Supply Chain Choice on the Internet

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Abstract

Internet companies extensively use the practice of drop-shipping, where the wholesaler stocks and owns the inventory and ships products directly to customers at retailers' request. Under the drop-shipping arrangement, the supply chain benefits from risk pooling since the inventory for multiple retailers is stocked at the same location, the wholesaler’s. Another more traditional channel alternative on the Internet is one in which retailers stock and own the inventory. These two supply chain structures, which predominate on the Internet, result in different inventory risk allocation, stocking decisions, and profits for channel members. Moreover, the two channel alternatives can be combined into a dual strategy whereby the retailer uses local inventory as a primary source and relies on drop-shipping as a backup. We model the dual strategy as a noncooperative game among the retailers and the wholesaler, analyze it, and obtain insights into the structural properties of the equilibrium solution to facilitate development of recommendations for practicing managers. Finally, we characterize situations in which each of three channels is preferable by specifying appropriate ranges of critical parameters, including demand variability, the number of retailers in the channel, wholesale prices and transportation costs.

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1 Introduction and literature survey

Alliance Entertainment Corp., Ingram Entertainment and Baker & Taylor are major players in the business of distributing home entertainment products, including CDs, videos, games and books to Internet as well as to brick-and-mortar retailers. One of the services they offer is drop-shipping, or consumer direct fulfillment, the practice whereby the wholesaler stocks and owns the product and ships it directly to customers at the retailer’s request. Under a drop-shipping arrangement, the retailer serves as a middleman who acquires customers and accepts orders, while the wholesaler owns and holds inventory and also fulfills orders. Drop-shipping offers two obvious benefits to the retailer: no investment into fulfillment capabilities is required, and the retailer takes no inventory risk (see the example of Spun.com [3]). Drop-shipping also benefits the wholesaler in that it allows her to charge a higher wholesale price and may provide access to a wider customer base (see Scheel [31], page 42). Finally, one would expect that the supply chain as a whole would benefit due to risk pooling and possible economies of scale in transportation in cases in which the wholesaler serves multiple retailers.

The invention of the Internet, a relatively cheap electronic medium, greatly reduced the transaction costs of drop-shipping by allowing a seamless integration of retailers and wholesalers. The Internet allows in-stock availability and prices to be communicated from the wholesaler to the retailer, and orders can be placed from the retailer to the wholesaler, all in real time. Before the Internet, such integration would have required implementing a rather expensive EDI system and hence was not economically viable in most channels. As a result of this integration of retailers and wholesaler, in a recent survey 30.6% of pure-play Internet retailers cited drop-shipping as their primary way of fulfilling orders. At the same time, 44.5% of Internet retailers relied primarily on stocking inventory internally (see [4]), while the rest either outsourced fulfillment to third parties or used the fulfillment capabilities of strategic partners.

Supply chain arrangements in which the retailers stock and own inventory have been extensively analyzed in the operations literature. But the practice of drop-shipping, although almost as popular among Internet retailers, has attracted only limited attention. The marketing literature offers some references to drop-shipping: although Scheel [31] provides extensive qualitative analysis of drop-shipping in the catalog business, he also notes that the practice is generally perceived as having somewhat limited potential. The goal of this paper is to analyze comparative advantages of inventory ownership in the traditional channel and risk pooling under drop-shipping.

1.1 Summary of results

We begin by presenting two alternative models: the traditional channel, in which each retailer stocks inventory locally, and the drop-shipping channel, in which the wholesaler stocks inventory centrally and ships directly to customers. We model both channels as two-stage supply chains with one wholesaler
and multiple identical retailers. In both models the wholesaler charges the retailers a fixed wholesale price (not necessarily the same). Additionally, based on our conversations with Internet companies, we assume that different channels may have different transportation costs due to economies of scale and possibly other cost differences.

When faced with a choice between the traditional and drop-shipping channels, the retailer trades off a higher wholesale price against inventory risk, and the wholesaler trades off the lower margin against inventory risk. Moreover, transportation cost differences play a role. By combining the two strategies, the retailer can first satisfy demand with the internally stocked inventory (purchased at a lower wholesale price) and then request the wholesaler to drop-ship the rest. For example, BlueLight.com, a major Internet retailer, uses the dual strategy for CDs: it stocks certain quantities of the top 40 CDs to increase profit margins and relies on drop-shipping in case of a stock-out (see [1]) while utilizing pure drop-shipping for the remaining CD titles. Also, several major retailers have recently installed Internet kiosks inside physical stores (see [2]); if a customer does not find a product on the store shelves, she can order it in store over the Internet and the product will be shipped directly to her from a central warehouse. Such an arrangement resembles the dual strategy we analyze.

The dual strategy is somewhat similar to the combination of make-to-stock and assemble-to-order practices discussed in the operations literature (see Rudi and Zheng [29] and Cattani et al. [15]) with one major difference: under the make-to-stock/assemble-to-order practice, the same company decides both how much to stock and how much to assemble-to-order, while under the dual strategy considered in this paper, the retailers and the wholesaler manage their inventories competitively such that a game-theoretic situation arises. We model the dual strategy problem as a noncooperative game in which retailers compete with the wholesaler for demand and, moreover, they compete with each other for the wholesaler’s inventory allocation. We analyze equilibria of this game, derive the structural properties of players’ best-response functions and use them to show the uniqueness of the symmetric equilibrium. One of our results is that in equilibrium under the dual strategy it may be optimal for the wholesaler or for retailers to stock nothing, and thus the dual channel converts into a pure traditional or a pure drop-shipping channel. We are able to identify simple conditions that lead to such boundary equilibria. Using properties of supermodular games and derived properties of the best-response functions, we conduct sensitivity analysis of the equilibrium to the changes in the wholesale price, the drop-shipping markup and transportation cost parameters.

We further compare the traditional, drop-shipping and dual channels using numerical experiments. We find that, when the number of retailers is large, the benefits of risk pooling make either the drop-shipping or the dual channel more attractive than the traditional channel to both retailers and the wholesaler. This finding is consistent with the practical observation that drop-shipping is often used for books and electronic products, two segments in which there are many retailers, most served by a few
large wholesalers. In the dual channel we find that changes in any cost-related parameters result in the reallocation of inventory between retailers and the wholesaler. For example, in this channel increasing the markup on drop-shipped products leads to higher retail inventory and lower wholesale inventory, whereas in the drop-shipping channel the same action would lead to higher wholesale inventory. Surprisingly, the impact of the transportation cost differential on profits in the dual channel is not always intuitive. In particular, as the transportation cost structure becomes more favorable for drop-shipping, the wholesaler’s profit initially drops and the retailers’ profit increases.

We find that the drop-shipping markup and the transportation cost differential are the main drivers of the choice of channel. We observe that both the drop-shipping and the dual channels have the potential to be Pareto-optimal choices, but the traditional channel does not. This finding indicates that most Internet retailers should strive to utilize drop-shipping at least to augment their internal inventory. However, in some industries wholesalers prefer the traditional channel and hence do not want to stock any inventory for drop-shipping, so this option is simply not available. In particular, this scenario arises when the drop-shipping markup is relatively small but transportation costs favor drop-shipping, so that the wholesaler does not make any money on shipping directly to customers even though the retailer prefers this option. In situations with a large set of problem parameters the dual channel is the Pareto-optimal choice, which is consistent with the proliferation of Internet kiosks and the increasing popularity of this arrangement. Due to the nonintuitive impact of transportation costs, the dual channel becomes the Pareto-optimal choice when the transportation costs are much higher in the drop-shipping channel than in the traditional channel. As a result, retailers carry a lot of inventory and rely on drop-shipping for only a small proportion of demand. Again, this finding is consistent with the existence of Internet kiosks in retail stores that already carry a lot of inventory. Overall, our results conform with the empirical findings of Randall et al. [27] that no channel option is uniformly preferred over the others and that channel choice depends on companies’ environments.

1.2 Relation to the literature

Our paper addresses two key issues: 1) inventory ownership and stocking decision rights and 2) risk pooling. From the inventory ownership/stocking decision rights point of view, the traditional channel in our paper is similar to that in most of the supply chain literature since the retailer both owns inventory and decides on stocking quantity (see, for example, Zipkin [38]). In contrast, in the drop-shipping channel both inventory ownership and stocking decisions rest with the wholesaler. The only related arrangement we are aware of is Vendor Managed Inventory (see, e.g., Bernstein et al. [8]), in which the wholesaler is endowed with stocking decision rights but inventory is still owned by the retailer. A combination of Vendor Managed Inventory with consignment transfers both inventory ownership and stocking decision rights to the wholesaler, a scenario somewhat similar to our drop-shipping model. The
important differences are that under consignment, inventory is stored at each retailer’s location so there are no benefits to risk pooling. Moreover, the structure of transportation costs is different.

In the dual channel model we consider, each echelon of the supply chain stocks inventory, a setting that parallels the multi-echelon literature. Typically, though, in this stream of literature all echelons are managed centrally (whereas in our paper retailers and the wholesaler are separate entities) and, moreover, sales in each period are limited by inventory at the lowest (retail) echelon (whereas in our model the wholesaler’s inventory is used as a backup). Cachon and Zipkin [14] and Lee and Whang [22] relax the former assumption by considering a supply chain with one retailer and each echelon locally managed. Bernstein and Federgruen [9], Cachon [11], and Chen et al. [16] further consider decentralized multi-echelon supply chains with multiple retailers. Lawson and Porteus [21] relax the second assumption by adding an option to expedite the product between stages but still assume that inventory is managed centrally. We are not aware of any papers that relax both of these assumptions. The dual strategy is also similar to the practice of subcontracting in case demand exceeds capacity (see Van Mieghem [36] and Plambeck and Taylor [25]). In another related paper, Cachon [12] models advance purchase discounts, a practice whereby the retailer can prebook inventory before the season at a lower per-unit price. His work is similar to ours in that both the retailer and the wholesaler are allowed to hold inventory. Cachon focuses on identifying Pareto-optimal price-only contracts and studies supply chain efficiency under such contracts. However, since there is only one retailer, the benefits of risk pooling are not reflected, and issues of inventory allocation among retailers do not arise.

Traditionally research on risk pooling focused on comparing the costs and benefits of building one central, versus many regional, warehouses for a single company (see Eppen [19]). Van Mieghem [35] and Rudi and Zheng [29] consider dual strategies under risk pooling by combining risk pooling at an extra cost per unit with the cheaper local inventory. All of these papers focus on the performance of a single firm and therefore do not compare the benefits of risk pooling for both the retailers and the wholesaler, whereas we do. Anupindi and Bassok [6] compare centralized and decentralized inventory management in a model with two retailers and one supplier while focusing on the effect of market search by customers in case of a stockout at one retailer. They compare the decentralized model with the model in which retailers form an alliance to centralize their inventories. Their work is similar to ours in that they consider the impact of risk pooling not only on downstream retailers, but also on the upstream supplier. The difference, however, is that in the case of centralized inventory management, Anupindi and Bassok [6] assume that the retailers still make stocking decisions, while in our drop-shipping model the wholesaler makes the stocking decisions, resulting in different channel dynamics. Furthermore, Anupindi and Bassok do not study the dual strategy that is the main focus of this paper.

