Positive vs. negative externalities in inventory management: implications for supply chain design*

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Abstract

In this paper we analyze the impact of supply-side externalities existing among downstream retailers on supply chain performance. Namely, multiple retail firms face stochastic demand, purchase the product from the upstream wholesaler, and make stocking decisions that affect all other retailers in the same echelon. Two sources of inefficiencies exist in such a supply chain: one is due to double-marginalization and the other due to externalities among retailers. While double-marginalization always leads to inventory understocking at the retail echelon, we find that the implications of externalities are more complex, since different externalities can improve or deteriorate supply chain performance (relative to the situation without externalities).

We show that the effect of externalities depends critically on whether the stocking decisions of retailers exhibit positive (complementarity) or negative (substitutability) externalities and whether retailers are managed centrally or competitively. Under complementarity, competing retailers tend to understock the product (compared to the centralized inventory management at the retail level), thus aggravating the double-marginalization effect. This is the opposite of what happens under substitutability, where competing retailers tend to overstock the product, thus compensating for the double-marginalization effect. Hence, we conclude that supply chain coordination between retailers and the wholesaler is most important when there is downstream competition that exhibits complementarity. From the wholesaler's point of view, competition among retailers is preferable over centralization of retailers when externalities are negative and vice versa when externalities are positive. Moreover, with competition on complements both retailers and the wholesaler have incentives to coordinate the supply chain.

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1 Introduction

The stocking decisions of retailers in the same supply chain echelon are often interdependent. For example, externalities may exist if retailers sell the same or related products. Both demand and supply-side externalities can be present: the former typically arises through prices (e.g., higher price for one product leads to higher demand for another product) while the latter arises through product availability. In this paper we focus on supply-side externalities.

We consider a situation in which multiple retailers carry either the same or interdependent products and make stocking decisions by purchasing said product(s) from the same upstream wholesaler. Retailers can be either centrally controlled or competing. Two sources of inefficiency exist in such supply chains. One is due to the presence of the wholesaler, who will sell the product to downstream retailers at a unit wholesale price that is higher than his marginal cost. This inefficiency is known as double-marginalization and leads to inventory understocking by all retailers. Another inefficiency may arise as a result of supply-side externalities among retailers. The goal of this paper is to show that supply chain efficiency can be improved or deteriorated depending on the type of externality. In particular, we consider four scenarios: with positive vs negative externalities and with competition vs centralized management of retail inventory. Each of these four situations results in different supply chain inefficiencies and incentives for coordination. As a result, the managerial implications for each type of externality differ as well.

The extant literature typically assumes that inventory decisions at retailers have *negative* externalities (also called *substitutability*). For example, a standard assumption is that an increase in the stocking quantity of one retailer reduces demand for other retailers. Such an effect may be a result of stock-out based substitution (if the product is out of stock, the customer may buy a substitute from another retailer) or may result from some other effect, e.g., the impact of the amount of displayed inventory (if product A has more shelf-space than product B, a customer may prefer product A) and the perceived service quality. The impact of such externalities on the inventory at the retail echelon is rather well understood: retailers overstock inventory under competition on substitutes compared to the centralized inventory management. The impact on supply chain performance has also recently been analyzed: it turns out that competition on substitutes is beneficial, since it compensates understocking due to double-marginalization and hence improves supply chain performance.

Surprisingly, however, little attention has been paid to the situation in which the stocking decisions of retailers exhibit *positive* externalities (also called *complementarities*), a situation frequently found in practice.[1] Namely, an increase in the stocking quantity of one product may increase demand for other products (either directly or through an increase in sales). Examples of settings in which stocking decisions of complementary products are centrally controlled include: a computer store that carries a large selection of printers and monitors whose sales are directly linked to computer sales; sales of video players that are influenced by TV sales at the same store; and cameras that are sold next to matching film and electronic devices next to batteries. Likewise, complementarities may be exhibited by products sold by independent (and hence competing[2]) companies. In the US video game market, Nintendo has a dominant market share

[1] From here on, we will use "substitutability" and "complementarity" rather than positive and negative externality.

[2] The term "competition" might be confusing here since it is typically used to indicate a rivalry among companies for the

largely due to better availability of game cartridges manufactured and sold by companies not related to Nintendo (Church and Gandal [9]). For companies selling music players, the availability of compatible pre-recorded discs is critical in creating demand for these players. When pre-recorded video tapes became popular, rental stores preferred to carry VHS tapes because more consumers owned VHS players. This complementarity allowed VHS to become a standard by 1988 (Economides [12]). Conversely, Micro Channel Architecture (MCA) introduced by IBM for new personal computers had significant difficulties being accepted due to poor availability of add-on boards manufactured by third parties (Economides [12]). Complementarity through the branding effect also has an impact; for example, a higher service level at one RadioShack store may induce higher demand for other stores in this retail chain. Such an effect within any franchise is called "spillover" in the marketing literature (see Lafontaine and Slade [14]) and its existence is widely acknowledged. Finally, new products or products that are relatively early in their life cycles often exhibit significant complementarities (e.g., telephone, telex, etc., see Economides [12]). Although, as we can see, there are a number of situations in which the availability of different products exhibits complementarities either under competition or under centralized inventory management, analysis of the impact of complementarity on operational decisions and supply chain performance has been rather modest.

In this paper our goal is to characterize inefficiencies existing in supply chains with complements, as well as to reconcile these results with corresponding analysis for substitutes in order to create a unifying approach to such problems. Our paper makes two primary contributions to understanding the impact that the nature of supply-side externalities has on supply chain performance. First, we provide an in-depth analysis of centralized and competitive inventory management with complementarities, and draw parallels and emphasize contrasts between the management of complements and substitutes. Namely, with minimal assumptions we show that competition with complements leads to inventory understocking by retailers compared to the setting in which retailers are centrally managed. This is the opposite of the effect of competition with substitutes, which typically leads to overstocking. This effect for complements, is, however, stronger: while there are cases in which overstocking for substitutes does not hold, understocking for complements always exists for all retail locations.

Second, we demonstrate that many results in the literature on retail competition rely on the (often implicit) substitutability assumption. To augment these studies, we are bridging two thus far separate streams of literature studying substitutability and complementarity on the supply-side by investigating the implications that the *type* and *level* of externalities have on supply chain performance and channel members' incentives. For this purpose, we model substitutes/complements within a common framework and compare supply chain performance. We introduce two models (with externality through inventory and with externality through sales) of supply-side externalities in which changing a single parameter b from -1 to 1 (with -1 corresponding to strong substitutes, 0 corresponding to independent retailers and 1 corresponding to strong complements) allows us to investigate the impact of externalities on supply chain performance. We find that the total inventory in the supply chain is monotone increasing in b, i.e., perfect substitutes result in the lowest inventory and perfect complements result in the highest inventory. Taken

same resource (e.g., demand). When companies manage complementary products there is no such rivalry; rather, competition in this case should be understood to mean "decentralized decision-making". However, for the sake of brevity, we will continue to use the word "competition".

together with the fact that inventory is overstocked under competition on substitutes and understocked under competition on complements, we conclude that retail competition with complementarity aggravates the double marginalization effect and results in the severe understocking of inventory at the retail level and the subsequent degradation of supply chain performance. At the same time, competition with substitutability compensates the understocking due to double marginalization and hence improves supply chain performance. This analysis leads to the key managerial insight that supply chain coordination is more important under competition with complementarity at the retail level than in any other situation. In fact, with just two retailers competing on substitutable products, the supply chain can be perfectly coordinated by choosing the wholesale price appropriately. Moreover, we observe that under substitutability the wholesaler prefers competition over centralization of retailers, since more of the product is demanded by retailers without coordination. However, under complementarity the wholesaler prefers centrally managed over competing retailers, which coincides with retailers' preferences. This finding reinforces our conclusion that coordinating contracts should be observed more often in the supply chain with competition on complements, since every party has an incentive to coordinate.

