CHAPTER 1

Rent Dissipation in R&D Races

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Abstract

The literature on R&D races suggests that noncolluding firms invest excessively in R&D. We show that this result depends critically on the winner-take-all assumption. Although rents continue to be dissipated once the winner-take-all assumption is relaxed because firms in general fail to provide the optimal R&D effort, the mechanisms behind this rent dissipation change with the degree of patent protection. We then illustrate how the patent system can be used to elicit the optimal R&D effort.

1. Introduction

Do firms invest too little or too much in R&D? The literature on R&D races suggests that noncolluding firms invest excessively in R&D (see Reinganum, 1989, for a survey). In this paper, we show that this result depends critically on the so-called winner-take-all assumption that is typically made to facilitate the welfare comparisons (e.g., Reinganum, 1981). According to this assumption, the firm that wins the race is awarded a patent, whereas the others receive nothing. The empirical evidence, however, suggests that patent protection is far from perfect and that there are benefits to imitation (losing the R&D race) as well as innovation (winning the R&D race) (e.g., Cohen et al., 2000). This casts serious doubts on the winner-take-all assumption.

We show that once the winner-take-all assumption is relaxed, it no longer can be ascertained that noncolluding firms invest excessively in R&D. In this more realistic setting, rents continue to be dissipated because firms in general fail to provide the optimal R&D effort. However, the mechanisms behind this rent dissipation change with the degree of patent protection.

We use this insight to show how the patent system can be used to elicit the optimal R&D effort. In particular, we show that, whether firms invest too much or too little in R&D, it is always possible to replicate the planner's solution by offering the appropriate rewards to the participating firms.

The literature has long recognized that the degree of patent protection is a crucial determinant of competitive behavior (e.g., Reinganum, 1982). Beath et

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al. (1989) show that whether a firm's reaction function in an R&D race is upward or downward sloping depends on the values of the firm's reaction function at zero ("profit incentive") and at infinity ("competitive threat"). Our results complement theirs in that we provide an alternative characterization of strategic complements and substitutes in terms of the relative magnitude of the value of continued play and the benefit to imitation. Going beyond Beath et al. (1989), we also draw the comparison between noncooperative equilibrium and collusive outcome.

The remainder of this paper is organized as follows. Section 2 develops a standard R&D race model (Lee and Wilde, 1980; Reinganum, 1981; Reinganum, 1982). Section 3 characterizes the noncooperative equilibrium, Section 4 the collusive outcome. Section 5 discusses the driving forces underlying rent dissipation. The policy implications are discussed in Section 6. Section 7 concludes. All proofs are relegated to the Appendix.

2. Model

Consider an R&D race in which two identical firms are simultaneously seeking a particular innovation. Firms compete to be the first to make the discovery. The date of a successful innovation is assumed to be random and influenced by firms' R&D efforts. Time is continuous and the horizon is infinite.

Let τ_i be the random date of a successful innovation by firm i. Following the literature, we assume that the distribution of τ_i is

$$\Pr(\tau_i \leqslant t) = 1 - e^{-\lambda u_i t}, \quad \lambda \geqslant 0.$$

It follows that firm i's hazard rate of successful innovation is $h_i = \lambda u_i$.[1]

The firm which makes the innovation first is awarded a patent of positive value $\overline{P} > 0$, whereas its rival receives nothing if patent protection is assumed to be perfect. This is the winner-take-all assumption that is typically made by the existing literature. On the other hand, if patent protection is imperfect as the empirical evidence suggests (e.g., Cohen et al., 2000), the loser receives a positive payoff $\underline{P}$, where $\overline{P} > \underline{P} > 0$. $\overline{P}$ is understood to be the expected net present value of all future revenues from marketing the innovation net of any costs the firm incurs in doing so. Similarly $\underline{P}$ is the expected net present value of the all future cash flows including costs of imitation. Hence, $\overline{P}$ and $\underline{P}$ implicitly depend on the length and breadth of patent protection and the particulars of product market competition (as modeled in Denicolo, 1996, see also Klemperer, 1990; Gallini, 1992, and Matutes et al., 1996).

To simplify the notation, we focus on firm 1 in what follows. The derivations for firm 2 are analogous. Let V_1 denote the expected value of the race to firm 1.

