Capacity Allocation Using Past Sales: When to Turn-and-Earn

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Consider a supplier selling to multiple retailers. Demand varies across periods, but the supplier's capacity and wholesale price are fixed. If demand is high, the retailers' needs exceed capacity, and the supplier must implement an allocation mechanism to dole out production. We examine how the choice of mechanism impacts retailer actions and supply chain performance. In particular, we analyze turn-and-earn allocation, a method commonly used in the automobile industry. This scheme bases current allocations on past sales and thus enables retailers to influence their future allocations; they compete for scarce capacity even if they do not compete for customers.

We show that turn-and-earn induces the retailers to increase their sales when demand is low and the supplier's capacity is otherwise underutilized. Supplier profits thus increase. The impact on the supply chain depends on how restrictive capacity is. With mildly tight capacity, the retailers' higher sales rate does not significantly lower their profits but does reduce the cost of idle capacity. Supply chain performance improves. With extremely tight capacity, the retailers' intense competition dissipates more profits than the supplier gains, and supply chain performance suffers. Consequently, turn-and-earn does not generally coordinate the system. It is best characterized as a means for the supplier to increase her profits at the expense of the retailers and potentially even the supply chain. Furthermore, these results hold even if the retailers can hold inventory in anticipation of scarce capacity.
(*Capacity Allocation*; *Bullwhip Effect*; *Supply Chain Management*; *Game Theory*)

1. Introduction

Consider a supply chain in which one supplier sells to multiple retailers and suppose that the sum of retailer orders exceeds the supplier's fixed capacity. To balance supply and demand, the supplier must employ an allocation mechanism, an algorithm for converting an infeasible set of orders into a feasible set of capacity assignments. We examine how the choice of allocation scheme affects the performance of the supply chain and the profits of individual participants.

Doling out capacity through allocation mechanisms has occurred in markets ranging from personal computers (Zarley and Damore 1996) to pharmaceuticals (Hwang and Valeriano 1992). We, however, focus on an industry in which the issue is particularly common: automobiles. Consider the case of the GMC Suburban. With a sticker price around $38,000, the gross profit to General Motors per Suburban sold is approximately $9,000, yet GM, despite multiple capacity expansions, has been unable to produce enough to meet demand. Shortages of Suburbans are so severe that customers must often wait months for their dealer to be allocated one that matches their preferences (Blumenstein 1996). There are many other examples of vehicles in short supply: BMW could not make enough Z3s after the roadster was hyped in the James Bond movie "Goldeneye" (Neil 1997); customers are forced to wait up to half a year to receive a Harley-Davidson motorcycle

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(Gottwald 1997); and Ford, until recently, prevented rural dealers from receiving any Lincoln Navigators (Harris 1997).

Automobile dealerships are quite concerned with the allocation process. A recent survey found that availability was the most important factor explaining why one dealer lost business to another (see Stalk et al. 1996). Indeed, some dealers believe that their businesses failed because they were not allocated a sufficient number of hot-selling vehicles (Sawyers 1995). Many feel that the allocation process favors urban dealerships at the expense of rural stores or established dealers over new ones (Lawrence 1996). Dealers have even resorted to illegal means to obtain better allocations (Jenkins 1996). Car makers have used the allocation process to discipline dealers; Porsche threatened to withhold vehicles from dealers that sold its new Boxster model above its list price (Healey 1997).

Most automobile companies use a system called *turn-and-earn* to allocate vehicles. (See Lawrence (1996) for a summary of the allocation systems used in the U.S. market.) In its simplest form, turn-and-earn means that a dealer earns the allocation of one vehicle next period by selling (i.e., turning) one vehicle this period. In practice, turn-and-earn implementations vary considerably, but they all share the common feature of linking a retailer's current allocation to his previous sales rate. Implementations differ in the degree to which they adjust allocations for special circumstances. For example, favorable allocations may be given to new dealerships to help them establish their businesses or to dealers running special promotions such as anniversary sales. Similarly, the allocations of those who experience poor sales for reasons beyond their control (e.g., floods and blizzards) are generally protected.

Although there is considerable interest in allocation rules among practitioners, there is essentially no academic research that relates directly to the topic. Cachon and Lariviere (1996) and Mallik and Harker (1997) study capacity allocation among retailers in a single-period model, so past sales cannot play a role in the allocation process. Lee et al. (1997) suggest that allocations based on past sales can reduce demand variability within a supply chain, but they neither test their conjecture in a model nor consider possible adverse consequences. Cachon (1997) studies inventory management in a two-echelon supply chain with multiple retailers and stochastic demand, but considers only first-come-first-serve allocation. There are numerous papers in the economics literature that study capacity allocation by price discrimination (e.g., Dewan and Mendelson 1990, Barro and Romer 1987, and Harris and Raviv 1981). In our setting, the supplier holds her wholesale price constant even when capacity is scarce, which is frequently observed in the automobile industry. There are several papers that consider the allocation of inventory among retailers in both competitive and cooperative settings, but all assume infinite supplier capacity (e.g., Anupindi and Bassok 1998; Deneckere et al. 1996; Hartman and Dror 1996; Lippman and McCardle 1997).

As far as we are aware, this is the first paper to model the impact of capacity allocation using past sales. Our model captures the primary issues associated with the allocation of scarce capacity. A single supplier's capacity and wholesale price are fixed throughout the game, but demand can vary from period to period. In some states of the world, capacity is sufficient to support the retailers' desired sales level, but the supplier is left with spare capacity and will wish that the retailers would sell more. In other states, the retailers' desired sales quantity exceeds capacity, so at least one retailer's sales must be below the level that would maximize his profits. We consider two alternative allocation procedures for this setting. Under fixed allocation, the supplier ignores past sales and each retailer may receive a fixed proportion of capacity. Turn-and-earn allocation guarantees a retailer with higher past sales a more favorable allocation.

Under fixed allocation each retailer orders his desired sales quantity in each period. When capacity binds, the retailer might receive only a portion of his order, but increasing the order will not increase his allocation. When capacity does not bind, selling more than the optimal quantity is pointless since it confers no advantage in future allocations. However, under turn-and-earn, a retailer has greater control over his own destiny. Selling more than would maximize current period profits could provide a better allocation in the future, when tight capacity might make a larger allocation quite valuable. By linking future allocations to current sales, turn-and-earn puts retailers in competition for scarce capacity even though they may not compete for consumers. A GMC dealer in Florida does not compete for customers with a dealer in Texas, but the former may receive an additional Suburban only if the latter does not. Thus, for a given wholesale price and capacity level, turn-and-earn allocation induces the retailers to sell at least as much as they would under fixed allocation, and in some cases they sell strictly more.

The retailers' sales increase surely raises the supplier's profits because her capacity is more fully utilized. Unfortunately for the retailers, we show that their profits decline. Retailers sell more under turn-and-earn, but we find that, in equilibrium, no one actually gains an advantage. The impact on the supply chain is ambiguous. The supplier's gain may or may not exceed the retailers' loss. In general, turn-and-earn likely benefits the supply chain when capacity is moderately constraining but harms the supply chain when capacity is very tight. Even if turn-and-earn improves system performance, it does not achieve the maximum profits for the supply chain. We conclude that turn-and-earn allocation is a scheme for the supplier to increase her profits, often at the expense of the retailers and even of overall supply chain performance. Furthermore, our conclusions remain essentially unchanged if the retailers are able to hold inventory in anticipation of possible future capacity shortages.

The next section details the model we study. Section 3 analyzes the game assuming the retailers cannot hold inventory from one period to the next, while §4 introduces the inventory option for the retailers. Section 5 discusses the results, highlights the critical assumptions of the model, and suggests future research opportunities. The last section summarizes our conclusions.

2. The Model

We consider a game with two periods and three players. There is one upstream player, the supplier, and two downstream players, the retailers. We assume the retailers are local monopolists, so one retailer's sale quantity and price have no effect on the other's market and profits. A retailer's focus is on winning a larger allocation of capacity, and not on snatching customers from other retailers. Such an assumption is reasonable when the retailers are geographically distant from each other (e.g., Florida and Texas). As we discuss in §5, further research is needed to explore the interaction between competition for better allocations and competition for customers. Our objective with this model is to derive insights by limiting the discussion to competition for better allocations.