Another relevant stream of literature considers transshipment, i.e., the practice whereby a retail location that is out of stock of certain goods can receive them from another location with excess
inventory. The most relevant papers in this stream are Rudi et al. [30] and especially Dong and Rudi [18] where both the retailers’ and the wholesaler’s profits are considered. Seifert and Thonemann [32] and Seifert et al. [33] model single-directional transshipments from physical to Internet retailers. Anupindi et al. [7] consider a very general decentralized transshipment model in which multiple retailers not only stock inventory internally but also stock it at multiple jointly owned warehouse locations (similar to Anupindi and Bassok [6]). They focus on inventory allocation mechanisms and use cooperative game theory to define appropriate allocations and separate ownership of stock using the notion of “claims.” The major difference between these papers and our model is that under transshipment the wholesaler does not take inventory risk and cannot satisfy customer demand, so the distinct features of drop-shipping do not apply.

A number of marketing papers explicitly focus on the issue of channel choice but not the issues of inventory ownership and inventory risk that are central to this paper. A notable exception is the marketing-operations interface paper by Porteus and Whang [26] where the authors use a principal-agent framework to model a single firm with multiple marketing managers responsible for promoting their products and a production manager responsible for mounting an effort to increase the available manufacturing capacity. The centralized decision-maker is endowed with the power to set up an efficient compensation scheme. Although this work includes some essential elements of our model, like demand uncertainty and inventory risk, it is clearly very different in motivation, modeling assumptions and goals pursued.

The rest of this paper is organized as follows. In Section 2 we present and analyze the traditional, drop-shipping, and dual strategy models. Section 3 contains a detailed analysis and comparison of the three channels. Section 4 discusses the impact of relaxing some of our assumptions and concludes with a summary of managerial insights and implications for practicing managers. All proofs are relegated to Appendix.

2 Supply chain models

We assume that there is a single product (or, equivalently, that there are multiple products but the problem is separable and solved for each product independently). The wholesaler either distributes the product to the retailers in the traditional channel (Model T) or ships directly to the customers at each retailer’s request in the drop-shipping channel (Model D). The two structures are combined under the dual strategy (Model C) with retailers’ inventory used as a primary source of stock. We use a single-period model that captures the trade-offs related to inventory risk while remaining sufficiently transparent to analysis and managerial insights. It is possible to extend our results into a multi-period model (see Netessine and Rudi [24]) using rather standard tools at the cost of additional notation that
makes the insights less transparent. We ignore the possible salvage value of leftover inventory which can easily be incorporated at the cost of introducing an extra parameter, but without any additional insights. There are \( n \) identical retailers, indexed by \( i = 1, \ldots, n \). Retailer \( i \) faces demand \( d_i \), and all demands \( d_i \) are random variables with identical distributions and a symmetric correlation structure; we therefore drop subscript \( i \) whenever appropriate. The symmetry assumption provides a basis for comparison among the models while still preserving the essence of the problem. We define \( D \) to be the total demand for the product, i.e., \( D = \sum_i d_i \), and we denote by \( f_X(\cdot) \) the density function of the random variable \( X \). We denote retailer \( i \)'s profit by \( \pi_i \) and the wholesaler's profit by \( \Pi \). Optimal stocking quantities and profits are marked with the superscript \( \ast \), while best responses are marked with superscript \( Br \) and bold variables indicate vectors.

Each retailer sells the product at an exogenously given unit price \( r' \) (which includes the shipping charge, if any), and the wholesaler buys the product from the upstream manufacturer at a fixed unit cost \( c \). At the beginning of the period, each retailer stocks \( q_{Ti} \) units of the product in the traditional model, the wholesaler centrally stocks \( Q_D \) units of the product in the drop-shipping model, and both parties stock \( q_{Ci} \) and \( Q_C \), respectively, under the dual strategy. Retailers and the wholesaler are all independently managed. Retailers pay a wholesale price \( w \) for each unit purchased to stock. Furthermore, retailers may request that the wholesaler drop-ship products. Since under a drop-shipping arrangement the wholesaler takes on the task of doing fulfillment and bears the inventory risk, a reasonable drop-shipping contract will have a higher wholesale price than when retailers buy the product for their own stock. In practice, the difference in wholesale prices is typically 10-20% (see Scheel [31], page 42). Hence, we denote the wholesale price under drop-shipping by \( w + \delta' \) where \( \delta' \) is the additional drop-shipping markup. We assume that the wholesaler exists in a competitive environment and thus does not possess price-setting power (e.g., books or CDs can be purchased from several major distributors). This assumption will be discussed in Section 4. Our analysis also ignores fixed costs, but we provide a discussion of their potential impact on the choice of channel in Section 4.

As described in the introduction, the structure of transportation costs widely defined (e.g., including picking and packing) can vary according to channels. The total price quoted to the customer typically includes a separate shipping charge. However, the amount a customer pays for shipping may or may not exceed the actual shipping cost incurred. For example, in the traditional channel the quantity \( q_T \) is shipped in bulk from the wholesaler to the retailer and later the quantity sold is shipped to customers (typically in single units). We will denote by \( t_I \) the inbound unit transportation cost, and by \( t_T \) the outbound unit transportation cost, both incurred by the retailer in the traditional channel. In contrast, in the drop-shipping channel, the wholesaler ships directly to customers. In this case, although the actual transportation costs are incurred by the wholesaler, the retailer receives the payment from the customer and transfers a part of this payment to the wholesaler. We will let \( t_D \) be the outbound
transportation cost incurred by the wholesaler. A part of this transportation cost is passed on to the retailer through the drop-shipping markup. For convenience, we define \( r = r' - t_T \), the net unit-revenue in the traditional channel, and \( \delta = \delta' - t_D \) as the net drop-shipping markup and \( \tau = t_T - t_D \), the transportation cost difference of the two channels. The combination of parameters \( \tau, t_I \) and \( \delta \) allows us to account for a variety of practical situations that can arise (e.g., both or neither channel members can make/lose money on transportation charges, and they can split costs/benefits arbitrarily). To avoid trivial situations, we assume that \( c < w \leq w + \delta < r + \tau \).

One of the key assumptions we make is that demand for each retailer is exogenously determined. The consequence of this assumption is that adding a retailer to the channel increases the total demand that the channel faces and does not cannibalize demand for the other retailers. Naturally, this assumption drives some of our results. A different approach might be to include the effect of cannibalization: while adding a retailer to the channel should increase total demand, demand for each specific retailer might go down. We believe, however, that our assumption is a reasonable one for many of the situations we have in mind: imagine that there are numerous supply chains of all three types (traditional, drop-shipping, and dual), and each retailer belongs to one of them. Then, if a retailer desires to switch between channels, he will bring his customer base without affecting demand for other retailers either in the channel that he leaves or in the channel that he joins. Hence, when we analyze the impact of the number of retailers on a channel’s profits, the reader should not think of an addition of a retailer as a new entry into the market; it is merely a shift from one channel to another. This assumption is especially appropriate if retailers are very small compared to the total market size.

In our model T there is no explicit justification for the existence of the wholesaler, just as there is no justification for the existence of the retailer in model D, i.e., it is not clear why the retailer or the wholesaler gets a cut of the channel’s profit since the only function each performs is the collection of the profit margin. In practice, there are many other reasons for the existence of the retailers and the wholesaler in a particular channel that we chose to ignore to keep our focus on inventory risk. Some reasons are access to a unique customer base, aggregation of different product lines, contractual obligations, etc. (see Randall et al. [28]).

2.1 The traditional channel

In the traditional channel, the retailers’ and the wholesaler’s profits are:

\[
\begin{align*}
\pi_T &= rE \min(d, q_T) - (w + t_I)q_T, \\
\Pi_T &= (w - c) n q_T.
\end{align*}
\]
Retailers make stocking decisions individually, each maximizing its expected profit, leading to the well-known newsvendor optimality condition \( \Pr(d < q_r^*) = (r - w - t_I) / r \).

### 2.2 The drop-shipping channel

In the drop-shipping channel there is an issue of allocating inventory among retailers. Various allocation mechanisms are possible (Cachon and Lariviere [13] and Anupindi et al. [7] focus on this issue). We assume that the allocation rule is efficient (i.e., inventory is always allocated if there is demand for it) and increasing (i.e., a retailer never receives less by requesting more). Cachon and Lariviere [13] provide extensive discussion of and the motivation behind these assumptions. Let \( A_i(q_C, Q_C, d) \) be the inventory allocation to the retailer \( i \) through drop-shipping. Then the expected profits of an arbitrary retailer and the wholesaler in the drop-shipping channel can be expressed as

\[
\pi_{Di} = (r + \tau - w - \delta) EA_i(q_C, Q_C, d),
\]

\[
\Pi_D = (w + \delta) E \min(D, Q_D) - cQ_D.
\]

In the drop-shipping channel, the inventory decision is made by the wholesaler, maximizing her expected profit and leading to the newsvendor optimality condition with respect to total demand and a set of cost/revenue parameters that is different from those of the traditional channel \( \Pr(D < Q_D^*) = (w + \delta - c) / (w + \delta) \).

### 2.3 The dual channel: combining drop-shipping and stocking decisions

A company does not necessarily have to choose between traditional and drop-shipping channels. The two pure strategies can also be combined to reap the benefits of both channels. Under such an arrangement, the retailer would stock a certain amount of inventory internally and use drop-shipping only in cases where demand could not be completely satisfied from his own inventory. In what follows, we begin by formulating the game and introducing an appropriate inventory allocation rule. We then demonstrate the existence of equilibrium in this game and analyze the best response functions. This analysis reveals that an equilibrium solution can be interior or boundary. We then conduct a parametric sensitivity analysis of the interior equilibrium solution with respect to problem parameters. This is done both to gain insights into the distribution of inventory between the retail and the wholesale echelons and to obtain intermediate results that help to demonstrate the uniqueness of the symmetric equilibrium in this game, as shown in the last result in this section.