[1] Doraszelski (2003) considers alternative distributions of success times.

It is implicitly given by

$$rV_1 = \max_{u_1 \geqslant 0} h_1(\overline{P} - V_1) + h_2(\underline{P} - V_1) - c(u_1),$$

where $r > 0$ is the interest rate and the cost incurred to exert R&D effort u_1 is $c(u_1) = \frac{1}{\eta} u_1^{\eta}$. The parameter $\eta > 1$ measures the elasticity of the cost function. The expected value V_1 can be interpreted as the asset or option value to firm 1 of participating in the race. This option is priced by requiring that the opportunity cost of holding it, rV_1, equals the current cash flow, $-c(u_1)$, plus the expected capital gain or loss flow. The latter is composed of two parts, namely the capital gain from winning the race, $\overline{P} - V_1$, times the likelihood of doing so, h_1, and the capital loss from losing the race, $\underline{P} - V_1$, times the likelihood of doing so, h_2.

Differentiating the above equation yields the FOC for an interior solution $u_1^* > 0$. Solving gives

$$u_1^* = \big(\lambda(\overline{P} - V_1)\big)^{\frac{1}{\eta - 1}}.$$

Since the objective function is strictly concave due to $\eta > 1$, the FOC is also sufficient for an interior solution.

3. Noncooperative equilibrium

Since firms are identical, we focus on symmetric Nash equilibria. Hence, $V_1 = V_2 = V$ and $u_1^* = u_2^* = u^*$. The following proposition establishes the existence of a unique symmetric Nash equilibrium.

PROPOSITION 1. *There exists a unique symmetric Nash equilibrium with* $0 < V < \overline{P}$ *characterized by*

$$0 = \lambda u^*(\overline{P} + \underline{P}) - \frac{1}{\eta}(u^*)^{\eta} - (r + 2\lambda u^*)V, \tag{1}$$

where

$$u^* = \big(\lambda(\overline{P} - V)\big)^{\frac{1}{\eta - 1}}. \tag{2}$$

There is a slight difference between our model and the one analyzed by Reinganum (1981, 1982). In her model, equilibrium strategies depend on time for two reasons. First, there is an exogenously given terminal date at which all competition ceases. Second, the costs of knowledge acquisition are discounted over time whereas the benefits accruing from innovation are not. Proposition 1 shows that if Reinganum's (1981, 1982) model is modified by discounting the benefits as well as the costs and if it is given an infinite horizon to eliminate end effects, then firms do not alter their R&D efforts over time (similar to Lee and Wilde's, 1980 model). Eliminating this somewhat artificial time dependency simplifies the characterization of the equilibrium and, in effect, enables us to obtain novel insights into the welfare properties of R&D races.

4. Collusive outcome

We study the welfare implications of our R&D race model by comparing the outcome of the noncooperative game to the collusive solution. The colluding firms strive to maximize the expected value W of making the discovery and receiving $Q = \overline{P} + \underline{P}$, which is implicitly given by

$$rW = \max_{u_1 \geqslant 0, u_2 \geqslant 0} (h_1 + h_2)(Q - W) - c(u_1) - c(u_2).$$

Carrying out the indicated maximization yields

$$u_1^{**} = u_2^{**} = u^{**} = \big(\lambda(Q - W)\big)^{\frac{1}{\eta-1}}.$$

As in the case of the noncooperative game, we establish the existence of a unique solution.

PROPOSITION 2. *There exists a unique solution with* $0 < W < Q$ *characterized by*

$$0 = 2\lambda u^{**} Q - \frac{2}{\eta}(u^{**})^{\eta} - (r + 2\lambda u^{**})W, \tag{3}$$

where

$$u^{**} = \big(\lambda(Q - W)\big)^{\frac{1}{\eta-1}}. \tag{4}$$

The collusive solution is a special case of the planner's solution in which a planning authority strives to maximize the benefits of the innovation to society. The difference is that in the planner's solution the social benefits Q need not be equal to the private benefits $\overline{P} + \underline{P}$.

