Olbers' Paradox, Wireless Telephones, and Poisson Random Sets
Is the universe finite?

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Abstract

Olbers' paradox is that if the universe is either infinite in age or extent, then the night sky should be bright. A related problem exists in terms of gravity, that an infinite universe full of stars should collapse on itself. Olbers' paradox has been used to support the "big bang" hypothesis. We find there is a simpler resolution of Olbers' paradox, that perhaps ought to be considered. We show that a standard theorem on convergence of infinite series of zero-mean independent random variables, due to Kolmogorov, can be viewed as saying that the sum of all the above contributions, *allowing destructive interference*, produces a finite sum even if the universe is infinite in age and extent. Thus our explanation of Olbers' paradox, is that if one thinks in terms of waves rather than of particles, the paradox disappears.

A new model for interference noise in wireless telephony has been given in the thesis of one of us [7] which is closely related to Chandrasekhar's model for stellar gravity. If we assume that the telephones which are sending signals to the base station at the origin are at the points of a planar Poisson random set, and that the $n\underline{th}$ most distant telephone from the origin is sending a signal X_n, then the total signal at any time received at the base station is

$$T = \sum_{n=1}^{\infty} \frac{X_n}{R_n^{\gamma}}$$

where R_n is the distance, $0 < R_1 < R_2 < \ldots$. The exponent, γ, is taken as 3.9 or 4 in engineering practice.

In cosmology the situation is similar but the Poisson points are in three dimensions rather than two and represent the stars or the galaxies, and T now represents either the total gravitational force on the earth or the total radiation at a frequency. In cosmology, γ is usually taken as 2 corresponding to an inverse square loss.

We will show first that under very general conditions the series for T is perfectly well convergent mathematically in both cases even if the sum extends over infinitely many summands. This seems to say that Olbers' paradox has an alternative explanation, namely that even if there are infinitely many sources of gravity and radiation the total contribution on earth has a specific finite value due to cancellation of the terms (interference). The sum of the squares of the terms above also converges although the series does not necessarily converge *absolutely* unless γ is large enough (one would need $\gamma > 3$) in 3 dimensions.

Remarkably, (a special case is due to Chandrasekhar) the distribution of T is always stable and there is only a two-parameter family of stable distributions, $S(\alpha, \beta)$, that T can possibly have. Not all stable distributions appear since $0 < \alpha < 2$ so, for example, the normal distribution is not among them.

1 Introduction

Stable distributions arise in several areas of physics and astronomy. In 1919, Holtsmark, when studying the spectral lines of stars, derived the probability distribution of the electrostatic force exerted on an atom by surrounding atoms [8]. The Holtsmark distribution is a symmetric stable law with index of stability, $\alpha = 1/2$. The Holtsmark distribution was used by Chandrasekhar to study the gravitational force acting on a star from the neighboring stars [2]. In his model, Chandrasekhar assumes the stars are distributed as a three dimensional Poisson point set, which is the same assumption we will use when considering the gravity or radiation of the stars.

Zolotarev [14] gave a more general model for these types of problems. He considered a collection of particles distributed as a Poisson set dispersed in some region, U, of n-dimensional Euclidean space. Each particle creates a "field of action;" the strength of this "field" at any point in space is given by an influence function. The examples considered in this paper fit his general model. In the case of cellular telephones the field is the radio waves transmitted by the phones, and the influence function is the power of the radio signal or the radio waves. When considering the stars, the field is the radiation emitted by the stars or the gravitational force of the stars. We will show that in both examples the sum converges to a stable distribution.

A stable distribution is characterized by four parameters. They are the index of stability, $0 < \alpha \leq 2$, the skewness parameter, $-1 \leq \beta \leq 1$, the scale parameter, σ, and the shift parameter, μ. For the examples considered in this paper $\mu = 0$. Any stable random variable with $0 < \alpha < 2$ can be represented as a convergent sum of random variables of the form $\frac{X}{\Gamma^{1/\alpha}}$ where $\{\Gamma_n\}$ are the points of a Poisson set. The series representation of an infinitely divisible random variable without a Gaussian component was first developed by Ferguson and Klass [5]. The stable distributions are a particular example of this.

