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The authors present a framework that enables researchers to differentiate better among a wide array of hedonic products. Specifically, the authors define and discuss characteristics of hedonic portfolio products and offer a joint segmentation model that is appropriate for understanding the sales dynamics of this class of products. The model offered in this article can accommodate a large degree of product heterogeneity through product clusters and model covariates. The basic premise is that several generic consumer segments exist and remain fixed across all albums, and each album (or each cluster of similar albums) can be viewed as drawing different proportions from each of these underlying segments. The authors also allow explanatory variables to have a differential impact on both components of the model—that is, accelerating purchase rates within a consumer segment and changing the proportions drawn from each consumer segment by each product cluster—thereby expanding or contracting the potential market size. The authors apply this model to music compact disc sales for 20 different albums and discuss the different effects of radio airplay and holiday buying on sales for a sample in the music industry.

Modeling Hedonic Portfolio Products: A Joint Segmentation Analysis of Music Compact Disc Sales

Recently, there has been increasing interest in the marketing of hedonic products, such as box office sales for new movies (e.g., Jedidi, Krider, and Weinberg 1998; Krider and Weinberg 1998; Radas and Shugan 1998; Sawhney and Eliaishberg 1996). Hedonic products have been defined as products whose consumption is primarily characterized by an affective experience (Dhar and Wertensbruch 2000). These products are driven by the experience the product provides rather than the utilitarian benefit offered by its bundle of attributes. As such, many hedonic products are not directly comparable or substitutable. Utilitarian product features can be mimicked by other manufacturers, whereas the abstract attributes provided by a hedonic product cannot easily be replicated. Consequently, many hedonic product categories contain a diverse array of products with few direct substitutes.

In addition, many hedonic products are purchased to be part of a portfolio. For example, books are added to a library, music compact discs (CDs) are added to a collection, and movies contribute to a repertoire of experiences. We further
categorize these items as hedonic portfolio products. To assemble this portfolio of products, repeat purchasing in the product category is common, but repeat purchasing of an individual item in the category is relatively rare. Although the process consumers undertake to assemble this portfolio of products is interesting in and of itself, it is not the focus of this article. Rather, we offer a method of decomposing commonly available aggregate sales data to enable managers to address several issues regarding purchasing behavior across a given set of portfolio products. For example, how similar are the products in the company’s portfolio to one another in the source of their sales? Are there consumer segments that differ dramatically from one another in their rates of adoption? How are sales affected by promotional support, seasonal effects, and so forth, and how do these effects vary across products and consumer segments?

In this article, we develop a new model to explain the sales patterns of hedonic portfolio products, specifically, music CDs. In a related endeavor, Jedidi, Kripper, and Weinberg (1998) develop a model that clusters movies on the basis of measurable characteristics of the movie itself, such as genre, rating, and whether or not it was a sequel, to name just a few of the attributes used. Our model, in contrast, goes beyond this purely cross-sectional analysis and captures each album’s buildup of sales over time, as well as acknowledging the heterogeneous segments of consumers from which each album draws its sales. In addition, because we believe that CDs are hedonic products with abstract, difficult-to-measure attributes, traditional product clustering techniques based on differences across product attributes are less feasible and less useful. Instead, we offer a model that allows the clustering of products to be based on factors such as the product’s potential market size, rate of purchase, response to radio airplay, and seasonal effects rather than any measurable product characteristics.

We also introduce covariates into our model in such a way that enables us to differentiate whether a given factor is increasing the rate of purchase of the product or expanding its potential market size. For example, because hedonic products are evaluated on the basis of the experience they provide rather than specific product attributes, product sampling is important in influencing purchase, and in the music industry, this influence occurs systematically through songs being played on the radio. In addition, there may also be seasonal influences, especially around the December holiday period. But a natural question regarding these factors (i.e., radio airplay and seasonal influences) is how do they influence sales? Does radio airplay increase awareness and therefore increase the potential market size, or does it simply speed up sales that would otherwise have occurred at a later time? The same question can be asked of seasonal effects. This question is of great interest to the music industry. The current industry belief is that getting the right album released for Christmas is key and will provide sales that are otherwise unattainable; in other words, it increases market size rather than accelerating purchases: “Despite the planning that labels put into their holiday marketing campaigns, who gets the coveted Christmas number one single slot is a lot like winning the lottery and comes as a windfall to the company lucky enough to distribute it” (Music Week 1996).

**DATA**

To illustrate our model, we use store-level sales data obtained from SoundScan Inc., a market research firm that plays a similar role in the music industry to the one Information Resources Inc. and ACNielsen play for consumer packaged goods. SoundScan’s computer system records and tabulates over-the-counter music sales soon after they are rung up at the cash register from more than 14,000 retail stores in the United States. This sample represents more than 85% of all over-the-counter record sales in the United States. The information recorded includes the name of the album sold, the number of units sold, the week of the sale, and the market in which the sale was made.

