Long-Term Contracts under the Threat of Supplier Default

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Contracting with suppliers prone to default is an increasingly common problem in some industries, particularly automotive manufacturing. We model this phenomenon as a two period contracting game with two identical suppliers, a single buyer, deterministic demand, and uncertain production costs. The suppliers are distressed at the start of the game and do not have access to external sources of capital, hence revenues from the buyer are crucial in determining whether or not default occurs. The production cost of each supplier is the sum of two stochastic components: a common term that is identical for both suppliers (representing raw materials costs, design specifications, etc.), and an idiosyncratic term that is unique to a given supplier (representing inherent firm capability). The buyer chooses a supplier and then decides on a single or two period contract. Comparing models with and without the possibility of default, we find that without the possibility of supplier failure, the buyer always prefers short-term contracts over long-term contracts, whereas this preference is typically reversed in the presence of failure. Neither of these contracts coordinates the supply chain. We also consider dynamic contracts, in which the contract price is partially tied to some index representing the common component of production costs (e.g., commodity prices of raw materials such as steel or oil), allowing the buyer to shoulder some of the risk from cost uncertainty. We find that dynamic long-term contracts allow the buyer to coordinate the supply chain in the presence of default risk. We also demonstrate that our results continue to hold under a variety of alternative assumptions, including stochastic demand, allowing the buyer the option of subsidizing a bankrupt supplier via a contingent transfer payment or loan, and allowing the buyer to unilaterally renegotiate contracts. We conclude that the possibility of supplier default offers a new reason to prefer long-term contracts over short-term contracts.

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1. Introduction

The relationship between manufacturers and suppliers in the American automotive industry is not always a cooperative one. American carmakers are on a perpetual quest to match the procurement costs of their competitors by increasing supply chain efficiency. Throughout the 1990's, an increased awareness of the value of cooperative buyer-supplier relationships sparked interest in fostering strategic partnerships between automotive manufacturers and suppliers. Nevertheless, despite the perceived value of collaborative behavior with suppliers, auto manufacturers often engage in adversarial and caustic supplier management tactics, typically employing one tool more than any other: direct pressure to reduce procurement prices.

In the 1990's, for example, Ford famously dictated an across-the-board 5% price decrease to all of its suppliers (Stallkamp 2005). In 2005, Lear, a key seat supplier to Chrysler, attempted to negotiate higher prices to cover recent sharp cost increases that plagued the automotive supplier base. When Lear threatened to cease shipping products to Chrysler, the automaker promptly took the supplier to court to enforce the terms of their contract, despite the fact that Lear posted a net loss of nearly $600 million in the fourth quarter of 2005 alone. This emphasis on low procurement prices has had a clear adverse effect on the suppliers, however: profit margins are low across the industry, and many suppliers routinely lose money (Wernle 2006a, Wynn 2006). In 2005, Delphi, the largest supplier of automotive parts in the country, was in bankruptcy, as were numerous smaller suppliers.

While the poor financial health of the suppliers is exacerbated by the low procurement costs demanded by automakers in response to the competitive North American automotive landscape, there are several additional factors that have played a key role in the current perilous state of the automotive supplier base. Bo Andersson, General Motors Vice President for Global Purchasing and Supply Chain, cites four critical issues: production cost increases, unstable domestic volume, legacy pension plans (resulting in larger overhead expenses), and difficult access to capital (Andersson 2006). The analysis that we present will touch upon each of these key factors.

From a buyer's point of view, losing a supplier to bankruptcy can have varying consequences: on one extreme, if the supplier ceases operations, the buyer may have to switch to a new supplier (possibly at a higher cost), whereas at another extreme, if the supplier continues normal operations without disruption, the buyer may have to help support the supplier financially, as was the case in some of the most visible supplier bankruptcies of 2005 (e.g., Delphi, see Nussel and Barkholz 2006). When interior parts supplier Collins & Aikman entered bankruptcy in 2005, automakers had little choice but to sustain the supplier–roughly 90% of vehicles made in North America have at least one component produced by the supplier–and the estimated total cost to the Big Three auto manufacturers was $532 million, resulting from the cancellation of loan repayments, parts price increases, operating subsidies, and legal fees (Barkholz and Sherefkin 2007).

Thus, the possibility of losing a supplier to failure may affect the decisions that a buyer makes, including the price and the length of contracts. Should a buyer pay more to avoid losing a supplier to bankruptcy, or should the buyer pay less because the supplier is risky? Should the buyer make a long-term commitment to the supplier, or should the firm minimize the length of its exposure to a risky partner? Should buyers bear some of the risk of cost uncertainty by compensating suppliers in a dynamic manner? These are the trade-offs we seek to capture, via an operationalized model of buyer-supplier relations under the threat of supplier failure. The motivation is the automotive industry, but the model is relevant to many scenarios: supply chains with members in financial distress, supply chains with start-ups prone to bankruptcy, etc. Based on the aforementioned examples from the automotive industry, we believe these types of relationships have several key characteristics:

1. **Uncertain Supplier Production Costs.** Tooling and capacity installation lead-times tend to be long and hence the buyer must commit to the supplier well in advance of the finalized design of the component. In addition, raw materials costs are uncertain and often have a large impact on the supplier's margins; Standard & Poor's June 2006 industry survey of autos and auto parts cites high raw materials costs as a key factor in the current distressed state of the supplier base (Levy and Ferazani 2006). Thus, the actual per unit production cost to the supplier is unknown at the time of contracting. We assume that this risk is not diversifiable because it derives from volatility not captured in any current futures market (e.g., outputs from higher tier suppliers or inherent uncertainty in design and production techniques). Even risk in raw materials prices cannot always be hedged–Sherefkin (2006) describes how Ford has struggled to create a futures market for automotive sheet steel.

2. **Strong Bargaining Power of the Buyer.** The buyer tends to be a large firm with most of the bargaining power while the supplier is the smaller firm at risk of default. For example, in the American auto industry, parts suppliers have few potential buyers and face weak bargaining positions and low profit margins, while in Europe the situation is far less bleak for the suppliers (Lewin 2006). Hence, the buyer has most of the bargaining power and offers the contract to the supplier, choosing both price and the length of the contract.

3. **Extended Sales Horizons.** The product being supplied will be used over several sales periods (e.g., a particular model of a car tends to be sold for 5-7 years before a major redesign). Since the cost of switching suppliers is typically high (due to large asset specific investments made by the manufacturers into a particular supplier), the financial health of a dedicated supplier over the sales horizon is critical to the buyer. Furthermore, the buyer has the option of contracting with a supplier for a single sale period, or for multiple periods; both practices are observed in the auto industry (Dyer 1996).

These three points form the core of our model. In what follows, the goal is to analyze and evaluate the performance of both long- and short-term contracts when suppliers face a risk of failure, and to determine under what conditions a particular contract type is preferred. We compare these results to a model with no supplier failure in which short-term contracts are always preferred, and find that under the threat of supplier default this preference is typically reversed. We further consider dynamic contracts, which partially compensate the suppliers for the realized value of production costs, and find that long-term dynamic contracts perform better than static contracts and are capable of achieving the centralized system optimal profit.

Our results complement the existing literature on the value of long-term contracts in supply chains by demonstrating their advantages despite controlling for many of the typical reasons a buyer would have to prefer these contracts. For example, the management literature often discusses long-term relationships or contracts as a method of developing trust between buyers and suppliers, but we do not consider any trust issues; indeed, the long-term contracts are preferred even though the buyer is completely self-interested. Similarly, long-term relationships are known to induce otherwise unsupportable actions in repeated games, thus increasing their value relative to short-term relationships; in our model, the value comes not from inducing actions but rather from reducing the expected cost of supplier default to the buyer. Overall, we offer a new reason to employ long-term contracts: to reduce the damage from supplier default. Our finding is consistent with current practices in the Japanese auto industry, in which long-term relationships are common and supplier defaults are, relative to the U.S., uncommon.

The remainder of this paper is organized as follows: the next section provides a brief literature review, and §3 describes the model. §4 analyzes a benchmark model with suppliers that never enter bankruptcy, and §5 considers suppliers prone to bankruptcy. §6 explores a class of contracts which partially compensate the suppliers for production costs in a dynamic fashion. §7 discusses three interesting extensions (demand uncertainty, contingent transfer payments, and normally distributed costs), and §8 concludes the paper.

2. Literature Review

There are three broad areas of research which are relevant to our paper. The first focuses specifically on understanding the nature of buyer-supplier relationships in the auto industry, while the second is primarily theoretical, concerning such issues as repeated contracting between firms, relational agreements, and contracting under cost uncertainty. The third area of research focuses on the effects of financial distress and supplier disruption risk.

There is a strong tradition, particularly in the management literature, of research into the types of relationships that exist between buyers and suppliers in the automotive industry. Tang (1999) provides an excellent discussion. Much of this work focuses on the historical and current differences between Japanese-style and American-style supply networks; see, for example, McMillan (1990), Dyer et al. (1998), Dyer (1996), and references therein. The traditional Japanese networks of suppliers, called *keiretsu*, are markedly different from their American counterparts. The former are characterized by fewer suppliers, long-term relationships, and heavy cooperation, while the latter traditionally utilized a large number of small suppliers, short-term contracts, and non-cooperative or adversarial behavior. Japanese firms are often heavily invested in their suppliers, wholly or partly owning their closest partners in many instances (Dyer et al. 1998), and even when buyers are not directly invested in suppliers, there is indirect investment via the value of the ongoing relationship. Such close networks of suppliers inextricably link the financial health of firms in the supply chain.

Empirically, there is evidence both that the financial health of suppliers matters to buyers and that collaborative relationships are beneficial to suppliers. Choi and Hartley (1996), in a survey of suppliers and automakers in the US industry, find that financial issues are a primary factor in supplier selection, and furthermore that greater importance is placed on financial health by downstream firms (i.e., the auto assemblers) than by upstream firms. Srinivasan and Brush (2006) find empirical evidence (outside of the auto industry) that buyer-supplier collaboration and target pricing benefit the financial performance of suppliers.

Thus, in recent years, American companies have attempted to emulate some aspects of the Japanese system, in particular the narrowing of supplier bases and longer term contracts; see, for example, Dyer (1996), Tang (1999), and more recently Wernle (2006b). Nevertheless, as Sako and Helper (1998) demonstrate, trust between supply chain partners is still higher among Japanese firms than among American firms. Furthermore, Rudambi and Helper (1998) find empirical evidence for non-cooperative behavior in the U.S. auto industry, suggesting that significant differences still persist between the American and Japanese auto industries. Consequently, a buyer only has incentive to keep suppliers in good financial standing if it benefits the buyer through lower costs. This is precisely the situation that we model: a non-cooperative supplier-buyer relationship in which the buyer is concerned with the failure of a supplier only to the extent that it might be costly for the buyer to switch suppliers.

We will examine two scenarios: a long-term contract that covers the entire horizon, and a series of repeated, short-term contracts. Li and Debo (2005) examine a similar model, quantifying the value of commitment to a single firm in a two period newsvendor context in which suppliers have private information about their production costs. The buyer runs an auction to pick a supplier, and may choose to run an auction in each period or to commit to a supplier in the first period. They find that a long-term contract increases the aggressiveness of supplier bidding, and thus helps to counteract the effect of information asymmetry. While the results of Li and Debo (2005) are driven by information asymmetries, our model is driven by failure risk and cost uncertainty; *ex ante*, there is no information asymmetry in our setting.

The short-term contracts in our model are essentially relational contracts, which constitute an emerging topic in the operations literature. The term "relational contract" refers to the fact that the enforcement of the contract comes from the value of the future relationship rather than from direct legal enforcement. Examples of related papers include Taylor and Plambeck (2003) and Atkins et al. (2005). These models contain single-buyer, single-supplier relationships, in contrast to our own model with two suppliers. Tunca and Zenios (2006), on the other hand, examine a model of repeated contracting with multiple suppliers and buyers. The authors compare relational contracts for high quality components with and without a secondary electronic market for low quality components. Their model, in contrast to ours, has a powerful high-quality supplier who both offers the contract to the buyer and does so before a group of low-quality suppliers (i.e., the high-quality supplier is a Stackelberg leader).

None of these papers considers the endogenous effect of one partner leaving the relationship due to failure, high production costs, low capital, etc. (although discount factors in repeated games can be thought of as an *exogenous* probability of relationship termination).

There is also a related literature comparing long- and short-term contracts under cost uncertainty, surveyed by Kleindorfer and Wu (2003). In much of this literature, long-term contracts refer to those signed prior to the realization of some random variable (e.g., cost or demand) and short-term contracts are those signed after the realization of this stochastic event, with demand usually occurring in a single period. One exception is the multiperiod model of Cohen and Agrawal (1999); however, in contrast to our own model, contract price is not a decision variable (although contract length is) and there is no chance of contract termination due to supplier default. For a general reference on contracting in supply chains, we refer to the survey by Cachon (2003).

Papers that have considered firm bankruptcy or failure in an operational context are Archibald et al. (2002) and Swinney et al. (2005). Both of these papers define failure in the same way as the present work: if capital falls below a prespecified level, the firm ceases to exist. The first paper looks at a monopoly setting, while the latter considers duopolies. Neither consider contracting effects between firms.

In a closely related paper, Babich et al. (2005) analyze a model with multiple suppliers and a single buyer, wherein suppliers face an exogenous probability of default and act as Stackelberg leaders in setting the wholesale price for a downstream newsvendor. Our model differs in that the default risk is endogenous (i.e., it is a function of the contract price between buyer and supplier, implying the buyer's business has a significant effect on the supplier's financial health) and the bargaining power lies with the buyer. Furthermore, Babich et al. (2005) consider a single period model, whereas our model considers contracting effects over multiple periods. Related papers include Babich (2006).

Tomlin (2006) considers methods of mitigating disruption risk when the buyer may contract with reliable and unreliable suppliers. The focus is on sourcing and contingency strategies to help mitigate the effects of disruption risk under uncertain demand, whereas our focus is on contract parameters that directly minimize the buyer's expenses from supplier failure under uncertain costs. In much of the disruption risk literature, the risk of failure or default is exogenous, whereas in our analysis, it is endogenously determined by the contract price between the buyer and the supplier. To summarize, we are not aware of any work that, like ours, uses contracting to mitigate the harmful effects of endogenous supplier default.

