THE IMPACT OF COMPETITIVE ENTRY IN A DEVELOPING MARKET UPON DYNAMIC PRICING STRATEGIES*

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This paper analyzes dynamic pricing strategies for new durable goods in a two-period context. The first period is characterized as a monopoly market structure for a new product having dynamic demand. The second period begins when a new firm enters the market, and thereby changes the market structure to a duopolistic one.

We begin by analyzing the pricing strategies of three types of monopolists: nonmyopic, myopic and "surprised". A nonmyopic monopolist is a first entrant who perfectly predicts the competitive entry. A myopic monopolist totally discounts the duopolistic period, and a "surprised" monopolist is a first entrant who has the longer time horizon of the nonmyopic monopolist, but who does not foresee the competitive entry. Our results indicate that the nature of these pricing strategies may be quite different. It is optimal for the nonmyopic firm to set its product at a higher price than the myopic monopolist. Additional results indicate under what circumstances the "surprised" monopolist will price too high during the monopoly period. The intuition behind these results is the fact that the myopic monopolist overestimates competition while the surprised monopolist underestimates it.

We also characterize the nature of the dynamic equilibrium prices that will prevail during the competitive period. For example, we show that products having higher prices (because of cost differences) will exhibit a more rapid rate of price decline. Moreover, a discontinuity in the first entrant's pricing strategy, as a response to a second entry—which is often observed in the market place—is also captured by our model.

Because these analyses are limited to situations where the time of second entry is predictable, a scenario that fits our model better would be the health-care equipment industry where a specified period of government observation and/or testing is required. Immediate extensions of our model should include a discount factor and learning effects on both costs and demand.

(New Product Competition; Market Saturation; Entry Deterrence; Dynamic Pricing)

1. Introduction

The diffusion of a new brand is an evolutionary process that calls for long-term planning and timely managing. Among the key strategic questions that product managers face over time are (1) should the price of the product be set at a high level initially, and reduced later, or vice versa?; (2) what is the pricing strategy that will best defend the product during its leadership period from possible competition?; (3) what should the

* An unabridged version of this paper is available from the authors. See Eliashberg-Jeuland (1984) in References.
leader's price level and market share be at the beginning of a competitive period?; and 4) what is an effective competitive pricing strategy during the competitive period? This paper addresses these questions.

More generally, the objective of this research is to start analyzing the impact of different factors (product differentiation, demand dynamics, fixed and variable costs, uncertainty, etc.) on optimal pricing strategies of new market entrants, given that the competitive environment is characterized by a dynamic process of sequential entries. The more specific objective of this particular paper is to investigate the case of a first entrant being in a monopoly position for some time and facing a potential competitor for a subsequent period. We limit ourselves to the case of a new durable so that buyers make only one purchase. The case of a durable is of particular significance in the context of sequential entry by firms: Given that buyers make only one purchase in the foreseeable future, the first entrant’s market penetration during the initial monopoly period translates into an equal reduction in volume of potential demand for the subsequent competitive period. Consequently, the question of optimality of the pricing strategy of the first entrant is important in this situation. An example of a market situation where a monopoly period was followed by a duopoly period is the launching of the SX-70 by Polaroid. Several years later, Kodak entered the market with its own version of the instant color camera. Due to lead times caused by government testing requirements or patent protection of known length many markets (e.g., medical equipment) start out as monopolies that evolve predictably toward oligopolies.

Significant portions of the marketing and economics literature deal with the dynamics of duopolistic and oligopolistic price competition. Recent research in marketing includes Clarke-Dolan [1983] and Rao-Bass [1985]. They study oligopolistic pricing decisions. These efforts have evolved from the area of optimal monopolistic pricing of new products. Some of these earlier papers are Robinson and Lakhani [1975], Bass [1980], Dolan-Jeuland [1981], Jeuland-Dolan [1982], Clarke et al. [1982] and more recently Kalish [1983]. Thompson and Teng [1984] develop an oligopolistic model of interaction between price and advertising where no new entry is assumed to take place. They also assume that there is a single selling price decided by the largest competitor. Dash and Rao [1983] also postulate an interaction model (price and advertising) but only for the monopoly case. The present paper deals with the same subject matter as Clarke-Dolan and Rao-Bass. It differs from the simulation approach of Clarke-Dolan in that we aim for analytical results that are precise, have generality, and are as powerful as possible. The basic difference with Rao-Bass is our assumption of differentiated products. Instead, Rao-Bass study a commodity-type product.

As far as the economics literature is concerned, the area most closely related to this paper is the limit pricing literature. The limit pricing literature has grown significantly over the past 15 years, some of the important papers being the dynamic models developed by Gaskins [1967] and Kamien and Schwartz [1971, 1972]. The limit pricing models treat the process of market entry endogenously: high prices and associated high profits during the early periods attract competitors and increase the likelihood of entry. While it certainly is true that the firm that first develops a new product can influence the time of introduction of a competitive product, there are also significant rigidities associated with the timing of market entries. New product development usually takes several years to complete from idea generation to actual commercialization. It typically involves concept testing, product development and testing, business analysis, pilot production, market testing and production capacity investments. Some phases may be more easily expedited than others. However, the overall shortening of time to introduction may not be very significant. Moreover, the time of entry of the competitor may be more a function of technical know-how of this competitor—and the extent to which the first entrant is protected by patents. Clearly, there are markets where the assumption of certain date of
potential competitive entry is more tenable. Furthermore, entry deterrence via lower prices is possible only if some conditions are satisfied (for example conditions on demand or uncertainty conditions—see Friedman 1979, Milgrom and Roberts 1982). Because our model is a complete information model, it can only handle issues of limit pricing or entry deterrence via demand considerations and not via uncertainty.

The paper is organized as follows. §2 states the problem under investigation, describes the model, and states our assumptions. §3 develops our key analytical results concerning the optimal pricing strategies. Several numerical simulations are also discussed. §4 addresses the question of extensions of the model. In our conclusion, the managerial implications and the significance of the results are discussed.

2. The Model

As briefly stated earlier, our major objective is to formulate the optimal pricing strategy of a firm that introduces a new durable first and anticipates competition in the future. This overall strategy involves two substrategies, one during the monopoly period, and another during the duopoly period. Also studied is the strategy of a second entrant during the duopoly period. Figure 1 represents the market scenario under investigation.

