A Model of Product Design and Information Disclosure Investments

Abstract

As online information availability for products and services is increasing and as consumers engage in more online search prior to purchase decisions, it is becoming more important for firms to know when to invest to reduce consumer uncertainty. This article argues that today's firms should view product design and investments to reduce consumer uncertainty as an integrated process, which is in turn heavily influenced by how much information consumers can obtain independently, for example, by reading other consumers' reviews or by accessing third party infomediaries. Using a game-theoretic model of a competitive market, we explain how product quality decisions influence future investments to reduce consumer uncertainty, and demonstrate how firms should take this dependency into account to avoid over-investing in quality. We also show that firms, and especially lower quality firms, can free ride on the product information already available in the market, and reduce their own disclosure investments. Finally we show that firms can take further advantage of such ambient information availability to ease the intensity of competition among them, and specifically to reduce their product quality investments.

1. Introduction

A major activity in markets is the transfer of product information by firms and third-party intermediaries to consumers. In recent years, technology has offered firms new ways with which to improve consumer knowledge about their products. Vendors can now showcase video instructions of product use as Apple does for its laptop computers; they can introduce interactive applications to help buyers find their perfect product fit, as prescription eyeglasses retailer Warby Parker does with its virtual try-on application (Miller, 2011); and can even employ technology that allows buyers to simulate the experience of owning the actual product, as gaming company Electronic Arts does with its playable demos and Microsoft has done with an HTML5 webpage that allows iPhone and Android users to experience the Windows Phone 7 interface. These technologies make use of interactive media to take advantage of increasing consumer familiarity with the internet.

Technology has also made it easy for consumers to acquire product information from independent third-parties (Montgomery et al 2004). These infomediaries offer expert product evaluations (Chen & Xie, 2005), aggregate consumer reviews (Koh, et al., 2010), or educate consumers about the product category in general. Many retailers, such as Amazon and J&R Electronics, also offer consumer reviews and expert product evaluations. The social commerce trend, currently exemplified by firms such as Facebook and Pinterest, also allows consumers to share product information and assessments with one another. The consequences of this "*ambient information*" made available by third parties is that consumers have an important alternative source of product information and need not rely exclusively on information provided by the manufacturers.

The economic and social consequences of these trends are significant. By 2009, 2% of total retail sales in the USA, and 10-15% of retail sales that involve the Internet, including Research Online-Purchase Offline – ROPO, were the direct result of consumers engaging in search for product information. The total value of search activity, including consumer surplus, exceeded $80 Billion (McKinsey Global Institute, 2011).

Although product videos, product simulators, playable demos and other similar technologies make it easier for firms to reveal product quality information to consumers, they can be expensive. For example, video game developers consider playable game demos as non-trivial investments (Crossley, 2010) and Electronic Arts has openly discussed the possibility of charging $10-$15 per comprehensive game demo (Martin, 2010), in order to justify the extra development costs. In fact, we collected data on 2196 PC games released between 1996 and 2005 and found that only 55% invested in creating a playable demo. Thus, firms have to determine when it is worthwhile to invest heavily in product information disclosure and, in particular, assess how ambient information available through infomediaries affects these decisions. Further, given the growing importance of information investments, these investments should be viewed as an integral part of the product design process. Firms have to determine how much to invest in quality given the subsequent cost of informing customers about these quality attributes. Consider a printer manufacturer who has recently released one high-end and one budget printer model. The firm needs to determine whether to invest in information disclosure for these products and, if so, whether to do so differently for the two models. Further, it needs to understand how to modify its decision in response to third-party expert reviews.

A recent real-world example is tied to problems facing computer processor designer AMD in 2008-09 (Vance, 2011) (Stokes, 2009). AMD was forced by changing consumer preferences to invest away from the "clock-speed" race, into designing products with improved mobile graphics and video. The firm believed that, should it go ahead with the investment, it would also need to invest heavily to inform customers about graphics and video support in its processor. For example, it would need to retrain its sales and marketing employees to be able to communicate the new processors' advantages to prospective customers. AMD needed to determine how much to invest in the new product design given the cost of retraining employees and rebranding the company. In also needed to know whether competition would intensify or weaken if the firm and its competitors started competing along a product dimension (graphics performance) for which limited ambient information exists, due to the absence of metrics that are widely familiar to end consumers.

The key managerial questions that we address in this paper are tied to product information disclosure in the presence of third-party infomediaries. First, we ask how firms should invest in informing consumers and, in particular, how product quality influences this decision. Second, we ask how firms should factor in the cost of future information disclosure when choosing product quality levels. Finally, we ask how ambient information from infomediaries affects the answers to the above questions.

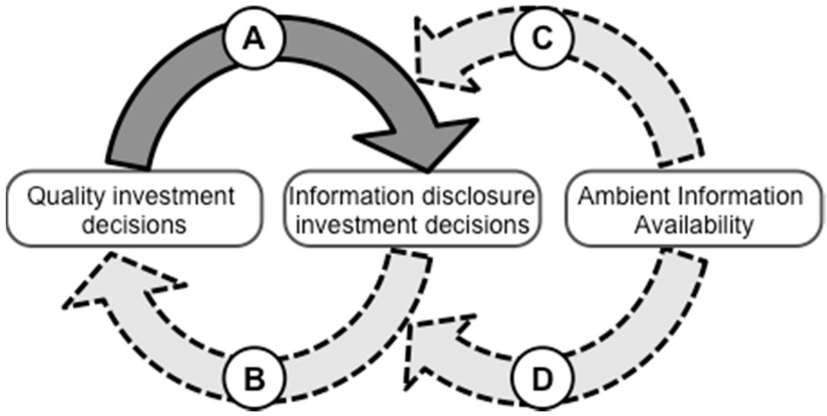


Figure 1: *Key interactions considered by our model*

These questions describe and give rise to the following four interactions (*Figure 1*).

A. Product quality choices influence future investments in informing consumers. Almost all prior work focuses on this interaction, depicted with a darker solid arrow, and many articles build on the “quality unraveling” argument, first presented by Grossman, Hart, and Milgrom (Milgrom, 1981) (Grossman & Hart, 1980) (Grossman, 1981), and discussed in more detail in Section 2.

B. The mechanism by which firms decide to invest and reduce consumer uncertainty, affects product information availability and should also affect product quality, because during the product design phase firms look forward to the future information regime, in order to decide how much to invest in product quality. This interaction is mostly ignored by prior models that study how firms invest in reducing consumer uncertainty but treat product quality as exogenous.

C. Firms should take into account ambient information availability, e.g., due to infomediaries, when investing to reduce consumer uncertainty, because information made available by third parties and information made available by the firms themselves are, to an extent, substitutes.

D. Finally, since ambient information availability affects information disclosure decisions, it must also impact quality investment decisions. Previous literature omits the last two interactions, with the notable exceptions of (Albano & Lizzeri, 2001), who study analytically the operation of a certification intermediary in a monopoly and (Chen & Xie, 2005), who study how firms should adjust their information investments in response to improved ambient information availability.

Considering the four interactions together makes our model more realistic and leads to findings that are not available to simpler models that study these interactions in a piecemeal fashion. More specifically, Section 4.1 focuses on interaction A and establishes the preliminary result that the ex-ante probability of quality disclosure increases smoothly with equilibrium quality. Section 4.2 builds upon this and focuses on interaction B: we show that firms should view product quality investments and information disclosure investments as an integrated process and that, by doing so, they should moderate quality investments. This is contrary to a naive argument that consumers might discount the quality less for those firms who have shown a willingness to invest in quality, thereby making it attractive to over-invest in quality. In Section 4.3 we focus on interaction C and find that information availability by third parties allows firms, and especially lower quality firms, to *free ride* (reduce their information disclosure investments). Others have argued that high quality firms benefit more from de-emphasizing product information (Chang & Wildt, 1994). A simplistic conclusion that it would be easier for a high-quality firm to de-emphasize product information when third parties already provide it is shown to be wrong. Finally Section 4.4 focuses on interaction D and shows that, surprisingly, information made available by third parties allows firms to reduce their product quality. That is, the intuitive argument that firms anticipating a better informed market will increase their quality investments is shown to be wrong.

2. Related Literature

Much of the current research on the role and availability of product information traces back to the work of Grossman, Milgrom, and Hart (Grossman & Hart, 1980) (Grossman, 1981) (Milgrom, 1981) and the "unraveling mechanism". The mechanism explains that high quality vendors first report their product quality in order to separate themselves from the group of vendors with unknown quality. That leaves another set of vendors as the highest quality vendors in the group that has not reported yet, giving them the incentive to report their quality in turn, and so on, until all but the lowest quality vendors have provided quality information. Grossman (1981) showed that a similar argument applies even to monopolists who will want to report all but the lowest possible product quality. Grossman & Hart argue that "*the buyers need not be particularly sophisticated or have repeated experience with the seller.*[…] *The buyer must just use the simple logic that the seller tries to be as optimistic as possible about his product subject to the constraint that he not lie"* (Grossman & Hart, 1980).

Jovanovic (1982) and Shavell (1994) studied how vendors decide to inform consumers about their quality, when disclosure entails a costly investment. They used the unraveling mechanism to show that vendors will invest in disclosure only above a quality threshold that depends on the investment cost. Empirical support for the positive impact of higher quality to quality disclosure investments has been provided by (Mathios, 2000). There are, however, arguments that claim the opposite, particularly when pricing is endogenous and can be an alternative signaling mechanism. Chang & Wildt (1994) designed a laboratory experiment where they show that "*price exerts a positive influence on perceived quality, […] moderated by the importance and amount of intrinsic information*". The role of pricing as a signaling mechanism is well known (Wolinsky, 1983) (Milgrom & Roberts, 1986), but Chang & Wildt show that it can substitute costly quality disclosure. The authors recommend that leading brands de-emphasize product information and focus more on using prices as a signaling mechanism. In support of this view, Sun (2011) points out that among the 100 best-selling magazines in Amazon.com in 2006, the 13 that won the National Magazine Award in General Excellence, offered fewer free trials than the rest, and that the more recent the award, and thus the more reliable the quality signal, the less likely was the magazine to offer a free trial.

There is also an extensive body of work on the impact of competition on quality disclosure. Jin (2005) observes that disclosure investments in the healthcare industry are lower in more competitive markets. However, Jin argues that because quality disclosure decisions are influenced by the underlying product quality, the manner in which competition impacts quality disclosure depends on the quality choices of vendors and manufacturers; since the literature makes "ambiguous predictions" on the impact of competition to product quality (Tirole, 1988), the relationship between competition and quality disclosure also depends on a complex combination of factors.

Indeed, one the one hand, there is work that suggests that the presence of multiple firms sharpens the incentives for information revelation. This may be because under intense competition even small changes in quality perceptions become important (Stivers, 2004), or because, when the market is unaware that the firms possess any new information, the announcement by one firm of news, signals to the market that other firms may also possess new information (Dye & Sridhar, 1995). On the other hand, some researchers argue that as the number of firms increases, vendors avoid disclosing information that may actually sharpen the competition between them, especially if the information is related to production costs or demand functions that are privately known to oligopolists (Okuno-Fujiwara, et al., 1990)[1], or if the information revealed educates the consumers about the product category in general, in which case firms in more competitive markets will be less likely to disclose in order to prevent competitors from free riding on their efforts (Jin, 2005).

On the same theme of firms trying to alleviate price competition by withholding product information, (Hotz & Xiao, 2010) and (Board, 2009) present duopoly models where, under certain conditions, a firm may choose to withhold quality information, even if it is costless and can actually improve consumers' perceptions about the firm's quality.

Table 1 lists the key relevant articles in the literature. In summary, the vast majority of papers in the literature on uncertain quality attributes has treated firms' quality choices as exogenous and has not considered the role of ambient information on firms' decision to disclose quality attributes. Two notable exceptions are the works by (Albano & Lizzeri, 2001), who present the first uncertainty model to include endogenous quality production in a monopoly setting, and (Chen & Xie, 2005), who investigate quality uncertainty and the role of infomediaries in a duopoly setting. However, in (Albano & Lizzeri, 2001) buyers cannot be informed by sellers but only by infomediaries, and the article's applicability is somewhat limited to markets that require a certification authority, e.g., an auditor. In (Chen & Xie, 2005) the authors focus solely on information disclosure and their model provides no insight to infomediaries' impact to product quality. Ours is the only model to include all the four key interactions identified in Figure 1.

[1] Oligopoly information sharing (Gal-Or, 1986) (Gal-Or, 1987) is often hard to extend to product information.

Authors	Year	Number of vendors	Info-med-iaries?	Vertical Differentiation: Quality attributes present?	Vertical Differentiation: Quality attributes endogenous?	Vertical Differentiation: Uncertainty over quality attributes?	Horizontal Differentiation: Taste attributes present?	Horizontal Differentiation: Taste attributes endogenous?	Horizontal Differentiation: Uncertainty over taste attributes?
Grossman, Hart	1980	N, 1	No	Yes	No	Yes	No	-	-
Grossman	1981	1	No	Yes	No	Yes	No	-	-
Milgrom	1981	N, 1	No	Yes	No	Yes	No	-	-
Jovanovic	1982	Large	No	Yes	No	Yes	No	-	-
Verrecchia	1983	1	No	Yes	No	Yes	No	-	-
Grossman, Shapiro	1984	N	No	No	-	-	Yes	Potentially	Yes
Wolinsky *	1984	N	No	No	-	-	Yes	Yes	Yes
Matthews, Postlewaite	1985	1	No	Yes	No	Yes	No	-	-
Dye	1986	1	No	Yes	No	Yes	No	-	-
Milgrom, Roberts	1986	N, 1	No	Yes	No	Yes	No	-	-
Okuno-F. et al **	1990	N	No	Yes	No	Yes	Yes***	No	Yes***
Economides *	1993	N	No	Yes	Yes	No	Yes	Yes	No
Shavell	1994	N	No	Yes	No	Yes	No	-	-
Meuer, Stahl	1994	1,2	No	No	-	-	Yes	No	Yes
Dye, Sridhar	1995	N	No	Yes	No	Yes	No	-	-
Bakos *	1997	N	No	No	-	-	Yes	Yes	Yes
Albano, Lizzeri	2001	1	Yes	Yes	Yes	Yes	No	-	-
Fishman, Hagerty	2003	1	No	Yes	No	Yes	No	-	-
Stivers	2004	N, 1	No	Yes	No	Yes	No	-	-
Cheong, Kim	2004	Large	No	Yes	No	Yes	No	-	-
Chen, Xie	2005	2	Yes	Yes	No	Yes	Yes	No	Yes****
Anderson, Renault	2006	1	No	No	-	-	Yes	No	Yes
Anderson, Renault	2009	2	No	Yes	No	No	Yes	No	Yes
Board	2009	2	No	Yes	No	Yes	No	-	-
Levin, Peck, Ye	2009	1,2	No	Yes	No	Yes	Yes	No	No
Markopoulos et al.	2010	N, 1	No	No	-	-	Yes	Yes	Yes
Hotz, Xiao	2010	2	No	Yes	No	Yes	Yes	No	No
Sun	2011	1	No	Yes	No	Yes	Yes	No	Yes
Current model	**2012**	**N**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**	**Yes**

* (Wolinsky 1984), (Economides 1993) & (Bakos 1997) are similar to our model but are not private information models.
****** We include this article as a representative of the literature on information sharing among oligopolists
*** Model does not deal with product attributes per-se, but rather with the idea of equilibrium payoffs being monotonic in beliefs of other firms' type. It may be able to encompass taste attributes, but the authors do not pursue the topic.
****Firms are not able to selectively inform the consumers about only their quality or only their taste product attributes

Table 1: *A comparison of previous models with codified features*

3. Analytical Model & Solution

3.1. Model and Game Set-up

We consider a market with N vendors in which products are characterized by one taste[2] and one quality characteristic. Following (Economides, 1993), we represent this in the form of a cylindrical market (see Figure 2). The unit size circumference represents the taste (type) choices, while the cylinder's height is the quality attribute space. Figure 2 depicts four vendors S_1,S_2,S_3 & S_4 with $(d_i, q_i), i = 1 \dots 4$, as the type and quality of their products respectively. Vendor enumeration proceeds sequentially and vendors with sequential indexes are neighbors in the taste space.

We assume that buyers can obtain perfect information on product prices and vendor identities, but not so for product attributes, unless vendors actively invest in informing them. If the vendors do not invest in information disclosure, consumers can rely on the information that is made available by

[2] The terms *Taste* and *Type* are used interchangeably throughout the article. Economists also use the terms *Variety* and *Horizontal Differentiation*.

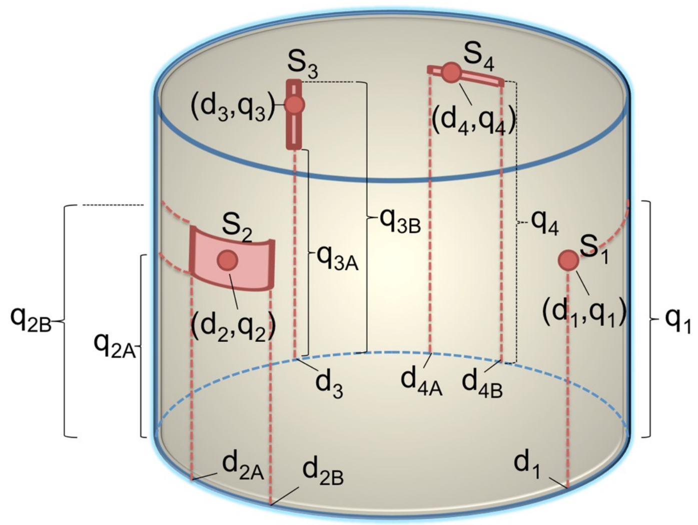


Figure 2: *Our model of product differentiation and consumer uncertainty*

third party intermediaries. Buyers' priors regarding these uncertain attributes are represented as a probability distribution whose support includes the product's actual location, in both product dimensions. For example, in Figure 2 consumers perceive the quality of vendor S_2 as a random variable $\overline{q_2}$ with PDF $f_{q2}(\cdot)$, CDF $F_{q2}(\cdot)$ and support $[q_{2A}, q_{2B}]$. The interval $[q_{2A}, q_{2B}]$ is termed the "*quality uncertainty interval*" for seller S_2 and it has size $\alpha_q \equiv q_{2B} - q_{2A}$, which is the same for all sellers. Similarly the information about the type of vendor S_2 is perceived by buyers as a random variable $\overline{d_2}$ with PDF $f_{d2}(\cdot)$, CDF $f_{d2}(\cdot)$ and support $[d_{2A}, d_{2B}]$. The interval $[d_{2A}, d_{2B}]$ is termed the "*type uncertainty interval*" for seller S_2, with size $\alpha_d \equiv d_{2B} - d_{2A}$, which is the same for all sellers[3]. Of course, $f_{qi}(\cdot)$ and $f_{di}(\cdot)$ are priors that may be rationally updated by the buyers, once they take into account additional information, such as vendor disclosure decisions. Thus, buyers can and do consider uncertainty intervals of different sizes prior to their purchase decision. Further we will assume that $f_{qi}(\cdot)$ and $f_{di}(\cdot)$ are uniform and independent for all $i = 1 \dots N$.

We do not require that uncertainty intervals are centered at the actual product location. For example a product whose actual quality is at the low end of its quality uncertainty interval, appears to prospective buyers to be better than it actually is. On the contrary, a product whose actual quality is at the high end of its quality uncertainty interval appears to be worse than it actually is. As more information about both products begins to become available, the expected quality of the first product would appear to reduce, while the expected quality of the second product would appear to increase. Both cases, as well as the case of expected product quality remaining constant when more information emerges, are easily observable in real markets. For example, in (Li & Hitt, 2008), the authors document the wide difference between how books sold on Amazon are initially perceived by

[3] We can show that the results are qualitatively similar, even if uncertainty intervals have different sizes from the beginning. In the Online Appendix, we explicitly account for unequal uncertainty interval sizes when we present the general equilibrium solution

consumers when information (i.e., reviews) about them is relatively sparse, and how the same books are perceived in the long run steady state, when enough reviews have accumulated. We view books with steep, downward-adjusting average review, as books whose true quality happened to be at the high end of their initial quality uncertainty intervals.

