Selling to Conspicuous Consumers: Pricing, Production, and Sourcing Decisions

Necati Tereyağolu
College of Management, Georgia Institute of Technology, Atlanta, Georgia 30308, necati.tereyagoglu@mgt.gatech.edu

Senthil Veeraraghavan
The Wharton School, University of Pennsylvania, Philadelphia, Pennsylvania 19104, senthilv@wharton.upenn.edu

Consumers often purchase goods that are “hard to find” to conspicuously display their exclusivity and social status. Firms that produce such conspicuously consumed goods such as designer apparel, fashion goods, jewelry, etc., often face challenges in making optimal pricing and production decisions. Such firms are confronted with precipitous trade-off between high sales volume and high margins, because of the highly uncertain market demand, strategic consumer behavior, and the display of conspicuous consumption. In this paper, we propose a model that addresses pricing and production decisions for a firm, using the rational expectations framework. We show that, in equilibrium, firms may offer high availability of goods despite the presence of conspicuous consumption. We show that scarcity strategies are harder to adopt as demand variability increases, and we provide conditions under which scarcity strategies could be successfully adopted to improve profits. Finally, to credibly commit to scarcity strategy, we show that firms can adopt sourcing strategies, such as sourcing from an expensive production location/supplier or using expensive raw materials.

Key words: strategic customer behavior; game theory; conspicuous consumption; pricing; scarcity; sourcing

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1. Introduction

Consumers looking to signal their uniqueness and exclusivity have often expressed themselves by consuming goods prominently to display their status. Firms that design and sell luxury products or innovative gadgets have often desired exclusivity in their looks and design. The prominent display of logos, limited availability, and expensive designs are some ways through which firms have displayed their exclusivity. For instance, the Big Pony apparel line designed by Ralph Lauren has more prominent logos that can be displayed conspicuously by the wearer.1

Many luxury watches with intricate designs, such as Piaget watches, are sold only through limited number of boutique stores and authorized retailers in the United States (http://www.piaget.com). Firms often face decisions on how to make production and pricing decisions when selling such conspicuous goods.

We study the decisions of a firm when there is conspicuous consumption, i.e., when some members of the population are motivated by invidious comparison (Bagwell and Bernheim 1996). Invidious comparison refers to situations in which a member of a social class consumes conspicuously to distinguish himself from other members. We examine the cases when some consumers seek, purchase, and consume hard-to-find products to display their distinction from the other consumers in the population. Consistent with the literature (Leibenstein 1950, Amaldoss and Jain 2005a), we term consumers that are driven by such invidious comparisons as engaging in snobbish behavior.

With increasingly unpredictable market demand conditions, many firms selling to conspicuous consumers face difficult trade-offs between profits and exclusivity, which puts them in a bind. Some firms adopt the strategy to compete on prices and hope to increase revenues through sales volume. Recently, retailers such as Nordstrom have attributed their increased revenues to slashed prices and increased inventory availability.2 On the other hand, other firms

1 A comparison between the Classic-Fit Polo and the more conspicuous Classic-Fit Big Pony Polo shirts on http://www.ralphlauren.com shows the Big Pony designs being sold at higher prices.

2 Nordstrom chief executive officer David Spatz argued for cutting prices of several products to respond effectively to the market. For instance, handmade Anyi Lu designer shoes sold at less than $400, instead of the regular retail price $595, which was accompanied by 69% increase in store inventory (Giacobbe 2009).
have chosen to limit their product availability by creating scarcity (Rigby et al. 2009), and such shortages for new products have been commonly observed (Gumbel 2007).

In general, a reduction in product availability leads to reduced sales, which may hurt firm profits. It is still unclear if the firms should or should not use scarcity strategies in selling goods and, if they do, when those strategies should be implemented. Thus, both from practitioner and research perspectives, it is imperative to understand how firms should make interconnected decisions such as how much of the good to produce, how to price those goods, and when to invest in innovative designs or use an expensive supplier, etc.

In this paper, we analyze a monopolist firm’s pricing and production decisions while selling a good to a market with uncertain demand from conspicuous consumers. When the demand is not deterministic, it is difficult to identify if scarcity occurred because of an unexpected high demand (a random realization) or because of decidedly low inventory (a strategic decision). Often, it is difficult to separate the two effects because of the lack of full information on the production process (unobservability). This is a key focal point of our approach. We show that scarcity strategy could emerge in equilibrium in markets with stochastic demand because of conspicuous consumers.

Our model uses the rational expectations (RE) framework (Muth 1961, Stokey 1981) to analyze the impact of conspicuous consumption on firm’s pricing, production, and sourcing decisions. The framework has been used in some recent papers (see, for instance, Jerath et al. 2010, Su 2007).

The scheme of our paper is as follows. We first position our contributions with respect to extant literature. In §2, we analyze the equilibrium pricing and production decisions of a firm selling to conspicuous consumers in a homogeneous market (in §2.1) and in a heterogeneous market (in §2.2). In §3, using our structural results, we consider strategic “scarcity” decisions. In §4, we show that our structural insights on scarcity hold under a variety of alternative models and conditions. In §5, we discuss how firms can commit to a scarcity strategy, either by limiting clearance pricing or sourcing expensively. Finally, in §6, we conclude and discuss future research.

1.1. Our Position in the Literature

Many new products—gadgets, fashion apparel, and goods (designer brands)—are often treated as vehicles of self-expression through which consumers exhibit their desire for exclusivity or conformity. Recently, there has been emergent interest in literature, on how production decisions are impacted by consumers’ decision-making behavior (within the rational framework). This paper studies the operational decisions of a firm when it sells to consumers involved in conspicuous consumption and notes how operational decisions in production, salvaging, and sourcing can be employed together with pricing and scarcity strategies.

Economists have pointed out how consumption could be beset with positive externalities due to social conformity in the context of restaurant choice (Becker 1991), due to network effects in the context of technology (Katz and Shapiro 1985), due to market frenzies (DeGraba 1995), or due to herd behavior (Bikhchandani et al. 1992).

However, the notion of consumers purchasing goods to be conspicuous dates back to Veblen (1899) who, in his “The Theory of the Leisure Class,” explained how individuals consumed highly conspicuous goods and services to advertise their wealth or social status. Leibenstein (1950) emphasized the significance of social factors in consumption and argued that price by itself might enhance utility. Bagwell and Bernheim (1996) argued that the relationship between price and demand should emerge in equilibrium, and derive conditions for such “Veblen effects” to arise in equilibrium. Corneo and Jeanne (1997) established that conspicuous consumption might emerge as a tool to signal wealth. Although economics literature has examined when Veblen effects may emerge in market equilibrium, production and pricing decisions of a firm facing conspicuous consumers have been relatively underexplored.

We believe that our paper is the first attempt on modeling the production, pricing, and sourcing decisions of a firm facing conspicuous consumption in a market with uncertain demand and strategic customer behavior. Along with demand uncertainty, the interest in modeling customer behavior and studying the effects on firms’ decisions is gaining increased attention. The impact of forward-looking or strategic customers on operational decisions has been considered in a variety of contexts such as seasonal goods (Aviv and Pazgal 2008), commitment in supply chain performance (Su and Zhang 2008), triggering early purchases (Liu and van Ryzin 2008), price-match guarantees (Lai et al. 2010), reservations (Cil and Lariviere 2010), and quick response strategy (Cachon and Swinney 2010). See Netessine and Tang (2009) and papers therein for an excellent overview of strategic consumer behavior literature. However, none of the above papers considers conspicuous consumption or studies intentional scarcity.

Little has been done to understand the customer-oriented reasons behind firms’ excessive pricing

3 Examples of such conspicuous products might include Christian Dior watches (Amaldoss and Jain 2005a), Yves Saint Laurent shoes (Brunner et al. 2005), and “Cherry Blossom” Murakami bags (Amaldoss and Jain 2008).
strategies used in parallel with scarce availability and addition of costly features to a product. In a series of papers, Amaldoss and Jain (2005a, b) showed that conspicuous consumers may exhibit an upward sloping demand curve only in a heterogeneous market, and the firm may charge extreme prices to extract that margin. Amaldoss and Jain (2008) showed that the addition of costly features to a product can increase profits in a market with reference group effects. Stock and Balachander (2005) provided a signaling reason behind product shortages to sell “hot products” in a market with quality uncertainty. Also, there has been some recent interest in understanding how inventory shortages (Debo and van Ryzin 2009) or long queues (Veeraraghavan and Debo 2009) may signal quality. In contrast, we believe that our work shows how scarcity strategies along with excessive pricing and addition of costly features could arise in equilibrium within a market with conspicuous consumption and uncertain demand.

- We build an analytical framework for a firm making operational decisions (viz. pricing, production quantity, and sourcing strategy) when selling to a market with uncertain demand and when consumers exhibit strategic purchasing behavior and/or conspicuous consumption. Our equilibrium results hold under general conditions of demand uncertainty. We show that demand uncertainty coupled with conspicuous consumption can indeed lead to market conditions where products are scarce and the firm makes higher profits.

- We show that when selling to markets with conspicuous consumption, because of increased margins, firms may overproduce goods compared to its production decision in a market without such conspicuous consumption. Therefore, surprisingly, there may be fewer stockouts in a market in which a sufficient number of consumers prefer exclusivity. For instance, if the market is composed of snobs, it may be optimal for the firm to overproduce, even more than it would produce in a market in which all strategic consumer behavior is ignored. This finding contrasts with the extant literature, which shows that strategic buying leads to a reduction in production quantities (see Su and Zhang 2008, p. 64).

- We show that scarcity strategy is beneficial to the firm when the fraction of consumers engaging in conspicuous consumption (“snobs”) is neither too high nor too low. When there are too few snobs in the market, the firm decides to sell to everyone at lower prices. When there are too many snobs in the market, the attractive profit margins trigger the firm to overcommit to large production quantities to minimize the “lost sales.” As a result, the product will not be scarce.

- Finally, firms that sell to consumers exhibiting conspicuous consumption may resort to expensive sourcing or increased production costs. In such cases, firms deliberately source the good from a more expensive location, or use a costlier supplier, and/or use more expensive raw material components in producing the good. Traditionally, sourcing exclusively from a more expensive supplier has been considered an unviable strategy unless the supplier has faster delivery times or better reliability (Tomlin 2006). In those cases, an expensive supply source is used sparingly as an expedient alternative. Often inventory commitment is not fully verifiable by consumers. A firm can credibly commit to its scarcity strategy by advertising its more expensive sourcing strategy. If products are produced through an expensive process, it is unlikely that the firm can invest in upfront costs to produce too many units of the good. Therefore, consumers believe that the product is likely to be scarce, which drives up the valuation for snobs in the market.

2. Pricing and Production Decisions

Our model involves a single monopolist firm who has two decisions to make—production quantity, $Q$, and the price charged per unit, $p$—before a random demand, $D$, is realized in a market composed of nonatomistic customers. The demand is distributed with cumulative distribution function $F_D$, with density function $f_D$. The firm incurs a constant marginal cost, $c$, per unit produced. If the firm produces more than the realized demand, it will be able to salvage the remaining leftover inventory at a lower price, $s$ ($<c$), at the end of the selling period (i.e., during the salvage period). Let $x^*$ denote $\max(x, 0)$. The firm’s expected profit can be written as $\Pi_N(Q, p) = \mathbb{E}[\min(D, Q) + s(Q - D)^+ - cQ]$. We assume that all the customers have the same valuation, $v$, for the product. In a market without any strategic behavior or conspicuous consumption, the optimal production quantity and price are set as per newsvendor decision (see Cachon and Terwiesch 2008): $p_0 = v$, $F_D(Q_0) = (p_0 - c)/(p_0 - s)$. Henceforth, the $p_0$ and $Q_0$ shall be referred to as the traditional newsvendor price and quantity.

However, customers are strategic, i.e., the customers recognize that if the product remains unsold it will be available in the salvage market at price $s$. We term such a customer a strategic customer (Su and Zhang 2008). The decision of the strategic customer is to choose whether to buy the product in the selling period or wait to buy the product later in the salvage.

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4 We assume that demand distribution $F_D$ has increasing generalized failure rate (Lariviere 2005). We suppress subscript $D$ and use $F(\cdot)$ to denote $F_D(\cdot)$ when it is unambiguous. Furthermore, let $\bar{F}(\cdot)$ denote $1 - F(\cdot)$.

5 To eliminate trivial outcomes, we assume that the customer will value a product more than its cost of production, i.e., $v > c$. 
period. The production quantity remains unobserved. Hence, each strategic customer has to form rational expectations of not being able to find the product, i.e., the stockout probability \( e_v \) during the selling period. Based on these expectations, the customer’s expected surplus if she faces an actual regular price \( p \) is 
\[
U_{\text{strategic}} = \max\{v - p, (1 - e_v)(v - s)\}.
\]
We apply rational expectations (Muth 1961) to solve for the equilibrium price and production quantity chosen by the firm in this environment.