The dynamic demand equations are postulated as:

\[
\dot{x}_i = (M - x_i)\alpha_i(1 - kp_i),
\]

\[x_i(0) = 0, \quad 0 \leq t \leq T_1 \quad \text{(Monopoly period),} \quad (1)\]

\[x_2(0) = 0, \quad (2)\]

\[\dot{x}_i = [m - (x - x_i(T_2))]\alpha_i(1 - kp_i) + \gamma(p_j - p_i)],
\]

\[i, j = 1, 2, \quad j \neq i, \quad T_1 \leq t \leq T_2, \quad \text{(Duopoly period),} \quad (2)\]

\[x = x_1 + x_2\]

\[x_1(T_1) > 0, \quad (3)\]

\[x_2(T_1) = 0, \quad (4)\]

with the notations defined as follows:

\[x_i \quad \text{cumulative sales of firm } i; \text{ first entrant is firm 1, second entrant is firm 2,} \]
\[T_1 \quad \text{time of second entry (zero is time of first entry),} \]
\[T_2 \quad \text{time horizon,} \]

FIGURE 1. Scenario of Sequential Market Entries.
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\[ \dot{x}_i = \frac{d}{dt}x_i, \]

\[ M \] total population size,

\[ m = M - x_i(T_1) \] uncaptured market at the beginning of the duopoly period,

\[ \alpha_i \] rate of diffusion of product \( i \),

\[ k \] price sensitivity,

\[ p_i \] price of product \( i \),

\[ \gamma \] effect of price differential between two products,

\[ x \] cumulative sales, \( x = x_1 + x_2 \).

The time argument has been omitted for notational simplicity.

The objective functions of the two firms are, respectively:

\[ \Pi_1 = \int_0^{T_2} (p_1 - c_1)\dot{x}_1 dt, \] (3)

\[ \Pi_2 = \int_{T_1}^{T_2} (p_2 - c_2)\dot{x}_2 dt, \] (4)

with \( c_i \) (\( i = 1, 2 \)) constant marginal costs of production. We turn now to an explicit discussion of the major assumptions underlying our model.

Assumptions

a. Dynamic demand equations. The dynamic demand equations combine two previously-used components. First, the growth component has been employed by Fourt and Woodlock [1960] and is also a component of the well-known diffusion model of Bass [1969]. Second, monopolist’s static linear demand function has been examined by Henderson and Quandt [1971, p. 236], Pashigian [1961, p. 20], McGuire, Weiss and Houston [1977, p. 133], Staelin and Winer [1976, pp. 672–3], Mansfield [1975, pp. 97–111] and Bulow [1982, p. 317]. The dynamic duopolistic demand equations (2) are a direct extension of the dynamic monopolistic demand function (1), and they are inspired by the static analyses of Wolf and Shubik [1978], and McGuire and Staelin [1983]. More recently those equations have also been adopted by Fershtman, Mahajan and Muller [1983] who show the conditions under which equilibrium market shares do not depend on the initial stock of goodwill of competing firms. In these equations, the parameter \( \alpha_i \) can be interpreted as overall marketing advantage—e.g., product availability and preference: For a given competitor’s price, as \( \alpha_i \) increases, demand at any given price \( p_i \) also increases. The parameter \( k \) reflects price sensitivity due to the fact that the two products compete with all other goods for the consumer’s dollars. The \( \gamma \) parameter reflects the amount of substitutability between the two products resulting from price differences.

b. Objective functions. Equations (3) and (4) have been constructed under the assumption of no discounting between time zero and the end of the finite time horizon, \( T_2 \), and complete discounting beyond \( T_2 \). This is made for mathematical tractability. In practice one would want the more general formulations \( \Pi_1 = \int_0^{T_2} (p_1 - c_1)\dot{x}_1 e^{-rt} dt \). However, this assumption does not alter the qualitative nature of our results because the step function used for discounting basically approximates the more appropriate negative exponential continuous function.

c. Constant marginal costs. Because our objective is to focus on the strategic consequences of demand dynamics, costs dynamics that would be reflected through experience curves are omitted in this paper. Again, the main consideration is mathematical tractability. Ultimately, it is an empirical question whether marginal costs of production are constant or not. Nicholson (1983, p. 247), for example, has cited some empirical evidence for constant marginal production costs. We feel that because our analysis is performed for a finite time horizon, the assumption of constant marginal costs is less severe.
d. Complete information. We assume complete information, i.e., both firms know each other’s costs and demand. In addition, it is assumed that the first entrant knows the time, $T_1$, when the second firm would enter (the earliest time when the latter can do so if it finds entry profitable). We also examine the situation where the incumbent firm is “surprised”, i.e. it does not foresee entry.

e. Noncooperative mode of behavior. Before formulating the profit maximization problem, one needs to define the criterion for optimality. We make the standard behavioral assumption that no cooperation is allowed during the duopoly period. Consequently, Nash equilibrium strategies will be sought. The Nash equilibrium solution of the two-player-game with objective functions $\Pi_1(p_1(t), p_2(t))$ and $\Pi_2(p_1(t), p_2(t))$ is the set of strategies, $\{p^*_1(t), p^*_2(t)\}$ satisfying $\Pi_1(p^*_1, p^*_2) \leq \Pi_1(p^*_1, p_2(t))$ and $\Pi_2(p^*_1, p^*_2) \leq \Pi_2(p_1(t), p^*_2)$. In other words, the Nash strategies are optimal in the sense that no competitor can obtain a better performance $\Pi$, for himself while his opponent continues to maintain his own Nash control strategy.

Before solving the complete problem, i.e., optimization over a monopoly period of length $T_1$ followed by a duopoly period of length $T_2 - T_1$, one needs to analyze the nature of the interrelationships between the strategies corresponding to the consecutive monopoly and duopoly periods. If the first entrant is nonmyopic and realizes that he will face competition after $T_1$, his strategy $p^*_1(t)$, $0 < t < T_1$, should reflect this foresight. Global optimality implies that a certain penetration $x^*(T_1) = x^*(T_1)$ will have been achieved at the end of the monopoly period. This optimal condition determines an optimal $m^*$, $m^* = M - x^*(T_1)$, the optimum uncaptured market available at the beginning of the duopoly period. We turn now to a detailed examination of these issues.

3. Optimal Pricing Strategies

This section first develops the optimal dynamic duopolistic strategies. We show that when these pricing strategies are designed at the beginning of the duopolistic period, they are independent of the initial conditions, $m$, although duopolistic profits obviously are not. For a nonmyopic monopolist (i.e., first entrant who perfectly predicts competitive entry), the optimum target penetration at the end of the monopoly period, $M - m^* = x^*(T_1)$, is characterized, as well as the optimal monopoly pricing strategy that achieves $x^*(T_1)$. The nonmyopic monopolistic pricing strategy is contrasted with the myopic and “surprised” monopolistic strategies. By myopic monopolist, we mean one who totally discounts the duopolistic period. The planning horizon of such firm is thus shorter. By a “surprised” monopolist we mean a first entrant who has the longer horizon of the nonmyopic monopolist but who does not foresee the competitive entry. We also prove that the globally optimum pricing strategy of the first entrant (the nonmyopic strategy case) displays a downward discontinuity at time of second entry.