If vendors are dissatisfied by the level of information that infomediaries provide, they can invest in information disclosure. In Figure 2, vendor S_2 has chosen to disclose neither his exact quality nor his exact type, vendor S_1 has disclosed his exact values on both dimensions (and hence depicted in the figure as a dot), vendor S_3 has chosen to disclose full taste information but no quality information and vendor S_4 has disclosed full quality but no taste information. These four choices represent the disclosure decision space of the vendors.

All buyers demand one unit of the product. Each consumer is defined by his most preferred product location in the taste space z. Consumers are uniformly distributed around the cylinder's circumference in this respect. The value to the consumer of purchasing a product with characteristics (d_i, q_i) and price p_i is $V(z, d_i, q_i, p_i) = v + \theta \cdot q_i - p_i - t \cdot |z - d_i|$, where v is the consumer's utility from consuming a product at the minimum possible quality that is located at his most preferred type, θ denotes the intensity of the consumer's preference for quality, and t, the fit cost parameter, denotes the disutility that the consumer experiences from consuming a product that is not his ideal. The parameters t and θ are common to all buyers. A usual additional assumption in spatial models is that v is large enough (or, alternatively, t is small enough) so that all the market is served (Hotz & Xiao, 2010) (Tirole, 1988). This assumption is needed so that none of the vendors become a monopolist – a case that requires substantially different analysis, and is in fact pursued in a separate article. We provide the exact requirement for v after we have introduced all relevant notation and before the solution to the symmetric equilibrium, below.

The expected utility to a consumer with preferred product location $z \notin [d_{iA}, d_{iB}]$ from purchasing from vendor S_i, is given by $E(V) = v + \theta \cdot E(\overline{q_i}) - p_i - t \cdot \left|z - E(\overline{d_i})\right|$ where $E(\cdot)$ denotes expected value.

Let $r_i \in \{0, c_q\}$ denote the quality disclosure investment cost for vendor S_i, which is zero if the vendor does not release quality information and $c_q > 0$ otherwise. We assume convex costs of quality of the form $C(q_i) = k{q_i}^2/2$. Similarly $s_i \in \{0, c_d\}$ denotes type disclosure investment cost for Firm S_i. In addition, we assume that a firm's type or quality uncertainty is fully resolved upon investment by the firm. The revenue and profit functions for firm S_i in vector notation, are given by:

$$R_i(\mathbf{p}, \mathbf{q}, \mathbf{d}) = p_i \cdot D_i(\mathbf{p}, \mathbf{q}, \mathbf{d}), \quad \Pi_i(\mathbf{p}, \mathbf{q}, \mathbf{d}) = p_i \cdot D_i(\mathbf{p}, \mathbf{q}, \mathbf{d}) - C(q_i) - r_i - s_i \qquad \textbf{[1]}$$

where $\boldsymbol{p} = (p_1, p_2, \ldots, p_N), \boldsymbol{q} = (E(\overline{q_i}), E(\overline{q_2}), \ldots, E(\overline{q_N})),\ \boldsymbol{d} = \left(E(\overline{d_i}), E(\overline{d_2}), \ldots, E(\overline{d_N})\right)$. Note that quality production cost depends on actual product quality, but demand depends on perceived quality.

We assume that vendors seek to maximize profits and that buyers seek to maximize their utility, and define our game as follows:

- **Stage 1**: All vendors choose their type.
- **Stage 2**: All vendors choose their level of quality investment (they thus choose their quality).
- Vendors learn how their own products are perceived by early users in pre-market trials. They thus learn the uncertainty intervals (chosen by nature) that will be associated with their products if they do not invest in information disclosure.
- **Stage 3**: All vendors make their decisions on whether to invest in information disclosure.
- All vendors and buyers learn the information of vendors who invested in disclosure and learn from infomediaries the uncertainty intervals of vendors who have not invested in disclosure, and update their expectations about product characteristics.
- **Stage 4**: All vendors choose their price.
- Buyers enter the market and make their purchase decisions.

Note that Stages 1& 2 are in effect being played simultaneously as vendors do not receive information on competitors' choices. In the Online Appendix, we also discuss alternative game structures.

While the main focus of this article is product quality, our model also includes taste-related product attributes. Besides making the model more realistic, the main reason for this is that the inclusion of a horizontal product dimension gives rise to multiple possible equilibria, so that it can reasonably be assumed that consumers are uncertain about product characteristics[4]. An additional advantage is that the model avoids Bertrand-type competition among sellers with very similar qualities. Models that exclude the horizontal dimension often lead to perverse incentives towards quality investments, with vendors wanting consumers to discount their quality, if by doing so they prevent consumers from learning that their quality is very similar to a competitor, all in their effort to reach a separable equilibrium with positive profit. A final reason is managerial: the horizontal dimension changes equilibrium dynamics by making profits convex on quality: a quality improvement allows a firm to increase price while capturing additional market share along the horizontal dimension. This, as we shall see, subtly changes the quality unraveling argument and leads to novel managerial insights.

We focus on the case where uncertainty about horizontal attributes is relatively low. For obtaining a closed form solution of the equilibrium that is symmetric on product types, it suffices to assume $\alpha_d < 1/(2N)$, which is simple to show that translates to assuming that horizontal uncertainty intervals will never overlap. We will provide the intuition behind each of our findings, without employing this assumption. Further, the assumption is relaxed in the Online Appendix and is not employed at all for quality uncertainty intervals, for which we only require that α_q is lower than the minimum product quality (buyers will never think that product quality is negative).

[4] In other words, there is no simple unique equilibrium for consumers to derive and reverse engineer product characteristics. Instead, we require minimum bounded rationality, in the sense of having assumed above that buyers do not have any priors for the likelihood that different equilibria are played, and will simply accept $f_{qi}(\cdot)$ and $f_{di}(\cdot)$ as priors that they can update, once they observe vendor disclosure decisions.

3.2. Analytical Solution

We next present the equilibrium solution to the subgame comprised of the last game stage (Stage 4, pricing). Then, we present the subgame perfect solution of the full game, leaving the analysis of Stages 1, 2 and 3 for the Appendix. The analysis of the last game stage is sufficient to intuitively understand the model results that follow, beginning in the next section.

Let the marginal consumer who is indifferent between vendors S_i and S_{i+1} have type z_i. Assuming $\alpha_d < 1/(2N)$, we can show that $z_i = \frac{E(\overline{d_i})+E(\overline{d_{i+1}})}{2} + \frac{p_{i+1}-p_i}{2t} + \frac{\theta(E(\overline{q_i})-E(\overline{q_{i+1}}))}{2t}$.

Repeating the calculation for z_{i-1}, we derive the demand $z_i - z_{i-1}$ for Firm S_i :

$$D_i = \frac{E(\overline{d_{i+1}}) - E(\overline{d_{i-1}})}{2} + \frac{p_{i+1} + p_{i-1} - 2p_i}{2t} + \frac{\theta(2E(\overline{q_i}) - E(\overline{q_{i+1}}) - E(\overline{q_{i-1}}))}{2t} \qquad \textbf{[2]}$$

Thus, a firm captures half the buyers that are in between its closest two competitors in the taste space (first term) adjusted for the price differential (second term) and the quality differential (third term). Note that a vendor's demand function (2) is independent of his own choice of type d_i except in the sense that it determines the two closest competitors. Also note that arithmetic in the taste space is done modulo the unit circle. For example, the neighbors of firm S_N are firms S_{N-1} and S_1, and a distance $d_{i+1} - d_i$ is taken to be $1 + d_{i+1} - d_i$, if $d_{i+1} < d_i$.

Differentiating Equation [1] with respect to price yields: $p_i - \frac{(p_{i-1}+p_{i+1})}{4} = \frac{t\left(E(\overline{d_{i+1}})-E(\overline{d_{i-1}})\right)}{4} + \frac{\theta(2E(\overline{q_i})-E(\overline{q_{i+1}})-E(\overline{q_{i-1}}))}{4}$. Substituting into Equation [2] we get $D_i^* = p_i^*/t$ and thus:

$$\Pi_i^*(p_i^*, \mathbf{q}, \mathbf{d}) = p_i^{*2}/t - C(q_i) - r_i - s_i \qquad \textbf{[3]}$$

In the pricing subgame, vendors have already chosen their product details and have made their disclosure decisions. Vendor S_i maximizes profits with respect to price p_i (Equation [1]) when: $p_i - \frac{p_{i-1}}{4} - \frac{p_{i+1}}{4} = \frac{t}{4}\left(E(\overline{d_{i+1}}) - E(\overline{d_{i-1}})\right) + \frac{\theta}{4}\left(2E(\overline{q_i}) - E(\overline{q_{i+1}}) - E(\overline{q_{i-1}})\right)$. In matrix form:

$$\mathbf{A} \cdot \mathbf{p} = t \cdot \mathbf{x} + \theta \cdot \mathbf{H} \cdot \mathbf{q} \qquad \textbf{[4]}$$

where matrix $\mathbf{A}$ has $\mathbf{A}_i = [A_{i,1} = 0, A_{i,2} = 0, \dots, A_{i,i-1} = -\frac{1}{4}, A_{i,i} = 1, A_{i,i+1} = -\frac{1}{4}, \dots, A_{i,N} = 0]$, $\mathbf{x}$ is the vector with $x_i = \left(E(\overline{d_{i+1}}) - E(\overline{d_{i-1}})\right)/4$, $\mathbf{q}$ is the vector with $\mathbf{q}_i = E(\overline{q_i})$, and $\mathbf{H} = \mathbf{A} - \mathbf{I}/2$, with $\mathbf{A}, \mathbf{H}$ circulant symmetric matrices of constants, which means that the inverse of $\mathbf{A}$ exist, as long as the first row sums up to a non-zero constant, as is the case here.

Since $\mathbf{A}, \mathbf{H}$ are invertible, Equation [4] has a unique solution[5] with

$$\mathbf{p}^*(\mathbf{q}, \mathbf{d}) = \mathbf{A}^{-1}(t \cdot \mathbf{x}(\mathbf{d}) + \theta \cdot \mathbf{H} \cdot \mathbf{q}) \equiv \mathbf{A}^{-1} \cdot \mathbf{e}(\mathbf{q}, \mathbf{d}) \qquad \textbf{[5]}$$

[5] As we indicated before, all arithmetic is done modulo the circle, so that, for example, summing up equilibrium prices, we have to account twice for the transition from position 1 to position 0, once for $E(\overline{d_1}) - E(\overline{d_{N-1}})$ and once for $E(\overline{d_2}) - E(\overline{d_N})$, yielding $\frac{\sum p_i}{2} = \frac{t}{4}(2) \Rightarrow \sum p_i = t$.

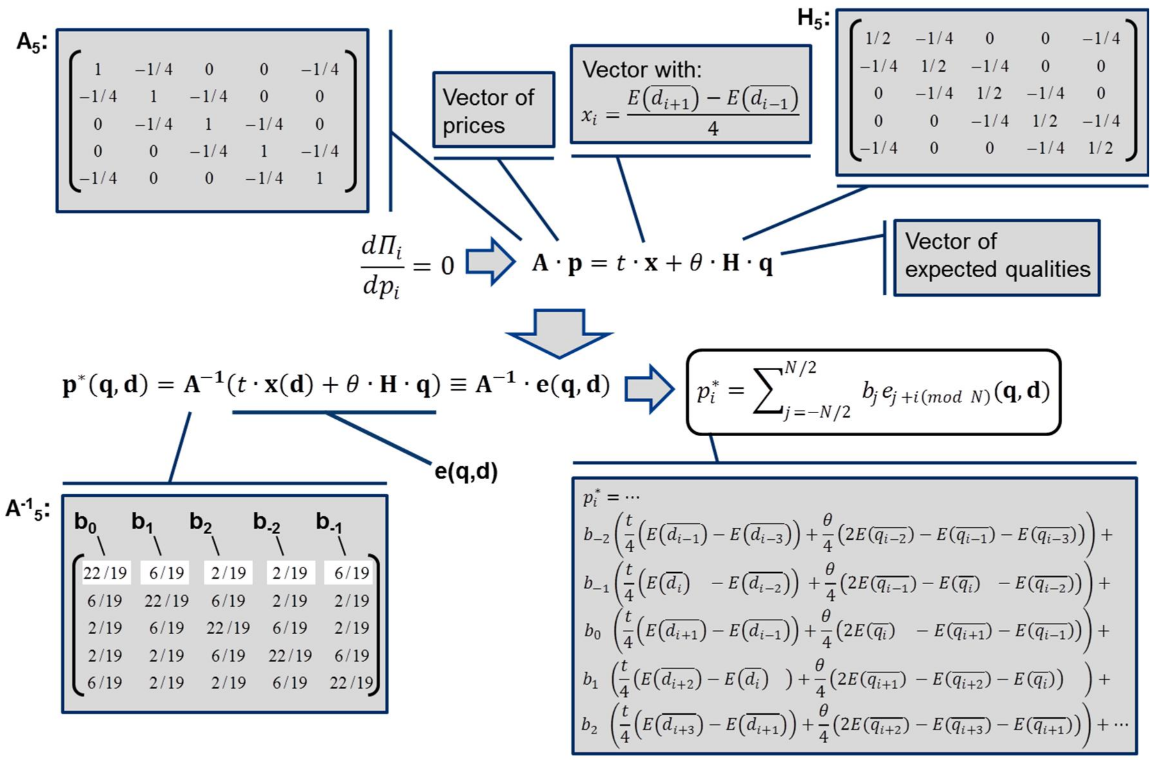


Figure 3: *Equilibrium solution of the pricing stage for $N = 5$*

Letting b_k denote the element of the diagonal at distance $|k|$ from the main diagonal of $\mathbf{A^{-1}}$, then:

$$p_i^* = \sum_{j=-N/2}^{N/2} b_j e_{j+i}(\mathbf{q}, \mathbf{d}) \qquad \textbf{[6]}$$

Figure 3 depicts the pricing equilibrium for $N = 5$ sellers, and helps us visualize p_i^* in general. The next two lemmas explain how equilibrium price responds to changes in expected quality. Intuitively, the lemmas show that an increase in expected quality allows a vendor to increase his equilibrium price, but an increase in the expected quality of a competitor, forces a vendor to reduce his equilibrium price by an amount that depends on how close a neighbor (in the taste space) that competitor is. Closer taste-space neighbors have greater impact on each other's prices.

Lemma 1: $dp_i^*/dE(\overline{q_i}) = \theta \cdot b/2$, *where* $b = b_0 - b_1$

Proof: $dp_i^*/dE(\overline{q_i}) = \sum_{j=N/2}^{N/2} b_j \big(de_{i+j}(\mathbf{q}, \mathbf{d})/dE(\overline{q_i})\big) = b_0\theta/2 - b_1\theta/4 - b_{-1}\theta/4 = \theta \cdot b/2 > 0$, since $b_i = b_{-i}$ and $b = b_0 - b_1$ is always positive (Economides, 1993).

Lemma 2: $dp_i^*/dE(\overline{q_j}) = -\theta \cdot b_m/2$, for $i \neq j$, *where* $m = k - i$

Proof: From Equation [6] (see also Figure 3), $dp_i^*/dE(\overline{q_j}) = \theta(b_m/2 - b_{m-1}/4 - b_{m+1}/4) = -\theta \cdot b_m/2$, because $4b_m = b_{m+1} + b_{m-1}, \forall\, m \neq 0$, by the properties of $\mathbf{A^{-1}}$.

Lemma 2 says that if the expected quality of vendor S_k, increases by ε, vendor S_i's equilibrium price would decrease by $\theta b_m \varepsilon/2$, where $m = k - i$ is the number of vendors between vendors *k* and *i*, in the taste space. This can be seen in Figure 3: if S_2 increased his price by ε, S_0 would have to respond to the price decreases of S_1 and S_3 that push S_0 price downwards by $-\theta\varepsilon(b_1/4 + b_{-2}/4)$, but S_0's price would also be somewhat pushed up by S_2's price increase (as per Lemma 1) by $\theta\varepsilon b_2/2$.

The analysis of the subgames of stages 3,2, and 1 is given in the Appendix. We present next the solution of one equilibrium instance that has firms choosing symmetric types, i.e., choosing types that are equidistantly positioned around the cylinder's circumference. The assumption that v is large enough for the entire market to be served translates to assuming that $v > \max_{\mathbf{q},\mathbf{d},\mathrm{i}} p_i + x(\mathbf{q}, \mathbf{d}) \cdot t - \theta \cdot q_i$, where $x(\mathbf{q}, \mathbf{d})$ is the maximum distance (for any market configuration $\mathbf{q}, \mathbf{d}$) between a buyer's most preferred product location and the two vendors with the closest product types. Finally, we assume that θ is not exceedingly high and thus seller profit is not competed away in high quality production costs, causing sellers to make a loss. In the Appendix, we provide the exact constraint on θ and show that it suffices for the roots of Γ and φ that appear in Theorem 1 to be positive.

The mathematical form for the equilibrium which includes firm response functions and asymmetric taste choices is provided in the Online Appendix.

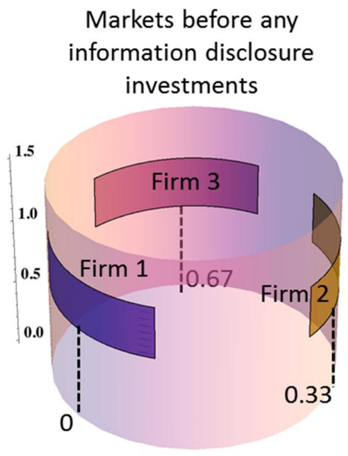
Theorem 1: *There exists an equilibrium that is symmetric on firm types, where:*

- ***Stage 1****: All firms choose symmetric types* $d_i - d_{i-1} = \frac{1}{N}$
- ***Stage 2****: All firms choose the same quality* $q^* = \begin{cases} \frac{\theta \cdot b}{k \cdot N} & , c_q > c_q{}' \\ \frac{\theta \cdot b}{k \cdot N} - \left(\frac{t}{N}\right)^2 \frac{8\left(\frac{t}{N}\right)^2 - 4t \cdot c_q - 8\Gamma}{\alpha_q \cdot k \cdot t \cdot \Gamma} & , c_q < c_q{}' \end{cases}$, where

$\Gamma = \frac{t}{N}\sqrt{\left(\frac{t}{N}\right)^2 - t \cdot c_q}$ and $c_q' = \frac{\alpha_q \cdot \theta \cdot b}{2t}\left(\frac{t}{N} - \frac{\alpha_q \cdot \theta \cdot b}{8}\right)$

- ***Stage 3****: Firms do not disclose their type. Firms disclose quality, iff their quality exceeds the lower limit of the quality uncertainty interval by* $\varphi = \frac{4}{\theta \cdot b \cdot \alpha_q}\left(\frac{t}{N} - \sqrt{\left(\frac{t}{N}\right)^2 - t \cdot c_q}\right)$. *The firms' ex-ante probability of disclosure is* $max\{0, 1 - \varphi\}$
- ***Stage 4:*** *Firms' ex-ante expected price is* $p^* = t/N$. *Actual prices depend on the realization of uncertainty intervals, chosen by nature, and are given by* $p_i^* = \sum_{j=-N/2}^{N/2} b_j e_{j+i}(q, d)$ □

Note that there is a move by Nature, before Stage 3, where Nature chooses quality and type uncertainty intervals so that vendor qualities are (ex-ante) equiprobable inside these intervals. Also note that, before Stage 4, buyers only observe type uncertainty intervals in the taste space, and in the quality space, buyers only know the qualities of the firms who disclosed quality in Stage 3.