Our data set contains weekly sales volume for a sample of 20 different music albums provided by Capitol Records. These albums are drawn largely from the adult contemporary and alternative music genres. The set of albums used for this analysis represents a convenience sample chosen by managers at Capitol Records in 1996 and therefore might not be fully representative of the complete portfolio of Capitol albums available at that (or any other) time. For this reason, we do not use our model’s parameter estimates to draw any firm, substantive conclusions about the music industry in general, but rather we view our data set as a detailed case study of how such a model might be applied in other managerial contexts.

Although this is an excellent and highly reliable data source, it suffers from two possible drawbacks (similar to Information Resources Inc. and Nielsen). One weakness is that SoundScan primarily contracts with major retail chains, so data from many small record stores are not collected, and such stores are often where newly introduced artists first begin to sell. Furthermore, SoundScan only records over-the-counter music sales. Sales through other distribution channels, such as record clubs and the Internet, are not captured within its system.

Two types of explanatory variables are included in our analysis: radio airplay and seasonal (holiday) indicators. Although radio airplay is not a controllable factor, as most marketing-mix variables are, record companies promote to radio stations in hopes of obtaining extra exposure. This added radio support is widely believed to enhance sales and has been empirically related to observed sales patterns through formal time-series analyses (Montgomery and Moe 1999).

Radio airplay data come from Broadcast Data System, a firm that identifies and tracks the airplay of songs by electronically monitoring more than 560 radio station signals across the United States. Broadcast Data System can recognize every song by matching a unique pattern present in each song’s digitized broadcast signal; the firm records the name of the song played, the exact time and date of the broadcast, the radio station that played the song, and the estimated size of the listening audience in gross ratings points (GRPs).

Figure 1 presents the data for a fairly typical album in our data set (Soup, by Blind Melon), shows the weekly sales for the album, and illustrates the role of the covariates. Sales tend to move with airplay levels, which suggests some relationship between the two. Also note that there is a sales increase around Week 19, corresponding with Christmas. There is also a peak at Week 11 that is not accounted for. This increase was in response to the death of the band’s lead singer due to a drug overdose. Several albums have similar blips, reflecting phenomena such as concert tours, award ceremonies, or other newsworthy events. But for the most part, the dominant feature for this and most other albums is the relatively smooth exponential decline that occurs over time.
In Table 1, we summarize the sales and airplay data for all 20 albums in our data set. As Table 1 shows, the albums differ widely in total units sold and the amount of radio airplay support (measured in GRPs). To make the albums directly comparable, we use only the first 21 weeks of data for each album in estimating our models. The last two columns of Table 1 display data for the first 21 weeks alone yet still show wide variance in terms of sales and GRPs across the albums.

The role of radio airplay is represented by two variables in our model. The variable CURAIR is simply the GRPs received on the radio each week for a given album. The variable AVGAIR is the average level of weekly GRPs the

**Figure 1**

BLIND MELON WEEKLY SALES AND RADIO AIRPLAY

![Graph showing weekly sales and radio airplay for Blind Melon](image)