Within a period, there are two possible demand states, "high" and "low." In the high-demand state a retailer can sell q units at price p, where

$$p = 1 - q,$$

whereas, in the low-demand state,

$$p = \alpha - q,$$

for some $0 \le \alpha < 1$. The demand state applies to both retailers' markets. It is best thought of as an indicator of overall economic conditions. For simplicity, we assume that the demand state in the first period is low.[1] In the second period, the demand state is high with probability ϕ. Let σ denote the second-period demand state, $\sigma \in \{h, l\}$, and for notational convenience define

$$z(\sigma) = \begin{cases} 1 & \sigma = h \\ \alpha & \sigma = l. \end{cases}$$

The assumption of only two-demand states is clearly a simplification of reality, but it is sufficient to express the supplier's basic problem: It is impossible to always match fixed capacity to variable demand. Further, an expanded state space introduces significant computational complexity. (See Van Miegham and Dada (1997) for a model of a single retailer facing linear demand

[1] Allowing high demand in the first period is not interesting. As we will see, capacity binds under high demand, which would eliminate the possibility of anyone gaining a first-period sales advantage and thus make the allocation scheme moot.

with the intercept determined by a continuous random variable.)

At the start of the game, the supplier chooses her capacity K. The supplier can produce up to K units each period. The supplier incurs a one-time cost c per unit of capacity built, but the subsequent marginal cost of production for any level less than K is normalized to zero. The supplier cannot adjust her capacity decision at any time during the game, a reasonable assumption for an industry such as the automobile industry, in which manufacturing is capital intensive. The supplier also specifies her allocation rule (i.e., how she will allocate capacity when total retailer orders exceed available supply) and her wholesale price w. We assume that under any allocation scheme she can never allocate to a retailer more than he has ordered and that the retailer must pay for all that he has been allocated. The supplier also cannot change her allocation rule or her wholesale price over the horizon. While price may be easier to adjust than capacity, it is nevertheless observed in many markets that firms are reluctant to adjust prices frequently, even for hot-selling products. (See Fishman (1992) for a discussion on the issue of price stickiness.)

At the start of each period everyone observes the prevailing demand state, making the state of the economy common knowledge. The retailers then simultaneously submit their orders. If the sum of retailer orders is less than capacity, the supplier fills all orders. If total orders exceed capacity, the supplier uses all of her capacity and allocates production using the announced procedure. After allocations are made, the retailers choose (1) how much to sell, (2) how much to hold in inventory for the next period, and (3) how much to dispose. There is a charge of $h \geq 0$ per unit of inventory held each period. Without loss of generality, all disposal costs or salvage values are set to zero. Since the game ends after the second period, no inventory is held in that period. The supplier cannot hold inventory.

Let $x_{\tau i}$ be retailer i's period τ order quantity and let q_i be his first-period sales quantity. The retailer with higher first-period sales is the *sales leader* while the other is the *sales laggard*. All players maximize their profits given their expectations of the other players' actions. All rules and parameters are common knowledge, there is no discounting, and all parties are risk neutral. Throughout, we focus on pure strategy Nash equilibria.

2.1. Allocation Mechanisms

Before detailing the two allocation mechanisms considered, it is useful to present two definitions that apply to both of them. Define a retailer's *guaranteed allocation* to be the amount of capacity that he is assured to receive if he wants it. Define a retailer's *maximum allocation* to be the retailer's largest possible allocation given the other retailer's order.

Now we describe the allocation mechanisms. In the first period there is no sales history, so the supplier implements *fixed allocation*. With this scheme, each retailer's guaranteed allocation equals a fixed proportion of capacity. Hence, a retailer can receive more than this fixed proportion only if the other retailer orders less than his portion. Since retailers are identical in this model, it is natural to assume that the supplier divides capacity equally between the two retailers, i.e., each retailer's guaranteed allocation equals $K/2$. (If retailers were not identical, the proportions could be adjusted to reflect their actual sizes. The subsequent analysis would not change qualitatively.) Formally, under fixed allocation in period τ, retailer i is allocated

$$\min\{x_{\tau i}, \tfrac{1}{2}K + (\tfrac{1}{2}K - x_{\tau j})^+\}. \tag{1}$$

Note that fixed allocation can also be applied in the second period. The second term in (1) is the retailer's maximum allocation.

Since sales histories are available in the second period, the supplier could implement *turn-and-earn allocation* in that period. Under turn-and-earn allocation the supplier reserves some second-period capacity for the first-period sales leader. Specifically, the supplier reserves

$$\psi = |q_i - q_j|.$$

The sales leader's guaranteed allocation equals his reserved capacity plus half of the unreserved capacity, $\psi + (K - \psi)/2$. The sales laggard's guaranteed allocation equals half of the unreserved capacity, $(K$

$-\ \psi)/2$. Therefore, under turn-and-earn, retailer i's second-period allocation, assuming retailer i is the sales leader, is

$$\min\{x_{2i},\ \psi + \tfrac{1}{2}(K-\psi) + (\tfrac{1}{2}(K-\psi) - x_{2j})^{+}\}, \quad (2)$$

and retailer j's allocation is

$$\min\{x_{2j},\ \tfrac{1}{2}(K-\psi) + (\psi + \tfrac{1}{2}(K-\psi) - x_{2i})^{+}\}. \quad (3)$$

In the above examples, the second term in each minimization is the retailer's maximum allocation. This rule captures the most important characteristic of turn-and-earn systems used in practice: Retailers with higher current sales receive more favorable future allocations. (One could define turn-and-earn such that the retailers' second-period guaranteed allocations equal their first-period sales plus an equal division of capacity that went unused in the first period. This definition yields the same results as our current one.)

3. Game Analysis Without Inventory

We now consider the game detailed in §2 but assume that holding costs are sufficiently high that carrying inventory is not an option for the retailers. When time horizons are sufficiently long, this is a reasonable assumption. For example, Harley-Davidson allocates stock once for each model year, so motorcycles not sold in one allocation period cannot be carried over to the next and offered as an equivalent product (Gottwald 1997). Ignoring inventory simplifies the analysis and allows us to highlight some qualitative results. In the next section, we confirm that these results still hold when retailers can hold inventory.

The analysis proceeds in reverse chronological order (beginning with the second period and ending with the supplier's decisions) because a player's current actions should be based on correct expectations of the game's evolution.

3.1. Second-Period Analysis

In the final period, retailers are only interested in maximizing current-period profits regardless of the allocation rule. Hence a retailer will never place an order that could result in his receiving more than his desired second-period sales quantity. Let $\pi_\sigma(q)$ be a retailer's single-period profits when σ is the demand state and the retailer sells q units,

$$\pi_\sigma(q) = (z(\sigma) - q - w)q.$$

Define q_σ as the sales quantity that maximizes $\pi_\sigma(q)$. It is straightforward to show that

$$q_\sigma = (z(\sigma) - w)^{+}/2.$$

Note that the product may not be viable in the low-demand state, i.e., $q_l = 0$ is possible.

Lemma 1. *Independent of the supplier's capacity or allocation procedure, each retailer will order q_σ in the second period.*

Proof. Let a be retailer i's maximum allocation. If $a > q_\sigma$, q_σ is the optimal order. If $a \le q_\sigma$, strict concavity of $\pi_\sigma(q)$ means any order a or larger, including q_σ, is optimal. □

3.2. First-Period Analysis

In the first period, the retailers choose their order quantities (x_{1i} and x_{1j}), receive their allocations, and then determine how much of their allocations to sell (q_i and q_j). A retailer will sell his entire allocation. If not, he could simply reduce his order and thus his allocation and payments to the manufacturer. Further, the retailers can correctly anticipate their allocation for any given pair of orders. It is therefore always possible to determine a pair of orders $\{x_{1i}, x_{1j}\}$ that leads to the sales $\{q_i, q_j\}$. Hence, we shall analyze the game as if the retailers choose their first-period sales quantities, q_i and q_j.

3.2.1. The Retailers' Problem Under Fixed Allocation. Under fixed allocation future allotments are independent of current sales. Consequently, the retailers choose first-period sales quantities solely to maximize current profits.

Lemma 2. *Under fixed allocation, the unique first-period Nash equilibrium has each retailer selling* $\min\{q_l, K/2\}$.

Proof. Each retailer's effective maximum allocation is $K/2$: If retailer i wants more than $K/2$ then retailer j must want more as well, keeping i from getting more than $K/2$. Since $\pi_l(q)$ is strictly concave, the sales quantity $\min\{q_l, K/2\}$ maximizes first-period profits. □

3.2.2. The Retailers' Problem Under Turn-And-Earn Allocation. There are two cases in which turn-and-earn produces the same equilibrium sales quantities as fixed allocation.

Lemma 3. *Assume turn-and-earn allocation. If* $K \leq 2q_l$ *or* $K \geq 2q_h$*, the unique first-period Nash equilibrium has each retailer selling* $\min\{q_l, K/2\}$.