		Firm 1	Firm 2	Firm 3
Product design Equilibrium	Type:	**0**	**0.33**	**0.67**
	Quality:	**0.87**	**0.87**	**0.87**
Uncertainty intervals (centers)	Type:	d_1	d_2+0.05	d_3
	Quality:	q_1+0.05	q_2	q_3
Quality disclosures & beliefs	Ex-ante, actual	[62%,No]	[62%,Yes]	[62%,Yes]
	Expected quality by buyers	0.80	0.87	0.87
[Prices, Profits]	Ex-ante:	[3.33,0.48]	[3.33,0.48]	[3.33,0.48]
	Actual:	[3.29,0.51]	[3.41,0.49]	[3.31,0.42]

Figure 4: *An example realization of the equilibrium that is symmetric in types, for* $N = 3$. *The values of the parameters used are* $\left[N = 3\ ,\ \alpha_q = 0.4\ ,\ c_q = 0.1\ , \alpha_d = 0.2\ , k = 1.5\ , t = 10\right]$

In Figure 4 we show one example realization of the subgame perfect symmetric equilibrium, which demonstrates the model mechanics. Firms choose the same quality level 0.87 and symmetric types, as per Theorem 1. Next, nature happens to draw uncertainty intervals whose centers match the firms' actual qualities and types, with two exceptions: the center of the type uncertainty interval of Firm 2 is 0.05 units closer to Firm 3 than Firm 2's actual type, and the center of the quality uncertainty interval of Firm 1 is 0.05 quality units higher than Firm 1's actual quality.

The impact of nature's draws of quality uncertainty intervals is as follows. Ex-ante (before the move by nature) the probability of disclosure for all firms is 62%. However, as per Theorem 1, and as we also explain in the next section, because Firm 1's actual quality is closer to the lower limit of its quality uncertainty interval, compared to Firms 2 and 3, it turns out to be below the threshold required for disclosure. Consequently Firm 1 does not disclose quality, while the other firms disclose and incur the disclosure cost. For Firms 2 and 3, buyers become informed about qualities (0.87), but buyers (as we discuss in the next section) rationally discount the quality of the non-disclosing Firm 1 and expect a quality of 0.80, which is actually 0.07 units lower than Firm 1's actual quality. This forces Firm 1 to price at 3.29 (Lemma 1) which is below its ex-ante expected price of 3.33. Firm 1 is content to operate this way: even if it is forced to lower its price, the fact that it has saved the cost of the quality investment, more than makes up for the lower price: its profit of 0.51 is higher than the ex-ante profit of 0.48. The intuition is that Firm 1 was fortunate to have buyers initially overestimate its quality (nature chose a quality uncertainty interval whose center was 0.05 units higher than its actual quality) and this allowed the firm to save on the quality disclosure cost. The buyers, observing non-disclosure, discount the firm's quality, but not severely enough to negate the disclosure cost savings.

The impact of nature's draws of type uncertainty intervals is as follows. Because Firm 2 appears closer to Firm 3 than it actually is, Firm 1 enjoys a larger potential market, and this allows it to somewhat raise its price (more accurately, it prevents Firm 1 from having to lower its price too much due to buyers discounting its quality). As per Equation [6] and [4], Firm 3 is forced to lower its price (3.31 versus ex-ante 3.33), in order to capture some of the market share that it loses to Firm 2. However, Firm 3 manages to avoid an even larger price decrease, because buyers discount Firm 1's quality. The net impact on Firm 3's profit is negative, with a profit of 0.42 versus an ex-ante expected profit of 0.48. Firm 2 is not affected by the fact that it appears to be moving closer to Firm 3, because its potential market share (the space between its two closest neighbors) has not changed (see proof of Lemma 3 in the Appendix). Thus, it would keep its ex-ante price of 3.33, was it not for the opportunity to significantly raise its price to 3.41, by taking advantage of the fact that buyers discount the quality of Firm 1 (see Lemma 2). However Firm 2 does not gain much in profitability (0.49 versus ex-ante 0.48) as its two competitors put pressure on its market share with their much lower prices.

In the next section, we focus on four key results that provide insight into the impact of uncertainty (and its resolution) on firms' investments in quality and its disclosure. In the Online Appendix we also compare our results with the symmetric equilibrium obtained under perfect product information in (Economides, 1993).

4. Results and Managerial Implications

A roadmap of the main results that are explored next is given in Table 2.

4.1. Information Disclosure Investments

Proposition 1: *The probability of disclosure can only increase with quality. Moreover the relationship is continuous*

Proof: Provided in the Appendix. The intuition is discussed below.

It is perhaps intuitively expected that the higher the quality of a product, the higher the vendor's profit, in case of disclosure. This is in general agreement with most literature on the topic (e.g. (Jovanovic, 1982), (Farrell, 1986), (Verrecchia, 1983), (Dye, 1986), (Shavell, 1994)). However, the model shows something more subtle: the higher the expected quality of a product, the more the buyers will discount its quality, should the vendor fail to invest in information disclosure. For example: assume that buyers expect ex-ante that a vendor's quality is uniformly distributed between 4 and 5 units (expected quality is 4.5). Upon observing that the vendor has not disclosed, buyers discount his quality and form new expectations in the range of 4 to $5-x$ units. Assume now that the buyers expect ex-ante that another vendor's quality is uniformly distributed between 2 and 3 units (expected quality is 2.5). Upon observing that the vendor failed to invest in quality disclosure, buyers discount the vendor's quality and form new expectations in the range 2 to $3-y$ range, but now $y<x$.

The intuition behind the finding that the higher the product quality the more the buyers discount it upon non-disclosure is as follows. Profits are convex in expected product quality because an increase in quality allows a vendor to increase his price (Equation [6]) while simultaneously gaining market share in the taste space (Equation [2]). With convexity, for a given quality improvement, a higher quality vendor increases profit more than a lower quality vendor does. Thus, a higher quality

Key question	Key Insight	Justification	Depiction & analysis	Inter-actions involved
How to account for quality when investing to reduce buyer uncertainty	Higher quality products should invest more in reducing quality uncertainty	Buyers rationally apply larger discount to higher quality products that do not disclose	§ 4.1 Proposition 1 Figure 5 & 6	A
How to account for future information investments when investing in quality	Firms should moderate quality investments, as future info investments affect quality ROI	Increasing quality also increases probability that a disclosure investment may be needed	§ 4.2 Proposition 2 Figure 7	A,B
How to account for ambient information availability when investing to inform consumers	External information availability enables firms, and especially lower quality firms, to free ride	Third party information reduces the maximum amount by which buyers discount non-disclosing products	§ 4.3 Proposition 3 Proposition 4 Figures 8 & 9	A,C
How to account for ambient information availability when investing in quality	Improvements in external information availability, generally enable firms to reduce quality investments	Quality becomes a less important determinant of profit as a quality increase is more likely to lead to future disclosure costs	§ 4.4 Proposition 5 Figure 10	A,B,C,D

Table 2: Roadmap for the discussion of our main results

vendor would invest to improve buyers' perception even by a small amount, or equivalently, would invest to prevent even a small decrease of buyers' expectations about its quality. In other words, buyers argue that a higher quality vendor would surely, if he could, disclose to prevent them from forming an expectation for quality close to the lower limit of the uncertainty interval, while a lower quality vendor would not necessarily do so. Thus, a non-disclosing higher quality vendor reveals that he cannot prevent buyers from expecting quality at the lower end of the uncertainty interval, because this is actually where his true quality lies, and a disclosure would not help in his case.

The mechanism of quality disclosure of our model extends the classic literature that describes how quality information "unravels" when disclosure is costly. For example in (Jovanovic, 1982) only vendors beyond a quality threshold (denoted by $\tilde{q}$ in Figure 5) disclose. Consequently, classic literature favors a 'quality cut-off' approach that separates disclosing from non-disclosing firms[6]. In the current model, while quality unraveling still occurs inside the uncertainty intervals and while vendors disclose if their quality exceeds a threshold inside the uncertainty interval, disclosure is probabilistic, since the process that determines the exact position of the uncertainty interval is probabilistic. Thus, it could happen that a lower quality vendor, such as vendor S_2 in Figure 5, exceeds its threshold and discloses, while a higher quality vendor, such as vendor S_1 does not exceed

[6] This is true, as long as quality probability distributions are reasonably well behaved. There may be multiple disclosure thresholds, as we discuss in the derivation of Equation [8] in the Appendix.

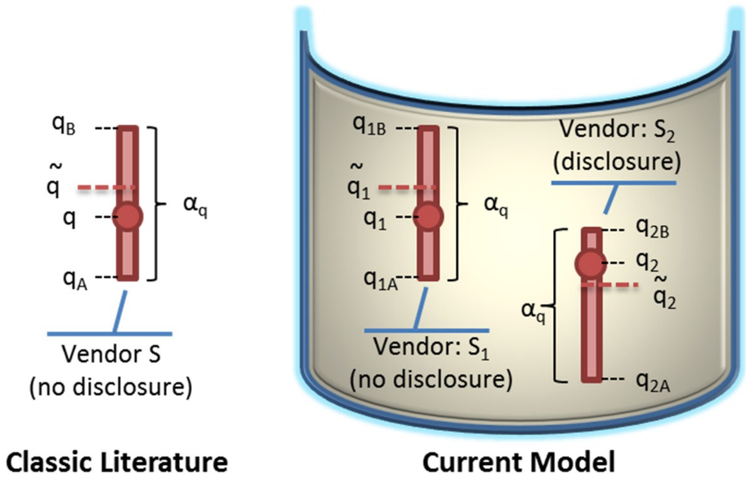


Figure 5: *The quality disclosure mechanism in our model, as a refinement of the classic literature*

its threshold and fails to invest to reduce consumer uncertainty. However, this is not the most likely outcome. As per our previous discussion, the higher a firms' quality, the more willing it would be to disclose and thus the lower the disclosure threshold of the firm inside its uncertainty interval. This means that as quality increases, the disclosure threshold decreases, and higher quality vendors are more likely to disclose than lower quality vendors. The probability of disclosure as a function of quality is given by Lemma 7 and Lemma 8 in the Appendix (leading to Proposition 1), where it can be seen that it also depends on the strength of consumers' preferences for quality (firms are more likely to disclose if buyers value quality more), as well as on the size of the uncertainty interval, as we will discuss in Section 4.3.

There is empirical support for Proposition 1. We gathered data on 2,196 PC-games released between 1996 and 2005, from the gaming website GameSpot. Game developers had a choice to invest in developing playable demo versions of their games, which could reduce buyer uncertainty about the quality of the game but were also expensive to develop. On the left side of Figure 6, we plot disclosure probability for a firm under different quality levels[7], and on the right side we have plotted the percent of PC-games that have invested in producing a playable demo version, for different game quality levels. The graph clearly shows that the higher the game quality, the greater the probability of the game developer having invested in the development of a playable demo version, and these results are robust to corrections for game genres and release dates.

4.2. Quality Production Decisions

Proposition 2: *Quality investment under uncertainty is at most equal to equilibrium quality under perfect information*[8]

[7] Different equilibrium quality levels are derived from the general equilibrium solution, as described in the Online Appendix. Other things equal, the further away a firm's neighbors, the higher the equilibrium quality of the firm.

[8] The idea of quality underinvestment under uncertainty was first discussed in (Jovanovic, 1982), but was first shown explicitly by Albano & Lizzeri (Albano & Lizzeri, 2001).

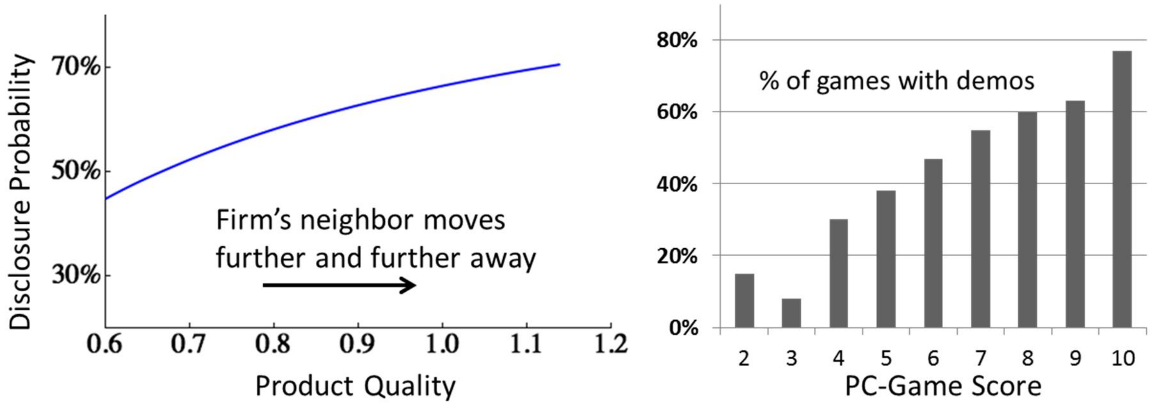


Figure 6: *Disclosure probability as a function of product quality in our model and in a real market. Parameter values used are [N=3 (which also implies b=0.8), t=10, θ=5, k=1.5,* c_q*=0.1,* α_q*=0.4,* α_d*=0.2]*

Proof: Provided in the Appendix. The comparison with the perfect information case, obtainable by setting $\alpha_q = \alpha_d = 0$, is further explored in the Online Appendix. It is also easy to see, that by setting $\alpha_q = 0$ in Theorem 1, quality can only increase. The intuition is discussed below.

In the previous section we saw that the probability of quality disclosure increases with product quality, because buyers discount the quality of a non-disclosing firm more at higher quality levels. Consequently, when a firm decides how much to invest in order to improve product quality, it has to account for its future expected quality disclosure costs. These future information disclosure investments act as a "hidden" quality production cost at the stage of product design and when taken into account, lead to lower optimal quality, compared with the quality that a firm would have chosen under a perfect information regime.

Figure 7 explains how quality production in our model differs from classical quality unraveling, where firms invest in quality disclosures only above a certain quality threshold. The figure shows the marginal benefit and the marginal cost of quality under perfect information and under uncertainty, where marginal benefit is defined simply as marginal profits minus the marginal quality disclosure cost, and are derived by differentiating ${p_i^*}^2/t - r_i - s_i$ (from Equation [3]) and $C(q_i) = k{q_i}^2/2$ with respect to quality. Obviously, for every firm, equilibrium quality production is determined by the point where the marginal benefit of an additional quality increase, matches the marginal quality production cost. Under classical quality unraveling, firms disclose quality only above a threshold $\tilde{q}$. Thus, on the left graph of Figure 7, the marginal benefit of increasing quality at any quality level below the disclosure threshold $\tilde{q}$, is zero, since quality below the threshold will not be disclosed and any quality improvement will go undetected with buyers assuming the worst possible product quality. However, the marginal benefit of increasing quality when already above the disclosure threshold $\tilde{q}$, is the same as the perfect information case, as quality will always be disclosed above the threshold. Therefore, with classical quality unraveling, optimal quality is either at its minimum level, or it is the

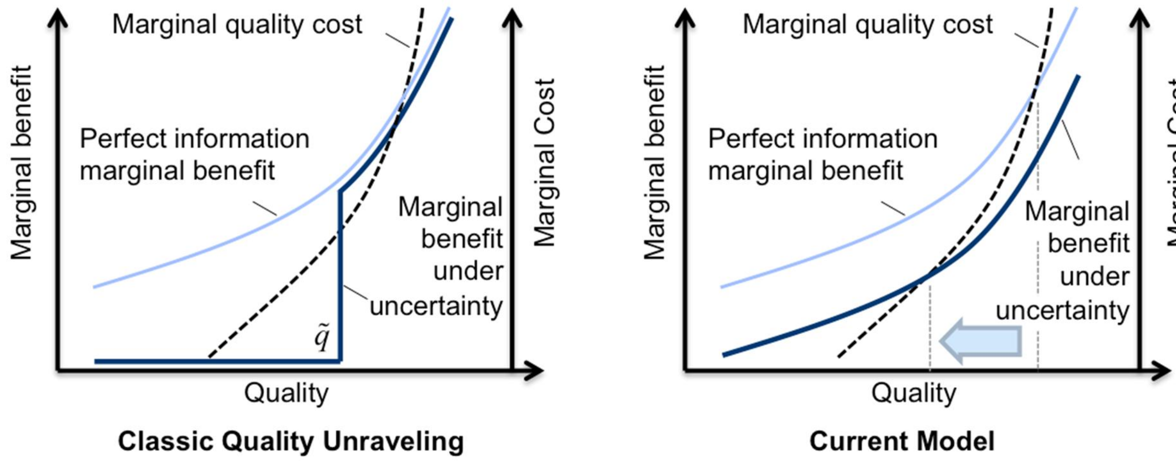


Figure 7: *The mechanism of quality production in our model, as a refinement of the classic literature*

same as the quality under perfect information, depending on whether or not the marginal cost curve meets the marginal benefit curve above the threshold.

In the current model, the marginal benefit of increasing quality lies below the marginal benefit under perfect information and non-zero disclosure probability, as we have seen in the previous section that any quality increase also increases the chances that a quality disclosure cost will be incurred. Therefore, compared to the perfect information case, our model predicts that firms will produce lower quality levels under uncertainty, as shown by the arrow on the right of Figure 7.

The managerial implication is that firms must include the cost of future information disclosure in their quality production 'Return on Investment' (ROI) calculations. Higher quality levels increase the probability that firms will also need a quality disclosure investment. In other words, quality is more expensive than it appears due to consumer uncertainty. Consider our prior example about AMD. When AMD chose to implement a significant quality improvement, it also ended up having to invest heavily in a yearlong program to retrain all of its retail sales representatives "*in the art of marketing visual performance*" (Vance, 2011).

An example where firms have been found to invest less in quality under uncertainty, concerns hygiene in the Los Angeles restaurant market, and has been documented in (Jin & Leslie, 2003). Prior to 1998, restaurants were not required to disclose their hygiene inspection results or to use any particular format when doing so. In 1998 a standardized report card was introduced that made reporting of hygiene inspections easily understood and comparable across all restaurants in all L.A. county cities. Each county city was then left to decide on whether or not to force their restaurants to prominently display these report cards. For mandatory disclosure cities, the result was a transition to a perfect information regime, since any quality disclosure costs become sunk as mandatory. For voluntary disclosure cities, the result was a reduction of the quality disclosure cost, since standardization had dramatically reduced the cost of explaining to consumers what the hygiene level of a restaurant was, compared with other restaurants in the area. This is equivalent to an upwards shift

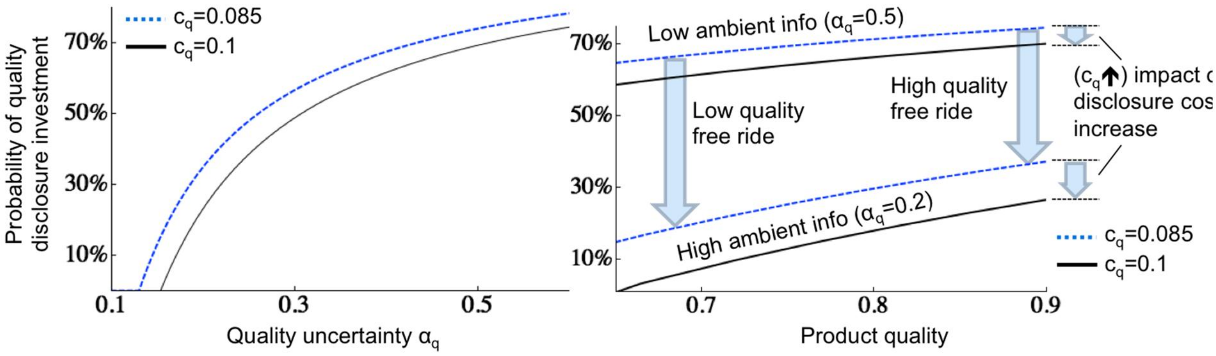


Figure 8: *Ex-ante probability of quality disclosure investments as a function of quality uncertainty in the symmetric equilibrium (left) and equilibrium quality in a general equilibrium (right). Parameter values used are [N=3 (which also implies b=0.8), t=10, θ=5, k=1.5, α_d=0.2]*

of the marginal benefit curve in the right hand side of Figure 7, very close to the perfect information marginal curve, which should also lead to an increase in quality (recall that disclosure costs are included in the marginal benefit). Indeed, Jin & Leslie documented the subsequent increase in the hygiene levels on both mandatory and voluntary disclosure cities, as captured by increasing inspection scores, and also by the reduction of food-related hospital admissions in L.A. county[9].