2.1. Modeling Conspicuous Consumption in a Homogeneous Market

Customers, in addition to being strategic (or forward looking), may also exhibit conspicuous consumption. As per Leibenstein (1950) and Amaldoss and Jain (2005a), we address these customers as snobs. In this section, we begin the analysis with a market composed solely of strategic customers who exhibit conspicuous consumption (the assumption is relaxed in §2.2). Snobs have a higher utility for consuming a product when they figure that other consumers are unable to consume the same product. Suppose that a firm produces a good in small quantities. If snobs acquire the product and consume it, they will be seen as the select few members in the market who consume such a scarce good (i.e., their consumption is conspicuously observed), which in turn increases their utility for such products (Brown 2001).

As before, we assume that the actual quantities produced by the firm for the market remain unobservable to the snobs.\(^6\) Thus, belief on product availability is one important factor that snobs can use to exhibit their conspicuous consumption. Based on their rational expectations, they seek out hard-to-find products and derive a higher utility in their exclusiveness.

A consumer might base her belief on availability based on the observation that inventory/shelf space dedicated to the product at a retailer is low. In general, the nonavailability of the product increases her utility for the product, although it might be equally hard for her to get the product. Mathematically, we integrate this snobbishness to her utility function based on the stockout belief \( e_v \) as
\[
U_{\text{snob}} = \max\{v + ke_v - p, (1 - e_v)(v - s)\},
\]
where \( k \) represents the sensitivity to stockouts.\(^7\) It measures a consumer’s responsiveness to the product scarcity.\(^8\) For a snob, the higher the value of \( k \), the higher the utility she gets from purchasing the product on the observation of a stockout.\(^9\) There is substantial evidence from literature regarding how stockouts may improve a customer’s utility or enhance her preference for the product (see Lynn 1991 and references therein).

The firm has to develop beliefs on the customers’ reservation price for the product. This belief may be accrued from sales/consumer data from a similar portfolio of products from the firm.\(^10\) We denote the firm’s (seller’s) belief on the reservation price as \( e_r \). Based on \( e_r \), it chooses the price optimally and will produce the corresponding optimal quantity to maximize its profits. A customer’s problem is to decide whether she should buy the product in the selling period or in the salvage period. She buys in the selling period if and only if \( v + ke_r - p \geq (1 - e_r)(v - s) \). This leads to the snob’s reservation price, 
\[
r = e_r(k + v - s) + s.
\]

In our case, given every agent’s rational expectations, the game between the firm and the consumers decomposes into two separate decision problems: for the consumers, a binary choice problem regarding whether to buy in the selling period or wait (based on stockouts), and for the firm, newsvendor profit maximization under demand uncertainty. Both of these problems have unique solutions. The firm needs to know the nature of the decision problem that customers are solving and vice versa. Therefore, once all players have rational beliefs, given market parameters, their optimal actions will settle into the unique rational equilibrium outcome. We are now ready to define the rational expectations equilibrium (RE equilibrium) conditions for our model.

**Definition 1.** An RE equilibrium \((p, Q, r, e_v, e_r)\) satisfies the following conditions: (i) \( p = e_v; \) (ii) \( Q = \arg\max_X \Pi_X(q, p); \) (iii) \( r = e_r(k + v - s) + s; \) (iv) \( e_r = \bar{F}_X(Q); \) (v) \( e_v = r. \)

Conditions (i)–(iii) assert that, under expectations \( e_v \) and \( e_r \), the firm and all consumers will rationally act to maximize their utilities. Condition (iv) specifies

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\(^6\) Many firms produce exclusive goods to sell to snobs. Sometimes, firms announce the exact quantities (Liverpool FC commemorative phones; Sung 2009). Often being proprietary, inventory and shipment quantities are often not easily verifiable information, because the production process remains unobserved by customers.

\(^7\) We keep the dependency on stockouts linear only for analytical and expository ease. As shown in §4.3, our results extend to utility functions that have nonlinear dependencies on the stockout belief.

\(^8\) There is evidence that some snobs wait for a good deal. Firms such as bluefly.com and Gilt Groupe specialize in salvage markets for luxury goods (Rice 2010). In any case, our results hold even if snobs are myopic and not forward looking in their purchase behavior (see §4.1).

\(^9\) We use one variable to connect both demand realization and production decision \((Q)\), i.e., the probability of stockouts, \( P[\text{buyers} > Q] \geq \bar{F}(Q) = e_r\) (based on rational expectations). Our results continue to hold even with alternate formulations in which the beliefs could be based on fill rate, expected sales, or the production quantity (see §§4.4–4.6).

\(^10\) Often the development process of these beliefs is unspecified. In contrast to evolutive justifications based on the learning offered by the repetitions of game, Guesnerie (1992) offered an eductive approach—forecasting “forecasting of others”—that also aligns well with the game we study.
that, in equilibrium, the stockout expectations $e_s$ must match with the actual probability of not being able to find the product (consistency conditions).

Consider a customer who is indifferent between buying in the first period and waiting to buy in the salvage period. Because she knows that every other customer is also strategic and snobbish, she develops a belief on the availability of the product. She rationalizes that other customers who are trying to buy the product face the same stockout probabilities as she does. We assume nonatomic decision makers, hence the mass of remaining customers is $D$ if the realization of the demand is $D$. Thus, she faces a possibility of stockout, $P(D > Q)$, which must be consistent with her belief $e_s$, as stated by (iv). Finally, condition (v) requires that the firm correctly predicts the snob’s reservation price.

Conditions in Definition 1 can be reduced to conditions in $p$ and $Q$ only: $p = F_0(Q)(k + v + s) + s$ and $Q = \arg\max_{s} \Pi_N(q, p)$. With the aforementioned conditions, we are ready to describe the RE equilibrium in Proposition 1.

**Proposition 1.** In the RE equilibrium, all customers can buy immediately, and the firm’s price and quantity choices are characterized by $p_s^* = s + 4\sqrt{(k + v - s)(c - s)}$ and $F_0(Q) = \sqrt{(c-s)/(k+v-s)}$, respectively.

All proofs are deferred to the appendix. We use $p_s^*$ and $Q_s^*$ (and subscript $s$ in general) to denote the equilibrium price and quantity decisions the firm makes when it chooses to sell the product based on snobs’ reservation prices. For the purposes of benchmarking, we compare the optimal decision when conspicuous consumption is present in the market to the decisions ($p_s^*$ and $Q_s^*$) when there is none ($k = 0$).

1. The equilibrium price when faced with snobs, $p_s^*$, turns out to be higher than the equilibrium price choice when faced with just strategic customers, $p_s^*$. This reaffirms our intuition.

2. Intriguingly, the equilibrium production quantity, when conspicuous consumption is present, $Q_s^*$, is higher than the equilibrium production, $Q_s^*$ (when there is no conspicuous consumption). The firm “overproduces” because of higher margins (under-age costs). Just because consumers exhibit conspicuous consumption does not imply that the consequent production quantities will be low. In fact, as customers become more snobbish (i.e., their valuation increases significantly due to stockouts), the equilibrium stockout probability falls. This is illustrated in Figure 1(b), where equilibrium stockout probability decreases steadily with sensitivity to stockouts.

3. Note from Figure 1(a) that the production quantity when customers are just strategic is lower than in the newsvendor case, i.e., $Q_s^* < Q_0$. However, because $Q_s^* > Q_0^*$, it is unclear whether, under conspicuous consumption, the production quantities are lower or higher than the newsvendor quantity. Comparing our equilibrium decisions with the newsvendor decisions, we find that $Q_s^* < Q_0$ and $p_s^* < p_0^*$ when $k < (v - s)(v - c)/(c - s)$, and $Q_s^* \geq Q_0$ otherwise (proofs are deferred to Lemma A1 in the appendix).

If $k$ is high, the firm produces more than it would in the traditional newsvendor setting. Thus, even though consumers exhibit strategic buying behavior and conspicuous consumption, we find that the firm may not necessarily produce less inventory. The higher margins that can be accrued from conspicuous consumers make the firm “overcommit” to higher production volume, more so than it would if those consumers were not conspicuous consumers. Thus,

\[ \text{Note:} \quad P(D > Q_s^*) > P(D > Q_0^*) \text{ because } F_0(Q_s^*) = \sqrt{(c-s)/(v-s)} > \sqrt{(c-s)/(k+v-s)} = F_0(Q_0^*). \]
accounting for customer behaviors—such as pricing under conspicuous consumption—impacts other areas of the firm and leads to distinct operational decisions. We now analyze the heterogeneous market that forms the base model for all extensions analyzed in this paper.

2.2. Conspicuous Consumption in a Heterogeneous Market (Snobs and Commoners)

The market is composed of two different types of customers, which we term snobs and commoners (see Leibenstein 1950). We use $\beta$ to denote the fraction of the customer population who are snobs. The rest of the population ($1 - \beta$) is composed of commoners. A commoner is distinguished from a snob in the following sense: A commoner does not exhibit any inclination for conspicuous consumption, but she may still be strategic in her decision making. Both types of customers (snobs and commoners) are willing to buy the product in the selling period as long as the firm does not charge a price higher than their own reservation price. Because there are two possible reservation prices within the market, the firm will have two possible consistent quantity choices, and this will in turn affect the equilibrium availability and beliefs ($e_r$).

Thus, there are two possible candidates for the RE equilibrium. The firm charges one of the reservation prices based on the percentage of snobs and produces an optimal quantity that will make the expectations of the customers consistent. Thereafter, the customers observe the price and decide whether to buy the product in the selling period.

Definition 2. When the firm charges the snob’s (commoners’) reservation price, an RE equilibrium $(p, Q, r, e_s, e_c)$ satisfies the following conditions: (i) $p = e_s$; (ii) $Q = \text{arg max}_{q} \Pi_N(q, p)$; (iii) $r = e_s(k + v - s) + s(r = e_s(v - s) + s)$; (iv) $e_s = \bar{F}_D(Q)$ ($e_c = \bar{F}_D(Q)$); (v) $e_r = r$.

The conditions imposed in Definition 2 are the same as those imposed in Definition 1 except for conditions (iii) and (iv). Those conditions relate to the beliefs on the reservation price and product availability.

If the firm charges the snobs’ reservation price, then only snobs are present in the market to purchase the product (because the high price rules out commoners buying the product). Thus, the random variable $D$ is rescaled from $D$ to $\beta D$, and stockout probability becomes $P(\beta D \geq Q)$, or simply $\bar{F}_{\beta D}(Q)$. The corresponding equilibrium production quantity is given by Proposition 2(i). On the other hand, if the firm charges the commoners’ reservation price, the demand remains identical to the initial demand distribution, because the offered price is lower than everyone’s reservation price. In this case, a possibility of stockout stays the same as in Definition 1, $\bar{F}_D(Q)$. This is indicated in Proposition 2(ii).

Proposition 2. (i) (Limited Production) In the RE equilibrium under limited production, only snobs can buy, and the firm’s price and quantity choices are characterized by $P(\beta : D > Q^*) = \sqrt{(c - s)/(k + v - s)}$ and $p^*_s = \sqrt{(c - s)/(k + v - s)} + s$.

(ii) (Regular Production) In the RE equilibrium, all customers (snobs and commoners) can buy, and the firm’s price and quantity decisions are characterized by $P(D > Q^*_c) = \sqrt{(c - s)/(v - s)}$ and $p^*_c = \sqrt{(c - s)(v - s)} + s$.

Note that given a specific market condition (fraction of snobs) and the strategies of other players, only one set of rational beliefs will emerge. Depending on the market parameters, the profit-maximizing firm would end up with one of the aforementioned strategies. Given a $\beta$, because consumers are rational and can correctly form expectations about firms’ strategies, the correct RE equilibrium would emerge. We investigate the two candidate strategies to see when limited production or regular production would be preferred by the firm. We use $\Pi^*_N$ and $\Pi^*_N$ to denote the firm’s optimal profits obtained under the limited production strategy (selling only to snobs) and regular production strategy (selling to snobs and commoners), respectively.

Proposition 3. There exists a unique threshold $\beta^*$ such that for $\beta \leq \beta^*$ in the RE equilibrium, $\Pi^*_N \geq \Pi^*_N$, the firm adopts the regular production strategy, and all customers can buy. If $\beta > \beta^*$, then $\Pi^*_N > \Pi^*_N$, the firm adopts the limited production strategy, and only snobs can buy.

Proposition 3 points out that for a given $\beta$, there is a unique rational expectations equilibrium. If the number of snobs in the market is low (i.e., $\beta \leq \beta^*$), the firm will price the product at the commoner’s reservation price, thus making it possible for all consumers to buy it. The additional profits accrued from the higher price premiums can be compensated by selling to a significantly larger market at a lower price. However, if there is a sufficient presence of snobs in the market ($\beta > \beta^*$), the firm adopts the limited production strategy by attempting to sell only to the snobs. On average, the firm can afford to sell to snobs at high prices, even though the volume of sales has been pushed down because of reduced market coverage.

We note that serving only to the snobs might also be perceived as a “scarcity” strategy. However, this is not necessarily true. In this case, the firm’s decisions are dictated by two counteracting factors. First, selling
only to snobs means that the average demand in the market is reduced—this means the production quantity will tend to reduce on average. However, selling only to snobs increases the underage cost or the product margin, because the product is now marketed to snobs at higher prices. This means that the production quantity will increase. Because of the higher underage cost, more units of the product are produced to avoid the opportunity cost of missing those high margin sales (lost sales). These two effects counteract each other. Thus, the resultant production quantity may be higher or lower than the production quantity when the firm sells to everyone in the market.

