

A Theory of Combative Advertising

Yuxin Chen¹ • Yogesh V. Joshi^{2*} • Jagmohan S. Raju² • Z. John Zhang²

¹The Leonard N. Stern School of Business, New York University, New York, NY.

(E-mail: ychen@stern.nyu.edu)

²The Wharton School, University of Pennsylvania, 3730 Walnut St, Philadelphia, PA 19104.

(E-mail: yogesh@wharton.upenn.edu, rajuj@wharton.upenn.edu, zjzhang@wharton.upenn.edu)

*Corresponding Author: Phone: 215 573 0541, Fax: 215 898 2534.

May 2006

Acknowledgement. We thank Eric Bradlow, Patti Williams, audience members at the 2005 INFORMS Marketing Science Conference session on Game Theory, seminar participants at Harvard and Yale, four anonymous reviewers, the AE and the Editor at Marketing Science for their helpful comments. ■

A THEORY OF COMBATIVE ADVERTISING

Abstract

We analyze the effects of combative advertising on market power. Combative advertising, a characteristic of mature markets, is defined as advertising that shifts consumer preferences towards the advertising firm, but does not expand the category demand. We propose a model for combative advertising where advertising changes the distribution of consumer preferences in a tug-of-war. We show that depending on the nature of consumer response, combative advertising can reduce price competition to benefit competing firms. However, it can also lead to a pro-competitive outcome where firms compete harder on price and make lower profits. Because of this pro-competitive effect, we further show that the cost of combative advertising could be a blessing in disguise – with a higher unit cost of advertising, the resulting equilibrium levels of advertising could be lower, leading to higher prices and profits. Further, we conduct a lab experiment to investigate how combative advertising by competing brands influences consumer preferences. Our analysis of the data thus generated offers strong support for our conclusions.

(Keyword: advertising; persuasion, game theory; competitive strategy; consumer behavior)

1 Introduction

In some of the earliest writings on the role of advertising in influencing firm prices and profitability, the renowned economist Alfred Marshall noted the following:

“the nation (America), which excels all others in the energy and inventive ability devoted to developing the efficiency of retail trade, is also the nation that pays the most dearly for the services of that trade.” – Marshall (1921), in ‘Industry and Trade’.

He believed that part of the reason why one observed high prices in many markets was the advertising efforts of firms; and the fact that much of this advertising spending was not ‘constructive’, but ‘combative’. He defined constructive advertising as all advertising designed to draw the attention of people to the opportunities of buying or selling, of which they may be willing to avail of themselves; and combative advertising as the iterative claims made by a firm in an effort to identify itself with consumers without expanding the market. His contention was that combative advertising helped a firm increase its market power and support a high price.

Over the past century, researchers have developed many theories to explain how advertising helps a firm enhance its market power. Three main views have emerged: the informative, complementary, and persuasive view of advertising (Bagwell 2005). As per the informative view, advertising works by increasing consumer awareness and reducing search costs. This results in an increase in price sensitivity and reduction in market power (the Stigler-Telser-Nelson school of thought). Of course, informative advertising can also enhance a firm’s market power if it merely informs consumers of product differences and hence increases product differentiation (Meurer and Stahl 1994). The complementary view recognizes the importance of advertising in the consumption process by providing additional utility to consumers – such as creating a feeling of greater social ‘prestige’ when the product is appropriately advertised, or by signalling high quality. Therefore, advertising is a ‘good’ or ‘bad’ complementary to

consumption. According to this view, advertising can increase a firm's market power if it enhances consumption utility, but need not always (Becker and Murphy 1993).

The view that is closest to capturing combative advertising described by Marshall is the persuasive view of advertising (the Kaldor-Bain-Comanor-Wilson school of thought). This view supports Marshall's thesis (Braithwaite 1928, Kaldor 1950, Comanor and Wilson 1967) and it holds that advertising alters consumer tastes, creates mostly spurious product differentiation, gives rise to entry barriers, and results in an outward shift of the demand curve (Gasmi, Laffont and Vuong 1992; Tremblay and Polasky 2002; Tremblay and Martins-Filho 2001). Ultimately, such advertising decreases consumer price sensitivity and causes demand to become less elastic, thus supporting higher prices and profits in the marketplace (Bagwell 2005).

As mature markets for consumer goods such as detergents and soft drinks are characterized by combative advertising, one would expect, according to the persuasive view, that as a firm increases its advertising expenditure, its demand will shift outward to the benefit of the advertising firm. However, research in marketing does not always corroborate with this view. While there is quite a bit of agreement that advertising in its various forms (rational, emotional, creative) helps shift consumers preferences towards the advertising firm, (Batra, Myers and Aaker, 1996), there is skepticism about advertising being of much help to firms in increasing short-term sales or profitability. Erickson (1985, 2003) suggests that when competing firms are equally-matched (symmetric), combative advertising is harmful (albeit necessary) for firms as it does not provide any additional increase in market share or profits. Tellis and Weiss (1995) further the skepticism of advertising's positive sales impact by showing empirically that the effect of advertising on current sales, based on disaggregate scanner data on detergents, is either weak or non-significant and it is present only when data are aggregated across consumers, or over time. This implies that any sales effect of advertising or any gain in market power may be spurious, most likely due to data aggregation. Other researchers have presented a more mixed view. Kaul and Wittink (1995) reviewed

14 empirical studies on the effects of non-price advertising¹ and concluded based on the majority results (9 out of 14 studies) that non-price advertising reduces price sensitivity of consumers (mostly for frequently purchased consumer goods). They report studies that suggest non-price advertising could increase price sensitivity particularly in the airline industry (Gatignon 1984) as well as for some categories of consumer goods such as aluminum foil and dog food (Kanetkar et. al. 1992). Albion and Farris (1981) and Lodish et. al. (1995) also have reported mixed evidence about the effect of advertising on sales, thus calling into question the prevalent belief that non-price (combative) advertising is primarily anti-competitive in nature.

These past findings suggest that while the existing theoretical research provides a strong support for the anti-competitive nature of combative advertising, existing empirical research offers at best a mixed support for this thesis. This raises two questions that are currently unaddressed in the literature:

- Is it possible that combative advertising, while altering consumer preferences to favor competing firms, could also lead to pro-competitive effects?
- If the answer to the previous question is yes, then through what mechanism does combative advertising lead to both pro- and anti-competitive effects?

The answers to these two related questions have many implications for our understanding of the role that advertising plays in competition, and for advertising research and practice. In this paper, we seek to provide some answers to these two questions by developing a model that incorporates the preference changing effects of combative advertising. In our model, advertising does not directly enhance a consumer's willing to pay for the advertised product, and it does not alter the advertising firm's positioning in the product space. Rather, the advertising we model is purely persuasive in nature: "each firm tries to convince consumers that what they *really* want is its particular variety" (von der Fehr and Stevik 1998). In other

¹In their nomenclature, they classify persuasive advertising as non-price advertising.

words, advertising changes the distribution of consumer ideal points. This sort of advertising is, arguably, most common in the mature markets for consumer goods, as noted previously, where mixed effects of advertising in enhancing a firm's market power are observed.

Two previous studies, von der Fehr and Stevik (1998) and Bloch and Manceau (1999), have investigated this preference transforming effect of advertising in a competitive context. von der Fehr and Stevik (1998) anchor their study in a Hotelling model where advertising by a firm moves consumers located along the Hotelling line uniformly toward the advertising firm. They conclude that in any competitive equilibrium, such advertising will not change the distribution of consumer ideal points and hence should not have any price or sales effect. They came to this conclusion by assuming that all consumers along the Hotelling line are unaffected by advertising whenever competing firms advertise at the same level. Bloch and Manceau (1999), however, suggests that persuasive advertisements may alter consumer preferences in a way to reduce or to intensify price competition. Their study is also anchored in a Hotelling model. According to this study, a firm's advertising can trigger more aggressive pricing by the competing, non-advertising firm, to the detriment of the advertising firm, if consumers already prefer the product of the advertising firm. In other words, persuasive advertising intensifies price competition only under the two conditions. First, when consumer preferences are biased in favor of one of the two competing firms' products, and the favored firm is the one that advertises. Second, the firm in consumers' disfavor cannot advertise and can only respond to the rival's advertising with price.² In our paper, we depart from both studies.

Similar to von der Fehr and Stevik (1998) and Bloch and Manceau (1999), the anchor to our analysis is the Hotelling model of a horizontally differentiated duopoly. Compared to von der Fehr and Stevik (1998), we offer a more general specification for the consumer prefer-

²Strictly speaking, Bloch and Manceau (1999) does not set up an equilibrium model for advertising decisions, and its main focus is on investigating a firm's incentives to advertise in as general a way as possible. As they point out in the paper, "at this level of generality, we are unable to analyze the firms' choice of the level of advertising expenses and cannot capture the effects of advertising spending by both firms on the distribution of consumers" (p. 565).

ence distribution and model combative advertising as causing a shift in consumer preference towards the advertising firm in a more heterogeneous way, consistent with empirical and experimental findings from the existing literature. We also incorporate the commonly observed diminishing returns to advertising in the sales response function. Compared to Bloch and Manceau (1999), we conduct an equilibrium analysis with competing firms setting both prices and the levels of advertising independently. This analysis allows us to identify the mechanism through which combative advertising may reduce or intensify price competition when all competing firms can advertise. Such a mechanism is absent in both studies. In our model, advertising always exerts a force pulling consumers closer to that firm. Surprisingly, however, the effect of such a force can be anti-competitive (all competing firms charge higher prices), or pro-competitive (all competing firms charge lower prices). We will discuss in detail how these pro- and anti-competitive effects of combative advertising come about and what mediates these different outcomes in Sections 3 and 4 after we set up the model in Section 2. Because of this pro-competitive effect, we can shed some new light on the effects of ever escalating advertising costs in Section 5: the rising costs of advertising might be a blessing in disguise. We demonstrate the possibility that with higher unit costs of advertising, the equilibrium levels of advertising could be lower, resulting in higher profits for competing firms. As all our conclusions hinge on the pro-competitive effect, we experimentally test the external validity of this pro-competitive effect in Section 6 and conclude in Section 7 with suggestions for future research.

