

PRODUCT LINE DESIGN AND PRODUCTION TECHNOLOGY*

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Product Line Design and Production Technology

ABSTRACT: In this paper we characterize the impact of production technology on the optimal product line design. We analyze a problem in which a manufacturer segments the market on quality attributes and offers products that are partial substitutes. Because consumers self-select from the product line, product cannibalization is an issue. In addition, the manufacturer sets a production schedule in order to balance production setups with accumulation of inventories in the presence of economies of scale. We show that simultaneous optimization of the product line design and production schedule leads to insights that differ significantly from the common intuition and assertions in the literature, which omits either the demand side or the supply side of the equation. In particular, we demonstrate that more expensive production technology always leads to lower product prices and may at the same time lead to higher-quality products. Further, a less efficient production technology does not necessarily increase total production costs or reduce consumer welfare. We also demonstrate that in the presence of production technology, the demand cannibalization problem may distort product quality upward or the number of products upward, which is contrary to the standard result.

Key words: product line; segmentation; cannibalization; EOQ; scale economies; marketing-manufacturing interface.

1 Introduction

Consumers seek product variety for a number of reasons (Hoch et al. 1999). First, greater variety ensures that a consumer finds exactly what he/she wants. Second, when preferences are uncertain, variety may offer an option value. Third, consumers may have an inherent desire to try different alternatives. Although there are exceptions in which variety can have a negative effect (see Gourville and Soman 2005), it is a generally accepted proposition that offering larger variety allows companies to increase both demand and market share (Kotler 2002). However, it has also been long recognized that a large number of products in the assortment is associated with dis-economies of scale and increases in production and distribution costs (Lancaster 1990, Quelch and Kenny 1994). For example, in a survey of product variety models, Lancaster (1990) points out that “scale economies mean that the potential welfare or revenue gain from greater product variety must be balanced against the lower unit production cost with fewer variants.” Hence, with the exception of a few firms (e.g., Dell) that are able to manufacture products to customer order and avoid large inventories, most companies face a trade-off between product variety and production costs, so that product-line decisions should be made with production costs in mind (Eliashberg and Steinberg 1993).¹ An intuitive corollary is that higher production costs should lead to a reduction in the number of products offered, higher prices (Bayus and Putsis 1999, Kekre and Srinivasan 1990) and lower quality.

Surprisingly, empirical literature in marketing has had mixed success in finding evidence of these effects (Kekre and Srinivasan 1990), and there are very few models that provide guidance with respect to the precise impact of production costs on product-line decisions. In particular, it is often assumed in the extant literature that adding a product to the assortment simply results in a fixed cost (see, e.g., Dobson and Kalish 1988, Yano and Dobson 1998). A simple example showing this approach to be problematic is that an increase in fixed cost does not affect product quality or prices (unless the increase is so high that a product is dropped), which is contrary to the common understanding that production costs should affect product quality and prices. To improve on this dimension we propose a dynamic model in which a product line and a production schedule are set simultaneously. We consider a manufacturer who segments the market based on quality attributes of the product, and, all else equal, consumers prefer higher quality. Examples of such product lines are the same model of automobile that may come with various levels of gas-mileage performance (e.g., hybrid or conventional engine), multiple variations of the same computer with different sizes of hard drives, processors of varying speeds, chemicals of different purity, and paper

¹For examples of companies whose product line decisions are informed by operational cost considerations see Womack et al. (1990) for the auto industry and McCormack (1998) for the apparel industry.

of different density. Consumers are heterogeneous in their preferences for quality and self-select from the product line. Hence, a lower-quality product can cannibalize the higher-quality product, an effect that the manufacturer must consider.

Consumers arrive dynamically and a firm sets a production schedule ensuring that an adequate amount of product is manufactured. Dis-economies of scale, inventory carrying costs and fixed production costs are the most frequently cited reasons to reduce product variety (see Lancaster 1990, Eliashberg and Steinberg 1993). For example, automotive manufacturers struggle with increasing product variety that results in lower demand per model and hence dis-economies of scale, higher inventories and higher overhead expenses (Hayes and Wheelwright 1984). We incorporate all three of these elements, for we also believe they are important determinants of optimal product-line decisions. In our model production occurs in batches which incur fixed costs. Batch sizes have to balance these costs with inventory carrying costs incurred between batches. Such a setup is common for many manufacturing firms that have to manufacture products in advance of customer demand.

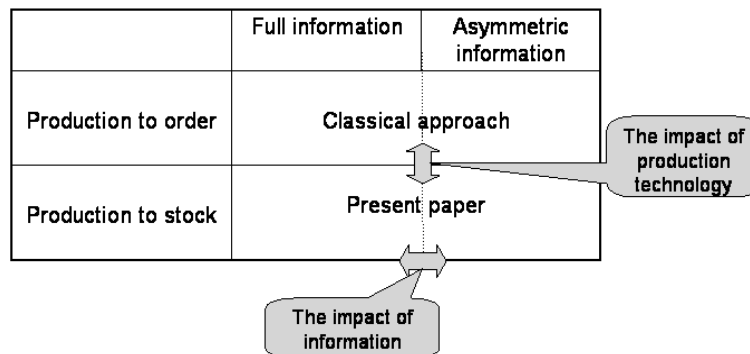


Figure 1. Paper positioning.

We focus on two important aspects of product line design (see Figure 1). The literature in this area generally assumes that firms produce to order (an approach we call classical). First, we analytically characterize *the impact of production technology* on product line design. This is done by comparing the production-to-order setup with the more elaborate model, which includes a production-to-stock setting, inventories and economies of scale. The second dimension of our analysis is *the impact of information* in the presence of production technology. This is done by comparing two settings: the benchmark setting with full information about consumer preferences and the setting with asymmetric information. In the full information case, the firm knows the preferences of individual consumers so that the cannibalization problem does not arise because the firm can segment consumers perfectly. In the asymmetric information case, the firm cannot observe the preferences of individual consumers and hence lower-quality products can cannibalize higher-quality products. We demonstrate that these two factors—production technology and information—

interact and have a major impact on the firm's product-line decisions. Our main findings are:

- Production costs reduce a firm's propensity to offer multiple products in favor of offering fewer products when production costs are large enough. This effect, however, is moderated by the (dis)similarity of consumer segments. When consumer segments are relatively close to each other, the firm offers one *composite* product designed to serve multiple segments. When consumer segments are far apart, the firm also reduces the number of products, but this time through not serving consumers at the low end of the market.
- More expensive production technology (in the sense of higher relevant cost parameters) can lead the firm to offer a product line of higher (average) quality at a lower (average) price. This occurs when the more expensive production technology makes it attractive to replace segment-specific product offerings with a composite product; economies of scale make it attractive to increase the quality level, but the product must be priced relatively low in order to appeal to the lower-end consumer segment. More generally, more expensive production technology *always* leads to lower product prices. Hence, using product prices as proxies for production costs, as is sometimes done in empirical studies, can be problematic because these variables can be inversely related. This occurs because a more expensive production technology makes producing lower quality products attractive.
- A firm offering more products may have lower total production costs than a firm offering fewer products. This counter-intuitive finding is consistent with (and may help explain) some empirical evidence on the relation between product variety and production costs (see Kekre and Srinivasan 1990). This result relies on the fact that firms may differ both in their production technology and in the markets they face. Another result is that a firm with less efficient production technology has lower total production costs. This occurs because less efficient production technology makes producing lower quality products attractive and these products are cheaper to produce. Finally, from the consumer welfare point of view, it can be beneficial to have more expensive production technology. Again, this occurs when the more expensive production technology makes it attractive to replace segment-specific product offerings with a composite product.
- Production technology alters the effect and value of information about consumer preferences. Under full information, the product line is "efficient" in that it maximizes firm profit and social welfare (Moorthy 1984). Typically (see Moorthy 1984, Desai 2001, Villas-Boas 2004, etc.) the firm facing the cannibalization problem due to information asymmetry produces

(weakly) fewer products which are of (weakly) lower quality than is efficient. Further, prices are distorted downward.² We show that when production technology is considered, these results may be reversed, i.e., the cannibalization problem may cause the firm to offer more products or they can be of higher quality than is efficient. Further, prices may be distorted upward. These results occur because the cannibalization problem may lead the firm to offer a segment-specific product line when a composite product is efficient.

- Production technology may help mitigate efficiency losses due to information asymmetry. While with production to order there are always distortions from the efficient product line (either in the number of products or in their quality), with production to stock these distortions may be eliminated. Hence, the value to the firm of information about individual consumer preferences may be zero. This occurs when the production technology makes serving the lower-end of the market unattractive under full information.

Overall, our results demonstrate that close attention should be paid to the interplay between the production technology and cannibalization problems associated with product line design. Simultaneous optimization of the product line design and production schedule leads to insights that differ significantly from the common intuition and assertions in the literature, which omits either the demand side or the supply side of the equation. Therefore, without clear understanding of the trade-offs involved, there is potential for managers to make serious judgement errors.

The paper proceeds as follows. §2 surveys the related literature. §3 models the product line design problem under full information and under asymmetric information. §4 compares the results of these models and describes their implications. §5 discusses the impact of relaxing some of the modeling assumptions and provides concluding remarks.

