibrium pricing condition capture the impact of the firm’s pricing decision
on its competitive position, its rival’s competitive position, and so on.

Our decomposition highlights the various incentives that a firm faces when it decides
on a price. Some of these incentives may be judged to be predatory while others reflect
the pursuit of efficiency. In this way, our decomposition mirrors the common practice of
antitrust authorities to question the intent behind a business strategy.

We establish formally that certain terms in our decomposition map into the definitions
of predation due to Ordover & Willig (1981) and Cabral & Riordan (1997). At the same time,
however, our decomposition makes clear that there is much latitude in where exactly to draw
the line between predatory pricing and mere competition for efficiency on a learning curve.

---

2Edlin (2002) provides a comprehensive overview of the current law on predatory pricing. Bolton, Brodley
& Riordan (2000) and Edlin (2010) provide excellent reviews of the theoretical and empirical literature.
Indeed, our decomposition lends itself to developing multiple alternative characterizations of a firm’s predatory pricing incentives.

For each of these characterizations, we show how to construct the corresponding sacrifice test for predatory pricing. As Edlin & Farrell (2004) point out, one way to test for sacrifice is to determine whether the derivative of a profit function that “incorporates everything except effects on competition” is positive at the price the firm has chosen (p. 510). A different characterization of the firm’s predatory pricing incentives is tantamount to a different operationalization of the everything-except-effects-on-competition profit function.

To further illustrate our decomposition and the multiple alternative sacrifice tests that follow from it, we first link the various terms in the decomposition to key features of the pricing decision. Then we gauge the consequences of applying sacrifice tests for industry structure and dynamics by way of an illustrative example. As antitrust authorities flag and prosecute an illegitimate profit sacrifice, they prevent a firm from pricing to achieve that sacrifice. This amounts to forcing firms to ignore the predatory incentives in setting their prices.

We avoid dealing with out-of-equilibrium adjustment processes and merely delineate what may happen in the counterfactual equilibria once firms are forced to ignore the predatory incentives. Because our goal is to show how to construct sacrifice tests in a modern industry-dynamics framework, and not to run a conclusive “horse race” between antitrust policies that are based on alternative characterizations of a firm’s predatory pricing incentives, we content ourselves with presenting equilibria and counterfactuals for a particular parameterization of the model. In practice, the impact of forcing firms to ignore the predatory incentives may differ across parameterizations, so that an antitrust authority set to apply a sacrifice test is well advised to first tailor the model to the institutional realities of the industry under study and estimate the underlying primitives.

The remainder of this paper is organized as follows. Section 2 lays out the model. Section 3 develops the decomposition of the equilibrium pricing condition and formalizes its relationship with the definitions of predation due to Ordover & Willig (1981) and Cabral & Riordan (1997). Section 4 uses the decomposition to develop multiple alternative characterizations of a firm’s predatory pricing incentives and construct the corresponding sacrifice tests. Section 5 exemplifies the link between our decomposition and equilibrium behavior and the impact of forcing firms to ignore the predatory incentives in setting their prices. Section 6 concludes.
2 Model

As a special case of Besanko et al. (2013), we consider a discrete-time, infinite-horizon dynamic stochastic game between two firms that compete in an industry characterized by learning-by-doing. At any point in time, firm \( n \in \{1, 2\} \) is described by its state \( e_n \in \{0, 1, \ldots, M\} \). A firm can be either an incumbent firm that actively produces or a potential entrant. State \( e_n = 0 \) indicates a potential entrant. States \( e_n \in \{1, \ldots, M\} \) indicate the cumulative experience or stock of know-how of an incumbent firm. By making a sale in the current period, an incumbent firm can add to its stock of know-how and, through learning-by-doing, lower its production cost in the subsequent period. Thus, competitive advantage is determined endogenously in our model. At any point in time, the industry’s state is the vector of firms’ states \( e = (e_1, e_2) \in \{0, 1, \ldots, M\}^2 \).

In each period, firms first set prices and then decide on exit and entry. As illustrated in Figure 1, during the price-setting phase, the industry’s state changes from \( e \) to \( e' \) depending on the outcome of pricing game between the incumbent firms. During the exit-entry phase, the state then changes from \( e' \) to \( e'' \) depending on the exit decisions of the incumbent firm(s) and the entry decisions of the potential entrant(s). The state at the end of the current period finally becomes the state at the beginning of the subsequent period. We model entry as a transition from state \( e'_n = 0 \) to state \( e''_n = 1 \) and exit as a transition from state \( e'_n \geq 1 \) to state \( e''_n = 0 \) so that the exit of an incumbent firm creates an opportunity for a potential entrant to enter the industry.

Before analyzing firms’ decisions and the equilibrium of our dynamic stochastic game, we describe the remaining primitives.

**Demand.** The industry draws customers from a large pool of potential buyers. One buyer enters the market each period and purchases one unit of either one of the “inside goods” that are offered by the incumbent firms at prices \( p = (p_1, p_2) \) or an “outside good” at an exogenously given price \( p_0 \). The probability that firm \( n \) makes the sale is given by the logit specification:

\[
D_n(p) = \frac{\exp\left(\frac{-p_n}{\sigma}\right)}{\sum_{k=0}^2 \exp\left(\frac{-p_k}{\sigma}\right)} = \frac{\exp\left(\frac{-p_n}{\sigma}\right)}{\sum_{k=0}^2 \exp\left(\frac{-p_k}{\sigma}\right)},
\]

where \( v \) is gross utility and \( \sigma > 0 \) is a scale parameter that governs the degree of product differentiation. As \( \sigma \to 0 \), goods become homogeneous. If firm \( n \) is a potential entrant, then we set its price to infinity so that \( D_n(p) = 0 \).

**Learning-by-doing and production cost.** Incumbent firm \( n \)’s marginal cost of production \( c(e_n) \) depends on its stock of know-how through a learning curve with a progress ratio
Figure 1: Possible state-to-state transitions.
\( \rho \in [0, 1] : \\
\begin{align*}
c(e_n) &= \begin{cases} 
\kappa \rho \log_2 e_n & \text{if } 1 \leq e_n < m, \\
\kappa \rho \log_2 m & \text{if } m \leq e_n \leq M.
\end{cases}
\end{align*}
\]

Because marginal cost decreases by \( 100(1 - \rho)\% \) as the stock of know-how doubles, a lower progress ratio implies a steeper learning curve. The marginal cost for a firm without prior experience, \( c(1) \), is \( \kappa > 0 \). The firm adds to its stock of know-how by making a sale. Once the firm reaches state \( m \), the learning curve “bottoms out,” and there are no further experience-based cost reductions.

**Scrap value and setup cost.** If incumbent firm \( n \) exits the industry, it receives a scrap value \( X_n \) drawn from a symmetric triangular distribution \( F_X(\cdot) \) with support \([\bar{X} - \Delta_X, \bar{X} + \Delta_X]\), where \( E_X(X_n) = \bar{X} \) and \( \Delta_X > 0 \) is a scale parameter. If potential entrant \( n \) enters the industry, it incurs a setup cost \( S_n \) drawn from a symmetric triangular distribution \( F_S(\cdot) \) with support \([\bar{S} - \Delta_S, \bar{S} + \Delta_S]\), where \( E_S(S_n) = \bar{S} \) and \( \Delta_S > 0 \) is a scale parameter. Scrap values and setup costs are independently and identically distributed across firms and periods, and their realization is observed by the firm but not its rival.

### 2.1 Firms’ decisions

To analyze the pricing decision \( p_n(e) \) of incumbent firm \( n \), the exit decision \( \phi_n(e', X_n) \in \{0, 1\} \) of incumbent firm \( n \) with scrap value \( X_n \), and the entry decision \( \phi_n(e', S_n) \in \{0, 1\} \) of potential entrant \( n \) with setup cost \( S_n \), we work backwards from the exit-entry phase to the price-setting phase. Because scrap values and setup costs are private to a firm, its rival remains uncertain about the firm’s decision. Combining exit and entry decisions, we let \( \phi_n(e') \) denote the probability, as viewed from the perspective of its rival, that firm \( n \) decides not to operate in state \( e' \): If \( e_n \neq 0 \) so that firm \( n \) is an incumbent, then \( \phi_n(e') = E_X[\phi_n(e', X_n)] \) is the probability of exiting; if \( e'_n = 0 \) so that firm \( n \) is an entrant, then \( \phi_n(e') = E_S[\phi_n(e', S_n)] \) is the probability of not entering.

We use \( V_n(e) \) to denote the expected net present value (NPV) of future cash flows to firm \( n \) in state \( e \) at the beginning of the period and \( U_n(e') \) to denote the expected NPV of future cash flows to firm \( n \) in state \( e' \) after pricing decisions but before exit and entry decisions are made. The price-setting phase determines the value function \( V_n(e) \) along with the policy function \( p_n(e) \); the exit-entry phase determines the value function \( U_n(e') \) along with the policy function \( \phi_n(e') \).