The rest of the paper is organized as follows. In Section 2 we survey the related literature. Section 3 introduces the general framework for the analysis of complementarity. In Section 4 we compare supply chain performance in cases of substitutes vs complements and centralized vs decentralized inventory management for the two demand models. The implications of our research are discussed in Section 5.

2 Literature survey

The investigation of the issue of inefficiencies in distribution channels dates back to work by Spengler [26], who introduced the term "double-marginalization" and demonstrated that in a decentralized channel with more than one echelon each firm relies on its own profit margin when making decisions and hence does not take into consideration the total margin that is earned by the supply chain. In operational terms, a typical result of such inefficiency is understocking of inventory by channel members. Lariviere and Porteus [15] analyze inefficiencies arising in the supply chain with one wholesaler selling to a single retailer facing stochastic demand (hence, horizontal externalities are not present). In Section 4 we generalize their model to incorporate multiple firms (either competing or centralized) at the retail level.

The papers that are most relevant to our work consider externalities among retailers in the same supply chain echelon. These papers often compare stocking decisions made by independent retailers with those made by centrally controlled retailers and sometimes analyze the impact of such decisions on supply chain performance. Using a model with fixed total industry demand that is allocated among retailers, Lippman and McCardle [16] demonstrate that horizontal competition leads to inventory overstocking (as compared to centralized inventory management at the retail level) and with a sufficiently large number of retailers may lead to zero profits for the whole industry. Mahajan and van Ryzin [18] confirm this finding using the dynamic customer choice model. Netessine and Rudi [21] find that, although in most situations competition leads to overstocking, there are cases in which some of the retailers may understock the product under competition. Netessine et al. [22] model horizontal competition with the stocking decisions of retailers affecting future demand, and show that higher levels of competition lead to more overstocking.

All these papers consider the implications of horizontal competition for a single echelon of the supply chain. Several studies put horizontal externalities into the supply chain framework by introducing an upstream wholesaler into the model. Wang and Gerchak [28] consider competition on product availability where demand is a function of displayed inventory and find that under competition the total inventory stocked by retailers is higher than when retailers are centrally controlled. Boyaci [4] considers downstream competition with the product sold through two channels that compete on product availability. He shows that both channels overstock due to competition. Using the dynamic customer choice model, Mahajan and van Ryzin [17] show that inefficiency due to horizontal competition can actually benefit the supply chain since competitive overstocking of inventory compensates understocking due to double-marginalization. In fact, competition at the retail level may result in a system-optimal performance when the number of retailers is large enough. Hence, Mahajan and van Ryzin [17] conclude that horizontal competition improves supply chain performance. Their findings are very much in line with the results of Anupindi and Bassok [1], who show that retail competition may benefit the manufacturer more than the pooling of retail inventories into a central location, which may be beneficial to retailers only (while some of their results are close in spirit to some results in our work, the focus is different: we do not consider the pooling of inventories into one location since only stocking decision rights can be centralized). Overall, the results of these papers either prove or strongly suggest that horizontal competition leads to overstocking and improves supply chain performance. However, an implicit assumption that is common in all these papers is that externalities existing among retailers are in the form of *substitutability*. As we will show, this is a crucial assumption since *complementarity* can lead to very different results.

Complementarities within the same supply chain echelon have been analyzed by Wang and Gerchak [29] in a single-period model for assembly systems. Zhang [31] and Bernstein and DeCroix [2] also consider a two-echelon assembly system under decentralized control but utilize an infinite-horizon periodic-review inventory model. Among other results, they show that the decentralized management of assembly systems leads to underinvestment in capacity, which is similar to the inventory understocking that we find. However, since an assembly system represents an extreme case of complementarity (one subassembly can only be used if the other subassembly is available), it is not immediately clear if understocking extends to imperfect complements. Moreover, the assembly system is quite different in nature from the supply chain we consider since, first, there is a separate assembler who also invests in capacity and, second, there is no wholesaler who introduces double-marginalization. We are not aware of any other operations papers analyzing complementarities, but there is a related stream of work in economics with the major difference that this literature typically analyzes *demand-side* externalities (e.g., the willingness to pay for one product increases when the price of the other product rises, see Economides [12] for a representative work), as opposed to the *supply-side* externalities analyzed in this paper. Recently, Siggelkow [25] found that it is generally more costly to misperceive complements than substitutes, and hence firms should invest more into the management of complementary activities, which parallels our finding that it is worthwhile to explore more costly coordinating contracts to manage positive externalities among retailers.

Other representative papers that investigate the impact of externalities arising from horizontal competition include Bernstein and Federgruen [3], Deneckere et al. [10] and Rudi et al. [24]. The first two papers allow retailers, in addition to making stocking decisions, to set selling prices. These two papers differ from our work since, first, we take retail prices as exogenous, and second, the externalities in these papers are through pricing (demand-side externalities) rather than through inventory decisions. Since consideration in our paper is limited to supply-side externalities while prices are exogenous, it is not clear if our results extend to these other problem settings; more research needs to be done. In the third paper, the nature of competition is different from ours as well: firms are able to transship inventory at extra cost from a location with excess inventory to a location with excess demand. As such, competition is on substitutes and externalities are on the supply side but the dynamics are different: a product stocked by one firm can benefit two firms and the extent of the benefit depends on the transshipment price. Hence, firms can overstock or understock compared to the system-optimal solution.

Our paper is also one of the few to take steps towards developing a theory of when complex supply-chain coordinating contracts might be preferred over simple contracts. Cachon [5], in the survey of coordinating contracts, notes that although multiplicity of contracts has been proposed in the literature, little is known about why some are used in practice and others are not. The only papers that shed light on this issue are Cachon and Lariviere [7], Lariviere and Porteus [15] and Cachon [6]; all three describe situations in which a simple price-only contract can achieve high levels of coordination. We supplement their findings by showing that a simple contract might be efficient when products are distributed through a channel with competing retailers that exhibit substitutability, while a complex contract might be appropriate in the case of complementarity.

3 Complementarity

We will now introduce a general model for analyzing the inventory management problem with complementarities at the retail echelon. Two decision-making frameworks are analyzed – decentralized and centralized retail inventory management – and the results are compared. In the decentralized case, each retail location is managed by a separate retailer/manager whose objective is to maximize her own profit. In the centralized case, a central planner controls the stocking levels of all retailers with the goal of maximizing the total profits in the retail echelon. Risk neutrality is assumed throughout, so expected profit maximization is an appropriate criterion. There are complementarities among retail locations that, as we have previously discussed, may have different interpretations. Whether these externalities are from selling the same product through different retail locations or from selling different products is not important, since both interpretations can be captured by the same model.

Consider a group of retailers indexed by i or j, $i, j = 1, 2, \cdots, N$. Retailer i sets the quantity Q_i to purchase at the beginning of a single selling season before demand is observed. There is no second chance for replenishment during the season. Retailer i obtains the product at a cost w_i from the upstream wholesaler and sells it at a price r_i to downstream consumers. Retail prices are fixed, which might be a result of the sales price maintenance that is frequently employed by some manufacturers (e.g., a game console produced by Sony costs the same at any retail outlet and the price is announced by Sony at the time of product introduction). There is no additional penalty for lost sales and inventory has no salvage value (again, to keep the analysis transparent, the introduction of these parameters does not affect results). Assume that parameters satisfy $0 < w_i < r_i$ to avoid trivial solutions. Retailers' profits are denoted by π_i.