2 Cellular Telephones

As radio waves travel from the transmitting phone (mobile station) to the receiving antenna (base station) they scatter off buildings, the ground, etc. The signal received at the base station is the sum of several scattered waves, and since they have traveled different paths they arrive with varying phases. These waves interfere and this is called multi-path fading. The signal arriving at the base station consists of an in phase component, T_1, and a quadrature component, T_2. The envelope of the signal, X, is defined as $X = (T_1^2 + T_2^2)^{1/2}$. The power of the received signal is proportional to the square of the envelope of the signal. At a given distance the distribution of the envelope is most often modeled as Rayleigh or Rician (after Stephen O. Rice.) Usually, Rayleigh is used when all received waves have been reflected and Rician when there is a direct wave plus reflected waves.

The power arriving at a base station from an individual mobile station at a distance of R is $\frac{cX^2}{R^\gamma}$. Where c is a constant which depends on transmitted power, antenna height and gain, and γ depends on the terrain and is usually taken to be 3.9 or 4. Neglecting the curvature of the earth the total power received at the base station is then

$$T = \sum_{i=1}^{\infty} \frac{cX_i^2}{R_i^\gamma}$$

Assume that the cellular telephones are located as the points of a two dimensional Poisson point set with rate λ. Then $R_1^2, R_2^2 - R_1^2, R_3^2 - R_2^2, \ldots$ are iid exponential random variables with mean $\frac{1}{\pi\lambda}$, and the sequence $R_1^2, R_2^2, \ldots$ is a Poisson point set with rate $\pi\lambda$.

It follows that one way to construct a two dimensional Poisson set is to use a one dimensional Poisson set $\Gamma_1, \Gamma_2, \ldots$ with rate $\pi\lambda$ and a sequence, $\{\Theta_k\}$, of iid uniform $[0, 2\pi)$

random variables. The sequence of points whose polar coordinates are $(\Gamma_n^{1/2}, \Theta_n)$ is a two dimensional Poisson set with rate λ [3].

The total power received at the base station is

$$T = (\pi\lambda)^{1/\alpha} \sum_{i=1}^{\infty} \frac{cW_i}{\Gamma_i^{1/\alpha}}$$

where $\Gamma_1, \Gamma_2, \ldots$ is a one dimensional Poisson point set with unit rate, and $W_i = X_i^2$. If $EW_i^\alpha < \infty$ then T is a stable random variable with parameters $\alpha = 2/\gamma, \beta = 1$ and $\sigma = c[\frac{\pi\lambda\Gamma(2-\alpha)cos(\frac{\pi\alpha}{2})EW_1^\alpha}{1-\alpha}]^{1/\alpha}$. The following proofs are given in [13] and are included here for completeness. Since γ is usually taken to be 3.9 or 4, we will only consider the case when $0 < \alpha < 1$.

First we will show that the series $\sum_{i=1}^{\infty} W_i \Gamma_i^{-1/\alpha}$ converges using Kolmogorov's three series theorem and then we will show T is stable. To show convergence of the series it is enough to show that $\sum_{i=1}^{\infty} W_i i^{-1/\alpha} < \infty$ a.s.

(i)

For $\lambda > 0$

$$\sum_{i=1}^{\infty} P(W_i i^{-1/\alpha} > \lambda) = \sum_{i=1}^{\infty} P(W_i^\alpha > i\lambda^\alpha) < \infty$$

since $EW_1^\alpha < \infty$.