---

3We obviously have to ensure \( e_n \leq M \). To simplify the exposition we abstract from boundary issues in what follows.
Exit decision of incumbent firm. To simplify the exposition we focus on firm 1; the
derivations for firm 2 are analogous. If incumbent firm 1 exits the industry, it receives the
scrap value $X_1$ in the current period and perishes. If it does not exit and remains a going
concern in the subsequent period, its expected NPV is

$$U_1(e') = E_X \left[ \max \left\{ \tilde{X}_1(e'), X_1 \right\} \right] = \beta \left[ V_1(e')(1 - \phi_2(e')) + V_1(e', 0)\phi_2(e') \right] + \phi_1(e') E_X \left[ X_1 | X_1 \geq \tilde{X}_1(e') \right],$$

where $\beta \in [0, 1)$ is the discount factor. Incumbent firm 1’s decision to exit the industry in
state $e'$ is thus $\phi_1(e', X_1) = 1 \left[ X_1 \geq \tilde{X}_1(e') \right]$, where $1 \left[ \cdot \right]$ is the indicator function and $\tilde{X}_1(e')$ the critical level of the scrap value above which exit occurs. The probability of incumbent
firm 1 exiting is $\phi_1(e') = 1 - F_X(\tilde{X}_1(e'))$. It follows that before incumbent firm 1 observes
a particular draw of the scrap value, its expected NPV is given by the Bellman equation

$$U_1(e') = E_X \left[ \max \left\{ \tilde{X}_1(e'), X_1 \right\} \right] = \beta \left[ V_1(e')(1 - \phi_2(e')) + V_1(e', 0)\phi_2(e') \right] + \phi_1(e') E_X \left[ X_1 | X_1 \geq \tilde{X}_1(e') \right],$$

where $E_X \left[ X_1 | X_1 \geq \tilde{X}_1(e') \right]$ is the expectation of the scrap value conditional on exiting the industry.

Entry decision of potential entrant. If potential entrant 1 does not enter the industry, it perishes. If it enters and becomes an incumbent firm (without prior experience) in the
subsequent period, its expected NPV is

$$S_1(e') = \beta \left[ V_1(1, e'_2)(1 - \phi_2(e')) + V_1(1, 0)\phi_2(e') \right].$$

In addition, it incurs the setup cost $S_1$ in the current period. Potential entrant 1’s decision
to not enter the industry in state $e'$ is thus $\phi_1(e', S_1) = 1 \left[ S_1 \geq \tilde{S}_1(e') \right]$, where $\tilde{S}_1(e')$ is the critical level of the setup cost. The probability of potential entrant 1 not entering is $\phi_1(e') = 1 - F_S(\tilde{S}_1(e'))$ and before potential entrant 1 observes a particular draw of the
setup cost, its expected NPV is given by the Bellman equation

$$U_1(e') = E_S \left[ \max \left\{ \tilde{S}_1(e') - S_1, 0 \right\} \right] = (1 - \phi_1(e')) \left\{ \beta[V_1(1, e'_2)(1 - \phi_2(e')) + V_1(1, 0)\phi_2(e')] - E_S \left[ S_1 | S_1 \leq \tilde{S}_1(e') \right] \right\},$$

where $\beta \in [0, 1)$ is the discount factor.
where \( E_S [S_1 | S_1 \leq \tilde{S}_1(e')] \) is the expectation of the setup cost conditional on entering the industry.\(^4\)

**Pricing decision of incumbent firm.** In the price-setting phase, the expected NPV of incumbent firm 1 is

\[
V_1(e) = \max_{p_1} \left( p_1 - c(e_1) \right) D_1(p_1, p_2(e)) + D_0(p_1, p_2(e)) U_1(e) + D_1(p_1, p_2(e)) U_1(e_1 + 1, e_2) + D_2(p_1, p_2(e)) U_1(e_1, e_2 + 1). \tag{3}
\]

Because \( D_0(p) = 1 - D_1(p) - D_2(p) \), we can equivalently formulate the maximization problem on the right-hand side of the Bellman equation (3) as \( \max_{p_1} \Pi_1(p_1, p_2(e), e) \), where

\[
\Pi_1(p_1, p_2(e), e) = (p_1 - c(e_1)) D_1(p_1, p_2(e)) + U_1(e) + D_1(p_1, p_2(e)) [U_1(e_1 + 1, e_2) - U_1(e)] - D_2(p_1, p_2(e)) [U_1(e) - U_1(e_1, e_2 + 1)] \tag{4}
\]

is the long-run profit of incumbent firm 1. Because \( \Pi_1(p_1, p_2(e), e) \) is strictly quasiconcave in \( p_1 \) (given \( p_2(e) \) and \( e \)), the pricing decision \( p_1(e) \) is uniquely determined by the first-order condition

\[
mr_1(p_1, p_2(e)) = c(e_1) + [U_1(e_1 + 1, e_2) - U_1(e)] + \mathcal{Y}(p_2(e)) [U_1(e) - U_1(e_1, e_2 + 1)] = 0, \tag{5}
\]

where \( mr_1(p_1, p_2(e)) = \frac{p_1}{1 - \frac{\sigma}{1 - D_1(p_1, p_2(e))}} \) is the marginal revenue to incumbent firm 1 or what Edlin (2010) calls inclusive price\(^5\) and \( \mathcal{Y}(p_2(e)) = \frac{D_2(p_1, p_2(e))}{1 - D_1(p_1, p_2(e))} = \frac{\exp(-\frac{p_2(e)}{\sigma})}{\exp(-\frac{p_1}{\sigma}) + \exp(-\frac{p_2(e)}{\sigma})} \) is the probability of firm 2 making a sale conditional on firm 1 not making a sale.

As discussed in Besanko et al. (2013), the pricing decision impounds two distinct goals beyond short-run profit: the **advantage-building motive** \( [U_1(e_1 + 1, e_2) - U_1(e)] \) and the **advantage-denying motive** \( [U_1(e) - U_1(e_1, e_2 + 1)] \). The advantage-building motive is the reward that the firm receives by **winning** a sale and moving down its learning curve. The advantage-denying motive is the penalty that the firm avoids by **preventing its rival from winning** the sale and moving down its learning curve. The advantage-building motive thus reflects the firm’s marginal benefit from becoming a more formidable competitor in the future while the advantage-denying motive reflects the firm’s marginal benefit from preventing its rival from becoming a more formidable competitor. Because it encompasses both the short

---

\(^4\)See the Online Appendix to Besanko et al. (2013) for closed-form expressions for \( E_X \left[ X_1 | X_1 \geq \tilde{X}_1(e') \right] \) in equation (1) and \( E_S \left[ S_1 | S_1 \leq \tilde{S}_1(e') \right] \) in equation (2).

\(^5\)\(mr_1(p_1, p_2(e))\) is marginal revenue with respect to quantity, i.e., the probability of making the sale, written as a function of price. See the Online Appendix to Besanko et al. (2013) for more details.
run and the long run, the pricing decision on our model is akin to an investment decision.

2.2 Equilibrium

Because our demand and cost specification is symmetric, we restrict ourselves to symmetric Markov perfect equilibria. Existence of a symmetric Markov perfect equilibrium in pure strategies follows from the arguments in Doraszelski & Satterthwaite (2010). In a symmetric equilibrium, the decisions taken by firm 2 in state \( e = (e_1, e_2) \) are identical to the decisions taken by firm 1 in state \( (e_2, e_1) \). It is therefore sufficient to determine the value and policy functions of firm 1.

3 Decomposition

To determine what constitutes an illegitimate profit sacrifice and isolate a firm’s predatory pricing incentives, we go beyond Besanko et al. (2013) and decompose the equilibrium pricing condition (5) with even more granularity by writing it as

\[
\text{mr}_1(p_1(e), p_2(e)) - c(e_1) + \left[\sum_{k=1}^{5} \Gamma_{1}^{k}(e)\right] + \eta(p_2(e)) \left[\sum_{k=1}^{4} \Theta_{1}^{k}(e)\right] = 0. \tag{6}
\]

\( \sum_{k=1}^{5} \Gamma_{1}^{k}(e) \) decomposes the advantage-building motive \( [U_1(e_1 + 1, e_2) - U_1(e)] \) and \( \sum_{k=1}^{4} \Theta_{1}^{k}(e) \) the advantage-denying motive \( [U_1(e) - U_1(e_1, e_2 + 1)] \). Each term in this decomposition has a distinct economic interpretation that we describe below.\(^6\)

**Advantage building.** The decomposed advantage-building motives summarized in Table 1 are the various sources of marginal benefit to the firm from winning the sale in the current period and moving further down its learning curve.

*Baseline advantage-building motive:*

\[
\Gamma_{1}^{1}(e) = (1 - \phi_1(e)) \beta [V_1(e_1 + 1, e_2) - V_1(e)].
\]

The baseline advantage-building motive is the firm’s marginal benefit from an improvement in its competitive position, assuming that its rival does not exit in the current period. It captures both the lower marginal cost and any future advantages (winning the sale, exit of rival, etc.) that stem from this lower cost.