2 A Model for Combative Advertising

We rely on the findings in the literature on advertising to develop a system of advertising response functions for a duopoly market. With these advertising response functions, we show that combative advertising can generate anti-competitive effects in equilibrium. However, the same forces that give rise to the anti-competitive effects of advertising can also, under

other conditions, generate pro-competitive effects.

Many studies show that advertising exposure has a favorable effect on consumer preferences towards the advertised product, and that the consumer response to advertising is concave (Winter 1973, Simon and Arndt 1980, Vakratsas and Ambler 1999). This concave nature of advertising response functions has been reported by Little 1979, Albion and Farris 1981, and also endorsed by Lilien, Kotler and Moorthy (1992), who note that *“while a good deal of discussion and modeling concerns S-shaped response, most of the empirical evidence supports concavity”*. Even when advertising response is S-shaped, typically the equilibrium advertising levels belong to the upper concave part of the S-curve, making the assumption of concavity more reasonable. We embed these important empirical findings in the familiar Hotelling linear city (1929) as the starting point of our analysis.

On the supply side, let two competing firms, 1 and 2 respectively, be located at the two ends of a linear city of unit length: firm 1 at $x = 0$ and firm 2 at $x = 1$. We assume, for simplicity, that both firms’ marginal costs of production are zero. Firms play a two-stage game. In the first stage, firms simultaneously decide advertising levels, k_i . We normalize k_i to be within $[0, 1]$ where $k_i = 0$ denotes no advertising and $k_i = 1$ denotes maximum advertising. We assume that advertising costs are quadratic, $\frac{1}{2}ck_i^2$ (Tirole 1988, Bagwell 2005), where c is a constant. In the second stage, after observing the other firm’s advertising level, each firm simultaneously decides its price p_i and then consumers optimally make their purchases.

On the demand side, each consumer on the Hotelling line buys at most one unit of the product and incurs a linear transportation cost for shopping at a firm. This transportation cost measures the dis-utility that a consumer suffers by consuming a non-ideal product. We denote the unit transportation cost by t . The reservation price of each consumer V is assumed to be sufficiently large compared to t so that the market is always covered in equilibrium, with or without advertising. This allows us to assume away any possible market expansion effect due to advertising in this market. Thus, a consumer located at x will consider firm 1’s

(firm 2's) product to be located at a distance x ($(1-x)$) away from her ideal product. Her utility for a product offered by firm 1 is then $(V - p_1 - tx)$ and $(V - p_2 - t(1-x))$ for the product offered by firm 2.

In the past, the effect of persuasive advertising in a Hotelling model has been modeled as increasing either the consumer reservation price V or the product differentiation parameter t , or as shifting the distribution of consumer ideal points (von der Fehr and Stevik 1998; Tremblay and Polasky 2002). We take the latter modeling approach to better capture combative advertising “as a tug-of-war in which each firm attempts to attract consumers by molding their preferences to fit the characteristics of its product” (von der Fehr and Stevik 1998). This means that the distribution of consumers along the Hotelling line depends on both firms' choices of advertising levels. We can write the consumer distribution density function as $f(k_1, k_2, x)$, with the total number of consumers in the market being normalized to 1.

It is important to note here that even though our modeling approach is similar to von der Fehr and Stevik (1998) and Bloch and Manceau (1999), we depart from these models by parameterizing the density function $f(k_1, k_2, x)$ based on the regularity conditions and the known effects of advertising to generate a tractable, but still fairly general model that lends itself to game-theoretic analysis.³ As a benchmark, we assume that in the absence of any advertising, consumers are uniformly distributed in the market, so that we have:

$$f(0, 0, x) = 1. \tag{1}$$

Consumers in this market are fully informed about the existence of the two firms, their locations, and prices; hence the effect of a firm's advertising is not to inform consumers about its existence, location, or price, but to engage in a combat with the competitor in shifting

³As briefly mentioned before, in Von der Fehr and Stevik (1998), the distribution function of consumer ideal points is uniform and if competing firms advertise at the same level, none of the consumers along the Hotelling line is affected so that the distribution function does not change. Bloch and Manceau (1999) allows for more general density functions. However, the model allows only one firm to advertise.

consumer preferences towards itself. Thus, under combative advertising, some consumers at location x may move towards firm i due to its advertising k_i . In other words, they consider the firm's product closer to their ideal product. Consequently, the distribution of consumers along the unit line will change from the uniform distribution to $f(k_1, k_2, x)$, with the mean location of consumers in this market being given by:

$$m(k_1, k_2) = \int_0^1 x f(k_1, k_2, x) dx \quad (2)$$

As a probability density function, $f(k_1, k_2, x)$ must satisfy the following regularity condition, for all k_1 and k_2 :

$$\int_0^1 f(k_1, k_2, x) dx = 1. \quad (3)$$

We further impose symmetry on the density function to assume away any firm-specific factors and to focus on the competitive effect of combative advertising. Algebraically, this means the following condition must hold:

$$f(k_1, k_2, x) = f(k_2, k_1, 1 - x). \quad (4)$$

This condition rules out the mechanism identified in Bloch and Manceau (1999) as the driving force for competitive effects. To introduce the preference-shifting effect of a firm's advertising, we assume:

$$\frac{\partial f(k_1, k_2, 0)}{\partial k_1} > 0, \frac{\partial f(k_1, k_2, 1)}{\partial k_2} > 0. \quad (5)$$

Equation (5) is the boundary conditions that ensure that at any given level of advertising by its rival, a firm can, through more of its own advertising, increase the number of customers who would consider its product as the ideal. Hence the density of consumers at $x = 0$ ($x = 1$) increases as firm 1 (firm 2) unilaterally increases its advertising intensity. We further impose

a boundary condition below to help us to parameterize the density function:

$$\frac{\partial f(k_1, 0, 1)}{\partial k_1} < 0, \frac{\partial f(0, k_2, 0)}{\partial k_2} < 0, \frac{\partial f^2(k_1, 0, 1)}{\partial^2 k_1} = \frac{\partial f^2(0, k_2, 0)}{\partial^2 k_2} = 0 \quad (6)$$

The conditions imposed in Equation (6) are essentially technical in nature and they ensure that the density of consumers at the extreme location ($x = 0$ or $x = 1$) who consider the respective firm's product as ideal will decrease with the rival's advertising level, and do so at a constant rate if the rival firm does not do any advertising at all. To scale such preference-shifting effects, we assume:

$$f(1, 0, 1) = 0, \quad f(0, 1, 0) = 0. \quad (7)$$

Equation (7) implies that the effect of a firm's maximum advertising is so strong that it can shift the preference of all consumers farthest away from it toward itself, if the other firm does not advertise at all.⁴ Finally, we assume:

$$\frac{\partial f(k_1, 0, x)}{\partial x} \leq 0, \quad \frac{\partial f(0, k_2, x)}{\partial x} \geq 0. \quad (8)$$

These two conditions simply state that if the rival firm does not advertise, a firm's advertising should skew the consumer distribution in its favor or should have favorable effects on consumer preferences. Together with (7), (8) ensures that the density of consumer distribution can never become negative.

To determine a specific density function that satisfies all conditions (1)-(8), let us assume that:

$$f(k_1, k_2, x) = v_1(k_1, x) + v_2(k_2, x), \quad (9)$$

i.e., $f(k_1, k_2, x)$ is of an additive form in terms of the impact of k_1 and k_2 on the shifts in the

⁴Note that as this assumption is only one of the many possible ways to scale the effect of advertising, we can easily relax this assumption by using any $0 \leq f(k_1, 0, 1) = f(0, k_2, 0) < 1$ without changing our main results.

distribution of preferences. While an additive form simplifies our analysis, our conclusions do not hinge on this assumption.⁵ Intuitively, v_i captures the effects of firm i 's advertising. We thus view the density at any given x , *i.e.*, $f(k_1, k_2, x)$, as resulting from two opposing advertising forces: consumers at x may be pulled away from x and at the same time consumers at other locations may be pulled to x when subject to combative advertising. We let v_1 take on the following, general quadratic form⁶ with respect to both x and k_1 :

$$v_1(k_1, x) = a_0 + a_1k_1 + a_2k_1^2 + a_3x + a_4k_1x + a_5k_1^2x + a_6x^2 + a_7k_1x^2 + a_8k_1^2x^2. \quad (10)$$

We can analogously specify $v_2(k_2, x)$. Now it is straightforward to use conditions (1)–(8) to derive the following parameterized functions:⁷

$$v_1(k_1, x) = \frac{1}{2} + k_1[(12a - 1) - 6(8a - 1)x + 6(6a - 1)x^2] - 12bk_1^2[1 - 4x + 3x^2] \quad (11)$$

$$v_2(k_2, x) = \frac{1}{2} + k_2[(12a - 1) - 6(8a - 1)(1 - x) + 6(6a - 1)(1 - x)^2] - 12bk_2^2[1 - 4(1 - x) + 3(1 - x)^2] \quad (12)$$

where $\frac{1}{8} < a < \frac{5}{12}$ and $\max\{0, a - \frac{1}{4}\} < b < \min\{a - \frac{1}{8}, \frac{a}{2} - \frac{1}{24}\}$. With these functions, we can easily write down the distribution density function under combative advertising by substituting v_1 and v_2 in Equation (9).

Note that this derived density function has only two parameters, a and b . To understand what they represent, we first derive the mean location of consumers in this market:

$$m(k_1, k_2) = \frac{1}{2} - a(k_1 - k_2) + b(k_1 - k_2)(k_1 + k_2). \quad (13)$$

⁵We can show that our main results still hold if we have an interaction term k_1k_2 in the specification of $f(k_1, k_2, x)$, *i.e.* $f(k_1, k_2, x) = v_1(k_1, x) + v_2(k_2, x) + k_1k_2v_3(k_1, k_2, x)$.