Each retailer independently and simultaneously sets the stocking level \( q_{Ci} \) by purchasing inventory from the wholesaler at unit price \( w \) before the uncertainty in demand is resolved. Once demand is known, each retailer requests drop-shipping at a unit price \( w + \delta \) for any demand in excess of \( q_{Ci} \). In Section 4.2
we briefly address the situation in which retailers’ decisions are coordinated. Note also that a variant of the problem in which the decisions of all retailers and the wholesaler are coordinated has been analyzed by Rudi and Zheng [29]. We denote by \( \bar{D} = \sum_i (d_i - q_{Ci})^+ \) the demand for drop-shipping consisting of the retailers’ residual demands. Further, we define \( L_i(q_C, Q_C, d) = (d_i - q_{Ci})^+ - A_i(q_C, Q_C, d) \), which is the amount of inventory that the retailer requests but does not get due to inventory shortages. This definition will be useful shortly in order to show the existence of the equilibrium. The objective functions of the players can be expressed as

\[
\pi_{Ci} = r E \min (d_i, q_{Ci}) + (r + \tau - w - \delta) (E (d_i - q_{Ci})^+ - E L_i(q_C, Q_C, d)) - (w + t_i) q_{Ci}, \forall i,
\]

\[
\Pi_C = (w - c) \sum_i q_{Ci} + (w + \delta) E \min (\bar{D}, Q_C) - cQ_C.
\]

In our situation the presence of inventory allocation adds significant complexity to the analysis because the allocation has to be done based on retailers’ residual demands rather than on retailers’ initial order quantities. (Extant literature on inventory allocation rules is restricted to the latter situation; see Cachon and Lariviere [13].) To guarantee the existence of a Nash equilibrium in this game, we must impose additional restrictions on the allocation rule as follows.

**Proposition 1.** If \( L_i(q_C, Q_C, d) \) is convex in \( q_{Ci} \), \( i = 1, ..., n \), then the best responses of all players are unique and hence the set of Nash equilibria is nonempty. Further, if \( L_i(q_C, Q_C, d) \) is symmetric across retailers, the subset of symmetric Nash equilibria is nonempty as well.

Notice that assuming convexity of \( L_i(q_C, Q_C, d) \) for results of Proposition 1 to hold is less restrictive than assuming concavity of \( A_i(q_C, Q_C, d) \), which would also be sufficient for the existence of the equilibrium. We verified that the result of Proposition 1 holds for at least two allocation rules commonly used in the literature: relaxed linear and proportional, which are defined as follows, respectively (see Cachon and Lariviere [13]):

\[
L_i^{Lin}(q_C, Q_C, d) = (\bar{D} - Q_C)^+ / n,
\]

\[
L_i^{Prop}(q_C, Q_C, d) = (d_i - q_{Ci})^+ (1 - Q_C / \bar{D})^+.
\]

**Corollary 1.** For the relaxed linear and proportional allocations defined above, the set of Nash equilibria is non-empty, and the subset of symmetric Nash equilibria is nonempty as well.

To obtain further insights, for the remainder of the paper we focus on the relaxed linear allocation rule as defined above because of its analytical tractability (see Section 4.4 for discussion of other allocation rules). The downside of this assumption is that it is an approximation of the linear allocation rule (as reflected by the word “relaxed”). Namely, the relaxed linear allocation rule may result in negative inventory allocations to retailers, especially when the wholesaler holds small amounts of inventory for
drop-shipping. Nevertheless, Cachon and Lariviere [13] show that in their scenarios the relaxed linear allocation is a remarkably good approximation. In most practical situations the wholesaler serves many retailers and stocks significant amounts of inventory, which suggests that this approximation will not affect our results significantly. Under the relaxed linear allocation rule, the retailer’s and the wholesaler’s expected profits are:

\[
\pi_{Ci} = rE \min (d_i, q_{Ci}) + (r + \tau - w - \delta) \left( E (d_i - q_{Ci})^+ - E (D - Q_C)^+ / n \right) - (w + t_I) q_{Ci}, \forall i, (1)
\]

\[
\Pi_C = (w - c) \sum_i q_{Ci} + (w + \delta) E \min (D, Q_C) - cQ_C. \tag{2}
\]

Next, we establish monotonicity properties of the best response functions and as well as obtain optimality conditions.

**Proposition 2.** 1) The game is submodular, so the best-response functions \( q_{Br Ci}^* (q_{C1}, \ldots, q_{C(i-1)}, q_{C(i+1)}, \ldots, q_{Cn}, Q_C), \forall i, \) and \( Q_{Pr q_C}^* (q_C) \) are decreasing.

2) Each equilibrium point is found from the following set of optimality conditions:

\[
\Pr (d_i < q_{Ci}^*) + \frac{r + \tau - w - \delta}{r} \left( \Pr (d_i > q_{Ci}^*) - \frac{\Pr (D > Q_C^*, d_i > q_{Ci}^*)}{n} \right) = \frac{r - w - t_I}{r}, \forall i, (3)
\]

\[
\Pr (D < Q_C^*) = (w + \delta - c) / (w + \delta). \tag{4}
\]

Proposition 2 indicates that, in the dual channel, an increase in stocking quantity by any player results in a decrease in stocking quantity of any other player, so that this becomes a game of strategic substitutes. In Lemmas 1 and 2 stated in the Appendix we proceed by carefully analyzing the best-response functions employed by the players. The best-response functions are implicitly defined by the players’ first-order conditions and hence cannot be found in a closed form. Nevertheless, we are able to identify several useful structural properties of the best responses. Note that pure channels always result in symmetric solutions (in terms of both inventories and profits). Hence, to enable proper comparison of all three channels, we focus on the symmetric equilibria in the dual channel as well (see Section 4.5 for discussion of this assumption). Main results of Lemmas 1 and 2 can be summarized as follows: as a best response, the wholesaler (retailers) may stock nothing if the retailers’ (wholesaler’s) stocking quantity is large enough. These results imply that there might be boundary equilibria in the game in which one of the channel members does not stock inventory. We now formally state conditions that are necessary and sufficient for two boundary equilibria to arise and discuss the intuition behind them. (These propositions follow from Lemmas 1 and 2).

**Proposition 3.** Define

\[
Q_0 = \{ Q : \Pr (D < Q) = (w + \delta - c) / (w + \delta) \},
\]
\[ Q_{\text{max}} = \{ Q : \Pr(D > Q) = \min \left[ n (\tau + t_I - \delta)^+ / (r + \tau - w - \delta), 1 \right] \}. \]

If \( Q_{\text{max}} \leq Q_0 \), then there exists one and only one boundary equilibrium in which the retailers stock nothing, and the channel members earn the same profits as under the pure drop-shipping strategy. Furthermore, if \( 0 \leq n (\tau + t_I - \delta) / (r + \tau - w - \delta) \leq 1 \), then this condition is equivalent to \( n (\tau + t_I - \delta) / (r + \tau - w - \delta) > c / (w + \delta) \).

We observe that this boundary equilibrium arises when the traditional channel has disadvantages compared to the drop-shipping channel: a) when cost/revenue parameters result in a relatively high margin for the wholesaler and a relatively low margin for the retailer (e.g., when \( w \) is large or when \( c \) or \( r \) are small), b) when there are significant benefits to risk pooling (i.e., when \( n \) is large) and c) when the transportation cost structure favors the drop-shipping channel (e.g., when \( t_I \) or \( \tau \) are large). (These results are simple to verify). As a result, the dual channel converts into the pure drop-shipping channel.

**Proposition 4.** Define
\[ q_0 = \{ q : \Pr(d < q) = (r - w - nt_I + (n - 1)(\delta - \tau)) / (r + (n - 1)(w + \delta - \tau)) \}, \]
\[ q_{\text{max}} = \{ q : \Pr(\delta_1 < q, ..., \delta_n < q) = (w + \delta - c) / (w + \delta) \}. \]

If \( q_0 \geq q_{\text{max}} \), then there exists one and only one boundary equilibrium in which the wholesaler stocks nothing, and the channel members earn the same profits as under the pure traditional strategy. Furthermore, if \( \delta_i \)’s are independent random variables, then this condition is equivalent to \( \sqrt{(w + \delta - c) / (w + \delta)} \leq (r - w - nt_I + (n - 1)(\delta - \tau)) / (r + (n - 1)(w + \delta - \tau)) \).

Notice the usefulness of the condition \( q_0 \geq q_{\text{max}} \): it allows us to characterize situations in which the wholesaler’s ability to drop-ship does not add value since the wholesaler, as a best response, will not stock anything anyway. Since drop-shipping might require significant investment into single-unit picking and packing capabilities, it is important for the wholesaler to understand what conditions justify such an investment. Proposition 4 gives us some intuitive ideas about when the dual strategy is not beneficial: a) when cost/revenue parameters result in a relatively high margin for the retailer and a relatively low margin for the wholesaler (e.g., when \( w \) is small or when \( c \) or \( r \) are large), b) when there is little benefit to risk pooling (i.e., when \( n \) is small) and c) when the transportation cost structure favors the traditional channel (e.g., when \( t_I \) or \( \tau \) are small). (These results are simple to verify). An easy way to check Proposition 4 is to let \( n = 1 \) so that the inequality that must be satisfied simply becomes \( (w + \delta - c) / (w + \delta) \leq (r - w - t_I) / r \). If this inequality does not hold, then it will not hold for any \( n > 1 \).

Results of Propositions 3 and 4 are best illustrated graphically. Three situations are possible (with parameters defined in Section 3 unless noted otherwise). In Figure 1 (\( n = 1, t_I = 6 \)) there is a

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1 It should be noted that the relaxed linear allocation rule results in underestimation of \( q_0 \) for \( n > 1 \). Hence, if we were to use the exact linear allocation rule, the boundary equilibrium would occur for a smaller range of problem parameters than implied by Proposition 4.
boundary equilibrium in which the retailers stock nothing, which happens when $Q_{\text{max}} \leq Q_0$ according to Proposition 3. In this case the dual channel in the equilibrium becomes a pure drop-shipping channel. In Figure 2 ($n = 1$, $t_I = 0$) there is an interior equilibrium in which the retailers and the wholesaler stock nonzero quantities. Such a situation arises when $Q_{\text{max}} > Q_0$ and $q_0 < q_{\text{max}}$. In Figure 3 ($n = 1$, $t_I = -3$) there is a different boundary equilibrium in which the wholesaler stocks nothing, which occurs if $q_0 \geq q_{\text{max}}$ according to Proposition 4. In this case the dual channel in the equilibrium becomes a pure traditional channel. Since we are mostly interested in situations when both retailers and the wholesaler stock inventory, for the remainder of this section we focus on the interior equilibria by assuming that problem parameters are such that $q_0 < q_{\text{max}}$ and $Q_{\text{max}} > Q_0$.