\(^6\)The decomposition applies to an industry with two incumbent firms in state \( e \geq (1, 1) \) and we focus on firm 1. We use equation (1) to express \( U_1(e) \) in terms of \( V_1(e) \). Because the terms \( \Gamma_{1}^{1}(e) \) and \( \Theta_{1}^{k}(e) \) are typically positive, we refer to them as marginal benefits. To streamline the exposition, we further presume monotonicity of the value and policy functions. For some parameterizations these assumptions fail.
Table 1: Decomposed advantage-building motives.

<table>
<thead>
<tr>
<th>Advantage-building motives</th>
<th>if the firm wins the sale and moves further down its learning curve, then the firm...</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_1^1(e)$ baseline</td>
<td>... improves its competitive position within the duopoly</td>
</tr>
<tr>
<td>$\Gamma_1^2(e)$ exit</td>
<td>... increases its rival’s exit probability</td>
</tr>
<tr>
<td>$\Gamma_1^3(e)$ survival</td>
<td>... decreases its exit probability</td>
</tr>
<tr>
<td>$\Gamma_1^4(e)$ scrap value</td>
<td>... increases its expected scrap value</td>
</tr>
<tr>
<td>$\Gamma_1^5(e)$ market structure</td>
<td>... gains from an improved competitive position as a monopolist versus as a duopolist</td>
</tr>
</tbody>
</table>

*Advantage-building/exit motive:*

$$\Gamma_1^2(e) = (1 - \phi_1(e)) [\phi_2(e_1 + 1, e_2) - \phi_2(e)] \beta [V_1(e_1 + 1, 0) - V_1(e_1 + 1, e_2)].$$

The advantage-building/exit motive is the firm’s marginal benefit from increasing its rival’s exit probability from $\phi_2(e)$ to $\phi_2(e_1 + 1, e_2)$.

*Advantage-building/survival motive:*

$$\Gamma_1^3(e) = [\phi_1(e) - \phi_1(e_1 + 1, e_2)] \beta [\phi_2(e_1 + 1, e_2)V_1(e_1 + 1, 0) + (1 - \phi_2(e_1 + 1, e_2))V_1(e_1 + 1, e_2)].$$

The advantage-building/survival motive is the firm’s marginal benefit from decreasing its exit probability from $\phi_1(e)$ to $\phi_1(e_1 + 1, e_2)$.

*Advantage-building/scrap value motive:*

$$\Gamma_1^4(e) = \phi_1(e_1 + 1, e_2)E_X \left[ X_1 | X_1 \geq \hat{X}_1(e_1 + 1, e_2) \right] - \phi_1(e)E_X \left[ X_1 | X_1 \geq \hat{X}_1(e) \right].$$

The advantage-building/scrap value motive is the firm’s marginal benefit from increasing its scrap value in expectation from $\phi_1(e)E_X \left[ X_1 | X_1 \geq \hat{X}_1(e) \right]$ to $\phi_1(e_1 + 1, e_2)E_X \left[ X_1 | X_1 \geq \hat{X}_1(e_1 + 1, e_2) \right]$.

*Advantage-building/market structure motive:*

$$\Gamma_1^5(e) = (1 - \phi_1(e))\phi_2(e)\beta \{[V_1(e_1 + 1, 0) - V_1(e_1, 0)] - [V_1(e_1 + 1, e_2) - V_1(e)]\}.$$

The advantage-building/market structure motive is the firm’s marginal benefit from an improvement in its competitive position as a monopolist versus as a duopolist.

**Advantage denying.** The decomposed advantage-denying motives summarized in Table 2 are the various sources of marginal benefit to the firm from winning the sale in the current period and, by doing so, preventing its rival from moving further down its learning curve. The decomposed advantage-denying motives differ from the decomposed advantage-building
If the firm wins the sale and prevents its rival from moving further down its learning curve, then the firm . . . prevents its rival from improving its competitive position within the duopoly.

\[ \Theta^1_1(e) = (1 - \phi_1(e))(1 - \phi_2(e_1, e_2 + 1))\beta [V_1(e) - V_1(e_1, e_2 + 1)] . \]

The baseline advantage-denying motive is firm’s the marginal benefit from preventing an improvement in its rival’s competitive position, assuming its rival does not exit in the current period.

Advantage-denying/exit motive:

\[ \Theta^2_1(e) = (1 - \phi_1(e))[\phi_2(e) - \phi_2(e_1, e_2 + 1)]\beta[V_1(e) - V_1(e_1, 0) - V_1(e)] . \]

The advantage-denying/exit motive is the firm’s marginal benefit from preventing its rival’s exit probability from decreasing from \( \phi_2(e) \) to \( \phi_2(e_1, e_2 + 1) \).

Advantage-denying/survival motive:

\[ \Theta^3_1(e) = [\phi_1(e_1, e_2 + 1) - \phi_1(e)] \beta [\phi_2(e_1, e_2 + 1)V_1(e_1, 0) + (1 - \phi_2(e_1, e_2 + 1))V_1(e_1, e_2 + 1)] . \]

The advantage-denying/survival motive is the firm’s marginal benefit from preventing its exit probability from increasing from \( \phi_1(e) \) to \( \phi_1(e_1, e_2 + 1) \).

Advantage-denying/scrap value motive:

\[ \Theta^4_1(e) = \phi_1(e)\beta[X_1|X_1 \geq \bar{X}_1(e)] - \phi_1(e_1, e_2 + 1)\beta[X_1|X_1 \geq \bar{X}_1(e_1, e_2 + 1)] . \]

The advantage-denying/scrap value motive is the firm’s marginal benefit from preventing its expected scrap value from decreasing in expectation from \( \phi_1(e) \) to \( \phi_1(e_1, e_2 + 1) \).
3.1 Economic definitions of predation

The decomposition in (6) relates to economic definitions of predation formulated in the existing literature.

**Cabral & Riordan (1997).** Cabral & Riordan (1997) call “an action predatory if (1) a different action would increase the probability that rivals remain viable and (2) the different action would be more profitable under the counterfactual hypothesis that the rival’s viability were unaffected” (p. 160). In the context of predatory pricing, it is natural to interpret “a different action” as a higher price $p_1 > p_1(e)$. To port the Cabral & Riordan definition from their two-period model to our infinite-horizon dynamic stochastic game, we take the “rival’s viability” to refer to the probability that the rival exits the industry in the current period. Finally, we interpret “the different action would be more profitable” in the spirit of Markov perfection to mean that by setting a higher price in the current period but returning to equilibrium play from the subsequent period onward, the firm can affect the evolution of the state to increase its expected NPV if it believed, counterfactually, that the probability that the rival exits the industry in the current period is fixed at $\phi_2(e)$.

With these interpretations, Proposition 1 formalizes the relationship between the Cabral & Riordan definition of predation and our decomposition (6):

**Proposition 1** Consider an industry with two incumbent firms in state $e \geq (1,1)$. Assume $\phi_1(e) < 1$, $V_1(e_1,0) > V_1(e)$, and $V_1(e_1 + 1,0) > V_1(e_1 + 1,e_2)$, i.e., exit by the firm is less than certain and the expected NPV of a monopolist exceeds that of a duopolist. (a) If $\Gamma_1^2(e) \geq 0$ and $\Theta_1^2(e) \geq 0$, with at least one of these inequalities being strict, and

$$\Gamma_1^2(e) + \left[ \Theta_1^1(e) - \Theta_1^1(e) \bigg|_{\phi_2 = \phi_2(e)} \right] + \left[ \Theta_1^2(e) - \Theta_1^2(e) \bigg|_{\phi_2 = \phi_2(e)} \right] > 0,$$

then the firm’s equilibrium price $p_1(e)$ in state $e$ is predatory according to the Cabral & Riordan (1997) definition. (b) If $p_1(e)$ is predatory according to the Cabral & Riordan definition, then $\Gamma_1^2(e) > 0$ or $\Theta_1^2(e) > 0$ and inequality (7) holds.

**Proof.** See Appendix.

**Ordover & Willig (1981).** According to Ordover & Willig (1981), “[p]redatory behavior is a response to a rival that sacrifices part of the profit that could be earned under competitive
circumstances were the rival to remain viable, in order to induce exit and gain consequent additional monopoly profit” (pp. 9–10). As Cabral & Riordan (1997) observe, the premise in the Ordover & Willig definition is that the rival is viable with certainty.\(^8\) We have:

**Proposition 2** Consider an industry with two incumbent firms in state \(e \geq (1,1)\). Assume \(\phi_1(e) < 1\), \(V_1(e_1, 0) > V_1(e)\), and \(V_1(e_1 + 1, 0) > V_1(e_1 + 1, e_2)\), i.e., exit by the firm is less than certain and the expected NPV of a monopolist exceeds that of a duopolist. (a) If \(\Gamma_1^2(e) \geq 0\) and \(\Theta_1^2(e) \geq 0\), with at least one of these inequalities being strict, and

\[
\Gamma_1^2(e) + \left[ \Gamma_1^3(e) - \Gamma_1^3(e)_{\phi_2=0} \right] + \Gamma_1^5(e) + \Theta_1^2(e) + \Theta_1^2(e)_{\phi_2=0} > 0, \tag{8}
\]

then the firm’s equilibrium price \(p_1(e)\) in state \(e\) is predatory according to the Ordover & Willig (1981) definition. (b) If \(p_1(e)\) is predatory according to the Ordover & Willig definition, then \(\Gamma_1^2(e) > 0\) or \(\Theta_1^2(e) > 0\) and inequality (8) holds.