Although most of the mathematical derivations are relegated to Appendices, we next provide an overview of the approach to obtain the optimal pricing strategy results. The analysis starts by deriving the necessary conditions for optimality by formulating the Hamiltonians of the two players (see Kamien and Schwartz 1981, Horsky and Simon 1983, Eliashberg and Chatterjee 1985, and Dolan, Jeuland and Muller 1985 for further details). The optimal dynamic pricing strategies $p^*_i(t)$ are the functions that maximize these Hamiltonians. Other necessary conditions are generated by formulating a set of differential equations. The derivations are given in Appendix A. The resulting Nash

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1 The second entrant starts with zero cumulative sales at time of entry. The monopolist’s additional sales during the duopoly period are $x_i = x_i(T_1)$ with initial conditions $x_i(T_1) = x_i(T_1) = 0$.

2 Piecewise continuous functions $p_i(t)$ are sought.
equilibrium strategies are the functions \( p_i(t) \) that satisfy a first-order quadratic differential equation system:\(^3\)

\[
\frac{dp_i}{dt} = A_i p_i^2 + B_i p_i p_j + C_i p_j^2 + D_i p_i + E_i p_j + F_i, \quad i, j = 1, 2, \quad j \neq i \quad (5)
\]

with end-conditions \( p_i(T_2) = p^* \) for \( i = 1, 2 \) being the Nash equilibrium prices corresponding to the static two-player game. Let each firm have the objective function

\[
\ln \Pi_i = (p_i - c_i)(a_i(l - k p_i) + \gamma(p_j - p_i)), \quad i, j = 1, 2, \quad j \neq i,\(^4\)
\]

then the static game prices \( p^* \) are obtained by solving the system of two equations \( \partial \Pi_i^S/\partial p_i = 0 \) for \( p_i, i = 1, 2. \)

Assuming the parametric condition: \( c_i < 1/k, \) and that an interior solution exists, i.e., \( p^*_i \) satisfies \( c_i < p^*_i < 1/k, \) one can then easily show that

\[
p^*_i = \frac{2(a_i k + \gamma)(c_i(a_i k + \gamma) + a_i)}{4(a_i k + \gamma)(a_i k + \gamma) - \gamma^2}, \quad i, j = 1, 2, \quad j \neq i. \quad (6)
\]

Although equation (6) appears to be a complex function of the basic parameters of our model, a number of points can be made:

(a) \( p^*_i \) is monotonically decreasing in \( \gamma. \)

(b) If \( c_1 = c_2, \) then the product that enjoys stronger customer response \( (\alpha_1 > \alpha_2) \) also will charge the higher terminal price.

(c) If \( \alpha_1 = \alpha_2, \) then the product which has the highest costs \( (c_1 > c_2) \) also will charge the higher terminal price.

Having discussed the boundary conditions (for \( t = T_2 \)) of the differential game of the duopoly period, we turn now to a discussion of the dynamic pricing implications (for \( T_1 < t < T_2 \)) of our model. The equilibrium pricing strategies satisfy the differential equations (5). We focus on various important properties of the optimal pricing strategies. This is done via a set of propositions and some proofs.\(^5\)

**PROPOSITION 1.** If the Nash equilibrium pricing strategies during the duopoly period are designed at the beginning of that period as functions of time only, they are independent of the penetration achieved by the first entrant at the time when the monopoly period ends and the duopoly period starts.

**PROOF.** In analyzing the dynamic game solution given by differential equations (5), note that the coefficients \( A_i \) through \( F_i \) are constants that are functions of six parameters of the model, namely \( \alpha_1, \alpha_2, c_1, c_2, k, \gamma. \) The specific expressions are given in Appendix A. The coefficients, however, do not depend upon \( m, \) market potential at the beginning of the duopoly period. The main implication of equations (5) with the end-conditions (6), which themselves do not depend upon \( m, \) is that the Nash strategies \( p_i^*(t) \) are independent of the initial state conditions \( x_i(T_1). \)  

**Q.E.D.**

The mathematical reason why the strategies are not dependent upon the initial state of the system is because of the particular separability assumption of equation (2). The

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\(^3\) It is worth noting that the general system \( dx/dt = F + Cx + Dp + Gx^3 + Hxy + Kx^2, dy/dt = E + Ax + By + Lx + Mxy + Ny^2 \) is a generalization of Volterra’s problem, of the growth of two conflicting populations [1931]. It has been the subject of extensive research by mathematicians dealing with nonlinear differential equations (see Davis 1960, Davies and James 1966). A thorough and general investigation of the system and its stability involves the identification of limit cycles and different types of singular points (nodal point, saddle point, focus vortex). This is beyond the scope of this paper.

\(^4\) Mathematically, this is a direct result of the zero-salvage conditions \( 4i(T_2) = 0 \) which imply \( H_i(T_2) = (p_i(T_2) - c_i)x_i(T_2). \) Because of the separability of the saturation term \( (M - x) \) and the term containing the control variables, \( H_i(T_2) \) is proportional to \( \Pi_i^S. \) The superscript \( S \) identifies \( \Pi_i^S \) as the static game profit function.

\(^5\) Detailed proofs of the more technical propositions are given in Appendix B which can be obtained by writing directly to the authors.
market saturation term \( m - (x - x_1(T_1)) \) can be factored out in the optimality equations \( \partial H_i/\partial p_i = 0 \) (see Appendix A) and this term is the only one that contains \( m \), the market size at the beginning of the duopoly period. The intuitive implication of Proposition 1 is as follows. When the first entrant designs, at \( t = T_1 \), for the duopoly period \((T_1, T_2)\), his equilibrium pricing strategy as a function of time only,\(^6\) he de facto ignores his behavior throughout the monopolistic period \([0, T_1]\). In particular, he ignores the penetration \( x_1(T_1) \) achieved at the end of the monopoly period. More complicated demand models would not necessarily result in this simplification.\(^7\)

By contrast, the pricing strategy that the first entrant designs at \( t = 0 \) for the monopoly period and that achieves a given penetration \( x_1(T_1) = x_1 \) at the end of the monopoly period is a function of both time and this penetration level \( x_1 \). We present this relationship in equation (7) (see proof in Appendix B). It applies to all three strategies of the nonmyopic, myopic and "surprised" monopolists. These three strategies vary only in the parameter \( x_1 \):

\[
p^*_H(t)|_{x_1} = \frac{1}{k} \left( 1 - \frac{2}{\alpha_t\{(T_1/(1 - \sqrt{1 - x_1/M})) - T_1\}} \right).
\]

Below we will discuss how \( x_1 \) changes as a function of whether the monopolist is nonmyopic, myopic or "surprised". The implication of equation (7) is that the monopolist's strategy during the monopoly period is a monotonically decreasing function\(^8\) of time. This is true even though no cost dynamics are postulated. Conditions under which price declines in monopolistic situations have also been presented in the literature (Kamien and Schwartz 1981 and Kalish 1983).