3. The Model

There are two identical suppliers (subscript $i = 1, 2$) and a single buyer (subscript b). The analysis is unchanged if we consider a pool of an arbitrary number of potential (*ex-ante* identical) suppliers. The buyer requires some critical component in each of two periods, and will contract with one supplier at a time in order to obtain the component. We assume that dual sourcing is too costly to be considered (i.e., there is a large fixed cost to doing business with any supplier); Tang (1999) describes how American companies have recently reduced supplier bases and increased the frequency of sole-sourcing to control costs. Demand is identical and known in each period and without loss of generality is normalized to one, consistent with the automotive industry, in which short-term forecasts of sales (and, in particular, of procurement quantities from suppliers) are fairly accurate for mature products. (We will relax the assumption of deterministic demand in §7.1.) The buyer sells the finished product for price s.

Each supplier has a linear unit production cost that is the sum of two independent stochastic components: common costs c_t, $t = 1, 2$, which are identical for both suppliers but may be time dependent, and idiosyncratic costs d_i, $i = 1, 2$ that are unique to a given supplier but may be correlated with one another, and are constant across time. Thus, the total production cost of supplier i at time t is $c_t + d_i$. For generality, we allow both random variables to have support in $(-\infty, \infty)$, though they may be restricted to some smaller interval. The uncertainty in common cost arises from stochastic elements that affect both firms: for example, raw materials costs or product design specifications. The uncertainty in a firm's idiosyncratic cost arises from factors unique to a given supplier, such as the efficiency of a supplier's production facilities or its level of technical expertise, reflecting an implicit notion of cost discovery in the production process; suppliers may have some estimate about their inherent efficiency, but until they physically produce a large number of units, the precise value of this cost is unknown.

All firms have identical beliefs that c_1 has distribution $F(\cdot)$, unimodal density $f(\cdot)$, and finite mean $\mu_1 > 0$. The second period common cost may depend on the realized value of the first period cost. The conditional mean is defined as $\mu_2(x) = \mathbb{E}(c_2|c_1 = x)$, and the unconditional mean is $\mu_2 > 0$, and both are assumed to be finite.

The idiosyncratic costs of the two suppliers may be correlated; however, since the suppliers are assumed to be *ex-ante* identical, the marginal distributions are identical as well, and all firms thus have beliefs that the marginal cdf is $G(x)$ and the marginal pdf is $g(x)$. The conditional mean is $\mu_d(x) = \mathbb{E}(d_2|d_1 = x)$ and the unconditional mean is $\mu_d > 0$, where both are assumed to be finite. The correlation coefficient between d_1 and d_2 is denoted ρ_d. For technical reasons, we assume that $x - \mu_d(x)$ is monotonically increasing. (The complementary case of $x - \mu_d(x)$ monotonically decreasing in x is impossible if the marginal distributions of d_1 and d_2 are identical.) For example, this assumption holds if d_1 and d_2 are bivariate normal.

There is no private information in the model; all parties learn the values of all random variables when they are realized. For example, if supplier 1 produces in the first period, then by the start of the second period both suppliers and the buyer know the values of c_1 and d_1. The second period common cost and d_2 are still unknown at this point, though the realized value of d_1 may convey some information about d_2 if the two have non-zero correlation. Our assumption of no private information comes for a variety of reasons reflective of the automotive industry: for example, sources of common cost uncertainty are clearly known well by the buyer (e.g., raw materials, design specifications), while the buyer is also likely to have a good estimate of the idiosyncratic capabilities of each supplier from existing (or previous) relationships (in the auto industry, given the limited number of suppliers, it is unlikely that a buyer has never worked with a given supplier in the past). Finally, most suppliers are public companies and much of the labor is unionized, meaning the general financial state of each supplier and labor costs are public information.

We assume that both suppliers are distressed at the start of the game; that is, they are already in danger of bankruptcy when the buyer offers a contract at the start of period one. Suppliers are already heavily leveraged at the time of contract signing, and hence cannot borrow additional funds from external sources (though it may be desirable to borrow or transfer funds from the buyer; this is discussed in §7.2). Payments on outstanding debt are made in alignment with the production time periods of our model. The cash flows to the suppliers are depicted in Figure 1. The total capital level of a supplier is the difference between the cash inflows and the cash outflows. If the total cash flow becomes negative, then the supplier is incapable of making the necessary debt payments and will enter bankruptcy. (Throughout, we use the terms "failure," "default," and "bankruptcy" synonymously for the sake of variety.) Without loss of generality, we normalize the sum of the first two components in each column to zero; that is, existing capital plus loans less interest payments and fixed operating expenses equal zero. Consequently, the supplier fails if, at the end of any period,

In	**Out**
Existing Capital	Interest Payment
Loans	Fixed Operating Expenses
Revenue from Buyer	Production Expenses

Figure 1. Cash flows to the suppliers in each period.

the total profit from operations with the buyer falls below the bankruptcy threshold of zero.

If the supplier fails, then the relationship with the buyer is broken, and the buyer must switch to a new supplier incurring a switching cost of $k \geq 0$. If the buyer chooses to switch suppliers after one period of production, the same switching cost k is incurred. Alternatively, k may be interpreted as a fixed setup cost incurred upon doing business with any supplier. This value also includes any additional costs incurred by the buyer to expedite production with the alternative supplier.

The expected profit to the buyer is π_b, and the expected profit to supplier i is π_i. We assume that the buyer maximizes expected profit, and that supplier i agrees to any contract with expected profit no less than the reservation level of zero. Non-zero reservation levels and bankruptcy thresholds merely add constant terms to the contract parameters and do not alter the qualitative nature of the results, and so for notational simplicity they are omitted.

We compare two types of contracts: short- and long-term. The short-term contract (Figure 2, top) provides legally enforceable terms for only one period of procurement, while the long-term contract (Figure 2, bottom) covers both periods. Since suppliers are *ex-ante* identical, without loss of generality, we assume that the buyer contracts first with supplier 1. The price offered to supplier i in period t is denoted p_{it}, thus the relevant prices are: p_{11} and p_{12} (respectively, first and second period prices with supplier 1) and p_{22} (second period price with supplier 2, if supplier 1 fails or if the buyer chooses to switch suppliers). We denote the optimal profits and prices in a long-term contract with the superscript l and the optimal prices and profits in a short-term contract with the superscript s. In all contracts, we seek prices that are subgame perfect–that is, the offered price at any given time must be optimal from the point of view of the buyer at that point in time. Initially, we assume that all contracts are static in nature (i.e., the prices cannot depend on the realized value of any random variables), as this is commonly the situation observed in the U.S. auto industry (McMillan 1990). We leave the analysis of less rigid contract forms to §6 and §7.

One might ask why the long-term contract is not subject to renegotiation at the start of

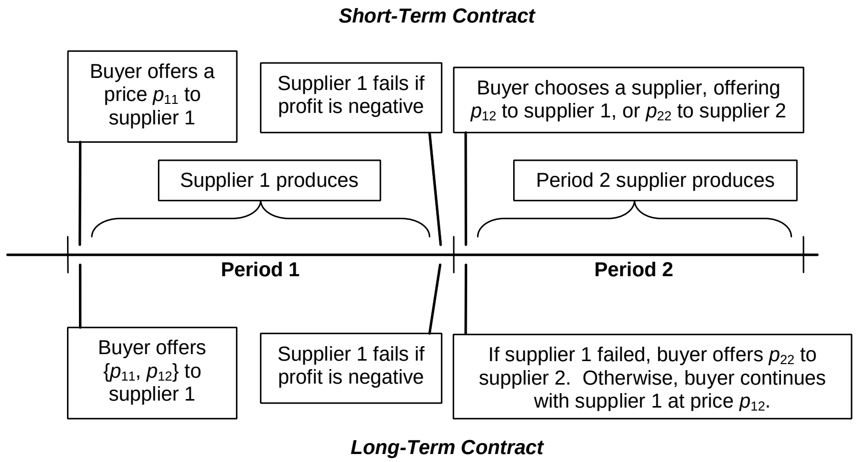

Figure 2. Sequence of events in a short-term (top) and long-term (bottom) contract.

the second period, provided the original supplier does not fail. Indeed, the susceptibility of long-term contracts to renegotiation may have a large effect of the performance of the contract and its ability to coordinate the supply chain; see, for example, Plambeck and Taylor (2007). There are two relevant types of renegotiation to consider: that initiated by the supplier (i.e., hold-up) and that initiated by buyer (exploiting his powerful bargaining position). We discuss buyer-initiated renegotiation in §7.3. As for supplier renegotiation, we explicitly exclude this possibility on the assumption that the buyer is powerful enough to thwart any attempt by a supplier at hold-up. A relevant and timely example is that of the Lear Corporation discussed in the introduction (Barkholz and Sherefkin 2006), in which Chrysler took Lear to court to enforce the contractual terms despite ample evidence of Lear's financial distress. Indeed, in response to Lear's attempts to raise prices, Chrysler replaced the firm with rival Johnson Controls as seat supplier for the Dodge Ram starting in 2008, a move that further imperiled the Lear's financial health. Thus, in the case of Lear, there are both court enforceable and relational effects in play: the court enforced the formal contract with Chrysler, and Chrysler initiated a relational punishment by switching suppliers on a later car model.

4. A Benchmark Model Without Failure

We first examine a model in which suppliers never default (i.e., negative profits do not force the buyer to switch suppliers). By comparing the results to a model with supplier failure, we will determine how the threat of default alters the behavior of the buyer and the suppliers. This model also serves as a benchmark, providing the maximum possible expected profit that the coordinated system can achieve.

The primary difference between the model with failure and the model without failure is that, in the latter model, the buyer never switches suppliers in a two period contract (and hence never incurs a switching cost). In a single period contract, the buyer only switches suppliers if it is cost effective for him to do so after taking into account the switching costs. Consequently, the model without failure provides an upper bound on the performance of the system with failure. This is an important observation, as we will derive a contract in a later section that achieves this upper bound even in the presence of suppliers prone to default.

The following theorem details the optimal contracts (both short- and long-term) for the no failure case. It is useful to define the following critical cost value: $\alpha = \{x : k + \mu_d(x) = x\}$, if such a solution exists, otherwise $\alpha = \infty$. Given that $x - \mu_d(x)$ is assumed to be increasing in x, there is at most one solution to this equation.

Theorem 1 *(i) In the absence of failure, the optimal short-term contract is* $p_{11}^s = \mu_d + \mu_1$, $p_{12}^s = d_1 + \mu_2(c_1)$, *and* $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$, *where* d_1 *is the realized value of supplier 1's idiosyncratic costs. The buyer switches suppliers in the second period if* $k + \mu_d(d_1) < d_1$ *(i.e., if* $\alpha < d_1$*). The resulting expected profit for the buyer is*

$$\pi_b^s = 2s - \mu_d - \mu_1 - \mu_2 - \mathbb{E}\min\left(k + \mu_d(d_1), d_1\right). \tag{1}$$

(ii) In the absence of failure, the optimal long-term contract is any pair $\{p_{11}^l, p_{12}^l\}$ *such that* $p_{11}^l + p_{12}^l = 2\mu_d + \mu_1 + \mu_2$, *and the resulting expected profit for the buyer is*

$$\pi_b^l = 2s - 2\mu_d - \mu_1 - \mu_2. \tag{2}$$

(iii) In the absence of failure, the buyer always prefers a short-term contract to a long-term contract. Furthermore, among single-sourcing contracts, the optimal short-term contract achieves the centralized system optimal profit, which we denote by $\overline{\pi}_b$.

Proof. All proofs appear in the technical appendix. ■

The reason why the buyer prefers the short-term contract is simple: when there is no possibility of failure, the buyer only switches suppliers when the alternative supplier has lower expected costs. The long-term contract eliminates the buyer's opportunity to switch suppliers, an option which always has value in the absence of supplier default.

Figure 3. Optimal second period action of the buyer as a function of the realized values of c_1 and d_1 in the short-term contract when there is no possibility supplier of failure.

The fact that the short-term contract achieves the system optimal (first best) profit is also intuitive. The short-term contract in Theorem 1 ensures that the buyer uses the most efficient supplier (in expectation, after accounting for switching costs) in each period, precisely the same goal that the centralized system has (i.e., if one firm controlled the buyer and both suppliers); thus, in a short-term contract without failure, the total profit in the system is maximized, so the contract is optimal among all (single-sourcing) contract types. Figure 3 details the second period actions of the buyer as a function of the realized values of the random variables.

5. Suppliers Prone to Default

In this section we consider contracts between a buyer and suppliers that are prone to default. Recall that suppliers default if, at the end of any period, their total profit is negative. For the purposes of the buyer, default only matters if it happens at the end of period 1, i.e., if $p_{11} < c_1 + d_1$; otherwise, supplier 1 survives.

5.1 Short-Term Contract

Under a short-term contract, the buyer may switch suppliers voluntarily (as in the no failure case) or involuntarily (if supplier 1 defaults). In determining the optimal second period action, the buyer finds himself in one of three separate cases, depending on the realized values of the cost parameters. In the first case (which we call Region I), supplier 1 survives the first period and the buyer chooses to continue with that supplier in the second period. This case occurs if total first period costs are small (i.e., below p_{11}) and if supplier 1's idiosyncratic costs are low. In the second case (Region II), supplier 1 survives but the buyer switches to supplier 2 in the second period. This region occurs if total first period costs are small but supplier 1's idiosyncratic costs are large. In the third case (Region III), supplier 1 defaults, and the buyer must switch to supplier 2 in the second period. This happens if total first period costs are high (i.e., above p_{11}). The buyer controls the size and shape of these regions via the offered prices; see Figure 4 for an illustration. Comparing this graph to Figure 3, we see that the addition of Region III forces the buyer to switch suppliers for a much larger region of the probability space.

The following lemma details when these regions occur, and characterizes the resulting optimal short-term contract. We adopt the convention that $\pi_b^s(p_{11})$ denotes the buyer's optimal expected profit in a short-term contract as a function of p_{11} (e.g., with all other prices set optimally), and hence $\pi_b^s = \max_{p_{11}} \pi_b^s(p_{11})$.

Lemma 1 *Define p_{11}^* as the solution to*

$$-1 + \int_{-\infty}^{\alpha} (\mu_d(x) + k - x) f(p_{11}^* - x) g(x) dx = 0.$$

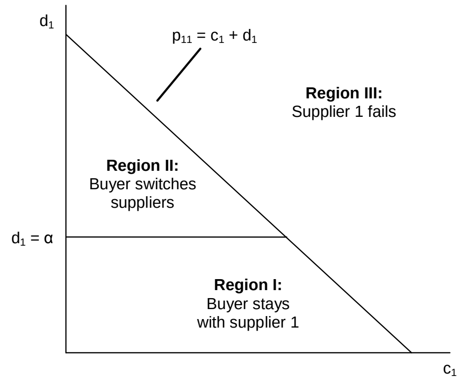

Figure 4. Optimal second period action of the buyer as a function of the realized values of c_1 and d_1 in the short-term contract when there is a risk of supplier failure.