The proof follows the same logic as the proof of Proposition 1 and is therefore omitted.

## 4 Sacrifice tests

To detect the presence of predatory pricing, antitrust authorities routinely ask whether a firm sacrifices current profit in exchange for the expectation of higher future profit following the exit of its rival. Sacrifice tests thus view predation as an “investment in monopoly profit” (Bork 1978).\(^9\)

As pointed out by Edlin & Farrell (2004), one way to test for sacrifice is to determine whether the derivative of a suitably defined profit function is positive at the price that the firm has chosen, which indicates that the chosen price is less than the price that maximizes profit. Moreover, “[i]n principle this profit function should incorporate *everything except effects on competition*” (p. 510, our italics).

To construct sacrifice tests, we therefore partition the profit function \(\Pi_1(p_1, p_2(e), e)\) in our model into an everything-except-effects-on-competition (EEEC) profit function \(\Pi_1^0(p_1, p_2(e), e)\)

---

\(^8\)This observation indeed motivates Cabral & Riordan (1997) to propose their own definition: “Is the appropriate counterfactual hypothesis that firm B remain viable with probability one? We don’t think so. Taking into account that firm B exits for exogenous reasons (i.e. a high realization of [the scrap value]) hardly means that firm A intends to drive firm B from the market” (p. 160).

\(^9\)Sacrifice tests are closely related to the “no economic sense” test that holds that “conduct is not exclusionary or predatory unless it would make no economic sense for the defendant but for the tendency to eliminate or lessen competition” (Werden 2006, p. 417). Both have been criticized for “not generally [being] a reliable indicator of the impact of allegedly exclusionary conduct on consumer welfare—the primary focus of antitrust laws” (Salop 2006, p. 313).
and a remainder \( \Omega_1(p_1, p_2(e), e) = \Pi_1(p_1, p_2(e), e) - \Pi_1^0(p_1, p_2(e), e) \) that by definition reflects the effects on competition. Because \( \frac{\partial \Pi_1(p_1(e), p_2(e), e)}{\partial p_1} = 0 \) in equilibrium, the sacrifice test \( \frac{\partial \Omega_1(p_1(e), p_2(e), e)}{\partial p_1} > 0 \) is equivalent to
\[
- \frac{\partial \Omega_1(p_1(e), p_2(e), e)}{\partial p_1} = \frac{\partial \Pi_1(p_1(e), p_2(e), e)}{\partial (-p_1)} > 0, \tag{9}
\]
\( \frac{\partial \Omega_1(p_1(e), p_2(e), e)}{\partial (-p_1)} \) is the marginal return to a price cut in the current period due to changes in the competitive environment. If profit is sacrificed, then inequality (9) tells us that these changes in the competitive environment are to the firm’s advantage. In this sense, \( \frac{\partial \Omega_1(p_1(e), p_2(e), e)}{\partial (-p_1)} \) is the marginal return to the “investment in monopoly profit” and thus a natural measure of the firm’s predatory pricing incentives.

The specification of the EEEC profit function determines what constitutes an illegitimate profit sacrifice—and thus predatory pricing—and there are as many sacrifice tests as there are possible specifications of the EEEC profit function. Because by construction \( \Omega_1(p_1, p_2(e), e) = \Pi_1(p_1, p_2(e), e) - \Pi_1^0(p_1, p_2(e), e) \), specifying an EEEC profit function is equivalent to specifying the firm’s predatory pricing incentives. Propositions 1 and 2 suggest starting from the predatory incentives to construct the corresponding EEEC profit function and sacrifice test. More generally, our decomposition (6) highlights the various incentives that a firm faces when it decides on a price. While some of these incentives may be judged to be predatory, others reflect the pursuit of efficiency. Using the decomposition, we therefore develop multiple alternative characterizations of a firm’s predatory pricing incentives and, for each of these characterizations, we construct the corresponding EEEC profit function and sacrifice test.

**Short-run profit.** In the quote above, Edlin & Farrell (2004) go on to point out that “in practice sacrifice tests often use short-run data, and we will often follow the conventional shorthand of calling [this profit function] short-run profit” (p. 510). Equating predatory pricing with a failure to maximize short-run profit implies that the firm’s predatory pricing incentives are its dynamic incentives in their entirety or, in other words, all decomposed advantage-building and advantage-denying motives. This then gives us our first definition of predatory incentives, which is identical to Definition 1 in Besanko et al. (2013):

**Definition 1 (short-run profit)** The firm’s predatory pricing incentives are \( [U_1(e_1 + 1, e_2) - U_1(e)] + \Upsilon(p_2(e)) [U_1(e) - U_1(e_1, e_2 + 1)] = \left\{ \sum_{k=1}^{8} \Gamma_k^*(e) \right\} + \Upsilon(p_2(e)) \left\{ \sum_{k=1}^{4} \Theta_k^*(e) \right\} \).

The EEEC profit function corresponding to Definition 1 is
\[
\Pi_1^{0,SRP}(p_1, p_2(e), e) = (p_1 - c_1(e_1))D_1(p_1, p_2(e)). \]
It follows from our decomposition (6) that \( \frac{\partial \Psi_{SRP}^{\#}(p_1(e),p_2(e),e)}{\partial (-p_1)} > 0 \) if and only if \( \left[ \sum_{k=1}^{5} \Gamma_1^k(e) \right] + \Upsilon(p_2(e)) \left[ \sum_{k=1}^{4} \Theta_1^k(e) \right] > 0. \)

The sacrifice test based on Definition 1 is equivalent to the inclusive price \( mr_1(p_1(e),p_2(e)) \) being less than short-run marginal cost \( c(e_1). \)^{10} Because \( mr_1(p_1(e),p_2(e)) \to p_1(e) \) as \( \sigma \to 0 \), in an industry with very weak product differentiation it is also nearly equivalent to the classic Areeda & Turner (1975) test that equates predatory pricing with below-cost pricing and upholds the current Brooke Group standard for predatory pricing in the U.S.

**Dynamic competitive vacuum.** By equating predatory pricing with a failure to maximize short-run profit, Definition 1 may be too broad for a dynamic environment like ours in which a firm has an incentive to price aggressively in order to realize experience-based cost reductions. Taking the resulting intertemporal trade-off into account, Farrell & Katz (2005) view an action as predatory only if it weakens the rival (see, in particular, p. 219 and p. 226). According to Farrell & Katz (2005), a firm should behave as if it were operating in a “dynamic competitive vacuum” by taking as given the competitive position of its rival in the current period but ignoring that its current price can affect the evolution of its rival’s competitive position beyond the current period. Our second definition of predatory incentives thus comprises all decomposed advantage-denying motives:

**Definition 2 (dynamic competitive vacuum)** The firm’s predatory pricing incentives are \[ [U_1(e) - U_1(e_1,e_2 + 1)] = \left[ \sum_{k=1}^{4} \Theta_1^k(e) \right]. \]

Definition 2 is identical to Definition 2 in Besanko et al. (2013). The corresponding EEEC profit function is

\[
\Pi_1^{0,DCV}(p_1,p_2(e),e) = (p_1 - c(e_1))D_1(p_1,p_2(e)) \\
+ U_1(e) + D_1(p_1,p_2(e)) [U_1(e_1 + 1,e_2) - U_1(e)],
\]

where we assume that from the subsequent period onward, play returns to equilibrium. To us, this best captures the idea that the firm is sacrificing something now in exchange for a later improvement in the competitive environment. It follows from our decomposition (6) that \( \frac{\partial \Omega_{DCV}^{\#}(p_1(e),p_2(e),e)}{\partial (-p_1)} > 0 \) if and only if \( \left[ \sum_{k=1}^{4} \Theta_1^k(e) \right] > 0. \)

The sacrifice test based on Definition 2 is equivalent to the inclusive price \( mr_1(p_1(e),p_2(e)) \) being less than long-run marginal cost \( c(e_1). \) Note that a lower bound on long-run marginal cost \( c(e_1) - \left[ \sum_{k=1}^{5} \Gamma_1^k(e) \right] \) is out-of-pocket cost at the bottom of the learning curve \( c(m) \) (Spence 1981). Hence, if \( mr_1(p_1(e),p_2(e)) < c(m) \), then \( mr_1(p_1(e),p_2(e)) <
\[ c(e_1) - \left[ \sum_{k=1}^{5} \Gamma_k^\delta(e) \right]. \] This provides a one-way test for sacrifice that can be operationalized given some basic knowledge of demand and cost.

**Rival exit in current period.** According to Definitions 1 and 2, the marginal return to a price cut in the current period may be positive not because the rival exits the industry in the current period but because the rival exits in some future period. The predatory incentives therefore extend to the possibility that the rival exits in some future period because the firm improves its competitive position in the current period. The economic definitions of predation formulated in the existing literature instead focus more narrowly on the immediate impact of a price cut on rival exit. Our remaining definitions of the firm’s predatory pricing incentives embrace this focus.