It has long been argued that, from society's point of view, $Q > \overline{P} + \underline{P}$ for several reasons. First, an important part of the value of an innovation are the benefits accruing to consumers. Second, R&D generates knowledge. This knowledge is valuable to the extent that it leads to spillovers across time or firms. For example, a firm's current R&D efforts may help it in making other discoveries in the future, or they may benefit firms in other industries that are engaged in similar R&D projects. Both consumer surplus and spillovers are neglected in $\overline{P} + \underline{P}$. Indeed, based on case studies of 17 industrial innovations, Mansfield et al. (1977) estimate a median social rate of return of 56% compared to a median private rate of return of 25%.

More recent evidence suggests that patent races reflect excessive patenting from a social perspective. In particular, the building of patent fences around some core innovation and the amassing of large patent portfolios are indicative of socially wasteful investment in R&D. Patent fences may not only preclude innovations that substitute for the core innovation but also innovations that improve upon it (see Scotchmer, 1991, for evidence and Scotchmer, 1996, and Denicolo, 2000, for models along this line). Similarly, the amassing of large patent portfolios may impede entry into the industry and the spur to innovative activity that usually accompanies it (Cohen et al., 2000). This suggests that $Q < \overline{P} + \underline{P}$.

In the next section, we focus on the special case where social and private benefits are equal ($Q = \overline{P} + \underline{P}$), and study the welfare implications of our R&D race model. In Section 6 we turn to the general case ($Q \neq \overline{P} + \underline{P}$).

5. Rent dissipation

If firms behave noncooperatively, then there is in general a misallocation of resources. In particular, since the colluding firms maximize the sum of the individual payoffs and are free to replicate the noncooperative outcome, the value of the race to the colluding firms must be at least as big as the combined value of the race to firms 1 and 2, i.e., $2V \leqslant W$. The reason for this rent dissipation, however, depends critically on the degree of patent protection.

To illustrate this, we provide two numerical examples. Our first example illustrates a winner-take-all situation in which the winning firm is awarded a patent of positive value whereas the losing firm receives nothing, $\overline{P} = 0.23$ and $\underline{P} = 0$. The remaining parameter values are $\lambda = 1$, $r = 0.05$, and $\eta = 2$. This ensures that the expected duration of the race is three years. Using Propositions 1 and 2 we obtain $2V = 0.1282 < 0.1455 = W$. The reason for this rent dissipation is that $u^* = 0.1667 > 0.0853 = u^{**}$, i.e., each firm invests excessively in R&D. Since the two firms compete for the same discovery, each additional dollar invested in R&D brings a firm closer to winning the race and, at the same time, brings its rival closer to losing the race. Hence, its R&D efforts impose a *negative externality* on its rival, and the firm consequently invests excessively in R&D.

Our second example illustrates the polar case in which the loser can costlessly and immediately imitate the winner and thus both firms receive the same payoff, $\overline{P} = \underline{P} = 0.23$. There is again rent dissipation ($2V = 0.3163 < 0.3326 = W$), but the reason is now that firms invest too little in R&D: $u^* = 0.0726 < 0.1290 = u^{**}$. In contrast to a winner-take-all situation, each additional dollar invested in R&D brings both firms closer to the finish line. Hence, a firm's R&D efforts impose a *positive externality* on its rivals, which causes the firm to underinvest in R&D.

To clarify the distinction between the two scenarios, let u^* denote firm 1's equilibrium strategy and let u denote an arbitrary strategy for firm 2. Then Equations (1) and (2) can be rewritten as

$$0 = \lambda u^* \overline{P} + \lambda u \underline{P} - \frac{1}{\eta}(u^*)^\eta - (r + \lambda u^* + \lambda u)V,$$

$$0 = \lambda(\overline{P} - V) - (u^*)^{\eta-1}.$$

Total differentiation yields

$$\frac{dV}{du} = \frac{\lambda(\underline{P} - V)}{r + \lambda u^* + \lambda u},$$

$$\frac{du^*}{du} = \frac{-\lambda^2(\underline{P} - V)}{(\eta - 1)(u^*)^{\eta-2}(r + \lambda u^* + \lambda u)}.$$