In fact, we find that if the fraction of snobs in the market is below a certain threshold, the product availability might be low compared to the case in which the firm sells the product to all consumer types. This is captured in Proposition 4.

**Proposition 4.** There exists a unique fraction of percentage of snobs, \( \beta_0 \), where \( Q^*_c < Q^*_s \) when \( \beta < \beta_0 \) and \( Q^*_c > Q^*_s \) when \( \beta > \beta_0 \). This threshold level is given by

\[
\beta_0 = \frac{1}{F_D^{-1}\left(\sqrt{c-s}/(v-s)\right) + F_D^{-1}\left(\sqrt{c-s}/(k+v-s)\right)}.
\]

Proposition 4 asserts that the strategy of restricting the sales only to snobs does not always imply low availability of the product. In fact, the product might be commonly available even though the firm covers only the snobs in the market. Consequent to Proposition 4, the product is scarce in the market only if (i) the product is limited to snobs (limited production) and (ii) the production quantities are lower than the quantities the firm produces when it sells to the whole market (i.e., \( Q^*_c < Q^*_s \)). Note that \( Q^*_s \) is the quantity available when serving the whole market, because all consumers (snobs and commoners) are strategic in our model. Thus scarcity exists only when \( \beta \in (\beta^*, \beta_0) \). We elaborate this interesting finding further in §3.

### 3. Analysis of Scarcity Strategies

To discuss scarcity strategies in the market, we first define the notion of “scarcity.” We define a product to be scarce when the optimal quantity available in the market when selling only to snobs is lower than the optimal quantity available when selling to all strategic consumers in the market with an identical demand distribution (i.e., \( Q^*_c < Q^*_s \)).

From Proposition 3, when \( \beta \leq \beta^* \), regular production is adopted, and \( Q^*_c \) units are available in the market. When the fraction of snobs in the population exceeds \( \beta^* \), the firm switches to selling only to snobs (i.e., limited production). As \( \beta \) increases, selling only to snobs continues to remain the optimal selling strategy. However, note that the production volume increases because the mean demand (i.e., the fraction of snobs in the market) is increasing. As a result, if the fraction of snobs in the market is higher than \( \beta_0 \) (from Proposition 4), the total production volume and the availability of products (in-stock probability) are both higher than in the case when customers are just strategic. Thus, the in-stock probability for the product is lower (i.e., the product is scarcer to find) in the intermediate region between \( \beta^* \) and \( \beta_0 \). Furthermore, Figure 2 also reveals that the extent of scarcity is the strongest when the fraction of snobs is just higher than \( \beta^* \).

#### 3.1. Increased Response to Stockouts

We now study the prevalence of scarcity as the snobs’ sensitivity to stockouts varies. In Figure 2, we study how scarcity decisions vary with the fraction of snobs in the market, as the sensitivity to stockouts increases (from \( k = 10 \) in Figure 2(a) to \( k = 45 \) in Figure 2(b)). We find that when the market is concentrated with snobs, who are highly sensitive to stockouts (high \( k \)), the firm might produce more quantity than the regular newsvendor quantity in equilibrium (even though the customers are strategic). Note that these results extend the observations from §2.1 that showed that the equilibrium production may exceed the newsvendor production quantity when the sensitivity to stockouts is high.

Furthermore, as the snobs become more sensitive to stockouts, we make two key observations:

1. The threshold \( \beta^* \) decreases with sensitivity to stockouts (\( k \)). If scarcity becomes more desirable to snobs, the firm is more likely to offer limited production.
2. On the other hand, the optimal equilibrium production quantity under the limited production strategy increases more steeply with the fraction of snobs in the market as \( k \) increases (i.e., the slope of the line under limited production strategy increases with \( k \)).

If the snobs respond strongly to stockouts, their reservation prices will be even higher, which results in a higher price (and an increased underage cost). As a result, the production quantities increase steeply despite the firm adopting a strategy of selling only to snobs. This has the effect of reducing the degree of scarcity (fewer stockouts).

**Lemma 1.** For higher \( k \), the equilibrium production quantity \( Q^*_c \) increases more steeply in \( \beta \).

Lemma 1 demonstrates that the optimal production quantity increases faster in \( \beta \) as the sensitivity to stockouts increases. As snobs become more sensitive to stockouts, the firm increases its production quantity...
It is unclear if the scarcity region that exists in the sales to snobs has also increased. Even though snobs are sensitive to stockouts, their willingness to pay more for exclusivity causes the firm to produce more goods, because the opportunity cost of losing a sale to such a customer would be very high. In other words, the firm is averse to losing a high margin sale (on those rare stockouts) and stocks up on inventory, even though it runs the risk of reduced exclusivity among the snobs. Lemma 2 summarizes the behavior of the thresholds with respect to sensitivity of snobs to stockouts.

**Lemma 2.** The threshold levels, $\beta^*$ and $\beta_Q^*$, decrease with increase in sensitivity to stockout, $k$.

Recall that the firm adopts the limited production strategy when the number of snobs in the market is more than $\beta^*$. Lemma 2 indicates that the firm would adopt the limited production strategy more often as the sensitivity to stockouts increases in the market for the snobbish customers. Conversely, Lemma 2 also states that $\beta_Q^*$ decreases in $k$. The more sensitive the snobs are to stockouts, the more likely the strategy of selling to snobs leads to overproduction (i.e., more than the equilibrium quantity produced when the good is available to the whole market). As seen in Lemma 1, the increased opportunity cost drives the firm to produce more goods. In other words, the cost of stockouts are high when the sensitivity of stockouts for snobs is high. As a result, the firm produces more goods, even though it is limiting its market to a fraction of customers (snobs). Aided with the results of Lemmas 1 and 2, we can now analyze the region of scarcity.

It is unclear if the scarcity region that exists in the region $\beta \in (\beta^*, \beta_Q^*)$ expands as snobs become more receptive to stockouts (i.e., as $k$ increases). Proposition 5 provides conditions under which the region of scarcity (i.e., $\beta_Q^* - \beta^*$) expands as snobs become more sensitive to stockouts.

**Proposition 5.** The scarcity region expands if and only if the generalized failure rate of the distribution is greater than a threshold, i.e., $g(Q^c_s/\beta) \geq (\beta_Q^*/\beta^*) \cdot M(Q^c_s/\beta)$, where $M$ is a constant dependent on $Q^c_s$ and $\beta$.

Proposition 5 provides a condition based on the demand distribution for the prevalence of the stockout strategy. When the snobs are very sensitive to stockouts, the scarcity strategy is often in equilibrium if the distribution of the uncertain demand has a high generalized failure rate (see Lariviere 2005). In §3.2, we focus on how the scarcity region varies with increasing demand variability for a given sensitivity to stockouts parameter.

### 3.2. Scarcity Strategy: The Effect of Uncertainty

In Figure 3, we hold the mean of the demand distribution constant and increase its variance. We hold $k$ constant and explore how the scarcity strategy region changes with increasing variability in demand. Figure 3 demonstrates that the following occur as the demand uncertainty increases:

(i) $\beta^*$ decreases. Applying the result on Proposition 3, the firm is less likely to adopt the regular production strategy as demand uncertainty increases.

(ii) $\beta_Q^*$ decreases. Applying the result from Proposition 4, the firm is more likely to overproduce if there are a sufficient number of snobs in the market as the demand uncertainty increases.

(iii) $\beta_Q^* - \beta^*$ decreases. Applying Proposition 5, the market proportion of consumers for which the
scarcity strategy is employed decreases. If demand uncertainty increases, the scarcity strategy is less likely.

(iv) $Q^*$ decreases (note that regular newsvendor quantity $Q_0$ increases). Applying Proposition 2(ii), as the demand uncertainty increases, the regular production quantity decreases.

(v) The slope of $Q^*$ increases. Applying Proposition 2(ii) for the same fraction of consumers in the market, more quantity is produced under the limited production strategy.

(vi) The degree of scarcity (i.e., $Q^* - Q^*_c(β)$) is reduced. As demand uncertainty increases, applying Proposition 2, the number of stockouts consumers face due to the scarcity strategy is reduced.

In summary, if the demand uncertainty increases, the firm commits to the scarcity strategy less often, and the degree of scarcity when it so commits is also reduced.

4. Extensions of the Base Model

We establish the robustness of our result, by showing that this “intermediate” scarcity profile continues to exist under a wide variety of modeling variations. Specifically, we consider the presence of the following cases: (i) myopic snobs, (ii) snobs who have a lower value for abundant products, and (iii) snobs who have a nonlinear utility dependence on stockouts. In addition, we consider other conspicuous consumption beliefs that snobs might possess: namely, the snobs could have beliefs based on (iv) fill rate, (v) expected sales, (vi) production quantity, and (vii) product prices (instead of beliefs based on stockouts). We show that our main conclusions continue to hold for all the variations. The firm adopts the regular production strategy when the proportion of snobs in the market ($β$) is low, and the limited production strategy when the fraction of snobs in the market is high. In particular, the scarcity strategy is adopted in the intermediate region of $β$. Most of the technical details for the subsections below are omitted due to page restrictions and they are available upon request. Results can be driven easily by applying the same methodology in the proof of each proposition of §2.2 after the suggested modification on snobs’ utility function in the subsections below.

4.1. Myopic Snobs

In this section, we analyze the case when customers exhibit conspicuous consumption but they are not strategic. Such snobs are willing to pay high prices for scarce products to distinguish themselves from others, and they may buy myopically (The Economist 2009). To account for the myopic nature of the snobs, we alter the conditions for the reservation price of snobs (condition (iii) in Definition 2, i.e., $v = v + ke_c$). Investigating the two candidate strategies for the firms, we find that the threshold similar to that established in §2.2 holds, with one exception: the threshold for the limited production strategy when considering myopic snobs is smaller than the threshold when snobs are strategic (i.e., $β^{\text{myopic}} ≤ β^*$).\(^\text{14}\) The firm adopts the limited production strategy in more scenarios compared to the case when snobs are strategic. As before, scarcity exists only in the intermediate region of $β$.

4.2. Snobs Have a Lower Value for Commonly Available Products

In §2, we assumed that snobs have weakly greater utility for (scarce) goods than commoners. We now extend that to consider the case when snobs have a strictly lower valuation than commoners while the product is widely available to everyone in the market. Let snobs’ valuation be $v_s + ke_s$, where $e_s$ is the belief on stockout probability. Furthermore, commoners’ valuation is $v$. We assume $v_s < v$. This would imply that if the stockout probability is lower than or equal to some threshold, the snobs will have a strictly lower utility than the commoners. Therefore, snobs could have a lower utility for widely available products that are consumed in large quantities. In other words, they suffer a disutility due to the lack of exclusivity, as in Balachander and Stock (2009). Nevertheless, they still possess snobbishness and enjoy higher utility from scarcer goods.

Thus, a snob’s utility function is $U_{\text{snob}} = \max\{v_s + ke_s - p, (1 - e_s)(v - s)\}$, which changes the reservation

\[^{14}\] $β^{\text{myopic}} = \frac{\sqrt{(v - s)^2 + 4K(v - s)}}{2} f_0^{-\frac{1}{2}}(\sqrt{(v - s)^2 + 4K(v - s)} - (v - s)) \frac{uf_s(u) du}{\int (v - s) + \sqrt{(v - s)^2 + 4K(v - s)} - (v - s)) - \int (v - s) + \sqrt{(v - s)^2 + 4K(v - s)} - (v - s))}$. 

Figure 3. As the Coefficient of Variation of the Demand Increases, $Q^*$ Decreases

Notes. Also note that both $β^*$ and $β_s$ also decrease. The scarcity region is further reduced by the increase of the slope of the line $Q^*_s$. Parameters are same as in Figure 2(a).
price of the snobs to $r = \varepsilon_s \cdot (k + v_s - s) + s$. In light of these modifications, condition (iii) in Definition 2 is suitably changed.

We can show that the firm adopts the limited production strategy when the fraction of snobs in the market is sufficiently large (i.e., $\beta > \beta^*$). Similarly, we can show that there is a unique threshold $\beta^*_Q$ below which we have $Q^*_s < Q^*_c$. We can show that these thresholds $\beta^*$ and $\beta^*_Q$ are both higher than in the base case in §2.2. The effect is qualitatively similar to Lemma 2. Thus, as a result of snobs having more disutility from the commonly available products, the firm adopts the Limited production strategy less often.

One can immediately notice that the scarcity strategies exist in intermediate range ($\beta^*, \beta^*_Q$). However, to adopt scarcity strategies viably, the firm also needs an increased number of snobs in the market. In addition, the firm has to offer a higher degree of scarcity (i.e., more stockouts) to enable the snobs to overcome their low valuation for commonly available goods.

Finally, the decreased valuation of snobs has some interesting effects when the fraction of snobs in the market is low ($\beta < \beta^*$), depending on the sensitivity of snobs to stockouts. When $k$ is sufficiently large, the conclusions remain identical to Proposition 2(ii). However, when $k$ is low (i.e., $k < (v - v_s)$), snobs have a much lower valuation than the commoners because of the wide availability of goods.