⁶ v_2 also takes on a similar form, given Equation (4).

⁷where $a = \frac{1+a_1}{12}$ and $b = -\frac{a_2}{12}$.

For additional details on the intermediate steps in this derivation, a technical appendix is available upon request from the corresponding author.

With assumptions (1)- (8), it is straightforward to show that the density function satisfies the following conditions:

$$\frac{\partial m(k_1, k_2)}{\partial k_1} < 0, \frac{\partial^2 m(k_1, k_2)}{\partial k_1^2} > 0; \text{ and } \frac{\partial m(k_1, k_2)}{\partial k_2} > 0, \frac{\partial^2 m(k_1, k_2)}{\partial k_2^2} < 0. \quad (14)$$

They imply that the average consumer location moves toward a firm as the firm increases its advertising intensity, given the rival's advertising level, but the rate of such movements diminishes as the firm's advertising level increases. In other words, a firm's increased advertising should, on average, make the firm's product closer to being the ideal product for the consumers in the market, given any level of the rival's advertising, but such preference-shifting effect diminishes as the firm does more and more advertising. This means that, holding the prices to be the same in the market, the sales response function for a firm's advertising is increasing and concave, as we set out to look for.

In Equation (13), we see clearly how these two effects come about. The term $(k_1 - k_2)$ captures the relative advertising intensity, while $(k_1 + k_2)$ captures the aggregate advertising intensity in the market. As a firm (say firm 1) increases its advertising intensity, consumers prefer firm 1's product so that the mean consumer location shifts towards firm 1 by the distance of a per unit of advertising intensity. This is the first order effect of advertising in shifting consumer preference. However, this mean shift is tempered by the second order effect of the diminishing returns to advertising efforts, captured in the term $b(k_1 + k_2)$. This second order effect depends on the aggregate advertising intensity in the marketplace as expected. Thus, a larger a means that the consumers are, on average, more responsive to advertising, and a larger b means that the consumers are more easily satiated with advertising appeals. If the two competing firms have the same advertising level, then the mean location of consumers, or the mean consumer preference, does not change and is fixed at $\frac{1}{2}$. This is rather expected in the context of combative advertising when the two competing firms are equally matched.

In this market, two competing firms engage in a tug-of-war in advertising. If only one firm has the ability to advertise, that firm will pull consumers over, gaining on the market share as pointed out in Bloch and Manceau (1999). If both firms advertise to the same extent and yet the distribution of consumer preferences is not altered as a result, the tug-of-war is at a dead draw everywhere along the Hotelling line. In that case, as pointed out by von der Fehr and Stevik (1998), combative advertising does not generate any price effect at all. Of course, when consumers in the market are all subject to two opposing forces, it is perhaps unrealistic to expect that the distribution of consumer preferences does not change at all. With this more general and empirically grounded consumer preference distribution function, we can conduct an explicit equilibrium analysis to investigate how the consumer distribution and hence price competition may change due to combative advertising.

3 Equilibrium Analysis

Let d be the location of marginal consumer who is indifferent between the two firms: $d = \frac{t-p_1+p_2}{2t}$. The demand functions for the two firms are: $D_1 = \int_0^d f dx = F(d)$; and $D_2 = \int_d^1 f dx = 1 - F(d)$. Given demand,⁸ the profit functions for the two firms are:

$$\pi_1 = p_1 D_1 - \frac{1}{2} c k_1^2; \text{ and } \pi_2 = p_2 D_2 - \frac{1}{2} c k_2^2 \quad (15)$$

We now proceed to derive the equilibrium using backwards induction.

In the second stage, both firms independently set their prices for any given pair of advertising levels (k_1, k_2) chosen in the previous stage. In equilibrium, both firms' prices must satisfy the following two first order conditions:

$$\frac{\partial \pi_1(p_1, p_2, k_1, k_2)}{\partial p_1} = 0; \frac{\partial \pi_2(p_1, p_2, k_1, k_2)}{\partial p_2} = 0 \quad (16)$$

⁸One can easily verify that the sales response to advertising in this market is concave for both firms.

As we show in Appendix 1, although the equilibrium prices $p_1^*(k_1, k_2)$ and $p_2^*(k_1, k_2)$ cannot be solved explicitly without any further simplifying assumptions, they exist and are unique. In other words, the second stage equilibrium always exists and is unique for any pair of (k_1, k_2) . When $k_1 = k_2$, we can write out the second stage price equilibrium explicitly below:

$$p^*|_{k_1=k_2=k} = \frac{t}{1+k-6ak+6bk^2}, \quad \pi^*|_{k_1=k_2=k} = \frac{t}{2(1+k-6ak+6bk^2)} - \frac{1}{2}ck^2. \quad (17)$$

The above expressions reveal what happens to price competition in this market when two competing firms are equally matched in advertising. Note that for $k = 0$, the above solutions reduce to the familiar Hotelling (1929) results as expected. Surprisingly, however, the relationship between advertising and price is not monotonic. Even more surprisingly, the market price may be lower because of combative advertising. In Figure 1,⁹ we illustrate the optimal price in the second stage as a function of advertising.

Proposition 1 *When competing firms are equally matched in advertising, equilibrium prices can be increasing or decreasing with the level of advertising in the market, dependent on the level of advertising ($\frac{\partial p^*}{\partial k} > 0$ when $k < \frac{6a-1}{12b}$). More importantly, combative advertising can intensify or moderate price competition in a market, relative to the case of no advertising.*

Given that empirical evidence in the literature largely suggests that manufacturer prices are increasing in advertising (Steiner 1973, Farris and Reibstein 1979, Tellis 2004, Reibstein, Joshi and Farris 2004), the anti-competitive effect of advertising in Proposition 1 is perhaps not so surprising. However, the pro-competitive effect in Proposition 1 is. At a high level of advertising ($k > \frac{6a-1}{12b}$), an increase in advertising would lead to a decrease, rather than an increase, in equilibrium prices. Indeed, if the advertising level is sufficiently high ($k > \frac{6a-1}{6b}$), the market price with advertising falls below that without any advertising. How does the pro-competitive effect come about? Equation (17) offers some clues. We can see that the equilibrium price increases as the advertising response becomes steeper (a increases), and

⁹The graph is plotted for sample values by setting $t = 1$, $a = 0.2$, $b = 0.055$.

decreases as the rate of diminishing returns becomes stronger (b increases), in addition to responding to the level of advertising. In other words, the shape of the advertising response function can have significant competitive implications.

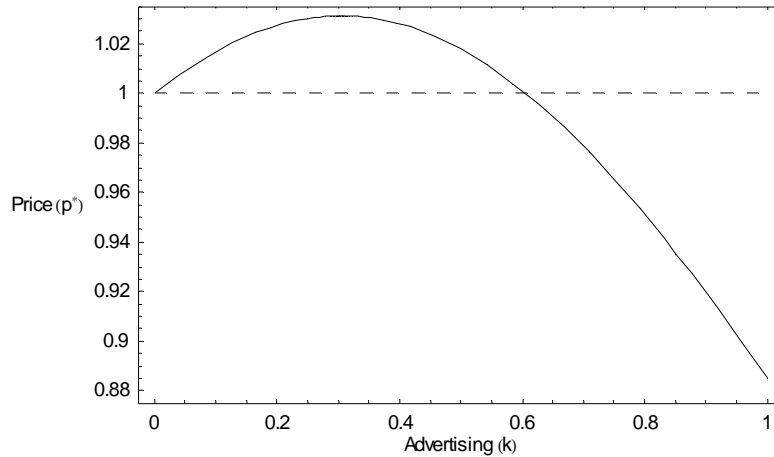


Figure 1: Effect of Combative Advertising on Equilibrium Price

Before we further discuss how a higher level of advertising in the market may lead to a lower price, we first show that competing firms may indeed optimally choose a level of advertising that would lead to a lower market price. We can do this by solving for the first stage equilibrium.

4 Competitive Effects of Combative Advertising

In the first stage, both firms set their advertising level, anticipating how their choice may affect their subsequent pricing decisions. Then, in the first stage equilibrium, the following two first order conditions must be satisfied:

$$\begin{aligned}
\frac{\partial \pi_1(p_1^*(k_1, k_2), p_2^*(k_1, k_2), k_1, k_2)}{\partial k_1} &= \frac{\partial \pi_1}{\partial k_1} + \frac{\partial \pi_1}{\partial p_1} \frac{\partial p_1^*}{\partial k_1} + \frac{\partial \pi_1}{\partial p_2} \frac{\partial p_2^*}{\partial k_1} = 0, \\
\frac{\partial \pi_2(p_1^*(k_1, k_2), p_2^*(k_1, k_2), k_1, k_2)}{\partial k_2} &= \frac{\partial \pi_2}{\partial k_2} + \frac{\partial \pi_2}{\partial p_1} \frac{\partial p_1^*}{\partial k_2} + \frac{\partial \pi_2}{\partial p_2} \frac{\partial p_2^*}{\partial k_2} = 0.
\end{aligned} \tag{18}$$

It is fairly tricky to solve for the equilibrium advertising levels based on the implicit expressions of $p_1^*(k_1, k_2)$ and $p_2^*(k_1, k_2)$ as defined by Equation (16). We first note that in the second stage equilibrium, we must have for any $k_i \in [0, 1]$ where $i = 1, 2$:

$$\frac{\partial \pi_1(p_1^*, p_2^*, k_1, k_2)}{\partial p_1} = 0, \quad \frac{\partial \pi_2(p_1^*, p_2^*, k_1, k_2)}{\partial p_2} = 0 \tag{19}$$