Hence, $\underline{P} < V$ if and only if $\frac{dV}{du} < 0$ if and only if $\frac{du^*}{du} > 0$. That is, if $\underline{P} < V$, then reaction functions are upward sloping and firms' R&D efforts are strategic complements. On the other hand, if $\underline{P} > V$, then reaction functions are downward sloping and firms' R&D efforts are strategic substitutes. It follows that a firm's R&D efforts impose a negative externality on its rival whenever $\underline{P} < V$ and a positive externality whenever $\underline{P} > V$. In other words, depending on whether or not the benefit to imitation $\underline{P}$ is less than the value of continued play V, the character of the R&D race changes from a *preemption game* into a *waiting game*. If patent protection is perfect and thus $\underline{P} = 0$, then the R&D race always has the character of a preemption game, whereas if imitation is costless and immediate and thus $\underline{P} = \overline{P}$, then the R&D race always has the character of a waiting game. Finally, if $0 < \underline{P} < \overline{P}$, then the preemption as well as the waiting incentive is operative.

The following proposition formally shows that there is indeed overinvestment (underinvestment) in R&D if and only if a firm's R&D efforts impose a negative externality (positive externality) on its rival.

PROPOSITION 3. *Let $Q = \overline{P} + \underline{P}$. Then $u^* \gtrless u^{**}$ if and only if $V \gtrless \underline{P}$.*

It follows that $\underline{P} = 0$ ($\underline{P} = \overline{P}$) implies $u^* > u^{**}$ ($u^* < u^{**}$). Hence, a sufficient condition for overinvestment (underinvestment) is that patent protection is perfect (imitation is costless and immediate).

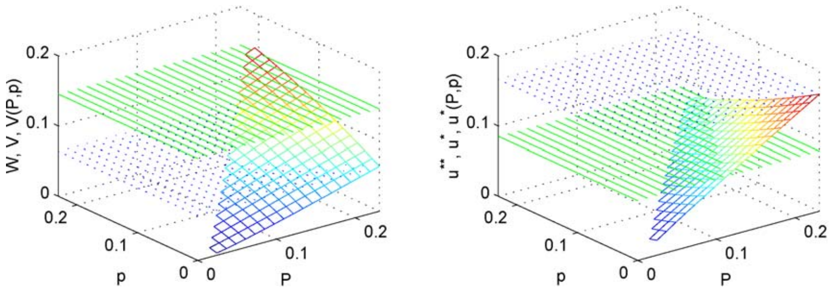
6. Policy implications

In this section, we study the policy implications of our R&D race model. Proposition 4 shows that a planning authority can always redistribute (part of) the social benefits of an innovation Q to replicate the planner's solution. That is, the planning authority can always choose rewards $\overline{P}$ and $\underline{P}$ with $\overline{P} + \underline{P} < Q$ that elicit the optimal R&D effort. Note that while we think of $\overline{P}$ and $\underline{P}$ as being implicitly determined by the patent system, choosing rewards is clearly tantamount to assigning property rights (as analyzed in Mortensen, 1982). To emphasize the dependence of the value and policy functions on the offered rewards, we write $V(\overline{P}, \underline{P})$ and $u^*(\overline{P}, \underline{P})$ in what follows.

PROPOSITION 4. *Given Q there exist $\overline{P}$ and $\underline{P}$ with $\overline{P} + \underline{P} < Q$ such that $u^*(\overline{P}, \underline{P}) = u^{**}$.*

In the remainder of this section, we provide a numerical example to show that there is in general more than one combination of $\overline{P}$ and $\underline{P}$ that leads to the optimal R&D effort. This enables the social planner to influence the firms' valuation of participating in the R&D race. We set $Q = 0.23$ and, for purposes

Figure 1. Replicating the planner's solution.



W and u^{**} refer to the planner's solution (dashed line), V and u^* to the noncooperative game in the absence of a social planner (dotted line). $V(P, p)$ and $u^*(P, p)$ are as defined in the text with $P = \overline{P}$ and $p = \underline{P}$ (solid line).

of comparison, $\overline{P} = Q$ and $\underline{P} = 0$. By Proposition 3 this leads to overinvestment in R&D. As the right panel of Figure 1 shows we can achieve $u^*(\overline{P}, \underline{P}) = u^{**}$ for any $\overline{P}$ and $\underline{P}$ such that $\overline{P} = 0.1122 + 0.6304\underline{P}$. As the left panel shows $V(\overline{P}, \underline{P})$ ranges from 0.0269 at (0.1122, 0) over 0.0727 at (0.1580, 0.0727) to 0.1455 at (0.23, 0.1881). Consequently, the social planner is free to choose the rewards to make firms either worse or better off than in the absence of a planning authority ($V = 0.0641$).