**Proposition 6 (Modified Regular Production).** In the RE equilibrium, when $k < v - v_s$, only commoners can buy, and the firm’s price and quantity decisions are characterized by $p^r = \sqrt{(c - s)/(v - s)} + s$. When $k > v - v_s$, Proposition 2(ii) holds.

Proposition 6 points out that given the low fraction of snobs and their lower valuation, it is optimal for the firm to price the products at commoners’ reservation price. Therefore, at low $\beta$ (and low $k$), snobs do not buy, and only commoners buy the product at the sold market price. The result is captured in Proposition 6, which acts as a modified proposition for the regular production case in Proposition 3. This result corresponds to the case wherein a product is sold at a low price and in large quantities (which keeps the snobs from buying the product, despite its low price). This result explains the market presence of many nonexclusive brands and low-end retailers who sell inexpensive goods to commoners in large quantities.

### 4.3. Nonlinear Utility from Stockouts

In §2.2, we assumed that the utilities of snobs are linearly dependent on the stockout probability. We now generalize the utility of snobs to nonlinear dependencies on stockout probability.

**Case A.** We consider the utility of snobs in the following functional form: $U_{\text{snob}} = \max[v + ke_{\text{lost}}^s - p, (1 - \varepsilon_s)((v - s))].$ We find that our conclusions remain unchanged.

There exists a threshold $\beta^*$ such that if the fraction of snobs is below $\beta^*$, the firm uses the regular production strategy. When $\beta > \beta^*$, the firm adopts limited production—only snobs can buy, and the firm’s equilibrium decisions are characterized by $k_F^e(Q^*_s)^n + F^e(Q^*_c)^n(v - s) = c - s$ and $p^*_s = k_F^e(Q^*_s)^n + F^e(Q^*_c)^n(v - s) + s$. In the case of regular production (i.e., $\beta < \beta^*$), all customers (snobs and commoners) can buy. The firm’s decision is characterized by $F_D(Q_s) = \sqrt{(c - s)/(v - s)}$ and $p^*_c = \sqrt{(c - s)/(v - s)} + s$ (same as in §2.2).

Again, as in §2, the scarcity strategies are employed in the intermediate region when the fraction of snobs $\beta \in (\beta^*, \beta^*_Q)$. Although the structure of the equilibrium remains the same, we find that, because of the nonlinearity of the snobs’ utility, their valuation for the product is higher, which causes the scarcity strategy to be employed even with a (relatively) lower percentage of snobs in the market. In addition, the degree of scarcity is also lower than it is in the linear case.

**Case B.** We also tested another version of nonlinearity specified by $U_{\text{snob}} = \max[v/(1 - \varepsilon_s)^n - p, (1 - \varepsilon_s)((v - s))].$ Our conclusions are identical to the above conclusions. The firm employs scarcity in the intermediate values of fraction of snobs in the market.

### 4.4. Beliefs Based on Fill Rate

In this section, we expanded our definition of conspicuous consumption from stockout probability to one related to fill rate, i.e., the fraction of customers who do not find the product (“lost sales”) available in equilibrium. The in-stock probability corresponds to the probability that all demand is fulfilled, whereas fill rate measures the fraction of demand that is fulfilled. Following Cachon and Terwiesch (2008), fill rate is $1 - (E[\text{Lost Sales}]/E[D])$.

Corresponding to beliefs based on stockout probability, we could represent the snobs’ utility using belief on the fraction of unfulfilled demand—$\varepsilon_{\text{lost}}$. Thus, $U_{\text{snob}} = \max[v + ke_{\text{lost}} - p, 0]$, where $\varepsilon_{\text{lost}}$ is the fraction of demand lost.

Because of the stochastic nature of the demand, the number of items left unsold from the selling period may vary. If there are items left over, then the demand in the selling period was lower than the production quantity, and therefore the corresponding fill rate would equal one. In this case, snobs have no utility from consuming the product in the second period. The assumption aids tractability. However, the property can be generalized and conclusions remain the same (just as in myopic snob case).
The condition for RE equilibrium remains the same as in Definition 2, except for conditions (iii) and (iv), which are now (iii) \( r = v + k e_{\text{lost}} \) and (iv) \( e_{\text{lost}} = \left( \int_{0}^{\infty} (u - Q) f_{p}(u) du \right)/\hat{F}[D] \), respectively.

Even if we use (1-fill rate) instead of the stockout probability in snobs’ utility function, we note that our conclusions remain identical. We can show that there exists a threshold \( \beta^* \) proportion of snobs in the market, below which the firm adopts the regular production, and above which the firm adopts the limited production strategy. It is straightforward to show that the equilibrium decisions when the firm engages in regular production are identical to the base case. However, when the firm adopts limited production, only snobs can buy because of higher prices; the firm’s price and quantity choices are characterized by

\[
\tilde{F}_{\text{BD}}(Q_*) = \frac{c - s}{v + k \cdot \int_{0}^{\infty} (u - Q_{\text{C}}) f_{\hat{D}}(u) du / \hat{E}[\hat{D}] - s}
\]

and \( p^* = (c - s) \tilde{F}_{\text{BD}}(Q_*) + s \). If we assume that the demand is distributed as \( U(0, N) \), then we obtain the following explicit results for the equilibrium price and quantity under the limited production strategy:

\[
p^* = (c - s)^{2/3} (k + v - s)^{1/3} + s,
\]

\[
Q^* = \beta N \left( 1 - \frac{c - s}{k + v - s} \right)^{1/3}.
\]

Finally, we show that there exists a unique level of percentage of snobs, \( \beta_{\Omega} \), where \( Q^*_c \leq Q^*_e \) if \( \beta \leq \beta_{\Omega} \) and \( Q^*_e > Q^*_c \) when \( \beta > \beta_{\Omega} \). Thus, scarcity strategies are adopted in the intermediate range of \( \beta \in (\beta^*, \beta_{\Omega}) \).

### 4.5. Beliefs Based on Expected Sales

We tested the robustness of our model by considering the dependency of conspicuous consumption on unit sales in which the snobs’ utility decreases in expected number of buyers. This is also an extension of the utility model of Amaldoss and Jain (2005a), if we ignore the production quantity mismatch. Accordingly, we redefine the utility function based on the belief on expected unit sales, \( e_{\text{sales}} \). The analysis proceeds as in the previous section. This analysis is tractable for distributions with finite support. When the belief on expected sales is \( e_{\text{sales}} \), the utility of snobs is \( U_{\text{snob}} = \max\{v + k/e_{\text{sales}} - p, 0\} \). Note that the utility of a snob is decreasing in the quantity of demand that is fulfilled (i.e., the number of buyers).

The condition for RE equilibrium remains the same as in Definition 2, except for conditions for snobs (iii) and (iv), which are now (iii) \( r = v + k/e_{\text{sales}} \) and (iv) \( e_{\text{sales}} = Q - \int_{0}^{Q} f_{p}(u) du \), respectively.

We note that our conclusions remain unaltered in this case. If the fraction of snobs in the market is below some threshold \( \beta^* \), the firm adopts regular production, and above \( \beta^* \) the limited production strategy is adopted. It is straightforward to show that the equilibrium decisions when the firm engages in regular production are identical to the base case. As in §2.2, when \( \beta \) exceeds the threshold \( \beta^* \), the firm adopts the limited production strategy, in which its price and quantity are characterized by

\[
\tilde{F}_{\text{BD}}(Q_*) = \frac{c - s}{v + k/(Q_{\text{C}} - \beta \int_{0}^{Q_{\text{C}}/\beta} \hat{F}_{D}(u) du) - s}
\]

and \( p^* = (c - s)/(\tilde{F}_{\text{D}}(Q_{\text{C}}/\beta) + s) \).

There exists a unique fraction of snobs, \( \beta_{\Omega} \), where \( Q^*_c < Q^*_e \) if \( \beta < \beta_{\Omega} \), and \( Q^*_e > Q^*_c \) otherwise. This leads to our final conclusion that the firm employs the scarcity strategy in the region \( (\beta^*, \beta_{\Omega}) \).

### 4.6. Beliefs Based on Firm’s Production Quantity

In line with our previous settings, we now extend our results to the case in which snobs value the product more when they expect that the firm produces the goods in low quantities. This section matches the results for when inventory is (credibly) announced to be limited. Thus, we now represent a snob’s utility using her belief about the production quantity, \( e_{\text{Q}} \).

**Definition 3.** When the firm charges the snob’s (commoner’s) reservation price, an RE equilibrium \((p, Q, r, e_{\text{Q}}, e_{r})\) satisfies the following conditions:

(i) \( p = e_{r} \); (ii) \( Q = \arg \max_{\text{sup}} \Pi_{\text{A}}(q, p) \); (iii) \( r = v + k/e_{\text{Q}} (r = e_{r}(v - s) + s) \); (iv) \( e_{\text{Q}} = Q \); (v) \( e_{r} = r \).

If the fraction of snobs in the market is below some threshold \( \beta^* \), the firm adopts regular production, and above this threshold the limited production strategy is adopted. The pricing and quantity decisions of the firm under regular production are identical to the base case. As before, when \( \beta \) exceeds the threshold, the firm adopts the limited production strategy, in which its price and quantity are now characterized by \( \tilde{F}_{\text{BD}}(Q^*) = (c - s)/(v + k/Q_0^* - s) \) and \( p^* = v + k/Q^*_e \).

There exists a unique level of percentage of snobs, \( \beta_{\Omega} \), where \( Q^*_c < Q^*_e \) if \( \beta < \beta_{\Omega} \), and \( Q^*_e > Q^*_c \) otherwise. This leads to our final conclusion that the scarcity strategy is exercised in the region \( (\beta^*, \beta_{\Omega}) \).

### 4.7. Price-Dependent Stockout Beliefs

It is very likely that consumers make inferences about the stockout probability for a product based on the selling price. For instance, it is quite possible for consumers to expect that less expensive items are more likely to be produced in large quantities and stay on shelves. On the other hand, even though there are fewer customers in the market for more expensive products, very few of such items are produced. Such
price-dependent snobbish behavior was proposed by Liebenstein (1950). Note that this assumption does not require the demand to increase in price. (In fact, this belief is consistent with price-dependent demand distributions that are first-order stochastically decreasing in price.) In this section, we develop the rational expectations equilibrium for the case when consumers develop their stockout beliefs (and their resultant conspicuous consumption) based on the product price.

**Definition 4.** An RE equilibrium \((p, r, r_s, e_s, p_s, e_r, e_r)\) satisfies the following conditions: (i) \(p = \arg \max_p \Pi(p, Q(p))\); (ii) \(r_s = e_s(p)(v + k - s) + s\); (iii) \(e_s(p) = G(p)\); (iv) \(e_r = r_s\). The price-dependent stockout beliefs can admit any general price–quantity relationship. In particular, the firm could be solving a newsvendor problem (which would be a specific optimization problem of the firm), in which case, the consistent stockout beliefs based on the expected reservation prices can be represented as \(G(p) = F(Q(p))\) if \(p \leq e_r\). Now, we are ready to derive the equilibrium price and quantity decisions.

**Proposition 7.** If \(\beta \leq \beta^*_s\), the firm’s price and quantity decisions are characterized by \(p^*_s = \sqrt{(c-s)(v-s)} + s\) and \(Q^*_s = F^{-1}_D(G(p^*_s)) = F^{-1}_D((c-s)/(p_c^*-s))\), and all customers can buy. However, if \(\beta > \beta^*_s\), the firm’s price and quantity decisions are characterized by \(p^*_s = \sqrt{(c-s)(v-k-s)} + s\) and \(Q^*_s = \beta F^{-1}_D(G(p^*_s)) = F^{-1}_D((c-s)/(p_c^*-s))\), and only snobs can buy.

Note that the RE equilibrium under this specification is identical to the base model. This is because, under RE equilibrium, the expectations are correctly formed with implicitly assumed price decisions. Again, we can show the existence of \(\beta_Q\) where the profits from limited production and regular production strategy are equal. Thus, the scarcity strategy is found between \((\beta^*_s, \beta_Q)\) (technical details are same as the proof of each proposition in §2.2).

## 5. Salvage Pricing and Sourcing Decisions

### 5.1. Endogenous Salvage Pricing

We now extend the results of our base model to quantity based salvage pricing. In §2.2, we had all leftover goods salvaged at an exogenous salvage price \(s\). Arguably, in many cases, the salvage value is dependent on the quantity that is left over. (If there are many items left over, the salvage prices would be expected to be very low.) We use a linear salvage price function decreasing in the proportion of leftover quantity to the product quantity. The salvage price function is \(s(L) = s_0 - s_1 \cdot L\), where \(L = (Q - D)^+ / Q\), and \(D\) is the realized demand. In light of endogenous salvage pricing, the firm’s expected profit can be written as

\[
\Pi_N(q, p) = (p - c)q - (p - s_0) \int_0^q (q - u) dF_D(u) - \frac{s_1}{q} \int_0^q (q - u)^2 dF_D(u).
\]

Consumers will base their decisions and the firm will make its choice not only based on the stockout probability, but also based on the probability that the goods will be available on the salvage market. We represent this belief by \(e_{\text{salvage}}\). We are ready to define the RE equilibrium conditions:

**Definition 5.** An RE equilibrium \((p, Q, r, e_s, e_r)\) satisfies the following conditions: (i) \(p = e_r\); (ii) \(Q = \arg \max \Pi_N(q, p)\); (iii) \(r_s = (k + v - e_{\text{salvage}}) + e_{\text{salvage}}\); (iv) \(e_s = F_D(Q)\); (v) \(e_{\text{salvage}} = E[s - s_1((Q - D)^+ / Q)]\); (vi) \(e_r = r_s\).