In light of Proposition 2 we have:

**Definition 3 (Ordover & Willig)** The firm’s predatory pricing incentives are

\[
\Gamma_1^2(e) + \left[ \Gamma_1^3(e) - \Gamma_1^3(e)_{\phi_2=0} \right] + \Gamma_1^5(e) + \Upsilon(p_2(e)) \left[ \left[ \Theta_1^1(e) - \Theta_1^1(e)_{\phi_2=0} \right] + \Theta_1^2(e) + \left[ \Theta_1^3(e) - \Theta_1^3(e)_{\phi_2=0} \right] \right].
\]

The Ordover & Willig definition of predation implies

\[
\Pi_{1,OW}^0(p_1, p_2(e), e) = \Pi_1(p_1, p_2(e), e)_{\phi_2=0},
\]

so that the EEEC profit function is the profit function under the counterfactual presumption that the probability that the rival exits the industry in the current period is zero.

In light of Proposition 1 we further have:

**Definition 4 (Cabral & Riordan)** The firm’s predatory pricing incentives are

\[
\Gamma_1^2(e) + \left[ \Gamma_1^3(e) - \Gamma_1^3(e)_{\phi_2=\phi_2(e)} \right] + \Upsilon(p_2(e)) \left[ \left[ \Theta_1^1(e) - \Theta_1^1(e)_{\phi_2=\phi_2(e)} \right] + \Theta_1^2(e) + \left[ \Theta_1^3(e) - \Theta_1^3(e)_{\phi_2=\phi_2(e)} \right] \right].
\]

The Cabral & Riordan definition of predation implies

\[
\Pi_{1,CR}^0(p_1, p_2(e), e) = \Pi_1(p_1, p_2(e), e)_{\phi_2=\phi_2(e)}.
\]

Our remaining definition of the firm’s predatory pricing incentives comes from partitioning the predatory incentives in Definitions 3 and 4 more finely by maintaining that the truly exclusionary effects on competition are the ones aimed at inducing exit by the firm winning
the sale and moving further down its learning curve as well as by the firm preventing its rival from winning the sale and moving further down its learning curve:

**Definition 5 (rival exit)** The firm’s predatory pricing incentives are \( \Gamma_1^2(e) + \Upsilon(p_2(e))\Theta_1^2(e) \).

Definition 5 is identical to Definition 3 in Besanko et al. (2013). The corresponding EEEC profit function is

\[
\Pi_{REX}^0(p_1, p_2(e), e) = (p_1 - c(e_1))D_1(p_1, p_2(e)) + U_1(e) + D_1(p_1, p_2(e)) \left[ \sum_{k \neq 2} \Gamma_k^1(e) \right] + D_2(p_1, p_2(e)) \left[ \sum_{k \neq 2} \Theta_k^1(e) \right].
\]

It follows from our decomposition (6) that \( \frac{\partial \Pi_{REX}^0(p_1, p_2(e), e)}{\partial (-p_1)} > 0 \) if and only if \( \Gamma_1^2(e) + \Upsilon(p_2(e))\Theta_1^2(e) > 0 \).

5 **Illustrative example**

To illustrate the types of behavior that can arise in our model, we compute the Markov perfect equilibria for the baseline parameterization in Table 1 in Besanko et al. (2013). Although this parameterization does not correspond to any specific industry, it is empirically plausible and in no way extreme. At the baseline parameterization there are three equilibria. For two of these three equilibria, Figure 2 shows the pricing decision of firm 1, the non-operating probability of firm 2, and the time path of the probability distribution over industry structures (empty, monopoly, and duopoly). The third equilibrium is essentially intermediate between the two shown in Figure 2 and is therefore omitted.

The upper panels of Figure 2 illustrate what is called an **aggressive equilibrium** in Besanko et al. (2013). As can be seen in the upper left panel, there is a deep well in the pricing decision in state \((1, 1)\) with \( p_1(1, 1) = -34.78 \). In the well, the firms engage in a preemption battle to determine which will be first to move down from the top of its learning curve. There is also a deep trench along the \( e_1 \)-axis, with \( p_1(e_1, 1) \) ranging from 0.08 to 1.24 for \( e_1 \in \{2, \ldots, 30\} \).\(^{11}\) A trench is a price war that the leader (firm 1) wages against the follower (firm 2), or an endogenous mobility barrier in the sense of Caves & Porter (1977). In the trench, the follower exits the industry with a positive probability of \( \phi_2(1, e_2) = 0.22 \) for \( e_2 \in \{2, \ldots, 30\} \) as can be seen in the upper middle panel. As long as the follower does not win a sale, it remains in this “exit zone.” If the follower exits, the leader raises its price and the industry becomes an

\(^{11}\)Because prices are strategic complements, there is also a shallow trench along the \( e_2 \)-axis with \( p_1(1, e_2) \) ranging from 3.63 to 4.90 for \( e_2 \in \{2, \ldots, 30\} \).
Figure 2: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from \( e = (1, 1) \) at \( T = 0 \) (right panels). Aggressive (upper panels) and accommodative (lower panels) equilibria.
entrenched monopoly. This sequence of events resembles conventional notions of predatory pricing. On the other hand, the industry evolves into a mature duopoly if the follower wins a sale while in the midst of the price war. However, this is unlikely to happen and, as can be seen in the upper right panel, a mature duopoly is much less likely than an entrenched monopoly.

The lower panels of Figure 2 are typical for an accommodative equilibrium. There is a shallow well in state (1, 1) with $p_1(1, 1) = 5.05$ as the lower left panel shows. Absent mobility barriers in the form of trenches, however, any competitive advantage is temporary and the industry evolves into a mature duopoly as the lower right panel shows.

The panel labeled “MPE” in Table 3 illustrates industry structure, conduct, and performance implied by the equilibria. The expected long-run Herfindahl index $HHI_\infty$ reflects that the industry is substantially more likely to be monopolized under the aggressive equilibrium than under the accommodative equilibrium. In the entrenched monopoly prices are higher as can be seen from the expected long-run average price $\bar{p}_\infty$. Finally, consumer and total surplus are lower under the aggressive equilibrium than under the accommodative equilibrium. The difference between the equilibria is smaller for expected discounted consumer surplus $CS^{NPV}$ than for expected long-run consumer surplus $CS_\infty$ because the former metric accounts for the competition for the market in the short run that manifests itself in the deep well and trench of the aggressive equilibrium and mitigates the lack of competition for the market in the long run.

In sum, predation-like behavior arises in aggressive equilibria. Aggressive equilibria often coexist with accommodative equilibria involving much less aggressive pricing. Aggressive equilibria involve more competition in the short run than accommodative equilibria but less competition in the long run.

### 5.1 Predation-like behavior and sacrifice tests

Our decomposition sheds light on the origins of the wells and trenches that are part and parcel of predation-like behavior and competition for the market. The upper panels of Table

---

12In this particular equilibrium, $\phi_2(e_1, 0) = 1.00$ for $e_1 \in \{2, \ldots, 30\}$, so that a potential entrant does not enter if the incumbent firm has moved down from the top of its learning curve.

13Following Cabral & Riordan (1994), we refer to an incumbent firm in state $e_n \geq m$ as a mature firm and an industry in state $e \geq (m, m)$ as a mature duopoly. In the same spirit, we refer to an incumbent firm in state $e_n = 1$ as an emerging firm and an industry in state $(1, 1)$ as an emerging duopoly.

14We use the policy functions $p_1$ and $\phi_1$ for a particular equilibrium to construct the matrix of state-to-state transition probabilities that characterizes the Markov process of industry dynamics. From this, we compute the transient distribution over states in period $T$ starting from state $(1, 1)$ in period 0. Depending on $T$, the transient distributions can capture short-run or long-run (steady-state) dynamics, and we think of period 1000 as the long run. To succinctly describe the equilibrium, we finally use the transient distributions to compute six metrics of industry structure, conduct, and performance. See Section III of Besanko et al. (2013) for details.
Table 3: Industry structure, conduct, and performance. Aggressive, intermediate, and accommodative equilibria without conduct restriction (panel labeled “MPE”) and with conduct restriction according to Definition 1 (panel labeled “SRP”), Definition 2 (panel labeled “DCV”), Definition 3 (panel labeled “OW”), Definition 4 (panel labeled “CR”), and Definition 5 (panel labeled “REX”).