7. *Conclusions*

The literature on R&D races suggests that noncolluding firms invest excessively in R&D. We show that this result depends critically on the winner-take-all assumption. Once the winner-take-all assumption is relaxed, it no longer can be ascertained that noncolluding firms always invest excessively in R&D. On the contrary, firms sometimes invest too little in R&D. We show that although rents continue to be dissipated because firms in general fail to provide the optimal R&D effort, the mechanisms behind this rent dissipation change with the degree of patent protection: As patent protection becomes less effective, the character of the R&D race changes from a preemption game with overinvestment into a waiting game with underinvestment in R&D.

Our results allow us to illustrate how the patent system can be used to elicit the optimal R&D effort. Starting from perfect patent protection in which the winner-take-all assumption is warranted, the misallocation of resources in the noncooperative game can be reduced by reducing the asymmetry in the rewards to winning and losing the R&D race. One way to accomplish this is to partially insure the participating firms against losing the R&D race, e.g., by making patent protection less than perfect. Another way is to "throw money" at all participating firms. In either case reducing the asymmetry in the rewards reduces the negative externality stemming from firms' R&D efforts. This in turn moves the R&D race away from a preemption game and overinvestment towards a waiting game and underinvestment in R&D.

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Appendix

PROOF OF PROPOSITION 1. Equations (1) and (2) follow from symmetry. Substitute Equation (2) into (1) to define

$$\Delta(V) = \big(\lambda(\overline{P} - V)\big)^{\frac{\eta}{\eta-1}}\left(1 - \frac{1}{\eta}\right) + \lambda\big(\lambda(\overline{P} - V)\big)^{\frac{1}{\eta-1}}(\underline{P} - V) - rV.$$

At $V = \overline{P}$, we have

$$\Delta(\overline{P}) = -r\overline{P} < 0.$$

At $V = 0$, we have

$$\Delta(0) = (\lambda\overline{P})^{\frac{\eta}{\eta-1}}\left(1 - \frac{1}{\eta}\right) + \lambda(\lambda\overline{P})^{\frac{1}{\eta-1}}\underline{P} > 0$$

since $\eta > 1$. Since $\Delta(V)$ is continuous in V, there exists a solution to $\Delta(V) = 0$ by the intermediate value theorem.

It remains to establish uniqueness of the solution. We have

$$\Delta'(V) = -2\lambda\big(\lambda(\overline{P} - V)\big)^{\frac{1}{\eta-1}} - \frac{\lambda^2}{\eta - 1}\big(\lambda(\overline{P} - V)\big)^{\frac{2-\eta}{\eta-1}}(\underline{P} - V) - r,$$

$$\Delta''(V) = \frac{3\lambda^2}{\eta - 1}\big(\lambda(\overline{P} - V)\big)^{\frac{2-\eta}{\eta-1}} + \frac{\lambda^3(2 - \eta)}{(\eta - 1)^2}\big(\lambda(\overline{P} - V)\big)^{\frac{3-2\eta}{\eta-1}}(\underline{P} - V).$$

Rearranging yields

$$\Delta''(V) = \frac{\lambda^2}{\eta - 1}\big(\lambda(\overline{P} - V)\big)^{\frac{2-\eta}{\eta-1}}\left(3 + \frac{2 - \eta}{\eta - 1}\frac{\underline{P} - V}{\overline{P} - V}\right).$$

Note that the term in parenthesis governs the sign of $\Delta''(V)$. Differentiating it yields

$$-\frac{2 - \eta}{\eta - 1}\frac{\overline{P} - \underline{P}}{(\overline{P} - V)^2},$$

which is nonnegative if $\eta \geqslant 2$ and nonpositive if $1 < \eta < 2$. Consider the case of $\eta \geqslant 2$ first. Then the term in parenthesis is nondecreasing in V and achieves its minimum of

$$3 + \frac{2-\eta}{\eta-1}\frac{\underline{P}}{\overline{P}} \geqslant 2$$

at $V = 0$, where the last inequality uses the facts that $0 \leqslant \frac{\underline{P}}{\overline{P}} \leqslant 1$ and $-1 \leqslant \frac{2-\eta}{\eta-1} \leqslant 0$ whenever $\eta \geqslant 2$. Hence, $\Delta''(V) \geqslant 0$ and the claim follows. Consider the case of $1 < \eta < 2$ next. Then the term in parenthesis is nonincreasing in V, achieves its maximum of