(ii)

For $i \geq 1$

$$E(W_i i^{-1/\alpha} 1(W_i i^{-1/\alpha} \leq 1)) \leq i^{-1/\alpha} \int_0^{i^{1/\alpha}} P(W_i > x) dx$$

and

$$\sum_{i=3}^{\infty} i^{-1/\alpha} \int_{3^{1/\alpha}}^{i^{1/\alpha}} P(W_1 > x)dx \;=\; \frac{1}{\alpha}\sum_{i=3}^{\infty} i^{-1/\alpha} \int_{3}^{i} y^{(1-\alpha)/\alpha} P(W_1^{\alpha} > y)dy$$
$$\leq\; C\int_{3}^{\infty} P(W_1^{\alpha} > y)dy < \infty$$

because $EW_1^{\alpha} < \infty$,

so

$$\sum_{i=1}^{\infty} E(W_i i^{-1/\alpha} 1(W_i i^{-1/\alpha} \leq 1)) < \infty.$$

(iii)

$$\sum_{i=1}^{\infty} E\left(\frac{W_i}{i^{1/\alpha}}(1\frac{W_i}{i^{1/\alpha}} \leq \lambda)\right)^2 \;=\; \sum_{i=1}^{\infty} i^{-2/\alpha}\int_0^{\infty} w^2 1\{w \leq \lambda i^{1/\alpha}\} f(w)dw$$
$$\leq\; C\int_0^{\infty} x^{-2/\alpha}dx \int_0^{\lambda x^{1/\alpha}} w^2 f(w)dw$$
$$=\; C\int_0^{\infty} w^2 f(w)dw \int_{\lambda^{-\alpha}w^{\alpha}}^{\infty} x^{-2/\alpha}dx$$
$$=\; C'\int_0^{\infty} w^{\alpha} f(w)dw < \infty$$

where C and C' are positive constants.

Therefore $\sum_{i=1}^{\infty} \frac{W_i}{\Gamma_i^{1/\alpha}} < \infty$.

We will now show that T is stable. Let $U_1, U_2, \ldots$ be a sequence of iid uniform (0,1) random variables which are independent of $W_1, W_2, \ldots$ and let $Y_i = \frac{W_i}{U_i^{1/\alpha}}$.

$$\lim_{\lambda\to\infty} \lambda^{\alpha} P(Y_i > \lambda) = EW_i^{\alpha} + \lim_{\lambda\to\infty} \lambda^{\alpha} P(W_i > \lambda) = EW_i^{\alpha},$$

so then $\frac{1}{n^{1/\alpha}}\sum_{i=1}^{n} Y_i \rightarrow X$ where X is a stable random variable with index of stability α, $\sigma = \left(\frac{EW_1^{\alpha}\Gamma(2-\alpha)cos(\pi\alpha/2)}{1-\alpha}\right)^{1/\alpha}$ and $\beta = 1$. Let $\Gamma_1, \Gamma_2 \ldots$ be a Poisson point set, the distribution of $\frac{\Gamma_1}{\Gamma_{n+1}}, \frac{\Gamma_2}{\Gamma_{n+1}}, \ldots$ given Γ_{n+1} is the distribution of n order statistics from U(0,1).

The conditional distribution of $\frac{\Gamma_1}{\Gamma_{n+1}}\frac{\Gamma_2}{\Gamma_{n+1}},\ldots$ given Γ_{n+1} is equal to the unconditional distribution.

So,

$$\begin{aligned}
\frac{1}{n^{1/\alpha}}\sum_{i=1}^{n} Y_i &= \frac{1}{n^{1/\alpha}}\sum_{i=1}^{n}\frac{W_i}{U_i^{1/\alpha}} \\
&\stackrel{d}{=} \frac{1}{n^{1/\alpha}}\sum_{i=1}^{n}\left(\frac{\Gamma_{n+1}}{\Gamma_i}\right)^{1/\alpha} W_i \\
&= \left(\frac{\Gamma_{n+1}}{n}\right)^{1/\alpha}\sum_{i=1}^{n}\frac{W_i}{\Gamma_i^{1/\alpha}} \stackrel{d}{\Rightarrow} X.
\end{aligned}$$

Γ_{n+1} is the sum of n+1 iid exponential random variables with mean 1 so then by the strong law of large numbers $P(\lim_{n\to\infty}\frac{\Gamma_{n+1}}{n}=1)=1$.