![Figure 1. $Q_{\text{max}} \leq Q_0$](image1.png) ![Figure 2. $q_0 < q_{\text{max}}$ and $Q_{\text{max}} > Q_0$](image2.png) ![Figure 3. $q_0 \geq q_{\text{max}}$](image3.png)

Before we can establish the uniqueness of the symmetric equilibrium in the game, we obtain a couple of other results. These results concern the monotonicity of equilibrium inventories in $w$, $\tau$ and $t_I$ (i.e., ordinal reactions to changes in $w$, $\tau$ and $t_I$). Moreover, we are able to compare relative changes in $Q_C^*$ and $q_C^*$ as a result of change in $w$, $\tau$ and $t_I$ or cardinal reactions to changes in $w$, $\tau$ and $t_I$.

**Proposition 5.** 1) Consider a collection of games parametrized by $w$. The set of symmetric equilibria is monotone in $w$, i.e., $Q_C^*(w)$ increases in $w$, while $q_C^*(w)$ decreases in $w$ and for $n \geq 2$: $|\partial q_C^* / \partial w| \leq |\partial Q_C^* / \partial w|$.

2) The set of symmetric interior equilibria is monotone in $\tau$, i.e., $Q_C^*(\tau)$ increases in $\tau$, while $q_C^*(\tau)$ decreases in $\tau$ and for $n \geq 2$: $1/n \leq |(\partial q_C^* / \partial \tau)/(\partial Q_C^* / \partial \tau)| \leq 1$.

3) The set of symmetric interior equilibria is monotone in $t_I$, i.e., $Q_C^*(t_I)$ increases in $t_I$, while $q_C^*(t_I)$ decreases in $t_I$ and for $n \geq 2$: $1/n \leq |(\partial q_C^* / \partial t_I)/(\partial Q_C^* / \partial t_I)| \leq 1$.

As we might expect, the higher wholesale price induces the retailers to stock less and the wholesaler to stock more, since the retail margin goes down and the wholesale margin goes up, resulting in reallocation of inventory between two echelons. An increase in the wholesaler’s inventory exceeds the corresponding decrease in each retailer’s inventory since the wholesaler stocks the product for multiple retailers and enjoys the benefits of risk pooling. The intuition for transportation costs is clear as well: higher values of $\tau$ (or $t_I$) make drop-shipping preferable and hence the wholesaler stocks more while
the retailers stock less. We also see that the increase in the wholesaler’s inventory is larger than the
decrease in inventory for each retailer, although it is also smaller than a decrease of inventory by all
retailers taken together.

It is rather straightforward to verify that the approach we use to show parametric monotonicity
in \( w, \tau \) and \( t_I \) fails to work for the drop-shipping markup since the retailers’ and wholesaler’s payoffs
fail to have increasing differences in \( \delta \). The intuition behind such non-monotonicity is as follows. An
increase in \( \delta \) may have two effects: the wholesaler may increase her stocking quantity \( Q^*_C \) since her
profit margin increases, and the retailer may increase his stocking quantity \( q^*_C \) since it is becoming more
expensive to drop-ship. However, since players’ stocking decisions are substitutes in the game, there
is also a second-order effect: one competitor’s decision to increase inventory induces the other player
to reduce inventory. Hence, extending the sensitivity analysis to the drop-shipping markup appears
difficult. We can still, however, observe some relations with respect to drop-shipping markup. For
example, under the argument above, we would not expect an increase in the drop-shipping markup
to lead to a simultaneous decrease in both players’ inventories. Moreover, one would think that in a
majority of situations, the total inventory in the system would increase as \( \delta \) increases due to the above
effects. As the next proposition shows, this intuition is correct.

**Proposition 6.** 1) Consider a collection of games parametrized by \( \delta \). It is never the case that the
retailers’ and the wholesaler’s inventories in any symmetric equilibrium simultaneously go down in \( \delta \).
2) If the retailers’ inventories go down and the wholesaler’s inventory goes up, then \(|\partial q^*_C / \partial \delta| < \)
\(|\partial Q^*_C / \partial \delta|\).
3) If the retailers’ inventories go up and the wholesaler’s inventory goes down, then \(|\partial q^*_C / \partial \delta| \times n > \)
\(|\partial Q^*_C / \partial \delta|\).

We are now ready to prove the last result of this section: the uniqueness of the symmetric equi-
librium. Although it is not immediately obvious, the by-products of the last two proofs enable the
characterization of the relative slopes of the players’ best-response curves at the equilibrium, which
ultimately leads to the guarantee of a unique symmetric equilibrium.

**Proposition 7.** There is a unique symmetric Nash equilibrium (boundary or interior) in the retailers-
wholesaler game.

### 3 Comparison of channels

The goal of this section is to gain insight into the issue of channel choice by channel members as well
as to see if/when Pareto-optimal channel choices exist. We begin the analysis by noting that the retailer
always prefers the dual strategy to the traditional strategy since the retailer obtains another “safety
stock” at the wholesaler’s location at no extra cost. At the same time it is not clear which channel the
wholesaler prefers. From a channel perspective, the dual strategy might be a reasonable compromise given additional considerations (e.g., working capital constraints or limited fulfillment capabilities). An observation can also be made regarding the stocking quantities. Clearly, under the dual strategy the retailer will stock less than he would in the traditional channel since now the wholesaler’s inventory represents a backup option. On the other hand, the wholesaler will not stock as much as she would in the drop-shipping channel since the retailer has first access to customer demand. It follows that the dual strategy is somewhere between the two pure strategies in terms of stocking quantities for individual players, which is also reflected in Propositions 3 and 4.

In this section we present results of extensive numerical experiments in order to compare all three channels. For the numerical experiments, we have selected the following problem parameters (unless otherwise noted): $n = 10$, $r = 20$, $w = 8$, $c = 3$, $\delta = 1$, $\tau = 0.5$ and $t_I = 0^2$. We believe that these parameters represent a reasonable practical situation: the number of retailers is large enough to achieve significant benefits of risk pooling and $\tau > 0$, meaning that the wholesaler in the drop-shipping channel can ship more economically due to economies of scale and negotiating power. Demand is Normal, symmetrical across retailers with mean $\mu = 100$, standard deviation $\sigma = 50$, and all coefficients of correlation $\rho = \rho_{ij} = 0$. We employ the Round-Robin Optimization algorithm (see Topkis [34], Algorithm 4.3.1), which guarantees convergence. To evaluate the probability terms in (3) and (4), we use Monte-Carlo integration. We always divide the wholesaler’s inventory and profit by the number of retailers so as to bring all graphs to comparable scale.

We begin by analyzing the impact of the number of retailers on channel choice by plotting the optimal stocking levels as a function of the number of retailers participating in the channel (Figure 4). In the dual channel increasing the number of retailers results in inventory reallocation from retailers to the wholesaler due to the benefits of risk pooling. However, inventories in all three models become rather insensitive to the number of retailers once $n \geq 10$ since most benefits of risk pooling are already achieved. As expected, the retailers and the wholesaler stock less in the dual model than in the traditional and drop-shipping models, respectively. Looking at the retailers’ and the wholesaler’s profits (Figures 5 and 6), we see how each additional retailer in the drop-shipping channel (almost always) benefits everyone: not only does the wholesaler’s total profit go up, but the profit per retailer goes up as well, and, moreover, as new retailers join the dual channel each retailer lowers inventory while increasing profits. The only exception is the wholesaler’s profit in the dual channel, whereby increasing the number of retailers from one to two diminishes the wholesaler’s profit. This effect is the direct outcome of the retail competition for the wholesaler’s inventory: when the number of retailers exceeds one, retailers greatly decrease the amount of inventory stocked locally and aggressively rely on drop-shipping.

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2The impact of the inbound transportation cost in the tradition channel $t_I$ is quite straightforward, so we let $t_I = 0$. 
the number of retailers reaches 5 to 10, adding retailers has a diminishing impact on profits. It is also clear that a larger number of retailers makes drop-shipping and dual channels significantly more attractive than the traditional channel. We note also that the difference between the drop-shipping and the dual channels is quite small for both the retailers and the wholesaler, so the number of retailers is not a major driving force in the channel choice (since in most practical situations \( n \) is rather large).

Next, we investigate the impact of wholesale price on channel choice. In the drop-shipping channel the wholesaler’s profit and inventory increase in \( w \) and the retailers’ profit decreases. Lariviere and Porteus [20] have demonstrated that in the traditional channel the wholesaler’s profit is unimodal in \( w \) while the retailer’s profit and inventory decrease in \( w \). Finally, for the dual channel some characterizations were given in Proposition 5 (that the wholesaler’s inventory is monotonically increasing and the retailer’s inventory is monotonically decreasing in \( w \)). Figures 7 to 9 illustrate these findings. Once again, the traditional channel does not appear to be a good alternative for channel members: drop-shipping and dual channels dominate the traditional channel for most values of \( w \). Remarkably, the wholesaler’s (as well as the retailers’) profits are essentially the same under the dual and drop-shipping strategies for any given wholesale price: hence, keeping everything else fixed, wholesale price is not a major driving force in channel choice either.

We now analyze the impact of drop-shipping markup on channel choice. Numerical experiments indicate (Figure 10) that inventories in the dual channel are quite sensitive to changes in \( \delta \). As drop-shipping becomes more expensive, the retailers and the wholesaler respectively increase or decreases inventory. This outcome indicates that the direct effect for the retailer (recall the discussion preceding Proposition 6) dominates: although seemingly higher \( \delta \) should lead to higher wholesale inventory, this does not happen because retailers shift away from drop-shipping towards stocking internally. As a result, the drop-shipping markup has a drastically different effect on inventory in the two pure channels and in the dual channel. Namely, the wholesaler’s inventory in the dual channel decreases while her inventory in the drop-shipping channel increases. Although this result for the drop-shipping channel is expected, in the dual channel the opposite happens due to reallocation of inventory from the wholesaler.
to retailers. Indeed, in the dual channel, retailers’ inventory increases with the drop-shipping markup: the retailers increasingly rely on their own inventory as it becomes more expensive to drop-ship.

Figure 7. Channel inventories  
Figure 8. Retailer’s profit  
Figure 9. Wholesaler’s profits

Retailers’ profits decrease (Figure 11) and the wholesaler’s profits increase (Figure 12) as the drop-shipping markup increases. Hence, even though in the dual channel retailers try to compensate for higher markup by stocking more locally, they still earn less as a result. The opposite is true for the wholesaler: since the retailers stock more internally for higher $\delta$, the wholesaler makes more money by selling the product to retailers than she loses due to reduced reliance on drop-shipping. To this end, we note that while changing $n$ and $w$ does not result in significant profit differences in the drop-shipping and dual channels, we can clearly observe that different values of $\delta$ can result in these two channels differing significantly in terms of profits. We explore this issue further at the end of this section.