By differentiating these two identities, we have:

$$\begin{aligned}
\frac{\partial^2 \pi_1}{\partial p_1 \partial k_1} + \frac{\partial \pi_1^2}{\partial p_1^2} \frac{\partial p_1^*}{\partial k_1} + \frac{\partial \pi_1^2}{\partial p_1 \partial p_2} \frac{\partial p_2^*}{\partial k_1} &= 0, \\
\frac{\partial^2 \pi_1}{\partial p_1 \partial k_2} + \frac{\partial \pi_1^2}{\partial p_1^2} \frac{\partial p_1^*}{\partial k_2} + \frac{\partial \pi_1^2}{\partial p_1 \partial p_2} \frac{\partial p_2^*}{\partial k_2} &= 0, \\
\frac{\partial^2 \pi_2}{\partial p_2 \partial k_1} + \frac{\partial \pi_2^2}{\partial p_1 \partial p_2} \frac{\partial p_1^*}{\partial k_1} + \frac{\partial \pi_2^2}{\partial p_2^2} \frac{\partial p_2^*}{\partial k_1} &= 0, \\
\frac{\partial^2 \pi_2}{\partial p_2 \partial k_2} + \frac{\partial \pi_2^2}{\partial p_1 \partial p_2} \frac{\partial p_1^*}{\partial k_2} + \frac{\partial \pi_2^2}{\partial p_2^2} \frac{\partial p_2^*}{\partial k_2} &= 0.
\end{aligned} \tag{20}$$

To obtain the symmetric equilibrium, we note that all second order derivatives in the above system of equations, *i.e.* $\frac{\partial^2 \pi_i}{\partial p_i \partial k_i}$, $\frac{\partial^2 \pi_i}{\partial p_i \partial k_j}$, $\frac{\partial \pi_i^2}{\partial p_i^2}$, and $\frac{\partial \pi_i^2}{\partial p_i \partial p_j}$, as well as the first order derivatives in Equation (18), *i.e.* $\frac{\partial \pi_i}{\partial p_i}$ and $\frac{\partial \pi_i}{\partial p_j}$, can be obtained directly from differentiating the profit expressions in Equation (15) and then substituting in $p^*|_{k_1=k_2=k}$ from Equation (17). This means that we can solve $\frac{\partial p_i^*}{\partial k_i}$ and $\frac{\partial p_j^*}{\partial k_j}$ from Equation (20) and substitute them into Equation (18). By imposing the symmetry condition $k_1 = k_2 = k$ in the first stage, the two independent first order conditions in Equation (18) will be reduced to one, leading to a unique symmetric solution for k^* , the candidate equilibrium we are looking for. In the case of $c = 0$, we obtain a closed form solution, which is given in Appendix 2. In Figure 2, we illustrate the parameter

space where we have $k^* \in [0, 1]$ for the case of $c = 0$. In the case of $c > 0$, we solve numerically for the equilibrium for all feasible parameters. In both cases, we numerically verify that the proposed equilibrium is indeed the equilibrium of the game, as neither firm has any incentive to deviate. For now, we will focus on the first case, as this is the simpler case that would give us all the qualitative conclusions. We will pick up the second case in Section 6.

Our analysis shows, as expected, that a firm's optimal advertising level k^* is increasing in a . This suggests that as consumers are more responsive to firms' advertising, firms will optimally choose to increase their advertising level. Also as expected, k^* is decreasing in b , suggesting that if the advertising response exhibits higher diminishing returns, firms will optimally choose to do less advertising. With this equilibrium value of k^* , we now assess the consequences of combative advertising on equilibrium pricing and profitability. The comparison of profits and prices with and without combative advertising is indicated in Figure 2. Firms' prices and profits fall in the vertically shaded region (Region 1), and they rise in the horizontally shaded region (Region 2).

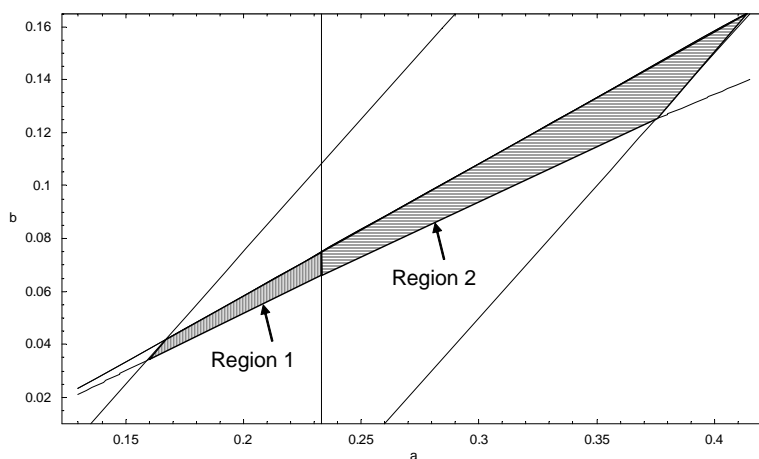


Figure 2: Combative Advertising and Equilibrium Profitability

Proposition 2 *Under combative advertising, competing firms' prices and profits are higher as compared to the case of no advertising if consumers in the market are sufficiently responsive to advertising (Region 2). However, prices and profits are lower when consumers are not sufficiently responsive (Region 1).*

Proposition 2, based on the first order effect, suggests that whether a firm can benefit from combative advertising will depend on whether consumers in the market are sufficiently impressionable so that a firm’s product becomes more ideal to them if the firm does more advertising. How does consumer responsiveness to advertising determine whether or not competing firms benefit from combative advertising? To better address that question, we need to investigate how combative advertising changes the distribution of consumer preferences in a market.

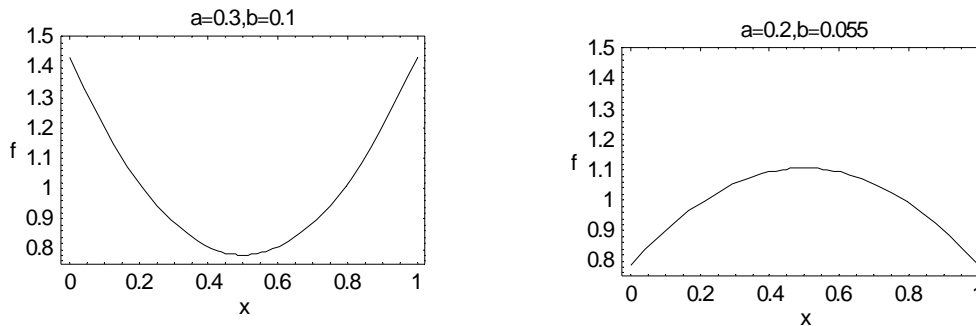


Figure 3: Effect of Combative Advertising on Distribution of Consumer Preferences

In Figure 3, we illustrate two different outcomes in the consumer preference distribution due to combative advertising. These outcomes depend on the responsiveness of consumers to advertising, as well as the extent of advertising done by firms in equilibrium. In the first case, the figure on the left, consumers are very responsive to advertising, and they are pulled strongly towards the firm that does the advertising. In this case, both firms pursue a high level of advertising. The net effect of high levels of combative advertising in this responsive market is to create “partisan consumers,” or what Braithwaite (1928) termed as “reputation monopolies,” leading to an accumulation of consumers near firms. As a result, the distribution density function f is convex in equilibrium, indicative of increased product differentiation in the minds of consumers. Increased product differentiation, in turn, leads to higher prices and profits. This conclusion is consistent with the traditional school of thought that combative advertising has an important anti-competitive effect (Braithwaite 1928, Kaldor 1950, Comanor and Wilson 1967, Krishnamurthi and Raj 1985, Pedrick and

Zufryden 1991, Bagwell 2005). In that regard, our model provides a distinct rationale and mechanism for this anti-competitive effect.

However, combative advertising is not always anti-competitive. As illustrated by the graph on the right in Figure 3, when consumers are not very responsive to advertising or the value of a is low, in equilibrium, firms spend less on advertising. In this case, consumers do not receive enough dosage of advertising to become partisan, and combative advertising simply generates more “indifferent customers.” This is manifested by a net accumulation of consumers in the middle of the market and fewer consumers located closer to firms as compared to a market without advertising. As a result, the distribution density function f in equilibrium is concave, implying reduced differentiation, hence lower prices and profits for firms competing in such a market.

While the profit implications of Proposition 2 are not surprising given price implications, the price implications themselves are surprising. Nevertheless, Proposition 2 seems consistent with what we frequently observe in the marketplace. As competing firms combat, engaging in extensive advertising campaigns touting the virtues of their own product, one of two things can happen. In the first instance, consumers are bombarded with so much advertising and become so convinced by their respective firms that they become partisan customers to the firm. In that case, it would take a lot of convincing for these consumers to consider purchasing a product from a rival firm.¹⁰ In the second instance, consumers receive a good dosage of advertising from both firms and they see merits in both firms’ products such that they feel more indifferent about buying from either firm, rather than partisan about buying from a specific firm. It is this latter possibility, the creation of indifferent customers, that the previous literature on advertising overlooks. Our model shows that the critical moderating factor for these two outcomes is how responsive consumers are to advertising in a market.

It is important to note here that the concave distribution function of consumer prefer-

¹⁰The case that best illustrates this possibility is, perhaps, from the political marketplace where competing candidates use media-based political campaigns to garner more votes. By and large, such combative political persuasion “mainly reinforces voters’ preexisting partisan loyalties” (Iyengar and Simon 2000).

ences in Figure 3 is an endogenous outcome of the equilibrium strategies and not assumed a priori. Given $a = 0.2$ and $b = 0.055$, the distribution function is concave only when the advertising intensity in the market is larger than 0.606061, as can be inferred from equation (17). When the advertising intensity is less than that cutoff point, the distribution of consumer preferences in the market is convex, as shown in Figure 4. However, competing firms are incentivized in this market to escalate their advertising intensities above and beyond that cutoff point and the equilibrium level of advertising intensity for both firms is 0.946128, resulting in the concave distribution function in Figure 3.