Proof. Under turn-and-earn, a retailer considers how his first-period sales influence his later allocation. When $K \geq 2q_h$, each knows he can receive q_h in the last period because the sales leader will never order more than q_h. With no value for reserved capacity, retailers maximize first-period profits by selling q_l. When $K/2 \leq q_l$, each retailer wants to sell more than his guaranteed first-period allocation even under fixed allocation. Hence, neither will be able to sell more than $K/2$. □

It remains to consider the retailers' sales quantities when $2q_l < K < 2q_h$. Here, capacity is sufficient to satisfy the retailers when second-period demand is low but insufficient to satisfy both retailers when demand is high. Reserved capacity is consequently advantageous in the high-demand state. Define $\pi_2(a)$ as a retailer's expected second-period profits when a is his expected maximum allocation in the second period,

$$\pi_2(a) = (1-\phi)\tfrac{1}{4}((\alpha - w)^+)^2 + \phi(1 - \min\{a, q_h\} - w)\min\{a, q_h\}.$$

The first term represents profits when second-period demand is low, allowing each retailer to receive his desired allocation q_l. When demand is high, a retailer sells $\min\{a, q_h\}$ since he will order only up to his desired sales quantity, q_h.

Define $a_t(q_i, q_j)$ as retailer i's expected maximum second-period allocation when turn-and-earn is implemented. From (2), the sales leader's expected maximum allocation, assuming the sales laggard orders at least his guaranteed allocation, is

$$\tfrac{1}{2}(K + \psi). \tag{4}$$

The sales leader can be allocated more than (4) only if the sales laggard ordered less than his guaranteed allocation. Note that when capacity binds, the sales laggard will never be allocated more than the sales leader. Hence, if the leader wants more than his maximum allocation, the laggard, facing an identical problem, will want more than his guaranteed allocation. Consequently, (4) is effectively the sales leader's maximum allocation; the sales leader can obtain more than $(K+\psi)/2$ only when he does not want more than $(K+\psi)/2$.

The sales leader will not sell his maximum allocation when $(K+\psi)/2 \geq q_h$ or, equivalently, when $\psi > \bar{\psi}$, where

$$\bar{\psi} = 1 - w - K.$$

Here, the leader can order and receive his desired allocation, q_h, leaving $K - q_h$ for the sales laggard. Therefore, the sales laggard's expected maximum allocation is $(K - \min\{\psi, \bar{\psi}\})/2$. To summarize,

$$a_t(q_i, q_j) = \begin{cases} \tfrac{1}{2}(K + q_i - q_j) & q_i \geq q_j \\ \tfrac{1}{2}(K - \min\{q_j - q_i, \bar{\psi}\}) & q_i < q_j \end{cases}.$$

Retailer i's expected second-period profits are thus $\pi_2(a_t(q_i, q_j))$. Define $\pi_r(q_i, q_j)$ as a retailer's expected total profits across both periods,

$$\pi_r(q_i, q_j) = \pi_l(q_i) + \pi_2(a_t(q_i, q_j)).$$

We now characterize the retailers' first-period actions for intermediate capacity values.

Theorem 4. *Assume* $2q_l < K < 2q_h$ *and turn-and-earn allocation. In the unique Nash equilibrium in sales quantities, each retailer sells* $\min\{x^*, K/2\}$ *in the first period, where* $x^* = \xi(K)^+$ *and*

$$\xi(K) = \tfrac{1}{2}(\alpha - w) + \frac{\phi}{4}(1 - w - K).$$

Proof. A retailer's marginal profit is

$$\frac{\partial \pi_r(q_i, q_j)}{\partial q_i} = \pi_l'(q_i) + \begin{cases} \dfrac{\phi}{2}(1 - w - (K + q_i - q_j)) & \psi < \bar{\psi} \\ 0 & \bar{\psi} \leq \psi \end{cases}.$$

Note that a retailer's profits are strictly concave in his sales quantity, holding the other retailer's sales quantity fixed. We first show that $\{x^*, x^*\}$ is a Nash equilibrium when capacity is not restrictive, i.e., $x^* \le K/2$. Since each sells the same quantity, $\psi = 0$. By construction, each retailer's first-order condition is satisfied. No asymmetric pair of sales quantities satisfies both retailers' first-order condition, so there are no asymmetric equilibria. Now suppose $x^* > K/2$. To confirm $\{K/2, K/2\}$ is an equilibrium, note that concavity implies

$$\frac{\partial \pi_r(K/2, K/2)}{\partial q_i} > 0.$$

Each retailer would like to sell more and neither wishes to sell less. □

3.3. The Supplier's Problem: Choosing the Allocation Rule

Combining Lemmas 2 and 3 indicates that the retailer decisions are unaffected by the allocation rule when capacity is either very tight ($K/2 \le q_l$), or very loose ($K/2 \ge q_h$). With intermediate capacities ($q_l < K/2 < q_h$), however, the supplier earns more with turn-and-earn allocation than she does with fixed allocation. This is our main result.

Theorem 5. *For any w and K, the supplier's profits under turn-and-earn are never less than under fixed allocation and are sometimes strictly greater.*

Proof. Holding w and K fixed, the supplier maximizes profits by maximizing expected orders from the retailers. In the final period, sales to the retailers are independent of the allocation rule. In the first period, under either allocation rule, the supplier sells K when $K \le 2q_l$ and $2q_l$ when $K \ge 2q_h$. Now assume that $2q_l < K < 2q_h$. Under fixed allocation the supplier sells $2q_l$, but under turn-and-earn sales are $\min\{K, 2x^*\}$. Noting that $2x^* \ge 2q_l$ completes the proof. □

Even though the retailers do not compete for customers, they do compete for better allocations when capacity might bind. When capacity is restrictive, the marginal value of obtaining a greater allocation in the second period is positive,

$$\pi_h'(q \mid q < q_h) > 0,$$

while the marginal cost of increasing first-period sales can be small. In fact, since profits are strictly concave, the marginal cost of increasing first-period sales above q_l is initially zero,

$$\pi_l'(q_l) = 0.$$

Hence, when there is a chance that final-period capacity might be restrictive, each retailer is *always* willing to sell more than his desired quantity in the initial period.

While the retailers' initial cost of increasing first-period sales is negligible, the supplier's marginal benefit of having each retailer raise his first-period order is w, which is positive. The boost in the retailers' first-period sales must then initially increase total supply chain profits. This likely provides little solace to the retailers.

Theorem 6. *For a given w and K, the retailers' profits are never higher under turn-and-earn than under fixed allocation and are sometimes strictly lower.*

Proof. Since in equilibrium neither retailer gains a sales advantage, the retailers' profits in the second period are the same under either allocation rule. In the first period, the retailers' profits are identical when capacity is either quite tight, $K \le 2q_l$, or quite ample, $K \ge 2q_h$. For intermediate levels, the retailers sell more than q_l under turn-and-earn but sell q_l under fixed allocation. Hence, they earn strictly less under turn-and-earn in these cases. □

This outcome indeed appears grim for the retailers. However, the stipulation of a given w and K is actually critical since the supplier's choice of capacity and wholesale price may well depend on the allocation rule.

3.4. The Supplier's Problem: Choosing the Wholesale Price and Capacity

The focus of this research is on the impact of the supplier's allocation rule on the performance of the supply chain. It is nevertheless interesting to explore the relationship between the supplier's allocation rule and her choice for w and K. We show that under either allocation mechanism the supplier can restrict her attention to a limited number of capacity choices for a given wholesale price.

Lemma 7. *Assume w is fixed. Under fixed allocation,* (*at least*) *one of the following three capacity choices maximizes the supplier's profits*: $\{0, 2q_l, 2q_h\}$. *Under turn-and-earn allocation*, (*at least*) *one of the following four choices maximizes the supplier's profits*: $\{0, 2q_l, k(w), 2q_h\}$, *where*

$$k(w) = \left(\frac{2\alpha + \phi}{2 + \phi} - w\right)^{+}.$$

Proof. See Appendix A.

According to Lemma 7, fixed allocation leads to one of three *capacity strategies*: build no capacity; build enough to cover only low-demand orders; build just enough to cover high-demand orders. The same three strategies are also viable under turn-and-earn allocation and yield identical profits to all firms as under fixed allocation (see §3.3). In addition, turn-and-earn admits a fourth strategy. When $K = k(w)$, the sum of the retailers' first-period desired sales quantities exactly equals the available capacity, so there is no idle capacity, i.e., $k(w) = 2x^*$. In the second period, $k(w)$ is sold only if demand is high, otherwise $2q_l$ is sold and some capacity is left idle.

For a given capacity it is possible to find the optimal wholesale price (details are in Appendix B). A search over each possible capacity strategy yields the best supplier choice.

