1 Product-Specific Prices

Figures 1–4 illustrate the results for the model with product-specific prices as detailed in Appendix B of the paper. They are analogous to Figures 2-5 in the paper. As can be seen, product-specific prices lead to minor changes in our results.

The by far most marked change is in the bottom left panel of Figure 4. Unlike the bottom left panel of Figure 5 in the paper, market segmentation occurs via niche firms, but no longer via full-line firms. To see what is going on, recall that the set of equilibria remains the same for a sufficiently large fixed cost $f$ because offering both products eventually becomes a dominated strategy. This critical value of $f$ turns out to be lower in the model with product-specific prices. Consequently, the bottom left of Figure 4 ($\beta = 4$ and $f = 0.3$) is the same as the bottom right panel ($\beta = 4$ and $f \geq 0.4$).
2 Distribution of Brand Preferences

To explore the robustness of our results to the functional form of the distribution of brand preferences, we have computed the NE of the pricing subgame for alternative distributions:

- Normal distribution:
  \[ F(\epsilon) = \Phi(\epsilon, 0, \sigma), \]
  where \( \Phi \) is the normal cdf with mean zero and variance \( \sigma^2 \). \( \sigma \) is chosen to equate the variance of the normal and logistic distributions, i.e.,
  \[ \sigma^2 = \frac{\pi^2 \beta^2}{3} \Rightarrow \sigma = \frac{\pi \beta}{\sqrt{3}}. \]

- Double exponential (Laplace) distribution:
  \[ F(\epsilon) = \begin{cases} 
  \frac{1}{2} e^{\frac{\epsilon}{\gamma}} & \text{if } \epsilon < 0, \\
  1 - \frac{1}{2} e^{-\frac{\epsilon}{\gamma}} & \text{if } \epsilon \geq 0. 
\end{cases} \]
  \( \gamma \) is chosen to equate the variance of the double exponential and logistic distributions, i.e.,
  \[ 2\gamma^2 = \frac{\pi^2 \beta^2}{3} \Rightarrow \gamma = \frac{\pi \beta}{\sqrt{6}}. \]

Figures 5 and 6 illustrate the results; they are analogous to Figure 1 in the paper. As can be seen, the shape of the profit function \( \pi^*_1(\Delta_a, \Delta_b) \) is preserved, especially in the case of a normal distribution (Figure 5). In case of a double exponential distribution (Figure 6), the profit function inherits the kink of the underlying pdf and the region of \((\Delta_a, \Delta_b)\)-space in which a pure-strategy NE exists is smaller. Nevertheless, the shape is similar to the case of a logistic distribution. Our conclusions regarding firms’ market segmentation strategies therefore remain unaltered.
Figure 1: Pareto-undominated equilibria in the model with product-specific prices for $\beta = 0.5$, $f \in \{0, 0.05\}$, and $f \geq 0.1$. $(GP, GP)$ is denoted by a blue star, $(A, B)$ (or $(B, A)$) by a red x-mark, and $(AB, AB)$ by a green plus.
Figure 2: Pareto-undominated equilibria in the model with product-specific prices for $\beta = 1$, $f \in \{0, 0.05\}$, and $f \geq 0.1$. $(GP, GP)$ is denoted by a blue star, $(A, B)$ (or $(B, A)$) by a red x-mark, and $(AB, AB)$ by a green plus.
Figure 3: Pareto-undominated equilibria in the model with product-specific prices for \( \beta = 2, f \in \{0, 0.05, 0.1\} \), and \( f \geq 0.2 \). \((GP, GP)\) is denoted by a blue star, \((A, B)\) (or \((B, A)\)) by a red x-mark, and \((AB, AB)\) by a green plus.
Figure 4: Pareto-undominated equilibria in the model with product-specific prices for \( \beta = 0.5, f \in \{0, 0.05, 0.1, 0.2, 0.3\} \), and \( f \geq 0.4 \). \((GP, GP)\) is denoted by a blue star, \((A, B)\) (or \((B, A)\)) by a red x-mark, and \((AB, AB)\) by a green plus.
Figure 5: Equilibrium market shares of firm 1 in segments $a$ and $b$, $F(\Delta_a + p_2(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$ and $F(\Delta_b + p_2^*(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$, equilibrium price of firm 1, $p_1^*(\Delta_a, \Delta_b)$, and equilibrium profit of firm 1, $\pi_1^*(\Delta_a, \Delta_b)$, for normal distribution with $\sigma = \frac{\pi}{\sqrt{3}}$ and $c = 0$. 
Figure 6: Equilibrium market shares of firm 1 in segments $a$ and $b$, $F(\Delta_a + p_2^*(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$ and $F(\Delta_b + p_2^*(\Delta_a, \Delta_b) - p_1^*(\Delta_a, \Delta_b))$, equilibrium price of firm $1$, $p_1^*(\Delta_a, \Delta_b)$, and equilibrium profit of firm $1$, $\pi_1^*(\Delta_a, \Delta_b)$, for double exponential distribution with $\gamma = \frac{\pi}{\sqrt{6}}$ and $c = 0$. 