Proposition 1 and equation (7) imply simple relationships between the market penetration achieved by the first entrant at the end of the monopoly period, \( x_1(T_1) = x_1 \), and the profits that the firms will achieve over the entire period \([0, T_2]\). These relationships are given in Corollary 1.

In order to state this corollary, recall that
- \( i = 1 \) denotes the first entrant and \( i = 2 \) the second entrant,
- \( x_1 \) denotes the penetration achieved at the end of the monopoly period.

In addition, let \( \Pi^*_i(x_1) \) denote the optimal profits obtained by the monopolist for a given target penetration \( x_1 \),
- \( \Pi_{12} \) denote the profits obtained by firm \( i \) during the duopoly period,
- \( x^*_i \) denote the optimal penetration (when \( \Pi = \Pi_{11} + \Pi_{12} \) is maximized) achieved at the end of the monopoly period,
- \( L_i \) be constants which are independent of \( x_1 \).

Then:

**COROLLARY 1.**

(i) \( \Pi_{12} = L_i(M - x_1) \) for \( i = 1, 2 \).

(ii) \( \Pi^*_i(x_1) = \frac{1}{k} \left( (1 - k\alpha_t)x_1 - \frac{4M}{\alpha_t T_1} (1 - \sqrt{1 - x_1/M})^2 \right) \) is concave in \( x_1 \).

(iii) \( \partial \Pi^*_i/\partial x_1 |_{x_1} = L_i > 0 \).

\(^6\) This means that we are seeking open-loop strategies and not closed-loop strategies that would be of the form \( p(x, t) \).

\(^7\) Clemhout and Wan [1974] have studied a class of differential games that also have this property. However, because of the term \( m - x_1 - x_2 \), our game is not a special case of their class of trilinear games.

\(^8\) Obviously, \( x_1(T_1) \) should not be so large that it cannot be achieved at feasible price strategies (i.e., \( p(t) \geq 0 \)) given the rate at which sales take place (\( \alpha_t \) determines the diffusion rate). The largest penetration achievable at \( T_1 \) is \( x_1(T_1) = m(1 - e^{-\alpha_t T_1}) \) with \( p(t) = 0, t < T_1 \). If one considers the lower bound for \( p(t) \) to be \( c_1 \), then for \( t \leq T_1 \), the largest penetration achievable is \( m(1 - e^{-\alpha_t (\log 2c_1)^2}) \). There are indeed some legal constraints concerning pricing below cost. If target \( x_1(T_1) \) is high enough, the strategy may involve pricing at the lowest admissible price for some interval of time ending at \( T_1 \).
PROOF. See Appendix B for proof of (i), (ii), and for formal definitions of $L_i$.\(^9\) To prove (iii), one simply needs to observe that the first entrant’s optimal total profits over both periods, given a market penetration of $x_1$ at $T_1$, is:

$$\Pi^*_{\text{F}}(x_1) = \Pi^*_{\text{F}}(x_1) + \Pi^*_{\text{D}}(x_1) = \Pi^*_{\text{F}}(x_1) + L_i(M - x_1).$$

As a result, the optimal monopoly period penetration, $x^*_1$, is a solution to the first-order necessary condition $\frac{\partial \Pi^*_{\text{F}}(x_1)}{\partial x_1} = 0$. Because $L_i$ is independent of $x_1$, (iii) results. Q.E.D.

Corollary 1(i) states that each firm’s duopoly period profits are functions of the market penetration $x_1$ achieved at the beginning of the duopoly period only through the term $M - x_1$, i.e., the market potential that remains at $t = T_1$.

The immediate consequence of Corollary 1(ii), (iii) is the relationship stated in Proposition 2 between $x^*_1$, the optimal penetration target at $T_1$ of the nonmyopic first entrant, and the penetration $x^{**}_1$ that would be achieved by a myopic monopolist, i.e. who completely discounts the duopoly period.

**Proposition 2.** The optimum target $x^*_1$ of a nonmyopic monopolist is less than the target $x^{**}_1$ of a myopic monopolist.

**Proof.** The optimization problem for the myopic monopolist is the optimal control problem over the period $[0, T_1]$ with free end-point. Consequently $x^{**}_1$ is given by the equation $\frac{\partial \Pi^*_{\text{F}}(x_1)}{\partial x_1} = 0$. According to Corollary 1, $x^*_1$ must satisfy $\frac{\partial \Pi^*_{\text{F}}(x_1)}{\partial x_1} > 0$. This and the concave behavior of $\Pi^*_{\text{F}}(x_1)$ stated in Corollary 1 result in $x^*_1 < x^{**}_1$. Q.E.D.

The intuitive interpretation of Proposition 2 is that the nonmyopic monopolist does not feel compelled to achieve as high a penetration level as a myopic firm. He views the competitive (duopoly) period as an opportunity to achieve additional profits.

We can now also compare the pricing strategies of the nonmyopic and the myopic monopolist. This comparison is stated formally in Proposition 3 and follows intuitively from Proposition 2.

**Proposition 3.** During the monopoly period, the nonmyopic monopolist prices higher than the myopic monopolist and does not decrease his price as rapidly.

**Proof.** Equation (7) specifies that the pricing strategy as a function of $x_1$ for a monopolist of any sort (nonmyopic, myopic or “surprised”) is of the form $p_t = A - B/[S(x_1) - t]$ where $A$ and $B$ are constants independent of $x_1$, $A = 1/k$, $B = 2/k \alpha_1$, $S(x_1)$ is a decreasing function of $x_1$. Again, the value of $x_1$ determines whether the strategy is globally optimal or is myopic. Consequently, the rate of change of prices satisfies $|p_t'(t)| < |p^*_t(t)|$ so that the rate of change of prices satisfies $|p_t'(t)| < |p^*_t(t)|$ and $|p_t'(t)/p^*_t(t)| < |p^*_t(t)/p^*_t(t)|$. Q.E.D.