Consequently,

$$\left(\frac{\Gamma_{n+1}}{n}\right)^{1/\alpha}\sum_{i=1}^{n}\frac{W_i}{\Gamma_i^{1/\alpha}} \to \sum_{i=1}^{\infty}\frac{W_i}{\Gamma_i^{1/\alpha}}.$$

And

$$T = (\pi\lambda)^{1/\alpha}\sum_{i=1}^{\infty}\frac{cW_i}{\Gamma_i^{1/\alpha}} \stackrel{d}{=} (\pi\lambda)^{1/\alpha} cX$$

so T has a stable distribution with $\sigma = c\left(\frac{EW_1^{\alpha}\pi\lambda\Gamma(2-\alpha)cos(\pi\alpha/2)}{1-\alpha}\right)^{1/\alpha}$.

3 Olbers' Paradox and Stellar Gravity

Edmund Hally first raised the question as to why the sky is dark at night in 1720. It was later discussed by HWM Olbers in 1823 and has become known as Olbers' paradox. Starting with the assumptions that the universe is infinitely old and infinite in extent and that the stars are uniformly distributed leads to the conclusion that the night sky should be uniformly bright. No matter which direction we look there should always be a star in our line of sight, and so the entire sky should have the same brightness as the surface of a star. Also, the energy carried by this radiation would cause the earth to have the same temperature as the surface of a star.

There have been several solutions proposed to resolve Olbers' paradox. In Olbers' time it was believed that dust and other matter absorbed the radiation en route to the earth. Generally, this is no longer accepted as a solution since the absorbing material would heat up and then radiate energy at the same rate it absorbed energy.

The expansion of the universe has also been used as a possible resolution of the paradox. The redshift, which is caused by the Doppler effect, has been observed in radiation from all galaxies. The energy from a quantum of light is given by $E = \frac{hc}{\lambda}$. The wavelength, λ, is increased by the Doppler effect, and therefore the energy of the quantum of light is decreased. Although, it is generally accepted that the expansion of the universe does decrease the energy of the light, some have argued that the decrease in energy of the radiation would not be great enough to resolve the paradox [6].

The darkness of the night sky can be explained if the assumption that the universe is infinitely old is dropped. In the bright sky model most of the radiation would have to come from very distant sources. Due to the finite speed of light, the radiation would have to be emitted long ago, on the order of 10^{24} years [11]. However, if the universe were not that old, light from these distant galaxies would not have reached the earth yet.

Still another possible explanation for the paradox is that the lifetime of the stars is too short to produce a bright sky. Harrison [6] calculates that the time required to attain a bright sky is approximately 10^{24} years. The luminous lifetime of the stars is approximated to be only 10^8 years, which would support this resolution. Hence, the lifetime of the stars is far too short to saturate the night sky with radiation.

It would seem that another possible explanation for the paradox should be considered. If the radiation from all of the stars is summed allowing for destructive interference then the sum converges. This would allow for a universe which is infinite in age and in extent with a dark night sky. This would imply that "Rather than proving the finiteness of the universe,the Olbers' paradox proves quantum mechanics [12]."

In the case of stellar gravity the situation is similar to Olbers' paradox, in an infinite universe with an infinite number of stars the total force exerted on the earth should be infinite. However, the gravitational force exerted from the stars in one region should be balanced by the gravitational force of stars in other regions. So, it would seem the earth should experience infinite forces in all directions, and the universe should be in a state of extreme "tension." The proposed resolutions to this problem are similar to those for Olbers' paradox.

When considering radiation, let R_i and W_i represent the distance and the magnitude of radiation from the i^{th} most distant star. To allow for interference assign the radiation a value of +1 or -1 with equal probability. Assuming an inverse square loss the total radiation from all of the stars is

$$T = \sum_{i=1}^{\infty} \frac{\epsilon_i W_i}{R_i^2}$$

where $\epsilon_1, \epsilon_2, \ldots$ is an iid sequence of Rademacher variables.