Figure 10. Channel inventories  
Figure 11. Retailer’s profit  
Figure 12. Wholesaler’s profit

Finally, we investigate the impact of differences in transportation costs. Recall that lower $\tau$ makes the traditional channel more efficient and higher $\tau$ favors the drop-shipping channel. While $\tau$ does not affect stocking quantities in pure channels, higher $\tau$ in the dual channel leads to increasing reliance on drop-shipping, as we proved in Proposition 5. Figures 13 to 15 demonstrate these findings. In terms of retailers’ and wholesaler’s profits, we see that the transportation cost is another major differentiator that significantly affects the comparison of the drop-shipping and dual channels. As we expect, higher values of $\tau$ make the drop-shipping and the dual channels preferable for retailers. Surprisingly, the wholesaler’s
profit in the dual channel is non-monotone in $\tau$ although it is mostly decreasing. We explain as follows: for very low values of $\tau$ the wholesaler benefits, because retailers purchase a lot of inventory through the traditional channel. As $\tau$ increases, the wholesaler actually loses money as she gains in transportation economies, a nonintuitive result that is driven by the fact that retailers stock less and less internally and rely more on drop-shipping, which is less profitable to the wholesaler. However, when $\tau$ is very high, then retailers stock almost nothing and purchase a lot through drop-shipping which is, again, very profitable for the wholesaler.

The preceding analysis sheds some light on the issue of channel choice. For convenience, we summarize all our analytical findings as well as conjectures based on numerical experiments in Table 1. We do not include sensitivity analysis with respect to mean demand and demand uncertainty because its results are obvious: higher mean demand (lower demand uncertainty) benefits all channel members in all models. Although many parameters affect channel choice, the preceding analysis also indicates that the main driving forces in channel choice are the drop-shipping markup $\delta$ and the transportation cost differential $\tau$. Since it is hard to envision a complete picture of channel choice by varying just one parameter at a time, in Figures 16 and 17 we plot the preferred channels for the retailers and the wholesaler, respectively, as functions of these two parameters, and in Figure 18 we plot Pareto-optimal choices.

<table>
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Table 1. The summary of parametric analysis.

In Figure 16 we see that the channel choice for retailers is quite straightforward: retailers prefer the drop-shipping channel when the traditional channel has a transportation cost disadvantage ($\tau$ is high)
while drop-shipping is not too expensive ($\delta$ is small). Otherwise, retailers prefer the dual channel. The choice for the wholesaler (Figure 17) is more complex: any of the three channels may be preferable. When the drop-shipping markup $\delta$ is small and the traditional channel’s transportation cost disadvantage $\tau$ is relatively high, the pure traditional channel is the best option for the wholesaler, because offering drop-shipping at this low value of the drop-shipping markup is not profitable; if the wholesaler were to offer drop-shipping, retailers would stock little internally, thus resulting in low profit for the wholesaler. However, as transportation costs under drop-shipping increase ($\tau$ decreases), the wholesaler (surprisingly) switches from the pure traditional to the dual channel. The reason for this counterintuitive effect is that, when transportation costs in the drop-shipping channel are high, the wholesaler stocks very little inventory and thus retailers stock a lot internally. Therefore, offering drop-shipping at low values of $\tau$ does not cannibalize very profitable sales through the traditional channel while adding to the wholesaler’s profit. When the drop-shipping markup is high, the wholesaler prefers either the dual or the drop-shipping channel. Interestingly, the dual channel is preferred when the transportation cost difference $\tau$ is either very low or very high. This behavior is directly related to the intuition underlying Figure 15, in which the wholesaler’s profit in the dual channel is highest at the extremes of the transportation cost difference parameter.

![Figure 16. Retailers’ channel choice](image1)

![Figure 17. Wholesaler’s channel choice](image2)

![Figure 18. Pareto-optimal channel](image3)

It is also clear that there are limited opportunities for Pareto-optimal choices (Figure 18). First, the dual channel can be preferred by both parties for negative values of $\tau$, something that we might see when the retailer is very large (e.g., Amazon.com) and therefore has better deals on transportation prices than the wholesaler does. Second, the drop-shipping channel can be preferred by both parties for relatively large values of $\delta$ and positive $\tau$, something we might observe when the wholesaler can transport for a lower price than the retailers can. For many problem parameters, however, the wholesaler and the retailers have different preferences.
4 Discussion of modeling assumptions and summary

4.1 Endogenous wholesale price and drop-shipping markup

One of our assumptions is that the wholesale price $w$ and the drop-shipping markup $\delta$ are exogenous, although in some cases a powerful wholesaler may be able to set both of these prices. In the traditional channel this problem has been solved by Lariviere and Porteus [20]. Even in this relatively simple problem, additional conditions are needed to ensure the uniqueness of the solution. In the drop-shipping channel, the wholesaler would set $w + \delta = r + \tau$ and extract all profits from the channel. A similar problem arises in the dual channel: the wholesaler has the opportunity to set $w, \delta$ to the highest possible values and extract most profits (as is evident from Figures 9 and 12). Clearly, this outcome is not reasonable in the eyes of the retailer. Cachon [12] considers situations in which the wholesaler does not have complete bargaining power by using the notion of a Pareto-improving set of contracts. Another possible approach commonly used in the literature is to introduce the participation constraint on the retailer’s (expected) reservation profit. The dual channel problem is then formulated as follows:

$$\max_{w \in [c, r-t_I], \delta \in [r_D, r+\tau-w]} \Pi_C^*(w, \delta)$$

S.t. $\pi_{Ci}^*(w, \delta) \geq U$,

where $\Pi_C^*(w, \delta)$ and $\pi_{Ci}^*(w, \delta)$ are profits resulting from the equilibrium in the inventory game and $U$ is the retailer’s reservation profit. We see that the result is a very complex Stackelberg game in which the wholesaler first sets prices, and then the Nash inventory game ensues. Below we demonstrate through numerical experiments the insights resulting from this game (Figures 19 to 21).

When the reservation profit constraint is absent, the wholesaler establishes a high price $w = r - t_I$ (in this case $w = 13.68$) and the highest possible markup $\delta = r + \tau - w$ (in this case $\delta = 5.82$) so that retailers make no money on drop-shipping. The high $w$ makes retailers stock little and rely extensively on drop-shipping. Interestingly, the wholesaler does not find it advantageous to set the
maximum possible $w$ but rather sets the maximum possible $\delta$. The intuition behind this decision comes from inventory risk. If the wholesaler were to set a very high $w$, retailers would not stock anything. Thus, most of the wholesaler’s profit would come from drop-shipping, which means taking a significant inventory risk. As $U$ increases, the wholesaler decreases $w$ while keeping $\delta$ at the highest possible value (so that $w + \delta = $constant). In response retailers increasingly shift towards stocking inventory and relying less on drop-shipping (Figure 20). Finally, Figure 21 indicates that the wholesaler’s profit decreases in $U$, as we would expect. As $U$ increases, the wholesaler must reduce her profit by increasingly shifting away from drop-shipping. Although an outcome of this model (namely, that retailers do not make any money on drop-shipping) is not practical, it is useful to be aware of the wholesaler’s incentives to inflate the drop-shipping markup. In practice there are transaction costs associated with forwarding orders to the wholesaler as well as with handling subsequent customer inquiries. Therefore, we might expect that the retailer will be unwilling to participate in the drop-shipping arrangement unless a certain per-transaction profit is ensured.

4.2 Coordinated retailers

We have so far focused on the variant of the problem in which all retailers act independently and maximize their individual profits. In practice, however, there are situations in which retailers’ stocking decisions are coordinated because, for instance, retailers belong to the same chain of stores and the parent company imposes centralized controls over individual stores’ inventory policies. When retailers are centrally managed, the objective function of the retail firm becomes the sum of individual retailers’ objective functions. Although the resulting optimality conditions are somewhat different, we have verified that all the equilibrium properties derived in Propositions 3 to 7 continue to hold when retailers’ stocking decisions are centrally managed and the attention is restricted to the symmetric equilibrium. The main difference is summarized in the next Proposition.

Proposition 8. Suppose the relaxed linear allocation rule is used. Then, in a symmetric interior equilibrium, under the coordination of the retailers’ stocking decisions, the retailers (the wholesaler) stock more (stocks less) than when retailers’ stocking decisions are not coordinated.

The last result can be explained as follows: when retailers are not coordinated, they compete for the allocation through drop-shipping. Retailers’ stocking quantities are substitutes in this game: if one retailer decides to increase his inventory, it will reduce this retailer’s inventory allocation and increase other retailers’ inventory allocations. Hence, other retailers will stock less and rely more on drop-shipping. As has been shown for other games in which retailers’ stocking decisions are substitutes, competing retailers tend to increase their share of the scarce resource (allocated inventory in this case) by stocking less and relying more on drop-shipping (see Netessine and Rudi [23]). The wholesaler’s
stocking quantity decreases in the retailers’ stocking quantities so the wholesaler responds by stocking more when retailers are not coordinated.

4.3 The impact of fixed costs

Throughout this paper, we do not take into account fixed costs, which can be quite substantial. We let \( k_T \), \( k_D \) and \( k_C \) denote the fixed costs of the three channels to the retailer and let \( K_T \), \( K_D \) and \( K_C \) denote the fixed costs of the three channels to the wholesaler. In the traditional channel, \( k_T \) will typically include larger costs than \( k_D \) for physical fulfillment capabilities (such as warehouses, shipping/packing equipment, and personnel). \( k_D \) might have a larger component than \( k_T \) in terms of expenses associated with the information technology that is needed to support order placement/tracking and communication with the wholesaler. Hence, the relative size of \( k_T \) and \( k_D \) is situation-dependent. On the other hand, in the traditional channel, \( K_T \) will typically include smaller costs than \( K_D \) for physical fulfillment capabilities as well as for information technology. Hence, it is reasonable to assume that \( K_T \leq K_D \). In the dual channel, both the retailers and the wholesaler need to spend money to support both channels (i.e., “everyone must be able to do everything”), so it can be argued that \( k_C > \max(k_T, k_D) \) and \( K_C > \max(K_T, K_D) \). From the above discussion it is clear that, in most reasonable situations, larger fixed costs will make the dual channel less attractive for channel members relative to pure channels. We note that fixed costs do not have an impact on order quantity decisions, but when comparing profit expressions in the three channels, the appropriate fixed costs must be subtracted from the expressions for variable profits. Overall, fixed cost considerations should augment our analysis of variable costs in the previous sections. However, one must be careful to remember that our analysis was conducted for a single product, while in practice fixed costs will be incurred for multiple products. Hence, the variety of products that the retailer/wholesaler offers in different channels will have an impact on the ultimate profitability (see Randall et al. [27]).