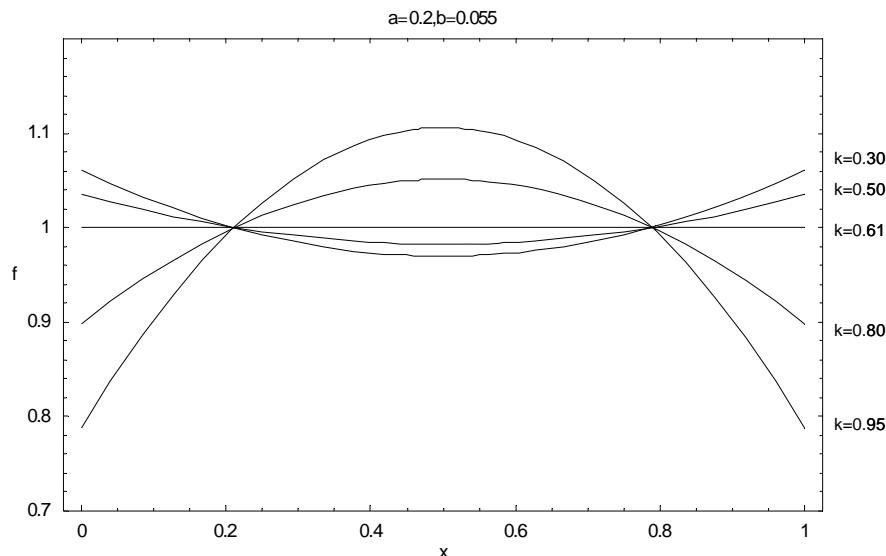


Figure 4: Variation in preference distribution with advertising.

5 The Role of Advertising Costs

In their study of informative advertising, Grossman and Shapiro (1984) found that an increase in c could have two effects: a direct increase in total advertising costs resulting in a reduction in profits; and also a reduction in the degree of competition as measured by demand elasticities, which results in higher profits. Their explanation was that advertising improves information available to consumers and reduces profits – with increased costs, the amount of advertising reduces, resulting in a reduction in the information available to consumers, as

a result allowing firms to charge higher prices.

In our analysis of combative advertising, an increase or decrease in advertising does not have any effect on the amount of information available to consumers. Hence, the above reasoning is not applicable to markets where consumers are already informed about the available products. Do the same relationships between costs and firms' prices and profits hold in the context of combative advertising?

To answer this question, we note that for any $c > 0$, we can no longer write down an explicit expression for k^* as a function of c ; however, we can numerically analyze this case. We find, as expected, that the equilibrium level of advertising decreases as the unit cost of advertising increases ($\frac{\partial k^*}{\partial c} < 0$). But what does this mean in terms of equilibrium profit levels? Note that from Equation (15), a firm's equilibrium profit can be written as $\pi^* = \pi(k^*(c), c)$. Therefore, we have:

$$\frac{d\pi^*}{dc} = \frac{\partial\pi(k^*(c), c)}{\partial k^*} \frac{\partial k^*(c)}{\partial c} + \frac{\partial\pi(k^*(c), c)}{\partial c} \quad (21)$$

The second term in Equation (21) is the direct effect of the advertising cost on a firm's profitability, which is always negative. The first term captures the indirect effect of advertising cost on a firm's profitability through advertising. As mentioned before, $\frac{\partial k^*(c)}{\partial c}$ in the first term is negative. Furthermore, it can be shown that $\frac{\partial\pi(k^*(c), c)}{\partial k^*}$ is also negative. This is because firms always advertise too much to maximize their profits, a feature of combative advertising. Thus, the first term, or the indirect effect, is always positive. This implies that a firm's profitability must decrease (increase) with the costs of advertising c , i.e. $\frac{d\pi^*}{dc} < 0$ ($\frac{d\pi^*}{dc} > 0$), if the direct effect is larger (smaller) than the indirect effect. The relative sizes of these two effects depend, in turn, on consumer responsiveness to advertising. The direct effect is larger (smaller) than the indirect effect, as we show in Figure 5, when consumers in the market are sufficiently responsive (unresponsive) so that firms are already (not) engaging in a high level of advertising.

Proposition 3 *With combative advertising, as the unit cost of advertising increases, equi-*

librium profit decreases when consumers are sufficiently responsive to advertising, and equilibrium profit increases when consumers are not sufficiently responsive to advertising.

Proposition 3 thus suggests that the intuition we have gained about the effect of advertising costs in the context of informative advertising does not always carry over to combative advertising. Higher advertising costs can indeed hurt the bottom line of competing firms, as much anecdotal evidence seems to suggest. In fact, as shown in Figure 5, firms that tend to get hurt by higher advertising cost are the firms in markets where the consumers are sufficiently responsive to advertising; and the firms that tend to benefit from higher advertising costs are the firms in markets where consumers are sufficiently unresponsive to advertising. This implication of our model is testable with suitable data.¹¹

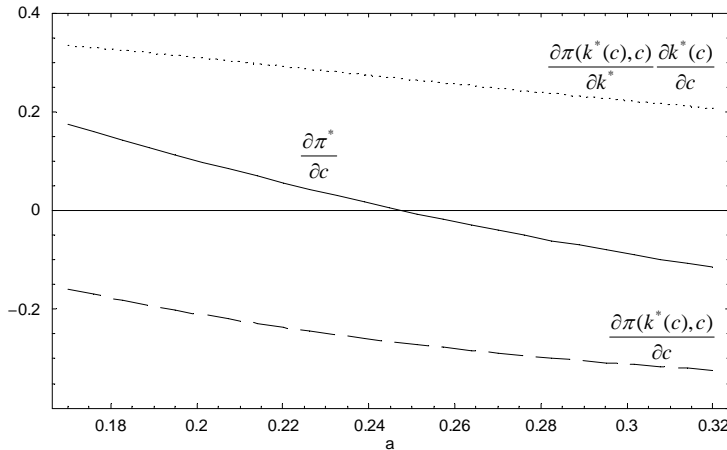


Figure 5: Effect of advertising costs

¹¹Understanding how the variation in profits with costs relates to variation in prices with costs could further facilitate conducting an empirical test. Since our modeling assumptions imply a fixed size market and symmetric market share, expected variation in profits would be positively correlated with the expected variation in prices. Thus, we expect that as the unit cost of advertising increases, equilibrium prices would decrease if consumers are sufficiently responsive to advertising, and equilibrium prices would increase if consumers were not sufficiently responsive to advertising. Since changes in prices and costs are relatively easier to monitor in a market than changes in profits and costs, these pricing implications of our cost analysis would be amenable to empirical testing in future research.

6 Effect of Combative Advertising on Consumer Preferences: Empirical Analysis

Our analysis so far has demonstrated that if combative advertising results in more indifferent consumers, it can result in lower prices and profits. While it is well documented that combative advertising can influence consumer preferences in favor of the advertising firms, to the best of our knowledge, there are no studies that demonstrate that combative advertising can result in more indifferent consumers. We therefore conducted experiments to investigate the effects of combative advertising on consumer preferences.¹² The objective of our empirical study is not to test the validity of our model's predictions but to merely demonstrate that combative advertising has the potential to result in more indifferent consumers, a key driver behind our pricing implications.

6.1 Description of the Study

We limited our attention to product categories with two equally matched competitors actively engaged in combative advertising to sway consumer preferences. While many product categories can fit this description, our choice was further limited by the availability of current TV commercials, and also whether these product categories were relevant to our experimental subjects – undergraduate students at a major east coast business school. The product categories and the brands chosen were as follows:

1. Credit Cards: Visa and Mastercard
2. Courier Services: Fedex and UPS
3. Batteries: Duracell and Energizer
4. Toothpastes: Colgate and Aquafresh
5. Cars: BMW and Audi

The study was conducted in a state of the art behavioral lab. 164 subjects participated

¹²We thank two anonymous reviewers for encouraging us to conduct these experiments.

in the study. The subject entered the lab, was greeted by the lab administrator, and was asked to fill out a questionnaire to report their preferences for each of the two brands in the five product categories. This was done by allocating 100 points between each brand pairs. The responses were collected and the subject was then exposed to advertising stimuli of the two brands in each of the five categories. The order in which the advertisements were shown was rotated across categories and also within each category.¹³ After viewing the TV commercials for a product category, the subjects were asked to report their preferences for the two brands again by allocating 100 points to the two brands – more to the one they preferred. The process was repeated for all five product categories.

6.2 Analysis and Results

The summary statistics are reported in Table 1. To analyze the effect of combative advertising on consumer preferences, we grouped subjects into two groups based on their reported preferences prior to being exposed to the stimuli. For example, those who assigned more than 50 points to Fedex are put in Group 1 and those who assigned less than 50 points to Fedex were put in Group 2. Those assigning exactly 50 points were randomly assigned to the two groups.¹⁴ We then computed the pre- and post-advertising preferences for each of these consumers. If advertising generates more indifferent consumers, we expect the conditional means of both these groups to move closer to 50. If advertising generates more partisan consumers, we expect the conditional means to move away from 50. The results are reported in Table 2.

INSERT TABLE 1 ABOUT HERE

Results in Table 2 suggest that for three of the five product categories (courier services, toothpaste and cars), the conditional means move closer to 50 and this change is significant at $p=0.05$. For credit cards, it is also in the same direction, but the results are not statistically

¹³The stimuli used in the study are available from the corresponding author's website.

¹⁴The reader may recognize that this is consistent with assigning the consumers to a Hotelling line.

significant ($p=0.10$). For the fifth product category (batteries), the conditional means move away from 50, implying more partisan consumers ($p=0.10$). The results do not depend on whether the two groups were formed based on mean or median.

One could argue that the movement of conditional means towards 50 could be because of regression to the mean (RTM). As we show in Appendix 3, if experimental subjects tend to give a small number after giving a large number or *vice versa*, the conditional means as we have calculated will move closer to the indifference point, regardless of whether advertising has any effect. In order to rule this out as a possible explanation, we conducted two additional statistical tests.