<table>
<thead>
<tr>
<th></th>
<th>HHI∞</th>
<th>( \bar{p}^\infty )</th>
<th>CS∞</th>
<th>TS∞</th>
<th>CS^{NPV}</th>
<th>TS^{NPV}</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>aggressive</td>
<td>0.96</td>
<td>8.26</td>
<td>1.99</td>
<td>6.09</td>
<td>104.18</td>
<td>110.33</td>
</tr>
<tr>
<td>intermediate</td>
<td>0.58</td>
<td>5.74</td>
<td>4.89</td>
<td>7.22</td>
<td>111.18</td>
<td>119.12</td>
</tr>
<tr>
<td>accommodative</td>
<td>0.50</td>
<td>5.24</td>
<td>5.46</td>
<td>7.44</td>
<td>109.07</td>
<td>120.14</td>
</tr>
<tr>
<td>SRP</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>accommodative</td>
<td>0.50</td>
<td>5.24</td>
<td>5.46</td>
<td>7.44</td>
<td>59.72</td>
<td>106.07</td>
</tr>
<tr>
<td>DCV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>accommodative</td>
<td>0.50</td>
<td>5.24</td>
<td>5.46</td>
<td>7.44</td>
<td>102.10</td>
<td>119.70</td>
</tr>
<tr>
<td>OW</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>aggressive</td>
<td>0.95</td>
<td>8.19</td>
<td>2.07</td>
<td>6.12</td>
<td>98.11</td>
<td>110.64</td>
</tr>
<tr>
<td>intermediate</td>
<td>0.63</td>
<td>6.07</td>
<td>4.51</td>
<td>7.07</td>
<td>109.79</td>
<td>118.19</td>
</tr>
<tr>
<td>accommodative</td>
<td>0.50</td>
<td>5.24</td>
<td>5.46</td>
<td>7.44</td>
<td>109.07</td>
<td>120.14</td>
</tr>
<tr>
<td>CR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>aggressive</td>
<td>0.92</td>
<td>8.04</td>
<td>2.24</td>
<td>6.18</td>
<td>98.84</td>
<td>111.25</td>
</tr>
<tr>
<td>intermediate</td>
<td>0.64</td>
<td>6.17</td>
<td>4.39</td>
<td>7.02</td>
<td>109.17</td>
<td>117.87</td>
</tr>
<tr>
<td>accommodative</td>
<td>0.50</td>
<td>5.24</td>
<td>5.46</td>
<td>7.44</td>
<td>109.07</td>
<td>120.14</td>
</tr>
<tr>
<td>REX</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>aggressive</td>
<td>0.95</td>
<td>8.19</td>
<td>2.07</td>
<td>6.12</td>
<td>98.11</td>
<td>110.64</td>
</tr>
<tr>
<td>intermediate</td>
<td>0.62</td>
<td>6.06</td>
<td>4.52</td>
<td>7.07</td>
<td>109.83</td>
<td>118.21</td>
</tr>
<tr>
<td>accommodative</td>
<td>0.50</td>
<td>5.24</td>
<td>5.46</td>
<td>7.44</td>
<td>109.07</td>
<td>120.14</td>
</tr>
</tbody>
</table>

4 illustrate the decomposition (6) for the aggressive equilibrium for a set of states where firm 2 is at the top of the learning curve. The competition for the market in the well in state \((1, 1)\) is driven mostly by the baseline advantage-building motive \(\Gamma_1(1, 1)\) and the advantage-building/exit motive \(\Gamma_2(1, 1)\). In contrast, the competition for the market in the trench in states \((e_1, 1)\) for \(e_1 \in \{2, \ldots, 30\}\) is driven mostly by the baseline advantage-denying motive \(\Theta_1(e_1, 1)\) and the advantage-denying/exit motive \(\Theta_2(e_1, 1)\). The predation-like behavior in the trench thus arises not because by becoming more efficient the leader increases the probability that the follower exits the industry but because by preventing the follower from becoming more efficient the leader keeps the follower in the trench and thus prone to exit. Another way to put this is that the leader makes the cost to the follower of attempting to move down its learning curve comparable to the benefit to the follower of doing so, so that exit is in the follower’s interest. Viewed this way, the aggressive pricing in the trench can be viewed as raising the rival’s cost of remaining in the industry. The decomposed advantage-denying motives remain in effect in states \((e_1, 1)\) for \(e_1 \in \{16, \ldots, 30\}\) where the leader has exhausted all learning economies.

As can be seen in lower panels of Table 4 for a set of states where firm 2 has already gained some traction, neither the advantage-building nor the advantage-denying motives are very large. To the extent that the price is below the static optimum this is due mostly to
the baseline advantage-building motive $\Gamma_1^1(e_1, 4)$ for $e_1 \in \{1, \ldots, 30\}$.

Table 5 complements Table 4 by illustrating the decomposition (6) for the accommodative equilibrium. The pricing decision is driven by the advantage-building/baseline and advantage-denying/baseline motives.

The various definitions of predatory incentives in Section 4 are ordered to hone in on ever fewer terms in our decomposition (6). Intuitively, they thus become narrower. The right panels of Tables 4 and 5 illustrate this point at the example of the aggressive and, respectively, accommodative equilibrium by marking states in which the predatory incentives according to a particular definition are positive. As noted above, the predatory incentives are positive if and only if the derivative of the EEEC profit function with respect to price is positive; hence, in the marked states firm 1 engages in an illegitimate profit sacrifice.

All sacrifice tests indicate predatory pricing in the deep well and trench of the aggressive equilibrium. The sacrifice tests according to Definitions 1 and 2 continue to indicate predatory pricing in other states involving much less aggressive pricing, such as state $(e_1, 4)$ for $e_1 \in \{5, \ldots, 30\}$ in which firm 1 charges a price above its marginal cost, but those according to Definitions 3–5 do not. We see the same pattern for the accommodative equilibrium.

### 5.2 Sacrifice tests and conduct restrictions

As antitrust authorities flag and prosecute an illegitimate profit sacrifice, they prevent a firm from pricing to achieve that sacrifice. In this way, applying a sacrifice test is akin to imposing a conduct restriction. The various definitions of predatory incentives in Section 4 indeed restrict the range of the firm’s price, e.g., Definition 1 prohibits the inclusive price, and thus also the actual price, from being less than marginal cost.

To gauge the consequences of applying a sacrifice test for industry structure and dynamics, we formalize a conduct restriction as a constraint $\Xi(p_1, p_2(e), e) = 0$ on the maximization problem on the right-hand side of the Bellman equation (3) that the firm solves in the price-setting phase. We form the constraint by rewriting our decomposition (6) as

$$
\begin{align*}
mr_1(p_1, p_2(e)) - c(e_1) + & \left[ \sum_{k=1}^{5} \Gamma^k_1(e) \pm \Gamma^3_1(e) \big|_{\phi_2=0} \pm \Gamma^3_1(e) \big|_{\phi_2=\phi_2(e)} \right] \\
+ & \left[ \sum_{k=1}^{4} \Theta^k_1(e) \pm \Theta^3_1(e) \big|_{\phi_2=0} \pm \Theta^3_1(e) \big|_{\phi_2=\phi_2(e)} \right] = 0
\end{align*}
$$

and “switching off” the predatory incentives according to a particular definition.\(^{15}\) For example, applying a sacrifice test according to Definition 2 in effect forces the firm to ignore