Taken together, Propositions 2 and 3 mean that the myopic monopolist with a short-time horizon (i.e., a monopolist who discounts the duopoly period) acts as if he would be at a strong disadvantage during the duopoly period and thus decides to leave available at the end of his short-time horizon a smaller fraction of the market. He achieves this by pricing lower. The lower price level results in a larger and faster penetration of the

\(^9\) $L_i$ is in fact equal to $-\psi_i(T_1)$, where $\psi_i(T_1)$ is the value of the adjoint variable for firm 1 at the beginning of the duopoly period (see Appendix A).

In Appendix A, the adjoint variables of the differential game have been defined as $\psi_1 = \psi_{11} = \psi_{12}$ and $\psi_2 = \psi_{21} = \psi_{22}$. The adjoint variable $\psi_i$ for firm $1$ satisfies

$$\dot{\psi}_i = (p_t - c_t + \psi_t) - \frac{x_i}{m - x} + \psi_t - \frac{x_i}{m - x} - \frac{\Pi_{12}}{m - x} + \psi_t - \frac{x}{m - x},$$

so that $\psi_1(x_1) = \Pi_{12} (p_t = c_t) x_i$. Consequently, after integration over $[T_1, T_2]$, $\psi_1(T_1) \times [M - x_i(T_1)] = \Pi_{12}$, since $\psi(T_2) = 0$. The constant $L_i$ is thus equal to $-\psi_i(T_1)$.\(^9\)
market that causes a steeper rate of change of price. In sum, the myopic monopolist de facto overestimates competition and prices lower as a consequence.

Another useful comparison is between the nonmyopic monopolist who envisions entry at \( T_1 \) and a "surprised" monopolist whose time horizon is \( T_2 > T_1 \) but who does not foresee entry at \( T_1 \). A priori, it is not obvious whether \( \bar{p}(t), t \in [0, T_1] \) for the nonmyopic monopolist is above or below the price of the "surprised" monopolist. Figure 2 illustrates the nature of this problem. The nonmyopic monopolist's strategy (who has time horizon \( T_2 \)) can be either \( C'C \) or \( B'B \) which correspond respectively to a higher or lower price strategy than the "surprised" monopolist with time horizon \( T'_2 \) (strategy \( E'E \)). As shown in equation (B-6) (Appendix B) the price charged by the "surprised" monopolist at point \( E \) corresponds to \( \bar{p}_E \) given by

\[
\bar{p}_E = \frac{1}{k} \left( 1 - \frac{2(1 - k_c)(T_2 - T_1)}{4 + \alpha_1(1 - k_c)(T_2 - T_1)} \right). \tag{8}
\]

On the other hand, \( p^*(T_1) = p(T_1, T_2) \) —the end of monopoly period price for the foresighted (nonmyopic) monopolist—can be obtained from equation (7) so that \( \bar{p}_E > p^*(T_1) \) if

\[
x^*(T_1) > M \left[ 1 - \frac{2}{4 + \alpha_1(1 - k_c)(T_2 - T_1)} \right]^2. \tag{9}
\]

Thus, determination of the magnitude of \( \bar{p}_E \) relative to \( p^*(T_1) \) depends on the particular parameter settings, and \( T_1 \) and \( T_2 \). Specific simulation results discussed later indicate that the "surprised" first entrant whose time horizon is the special case \( T'_2 = T_2 \) would

---

**Figure 2.** Foresighted monopolist who predicts entry at \( T_1 \) (and has time horizon \( T_2 \)) and surprised monopolist with time horizon \( T'_2 \) (who does not predict entry at \( T_1 \)).

---

\( ^{10} \) The curves in Figure 2 are all drawn with some concavity. The reason is that price strategies satisfy the equation

\[
\bar{p}_i = \frac{-\alpha}{2} \left( \frac{1}{k} - p_i \right)^2
\]

(see Appendix B) which implies \( \bar{p}_i = \kappa \alpha (1/k - p_i) \bar{p}_i < 0 \).
price higher during the monopoly period than the nonmyopic first entrant. Intuitively, the "surprised" monopolist expects a longer monopoly period than the nonmyopic monopolist and therefore follows a slower penetration strategy than is optimal. The nonmyopic monopolist, anticipating entry, has an incentive to gain greater sales of the durable good early on rather than split these sales with a competitor in the duopoly period.

We now focus on the discontinuity at \( T_1 \) in the nonmyopic first entrant's pricing strategy. Indeed, many sharp price declines, especially in response to a discrete event like the entry of a new competitor, are observed in practice. Abell and Hammond [1979, p. 115] call these periods of chaotic competition "shakeout" periods. The present model is consistent with the existence of price discontinuities at the time of a new entry. The direction of the price change is downward. The magnitude of the discontinuity is a function of the degree of competition represented in the model by the parameter \( \gamma \) of price differential. Proposition 4 formalizes this contention.

**Proposition 4.** The price charged by the first entrant—who correctly anticipates entry—immediately at the beginning of the duopoly period is lower than that charged by him at the end of the monopoly period. However, if there is no differential price effect \( (\gamma = 0) \), the first entrant's price is continuous at time of entry \( T_1 \).

**Proof.** See Appendix B.

Mathematically, the price drop is due to the discontinuity in the state equation which disappears when \( \gamma = 0 \). Intuitively, when \( \gamma = 0 \), it is reasonable that the first entrant's price is continuous at the time of entry because, in this case, the second entrant is not really a "competitor" of the first entrant. In fact, for \( \gamma = 0 \), the price path the nonmyopic monopolist chooses should be identical to the price path chosen by a monopolist with time horizon \( T_2 \) and no expectation of entry.

Finally, the last three propositions deal specifically with the dynamics of pricing during the duopoly period. They are useful for two major purposes. First, they can be used to assess the effect of market structure (monopoly vs. duopoly) on industry prices. Second, they can provide the marketing scientist with parameterizations of price dynamics that can be estimated on real data. We note cautiously, however, that before any empirical study is undertaken, the researcher must consider carefully the correspondence between the data and our theoretical model.

**Proposition 5.** For two similar competitors (i.e., \( \alpha_1 = \alpha_2 = \alpha \) and \( c_1 = c_2 = c \)), the identical Nash equilibrium pricing strategies are monotonically declining over the finite time horizon.