4.4 The impact of the allocation rule choice

Cachon and Lariviere (1999) demonstrate that different allocation rules result in different incentives in supply chains and therefore lead to different equilibrium inventories. To understand how different allocation rules might alter our results, we conducted numerical experiments comparing inventory decisions and profits under three allocation rules: the relaxed linear (Lin), the lexicographic (Lex) and the uniform (Uni). For the lexicographic allocation rule we assume that each retailer has a 50% chance of obtaining access to the wholesaler’s inventory first and, thus, a 50% chance of obtaining access to the wholesaler’s inventory second. Under the uniform allocation rule each retailer is guaranteed access to 50% of the wholesaler’s inventory plus any of the other 50% of inventory unused by the other retailer.
Note that the uniform allocation rule favors retailers with small residual demands (see Cachon and Lariviere 1999), which is the opposite of the relaxed linear allocation rule, which favors retailers with high residual demands. The lexicographic allocation rule is purely random and, in this sense, falls in between the other two allocation rules. Thus, by considering these three allocation rules we cover a variety of incentives that different allocation rules might create. Under the assumption that there are only two retailers in the channel, we are able to obtain analytical expressions for optimality conditions for each of these allocation rules, and we use them to calculate the equilibria of the game. In all numerical experiments with the above three allocation rules, we observe that $q_{C, \text{Lin}} \leq q_{C, \text{Lex}} \leq q_{C, \text{Uni}}$ and $Q_{C, \text{Lin}} \geq Q_{C, \text{Lex}} \geq Q_{C, \text{Uni}}$ thus confirming our intuition. For the vast majority of the problem parameters we experimented with, gaps between inventories were rather small and rarely exceeded 10%. Gaps between profits were even smaller, rarely exceeding 3%. The typical rank ordering of profits was the opposite of the rank ordering of inventories, namely, $\pi_{C, \text{Lin}} \geq \pi_{C, \text{Lex}} \geq \pi_{C, \text{Uni}}$ and $\Pi_{C, \text{Lin}} \leq \Pi_{C, \text{Lex}} \leq \Pi_{C, \text{Uni}}$. More importantly, in all of our experiments we observe that insights from Propositions 1-7 as well as insights from numerical experiments using the relaxed linear allocation rule continue to hold. Thus, we conclude that our findings are generally robust to the allocation rule specification.

4.5 The impact of the symmetry assumption

In the analysis so far we have focused on studying symmetric equilibria, although we are unable to prove that only symmetric equilibria can exist in this game. In our extensive numerical experiments with symmetric problem parameters, we never encountered asymmetric equilibria and therefore we conjecture that asymmetric equilibria are unlikely to exist in a problem with symmetric parameters. A related question is whether our insights continue to hold when problem parameters are asymmetric across retailers and therefore only asymmetric equilibria exist. To address this issue we conducted extensive numerical experiments with asymmetric problem parameters and two retailers. We introduced asymmetry through revenue parameters $r_i$ as well as through demand parameters $\mu_i$ and compared results with the base case in which $r = (r_1 + r_2) / 2$ and $\mu = (\mu_1 + \mu_2) / 2$ for both retailers. In these experiments we did not encounter multiple equilibria. Overall, we found that, as expected, the retailer with the higher retail price or higher mean demand stocks more inventory, and the impact of change in mean demand is stronger than the impact of a (similar in relative terms) change in the retail price. The reason might be that changes in mean demand affect both each retailer’s inventory and his inventory allocation by the wholesaler. However, the total retail inventory and inventory held by the wholesaler are largely unaffected by the presence of two asymmetric retailers. Moreover, we observed that all of our insights continue to apply to the asymmetric problem as well. Therefore, we believe that our results are representative of results for asymmetric problem parameters as well.
4.6 Summary and discussion

The practice of drop-shipping that has become very popular among Internet retailers is rather new to the operations literature. In this paper, we further advance the study of drop-shipping by modeling a supply chain with multiple retailers. In addition to the models for the traditional and drop-shipping supply chains, we also present a dual strategy that combines the two channel alternatives: the retailer stocks some inventory internally and drop-ships whenever local inventory is depleted. This is a competitive model, since the retailers and the wholesaler make their decisions independently. We compare all three channel alternatives and find that the drop-shipping and dual channels can be viable choices for both the retailer and the wholesaler, and that both of these channels can be Pareto-optimal (whereas the dual channel is always preferable to the traditional channel for retailers). In summary, we show that the dual channel often offers advantages to both retailers and the wholesaler, the finding that is in line with the prediction of industry analysts (see [5]) that in the near future most Internet retailers will operate using a combination of internal inventory and drop-shipping. By comparing the supply chain alternatives on the Internet, we provide practicing managers with specific guidelines for choosing an appropriate channel structure for Internet retailing. Our findings are in line with the empirical results of Randall et al. [27], who found that no single channel option is uniformly preferred: the choice of channel depends on the characteristics of the competitive environment.

The main focus of this paper is on economic insights into managerial decisions of channel choice, so, as with any economic modeling, in specifying the model we face the challenge of capturing the relevant managerial trade-offs without diffusing insights. Hence, we make several simplifying assumptions. First and foremost, we focus on the operational side of business, namely the allocation of the risk associated with carrying inventory and the related effects of risk pooling. Thus, we ignore the other aspects of channel choice that are extensively described in the marketing literature (see an excellent account in Coughlan et al. [17]). We also abstract away issues of real-time inventory allocation and the related problem of inventory rationing. Thus, our model is meant to aid managerial understanding of inventory ownership issues under drop-shipping and should be used with these and other limitations in mind (see Randall et al. [28] for other practical considerations that are hard to incorporate in a formal analytical model). Moreover, both the present work and a paper by Netessine and Rudi [24] consider single-product situations. An important open question is, Should the same channel strategy be used for each of the multiple products with different cost/revenue/demand characteristics? We have already found that demand variability influences the decision to select a particular channel and hence our model provides a framework to classify different products into appropriate channel structures. Another major area for future research is pricing decisions, the direction pursued in Biyalogorsky and Koenigsberg [10] for a single product/retailer.
References


Appendix: Proofs

**Proof of Proposition 1:** It is straightforward to verify that the retailers’ objective functions are concave in their respective decision variables, which is sufficient to prove the existence of pure-strategy equilibria. Furthermore, the symmetry of the problem implies that at least one equilibrium is symmetric.

**Proof of Corollary 1:** For the relaxed linear allocation we note that $\overline{d} = \sum_j (d_j - q_{Cj})^+$ is convex in $q_{Ci}$, $i = 1, \ldots, n$. Furthermore, $(\cdot)^+$ is a convex increasing function. The increasing convex function of a convex function is itself convex. Hence, $(\overline{d} - Q_C)^+ / n$ is convex in $q_{Ci}$, $i = 1, \ldots, n$. For the proportional allocation, we note that $L^\text{Prop}_i (q_C, Q_C, d)$ is a multiplication of two functions. It can be shown that a multiplication of two positive, decreasing, convex functions is itself convex. Since $(d_i - q_i)^+$ is convex decreasing, it suffices to show that $(1 - Q_C / \overline{d})^+$ is convex decreasing. We note that $1 - Q_C / \overline{d} = 1 + Q_C / \left( -\sum_{j \neq i} (d_j - q_{Cj})^+ - (d_i - q_{Ci})^+ \right)$ is convex decreasing in $q_{Ci}$ on $[0, d_i]$ and constant on $[d_i, \infty]$, implying that $1 - Q_C / \overline{d}$ is convex decreasing. Finally, $(1 - Q_C / \overline{d})^+$ is convex as a convex increasing transformation of a convex function.

**Proof of Proposition 2:** First derivatives are:

$$
\frac{\partial \pi_{Ci}}{\partial q_{Ci}} = r \Pr (d_i > q_{Ci}) - (r + \tau - w - \delta) \left( \Pr (d_i > q_{Ci}) - \Pr (\overline{d} > Q_C, d_i > q_{Ci}) / n \right) - (w + t), \forall i,
$$

$$
\frac{\partial \Pi_C}{\partial Q_C} = (w + \delta) \Pr (\overline{D} > Q_C) - c.
$$

The resulting optimality conditions follow. To demonstrate the submodularity of the objective functions, it is sufficient to show that the second-order cross-partial derivatives are negative (see Topkis [34]):

$$
\frac{\partial^2 \pi_{Ci}}{\partial q_{Ci} \partial q_{Cj}} = -(r + \tau - w - \delta) f_{\overline{d}(d_i > q_{Ci}, d_j > q_{Cj})} (Q_C) \Pr (d_i > q_{Ci}, d_j > q_{Cj}) / n < 0,
$$

$$
\frac{\partial^2 \pi_{Ci}}{\partial q_{Cj} \partial Q_C} = -(r + \tau - w - \delta) f_{\overline{d}(d_i > q_{Ci})} (Q_C) \Pr (d_i > q_{Ci}) / n < 0,
$$

$$
\frac{\partial^2 \Pi_C}{\partial Q_C \partial q_i} = -(w + \delta) f_{\overline{d}(d_i > q_{Ci})} (Q_C) \Pr (d_i > q_{Ci}) < 0,
$$

and result 1) is verified.

**Lemma 1. Define**

$Q_0 = \{ Q : \Pr (D < Q) = (w + \delta - c) / (w + \delta) \}$,

$q_{\max} = \{ q : \Pr (d_1 < q, \ldots, d_n < q) = (w + \delta - c) / (w + \delta) \}.$

Then the wholesaler’s best-response function $Q^\text{Br}_C (q_C)$ to a set of symmetric retailers’ strategies (i.e., the wholesaler responds to the simultaneous change in the stocking quantity by all retailers, $q_C = q_{Ci}, \forall i$) possesses the following properties:

1) $Q^\text{Br}_C (0) = Q_0$.

2) On the interval $q_C \in [0, q_{\max})$, $Q^\text{Br}_C (q_C)$ is defined by optimality condition (4). It is a decreasing
function with slope \(-1 \leq \partial Q^B_C(q_C)/\partial q_C \leq 0\) for \(n = 1\), and \(-n \leq \partial Q^B_C(q_C)/\partial q_C \leq -1\) for \(n \geq 2\).

3) On the interval \(q_C \in [q_{\text{max}}, \infty)\), \(Q^B_C(q_C) = 0\).