2 Literature Survey

The literature on product line design has a long history which is surveyed in Lancaster (1990), Eliashberg and Steinberg (1993), Krishnan and Ulrich (2001) and Ramdas (2003). The stream of work most relevant to our paper considers quality as a differentiating dimension along with price, and allows consumers to self-select from variants offered. We utilize the quality differentiation approach for two reasons. First, production and inventory carrying costs typically depend on product quality but may not be affected by other differentiating dimensions (e.g., color). Second, we are interested in studying the interaction between the cannibalization effect and production costs.

²These results hold under the commonly invoked restriction that consumer utility functions satisfy the single-crossing property (Moorthy 1984).

The stream of research on quality differentiation was pioneered by Mussa and Rosen (1978), and Moorthy (1984) was the first to introduce this framework into the marketing literature. In Moorthy (1984) the monopolist offers a menu of products with higher-quality products sold at higher prices. Due to the cannibalization effect, products are priced so that only consumers with the highest valuation for quality receive their efficient quality level, while all others receive products of lower than their efficient quality. Prices are distorted downward as well. Moreover, if the cannibalization effect is strong enough, the firm may choose not to serve low-valuation segments of consumers by offering less than the efficient number of products. As noted above, these classical results appear in many subsequent papers. We show that these well-accepted results may be reversed when the production technology is considered explicitly. Other papers modeling product variety with quality differentiation incorporate channel considerations (see Villas-Boas 1998), marketing costs of communicating with consumers (see Villas-Boas 2004) and competition (see Desai 2001), issues we do not explore here. In all of these papers the marginal cost of production is assumed to be convex increasing in product quality and fixed costs/inventories are not modeled. Complementing this analytical work, empirical researchers have examined the impact of variety on sales (see, e.g., Borle et al. 2005).

On the supply side, economists have long recognized that production costs often exhibit economies of scale that will typically affect the number of products offered by a monopolist (see Dixit and Stiglitz 1977, Panzar and Willig 1977). Much progress has been made on coordinating marketing and operations decisions (see Eliashberg and Steinberg 1993 for a survey and Ho and Tang 1998 for a representative set of articles, among which Chen et al. 1998 is the most relevant). Eliashberg and Steinberg (1993) identify the combination of the joint product mix and economic lot-scheduling problem (of which our model is one example) as a fruitful direction for future research.

The papers that are most relevant to our work examine the product line design problem, but with particular attention to the details of the production technology and associated costs. In a product line design setting, Dobson and Kalish (1988) consider product-specific variable and fixed costs. However, the model is static (so inventory is not modeled) and these costs are independent of other decision variables and demand volume. Even with these simplifications their formulation leads to a complex mixed-integer program that must be solved numerically. While the goal in Dobson and Kalish (1988) is to develop solution methodology for practically-sized problems, we limit the size of the problem to obtain analytical solutions that lend themselves to analysis and interpretations.

In Kim and Chhajed (2000) the firm utilizes component commonality to mitigate the cost of product variety. Desai et al. (2001) and Heese and Swaminathan (2005) generalize this setup to

allow the manufacturing cost to be mitigated by exerting the design effort. These papers demonstrate that the benefits of component commonality have to be balanced against dilution in model differentiation. This finding is roughly in the same spirit as one of our results (the benefits of product variety have to be balanced with supply-side considerations), although their focus on component commonality is distinct. Van Ryzin and Mahajan (1999) and Hopp and Xu (2004, 2005) use logit models of demand and examine cost effects on product-line decisions. Van Ryzin and Mahajan (1999) use a “newsvendor” model to capture inventory costs that increase with the number of products offered, and they analyze the resulting revenue-cost trade-off. Hopp and Xu (2004, 2005) consider production systems that exhibit economies of scope but not scale and show that greater component commonality increases the number of products offered.

Finally, empirical researchers have analyzed linkages between variety and production costs but have arrived at contradicting conclusions. Kekre and Srinivasan (1990) find that manufacturing and inventory costs decrease when product variety is expanded (because of the economies of scale due to the market share increase) while Bayus and Putsis (1999) find that the cost increases associated with a broader product line dominate any potential demand increases. Our analytical model may help in reconciling these conflicting results; we also discuss other considerations that may improve future econometric studies.

3 Model

To examine a monopolist’s marketing and production decisions, we consider a model which combines the product line design problem proposed by Moorthy (1984) with the classical EOQ (Economic Order Quantity) production cost model. We chose to analyze simple functional forms to facilitate closed-form characterizations of the optimal product line that lead to clean analytical results. We believe that our main insights are robust to our particular model specification, and we discuss the implications of generalizing the model in §5.

On the demand side, we assume that consumers³ belong to one of two segments. Consumers of type $t = \{L, H\}$ have valuation $\theta_t q$ for the product, where q represents product quality and $\theta_H > \theta_L$. Although throughout the paper we interpret q as product quality, it can be thought of as a combination of many of the product’s characteristics. In our analysis we follow the traditional analytical literature on vertical differentiation in assuming that quality is one-dimensional, although in practice there might be several quality dimensions that cannot be uniquely ordered. Each consumer’s utility for the product is $\theta_t q - p$, where p is the product’s price. If a consumer does not

³Although we refer to the firm’s customers as “consumers” throughout the paper, these can also be other firms.

purchase a product she receives her reservation utility, which is normalized to zero. Consumers of type t exogenously arrive at a deterministic rate of λ_t per unit time. Upon arrival, each consumer decides which product, if any, to purchase based on the offered products' qualities and prices. Thus, the actual demand rate experienced for a specific product depends on the qualities and prices of all the offered products. This is perhaps the simplest model that can capture this dynamic. Although our setup assumes that the firm does not influence the size (arrival rate) of each consumer segment, §5 describes the extension to a continuum of types, in which case the size of consumer segments is made endogenous.

On the supply side, we explicitly model the production costs using the EOQ model, which is classical in operations management (see, for example, Lal and Staelin 1984, De Groote 1994, Cachon and Terwiesch 2004). The EOQ model is dynamic and assumes that the firm produces each product in batches, carries inventory between batch production and has to perform costly production setups for each batch. Moreover, production costs depend on product characteristics because higher-quality products are more expensive to hold in inventory. Finally, the EOQ model exhibits economies of scale: the per-unit production cost is decreasing in the demand rate.

We assume that the cost of building one unit of product with quality q is aq^2 , which we refer to as the quality cost. The quadratic form reflects the increasing marginal quality cost; a can be interpreted as the costliness of quality, where $a > 0$. Each product the firm manufactures is instantly produced to stock in batches of size Q . The batch size essentially determines the production schedule. The firm incurs fixed setup cost $K \geq 0$ for each batch.⁴ Once the batch is produced, the firm sells inventory to consumers until it is depleted, at which point a new batch is produced. Each unit of inventory incurs a holding cost that is proportional to the product's cost iaq^2 ; $i \geq 0$ is the cost of capital. Together $\{a, i, K\}$ characterize the production technology, and we refer to these parameters as "production cost parameters." For convenience, we denote $Z = \sqrt{2aiK}$, which can be interpreted as an aggregate measure of the production cost parameters, with higher Z corresponding to less efficient production technology. The firm determines the number of products to offer and selects the quality q , price p and production batch size Q for each product to maximize the profit rate per unit time. Because the arrival rates are constant, these three decisions can be made once in the beginning. Whether these decisions are made sequentially or simultaneously does not affect the solution; for simplicity, we present the simultaneous decision case.

⁴Although we describe the firm as producing its own products, the model also captures the situation in which the firm instead purchases from an outside supplier, incurring the fixed cost K each time it places an order in addition to the quadratic per-unit cost.

3.1 Full Information

We begin our analysis by studying the full information case. Here, the firm can observe consumer types and can tailor its product offering and price to each individual consumer (perfect market segmentation) and hence products do not cannibalize each other. In particular, the firm can customize its price by consumer, so that different consumers pay may different prices for the same product. We study the full information case for two reasons. The primary reason is that the solution to this case is “efficient” in that it maximizes firm profit (in this idealized case) and social welfare. Comparing the results in this setting to the considerably more common case, in which the firm is unable to observe the preferences of its consumers, reveals the value of information about consumer preferences and the way information asymmetry distorts decisions away from the efficient decision.

The second reason is that there may be practical situations where a firm can tailor its product offering and price to each individual consumer. This may occur when the consumer is ill-informed and so relies on the firm to direct it to an appropriate product. Over the course of the sales process, a skilled salesperson may be able to decipher the consumer’s preferences and direct the consumer to a specific price-product offering. This will be facilitated if the usual practice is for prices to be “negotiated.” However, if consumers are well-informed, the prospect of arbitrage among consumers would prevent the firm from pursuing this approach. Further, when a firm’s consumers are businesses that compete in a common market, differential pricing violates U.S. law when “the effect of such discrimination may be to substantially lessen competition” amongst these downstream competitors (Dolan and Simon 1996).