4.3. Impact of Ambient Information on Firm's Information Investments

Proposition 3: *Firms disclose less as α_q decreases (i.e., as ambient market information increases)*

Proof: Follows directly from Lemma 7, in the Appendix. The intuition is discussed below.

Firms' investments to reduce consumer uncertainty naturally depend on the amount of ambient information available in the market. Infomediaries, consumer reviews, press coverage, and other third party sources of product information contribute to consumer informedness, often allowing firms to free ride on this ambient information, and forgo their own information disclosure investments. However, as infomediaries provide more and more information, the sizes of uncertainty intervals that consumers can assign to different products reduces, and the potential for consumers to discount the quality of a vendor who does not disclose diminishes, making non-disclosure less costly.

On the left side of Figure 8 we plot the (ex-ante) probability that a firm will invest in information disclosure for the case of symmetric firm types, as given in Theorem 1 and Lemma 7. No firm discloses when α_q is below a certain threshold relative to the quality disclosure cost, given by Lemma 8 and also shown in Theorem 1[10].

Further, our model shows that the opportunity for firms to free ride on ambient information and forgo their own information disclosure investments is relatively greater for lower quality vendors,

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[9] The authors carefully excluded many potential alternative explanations, such as consumer sorting (consumers avoiding food-related poisoning by shifting to more hygienic restaurants).

[10] The threshold, given in terms of c_q in Theorem 1, by $c_q' = \frac{\alpha_q \cdot \theta \cdot b}{2t}\left(\frac{t}{N} - \frac{\alpha_q \cdot \theta \cdot b}{8}\right)$ decreases in N, so that fewer firms disclose with more competition. This agrees with (Okuno-Fujiwara, et al., 1990), (Cheong & Kim, 2004) and (Hotz & Xiao, 2010) and is empirically supported by Jin (Jin, 2005), as discussed in Section 2. In contrast, for type disclosures, as we show in the Online Appendix, increasing the number of vendors increases the disclosure rate for type information.

than it is of higher quality vendors. This is depicted on the right side of Figure 8, where we can see that as we increase the amount of ambient information in the market (α_q reduces from 0.5 to 0.2) the (ex-ante) probability of a disclosure investment decreases, but more so for lower quality vendors. In other words, it is easier for lower quality vendors to free ride on third party information. Formally:

Proposition 4: *The reduction in the vendors' (ex-ante) probability of quality disclosure as α_q decreases, is greater (in absolute value) at lower product qualities*

Proof: From Lemma 7 in the Appendix, and also using Lemma 1, it can easily be shown by differentiation that at all product quality levels the derivative of the probability of disclosure with respect to product quality, decreases as α_q increases. Proposition 4 is now implied, as the probability of disclosure as a function of quality will always be steeper at lower α_q.

The intuition behind this result is that any increase in quality reduces a firm's disclosure threshold inside its quality uncertainty interval. When the uncertainty interval is relatively small (high ambient information) even a small decrease in the position of the disclosure threshold can significantly affect the probability that a firm's quality will be below (or above) the threshold. Thus, the probability of a disclosure investment becomes more sensitive on quality when α_q is low, as can be seen on the right side of Figure 8 where the bottom pair of lines is steeper. But, obviously, this means that the two pairs of lines should converge towards higher qualities, which means that free riding on ambient information should be less pronounced in higher qualities.

The managerial implication is that firms that plan their information disclosure investments, should account for ambient information availability and be aware of an opportunity to free ride.

This fact was nicely demonstrated by Chen & Xie (2005) who looked at how firms adjust their advertising spending as a response to an independent product review published in a magazine (see Figure 9). They observed reviews that discuss product quality and end in a recommendation (or not). As per our analysis, we would expect firms to respond to a third party review by reducing the amount of information they disclose. Further, lower quality firms should reduce their disclosure spend, more than higher quality firms. This is indeed what Chen & Xie found. Higher quality printers did not significantly change their advertising spend, while lower quality printers did. In a different product category, lower quality running shoes reduced their advertising spend, as a response to a product review, by 71%, compared to 30% reduction for high quality running shoes.

4.4. Impact of Ambient Information on Quality

Proposition 5: *If information disclosure costs are not exceedingly high to completely prevent all firms from disclosing (exact threshold given in Lemma 8), equilibrium quality reduces with more ambient product information (i.e., as α_q decreases)*

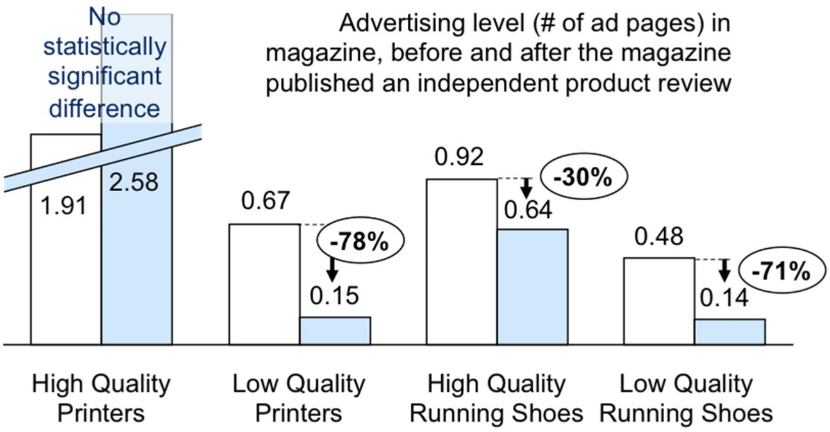

Figure 9: *Impact of 3rd party disclosures to firms' own disclosure investments. (Chen & Xie, 2005).*

Proof: Follows directly from Lemma 11 in the Appendix. The intuition is discussed below.

Proposition 5 says that as quality uncertainty α_q reduces, product quality also reduces (see Arrow A in Fig 10) because firms compete less in quality. To understand this somewhat counterintuitive result, we must recall that the probability of an information disclosure is sensitive to product quality. This sensitivity increases at higher levels of ambient information (higher slope of lines for lower α_q in Figure 8b), where even a small increase in quality can significantly increase expected future information investment costs. Consequently, as disclosure probability becomes more sensitive to quality, profits become less elastic on quality and firms compete less and less on their product quality, exactly as Proposition 5 states (Arrow A in Fig 10).

The impact of ambient information availability on product quality can be significant, even under moderate information disclosure costs. For example, for the dark thick line of the quality graph of Figure 8, while quality investment cost is 0.5, or more than 7 times lower than quality production cost under perfect information (which is $1/2 \times 2.67^2 \approx 3.6$), when firms in the market take this cost and its impact on profitability into account, equilibrium quality investment can be as much as 17% lower ($1/2 \times 2.46^2 \approx 3.0$). In the next section, we show that this result has significant implications for firm profitability in markets that differ in the amount of ambient information that they include.

Finally, note that when there is so much ambient information available in the market (very low a_q) that no firm wants to incur additional disclosure cost (disclosure probability is zero), quality no longer depends on uncertainty a_q, because disclosure probability (at zero) is no longer sensitive to quality. Thus profits are as elastic on quality as they are under perfect product information (arrow B). This threshold (given in Lemma 8 in the Appendix) is easier to exceed as disclosure costs increase.

4.5. Impact on Firm Profitability

The right side of Figure 10 shows how ambient information and information disclosure costs impact firm profitability. Under very high quality uncertainty, firms almost always disclose (see Figure 8), and the market resembles a market of perfect information with firms always incurring the disclosure

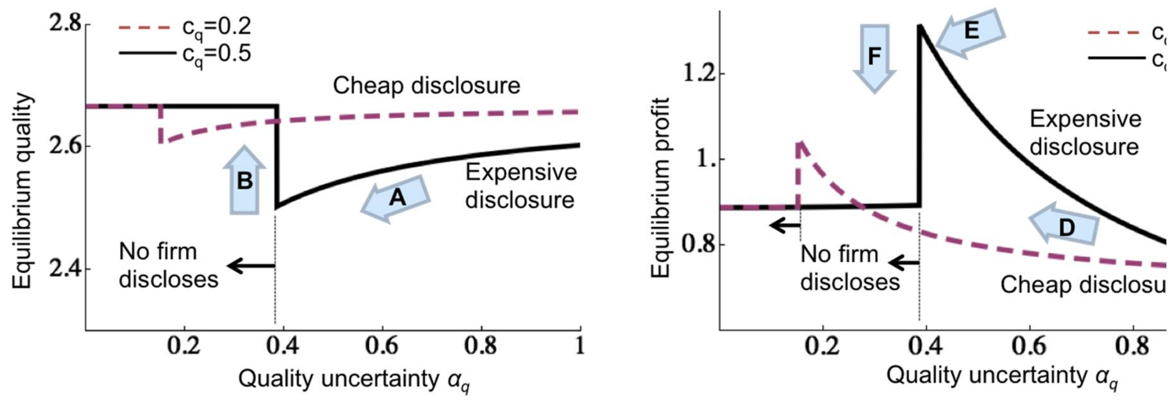

Figure 10: *Equilibrium quality (left, given in Theorem 1) and expected equilibrium profit (right, derived in the Online Appendix) as a function of quality uncertainty in the symmetric equilibrium. Parameter values used are [N=3 (which also implies b=0.8),*

cost c_q. Consequently, under high quality uncertainty α_q, equilibrium profit approaches profit under perfect quality information minus c_q (see Arrow C). As α_q decreases, firms invest less in quality (Arrow A) and firms save both on quality production and on expected information investment cost (disclosure probability reduces) so that profits begin to increase (Arrow D) and can exceed profits under perfect information. Arrow E shows that maximum firm profitability is obtained when α_q is such that only few firms invest in quality disclosure, but the probability of disclosure is not yet zero. That is the point where profits are least elastic in quality so that quality level is minimum and the ex-ante probability of a quality disclosure investment is very low. When third parties have provided enough information for the probability of disclosure to drop to zero, quality production jumps up to its high perfect-information-market level (Arrow B) and profitability reduces significantly (Arrow F). However, it is still slightly above the level obtainable at $\alpha_q = 0$ (what appears as a horizontal profit line for low α_q in Figure 10, is in fact an increasing line segment), because due to quality and type uncertainty, there is a small utility transfer from buyers to sellers[11].

One key managerial implication is related to the stylized example on AMD in the article's Introduction, and concerns the dependence of profitability on ambient information. Recall that AMD, following evolving consumer preferences, decided to move away from the "clock-speed" processor race and compete instead on the users' graphics experience. However, this is a dimension for which much less ambient information exists, due to the absence of metrics that are widely familiar to end consumers. Our model predicts that in this stylized setting, the rivalry among computer processor

[11] Intuitively, one reason is that profit is convex in product quality. Firms whose quality buyers overestimate enjoy both higher than average price and higher than average market share. Thus, when buyers overestimate quality they overpay more so than they underpay when they underestimate quality. Type uncertainty also induces a utility transfer. The magnitude of both transfers is given in the Online Appendix

manufacturers would become less intense and competitive pressures to invest in constant quality improvements would ease out, as any product quality improvement would also require considerable information disclosure investments.

5. Concluding Remarks

The rising popularity of electronic markets has increased the significance of firms' investments to reduce consumer uncertainty about their products. In this paper, we investigate an integrated process of product design and product information investments and argue that both decisions are heavily influenced by the availability of ambient market information, i.e. information that consumers can access from other consumers' reviews and third party infomediaries. Specifically we employ a game theoretic model of an oligopolistic market where product design and product information investments are endogenous decision variables, considered in the context of changing information availability by third parties.

We showed that buyers discount the quality of firms who do not invest to reduce consumer uncertainty and that, in the case of high quality vendors, buyer discount for non-disclosure is even higher. We also showed that when vendors decide their quality investment level, they should always account for the possibility that they will, in the future, have to also invest in quality disclosure, as the higher the level of the quality that they produce, the more likely it is that they will incur a future quality disclosure cost. This leads forward-looking firms to moderate their level of quality in the presence of uncertainty.

Both quality investments and information disclosure investments are influenced by the amount of information that buyers can receive from third parties. We showed that firms, especially those with lower quality products, can forgo information disclosure investments, and free ride on ambient information availability. Furthermore, improvements in product information made available by third parties, were shown to blunt competitive pressures to produce higher quality products, and thereby allow firms to improve their profitability.

While the idea that the processes of product design and information disclosure investments should be integrated and account for information availability by third parties, has been demonstrated to be theoretically sound, much more work is needed before it can actually inform managerial decisions. Detailed empirical and experimental investigation is required to verify the applicability of the article's theoretical recommendations, and much additional work is required to specify how these recommendations can be made actionable and practical.

A. Appendix of symbols used in notation

Symbol	Definition
Decision Variables	
q_i, d_i, p_i	Firm S_i's chosen quality, type, and price respectively
r_i, s_i	Firm S_i's quality & type disclosure cost. $r_i \in \{0, c_q\}$, $s_i \in \{0, c_d\}$
Model parameters	
N	Number of firm in the market
v	Buyer utility for product of zero quality located at his most preferred type
$\overline{q_i}$, $\overline{d_i}$	Random variables that correspond to q_i and d_i under imperfect information
$f_{qi}, f_{di}, F_{qi}, F_{di}$	PDFs & CDFs of $\overline{q_i}$ and $\overline{d_i}$
$[q_{iA}, q_{iB}], [d_{iA}, d_{iB}]$	Quality & type uncertainty intervals. Non-zero regions (support) of f_{qi}&f_{di}
c_q, c_d	Cost for disclosing quality & type, respectively
θ, t	The intensity of consumer preference for quality & type, respectively
$C(q_i), k$	$C(q_i)$ is the quality production cost and equals: $C(q_i) = kq_i{}^2/2$
Derived values	
α_q, a_d	Sizes of the quality & type uncertainty intervals, respectively.
z_i	Most preferred location of buyer who is indifferent between Firms i and i+1
q, d	Vectors of expectations for q_i and d_i for all vendors
$\widetilde{q_i}$, $\underline{q_i}$	Value of q_i, above which quality is disclosed, and $E(\overline{q_i} \mid \overline{q_i} < \widetilde{q_i})$
δ_i	$\delta_i = \widetilde{q_i} - \underline{q_i}$. Equals half the disclosure range for uniform f_{qi}
p_i, R_i, D_i, Π_i	Firm S_i's product price, revenue, unit demand, and profit, respectively
p	Vector of prices for all firms
x	Vector with $x_i(\mathbf{d}) = \left(E\left(\overline{d_{i+1}}\right) - E\left(\overline{d_{i-1}}\right)\right)/4$. Used for notational convenience
$\mathbf{A}, \mathbf{A}^{-1}$	Symmetric circulant matrices. $\boldsymbol{A}[i,i] = 1, \boldsymbol{A}[i+1,i] = -1/4$ & zeros elsewhere
H	Matrix $\mathbf{H} = \mathbf{A} - \mathbf{I}/2$. Used for notational convenience
b_i, b	b_i is element of the diagonal of $\mathbf{A}^{-1}$ at distance *I* from main diagonal; b is $b_0 - b_1$
e	Vector defined as $\mathbf{e} \equiv t \cdot \mathbf{x}(\mathbf{d}) + \theta \cdot \mathbf{H} \cdot \mathbf{q}$. Used for notational convenience
$Pr_i(\cdot)$	The probability that firm S_i will invest in quality disclosure, when uncertainty intervals are not yet known
$c_q'(\cdot)$	Value of c_q beyond which $Pr_i(\cdot) = 0$ (no firm discloses)

B. Appendix of Proofs and derivations

i. The Pricing Stage

Stage 4 of the game with symmetric firm types has been analyzed in Section 3.2. The Online Appendix includes the analysis for the general equilibrium form that includes non-symmetric firms.

ii. The Information Disclosure Investment Stage

In the information disclosure subgame, vendors have chosen their product details and have received information about how their own uncertainty intervals will look if they decide to withhold product information. They expect that by withholding information, they may cause buyers to reevaluate the uncertainty intervals, taking into account the fact that vendors have withheld information, and finally they anticipate that in the pricing stage they will price according to Equations [5] and [6].

[Taste-related Information] When vendor S_i withholds type information, vendors will form some expectation $E(\overline{d_i})$ on his product type which, though Equation [6] will affect the vendor's price. If the vendor releases product type information, buyers will know his product type with certainty, so that $E(\overline{d_i}) = d_i$. The result is that the act of disclosing type, changes the value of $E(\overline{d_i})$ by some constant $z \in [d_i - d_{iB,} d_i - d_{iA,}]$. First, we show that disclosure of type information by seller S_i will not affect S_i's equilibrium price, by looking at the rate of change of equilibrium price with respect to changes in perceived expected vendor location. This is given by $dp_i^*/dE(\overline{d_i}) = \sum_{j=-N/2}^{N/2} b_j \left(de_{i+j}(\mathbf{q},\mathbf{d})/dE(\overline{d_i})\right) = t \cdot (b_{-1} - b_{+1})/4 = 0$, since $de_{i+j}(\mathbf{q},\mathbf{d})\,/dE(\overline{d_i}) = 0$ for $j \neq \pm 1$, $b_j \left(de_{i+j}(\mathbf{q},\mathbf{d})/dE(\overline{d_i})\right) = -t \cdot b_{+1}/4$, for $j = +1$, and $b_j \left(de_{i+j}(\mathbf{q},\mathbf{d})/dE(\overline{d_i})\right) = +t \cdot b_{-1}/4$ for $j = -1$ and $b_{+1} = b_{-1}$ because $\mathbf{A^{-1}}$ is symmetric circulant. From Equations [1],[2] revenue change is: $dR_i(\mathbf{p},\mathbf{q},\mathbf{d})/dE(\overline{d_i}) = dp_i/dE(\overline{d_i}) \cdot D_i(\mathbf{p},\mathbf{q},\mathbf{d}) + p_i \cdot dD_i(\mathbf{p},\mathbf{q},\mathbf{d})/dE(\overline{d_i}) = 0 + p_i\big(dp_{i+1}/dE(\overline{d_i}) + dp_{i-1}/dE(\overline{d_i}) - 2dp_i/dE(\overline{d_i})\big)/(2t) = p_i(t \cdot b_{+1} - t \cdot b_{-1} - 0)/(8t) = 0$. Thus, from Equation [1], the impact to profit from disclosing type information is just $-c_d$, leading to:

Lemma 3: *Vendors do not release product type information for any* $c_d > 0$

The intuition is that a vendor's type disclosure which changes $E(\overline{d_i})$ by some constant z, causes one neighbor in the type space to lower price by $z \cdot t \cdot b_{+1}$ (the vendor to which he is now perceived to be closer), while causing the neighbor to which he is perceived to be moving away from, to raise his price by $z \cdot t \cdot b_{-1}$. The net effect of these changes on the vendor's profit is zero minus the disclosure cost c_d. A similar result was obtained in (Markopoulos, et al., 2010) in a model that also allows for vendors to charge buyers in order to provide them with the information that they would not release in the absence of such a payment.