---

\(^{15}\)The notation $\pm$ means that we add and subtract the relevant term.
<table>
<thead>
<tr>
<th>e</th>
<th>$p_1(e)$</th>
<th>$c(e_1)$</th>
<th>advantage building</th>
<th>advantage denying</th>
<th>sacrifice tests</th>
<th>SRP</th>
<th>DCV</th>
<th>OW</th>
<th>CR</th>
<th>REX</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>-34.78</td>
<td>10.00</td>
<td>39.45 6.44 0.02 0.00 -0.01</td>
<td>0.93 0.03 0.44 -0.51</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,1)</td>
<td>0.08</td>
<td>7.50</td>
<td>4.27 0.02 0.00 0.00 -0.20</td>
<td>32.93 6.45 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,1)</td>
<td>0.56</td>
<td>6.34</td>
<td>2.94 0.01 0.00 0.00 -0.12</td>
<td>33.96 6.27 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,1)</td>
<td>0.80</td>
<td>5.63</td>
<td>2.20 0.01 0.00 0.00 -0.08</td>
<td>34.54 6.17 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5,1)</td>
<td>0.95</td>
<td>5.13</td>
<td>1.71 0.01 0.00 0.00 -0.05</td>
<td>34.91 6.10 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6,1)</td>
<td>1.05</td>
<td>4.75</td>
<td>1.36 0.00 0.00 0.00 -0.04</td>
<td>35.17 6.06 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7,1)</td>
<td>1.11</td>
<td>4.46</td>
<td>1.09 0.00 0.00 0.00 -0.03</td>
<td>35.35 6.02 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,4)</td>
<td>4.41</td>
<td>10.00</td>
<td>5.21 0.00 1.92 -0.52 0.00</td>
<td>0.00 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,4)</td>
<td>6.06</td>
<td>7.50</td>
<td>2.87 0.00 0.00 0.00 0.00</td>
<td>0.16 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,4)</td>
<td>5.79</td>
<td>6.34</td>
<td>2.12 0.00 0.00 0.00 0.00</td>
<td>0.24 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,4)</td>
<td>5.65</td>
<td>5.63</td>
<td>1.66 0.00 0.00 0.00 0.00</td>
<td>0.31 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5,4)</td>
<td>5.56</td>
<td>5.13</td>
<td>1.34 0.00 0.00 0.00 0.00</td>
<td>0.35 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6,4)</td>
<td>5.49</td>
<td>4.75</td>
<td>1.10 0.00 0.00 0.00 0.00</td>
<td>0.39 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7,4)</td>
<td>5.45</td>
<td>4.46</td>
<td>0.90 0.00 0.00 0.00 0.00</td>
<td>0.41 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(14,1)</td>
<td>1.24</td>
<td>3.34</td>
<td>0.09 0.00 0.00 0.00 0.00</td>
<td>35.71 5.96 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(15,1)</td>
<td>1.24</td>
<td>3.25</td>
<td>0.00 0.00 0.00 0.00 0.00</td>
<td>35.71 5.96 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(16,1)</td>
<td>1.24</td>
<td>3.25</td>
<td>0.00 0.00 0.00 0.00 0.00</td>
<td>35.71 5.96 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(30,1)</td>
<td>1.24</td>
<td>3.25</td>
<td>0.00 0.00 0.00 0.00 0.00</td>
<td>35.71 5.96 0.00 0.00</td>
<td>√√ √√ √√ √√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1,4)</td>
<td>4.41</td>
<td>10.00</td>
<td>5.21 0.00 1.92 -0.52 0.00</td>
<td>0.00 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2,4)</td>
<td>6.06</td>
<td>7.50</td>
<td>2.87 0.00 0.00 0.00 0.00</td>
<td>0.16 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3,4)</td>
<td>5.79</td>
<td>6.34</td>
<td>2.12 0.00 0.00 0.00 0.00</td>
<td>0.24 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4,4)</td>
<td>5.65</td>
<td>5.63</td>
<td>1.66 0.00 0.00 0.00 0.00</td>
<td>0.31 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5,4)</td>
<td>5.56</td>
<td>5.13</td>
<td>1.34 0.00 0.00 0.00 0.00</td>
<td>0.35 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6,4)</td>
<td>5.49</td>
<td>4.75</td>
<td>1.10 0.00 0.00 0.00 0.00</td>
<td>0.39 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7,4)</td>
<td>5.45</td>
<td>4.46</td>
<td>0.90 0.00 0.00 0.00 0.00</td>
<td>0.41 0.00 0.00 0.00</td>
<td>√√ √√</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Decomposed advantage-building and advantage-denying motives and sacrifice tests according to Definitions 1–5. √√ means that the predatory incentives are larger than 0.5, √ that they are between 0 and 0.5, and a blank that they are smaller or equal to 0. Aggressive equilibrium.
Table 5: Decomposed advantage-building and advantage-denying motives and sacrifice tests according to Definitions 1–5. ✓✓ means that the predatory incentives are larger than 0.5, ✓ that they are between 0 and 0.5, and a blank that they are smaller or equal to 0. Accommodative equilibrium.
Figure 3: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $e = (1, 1)$ at $T = 0$ (right panels). Accommodative equilibrium with conduct restriction according to Definition 1.

\[
\sum_{k=1}^{4} \Theta_k^1(e) \quad \text{in setting its price, so the constraint is } \Xi_1(p_1, p_2(e), e) = mr_1(p_1, p_2(e)) - c(e_1) + \left[ \sum_{k=1}^{6} \Gamma_k^1(e) \right] = 0.
\]

We compute the Markov perfect equilibria of the counterfactual game with a conduct restriction (according to a particular definition) in place. For the conduct restrictions according to Definitions 1 and 2, respectively, Figures 3 and 4 show the pricing decision of firm 1, the non-operating probability of firm 2, and the time path of the probability distribution over industry structures. Comparing the left panels in Figures 3 and 4 to the left panels in Figure 2, we see that there are neither deep wells nor trenches in the pricing decision and that the counterfactuals are accommodative in nature. This is because the intense competition for the market in the trench of an aggressive equilibrium is driven almost entirely by the baseline advantage-denying motive and the advantage-denying/exit motive (see Table 4). By shutting down the advantage-denying motive in its entirety, the conduct restrictions according to Definitions 1 and 2 eliminate a trench and thus the mobility barrier that is likely to lead to an entrenched monopoly over time. As further discussed in Besanko et al. (2013), it is as if these conduct restrictions eliminate the aggressive (as well as the intermediate) equilibrium of the original game.

Just as there are multiple equilibria in the original game, there are multiple equilibria in the counterfactual game with a conduct restriction according to Definitions 3–5. For
two of these three equilibria, Figures 5–7 show the pricing decision of firm 1, the non-operating probability of firm 2, and the time path of the probability distribution over industry structures. The counterfactuals in the upper panels are aggressive in nature while those in the lower panels are accommodative.

Further comparing industry structure, conduct, and performance between counterfactuals and equilibria tells us how much bite the conduct restrictions have. The panels labeled “SRP”, “DCV”, “OW”, “CR”, and “REX” in Table 3 illustrate industry structure, conduct, and performance implied by the equilibria of the counterfactual game with a conduct restriction according to Definitions 1–5.

Table 3 shows little changes between counterfactuals and equilibria, holding fixed the type of equilibrium behavior. To the extent that there are changes, they are sometimes for the better but sometimes for the worse. Compared to the intermediate equilibrium of the original game, the conduct restrictions according to Definitions 3–5 increase concentration and prices and decrease expected long-run consumer surplus $CS^\infty$. The most striking feature of Table 3 is though that the conduct restrictions according to Definitions 1–5 decrease expected discounted consumer surplus $CS^{NPV}$, sometimes substantially so.

The conduct restriction according to Definition 1 substantially decreases $CS^{NPV}$ because, by shutting down the dynamic incentives in their entirety, it denies the efficiency gains from
Figure 5: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $e = (1, 1)$ at $T = 0$ (right panels). Aggressive (upper panels) and accommodative (lower panels) equilibria with conduct restriction according to Definition 3.
Figure 6: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $e = (1, 1)$ at $T = 0$ (right panels). Aggressive (upper panels) and accommodative (lower panels) equilibria with conduct restriction according to Definition 4.
Figure 7: Pricing decision of firm 1 (left panels), non-operating probability of firm 2 (middle panels), and time path of probability distribution over industry structures, starting from $e = (1, 1)$ at $T = 0$ (right panels). Aggressive (upper panels) and accommodative (lower panels) equilibria with conduct restriction according to Definition 5.
pricing aggressively in order to move down the learning curve. In addition, the conduct restriction according to Definition 1 annihilates competition for the market. As can be seen by comparing the left panel of Figure 3 with the lower left panel panel of Figure 2, the shallow well in the accommodative equilibrium of the original game is absent. In contrast, the conduct restriction according to Definition 2 allows a shallow well, as can be seen in the left panel of Figure 4. Because it preserves a modicum of competition for the market, the conduct restriction according to Definition 2 decreases expected discounted consumer surplus much more modestly.

The conduct restrictions according to Definitions 3–5 are similar, as may be expected given their more narrow focus on the immediate impact of a price cut on rival exit. These conduct restrictions, in particular, force the firm to ignore the advantage-building/exit motive—thereby limiting the competition for the market in the well of an aggressive equilibrium—and the advantage-denying/exit motive—thereby limiting the competition for the market in the trench. Especially because the well is less deep, the conduct restrictions according to Definitions 3–5 decrease expected discounted consumer surplus $CS_{NPV}$ compared to the aggressive equilibrium of the original game.

6 Concluding remarks

To detect the presence of predatory pricing, antitrust authorities routinely ask whether a firm sacrifices current profit in exchange for the expectation of higher future profit following the exit of its rival. Because predatory pricing is an inherently dynamic phenomenon, we show in this paper how to construct sacrifice tests for predatory pricing in a modern industry-dynamics framework along the lines of Ericson & Pakes (1995). In particular, we adapt the definitions of predation due to Ordover & Willig (1981) and Cabral & Riordan (1997) to this setting and construct the corresponding sacrifice tests.

To do so, we build on Besanko et al. (2013) and decompose the equilibrium pricing condition in a model of learning-by-doing. Our decomposition highlights the various incentives that a firm faces when it decides on a price. Some of these incentives may be judged to be predatory while others reflect the pursuit of efficiency. We establish formally that certain terms in our decomposition map into the definitions of predation due to Ordover & Willig (1981) and Cabral & Riordan (1997). We furthermore use our decomposition to develop multiple alternative characterizations of a firm’s predatory pricing incentives and construct the corresponding sacrifice tests.

In a dynamic pricing model like ours, consumers benefit in the short run from competition for the market and in the long run from competition in the market. An antitrust policy boosting both seems ideal. To gauge the consequences of applying sacrifice tests, we note
that as antitrust authorities flag and prosecute an illegitimate profit sacrifice, they prevent a firm from pricing to achieve that sacrifice. An illustrative example shows that, to the extent that forcing firms to ignore the predatory incentives in setting their prices has an impact, this impact arises largely because equilibria involving predation-like behavior are eliminated. The example finally illustrates that applying sacrifice tests may limit competition for the market and may thus harm consumers, at least in the short run.