Then the buyer's optimal short-term contract consists of $p_{12}^s = d_1 + \mu_2(c_1)$, $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$, *and*

$$p_{11}^s = \begin{cases} p_{11}^* & \text{if } p_{11}^* \geq \mu_d + \mu_c \text{ and } \pi_b^s(p_{11}^*) \geq \pi_b^s(\mu_d + \mu_c) \\ \mu_d + \mu_1 & \text{otherwise} \end{cases},$$

where $\pi_b^s(p_{11})$ *is given by,*

$$\begin{aligned} \pi_b^s(p_{11}) &= 2s - p_{11} - \mu_2 - \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(x) + k \,|c_1 + d_1 > p_{11}) \\ &\quad - \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(\min(d_1, \mu_d(x) + k) \,|c_1 + d_1 \leq p_{11}). \end{aligned} \tag{3}$$

The buyer switches suppliers in period 2 if $k + \mu_d(d_1) < d_1$.

The key to understanding the form of the optimal contract in Lemma 1 lies in understanding the shape of the expected profit function in Figure 5. The slope of $\pi_b^s(p_{11})$ asymptotically approaches -1 as p_{11} goes to $\pm\infty$, and π_b^s has a convex-concave shape. Thus, $\pi_b^s(p_{11})$ either has a local maximum (which is p_{11}^* from the lemma) or it is decreasing everywhere. Ensuring that supplier 1's participation constraint holds limits the buyer to a feasible region consisting of $p_{11} \geq \mu_d + \mu_1$. Thus, the optimal first period price is either $\mu_d + \mu_1$ or p_{11}^*.

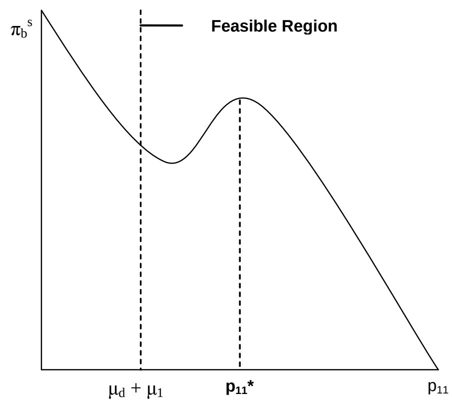

Figure 5. An example of the buyer's expected profit as a function of p_{11} in the single period contract with the risk of failure.

The intuition behind this result is that, if switching costs are small, the buyer's profit is likely to be decreasing in p_{11}; there is little consequence to failure, hence the buyer offers the lowest possible price. However, if switching costs are high, $\pi_b^s(p_{11})$ has a shape like that depicted in Figure 5. If p_{11} is very small, increasing it slightly does nothing to lower the chance of default and only costs the buyer more; likewise, if p_{11} is very large, then a slight increase does little to affect the probability of default. However, if p_{11} is intermediate in value then a small change may result in a large decrease in the probability of failure, outweighing the excess cost to the buyer. Thus, it may be optimal to offer a price that is higher than the supplier's minimum acceptable price in order to lower the probability of default.

It is interesting that the optimal contract derived in Lemma 1 is identical to the optimal contract in Theorem 1, except for the first period price p_{11}^s, because failure is irrelevant (from the buyer's point of view) in the second period. Furthermore, with failure, the first period price is greater than the first period price without failure, since the minimum possible price from Lemma 1 is equal to the optimal price in Theorem 1. Thus, comparing short-term contracts, we see that the buyer pays more when suppliers face an endogenous default risk than when suppliers have no risk of default, in order to reduce the probability of failure and hence the chance of incurring the switching cost k. In addition, by applying the Envelope Theorem, it can be shown that the buyer's optimal profit is decreasing in k; intuitively, the more expensive it is to switch suppliers in the middle of the product's sale horizon, the lower the buyer's expected profit.

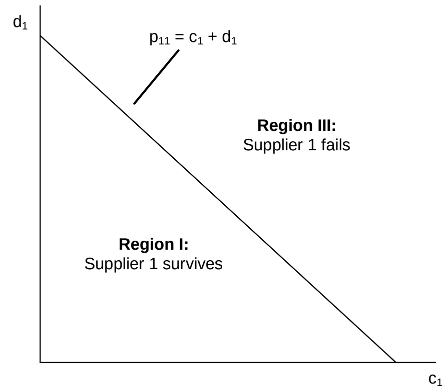

Figure 6. Optimal second period action of the buyer as a function of the realized values of c_1 and d_1 under the long-term contract.

5.2 Long-Term Contract

In a long-term static contract, the buyer offers a fixed set of prices $\{p_{11}, p_{12}\}$ to the first supplier. If the supplier accepts the contract, the buyer only switches suppliers in the event of supplier 1's bankruptcy. Thus, in contrast to the short-term contract, there are two rather than three regions of interest. In Region I, supplier 1 survives the first period, and the buyer continues to do business with that supplier at the agreed upon price of p_{12} in the second period. In Region III, supplier 1 fails, leaving the buyer with only one alternative: to switch to supplier 2. See Figure 6 for a graphical representation of these regions. The slope of the line is fixed and cannot be controlled by the buyer, hence he is incapable of replicating the optimal switching (found in Figure 3) if supplier 1 turns out to be high-cost.

In what follows, it will be useful to define the following function: let $p_{12}(p_{11})$ be the second period price when the first period price is p_{11} and supplier 1's participation constraint binds.

Lemma 2 *Let p^l_{11} be the solution to*

$$\int_{-\infty}^{\infty} (\mu_d(x) + k - x) f(p^l_{11} - x) g(x) dx = 0. \tag{4}$$

Then, the optimal long-term contract under the threat of default is p^l_{11}, $p^l_{12} = p_{12}\left(p^l_{11}\right)$, and $p^l_{22} = \mu_d + \mu_2(c_1)$.

Note that in Lemma 2, we have not restricted p^l_{12} to be non-negative. Numerically, it is rare to observe $p^l_{12} < 0$, but not impossible. For this to occur, the switching cost must be very large (e.g., an order of magnitude greater than the average total production cost). The economic interpretation of a negative second period price is that, if the expected cost incurred due to default is extremely large (i.e., if the chance of default is high or k is large), it is optimal for the buyer to heavily subsidize the supplier, and in return, if costs turn out to be low, the supplier reimburses the buyer in the second period for insuring the firm against default.

5.3 Contract Choice

To determine which contract the buyer prefers, we must compare expected profits under the optimal contract in each case.

Theorem 2 *In the presence of failure risk, (i) $\pi^s_b, \pi^l_b \leq \overline{\pi}_b$ and (ii) there exists some k^* such that, for all $k > k^*$, $\pi^s_b \leq \pi^l_b$.*

In other words, the long-term contract is preferred to the short-term contract if switching costs are high, but neither contract achieves the system optimal profit $\overline{\pi}_b$. The first part of theorem is intuitive, as we expect the system to perform no better under the threat of default than a system without failure. The second part of theorem demonstrates that long-term contracts are preferred when switching costs are high. Essentially, long-term contracts allow the buyer to shift more of the total compensation to the first period, thus lowering the chance of supplier failure. This comes at the expense of losing the option to voluntarily switch suppliers, and hence the buyer only prefers the long-term contract if the savings due to the decreased chance of default outweigh the lost option to switch suppliers. Numerically, we observe that the threshold k^* is typically very small in relation to the average production costs in the system (e.g., an order of magnitude or more), and in many cases the long-term contract is preferred for all non-negative values of k. In a numerical study consisting of 243 sets of parameters (see §5 of the appendix for details), we found that on average, k^* was 27% of the total mean per unit production cost. Thus, for many reasonable parameters (i.e., moderately significant switching costs satisfying $k \gtrsim 0.3\,(\mu_c + \mu_1)$), the buyer prefers the long-term contract.

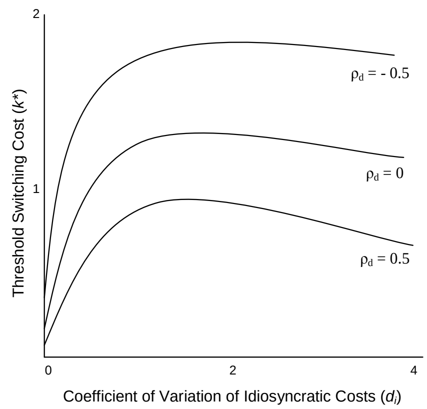

Figure 7. An example of the general behavior of the threshold switching cost as a function of variability in the idiosyncratic production cost observed in numerical experiments. Both idiosyncratic and common costs are normally distributed with mean 3.

It is also interesting to note how the value of k^* changes as a function of the variability in the idiosyncratic cost parameter. Figure 7 provides an example of the typical behavior observed in numerical experiments using normally distributed costs. First, we note that for any given value of the coefficient of variation, the value of k^* is decreasing in ρ_d. This is due to the fact that the value of the option to voluntarily switch suppliers is also decreasing in ρ_d, and hence the relative value of the short-term contract compared to the long-term contract is decreasing as costs become more correlated. Thus, the buyer is more likely to prefer the short-term contract for smaller switching costs. In addition, for fixed ρ_d, the threshold k^* has a quasi-concave shape (although we note that, depending on the problem parameters, the functions do not necessarily have a decreasing portion). This is due to the fact that there are two competing forces at play affecting the value of the contracts. As the variability of d_i increases, the value of the option to switch suppliers in the short-term contract increases; the buyer is able to exploit the low realizations of d_i more efficiently in the short-term contract, hence k^* is increasing. On the other hand, as the variability of d_i increases, the chance of default does as well, which increases the relative attractiveness of the long-term contract. If the variability is very high, failure is frequent and costly, hence the default effect dominates the option effect and k^* is decreasing; the opposite holds if the variability is very low because the chance of failure is small.

6. Dynamic Contracts

Now that we have shown that the contracts described in the previous sections perform worse than the centralized system, we move on to a class of contracts that does coordinate the channel. This class of contracts is "dynamic" or state-dependent, as opposed to the previous "static" contracts. The sequence of events is precisely the same in a dynamic contract as in a static contract. The difference between the two is that the prices in a static contract are fixed, whereas prices in a dynamic contract may be tied to some index in order to help insulate the supplier against failure by shifting some of the risk to the buyer. For example, suppose the uncertainty in the common cost c_t is primarily due to fluctuations in the global petroleum market. Then in forming a dynamic contract, the buyer might tie the contract price to the commodity price of oil, compensating the supplier for part or all of the variation in the common cost. These types of contracts are frequently observed in Japanese industry, contrasting with traditionally static contracts in the U.S. industry. For example, McMillan (1990) describes how Japanese manufacturers typically do not specify prices in initial contracts, but rather update prices every six months based on a review of the supplier's production costs, considering separately such issues as labor, raw materials, design changes and energy costs. Buyers typically allow unavoidable cost increases, such as raw materials, to be reflected in the contract price, but are less likely to allow increases due to controllable costs, such as labor.

For the purpose of our analysis, we assume that the dynamic contract compensates the supplier perfectly for c_t. Effectively, this assumption removes c_t from the contracting problem by having the buyer bear this cost in its entirety. The decision of the buyer is then how to compensate the supplier for idiosyncratic costs, d_i. Since this contract will be shown to coordinate the channel, there is no loss of generality in restricting attention to this particular form of cost compensation as opposed to some other form (e.g., dynamically compensating for idiosyncratic costs or some combination of the two components).

We assume that there is no additional cost to implementing a dynamic contract; however, dynamic contracts may indeed be difficult or costly to administer, factors which may reduce their relative attractiveness. Administrative costs are not explicitly modeled here, but might include, for example, tracking and verification of the pricing index which determines the supplier's compensation.

6.1 Dynamic Contracts without Failure

The following lemma details the optimal contracts for the no failure case.

Lemma 3 *(i) The optimal short-term dynamic contract is* $p_{11}^s = \mu_d + c_1$, $p_{12}^s = d_1 + c_2$, *and* $p_{22}^s = \mu_d(d_1) + c_2$. *The buyer switches suppliers in the second period if* $k + \mu_d(d_1) < d_1$.

(ii) The optimal long-term dynamic contract is any pair $\{p_{11}^l, p_{12}^l\}$ *such that* $p_{11}^l + p_{12}^l = 2\mu_d + \mu_1 + \mu_2$.

(iii) The expected profit in each dynamic contract is equal to the expected profit in their static counterparts in Theorem 1.

The preceding lemma provides an interesting result: in the absence of failure risk, in terms of expected profit, the dynamic contracts are equivalent to static contracts of the same length. In other words, to a risk neutral buyer, choosing a dynamic contract offers no advantage.

6.2 Dynamic Contracts with Failure

Dynamic contracts transfer the risk of common cost uncertainty from the supplier to the buyer, thus lowering the probability of failure due to high common costs. Since we assume that the buyer is a large, risk-neutral firm, transferring this risk increases the relative attractiveness of the dynamic contracts to the buyer when supplier failure is a possibility. The following lemma demonstrates this fact.

Lemma 4 *(i) The optimal long-term dynamic contract is* $p_{11}^l = \alpha + c_1$, $p_{12}^l = p_{12}\left(p_{11}^l\right)$, *and* $p_{22}^l = \mu_d(d_1) + c_2$, *where* $p_{12}(p_{11}^l)$ *is the dynamic second period price for which the supplier's participation constraint binds.*

(ii) The optimal short-term dynamic contract is given by $p_{11}^s = c_1 + \max(x^*, \mu_d)$, $p_{12}^s =$

$d_1 + c_2$, and $p_{22}^s = \mu_d(d_1) + c_2$, where x^* is the solution to

$$-1 + g(x^*)\left(-x^* + \mu_d(x^*) + k\right) = 0.$$

(iii) The long-term dynamic contract is preferred to the short-term dynamic contract, and yields expected profit equal to the system optimal expected profit without failure risk.