**Table 1**

SALES AND AIRPLAY DATA FOR ALL 20 ALBUMS

| Album Name                          | Launch Week | Number of Weeks | for all available weeks | | for 21 weeks only | |
|-------------------------------------|-------------|-----------------|-------------------------| |-------------------|------|
|                                     |             |                 | Sales                   | Airplay GRPs (in 1000s) | Sales | Airplay GRPs (in 1000s) |
| Adam Ant Wonderful                  | 3/12/95     | 56              | 132,804                 | 295,164                   | 110,516 | 214,047 |
| Beastie Boys III Communication      | 6/5/94      | 96              | 1,517,575               | 226,586                   | 1,008,788 | 103,415 |
| Blind Melon Soup                    | 8/20/95     | 33              | 201,693                 | 131,499                   | 184,411 | 125,213 |
| Bob Seger It's a Mystery            | 10/29/95    | 23              | 420,413                 | 91,929                    | 416,783 | 85,681 |
| Bonnie Raitt Longing in Their Hearts| 3/27/94     | 106             | 1,499,606               | 979,389                   | 1,072,787 | 571,076 |
| Bonnie Raitt Road Tested            | 11/12/95    | 21              | 352,575                 | 59,108                    | 352,375 | 59,108 |
| Charles & Eddie Chocolate Milk      | 9/3/95      | 31              | 3000                    | 1502                      | 2674   | 1423   |
| Cocteau Twins Four Calendar Café    | 11/7/93     | 126             | 147,681                 | 23,173                    | 99,810 | 10,654 |
| Dink Dink                           | 11/20/94    | 72              | 70,229                  | 40,300                    | 49,143 | 36,420 |
| Everclear Sparkle & Fade            | 5/28/95     | 45              | 471,748                 | 581,585                   | 51,265 | 69,251 |
| Heart Desire Walks On               | 11/21/93    | 123             | 326,953                 | 325,363                   | 251,615 | 271,849 |
| John Hiatt Walk On                  | 10/29/95    | 23              | 185,094                 | 59,054                    | 179,068 | 55,525 |
| Luscious Jackson Natural Ingredients| 8/28/94     | 84              | 180,437                 | 28,529                    | 96,964 | 27,723 |
| Radiohead The Bends                 | 4/9/95      | 52              | 272,143                 | 236,045                   | 114,171 | 84,195 |
| Richard Marx Paid Vacation           | 2/13/94     | 112             | 493,610                 | 1,938,426                 | 326,032 | 840,282 |
| Robbie Robertson Music for the Native Americans | 10/9/94 | 78              | 152,759                 | 152                        | 108,124 | 138 |
| Smoking Popes Born to Quit          | 7/9/95      | 39              | 33,720                  | 36,409                    | 28,699 | 34,964 |
| Sparklehorse Someday                | 10/1/95     | 27              | 4160                    | 32,516                    | 2358   | 1220   |
| Supergrass I Should Coco            | 7/23/95     | 37              | 25,139                  | 8,334                      | 17,596 | 8230 |
| Tom Cochrane Ragged Ass Road        | 11/12/95    | 21              | 11,225                  | 4956                       | 11,225 | 4956 |
album has received since it was introduced on the radio. The motivation behind the second airplay variable is twofold. First, we observe that for several albums, radio stations began playing the songs on the air before the product was available for sale at retail stores. The AVGAIR variable enables the model to reflect this prelaunch airplay support. The variable also reflects lagged effects of airplay, which may have a substantial impact for certain consumer segments and/or product clusters.1 Thus, the CURAIR covariate represents the contemporaneous effect of airplay on sales, whereas the AVGAIR covariate will allow for more long-term effects.

Two holiday dummy variables were also created. The variable SEASON represents the four weeks around Christmas, capturing the overall elevated purchasing that occurs around the holiday gift-giving season. A more careful look at the data from this time period shows that Christmas week itself tends to be associated with an even larger increase in sales. Therefore, a second dummy variable, XMAS, was created for Christmas week alone.

Before moving on with our model development, we briefly consider how this data set compares with others that might exist for different types of hedonic portfolio products. Naturally, the specific details of the data collection process and our particular set of covariates are unique to the music industry. But the overall structure of the data set will commonly exist in other contexts. Basically, we have a 20 × 21 matrix that reflects the sales level for each of our 20 albums during each of the 21 weeks in our calibration period. (Each covariate is captured by a similarly shaped matrix that shows how these marketing effects vary across albums and over time.) It is easy to imagine how an equivalent sales matrix could be constructed for a given set of books, collectible items, videotapes, or any other type of item that conforms to the broad class of hedonic portfolio products. Therefore, as we now develop the model to be estimated and analyzed in this particular context, it is important to keep in mind that the same basic framework shown here should be widely applicable in other settings.

MODEL DEVELOPMENT

Modeling Individual Products

Most albums in our data set have sales that decline exponentially, similar to those for Blind Melon's Soup in Figure 1. Therefore, we begin with the simple but robust assumption of a constant hazard rate, \( \lambda \), which implies an exponential process, though the fully heterogeneous model will allow for increasing or decreasing hazard rates over time. Under the exponential specification, the consumer population is viewed as homogeneous, each with an individual purchase rate (or hazard rate) of \( \lambda \). This process can be derived from the continuous-time version of the classic Fourt-Woodlock (Fourt and Woodlock 1960) model:2

\[
F(t) = p(1 - e^{-\lambda t}),
\]

where

\[
F(t) = \text{cumulative probability of having purchased by time } t; \quad t = \text{time};
\]

\[
p = \text{market penetration parameter}, \quad 0 < p \leq 1; \quad \text{and}
\]

\[
\lambda = \text{rate of purchase parameter}, \quad \lambda > 0.
\]

Because our data are measured at the weekly level rather than on a continuous-time basis, our incremental sales function is derived by taking differences on the cumulative function:

\[
f(t) = F(t) - F(t-1) = p[(1 - e^{-\lambda t}) - (1 - e^{-\lambda (t-1)})]
\]

\[
= p(e^{-\lambda (t-1)} - e^{-\lambda t}).
\]

Decomposing Sales as a Mixture of Hazard Rates

Sales diffusion for a given album can be decomposed into a mixture of hazard rates, which can be interpreted as a product drawing sales from a mixture of consumer segments. These segments are differentiated by their purchasing behavior, in this case, different purchase rates (\( \lambda_i \)). The relative size of these segments for a given album, or the amount each segment contributes to overall product sales, can be represented by the penetration parameter (\( p \)).