The total gravitational force at a point per unit mass is

$$\mathbf{F} = \sum_{i=1}^{\infty} \frac{GM_i \mathbf{R_i}}{|\mathbf{R_i}|^3}$$

where $\mathbf{R}_i$ is the position of the i^{th} most distant star. The vector $\frac{\mathbf{R}_i}{|\mathbf{R}_i|}$ is uniformly distributed on the unit sphere and independent of $|\mathbf{R}_i|$. We will consider the projection of the gravitational force onto a given vector $\mathbf{v}$. Let $W_i = GM_i \left| P_{\mathbf{v}} \left(\frac{\mathbf{R_i}}{|\mathbf{R}_i|} \right) \right|$ and let $\epsilon_i = cos\theta$ where θ is the angle between $\mathbf{v}$ and $P_{\mathbf{v}} \left(\frac{\mathbf{R_i}}{|\mathbf{R}_i|} \right)$. The gravitational force in the direction of $\mathbf{v}$ is then

$$T = \sum_{i=1}^{\infty} \frac{\epsilon_i W_i}{R_i^2}$$

where $R_i = |\mathbf{R}_i|$.

We will consider the stars to be located at the points of a three dimensional Poisson set with rate λ. The sequence $R_1^3, R_2^3 - R_1^3, R_3^3 - R_2^3, \ldots$ are iid exponential random variables with mean $(\frac{4}{3}\pi\lambda)^{-1}$, and $R_1^3, R_2^3, \ldots$ is a one dimensional Poisson set with rate $\frac{4}{3}\pi\lambda$.

So,

$$T = \left(\frac{4}{3}\pi\lambda\right)^{2/3} \sum_{i=1}^{\infty} \frac{\epsilon_i W_i}{\Gamma_i^{2/3}}$$

where $\Gamma_1, \Gamma_2, \ldots$ is a Poisson set with unit rate. If $E|W_1|^{3/2} < \infty$, then T is a symmetric stable random variable with $\alpha = 3/2$ and $\sigma = 2\pi \left(\frac{2\lambda}{3} E W_1^{3/2}\right)^{2/3}$.

As before, the following proofs are given in [13] and are included for completeness. First we will show convergence of the series using Kolmogorov's three series theorem. It is enough to show that $\sum \frac{\epsilon_i W_i}{\Gamma_i^{2/3}}$ converges for each fixed sequence $\{\Gamma_i\}$. For a fixed sequence the summands $\epsilon_i \Gamma_i^{-2/3} W_i$ are independent and$C_1 i \leq \Gamma_i \leq C_2 i$ where C_1 and C_2 are positive constants.

(i)

For $\lambda > 0$

$$\begin{aligned}\sum_{i=1}^{\infty} P\left(\frac{W_i}{\Gamma_i^{2/3}} > \lambda\right) &= \sum_{i=1}^{\infty} P(W_i^{3/2} > \lambda^{3/2}\Gamma_i) \\ &\leq \sum_{i=1}^{\infty} P(W_i^{3/2} > \lambda^{3/2} C_1 i) < \infty\end{aligned}$$

since $E|W_i|^{3/2} < \infty$.

(ii)

$$\sum_{i=1}^{\infty} E\left(\frac{\epsilon_i W_i}{\Gamma_i^{2/3}} 1(\frac{W_i}{\Gamma_i^{2/3}} \leq \lambda)\right) = 0$$

since each summand equals 0.