In the classical full information problem (see Moorthy 1984) the ability to perfectly price discriminate leads the firm to design one product for each consumer segment. When the firm sells one product to each segment, the firm’s product line design and production scheduling problem is

$$\begin{aligned} \max_{q_L, q_H, Q_L, Q_H, p_L, p_H} \pi^{FI2} &= \sum_{t=L, H} \left[(p_t - a q_t^2) \lambda_t - \frac{\lambda_t K}{Q_t} - \frac{ia q_t^2 Q_t}{2} \right], \\ \text{s.t. } \theta_t q_t - p_t &\geq 0, \quad t = L, H, \end{aligned} \tag{1}$$

where type- t consumers purchase the product of quality q_t and pay price p_t . The objective function in (1) represents the profit rate per unit time; the first term is revenue less quality cost rate, the second term is the setup cost rate (with λ_t/Q_t setups per unit time) and the third term is the inventory holding cost rate (with an average inventory level of $Q_t/2$). Therefore, a larger batch size Q_t reduces setup costs but increases inventory holding costs and vice versa. Although it is possible to offer two products, as we show below, due to the economies of scale in production, it might be more profitable for the firm to design just one product and sell it to both consumer segments at

different prices. We call such a product “composite” and denote its quality, batch size and price by a subscript “ C ”. When selling the composite product, the firm’s problem is

$$\begin{aligned} \max_{q_C, Q_C, p_{CL}, p_{CH}} \pi^{FI1} &= \sum_{t=L,H} [(p_{Ct} - aq_C^2) \lambda_t] - \frac{(\lambda_L + \lambda_H) K}{Q_C} - \frac{iaq_C^2 Q_C}{2}, \\ \text{s.t. } \theta_t q_C - p_{Ct} &\geq 0, t = L, H, \end{aligned} \quad (2)$$

where p_{Ct} denotes the price charged type- t consumers.

Denote the optimal solution that solves $\max \{\pi^{FI1}, \pi^{FI2}\}$ with the superscript “*”. The following proposition summarizes the solution under full information. In the appendix all proofs are provided and the constant z^{FI} is defined in closed-form.

Proposition 1 *Under full information, the optimal product line, batch sizes, prices, and resulting profit are as follows:*

(i) *If $Z < \min(\theta_L \sqrt{\lambda_L}, z^{FI})$, then the firm offers two products and*

$$\begin{aligned} q_t^* &= \frac{\theta_t - Z/\sqrt{\lambda_t}}{2a}, t = L, H, \\ Q_t^* &= \sqrt{\frac{2K\lambda_t}{ia}} \frac{1}{q_t^*}, t = L, H, \\ p_t^* &= \theta_t q_t^*, t = L, H, \\ \pi^* &= \frac{1}{4a} \sum_{t=L,H} (\theta_t \sqrt{\lambda_t} - Z)^2. \end{aligned}$$

(ii) *If $\theta_H < \theta_L(1 + \sqrt{1 + \lambda_L/\lambda_H})$ and $z^{FI} \leq Z < (\theta_L \lambda_L + \theta_H \lambda_H)/\sqrt{\lambda_L + \lambda_H}$, then the firm offers one composite product that serves both consumer segments and*

$$\begin{aligned} q_C^* &= \frac{(\theta_L \lambda_L + \theta_H \lambda_H) / (\lambda_L + \lambda_H) - Z/\sqrt{\lambda_L + \lambda_H}}{2a}, \\ Q_C^* &= \sqrt{\frac{2K(\lambda_L + \lambda_H)}{ia}} \frac{1}{q_C^*}, \\ p_{Ct}^* &= \theta_t q_C^*, t = L, H, \\ \pi^* &= \frac{[(\theta_L \lambda_L + \theta_H \lambda_H) / \sqrt{\lambda_L + \lambda_H} - Z]^2}{4a}. \end{aligned}$$

(iii) *If $\theta_H > \theta_L(1 + \sqrt{1 + \lambda_L/\lambda_H})$ and $\theta_L \sqrt{\lambda_L} \leq Z < \theta_H \sqrt{\lambda_H}$, then $q_L^* = Q_L^* = p_L^* = 0$, the firm*

offers one product that serves high-valuation consumer segment and

$$q_H^* = \frac{\theta_H - Z/\sqrt{\lambda_H}}{2a},$$

$$Q_H^* = \sqrt{\frac{2K\lambda_H}{ia} \frac{1}{q_H^*}},$$

$$p_H^* = \theta_H q_H^*,$$

$$\pi^* = \frac{(\theta_H \sqrt{\lambda_H} - Z)^2}{4a}.$$

Otherwise, no product is offered.

The solution in Proposition 1 is best explained using the graphic representation in Figure 2, which depicts the solution as a function of θ_H , which can be interpreted as how far apart consumer segments are, and Z , which is an aggregate measure of the production cost parameters. Although the figure is plotted for a specific set of problem parameters (in particular, $\lambda_L = \lambda_H$), all insights are not parameter-specific because only the relative size of the regions changes and not the solution structure. The boundaries between the regions correspond to the closed-form thresholds in Proposition 1.

Consider the solution at $Z = 0$ (meaning that either $K = 0$ or $i = 0$) which corresponds to the classical full information problem (see Moorthy 1984), where setup and inventory costs are not considered. In this case it is always optimal to offer two products with qualities $q_L^* = \theta_L/(2a) < \theta_H/(2a) = q_H^*$, which is called a “separating” solution (because different consumer types buy separate products). If production costs are relatively small as in area (i), the firm still finds it profitable to manufacture two products, each targeting a different consumer segment. However, as production costs increase, a different solution emerges.

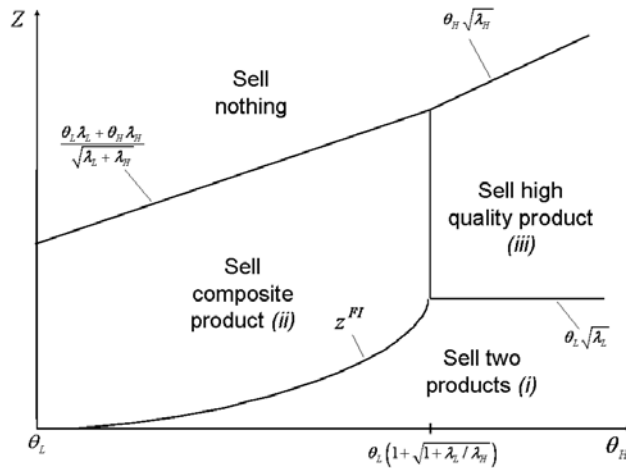


Figure 2. Full information.

When consumer segments are relatively close to each other in preferences and production costs are relatively significant as in area (ii), the firm finds it profitable to offer just one composite product that is sold to both high-valuation and low-valuation consumers. This solution is termed “pooling” (see Moorthy 1984) and it does not arise in the absence of setup and holding costs.⁵ In the absence of such costs, the firm would tailor the *quality* of each product to the segment it serves. On the one hand, producing a single product reduces the contribution (revenue less quality cost) from the product line, because such tailoring is impossible. On the other hand, producing a single product allows the firm to take advantage of economies of scale in production and so reduce total production costs. The gain in contribution from tailoring the products’ quality levels is relatively small when the consumers have similar preferences, and so in this region the latter effect dominates: offering a single composite product is optimal. This region is relatively large because even when the firm sells a single product (and so is not able to tailor its quality levels), it is able to tailor its *prices* to each segment.

Finally, if consumer segments are far apart in their preferences and production costs are significant as in area (iii), the firm decides not to serve low-valuation consumers and only serves high-valuation consumers. Because the segments are far apart in their preferences, tailoring the quality levels to each segment is more attractive than serving both segments with a single product. However, offering a product aimed solely at low-valuation consumers would entail incurring a loss (because of high Z); the higher willingness to pay of the high-valuation segment makes a product aimed at this segment economically viable. Hence, the firm implements an “exclusion” policy, in which it does not serve one consumer segment. Again, this solution does not arise in the absence of setup and holding costs under full information. Finally, with high enough production costs, no product is offered. Naturally, the higher the θ_H , the smaller the area in which no product is offered.

3.2 Asymmetric Information

We now turn our attention to the setting with asymmetric information arising when the firm is unable to observe the preferences of individual consumers. The firm may offer a product of quality and price (q_H, p_H) designed for the high-valuation segment and product (q_L, p_L) designed for the low-valuation segment. However, a consumer may find a product designed for another segment more attractive than the product designed for her own segment, and so the firm faces a cannibalization problem. To prevent this cannibalization, two incentive compatibility constraints are added to the

⁵The pooling solution may arise if the utility function does not satisfy the single-crossing property. Our utility function is multiplicatively separable, so it satisfies this property.

formulation (1):

$$\max_{q_L, q_H, Q_L, Q_H, p_L, p_H} \pi^{AI2} = \sum_{t=L, H} \left[(p_t - aq_t^2) \lambda_t - \frac{\lambda_t K}{Q_t} - \frac{iaq_t^2 Q_t}{2} \right], \quad (3)$$

$$\text{s.t. } \theta_L q_L - p_L \geq 0, \quad (4)$$

$$\theta_H q_H - p_H \geq 0, \quad (5)$$

$$\theta_L q_L - p_L \geq \theta_L q_H - p_H, \quad (6)$$

$$\theta_H q_H - p_H \geq \theta_H q_L - p_L. \quad (7)$$

Constraints (6) and (7) ensure that, correspondingly, low-valuation (high-valuation) consumers do not find it more attractive to buy the product designed for high-valuation (low-valuation) consumers. Similar to the full information case, we also need to consider the alternative in which a single product is sold to both consumer segments. Note that in this case it suffices to satisfy the reservation utility constraint of the low-valuation consumers only and there are no incentive-compatibility constraints. The resulting formulation is:

$$\max_{q_C, Q_C, p_C} \pi^{AI1} = (p_C - aq_C^2) (\lambda_L + \lambda_H) - \frac{(\lambda_L + \lambda_H) K}{Q_C} - \frac{iaq_C^2 Q_C}{2}, \quad (8)$$

$$\text{s.t. } \theta_L q_C - p_C \geq 0. \quad (9)$$

Denote the optimal solution that solves $\max \{ \pi^{AI1}, \pi^{AI2} \}$ with the superscript “#”. The next proposition summarizes the solution under asymmetric information. The constant z^{AI} is defined in the appendix.