Therefore, sales for each album can be represented as a discrete mixture of the homogeneous FourtnWoodlock model from Equation 1:

\[
F(t) = p_1(1 - e^{-\lambda_1 t}) + p_2(1 - e^{-\lambda_2 t}) + \ldots + p_S(1 - e^{-\lambda_S t})
\]

\[
= \sum_{i=1}^{S} p_i(1 - e^{-\lambda_i t}),
\]

where

\[
p_i = \text{size of Segment } i, \quad \text{and } \sum_i p_i \leq 1;
\]

\[
\lambda_i = \text{rate of purchase for Segment } i, \quad \lambda_i > 0; \quad \text{and}
\]

\[
S = \text{number of segments}.
\]

In this generalization, the \( p_i \) parameters add up to the total eventual penetration, and therefore \( 1 - \sum_i p_i \) represents the fraction of hard-core nontriers. Although there are more parsimonious ways of introducing consumer heterogeneity, such as a gamma distribution across the possible values of \( \lambda \), there are several reasons supporting the proposed discrete mixture: (1) It enables us subsequently to incorporate segment-specific covariates and consequently to identify differences among the response coefficients in addition to the \( \lambda \)'s, and (2) it allows the possibility of multiple modes in the distribution of \( \lambda \).

Modeling Multiple Products

To deal with multiple products, we could, in theory, fit the model from Equation 3 (with one or more segments) to each of the products individually. In that case, each product (or album) would have its own unique vector of parameters, \( p \) and \( \lambda \).3 This would suggest that each product faces an

\[3\text{In practice, however, there would likely be severe parameter identification problems in trying to sort out separate segments as well as the market penetration parameter } p \text{ (and its covariates) for a single product. By modeling across multiple products, we avoid identification issues.} \]
entirely different population of consumers that is characterized by different purchasing rates ($\lambda$) and that there is no shared characteristics across these groups. There is ample reason to doubt this notion of complete independence across products. It is more likely that there is a single (segmented) consumer population that all albums face and that sales differences arise from albums attracting consumers in different proportions. Therefore, a more intuitive and parsimonious assumption is that all albums derive their sales from a common set of consumer segments, but in different proportions. (See the schematic diagram in Figure 2.)

The perspective here is that the segment-by-segment characteristics of consumers are invariant across all products, but the relative sizes of the segments may differ on a product-by-product basis. This assumption also results in a much more parsimonious model. Similar approaches have been used by Gupta and Bodapati (1999) and ter Hofstede, Steenkamp, and Wedel (1999) to explain differences across geographical areas. For example, ter Hofstede, Steenkamp, and Wedel derive four fixed consumer segments from a data set that covers nearly 3000 consumers in 11 European nations. They then assume that each nation can be represented as a unique blend of those segments.

Our model, similar to these others in this one respect, allows products to differ on the basis of the way they differentially draw from the set of fixed segments. If we fit the model to each product individually, each product would be characterized by a vector $p_1$, the proportion of sales drawn from each consumer segment (this is the basic idea used by ter Hofstede, Steenkamp, and Wedel [1999]). But rather than treat each product individually, we cluster products together on the basis of their p parameters (similar in spirit to Gupta and Bodapati [1999]). For example, a product in Cluster 1 is associated with a particular set of p's and $\lambda$'s. A product in Cluster 2, however, is associated with a different set of p's but the same set of $\lambda$'s as in Cluster 1 (because the $\lambda$'s are consumer-segment specific and not product specific).

Finally, to complete our description of the model, we now broaden its structure to incorporate the effects of explanatory variables.

**Adding Explanatory Variables**

Explanatory variables, such as marketing-mix activities, can affect the consumer decision process in two different ways. First, they can boost the hazard rate ($\lambda$), thereby accelerating the purchase timing among the remaining potential buyers. Second, these covariates can alter the probability of a consumer being in the market for the product at all. Increased marketing efforts help boost product awareness and therefore could lead to a larger market penetration, p, above and beyond any purchase acceleration effects. We deal with each of these two influences in turn.

To incorporate purchase rate ($\lambda$) covariates, we use the well-established proportional hazards modeling paradigm (e.g., Kalbfleisch and Prentice 1980; Lawless 1982). For more details, see the original sources or more recent applications (e.g., Radas and Shugan 1998; Vanhuele et al. 1995). In effect, the existence of explanatory variables acts as if it speeds up time, causing consumers to purchase sooner than they would in a no-covariate world. For example, a consumer who was planning to purchase a product at the end of the month may decide to do so earlier after hearing an advertisement on the radio. In this case, the advertisement speeds up the consumer's normal rate of purchase. More formally, covariates enter the purchase timing model as follows:

$$F(t) = p[1 - e^{-\lambda(t)}]$$

where

$$B(t) = \sum_{s=1}^{S} \lambda(t),$$

$\lambda(t)$ = vector of response parameters, and

$x(t)$ = vector of covariates at time t.