3.5. A Numerical Study

Numerical examples illustrate several qualitative results regarding the supplier's choice of capacity and price. Table 1 displays 36 scenarios which represent all combinations of the following parameter values: $\phi \in \{0.15, 0.50, 0.85\}$; $c \in \{0.1, 0.4, 0.7\}$; and $\alpha \in \{0.2, 0.4, 0.6, 0.8\}$. The table gives the supplier's w and K choice under either allocation rule. Since three capacity strategies are viable for both allocation rules, for some scenarios the choices are identical. Table 2 facilitates the comparison of the two allocation mechanisms and provides data on integrated supply chain profits, i.e., the maximum profits that could be achieved if the system were run as one firm. (Appendix C provides details for evaluating a retailer's expected profits. Appendix D evaluates the integrated supply chain profits.) Several observations follow from these tables.

Turn-and-Earn Allocation Can Have a Substantial Impact on Supplier Profits. In 19 scenarios, the supplier exploits turn-and-earn to increase her profits by choosing $K = k(w)$. The increase can be substantial; in ten scenarios, supplier profits increase by more than 10%, and in four scenarios profits rise by more than 50%.

When the Supplier Can Choose w and K, Turn-and-Earn Allocation Does Not Necessarily Lower the Retailers' Profits. As suggested by Theorem 6, in many scenarios the retailers are worse off with turn-and-earn allocation, but in four scenarios they are actually better off. This can occur because the supplier may either build more capacity and/or lower her wholesale price. Cachon and Lariviere (1996) observe a similar result in a one-period setting with capacity allocation: For a fixed capacity, the competition for allocation hurts retailer profits, but this competition induces the supplier to build more capacity, benefiting the retailers. As this numerical example indicates, the latter effect can dominate the former.

Turn-and-Earn Allocation Can Lower Total Supply Chain Profits. Supply chain profits will clearly rise in those scenarios in which the retailers are better off with turn-and-earn. The supply chain can even be better off when the retailers suffer as long as the supplier's gains are larger than the retailers' losses. However, in three scenarios, the supply chain is worse off under turn-and-earn than under fixed allocation. In these cases the retailers' competition for better allocations destroys more profits than the supplier gains, thereby yielding a net loss for the supply chain.

Neither Fixed Nor Turn-and-Earn Allocation Guarantees the Integrated Supply Chain Profits. Table 2 shows that turn-and-earn does not coordinate the supply chain to maximize total profits. Instead, turn-and-earn should be viewed as a mechanism for the supplier to increase her profits, often at the expense of the retailers and sometimes even at the expense of total supply chain performance.

4. Game Analysis with Inventory

We now investigate whether the supplier can benefit from turn-and-earn allocation when the retailers can hold inventory economically. To simplify the

Table 1 Supplier Wholesale Price, Capacity and Allocation Decisions

			Even Allocation					Turn-and-Earn Allocation				
					Expected Profits					Expected Profits		
ϕ	c	α	w	K	Supplier	One Retailer	Total Chain	w	K	Supplier	One Retailer	Total Chain
0.15	0.1	0.2	0.125	0.075	0.0113	0.0073	0.0259	0.141	0.115	0.0142	0.0077	0.0297
0.15	0.1	0.4	0.225	0.175	0.0613	0.0232	0.1076	0.237	0.205	0.0682	0.0220	0.1122
0.15	0.1	0.6	0.325	0.275	0.1513	0.0461	0.2434	0.333	0.295	0.1590	0.0443	0.2475
0.15	0.1	0.8	0.425	0.375	0.2813	0.0759	0.4331	0.429	0.385	0.2867	0.0745	0.4358
0.15	0.4	0.2	0.000	0.000	0.0000	0.0000	0.0000	0.000	0.000	0.0000	0.0000	0.0000
0.15	0.4	0.4	0.300	0.100	0.0200	0.0095	0.0390	0.300	0.100	0.0200	0.0095	0.0390
0.15	0.4	0.6	0.400	0.200	0.0800	0.0260	0.1320	0.408	0.220	0.0818	0.0248	0.1314
0.15	0.4	0.8	0.500	0.300	0.1800	0.0495	0.2790	0.504	0.310	0.1825	0.0484	0.2793
0.15	0.7	0.2	0.000	0.000	0.0000	0.0000	0.0000	0.000	0.000	0.0000	0.0000	0.0000
0.15	0.7	0.4	0.375	0.025	0.0013	0.0014	0.0041	0.375	0.025	0.0013	0.0014	0.0041
0.15	0.7	0.6	0.475	0.125	0.0313	0.0116	0.0544	0.475	0.125	0.0313	0.0116	0.0544
0.15	0.7	0.8	0.575	0.225	0.1013	0.0287	0.1586	0.575	0.225	0.1013	0.0287	0.1586
0.5	0.1	0.2	0.600	0.400	0.0800	0.0200	0.1200	0.600	0.400	0.0800	0.0200	0.1200
0.5	0.1	0.4	0.300	0.700	0.0800	0.0650	0.2100	0.270	0.250	0.0938	0.0406	0.1749
0.5	0.1	0.6	0.375	0.625	0.1813	0.0678	0.3169	0.355	0.325	0.1841	0.0601	0.3043
0.5	0.1	0.8	0.450	0.550	0.3050	0.0838	0.4725	0.450	0.550	0.3050	0.0838	0.4725
0.5	0.4	0.2	0.900	0.100	0.0050	0.0013	0.0075	0.900	0.100	0.0050	0.0013	0.0075
0.5	0.4	0.4	0.300	0.100	0.0200	0.0200	0.0600	0.345	0.175	0.0301	0.0224	0.0748
0.5	0.4	0.6	0.400	0.200	0.0800	0.0400	0.1600	0.430	0.250	0.0978	0.0371	0.1719
0.5	0.4	0.8	0.500	0.300	0.1800	0.0600	0.3000	0.515	0.325	0.1945	0.0563	0.3070
0.5	0.7	0.2	0.000	0.000	0.0000	0.0000	0.0000	0.000	0.000	0.0000	0.0000	0.0000
0.5	0.7	0.4	0.375	0.025	0.0013	0.0041	0.0094	0.375	0.025	0.0013	0.0041	0.0094
0.5	0.7	0.6	0.475	0.125	0.0313	0.0203	0.0719	0.505	0.175	0.0341	0.0196	0.0733
0.5	0.7	0.8	0.575	0.225	0.1013	0.0366	0.1744	0.590	0.250	0.1082	0.0340	0.1761
0.85	0.1	0.2	0.559	0.441	0.1654	0.0414	0.2482	0.559	0.441	0.1654	0.0414	0.2482
0.85	0.1	0.4	0.559	0.441	0.1654	0.0414	0.2482	0.559	0.441	0.1654	0.0414	0.2482
0.85	0.1	0.6	0.410	0.590	0.2362	0.0907	0.4175	0.410	0.590	0.2362	0.0907	0.4175
0.85	0.1	0.8	0.468	0.533	0.3371	0.1114	0.5599	0.468	0.533	0.3371	0.1114	0.5599
0.85	0.4	0.2	0.735	0.265	0.0596	0.0149	0.0893	0.735	0.265	0.0596	0.0149	0.0893
0.85	0.4	0.4	0.735	0.265	0.0596	0.0149	0.0893	0.383	0.196	0.0614	0.0354	0.1322
0.85	0.4	0.6	0.400	0.200	0.0800	0.0540	0.1880	0.455	0.264	0.1267	0.0488	0.2243
0.85	0.4	0.8	0.543	0.458	0.1886	0.0751	0.3389	0.528	0.332	0.2128	0.0637	0.3402
0.85	0.7	0.2	0.912	0.088	0.0066	0.0017	0.0099	0.912	0.088	0.0066	0.0017	0.0099
0.85	0.7	0.4	0.912	0.088	0.0066	0.0017	0.0099	0.458	0.121	0.0178	0.0176	0.0531
0.85	0.7	0.6	0.475	0.125	0.0313	0.0291	0.0894	0.530	0.189	0.0587	0.0280	0.1147
0.85	0.7	0.8	0.575	0.225	0.1013	0.0444	0.1901	0.603	0.257	0.1245	0.0397	0.2039

analysis, we assume that second-period demand is high with certainty. The main insights from this section apply even if stochastic demand is maintained, but the analysis of the model is more cumbersome. In addition, we assume the supplier's capacity and wholesale prices are fixed. As in the previous section, the analysis follows a reverse chronological order.