It is interesting that in Lemma 4 we have precisely the opposite result from Theorem 1: long-term dynamic contracts are always preferred to short-term dynamic contracts. The reason is that long-term contracts allow the buyer to switch suppliers in the optimal manner; by setting $p_{11} = \alpha + c_1$, supplier 1 fails (and hence the buyer switches suppliers) if $\mu_d(d_1) + k < d_1$, exactly the same action the centralized system would make. The second period price then serves as a compensation mechanism to ensure supplier 1 has a binding participation constraint. In the short-term contract, however, second period prices must be subgame perfect. Thus, the buyer cannot promise a price that justifies setting $p_{11} = \alpha + c_1$ as the first period price, and hence he cannot switch suppliers in the system optimal manner. Consequently, the first period price is strictly less than $\alpha + c_1$ and total profits are lower than in the long-term contract.

The fact that long-term dynamic contracts in the presence of default risk achieve the system optimal solution *without failure* is intriguing. It means that buyers can simultaneously lock-in suppliers, form lasting relationships built on trust and dynamic cost compensation, yet still achieve the proper filtering of costly suppliers in order to maximize their own profits. Indeed, if we were to plot the actions resulting from the optimal dynamic long-term contract it would look exactly like Figure 3, with the only difference being that the regions in the dynamic contract case are determined by failure rather than by the buyer's choice to switch suppliers. Furthermore, since a first period price of $\alpha + c_1$ is never optimal in the short-term dynamic contract, this contract is incapable of replicating the switching in Figure 3 and hence cannot coordinate the channel.

The value of a dynamic contract lies in its ability to remove the stochastic element that affects both suppliers from the factors affecting default. With static contracts, a potentially efficient supplier (i.e., a supplier with lower idiosyncratic costs than the expected costs of the alternative supplier) may fail due to high common costs, which reduces overall supply chain efficiency; with dynamic contracts, on the other hand, suppliers only fail if their idiosyncratic costs are high, precisely the situation in which the buyer would switch suppliers voluntarily. Hence, dynamic contracts eliminate a harmful source of stochasticity (common costs) while retaining a potentially useful source of stochasticity (idiosyncratic costs), from the supply chain's point of view.

7. Extensions

In this section we analyze four independent extensions to the core model that allow us to comment further on the scope of our results. In the first subsection we consider the effects of demand uncertainty. The second subsection addresses contingent transfer payments– that is, transfer payments made from the buyer to a supplier (possibly dependent on cost realizations) intended to subsidize the supplier and prevent bankruptcy. The third subsection addresses the issue of renegotiation. The final subsection discusses the special case of normally distributed costs, using the increased specificity of the model to answer several interesting questions concerning the performance of the various contracts as a function of cost correlation and uncertainty.

7.1 Demand Uncertainty

We have assumed throughout the paper that the buyer's demand is deterministic and equal to one in each period. In practice, suppliers may face demand uncertainty in addition to cost uncertainty. However, because we have explicitly incorporated cost uncertainty into the model, this assumption is equivalent to a make-to-order system with uncertain demand. To see this, consider a supplier that only produces units that are purchased by the buyer. The total size of the buyer's order in period t is a random variable D_t that is independent of both idiosyncratic costs, d_i, and common costs, c_t. The supplier may face exogenous capacity constraints, in which case D_t is truncated at the capacity level of the supplier with mass added to the endpoint. The profit to supplier i in period t is then $D_t\,(p_t - d_i - c_t)$.

Assume that the supplier must make a minimum profit of K_t in period t in order to survive. K_t may represent the supplier's annual (fixed) operating costs, interest payments on outstanding loans, capital outlay for the new production process, legacy pension expenses, etc. Note that K_t is allowed to be negative, i.e., the supplier may be allowed to lose some money and still survive, and may evolve across time; however, because suppliers are *ex-ante* identical, K_t is the same for both suppliers in each period. Thus, supplier i survives in period t if $D_t(p_t - d_i - c_t) \geq K_t$. Since all the random variables are independent from one another, we may write this as $p_t - d_i - c_t \geq K_t/D_t$. Redefining the common cost to incorporate the demand term, $c'_t = c_t + K_t/D_t$, we see that supplier i survives if and only if profit is non-negative, i.e.,

$$p_t - d_i - c'_t \geq 0. \tag{5}$$

In addition, we may consider the case in which the supplier only accepts a contract that yields expected profits of at least K_t. In that case, the same logic yields the result that the supplier accepts any contract in which (5) holds in expectation. Both of these conditions–supplier survival and contract acceptance–are identical to the case with deterministic demand in each period, so long as the common cost terms are properly defined; consequently, our assumptions of deterministic demand and no fixed operating expenses are made without loss of generality in a make-to-order production system with exogenous capacity constraints.

This equivalence also yields insight into why the common cost term is not diversifiable and hence cannot be hedged; because the common cost term may be thought of as capturing demand risk as well as common cost risk, it is unlikely that this risk could be mitigated. Furthermore, because the cost term also depends on materials prices that cannot be procured from any industrial exchange (e.g., the outputs of upstream suppliers), our implicit assumption that cost uncertainty cannot be hedged is justified.

It is important to note that our extension to the case of demand uncertainty is only valid if the supplier is uncapacitated or faces an exogenous capacity constraint. This scenario is reflective of our motivating example (the auto industry) in which the capacity is sometimes dictated by the buyer, suppliers may not have sufficient capital for creating excess capacity, capacity constraints on components are typically not tight (i.e., it is more likely that assembly capacity is binding) and, if capacity is likely to be tight, components are multi-sourced. By focusing on a single-sourced component, we have implicitly assumed that capacity constraints are not an issue for the supplier; a model in which capacity is tight may be more suitable to an analysis of multi-sourcing, see Tomlin (2006). In any event, endogenous capacity decisions are likely to provide further reasons to favor long-term contracts, as long-term relationships are known to stimulate capacity investment (Taylor and Plambeck 2003), hence it is unlikely that endogenizing the capacity decision would significantly alter our results.

7.2 Contingent Transfer Payments and Loans

The contract types we have discussed thus far are fairly simple, consisting of either fixed prices or prices that are a function of one of the stochastic elements (i.e., the common cost). When a supplier enters bankruptcy, we provide no recourse to the buyer to help alleviate the supplier's financial distress. This may be an acceptable assumption if the buyer is unable or unwilling to subsidize the supplier in the event of bankruptcy (e.g., if capital is expensive to the buyer, or if the buyer is also in financial distress); however, in some situations the buyer may prefer to make a transfer payment that allows the supplier to avoid bankruptcy and continue operations with the buyer. For example, the Big Three Detroit automakers provided $100 million in direct operating subsidies to support Collins & Aikman in bankruptcy in 2006 (Barkholz and Sherefkin 2007). In this section, we discuss precisely this scenario.

We consider the following modification of the core model: if the first supplier enters bankruptcy, then at the start of the second period, upon observing the realized value of all costs, the buyer has the option of either switching suppliers or making a transfer payment to the first supplier that raises the capital level to zero and prevents bankruptcy. In making this decision, the buyer takes into account the size of the necessary transfer payment as well as the costs of contracting with each supplier and any switching costs. The resulting transfer payment thus depends upon the realized value of *both* first period cost terms, hence it is termed a contingent transfer payment. In addition to allowing direct operating subsidies that are not reimbursed, we also consider loans from the buyer to the supplier that may be partially for fully repaid (with interest). We refer to either scenario (no repayment, or repayment) as a transfer payment.

For the details of our model of transfer payments, we refer the reader to §2 of the technical appendix. We state here, however, a critical assumption concerning the timing of the transfer payment. In what follows, we assume any transfer payment occurs at the start of the second period, and that such payments are not considered in the short-term (i.e., one period) participation constraint of the supplier. The implication of this assumption is that the supplier is unwilling to accept a lower first period price if the buyer has the option of offering a transfer payment when bankruptcy occurs. The short-term participation constraint represents the outside option of the supplier over the immediate future; given the financial distress of the firm, the primary concern (particularly when engaging in a short-term contract without the promise of future business) is likely short-term financial performance, and hence the supplier is likely unwilling to sacrifice short-term profit for a potential future transfer payment, especially when this may greatly increase the chance of bankruptcy. The consequences of this assumption are discussed in more detail below.

It is clear that the buyer does at least as well when allowed to make a transfer payment as he does in the simpler contracts we have discussed previously, since the transfer payment is entirely optional. Thus, contracts with transfer payments are preferred to contracts without such payments. The questions we then seek to address are the following. Firstly, does the result of Theorem 2 hold with transfer payments? In other words, are long-term contracts preferred when switching costs are high, even if transfer payments are available under both short- and long-term contract types? Secondly, does either contract type coordinate the system (i.e., achieve the first-best solution that the long-term dynamic contract without transfer payments achieves)?

The following theorem answers both of the questions, essentially demonstrating that all of the results of Theorem 2 hold even when the buyer is given the option of subsidizing distressed suppliers.

Theorem 3 *If the buyer is allowed to make contingent transfer payments or loans, then (i) neither the short-term nor the long-term contract achieves the first best profit ($\overline{\pi}_b$) and (ii) there exists some k^* such that, for all $k > k^*$, the long-term contract is preferred to the short-term contract.*

The intuition behind this result is that transfer payments and loans allow the buyer to avoid switching suppliers in the event of failure, provided the incumbent supplier is efficient enough to warrant a subsidy. However, this benefit of transfer payments applies to both short- and long-term contracts in different ways. Transfer payments are valuable with short-term contracts because these contracts typically involve lower first period prices and higher failure rates than long-term contracts, hence the recourse provided in subsidizing a bankrupt supplier is greater with short-term contracts because failure happens more often. On the other hand, transfer payments introduce an option to stay with an efficient (albeit bankrupt) supplier in long-term contracts, hence there is an increase in value due to the second period option effect. Numerically, we observe that neither benefit dominates, and the effect on k^* (compared to the case of no transfer payments) is ambiguous: in a numerical study consisting of 243 sets of parameters (see §5 of the appendix for details), we found that on average, k^* was 18% of the total mean per unit production cost, compared to 27% without transfer payments. In 42 out 243 cases, k^* increased due to the presence of transfer payments (i.e., the transfer payment provided more value to the short-term contract than to the long-term contract), while in the remaining 201 cases, k^* was lower with a transfer payment.

Recall that we assumed that any transfer payment is made at the start of the second period (or, more importantly, that the supplier does not consider the transfer payment in his short-term contract participation constraint). This is a key assumption: if the supplier *does* take the transfer payment into account, for large k the short-term contract essentially transforms into a long-term contract in the following sense. If switching costs are high, the buyer pays the supplier nothing for first period production. Consequently, the supplier always fails. The buyer never switches suppliers, however, preferring instead to make a transfer payment to the bankrupt supplier and avoid high switching costs. The short-term contract has effectively become a long-term contract in which the buyer promises to work with supplier 1 in the second period and pay all production costs ex-post after the supplier enters bankruptcy, and as a result the expected profit is the same with a long-term and short-term contract as k becomes large, hence there is not a strict preference between the two. (For low switching costs, as in all other cases we have considered, it remains true that the short-term contract is preferred to the long-term contract.)

This equivalence is eliminated and a strict preference for long-term contracts is restored if any of several complications arise, including: if the supplier is unwilling to accept certain bankruptcy (because of a contract price of zero) in the first period, i.e., there is a minimum acceptable contract price; if there is some fixed cost to making a transfer payment (e.g., the buyer must pay the supplier's bankruptcy or default penalty); or if the supplier discounts future revenues (which implies the supplier will not accept a contract price of zero even if a transfer payment is made in the future). Thus, although the timing of the transfer payment is important to the result of Theorem 3, this assumption may be relaxed while preserving the result if a one of a variety of alternative conditions holds.

While we have assumed that the suppliers in the current analysis are incapable of securing external funds in the event of bankruptcy between periods one and two, it is interesting to consider this case. If the supplier is capable of borrowing enough funds in any scenario to avert bankruptcy, then clearly failure has no effect to the buyer; the supplier always avoids bankruptcy and hence the model is equivalent to the model without default. If the supplier is capable of borrowing limited funds, however, and the chance of default remains, then the core results of the model are preserved; it is still true that long-term contracts hold value in allowing the buyer to rearrange the cash flows and helping the supplier avoid default.

7.3 Renegotiation

We explicitly excluded the possibility of supplier renegotiation (i.e., supplier hold-up) in long-term contracts due to the strong bargaining power of the buyer. However, there is a possibility of *buyer* initiated renegotiation, as in the Ford example discussed in the introduction. It is possible to show (see §3 of the technical appendix) that, if the buyer is allowed to renegotiate a long-term contract in the second period, all of the results are preserved. This result critically depends on the fact that, with or without renegotiation, the supplier's participation constraint is binding in the optimal contract. Thus, the buyer extracts all surplus from the supplier, and if renegotiation occurs (and the supplier anticipates the renegotiation) the buyer must compensate the supplier via a higher first period price to satisfy the supplier's participation constraint. In other words, any additional dollar extracted via renegotiation must be compensated for via the contract price. The net result is that the buyer's expected profit is the same regardless of whether renegotiation occurs, hence the preference between contracts remains identical to the cases already discussed in §§3–6.

Interestingly, if the supplier does not anticipate renegotiation in a long-term contract (i.e., if renegotiation is not taken into account in the supplier's participation constraint), then the buyer enjoys strictly greater profits in a long-term contract than in a model without renegotiation because the buyer need not compensate the supplier with a higher contract price. As a result, long-term contracts have even greater value than previously discussed, and they are thus preferred for large switching costs. Hence, the results of the paper hold even when the buyer is allowed to unilaterally renegotiate in long-term contracts, regardless of whether the supplier anticipates that the contract will be renegotiated.

7.4 Normally Distributed Costs

By assuming that costs are normally distributed, we are able to derive further insights into the behavior of the various contracts. In what follows we consider three contract types in the presence of default risk: the long-term static and dynamic contracts and the short-term static contract. Since the long-term dynamic contract dominates the short-term dynamic contract, the latter case is uninteresting and is hence omitted. Let c_1, c_2 be identically distributed (possibly correlated) $N(\mu_c, \sigma_c)$ random variables, and let d_1, d_2 be bivariate normal with identical mean and variance μ_d and σ_d^2, and correlation ρ_d. From the properties of the bivariate normal distribution, the expected value of d_2 conditional on $d_1 = x$ is $\mu_d(x) = (1 - \rho_d)\mu_d + \rho_d x$. Hence, it is optimal to switch suppliers in the second period if $d_1 > \mu_d + k/(1 - \rho_d)$. Since α is increasing in ρ_d, it is apparent that the buyer is less likely to switch suppliers if costs are highly correlated. The following theorem further describes the behavior of the contracts as a function of ρ_d and σ_d.