Note how the reservation price of the snobs is dependent on both \(e_s\) and \(e_{\text{salvage}}\). The snobs have a higher utility for the product that is more likely to be stocked out and less likely to be salvaged. Our main insights continue to hold.

**Proposition 8.** (i) (Limited Production) If \(\beta > \beta^*_s\), in the RE equilibrium, only the snobs can buy, and the firm’s price and quantity decisions are characterized by

\[
p^*_s = \frac{F_D(Q^*_s)}{\beta} \left( k + v - s_0 + s \int_0^{Q_c^*/Q^*_s} \frac{Q_c^*-u}{Q^*_s} dF_D(u) \right)
+ s_0 - s_1 \int_0^{Q_c^*/Q^*_s} \frac{Q_c^*-u}{Q^*_s} dF_D(u),
\]

\[
\frac{s_1}{Q^*_s} \int_0^{Q_c^*/Q^*_s} \beta u^2 dF_D(u) + \frac{F_D(Q^*_s)}{\beta} \left( \frac{F_D(Q^*_s)}{\beta} \right) (v + k - s_0)
+ s_1 - s_1 F_D(Q^*_s) \int_0^{Q_c^*/Q^*_s} \frac{Q_c^*-u}{Q^*_s} dF_D(u)
= c - (s_0 - s_1).
\]

(ii) (Regular Production) If \(\beta \leq \beta^*_s\), in the RE equilibrium, all customers (snobs and commoners) can buy, and the firm’s price and quantity decisions are characterized by

\[
p^*_s = \frac{F_D(Q^*_s)}{\beta} \left( v - s_0 + s \int_0^{Q_c^*/Q^*_s} \frac{Q_c^*-u}{Q^*_s} dF_D(u) \right)
+ s_0 - s_1 \int_0^{Q_c^*/Q^*_s} \frac{Q_c^*-u}{Q^*_s} dF_D(u),
\]

\[
\frac{s_1}{Q^*_s} \int_0^{Q_c^*/Q^*_s} u^2 dF_D(u) + \frac{F_D(Q^*_s)}{\beta} \left( \frac{F_D(Q^*_s)}{\beta} \right) (v - s_0) + s_1
- s_1 F_D(Q^*_s) \int_0^{Q_c^*/Q^*_s} \frac{Q_c^*-u}{Q^*_s} dF_D(u)
= c - (s_0 - s_1).
\]
Note from Proposition 8 that there exists a threshold $\beta^*$ above which there is limited production in the market, and there is an increased stockout probability (i.e., $Q^*_t < Q^*_0$). However, we can also show that there exists another unique threshold, $\beta^*_0$, where $Q^*_t < Q^*_0$ when $\beta < \beta^*_0$, and $Q^*_t > Q^*_0$ when $\beta > \beta^*_0$. Thus, scarcity occurs in the intermediate region of the fraction of snobs in the market. We note that with added salvage flexibility, the firm increases the extent and degree of stockouts.

5.2. Clearance Pricing Model

In this section, we explore the optimal price and quantity decisions of the firm using a clearance pricing model (based on the model introduced by Cachon and Kok 2007) that consists of two periods. In period 1, fraction $(1-\beta)$ of commoners and $\beta$ snobs make their strategic decision of whether to buy or not. In period 2, because all the products offered are leftovers from period 1, all consumers in the market have a low utility for the good. Consumers buy the product at the clearance price offered by the firm if they receive a net positive utility.

To keep the multiperiod model tractable, we use a clearance price function linear in the quantity salvaged in the second period ($s_2$). Let this function be $p_2(s_2) = p_1 - p_2 s_2$ for period 2 demand. Let $R_2(s_2) = s_1(p_1 - p_2 s_2)$ be the unconstrained revenue function in period 2, which is concave in $s_2$.

In period 1, the firm has to make two decisions—production quantity, $Q$, and the price charged per unit, $p_1$—before a random demand, $D$, is realized. In period 2, the firm has to make only one decision, $s_2(D)$, i.e., how much of the remaining units from period 1 to sell to maximize the period 2 revenue. In line with the extant models, the cost of production is higher than the maximum possible unit revenue that a firm can obtain in period 2, i.e., $c > p_2$. The firm’s expected profit can be written as $\Pi(Q, p_1, s_2(D)) = E[p_1 \min(Q, D) - cQ + p_2(s_2(D))s_2(D)]$.

Based on their beliefs on product availability in period 1 and on period 2 clearance price, snobs buy products and derive a higher utility in their exclusiveness. For snobs, the utility based on the stockout belief is $u_s = v + k e_s - p_1$, where $k$ represents the sensitivity to stockouts. A consumer develops stockout sensitivity to stockouts. A consumer develops stockout sensitivity to stockouts.

We denote the firm’s belief on the reservation price in period 2 as $e_{r_2}(q, D)$. The price in period 2 is a direct mapping to the number of units sold in period 2 through the clearance price function. To increase a snob’s utility for the product, the expected period 2 price needs to be high (and the number of items sold in clearance needs to be low). We represent the customer’s overall utility as $U_{\text{snob}}(q) = \max[v + k e_s - p_1, E[(1 - e_s)(v - e_{r_2}(q, D))]]$ when the firm chooses to produce $q$ number of units in period 1.

Similarly, we denote the firm’s reservation price belief in period 1 as $e_{r_1}$. Based on $e_{r_1}$, the firm chooses the price optimally and will produce the optimal quantity to maximize its profits. Every snob decides whether she should buy the product in the first period or not. She buys in the first period if and only if $v + k e_s - p_1 \geq E[(1 - e_s)(v - p_2(s_2, D))]$. This condition then leads to snobs’ reservation price, $r_1 = e_s(v + k - E[p_2(s_2, D)] + E[p_2(s_2, D)])$. The commoners make their decisions in the same manner.

After the realization of the demand and sales in period 1, the firm chooses the number of remaining units to sell, and sets the period 2 clearance price through the clearance price function. To achieve this, the firm has to solve a constrained optimization problem, where the sales quantity is bounded above by the quantity of items left over from period 1. Condition (iii) below discusses this optimization; if the firm sold goods at the commoners’ reservation price, the demand in period 1 was $D$, the optimal sales quantity in period 2 is $s_2^*(q, D) = \arg \max_{s_2} \{s_2 p_2(s_2) \mid s_2 \leq (q - D)^+\}$. To this end, the firm develops reservation price beliefs in period 2 for every possible realization of the demand in period 1, before making the decision on how much to sell in period 2. Because every player in the market in period 1 has the same information set, the firm anticipates customers’ reservation price in period 2 as $e_{r_1}(q, D)$. Based on this reservation price and the quantity left unsold from period 1, the firm chooses the optimal sales quantity in period 2.

**Definition 6.** When the firm charges the snob’s (commoner’s) reservation price, an RE equilibrium $(p_1, p_2^*(q, D), Q, r_1, r_2(q, D), e_s, e_{r_1}, e_{r_2}(q, D))$ satisfies the following conditions:

(i) $p_1 = e_{r_1}$ and $p_2^*(q, D) = e_{r_2}(q, D)$;

(ii) $Q = \arg \max_{Q} \{E[p_1 \min(Q, D) - cQ + p_2^*(q, D) s_2^*(q, D))] \mid (Q = \arg \max_{Q} \{E[p_1 \min(Q, D) - cQ + p_2^*(q, D) s_2^*(q, D)]\})$;

(iii) $s_2^*(q, D) = \arg \max_{s_2} \{s_2 p_2(s_2) \mid s_2 \leq (q - D)^+\}$;

(iv) $r_1 = e_s(v + k - E[e_{r_2}(Q, D))] + E[e_{r_2}(Q, D)]$;

(v) $e_s = \tilde{E}_{\text{d}}(Q)$ ($e_s = \tilde{E}_{\text{d}}(Q)$);

(vi) $e_{r_1} = r_1; e_{r_2}(q, D) = r_2(q, D)$.

Conditions (i)–(iv) assert that under the expectations $e_{r_1}, e_{r_2}(q, D)$, and $e_s$ the firm and all consumers will rationally act to maximize their utilities. Condition (v) specifies that, in equilibrium, the stockout probability belief $e_s$ must match with the actual probability of not being able to find the product. Finally, condition (vi) requires that the firm correctly
predicts customers’ reservation price in both periods. The conditions imposed in Definition 6 relate to the production decision in period 1, remaining units to sell decision in period 2, and beliefs on the reservation price and product availability.

When there is a sufficient presence of snobs in the market, the firm prices the product based on its belief of snobs’ reservation price in period 1, and thus the random variable \( D \) is scaled down to \( \beta D \), and stock-out probability becomes \( \tilde{F}_{\beta D}(Q) \). The corresponding equilibrium production quantity in period 1 is given by Proposition 9(i). On the other hand, if the fraction of snobs in the market is low, the firm charges the commoner’s reservation price. In this case, the stock-out probability stays the same at \( \tilde{F}_{D}(Q) \). This is indicated in Proposition 9(ii).

**Proposition 9.** (i) **(Limited Production)** When \( \beta > \beta^* \), in the RE equilibrium, only snobs can buy, and the firm’s price and quantity choices are characterized by \((p_{1}, q_{1}^{s}, s_{1}^{s}, (q_{1}, D), p_{2}^{s}, (q_{1}, D))\).

(ii) **(Regular Production)** When \( \beta \leq \beta^* \), in the RE equilibrium, all customers (snobs and commoners) can buy, and the firm’s price and quantity choices are characterized by \((p_{1}, q_{1}^{s}, s_{1}^{s}, (q_{1}, D), p_{2}^{c}, (q_{1}, D))\).

The terms for the equilibrium strategies are presented in the proofs in the appendix. When the expressions are implicit, the equilibrium values can be calculated using numerical computations. In our clearance pricing model, we find that when the firm adopts the scarcity strategy, the clearance prices are lower than corresponding clearance price when there is no scarcity. We also establish that scarcity strategies, again as before, are applied in the intermediate value of \( \beta \), as supported by a representative numerical example in Figure 4.

### 5.3. Commitment to Scarcity Through Sourcing

Although it is true that scarcity strategies could be adopted to generate more revenues, it is far from certain that such shortage information is credible, especially because the production decisions are often unobservable. Amaldoss and Jain (2008, p. 939) correctly observe that “limited edition strategy is constrained by the firms’ ability to credibly convince consumers that it will not sell a higher quantity . . . (since it is ex post profitable to do so).” In this section, we study how firms may commit to scarcity strategies credibly. We show that in equilibrium, the firms may end up with lower production volume (depending on the market structure) because of higher upfront investments in sourcing costs. In our model, the firm does not have any additional utility to produce more goods after the demand is realized, because the reservation price for the remaining consumers is reduced to salvage value \( s \) (i.e., overage cost is incurred on additional units).

In particular, we examine how sourcing strategies of a firm can be employed to commit to exclusivity. Such strategies are not uncommon in practice. Nearly all European luxury goods companies still produce their goods in Europe, despite the much higher production costs in Europe (Bruner et al. 2005). Also, many luxury apparel firms advertise their products to be handmade or Italian leather, signaling higher value to the customer. In addition, many firms that produce conspicuous products, such as Timbuk2, prominently advertise their expensive sourcing decisions to sell the goods at a higher premium. We argue that in some cases such information may not be, in itself, intrinsically valuable to snobs (i.e., snobs may have no additional utility in handmade bags or shoes made of imported Italian leather). However, such information may be processed by snobs as being indicative of the firm’s cost commitment to the product.

Consider a firm that makes a sourcing or production decision for a conspicuous good before the decisions are made on price and production quantity. The sourcing decision will distinguish the product from the functionality equivalent product sourced elsewhere. For simplicity, we assume two sourcing methods. The cheaper sourcing method has a...
marginal cost \( c_{L*} \), and the more expensive method involves \( c_{H*} \), i.e., \( c_{H*} > c_{L*} \). The more expensive source might involve a combination of factors that increase the marginal cost of production—an in-sourced supplier whose assembling wages are higher, or the utilization of more expensive raw materials, or using a more intricate production process.

We consider the decision of the firm and consumers in a multiperiod game. In the first period, the firm makes its sourcing decisions. In the second period, pricing and quantity decisions are made by the firm before demand is known, and the consumers make their purchase decisions. This is followed by the period in which leftover goods are salvaged. We derive the equilibrium through backward induction. In the second period, given the sourcing decision, the subgame proceeds exactly as analyzed in the previous sections (except that \( c_1 \) or \( c_{ij} \) replaces \( c \)). Then, in the first period, the firm compares the profits obtained from each sourcing decision and chooses the profit-maximizing source.

Following Proposition 3, the firm chooses limited production when \( \beta \geq \beta^{*}_{L} \), when the sourcing cost is \( c_{L*} \), and when \( \beta \geq \beta^{*}_{H} \) when the sourcing cost is \( c_{H*} \). Proposition 10 shows that this unique threshold level is decreasing with the marginal cost of supply.