**Proof:** Result 1 is straightforward. To see Result 3, we observe that \(\tilde{D} = \sum_i (d_i - q)^+\) has a probability distribution with a mass point at zero. The probability of this mass point is calculated as follows:

\[
\Pr(\tilde{D} = 0) = \Pr(\sum_i (d_i - q)^+ = 0) = \Pr(d_1 < q_C, \ldots, d_n < q_C).
\]

From the wholesaler’s optimality condition (4) we know that as long as \(\Pr(d_1 < q_C, \ldots, d_n < q_C) \geq (w + \delta - c)/(w + \delta)\), or in other words as long as \(q_C > q_{\text{max}}\), the wholesaler will stock nothing, leaving us with Result 3. Finally, to see that Result 2 holds, we note that for \(q_C \leq q_{\text{max}}\) and for any \(n\) we find from implicit differentiation that

\[
\left|\frac{\partial Q^B_C}{\partial q_C}\right| = \sum_i f_{\tilde{D}|d_i > q_C}(Q_C) \Pr(d_i > q_C) / f_{\tilde{D}}(Q_C) \leq n f_{\tilde{D}}(Q_C) / f_{\tilde{D}}(Q_C) = n.
\]

To prove that \(\left|\frac{\partial Q^B_C}{\partial q_C}\right| \geq 1\) for \(n \geq 2\), it is sufficient to show that \(\sum_i f_{\tilde{D}|d_i > q_C}(Q_C) \Pr(d_i > q_C) / f_{\tilde{D}}(Q_C) \geq 0\). This is done by mathematical induction. We first show that the statement holds for \(n = 2\), then assume that it holds for any \(n\) and prove that it will also hold for \(n + 1\). First, for \(n = 2\),

\[
\sum_{i=1}^{2} f_{\tilde{D}|d_i > q_C}(Q_C) \Pr(d_i > q_C) = f_{\tilde{D}|d_1 > q_C}(Q_C) \Pr(d_1 > q_C) + f_{\tilde{D}|d_2 > q_C}(Q_C) \Pr(d_2 > q_C) - f_{\tilde{D}}(Q_C)
\]

\[
= f_{\tilde{D}|d_1 > q_C}(Q_C) \Pr(d_1 > q_C) + f_{\tilde{D}|d_2 > q_C}(Q_C) \Pr(d_2 > q_C) - f_{\tilde{D}}(Q_C)
\]

\[
= f_{\tilde{D}|d_1 > q_C}(Q_C) \Pr(d_1 > q_C) - f_{\tilde{D}|d_2 < q_C}(Q_C) \Pr(d_2 < q_C) - f_{\tilde{D}|d_1 < q_C}(Q_C) \Pr(d_1 < q_C)
\]

\[
= f_{\tilde{D}|d_2 > q_C}(Q_C) \Pr(d_2 > q_C) - f_{\tilde{D}|d_1 < q_C}(Q_C) \Pr(d_1 < q_C)
\]

\[
= f_{\tilde{D}|d_1 > q_C,d_2 > q_C}(Q_C) \Pr(d_1 > q_C, d_2 > q_C) + f_{\tilde{D}|d_1 < q_C,d_2 > q_C}(Q_C) \Pr(d_1 < q_C, d_2 > q_C)
\]

\[
- f_{\tilde{D}|d_1 < q_C,d_2 < q_C}(Q_C) \Pr(d_1 < q_C, d_2 < q_C) - f_{\tilde{D}|d_2 < q_C,d_2 < q_C}(Q_C) \Pr(d_1 < q_C, d_2 < q_C)
\]

\[
= f_{\tilde{D}|d_1 > q_C,d_2 > q_C}(Q_C) \Pr(d_1 > q_C, d_2 > q_C) \geq 0.
\]

The last equality follows from the fact that \(f_{\tilde{D}|d_1 < q_C,d_2 < q_C}(Q_C) \Pr(d_1 < q_C, d_2 < q_C) = 0\), since \(Q_C > 0\) on the studied interval. This completes the proof for \(n = 2\). For convenience, we denote \(\tilde{D}^n = \sum_{i=1}^{n} (d_i - q_C)^+\). We now assume that the inequality in question holds for some \(n\), that is,

\[
\sum_{i=1}^{n} f_{\tilde{D}^n|d_i > q_C}(Q_C) \Pr(d_i > q_C) - f_{\tilde{D}^n}(Q_C) \geq 0.
\]

(5)

We need an additional lemma that can be proven trivially.

**Lemma.** If (5) is true, then for a random variable \(d_{n+1}\) and any \(n \geq 2\) it is also true that

\[
\sum_{i=1}^{n} f_{\tilde{D}^n|d_i > q_C,d_{n+1} < q_C}(Q_C) \Pr(d_i > q_C, d_{n+1} < q_C) - f_{\tilde{D}^n|d_{n+1} < q_C}(Q_C) \Pr(d_{n+1} < q_C) \geq 0.
\]
With this lemma in mind, we are finally ready to show that for $n + 1$:

$$
\sum_{i=1}^{n+1} f_{D^{n+1}|d_i>q_C} (Q_C) \Pr (d_i > q_C) = f_{D^{n+1}} (Q_C)
$$

$$
\sum_{i=1}^{n+1} f_{D^{n+1}|d_i>q_C, d_{n+1} > q_C} (Q_C) \Pr (d_i > q_C, d_{n+1} > q_C)
$$

$$
+ \sum_{i=1}^{n+1} f_{D^{n+1}|d_i>q_C, d_{n+1} < q_C} (Q_C) \Pr (d_i > q_C, d_{n+1} < q_C)
$$

$$
- f_{D^{n+1}|d_{n+1} > q_C} (Q_C) \Pr (d_{n+1} > q_C) - f_{D^{n+1}|d_{n+1} < q_C} (Q_C) \Pr (d_{n+1} < q_C)
$$

$$
= \sum_{i=1}^{n} f_{D^{n}|d_i>q_C, d_{n+1} > q_C} (Q_C) \Pr (d_i > q_C, d_{n+1} > q_C)
$$

$$
+ \sum_{i=1}^{n} f_{D^{n}|d_i>q_C, d_{n+1} < q_C} (Q_C) \Pr (d_i > q_C, d_{n+1} < q_C)
$$

$$
- f_{D^{n}|d_{n+1} > q_C} (Q_C) \Pr (d_{n+1} > q_C) - f_{D^{n}|d_{n+1} < q_C} (Q_C) \Pr (d_{n+1} < q_C)
$$

$$
= \sum_{i=1}^{n} f_{D^{n+1}|d_i>q_C, d_{n+1} > q_C} (Q_C) \Pr (d_i > q_C, d_{n+1} > q_C)
$$

$$
+ \sum_{i=1}^{n} f_{D^{n}|d_i>q_C, d_{n+1} < q_C} (Q_C) \Pr (d_i > q_C, d_{n+1} < q_C)
$$

$$
- f_{D^{n}|d_{n+1} < q_C} (Q_C) \Pr (d_{n+1} < q_C)
$$

$$
\geq \sum_{i=1}^{n} f_{D^{n+1}|d_i>q_C, d_{n+1} > q_C} (Q_C) \Pr (d_i > q_C, d_{n+1} > q_C) \geq 0,
$$

where the first inequality follows from lemma above. This completes the proof. □

**Lemma 2.** Define

$$
q_0 = \{ q : \Pr (d < q) = (r - w - nt_I + (n - 1)(\delta - \tau))/(r + (n - 1)(w + \delta - \tau)) \},
$$

$$
q_\infty = \{ q : \Pr (d < q) = \delta - \tau-t_I + (w + \delta - \tau) \text{ if } \delta > \tau + t_I \},
$$

$$
Q_{\text{max}} = \{ Q : \Pr (D > Q) = \min [n(\tau + t_I - \delta)/(r + \tau - w - \delta), 1] \}.
$$

The retailers’ (symmetric) best-response to the wholesaler’s strategy (implying that all retailers simultaneously respond to a change in the stocking quantity of the wholesaler) $q_C^{Br} (Q_C)$ possesses the following properties:

1) $q_C^{Br} (0) = q_0$.

2) On the interval $Q_C \in [0, Q_{\text{max}}]$, $q_C^{Br} (Q_C)$ is defined by the optimality condition (3). It is a decreasing function with slope $-1 \leq \partial q_C^{Br} (Q_C)/ \partial Q_C \leq 0$.

3) If $Q_{\text{max}} < \infty$, then on the interval $Q_C \in (Q_{\text{max}}, \infty)$, $q_C^{Br} (Q_C) = 0$. Otherwise, $\lim_{Q_C \to \infty} q_C^{Br} (Q_C) = q_\infty$.

**Proof:** The Results 1 and 3 can be obtained by substituting $Q_C = 0$ and $Q = \infty$ into (3) and simplifying. To obtain Result 2 we note that the slope of the retailer’s best-response function can be
bounded using implicit differentiation as follows:

\[
|\partial q_C^{Br} / \partial Q_C| = \left[ (r + \tau - w - \delta) f_{D_i > q_C} (Q_C) \Pr(d_i > q_C) / n \right] / \left[ r f_{d_i} (q_C) + (r + \tau - w - \delta) (-f_{d_i} (q_C) + f_{d_i} (\bar{D} > Q_C) / n + \sum_j f_{D_i > q_C, d_j > q_C} (Q_C) \Pr(d_i > q_C, d_j > q_C) / n \right] \\
\leq \frac{f_{d_i} (\bar{D} > q_C) (Q_C) \Pr(d_i > q_C)}{\sum_j f_{D_i > q_C, d_j > q_C} (Q_C) \Pr(d_i > q_C, d_j > q_C) + f_{D_i > q_C} (Q_C) \Pr(d_i > q_C) - 1, i, j = 1, ..., n.}
\]

Note that the sign of the slope of the best-response function is negative. This completes the proof. ■

PROOF OF PROPOSITION 5: To show results in this Proposition we construct a fictitious game between one wholesaler and one retailer in which the payoffs are defined as follows (notice the \(n^2\) coefficient):

\[
\bar{\pi}_C = r E \min(d_i, q_C) + (r + \tau - w - \delta) \left( E(d_i - q_C)^+ - E(\bar{D} - Q_C)^+ \right) / n^2 - (w + t_1) q_C, \\
\bar{\Pi}_C = (w - c) n q_C + (w + \delta) E \min(\bar{D}, Q_C) - c Q_C.
\]

It is straightforward to verify that the optimality conditions for this game coincide with the optimality conditions for the symmetric equilibrium in the original game. Hence, the sets of equilibria in these two games coincide as well and any equilibrium properties that we can show for the fictitious game will carry over to the symmetric equilibrium of the original game. If we redefine \(\hat{Q}_C = -Q_C, \hat{w} = -w\) and consider first derivatives of the fictitious game,

\[
\frac{\partial \bar{\pi}_C}{\partial q_C} = r \Pr(d_i > q_C) - (r + \tau + \hat{w} - \delta) \left( \Pr(d_i > \hat{Q}_C) - \Pr(\bar{D} > -\hat{Q}_C, d_i > \hat{Q}_C) / n \right) - (\hat{w} + t_1), \\
\frac{\partial \bar{\Pi}_C}{\partial \hat{Q}_C} = -(\hat{w} + \delta) \Pr(\bar{D} > -\hat{Q}_C) + c.
\]