Theorem 4 *(i) The optimal expected profit under all contract types is decreasing in ρ_d.*

(ii) The difference between the system optimal (long-term dynamic) profit and the profit under the long-term static contract is decreasing in ρ_d. In the limit as $\rho_d \to 1$, profits are equal.

(iii) The centralized system optimal expected profit is increasing in σ_d.

Intuitively, from part (i), the profit of the system is lower if the idiosyncratic costs of the two firms are highly correlated; there is little value contained in the option to switch suppliers, so the overall expected profit of the buyer is higher when suppliers have negatively correlated idiosyncratic costs.

Part (ii) demonstrates how the relative advantage of the dynamic long-term contract varies as a function of ρ_d. If costs are strongly negatively correlated, then the dynamic long-term contract performs quite well; in this case, the dynamic contract is effective at switching suppliers in the optimal manner, while the static contract is less efficient. If costs are strongly positively correlated, however, the value of switching suppliers is lower, hence the performance gap between the two contract types is much smaller, although dynamic contracts offer value for any $\rho_d < 1$. Consequently, long-term dynamic contracts are most valuable in situations wherein suppliers have negatively correlated costs.

Part (iii) describes the behavior of the system optimal expected profit as a function of variability in the suppliers' private cost. Intuitively, the more variable the costs of the suppliers, the more likely a low cost realization is; the buyer is shielded from high cost realizations by the option to switch suppliers. Thus, for fixed ρ_d and μ_d, increased variability in the idiosyncratic costs of the suppliers allows the buyer to exploit his option to switch.

8. Discussion

In this paper, we have presented a model of buyer-supplier contracting primarily characterized by three features: uncertain production costs, extended sales horizons, and the strong bargaining power of the buyer. Within this context, we have shown that the threat of supplier failure can increase the buyer's preference for long-term contracts. Furthermore, dynamic contracts which compensate the suppliers for common costs (materials, etc.) achieve the system optimal profit. This feature helps to explain the adoption of these contracts in the Japanese auto industry (McMillan 1990).

We did not model risk-averse firms because discussing supply chain coordination in such a setting adds another level of complexity that is outside the scope of our work (Gan et al. 2004). However, the true value of dynamic contracts may depend on the risk-neutrality (or lack thereof) of the buyer. In practice, auto manufacturers do not always reimburse suppliers for the full amount of common costs; one potential explanation for this behavior is that the buyer is not risk-neutral, and hence does not seek to bear all of the risk associated with the variability in raw materials costs. Another possible explanation may be that the variability associated with c_t is not completely outside of the supplier's control, hence the buyer needs to leave some of the risk with the supplier in order to induce the proper actions (e.g., negotiating low prices from second tier suppliers, etc.). Finally (and perhaps most importantly), dynamic contracts are difficult to administer and are significantly less formal than static contracts, and trust between partners is key to their implementation. There has historically been a severe lack of trust between U.S. auto manufacturers and suppliers, perhaps helping to explain why both parties are hesitant to engage in dynamic contracting agreements.

Throughout the analysis, we have ignored the effects of learning curves, seen for example in Spence (1981). In much of the multiperiod contracting literature, learning curves are modeled as cooperative improvements in design and production processes that reduce costs in long-term relationships (e.g., Cohen and Agrawal 1999), whereas short-term relationships offer lesser (or no) opportunities for cost reduction. If such a learning curve were present in our model, it would serve to increase the profitability of the long-term contract, thus providing further incentive for a buyer to choose this contract type. Qualitatively, our main results should be strengthened by this complication. In addition, considering asymmetric suppliers yields a similar result: if one supplier's costs dominate the other supplier, the buyer will seek to contract in the first period with the more efficient supplier. The long-term contract then serves as a tool to both mitigate switching costs and prevent switching to a less efficient supplier; thus, the value of the long-term contract is increased.

An interesting counterpart to the case of contingent transfer payments are contracts that explicitly allow the buyer to end the relationship should the production costs of the supplier exceed a certain threshold (i.e., the buyer is provided a means of breaking the relationship if a supplier is very inefficient). Such a contract, which would provide the buyer the option of switching suppliers if the first supplier survives (rather than fails) increases the profitability of long-term relationships and hence increases the relative attractiveness of a long-term contract compared to a short-term contract, leaving our main results intact. Such contracts are unable to coordinate the system, however, because of the forced switching of suppliers that occurs if the incumbent supplier fails. Thus, dynamic contracts still perform better by achieving coordination.

There are two complications that may increase the relative value of short-term contracts: discounting of the buyer's second period profit, and non-zero bankruptcy costs for the suppliers. The first complication lessens the impact of supplier default and the cost of voluntary switching: since both of these costs are incurred in the second period, discounting decreases their relative contribution to the total expected profit. The second complication effectively increases the switching costs due to default, while leaving unchanged the switching costs due to voluntary changes in the second period supplier. Intuitively, if the suppliers incur some bankruptcy penalty, the buyer must compensate the supplier more in order to satisfy the participation constraint; thus, if the participation constraint is binding, the buyer's profit function includes an extra term penalizing for the bankruptcy costs if default occurs, which is essentially the same as increasing the switching cost k. The extra term is not present, however, when the supplier survives but the buyer voluntarily switches suppliers, hence the relative value of the short-term contract is increased because voluntary switching is less costly. Both of these complications essentially increase the threshold k^* from Theorem 2, but other results remain qualitatively unchanged.

Our model is one of partial equilibrium; in reality, as distressed suppliers enter bankruptcy and exit the market, new suppliers may enter, perhaps in better financial standing. This effect is important in the automotive industry, but occurs over long time scales (e.g., it may take years for a newly created supplier to build the necessary technology to produce at the scale and quality level of a large, existing supplier such as Delphi). Our model analyzes the medium-term issue of dealing with a distressed supplier base. In this time frame, it is often impractical for a buyer to work with an emerging supplier: particularly in the automotive industry, asset specific investments in suppliers are quite large and hence switching costs are prohibitive.

With no information asymmetry for suppliers to use as leverage, we have created a situation wherein buyers extract all of the surplus in the supply chain. This assumption has provided the strongest incentive for the buyer to engage in a short-term contract in order to exploit the option to voluntarily switch suppliers, and still we find that long-term commitment is very often the more profitable choice; unlike the results in Li and Debo (2005), the attractiveness of the long-term contract in our model is not driven by increased price competition in the supplier's bidding strategies, but rather by the decreased chance of failure resulting from a long-term commitment.

The main conclusion of our analysis is that long-term contracts may be even more valuable to buyer-supplier relations–particularly within the American automotive industry–than previously thought. The literature abounds with reasons for firms to prefer long-term contracts: as a manner of developing trust and cooperation between partners (Dyer 1996), as a tool to increase price competition in auction scenarios (Li and Debo 2005), and as relational tools to enforce actions which are otherwise unsupportable or uncontractable in short-term situations (Taylor and Plambeck 2003). We add to those reasons by demonstrating the value of long-term relationships when suppliers face the threat of failure.

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Technical Appendix to "Long-Term Contracts under the Threat of Supplier Default"

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1. Proofs

Theorem 1 *(i) In the absence of failure, the optimal short-term contract is* $p_{11}^s = \mu_d + \mu_1$, $p_{12}^s = d_1 + \mu_2(c_1)$, *and* $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$, *where* d_1 *is the realized value of supplier 1's idiosyncratic costs. The buyer switches suppliers in the second period if* $k + \mu_d(d_1) < d_1$ *(i.e., if* $\alpha < d_1$*). The resulting expected profit for the buyer is*

$$\pi_b^s = 2s - \mu_d - \mu_1 - \mu_2 - \mathbb{E}\min(k + \mu_d(d_1), d_1). \tag{1}$$

(ii) In the absence of failure, the optimal long-term contract is any pair $\{p_{11}^l, p_{12}^l\}$ *such that* $p_{11}^l + p_{12}^l = 2\mu_d + \mu_1 + \mu_2$, *and the resulting expected profit for the buyer is*

$$\pi_b^l = 2s - 2\mu_d - \mu_1 - \mu_2. \tag{2}$$

(iii) In the absence of failure, the buyer always prefers a short-term contract to a long-term contract. Furthermore, among single-sourcing contracts, the optimal short-term contract achieves the centralized system optimal profit, which we denote by $\overline{\pi}_b$.

Proof. (i) In a short-term contract, prices must be subgame perfect. Thus, it is easy to see that in the second period the buyer will offer the lowest prices that satisfy the participation constraints of the suppliers, i.e., $p_{22}^s = \mu_2(c_1) + \mu_d(d_1)$ for supplier 2 and $p_{12}^s = d_1 + \mu_2(c_1)$ for supplier 1 (since d_1 is known at the end of the first period). Recalling that the buyer must also incur a per unit switching cost of k if he decides to switch to supplier 2, we see that the buyer will switch suppliers if $k + \mu_d(d_1) < d_1$. Thus, in choosing the optimal first period price, the buyer maximizes his total expected profit,

$$\begin{aligned} \pi_b &= \max_{p_{11}} 2s - p_{11} - \mu_2 - \mathbb{E}\min(d_1, k + \mu_d(d_1)), \\ &\quad \text{s.t. } \mathbb{E}(p_{11} - c_1 - d_1) \geq 0 \end{aligned}$$

Note that the supplier earns, in expectation, zero in the second period, as the buyer knows the realized value of costs. Hence, the supplier is effectively myopic, agreeing to any contract with $\mathbb{E}(p_{11} - c_1 - d_1) \geq 0$. Thus, the buyer's optimal first period price is $p_{11}^s = \mu_d + \mu_1$. This implies the expected profit is given by (1).

(ii) In a long-term contract, the buyer maximizes $\pi_b = 2s - p_{11} - p_{12}$ subject to $p_{11} + p_{12} - 2\mu_d - \mu_1 - \mu_2 \geq 0$. The optimal contract is found by solving for the binding participation constraint.

(iii) Follows from parts (i) and (ii). ■

Lemma 1 *Define p_{11}^* as the solution to*

$$-1 + \int_{-\infty}^{\alpha} \left(\mu_d\left(x\right) + k - x\right) f(p_{11}^* - x) g(x) dx = 0.$$

Then the buyer's optimal short-term contract consists of $p_{12}^s = d_1 + \mu_2\left(c_1\right)$, $p_{22}^s = \mu_d\left(d_1\right) + \mu_2\left(c_1\right)$, and

$$p_{11}^s = \begin{cases} p_{11}^* & \text{if } p_{11}^* \geq \mu_d + \mu_c \text{ and } \pi_b^s\left(p_{11}^*\right) \geq \pi_b^s\left(\mu_d + \mu_c\right) \\ \mu_d + \mu_1 & \text{otherwise} \end{cases},$$

where $\pi_b^s\left(p_{11}\right)$ is given by,

$$\begin{aligned} \pi_b^s\left(p_{11}\right) &= 2s - p_{11} - \mu_2 - \Pr\left(c_1 + d_1 > p_{11}\right) \times \mathbb{E}\left(\mu_d\left(x\right) + k \left| c_1 + d_1 > p_{11}\right.\right) \\ &\quad - \Pr\left(c_1 + d_1 \leq p_{11}\right) \times \mathbb{E}\left(\min\left(d_1, \mu_d\left(x\right) + k\right) \left| c_1 + d_1 \leq p_{11}\right.\right). \end{aligned} \tag{3}$$

The buyer switches suppliers in period 2 if $k + \mu_d\left(d_1\right) < d_1$.

Proof. As in the proof of Theorem 1, we have $p_{22}^s = \mu_d\left(d_1\right) + \mu_2\left(c\right)$ and $p_{12}^s = d_1 + \mu_2\left(c\right)$. As in the no failure case, the buyer switches suppliers voluntarily if $\mu_d\left(d_1\right) + k < d_1$. However, the buyer involuntarily switches suppliers if the first supplier fails, i.e., if $d_1 + c_1 > p_{11}$. Thus we have a partitioning of the buyer's second period actions according to the realized values of d_1 and c_1 in the following manner (Figure 5 in the main paper): in Region I, the second period price is $d_1 + \mu_2\left(c_1\right)$; in Regions II and III, it is $\mu_d\left(d_1\right) + \mu_2\left(c_1\right)$. Thus, the buyer's expected profit is given by (3).

We next use the following identity:

$$\begin{aligned} & \Pr\left(c_1 + d_1 > p_{11}\right) \times \mathbb{E}\left(\mu_d\left(x\right) + k \left| c_1 + d_1 > p_{11}\right.\right) \\ = \; & \mu_1 + \mu_d - \Pr\left(c_1 + d_1 \leq p_{11}\right) \times \mathbb{E}\left(\mu_d\left(x\right) + k \left| c_1 + d_1 \leq p_{11}\right.\right) \end{aligned}$$

to rewrite the profit function as

$$\begin{aligned} \pi_b^s\left(p_{11}\right) &= 2s - p_{11} - \mu_1 - \mu_d - \mu_2 \\ &\quad - \Pr\left(c_1 + d_1 \leq p_{11}, d_1 < \alpha\right) \times \mathbb{E}\left(d_1 - \mu_d\left(x\right) - k \left| c_1 + d_1 \leq p_{11}, d_1 < \alpha\right.\right). \end{aligned}$$

Differentiating this expression with respect to p_{11}, we have, since d_1 and c_1 are independent and the joint density function is simply the product of the independent densities,

$$\frac{d\pi_b^s\left(p_{11}\right)}{dp_{11}} = -1 + \int_{-\infty}^{\alpha} \left(\mu_d\left(x\right) + k - x\right) f(p_{11} - x) g(x) dx.$$

Since all costs have finite positive means, it is true that $\lim_{x \to \pm\infty} f(x) = 0$ and $\lim_{x \to \pm\infty} g(x) =$

0. Thus, $\lim_{p_{11}\to\pm\infty}\frac{d\pi_b^s(p_{11})}{dp_{11}} = -1$. Define p_{11}^* to be the solution to $\left.\frac{d\pi_b^s(p_{11})}{dp_{11}}\right|_{p_{11}=p_{11}^*} = 0$ (if such a solution exists). As long as the density f is unimodal, the second derivative

$$\frac{d^2\pi_b^s(p_{11})}{dp_{11}^2} = \int_{-\infty}^{\alpha} (\mu_d(x) + k - x) f'(p_{11} - x)g(x)dx$$

is positive then negative, i.e., the profit function is convex-concave. Combined with the fact that the slope tends to -1 as p_{11} becomes large or small, the profit function is either always decreasing or has a local unconstrained maximum; see Figure 6 in the main paper. As in the no failure case, the supplier earns zero profit in expectation in the second period, thus the feasible region is any $p_{11} \geq \mu_d + \mu_1$, i.e., any p_{11} for which the supplier's participation constraint binds. Thus, there are three possibilities: (1) $p_{11}^* < \mu_d + \mu_1$, in which case the optimal price is $\mu_d + \mu_1$; (2) $p_{11}^* \geq \mu_d + \mu_1$ and $\pi_b^s(p_{11}^*) \geq \pi_b(\mu_d + \mu_1)$, in which case the optimal solution is p_{11}^*; and (3) $p_{11}^* \geq \mu_d + \mu_1$ and $\pi_b^s(p_{11}^*) < \pi_b^s(\mu_d + \mu_1)$, in which case the optimal price is $\mu_d + \mu_1$. ■

Lemma 2 *Let* p_{11}^l *be the solution to*

$$\int_{-\infty}^{\infty} (\mu_d(x) + k - x) f(p_{11}^l - x)g(x)dx = 0. \tag{4}$$

Then, the optimal long-term contract under the threat of default is p_{11}^l, $p_{12}^l = p_{12}\left(p_{11}^l\right)$, *and* $p_{22}^l = \mu_d + \mu_2(c_1)$.