$$3 + \frac{2-\eta}{\eta-1}\frac{\underline{P}}{\overline{P}} \geqslant 3$$

at $V = 0$, and approaches $-\infty$ as V approaches $\overline{P}$. By continuity it follows that $\Delta''(V) \geqslant 0$ around $V = 0$ and $\Delta''(V) \leqslant 0$ around $V = \overline{P}$. Since the term in parenthesis changes sign at most once, so does $\Delta''(V)$, and the claim follows. □

PROOF OF PROPOSITION 2. Equations (3) and (4) follow from symmetry. Define

$$\Delta(W) = 2\big(\lambda(Q-W)\big)^{\frac{\eta}{\eta-1}}\left(1-\frac{1}{\eta}\right) - rW.$$

At $W = Q$, we have

$$\Delta(Q) = -rQ < 0.$$

At $W = 0$, we have

$$\Delta(0) = 2(\lambda Q)^{\frac{\eta}{\eta-1}}\left(1-\frac{1}{\eta}\right) > 0$$

since $\eta > 1$. Since $\Delta(W)$ is continuous in W, there exists a solution to $\Delta(W) = 0$ by the intermediate value theorem. Uniqueness of the solution follows from noting that $\Delta(W)$ is decreasing in W. □

PROOF OF PROPOSITION 3. We have $u^* = (\lambda(\overline{P}-V))^{\frac{1}{\eta-1}} \gtrless (\lambda(\overline{P}+\underline{P}-W))^{\frac{1}{\eta-1}} = u^{**}$ if and only if $W \gtrless V + \underline{P}$. From the proofs of Propositions 1 and 2, we know that the solution to the noncooperative game and to the planner's problem are characterized by the zeros of

$$\Delta^V(V) = \big(\lambda(\overline{P}-V)\big)^{\frac{\eta}{\eta-1}}\left(1-\frac{1}{\eta}\right) + \lambda\big(\lambda(\overline{P}-V)\big)^{\frac{1}{\eta-1}}(\underline{P}-V) - rV$$

and

$$\Delta^W(W) = 2\big(\lambda\big((\overline{P}+\underline{P})-W\big)\big)^{\frac{\eta}{\eta-1}}\left(1-\frac{1}{\eta}\right) - rW,$$

respectively. Let V solve $\Delta^V(V) = 0$ and consider $W = V + \underline{P}$ as a candidate solution to $\Delta^W(W) = 0$. Rewriting yields

$$\Delta^W(V + \underline{P}) = 2\Delta^V(V) + \left(r + 2\lambda\left(\lambda(\overline{P} - V)\right)^{\frac{1}{\eta-1}}\right)(V - \underline{P}).$$

Since $\Delta^V(V) = 0$, $\Delta^W(V + \underline{P}) \gtreqless 0$ if and only if $V \gtreqless \underline{P}$. Since $\Delta^W(W)$ is decreasing, this implies that the actual solution to $\Delta^W(W) = 0$ satisfies $W \gtreqless V + \underline{P}$ if and only if $V \gtreqless \underline{P}$. □

Proof of Proposition 4. Set $\underline{P} = 0$. Hence, $0 \leqslant \overline{P} \leqslant Q$. We have $V(0,0) = 0$ and $u^*(0,0) = 0 \leqslant u^{**}$. Since the value of the race to the planner exceeds the combined value of the race to firms 1 and 2 whenever $Q = \overline{P} + \underline{P}$, we have $2V(Q,0) \leqslant W$ which, in conjunction with $0 < V(Q,0)$, gives $V(Q,0) < W$. We therefore have $Q - V(Q,0) > Q - W$ and $u^*(Q,0) = (\lambda(Q - V(Q,0)))^{\frac{1}{\eta-1}} > (\lambda(Q - W))^{\frac{1}{\eta-1}} = u^{**}$. Since $u^*(\overline{P},0)$ is continuous in $\overline{P}$, there exists a solution to $u^*(\overline{P},0) = u^{**}$ by the intermediate value theorem. □

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