INSERT TABLE 2 HERE

The first test we conduct is an F-test on the variance of pre- and post-advertising data. The basic idea is as follows: the variance in the data consists of two parts – the true variance in the data (σ_B or σ_A) and the error variance (σ_0) that causes the RTM drift (Morrison 1973). As the latter variance should be the same for both the pre- and post- advertising data, it is easy to see that any significant difference in the measured total variance between pre- ($\sigma_B + \sigma_0$) and post-advertising data series ($\sigma_A + \sigma_0$) must imply that σ_B and σ_A are significantly different. The difference must be due to advertising exposures, not due to any RTM effect. However, when the difference between ($\sigma_B + \sigma_0$) and ($\sigma_A + \sigma_0$) is not significant by the F-test, the difference between σ_B and σ_A can still be significant. In other words, the F-test here is an overly restrictive test for our purposes. Table 3 shows that in the categories of cars and credit cards, subjects become significantly more indifferent due to advertising exposures, as the post-advertising variance is significantly smaller than the pre-advertising variance (at the 5% level). In the toothpaste category, subjects become more indifferent at the 10% significance level. In the battery category, the variance moves in the direction that would indicate more partisan customers, however, the difference is not statistically significant.

INSERT TABLE 3 HERE

Another test that incorporates a clearer indifference point is the distance dispersion test. With this test, we can also statistically eliminate the preference bias in favor of a brand and control for the possible differences in commercials' potency. To do this test, we first compute the mean for both the pre- and post-advertising preferences. Then, we subtract the respective mean from each pre- and post-advertising observations to de-mean the two data-sets. In other words, we statistically center both the pre- and post-advertising preference distributions at zero. Then, for each subject i , we compute the distance of his or her preference from zero in absolute value, respectively for both pre- and post-advertising cases. The difference in the pre- and the post-advertising distances then measures the change in a subject's preference: the subject's preference moves closer to the center when the difference is positive and further away from the center when the difference is negative. We can compute such a difference measure for all experimental subjects and find the mean difference for the population. As we show in Appendix 4, this procedure should eliminate the RTM effect. We expect that the mean difference for the population is not zero due to the effect of the advertisements. Table 4 reports the results of this analysis.

INSERT TABLE 4 HERE

Consistent with the analysis contained in Tables 2 and 3, Table 4 shows that the dispersion in preferences for the two product categories of credit cards and cars shrinks even after controlling for any mean shift. The shrinkage is strongly significant statistically, implying that subjects become more indifferent between two brands in those two product categories after the ad exposure. For toothpaste, the change in subjects' preferences is in the same direction of indifference, but it is statistically significant only at the 10% level as in the F-test. However, with this test, the dispersion of subjects' preferences increases for the battery category with a strong statistical significance. Overall, this test once again confirms that combative advertisements can produce indifferent customers.

6.3 Pricing Implications

Although preferences may change due to advertisements in a statistically significant way, one may still question whether the change is economically significant so as to modify a brand’s pricing behavior as predicted by our model.¹⁵ We address this question in the context of our experiment by fitting continuous cumulative distribution functions to the preference data within a product category. We estimate two such functions for each category, using respectively the pre- and post-advertising data. Of course, we expect these functions to be highly non-linear. To accommodate non-linearity, we specify each function as a Taylor series to the fifth power to achieve the proper goodness-of-fit.¹⁶ We subsequently assume a common reservation price as in the Hotelling model and set the unit transportation cost t (a scaling factor) to 1, and marginal costs for both brands to zero. This allows us to numerically solve for the equilibrium in the pre- and post-advertising pricing games, and see if the post-advertising price competition intensifies in a product category, as predicted by our model.

As illustrated in Figure 6, the pre-advertising price equilibrium for the category of cars is 0.68 for *BMW* and 0.48 for *Audi* and the post-advertising equilibrium prices are respectively 0.52 and 0.41. In other words, price competition intensifies in the post-advertising market, because combative advertisements have generated more indifferent customers.

Correspondingly, the equilibrium profit for BMW decreases from 0.3956 to 0.2924 and for Audi from 0.2007 to 0.1795. In Table 4, we report the equilibrium analysis for the rest of the product categories. Note that the post-advertising equilibrium prices go down for two additional categories – credit cards and toothpaste, and they increase for the batteries category, consistent with the preference shifts in Table 3. Interestingly, even if equilibrium prices decrease (increase) for both competing brands, not every brand is worse off (better

¹⁵We thank an anonymous reviewer for suggesting this analysis.

¹⁶We need to fit a continuous *cdf* to guarantee the existence of price equilibrium. Otherwise, a brand’s pricing strategy is continuous, but its demand is not, such that a pricing equilibrium may not exist. For all ten estimated functions, the minimum R^2 we get is 0.81 and the highest 0.96, with the average being 0.92.

off), as shown in the case of Mastercard (Energizer), because of preference-induced market share changes.¹⁷

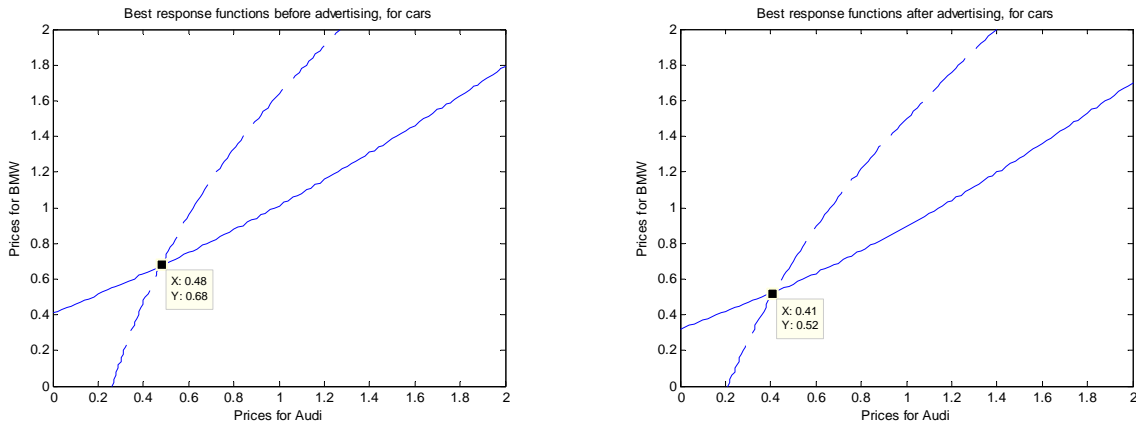


Figure 6a: Pre-Advertising Price Equilibrium Figure 6b: Post-Advertising Price Equilibrium

INSERT TABLE 5 ABOUT HERE

Overall, our experimental analysis demonstrates that combative advertisements can indeed generate indifferent customers, and lead to intensified price competition.

7 Conclusions and Future Research

Our objective in this research is to examine the competitive implications of combative advertising. We show that although from an individual firm's perspective, combative advertising can change consumer preferences in its favor, through competitive interactions in the marketplace, such advertising may not favor the firm. We show that combative advertising can generate either more partisan customers or more indifferent customers. In the

¹⁷While our analysis in this section confirms the possibility of intensified/reduced price competition as a consequence of combative advertising, it would be worthwhile to empirically observe this intensified/reduced price competition in test markets, given changes in advertising levels and differences in consumer responsiveness. Such an analysis is outside our current scope, but we believe it would be an interesting avenue for future research.

former case, more consumers will have a stronger preference for a particular firm's product and hence reduce price competition. In the latter case, more consumers feel indifferent about buying from either firm so as to intensify price competition through reduced product differentiation. Our analysis shows that what mediates the two different outcomes is how responsive consumers are to advertising. In markets where consumers are sufficiently responsive, heavy combative advertising will create more partisan customers to the benefit of competing firms. In contrast, in markets where consumers are not sufficiently responsive, combative advertising might simply create more indifferent customers. As a result, price competition in the market could intensify.

Looking at it from a slightly different perspective, consumers in the market are subject to forces from opposite directions when firms engage in combative advertising. Our analysis reveals that depending on the strengths of these opposing forces, consumers are either pulled away from the middle (more partisan customers) or pulled towards the middle (more indifferent customers). To the best of our knowledge, our model is the first in the literature that studies the mechanics of combative advertising, analyzing the full range of competitive implications of the preference-changing effects of advertising. The conclusion about indifferent customers is surprising, but plausible. Indeed, our experiment shows that the outcome we have uncovered is not just a theoretical curiosity, it can happen with real people watching real commercials and evaluating real products.

Our analysis, informed by empirical research on advertising, also provides a number of insights that may guide practice in, and future research on, advertising. First, if combative advertising can have anti-competitive effects in one case, but pro-competitive effects in the other, it is important for us to ascertain the mediators for these two kinds of effects. Our model suggests that the mediators are most likely related to the consumer responsiveness to advertising, which is, in turn, related perhaps to consumer characteristics, product characteristics, and message effectiveness. Future research, most suitably experimental research, can further explore those mediators. Second, by recognizing the phenomenon of combative

advertising generating partisan and indifferent customers, we find an intuitive, measurable way of gauging its competitive effects in practice. As the creation of partisan or indifferent consumers is a phenomenon that could be observed even in the presence of more than two firms in the market, we are inclined to believe that the findings from our model are easily generalizable to product categories where oligopolistic competition is more prevalent as compared to duopolistic competition. Third, the competitive implications of advertising may differ in different markets depending on how responsive consumers are to advertising. This means that lumping different product categories together to measure competitive effects may not be advisable. Finally, in the context of combative advertising, lower advertising costs may or may not be a blessing for a firm. In markets where advertising creates indifferent customers, firms may want to embrace, rather than fight, the increase in advertising costs.

As a first attempt to study competitive implications of combative advertising, our model obviously can be improved in many ways. For instance, the model needs to allow for other marketing decision variables.¹⁸ Also, it might be desirable to have a more detailed model at the micro-level, once more consensus is formed about how different types of consumers may be affected differently by unique competing messages. Another possibility is to introduce dynamic advertising competition between firms over time. We hope that the first step we have taken here shows the fruitfulness of research on combative advertising.

¹⁸One could argue that in place of our functional specification for f , a traditional beta distribution (as typically used in marketing to specify consumer preferences) could be used to generate the two distributions illustrated in Figure 3. Although that is a possibility, the beta distribution was not amenable to closed-form solutions for problem setup considered in this paper.