Appendix

Proof of Proposition 1. The probability that firm 2 exits the industry in the current period (given \( p_2(e) \) and \( e \)) is

\[
\Phi_2(p_1, p_2(e), e) = \phi_2(e) D_0(p_1, p_2(e)) + \phi_2(e_1 + 1, e_2) D_1(p_1, p_2(e)) + \phi_2(e_1, e_2 + 1) D_2(p_1, p_2(e)) \\
= [\phi_2(e_1 + 1, e_2) - \phi_2(e)] D_1(p_1, p_2(e)) - [\phi_2(e) - \phi_2(e_1, e_2 + 1)] D_2(p_1, p_2(e)).
\]

We say that \( p_1(e) \) is predatory according to the Cabral & Riordan (1997) definition if there exists a price \( \bar{p}_1 > p_1(e) \) such that (1) \( \Phi_2(p_1(e), p_2(e), e) > \Phi_2(\bar{p}_1, p_2(e), e) \) and (2) \( \Pi_1(p_1(e), p_2(e), e)|_{\phi_2=\phi_2(e)} < \Pi_1(\bar{p}_1, p_2(e), e)|_{\phi_2=\phi_2(e)} \).

Part (a): Let \( \bar{p}_1 = \arg \max_{p_1} \Pi_1(p_1, p_2(e), e)|_{\phi_2=\phi_2(e)} \). Then \( \bar{p}_1 \) is uniquely determined by

\[
mr_1(\bar{p}_1, p_2(e)) - c(e_1) + \left[ \Gamma_1^1(e) + \Gamma_1^3(e) \bigg|_{\phi_2=\phi_2(e)} + \Gamma_1^4(e) + \Gamma_1^4(e) \right] \\
+ \Upsilon(p_2(e)) \left[ \Theta_1^1(e) \bigg|_{\phi_2=\phi_2(e)} + \Theta_1^3(e) \bigg|_{\phi_2=\phi_2(e)} + \Theta_1^4(e) \right] = 0.
\]

Subtracting equation (6) from equation (11), we have

\[
mr_1(\bar{p}_1, p_2(e)) - mr_1(p_1(e), p_2(e)) = \left[ \Gamma_1^2(e) + \left[ \Gamma_1^3(e) - \Gamma_1^3(e) \bigg|_{\phi_2=\phi_2(e)} \right] \right] \\
+ \Upsilon(p_2(e)) \left[ \left[ \Theta_1^1(e) - \Theta_1^1(e) \bigg|_{\phi_2=\phi_2(e)} \right] + \Theta_1^3(e) + \left[ \Theta_1^3(e) - \Theta_1^3(e) \bigg|_{\phi_2=\phi_2(e)} \right] \right] > 0
\]

per inequality (7). Because \( mr_1(p_1, p_2(e)) \) is strictly increasing in \( p_1 \), it follows that \( \bar{p}_1 > p_1(e) \).

Because \( \Gamma_1^1(e) \geq 0 \) and \( \Theta_1^3(e) \geq 0 \), with at least one of these inequalities being strict, under the maintained assumptions of Proposition 1 it follows that \( \phi_2(e_1 + 1, e_2) - \phi_2(e) \geq 0 \) and \( \phi_2(e) - \phi_2(e_1, e_2 + 1) \geq 0 \), with at least one of these inequalities being strict. Using equation (6),

\[
\frac{\partial \Phi_2(p_1, p_2(e), e)}{\partial p_1} = [\phi_2(e_1 + 1, e_2) - \phi_2(e)] \frac{\partial D_1(p_1, p_2(e))}{\partial p_1} + [\phi_2(e) - \phi_2(e_1, e_2 + 1)] \frac{\partial D_2(p_1, p_2(e))}{\partial p_1} < 0
\]

since \( \frac{\partial D_1(p_1, p_2(e))}{\partial p_1} < 0 \) and \( \frac{\partial D_2(p_1, p_2(e))}{\partial p_1} > 0 \). Thus, \( \Phi_2(p_1(e), p_2(e), e) > \Phi_2(\bar{p}_1, p_2(e), e) \).

This establishes part (1) of the Cabral & Riordan definition above.
To establish part (2), recall that by construction $\Pi_1(p_1(e), p_2(e), e)|_{\phi_2=\phi_2(e)} \leq \Pi_1(\tilde{p}_1, p_2(e), e)|_{\phi_2=\phi_2(e)}$. Moreover, this inequality is strict because $\Pi_1(p_1(e), p_2(e), e)|_{\phi_2=\phi_2(e)}$ is strictly quasiconcave in $p_1$.

**Part (b):** Because $p_1(e)$ is predatory according to the Cabral & Riordan definition, there exists a higher price $\tilde{p}_1 > p_1(e)$ such that (1) $\Phi_2(p_1(e), p_2(e), e) > \Phi_2(\tilde{p}_1, p_2(e), e)$ and (2) $\Pi_1(p_1(e), p_2(e), e)|_{\phi_2=\phi_2(e)} < \Pi_1(\tilde{p}_1, p_2(e), e)|_{\phi_2=\phi_2(e)}$. Thus we have

$$\Phi_2(p_1(e), p_2(e), e) - \Phi_2(\tilde{p}_1, p_2(e), e) = [D_1(p_1(e), p_2(e)) - D_1(\tilde{p}_1, p_2(e))][\phi_2(e_1 + 1, e_2) - \phi_2(e)] - [D_2(p_1(e), p_2(e)) - D_2(\tilde{p}_1, p_2(e))][\phi_2(e) - \phi_2(e_1, e_2 + 1)] > 0. \tag{12}$$

Because $\frac{\partial D_1(p_1, p_2(e))}{\partial p_1} < 0$ and $\frac{\partial D_2(p_1, p_2(e))}{\partial p_1} > 0$, $D_1(p_1(e), p_2(e)) - D_1(\tilde{p}_1, p_2(e)) < 0$ and $D_2(p_1(e), p_2(e)) - D_2(\tilde{p}_1, p_2(e)) < 0$. The only way for inequality (12) to hold is thus that $\phi_2(e_1 + 1, e_2) - \phi_2(e) > 0$ or $\phi_2(e) - \phi_2(e_1, e_2 + 1) > 0$ which, in turn, implies $\Gamma_2^2(e) > 0$ or $\Theta_2^2(e) > 0$.

Like $\Pi_1(p_1, p_2(e), e)$, $\Pi_1(p_1, p_2(e), e)|_{\phi_2=\phi_2(e)}$ is strictly quasiconcave. It follows from $\tilde{p}_1 > p_1(e)$ and $\Pi_1(p_1(e), p_2(e), e)|_{\phi_2=\phi_2(e)} < \Pi_1(\tilde{p}_1, p_2(e), e)|_{\phi_2=\phi_2(e)}$ that $\Pi_1(p_1, p_2(e), e)$ is strictly increasing in $p_1$ at $p_1(e)$. (If not, then either $p_1(e)$ or $\Pi_1(p_1, p_2(e), e)$ is strictly decreasing in $p_1$ at $p_1(e)$. In the latter case, $\tilde{p}_1 > p_1(e)$ then implies $\Pi_1(p_1(e), p_2(e), e)|_{\phi_2=\phi_2(e)} > \Pi_1(\tilde{p}_1, p_2(e), e)|_{\phi_2=\phi_2(e)}$ because $\Pi_1(p_1, p_2(e), e)$ is a single-peaked function, again contradicting the hypothesis that $\tilde{p}_1$ is more profitable than $p_1(e)$.) Thus,

$$\frac{\partial}{\partial p_1} \Pi_1(p_1(e), p_2(e), e)|_{\phi_2=\phi_2(e)} = \frac{\partial D_1(p_1(e), p_2(e))}{\partial p_1} \left\{ mr_1(p_1(e), p_2(e)) - c(e_1) + \left[ \Gamma_1^1(e) + \Gamma_1^3(e) \right]_{\phi_2=\phi_2(e)} + \left[ \Gamma_1^4(e) + \Gamma_1^5(e) \right] \right\} + \Upsilon(p_2(e)) \left\{ \Theta_1^1(e)|_{\phi_2=\phi_2(e)} + \Theta_1^3(e)|_{\phi_2=\phi_2(e)} + \Theta_1^4(e) \right\} > 0$$

or

$$mr_1(p_1(e), p_2(e)) - c(e_1) + \left[ \Gamma_1^1(e) + \Gamma_1^3(e) \right]_{\phi_2=\phi_2(e)} + \left[ \Gamma_1^4(e) + \Gamma_1^5(e) \right] \right\} + \Upsilon(p_2(e)) \left\{ \Theta_1^1(e)|_{\phi_2=\phi_2(e)} + \Theta_1^3(e)|_{\phi_2=\phi_2(e)} + \Theta_1^4(e) \right\} < 0. \tag{13}$$

Subtracting inequality (13) from equation (6) then yields

$$\Gamma_1^2(e) + \left[ \Gamma_1^3(e) - \Gamma_1^4(e) \right]_{\phi_2=\phi_2(e)} + \Upsilon(p_2(e)) \left\{ \Theta_1^2(e)|_{\phi_2=\phi_2(e)} + \Theta_1^3(e) + \left[ \Theta_1^2(e) - \Theta_1^3(e) \right]_{\phi_2=\phi_2(e)} \right\} > 0.$$
References