**Proof.** See Appendix B.

The significance of the proposition is that it shows that even in the absence of cost dynamics, declining prices over time are still optimal in duopolistic markets as long as the horizon is finite. This is similar to the monotonically declining monopoly pricing strategies discussed earlier. In addition, as shown in Appendix B, it is possible to derive the equilibrium pricing strategy for the identical duopolists. This function is:

\[
p^*(t) = p^*(T_2) = \frac{Qe^{RT_2 - 0} + S}{Ue^{RT_2 - 0} + V} \quad \text{where} \quad (10)
\]

\[
Q = (2\alpha + akc + 2\gamma c)(ak + \gamma), \quad R = \alpha(ak + 2\gamma)(1 - kc)/(2ak + \gamma), \quad S = -\alpha^2 k, \quad U = (3ak + 2\gamma)(ak + \gamma), \quad V = -\alpha^2 k^2.
\]

11 Identical \( \alpha \)'s does not mean identical preference and availability since one can compensate for the other.

12 As several numerical simulations that we have run show, the declining price result applies to more general situations than the case of similar competitors. Additional simulation results, which illustrate price strategies, penetrations and profits for varying degrees of the \( \gamma \) parameters, for nonmyopic and "surprised" monopolists, are provided in Eliashberg and Jeuland (August 1984).
It declines from a maximum at most equal to \((\lambda_0/k) + (1 - \lambda_0)c\) with \(\lambda_0 = 2ak/(3ak + 2\gamma)\) (this maximum is reached in the limit when the duopoly period becomes infinite in which case pricing over time is constant and equal to this maximum) to a low value of \((\lambda_1/k) + (1 - \lambda_1)c\) with \(\lambda_1 = ak/(2ak + \gamma)\) (reached at \(T_2\)). The monopolist with the same diffusion parameter \(\alpha\), price sensitivity parameter \(k\), marginal cost \(c\), and time horizon \(T_2\) would price according to the function (see (B-6)):

\[
p^{\ast\ast}(t) = \frac{1}{k} \left(1 + \frac{2(1 - kc)}{4 + \alpha(1 - kc)(T_2 - t)}\right).
\]  

(11)

It can be shown that (see Appendix B)

\[
p^{\ast\ast}(t) > p^{\ast\ast}(t) \quad \text{for} \quad 0 \leq t \leq T_2.
\]

(12)

The equality may only take place at \(t = T_2\).

Table 1 illustrates, for several values of the differential effect parameter \(\gamma\), the difference between the duopolistic and monopolistic dynamic prices.

Thus, the duopolistic environment causes prices to be lower and to decline less rapidly over time than in the monopolistic environment (smaller absolute first derivatives of price). We next explore the effects of the product differentiation parameter \(\gamma\) on prices for the case of equal marginal costs, but different diffusion parameters \(\alpha_i\).

**Proposition 6.** For two competitors with identical production costs \((c_i = c_j = c)\), but different customer response parameters \((\alpha_i \neq \alpha_j)\), when price differential parameter, \(\gamma\), becomes large, equilibrium strategies are always to price at cost \(c\).

**Proof.** See Appendix B.

This is the Bertrand solution (Bertrand 1883) but in a dynamic context. Since \(\gamma\) measures the intensity of the competition between the two products, the difference in prices charged (at any given point) by the two firms should decrease as \(\gamma\) increases, irrespective of any "\(\alpha\)-advantage" that one may have over the other. In the limit, this difference

**Table 1**

<table>
<thead>
<tr>
<th>(t = T_1)</th>
<th>(t = T_1 + 1)</th>
<th>(t = T_1 + 5)</th>
<th>(t = T_2 = T_1 + 10)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MONOPOLY</strong></td>
<td>(p^{\ast\ast})</td>
<td>(\frac{dp^{\ast\ast}}{dt})</td>
<td>(\frac{dp^{\ast\ast}}{dt})</td>
</tr>
<tr>
<td>(\gamma = 10^{-3})</td>
<td>$1368</td>
<td>$1358</td>
<td>$1314</td>
</tr>
<tr>
<td>(\gamma = 10^{-4})</td>
<td>$1369</td>
<td>$1358</td>
<td>$1314</td>
</tr>
<tr>
<td>(\gamma = 10^{-5})</td>
<td>$1369</td>
<td>$1358</td>
<td>$1314</td>
</tr>
<tr>
<td><strong>DUOPOLY</strong></td>
<td>(p^{\ast})</td>
<td>(\frac{dp^{\ast}}{dt})</td>
<td>(\frac{dp^{\ast}}{dt})</td>
</tr>
<tr>
<td>(\gamma = 10^{-3})</td>
<td>$569</td>
<td>$569</td>
<td>$569</td>
</tr>
<tr>
<td>(\gamma = 10^{-4})</td>
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<tr>
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<td>$569</td>
</tr>
<tr>
<td>(\gamma = 10^{-7})</td>
<td>$1324</td>
<td>$1318</td>
<td>$1291</td>
</tr>
</tbody>
</table>

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should be reduced to zero and the prices charged will equal the marginal cost. This is because the demand is extremely sensitive to price differentials (in the limit, the products are perfect substitutes), and hence, none of the firms can afford to charge more than the lowest price that is offered by the other firm. Otherwise they lose all sales. Note that the assumption of noncooperation between the two duopolists is key to this result.

**Proposition 7.** For two competitors with identical customer response parameters \((\alpha_i = \alpha_j = \alpha)\) but different costs \((c_i > c_j)\), the high cost manufacturer’s price is higher and declines more rapidly than the low cost manufacturer’s price. Mathematically, \(p_i(t) > p_j(t)\) and \(\dot{p}_i(t) < \dot{p}_j(t)\).

**Proof.** See Appendix B.

Proposition 7 implies that a firm with higher price level due to higher marginal costs will exhibit a more rapid rate of price decline.

Because the above propositions do not completely characterize the optimal strategies due to lack of closed form expressions for quantities like \(p_i(t), p_j(t), L_1\) and \(x^*(T_1)\), for general values of our parameter set, several numerical simulations were performed. Due to space limitations these results cannot be reported in full here, but can be found in Eliashberg-Jeuland (1984). See also Appendix C (available from the authors upon request) for a detailed description of the numerical solution to our differential game.

In brief, these simulations indicate that, as the price differential effect \((\gamma)\) increases, duopolistic price levels decrease and the gap between monopoly period prices and duopoly prices increases. Prices always decline over time, a phenomenon that is due to the progressive saturation of the market. Also, as the intensity of the competition between the two products increases, market penetration at \(T_2\) by both firms increases. The reason for this increase is the lower price levels that result from the more intense competition. The effect of stronger competition is also seen in the smaller profit levels achieved by the firms in the duopoly period.

Other simulations involved a “surprised” (who does not foresee entry at \(T_1\)) but adaptive first entrant whose time horizon is also \(T_2\). Once he observes entry he alters his strategy. Consequently, during period \((0, T_1)\), he follows the pricing strategy defined earlier in the paper as \(p_i^*(t)\). After observing entry at \(T_1\), the equilibrium duopolistic pricing strategies are followed by the two firms. Remember that duopolistic Nash strategies are only function of the time remaining in the game. In this scenario, the simulation results show that the “surprised” first entrant charges a price higher than he should during the monopoly period, and thus achieves a lower penetration than is optimal. This lower penetration at \(T_1\) results in a lower penetration at \(T_2\). The lower penetration achieved by the “surprised” first entrant at \(T_1\) leaves a larger market available for the second entrant. The latter thus achieves higher penetration than he would against an incumbent acting “globally” in an optimal fashion. Both firms makes more profits in the duopoly period. However, for the first entrant, this increase does not compensate for his lower profits in the monopoly period.