Let $Q = (Q_1, Q_{2,} \cdots, Q_N)$ be the quantity vector reflecting the stocking decisions of all retailers and $Q_{-i} = (Q_1, \cdots, Q_{i-1}, Q_{i+1}, \cdots, Q_N)$ be the quantity vector without the ith element. Define $S = \{x \in \Re^{N-1} : x \geq 0\}$ so S is a lattice and $Q_{-i} \in S$. Define partial ordering on S as follows: for $x, y \in S$, $x \geq y$ if and only if the inequality holds component-wise. Let $D_i^e(Q_{-i})$ be the effective demand for retailer i, which depends on the stocking levels of retailers other than i. $D_i^e(Q_{-i})$ is a random variable with a continuous distribution function (unless otherwise stated) and a finite mean for all Q_{-i}. We say that a random variable X is stochastically greater than a random variable Y, written as $X \geq_s Y$, if and only if $\Phi_Y(x) \geq \Phi_X(x)$ for all x, where $\Phi_X(x)$, $\Phi_Y(x)$ are cumulative distribution functions of random variables X and Y. It is obvious that $X \geq_s Y$ implies $EX \geq EY$. Next, we define what we mean by complementarity (substitutability).

DEFINITION: **If for all i, $Q'_{-i} \geq Q_{-i}$ implies $D_i^e(Q'_{-i}) \geq_s D_i^e(Q_{-i})$, then $Q_1, ..., Q_N$ are complementary. Analogously, if for all i, $Q'_{-i} \geq Q_{-i}$ implies $D_i^e(Q'_{-i}) \leq_s D_i^e(Q_{-i})$, then $Q_1, ..., Q_N$ are substitutable.**

The Definition of complementarity (substitutability) says that the effective demand of retailer i is increasing (decreasing) in Q_{-i} in a stochastic sense[3]. In particular, increasing (decreasing) the stocking level of other retailers raises (lowers) the expected demand for retailer i. Note also that this effect can occur directly through stocking quantity or indirectly through sales (i.e., stocking quantity impacts sales which, in turn, impacts demand for other products). For concision, denote $D_i^e(Q_{-i})$ by D_i^e when no confusion arises. The Definition is in the spirit of, but slightly more general than, the assumption made in Lippman and McCardle [16] in the analysis of the problem with demand substitution. We will see shortly that the Definition above is the only major requirement for many results in this section to hold. Some of the specific demand models that satisfy the Definition for complements are:

(i) Demand model with externality through sales. Suppose the original demand for each retailer is D_i. Additionally, a deterministic portion $\alpha_{ij} \geq 0$ of customers willing to purchase from retailer i will also purchase from retailer j but only if retailer i has the product in stock. Hence, $D_i^e = D_i + \sum_{j \neq i} \alpha_{ji} \min(D_j, Q_j)$. This model is a counterpart of the demand substitution model used by Parlar [23], Wang and Parlar [30], Netessine and Rudi [21], Boyaci [4] and others.

(ii) Additive demand model $D_i^e = A_i + g_i(Q_{-i}) + \varepsilon_i$, where A_i is a real number, $g_i(Q_{-i})$ is an increasing real-valued function and ε_i is a random variable. Netessine et al. [22] use a special case of such a model in the demand substitution setting with linear g_i. Additive externality is extensively used in the literature studying demand-side externalities (see, for example, Economides [12]).

(iii) Multiplicative demand model $D_i^e = \varepsilon_i g_i(Q_{-i})$, where $g_i(Q_{-i})$ is an increasing real-valued function and ε_i is a random variable with a positive mean. A similar form of demand is employed by Bernstein and Federgruen [3] to model retail competition on prices.

(iv) Demand model with perfect complements $D_i^e = \min(Q_1, Q_2, ..., Q_{i-1}, Q_{i+1}, ..., Q_n, D)$. This model is used by Wang and Gerchak [28] to analyze an assembly system. D is the demand for an assembly and Q_i is the inventory of component i.

[3]Throughout the paper we use "increasing" and "decreasing" in the weak sense.

3.1 Competition at the retail level

In the decentralized problem, each retail location is managed independently. We consider the case in which all firms simultaneously choose their stocking levels so the problem is an N-player static non-cooperative game with complete information. Denote the game by $(A_i, \pi_i^d : i \in N)$, where $A_i = [0, M]$ is the action space of player i and M is a sufficiently large number. Let $\pi_i^d(Q)$ be firm i's expected profit given that stocking quantity vector Q is chosen, then

$$\pi_i^d(Q) = E\left[r_i \min(Q_i, D_i^e) - w_i Q_i\right]. \tag{1}$$

Much of our analysis is based on the fact that the above definition of complements is sufficient for the game to be supermodular so that some of the properties of supermodular games can be exploited. The next proposition makes this statement precise (see the Appendix for proofs).

Proposition 1 *Under decentralized inventory management and assuming that the products are complementary:*

1) The game is supermodular.

2) Let E be the set of Nash equilibria of the game. Then E is nonempty and has largest and smallest elements $\overline{Q}$ and $\underline{Q}$, respectively. In addition, $\overline{Q}$ is a Pareto-optimal equilibrium.

3) A vector Q^d is a Nash equilibrium if the following equations are satisfied:

$$\Pr\left(D_i^e \le Q_i^d\right) = (r_i - w_i)/\, r_i,\; i = 1, ..., N. \tag{2}$$

If D_i^e has continuous distribution, the condition (2) is also necessary.

To facilitate intuition, let $D_i = D_i^e(0, ..., 0)$. If D_i and D_i^e are defined on the same probability space and $D_i^e \ge D_i$ almost surely (these conditions are satisfied by examples $(i) - (iv)$), then the above FOC can be conveniently written as

$$\Pr(D_i \le Q_i^d) - \Pr(D_i \le Q_i^d \le D_i^e) = (r_i - w_i)\,/r_i. \tag{3}$$

Without the second term on the left-hand side, the complementarity effect, equation (3) gives the familiar newsvendor quantity. Hence with complementarity, the inventory level for every retailer is greater than the newsvendor quantity. The optimality condition for substitutes differs only in the functional form of the effective demand D_i^e. It is also easy to see that with complementarity, the equilibrium inventory levels are higher than the newsvendor solution for all retailers (the same holds with the substitutability model of Netessine and Rudi [21]). Multiple equilibria may exist in this game, that is, there could be multiple solutions to the above optimality conditions. One example with multiple equilibria is reported by Wang and Gerchak [28], who use a demand model as in (iv) above.

3.2 Centralization at the retail level

Under centralized inventory management, all retail locations are managed centrally. The expected total profit can be written as

$$\pi\left(Q\right)=\sum\nolimits_{i}\pi_{i}\left(Q\right)=E\sum\nolimits_{i}\left[r_{i}\min(Q_{i},D_{i}^{e})-w_{i}Q_{i}\right]. \tag{4}$$

For further analysis, let $D_i^e(Q_{-i},\omega)$ denote the demand for a particular sample path $\omega\in\Omega$ (Ω is the sample space). To ensure that the model is well-behaved, we provide a condition sufficient for the uniqueness of the solution.