Proof. We first show that at optimality, supplier 1's participation constraint is binding. First, note that as in the short-term contract $p_{22}^l = \mu_d(d_1) + \mu_2(c_1)$. Then the buyer's profit is

$$\begin{aligned}\pi_b^l(p_{11}, p_{12}) &= 2s - p_{11} - \Pr(c_1 + d_1 \leq p_{11}) \times p_{12} \\ &\quad - \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(d_1) + \mu_2(c_1) + k | c_1 + d_1 > p_{11}),\end{aligned}$$

and supplier 1 accepts any contract with

$$\mathbb{E}(p_{11} - c_1 - d_1) + \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(p_{12} - c_2 - d_1 | c_1 + d_1 \leq p_{11}) \geq 0.$$

Constructing the Lagrangean $\mathcal{L}$ with multiplier λ and taking the derivative with respect to p_{12}, we see $\frac{d\mathcal{L}}{dp_{12}} = (\lambda - 1)\Pr(c_1 + d_1 \leq p_{11})$. This implies that at optimality we must have $\lambda = 1$, hence the constraint is binding. Then we may rewrite π_b^l, substituting the binding participation constraint, to yield

$$\begin{aligned}\pi_b^l(p_{11}) &= 2s - \mathbb{E}(c_1 + d_1) - \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(c_2 + d_1 | c_1 + d_1 \leq p_{11}) \\ &\quad - \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(d_1) + \mu_2(c_1) + k | c_1 + d_1 > p_{11}).\end{aligned}$$

Since

$$\begin{aligned} & \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(c_2 + d_1 | c_1 + d_1 \leq p_{11}) \\ = \; & \mu_d + \mu_2 - \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(c_2 + d_1 | c_1 + d_1 > p_{11}), \end{aligned}$$

we have

$$\pi_b^l(p_{11}) = 2s - \mu_1 - \mu_2 - 2\mu_d + \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(d_1 - \mu_d(d_1) - k | c_1 + d_1 > p_{11}).$$

Taking derivatives with respect to p_{11}, we get

$$\begin{aligned} \frac{d\pi_b^l(p_{11})}{dp_{11}} &= \int_{-\infty}^{\infty} (\mu_d(x) + k - x) f(p_{11} - x) g(x) dx, \\ \frac{d^2\pi_b^l(p_{11})}{dp_{11}^2} &= \int_{-\infty}^{\infty} (\mu_d(x) + k - x) f'(p_{11} - x) g(x) dx \end{aligned}$$

Note that $\lim_{p_{11}\to\pm\infty} \frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$ by the assumption that $\lim_{x\to\pm\infty} f(x), g(x) = 0$. Also note that, since f is unimodal, $\frac{d^2\pi_b^l(p_{11})}{dp_{11}^2}$ is positive, then negative, then positive again, i.e., π_b is convex-concave-convex. Since $\lim_{p_{11}\to\pm\infty} \frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$, the sign of $\frac{d\pi_b^l(p_{11})}{dp_{11}}$ is $+/-$, i.e. π_b is quasi-concave. Thus, the optimal first period price is the solution to (4). ■

Theorem 2 *In the presence of failure risk, (i) $\pi_b^s, \pi_b^l \leq \overline{\pi}_b$ and (ii) there exists some k^* such that, for all $k > k^*$, $\pi_b^s \leq \pi_b^l$.*

Proof. (i) In the short-term static contract, $p_{11} \geq \mu_d + \mu_1$, so the expected profit is bounded above by

$$\begin{aligned} \pi_b^s(p_{11}) \leq \; & 2s - \mu_1 - \mu_2 - \mu_d - \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(d_1) + k | c_1 + d_1 > p_{11}) \\ & - \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(\min(d_1, \mu_d(d_1) + k) | c_1 + d_1 \leq p_{11}). \end{aligned}$$

Recall that the upper bound on profit $\overline{\pi}_b$ is given by (1), and since

$$\begin{aligned} \mathbb{E}(\min(d_1, \mu_d(d_1) + k)) \leq \; & \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(d_1) + k | c_1 + d_1 > p_{11}) \\ & + \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(\min(d_1, \mu_d(d_1) + k) | c_1 + d_1 \leq p_{11}) \end{aligned}$$

we have $\pi_b^s(p_{11}) \leq \overline{\pi}_b$ for all feasible p_{11}. The expected profit from the long-term static contract is, as a function of p_{11},

$$\begin{aligned} \pi_b^l(p_{11}) = \; & 2s - \mu_1 - \mu_2 - \mu_d - \Pr(d_1 + c_1 \leq p_{11}) \times \mathbb{E}(d_1 | d_1 + c_1 \leq p_{11}) \\ & - \Pr(d_1 + c_1 > p_{11}) \times \mathbb{E}(\mu_d(d_1) + k | d_1 + c_1 > p_{11}). \end{aligned}$$

Note that

$$\begin{aligned} \mathbb{E}(\min(d_1, \mu_d(d_1) + k)) \leq \; & \Pr(c_1 + d_1 > p_{11}) \times \mathbb{E}(\mu_d(d_1) + k | d_1 + c_1 > p_{11}) \\ & + \Pr(c_1 + d_1 \leq p_{11}) \times \mathbb{E}(d_1 | d_1 + c_1 \leq p_{11}) \end{aligned}$$

so $\pi_b^l(p_{11}) \leq \overline{\pi}_b$ for all p_{11}.

(ii) First, note that in the limit as $k \to \infty$, the buyer's expected profit in the optimal short-term contract satisfies

$$\begin{aligned}\lim_{k\to\infty} \pi_b^s &= \lim_{k\to\infty} 2s - p_{11}^s - \mu_2 - \Pr(c_1 + d_1 > p_{11}^s) \times \mathbb{E}(\mu_d(d_1) + k | c_1 + d_1 > p_{11}^s) \\ &\quad - \Pr(c_1 + d_1 \leq p_{11}^s) \times \mathbb{E}(\min(d_1, \mu_d(d_1) + k) | c_1 + d_1 \leq p_{11}^s).\end{aligned}$$

There are two possibilities: either (1) $\lim_{k\to\infty} p_{11}^s < \infty$, in which case $\lim_{k\to\infty} \pi_b^s = -\infty$ since the switching cost term dominates, or (2) $\lim_{k\to\infty} p_{11}^s = \infty$, in which case we also have $\lim_{k\to\infty} \pi_b^s(p_{11}) = -\infty$ since the p_{11}^s term dominates. In a long-term contract, on the other hand,

$$\begin{aligned}\lim_{k\to\infty} \pi_b^l &= \lim_{k\to\infty} 2s - \mu_1 - \mu_2 - \mu_d - \Pr\left(d_1 + c_1 \leq p_{11}^l\right) \times \mathbb{E}\left(d_1 | d_1 + c_1 \leq p_{11}^l\right) \\ &\quad - \Pr\left(d_1 + c_1 > p_{11}^l\right) \times \mathbb{E}\left(\mu_d(d_1) + k | d_1 + c_1 > p_{11}^l\right).\end{aligned}$$

Since p_{11}^l satisfies

$$\int_{-\infty}^{\infty} (\mu_d(x) + k - x) f(p_{11}^l - x) g(x) dx = 0,$$

it is easy to see that $\lim_{k\to\infty} p_{11}^l = \infty$, and hence $\lim_{k\to\infty} \pi_b^l = 2s - \mu_1 - \mu_2 - 2\mu_d$, i.e., the contract is equivalent to a long-term contract without failure (since the buyer pays such a high price that switching never occurs). In other words,

$$\lim_{k\to\infty} \pi_b^s = -\infty \text{ and } \lim_{k\to\infty} \pi_b^l = 2s - \mu_1 - \mu_2 - 2\mu_d > -\infty$$

hence for very large switching costs, the long-term contract is preferred. Thus, either the long-term contract is preferred for all k, or there must be some k^* for which $\pi_b^s < \pi_b^l$ for all $k > k^*$. ■

Lemma 3 *(i) The optimal short-term dynamic contract is* $p_{11}^s = \mu_d + c_1$, $p_{12}^s = d_1 + c_2$, *and* $p_{22}^s = \mu_d(d_1) + c_2$. *The buyer switches suppliers in the second period if* $k + \mu_d(d_1) < d_1$.

(ii) The optimal long-term dynamic contract is any pair $\{p_{11}^l, p_{12}^l\}$ *such that* $p_{11}^l + p_{12}^l = 2\mu_d + \mu_1 + \mu_2$.

(iii) The expected profit in each dynamic contract is equal to the expected profit in their static counterparts in Theorem 1.

Proof. Omitted; similar to Theorem 1. ■

Lemma 4 *(i) The optimal long-term dynamic contract is* $p_{11}^l = \alpha + c_1$, $p_{12}^l = p_{12}\left(p_{11}^l\right)$, *and* $p_{22}^l = \mu_d(d_1) + c_2$, *where* $p_{12}(p_{11}^l)$ *is the dynamic second period price for which the supplier's participation constraint binds.*

(ii) The optimal short-term dynamic contract is given by $p_{11}^s = c_1 + \max(x^*, \mu_d)$, $p_{12}^s = d_1 + c_2$, *and* $p_{22}^s = \mu_d(d_1) + c_2$, *where* x^* *is the solution to*

$$-1 + g(x^*)(-x^* + \mu_d(x^*) + k) = 0.$$

(iii) The long-term dynamic contract is preferred to the short-term dynamic contract, and yields expected profit equal to the system optimal expected profit without failure risk.

Proof. (i) With a dynamic long-term contract, the buyer offers $p_{11} = c_1 + \delta_1$, $p_{12} = c_2 + \delta_2$, and $p_{22} = c_2 + \delta_3$, optimizing over δ_i, $i = 1, 2, 3$. Failure occurs if $\delta_1 < d_1$. For the same reasons as in the previous lemmas, $p_{22}^l = c_2 + \mu_d(d_1)$. Thus, the buyer's profit is

$$\pi_b = 2s - \mu_1 - \mu_2 - \delta_1 - \Pr(\delta_1 \geq d_1) \times p_{12} - \Pr(\delta_1 < d_1) \times (\mu_d(d_1) + k),$$

maximized subject to

$$\delta_1 - \mu_d + \Pr(\delta_1 \geq d_1) \times \mathbb{E}(\delta_2 - d_1 | \delta_1 \geq d_1) \geq 0. \tag{5}$$

As in Lemma 4, one can show that the constraint is binding at any optimal solution by constructing the Lagrangean. Thus, by substituting a binding constraint into the objective function, we eliminate δ_2 and write the buyer's profit as

$$\pi_b = 2s - \mu_1 - \mu_2 - \mu_d - \int_{-\infty}^{\delta_1} x g(x) dx - \int_{\delta_1}^{\infty} (\mu_d(x) + k) g(x) dx.$$

Optimizing over δ_1, we have

$$\frac{d\pi_b}{d\delta_1} = g(\delta_1)(-\delta_1 + \mu_d(\delta_1) + k)$$

Since $g(x) > 0$ for all feasible x, π_b is quasiconcave, and is maximized by $\delta_1 = \mu_d(\delta_1) + k$, or $\delta_1 = \alpha$. Letting $\delta_2(\delta_1)$ be the value of δ_2 such that (5) holds, we have $p_{11}^l = c_1 + \alpha$, $p_{12}^l = c_2 + \delta_2(\delta_1)$, and $p_{22}^l = c_2 + \mu_d(d_1)$. Defining $p_{12}(p_{11}) = c_2 + \delta_2(\delta_1)$ yields the result.

(ii) In a dynamic short term contract, the buyer offers $p_{11} = c_1 + \delta_1$, $p_{12} = c_2 + \delta_2$, and $p_{22} = c_2 + \delta_3$, optimizing over the various δ. Failure occurs if $\delta_1 < d_1$. For the same reasons as in the previous lemmas, $\delta_3 = \mu_d(d_1)$ and $\delta_2 = d_1$, and the buyer switches suppliers if $d_1 > \mu_d(d_1) + k$. This implies

$$\begin{aligned}
\pi_b = {} & 2s - \mu_1 - \mu_2 - \delta_1 - \Pr(\delta_1 \geq d_1, \mu_d(d_1) + k \geq d_1) \times \mathbb{E}(d_1 | \delta_1 \geq d_1, \mu_d(d_1) + k \geq d_1) \\
& - \Pr(\delta_1 < d_1) \times \mathbb{E}(\mu_d(d_1) + k | \delta_1 < d_1) \\
& - \Pr(\mu_d(d_1) + k < d_1) \times \mathbb{E}(\mu_d(d_1) + k | \mu_d(d_1) + k < d_1).
\end{aligned}$$

Supplier 1 accepts any contract with $\delta_1 \geq \mu_d$. Clearly there are two cases. In the first, $\delta_1 > \alpha$, so

$$\begin{aligned}
\pi_b = {} & 2s - \mu_1 - \mu_2 - \delta_1 - \Pr(\alpha \geq d_1) \times \mathbb{E}(d_1 | \mu_d + k \geq d_1) \\
& - \Pr(\alpha < d_1) \times \mathbb{E}(\mu_d(d_1) + k | \alpha < d_1).
\end{aligned}$$

The derivative is thus $\frac{d\pi_b}{d\delta_1} = -1$ and the optimal solution is $\delta_1 = \alpha$. In the second case,

$\delta_1 \leq \alpha$, so

$$\pi_b = 2s - \mu_1 - \mu_2 - \delta_1 - \int_{-\infty}^{\delta_1} xg(x)dx - \int_{\delta_1}^{\infty} \left(\mu_d\left(x\right) + k\right) g(x)dx$$

and $\frac{d\pi_b}{d\delta_1} = -1 + g(\delta_1)\left(-\delta_1 + \mu_d\left(\delta_1\right) + k\right)$. Since $g(x) > 0$ for all x, the second term in this expression is negative for $\delta_1 \geq \alpha$, then positive for $\delta_1 < \alpha$, thus $\frac{d\pi_b}{d\delta_1}$ is either always negative (resulting in the optimal solution being boundary, $\delta_1 = \mu_d$) or has a unique zero in the interval $\mu_d \leq \delta_1 < \alpha$.