References

- Albion, M. S. and P. W. Farris (1981), *The Advertising Controversy: Evidence on the Economic Aspects of Advertising*, Boston, MA: Auburn House Publishing Company.
- Bagwell, K. (2005), "The Economic Analysis of Advertising," Forthcoming in M. Armstrong and R. Porter.(Eds.), *Handbook of Industrial Organization*, Volume 3, North-Holland, Amsterdam.
- Batra, R., J. G. Myers and D. A. Aaker (1996), *Advertising Management*, Upper Saddle River, NJ: Prentice Hall.
- Becker, G. S. and K. M. Murphy (1993), "A Simple Theory of Advertising as a Good or Bad," *The Quarterly Journal of Economics*, 108(4), 941-64.
- Bloch, F. and Manceau (1999), "Persuasive Advertising in Hotelling's Model of Product Differentiation," *International Journal of Industrial Organization*, 17, 557-74.
- Braithwaite, D. (1928), "The Economic Effects of Advertisement," *Economic Journal*, 38 (Mar), 16-37.
- Comanor, W. S. and T. A. Wilson (1967), "Advertising, Market Structure and Performance," *The Review of Economics and Statistics*, 49, 423-440.
- Erickson, G. M. (1985), "A Model of Advertising Competition," *Journal of Marketing Research*, 22, 297-304.
- Erickson, G. M. (2003), *Dynamic Models of Advertising Competition*, Norwell, MA: Kluwer Academic Publishers.
- Farris, P. W. and D. J. Reibstein (1979), "How Prices, Ad Expenditures, and Profits are Linked," *Harvard Business Review*, 57, November/December, 173-184.
- Gasmi, F., J. J. Laffont and Q. Vuong (1992), "Econometric Analysis of Collusive Behavior in a Soft-Drink Market," *Journal of Economics and Management Strategy*, 1.2, Summer, 277-311.
- Gatignon, H. (1984), "Competition as a Moderator of the Effects of Advertising on Sales," *Journal of Marketing Research*, 21, November, 387-398.

Grossman, G. M. and C. Shapiro (1984), "Informative Advertising with Differentiated Products," *The Review of Economic Studies*, 51.1, January, 63-81.

Hotelling, H. (1929), 'Stability in Competition,' *Economic Journal*, 39, 41-57.

Iyengar, S. and A. F. Simon (2000), 'New Perspectives and Evidence on Political Communication and Campaign Effects,' *Annual Review of Psychology*, 51, 149-169.

Kaldor, N. V. (1950), "The Economic Aspects of Advertising," *Review of Economic Studies*, 18, February, 1-27.

Kanetkar, V., C. B. Weinberg, and D. L. Weiss (1992), "Price Sensitivity and Television Advertising Exposures: Some Empirical Findings," *Marketing Science*, 11(4), 359-371.

Kaul, A. and D. R. Wittink (1995), "Empirical Generalizations About the Impact of Advertising on Price Sensitivity and Price," *Marketing Science*, 14(3-2), G151-160.

Krishnamurthi, L. and S. P. Raj (1985), "The Effect of Advertising on Consumer Price Sensitivity," *Journal of Marketing Research*, 22(May), 119-129.

Lilien, G. L., P. Kotler and K. S. Moorthy (1992), *Marketing Models*, Upper Saddle River, NJ: Prentice Hall.

Little, J. D. C. (1979), "Aggregate Advertising Models: The State of the Art," *Operations Research*, 24.4, 629-667.

Lodish, L. M., M. M. Abraham, S. Kalmenson, J. Livelsberger, B. Lubetkin, B. Richardson and M. E. Stevens (1995), "How T.V. Advertising Works: A Meta-Analysis of 389 Real World Split Cable T.V. Advertising Experiments," *Journal of Marketing Research*, 32, 125-139.

Marshall, A. (1921), *Industry and Trade: A study of Industrial Technique and Business Organization; and of Their Influences on the Conditions of Various Classes and Nations*, London: MacMillan and Co.

Meurer, M. and D. Stahl (1994), "Informative Advertising and Product Match," *International Journal of Industrial Organization*, Elsevier, 12(1), 1-19.

Morrison, D. G. (1973) "Reliability of Tests: A Technique Using the "Regression to the Mean" Fallacy," *Journal of Marketing Research*, 10-1, 91-93.

Pedrick, J. H. and F. S. Zufryden (1991), "Evaluating the Impact of Advertising Media Plans: A Model of Consumer Purchase Dynamics Using Single-Source Data," *Marketing Science*, 10(2), 111-130.

Reibstein, D. J., Y. V. Joshi and P. W. Farris (2004), "Marketing Costs and Prices, An Expanded View", in *The Profit Impact of Marketing Strategy Project: Retrospect and Prospects*, Ed. P. W. Farris and M. J. Moore, Cambridge University Press, 124-152.

Simon, J. L. and J. Arndt (1980), "The Shape of the Advertising Response Function," *Journal of Advertising Research*, 20(4), 11-28.

Steiner, R. L. (1973), "Does Advertising Lower Consumer Prices?," *Journal of Marketing*, 37, October, 19-26.

Tellis, G. J. and D. L. Weiss (1995), "Does TV Advertising Really Affect Sales? The Role of Measures, Models and Data Aggregation," *Journal of Advertising*, 24.3, 1-12.

Tellis, G.J. (2004), *Effective Advertising: Understanding When, How and Why Advertising Works*, Thousand Oaks, CA: Sage Publications.

Tirole, J. (1988), *The Theory of Industrial Organization*, Prentice Hall of India, New Delhi.

Tremblay, V. J. and C. Martins-Filho (2001), "A Model of Vertical Differentiation, Brand Loyalty, and Persuasive Advertising," in 'Advertising and Differentiated Products', Edited by Baye and Nelson, Elsevier Publications.

Tremblay, V. J. and S. Polasky (2002), "Advertising with Subjective Horizontal and Vertical Product Differentiation," *Review of Industrial Organization*, 20, 253-265.

Vakratsas, D. and T. Ambler (1999), "How Advertising Works: What Do We Really Know?," *Journal of Marketing*, 63, January, 26-43.

von der Fehr, N.-M. and K. Stevik (1998), "Persuasive Advertising and Product Differentiation," *Southern Economic Journal*, 65-1, 113-126.

Winter, F. W. (1973), "A Laboratory Experiment of Individual Attitude Response to Advertising Exposure," *Journal of Marketing Research*, 10, May, 130-140.

Appendix 1

We first establish the existence and uniqueness of the second stage pricing equilibrium.

From the first order conditions (Section 3), we have:

$$\begin{aligned} \frac{\partial [p_1 F(d)]}{\partial p_1} = 0, \quad \frac{\partial [p_2(1-F(d))]}{\partial p_2} = 0; \quad \Rightarrow p_1^* &= \frac{2tF(d^*)}{f(d^*)}, \quad p_2^* = \frac{2t[1-F(d^*)]}{f(d^*)}; \\ \text{where, } d &= \frac{t-(p_1-p_2)}{2t}; \quad d^* = \frac{t-(p_1^*-p_2^*)}{2t} \\ \Rightarrow p_1^* - p_2^* &= t - 2td^* = \frac{2tF(d^*)}{f(d^*)} - \frac{2t[1-F(d^*)]}{f(d^*)} \\ \Rightarrow g(d^*) &= 4F(d^*) - (1 - 2d^*)f(d^*) - 2 = 0 \end{aligned}$$

We now show that $g(d)$ has a unique solution for $d \in [0, 1]$. First, note that $g(0) < 0$ and $g(1) > 0$; hence a solution exists for $d \in (0, 1)$. Given our specification of $f(x)$, $F(d)$ is of the form $F(d) = c_1d + c_2d^2 + c_3d^3$. With this functional form for $F(d)$, solving for $g(d) = 0$ (in *Mathematica*) gives us three roots; of which two are imaginary. Therefore, $g(d) = 0$ has a unique solution on $d \in (0, 1)$.

We can show that p_i^* solved from the unique solution for d^* from $g(d^*) = 0$ constructs the unique equilibrium. p_i^* is unique because d^* is unique. We now argue that p_i^* is an equilibrium solution. Let us consider the case where it is not the equilibrium solution: then p_i^* is a local minimum. From the uniqueness of p_i^* , p_i^* must then be the unique local minimum, and this implies that the profit at $p_i = 0$ is higher than the profit at p_i^* given the same p_j^* . However, we know $p_i D$ is 0 at $p_i = 0$ but $p_i^* D$ is positive. Hence, p_i^* cannot be a local minimum, thus it has to be a local maximum. The uniqueness of p_i^* then implies that p_i^* is also the unique global maximum. This completes the proof of the existence and uniqueness of the second stage pricing equilibrium.

Solving the FOCs for the equilibrium prices under symmetric advertising conditions ($k_1 = k_2 = k$), we get $p^*|_{k_1=k_2=k} = \frac{t}{1+k-6ak+6bk^2}$.

We establish uniqueness of the first stage advertising equilibrium numerically, by checking for deviations in the entire advertising strategy space followed by competitive price setting by both firms. Such a deviation is never profitable, establishing that the first stage equilibrium exists and is unique.

Appendix 2

The expression for k^* is

$$k^* = \frac{1}{1152b^2} \left(\frac{64(2^{\frac{2}{3}})(9(1-3a)a+45b-1)b^2}{\beta_1} + 64(9a-1)b - 32(2^{\frac{1}{3}})\beta_1 \right)$$

where,

$$\beta_1 = \sqrt[3]{-27b^4 + 2(9a-2)(9a-1)b^3 + \beta_2}$$

$$\beta_2 = \sqrt{b^6(16(9(1-3a)a+45b-1)^3 + (54a(3a-1) - 27b+4)^2)}$$

Appendix 3

Assume the individual true mean as x_i^* and x_{1i} and x_{2i} are the values for individual i pre- and post-advertising. If the only effect presented is a regression to mean, x_i^* will be unchanged pre- and post-advertising. We can define $x_{1i} = x_i^* - e_i$ and $x_{2i} = x_i^* + e_i$, where e_i can be negative or positive with 0 mean and follows a symmetric distribution (e.g. a normal distribution).