Proposition 2 *If $D_i^e(Q_{-i},\omega)$ is concave in Q_{-i} for all $\omega\in\Omega$, then $\pi(Q)$ is jointly concave in Q. In addition, if D_i^e has a continuous distribution, then $\pi(Q)$ is differentiable and the first-order conditions are sufficient for optimality.*

It is rather straightforward to demonstrate that demand models (i) and (iv) satisfy the condition of Proposition 2 and demand models (ii) and (iii) satisfy it with the mild additional assumption that the function g_i is concave in Q_{-i} for each i, which is a natural assumption implying decreasing marginal returns to an increase in any one stocking quantity. Hence, under all these demand models there is a unique inventory policy when retailers are centrally managed. It is interesting to note that when there is substitutability among retailers, a unique solution under centralized inventory management is not guaranteed: in fact Netessine and Rudi [21] and Mahajan and van Ryzin [19] demonstrate non-quasi-concave objective functions in their models. From the previous result we see that complementarities are somewhat easier to handle.

3.3 Comparison of centralized and decentralized inventory management

We will now state and prove the first key result of this paper: the comparison of inventory policies under centralized and decentralized control for a single supply chain echelon. It has been established in the extant literature that competition on substitutes *typically* leads to inventory overstocking by all retailers. The intuition behind this result is as follows: independent retailers ignore substitutability and hence tend to stock more than is centrally optimal. However, Netessine and Rudi [21] show through a counter-example that under substitution some retailers can actually understock. Remarkably, in the next proposition we are able to demonstrate that under competition on complements, *all* retailers *always* understock with no exceptions.

Proposition 3 *Suppose the centralized optimal solution Q^c is unique. Then for any equilibrium Q_i^d, $Q_i^c\geq Q_i^d$ for all i, all firms understock under competition. Further, under competition each retailer receives less profit than in the centralized case: $\pi_i\left(Q^d\right)\leq\pi_i\left(Q^c\right)$.*

Proof. The proof is by contradiction. Take Q^d as the largest equilibrium of the decentralized game and suppose $Q_i^c\geq Q_i^d$ for all i is not true. We can partition the index set $\{1,2,\cdots,N\}$ into two subsets A and B, where A is nonempty, such that $Q_i^c<Q_i^d$ for $i\in A$, $Q_i^c\geq Q_i^d$ for $i\in B$. Below we show that this assumption leads to a contradiction. The proof proceeds in two steps.

1. $B = \emptyset$. That is, assume $A = \{1, 2, \cdots, N\}$ or in other words $Q_i^c < Q_i^d$ for all i. Note that in the centralized case each retailer earns

$$\pi_i(Q^c) = E[r_i \min(Q_i^c, D_i^e(Q_{-i}^c)) - w_i Q_i^c]$$

and under competition each retailer earns

$$\pi_i(Q^d) = \max_{Q_i} E[r_i \min(Q_i, D_i^e(Q_{-i}^d)) - w_i Q_i].$$

Since $Q_{-i}^c < Q_{-i}^d$ and $D_i^e(Q_{-i})$ is stochastically increasing in Q_{-i}, we know

$$E[r_i \min(Q_i, D_i^e(Q_{-i}^c)) - w_i Q_i] \leq E[r_i \min(Q_i, D_i^e(Q_{-i}^d)) - w_i Q_i]$$

for all Q_i. Moreover, Q_i^d is a maximizer of $\pi_i(Q_i, Q_{-i}^d)$ so it follows that $\pi_i(Q^c) \leq \pi_i(Q^d)$ for all i. If the strict inequality holds for any i, then $\pi(Q^c) < \sum_i \pi_i(Q^d)$. This is a contradiction because profit under centralized inventory management should be no less than that in the decentralized case. If the equality holds for all i, then it implies that Q_i^d is also a centralized optimal solution. This contradicts the assumption that Q_i^c is the unique optimal solution.

2. $B \neq \emptyset$. Use Q_A for the quantity vector of the retailers in A, and define Q_B similarly. Consider a new decentralized game in which $\widehat{Q}_B^d = Q_B^c$ are fixed. That is, in the new game, players belonging to B have fixed stocking levels as in the centralized solution, while players in A can simultaneously choose their stocking levels. The game (essentially only for players in A) is still supermodular. Denote player i ($i \in A$)'s payoff function in this new game by $\widehat{\pi}_i(Q_i, Q_{A\setminus\{i\}})$. Then $\widehat{\pi}_i(Q_i, Q_{A\setminus\{i\}})$ has increasing differences in Q_i and $\widehat{Q}_B^d$ for each $Q_{A\setminus\{i\}}$. By Theorem 4.2.2 in Topkis [27], the (largest) equilibrium of the new game, $\widehat{Q}_A^d$, is increasing in $\widehat{Q}_B^d$. Since $\widehat{Q}_B^d = Q_B^c \geq Q_B^d$, we know $\widehat{Q}_A^d \geq Q_A^d > Q_A^c$. Now we have $\widehat{Q}^d > Q^c$ again, which leads to contradiction by an argument similar to step 1. ■

In the decentralized setting, players ignore the complementarity effect that their stocking decisions have on other players, which leads to the understocking of inventory. The underlying reason for this result is the supermodularity of the game which enables monotonicity of decisions. For further intuition consider the following reasoning. Suppose each player has some amount of initial inventory and players can adjust their inventories sequentially. Suppose that player 1 decides to increase his stocking quantity. Clearly, such an increase leads to more demand for all other players, thus inducing all retailers to stock more, which in turn increases demand for player 1 and induces him to further increase stocking quantity and so on. As a result, we can argue that all stocking quantities under complementarity will move in the same direction regardless of the initial inventory that players get. Now consider competition on substitutes. Increasing the stocking quantity of player 1 reduces demand for all other players and persuades them to decrease their stocking quantities. This move further increases demand for player 1 so that the stocking quantities of player 1 and of all other players move in opposite directions. Hence, the stocking quantities of players behave differently: some increase and some decrease. From a managerial perspective, the impact of competition on inventory management with complementarities appears more predictable and transparent than the impact of competition on inventory management with substitutes. Finally, the incentives of all retailers are aligned under complementarity: they prefer centralized inventory control. Clearly, this may not be the case under substitutability: just as some retailers may overstock and others understock, some retailers may not benefit from the centralization of inventory decisions.

4 Complements vs substitutes and competition vs centralized control: a duopoly analysis

In this section we consider the key issue of this paper: comparative analysis of the two types of horizontal supply-side externalities – complements/substitutes and two types of decision-making frameworks – centralized/decentralized retail inventory control. While the previous section allowed us to outline certain parts of this comparison, not all situations in practice can be simply characterized by complementarity or substitutability, since both the direction of externality and the strength of externality have effects. Hence, the question is: how can situations with these different levels of externalities be compared?

From practical considerations, two distinct forms of supply-side externalities can arise. The first form is the externality through sales whereby an increase in sales (which is a function of both demand and supply) of any product affects demand for other products. This form of externality may arise between a TV and a VCR (demand for VCRs is higher if more TVs are sold, not merely if more TVs are stocked), CDs and CD players, GAP and CK jeans (the jeans-wearing population is relatively fixed so if a person buys one brand, he/she in not likely to buy the other), etc. The second form of supply-side externality occurs directly through stocking quantity, i.e., the inventory of one product affects the demand for other products. For example, in Cachon et al. [8], the mere presence of other products can increase the demand for a product since the customer is less likely to search for alternatives elsewhere. We analyze these two forms of supply-side externalities separately. Evidently, a shortcoming of considering externalities through sales and through stocking quantity separately is that both forms of externalities might be present at the same time. However, we chose to restrict consideration to one form of externality at a time in order to simplify the problem and develop intuition. Since most of our results coincide for both forms of externality, it is plausible that such a simplification is not critical.