(iii) The profit from the short-term contract is

$$\begin{aligned} \pi_b &= 2s - \mu_1 - \mu_2 - \delta_1 - \int_{-\infty}^{\delta_1} xg(x)dx - \int_{\delta_1}^{\infty} \left(\mu_d\left(x\right) + k\right) g(x)dx \\ &\leq 2s - \mu_1 - \mu_2 - \mu_d - \int_{-\infty}^{\delta_1} xg(x)dx - \int_{\delta_1}^{\infty} \left(\mu_d\left(x\right) + k\right) g(x)dx, \end{aligned}$$

since $\mu_d \leq \delta_1 < \alpha$. Comparing this to the long-term contract's profit of $\overline{\pi}_b$, we see that the difference between these two is

$$\overline{\pi}_b - \pi_b \geq \Pr\left(\delta_1 \leq d_1 < \alpha\right) \times \mathbb{E}\left(\mu_d\left(d_1\right) + k - d_1 | \left(\delta_1 \leq d_1 < \alpha\right)\right) > 0$$

hence the long-term dynamic contract is always preferred. ■

2. Contingent Transfer Payments and Loans

In what follows, we allow the buyer the option of making a transfer payment T to the supplier in the second period. The purpose of this transfer payment is to help support the supplier in the event that bankruptcy occurs at the end of the first period; that is, following a loss in the first period, if the buyer raises the total capital level of the supplier to zero or higher the supplier is spared from bankruptcy and may continue to do business with the buyer in the second period. See Babich (2006) for an analysis of why such threshold transfer payments may be optimal when suppliers face default risk.

We assume that some proportion $r \geq 0$ of the transfer payment T is repaid to the buyer at the end of the second period. If $r = 0$, then no amount is paid to the buyer; in this case, T is a direct operating subsidy. If $r > 0$, then T is a loan, some fraction of which is repaid to the buyer. If $r > 1$, then the buyer charges interest on the loan. For simplicity, we assume that r is exogenously determined (i.e., the firms do not bargain over the interest rate or repayment percentage of any transfer payment or loan). We also assume that the supplier only repays what he can based on profits in the second period.

It is easy to see that if the supplier does not enter bankruptcy, it is optimal to offer no transfer payment, $T = 0$. If the supplier does enter bankruptcy, then let $T(y) = -p_{11} + c_1 + d_1 + y$, where $y \geq 0$ is the excess cash that the buyer provides in the transfer payment that raises the supplier's capital level above zero. Let $R = \min\left(rT, \pi_{12}^+\right)$ be the repayment amount, where $\pi_{12}^+ = (p_{12} - c_2 - d_1 + y)^+$ is the positive part of the incumbent supplier's total profit at the end of the second period. Conditional on a transfer payment $T(y)$ having been made to a bankrupt supplier, the buyer's optimization problem in contracting with the incumbent supplier in the second period is

$$\max_{y,p_{12}} s - p_{12} + \mathbb{E}R$$
$$\text{s.t. } p_{12} - d_1 - \mathbb{E}\left(c_2 + R\right) + y \geq 0.$$

Differentiating the objective with respect to p_{12}, we see

$$\frac{d\left(s - p_{12} + \mathbb{E}R\right)}{dp_{12}} = -1 + \Pr\left(rT > p_{12} - c_2 - d_1 + y > 0\right) < 0,$$

i.e., profit is a decreasing function of price, so the participation constraint will be binding. By inserting the binding participation constraint into the objective function, we see that the buyer's expected profit from contracting with the incumbent supplier in the second period is, after subtracting the cost of the transfer payment,

$$s - d_1 - \mathbb{E}\left(c_2\right) + y - T.$$

The y term in this expression sums to zero with the y term in T, thus, in general, the optimal transfer payment satisfies $y = 0$, implying

$$T = \left(-p_{11} + c_1 + d_1\right)^+,$$

while the cost of contracting with a bankrupt supplier and providing an operating subsidy is $d_1 + \mu_2(c_1) + T$. Note that this expression is independent of r, i.e., it does not depend on whether or not the loan is repaid to the buyer. The reason for this is that the buyer extracts all of the supplier's surplus whether or not the loan is repaid, making the participation constraint binding; hence, if the supplier is to repay the buyer at the end of the second period, the buyer must compensate the supplier by paying a higher contract price. Since the buyer is already extracting all surplus from the supplier, he cannot extract further surplus, and his expected profit from contracting with the bankrupt supplier is thus independent of the magnitude of repayment. Thus, for the remainder of the analysis, we may ignore the precise value of r, and assume that the buyer pays a price of $p_{12} = d_1 + \mu_2(c_1) + T$ to subsidize and do business with a bankrupt supplier in period 2.

It may not be in the best interests of the buyer to support the bankrupt supplier. In particular, if the total expected procurement cost plus the necessary transfer payment exceed the total cost of switching to a new supplier, the buyer will choose the alternative supplier. In other words, if

$$d_1 + T \geq \mu_d(d_1) + k, \tag{6}$$

then it is optimal for the buyer to switch suppliers. We now define the following function:

$$\omega\left(p_{11}, c_1\right) \equiv \left\{x : 2x - \mu_d(x) = p_{11} - c_1 + k\right\},$$

which represents the value of the idiosyncratic cost d_1 such that (6) holds with equality for given values of p_{11}, c_1, and k. Since, by assumption, $2x - \mu_d(x)$ is monotonically increasing in x, there is a unique value of $\omega\left(p_{11}, c_1\right)$ for each p_{11} and c_1. Implicitly differentiating

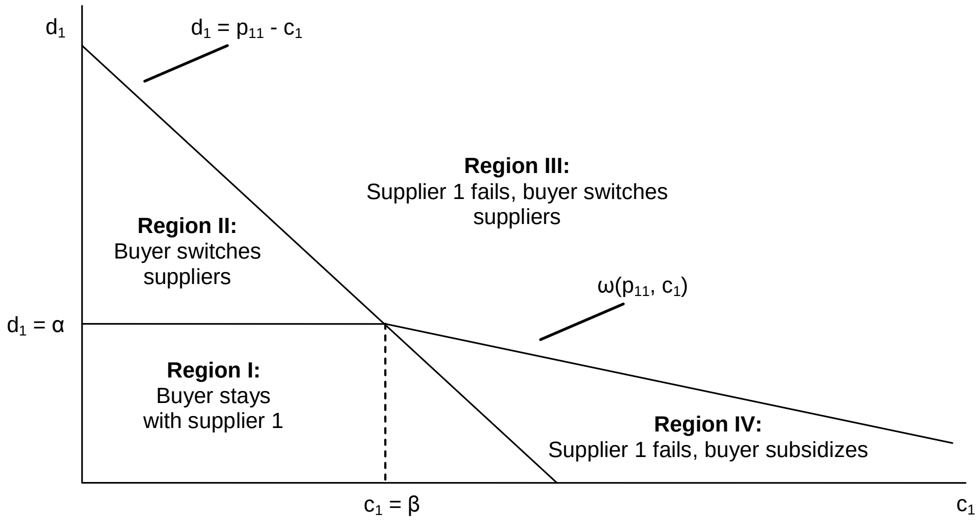

Figure 1. Optimal second period action of the buyer as a function of the realized values of c_1 and d_1 in the short-term contract with a contingent transfer payment in the second period.

$\omega(p_{11}, c_1)$ yields

$$\frac{d\omega(p_{11}, c_1)}{dc_1} = -\frac{d\omega(p_{11}, c_1)}{dp_{11}} = \frac{-1}{2 - \mu_d'(\omega(p_{11}, c_1))} \in [-1, -1/3] .$$

The curves dictated by $\omega(p_{11}, c_1)$ and $d_1 = p_{11} - c_1$ intersect when

$$d_1 - \mu_d(d_1) = k,$$

i.e., when $d_1 = \alpha$. Because the slope (as a function of c_1) of ω is always greater than the slope of $d_1 = p_{11} - c_1$, it must be true that there exists some β such that $\beta = p_{11} - \alpha$, and for all $c_1 > \beta$, $\omega(p_{11}, c_1) > p_{11} - c_1$, while for all $c_1 < \beta$, the opposite inequality holds. Figure 1 depicts this feature graphically in the case of short-term contracts.

Essentially, the addition of a transfer payment to the buyer's action space introduces Region IV to the diagram; there is now a region of the probability space for which it is optimal to subsidize a bankrupt supplier. In particular, the buyer chooses to support the distressed supplier if the idiosyncratic costs of the first supplier are small but the common cost component is high, as the figure shows. In this case, switching to the second supplier is more expensive than helping first supplier avoid bankruptcy.

The following proposition mirrors Lemma 1 in deriving the form of the optimal short-term contract.

Proposition 1 *With the option of contingent transfer payments, the buyer's optimal short-term contract consists of* $p_{11}^s = \mu_d + \mu_1$, $p_{12}^s = d_1 + \mu_2(c_1) + T$, *and* $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$. *The buyer switches suppliers in period 2 if* $d_1 \geq \min(\alpha, \omega(p_{11}, c_1))$.

Proof. It is easy to see that $p_{12}^s = d_1 + \mu_2(c_1) + T$ and $p_{22}^s = \mu_d(d_1) + \mu_2(c_1)$. Substituting these expressions into the buyer's profit function, we see that profit in the optimal short term contract as a function of p_{11} is

$$\pi_b^s(p_{11}) = 2s - \mu_2 - p_{11} - \mathbb{E}\min(d_1 + T, \mu_d(d_1) + k),$$

where $T = (c_1 - d_1 - p_{11})^+$. Differentiating this expression with respect to p_{11}, we have

$$\frac{d\pi_b^s(p_{11})}{dp_{11}} = -1 + \Pr(\text{Region IV}) \leq 0,$$

where $\Pr(\text{Region IV})$ denotes the probability of Region IV in Figure 1, i.e. the probability that $c_1 > \beta$ and $\omega(p_{11}, c_1) > d_1 > p_{11} - c_1$. Since profit is decreasing in p_{11}, it is optimal to set p_{11} as small as possible such that the supplier's participation constraint is satisfied, i.e., $p_{11}^s = \mu_d + \mu_1$. ■

The following proposition describes the optimal two-period contract with contingent transfer payments. In the analysis below, we assume that long-term contracts are renegotiated if bankruptcy occurs and the buyer subsidizes the bankrupt supplier. This is reflective of the fact that often, when working with bankrupt suppliers, buyers must pay court-ordered price increases rather than continue with the terms of existing (likely unprofitable) contracts (see, for example, the case of Collins & Aikman discussed in Barkholz and Sherefkin 2007). We note that our results continue to hold even if prices are not renegotiated (see §3 of this appendix, which can be shown to apply to the case of contingent transfer payments as well) although the first order condition changes slightly (in particular, the expression for $\omega(p_{11}, t)$ changes).

Proposition 2 *Let p_{11}^l be the solution to*

$$\int_{p_{11}-\alpha}^{\infty} f(t)\, g(\omega(p_{11}, t)) \frac{\partial \omega(p_{11}, t)}{\partial p_{11}} (\omega(p_{11}, t) + t - p_{11})\, dt \qquad (7)$$
$$+ \int_{\alpha}^{\infty} (k + \mu_d(x) - x)\, f(p_{11} - x)\, g(x)\, dx = 0.$$

Then, the optimal long-term contract under the threat of default is p_{11}^l, $p_{12}^l = p_{12}(p_{11}^l)$, and $p_{22}^l = \mu_d + \mu_2(c_1)$.