PROPOSITION 10. The threshold level for limited production decreases with the marginal cost \( c \) of the supply source. Therefore, \( \beta^{*}_{L} > \beta^{*}_{H} \).

Because the threshold \( (\beta^{*}_{L}) \) under the more expensive supply is lower than the threshold level \( (\beta^{*}_{H}) \) under the cheaper supply, we note that limited production is more prevalent when the supplier is expensive. This yields three possible positions for the fraction of snobs within the population: (i) \( \beta < \beta^{*}_{L} \leq \beta^{*}_{H} \), the firm prefers to use regular production when using either source; (ii) \( \beta^{*}_{L} < \beta < \beta^{*}_{H} \), the firm prefers to use limited production for the expensive source, and the regular production strategy for the cheaper source; (iii) \( \beta^{*}_{L} < \beta^{*}_{H} < \beta \), the firm prefers to use limited production for both sources.

We focus our attention on the most interesting case (case (iii)), when the firm adopts limited production with either source. The other cases offer the same qualitative conclusions and are omitted for the sake of brevity.

To analyze the sourcing decisions, we study the profits of the firm as a function of the expensive source cost \( c_{H*} \) (holding the cost of the cheaper source \( c_{L*} \) constant). We show that this profit function is unimodal and attains the global maximum at \( c_{H*} = c^* \in [s, v] \) (the unique global minimum is at \( v \)). Furthermore, at \( c_{H*} = c_{\text{equal}} \), the profit using expensive supply matches the profit using the low cost source.\(^{18}\) This property of the profit function helps us to provide equilibrium results for a general demand distribution and product costs in Proposition 11.

We present the equilibrium result for case (iii), when the firm prefers to adopt limited production for either of the two supply sources, but the results for low intensity and medium intensity are qualitatively similar. Proposition 11 provides conditions under which an expensive option is chosen by the firm. The specific sourcing decisions are indicated in Figure 5.

PROPOSITION 11. When \( \beta > \beta^{*}_{L} (\geq \beta^{*}_{H}) \), the firm chooses the expensive source when (i) \( c_{L*} < c_{H*} \leq c^* \) (Region A) or (ii) \( c_{L*} < c^* < c_{\text{equal}} < c_{H*} \) (Region B), and chooses the cheaper source when (i) \( c_{L*} < c^* < c_{\text{equal}} < c_{H*} \) (Region C) or (ii) \( c_{L*} \geq c^* \) (Region D).

Figure 5 shows that a more expensive source is chosen (i) when the low-cost source is cheap (i.e., \( c_{L*} \) is low) and (ii) when the expensive source is (comparatively) not too costly (i.e., \( c_{H*}/c_{L*} < c_{\text{equal}}/c_{L*} \)). The latter point is intuitive. We focus on the intriguing first condition (condition (i)). It is interesting to note that the firm avoids sourcing from the cheaper source when the source is at its cheapest cost. In other words, a sufficiently cost-efficient production process or supply source will be unused in equilibrium. This is because the firm is perceived by the market as committing to a high volume of production (low exclusivity) when it uses a low-cost source. This result mirrors the higher marginal cost result of Amaldoss and Jain (2008), which shows, using reference group effects, that increased marginal costs can improve the profits of a firm. In their paper, the increased costs make the product less attractive to followers; thus the leaders (to differentiate themselves) adopt the product at a lower price (at a high volume of sales).

\(^{18}\) See the appendix for technical details (proof of Proposition 11).
Our explanations are based on demand uncertainty. Given that the firm has to make a “bet” on optimal quantity in an uncertain demand market, the firm with the higher sourcing costs produces less goods, because the marginal cost of unsold goods \(c - s\) is high. This low inventory in turn increases the valuation for snobs, and hence the equilibrium price. Thus, in equilibrium, the firm with higher costs produces fewer quantities sold at a higher price. Consumers can rationalize that given the uncertain demand environment, the firm’s increased investment and production costs can only be recouped by producing a few exclusive items and selling each of those items at a high margin. Even if the product produced using the cheaper source is indistinguishable in terms of performance quality, a firm selling conspicuous goods may prefer to use an expensive source to commit to exclusivity.

6. Conclusions

This paper attempts to study how operational decisions such as production, salvaging, and sourcing can be employed together with pricing and scarcity decisions in a market with uncertain demand and conspicuous consumption. In particular, we model the role that stockouts play in the decisions of a firm. Su and Zhang (2008) show how the cost of customers of not being able to find the product might force firms to provide availability guarantees to allay scarcity fears. Our paper takes a different approach. Just as scarcity may be a signal of product quality (Stock and Balachander 2005), we show how scarcity may, in markets with uncertain demand, also be used to influence demand and consumer valuations, especially when some consumers’ decisions are affected by the desire for exclusivity.

We show through an analytical model that the existence of conspicuous consumption, by itself, does not guarantee scarcity and low production volumes. In fact, if there is a sufficient number of snobs in the market, the firm may be driven by high margins to produce more goods (because the cost of losing a sale is high). We provide an explanation for why some firms may limit their production before introducing their product to the market and others do not, even in an uncertain market where demand remains unobserved. Unlike the extant results, our results on limited production are ex post consistent, i.e., the commitment the firm can make to limited quantities is credible, and the firm cannot produce and sell more items after purchasing occurs. Using the limited production results, we consider when and how firms should adopt the scarcity strategy, and how it is dependent on market parameters and demand uncertainty.

We find that the scarcity strategy is applied when the fraction of snobs in the market is neither too high nor too low. For a market with a low percentage of snobs, it is not worth excluding the commoners by charging the snob’s reservation price because the number of snobs is insufficient to overcome the revenue lost by excluding commoners. When the fraction of snobs in the market is too high, the firm is influenced by the high margins/underage costs to overproduce goods. We provide an analytical identification of the interval of the percentage of snobs in the market for which scarcity is a profitable strategy. This scarcity region decreases as the demand becomes more variable, thus making it harder for a firm to commit to the scarcity strategy under uncertain demand.

We explore how firms adopt more expensive sourcing decisions, incurring higher up-front costs to produce a functionally equivalent good when selling to conspicuous consumers. The higher-cost investment in turn helps the firm in distinguishing its product in terms of exclusivity, even though the consumer utility of the product itself remains unaltered. Intriguingly, we find that the choice of a more expensive sourcing may or may not be employed in conjunction with scarcity strategies.

Our model is not without its limitations. In reality, the firms and consumers could make product decisions periodically over multiple periods. In such a dynamic model, learning about stockouts may play a role in how snobs and commoners make their future decisions.

Some future directions include empirically testing our analytical findings using secondary data from natural or laboratory experiments. We hope that our detailed analytical conclusions for the uncertain demand market can provide interesting hypotheses for empirical research. Manski (2004) pointed out the paucity of empirical research on expectations formation and that studying revealed preferences alone is insufficient to draw conclusions, and researchers need to measure consumer expectations while those choices are being made. In our context, a careful empirical research would have to differentiate among various factors (conspicuous consumption, herding effects, network effects, or learning/quality effects) when considering choice and expectations for those consumers buying an expensive product based on externalities. This differentiation might be quite involved (Manski 1993). We believe that a careful empirical analysis of the relationship between consumer characteristics and the impact of stockouts on their buying behavior (see Anderson et al. 2006) or an experimental approach (see Amaldoss and Jain 2010) would help us understand how exclusive goods are sold and bought in a market with conspicuous consumption.
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Appendix. Proofs

Proof of Proposition 1. The RE equilibrium conditions reduce to \( p = \bar{F}_D(Q^*_v) \cdot (k + v - s) + s \). The firm will obtain the critical fractile quantity choice as

\[
\frac{\partial \bar{F}_D}{\partial Q} = (p - s) \Pr(D > Q^*_v) - (c - s) = 0
\]

\[
\Rightarrow \bar{F}_D(Q^*_v) = \frac{c - s}{p - s}.
\]  (5)

Plugging \( p \) and solving (5) provides the equilibrium quantity:

\[ \bar{F}_D(Q^*_v) = \sqrt{\frac{c - s}{k + v - s}}. \]

Putting \( \bar{F}_D(Q^*_v) \) back in \( p \) provides the equilibrium price:

\[ p^*_v = \sqrt{(c - s)(k + v - s) + s}. \]

Lemma A1. (i) \( Q^*_v < Q_0 \) and \( p^*_v < p_0 \) when \( k < (v - c)/(c - s) \). (ii) \( Q^*_v > Q_0 \) and \( p^*_v > p_0 \) when \( k > (v - c)/(c - s) \).

Proof of Lemma A1. We define \( \Delta(Q) = Q^*_v(k) - Q_0 = \bar{F}_D^{-1}(\sqrt{(c - s)/(k + v - s)}) - \bar{F}_D^{-1}((c - s)/(v - s)) \). Showing that \( \Delta(Q)(k) \) is negative at \( k = 0 \) and \( \Delta(Q)(k) \) strictly increases with \( k \) is sufficient to show that there exists a unique \( k^* \) such that \( \Delta(Q)(k^*) = 0 \):

\( \Delta(Q)(k) < 0 \): Note that \( (c - s)/(v - s) > (c - s)/(v - c) \) since \( v > c > s \). Then, \( \bar{F}_D^{-1}(\sqrt{(c - s)/(v - s)}) < \bar{F}_D^{-1}((c - s)/(v - s)) \), which confirms that \( \Delta(Q)(0) = \bar{F}_D^{-1}((c - s)/(v - s)) - \bar{F}_D^{-1}((c - s)/(v - s)) < 0 \).

\( \Delta(Q)(k)/\partial k > 0 \):

\[
\frac{\Delta(Q)(k)}{\partial k} = \frac{\partial Q^*_v(k)}{\partial k} = -\frac{1}{2} \left( \frac{\sqrt{c - s}}{(k + v - s)^{3/2}} - \frac{1}{\bar{F}_D^{-1}(\sqrt{(c - s)/(k + v - s)})} \right)
\]

\[
= \frac{1}{2} \left( \frac{\sqrt{c - s}}{(k + v - s)^{3/2}} - \frac{1}{\bar{F}_D^{-1}(\sqrt{(c - s)/(k + v - s)})} \right).
\]

Note that \( \sqrt{c - s} > 0 \) and \( k + v - s > 0 \) because \( v > c > s \) and \( k > 0 \). Then, \( \Delta(Q)(k)/\partial k > 0 \).

Thus, there exists a unique \( k^* \) such that \( \Delta(Q)(k^*) = 0 \): \( k^* = (v - c)/(v - c)/(c - s) \). Observe that \( \Delta(Q)(0) < 0 \) and \( \partial Q^*_v(k)/\partial k > 0 \). This implies that \( \Delta(Q)(k) \) changes sign only once at \( k^* \) as \( k \) increases. It is easy to show that the same threshold, \( k^* \), holds for the relation between \( p^*_v \) and \( p_0 \) when \( k < (v - c)/(v - c)/(c - s) \), and (ii) \( Q^*_v > Q_0 \) and \( p^*_v > p_0 \) when \( k > (v - c)/(v - c)/(c - s) \). □

Proof of Proposition 2(ii). The firm sets the price at the reservation price of snobs, and the commoners are excluded because of their lower reservation price (\( \beta D \) instead of \( \beta \)). The RE equilibrium conditions reduce to \( p = \bar{F}_D(Q^*_v) \cdot (k + v - s) + s \). The firm’s critical fractile quantity choice is

\[
\frac{\partial \Pi}{\partial Q} = (p - s) \Pr(\beta D > Q^*_v) - (c - s) = 0
\]

\[
\Rightarrow \bar{F}_D(Q^*_v) = \frac{c - s}{p - s}.
\]  (6)

Solving \( p \) and (6) provides the equilibrium quantity: \( \bar{F}_D(Q^*_v) = \sqrt{(c - s)/(k + v - s)} \). Putting \( \bar{F}_D(Q^*_v) \) back in \( p \) provides the equilibrium price: \( p^*_v = \sqrt{(c - s)(k + v - s) + s} \). □

Proof of Lemma A2. We show that the difference between profits obtained from limited production and regular production changes sign at a unique threshold of snobs, \( \beta^* \), as \( \beta \) increases. Recall the implicit formulation of \( Q^*_v \) from Proposition 2(ii). Then, the optimal profit for the firm is \( \Pi_{N_s} = \bar{F}_D^{-1}((c - s)/(k + v - s))\beta f_{D}(u) \) du. We define \( \Delta\Pi(\beta) \) to represent the firm’s optimal profit function when limited production strategy is applied given that the percentage of snobs is \( \beta \). We assume that \( Q^*_v(k^*) > 0 \) except for \( \beta = 0 \) and \( Q^*_v > 0 \) without loss of generality. Also, we define \( \Delta\Pi(\beta) = \Pi_{N_s}(\beta) - \Pi_{N_s}(\beta^*) \). Note that \( \beta \) is in the domain \([0, 1]\). Then, showing that \( \Delta\Pi(\beta) \) is negative at \( \beta = 0 \), \( \Delta\Pi(\beta) \) is positive at \( \beta = 1 \), and \( \Delta\Pi(\beta) \) strictly increases in \( \beta \) is sufficient to prove that there exists \( \beta^* \) such that \( \Delta\Pi(\beta^*) = 0 \):

\( \Delta\Pi(0) < 0 \):

\[
\Delta\Pi(0) = -\sqrt{(c - s)/(v - s)} \int_0^{\bar{F}_D^{-1}(\sqrt{(c - s)/(v - s)})} f_{D}(u) \ du.
\]

\( \int_0^{\bar{F}_D^{-1}(\sqrt{(c - s)/(v - s)})} f_{D}(u) \ du \)

The unique threshold is

\[
\beta^* = \sqrt{\frac{v - s}{k + v - s}} \int_0^{\bar{F}_D^{-1}(\sqrt{(c - s)/(v - s)})} f_{D}(u) \ du.
\]

\[
\int_0^{\bar{F}_D^{-1}(\sqrt{(c - s)/(v - s)})} f_{D}(u) \ du.
\]
The term \(\sqrt{(c-s)(v-s)}\) is positive since \(v > c > s\). The support of \(D\) is nonnegative; and \(Q^* > 0\). Then, the above term must be nonpositive.