(iii)

$$\begin{aligned}\sum_{i=1}^{\infty} E\left(\frac{\epsilon_i W_i}{\Gamma_i^{2/3}} 1(\frac{W_i}{\Gamma_i^{2/3}} \leq \lambda)\right)^2 &\leq C_1^{-4/3} \sum_{i=1}^{\infty} i^{-4/3} \int_0^{\infty} w^2 1(w \leq \lambda C_2^{2/3} i^{2/3}) f(w) dw \\ &\leq C \int_0^{\infty} x^{-4/3} dx \int_0^{\lambda C_2^{2/3} x^{2/3}} w^2 f(w) dw \\ &= C \int_0^{\infty} w^2 f(w) dw \int_{\lambda^{-3/2} C_2^{-1} w^{3/2}}^{\infty} x^{-4/3} dx \\ &= C' \int_0^{\infty} w^{3/2} f(w) dw < \infty\end{aligned}$$

where C and C' are positive constants.

Next we will show T is a stable random variable. Let $U_1, U_2, \ldots$ be a sequence of iid uniform (0,1) random variables, and let $Y_i = \frac{\epsilon_i W_i}{U_i^{2/3}}$. As before, $lim_{\lambda\to\infty} \lambda^{\alpha} P(|Y_i| > \lambda) = EW_i^{\alpha}$, and so $\frac{1}{n^{2/3}} \sum_{i=1}^{n} Y_i \to X$, where X is stable $\alpha = 3/2$, $\sigma = (2\pi)^{1/3}(EW_1^{3/2})^{2/3}$ and $\beta = 0$.

So,

$$\frac{1}{n^{2/3}} \sum_{i=1}^{n} Y_i = \frac{1}{n^{2/3}} \sum_{i=1}^{n} \frac{\epsilon_i W_i}{U_i^{2/3}} \stackrel{d}{=} \left(\frac{\Gamma_{n+1}}{n}\right)^{2/3} \sum_{i=1}^{n} \frac{\epsilon_i W_i}{\Gamma_i^{2/3}} \to X.$$

And by the strong law of large numbers $P(\lim_{n\to\infty}\frac{\Gamma_{n+1}}{n}=1)$ so then

$$\sum_{i=1}^{\infty}\frac{\epsilon_i X_i}{\Gamma^{2/3}} \stackrel{d}{=} X.$$

And $T=(\frac{4}{3}\pi\lambda)^{3/2}X$, so T has a stable distribution.

For stellar gravity, $\mathbf{F}=(F_x\hat{i},F_y\hat{j},F_z\hat{k})$, we have shown that each of F_x, F_y, and F_z has a stable distribution with $\alpha=3/2$. Since the projection of $\mathbf{F}$ in any direction is stable and $\alpha\geq 1$, then $\mathbf{F}$ is a stable vector, this follows from theorem 2.1.5(c) in [13]. We will now obtain the characteristic function for $\mathbf{F}$,

$$Ee^{i\mathbf{F}\cdot\mathbf{z}}=Ee^{i|\mathbf{z}|\left(\mathbf{F}\cdot\frac{\mathbf{z}}{|\mathbf{z}|}\right)}=Ee^{i|\mathbf{z}|T}.$$

Where T is the projection of $\mathbf{F}$ onto $\mathbf{z}$, T was shown to be a symmetric stable random variable with $\alpha=3/2$, so

$$Ee^{i|\mathbf{z}|T}=e^{-\sigma^{3/2}|\mathbf{z}|^{3/2}}$$

with $\sigma^{3/2}=(2\pi)^{3/2}\left(\frac{2\lambda}{3}EW_1^{3/2}\right)=\frac{2\lambda}{3}(2\pi G)^{3/2}EM_1^{3/2}E\left(\left|P_{\mathbf{z}}\frac{\mathbf{R}_1}{|\mathbf{R}_1|}\right|^{3/2}\right)=\frac{4}{15}\lambda(2\pi G)^{3/2}EM_1^{3/2}$
The characteristic function for $\mathbf{F}$ is

$$Ee^{i\mathbf{z}\cdot\mathbf{F}}=e^{\frac{4}{15}\lambda(2\pi G)^{3/2}EM_1^{3/2}|\mathbf{z}|^{3/2}}.$$