Proposition 2 *Under asymmetric information, the optimal product line, batch sizes, prices, and resulting profit are as follows:*

(i) *If $Z < \min((\theta_L \lambda_L - (\theta_H - \theta_L) \lambda_H) / \sqrt{\lambda_L}, z^{AI})$, then the firm offers two products and*

$$\begin{aligned} q_L^\# &= \frac{\theta_L - (\theta_H - \theta_L) \lambda_H / \lambda_L - Z / \sqrt{\lambda_L}}{2a}, \quad q_H^\# = \frac{\theta_H - Z / \sqrt{\lambda_H}}{2a}, \\ Q_t^\# &= \sqrt{\frac{2K\lambda_t}{ia} \frac{1}{q_t^\#}}, \quad t = L, H, \\ p_L^\# &= \theta_L q_L^\#, \quad p_H^\# = \theta_L q_L^\# + \theta_H (q_H^\# - q_L^\#), \\ \pi^\# &= \frac{[(\theta_L \lambda_L - (\theta_H - \theta_L) \lambda_H) / \sqrt{\lambda_L} - Z]^2 + (\theta_H \sqrt{\lambda_H} - Z)^2}{4a}. \end{aligned}$$

(ii) *If $\theta_H < \theta_L \sqrt{1 + \lambda_L / \lambda_H}$ and $z^{AI} \leq Z < \theta_L \sqrt{\lambda_L + \lambda_H}$, then the firm offers one composite product*

that serves both consumer segments and

$$\begin{aligned}
q_C^\# &= \frac{\theta_L - Z/\sqrt{\lambda_L + \lambda_H}}{2a}, \\
Q_C^\# &= \sqrt{\frac{2K(\lambda_L + \lambda_H)}{ia}} \frac{1}{q_C^\#}, \\
p_C^\# &= \theta_L q_C^\#, \\
\pi^\# &= \frac{(\theta_L \sqrt{\lambda_L + \lambda_H} - Z)^2}{4a}.
\end{aligned}$$

(iii) If $\theta_H > \theta_L \sqrt{1 + \lambda_L/\lambda_H}$ and $(\theta_L \lambda_L - (\theta_H - \theta_L) \lambda_H) / \sqrt{\lambda_L} \leq Z < \theta_H \sqrt{\lambda_H}$, then $q_L^\# = Q_L^\# = p_L^\# = 0$, the firm offers one product that serves high-valuation consumer segment and

$$\begin{aligned}
q_H^\# &= \frac{\theta_H - Z/\sqrt{\lambda_H}}{2a}, \\
Q_H^\# &= \sqrt{\frac{2K\lambda_H}{ia}} \frac{1}{q_H^\#}, \\
p_H^\# &= \theta_H q_H^\#, \\
\pi^\# &= \frac{(\theta_H \sqrt{\lambda_H} - Z)^2}{4a}.
\end{aligned}$$

Otherwise, no product is offered.

Similar to the full information case, the intuition behind this Proposition can be explained using graphical representation of the solution in Figure 3. Again, although the figure is plotted for a specific set of problem parameters (in particular, $\lambda_L = \lambda_H$), all insights are not parameter-specific because only the relative size of the regions changes and not the solution structure. First, consider the solution at $Z = 0$ (meaning that either $K = 0$ or $i = 0$), which corresponds to the classical problem with asymmetric information in Moorthy (1984). Consistent with Moorthy (1984), there are two possibilities. If consumer segments are not too far apart, it is optimal to offer two products with qualities $q_H^\# = \theta_H/(2a)$ and $q_L^\# = \theta_L/(2a) - (\theta_H - \theta_L) \lambda_H/(2a\lambda_L)$, a separating solution. However, when consumer segments are far apart, the firm implements an exclusion policy: it only serves the segment with high valuations. This happens because when two products are offered high-valuation consumers capture utility in excess of their reservation utilities (which is called “informational rent”), while low-valuation consumers only receive their reservation utilities (see the proof of Proposition 2). Hence, the firm has to give up some profit in order to elicit consumer types when information asymmetry (product cannibalization) is present. If consumer segments are very far apart, this informational rent becomes too high and the firm prefers not to serve low-valuation

consumers. In this case the single product is priced at the high-valuation consumers' reservation level, which makes it too expensive for low-valuation consumers. Because high-valuation consumers no longer extract informational rent, more profit is left for the firm.

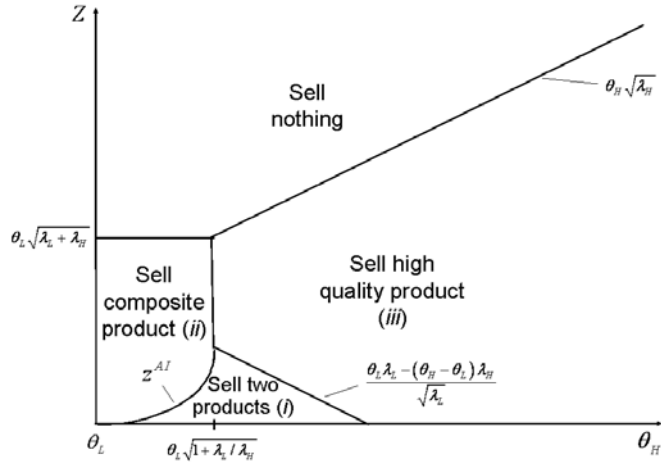


Figure 3. Asymmetric information.

As production costs increase, the region (i) in which the firm offers two products shrinks more quickly than without asymmetric information (in Figure 2). This happens because, in addition to dis-economies of scale in production, the firm also has to give up informational rent due to information asymmetry. Hence, economies of scale in production are *reinforced* by the cannibalization problem. The firm can respond by either decreasing the production cost through manufacturing just one (composite) product or by reducing informational rent through excluding low-valuation consumers. The former is preferred when consumer segments are close together, and the latter is preferred when consumer segments are far apart.

When consumer segments are relatively close to each other in preferences and production costs are significant as in the area (ii), the firm finds it profitable to offer just one composite product that is sold to both high- and low-valuation consumers. This product must be priced to make it attractive for the low-valuation consumer segment. When consumer segments are close together in preferences, there is little loss in revenue in doing so, and by selling one product to both segments the firm takes advantage of economies of scale in production. When consumer segments are far apart in their preferences as in the area (iii), the firm decides not to serve low-valuation consumers in order to avoid giving up informational rents to high-valuation consumers and in order to avoid selling the low-quality product at a loss. Recall that only the latter reason was present with full information. Hence, the informational rents are responsible for the much larger area (iii) under asymmetric information than under full information. Finally, for very high production costs no product is offered.

4 Comparisons and Implications

This section investigates how the interplay between production technology and information asymmetry affects the number of products, their qualities and prices, consumer welfare and total production costs. In particular, we describe how the optimal product line changes in response to the exogenous production cost parameters. We want to emphasize that our model takes the exogenous parameters as being static and so does not describe how a firm should dynamically change its product line over time. Accordingly, our comparative statics results should be interpreted as describing how the one-time product-line decision changes in response to changes in the underlying parameters. For graphical illustrations we continue to assume that $\lambda_L = \lambda_H$, but all conclusions are easily verified for a more general case with $\lambda_L \neq \lambda_H$. We state our findings in the form of results without proofs: although some of these proofs are quite tedious, they are relatively straightforward and follow from Propositions 1 and 2. Unless stated otherwise, all results hold for both the full and asymmetric information cases.

4.1 Impact on Number of Products

By examining Figures 2 and 3 we clearly see that the number of products offered to the market under both full and asymmetric information decreases in the production cost parameters (Z). This result is seemingly intuitive in view of the economies of scale, which favor offering fewer products. This is, however, only part of the story. As discussed above, economies of scale in production lead to product consolidation only when consumer segments are sufficiently close to each other. Otherwise, the shift towards fewer products happens through excluding low-valuation consumers.

Result 1 *As production cost parameters increase, the number of products offered (weakly) decreases. When consumer segments are close (far), this happens without (with) excluding low-valuation consumers.*

Further, in the absence of production technology considerations, we know (Moorthy 1984) that the firm offers no more products under asymmetric information than under full information (recall that the full information solution is efficient in that it maximizes social welfare). That is, due to the cannibalization problem, the firm may opt to offer a single product (and thus exclude low-valuation consumers), while under full information this does not happen. The situation changes significantly when the impact of production technology is accounted for, and Figure 4 concisely summarizes the differences in the number of products offered to the market,⁶ where N^* denotes the optimal number of products under full information and $N^\#$ denotes this quantity under asymmetric information.

⁶Even when the number of products is the same, different markets can be served, an issue we discuss in §4.2.

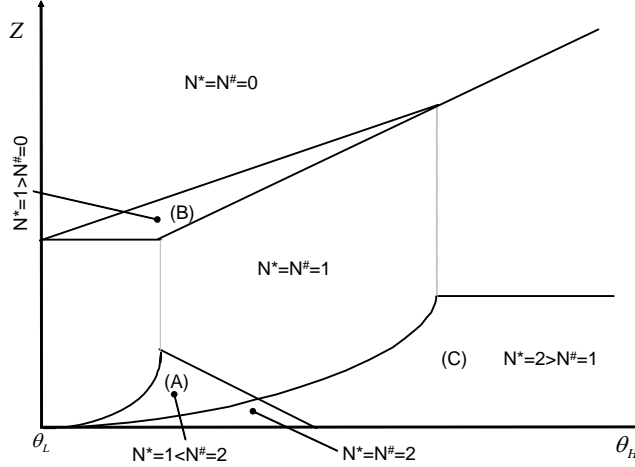


Figure 4. The impact of information asymmetry on the number of products.