As far as the sensitivity of the optimal strategies to marginal production cost differences as well as to differences in the product diffusion parameters are concerned, our simulations show that when the second entrant has a cost advantage over the first entrant, the first entrant should price lower (than without this cost disadvantage) during the duopoly period, and make more profits in the monopoly period. He does so by charging lower prices and selling more during monopoly period. (An industry where the production technology is constantly improving over time is one where a cost advantage for later entrants may be expected.) Under the cost disadvantage scenario, the first entrant’s total profits are lower, however, because his duopoly profits decrease more than his monopoly profits can increase. When the second entrant does not have a cost advantage (an example is the first entrant tying up the best and cheapest sources of supply of inputs for the
production of the product) and has a weaker product (smaller diffusion parameter \( \alpha_2 \)), the first entrant finds it in his best self-interest to increase the price level in the monopoly period (relative to acting under cost disadvantage or the two identical competitors case) and thus achieves lower penetration at the time of second entry. His lower monopoly profits are more than compensated by higher duopoly profits achieved by lower duopolistic prices. As is expected, total penetration achieved by the first entrant is higher when he enjoys a product advantage than when he does not. His total penetration is lower when his competitor has a cost advantage than when he does not. Finally in the situation where the second entrant introduces a superior product (\( \alpha_2 > \alpha_1 \)) which is more costly to manufacture (\( c_2 > c_1 \)), our simulation results for profitability and penetration indicate that this is a very viable strategy for late entry. In fact, this situation may be a more likely scenario than the cost advantage. Indeed, it may be difficult for the second entrant to bring to market a lower cost product that is successfully differentiated: In many instances one may expect that achieving strong differentiation (low \( \gamma \)) requires additional costs (higher \( c_2 \)). In other words, one may expect a positive correlation between \( c_2 \) and \( \gamma \).

To sum up, the analytical as well as simulation results we studied have all assumed that entry is exogenous. The situations examined have been (1) the “nonmyopic” or “foresighted” monopolist who anticipates entry, (2) the “myopic” monopolist who completely discounts the period after entry, (3) the “surprised” monopolist who does not anticipate entry but, once entry has occurred, either does not adopt or adopts the optimal noncooperative Nash strategy during the duopoly period.

We have also obtained results for the situation where the first entrant realizes that effective deterrence of entry can be achieved by sufficiently penetrating the finite market for the durable good. The reason is that the market left at time of entry, may be made too small to justify the fixed cost of entry.\(^{13}\)

4. Conclusions and Further Extensions

The specification of optimal new product entry strategies over time is an important substantive area of marketing. Formal dynamic marketing models are being developed and, as they become more realistic, will provide a basic tool for strategic marketing decision-making.

In this paper, we focused on a two-period dynamic problem. The first period market structure is a monopoly and the second, a duopoly. Our results characterize the Nash equilibrium dynamic pricing strategies that will prevail in a duopoly market. When comparing industry prices for a duopolistic environment with the price of the single firm industry (monopoly), we have found that the monopoly price is higher and drops more rapidly. Also, lower product differentiation drives prices toward cost. A higher cost manufacturer prices higher than the other (lower cost) manufacturer, and his price drops more rapidly over time. If the product differentiation is insufficient, the higher cost manufacturer’s margin may be driven to zero. In this case, the higher cost manufacturer would not leave the industry if he intends to depress the other manufacturer’s profits.\(^{14}\)

In analyzing the monopoly period, we have found that a nonmyopic monopolist, who

\(^{13}\) The appropriate deterrence target \( x_0(T_i) > x^* \) (\( x^* \) is defined in Proposition 2) is the following function of fixed cost of entry \( F_2 \) of second firm: \( x_0(T_i) = 1 - (1 - x^*F_2)/\Omega_2(x^*) \). If \( F_2 \) is not too low, this entry deterrence strategy can be more profitable than the strategy that does not attempt to prevent entry. This deterrence strategy is credible because it is based on the irreversible penetration of the market. The profitability of entry deterrence is found to decrease as time of entry \( T_i \) is earlier and as degree of differentiation of second entrant is higher (\( \gamma \) smaller). Dixit (1983) has also analyzed the role of fixed costs and product differentiation as entry barriers. That low differentiation entry may be more vulnerable to entry deterrence may explain why me-too strategies are rarely encountered in practice.

\(^{14}\) The mathematical issue here is the possibility of obtaining corner solutions instead of interior solutions \( c_i < p_i < 1/k \) as assumed in the derivations of §§2 and 3.
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anticipates and is prepared for a competitive entry, will price his product higher than the myopic monopolist who perfectly discounts the duopoly period and de facto overestimates competition. In addition, our simulation results indicate that the nonmyopic firm prices lower than the “surprised” monopolist with an identical long time horizon but who fails to foresee entry. The reason is that this “surprised” monopolist de facto underestimates competition. More specifically, our simulations have shown that the nonmyopic monopolist with time horizon $T_2$ has, as lower bound, the myopic monopolist’s strategy and, as upper bound, the “surprised” monopolist’s strategy corresponding to the same time horizon of $T_2$.

The impact of the new entry on the nonmyopic monopolist’s behavior has been shown to take the form of a sudden drop in prices at the time when the entry occurs. Our simulations show that the magnitude of the price drop is decreasing with the level of product differentiation. In fact, one analytical result indicates that there is no price discontinuity at time of second entry if the intensity of competition parameter $\gamma$ is zero (i.e., perfect product differentiation exists). The model predicts declining prices in both the monopoly period and the duopoly period, even though no cost dynamics are postulated. This means that, as the market saturates, lower price is needed to stimulate additional demand. One should thus be cautious in interpreting declining prices as evidence of, for example, experience curve effects. Finally, we have generated additional simulations which indicate that entry deterrence can be a feasible strategy for the incumbent firm. It may even be more profitable than the optimal strategy that does not attempt to prevent entry. This is more likely if cost of entry is large enough and if product differentiation is not too strong.