We can verify that the fictitious game is supermodular because its cross-partial derivatives are positive:

\[
\frac{\partial^2 \bar{\pi}_C}{\partial q_C \partial \hat{Q}_C} = (r + \tau + \hat{w} - \delta) f_{D_i > q_C} (\hat{Q}_C) \Pr(d_i > q_C) / n > 0, \\
\frac{\partial^2 \bar{\Pi}_C}{\partial \hat{Q}_C \partial q_i} = (\hat{w} + \delta) f_{D_i > q_C} (\hat{Q}_C) \Pr(d_i > q_C) / n > 0.
\]

To demonstrate the parametric monotonicity of the set of equilibrium inventories \((q_C^*(\hat{w}), \hat{Q}_C^*(\hat{w}))\) in \(\hat{w}\), we must show that the retailer’s payoff function has increasing differences in \((q_C, \hat{w})\) and the wholesaler’s payoff function has increasing differences in \((\hat{Q}_C, \hat{w})\) (see Topkis [34], Theorem 4.2.2), which in our case is equivalent to showing that second-order cross-partial derivatives are nonnegative (see Topkis [34]).
Thus, the set of equilibria \((q^*_C, \hat{Q}_C^*)\) increases in \(\hat{w}\). The rest of the proof relies on the Implicit Function Theorem (IFT). When applied to the optimality conditions of the fictitious game, the IFT can be stated as follows. At the equilibrium we have

\[
\begin{align*}
F &= \Pr(d_i > q_C) - (r + \tau - w - \delta) \left( \Pr(d_i > q_C) - \Pr(\overline{D} > Q_C; d_i > q_C) / n \right) / r - (w + t_f) / r = 0, \\
G &= \Pr(\hat{D} > Q_C) - c / (w + \delta) = 0,
\end{align*}
\]

so that

\[
\begin{pmatrix}
\partial q^*_C / \partial w \\
\partial Q^*_C / \partial w
\end{pmatrix} = \begin{vmatrix} F_q & F_Q \\
G_q & G_Q \end{vmatrix}^{-1} \begin{vmatrix} F_w \\
G_w \end{vmatrix},
\]

where subscripts denote partial derivatives. We can find each term analytically as follows:

\[
\begin{align*}
F_q &= -f_{d_i}(q_C) - \frac{(r + \tau - w - \delta)}{r} \left( -f_{d_i}(q_C) + f_{d_i, \overline{D} > Q_C}(q_C) \Pr(\overline{D} > Q_C) / n \right) \\
&\quad + \sum_j f_{D|d_i > q_C, d_j > q_C}(Q_C) \Pr(d_i > q_C, d_j > q_C) / n \right) / r \\
&\leq - \frac{(r + \tau - w - \delta)}{r} \sum_j f_{D|d_i > q_C, d_j > q_C}(Q_C) \Pr(d_i > q_C, d_j > q_C) / n < 0, \\
F_Q &= - (r + \tau - w - \delta) f_{\overline{D}|d_i > q_C}(Q_C) \Pr(d_i > q_C) / r < 0, \\
G_q &= - \sum_i f_{\overline{D}|d_i > q_C}(Q_C) \Pr(d_i > q_C) < 0, \\
G_Q &= -f_{\overline{D}}(Q_C) < 0, \\
F_w &= \Pr(d_i > q_C) / r - \Pr(\overline{D} > Q_C; d_i > q_C) / (rn) - 1 / r < 0, \\
G_w &= c / (w + \delta)^2 > 0.
\end{align*}
\]

From the IFT, we have the following equality: \(G_w \partial q^*_C / \partial w + G_Q \partial Q^*_C / \partial w = -G_w\). Using the established fact that \(G_w > 0\) and the result from the proof of Lemma 1 that, for \(n \geq 2\), \(|G_Q| < |G_q|\), we have \(|(\partial q^*_C / \partial w) / (\partial Q^*_C / \partial w)| < G_Q / G_q < 1\). This completes the proof of Result 1. To prove Result 2, we note that

\[
\begin{align*}
F_r &= - \left( \Pr(d_i > q_C) - \Pr(\overline{D} > Q_C; d_i > q_C) / n \right) / r < 0, \\
G_r &= 0.
\end{align*}
\]
so that \( (\partial q_C^*/\partial \tau)/(\partial Q_C^*/\partial \tau) = -G_Q/G_q \). Recall that \( 1/n \leq G_Q/G_q \leq 1 \) for \( n \geq 2 \) and the result follows. Result 3 is proven similarly.

**Proof of Proposition 6:** Similar to the proof of Proposition 5, we can apply the IFT to the optimality conditions of the fictitious game. It is readily verified that (consistent with the notation from the proof of Proposition 5) \( F_\delta > 0 \) and \( G_\delta > 0 \). Using the IFT similar to the proof of Proposition 5 we obtain

\[
\begin{align*}
F_q(\partial q_C^*/\partial \delta) + F_Q(\partial Q_C^*/\partial \delta) &< 0, \quad (7) \\
G_q(\partial q_C^*/\partial \delta) + G_Q(\partial Q_C^*/\partial \delta) &< 0.
\end{align*}
\]

Also, from the proof of Lemma 1 we have, for any \( n \), \( |G_q| < n |G_Q| \) and from the proof of Lemma 2 we have \( |F_q| > |F_Q| \). Using these results, we consider three possibilities:

1) \( \partial q_C^*/\partial \delta, \partial Q_C^*/\partial \delta < 0 \). This is clearly impossible since (7) is not satisfied.
2) \( \partial q_C^*/\partial \delta < 0, \partial Q_C^*/\partial \delta > 0 \). Then we have \( \left| (\partial q_C^*/\partial \delta)/(\partial Q_C^*/\partial \delta) \right| < F_Q/F_q < 1 \).
3) \( \partial q_C^*/\partial \delta > 0, \partial Q_C^*/\partial \delta < 0 \). Then we have \( \left| (\partial q_C^*/\partial \delta)/(\partial Q_C^*/\partial \delta) \right| > G_Q/G_q > 1/n \).

This completes the proof.

**Proof of Proposition 7:** We first establish that there is at most one interior equilibrium. By solving the matrix equation (6) and invoking the result of Proposition 5 we obtain:

\[
\partial q_C^*/\partial w = (-G_wF_Q + F_wG_Q)/(G_qF_Q - F_qG_Q) < 0.
\]

Note that \( -G_wF_Q + F_wG_Q > 0 \), and thus it must be that \( G_qF_Q - F_qG_Q < 0 \) or, similarly,

\[
\frac{G_q}{G_Q} \frac{F_Q}{F_q} < 1 \Rightarrow \frac{\partial q_C^*/\partial q_C}{\partial Q_C^*/\partial Q_C} < 1,
\]

so that at any interior equilibrium, \( (\partial q_C^*/\partial Q_C \times \partial Q_C^*/\partial q_C)_{(q_C^*,Q_C^*)} < 1 \). We define the inverse of \( q_C^*(Q_C) \) as \( Q_C(q_C) \). Our goal is to show that there is only one unique \( q_C^* \) such that \( Q_C(q_C) = Q_C(q_C^*) \).

If we consider an auxiliary function \( \gamma(q_C) = Q_C(q_C) - Q_C(q_C^*) \), we find that proving uniqueness of the equilibrium is then identical to showing that \( \gamma(q_C) \) crosses 0 once and only once (since we are assured of the existence of the equilibrium, we do know that there is at least one crossing). This technique is known as the index approach (see Vives [37], page 48). We then consider the derivative of \( \gamma(q_C) \) w.r.t. \( q_C \) evaluated at the equilibrium:

\[
(\partial \gamma(q_C)/\partial q_C)_{q_C^*} = \left( \partial Q_C(q_C)/\partial q_C \right)_{Q_C^*} - \left( \partial Q_C^*(q_C)/\partial q_C \right)_{Q_C^*} = \left( 1/(\partial q_C^*(Q_C)/\partial Q_C) \right)_{Q_C^*} - \left( \partial Q_C^*(q_C)/\partial q_C \right)_{q_C^*} > 0.
\]

That is, every time \( \gamma(q_C) \) crosses 0, it does so from below. Clearly, since \( \gamma(q_C) \) is a continuous function, there can be only one such crossing. Therefore, there can be at most one interior equilibrium. Further-
more, we note that, if there is a boundary equilibrium as in Figure 1, then there could not be another equilibrium because, based on Lemmas 1 and 2, the slopes of the best responses are such that, once \( Q_{\text{max}} \leq Q_0 \), the best responses will never cross. Based on the same argument, if there is a boundary equilibrium as in Figure 3, then there could not be another equilibrium.

**Proof of Proposition 8:** When the retailers’ stocking decisions are coordinated and the relaxed linear allocation rule is used, the total profit of the retail channel can be written as follows:

\[
\pi_C = r \sum_i E(\min(d_i, q_{Ci})) + (r + \tau - w - \delta) \left( \sum_i E(d_i - q_{Ci})^+ - E(\overline{D} - Q_C)^+ \right) - (w + t_I) \sum_i q_{Ci},
\]

while the wholesaler’s profit is unchanged. To this end, we consider a fictitious game parametrized by \( k \in [0, n] \), with equilibria found from the following set of optimality conditions:

\[
\frac{\partial \pi_C}{\partial q_C} = r \Pr(d_i > q_C) - (r + \tau - w - \delta) \left( \Pr(d_i > q_C) - k \Pr(\overline{D} < Q, d_i > q_C) / n \right) - (w + t_I) = 0,
\]
\[
\frac{\partial \Pi_C}{\partial Q_C} = (w + \delta) \Pr(\overline{D} > Q_C) - c = 0.
\]

We note that when \( k = 1 \), this system of equations coincides with optimality conditions for the fictitious game introduced in the proof of Proposition 5 and thus its solutions will coincide with the symmetric solutions to the problem with uncoordinated retailers. At the same time, when \( k = n \) it can be verified that the solution to this system of equations coincides with the symmetric solution when the retailers’ stocking decisions are coordinated. We now show that the set of equilibria satisfying this system of equations is monotone in \( k \), which allows us to compare stocking decisions with and without coordination at the retail level. It is readily verified that this fictitious game is supermodular in \((q_C, -Q_C), (q_C, k)\) and in \((-Q_C, k)\). Hence, the set of equilibria \((q_C, -Q_C)\) increases monotonically in \( k \). The result follows.