4.1. Stock Decision

The inventory option introduces an additional decision variable for each retailer: how much stock to

Table 2 Comparison of Fixed Allocation, Turn-And-Earn Allocation and Integrated Chain Profits

			% Change When Switching from Fixed Allocation to Turn-And-Earn Allocation					Expected Chain Profits As a Percentage of Integrated Chain Profits	
					Expected profits				
ϕ	c	α	w	K	Supplier	One Retailer	Chain	Fixed Allocation	Turn-and-Earn Allocation
0.15	0.1	0.2	12.8%	53.0%	26.3%	5.7%	14.6%	57%	65%
0.15	0.1	0.4	5.3%	17.0%	11.3%	−5.2%	4.2%	69%	72%
0.15	0.1	0.6	2.5%	7.2%	5.1%	−3.9%	1.7%	72%	74%
0.15	0.1	0.8	0.9%	2.7%	1.9%	−1.8%	0.6%	74%	74%
0.15	0.4	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	0%	0%
0.15	0.4	0.4	0.0%	0.0%	0.0%	0.0%	0.0%	65%	65%
0.15	0.4	0.6	2.0%	9.9%	2.3%	−4.6%	−0.5%	71%	71%
0.15	0.4	0.8	0.8%	3.3%	1.4%	−2.2%	0.1%	74%	74%
0.15	0.7	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	100%	100%
0.15	0.7	0.4	0.0%	0.0%	0.0%	0.0%	0.0%	46%	46%
0.15	0.7	0.6	0.0%	0.0%	0.0%	0.0%	0.0%	69%	69%
0.15	0.7	0.8	0.0%	0.0%	0.0%	0.0%	0.0%	73%	73%
0.5	0.1	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	63%	63%
0.5	0.1	0.4	−10.0%	−64.3%	17.3%	−37.6%	−16.7%	75%	62%
0.5	0.1	0.6	−5.3%	−48.0%	1.5%	−11.4%	−4.0%	74%	71%
0.5	0.1	0.8	0.0%	0.0%	0.0%	0.0%	0.0%	74%	74%
0.5	0.4	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	19%	19%
0.5	0.4	0.4	15.0%	75.0%	50.3%	11.8%	24.6%	49%	61%
0.5	0.4	0.6	7.5%	25.0%	22.3%	−7.4%	7.4%	64%	69%
0.5	0.4	0.8	3.0%	8.3%	8.0%	−6.2%	2.3%	71%	73%
0.5	0.7	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	0%	0%
0.5	0.7	0.4	0.0%	0.0%	0.0%	0.0%	0.0%	23%	23%
0.5	0.7	0.6	6.3%	40.0%	9.0%	−3.4%	1.9%	59%	60%
0.5	0.7	0.8	2.6%	11.1%	6.9%	−7.1%	1.0%	70%	70%
0.85	0.1	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	70%	70%
0.85	0.1	0.4	0.0%	0.0%	0.0%	0.0%	0.0%	59%	59%
0.85	0.1	0.6	0.0%	0.0%	0.0%	0.0%	0.0%	78%	78%
0.85	0.1	0.8	0.0%	0.0%	0.0%	0.0%	0.0%	80%	80%
0.85	0.4	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	63%	63%
0.85	0.4	0.4	−47.9%	−25.9%	3.2%	137.5%	47.9%	42%	63%
0.85	0.4	0.6	13.8%	32.1%	58.3%	−9.6%	19.3%	58%	69%
0.85	0.4	0.8	−2.7%	−27.4%	12.8%	−15.3%	0.4%	72%	73%
0.85	0.7	0.2	0.0%	0.0%	0.0%	0.0%	0.0%	27%	27%
0.85	0.7	0.4	−49.8%	37.3%	168.9%	965.9%	434.6%	11%	57%
0.85	0.7	0.6	11.6%	51.3%	87.7%	−3.6%	28.3%	51%	65%
0.85	0.7	0.8	4.8%	14.2%	22.9%	−10.7%	7.2%	66%	71%

carry into the second period. The stock decision will clearly depend on the retailer's expected maximum allocation in the second period, a. The more a retailer expects to be able to purchase in the final period, the less he needs to stock in the first period. The stock decision also depends on whether the retailer's first-period allocation is limited. In that case an increase in inventory will lower first-period sales. We consider this latter effect in §4.2 and, for now, focus on a retailer's stock decision when he can obtain as much stock as he desires in the first period.

Let s be the amount of inventory a retailer holds and

$\pi_2(a, s)$ be a retailer's second-period profits plus the purchase and holding costs incurred on inventory,

$$\pi_2(a, s) = -sh + (1 - w - \min\{a + s, q_h\}) \times \min\{a + s, q_h\}.$$

Lemma 8. *A retailer's optimal stock level is $s(a)$,*

$$s(a) = (q_h - a - h/2)^+,$$

assuming the retailer's first-period allocation is sufficient to cover $s(a)$ and his desired first-period sales quantity.

Proof. If $a > q_h$, the retailer chooses $s = 0$ because he does not need inventory to sell his desired final-period quantity q_h. Suppose $a < q_h$, so

$$\frac{\partial \pi_2(a, s \mid a < q_h)}{\partial s} = 1 - w - h - 2(a + s).$$

Profits are strictly concave on the interval $s \in [0, q_h - a]$, so the result follows from the first-order condition. □

Inventory is not free, so even when $a < q_h$, the retailer may not carry inventory. Indeed, when $q_h - h/2 \leq a \leq q_h$, the marginal increase in profits from increasing sales in the second period is less than the marginal cost of holding inventory,

$$\frac{\partial \pi_2(a, s \mid q_h - h/2 < a)}{\partial s} < h.$$

When $a < q_h - h/2$, the retailer would like to stock a sufficient amount to increase second-period sales only to $q_h - h/2$, the sales level at which the marginal benefit of stock exactly equals his marginal cost,

$$\frac{\partial \pi_2(q_h - h/2, s)}{\partial s} = h.$$

4.2. First-Period Decisions with Very Tight Capacity

Suppose

$$K/2 < q_l + s(K/2). \tag{5}$$

Here, capacity is insufficient to cover both a retailer's desired first-period sales quantity and his stock quantity, assuming the retailer expects $K/2$ is his maximum second-period allocation. In this case, the supplier is assured of selling her full production in each period.

Lemma 9. *When $K/2 < q_l + s(K/2)$, the supplier sells K in each period.*

Proof. It is sufficient to show that result holds for fixed allocation because fixed allocation sales are never greater than turn-and-earn sales. When $s(K/2) > 0$, each retailer will order at least $K/2$ since both want $q_l + s(K/2)$ in the first period. From Lemma 8, a retailer holds inventory only if he expects to purchase his maximum allocation in the second period, $K/2$. Total purchases in the second period are then K. If $s(K/2) = 0$, by assumption $K/2 < q_l$, so $K/2 < q_h$, and the supplier again sells K in the second period. □

Since capacity is fully utilized, each retailer will order at least $K/2$ in the first period and receive $K/2$. Since retailer i receives only $K/2$ units, his sales quantity is bounded, i.e., $q_i \leq K/2$. Further, whatever is not sold is put in inventory, making $K/2 - q_i$ retailer i's stock level. Hence, a retailer really has only a single first-period decision: how much to sell.

4.2.1. Fixed Allocation. Suppose the supplier implements fixed allocation. In the final period, neither retailer has an advantage, so each orders at least $K/2$ and receives $K/2$ units. Retailer i can sell up to the sum of his stock level and his second-period allocation, $K - q_i$. Define $\pi_f(q_i)$ as retailer i's profits over both periods,

$$\pi_f(q_i) = \pi_l(q_i) + \pi_h(K - q_i) - h(K/2 - q_i).$$

Lemma 10. *Assume fixed allocation. In the unique Nash equilibrium, each retailer sells q_f in the first period, where*

$$q_f = \min\{\tfrac{1}{4}(2K - (1 - \alpha) + h)^+, K/2\}.$$

Proof. Differentiate $\pi_f(q_i)$,

$$\pi_f'(q_i) = \pi_l'(q_i) - \pi_h'(K - q_i) + h. \tag{6}$$

Over the feasible range, $\pi_f(q_i)$ is strictly concave in q_i. The optimal sales quantity satisfies the first-order condition or lies on the boundary of the feasible range,

$$q_f = \min\{\tfrac{1}{4}(2K - (1 - \alpha) + h)^+, K/2\}.$$

The sales quantity only equals $K/2$ when $K/2 < q_l$ since for extremely tight capacity $q_f < q_l$ (i.e., retailers sacrifice some first-period sales when capacity is insufficient to cover the desired stock level). The equilibrium is unique since neither retailer has an impact on the other, given that both order at least $K/2$ each period. □

Inspection of (6) shows that there is a marginal benefit to increasing first-period sales for two reasons: Sales are below the level that would maximize first-period profits and increasing sales reduces inventory holding costs. The marginal cost of increasing sales is forgone profits in the second period, in which the marginal value of selling a unit is higher. Inventory thus allows the retailers to make intertemporal substitutions between periods. Sales in a down market are sacrificed so inventory can be carried into a high-demand period.