For any symmetric distribution of preferences, its mean m is equal to its median so that there are an equal number of people on both sides of the mean. Also this mean should not change pre- and post-advertising.

The conditional mean change for those with pre-advertising preference less than m is

$$\begin{aligned}
 \Delta m_c(\text{post} - \text{pre}) &= \left[\sum_{x_i^* - e_i \leq m} (x_i^* + e_i) - \sum_{x_i^* - e_i \leq m} (x_i^* - e_i) \right] / (n/2) \\
 &= \left[2 \sum_{x_i^* - e_i \leq m} e_i \right] / (n/2) \\
 &= 4 \left(\sum_{x_i^* - e_i \leq m} e_i \right) / n \\
 &= 4 \left(\sum_{e_i \geq x_i^* - m} e_i \right) / n \\
 &> 0,
 \end{aligned}$$

where n is the number of people. In the above proof, we used the fact $\sum_{e_i \geq -\infty} e_i = 0$ so that $\sum_{e_i \geq x_i^* - m} e_i > 0$ given the symmetry in the distribution of e_i . Similarly, we can show that the conditional mean change for those with pre-advertising preference larger than m is less than 0. Thus, together they imply that conditional means can move to the middle purely due to the RTM effect.

Appendix 4

Assume that an individual's true mean is x_i^* and x_{1i} and x_{2i} are the values for individual i pre- and post-advertising. If only the RTM effect is present, x_i^* will be unchanged before and after advertising exposures. We can define $x_{1i} = x_i^* - e_i$ and $x_{2i} = x_i^* + e_i$, where e_i can be negative or positive with 0 mean and follows a symmetric distribution (e.g. normal distribution).

For any symmetric distribution of preferences, its mean is equal to its median so that there are equal number of people on both sides. Also this mean does not change pre- and post-advertising. The total pre-advertising distance to this mean m across all people is

$$\begin{aligned}
 D_{before} &= \sum_i |x_i^* - e_i - m| \\
 &= \sum_{x_i^* - e_i > m} (x_i^* - e_i - m) + \sum_{x_i^* - e_i < m} (m - x_i^* + e_i) \\
 &= \sum_{x_i^* - e_i > m} -e_i + \sum_{x_i^* - e_i < m} e_i + \sum_{x_i^* - e_i > m} x_i^* + \sum_{x_i^* - e_i < m} -x_i^*
 \end{aligned}$$

In the above proof, we used the fact that the preference distribution is symmetric so that the number of people with $x_i^* - e_i > m$ is equal to the number of people with $x_i^* - e_i < m$.

Similarly, the total post-advertising distance to this mean m across all people is

$$\begin{aligned}
 D_{after} &= \sum_i |x_i^* + e_i - m| \\
 &= \sum_{x_i^* + e_i > m} (x_i^* + e_i - m) + \sum_{x_i^* + e_i < m} (m - x_i^* - e_i) \\
 &= \sum_{x_i^* + e_i > m} e_i + \sum_{x_i^* + e_i < m} -e_i + \sum_{x_i^* + e_i > m} x_i^* + \sum_{x_i^* + e_i < m} -x_i^*
 \end{aligned}$$

Therefore, $D_{before} - D_{after}$ is

$$\begin{aligned}
& \left(\sum_{x_i^* - e_i > m} -e_i + \sum_{x_i^* - e_i < m} e_i - \sum_{x_i^* + e_i > m} e_i - \sum_{x_i^* + e_i < m} -e_i \right) \\
& + \left(\sum_{x_i^* - e_i > m} x_i^* - \sum_{x_i^* - e_i < m} x_i^* - \sum_{x_i^* + e_i > m} x_i^* + \sum_{x_i^* + e_i < m} x_i^* \right) \\
= & \left(- \sum_{x_i^* - e_i > m, x_i^* + e_i > m} 2e_i + \sum_{x_i^* - e_i < m, x_i^* + e_i < m} 2e_i \right) \\
& + \left(\sum_{x_i^* - e_i > m, x_i^* + e_i < m} 2x_i^* - \sum_{x_i^* - e_i < m, x_i^* + e_i > m} 2x_i^* \right) \\
= & \left(- \sum_{x_i^* - m > e_i, x_i^* - m > -e_i} 2e_i + \sum_{x_i^* - m < e_i, x_i^* - m < -e_i} 2e_i \right) \\
& + \left(\sum_{x_i^* - m > e_i, x_i^* - m < -e_i} 2x_i^* - \sum_{x_i^* - m < e_i, x_i^* - m > -e_i} 2x_i^* \right) \\
= & \left[\sum_{e_i > 0} \left(- \sum_{x_i^* - m > e_i} 2e_i + \sum_{x_i^* - m < -e_i} 2e_i \right) \right. \\
& \left. + \sum_{e_i < 0} \left(- \sum_{x_i^* - m > -e_i} 2e_i + \sum_{x_i^* - m < e_i} 2e_i \right) \right] \\
& + \left[\sum_{e_i > 0} \left(\sum_{|x_i^* - m| > |e_i|} 2x_i^* - \sum_{|x_i^* - m| < |e_i|} 2x_i^* \right) + \right. \\
& \left. \sum_{e_i < 0} \left(\sum_{|x_i^* - m| < |e_i|} 2x_i^* - \sum_{|x_i^* - m| > |e_i|} 2x_i^* \right) \right] \\
= & \left[\sum_{e_i > 0} \left(- \sum_{x_i^* - m > e_i} 2e_i \right) + \sum_{e_i < 0} \left(- \sum_{x_i^* - m > -e_i} 2e_i \right) \right] + \\
& \left[\sum_{e_i > 0} \sum_{x_i^* - m < -e_i} 2e_i + \sum_{e_i < 0} \sum_{x_i^* - m < e_i} 2e_i \right] + \\
& \left[\sum_{e_i > 0} \sum_{|x_i^* - m| > |e_i|} 2x_i^* - \sum_{e_i < 0} \sum_{|x_i^* - m| > |e_i|} 2x_i^* \right] + \\
& \left[\sum_{e_i < 0} \sum_{|x_i^* - m| < |e_i|} 2x_i^* - \sum_{e_i > 0} \sum_{|x_i^* - m| < |e_i|} 2x_i^* \right] \\
= & \left[\sum_{e_i < 0} \sum_{x_i^* - m > -e_i} 2e_i + \sum_{e_i < 0} \left(- \sum_{x_i^* - m > -e_i} 2e_i \right) \right] + \\
& \left[\sum_{e_i < 0} \left(- \sum_{x_i^* - m < e_i} 2e_i \right) + \sum_{e_i < 0} \sum_{x_i^* - m < e_i} 2e_i \right] + 0 + 0 \\
= & 0 + 0 + 0 + 0 = 0
\end{aligned}$$

The last two steps are due to fact that e is symmetrically distributed with zero mean.

What this means is that by looking at the difference in the distances across the subjects, we cancel out any RTM effect so that any detected difference in distances can only be due to advertising exposures.

Table 1: Summary Statistics

	FedEx	UPS	Aquafresh	Colgate	Mastercard	Visa	Audi	BMW	Duracell	Energizer
Mean Before	53.08	46.92	39.18	60.82	38.60	61.40	36.74	63.26	50.67	49.33
Mean After	48.74	51.26	38.82	61.18	45.60	54.40	41.77	58.23	60.73	39.27
Median Before	50	50	40	60	50	50	30	70	50	50
Median After	50	50	40	60	50	50	40	60	60	40
σ Before	16.58	16.58	23.51	23.51	20.80	20.80	20.26	20.26	17.58	17.58
σ After	17.05	17.05	21.00	21.00	17.34	17.34	16.75	16.75	18.90	18.90

Table 2: Analysis of Conditional Means

	FedEx	UPS	Aquafresh	Colgate	Mastercard	Visa	Audi	BMW	Duracell	Energizer
Mean Before	37.84	60.07	34.43	73.42	43.06	70.42	37.38	72.17	38.81	60.38
Mean After	48.68	55.00	48.21	67.37	47.50	57.80	48.71	61.50	34.82	43.94
# Observations	97	67	53	111	54	110	42	122	84	80
t-statistic	-6.40	2.19	-4.66	3.72	-1.81	7.18	-5.20	7.04	1.94	7.40
p-value (2-tailed)	0.0000	0.0323	0.0000	0.0003	0.0762	0.0000	0.0000	0.0000	0.0554	0.0000

Table 3: Analysis of Variance in the Pre- and Post-Exposure Preferences, using the F-test

	Courier Services	Toothpastes	Credit Cards	Cars	Batteries
Pre-Advertising Variance	275.0	552.8	432.5	410.6	309.1
Post-Advertising Variance	290.6	441.1	302.0	280.5	357.4
F-statistic	0.9463	1.2532	1.4321	1.4638	0.8649
p-value	0.372	0.076	0.011	0.008	0.188

Table 4: Analysis of Pre- and Post-Exposure Distances

	Courier Services	Toothpastes	Credit Cards	Cars	Batteries
Difference in Pre- and Post-Exposure Distances	-0.528257	1.996356	3.805399	3.349048	-3.21163
t-Statistic	-0.492479	1.728975	3.664976	3.302581	-2.867211
p-Value (2-tailed)	0.623043	0.085707	0.000334	0.001177	0.004688

Table 5: Equilibrium Analysis

	Visa	Mastercard	Engergizer	Duracell	Colgate	Aquafresh	UPS	FEDEX
Price Before	0.52	0.42	0.36	0.38	0.74	0.6	0.4	0.44
Price After	0.42	0.39	0.42	0.58	0.68	0.53	0.42	0.41
Profit Before	0.2889	0.1866	0.1796	0.1904	0.4083	0.2689	0.1852	0.2363
Profit After	0.2181	0.1875	0.1750	0.3384	0.3823	0.2321	0.2142	0.2009