Figure 4 shows that the impact of information depends on the cost of production technology. Under information asymmetry the firm may introduce a larger (area A), smaller (areas B and C), or equal (all other areas) number of products compared to the efficient product line. Perhaps the most surprising result occurs when production costs are relatively small and market segments are moderately distinct in their preferences (area A). Here the firm actually offers *more* products under information asymmetry than is efficient. This result occurs because under full information the firm is able to tailor the price to each segment even while it takes advantage of economies of scale by offering a single composite product; when segments are similar enough and production costs are large enough (as they are in area A), this is optimal. In contrast, under asymmetric information, if the firm is to charge the segments different prices, it must offer two distinct products. When consumer segments are moderately heterogenous and production costs are relatively small, tailoring the prices is more important than taking advantage of economies of scale, and so the firm offers two products.

Result 2 *The optimal number of products when the cannibalization problem is present can be larger, the same, or smaller than is efficient.*

4.2 Impact on Prices, Product Qualities and Product Line Length

In this section we consider the impact of production technology and information asymmetry on both the quality and price paid by each consumer segment as well as on *average* quality and price, where these terms denote averages weighted by demand rates. One would expect that as the costliness of quality, a , increases, product quality decreases. This is the case when no other production costs are present. When setup and inventory costs are present, it continues to be true in most situations, as can be verified from Propositions 1 and 2. There is, however, an exception. Due to the rise

in a , the firm may switch from offering two products to offering one composite product. At the moment this happens, one can verify for both full and asymmetric information that the average quality *increases*:

$$q_C > \frac{\lambda_L q_L + \lambda_H q_H}{\lambda_L + \lambda_H} \quad (10)$$

and that the quality of the composite product lies between that of the individual products:

$$q_L < q_C < q_H. \quad (11)$$

Thus, surprisingly, as the costliness of quality increases, both the quality of the product purchased by low-valuation consumers and the average quality purchased actually *increase*. This result occurs because switching to a composite product pushes down the cost of providing a given level of quality because of economies of scale, which makes it attractive to increase the average quality level (10).

Result 3 *The quality of the product purchased by high-valuation consumers is decreasing in the costliness of quality. The quality of the product purchased by low-valuation consumers may increase in the costliness of quality. Further, the average quality may increase in the costliness of quality.*⁷

It is generally assumed that higher production costs should increase prices (see Bayus and Putsis 1999), the intuition being that the firm will pass on higher production costs to consumers in the form of higher prices. On the other hand, higher production costs will typically result in lower quality, which favors lower prices. In our model, the prices the firm can charge are solely a function of the quality levels of the products (consumers' willingness to pay is unaffected by how costly production is for the firm), so this second effect dominates: prices are *decreasing* in the production cost parameters.

In general, there would seem to be a clear relationship between quality and prices, so that as quality increases, prices do as well. At the moment the production cost hits the threshold where it is more attractive to offer one composite product, average quality increases (10), so following this logic, one would expect average price to increase as well. In fact, the opposite happens (both under full and asymmetric information):

$$\frac{\lambda_L p_{CL}^* + \lambda_H p_{CH}^*}{\lambda_L + \lambda_H} < \frac{\lambda_L p_L^* + \lambda_H p_H^*}{\lambda_L + \lambda_H} \quad \text{and} \quad p_C^\# < \frac{\lambda_L p_L^\# + \lambda_H p_H^\#}{\lambda_L + \lambda_H}.$$

To see the intuition, observe that when the firm shifts from offering two products to a composite product, the price paid by the low-valuation segment increases and the price paid by the high-valuation segment decreases. Because changes in quality lead to a larger change in the willingness

⁷Result 3 also holds when “costliness of quality” is replaced by the more general term “production cost parameters.”

to pay for the high-valuation segment, the latter effect dominates. Thus, surprisingly, for the optimal product line, the average quality and price can move in opposite directions in response to changes in the underlying production cost parameters.

Result 4 *The individual product prices as well as the average price are decreasing in the production cost parameters. An increase in the production cost parameters can simultaneously increase the average quality and decrease the average price.*

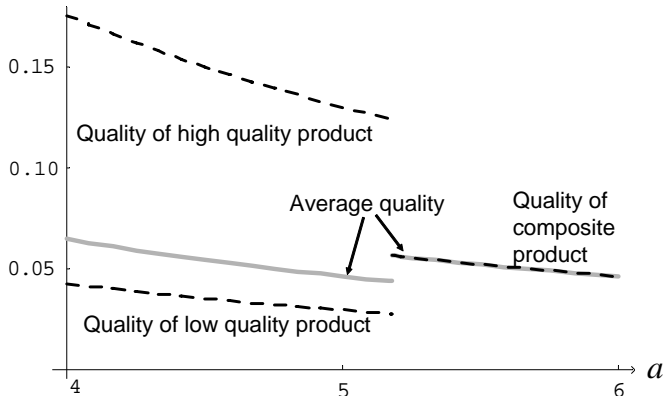


Figure 5. Quality levels.

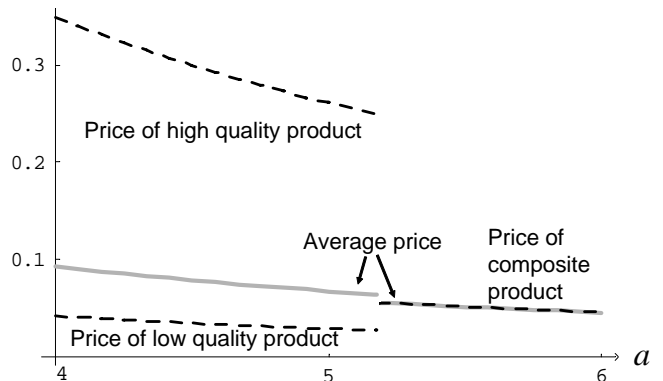


Figure 6. Prices.

Figures 5 and 6 illustrate Results 3 and 4 by depicting the qualities and prices for individual products and on average under asymmetric information as a function of the costliness of quality a for parameters $\lambda_H = 1.0$, $\lambda_L = 5.0$, $\theta_H = 2.3$, $\theta_L = 1.0$, $K = 1.0$ and $i = 0.1$. When the costliness of quality a crosses the threshold such that offering a composite product becomes attractive, the average quality jumps up 30% and the average price jumps down 12%.

We now turn to the impact of information on quality and prices, beginning with quality. The impact of information on product quality is best illustrated graphically (see Figure 7). A standard restriction, which our model satisfies, is that valuation function V satisfy the single-crossing property $\partial^2 V / \partial q \partial \theta > 0$: higher-valuation consumers are willing to pay more for an increase in quality than lower-valuation consumers. A classical result (see Moorthy 1984) asserts that, with this restriction, product quality under asymmetric information is not higher than under full information. Namely, the quality of the product targeting the highest-valuation consumers is the same (“no distortion at the top”), but the quality of the product targeting lower-valuation consumers is lower under asymmetric information. This result is due to the incentive compatibility constraint (7) forcing the firm to distort the quality of the product targeting lower-valuation consumers downwards to ensure that higher-valuation consumers do not find it attractive and the low-quality product does not cannibalize the high-quality product. This logic is manifested in our case when $Z = 0$. But, just as is the case with the number of products, product quality under asymmetric information can be higher than, equal to, or lower than product quality under full information, depending on the

combination of problem parameters. We will discuss two of the most interesting cases.

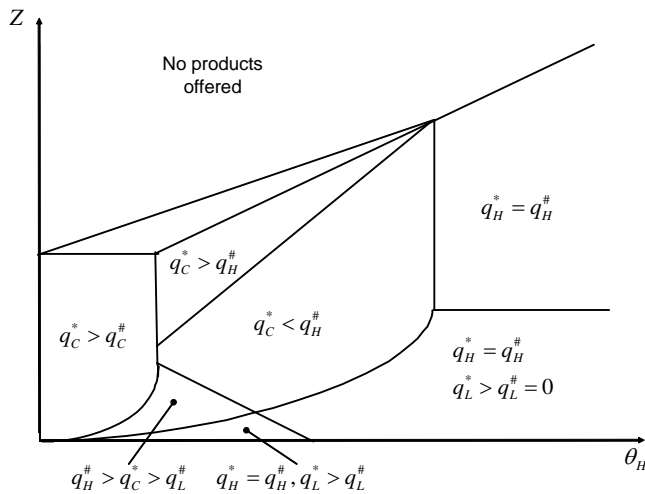


Figure 7. Distortions of product quality.

First, consider the scenario in the upper-right-hand corner of Figure 7, which occurs when production technology is expensive and consumer segments are far apart. In this case the distortion introduced by the cannibalization problem disappears: the quality of the only product on the market is the same with or without information asymmetry. This happens because the firm finds it profitable to sell just one product to high-valuation consumers, whether information is asymmetric or not, allowing the firm to extract all utility from these consumers and hence achieve the efficient outcome. Therefore, we have the somewhat surprising result that the presence of inferior (more costly) production technology may actually reduce the firm's value of information about individual consumer preferences to zero.