4.2.2. Turn-And-Earn Allocation. We now consider turn-and-earn allocation. When retailer i enters the second period as the sales leader, his maximum allocation is $(K + q_i - q_j)/2$. Including his stock, $K/2 - q_i$, yields his maximum second-period sales, $K - (q_i + q_j)/2$. (In equilibrium the sales leader will order at least his maximum allocation. Otherwise, the leader could sell more and stock less in the first period without changing his second-period sales.) Since the sales leader orders at least his maximum allocation, when retailer j is the sales laggard his maximum allocation is $(K - q_i + q_j)/2$, and his maximum second-period sales are $K - (q_i + q_j)/2$. Hence, define $\pi_t(q_i, q_j)$ as the sum of retailer i's first- and second-period profits

$$\pi_t(q_i, q_j) = \pi_l(q_i) + \pi_h(K - (q_i + q_j)/2) - h(K/2 - q_i),$$

assuming $K - (q_i + q_j)/2 \le q_h$. (Note that $\pi_t(q_i, q_j)$ applies whether retailer i is the sales leader or laggard because their maximum allocations have the same functional form.)

Lemma 11. *Assume turn-and-earn allocation. In the unique Nash equilibrium each retailer sells q_t in the first period, where*

$$q_t = \min\{\tfrac{1}{3}(K - (1/2 - \alpha) - w/2 + h)^+, K/2\}.$$

Proof. Assuming $K - (q_i + q_j)/2 \le q_h$ (otherwise the pair $\{q_i, q_j\}$ could not be an equilibrium), a retailer's marginal profits are

$$\pi_t'(q_i, q_j) = \pi_l'(q_i) - \tfrac{1}{2}\pi_h'\left(K - \frac{q_i + q_j}{2}\right) + h. \quad (7)$$

Since the marginal profits are identical for both the sales leader and the sales laggard, no asymmetric equilibrium exists because at least one retailer would have an incentive to deviate. By construction, q_t either satisfies the first-order condition or equals the boundary of the feasible interval. □

While Lemma 9 indicates that the allocation mechanism has no impact on the supplier when capacity is tight, the allocation mechanism still influences retailer behavior.

Theorem 12. *Turn-and-earn induces the retailers to sell more in the first period than fixed allocation, i.e., $q_f \le q_t$. Further, retailer profits are never higher with turn-and-earn than with fixed allocation and may be lower.*

Proof. In equilibrium, each retailer sells $K - q_t$ in the second period under turn-and-earn while under fixed allocation each sells $K - q_f$. In both cases the retailers sell no more than q_h in the second period, so $\pi_h'(K - q) \ge 0$, where $q = q_f$ or $q = q_t$. Since each retailer wants to sell more in the final period, comparison of (6) with (7) reveals

$$\pi_t'(q_f, q_f) = \pi_l'(q_f) - \tfrac{1}{2}\pi_2'(K - q_f) + h \ge 0,$$

so it holds that $q_t \ge q_f$. Since (6) is a retailer's marginal profit, choosing q_t over q_f lowers profits when $q_t > q_f$. □

Combining Theorem 12 with Lemma 9 paints a poor picture for turn-and-earn. When capacity is extremely tight, the supplier gains nothing by choosing turn-and-earn over fixed allocation, but the retailers are worse off. Hence, turn-and-earn lowers total supply chain profits. Turn-and-earn is problematic in this situation for two reasons. First, since the supplier is already selling her full capacity, the supplier garners no benefit from inducing larger orders. Second, the competition to defend their allocation distorts the retailers' intertemporal substitution between first- and second-period sales; in particular, it biases the retailers toward selling too much in the low-demand period. This may be a serious problem in the auto industry. Dealerships might offer ridiculous discounts at the end of the month to make sure they defend their position in the next allocation cycle even though the vehicles could be sold without the discounts within a few days.

4.3. First-Period Decisions with Moderately Tight Capacity

We now consider first-period actions when capacity, while more plentiful than in the previous section, is still somewhat restrictive. Specifically, suppose

$$q_h \geq K/2 > q_l + s(K/2).$$

When the supplier implements fixed allocation, each retailer will order $q_l + s(K/2)$ in the first period and receive his order. Further, each retailer correctly anticipates that $K/2$ is his maximum allocation in the second period since capacity remains constraining, $K < 2q_h$. Thus, with fixed allocation some of the supplier's capacity is left idle in the first period. The main question is whether turn-and-earn allocation induces the retailers to order more in the first period even though they have the option of holding inventory.

Assume the supplier implements turn-and-earn allocation. As in the no-inventory analysis, we must determine the retailers' expected maximum second-period allocations.

Lemma 13. *When the supplier implements turn-and-earn allocation and the retailers can inventory units, $a_t(q_i, q_j)$ is a retailer's expected maximum second-period allocation.*

Proof. The sales leader's maximum allocation is $(K + \psi)/2$ whether the sales leader carries stock or not. From Lemma 8, the sales leader requests his maximum allocation a, whenever $a \leq q_h$, otherwise he holds no stock and requests q_h. Hence, the sales leader requests only a portion of his maximum allocation when $q_h < (K + \psi)/2$, which is the same condition as $\psi > \bar{\psi}$. The sales laggard's maximum allocation is then $(K - \psi)/2$ when the leader takes his maximum allocation. Otherwise the laggard's maximum allocation is $(K - \bar{\psi})/2$. □

Consequently, the inventory option does not affect the retailers' expected maximum second-period allocations. Define $\hat{\pi}_r(q_i, q_j)$ as a retailer's expected profits over both periods when the retailers can carry inventory,

$$\hat{\pi}_r(q_i, q_j) = \pi_l(q_i) + \begin{cases} \pi_h(q_h - h/2) - h(q_h - h/2 - a_t(q_i, q_j)) & a_t(q_i, q_j) < q_h - h/2 \\ \pi_h(\min\{a_t(q_i, q_j), q_h\}) & a_t(q_i, q_j) \geq q_h - h/2 \end{cases},$$

where $(q_h - h/2 - a_t(q_i, q_j))^+$ is the inventory the retailer holds.

Theorem 14. *Assume turn-and-earn allocation. Define*

$$\hat{\xi}(K) = \tfrac{1}{2}(\alpha - w) + \tfrac{1}{4}\min\{h, 1 - w - K\}.$$

Suppose $q_h > K/2 \geq y^* + s(K/2)$, where $y^* = \hat{\xi}(K)^+$. In the unique first-period Nash equilibrium each retailer sells y^* and stocks $s(K/2)$.

Proof. Assume first-period capacity is not binding, so each retailer sells and stocks as he wishes. The stock decision is thus independent of the sales decision. Consider the following marginal profits, which apply to either the sales leader or sales laggard,

$$\frac{\partial \hat{\pi}_r(q_i, q_j)}{\partial q_i} = \pi_l'(q_i) + \begin{cases} \frac{h}{2} & a_t(q_i, q_j) < q_h - h/2 \\ \tfrac{1}{2}(1 - w - (K + q_i - q_j)) & q_h - h/2 \leq a_t(q_i, q_j) < q_h \\ 0 & q_h \leq q_t(q_i, q_j) \end{cases}.$$

A retailer's expected profits are strictly concave in his first-period sales quantity, holding the other's sales fixed. By construction, $\hat{\xi}(K)$ satisfies the first-order condition. Further, asymmetric sales quantities cannot satisfy both first-order conditions, so there are no asymmetric equilibria. Given that both sell the same amount in the first period, each expects a maximum allocation of $K/2$ in the second. Since $K/2 \geq y^*$

$+ s(K/2)$, capacity in the first period is indeed not constraining. □

The main theorem of this section follows immediately.

Theorem 15. *Assume $q_h > K/2 \geq y^* + s(K/2)$ and $h > 0$, so inventory is not free. In equilibrium, the retailers sell more in the first period when the supplier implements turn-and-earn allocation than when the supplier implements fixed allocation. When inventory is free, the allocation rules produce the same first-period supplier sales.*

Proof. Given the condition on capacity ($K < 1 - w$), from Theorem 14,

$$y^* = \hat{\xi}(K)^+ > (\alpha - w)^+/2 = q_l$$

when $h > 0$, and $y^* = q_l$ when $h = 0$. □

Turn-and-earn can therefore increase a supplier's sales (and hence profits) when inventory is not free. The result occurs because each retailer is interested in increasing his final-period sales (since capacity will be constraining). Greater second-period sales can be achieved by carrying more inventory into the second period or by receiving a greater second-period allocation. The marginal cost of increasing inventory is h, but a retailer's marginal cost of increasing his second-period allocation is initially zero. A retailer is thus better off increasing first-period sales rather than holding more inventory. (Note that this logic applies even if second-period demand was stochastic.) Unfortunately for the retailers, since they compete for capacity, neither retailer actually obtains a better allocation, i.e., in equilibrium they increase their early sales merely to defend, not to expand, their later allocations.