Proof. We first show that at optimality, supplier 1's participation constraint is binding. Note that as in the short-term contract, $p_{22}^l = \mu_d(d_1) + \mu_2(c_1)$, and furthermore due to the renegotiation of the long-term contract if failure occurs, the cost of contracting with a bankrupt supplier is $d_1 + \mu_2(c_1) + T$ (effectively, regardless of the amount of loan repayment). Then the buyer's profit is

$$\begin{aligned}\pi_b^l(p_{11}, p_{12}) \;=\;& 2s - p_{11} - \Pr(\text{Region I,II})\,\mathbb{E}(p_{12}|\text{Region I,II}) \\ & - \Pr(\text{Region IV})\,\mathbb{E}(d_1 + \mu_2(c_1) + T|\text{Region IV}) \\ & - \Pr(\text{Region III})\,\mathbb{E}(\mu_d(d_1) + \mu_2(c_1) + k|\text{Region III}),\end{aligned}$$

where $T = (-p_{11} + c_1 + d_1)^+$, and supplier 1 accepts any contract with

$$\begin{aligned} & \mathbb{E}\left(p_{11} - c_1 - d_1\right) + \Pr\left(\text{Region I,II}\right)\mathbb{E}\left(p_{12} - c_2 - d_1|\text{Region I,II}\right) \\ + \quad & \Pr\left(\text{Region IV}\right)\mathbb{E}\left(T|\text{Region IV}\right) \geq 0. \end{aligned}$$

Note that since the participation constraint of the supplier now considers two periods, the transfer payment T (and potential repayment R) are taken into account. Constructing the Lagrangean $\mathcal{L}$ with multiplier λ and taking the derivative with respect to p_{12}, we see

$$\frac{d\mathcal{L}}{dp_{12}} = (\lambda - 1)\Pr\left(\text{Region I,II}\right).$$

This implies that at optimality we must have $\lambda = 1$, hence the constraint is binding. Then we may rewrite π_b, substituting the binding participation constraint, which yields

$$\begin{aligned} \pi_b^l\left(p_{11}\right) = \; & 2s - \mu_1 - \mu_2 - \mu_d - \Pr\left(\text{Regions I,II,IV}\right)\mathbb{E}\left(d_1|\text{Regions I,II,IV}\right) \\ & - \Pr\left(\text{Region III}\right)\mathbb{E}\left(\mu_d\left(d_1\right) + k|\text{Region III}\right). \end{aligned}$$

Differentiating with respect to p_{11}, we have

$$\begin{aligned} \frac{d\pi_b^l\left(p_{11}\right)}{dp_{11}} = & \int_{p_{11}-\alpha}^{\infty} f\left(t\right)g\left(\omega\left(p_{11},t\right)\right)\frac{\partial\omega\left(p_{11},t\right)}{\partial p_{11}}\left(\omega\left(p_{11},t\right) + t - p_{11}\right)\,dt \\ & + \int_{\alpha}^{\infty}\left(k + \mu_d\left(x\right) - x\right)f\left(p_{11} - x\right)g\left(x\right)\,dx \end{aligned}$$

The first term is positive (since $\frac{\partial\omega(p_{11},t)}{\partial p_{11}}$ is positive) while the second term is negative. Furthermore, note that $\lim_{p_{11}\to\pm\infty}\frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$ by the assumption that $\lim_{x\to\pm\infty} f(x), g(x) = 0$, together with the fact that

$$\lim_{p_{11}\to\pm\infty}\omega\left(p_{11}, c_1\right) = \lim_{p_{11}\to\pm\infty}\left\{x : 2x - \mu_d(x) = p_{11} - c_1 + k\right\} = \pm\infty.$$

Differentiating once more, we have

$$\begin{aligned} \frac{d^2\pi_b^l\left(p_{11}\right)}{dp_{11}^2} = & \int_{p_{11}-\alpha}^{\infty} f\left(t\right)g\left(\omega\left(p_{11},t\right)\right)\frac{\partial\omega\left(p_{11},t\right)}{\partial p_{11}}\left(\frac{\partial\omega\left(p_{11},t\right)}{\partial p_{11}} - 1\right)\,dt \\ & + \int_{p_{11}-\alpha}^{\infty} f\left(t\right)g\left(\omega\left(p_{11},t\right)\right)\frac{\partial^2\omega\left(p_{11},t\right)}{\partial p_{11}^2}\left(t - p_{11} + \omega\left(p_{11},t\right)\right)\,dt \\ & + \int_{p_{11}-\alpha}^{\infty} f\left(t\right)g'\left(\omega\left(p_{11},t\right)\right)\frac{\partial\omega\left(p_{11},t\right)}{\partial p_{11}}^2\left(t - p_{11} + \omega\left(p_{11},t\right)\right)\,dt \\ & + \int_{\alpha}^{\infty}\left(\mu_d\left(x\right) + k - x\right)f'\left(p_{11} - x\right)g\left(x\right)dx. \end{aligned}$$

Note that in the limit as $p_{11} \to \infty$, the first three integrals converge to zero, and hence

$$\lim_{p_{11}\to\infty} \frac{d^2\pi_b^l(p_{11})}{dp_{11}^2} = \lim_{p_{11}\to\infty} \int_\alpha^\infty (\mu_d(x) + k - x) f'(p_{11} - x) g(x)\, dx \geq 0$$

Thus, as p_{11} becomes large, $\frac{d\pi_b^l(p_{11})}{dp_{11}}$ tends to be increasing towards zero, i.e., it is negative, and hence the optimal first period price solves the first order condition, $\frac{d\pi_b^l(p_{11})}{dp_{11}} = 0$. ■

Finally, Proposition 3 demonstrates that the primary result of the paper holds: namely, long-term contracts are preferred if switching costs are high, and neither contract type coordinates the system in general. This proposition is referenced in Theorem 3 of the paper.

Proposition 3 *In the presence of failure risk with contingent transfer payments or loans, (i)* $\pi_b^s, \pi_b^l \leq \overline{\pi}_b$ *and (ii) there exists some* k^* *such that, for all* $k > k^*$, $\pi_b^s \leq \pi_b^l$.

Proof. (i) In the case of the short-term contract, by substituting the binding participation constraint of supplier 1 into the buyer's profit function, we see that the buyer's profit under the optimal single period contract is

$$\pi_b^s(p_{11}) = 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}\min(d_1 + T, \mu_d(d_1) + k),$$

Since $T \geq 0$, it is easy to see that

$$\pi_b^s \leq 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}(\min(d_1, \mu_d(d_1) + k)) = \overline{\pi}_b.$$

The expected profit from the long-term static contract is

$$\begin{aligned} \pi_b^l &= 2s - \mu_1 - \mu_2 - \mu_d - \Pr(\text{Regions I,II,IV})\, \mathbb{E}(d_1|\text{Regions I,II,IV}) \\ &\quad - \Pr(\text{Region III})\, \mathbb{E}(\mu_d(d_1) + k|\text{Region III}). \end{aligned}$$

Again, it follows by definition that

$$\pi_b^l \leq 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}(\min(d_1, \mu_d(d_1) + k)) = \overline{\pi}_b.$$

(ii) In the case of the short-term contract, as $k \to \infty$, we have

$$\begin{aligned} \lim_{k\to\infty} \pi_b^s &= \lim_{k\to\infty} 2s - \mu_1 - \mu_2 - \mu_d - \mathbb{E}\min(d_1 + T, \mu_d(d_1) + k) \\ &= \lim_{k\to\infty} 2s - \mu_1 - \mu_2 - 2\mu_d - \mathbb{E}\left((d_1 + c_1 - \mu_1 - \mu_d)^+\right). \end{aligned}$$

In the long-term contract, on the other hand, from (7) we see that $\lim_{k\to\infty} p_{11}^l = \infty$, implying $\lim_{k\to\infty} \Pr(\text{Regions III,IV}) = 0$, and hence

$$\lim_{k\to\infty} \pi_b^l = 2s - \mu_1 - \mu_2 - 2\mu_d.$$

Thus, for large k, the long-term contract is preferred to the short-term contract, and the proposition holds. ■

As discussed in the main text of the paper, the assumption that any transfer payment is made at the start of the second period is key. However, if this assumption is relaxed, the results of the above theorem continue hold if one of several alternative conditions are met. We refer the reader to §7.2 of the main text for a discussion of this point.

3. Renegotiation

If the buyer is capable of renegotiating the long-term contract, then he will do so whenever the renegotiated price is lower than the contract price. Assume that during the renegotiation process, the supplier will accept any contract with non-negative profit in period 2. In that case, the renegotiated price is $d_1 + \mu_2(c_1)$. The buyer then pays the supplier $\min(p_{12}, d_1 + \mu_2(c_1))$ in the second period, where p_{12} is the second period contract price.

The profit to the buyer is thus

$$\begin{aligned}\pi_b &= 2s - p_{11} - \Pr(p_{11} \geq c_1 + d_1) \times \mathbb{E}(\min(p_{12}, d_1 + \mu_2(c_1)) | p_{11} \geq c_1 + d_1) \\ &\quad - \Pr(p_{11} < c_1 + d_1) \times \mathbb{E}(\mu_d(d_1) + \mu_2(c_1) + k | p_{11} < c_1 + d_1).\end{aligned}$$

There are subsequently two cases: either the supplier recognizes that the buyer will renegotiate the contract (and hence takes this into account in the participation constraint), or the supplier does not recognize that renegotiation will occur. In the former case, supplier 1 accepts any contract with

$$\mathbb{E}(p_{11} - c_1 - d_1) + \Pr(p_{11} \geq c_1 + d_1)\, \mathbb{E}(\min(p_{12}, d_1 + \mu_2(c_1)) - c_2 - d_1 | p_{11} \geq c_1 + d_1) \geq 0.$$

As in the proof of Lemma 2, we may construct the Lagrangean $\mathcal{L}$ with multiplier λ and take the derivative with respect to p_{12} to see that $\frac{d\mathcal{L}}{dp_{12}} = (\lambda - 1)\Pr(p_{11} \geq c_1 + d_1, p_{12} \leq d_1 + \mu_2(c_1))$. This implies that at optimality it must be true that $\lambda = 1$, and the constraint is binding. We may then rewrite π_b, substituting the binding participation constraint, to yield

$$\pi_b^l(p_{11}) = 2s - \mu_1 - \mu_2 - 2\mu_d + \int_{-\infty}^{\infty}\int_{p_{11}-x}^{\infty} (x - \mu_d(x) - k) f(t) g(x)\, dt\, dx,$$

which is precisely the same expression as the model without buyer renegotiation.

On the other hand, if the supplier does not recognize that renegotiation will occur, then the buyer's profit is clearly higher than in the no-renegotiation case. Thus, the profitability of the long-term contracts is increased relative to the value of short-term contracts, and the results of the model continue to hold.

4. Normally Distributed Costs

Theorem 4. *(i) The optimal expected profit under all contract types is decreasing in* ρ_d.

(ii) The difference between the system optimal (long-term dynamic) profit and the profit under the long-term static contract is decreasing in ρ_d. *In the limit as* $\rho_d \to 1$, *profits are equal.*

(iii) The centralized system optimal expected profit is increasing in σ_d.

Proof. (i) Recall that, from the properties of the bivariate normal distribution, the expected value of d_2 conditional on $d_1 = x$ is

$$\mu_d(x) = (1-\rho_d)\,\mu_d + \rho_d x. \tag{8}$$

Using (8) and differentiating $\overline{\pi}_b$, we see that for the long-term dynamic contract,

$$\frac{d\overline{\pi}_b}{d\rho_d} = -\mu_d + \mathbb{E}\min\left(d_1, \mu_d + k/\left(1-\rho_d\right)\right) - \frac{k}{1-\rho_d}\overline{G}\left(\mu_d + k/\left(1-\rho_d\right)\right) < 0.$$

Now turning to the long-term static contract, by differentiating $\pi_b^l(p_{11})$ and using the Envelope Theorem,

$$\begin{aligned}\frac{d\pi_b^l}{d\rho_d} &= \frac{\partial\pi_b^l}{\partial\rho_d} = \int_{-\infty}^{\infty}\int_{p_{11}^l-x}^{\infty}\left(-x+\mu_d\right)f(t)g(x)dtdx \\ &= \mathbb{E}\left(-d_1+\mu_d|d_1 > p_{11}^l - c_1\right)\Pr\left(d_1 > p_{11}^l - c_1\right) < 0.\end{aligned}$$

Finally, consider the short-term static contract. Applying the Envelope Theorem to $\pi_b^s\left(p_{11}\right)$,

$$\frac{d\pi_b^s}{d\rho_d} = \frac{\partial\pi_b^s}{\partial\rho_d} = \int_{\alpha}^{\infty}\left(\mu_d - x\right)g(x)dtdx + \int_{-\infty}^{\alpha}\int_{p_{11}^s-x}^{\infty}\left(\mu_d - x\right)f(t)g(x)dtdx \le 0.$$

(ii) Consider the effect of taking the limit as $\rho_d \to 1$. Then, $\overline{\pi}_b = 2s - 2\mu_c - 2\mu_d$, and since $\lim_{\rho_d\to 1} p_{11}^* = \infty$, $\pi_b^l(p_{11}) = 2s - 2\mu_c - 2\mu_d$ as well, thus the contracts are equivalent. To prove the first part of the proposition, by examining $\frac{d\overline{\pi}_b}{d\rho_d}$ and $\frac{d\pi_b^l}{d\rho_d}$ and noting that in the long-term static contract $p_{11}^* = \mu_c + \mu_d + k/\left(1-\rho_d\right)$, it can be shown that $\frac{d^2\overline{\pi}_b}{d\rho_d^2} > 0$ and $\frac{d^2\pi_b^l}{d\rho_d^2} > 0$. Since $\overline{\pi}_b > \pi_b^l$ for $\rho_d < 1$, both functions are convex and decreasing, and they converge when $\rho_d = 1$, it must be true that $\frac{d\left(\overline{\pi}_b - \pi_b^l\right)}{d\rho_d} < 0$, i.e., the functions are smoothly converging to one another.

(iii) Using (8) and differentiating $\overline{\pi}_b$, we see that for the long-term dynamic contract,

$$\frac{d\overline{\pi}_b}{d\sigma_d} = -\frac{d}{d\sigma_d}\left(\mathbb{E}\min\left(d_1, \mu_d + k/\left(1-\rho_d\right)\right)\right).$$

This expression is merely the negative of a newsvendor expected sales function. It is well known that expected sales decrease as a function of standard deviation in a newsvendor with normally distributed demand, hence $\frac{d}{d\sigma_d}\left(\mathbb{E}\min\left(d_1, \mu_d + k/\left(1-\rho_d\right)\right)\right) < 0$ and thus $\frac{d\overline{\pi}_b}{d\sigma_d} > 0$.

5. Numerical Study

To study the magnitude of k^* (the switching cost above which long-term contracts are preferred), we calculated this critical switching cost both with and without transfer payments for every combination of the following parameters: $\mu_d = \{1,2,3\}$, $\mu_1 = \mu_2 = \{1,2,3\}$, $\sigma_d = \{1,3,6\}$, $\sigma_1 = \sigma_2 = \{1,3,6\}$, and $\rho_d = \{-0.5, 0, 0.5\}$, where costs are normally distributed and the common cost component is i.i.d. across time. The result is 243 distinct sets of parameters with coefficient of variation of idiosyncratic and common costs ranging from 0.333 to 6. The average total production cost in each period was 4. Our results are as follows:

Case	Average k^*	Average $k^*/(\mu_d + \mu_1)$	Median k^*
No Transfer Pmt	0.99	27.3%	0.81
Transfer Pmt	0.56	18.3%	0.00

In 42 of 243 cases, k^* increased as a result of allowing a transfer payment, while in the remainder, k^* decreased. Breaking down the results by coefficient of variation, we see:

CV of d_1	Avg $k^*/(\mu_d + \mu_1)$ with no Transfer	Avg $k^*/(\mu_d + \mu_1)$ with Transfer
0.3	13.8%	0.00%
0.5	17.5%	0.00%
1.0	23.0%	0.00%
1.5	27.8%	0.00%
2	25.6%	0.04%
3	35.4%	0.24%
6	44.8%	1.10%

In other words, k^* is smallest when there is low variability in the idiosyncratic cost term. This is intuitive, as the option to switch suppliers contains the most value (and hence short-term contracts contain the most value) when d_1 is highly variable. (Note that the behavior in the above table appears to be non-monotonic due to the fact that there are various parameter combinations–perhaps unequal in number–for each particular CV.)

References

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