To provide a sound description of supply-side externalities, we need models that cope with reality, making simplifications but with careful and explicit justification. In what follows, we focus our analysis on the demand model similar to (*ii*) above with externality through inventory. We have also verified that most of our results for this model continue to hold when externality occurs through sales (as in (*i*) above) – details of this analysis are provided in the On-line Supplement[4]. These modeling assumptions do not apply equally well to all practical situations described previously but rather represent a trade-off between analytical tractability and realism. However, since it will be demonstrated that the same main results hold for these two distinct models, our results and conclusions are relatively robust to the model specification. Although we make no claim that the assumptions about the analytical form of demand we chose to analyze are entirely realistic, we maintain that they capture the essence of the strategic interaction that takes place when supply-side externalities are involved.

[4]Available at http://www.msom.org.

In what follows, our goal is to compare the inventory policies for various values of externality in three models: a model with retailers and the wholesaler centrally controlled (we will call it the *supply chain optimal* model with optimal order quantities Q_i^o), a model with competing retailers (correspondingly, the *decentralized* model characterized by Q_i^d, π_i^d and Π^d) and a model with centrally controlled retailers but the wholesaler as a separate entity (correspondingly, the *centralized* model characterized by Q_i^c, π_i^c and Π^c). We now analyze the model in which the externality occurs directly through stocking quantity. For analytical transparency we focus on the model with only two retailers but we are able to obtain analytical analysis for a rather general functional form of externality. Namely, we utilize the demand model (*ii*) outlined earlier:

$$D_i^e(Q_j) = A_i + g_i(bQ_j) + \varepsilon_i, \; i = 1, 2 \tag{5}$$

where A is a constant, $g_i(\cdot)$ is a strictly increasing ($g_i' > 0$), concave ($g_i'' \leq 0$), real-valued function with $g_i(0) = 0$ (so that when $b = 0$, demand does not depend on the competitor's inventory) and ε_i is a random variable so that the random shock is additive. We denote the density function of D_i^e by $f_{D_i^e}(\cdot)$. We also assume that $|bg_i'| < 1$, since otherwise the model might not be stable: increasing the stocking quantity to infinity or reducing it to zero might be optimal.

We first assume that the wholesaler does not have price-setting power but we later relax this assumption. In the centralized and decentralized models the wholesaler earns the following profit:

$$\Pi^k = \sum\nolimits_i (w_i - c) Q_i^k, \; k = c, d, \tag{6}$$

which is deterministic. We begin by considering the decentralized model with competing retailers.

Proposition 4 *Under decentralized inventory management with demand model (5)*

1) The game is supermodular for $b > 0$ and submodular for $b < 0$.

2) The game possesses a unique globally stable Nash equilibrium.

3) The total inventory stocked by the two competing retailers in equilibrium (and hence the wholesaler's profit) is monotonically increasing in b.

We now consider the same problem but under centralized inventory management at the retail level. By assuming that the retailers are symmetric and $g(\cdot)$ is not too concave, we are able to prove that the total inventory is increasing in b.

Proposition 5 *Under the centralized inventory management with demand model (5)*

1) The unique global optimum is found from the following optimality conditions:

$$\Pr\left(D_i^e \leq Q_i\right) = \left(r_i - w_i + \left(r_j - w_j\right) b g_j'\right) / r_i \left(1 - b^2 g_i' g_j'\right), \; i, j = 1, 2. \tag{7}$$

2) The objective function is supermodular when $b > 0$ and submodular when $b < 0$.

3) In a symmetric problem, $Q_i^c, i = 1, 2$ are monotonically increasing in b as long as $-\,bQg''/\,g' < 1$, and hence the wholesaler's profit is increasing as well.

The assumption in item 3) has analogs in economic theory. For any utility function $u(x)$, a measure $r_R = -xu''(x)/u(x)$ is called the coefficient of relative risk aversion (see Mas-Collel et al. [20] page 194 for the definition and interpretations). One can notice that this condition in the Proposition ensures that function $g(\cdot)$ is not too concave, which is satisfied, for example, for any $b < 0$ or for any linear $g(\cdot)$. We conjecture at this point that total inventory will be increasing in b even when the problem is not symmetric, which we also verified through extensive numerical experiments.

It is quite clear that the supply chain optimal model can be analyzed in exactly the same way as the model with centralized retailers: the only difference is that $w_i = c$. One observation we can make immediately is that $Q_i^o \geq Q_i^c$: due to double-marginalization, retailers understock in the centralized model as compared to the system-optimal quantity. The next step is to compare stocking policies under centralization Q^c and decentralization Q^d at the retail level, and to further compare Q_i^o and Q_i^d. From Proposition 3 we know that both retailers will understock in the decentralized model under complementarity (i.e., when $b > 0$) as compared to the centralized model. We will also verify that in this model retailers will overstock under substitutability (i.e., when $b < 0$). While this result has been demonstrated previously in other models, it has not been established in general (and hence it is not clear if it holds for our model) and moreover, examples exist in which overstocking does not occur (see Netessine and Rudi [21]). In the next proposition we confirm that the retailers indeed overstock under substitution and moreover, we show that this overstocking can compensate for the double-marginalization effect and can result in a system-optimal performance.

Proposition 6 *The following holds under demand model (5):*
1) $Q_i^c \geq Q_i^d$ *for* $b > 0$, $Q_i^c = Q_i^d$ *for* $b = 0$, *and* $Q_i^c \leq Q_i^d$ *for* $b < 0$.
2) $\Pi^c \geq \Pi^d$ *for* $b > 0$, $\Pi^c = \Pi^d$ *for* $b = 0$, *and* $\Pi^c \leq \Pi^d$ *for* $b < 0$.
3) In the decentralized model for $b \leq 0$ *there always exist wholesale prices* $c \leq w_i \leq r_i$ *that achieve channel coordination under a simple wholesale price contract. No wholesale price contract exists that coordinates the supply chain for* $b > 0$.

As the last proposition indicates, competition at the retail level is beneficial for the supply chain in the case of substitutes, but is detrimental in the case of complements, and retail competition on substitutes can coordinate the supply chain (if wholesale prices are chosen appropriately) since it compensates for the effect of double-marginalization.

To illustrate our results graphically, we assume that retailers are symmetric and demand is a linear function of the competitors' stocking decisions, $D_i^e(Q_j) = A_i + bQ_j + \varepsilon_i$. Further, we assume that ε has Normal distribution with $\mu = 0$, $\sigma = 50$, $\rho = 0$, and that other problem parameters are $r = 150$, $w = 110$, $c = 80$, $A = 150$. Under these parameters, only values of $b < 0.53$ result in non-trivial solutions (as can be verified from optimality conditions). First, we plot the absolute inventory as a function of b (Figure 1). In all three models total inventory is increasing in b, and when retailers compete, the magnitude of the double marginalization effect is alleviated by substitution and is aggravated by complementarity. As the figure indicates, at $b = -0.8$ the decentralized model results in the same performance as the supply chain optimal model and for $b < -0.8$ decentralized retailers even stock more than is system-optimal. To visually represent supply chain losses, we plot inefficiencies arising in the decentralized and the centralized models measured as a percentage of the supply chain optimal solution in Figure 2. The number on the vertical axis is calculated as

$$\frac{\text{supply chain optimal profit - profit for the scenario}}{\text{supply chain optimal profit}} * 100.$$

It is interesting to note that profit losses in the centralized model are essentially independent of the level of externality but not so in the decentralized model.