Lemma 3 allows us to derive the distribution of $E(\overline{d_i})$, before horizontal uncertainty intervals realize prior to Stage 4. For uniform $f_{dj}(\cdot)$, $E(\overline{d_i})$ is simply the center of a vendor's horizontal uncertainty interval. Since d_{iA} can range from d_i to $d_i + \alpha_d$, the center of the uncertainty interval can range from $d_i - \alpha_d/2$ to $d_i + \alpha_d/2$. Further, since no vendor chooses to disclose type information, buyers do not receive any signal from vendors who do not disclose type. Thus:

Lemma 4: *Before uncertainty intervals realize, $E(\overline{d_i})$ is a random variable $E(\overline{d_i}) = d_i + \varepsilon$, where ε is a random variable which is uniformly distributed with support $[-\alpha_d/2, \alpha_d/2]$. Thus, $dE(\overline{d_i}) = dd_i$*

For *c* constant, we obtain the following properties for ε:

$$\int_{-\alpha_d/2}^{\alpha_d/2} \frac{\varepsilon}{\alpha_d} d\varepsilon = 0, \qquad \int_{-\alpha_d/2}^{\alpha_d/2} c\frac{1}{\alpha_d} d\varepsilon = c, \qquad \int_{-\alpha_d/2}^{\alpha_d/2} \varepsilon^2 \frac{1}{\alpha_d} d\varepsilon = \frac{{\alpha_d}^2}{12} \qquad \textbf{[7]}$$

We will contrast this with quality information, where buyers do receive a signal from vendors who do not to disclose and use the signal to reduce their uncertainty about these buyers' quality.

[Quality Information] Using Equation [3] and Lemma 1, a vendor's profit (minus the required disclosure cost) is increasing in perceived quality: $\frac{d(\Pi_i^* - r_i)}{dE(\overline{q_i})} = \frac{dR_i^*}{dE(\overline{q_i})} - \frac{dC(q_i)}{dE(\overline{q_i})} = \frac{2p_i^*}{t} \cdot \frac{dp_i^*}{dE(\overline{q_i})} - 0 = \frac{p_i^* \cdot \theta \cdot b}{t} > 0$ (the quality production cost has already been incurred in Stage 2, so that now $dC(q_i)/dE(\overline{q_i}) = 0$). Further, buyers know that vendors have knowledge of their uncertainty intervals and disclosure of quality information entails a cost c_q. Thus, our model satisfies the assumptions of the classic analysis of costly quality disclosure (Verrecchia, 1983) (Fishman & Hagerty, 2003) so that quality information unravels above a threshold:

Lemma 5: *Given seller S_i's quality uncertainty interval $[q_{iA,}q_{iB}]$, there exists a quality level $\widetilde{q_i} \in [q_{iA,}q_{iB}]$, so that iff $q_i > \widetilde{q_i} \Leftrightarrow q_i \in [\widetilde{q_i}, q_{iB}]$, S_i will disclose*

If the seller does not disclose the buyers will reduce their expectation of the vendor's quality to:

$$\underline{q_i} = \int_{q_{iA}}^{\widetilde{q_i}} \frac{x f_{qi}(x)}{F_{qi}(\widetilde{q_i}) - F_{qi}(q_{iA})} dx = \int_{q_{iA}}^{\widetilde{q_i}} \frac{x f_{qi}(x)}{F_{qi}(\widetilde{q_i})} dx = \frac{q_{iA} + \widetilde{q_i}}{2} \qquad \textbf{[8]}$$

$\widetilde{q_i}$ is such that if $\delta_i \equiv \widetilde{q_i} - \underline{q_i}$, then $R_i^*(\widetilde{q_i}) - c_q = R_i^*(\widetilde{q_i} - \delta_i)$, so that the revenue that a vendor with quality $\widetilde{q_i}$ gives up by withholding quality information, is offset by disclosure cost savings. In general $\widetilde{q_i}$ may not be unique. However, the uniform $f_{qi}(\cdot)$ is a log-concave function, so that, given q_{iA}, δ_i is increasing in $\widetilde{q_i}$ (Bagnoli & Bergstrom, 2005), and thus $\widetilde{q_i}$ is unique, given $f_{qi}(\cdot)$ [12].

We can also define $Pr_i(q_i)$ to be the probability that vendor S_i will disclose, given quality q_i:

$$Pr_i(q_i) = Prob(q_i > \widetilde{q_i}) = 1 - 2\delta_i/\alpha_q \qquad \textbf{[9]}$$

Before we are able to calculate the equilibrium value of δ_i, we must know the distribution of $E(\overline{q_i})$, before the uncertainty intervals realize. In the Online Appendix we prove:

[12] See (Cheong & Kim, 2004). *f* is log-concave if *ln(f)* is concave in its support (e.g., uniform, normal, etc.)

Lemma 6: *Before uncertainty intervals realize,* $E(\overline{q_i})$ *is a random variable* $E(\overline{q_i}) = q_i + \varphi$*, with* φ *a random variable with support* $[-\delta_i, \delta_i]$ *and PDF* f'_{qi} *given by:* $Pr_i(q_i) \cdot \frac{1}{2\delta_i} + \left(1 - Pr_i(q_i)\right) \cdot \delta(\varphi)$*, where* $\delta(\varphi)$ *is Dirac's delta function.*

For *c* constant, and by using Equation [9], we obtain the following properties for f'_{qi}:

$$\int_{-\delta_i}^{\delta_i} \varphi \cdot f'_{qi}(\varphi) \cdot d\varphi = 0, \quad \int_{-\delta_i}^{\delta_i} c \cdot f'_{qi}(\varphi) \cdot d\varphi = c, \quad \int_{-\delta_i}^{\delta_i} \varphi^2 \cdot f'_{qi}(\varphi) \cdot d\varphi = \frac{{\alpha_q}^2\left(1-Pr_i(q_i)\right)^3}{12} \qquad \textbf{[10]}$$

In equilibrium, all firms will choose their quality disclosure strategy according to Lemma 5 and vendor S_i will choose his optimal quality disclosure threshold $\widetilde{q_i}$, consistent with every other vendor S_j optimally choosing quality disclosure threshold $\widetilde{q_j}$. We can now show that $\delta_i \equiv \widetilde{q_i} - \underline{q_i}$ is given as a function of q_i by: (see Online Appendix):

$$\delta_i = min\left[\frac{2}{\theta \cdot b}\left(p_i^*\left(q_i \middle| \forall j \neq i: E\left(\overline{q_j}\right) = q_j\right) - \sqrt{p_i^*\left(q_i \middle| \forall j \neq i: E\left(\overline{q_j}\right) = q_j\right)^2 - t \cdot c_q}\right), \alpha_q/2\right] \qquad \textbf{[11]}$$

Buyers discount non-disclosing products by $\alpha_q/2 - \delta_i > 0$, since buyers update their expectation for the product's expected quality from $q_{iA} + \alpha_q/2$ to $q_{iA} + \delta_i$, as δ_i was defined to be $\delta_i \equiv \widetilde{q_i} - \underline{q_i}$.

Note that vendor S_i's equilibrium price in Equation [11], is the price that vendor S_i would have chosen if for all other vendors $E\left(\overline{q_j}\right)$ exactly matched q_j. In other words, the formula says that a vendor who must choose his quality disclosure threshold $\widetilde{q_i}$, optimizes by assuming that for every competitor S_j, $E\left(\overline{q_j}\right)$ will neither overestimate nor underestimate the competitor's true quality q_j.

For simplicity, the remaining of the article will somewhat abuse the notation and use $p_i^*(q_i)$ to mean: $p_i^*\left(E(\overline{q_i}) = q_i \middle| E\left(\overline{q_{j\neq i}}\right) = q_j\right)$.

We can now use Equation [9] to immediately derive the exact form of $Pr_i(q_i)$ as well as the quality investment cost, beyond which no vendor would wish to invest in quality disclosure:

Lemma 7: $Pr_i(q_i) = max\left[0, \;\; 1 - \frac{4}{\theta \cdot b \cdot \alpha_q}\left(p_i^*(q_i) - \sqrt{p_i^*(q_i)^2 - t \cdot c_q}\right)\right]$

Lemma 8: $Pr_i(q_i) = 0$**,** *when* $c_q > \frac{\alpha_q \cdot \theta \cdot b}{2t}\left(\left(p_i^*(q_i)\right) - \frac{\alpha_q \cdot \theta \cdot b}{8}\right)$

To summarize, vendors will disclose quality information if and only if their quality exceeds a threshold within their quality uncertainty interval and the probability of disclosure can be zero, for high enough c_q. In general, it may happen that a lower quality vendor exceeds this quality threshold inside her own uncertainty interval, but that a higher quality vendor does not, as we saw in Figure 5.

Proof of Proposition 1: The proposition can be proven either by starting from Equation [11], or by starting from Lemma 7. In the first approach δ_i decreases in equilibrium price p_i^*: $d\delta_i/dp_i^* = \cdots = \frac{2}{\theta b}\left(1 - p_i^*/\sqrt{{p_i^*}^2 - t \cdot c_q}\right) < 0$. Thus, $\frac{d\delta_i}{dE(\overline{q_i})} = \left(\frac{d\delta_i}{dp_i^*}\right) \cdot \left(\frac{dp_i^*}{dE(\overline{q_i})}\right) = \left(\frac{d\delta_i}{dp_i^*}\right) \cdot \theta \cdot \frac{b}{2} < 0$. Further, given q_{iA}, δ_i is 1-1 with $\widetilde{q_i}$ and it is also true that $dE(\overline{q_i})/dq_i > 0$. Thus we obtain $d\delta_i/dE(\overline{q_i}) < 0 \Leftrightarrow$

$d(\tilde{q_i} - q_{iA})/dE(\overline{q_i}) < 0 \Leftrightarrow d(\tilde{q_i} - q_{iA})/dq_i < 0$. Fixing q_{iA} also fixes q_{iB}, thus $d(q_{iB} - \tilde{q_i})/dq_i > 0$, so that $Prob(q_i > \tilde{q_i})$ is increasing in q_i. Thus, we have shown that the probability of quality disclosure in Stage 3, is an increasing and continuous function of the vendor's choice of quality q_i in Stage 2, which is another way to state Proposition 1.

iii. The Quality and Type Choice Stages

Since firms choose quality and type before they learn competitors' choices, they are, in effect, choosing quality and type simultaneously. Firms anticipate that in information disclosure stage they will not disclose type information, and that the probability that they will disclose quality information is increasing with their quality choice at the present stage. Firms further anticipate that in the final game stage, prices will follow the non-cooperative equilibrium given in vector form by Equation [5].

For equilibrium derivation, only the conditional expected profit of Firm S_i is required, i.e., the expected profit that takes competitor choices as given (we derive the unconditional expected profit in the Online Appendix). From Lemma 1 and Equations [3] and [10] conditional expected profit is:

$$E(\Pi(q_i)) = E\left(\frac{p_i^*(E(\overline{q_i}))^2}{t} - \frac{k \cdot q_i^2}{2} - c_q Pr_i(q_i)\right) = \int_{-\delta_i}^{\delta_i}\left(\frac{p_i^*(q_i+\varphi)^2}{t} - \frac{k \cdot q_i^2}{2} - c_q Pr_i(q_i)\right) f'_{qi}(\varphi) d(\varphi) =$$

$$\int_{-\delta_i}^{\delta_i} \frac{\left(p_i^*(q_i) + \frac{\theta \cdot b \cdot \varphi}{2}\right)^2}{t} f'_{qi}(\varphi) d(\varphi) - \frac{k \cdot q_i^2}{2} - c_q Pr_i(q_i) = \frac{p_i^*(q_i)^2}{t} + 0 + \int_{-\delta_i}^{\delta_i} \frac{(\theta \cdot b \cdot \varphi)^2}{4t} \cdot f'_{qi}(\varphi) d(\varphi) - \frac{k \cdot q_i^2}{2} - c_q \cdot Pr_i(q_i)$$

$$\Leftrightarrow \cdots \Leftrightarrow E(\Pi(q_i)) = \frac{p_i^*(q_i)^2}{t} - \frac{k \cdot q_i^2}{2} - c_q \cdot Pr_i(q_i) + \frac{\theta^2 b^2 {\alpha_q}^2 (1 - Pr_i(q_i))^3}{48 \cdot t} \quad \textbf{[12]}$$

Since a firm simultaneously optimizes on quality and type, $\partial q_i / \partial d_i = 0$. Equation [6] yields $\frac{\partial p_i}{\partial d_i} = \frac{\partial p_i}{\partial E(\overline{d_i})} \cdot \frac{\partial E(\overline{d_i})}{\partial d_i} = t \cdot \frac{(b_{-1} - b_{+1})}{4} \cdot 1 = 0$. This is because $\partial e_{i+j}(\mathbf{q}, \mathbf{d}) / \partial E(\overline{d_i}) = 0$ for $j \neq -1, +1$, $b_j\left(\partial e_{i+j}(\mathbf{q}, \mathbf{d}) / \partial E(\overline{d_i})\right) = -t b_{+1}/4$, for $j = +1$, $b_j\left(\partial e_{i+j}(\mathbf{q}, \mathbf{d}) / \partial E(\overline{d_i})\right) = -t b_{-1}$ for $j = -1$, $b_{+1} = b_{-1}$ because $\mathbf{A^{-1}}$ is symmetric circulant, and finally, from Lemma 4, $\frac{\partial E(\overline{d_i})}{\partial d_i} = 1$. Using $\frac{\partial p_i}{\partial d_i} = 0$ and $\frac{\partial q_i}{\partial d_i} = 0$, Equation [12] yields $\partial E(\Pi(q_i)) / \partial d_i = 0$. Thus, if there exists a market equilibrium, then there also exists an equilibrium with symmetric types, with $d_j - d_{j-1} = 1/N, \forall j$.

We will also make use of the following two Lemmata (see Online Appendix):

Lemma 9: $q_i^* = argmax_{q_i} \Pi\left(q_i \middle| \forall j \neq i: E(\overline{q_j}) = q_j\right)$

Lemma 10: $q_i^* = argmax_{q_i} \Pi\left(d_i \middle| \forall j \neq i: E(\overline{d_j}) = d_j\right)$

What these two propositions say is that a vendor's optimal quality is the same as the quality that the vendor would have chosen, if all of his competitors quality and type choices were to become exactly known to the consumers, or alternatively, that a vendor should neither overestimate nor underestimate his competitors' expected qualities, compared to their actual choices.

Using Lemma 9 and Lemma 10 and working with the actual choices of competitors d_j and q_j, instead of the random variables $\overline{d_j}$ and $\overline{q_j}$, and assuming that initially all vendors have chosen symmetric types: $d_j - d_{j-1} = \frac{1}{N}, \forall j$ and that all vendors except vendor S_i have chosen qualities $q_j = q, \forall j \neq i$, then from Equation [6],

$p_i^* \left(q_i \middle| \forall j \neq i: E(\overline{q_j}) = q, \forall m: E(\overline{d_m}) - E(\overline{d_{m-1}}) = \frac{1}{N} \right) = \sum_{j=-N/2}^{N/2} \left(b_j \frac{t}{2N} \right) + \frac{b \cdot \theta}{2}(q_i - q) =$

$= \frac{t}{2N} \left(\sum_{j=-N/2}^{N/2} b_j \right) + \frac{b \cdot \theta}{2}(q_i - q) = \frac{t}{N} + \frac{b \cdot \theta}{2}(q_i - q)$, because the sum of elements of any row of $\mathbf{A^{-1}}$ equals 2. Vendor S_i, optimizes quality when $\partial E(\Pi(q_i))/\partial q_i = 0 \Leftrightarrow$

$$q_i^* = \frac{b \cdot \theta \cdot p_i^*}{k \cdot t} - \theta \cdot b \cdot \frac{16 \cdot c_q \cdot t + b^2 \cdot \theta^2 \cdot \alpha_q{}^2 \cdot (1 - Pr_i(q_i))^2}{32 \cdot k \cdot t} \cdot \left(\frac{dPr_i(q_i)}{q_i} \bigg|_{q_i = q_i^*} \right) \qquad [13]$$

In the Online Appendix we show that Equation [13] leads to:

Lemma 11: *There exists a symmetric equilibrium in qualities and types, where vendors place their products symmetrically in the type space* $d_i - d_{i-1} = 1/N$, *and:*

$$q^* = \begin{cases} \dfrac{\theta \cdot b}{k \cdot N} & , c_q > c_q{}' \\[2ex] \dfrac{\theta \cdot b}{k \cdot N} - \left(\dfrac{t}{N}\right)^2 \dfrac{8\left(\frac{t}{N}\right)^2 - 4t \cdot c_q - 8\Gamma}{\alpha_q . k \cdot t \cdot \Gamma} & , c_q < c_q{}' \end{cases}$$

where $\Gamma = \frac{t}{N}\sqrt{\left(\frac{t}{N}\right)^2 - t \cdot c_q}$ and $c_q' = \frac{\alpha_q \cdot \theta \cdot b}{2t}\left(\frac{t}{N} - \frac{\alpha_q \cdot \theta \cdot b}{8}\right)$

Proof of Proposition 2: follows directly from Lemma 11, by observing that $\alpha_q = 0$ implies $c_q' = 0$,

iv. Derivation of the Symmetric Equilibrium

Combining Equation [6], Lemma 3, Equation [11], and Lemma 11, we can characterize firms' choices in the equilibrium that is symmetric in product types, as given by Theorem 1. The ex-ante expected price is calculated by integrating for all possible realizations of all uncertainty intervals.

Finally, notice that from Equation [12], if $\frac{t}{N^2} - \frac{\theta^2 b^2}{2k \cdot N^2} - c_q > 0$, then expected firm profit is positive. θ cannot be too high, as in that case competition would lead to the production of quality levels that would force firms to take a loss. It is easy to show that if the previous quantity is positive then so is $\left(\frac{t}{N}\right)^2 - c_q \cdot t$, that appears inside the roots of Γ and φ, used in Theorem 1.

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C. Online Appendix

This Online Appendix that accompanies the article "A Model of Product Design and Information Disclosure Investments" aims to expand and improve the analysis of the model presented in the article.

Section i completes the study of the model by providing the social welfare analysis of the market, and by comparing previous results (Economides, 1993) obtained in the cylindrical market under perfect market information. The social welfare analysis in Section i.1 is interesting as it reveals that product uncertainty reduces total market surplus but that actions by infomediaries that reduce uncertainty do not necessarily increase market surplus. This can happen because, as per Proposition 3, some firms choose to free ride on the availability of third-party information, and thus reduce their own information disclosure investments. The comparison of the model results with the perfect information case in Section i.2 is also interesting as it helps clarify the impact of product uncertainty to firm quality, price, and profit.

Section ii explores the model results under alternative modeling assumptions. More specifically, we provide the general equilibrium solution that includes consideration of non-symmetric firm types, we study firm investments in disclosing taste-related information when overlapping uncertainty intervals are allowed, and we explore an alternative timing of the game stages where firms must decide on disclosure investments prior to learning about the realization of their uncertainty interval. The description of the general equilibrium solution in Section ii.1 no longer assumes that firms have chosen symmetric types. In the main article we argued that assuming symmetric types does not affect the intuition behind our main results, but that it is merely analytically convenient. Here, we will study an instance of an asymmetric equilibrium, and we will show that the overall dynamics of the model are also the same. Dropping the assumption of non-overlapping type uncertainty intervals in Section ii.2, by considering “very large” product type uncertainty, we are able to show that firms may actually be driven to invest in type information disclosures. While the main article focused on quality-related information disclosures, we show here that taste-related information disclosures also have interesting dynamics: firms can free ride not only on type information revealed by third party infomediaries, but also on information disclosure investments made by competitors. Section ii concludes with the study of an alternative timing of the game stages. Specifically, by forcing disclosure decisions to be made prior to firms’ receiving any information about the realization of their uncertainty intervals, we show that firms will fail to invest in quality disclosure. We thus offer a cautionary note on the sensitivity of information disclosures on what exactly different players know at the time of the disclosure investment.

Finally, Section iii provides the proofs and derivations of certain technical results that were omitted from the main article due to space considerations.