**Figure 2**

**MULTISEGMENT MODEL WITH MULTIPLE PRODUCTS**

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Notes: $\lambda$ is Segment i's rate of purchase, and $p_j$ is the proportion of consumers that any product in Cluster j draws from Segment i.
Note that if all the variables are removed from the model, or if all the \( B \)'s are equal to zero, then the \( B(t) \) term in Equation 4 collapses down to ordinary time \( t \) (in Equation 1). As the covariates and the \( B \)'s become larger, the \( B(t) \) term acts as if time were passing more quickly, thereby allowing for more purchasing to take place within a fixed period of real time. Note that the set of \( B \)'s, similar to \( \lambda \), is segment-specific because consumer segmentation is based on both the consumers' underlying purchase rates and their responsiveness to explanatory variables.

To incorporate market size \( p \) covariates, we include explanatory variables in a response function that determines long-term market penetration:

\[
p(t) = p_0 \times f[x(t)] = p_0 \times \frac{e^{\gamma x(t)}}{1 + e^{\gamma x(t)}},
\]

where

\( p_0 \) = maximum market penetration level,
\( f[x(t)] \) = response function,
\( \gamma \) = vector of response parameters including a constant \( \gamma_0 \), and
\( x(t) \) = vector of explanatory variables at time \( t \).

Similar to many market response models (e.g., Blattberg and Jeuland 1981; Little 1979), this formulation for market penetration suggests an S-shaped response function. Low levels of marketing-mix support would have limited impact in penetrating the market. Only when critical mass is achieved does the market significantly respond. But any more support beyond that point may not necessarily benefit as the maximum penetration level is asymptotically approached. To the best of our knowledge, this type of logistic function has never been used in a duration modeling context, though it has been successfully employed for zero-inflated Poisson (counting) models (Lambert 1992). This seems to be an excellent mechanism to reflect the potential variations in the number of eligible buyers over the course of an album's lifetime and to capture the influences of covariates on this dynamic process.

For the case of music album sales, increased radio exposure may increase the awareness level for a particular album, thus growing its market potential through the penetration function, \( p(t) \). Conversely, the loss of airplay support may result in the consumers gradually forgetting about the album and removing it from their consideration sets, which thereby decreases market potential. Therefore, \( p(t) \) can both rise and fall with changes in the covariates. Each album's penetration level has a lower bound, \( (p_0, e^{\gamma t})/(1 + e^{\gamma t}) \), which would apply if there were absolutely no support from the explanatory variables. The maximum penetration level an album can attain with full (infinite) support is \( p_0 \). Using a response function for \( p(t) \) rather than fixing it as a constant allows an album to attract more new potential consumers over time. This is especially important for a fragmented market (such as the music industry), in which products from one genre attempt to cross over to new genres (and segments of consumers) on a regular basis.

With covariates, our multiproduct model can be written as follows:

\[
F(t|\text{Cluster } i) = p_1(t) \times \left[ 1 - e^{-\lambda_1 B_1(t)} \right] \\
+ p_2(t) \times \left[ 1 - e^{-\lambda_2 B_2(t)} \right] \\
+ \ldots + p_S(t) \times \left[ 1 - e^{-\lambda_S B_S(t)} \right].
\]

\[
F(t|\text{Cluster } j) = p_1(t) \times \left[ 1 - e^{-\lambda_1 B_1(t)} \right] \\
+ p_2(t) \times \left[ 1 - e^{-\lambda_2 B_2(t)} \right] \\
+ \ldots + p_S(t) \times \left[ 1 - e^{-\lambda_S B_S(t)} \right].
\]

where

\[
F(t|\text{Cluster } j) = \text{cumulative probability of buying at time } t \\
\text{for a product in Cluster } j,
\]

\( p_i(t) \) = proportion of Cluster \( j \)'s consumers accounted for by Segment \( i \), and
\( C \) = number of clusters.

\[
F(t) = \sum_{j=1}^{C} \text{P(Cluster } j) \times F(t|\text{Cluster } j)
\]

\[
= \sum_{j=1}^{C} \text{P(Cluster } j) \times \sum_{i=1}^{S} p_i[t(1 - e^{-\lambda_i B_i(t)})].
\]

where \( \text{P(Cluster } j) \) = probability that a given product belongs in product Cluster \( j \).