As a more extreme example, consider the scenario in the center of Figure 7 in which the firm offers a composite product under full information and one high-quality product under asymmetric information. In this region, the product quality offered by the firm in the presence of asymmetric information is *higher* than the efficient level, the opposite of what we know from the classical analysis of the same problem. The intuition for why the inclusion of production costs reverses the classical result is as follows: Without economies of scale in production, the firm would tailor the quality of each product to the segment it serves. Economies of scale make it attractive to pool consumer segments, which entails offering a lower quality level than would be offered if the product were tailored to the high-valuation segment. Further, just as in the case without production costs, information asymmetry makes it more attractive to exclude low-valuation consumers. Hence, when production cost and consumer heterogeneity are moderate, information asymmetry causes the firm to offer a single product targeted to high-valuation consumers, which is consequently of high quality.

In contrast, without information asymmetry, the firm serves both markets with a single product, which entails offering a lower-quality product. The reversal of the classical result applies only to high-valuation consumers; as in the case without setup and holding costs, low-valuation consumers always receive lower quality under asymmetric information.

Result 5 *The impact of information asymmetry is to distort the quality and price of the product serving low-valuation consumers downward from the efficient level. The quality and price of the product serving high-valuation consumers can be distorted upward, downward or not at all. The average quality and average price can be distorted upward, downward or not at all.*

A final observation related to this result is that the impact of production cost on distortions in product quality is not monotone. For example, in the middle area of Figure 7 the quality distortion $q_C^* - q_H^\#$ is first decreasing in Z until $q_C^* = q_H^\#$ (there is no distortion at all), at which point the distortion starts increasing.

The previous section noted that as production costs exceed a threshold, the product line is shortened in that it is optimal to replace a two-product line with a single product. The remainder of this section focuses on the impact of production costs on the product line length when the optimal product line has more than one product, i.e., $q_H - q_L$. As the production cost parameters increase, the quality levels of both products decrease, but the impact on each product's quality differs. In the absence of setup and holding costs, it is intuitive that an increase in the costliness of quality a has a larger impact on the quality of the high-quality product, so that the product line length shortens. This intuition extends to the case with setup and holding costs.

More interesting is the impact of the cost of capital i and the setup cost K on the length of the product line. Because these costs reflect economies of scale in production, the impact of changes in these parameters on the optimal quality level of a product is small when the product is targeted at a consumer segment with large demand. Conversely, when the targeted segment is small, quality is very sensitive to changes in these cost parameters. Consequently, whether changes in these parameters lead to a shortening of the product line (due to a larger decrease in the quality of the high-quality product) or a lengthening of the product line (due to a larger change for the low-quality product) depends on the relative size of the high- and low-valuation segments. When the high-valuation segment is larger (i.e., $\lambda_H > \lambda_L$), an increase in these cost parameters lengthens the product line; when this segment is smaller, the opposite results.

Result 6 *When the product line includes two products, the product line length is increasing in the cost of capital i and the setup cost K if and only if the high-valuation segment is larger than the low-valuation segment. The product line length is decreasing in the costliness of quality a .*

4.3 Impact on Total Production Cost

The extant literature proposed that a higher number of products leads to higher total production cost (see e.g., Eliashberg and Steinberg 1993, Ho and Tang 1998). Given this consideration, previous papers modeled fixed costs associated with including a product in the product line (see e.g., Dobson and Kalish 1988). However, empirical literature has had difficulty confirming this assertion. For example, Kekre and Srinivasan (1990) found a “lack of strong negative impact of broader product line on . . . manufacturing costs.” Our stylized model may offer a plausible explanation of this contradiction, although we do not account for certain effects that are present in their paper.

To examine total production costs, consider the full information case with two products offered (all other cases can be shown to follow the same pattern). The total profit can be written as follows:

$$\pi^* = \frac{1}{2a} \sum_{t=L,H} \left(\theta_t^2 \lambda_t - \theta_L Z \sqrt{\lambda_L} \right) - \frac{1}{4a} \sum_{t=L,H} \left(\theta_t^2 \lambda_t - Z^2 \right).$$

The first summation term represents revenues while the second summation term represents total production cost. Note that, surprisingly, total production cost is *decreasing* in all cost-related parameters (a , i and K). This happens because, as these parameters increase, the firm responds by offering products of lower quality and at lower prices which reduces both revenues *and* costs. This finding is in sharp contrast with both the common intuition and predictions following from the classical EOQ model. As another observation, note that higher θ_H (indicating more dis-similar consumer segments) is associated with higher total production cost. Thus, a firm offering one composite product and facing larger cost-related parameters as well as more dis-similar consumer segments may have higher total production cost than a firm offering two products but facing smaller cost-related parameters as well as more similar consumer segments. This happens if the reduction in total production cost due to higher cost-related parameters is dominated by the increase in total production costs due to the larger dis-similarity of consumer segments.

Result 7 *Total production cost is decreasing in the cost-related parameters a , i and K , so that a less efficient firm has lower total production cost. Moreover, a firm with a higher number of products may have lower total production cost than a firm with a smaller number of products.*

4.4 Impact on Consumer Welfare

So far our analysis has focused on the firm’s profits, but now we consider consumer welfare. Under full information consumer welfare is always zero because the firm extracts all the consumers’ utility. In contrast, under asymmetric information consumers with high valuations may earn informational rent, while consumers with low valuations always receive only their zero reservation utility. In

the absence of setup and holding costs, consumer welfare is decreasing in the costliness of quality, a : the firm responds to higher quality costs by producing lower-quality products, which reduces the utility captured by high-valuation consumers. When setup and holding costs are considered, this result can be reversed. To see why, observe that with these costs the firm may offer one composite product. High-valuation consumers receive positive utility when either two products or one composite product is offered, but receive zero utility when one high-quality product is offered. If the number of products does not change, consumer welfare decreases monotonically in production costs. If, however, the number of products changes from two products to one composite product as a increases, consumer welfare jumps up. The intuition is that the utility (informational rent) that a high-valuation consumer receives is the utility she would receive if she purchased the product intended for the low-valuation segment. With q denoting the quality of this product, a high-valuation consumer receives utility $\theta_H q - \theta_L q$, where the first term represents her valuation and the second term represents the price. Consumer welfare jumps up at the point when it becomes attractive to offer the composite product because the composite product is of higher quality than the low-quality product (11) and high-valuation consumers value an increase in quality more highly than low-valuation consumers: $\theta_H(q_C - q_L) > \theta_L(q_C - q_L)$, or equivalently $\theta_H q_C - \theta_L q_C > \theta_H q_L - \theta_L q_L$. This logic also applies to the impact of increases in production cost parameters i and K .

Result 8 *Consumers may prefer a firm with higher production cost parameters to a firm with lower production cost parameters.*

5 Summary and Discussion of Assumptions

We have analyzed the problem of designing the product line for a firm that has to explicitly consider associated production costs. We used a stylized model that allowed us to obtain several insights that expand our understanding of joint marketing-manufacturing decisions, as well as our understanding of the impact of information asymmetry with respect to consumer preferences on a firm's decisions. These insights often go counter to our intuition and sometimes go counter to the classical results in the economics and marketing literatures, where setup and holding costs are not considered. We have demonstrated that complex interactions exist between product line design and production scheduling decisions: the number of products affects the extent of the economies of scale, product quality affects inventory holding costs and vice versa, and together they affect setup costs and prices. As a result, common intuition may not predict the direction of these interactions correctly.

Our work suggests that heterogeneity in consumer tastes, relative sizes of consumer segments, and information about consumer preferences be considered in empirical efforts. All these environ-

mental variables impact product-line decisions and corresponding production costs, and omitting them from econometric tests may lead to incomplete results. For example, a firm with fewer products of higher quality may have production costs that are higher than a firm with more products of lower quality. Furthermore, we show that product price may not be a good proxy for production cost parameters (in fact, these two are inversely related in our model) so there is a need to directly estimate production costs, although this is a challenging task (see Bayus and Putsis 1999). Finding the right proxies for these variables and studying the relationship between product line design and production costs empirically should prove to be a fruitful direction for future research.

Practical examples appear to be consistent with the product line structure predicted by our model. For example, in the auto industry, Toyota is well-known for having developed a flexible production process with low setup costs (in our terminology, less expensive production technology). In the sedan and sport utility categories, Toyota produces both middle-market Toyota and premium Lexus vehicles in the same plant. This is consistent with our model in that within each of these vehicle categories there is substantial consumer heterogeneity. This, coupled with Toyota’s efficient production technology, leads to broad product variety where individual models are tailored to specific consumer segments (area *(i)* of Figure 3). In the European sports car category, manufacturers (e.g., Porsche) typically do not offer low-end versions of their products, partially because they target high-end consumer segments and do not want to cannibalize sales of high-end products. Note that, even though these companies employ manual production processes with limited flexibility, they still offer products of very high quality (area *(iii)* of Figure 3 rather than area *(ii)*) so that the main driving force behind low product variety is probably cannibalization, not high production costs. Finally, in the minivan category, consumers are more homogenous, and so each auto-brand offers essentially one model of vehicle in this category and that model serves a broad range of the potential market (area *(ii)* of Figure 3). Although there are issues at play in these examples that are not captured in our model, we believe that insights offered by our model are nonetheless relevant.