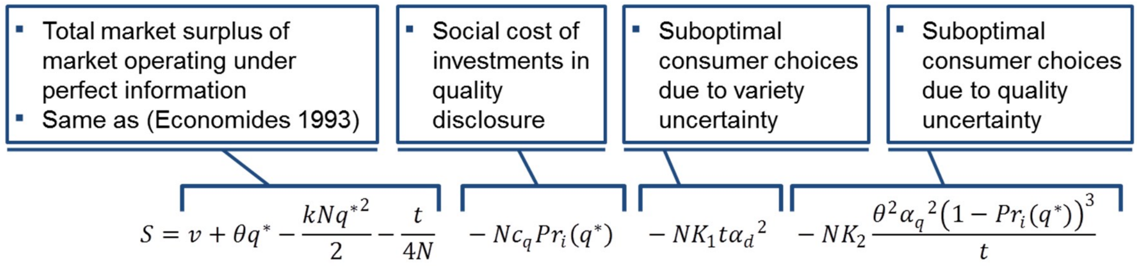


Figure 11: *Composition of the Total Market Surplus formula from Equation* [17]

i. Additional analysis of the game theoretical model

1. Social welfare analysis

We will first calculate the total market surplus. In the symmetric equilibrium, firms choose symmetric types, product qualities as shown in Theorem 1, do not invest in type disclosure, and choose their quality disclosure investment and price taking into account the realization of their uncertainty intervals. Lemmata 4 and 6 show that before realization $E(\overline{d_i}) = d_i + \varepsilon_i$ and $E(\overline{q_i}) = q_i + \varphi_i$ are random variables with well-defined distributions. Thus, a specific market outcome is fully determined by the realization of $\mathbf{x}' = (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_N)$ and of $\mathbf{q}' = (\varphi_1, \varphi_2, \dots, \varphi_N)$.

We first consider the surplus generated by buyers whose ideal type lies between vendors S_i and S_{i+1}, and who purchase from vendor S_i. We omit monetary payments as these cancel out between vendors and buyers. Given a specific market outcome, determined by a realization of **x'** and **q'**, the distance from vendor S_i of the marginal buyer who is indifferent between S_i and S_{i+1} is

$$m_i(\mathbf{x}', \mathbf{q}') = \frac{1/N + \varepsilon_i + \varepsilon_{i+1}}{2} + \frac{\theta}{2t}(\varphi_i - \varphi_{i+1}) + \frac{p_{i+1}^*(\mathbf{x}', \mathbf{q}') - p_i^*(\mathbf{x}', \mathbf{q}')}{2t} \qquad \textbf{[14]}$$

Similarly, the distance from vendor S_i of the marginal buyer who is indifferent between S_i and S_{i-1} is

$$m_{i-1}(\mathbf{x}', \mathbf{q}') = \frac{1/N + \varepsilon_i + \varepsilon_{i-1}}{2} + \frac{\theta}{2t}(\varphi_i - \varphi_{i-1}) + \frac{p_{i-1}^*(\mathbf{x}', \mathbf{q}') - p_i^*(\mathbf{x}', \mathbf{q}')}{2t} \qquad \textbf{[15]}$$

Given **x'**, **q'**, the surplus generated by customers of vendor S_i who prefer product types between S_i and S_{i+1} is: $S_{i,i+1}(\mathbf{x}', \mathbf{q}') = \int_0^{m_i(\mathbf{x}', \mathbf{q}')} (v + \theta \cdot q^* - t \cdot x) dx = m_i(\mathbf{x}', \mathbf{q}') \cdot (v + \theta \cdot q^*) - t \cdot m_i(\mathbf{x}', \mathbf{q}')^2 / 2$. Taking into account buyers with ideal product types between S_i and S_{i-1}, the surplus due to vendor S_i is:

$$S_i(\mathbf{x}', \mathbf{q}') = \left(m_i(\mathbf{x}', \mathbf{q}') + m_{i-1}(\mathbf{x}', \mathbf{q}')\right) \cdot (v + \theta \cdot q^*) - \frac{t \cdot m_i(\mathbf{x}', \mathbf{q}')^2}{2} - \frac{t \cdot m_{i-1}(\mathbf{x}', \mathbf{q}')^2}{2} - k\frac{q^{*2}}{2} - c_q Pr_i(\cdot)$$

Using $\iiint(Z)$ to denote $\int_{\varepsilon_1 = -\frac{\alpha_d}{2}}^{\frac{\alpha_d}{2}} \dots \int_{\varepsilon_N = -\frac{\alpha_d}{2}}^{\frac{\alpha_d}{2}} \int_{\varphi_1 = -\delta_i}^{\delta_i} \dots \int_{\varphi_N = -\delta_i}^{\delta_i} (Z) \frac{f'_{q1} \dots f'_{qN}}{\alpha_d{}^N} d\varepsilon_1 \dots d\varepsilon_N d\varphi_1 \dots d\varphi_N$

the total expected surplus due to vendor S_i is:

$$S_{itot} = \iiint S_i(\mathbf{x}', \mathbf{q}') = \frac{v + \theta \cdot q^*}{N} - \frac{k \cdot q^{*2}}{2} - c_q Pr_i(q^*) - \frac{t}{2}\left(\iiint(m_i(\mathbf{x}', \mathbf{q}')^2) + \iiint(m_{i-1}(\mathbf{x}', \mathbf{q}')^2)\right) \qquad \textbf{[16]}$$

We use Equation [16] to show in Section iii.2 of this Appendix:

$$S_{itotal} = \frac{v + \theta \cdot q^*}{N} - \frac{k \cdot q^{*2}}{2} - \frac{t}{4N^2} - c_q Pr_i(q^*) - K_1 \cdot t \cdot \alpha_d{}^2 - K_2 \cdot \frac{\theta^2 \cdot \alpha_q{}^2 \cdot (1 - Pr_i(q^*))^3}{t},$$

so that the total market surplus is:

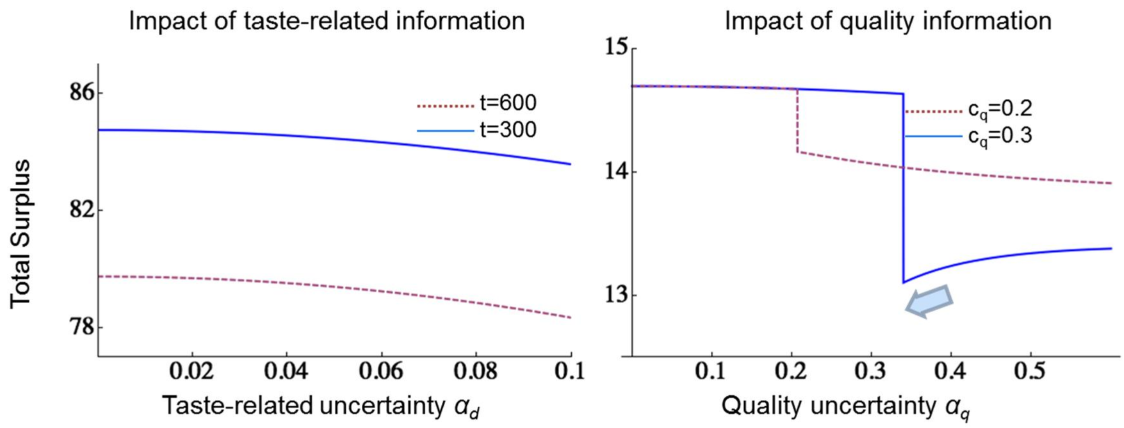


Figure 12: *Total market surplus (Equation* [*17*]*) as intermediaries provide varying degrees of type and quality information. The values of the parameters used for the provision of taste-related information are [N=5 (which also implies b=16/19 and K1≈0.047), θ=10, k=1,* c_d*=0.4, v=100,* α_q*=0], and t as shown in the legend. The values used for the provision of quality information are [N=5 (which also implies b=16/19 and K2≈0.0077), θ=13, k=5, t=12, v=12,* α_d*=0], and* c_q *as shown in the legend*

$$S = v + \theta q^* - \frac{kNq^{*2}}{2} - \frac{t}{4N} - Nc_q Pr_i(q^*) - NK_1 t\alpha_d^{\,2} - NK_2 \frac{\theta^2 \alpha_q^{\,2}\left(1 - Pr_i(q^*)\right)^3}{t} \qquad \textbf{[17]}$$

with K_1,K_2 positive constants depending on *N*. Figure 11 helps us interpret the different terms in Equation [17].

Under perfect information ($\alpha_d = \alpha_q = 0$) Equation [17] yields $v + \theta q^* - kN\frac{q^{*2}}{2} - \frac{t}{4N}$, which matches the market surplus computed in (Economides, 1993). In reality Equation [17] overestimates market surplus, as our model ignores the operating costs of the infomediaries.

Proposition 6: *If infomediaries provide enough type information so that products appear distinct to consumers (no overlapping type uncertainty intervals), any additional provision of type information always increases social welfare*

*Proof***:** Reducing α_d in Equation [17] always increases total surplus.

We next turn our attention to the impact of quality uncertainty. In general total market surplus is somewhat resilient to quality uncertainty and the constant *K2* is relatively small (see Section iii.2 of this Appendix). The resilience is due to the fact that the firms somewhat counterbalance the impact of quality uncertainty to consumers, through their choice of prices. For example, if in the presence of uncertainty infomediaries overstate the true quality of a vendor and buyers risk making a suboptimal product decision, from Lemma 2, neighboring vendors will respond by lowering their prices and capture back some of the consumers that would have otherwise made a suboptimal choice.

In Figure 12 we plot the impact of infomediaries that provide type and quality information, having chosen the parameter values so as to demonstrate the possibility of total welfare reduction as infomediaries increase the level of quality information that they provide. We see that, as per

Proposition 6, the provision by infomediaries of type information is always beneficial to society, when it exceeds the threshold that makes product types appear distinct, as vendors themselves will not release product type information after this threshold is achieved.

The equivalent of Proposition 6 does not hold for quality information. As infomediaries release more quality information they may actually reduce total market welfare. Figure 12 shows that, when $c_q = 0.3$, reducing α_q from 0.5 to anything more than ~0.34, actually reduces market surplus. The intuition as follows: as per Proposition 3, when infomediaries provide more information and help consumers improve their choices and reduce their fit costs, they may also cause many vendors that would have invested in quality information disclosure to refrain from doing so. This occurs in the region where $c_q < c_q'$, where c_q' is given by $c_q' = \frac{\alpha_q \cdot \theta \cdot b}{2t}\left(\frac{t}{N} - \frac{\alpha_q \cdot \theta \cdot b}{8}\right)$. Thus, in contrast to the case of taste-related information, the provision of more quality information by infomediaries may not be socially beneficial. The effect would be even more pronounced if one accounts for the cost of the operation of the infomediaries. However, it is always true that when infomediaries provide perfect quality information the social surplus is maximal, as buyers would always avoid suboptimal decisions due to quality uncertainty.

The combination of the market's resilience to quality uncertainty, as discussed above, and the free riding of vendors on infomediaries' provision of quality information, makes it hard to argue in favor of the social benefit of infomediaries in reducing consumer uncertainty about quality product attributes, even when ignoring the social cost of the operation of the infomediaries themselves (unless the reduction of consumer quality uncertainty is such that it shifts the market's equilibrium regime from $c_q < c_q'$ to $c_q > c_q'$). Society may be better served if infomediaries focus instead in complementing, rather than substituting vendor's information. A complementary role could consist, for example, of a focus in fact checking, in educating consumers about the product category in general, in rating the dependability of the sellers (Pavlou & Gefen, 2004) though the operation of feedback mechanisms (Dellarocas, 2003) or in providing information with a social value component (Hirshleifer, 1971), such as advice on how to use a product in such a way so that a consumer can maximize her utility (e.g., product care instructions or advanced product features).

2. Comparison with perfect information case

Table 3 depicts the market outcomes for the equilibrium that is symmetric in firm types (shown in Theorem 1 and Figures 8 and 10) in a way that facilitates comparison with the perfect information case, obtainable by setting $\alpha_q = \alpha_d = 0$. The latter case is essentially equivalent with the perfect information model studied in (Economides, 1993). We are comparing three cases to the perfect information base case: the case where quality disclosure is costless, the case $c_q < c_q'$ (with c_q' as shown in Theorem 1) where quality disclosure cost is such that each firm has a nonzero probability of

	Base Case ($\alpha_q = \alpha_d = 0$) (Economides 1993)	$c_q = 0$	$c_q < c'_q$	$c_q > c'_q$
Probability of Type Disclosure	N/A	0	0	0
Probability of Quality Disclosure	N/A	1	$1 - 4\frac{\frac{t}{N} - \sqrt{\left(\frac{t}{N}\right)^2 - t \cdot c_q}}{\theta \cdot b \cdot \alpha_q}$	0
Product Quality	Qbase: $\frac{\theta \cdot b}{k \cdot N}$	= Qbase	< Qbase	= Qbase
Product Price	Pbase: $\frac{t}{N}$	Avg price=Pbase	Avg price=Pbase	Avg price=Pbase
Vendor Profit	Πbase: $\frac{t}{N^2} - \frac{\theta^2 \cdot b^2}{2k \cdot N^2}$	Avg profit = Πbase $+\propto \alpha_d{}^2$	Depends on parameters	Avg profit = Πbase $+\propto \alpha_q{}^2 +\propto \alpha_d{}^2$

Table 3: *Comparison of market outcomes for the symmetric equilibrium with the perfect information case, studied in* (Economides, 1993)

disclosure, and finally the case $c_q > c'_q$ where quality disclosure cost is high enough for no firm to ever want to disclose.

The case where quality disclosure is costless closely resembles the way that the market operates under perfect information, as firms chose similar qualities. However, the price is only on average equal to the perfect information price of t/N, as it depends on the exact realizations of the type uncertainty intervals, as per Theorem 1. Also, the average profit is somewhat larger than the profit under perferct information, as, due to type uncertainty, there is some welfare transfer from buyers to sellers (see Section iii.3). It is however clear that if we also set $c_d = 0$, then our model exactly reproduces the results obtained in (Economides, 1993).

In the case where $c_q < c'_q$, we already know from Proposition 2 that firms underinvest in quality, and that their expected profit may be lower or greater than the perfect information case (Figure 10 and Section 4.5). Profit is maximized when c_q approaches c'_q so that firms save on quality production costs, while only some of them incur information disclosure costs.

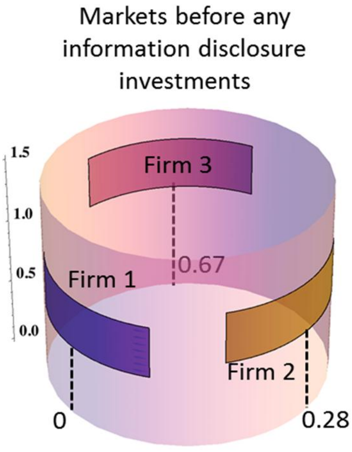
Finally, in the case where $c_q < c'_q$ firms never invest in reducing type and quality uncertainty, but qualities and average prices are the same as the perfect information case. As per Section iii.3, there is a small utility transfer from buyers to sellers that somewhat boosts seller profit compared to the perfect information case.

ii. Relaxing modeling assumptions

1. General Equilibrium Solution

Theorem 2: *The general subgame-perfect solution to the multi-stage game is as follows:*

- ***Stages 1&2***: *Any type configuration* $\mathbf{d}_{init} = [d_1^*, d_2^* \ldots d_N^*]$ *gives rise to a unique type-quality equilibrium, that is the solution to the system:*



		Firm 1	Firm 2	Firm 3
Product design Equilibrium	Type:	**0**	**0.28**	**0.67**
	Quality:	**0.69**	**0.88**	**1.05**
Uncertainty intervals (centers)	Type:	d_1	d_2	d_3
	Quality:	q_1+0.05	q_2	q_3
Quality disclosures & beliefs	Ex-ante, actual	[51%,No]	[62%,Yes]	[68%,Yes]
	Expected quality by buyers	0.63	0.88	1.05
[Prices, Profits]	Ex-ante:	[2.67,0.31]	[3.36,0.49]	[3.96,0.68]
	Actual:	[2.58,0.31]	[3.41,0.48]	[4.02,0.69]

Figure 13: *An example realization of an equilibrium that is not symmetric in types, for* $N = 3$. *The values of the parameters used are* $\left[N = 3\,, \alpha_q = 0.4\,,\ c_q = 0.1\,, \alpha_d = 0.2\,, k = 1.5\,, t = 10\right]$

$\forall i: q_i^* = \frac{b\cdot\theta\cdot p_i(\mathbf{d}_{init},\mathbf{q}_{init})}{k\cdot t} - \theta\cdot b\,\frac{16c_q\cdot t + b^2\theta^2{\alpha_q}^2\left(1-Pr(q_i^*)\right)^2}{32k\cdot t}\left(\frac{dPr_i(q_i)}{q_i}\bigg|_{q_i=q_i^*}\right)$, where $p_i(\mathbf{d}_{init},\mathbf{q}_{init})$ and $Pr(q_i^*)$ are increasing functions of q_i^* that also depend $\mathbf{d}_{init}$ and on all other $q_j^*, j \neq i$

- ***Stage 3****: Firm* i *will disclose quality if and only if its quality exceeds the lower limit of its quality uncertainty interval by* φ*. Furthermore,* φ *decreases in quality* q_i^**. Specifically:* $\varphi = \frac{4}{\theta\cdot b\cdot\alpha_q}\left(p_i(\mathbf{d}_{init},\mathbf{q}_{init}) - \sqrt{p_i(\mathbf{d}_{init},\mathbf{q}_{init})^2 - t\cdot c_q}\right)$*. Firms do not disclose their (horizontal) type for any* $c_d > 0$.
- ***Stage 4:*** *In matrix form:* $\mathbf{p}^*(\mathbf{q},\mathbf{d}) = \boldsymbol{A}^{-1}(t\cdot\mathbf{x}(\mathbf{d}) + \theta\cdot\mathbf{H}\cdot\mathbf{q})$ *where* $\mathbf{d},\mathbf{q}$ *are vectors that contain* $E(\bar{d}_\iota)$ *and* $E(\bar{q}_\iota)$ *respectively,* $\mathbf{x}$ *is a vector with elements* $x_i = \frac{\left(E(\bar{d}_{\iota+1}) - E(\bar{d}_{\iota-1})\right)}{4}$*,* $\boldsymbol{A}^{-1}$ *is the inverse of matrix* $\boldsymbol{A}$ *with rows* $\mathbf{A}_i = [A_{i,1} = 0, \dots, A_{i,i-1} = -\frac{1}{4}, A_{i,i} = 1, A_{i,i+1} = -\frac{1}{4}, \dots, A_{i,N} = 0]$

Note that there is a move by Nature before Stage 3: nature chooses quality and type uncertainty intervals so that vendor qualities are (ex-ante) equiprobable inside these intervals. Also note that, before Stage 4, buyers only observe type uncertainty intervals in the taste space, and in the quality space, buyers only know the qualities of the firms who disclosed in Stage 3.

It is easy to also accommodate quality uncertainty intervals of unequal sizes in the general equilibrium solution. In an extended model where Firm i has associated quality uncertainty interval α_{qi}, the analysis of sections B.ii and B.iii in the Appendix is simply repeated with α_{qi} substituted for α_q, a substitution that carries over to Theorem 2. The intuition behind all the basic results (Propositions 1-5) would remain the same, and the model and results are qualitatively the same. However, we can no longer solve analytically for the case where product types are symmetric $d_i - d_{i-1} = 1/N$, because it is no longer the case that either $Pr_i(q_i) > 0$ for all vendors or $Pr_i(q_i) = 0$ for all vendors (see proof of Lemma 11, below).