The impact of allowing the retailers to hold inventory may be summarized as follows. When capacity is constraining in the second period but not in the first (for either allocation rule), turn-and-earn allocation induces the retailers to sell more in the first period than they would under fixed allocation. Here, turn-and-earn does not influence the retailers' stocking decision, i.e., they stock $s(K/2)$ under either allocation rule. When capacity is constraining in the first period, turn-and-earn also impacts the retailers' stocking decision, i.e., they stock less and sell more under turn-and-earn than they would under fixed allocation. This effect persists even when capacity is so tight that the supplier sells her full production in each period. In that case, turn-and-earn provides no benefit to the supplier. She already sells her full production, so turn-and-earn destroys some supply chain profits by forcing the retailers to sell more in the low-demand market than they should.

For completeness, when $y^* + s(K/2) > K/2 > q_l + s(K/2)$, capacity is constraining in the first period under turn-and-earn but not under fixed allocation. Now, turn-and-earn causes the retailers to order more than under fixed allocation (in fact, they order all of capacity). The retailers' intertemporal substitution between sales and inventory is again distorted, as they sell more and stock less in the first period.

5. Discussion

We have studied turn-and-earn allocation in a stylized model. In this section we comment on some of our assumptions (1) to conjecture on the degree to which an assumption influences our qualitative findings and (2) to highlight questions that require future exploration. While we primarily address modeling issues, empirical validation of our findings is a useful endeavor.

Wholesale Price Stickiness. Some degree of wholesale price rigidity is essential to the model. Suppose the supplier could freely adjust the period's wholesale price after observing the demand state. The supplier never prices so that retailer demand exceeds available capacity, otherwise her price could rise without lowering her sales. If the supplier never allows capacity to bind, the retailers need not be concerned with securing better allocations. In other words, allocation is only relevant if there is some expectation that capacity can bind. In fact, because periods of allocation are observed in practice, one must conclude that some degree of wholesale price stickiness exists.

Nonstationary Demand. If demand were constant in every period, then capacity would either always or never bind. Either case makes allocation moot. With binding capacity neither retailer could ever secure a better allocation while with unconstrained capacity neither retailer would ever need a better allocation.

However, demand need not be perfectly correlated across markets, since retailers would still anticipate that future demand could exceed capacity. Hence, the incentive to secure a higher future allocation remains.

Retailer Competition for Customers. If retailers competed directly for customers, a given retailer would be better off, all else being equal, when his competitor had lower inventory. (See Balachander and Farquhar 1994.) A competitor with fewer units to sell, for example, would likely price less aggressively. This suggests that a competitive market should enhance each retailer's incentive to increase his future allocation because capacity allocation is a zero-sum game; a higher future allocation for one retailer also implies lower future allocations for his competitors.

Stochastic Retailer Demand. We have assumed that each retailer faces a deterministic demand function given the observed state of demand. In reality, a retailer may know the state of demand (a good or bad economy) but still face stochastic demand. Even with stochastic demand, turn-and-earn provides an incentive for the retailer to try to increase his current-period sales. However, a retailer probably cannot guarantee that he will always sell his entire stock. We conjecture that in this environment turn-and-earn allocation favors retailers with lower demand variance relative to their mean demand and thus might favor larger markets over smaller ones.

Longer Horizons. We have assumed only two periods, so turn-and-earn impacts only the last period's allocation. With additional periods, retailers will still have incentives to increase current sales to secure higher future allocations, but multiple periods introduces other issues. For example, how would the retailer with the highest allocation behave in a low-demand period? Specifically, will he "defend" his position? This has interesting policy implications. Under anti-trust laws, suppliers are not supposed to favor one retailer over another, but turn-and-earn allocation may in effect give a high-allocation retailer a permanent advantage because a sales laggard may be unable to improve his allocation. In essence, you cannot sell if you do not have, but you cannot have if you do not sell. There is some anecdotal evidence to suggest that this issue is indeed relevant. Dave Smith Motors is the number two Dodge dealer in the country, selling 4,000 Dodge trucks per year even though it is located in Kellogg, Idaho, a town of 3,000. In comparison, Sundance Dodge in Boise, Idaho, receives about 720 vehicles per year. Dave Smith Motors is able to sell so many vehicles and maintain its allocation because it prices aggressively, advertises widely, and draws customers from far away (Jackson 1997). This example raises several questions: Can Sundance Dodge increase its allocation if it does not have the inventory to sell? Is it in Dodge's interest to allocate so many vehicles to a dealership located in a remote area? Should Dave Smith Motors' allocation be increasing in its sales to customers from outside its designated market area?

Supplier Inventory. Suppose the supplier could carry inventory from period to period. The retailers' allocation would then depend on supplier's available capacity and stock. As long as total supply is less than the retailers' possible needs, the retailers should anticipate that their future allocations could be constrained. The incentive to secure a higher future allocation remains. It is also worthwhile to ask if the supplier would even want to hold inventory. The supplier is indifferent between selling a unit in the first or second period since the wholesale price is constant. A unit held into the last period lowers that period's capacity constraint, implying that the retailers will not sell as much in the first period.

The Selling Process. In our model, the supplier only wants to increase her sales for a given wholesale price. A supplier might be concerned with other aspects of the selling process. In the automobile industry, customers generally hate pushy salespeople. While turn-and-earn does give a dealer the incentive to "move the metal," it may also lead them to adopt high pressure sales tactics, to the possible detriment of the supplier's long run brand image. Toyota, which uses turn-and earn, is concerned about this issue. While customers are very happy with their vehicles, Toyota dealerships routinely rank among the worst in the industry for customer satisfaction. Some argue that this could become Toyota's Achilles' heel, much as how poor quality hurt the American automobile manufacturers (Mateja 1997).

Alternative Allocation Procedures. While turn-and-earn allocation is common, other techniques are also used in practice. Some are related to turn-and-earn, and others are quite different. Many companies base allocations on *national balanced days' supply*. That is, they attempt to dole out vehicles so that every dealer's inventory equals the same number of days of average demand. For example, allotments may be set so that every dealer has 20 days of inventory. To convert 20 days of inventory into an actual number of vehicles, of course, requires the retailer's average-demand rate. Since the demand rate is estimated with past sales, it is not clear how this approach yields different results than a turn-and-earn system. Infinity uses turn-and-earn allocation for new vehicles based on past sales of both new and used vehicles (*Chicago Tribune* 1996). Many companies are considering allocations based on customer satisfaction scores (Mateja 1996, 1997). Past research suggests that a properly designed incentive scheme can induce retailers to provide better customer service (Chu and Desai 1995). Mercedes allocates its new sport utility vehicle based on the degree to which a dealer has invested in improvements to its facility, with some dealerships being explicitly excluded (Henry 1997). Saturn considers each dealer's market potential in its allocation. In each of these cases, the supplier dictates the allocation procedure, but there has been tremendous consolidation among automobile dealers. In the future some dealerships may be able to dictate their own allocation (Taylor 1997). Alternatively, some companies envision a future in which allocation is not an issue because consumers will either special order their vehicles from the factory or will choose their vehicle from a large centralized facility (Christian 1997).

Discounting. Turn-and-earn allocation works only if the retailers are concerned about their future allocations. This logic applies even in a model with discounting, but the effectiveness of turn-and-earn should decrease as retailers discount the future more.

6. Conclusion

Using a stylized model we have demonstrated that even though retailers may not compete for customers, turn-and-earn allocation induces them to compete for scarce capacity. Aware that capacity may be insufficient, retailers increase the stock available to support future sales either by securing a better allocation or by holding inventory. Under turn-and-earn, a better allocation is secured by increasing current sales. When capacity is not restrictive (because demand is low), the initial marginal cost of increasing current sales is zero and thus lower than the marginal cost of holding inventory. Therefore, it is always worthwhile for a retailer to increase current sales to ensure a better future allocation. Higher retailer sales lead to higher supplier profits.

Unfortunately for the retailers, all retailers increase their current sales in equilibrium, so no one actually obtains a better allocation. Higher sales merely defend one's allocation, not improve it. Since the retailers gain nothing from turn-and-earn but sell more than their optimal quantity, their profits fall. A fixed wholesale price and capacity are key to this conclusion. When the supplier can choose her wholesale price and capacity at the start of the game, the anticipation of higher sales may lead her to build more capacity and/or charge a lower wholesale price, either of which benefits the retailers.

Turn-and-earn's impact on supply chain performance is ambiguous. The initial sales increase is essentially costless to the retailers but provides a significant benefit to the supplier in the form of less idle capacity. Overall performance improves. However, when turn-and-earn induces the retailers to sell too much, supply chain performance deteriorates.