Like all models, ours has limitations. First, our results rely on the assumptions that each production batch incurs a fixed cost and that inventoried products incur holding costs that are proportional to the unit production cost (which is a quality-dependent). These assumptions may not hold. For example, a firm may choose to have a “setup crew” available so that the incremental setup cost becomes negligible. Alternately, a firm’s holding costs might be independent of product quality (e.g., if holding costs are driven by fixed costs associated with operating warehouses rather than the capital cost associated with inventory). In either situation, the assumptions of our model would be violated and therefore key insights would change as well. Second, our model ignores competition. In particular, our model is concerned with short-term profit maximization in that it ignores more

strategic considerations of market share or sales growth; such considerations may favor a product line that sacrifices near-term profit in exchange for greater profit in the longer-term. Third, our model assumes the demand is deterministic and the production is instantaneous. Although our results continue to hold when production is not instantaneous, provided that the production rate is sufficiently high, incorporating stochastic demand together with non-zero production lead times would require a substantially different setup. We believe that incorporating stochastic demand and/or competition may prove to be fruitful in future research.

Finally, we have assumed that there are only two consumer segments, the valuation function is multiplicatively separable, and the quality cost is quadratic. The remainder of the paper describes the extent to which our results continue to hold when these assumptions are relaxed; the details behind these extensions are provided in Netessine and Taylor (2006).

More Than Two Consumer Segments

It is rather straightforward to extend our problem formulation to more than two consumer segments. The key product-line decisions are still which consumer segments to serve and how segments should be pooled, if at all. For any such segmentation, following the approach in §3, it is straightforward to obtain optimal prices, production lot sizes and consequent profits in closed form. Figures 8 and 9 display the optimal product line under full and asymmetric information, respectively, for the case with three segments in which consumer segment valuations form a geometric progression $\theta_t = \phi\alpha^t$, where $\phi > 0$ and $\alpha > 1$ for $t = 1, 2, 3$ and consumer segments are of equal size $\lambda_t = \lambda$. The figures depict the optimal segmentation as a function of the aggregate measure of the production cost parameters Z and consumer heterogeneity α . For example, $\{1\}\{2, 3\}$ corresponds to offering two products so that the lowest valuation segment buys one product and the two higher valuation segments buy the second product. The figures depict the structure of the solution, which is invariant to the choice of ϕ and λ (changing ϕ or λ corresponds to simply rescaling the vertical axis).

The figures demonstrate that the essential insights from §3 extend when there are more than two consumer segments: under full and asymmetric information, if consumer segments are similar and production costs are high, it is optimal to pool segments by offering a composite product. If consumer segments are very different, it is optimal to offer distinct products to each segment served. As consumer heterogeneity increases, it is optimal to abandon low-end consumer segments more quickly under asymmetric information. The number of products offered may be lower or higher under asymmetric information. As in §3, information asymmetry distorts the number of products higher when consumer heterogeneity is moderate and production costs are small.

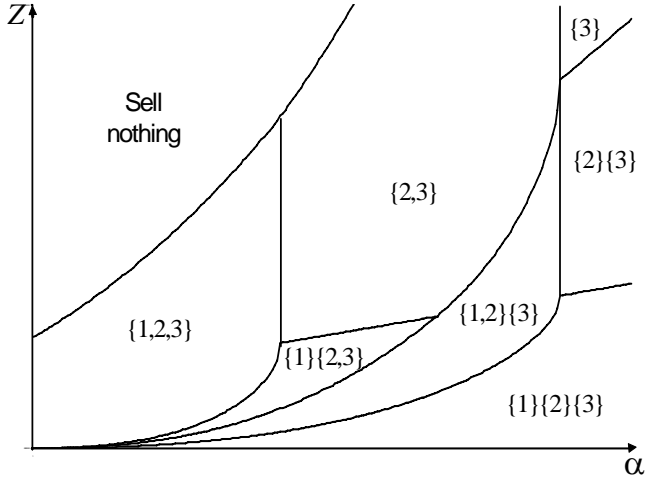


Figure 8. Full information.

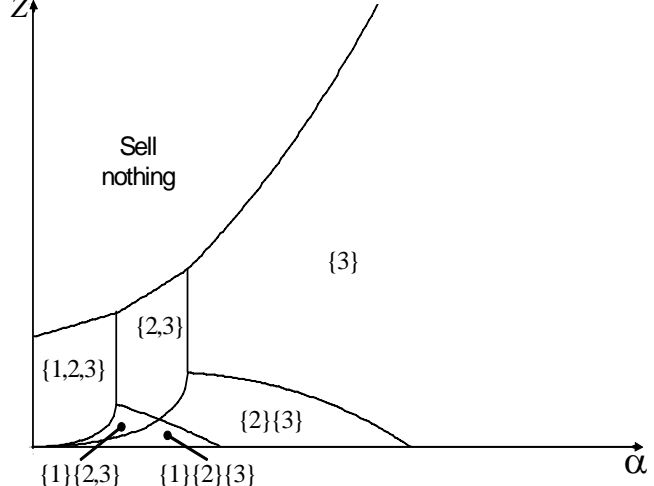


Figure 9. Asymmetric information.

Just as Figures 2 and 3 graphically illustrate the analytical results providing the optimal product-line decisions in closed form in Propositions 1 and 2, so Figures 8 and 9 correspond to closed-form analytical characterizations of the optimal product line. When the number of consumer segments is large, the optimal product line is the solution to a large-scale combinatorial optimization problem and a numerical solution along the lines of the approach in Dobson and Kalish (1988) might be needed. Further complexity is added in the limiting case in which one faces a continuum of consumer types. Then, for example, as the production cost parameters Z go to zero, the number of products offered under full information goes to infinity, which makes characterizing the optimal product line for general Z difficult. Further, when the number of products is unbounded it is difficult to compare the outcome with our previous results. To facilitate a comparison, we focus on the case where the firm offers at most two products and where consumer types θ are distributed uniformly on $[A, B]$. The restriction on the number of products is without loss of generality when the production cost parameters Z are sufficiently large, because then it is optimal to offer at most two products. With a continuum of consumer types, the sizes of the segments are endogenous. In particular, it is optimal to segment the market so that consumers of type $[\theta_H, B]$ purchase a higher quality product and consumers of type $[\theta_L, \theta_H)$ purchase a lower quality product, where θ_L and θ_H are decision variables and $\theta_L \leq \theta_H$.

To characterize the extent to which our main insights continue to hold, we conducted a numerical study of the 36,800 parameter combinations of $A = \{1, 2, 4, 8\}$, $B = \{A+0.2, A+0.4, \dots, A+2.0\}$, and $Z = \{0.00, 0.02, \dots, 9.00\}$, where λ is normalized to unity. (This parameter set allows for considering the impact of the cost-related parameters a , i and K individually; to examine the impact of a , when i and K are fixed, we fixed the product iK at unity.) Because the segments are endogenous, the second part of Result 1, Result 5, and the first part of Result 6 are not meaningful. All of the

remaining assertions in Results 1-8 are consistent with the numerical results. However, there are some caveats. In contrast to the two-type case, the value of information about consumer preferences will always be positive. Further, some of the results change when $A = 0$, so that some consumers have very low valuations. In this case, one can characterize the optimal product line analytically; it is never optimal to serve the entire market with a single product (i.e., offer a composite product), so the structure of the solution differs. Further, the results in §4 that rely on the firm offering a composite product for at least some parameters do not extend to the case with $A = 0$. We conclude that the extent to which our results extend when consumer types are distributed uniformly depends on the support of the distribution, but that our extensive numerical study suggests that, when no consumers have extremely small valuations, the main insights continue to hold.

Generalized Valuation and Quality Cost Functions

We have assumed that the valuation function is multiplicatively separable and that the quality cost is quadratic largely for analytical convenience: under these assumptions we are able to solve the problem in closed form. Without these assumptions, all solutions become implicit rather than explicit, which limits insights that can be derived from the model. However, we have verified that most of our insights remain unchanged when assumptions about the specific functional forms are generalized as follows. We assume that consumers have a continuous, twice-differentiable valuation function $V(\theta, q)$ which satisfies the following (rather standard) technical assumptions: $V_q > 0$, $V_\theta > 0$, $V_{q\theta} > 0$ and $V(0, q) = V(\theta, 0) = 0$, where subscripts denote partial derivatives. Furthermore, we assume that the quality cost $g(q)$ is a continuous, twice-differentiable and increasing. Finally, we assume that the profit functions are unimodal in q_L , q_H and q_C (a simple sufficient condition for this would be $2g''(q)g(q) > (g'(q))^2$) and that $V_q(\theta_L, 0) > g'(0)$, so that the low-type segment is viable.

Under these assumptions, the structure of the optimal product line as reflected in Figures 2-3 is preserved with minor exceptions: namely, in both figures the boundaries between areas (ii) and (iii) and boundaries of the “Sell nothing” area need not be linear, and moreover, in Figure 3 the boundary between areas (i) and (iii) need not be linear either. Furthermore, Results 1, 2, and 8 hold without changes. The surprising part of Result 3, that the quality of the product purchased by low-valuation consumers may increase in production cost parameters, continues to hold, and Result 4 holds for individual product prices. Result 5 holds for the products purchased by the low- and high-valuation segments, with the sole exception that the price of the product serving high-valuation customers might not be distorted upward. However, it is hard to verify Results 3, 4 and 5 for the average quality and prices because all solutions are implicit. We believe that Results 6 and 7 may depend on the specific functional forms. We conclude that, allowing for a few exceptions, our

results are quite robust to the assumptions about specific functional forms of the valuation function and quality cost.