In Figure 13 we show one example realization of the subgame perfect equilibrium. The case differs from the symmetric case of Figure 4 as Firm 2 is 0.05 units closer $(0.33 - 0.28)$ to Firm 1, breaking the symmetry in product types. In this case Firm 1 is hurt as its potential market share shrinks, and Firm 3 gains as its potential market share grows. Firm 1's uncertainty interval happens to be such that the buyers initially believe its expected quality to be 0.05 units better than its actual quality. This is just high enough for Firm 1 to prefer not to disclose its quality, causing buyers to discount its quality to 0.63 (0.06 units less than its actual quality) but allowing Firm 1 to save on disclosure costs. Because of lower perceived quality, Firm 1 is forced to lower its price (compared to its ex-ante expected price) and obtains a profit of 0.31 which (ignoring rounding errors) is the same as its ex-ante expected profit. Notice the effect of profits being convex in expected product quality: buyers initially overestimate Firm 1's quality by 0.05 but the firm still does not manage to exceed its ex-ante expected profit. This is because the ex-ante expected profit is inflated by the less frequent cases when buyers overestimate its quality by even larger values (average profit is higher than median profit). Firms 2 and 3 gain from the fact that buyers underestimate the quality of their rival Firm 1, and can charge higher than their ex-ante expected prices. However, due to Firm 1's relatively low price, they just manage to make their ex-ante expected profits.

2. Disclosure of horizontal information

Here, we study the case where initial product type uncertainty is very large, so that type uncertainty intervals of neighboring sellers overlap. This is an interesting case to study for two reasons. First, when type uncertainty intervals overlap, firms' incentives regarding the dissemination of taste-related information change and firms may choose to invest in disclosing their type information. This contrasts with the case of non-overlapping type uncertainty intervals where we showed that firms never disclose type information. The second reason is the emergence of new market dynamics that are not present in the case of quality disclosure. More specifically, it is now the case that firms can free ride, not only on information disclosures by third-party infomediaries, but also on information disclosure investments by competitors. We study both phenomena in turn and show that they lead to firms adopting mixed strategies in the disclosure subgame.

It should perhaps be intuitively expected that under very large product type uncertainty product differences begin to "blur" and firms that do not disclose are forced to compete more and more on product price. In the limit, where type uncertainty is high enough for the firm's type uncertainty interval to span the entire circle, one can easily show that if firms do not disclose they would have to engage in Bertrand-type competition. We should thus expect that firms may start to disclose type information if prior taste-related uncertainty becomes too great.

The exact mechanism by which this happens involves the marginal buyer – the buyer that is indifferent between two firms. When uncertainty intervals begin to overlap the marginal buyer may be situated inside the firms' uncertainty intervals, and this causes demand to become more sensitive to price changes. To see why, we must re-derive firm demand (Equation [2]). We first notice that buyer fit costs may no longer be linear in the distance x between the buyer's most desired product location and the center of a vendor's uncertainty interval, which has size α_d. The fit cost is calculated as follows: if $x > \alpha_d$, then fit costs are given by $\int_{x-a/2}^{x+a/2} \frac{y \cdot t}{\alpha} dy = x \cdot t$ and if $x < \alpha_d$, fit costs are $\int_0^{\alpha_d/2-x} \frac{y \cdot t}{\alpha_d} dy + \int_0^{\alpha_d/2+x} \frac{y \cdot t}{\alpha_d} dy = \frac{\left(\frac{\alpha_d}{2}\right)^2 + x^2}{\alpha_d} t$. In summary:

$$fit\ cost = \begin{cases} x \cdot t & , \quad x \geq \alpha_d \\ \dfrac{\left(\frac{\alpha_d}{2}\right)^2 + x^2}{\alpha_d} t & , \quad x < \alpha_d \end{cases} \tag{18}$$

Equation [3] is depicted in Figure 14, where we can see that expected fit cost can never be zero, when there is uncertainty in the market. We can now calculate the demand facing Firm S_i.

If the marginal buyer between Firms S_i and S_{i+1} is outside any uncertainty interval (because uncertainty intervals do not overlap, or because one or both firms are disclosing taste-related information), the demand facing Firm S_i on the side of the market that is closer to Firm S_{i+1} is given by:

$$D_{i+1}^i = \frac{E\left(\overline{d_{i+1}}\right) - E\left(\overline{d_i}\right)}{2} + \frac{p_{i+1} - p_i}{2t} + \frac{\theta\left(E(\overline{q_i}) - E(\overline{q_{i+1}})\right)}{2t} \tag{19}$$

which is identical to Equation [2].

If the marginal buyer between Firms S_i and S_{i+1} is outside the uncertainty interval of Firm S_i but inside the uncertainty interval of Firm S_{i+1}, the demand facing Firm S_i on the side of the market that is closer to Firm S_{i+1} is given by:

$$D_{i+1}^i = \frac{\alpha_d}{2} + E\left(\overline{d_{i+1}}\right) - E\left(\overline{d_i}\right) + \frac{\sqrt{\alpha_d \cdot t \cdot \left(p_{i+1} - p_i + \theta\left(E(\overline{q_i}) - E(\overline{q_{i+1}})\right) - t\left(E\left(\overline{d_{i+1}}\right) - E\left(\overline{d_i}\right)\right)\right)}}{t} \tag{20}$$

If the marginal buyer between Firms S_i and S_{i+1} is inside the uncertainty interval of Firm S_i but outside the uncertainty interval of Firm S_{i+1}, the demand facing Firm S_i on the side of the market that is closer to Firm S_{i+1} is given by:

$$D_{i+1}^i = -\frac{\alpha_d}{2} + \frac{\sqrt{\alpha_d \cdot t \cdot \left(p_{i+1} - p_i + \theta\left(E(\overline{q_i}) - E(\overline{q_{i+1}})\right) + t\left(E\left(\overline{d_{i+1}}\right) - E\left(\overline{d_i}\right)\right)\right)}}{t} \tag{21}$$

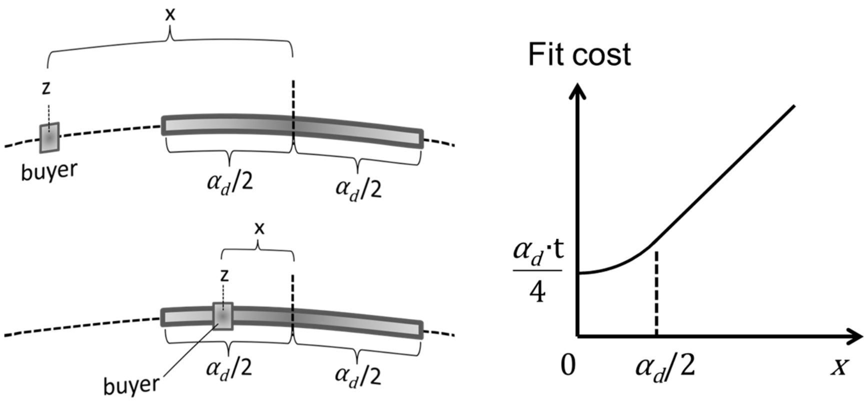


Figure 14: *Buyer fit cost as a function of the distance x from the center of a firm's uncertainty interval*

Finally, if the marginal buyer between Firms S_i and S_{i+1} is inside the uncertainty interval both Firm S_i and Firm S_{i+1}, the demand facing Firm S_i on the side of the market that is closer to Firm S_{i+1} is given by:

$$D_{i+1}^{i} = \frac{\alpha_d\left(p_{i+1} - p_i + \theta\left(E(\overline{q_i}) - E(\overline{q_{i+1}})\right) + t\left(E\left(\overline{d_{i+1}}\right) - E\left(\overline{d_i}\right)\right)^2\right)}{2t\left(E\left(\overline{d_{i+1}}\right) - E\left(\overline{d_i}\right)\right)} \qquad \textbf{[22]}$$

We similarly calculate the demand D_{i-1}^{i} demand facing Firm S_i on the side of the market that is closer to Firm S_{i-1}, which again depends on whether or not the marginal buyer is inside the uncertainty intervals of the two firms. Obviously, the total demand facing Firm S_i is given by:

$$D_i = D_{i+1}^{i} + D_{i-1}^{i} \qquad \textbf{[23]}$$

We can immediately expect price competition to be more intense with these demand functions, as firms capture more buyers for a given price decrease, i.e., demand is more sensitive to price changes. For example when the marginal buyer is outside any uncertainty intervals, differentiating Equation [2] with respect to price, we get $-1/(2t)$. However, if the marginal buyer is inside both firms' uncertainty intervals, then differentiating Equation [22] with respect to price, we get $-a_d/\left(2tE(\overline{d_{i+1}}) - E(\overline{d_i})\right)$, which is greater than $-1/(2t)$, since necessarily $a_d > E(\overline{d_{i+1}}) - E(\overline{d_i})$, or it would not be possible for the uncertainty intervals to overlap. The same is also easy to show for total demand and for the remaining cases. Specifically we can show that differentiating Equations [20] and [21] with respect to price, we get results in-between the previous two values.

Since quality production and disclosure decisions represent sunk costs in the pricing stage, the pricing equilibrium is obtained by solving the system $\frac{d(p_i D_i)}{dp_i} = 0, \forall i$. With demand more sensitive to price changes, the above system will always yield lower prices than the case where uncertainty intervals do not overlap, hence there is now an incentive for firms to disclose product type information, so that they can command higher prices in the last game stage.

Firm Disclosure Strategies (0=No, 1=Yes) Firm1, Firm2, Firm3	Expected profit Profit1, Profit2, Profit3
0,0,0	2.14,2.14,2.14
0,0,1	2.29,2.29,2.20
0,1,0	2.29,2.20,2.29
0,1,1	2.40,2.30,2.30
1,0,0	2.20,2.29,2.29
1,0,1	2.30,2.40,2.30
1,1,0	2.30,2.30,2.40
1,1,1	2.34,2.34,2.34

Table 4: *Expected payoff matrix for a case where product type uncertainty is large enough (a_d=0.24) for uncertainty intervals to overlap, in a market with symmetric product types. The values of the parameters are [N=3 (which also implies b=0.8), k=1, t=30, θ=5, α_q=0 and a_d=24, c_d=0.1]*

We have so far shown that when product type uncertainty is large enough for firms to no longer appear "distinct" (type uncertainty intervals overlap) firms have an incentive to invest in disclosing their type information. We will now see that firms can free ride on the information disclosures made by their competitors and may be able to forgo their own information disclosure investments if their neighbors are disclosing instead. This happens because, consistently to our previous discussion, demand is maximally sensitive to price changes when a marginal buyer is inside both (closest) competitors' uncertainty intervals, it is minimally sensitive to price changes when a marginal buyer is outside both (closest) competitors' uncertainty intervals, and it is somewhere in between for both firms when the marginal buyer is inside only one firm's uncertainty interval. Thus, a competitor's disclosure may force the marginal buyer outside the competitor's uncertainty interval, lower the sensitivity of demand on prices for both firms, and improve equilibrium prices in the last game stage.

We can observe this behavior in Table 4 for $N = 3$ firms. On the left column we see all potential combinations of firm choices in Stage 3, and on the right column we see expected firm payoffs for each combination. Since firms must decide on whether or not to invest in disclosure before they learn the realization of their competitors' uncertainty intervals, the expected profit is calculated by simulating a large number of possible uncertainty interval realizations. Type uncertainty intervals begin to overlap for $N = 3$, when $a_d = 1/6$. At $a_d = 1/6$, there is a non-zero probability that two neighboring firms may have overlapping uncertainty intervals, and at $a_d = 1/3$ the probability becomes one.

We see in Table 4 that if no firm decides to disclose, they each expect a profit of 2.14. In that case, a firm has an incentive to disclose, in which case it improves its expected profit to 2.20. However, its competitors are even happier, as they avoid the information disclosure cost themselves, benefit from overall higher prices in the pricing stage, and manage to get expected profit 2.29. Furthermore, if only one firm discloses, one of the two non-disclosing competitors can improve expected profit from 2.29 to 2.30 by disclosing. But now the remaining non-disclosing firm does even better and expects a profit of 2.40. In fact it is easy to observe that this subgame has 3 non-symmetric

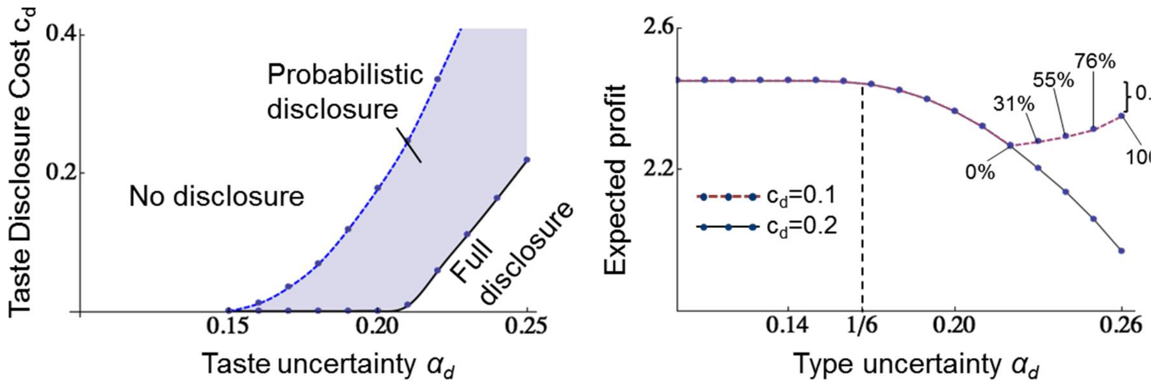


Figure 15: *Left: Type disclosure investments as a function of disclosure cost and ambient type information availability. The values of the parameters are [N=3 (which also implies b=0.8), k=1, θ=5, t=100, α_q=0] Right: Expected firm profits as a function of the type information available from infomediaries. The values of the parameters are [N=3 (which also implies b=0.8), k=1, t=30, θ=5, α_q=0 and a_d as shown]. Percentages denote probability of disclosure in mixed strategy*

equilibria, where only two of the three firms disclose and where the non-disclosing firm obtains maximum profit. Since each of the three firms want to benefit and free ride on the others' disclosures, the only symmetric equilibrium in this case is to disclose with probability ~55%.

On the left graph of Figure 15 we see that there is a region where firms play mixed strategies. For very low quality disclosure costs, mixed strategies begin slightly above $a_d = 1/6$ when uncertainty intervals begin to overlap. As disclosure costs increases, firms require higher and higher a_d in order to consider abandoning non-disclosure. At high a_d, where overlaps are common and large, firms choose to always disclose.

On the right graph of Figure 15 we plot expected firm profit for different sizes of the type uncertainty interval a_d, for two different disclosure costs. As long as a_d is lower than $1/6$, all buyers are always outside firms' uncertainty intervals, prices are relatively high, and profits do not depend on a_d. In this region firms are free riding on the information that is being made available by third party infomediaries. When a_d exceeds $1/6$, uncertainty intervals begin to overlap, prices decrease, and profits fall. However, between $1/6$ and 0.22 firms initially choose to maintain their non-disclosure strategy. At about $a_d = 0.22$, for $c_d = 0.1$, firms start to disclose by adopting a mixed strategy, price competition begins to ease, and profits begin to recover. By the time when $a_d = 0.26$, firms always disclose, marginal buyers are always outside uncertainty intervals, and prices are back to the level observed when a_d was lower than $1/6$. Profits are now exactly the same as profit under low a_d, minus the type disclosure cost c_d. For $c_d = 0.2$ this behavior is observed for higher values of a_d, beyond the right edge of the graph, and thus not shown.

To summarize, our model shows that firms may begin to disclose type information when initial type uncertainty becomes too large and that firms adopt complex strategies free riding not only on disclosures by third party infomediaries, but also on the disclosure investments of their competitors.

3. Alternative game structures

Our model explored the outcome of the multi-stage game in the case where vendors make their decision to invest in information disclosure, knowing the uncertainty that is associated with their own product. It is also interesting to understand vendor incentives to invest in information disclosure when these investments are made prior to receiving any information on product uncertainty intervals.

In the case where vendors have to make disclosure investment decisions prior to receiving information on any uncertainty intervals (including their own), and they can credibly claim to the consumers that this is indeed so, then our model fulfills the three key assumptions of the model studied by Matthews and Postlewaite (Matthews and Postlewaite 1985): First, vendors do not a-priori know if the market will underestimate or overestimate the true quality of their product. If they can credibly claim to consumers that disclosure investment decisions were made prior to observing the information that third parties provide about their product, then the second Matthews & Postlewaite assumption is fulfilled and consumers have no grounds to discount the quality of non-disclosing. Finally, as we see from Table 3, firm profit in a regime of uninformed consumers dominates expected profit under disclosure by the factor $c_q + \propto {\alpha_q}^2 + \propto {\alpha_d}^2$. Mathews and Postlewaite have shown that these assumptions are sufficient for vendors to refrain from investing in quality information disclosure for any positive quality disclosure cost c_q.

This result provides a cautionary tale on the sensitivity of information disclosure investments on the exact time at which these are made. Even small differences on what firms know at the moment when they decide on disclosure investments can lead to very different investment decisions.

iii. Technical proofs and derivations

1. Proof of Lemma 6

From Lemma 5 the combination of the vendor's choice of $\mathrm{q_i}$ in Stage 2 and nature's choice of uncertainty intervals afterwards determines a disclosure threshold $\tilde{\mathrm{q}}_\mathrm{i}(\mathrm{q_i}, \mathrm{f_{qi}})$ for the vendor, so that only if $\mathrm{q_i} < \tilde{\mathrm{q}}_\mathrm{i}$ will the vendor invest in quality disclosure in Stage 3. If $\mathrm{q_i} > \tilde{\mathrm{q}}_\mathrm{i}$ the vendor will not release quality information and buyers will update their beliefs about the vendor's expected quality $\mathrm{E}(\overline{\mathrm{q_i}})$ according to Equation [8]. This value will either overestimate or underestimate the vendor's true quality by some value $\varepsilon_\mathrm{i} \in \left[\mathrm{q_{iA}} - \underline{\mathrm{q_i}}, \tilde{\mathrm{q}}_\mathrm{i} - \underline{\mathrm{q_i}}\right] = [-\delta_\mathrm{i}, \delta_\mathrm{i}]$. To recapitulate, with probability $\mathrm{Pr_i}(\mathrm{q_i})$ the buyers learn the vendor's true quality so that $\mathrm{E}(\overline{\mathrm{q_i}}) = \mathrm{q_i}$, and with probability $1 - \mathrm{Pr_i}(\mathrm{q_i})$, the buyers, seeing that the vendor did not disclose set $\mathrm{E}(\overline{\mathrm{q_i}}) = \underline{\mathrm{q_i}} \in [\mathrm{q_i} - \delta_\mathrm{i}, \mathrm{q_i} + \delta_\mathrm{i}]$.