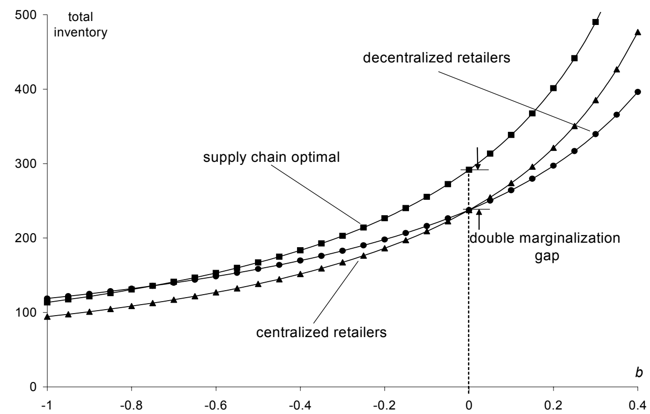

Figure 1. Inventory in the channel.

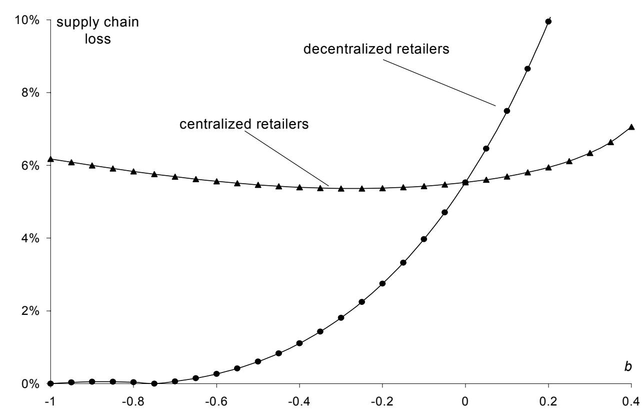

Figure 2. Supply chain efficiency loss.

We have demonstrated that retailers stock more under complementarity than under substitutability. However, a major underlying assumption that leads to such a conclusion is that the wholesaler will charge retailers the same wholesale price for any value of b. This assumption is acceptable if the wholesaler exists in a competitive environment and does not possess any price-setting power. Otherwise, one might expect that the wholesaler will establish a higher wholesale price for higher values of b since more is demanded by retailers. As a result, it will no longer be obvious if complementarity will still lead to higher inventory and whether the comparison of inventory decisions made in Proposition 6 will be preserved, since wholesale prices can differ under centralized and decentralized decision making. It turns out that the results of this section continue to hold if the wholesale price is endogenous but under some additional simplifying assumptions. The model with a wholesaler selling to a single newsvendor has been addressed only recently by Lariviere and Porteus [15]. The only papers we are aware of that consider a price-setting wholesaler selling to multiple newsvendors are Dong and Rudi [11] and Anupindi and Bassok [1] (with centralized and competing retailers, correspondingly). To achieve tractability, additional assumptions had to be made in these papers (Normal demand distribution and Normal demand distribution with approximations, respectively). Naturally, given the difficulty of the problem, we have to make some additional simplifying assumptions as well: namely, we consider only a symmetric problem with a linear demand function $D^e(Q) = A + bQ + \varepsilon$. The wholesaler maximizes (6) with respect to the wholesale price. Proposition 7 establishes that most of our results derived for a fixed wholesale price continue to hold in the presence of vertical competition and under mild distributional assumptions. Note that our analysis has the model of Lariviere and Porteus [15] as a special case for $b = 0$.

Proposition 7 *Suppose ε has an IFR (Increasing Failure Rate) distribution. Then, for the symmetric problem and the linear demand function*

1) *There are unique stocking quantities* Q^d, Q^c *(and hence wholesale prices* w^d, w^c*) maximizing the wholesaler's respective objective functions.*

2) *Both* Q^d *and* Q^c *are monotone increasing in* b.

3) $Q^c \geq Q^d$ *for* $b > 0$, $Q^c = Q^d$ *for* $b = 0$, *and* $Q^c \leq Q^d$ *for* $b < 0$.

5 Managerial implications for supply chain design

Supply-side externalities are a subject of extensive research in operations management. So far, however, supply-side externalities have been treated almost exclusively in the setting with substitutable products. We provide the first comprehensive treatment of the complementarities and bridge these two topics by highlighting the similarities and differences between them.

Our first main result is that under competition on complements retailers understock inventory, thus aggravating the double-marginalization effect. Understocking of inventory by all retailers is a quite general result that does not rely on the specific form of complementarity (other than that demand for any retailer should be stochastically increasing in any other retailers' inventory). This result should be contrasted with overstocking of inventory in the case of competition on substitutes, which holds most of the time but not always. If we go back to the examples with competition on complements cited in the introduction, several parallels can be drawn between our results and these examples (although, undoubtedly, our model is too simple to serve as the only explanation of these phenomena). For instance, when Philips introduced a new digital music system (Digital Compact Cassette) it obtained commitments from various record companies (independent from Philips) that up to 500 titles would be available in DCC format (Church and Gandall [9]). This case seems to point out that, absent these pre-commitments, availability of compatible music media would be lacking, which is akin to the understocking we predicted. The failure of the Betamax video cassette recorder due to the lack of compatible tapes at video stores also points towards understocking. Finally, the shortage of add-on boards for MCC architecture introduced by IBM may, at least in part, be due to decentralized decision making on the part of add-on manufacturers/retailers.

Our second insight relates to the issue of retail centralization vs competition: when is it preferable to centralize retail control and when does it make sense to allow independent retail store operation? Seemingly, coordination (even for a part of the supply chain) should always be beneficial. The following table summarizes the incentives of channel members.

Type of competition	Retailer's preferences	Wholesaler's preferences	System's profit perspective
Substitutes	Vary	Retail Competition	Retail Competition
Complements	Retail Centralization	Retail Centralization	Retail Centralization

Table 1. Centralization vs competition at the retail level.

An important take-away here is that under complementarity all channel members have an incentive to centralize retail operation: even though system-optimal performance will not be achieved, under centralized retail control the supply chain only suffers from double-marginalization. On the other hand, under substitution at least the wholesaler always benefits from competition among retailers.

Our third insight concerns the comparative performance of the four situations: competition vs centralization at the retail level and complements vs substitutes. We find that retail competition with complements results in the highest supply chain inefficiency, followed by centralized retail operation (either with complements or with substitutes), and retail competition with substitutes results in the lowest inefficiency. Hence, we can conclude that complex coordinating contracts that involve high administrative costs are most likely worth using when retailers are competing on complements. On the other hand, when retailers compete on substitutes, a simple wholesale price only contract might perform well. Cachon [5] notes that, although numerous complex coordinating contracts have been proposed in academic literature, simple wholesale price contracts are more often observed in practice. Our finding provides partial intuition that may help explain this phenomenon: since, perhaps, in most practical cases retailers compete with substitutable products, simple wholesale price contracts may be very efficient. Based on our results, one could also argue that complex coordinating contracts should be applied early in the product life cycle when there are significant complementarities among retail locations: if more is sold by one store, demand for other stores is higher (akin to the network externality effect, see Economides [12]). Later in the life cycle the product matures and market saturation occurs so different retail stores become substitutes; if more is purchased at one store, fewer customers will buy from other stores. Hence, our results can be interpreted as follows: supply chain coordination is much more important in the early stages of product introduction since inefficiencies are greatest at this stage. Our findings are in line with anecdotal evidence existing in the literature that before a product launch the manufacturer typically uses various tools to ensure that retailers stock sufficient quantities of the product.

This insight can also be applied to retail franchises (such as RadioShack, Hallmark, etc.). Consider externalities arising between two retailers belonging to the same chain of stores as a function of the geographical distance between them. When retailers are close together, they serve essentially the same market so substitution effects dominate. On the other hand, when retailers are far apart, complementarities through branding and word-of-mouth will prevail. In the first case inefficiencies in the supply chain will be lower. If the manufacturer has a rule of granting exclusive territories to retailers (as is the case with RadioShack and many other franchised retailers), he has to be aware of the inefficiencies that will arise due to complementarities and resulting understocking. Consistent with this reasoning, Lafontaine and Slade [14] point out that contract clauses such as exclusive territories occur together with various forms of quantity forcing. That is, the franchiser might be aware of understocking behavior caused by granting exclusive territories and hence requires retailers to maintain certain inventory availability.