Perhaps experiments could be devised to test whether or not small fluctuations in the earth's gravitational pull are consistent with the stable law hypothesis. It might be of some interest to calculate the probability that the net gravitational field at a given point is *away from* the direction of the nearest Poisson star. For example, the probability that the gravitational force on earth is away from the sun is

$$P\left(\frac{GM_s}{r^2}\leq T\right)=P\left(\frac{GM_s}{\sigma r^2}\leq X\right)=P\left(\frac{M_s}{2\pi r^2(\frac{4}{15}\lambda EM_1^{3/2})^{2/3}}\leq X\right)$$

where $T = \sigma X$, X is a stable random variable with $\alpha = 3/2$ and $\sigma = 1$, and r is the distance from the earth to the sun.

As given in Zolotarev [14]

$$P\left(\frac{M_s}{2\pi r^2(\frac{4}{15}\lambda E M_1^{3/2})^{2/3}} \leq X\right) = \frac{1}{\pi}\int_0^{\pi/2} exp\{-\left(\frac{M_s}{2\pi r^2(\frac{4}{15}\lambda E M_1^{3/2})^{2/3}}\right)^3 U(\gamma)\}d\gamma$$

$$U(\gamma) = \left(\frac{sin\frac{3}{2}\gamma}{cos\gamma}\right)^3 \left(\frac{cos\frac{1}{2}\gamma}{cos\gamma}\right).$$

It should be noted that an objection to this argument was raised in the case of radiation [9]. A light-wave is comprised of an electric field, E, and a magnetic field, B, and the intensity is proportional to the cross product of E and B. Since intensity drops off with $\gamma = 2$ then the electric field (or magnetic field) perhaps should drop off with $\gamma = 1$ not $\gamma = 2$.

4 Poisson Model

For cellular telephones, as the users move form place to place, Doob [4] give conditions under which the Poisson distribution will be preserved. The distribution of mobile phones will remain Poisson if the following conditions are satisfied. For t>0, the classes of random variables

$$\{x_i(t) - x_i(0), -\infty < i < \infty\}, \qquad \{x_i(0), -\infty < i < \infty\}$$

are mutually independent, and the random variables in the first class are mutually independent, where $x_i(t)$ represents the position of the i^{th} mobile phone at time t.

In the case of the stars, assume that the initial position and momentum of the stars are distributed as a Poisson point set in 6 dimensions. To ensure that there is not an infinite number of stars in a finite region condition on a given energy density. For any finite region of space, the initial position and momentum of the N stars in that region can be described by a point in 6N-dimensional phase space, and the motion of the stars is described by a trajectory through phase space. It would seem that since Lebesgue measure is preserved in phase space due to Liouville's theorem, the stars should continue to have a Poisson distribution as they move. And taking the limit as the size of the region goes to infinity would seem to suggest that the stars in an infinite region should remain Poisson.

The above argument implies that an infinite homogeneous gravitating system can be in equilibrium, which according to Binney and Tremaine [1] cannot be the case. Binney and Tremaine state that the way to construct an infinite homogeneous equilibrium is to perpetrate the "Jeans' swindle." Known as such because, until recently, it was believed that there is no formal justification for Jeans' relation. Kiessling [10] gives a derivation of Jeans' relation, showing that it is not a "swindle." Both Kiessling's derivation and the above argument show that it is possible to have an infinite homogeneous gravitating system in equilibrium.

5 Conclusion

We have modeled both the locations of cellular telephones and the locations of the stars as Poisson point sets and found the distribution of the total power of the radio signal and the total stellar radiation and gravity. In the case of cellular telephones we have shown that the total power received at the base station has a skewed α stable distribution where typical values for α are about .5 and $\beta = 1$. The total radiation or total gravitational force on earth has a symmetric stable distribution with $\alpha = 3/2$.

Using this Poisson model for stars and allowing for interference leads to an alternative explanation of Olbers' paradox. A universe which is infinite in age and extent can have a dark night sky since the sum of the radiation will converge.

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