**Summary of Model Development**

To help focus on the motivations and justifications for our various model components, we have developed the model specification in a piecewise manner. For completeness, we present the full log-likelihood function with a brief recap of its various properties:

\[
\text{LL(buyer)} = \sum_{t=1}^{T} \text{Sales}_t \\
\times \log \left[ \sum_{j=1}^{C} \text{P(Cluster } j) \times \sum_{i=1}^{S} p_i(t e^{-\lambda_i B_i(t-1)} - e^{-\lambda_i B_i(t)}) \right],
\]

\[
\text{LL(nonbuyer)} = \left( N - \sum_{t=1}^{T} \text{Sales}_t \right) \\
\times \log \left[ 1 - \sum_{j=1}^{C} \text{P(Cluster } j) \times \sum_{i=1}^{S} p_i(t) [1 - e^{-\lambda_i B_i(t)}] \right],
\]

and

\[
\text{LL} = \text{LL(buyer)} + \text{LL(nonbuyer)}.
\]
where Sales\(_i\) is the number of units sold in week \(t\), \(N\) is the number of consumers in the market, \(T\) is the number of weeks being modeled, \(B_i(t) = \sum_j y_0 x_j(t)\), and \(p_j(t) = p_j(0) \times \exp(x_j(t)) / (1 + \exp(x_j(t)))\).

Consumer segments (indicated by \(i\)) are represented by their rate of purchase (\(\lambda_i\)) and their response to explanatory variables (\(\beta\)). Because we assume that the set of consumer segments is invariant across all products, these parameter estimates describe the consumer segments and do not vary across the product clusters. For example, if we model only two consumer segments, we have only two sets of \(\lambda\)'s and \(\beta\)'s regardless of the number of products or product clusters modeled.

Product-cluster characteristics, in contrast, are captured within \(p(t)\). Remember from Equation 5 that \(p(t)\) is driven by a maximum penetration parameter (\(p_0\)) and a logistic response function of coefficients that incorporate covariate effects. The \(p_0\) and \(\gamma\) parameters combine to yield the relative proportions of the consumer segments that each product cluster would attract in the absence of any covariate effects. The remaining \(\gamma\) parameters illustrate how sales for products in that particular cluster tend to respond to covariate activity.

The total number of parameters for the model will depend on the number of covariate effects, the number of consumer segments, and the number of product clusters specified:

\[
\begin{align*}
    k &= SC + (S + v_S) + C(1 + v_C),
\end{align*}
\]

where \(k\) is the number of parameters, \(S\) is the number of consumer segments, \(C\) is the number of product clusters, and \(v_S\) and \(v_C\) are the number of covariates associated with the consumer segments and product clusters, respectively.

In addition, notice that the log-likelihood function consists of two components, one for the buyers and one for the nonbuyers. The actual sales observed represent the buyers, but it is also important to recognize the probability of not purchasing each album for the many households that do just that. The number of nonbuyers is the total population, \(N\), less the cumulative number of sales through any given time period. The population, \(N\), is essentially a scaling constant that will affect the log-likelihood calculations but will not change the basic structure of the model. Without loss of generality, we set \(N = 10\) million, which represents the order of magnitude of domestic sales for an extraordinarily successful album (e.g., the *Titanic* soundtrack). This provides a practical and effective upper bound and has little consequence for the implications of our model.

**MODEL ESTIMATION**

We obtain our parameter estimates using the constrained nonlinear optimization algorithms available in the MATLAB software package and an expectation-maximization (E-M) algorithm. The E-M algorithm, commonly used in the estimation of latent structure models, is composed of two steps, the expectation step and the maximization step. In our case, however, the data are such that the cluster membership probabilities tend toward zero and one, primarily because of the large sample sizes for each of the products. Thus, for computational efficiency (but with virtually no impact on the estimated parameters), we modify the E-M algorithm to use discrete (rather than probabilistic) assignment for the clusters (see Banfield and Bassill 1977; Wedel and Kistemaker 1989). The steps are similar to those of the E-M algorithm except that the researcher, instead of calculating the probability of a product being in a particular cluster, discretely assigns the product to the cluster with the parameters that maximize its likelihood:

1. Randomly assign the products into clusters.
2. Optimize the parameters for each cluster given the particular product-cluster assignments.
3. Reassign the products into clusters that maximize the likelihood of that product's sales being observed.
4. Repeat Steps 2 and 3 until improvement in likelihood is less than some convergence criterion.

This process is repeated several times (using different random starting points), which helps gauge and avoid possible problems with local optima. Our experience with this algorithm, at least for this particular data set, revealed no problems with local optima or parameter instability. A multitude of different starting values result in the same final parameter estimates.

Before arriving at a final specification, several model structure questions need to be answered. How many consumer segments and product clusters should be modeled? What are the roles of our explanatory variables? That is, should airplay and the holiday variables serve as purchase rate covariates or market size covariates? To answer these questions in a careful manner, we use the following three-stage process.

**Stage 1**

Without any covariates, we estimate the segment-cluster model across all 20 albums for varying combinations of segment numbers and cluster numbers. Using standard likelihood-based model selection criteria (e.g., consistent Akaike's information criterion [CAIC]), the most appropriate model specification for this data set appears to be one with two consumer segments and four product clusters.