We have demonstrated that supply-side externalities can have profound implications for various managerial situations. However, as with any other model, care should be taken in applying our results, which rely on several simplifying assumptions. First, we only considered supply-side externalities while other types of externalities exist: advertising and pricing are two other important decisions that are known to create externalities. More research is needed to find out how these other externalities affect supply chain performance. Furthermore, in many practical situations it can prove difficult to figure out if complementarity or substitutability is present. For example, at which point in the product life cycle does substitutability begin to dominate? To answer this question one would have to use a Bass diffusion model to derive externalities more precisely (see Kini et al. [13], who derive demand-side externalities arising between competing firms in a Bass-like setting but without supply-side considerations). Finally, confirmation of our findings through empirical data is needed. For example, there is extensive empirical research on franchising agreements but, to the best of our knowledge, none of it analyzes the supply chain inefficiencies that arise as a result of such agreements.

Appendix: Proofs

Proof. [Proposition 1] 1) By the definition of supermodular games (Topkis [27]), we need to show for all i: A_i is a compact lattice, π_i is upper semi-continuous, π_i is supermodular in Q_i given fixed Q_{-i}, and π_i has increasing differences in (Q_i, Q_{-i}). Since π_i is continuous (due to the continuity of the distribution function of D_i^e), it is also upper semi-continuous. π_i is trivially supermodular in Q_i because the action space is one-dimensional. It remains to show supermodularity in Q_{-i}. Fix $Q_i' \geq Q_i$, and define

$$f(Q_{-i}) = \pi_i^d((Q_i', Q_{-i})) - \pi_i^d((Q_i, Q_{-i})),$$

then we need to show that $f(Q_{-i})$ is increasing in Q_{-i}. Define

$$g(x) = \min[Q_i', x] - \min[Q_i, x],$$

then $g(x)$ is an increasing function in x, and

$$f(Q_{-i}) = r_i Eg(D_i^e(Q_{-i})) - w_i\left(Q_i' - Q_i\right).$$

Since $D_i^e(Q_{-i})$ is stochastically increasing in Q_{-i}, we know $f(Q_{-i})$ is increasing in Q_{-i} as well. Thus we have shown that the decentralized problem is a supermodular game.
2) Follows from Theorem 3.1 of Topkis [27] for a supermodular game. In a supermodular game, if player i's payoff function is increasing in other players' strategies, then the largest equilibrium is Pareto-optimal.
3) Similar to Lippman and McCardle [16], we first show sufficiency of the condition. Assume (2) is satisfied. Consider the profit function of firm i given in (1). If firm i increases Q_i^d by a small amount δ, then the cost part $(w_i Q_i)$ will increase by $w_i\delta$, while the revenue part $r_i \min(Q_i, D_i^e)$ will increase by at most $r_i\delta\frac{w_i}{r_i} = w_i\delta$ so firm i has no incentive to raise its stocking level. Similarly, we can show that firm i has no incentive to decrease its stocking level. Thus the condition is sufficient. If D_i^e has continuous distribution, then $\pi_i^d(Q)$ is differentiable in Q_i. Note that $\pi_i^d(Q)$ is concave in Q_i, so firm i's best response must satisfy the first-order condition:

$$\partial\pi_i^d(Q)/\partial Q_i = r_i \Pr(D_i^e \geq Q_i^d) - w_i = 0. \tag{8}$$

■

Proof. [Proposition 2] First we show that $r_i \min(Q_i, D_i^e) - w_i Q_i$ (the term inside the summation) is concave in Q for each ω. Since $w_i Q_i$ is linear and $r_i > 0$, we only need to prove the concavity of $\min(Q_i, D_i^e)$. Since $D_i^e(Q_{-i}, \omega)$ is concave in Q_{-i} and $\min(f, g)$ is concave if both f and g are concave functions, we know $\min(Q_i, D_i^e)$ is concave for each particular ω. The rest follows by using the fact that summation and expectation preserve concavity.

■

Proof. [Proposition 4] 1) To verify complementarity/substitutability, take the second derivatives

$$\frac{\partial^2\pi_i^d(Q_i, Q_j)}{\partial Q_i \partial Q_j} = r_i b g_i' f_{D_i^e}(Q_i) \begin{cases} \geq 0 \; if \; b \geq 0, \\ < 0 \; if \; b < 0. \end{cases}$$

2) To assure uniqueness, it is sufficient to show that the slopes of best-response functions are less than one in the absolute value. Indeed, $\left|\partial Q_i^d \middle/ \partial Q_j\right| = |b g_i'| < 1$.
3) We apply the Implicit Function Theorem to the optimality conditions (8) and obtain the system of

equations (9). The resulting matrix equality that has to hold at the equilibrium is

$$\left| \begin{array}{cc} \frac{\partial^2 \pi_1}{\partial Q_1^2} & \frac{\partial^2 \pi_1}{\partial Q_1 \partial Q_2} \\ \frac{\partial^2 \pi_2}{\partial Q_1 \partial Q_2} & \frac{\partial^2 \pi_2}{\partial Q_2^2} \end{array} \right| \times \left| \begin{array}{c} \frac{\partial Q_1^*}{\partial b} \\ \frac{\partial Q_2^*}{\partial b} \end{array} \right| = - \left| \begin{array}{c} \frac{\partial^2 \pi_1}{\partial Q_1 \partial b} \\ \frac{\partial^2 \pi_2}{\partial Q_2 \partial b} \end{array} \right| . \tag{9}$$

We can find some entries:

$$\partial^2 \pi_i^d / \partial Q_i^2 = -r_i f_{D_i^e}(Q_i) < 0, \; \partial^2 \pi_i^d / \partial Q_i \partial b = r_i Q_j g_i' f_{D_i^e}(Q_i) > 0,$$

so that the matrix equation can be solved explicitly as follows:

$$\partial Q_i^d / \partial b = g_i' \left(Q_j + Q_i b g_j' \right) / \left(1 - b^2 g_i' g_j' \right) , \; i, j = 1, 2.$$

From these expressions we can obtain the reaction of the total inventory in the supply chain as a response to a change in b

$$\partial \left(Q_i^d + Q_j^d \right) / \partial b = \left(Q_i g_j' \left(1 + b g_i'\right) + Q_j g_i' \left(1 + b g_j' \right) \right) / \left(1 - b^2 g_i' g_j' \right) .$$

First, note that the denominator is positive due to the assumption that $|b g_i'| < 1$. Second, the numerator is clearly positive due to the same assumption. Hence, we are assured that the total stocking quantity is increasing in b. The wholesaler's profit is clearly increasing in b if $\left(Q_1^d + Q_2^d \right)$ is increasing.
■

Proof. [Proposition 5] 1) Global concavity follows from Proposition 2. First derivatives are

$$\partial \pi^c (Q_i, Q_j) / \partial Q_i = r_i \Pr(D_i^e \geq Q_i^c) + r_j b g_j' \Pr(D_j^e \leq Q_j^c) - w_i, \; i, j = 1, 2. \tag{10}$$

After equating to zero and solving simultaneously, we obtain the solution.
2) To check substitutability/complementarity we can find the second derivatives

$$\frac{\partial^2 \pi^c}{\partial Q_i \partial Q_j} = r_i b g_i' f_{D_i^e}(Q_i) + r_j b g_j' f_{D_j^e}(Q_j) \left\{ \begin{array}{l} > 0 \, if \, b > 0, \\ < 0 \, if \, b < 0. \end{array} \right.$$