Our conclusions differ from those in Lee et al. (1997). They focus on how turn-and-earn allocation can limit strategic order inflation, improving the transmission of information and allowing the system to operate more efficiently. Our model does not address information sharing (e.g., we assume common knowledge), and thus our results are not directly comparable. Nonetheless, we do find that strategic behavior is not eliminated by basing allocations on past sales.

Further, our supplier actually prefers that the retailers act strategically.

While this model provides insights into the impact of turn-and-earn allocation on supply chain performance, many important questions remain unanswered. In a theoretical model, it is particularly important to evaluate the impact of retailer competition, longer time horizons and stochastic demand. There are also many questions which are best answered through empirical investigation: Does turn-and-earn allocation contribute to high pressure sales tactics and lower customer satisfaction? How should the allocation rule reflect special circumstances like unusual weather or changing demand patterns? Does turn-and-earn allocation favor one class of dealers over others? And can turn-and-earn allocation threaten the financial viability of some dealerships?[2]

[2] The authors thank Don Gottwald for his valuable research assistance. They are also grateful for the helpful comments of Jim Anton, Charlie Fine, Rick Staelin, and seminar participants at the Fuqua School of Business, The Wharton School, and the 1998 Manufacturing and Service Operations Management Conference in Seattle.

Appendix A. Proofs

Lemma 7 Proof. Assume fixed allocation. The supplier earns

$$\pi_s^f(w, K) = -Kc + \begin{cases} 2Kw & K < 2q_l \\ (2q_l + \phi K + (1-\phi)2q_l)w & 2q_l \le K \le 2q_h \\ (2q_l + \phi 2q_h + (1-\phi)2q_l)w & 2q_h < K \end{cases}.$$

Let $K^f(w)$ be a capacity that maximizes the supplier's profits for a given wholesale price and fixed allocation,

$$K^f(w) = \begin{cases} 0 & 0 \le w \le \dfrac{c}{2} \\ 2q_l & \dfrac{c}{2} \le w \le \dfrac{c}{\phi} \\ 2q_h & \dfrac{c}{\phi} \le w \end{cases}.$$

Assume turn-and-earn allocation. The supplier earns

$$\pi_s^t(w, K) = -Kc + \begin{cases} 2Kw & 0 \le K < 2q_l \\ (K + \phi K + (1-\phi)2q_l)w & 2q_l \le K < 2\xi(K) \\ (2\xi(K)^+ + \phi K + (1-\phi)2q_l)w & 2\xi(K) \le K < 2q_h \\ (2q_l + \phi 2q_h + (1-\phi)2q_l)w & 2q_h \le K. \end{cases}$$

When $w \ge 2c/\phi$ the supplier is better off with $K = 2q_h$, otherwise the supplier is better off with $K = 2\xi(K)^+$, i.e., the capacity at which the turn-and-earn first-period sales quantity just binds. Let $k(w)$ equal the K that solves $K = 2\xi(K)^+$,

$$k(w) = ((2\alpha + \phi)/(2+\phi) - w)^+.$$

Hence, the supplier will only sell in the high-demand state when $(2\alpha+\phi)/(2+\phi) \le w$. When $\alpha \le w < (2\alpha+\phi)/(2+\phi)$ the supplier sells in the low-demand state only in the first period. Finally, when $w \le \alpha$, the supplier sells in all demand states. Let $K^t(w)$ be a capacity that maximizes the supplier's profits given the wholesale price w and turn-and-earn allocation,

$$K^t(w) = \begin{cases} \begin{cases} 0 & w < \dfrac{c}{\phi} \\ 2q_h & w \ge \dfrac{c}{\phi} \end{cases} & \dfrac{2\alpha+\phi}{2+\phi} \le w \\ \begin{cases} 0 & w < \dfrac{c}{1+\phi} \\ k(w) & \dfrac{c}{1+\phi} \le w < \dfrac{2c}{\phi} \\ 2q_h & \dfrac{2c}{\phi} \le w \end{cases} & \alpha \le w < \dfrac{2\alpha+\phi}{2+\phi} \\ \begin{cases} 0 & w < c/2 \\ 2q_l & c/2 \le w < \dfrac{c}{1+\phi} \\ k(w) & \dfrac{c}{1+\phi} \le w < \dfrac{2c}{\phi} \\ 2q_h & \dfrac{2c}{\phi} \le w \end{cases} & w \le \alpha \end{cases}. \quad \square$$

Appendix B. The Supplier's Wholesale Price

It is sufficient to evaluate the supplier's optimal wholesale price for each of the four possible capacity strategies, $K \in \{0, 2q_l, k(w), 2q_h\}$, and then to choose the best among that group. If $K = 2q_l$, the supplier's profits are $\pi_s(w) = 2q_l(2w - c)$, and the optimal wholesale price is $w_l = \min\{(\alpha + c/2)/2, \alpha\}$. If $K = 2q_h$ and $w \le \alpha$ (so the supplier sells both in the high- and low-demand states), the supplier's profits are

$$\pi_s(w \mid w \le \alpha) = -2q_h c + (2q_l + \phi 2q_h + (1-\phi)2q_l)w$$

and the optimal wholesale price is $w_{lh} = \min\{(\alpha + c/2 + \phi(1-\alpha)/2)/2, \alpha\}$. If $K = 2q_h$ and $w > \alpha$, the supplier's profits are

$$\pi_s(w \mid 1 \ge w > \alpha) = -2q_h c + \phi 2q_h w$$

and the optimal wholesale price is $w_h = \min\{(\phi + c)/2\phi, 1\}$.

If the supplier implements turn-and-earn and $K = k(w)$, the supplier's profits are

$$\pi_s(w \mid k(w) \ge 0) = -k(w)c + (k(w) + \phi k(w) + (1-\phi)(\alpha - w))w,$$

and the optimal wholesale price is

$$w_t = (\phi(1+\phi) + c(2+\phi) + \alpha(4 + \phi - \phi^2))/(8 + 4\phi),$$

assuming $k(w_t) \ge 0$. (It necessarily holds that $k(w) \le 2q_h$.)

Appendix C. Expected Retailer Profits

Expected retailer profits, $\pi_r(w)$, when the supplier chooses $K \in \{2q_l, 2q_h\}$:

$$\pi_r(w) = \begin{cases} (2-\phi)(\alpha - q_l - w)q_l + \phi(1 - q_l - w)q_l & K = 2q_l \\ (1+\phi)\dfrac{(\alpha - w)^2}{4} + \phi\dfrac{(1-w)^2}{4} & K = 2q_h,\ w \le \alpha \\ \phi\dfrac{(1-w)^2}{4} & K = 2q_h,\ w > \alpha \end{cases}.$$

If the supplier implements turn-and-earn allocation and $K = k(w)$,

$$\pi_r(w) = (1-\phi)(\alpha - w)^2/4 + (\alpha - k(w)/2 - w)k(w)/2$$
$$+ \phi(1 - k(w)/2 - w)k(w)/2.$$

Appendix D. Integrated Supply Chain

What choices would the integrated supply chain make? Let Q_l and Q_h be the optimal per period sales quantities at each retailer. Since there is a zero marginal cost of production, $Q_l = \alpha/2$, $Q_h = 1/2$. (Note that these are the same as q_l and q_h when $w = 0$.) The supply chain will choose capacity from one of two intervals: $K \in [0, 2Q_l]$, and $K \in (2Q_l, 2Q_h]$. Capacity larger than $2Q_h$ can immediately be ruled out since capacity could be lowered to $2Q_h$ without reducing revenues. Suppose the chain chooses $K \le 2Q_l$, so capacity binds in either state. In this case profits are

$$\Pi_I(K \mid K \le \alpha)$$
$$= -cK + 2((2-\phi)(\alpha - K/2)K/2 + \phi(1 - K/2)K/2),$$

which are maximized with capacity K_l

$$K_l = \min\{\tfrac{1}{2}((2-\phi)\alpha + \phi - c)^+, \alpha\}.$$

Suppose the chain chooses $2Q_l < K \le 2Q_h$, so capacity binds in the high-demand state. In this case profits are

$$\Pi_I(K \mid \alpha < K \le 1) = (2-\phi)\alpha^2/2 + 2\phi(1 - K/2)K/2 - cK,$$

which are maximized with capacity K_h

$$K_h = \max\{(\phi - c)/\phi, \alpha\}.$$

Combining these results,

$$\Pi_I = \begin{cases} 0 & c > 2\alpha + \phi(1-\alpha) \\ \tfrac{1}{4}((2-\phi)\alpha + \phi - c)^2 & 2\alpha + \phi(1-\alpha) \ge c > \phi(1-\alpha) \\ \dfrac{1}{2}\left((2-\phi)\alpha^2 + \dfrac{(\phi - c)^2}{\phi}\right) & \phi(1-\alpha) > c. \end{cases}$$

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