**Stage 2**

The purpose of this stage is to identify the role of our covariates. Specifically, we allow each pair of covariates (airplay and seasonal) to affect each component of the model (purchase timing and market potential) separately. Using the two-segment, four-cluster structure from Stage 1, we estimate and compare four separate models with different covariate specifications:

- Model A: Airplay and seasonal variables serve as purchase rate covariates only.
- Model B: Airplay variables are modeled as purchase rates covariates, and seasonal variables are modeled as market size covariates.
- Model C: Seasonal variables are modeled as purchase rates covariates, and airplay variables are modeled as market size covariates.
- Model D: Airplay and seasonal variables serve as market size covariates only.

Table 2 provides the results from the four estimated models. Overall, the best-fitting model is Model C, implying that the appropriate model specification is one that allows airplay to affect market size and seasonal indicators to affect purchase rates.

**Stage 3**

When the covariate structure is determined from Stage 2, we need to confirm that the original segment-cluster structure is still appropriate when covariates are included in the
model, because the covariates allow the model to account for more heterogeneity than the original model in Stage 1. Therefore, with the aforementioned covariate structure, we reestimate a variety of models to determine the best segment-cluster structure again. In this case, we find that even with the covariates included, the best model is still one with two segments and four clusters.

**EMPIRICAL ANALYSIS**

Figure 3 provides the parameter estimates from Model C with two segments and four clusters. The consumers belonging to the first segment have a relatively high rate of purchase ($\lambda = .64$) but are not responsive to the explanatory variables we have included in our model. In contrast, the consumers in the second segment are slower to adopt the product ($\lambda = .06$) under most circumstances but are highly motivated by the Christmas season.

On the basis of Equation 4, Segment 2 consumers purchase during peak season at an effective rate of .16, more than double that of off-peak purchasing ($\lambda = .06$). In contrast, radio airplay serves mainly to grow the potential market, as evidenced by the sizeable AVG AIR effects (as well as some contemporaneous effects seen in the CUR AIR covariate) in the market size component of the model. This suggests that radio exposure increases consumer awareness and therefore attracts new buyers into the market.²

²To help facilitate parameter inference, we rescale the airplay variables as (thousands of GRPs) in our analysis.
Table 3
MINIMUM AND MAXIMUM MARKET PENETRATION BY CLUSTER

<table>
<thead>
<tr>
<th>Cluster 1</th>
<th>Cluster 2</th>
<th>Cluster 3</th>
<th>Cluster 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Segment 1</td>
<td>Segment 2</td>
<td>Segment 1</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>.0313</td>
<td>.1270</td>
<td>.0006</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>-4.3877</td>
<td>-4.3877</td>
<td>-5.7831</td>
</tr>
<tr>
<td>Minimum p</td>
<td>.0004</td>
<td>.0016</td>
<td>.0000</td>
</tr>
<tr>
<td>Maximum p</td>
<td>.0313</td>
<td>.1270</td>
<td>.0006</td>
</tr>
<tr>
<td>Minimum penetration</td>
<td>20,000</td>
<td>1000</td>
<td>116,000</td>
</tr>
<tr>
<td>Maximum penetration</td>
<td>1,583,000</td>
<td>480,000</td>
<td>243,000</td>
</tr>
<tr>
<td>Minimum/maximum ratio</td>
<td>.0123</td>
<td>.0031</td>
<td>.4780</td>
</tr>
</tbody>
</table>

To understand better how the potential market sizes can vary across the clusters, we examine the \( p_0 \) and \( \gamma \) parameters more closely. As discussed previously, these estimates can be combined to derive the upper and lower bounds on the relative proportions that each product cluster draws from each consumer segment. Table 3 shows the maximum and minimum penetration levels for each cluster. It is evident that albums in Cluster 3 have the smallest maximum potential market size, because they theoretically can reach a total of only 243,000 sales (2.4% from Segment 1 and 2.19% from Segment 2 among the assumed universe of 10,000,000 potential buyers), whereas Cluster 1 has the largest reach, with a potential of 1,583,000 in sales penetration. Both Clusters 1 and 4 have large maximum penetration levels. However, they differ significantly in both their minimum penetration and the rate at which changes in airplay will move each cluster toward its maximum penetration. Cluster 4 has the benefit of having a fairly large minimum penetration level, so even without airplay support, sales should be substantial. Furthermore, the high values of the airplay coefficients for Cluster 4 help its album move rapidly from the minimum toward the maximum penetration level with relatively modest amounts of radio airplay.

Cluster 1 albums, in contrast, start with a much lower penetration level and experience much slower increases in market potential due to airplay. The ratio of the minimum penetration to the maximum penetration summarizes the extent to which airplay can increase the potential market for albums in any cluster. The smaller the ratio, the more albums in the cluster need airplay support. Both Clusters 1 and 2 need significant airplay support to approach their maximum potential, whereas Clusters 3 and 4 can obtain nearly 50% of their maximum penetration without any airplay at all.