3) If the problem is symmetric, it has a symmetric solution that we can write as follows (subscripts can be omitted):

$$\Pr(D^e \leq Q^c) = (r - w_i) / (r \left(1 - b g'\right)) . \tag{11}$$

From the implicit differentiation we find

$$\frac{\partial Q^c}{\partial b} = - \frac{- \left(g' + b Q g''\right) \Pr(D^e \leq Q^c) - \left(1 - b g'\right) Q g' f(Q^c)}{-b^2 g'' \Pr(D^e \leq Q^c) + \left(1 - b g'\right)^2 f(Q^c)} \geq 0$$

as long as $(- \, b Q g'' / \, g') < 1$.
■

Proof. [Proposition 6] 1) The result for $b > 0$ follows immediately from Proposition 3. Suppose that $b < 0$ (the substitution case). From the optimality conditions it is straightforward to see that $\Pr(D_i^e > Q_i^c) > \Pr(D_i^e > Q_i^d)$ so that

$$\Pr(\varepsilon_i > Q_i^c - A_i - g_i(b Q_j^c)) \geq \Pr(\varepsilon_i > Q_i^d - A_i - g_i(b Q_j^d)), \; i, j = 1, 2,$$

and it immediately follows that

$$Q_i^c - Q_i^d \leq g_i\left(bQ_j^c\right) - g_i\left(bQ_j^d\right),\ i,j = 1,2.$$

Consider all possible scenarios:
1. $Q_1^c < Q_1^d, Q_2^c > Q_2^d$. Second inequality ($i = 2,\ j = 1$) does not hold.
2. $Q_1^c > Q_1^d, Q_2^c < Q_2^d$. First inequality ($i = 1,\ j = 2$) does not hold.
3. $Q_1^c > Q_1^d, Q_2^c > Q_2^d$. Then re-write the inequalities as follows:

$$\frac{Q_i^c - Q_i^d}{Q_j^c - Q_j^d} \leq \frac{g_i(bQ_j^c) - g_i(bQ_j^d)}{Q_j^c - Q_j^d},\ i,j = 1,2.$$

Clearly, the expressions on the right are derivatives of g_i w.r.t. Q that are assumed to be less than one so we get

$$\frac{Q_1^c - Q_1^d}{Q_2^c - Q_2^d} \leq 1, \frac{Q_2^c - Q_2^d}{Q_1^c - Q_1^d} \leq 1,$$

but these two inequalities cannot hold together. The only remaining possibility is $Q_1^c < Q_1^d$, $Q_2^c < Q_2^d$.
2) Follows immediately from the previous result.
3) It is straightforward to verify that the system-optimal solution must satisfy the following optimality conditions:

$$\frac{\partial \pi^o}{\partial Q_i} = r_i \Pr\left(D_i^e\left(Q_j^o\right) \geq Q_i^o\right) + r_j b g_j'\left(bQ_i^o\right) \Pr\left(D_j^e\left(Q_i^o\right) \leq Q_j^o\right) - c = 0,\ i,j = 1,2. \tag{12}$$

Recall that the solution in the case with competing retailers is $w_i = r_i \Pr\left(D_i^e\left(Q_j^d\right) \geq Q_i^d\right)$, $i,j = 1,2$. We need to show that we can choose $c \leq w_i \leq r_i$ such that $w_i = r_i \Pr\left(D_i^e\left(Q_j^o\right) \geq Q_i^o\right)$. This is achieved as long as $b \leq 0$ (since otherwise $r_i \Pr\left(D_i^e\left(Q_j^o\right) \geq Q_i^o\right) < c$ so that $w_i < c$). Hence, if $b \leq 0$ the supply chain optimal solution can always be achieved by some $c \leq w_i \leq r_i$.
■

Proof. [Proposition 7] Denote $\Gamma(Q) = f_{D^e}(Q)Q/\Pr(D^e \geq Q)$, generalized failure rate, and consider decentralized inventory management first.
1) From (8), we can find the wholesale price as a function of the stocking quantity as $w^d = r\Pr(D^e \geq Q)$ and substitute into the wholesaler's objective function:

$$\Pi^d = 2(w^d - c)Q = 2(r\Pr(D^e \geq Q) - c)Q.$$

After differentiating w.r.t. Q and rearranging we get

$$\frac{\partial \Pi^d}{\partial Q} = 2r\Pr(D^e \geq Q)\left[1 - (1-b)\Gamma(Q)\right] - 2c.$$

Clearly, if ε has IFR distribution then the first derivative is decreasing in Q monotonically so that there is a unique Q^d (and hence a unique w^d) that is optimal for the wholesaler.
2) Since the objective function is concave around the optimum, it is sufficient to show that $\partial^2\Pi^d \big/ \partial Q\partial b \geq 0$:

$$\begin{aligned}\frac{\partial^2 \Pi^d}{\partial Q \partial b} &= 2r f_{D^e}(Q)Q\left[1 - (1-b)f_{D^e}(Q)Q/\Pr(D^e \geq Q)\right] \\ &+ 2r\Pr(D^e \geq Q^d)\left[\Gamma(Q) - (1-b)\partial\Gamma(Q)/\partial b\right] \geq 0,\end{aligned}$$

since the first term is positive at optimality and the term $\partial\Gamma(Q)/\partial b$ is negative due to the IFR assumption.
Next, consider centralized inventory management.
1) From (11) we can find the wholesale price as a function of the stocking quantity as $w^c = r((1-b)\Pr(D^e \geq Q^c) -$
and substitute it into the wholesaler's objective function:

$$\Pi^c = 2(w^c - c)Q = 2(r((1-b)\Pr(D^e \geq Q) + b) - c)Q.$$

After differentiating w.r.t. Q and rearranging we get

$$\frac{\partial \Pi^c}{\partial Q} = 2r(1-b)\Pr(D^e \geq Q)[1-(1-b)\Gamma(Q)] - 2(c - rb).$$

Clearly, if ε has IGFR distribution then the first derivative is decreasing in Q monotonically so that there is a unique Q^c (and hence a unique w^c) that is optimal for the wholesaler.
2) Once again, it is sufficient to show that $\partial^2\Pi^d \big/ \partial Q\partial b \geq 0$:

$$\begin{aligned} \frac{\partial^2 \Pi^c}{\partial Q \partial b} &= 2r + 2r((1-b)f_{D^e}(Q)Q - \Pr(D^e \geq Q))[1-(1-b)\Gamma(Q)] \\ &+ 2r(1-b)\Pr(D^e \geq Q)[\Gamma(Q) - (1-b)\partial\Gamma(Q)/\partial b]. \end{aligned}$$

Again, the first line is positive at optimality and the second line in positive since $\partial\Gamma(Q)/\partial b$ is negative due to the IFR assumption.
Finally, we compare centralized and decentralized inventory management.
3) Consider the first derivative of the centralized objective function $\partial\Pi^c / \partial Q$ evaluated at Q^d and use $\partial\Pi^d \big/ \partial Q = 0$ to simplify it

$$\left.\frac{\partial \Pi^c}{\partial Q}\right|_{Q^d} = 2r(1-b)\Pr(D^e \geq Q^d)\left[1-(1-b)\Gamma\left(Q^d\right)\right] - 2(c-rb) = 2b(r-c).$$

Hence, for $b > 0$ we have $\partial\Pi^c/\partial Q|_{Q^d} > 0$ so that due to the concavity of Π^c it follows that $Q^d < Q^c$. The rest follows trivially. ■

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