- \(\Delta \Pi(1) > 0:\)
  \[
  \Delta \Pi(1) = \frac{Q_{4'}^{1/2} \sqrt{(c-s)/(k+v-s)}}{v-s} \int_0^x u f_D(u) \, du - \frac{Q_{4'}^{1/2} \sqrt{(c-s)/(v-s)}}{v-s} \int_0^x u f_D(u) \, du.
  \]

Note that \(\sqrt{(c-s)/(v-s)} \leq \sqrt{(c-s)/(k+v-s)}\) since \(k \geq 0\). Also, \(F_D^{-1}(\sqrt{(c-s)/(v-s)}) \leq F_D^{-1}(\sqrt{(c-s)/(k+v-s)})\), since \(\sqrt{(c-s)/(v-s)} \geq \sqrt{(c-s)/(k+v-s)}\). Then, the above term must be positive.

- \((\partial \Delta \Pi(1))/\partial \beta > 0:\)
  \[
  \frac{\partial \Delta \Pi(1)}{\partial \beta} = \frac{Q_{4'}^{1/2} \sqrt{(c-s)/(k+v-s)}}{v-s} \int_0^x u f_D(u) \, du.
  \]

The first term of the above term is positive since \(v > c > s\) and \(k \geq 0\). The support of \(D\) is nonnegative, and \(Q^*(\beta)/\beta > 0\). Then, the above term must be positive.

Then, there exists a unique root \(\beta^*\) such that \(\Delta \Pi(\beta^*) = 0:\)

\[
\beta^* = \sqrt{\frac{v-s}{k+v-s}} \int_0^\infty \sqrt{\frac{(c-s)/(k+v-s)}} \frac{uf_D(u) \, du}{uf_D(u) \, du}.
\]

These findings \(\Delta \Pi(0) < 0, \Delta \Pi(1) > 0, \) and \(\partial \Delta \Pi(1)/\partial \beta > 0\) imply that \(\Delta \Pi(\beta)\) changes sign only at \(\beta^*\) as \(\beta\) increases. This leads to the following result: (i) \(\Pi_{1,\epsilon} > \Pi_{1,\epsilon}^*\) when \(\beta = \beta^*\), and (ii) \(\Pi_{1,\epsilon} > \Pi_{1,\epsilon}^*\) when \(\beta > \beta^*.\)

**Proof of Proposition 3.** The proof follows from the results of Lemma A2.

**Proof of Proposition 4.** We show that the difference between \(Q^*\) and \(Q^*_c\) changes sign only at a particular threshold level, \(\beta_0\), as \(\beta\) increases. We define \(Q^*(\beta)\) as the equilibrium quantity choice under the limited production strategy when snobs represent a fraction \(\beta\) of the market. We assume that \(Q^*(\beta) > 0\) except for \(\beta = 0\) and \(Q^*_c > 0\) without loss of generality. Also, we define

\[
\Delta Q(\beta) = Q^*(\beta) - Q^*_c = -\frac{Q_{4'}^{1/2} \sqrt{(c-s)/(k+v-s)}}{v-s} - F_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}) - F_D^{-1}(\sqrt{\frac{c-s}{v-s}}).
\]

Then, showing that \(\Delta Q(0) < 0, \Delta Q(1) > 0, \) and \(\partial \Delta Q(\beta)/\partial \beta > 0\) is sufficient to say there exists unique \(\beta_0\) such that \(\Delta Q(\beta_0) = 0:\)

- \(\Delta Q(0) < 0:\)
  \[
  \Delta Q(0) = -F_D^{-1}(\sqrt{\frac{c-s}{v-s}}) - F_D^{-1}(\sqrt{\frac{c-s}{v-s}}).
  \]

is positive because we assume that the support of \(D\) is nonnegative and \(Q^*_c > 0\). Then, \(\Delta Q(0) < 0.\)

- \(\Delta Q(1) > 0:\)
  \[
  \Delta Q(1) = F_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}) - F_D^{-1}(\sqrt{\frac{c-s}{v-s}}).
  \]

We show that

\[
F_D^{-1}(\sqrt{\frac{c-s}{k+v-s}}) > F_D^{-1}(\sqrt{\frac{c-s}{v-s}})
\]

within the proof of Lemma A2. Then, \(\Delta Q(1) > 0.\)

- \(\partial \Delta Q(\beta)/\partial \beta > 0:\)
  \[
  \frac{\partial \Delta Q(\beta)}{\partial \beta} = \frac{\partial Q^*_c}{\partial \beta} = F_D^{-1}(\sqrt{c-s/(k+v-s)}) > 0.
  \]

The inequality follows directly from the assumption that the support of \(D\) is nonnegative and \(Q^*_c(\beta)/\beta > 0\). Then, \(\partial \Delta Q(\beta)/\partial \beta > 0.\)

Then, there exists a unique root \(\beta_0\) such that \(\Delta Q(\beta_0) = 0:\)

\[
\beta_0 = F_D^{-1}(\sqrt{c-s/(k+v-s)}).
\]

These findings \(\Delta Q(0) < 0, \Delta Q(1) > 0, \) and \(\partial \Delta Q(\beta)/\partial \beta > 0\) imply that \(\Delta Q(\beta)\) changes sign only at \(\beta_0\) as \(\beta\) increases. This leads to the following result: \(Q^*_c < Q^*\) when \(\beta < \beta_0\), and \(Q^*_c > Q^*\) when \(\beta > \beta_0.\)

**Proposition A3.** \(\beta_0\) is larger than or equal to \(\beta^*\) when \(x/y \geq F_D(y)/F_D(x).\)

**Proof of Proposition A3.** Using the closed form expressions for \(\beta_0\) from Proposition 4 and \(\beta^*\) from Lemma A2, we show that \(\beta_0\) is larger than or equal to \(\beta^*\) for given parameters \((s, c, v, k)\) under a particular condition. We define \(q = F_D^{-1}(\sqrt{c-s/(v-s)})\) and \(\omega = F_D^{-1}(\sqrt{c-s/(k+v-s)})\) for simplification of the system. Rewriting \(\beta^*\) and \(\beta_0\) with the new notation leads to the following equations:

\[
\beta_0 = q/\omega, \quad \beta^* = \left[F_D(q)/F_D(q)\right] \int_{q}^{\infty} f_D(u) \, du / \int_{q}^{\infty} f_D(u) \, du.
\]

\(\beta^*\) can be further reduced to the following equation by applying integration by parts on the numerator and the denominator of the second term: \(\beta^* = \left[F_D(q)/F_D(q)\right] \cdot \left[\int_{q}^{\infty} f_D(u) \, du - qF_D(q)\right] / \left[\int_{q}^{\infty} f_D(u) \, du - qF_D(q)\right].\)

Showing that \(\beta_0 - \beta^* \geq 0\) is sufficient for the validity of the claim. Recall that, in the proof of Lemma A2, we showed both the first and the second term of \(\beta^*\) are less than or equal to 1 and nonnegative. Thus, eliminating the second term will provide us a lower bound for \(\beta_0 - \beta^*\): \(\beta_0 - \beta^* \geq q/\omega - F_D(q)/F_D(q).\) Therefore, we have shown that \(\beta_0\) is larger than or equal to \(\beta^*\) if \(q/\omega \geq F_D(q)/F_D(q)\) for given parameters \((s, c, v, k)\).

**Proof of Lemma 1.** We show that for higher \(k\), \(Q^*_c\) increases more steeply in \(\beta\). Recall

\[
Q^*_c = \frac{c-s}{k+v-s} > 0
\]

from the proof of Lemma A2. In Proposition 4, we show that

\[
\frac{\partial Q^*_c}{\partial \beta} = F_D^{-1}(\sqrt{c-s/(k+v-s)}).
\]

What we need to show now is that

\[
\frac{\partial^2 Q^*_c}{\partial \beta^2} = \frac{1}{(k+v-s)^3} \left[F_D^{-1}(\sqrt{c-s/(k+v-s)})\right] > 0.
\]

The term \((\sqrt{c-s})/(k+v-s)^3/2\) is positive by the assumptions \(v > c > s\) and \(k > 0\). Then, \(\partial^2 Q^*_c/\partial \beta^2 > 0.\) Therefore, we have shown that for higher \(k\), \(Q^*_c\) increases more steeply in \(\beta.\)
Proof of Lemma 2. The proof follows directly from showing that the first derivatives of both threshold levels (β′ and β̂) with respect to k are negative. □

Proof of Proposition 5. We derive the conditions under which the region of scarcity (i.e., β̂−β′) expands as snobs become more sensitive to stockouts. We define Δβ′(k) = ∂β′/∂k − ∂β̂/∂k. We will use the same notation, ̂p and ω, that we used in the proof of Proposition A3. Showing that Δβ′(k) is negative for all k in [0, ∞) is sufficient for proving the proposition:

Δβ′(k) = \frac{1}{2k + v - s} \left( β′ \left( \frac{\int_0^u \tilde{F}_D(u) \, du}{\tilde{F}_D(u) \, du} - ω \right) = \bar{F}_D(ω) \right).

The above term is less than or equal to 0 if and only if the equation in parentheses is nonnegative. We know from the proofs of Lemma A2 and Proposition 4 that all terms within the parentheses are nonnegative. Thus, we require a sufficient condition that will make the equation in parentheses nonnegative:

⇒ \omega(ω) = \bar{F}_D(ω) \int_0^u \tilde{F}_D(u) \, du (h(ω) = f(ω)/\tilde{F}_D(ω); the hazard rate of D)

= \frac{β̂}{β} Q_0/β \text{ (since } ω = Q_0/β). \]

We define the hazard rate of D above as h(•) (see Bryson and Siddiqui 1969 for details). The second term on the right-hand side of the inequality is less than or equal to 1 since \int_0^u \tilde{F}_D(u) \, du − ω \tilde{F}_D(ω) ≤ \int_0^u \tilde{F}_D(u) \, du and ω ≥ 0. The right-hand side of the inequality can be more than or less than or equal to 1 depending on the relationship between the first and second terms. Implicit sufficient conditions can be similarly obtained in this case. □

Proof of Proposition 6. The firm sets the reservation price of commoners so the snobs are excluded from consideration ((1−β)D instead of β; recall that this is the case where the snobs have a lower valuation than the commoners because of abundant availability). Replace D with (1−β)D in the proof of Proposition 2(ii) and the results follow. □

Proof of Proposition 7. The proof follows the technique used in the proof of Proposition 2. In this case, limited production follows by the firm’s action toward setting the price to e_1, < p < e, so BD instead of β. The RE equilibrium conditions would then reduce to p^*_D = \sqrt{(c - s)(k + v - s) + s}. Regular production follows by the firm’s action toward setting the price to p ≤ e. Then, the RE equilibrium conditions would result in p^*_D = \sqrt{(c - s)(v - s) + s}. The firm sets the same equilibrium price and produces the same equilibrium quantity as in Proposition 2 for the corresponding selling strategies; hence the result follows from the proof of Proposition 3. □

Proof of Proposition 8. The firm sets the price of the good at the reservation price of snobs so the commoners are excluded from the consideration (BD instead of β). The RE equilibrium conditions for price and quantity reduce to

p = \tilde{F}_D(Q/β) \left( k + v - s_0 + \frac{s_1}{Q/β} \int_0^{Q/β} ((Q - βu)/Q) \, dF_D(u) \right)

+ s_0 - \frac{s_1}{Q/β} \int_0^{Q/β} ((Q - βu)/Q) \, dF_D(u) \right)

(8)

Putting Q^*_0 back in (8) provides the equilibrium price p^*_D. The proof for part (ii) (regular production) follows by setting β = 1 and k = 0 above. □

Proof of Proposition 9. When β > β′, RE equilibrium conditions reduce to

s^*_2(q, D) = \text{arg max}_{s_2} \{ s_2 \cdot p_2(s_2) | s_2 \leq (q - βD)^+ \};

p^*_2(q, D) = p_2(s^*_2(q, D));

p_1 = \tilde{F}_D(\frac{q}{β}) \cdot (v + k - E_0[p^*_2(q, D)] + E_0[p^*_2(q, D)].