Table 4 summarizes the clusters in terms of their maximum potential penetration levels and the effectiveness of airplay in achieving that penetration. Each of the four clusters is quite distinct from the others along these two dimensions. From a managerial perspective, an album in the high potential penetration and high airplay effectiveness cell (i.e., Cluster 4) is in an attractive position. High sales can be obtained with very little effort targeted at acquiring airplay support. In contrast, an album in the low–low cell (Cluster 2) is in a poor position. Efforts to increase market potential through radio airplay will be relatively expensive and will ultimately be limited by the low ceiling on maximum potential for this cell. Perhaps this is why the two lowest selling albums are found in this cell.

It is also interesting to compare the low–high and high–low cells (Clusters 1 and 3, respectively). Without any airplay, a Cluster 1 album will have a comparatively low ultimate penetration level of roughly 20,000 sales, which pales in comparison with Cluster 3’s lower bound penetration of 116,000. However, with substantial airplay support, Cluster 1 albums will greatly outperform those in Cluster 3, having a penetration as high as 1,583,000, six times higher than that of Cluster 3. These upper bounds are theoretical, because they assume the presence of infinitely high levels of radio airplay. Although Cluster 1 is far larger than Cluster 3 in terms of potential market size (\( p_0 \)), several albums in Cluster 3 (e.g., Soup by Blind Melon) outperform most of the albums in Cluster 1 because of significant differences in the airplay support they receive. Similar patterns can be found across several other pairs of clusters. Examining and understanding these trade-offs and apparent anomalies provides a strong testimonial in favor of the usefulness and validity of the joint segmentation approach used here.

**DISCUSSION AND CONCLUSIONS**

The motivating issues and methodological approaches raised in this article are by no means limited to the music industry. Many other product categories demonstrate similar characteristics and are well-suited for this type of mod-

Table 4
CLUSTER CHARACTERISTICS

<table>
<thead>
<tr>
<th>Effectiveness of Airplay in Achieving Maximum Penetration Low</th>
<th>High</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Cluster 2</td>
<td>Cluster 1</td>
</tr>
<tr>
<td>Bob Seger (416,783)</td>
<td>Bonnie Raitt (1,072,787)</td>
</tr>
<tr>
<td>Bonnie Raitt 2 (352,575)</td>
<td>Heart (251,615)</td>
</tr>
<tr>
<td>Richard Marx (326,032)</td>
<td>Adam Ant (110,516)</td>
</tr>
<tr>
<td>Dink (49,143)</td>
<td>Everyclear (51,265)</td>
</tr>
<tr>
<td>Charles &amp; Eddie (2674)</td>
<td>Smoking Popes (28,699)</td>
</tr>
<tr>
<td>Sparklehorse (2358)</td>
<td>Supergrass (17,596)</td>
</tr>
<tr>
<td>Tom Cochrane (11,225)</td>
<td></td>
</tr>
<tr>
<td>High Cluster 3</td>
<td>Cluster 4</td>
</tr>
<tr>
<td>Blind Melon (184,411)</td>
<td>Beastie Boys (1,008,788)</td>
</tr>
<tr>
<td>John Hiatt (179,068)</td>
<td></td>
</tr>
<tr>
<td>Radiohead (114,171)</td>
<td></td>
</tr>
<tr>
<td>Robbie Robertson (108,124)</td>
<td></td>
</tr>
<tr>
<td>Cocteau Twins (99,810)</td>
<td></td>
</tr>
<tr>
<td>Luscious Jackson (96,964)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Numbers in parentheses represent the cumulative sales for the 21-week period modeled.
eling approach, especially other hedonic products for which traditional attribute-based methods would be of little use.

Despite the high degree of heterogeneity across albums in our data set, our model is extremely parsimonious, requiring only 26 parameters to capture the sales patterns across 20 different albums. Our data set could probably continue to grow in size (number of weeks and/or number of albums) to some extent without requiring any additional parameters. As long as additional albums conform to the basic cluster structure already in place, there may be no reason to increase the size of the model or even reestimate its parameters at all.

One important feature (and possibly a limitation) of our model is its reliance on aggregate data. If we had detailed information about the exact set (or portfolio) of products each household had purchased over time, our modeling approach might have been dramatically different than the structure developed here. But such panel data are rarely available for the vast majority of products sold through conventional retail channels. Perhaps the explosive growth of the Internet will eliminate this constraint at some point, but for the foreseeable future a collection of aggregate sales curves will continue to be the only reliable source to accomplish the type of study performed here.

Finally, a natural extension of this modeling framework would be to produce pretax (or early posttax) sales forecasts for new items that might be added to an existing portfolio. Because we are simultaneously modeling multiple items (i.e., albums), we can leverage the information from existing products and apply it to a new release. This can provide an extraordinarily valuable decision-support tool to the many hedonic product markets (such as the recorded music industry) that historically have made virtually no effort to predict future sales formally or perform "what-if" policy scenarios to fine-tune their marketing activities.

REFERENCES