Appendix

Define

$$\begin{aligned}
z^{FI} &= \theta_L \sqrt{\lambda_L} + \theta_H \sqrt{\lambda_H} - \frac{\theta_L \lambda_L + \theta_H \lambda_H}{\sqrt{\lambda_L + \lambda_H}} \\
&\quad - \sqrt{\left[\left(\theta_L \sqrt{\lambda_L} + \theta_H \sqrt{\lambda_H} - \frac{\theta_L \lambda_L + \theta_H \lambda_H}{\sqrt{\lambda_L + \lambda_H}} \right)^2 - (\theta_H - \theta_L)^2 \frac{\lambda_L \lambda_H}{\lambda_L + \lambda_H} \right]^+}, \\
z^{AI} &= \frac{1}{\sqrt{\lambda_L}} \left[\theta_L \left(\lambda_L + \lambda_H - \sqrt{\lambda_L (\lambda_L + \lambda_H)} \right) + \theta_H \left(\sqrt{\lambda_L \lambda_H} - \lambda_H \right) \right. \\
&\quad \left. - \sqrt{\left[\left(\theta_L \left(\lambda_L + \lambda_H - \sqrt{\lambda_L (\lambda_L + \lambda_H)} \right) + \theta_H \left(\sqrt{\lambda_L \lambda_H} - \lambda_H \right) \right)^2 - (\theta_H - \theta_L)^2 \lambda_H (\lambda_L + \lambda_H) \right]^+} \right],
\end{aligned}$$

where $[x]^+ \equiv \max[x, 0]$.

Proof of Proposition 1: We solve the optimization problem (1) first. We begin by reducing this six-variable optimization problem to a two-variable problem by fixing the quality levels (q_L, q_H) and solving for the optimal batch sizes and prices. Because the objective function is concave in Q_t , $t = L, H$, we can solve for the optimal batch sizes $Q_t = \sqrt{2K\lambda_t/(iaq_t^2)}$. Furthermore, the firm can extract all profit from consumers by pricing at $p_t = \theta_t q_t$, $t = L, H$. Substituting these expressions back into the objective function we obtain

$$\max_{q_L, q_H} \pi^{FI2} = \sum_{t=L, H} \left[(\theta_t q_t - a q_t^2) \lambda_t - q_t \sqrt{2Kia\lambda_t} \right].$$

It is straightforward to verify that the solution to this problem is described in (i), provided that $Z < \theta_t \sqrt{\lambda_t}$; otherwise, $q_t = Q_t = p_t = 0$. Finally, substituting this solution back into the objective function, we obtain the following expression for the optimal profit:

$$\pi^{FI2} = \frac{1}{4a} \sum_{t=L, H} \left(\left[\theta_t \sqrt{\lambda_t} - Z \right]^+ \right)^2.$$

We can similarly solve the optimization problem (2) with the exception that now the firm sells the same product at two different prices. The resulting solution is as in (ii), provided that $Z < (\theta_L \lambda_L + \theta_H \lambda_H) / \sqrt{\lambda_L + \lambda_H}$; otherwise, $q_C^* = Q_C^* = p_C^* = 0$. The optimal profit is

$$\pi^{FI1} = \frac{\left(\left[(\theta_L \lambda_L + \theta_H \lambda_H) / \sqrt{\lambda_L + \lambda_H} - Z \right]^+ \right)^2}{4a}.$$

The next step is to compare π^{FI1} with π^{FI2} which can be done by examining the function $f(Z) \equiv \pi^{FI1} - \pi^{FI2}$. Note that $\theta_H > \theta_L(1 + \sqrt{1 + \lambda_L/\lambda_H})$ implies $f(Z) < 0$ and $\theta_L\sqrt{\lambda_L} < z^{FI}$. Therefore, if $Z < \theta_L\sqrt{\lambda_L}$, the solution is (i); if $\theta_L\sqrt{\lambda_L} \leq Z < \theta_H\sqrt{\lambda_H}$, the solution is (iii); if $Z \geq \theta_H\sqrt{\lambda_H}$, the solution is to offer no product. Suppose instead that $\theta_H < \theta_L(1 + \sqrt{1 + \lambda_L/\lambda_H})$. Then $f(Z)$ is strictly concave on $Z \in [0, \min(\theta_L\sqrt{\lambda_L}, \theta_H\sqrt{\lambda_H})]$ with $f(0) < 0 < f(\min(\theta_L\sqrt{\lambda_L}, \theta_H\sqrt{\lambda_H}))$; $f(Z)$ is convex, decreasing on $Z \in [\min(\theta_L\sqrt{\lambda_L}, \theta_H\sqrt{\lambda_H}), (\theta_L\lambda_L + \theta_H\lambda_H)/\sqrt{\lambda_L + \lambda_H}]$ with $f((\theta_L\lambda_L + \theta_H\lambda_H)/\sqrt{\lambda_L + \lambda_H}) = 0$. Therefore $f(Z) = 0$ has one root, z^{FI} , and $f(Z) < 0$ if and only if $Z < z^{FI}$. Because $z^{FI} < \theta_L\sqrt{\lambda_L}$, the result follows. ■

Proof of Proposition 2: We solve the optimization problem (3) first. Notice that the objective function is concave in Q_t , $t = L, H$ so we can solve for the optimal batch sizes $Q_t = \sqrt{2K\lambda_t/(iaq_t^2)}$. Thus, (3) simplifies to

$$\begin{aligned} \max_{q_L, q_H, p_L, p_H} \pi^{AI2} &= \sum_{t=L, H} \left[(p_t - aq_t^2) \lambda_t - q_t \sqrt{2Kia\lambda_t} \right] \\ \text{s.t. (4), (5), (6) and (7).} \end{aligned}$$

Consider the relaxed problem that includes (4) and (7) and excludes (5) and (6). Clearly, constraint (4) must bind, as otherwise one can simultaneously increase p_L and p_H by the same amount without violating any constraint. Clearly, (7) must bind, as otherwise one can increase p_H . It is straightforward to verify that the solution to this problem is

$$q_L = \frac{[\theta_L - (\theta_H - \theta_L) \lambda_H/\lambda_L - Z/\sqrt{\lambda_L}]^+}{2a}, \quad q_H = \frac{[\theta_H - Z/\sqrt{\lambda_H}]^+}{2a}$$

and that the solution to the relaxed problem satisfies (5) and (6). Finally, substituting the solution back into the objective function we obtain the following expression for optimal profit:

$$\pi^{AI2} = \frac{\left([(\theta_L\lambda_L - (\theta_H - \theta_L)\lambda_H)/\sqrt{\lambda_L} - Z]^+ \right)^2}{4a} + \frac{\left([\theta_H\sqrt{\lambda_H} - Z]^+ \right)^2}{4a}.$$

Next we solve the optimization problem (8). Clearly, constraint (9) must bind. By the argument in the proof of Proposition 1, the solution is as in (ii), provided that $Z < \theta_L\sqrt{\lambda_L + \lambda_H}$; otherwise, $q_C = Q_C = p_C = 0$. The optimal profit is

$$\pi^{AI1} = \frac{\left([\theta_L\sqrt{\lambda_L + \lambda_H} - Z]^+ \right)^2}{4a}.$$

The next step is to compare π^{AI1} with π^{AI2} , which can be done by examining the function $F(Z) \equiv \pi^{AI1} - \pi^{AI2}$. Note that $\theta_H > \theta_L\sqrt{1 + \lambda_L/\lambda_H}$ implies $F(Z) < 0$ and $(\theta_L\lambda_L - (\theta_H - \theta_L)\lambda_H)/\sqrt{\lambda_L} < z^{FI}$. Therefore, if $Z < (\theta_L\lambda_L - (\theta_H - \theta_L)\lambda_H)/\sqrt{\lambda_L}$, the solution is (i); if $(\theta_L\lambda_L - (\theta_H - \theta_L)\lambda_H)/\sqrt{\lambda_L} \leq Z < \theta_L\sqrt{\lambda_L + \lambda_H}$, the solution is (ii); if $Z \geq \theta_L\sqrt{\lambda_L + \lambda_H}$, the solution is to offer no product.

$Z < \theta_H \sqrt{\lambda_H}$, the solution is (iii); if $Z \geq \theta_H \sqrt{\lambda_H}$, the solution is to offer no product. Suppose instead that $\theta_H < \theta_L \sqrt{1 + \lambda_L / \lambda_H}$. Then $F(Z)$ is strictly concave on $Z \in [0, \min((\theta_L \lambda_L - (\theta_H - \theta_L) \lambda_H) / \sqrt{\lambda_L}, \theta_H \sqrt{\lambda_H})]$ with $F(0) < 0 < F(\min((\theta_L \lambda_L - (\theta_H - \theta_L) \lambda_H) / \sqrt{\lambda_L}, \theta_H \sqrt{\lambda_H}))$; $F(Z)$ is convex, decreasing on $Z \in [\min((\theta_L \lambda_L - (\theta_H - \theta_L) \lambda_H) / \sqrt{\lambda_L}, \theta_H \sqrt{\lambda_H}), \theta_L \sqrt{\lambda_L + \lambda_H}]$ with $F(\theta_L \sqrt{\lambda_L + \lambda_H}) = 0$. Therefore $F(Z) = 0$ has one root, z^{AI} , and $F(Z) < 0$ if and only if $Z < z^{AI}$. Because $z^{AI} < (\theta_L \lambda_L - (\theta_H - \theta_L) \lambda_H) / \sqrt{\lambda_L}$, the result follows. ■

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