2. Derivation of Equation [17]

There are six terms in $\iiint {m_i}^2 = \iiint \left(\frac{1/N+\varepsilon_i+\varepsilon_{i+1}}{2}\right)^2 + \iiint \left(\frac{\theta}{2t}(\varphi_i - \varphi_{i+1})\right)^2 + \iiint \left(\frac{p^*_{i+1}-p^*_i}{2t}\right)^2 +$

$$+2 \iiint \frac{1/N+\varepsilon_i+\varepsilon_{i+1}}{2} \frac{\theta}{2t}(\varphi_i - \varphi_{i+1}) + 2 \iiint \frac{1/N+\varepsilon_i+\varepsilon_{i+1}}{2} \cdot \frac{p^*_{i+1}-p^*_i}{2t} + 2 \iiint \frac{\theta}{2t}(\varphi_i - \varphi_{i+1}) \cdot \frac{p^*_{i+1}-p^*_i}{2t}$$

and similarly for $\iiint m_{i-1}{}^2$. We proceed to calculate term. It is useful to have a picture of $p_{i+1}^* - p_i^*$. (the difference $p_{i-1}^* - p_i^*$ that appears in $\iiint m_{i-1}{}^2$ is similar). From Equation [6]:

$$p_{i+1}^* - p_i^* = \sum_{j=-N/2}^{N/2} \left(b_j - b_{j+1} \right) e_{j+i+1} = \ldots$$

$$(b_{-3} - b_{-2}) \cdot \left(\frac{t}{4}\left(\frac{2}{N} + \varepsilon_{i-1} - \varepsilon_{i-3} \right) + \frac{\theta}{4}\left(2\varphi_{i-2} - \varphi_{i-1} - \varphi_{i-3} \right) \right) +$$

$$(b_{-2} - b_{-1}) \cdot \left(\frac{t}{4}\left(\frac{2}{N} + \varepsilon_{i} - \varepsilon_{i-2} \right) + \frac{\theta}{4}\left(2\varphi_{i-1} - \varphi_{i} - \varphi_{i-2} \right) \right) +$$

$$(b_{-1} - b_{0}) \cdot \left(\frac{t}{4}\left(\frac{2}{N} + \varepsilon_{i+1} - \varepsilon_{i-1} \right) + \frac{\theta}{4}\left(2\varphi_{i} - \varphi_{i+1} - \varphi_{i-1} \right) \right) +$$

$$(b_{0} - b_{1}) \cdot \left(\frac{t}{4}\left(\frac{2}{N} + \varepsilon_{i+2} - \varepsilon_{i} \right) + \frac{\theta}{4}\left(2\varphi_{i+1} - \varphi_{i+2} - \varphi_{i} \right) \right) +$$

$$(b_{1} - b_{2}) \cdot \left(\frac{t}{4}\left(\frac{2}{N} + \varepsilon_{i+3} - \varepsilon_{i+1} \right) + \frac{\theta}{4}\left(2\varphi_{i+2} - \varphi_{i+3} - \varphi_{i+1} \right) \right) + \ldots$$

The 1st term $\iiint \left(\frac{1/N + \varepsilon_i + \varepsilon_{i+1}}{2} \right)^2$ yields $\frac{1}{4} \iiint \left(\frac{1}{N^2} + \varepsilon_i{}^2 + \varepsilon_{i+1}{}^2 \right) + \iiint \left(\frac{1}{N}\varepsilon_i + \frac{1}{N}\varepsilon_{i+1} + \frac{1}{N}\varepsilon_i\varepsilon_{i+1} \right)$. By Equation [7]: $\iiint \left(\frac{1}{N}\varepsilon_i + \frac{1}{N}\varepsilon_{i+1} + \frac{1}{N}\varepsilon_i\varepsilon_{i+1} \right) = 0$ and $\frac{1}{4} \iiint \left(\frac{1}{N^2} + \varepsilon_i{}^2 + \varepsilon_{i+1}{}^2 \right) = \frac{1}{4N^2} + \frac{\alpha_d{}^2}{24}$. The similar term from $\iiint m_{i-1}{}^2$ also yields $\frac{1}{4N^2} + \frac{\alpha_d{}^2}{24}$.

By Equation [10]:, the 2nd term $\iiint \left(\frac{\theta}{2t}(\varphi_i - \varphi_{i+1}) \right)^2$ yields $\frac{\theta^2}{4t} \iiint \varphi_i{}^2 + \varphi_{i+1}{}^2 + 2\varphi_i\varphi_{i+1} = = \frac{\theta^2 \alpha_q{}^2 \left(1 - Pr_i(q^*)\right)^3}{24t^2}$, as $\iiint \varphi_i\varphi_{i+1} = 0$. The similar term from $\iiint m_{i-1}{}^2$ also yields $\frac{\theta^2 \alpha_q{}^2 \left(1 - Pr_i(q^*)\right)^3}{24t^2}$.

The 3rd term $\iiint \left(\frac{p_{i+1}^* - p_i^*}{2t} \right)^2$ includes many terms which we proceed to group. Terms where some ε_i is multiplied to anything but itself are zero, and so are the terms where some φ_i is multiplied to anything but itself. Terms that do not involve φ_i or ε_i are

$$\frac{t}{2N^2}\left(\ldots + (b_{-1} - b_0) \sum_{j=-N/2}^{N/2} \left(b_j - b_{j+1} \right) + (b_0 - b_1) \sum_{j=-N/2}^{N/2} \left(b_j - b_{j+1} \right) + \ldots \right) =$$

$$\frac{t}{2N^2} \sum_{j=-N/2}^{N/2} \left(b_j - b_{j+1} \right) \cdot \sum_{j=-N/2}^{N/2} \left(b_j - b_{j+1} \right) = \frac{t}{2N^2} \cdot 0 \cdot 0 = 0$$

The only non zero terms are those that include $\varphi_i{}^2, \forall i$ and $\varepsilon_i{}^2, \forall i$. The terms that include $\varphi_i{}^2$, $\forall i$ are

$$\frac{1}{4^2}\left(4\varphi_i{}^2 (b_{-1} - b_0)^2 + \varphi_i{}^2 (b_{-2} - b_{-1})^2 + \varphi_i{}^2 (b_0 - b_1)^2 - 4\varphi_i{}^2 (b_{-1} - b_0)(b_{-2} - b_{-1}) - \right.$$

$$\left. -4\varphi_i{}^2 (b_{-1} - b_0)(b_0 - b_1) + 2\varphi_i{}^2 (b_0 - b_1)(b_{-2} - b_{-1}) \right) = \frac{\varphi_i{}^2}{16}\left(9b_{-1}{}^2 + 9b_0{}^2 + b_{-2}{}^2 + b_1{}^2 - \right.$$

$\left. 18b_{-1}b_0 + 6b_{-2}b_0 - 6b_0b_1 - 6b_{-2}b_{-1} + 6b_{-1}b_1 - 2b_{-2}b_1 \right)$. Since $\iiint \varphi_i{}^2 = \iiint \varphi_j{}^2$, when we add these terms up, $\forall i$, we will have $(9 + 9 + 1 + 1)b_i = 20b_i$, $(-18 - 6 - 6)b_ib_{i+1} = -30b_ib_{i+1}$, $(6 + 6)b_ib_{i+2} = 12b_ib_{i+2}$ and $-2b_ib_{i+3}$. The summation of all such terms ($N \geq 4$) yields: $\frac{\theta^2}{16 \cdot 4t^2} B_1 \iiint \varphi_i{}^2 = \frac{\theta^2 \alpha_q{}^2 \left(1 - Pr_i(q^*)\right)^3}{16 \cdot 48t^2} B_1$, with $B_1 = \sum_{j=-N/2}^{N/2} \left(20b_j{}^2 - 30b_jb_{j+1} + 12b_jb_{j+2} - 2b_jb_{j+3} \right)$. The similar term from $\iiint m_{i-1}{}^2$ also yields the same result. Using exactly the same process, the summation of all terms that include $\varepsilon_i{}^2$ yields $\frac{\cancel{t^2}}{4 \cdot \cancel{t^2} \cdot 16} B_2 \iiint \varepsilon_i{}^2 = B_2 \cdot \frac{\alpha_d{}^2}{768}$, where $B_2 =$

$\sum_{j=-N/2}^{N/2}\left(4b_j^{\,2}-2b_jb_{j+1}-4b_jb_{j+2}+2b_jb_{j+3}\right)$. The similar term from $\iiint m_{i-1}^{\,2}$ also yields the same result.

The 4th term $2\iiint \frac{1/N+\varepsilon_i+\varepsilon_{i+1}}{2}\frac{\theta}{2t}(\varphi_i-\varphi_{i+1})$ yields zero from Equation [10]:

The 5th term $2\iiint \frac{1/N+\varepsilon_i+\varepsilon_{i+1}}{2}\cdot\frac{p_{i+1}^*-p_i^*}{2t}$ includes terms which we group as follows. The terms where some ε_i is multiplied to anything but itself are 0, and so are the terms where some φ_i is multiplied to anything but itself. Terms not involving φ_i or ε_i are $2\iiint\left(\frac{1}{2N}\cdot\frac{1}{4N}\cdot\sum_{j=-N/2}^{N/2}\left(b_j-b_{j+1}\right)\right)=0$. Finally, the non-zero terms are: $\iiint(\varepsilon_i+\varepsilon_{i+1})\left(\frac{t/4}{2t}\right)\left((b_{-2}-b_{-1})\varepsilon_i+(b_{-1}-b_0)\varepsilon_{i+1}-(b_0-b_1)\varepsilon_i-(b_1-b_2)\varepsilon_{i+1}\right)=\cdots=-\frac{b_0-b_2}{2}\cdot\frac{\alpha_d^{\,2}}{4\cdot 6}$. In this case, the similar term from $\iiint m_{i-1}^{\,2}$ yields the opposite: $+\frac{b_0-b_2}{2}\cdot\frac{\alpha_d^{\,2}}{4\cdot 6}$, so that the contribution of this term in $\iiint m_i^{\,2}+\iiint m_{i-1}^{\,2}$ is zero.

Finally, the 6th term $2\iiint\frac{\theta}{2t}(\varphi_i-\varphi_{i+1})\frac{p_{i+1}^*-p_i^*}{2t}$ includes terms which we group as follows. The terms where some ε_i is multiplied to anything but itself are 0, and so are the terms where some φ_i is multiplied to anything but itself. There are no terms not involving φ_i or ε_i. The non-zero terms are:

$$\iiint\frac{\theta^2(\varphi_i-\varphi_{i+1})}{2t^2}\left((b_{-1}-b_0)(2\varphi_i-\varphi_{i+1})-(b_{-2}-b_{-1})\varphi_i+(b_0-b_1)(2\varphi_{i+1}-\varphi_i)-(b_1-b_2)\varphi_{i+1}\right)$$

$=\cdots=\frac{\theta^2\alpha_q^{\,2}\left(1-Pr_i(q^*)\right)^3}{48t^2}\left((b_1-b_2)-3(b_0-b_1)\right)$. The similar term of $\iiint m_{i-1}^{\,2}$ is also the same.

In summary: $\iiint m_i^{\,2}+\iiint m_{i-1}^{\,2}=\frac{1}{2N^2}+\left(\frac{1}{12}+\frac{B_2}{384}\right)\alpha_d^{\,2}+\frac{\theta^2\alpha_q^{\,2}\left(1-Pr_i(q^*)\right)^3}{t^2}\left(\frac{1}{12}+\frac{B_1}{384}-\frac{B_3}{24}\right)$

where $B_3=3(b_0-b_1)-(b_1-b_2)$, $B_1=\sum_{j=-N/2}^{N/2}\left(20b_j^{\,2}-30b_jb_{j+1}+12b_jb_{j+2}-2b_jb_{j+3}\right)$ and $B_2=\sum_{j=-N/2}^{N/2}\left(4b_j^{\,2}-2b_jb_{j+1}-4b_jb_{j+2}+2b_jb_{j+3}\right)$.

3. Derivation of unconditional expected profit

Using $\iiint(Z)$ to denote $\int_{\varepsilon_1=-\frac{\alpha_d}{2}}^{\frac{\alpha_d}{2}}\cdots\int_{\varepsilon_N=-\frac{\alpha_d}{2}}^{\frac{\alpha_d}{2}}\int_{\varphi_1=-\delta_i}^{\delta_i}\cdots\int_{\varphi_N=-\delta_i}^{\delta_i}(Z)\frac{f'_{q1}\cdots f'_{qN}}{\alpha_d^{\,N}}d\varepsilon_1\ldots d\varepsilon_N d\varphi_1\ldots d\varphi_N$ the expected profit of vendor S_i is: $\iiint\left(\frac{p_i^{*2}}{t}-\frac{k\cdot q_i^2}{2}-c_q\cdot Pr_i(q_i)\right)$. We first integrate over all possible realizations of quality uncertainty intervals. Using Lemmata 1 and 2 and Equation [10], we first obtain $\frac{p_i^{*2}}{t}-\frac{k\cdot q_i^2}{2}-c_q\cdot Pr_i(q_i)+\frac{\alpha_q^{\,2}\theta^2\left(b^2\left(1-Pr_i(q_i)\right)^3-\sum_{j\neq i}b_m^{\,2}\left(1-Pr_j(q_j)\right)^3\right)}{48\cdot t}$. We next integrate over all possible realizations of type uncertainty intervals, which only affect the term p_i^{*2}/t. Following the process used in the previous section, we obtain $\frac{t}{16}\sum_{j=1}^{N}\left(\sum_{k=-N/2}^{N/2}\left(d_{j+k}-d_{j+k-2}\right)\cdot b_{k-1}\right)^2+t\cdot\frac{\sum_{k=-N/2}^{N/2}(b_{k+2}-b_k)^2}{16\cdot 12}\sum_{j=1}^{N}a_j^{\,2}-\frac{k\cdot q_i^2}{2}-c_q\cdot Pr_i(q_i)+\frac{\alpha_q^{\,2}\theta^2\left(b^2\left(1-Pr_i(q_i)\right)^3-\sum_{j\neq i}b_m^{\,2}\left(1-Pr_j(q_j)\right)^3\right)}{48\cdot t}$. For symmetric firm types this simplifies to:

$$\text{Expected profit} = \frac{t}{N^2} - \frac{\theta^2 \cdot b^2}{2k \cdot N^2} - c_q \cdot Pr_i(q_i) + \propto {\alpha_q}^2 + \propto {\alpha_d}^2 \qquad \textbf{[24]}$$

where $\propto$ denotes proportionality. The term $\frac{t}{N^2} - \frac{\theta^2 \cdot b^2}{2k \cdot N^2}$ equals expected firm profit under perfect information, $-c_q \cdot Pr_i(q_i)$ is the expected disclosure cost and the last two terms represent utility transfers from buyers to sellers, due to uncertainty. Utility transfer due to quality uncertainty is $\frac{{\alpha_q}^2\theta^2\left(b^2\left(1-Pr_i(q_i)\right)^3 - \sum_{j\neq i} {b_m}^2\left(1-Pr_j(q_j)\right)^3\right)}{48\cdot t}$ and due to type uncertainty is $\frac{\sum_{k=-N/2}^{N/2}(b_{k+2}-b_k)^2}{16\cdot 12}\sum_{j=1}^{N} {a_j}^2$.

4. Proof of Lemma 9

$$E\left(\Pi\left(q_i \middle| \forall j \neq i: E(\overline{q_j})\right)\right) = \frac{\left(p_i^*(q_i = E(\overline{q_\iota}) | \forall j \neq i: E(\overline{q_j}))\right)^2}{t} - \frac{k\cdot {q_i}^2}{2} - c_q \cdot Pr_i(q_i) + \frac{\theta^2 b^2 {\alpha_q}^2 \left(1-Pr_i(q_i)\right)^3}{48\cdot t}$$

A vendor S_i optimizes quality by: $q_i^* = argmax_{q_i} E\left(\Pi\left(q_i \middle| \forall j \neq i: E(\overline{q_j})\right)\right) = argmax_{q_i}[$
$\int_{\varepsilon_1=-\delta_1}^{\delta_1} \dots \int_{\varepsilon_{j\neq i}=-\delta_i}^{\delta_i} \dots \int_{\varepsilon_N=-\delta_N}^{\delta_N} E\left(\Pi(q_i|\forall j \neq i: E(\overline{q_j}) = q_j + \varepsilon_j\right) f'_{q1}(\varepsilon_1) \dots f'_{qj}(\varepsilon_j) \dots f'_{qN}(\varepsilon_N) d\varepsilon_N \dots d\varepsilon_{j\neq i} \dots d\varepsilon_1 \,]$
as vendors optimize quality over all possible realizations of competitors' uncertainty intervals. However, quality uncertainty intervals are private information at the stage of information disclosure, and the realization of competitors' uncertainty intervals is not known until the pricing stage, so that the terms $-\frac{k\cdot {q_i}^2}{2} - c_q \cdot Pr_i(q_i) + \frac{\theta^2 b^2 {\alpha_q}^2 \left(1-Pr_i(q_i)\right)^3}{48\cdot t}$ are not affected by different realizations of competitors' uncertainty intervals, since they only depend on actual product qualities. Denoting these three terms as Y, and using Equation [10] and Lemma 2:

$$q_i^* = argmax_{q_i}\left[\frac{1}{t}\iiint_{\varepsilon_{j\neq i}=-\delta_j}^{\varepsilon_{j\neq i}=\delta_j} \left(p_i^*\left(q_i \middle| \forall j \neq i: E(\overline{q_j}) = q_j + \varepsilon_j\right)\right)^2 + Y\, f'_{q1}(\varepsilon_1) \dots f'_{qj}(\varepsilon_j) \dots f'_{qN}(\varepsilon_N) d\varepsilon_N \dots d\varepsilon_{j\neq i} \dots d\varepsilon_1\right]$$

$$= argmax_{q_i}\left[\frac{1}{t}\iiint_{\varepsilon_{j\neq i}=-\delta_j}^{\varepsilon_{j\neq i}=\delta_j} \left(p_i^*\left(q_i \middle| \forall j \neq i: E(\overline{q_j}) = q_j\right) - \sum_{m=-\frac{N}{2}, j\neq 0}^{\frac{N}{2}} \frac{\theta b_m \varepsilon_{m+i}}{2}\right)^2 f'_{q1}(\varepsilon_1) \dots f'_{qj}(\varepsilon_j) \dots f'_{qN}(\varepsilon_N) d\varepsilon_N \dots d\varepsilon_{j\neq i} \dots d\varepsilon_1\right] + Y$$

$$= argmax_{q_i}\frac{1}{t}\left(p_i^*\left(q_i \middle| \forall j \neq i: E(\overline{q_j}) = q_j\right)\right)^2 + Y + \text{terms that are either zero or constant}$$

$$\Rightarrow q_i^* = argmax_{q_i}\Pi\left(q_i \middle| \forall j \neq i: E(\overline{q_j}) = q_j\right)$$

5. Proof of Lemma 10

Follows exactly the proof of Lemma 9

6. Proof of Lemma 11

Using ${p_i}^* = \frac{t}{N} + \frac{b\cdot\theta}{2}(q_i - q)$ in Equation [13], setting $q_i^* = q$ and solving for q, we find the symmetric equilibrium in qualities, to which every vendor is responding optimally to all other vendors having chosen quality level q.

There exist two different cases, depending on whether or not $Pr_i(q_i) > 0$. If for all vendors $1 - \frac{4}{\theta\cdot b\cdot\alpha_q}\left(p_i^* - \sqrt{{p_i^*}^2 - t\cdot c_q}\right) < 0$, which, from Lemma 7, implies $Pr_i(q_i) = 0$, then $\partial Pr_i(q_i)/\partial q_i = 0$. From Equation [13], setting $q_i^* = q$ yields $p_i^* = t/N$ and $q_i^* = (b\cdot\theta)/(k\cdot N)$. The requirement $1 < \frac{4}{\alpha_q \cdot \theta \cdot b}\left(p_i^* - \sqrt{p_i^{*2} - t \cdot c_q}\right)$ translates to $c_q > \frac{\alpha_q \cdot \theta \cdot b}{2t}\left(\frac{t}{N} - \frac{\alpha_q \cdot \theta \cdot b}{8}\right) = c_{q\_critical}(t/N)$. If on the other hand, for all vendors, $Pr_i(q_i) > 0$, we similarly find that as long as $c_q < c_{q\_critical}(t/N)$, then $q_i^* = \frac{b \cdot \theta}{k \cdot N} - \frac{t}{N} \cdot \frac{8\left(\frac{t}{N}\right)^2 - 4c_q \cdot t - 8\frac{t}{N}\sqrt{\left(\frac{t}{N}\right)^2 - c_q \cdot t}}{\alpha_q \cdot k \cdot t \cdot \sqrt{\left(\frac{t}{N}\right)^2 - c_q \cdot t}}$. This covers all possible cases, as depending on the value of c_q, either $Pr_i(q_i) > 0$ for all vendors or $Pr_i(q_i) = 0$ for all vendors (there can be no symmetric equilibrium where $Pr_i(q_i) = 0$ is only true for some vendors). The case of $c_q = c_{q\_critical}(t/N)$ does not yield an equilibrium as the derivative $dPr_i(q_i)/q_i|_{q_i = q_i^*}$ is not defined.