We use the backward induction method to find the RE equilibrium of this game. First, we start with period 2. We find the equilibrium number of remaining units to sell, s^*_2(q, •), and hence the equilibrium price set, p^*_2(q, •), for every possible realization of demand from period 1 (ξ) and number of units produced in period 1 (q). Let R^*_2 = s^*_2(p_2 - p_0) be the unconstrained revenue function of the firm from period 2. Note that ∂^2 R^*_2(ξ)/∂s^*_2 ≥ -2p_0 < 0; hence, R^*_2 is concave in s^*_2. Then, s^*_2 = p_2/(2p_0) that sets ∂^2 R^*_2(ξ)/∂s^*_2 to 0 maximizes the period 2 revenue. However, number of units to sell in period 2 is constrained by the number of remaining units from period 1, i.e., (q - βD^+). If s^*_2 turns out to be higher than (q - βD^+), the firm earns the most if it sells the whole remaining inventory from period 1, i.e., (q - βD^+), because of the concave structure of R^*_2(s^*_2). We are ready to write the corresponding optimal selling quantity, which also provides the optimal clearance price in period 2 through the clearance price function as

s^*_2,1(q, D) = \begin{cases} p_2/(2p_0) & \text{if } p_2/(2p_0) \leq (q - βD)^+, \\
(q - βD)^+ & \text{if } p_2/(2p_0) > (q - βD)^+. \end{cases}

p^*_2(q, D) = \begin{cases} p_2/2 & \text{if } p_2/(2p_0) \leq (q - βD)^+, \\
p_2 - p_0(q - βD)^+ & \text{if } p_2/(2p_0) > (q - βD)^+. \end{cases}

We move to period 1 after finding the period 2 equilibrium price and selling quantity decisions. The firm’s profit can be written as

\Pi(q, p_1) = E_0[p_1 \min\{q, BD\} - c + p^*_2(q, D) s^*_2(q, D)]

= (p_1 - c)q

- \int_0^{q/β} p_1(q - βu) \, dF_D(u) + \frac{p_1^2}{4p_0} \tilde{F}_D(q - p_1/2p_0)

+ \int_0^{q/β} (p_2 - p_0(q - βu))(q - βu) \, dF_D(u).
Note that \( \partial \Pi(q, p_1) / \partial p_1 = q - f_0(q - \beta u) dF_2(u) > q(1 - E_2(q/\beta)) > 0 \). Then, the firm sets the reservation price to the maximum possible value that is the reservation price of the customers. The firm’s period 1 price choice can be written as

\[
p_1^* = \int_0^{Q_1/\beta} \left( \frac{Q_1}{\beta} \right) (v + k - p_2^*(Q_1^*, u)) + p_2^*(Q_1^*, u)) dF_2(u). \tag{9}
\]

The firm will obtain the equilibrium production quantity as

\[
\Rightarrow \frac{\partial \Pi(p_1, q)}{\partial q} = (p_1^* - c) - p_1^* F_2\left( \frac{Q_1}{\beta} \right) + \int_0^{Q_1/\beta} (p_2 - 2p_2(Q_1^* - \beta u)) dF_2(u) = 0.
\tag{10}
\]

Solving equations (9) and (10) provides \( Q_1^* \) characterized by the implicit equation

\[
f_D\left( \frac{Q_1}{\beta} \right) F_2\left( \frac{Q_1}{\beta} \right) \left( (v + k - p_2) F_2\left( \frac{Q_1}{\beta} \right) + p_2 \right) - f_D\left( \frac{Q_1}{\beta} \right) F_2\left( \frac{Q_1}{\beta} \right) - F_2\left( \frac{Q_1}{\beta} \right) + 2p_2 \int_0^{Q_1/\beta} (Q_1^* - \beta u) dF_2(u) - c
+ p_2 \cdot F_2\left( \frac{Q_1}{\beta} \right) - F_2\left( \frac{Q_1}{\beta} \right) - F_2\left( \frac{Q_1}{\beta} \right) - (p_2 - 2p_2(Q_1^* - \beta u)) dF_2(u) = 0.
\]

The proof for regular production (\( \beta < \beta^* \)) follows by setting \( \beta = 1 \) and \( k = 0 \) above.

\( \square \)

**Proof of Proposition 10.** We show that the threshold level \( \beta^* \) is decreasing in \( c \). Recall that \( \beta^* \) is defined \( \forall c \in [s, v] \) from Lemma A2. Showing that the first derivative of \( \beta^* \) with respect to \( c \) is nonpositive will be sufficient for the argument:

\[
\frac{\partial \beta^*}{\partial c} = \sqrt{\frac{v - s}{k + v - s}} \left( \frac{1}{f_D^{-1}\left( \frac{s}{\beta} \right)} \int_0^{s/\beta} \frac{1}{2\sqrt{c - s}} \right. \\
\left. \frac{1}{f_D^{-1}\left( \frac{s}{\beta} \right)} \int_0^{(c - s)/(k + v - s)} \sqrt{\frac{k + v - s}{v - s}} dF_2(u) du \right) \int_0^{v/\beta} \frac{1}{2\sqrt{c - s}} \\
\frac{1}{f_D^{-1}\left( \frac{s}{\beta} \right)} \int_0^{(c - s)/(k + v - s)} \sqrt{\frac{k + v - s}{v - s}} dF_2(u) du.
\]

Terms outside the parentheses are positive because \( s < c < v \) and \( D \) has a nonnegative support. Thus, the above term is nonpositive if and only if terms in the parentheses give nonpositive value. The analysis of the terms in parentheses will provide the sufficient and necessary condition

\[
\sqrt{\frac{v - s}{k + v - s}} \int_0^{s/\beta} \frac{1}{f_D^{-1}\left( \frac{s}{\beta} \right)} \int_0^{(c - s)/(k + v - s)} \sqrt{\frac{k + v - s}{v - s}} dF_2(u) du
\leq \frac{f_D^{-1}\left( \frac{c - s}{(k + v - s)} \right)}{f_D^{-1}\left( \frac{c - s}{(k + v - s)} \right)} \Rightarrow \beta^* \leq \beta_{Q^*}.
\]

Recall that we provide one sufficient condition in Proposition A3 for the above term to hold. Then, this is sufficient to say that \( \beta^* / \beta c \leq 0 \). We have shown that the threshold level for limited production decreases with the marginal cost of the supply \( c \). Therefore, the threshold level for the more expensive source, \( \beta_{Q^*} \), is less than the threshold level for the cheaper source, \( \beta_{Q^*} \). Simply, \( \beta_{Q^*} < \beta_{Q^*} \) since \( c_1 < c_2 \). \( \square \)

**Proof of Proposition 11.** We derive the conditions that dictate the choice of the source by the profit-maximizing firm for the high-intensity region. We define \( v' = v + k \) without loss of generality. (Proof for the low-intensity region can be obtained by setting \( k = 0 \) and \( \beta = 1 \).) To generalize our results for all demand distributions, we derive the structure of the profit function for changing cost of the supplier. We show that there exists a global maximum at \( c^* \), at least one inflection point in \((c^*, v')\), and a global minimum at \( v' \).

Recall \( \Pi_{Q^*}(c) \) from the proof of Lemma A2 that stands for the optimal profit of the firm experiencing high intensity of snobs under the limited production strategy:

\[
\Pi_{Q^*}(c) = \sqrt{(c - s)(v' - s)} \int_0^{f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right)} dF_2(u) du;
\]

\( \Pi_{Q^*}(c) \) is a continuous function on the closed interval \([s, v']\) and differentiable on the open interval \((s, v')\), where \( s < v \). Note that \( \Pi_{Q^*}(s) = 0 \) and \( \Pi_{Q^*}(v) = 0 \). Then, there exists at least one \( c^* \) in \((s, v')\) such that \( \partial \Pi_{Q^*}(c^*) / \partial c = 0 \) by the mean value theorem. Now that we show there must be at least one extreme point within \((s, v')\), the next step is to show that there can only be one extreme point that is a global maximum in \((s, v')\). We check the first derivative of \( \Pi_{Q^*}(c) \) with respect to \( c \):

\[
\frac{\partial \Pi_{Q^*}(c)}{\partial c} = \frac{\beta}{2} \sqrt{\frac{v - s}{c - s}} \int_0^{f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right)} dF_2(u) du
- \beta f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right).
\]

Any extreme point \( c^* \) in \([s, v']\) must satisfy \( \partial \Pi_{Q^*}(c^*) / \partial c = 0 \):

\[
\int_0^{f_D^{-1}\left( \frac{(c^* - s)/(v' - s)}{\beta} \right)} dF_2(u) du = 2 f_D^{-1}\left( \frac{c^* - s}{v' - s} \right) \sqrt{\frac{c^* - s}{v' - s}}. \tag{11}
\]

We check the sign of the second derivative of \( \Pi_{Q^*}(c) \) with respect to \( c \) at the extreme points:

\[
\frac{\partial^2 \Pi_{Q^*}(c^*)}{\partial c^2} = \frac{1}{4} \left( \frac{c^* - s}{v' - s} \right) \left( \frac{2 f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right)}{\sqrt{(c - s)/(v' - s)}} + \frac{1}{f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right)} \right) \tag{by (11)}.
\]

Terms outside the parentheses are positive because \( s < c < v' \). Thus, the sign of the above term is dictated by the terms in the parentheses. We define \( f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right) = \xi^* \). Note that \( \xi^* / \beta c \leq 0 \). The analysis of the terms in the parentheses reveals the following result:

\[
-\frac{2 f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right)}{\sqrt{(c - s)/(v' - s)}} + \frac{1}{f_D^{-1}\left( \frac{(c - s)/(v' - s)}{\beta} \right)} = -\frac{2 \xi^*}{f_D(\xi^*)} + \frac{1}{f_D(\xi^*)} = \frac{1}{f_D(\xi^*)} \left( -\frac{2 f_D(\xi^*) \xi^*}{f_D(\xi^*)} + 1 \right).
\]
Hence, the structure of the function at the potential extreme point is dictated by the following conditions:

- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} < 0$ if and only if $f_0(\xi^*) / f_0(\xi^*) > \frac{1}{2}$;
- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} > 0$ if and only if $f_0(\xi^*) / f_0(\xi^*) < \frac{1}{2}$;
- $\frac{\partial^2 \Pi_{N,s}(c^*)}{\partial c^2} = 0$ if and only if $f_0(\xi^*) / f_0(\xi^*) = \frac{1}{2}$.

Recall that the demand in our model has an increasing generalized failure rate property. Then, when we move from a potential extreme point to a higher potential extreme point, $\xi^*$ must decrease because $\xi^*$ decreases with $c^*$. This property eliminates the possibility of more than one combination of local maximum and local minimum in $(s,v')$. We know that $\Pi_{N,s}(0) = 0$, $\Pi_{N,s}(v') = 0$, and $\Pi_{N,s}(c) > 0$ in $(s,v')$. Note that $\frac{\partial \Pi_{N,s}(s)}{\partial c}$ and $\frac{\partial \Pi_{N,s}(s)}{\partial c^2}$ are undefined so the function might be tangent at $s$ because we know that the function is continuous and differentiable in $(s,v')$. Because $\lim_{c\to c^*} \frac{\partial^2 \Pi_{N,s}(c)}{\partial c^2} < 0$, the function can only be increasing concave after $s$. Having increasing concave structure implies that the first extreme point in $(s,v')$ can either be an inflection point or a local maximum.

It is easy to show that the first extreme point cannot be an inflection point by contradiction. Having inflection point first as an extreme point would imply that there exists no local maximum because $\xi^*$ cannot attain values larger than $1/2$ anymore. This contradicts with the fact that function returns back to 0 at $v'$. Thus, the first extreme point must be a local maximum, which we denote as $c^*$. Because there is no possibility of more than one combination of local maximum and local minimum in $(s,v')$, the next possible set of extreme points after $c^*$ is a set of inflection points plus a local minimum point. In fact, it can be immediately shown that there exists at least one inflection point in $(c^*,v')$ by the mean value theorem. Hence, the unique local minimum is $v'$ since $\frac{\partial \Pi_{N,s}(v')}{\partial c} = 0$ and $\frac{\partial^2 \Pi_{N,s}(v')}{\partial c^2} > 0$.

We have shown that $\Pi_{N,s}(c^*)$ reaches a global maximum at $c^*$, has at least one inflection point in $(c^*,v')$, and reaches a global minimum at $v'$. Therefore,

1. if $c_l > c_l \geq c^*$, then $\Pi_{N,s}(c_l) \geq \Pi_{N,s}(c^*)$ (Region D);
2. if $c_l < c_l \leq c^*$, then $\Pi_{N,s}(c_l) \geq \Pi_{N,s}(c^*)$ (Region B);
3. if $c_l < c^* < c_H < c_l = c_{equal}$, then $\Pi_{N,s}(c_l) \leq \Pi_{N,s}(c_H)$ (Region C),
4. if $c_l < c^* < c_{equal} < c_H$, then $\Pi_{N,s}(c_l) \geq \Pi_{N,s}(c_H)$ (Region C),

where $\Pi_{N,s}(c_l) = \Pi_{N,s}(c_H)$ when $c_H = c_{equal}$.

References