Clearly, all these results are significant for new product strategy decisions. For example, when marketing managers must choose between the alternatives of “me-too” versus differentiation strategies: they could investigate this issue through a model such as ours which recognizes explicitly the relationship between the parameters $\gamma$ and $c_i$. Usually, one would expect that the second entrant would be able to achieve a lower cost if he chooses the “me-too” strategy instead of the differentiation strategy. The fixed costs are also important, in particular in planning entry-deterrence: fixed costs affect the overall profitability and thus the choice of an entry strategy. Furthermore, a differentiation strategy may mean higher fixed costs. Whether the firm should focus on achieving a marketing advantage or a production cost advantage can also be studied via the parameters of diffusion $\alpha$. A marketing advantage, be it higher product quality or higher advertising (brand recognition) or superior availability, would translate into a higher rate of diffusion $\alpha$. Models similar to the one presented in this paper can be used to study whether a firm should aim for a marketing advantage ($\alpha$-advantage) or a production cost advantage.

Finally, the model presented here is very parsimonious and may provide a useful framework for researchers who want to devise new measures for the early assessment of new product strategies. Intentions to purchase may be a valuable input to quantify the rate of diffusion $\alpha$ (Urban and Hauser 1981, pp. 480–481). Concept testing combined with price sensitivity analysis may be useful to assess the differentiation parameter $\gamma$ (see Gabor and Granger 1965). Also, given actual time series data on sales and prices in a developing market, a model of the type developed here may prove valuable for estimation purposes.

However, this model should not be oversold as it does have a number of limitations. The present paper can only be a preliminary attempt at investigating new product entry strategies. Immediate extensions should include a discount factor and learning effects on the cost side through the experience curve, as well as on the demand side through imitation effects (see Bass 1969). Incorporating other variables besides price, and the interaction between marketing mix strategies is also needed. The extension of the model to more than two periods will lead to the analysis of oligopolistic markets. The other demand
situation of nondurables should also be analyzed. Other modes of competitive behavior besides Nash are possible in practice and should be analyzed: In particular, when a very small number of firms are involved, cooperation—at least partial—may be possible.

Moreover, allowing for uncertainty about demand, cost and timing of entries is essential. As the work of Kamien and Schwartz [1971], Milgrom and Roberts [1982], and others show, uncertainty may be essential for entry deterrence to be rational. However, entry-deterrence is rational in our model, even in the absence of uncertainty, because of the dynamics of demand, i.e., the saturation effect.

In sum, we feel that we have just begun an analysis of an important strategic problem and continuing efforts should be made in the directions mentioned above. This research agenda is a tough challenge, however, as most of the extensions suggested greatly complicate the “coupling” between the initial monopoly period and the subsequent competitive periods.15

15 This paper was received December 1982 and has been with the authors for 4 revisions.

### Appendix A. Derivation of Nash Equilibrium Duopoly Strategies

The formulation of the differential game is as follows. The dynamics of the state variables are given by the system of differential equations:

\[ x_i = (M - x)(\alpha_i(1 - k p_i) + \gamma(p_j - p_i)), \quad i, j = 1, 2, \quad j \neq i, \]

\[ x_i(T_1) > 0, \quad x_j(T_2) = 0, \quad t \in [T_1, T_2]. \quad (A-1) \]

One Hamiltonian function is defined for each firm:

\[ H_i = (p_i - C_i)x_i + \psi_i x_i + \psi_{i2} x_2, \quad i = 1, 2, \quad (A-2) \]

where the adjoint variables (shadow prices or generalized Lagrangian multipliers) satisfy the end-conditions:

\[ \psi_i(T_2) = 0. \]

The adjoint variables satisfy the system of differential equations \[ \psi_i = -(dH_i/dx_j). \]

The optimal strategies are the \[ p^*(t) \] functions that optimize the corresponding Hamiltonian function, given \( A, \psi_i, x_i, x_2. \)

The first-order necessary conditions \[ p^* = \partial H_i/\partial p = 0 \] imply:

\[ \alpha_i(1 - k p_i) + \gamma(p_j - p_i) - (p_i - C_i + \psi_i(\alpha_i k + \gamma) + \gamma \psi_i) = 0, \quad i, j = 1, 2, \quad j \neq i. \quad (A-3) \]

The equations \[ \psi_i = -(dH_i/dx), \quad x = x_i + x_2 \] imply:

\[ \psi_i = (p_i - C_i + \psi_i(\alpha_i(1 - k p_i) + \gamma(p_j - p_i)) + \psi_i(\alpha_i(1 - k p_i) + \gamma(p_i - p_j)), \quad i, j = 1, 2, \quad i \neq j. \quad (A-4) \]

Differentiating (A-3) with respect to \( t \) and solving for \( \dot{p}_i \), one obtains:

\[ \dot{p}_i = -\frac{k}{\Delta} \{2\alpha_i(\alpha_i k + \gamma)\psi_i + \alpha_i \gamma \psi_i \} \quad \text{where} \]

\[ \Delta = 4(\alpha_i k + \gamma(\alpha_i k + \gamma)) - \gamma^2, \quad i, j = 1, 2, \quad i \neq j. \]

Rearranging terms, one obtains:

\[ \dot{p}_i = -\frac{k}{\Delta} \{A_i p_i^2 + B_i p_i p_j + C_i p_j^2 + D_i p_i + E_i p_j + F_i \}, \quad \text{where} \]

*Because no salvage term is assumed.

The second order conditions \[ \partial^2 H_i/\partial p_i^2 = -2(\alpha_i k + \gamma) < 0 \] indicate that the solution of the first order condition is a maximum (assuming an interior solution). Furthermore, prices \( p_i \) and adjoint variables \( \psi_i \) are independent of the state variables \( x_i \). Therefore, the maximized Hamiltonians are linear and hence concave in the state variables. Therefore, a solution to the necessary conditions is also a solution to the optimization problem (see Kamien and Schwartz 1981, p. 207).
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\[ A_i = \alpha_i(2(\alpha_i k + \gamma)(\alpha_i k + \gamma) - \gamma^2), \]
\[ B_i = 2(\alpha_i k + \gamma)2\alpha_i(\alpha_i k + \gamma) + \gamma\alpha_i, \]
\[ C_i = -\gamma\alpha_i(\alpha_i k + \gamma), \]
\[ D_i = -2(\alpha_i k + \gamma)(\gamma(\alpha_i + \alpha_i) + \alpha_i K)(1 + k_c). \]
\[ E_i = -2(\alpha_i k + \gamma)(\gamma(\alpha_i + \alpha_i) + \alpha_i K)(1 + k_c). \]
\[ F_i = 2\alpha_i(\alpha_i k + \gamma)^{\alpha_i(1 + k_c) + 2\gamma} \]
\[ + a_i \gamma^{-\alpha_i(1 + k_c) + 2\gamma} \]
\[ + i, j = 1, 2, i \neq j. \]

References


