AN ANALYTIC HIERARCHY APPROACH TO THE ALLOCATION OF RESOURCES WITHIN
A TARGET PRODUCT/MARKET/DISTRIBUTION PORTFOLIO

Yoram Wind and Daniel Gross*

Introduction

The analytic hierarchy modeling and measurement process (Saaty, 1972, 1977b) is a recent addition to the various approaches used to determine the relative importance of a set of activities or criteria. The novel aspect and major distinction of this approach is that it structures any complex, multiperson, multicriterion, and multiperiod problem hierarchically. Using a method for scaling the weights of the elements in each level of the hierarchy with respect to an element (e.g., criterion) of the next higher level, a matrix of pairwise comparisons of the activities can be constructed where the entries indicate the strength with which one element dominates another with respect to a given criterion. This scaling formulation is translated into a largest eigenvalue problem which results in a normalized and unique vector of weights for each level of the hierarchy (always with respect to the criterion in the next level), which in turn (by a principle of hierarchical composition) via a series of multiplications results in a single composite vector of weights for the entire hierarchy. This vector measures the relative priority of all entities at the lowest level that enables the accomplishment of the highest objective of the hierarchy. These relative priority weights can provide the guidelines for the allocation of resources among the entities at the lower levels of the hierarchy. When a hierarchy is designed to reflect likely environmental scenarios, corporate objectives, and alternative product, market, and distribution options, the analytic hierarchy process (AHP) can provide a framework and methodology for the determination of the firm's target product/market/distribution portfolio.

The purpose of this paper is to describe the application of the analytic hierarchy approach to the product portfolio problem.

More specifically, the paper focuses on four areas:

1. The analytic hierarchy approach -- what it is, its conceptual foundation, assumptions, required inputs, analytical processes, output, and areas of application.

2. An analytic hierarchy approach to the product/market/distribution portfolio decisions -- what are the conceptual dimensions of this application?

3. An illustrative application -- an application of the analytic hierarchy approach to the portfolio decision of a large insurance firm.

4. Discussion and directions for future research -- suggestions for a number of other marketing applications of the analytic hierarchies methodology, such as consumer concept/product testing, and needed directions for future research on product portfolios.

*Yoram Wind is Professor of Marketing, The Wharton School, and Daniel Gross is President, Kramer Capitol Group, New York.
The Analytic Hierarchy Process

The analytic hierarchy process developed by Saaty (1972, 1977b) is a framework and measurement system designed to analyze complex decisions and systems and evaluate alternative courses of actions in terms of their priorities for achieving higher objectives of the systems.

The approach focuses on dominance matrices and their corresponding measurement -- ignored areas of research compared with the more popular proximity, profile, and conjoint measurement approaches (Shepard, 1972). It goes beyond Thurstone's (1927) comparative judgment approach (by relaxing the assumption of normality on the parameters; e.g., equal variance and zero covariance and restriction of the type of comparisons). Similar to the conjoint analysis approaches, the analytic hierarchy process is based on the trade-off concept. Yet, whereas in the conjoint analyses, the respondent is confronted with the trade-off task, the analytic hierarchy develops the trade-off in the course of structuring and analyzing a series of simple reciprocal pairwise comparison matrices.

The approach is based on three major components:

a. A structure of the complex decision system as a hierarchy reflecting the decision-maker's perceived chain relationship among the strata (levels) of the hierarchy and among specific elements of the various strata.

b. A measurement methodology to establish priorities based on reciprocal pairwise comparisons of elements within each stratum with respect to an element in a higher stratum. This measurement methodology provides the framework for data collection and analysis and constitutes the heart of the analytic hierarchy process. The priorities comprise the principal eigenvector of the matrix.

c. A measurement theory to establish the consistency of the judgmental data provided by the respondents.

The basic premise of the analytic hierarchy process is that measurement evolves out of comparisons, particularly pairwise comparisons. Let us suppose that we have n objects $A_1, \ldots, A_n$ whose vector of corresponding weights $w = (w_1, \ldots, w_n)$ is known. Let us form the matrix of pairwise comparisons of weights

$$A = \begin{pmatrix} w_1/w_1 & \cdots & w_1/w_n \\ \vdots & \ddots & \vdots \\ w_n/w_1 & \cdots & w_n/w_n \end{pmatrix}.$$

We note that we can recover the scale of weights $w_1, \ldots, w_n$ by multiplying $A$ on the right by $w$, obtaining $nw$, and then solving the eigenvalue problem $Aw = nw$, which has a nontrivial solution since $n$ is the largest eigenvalue of $A$. (The matrix $A$ has unit rank; hence, all but one of its eigenvalues $\lambda_1, \ldots, \lambda_n$ are zero. Since

$$\sum_{i=1}^{n} \lambda_i = \text{trace} (A) = n,$$

$n$ is the maximum eigenvalue.)
In general, we do not know the ratios \( w_i/w_j \), but we may have estimates of them from data and experiments or even from experienced judges. We would elicit a judgment and automatically enter its reciprocal in the transpose position. In that case we have perturbations of \( A \) which lead to perturbations in the eigenvalues of \( A \). We can show that now we must solve the problem \( A\bar{w} = \lambda_{\text{max}} w \) to obtain an estimate of the weights \( w \). We can prove that \( \lambda_{\text{max}} > n \) always and that \((\lambda_{\text{max}} - n)/(n-1)\) serves as a consistency index which gives the departure from consistency in estimating the ratios \( w_i/w_j \), with consistency obtaining if and only if \( \lambda_{\text{max}} = n \). By consistency here we mean a relation stronger than transitivity. It is defined by the relation between the entries of \( A \): \( a_{ij}a_{jk} = a_{ik} \), which means that if we have \( n \) entries that form a spanning tree, the remainder of the matrix can then be generated from them. In our approach to measurement, inconsistency is admissible provided we can specify its effect on the final results.

To provide numerical judgments in making pairwise comparisons, we need a reliable and workable scale.\(^1\) We assume that the elements involved in the comparison are of the same order of magnitude; i.e., their relative weights do not differ by more than 9. If they do, they are separated into different clusters. The 9-point scale used in typical analytical hierarchy studies is presented in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Intensity of Importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1(^a)</td>
<td>Equal importance</td>
<td>Two activities contribute equally to the objective</td>
</tr>
<tr>
<td>3</td>
<td>Weak importance of one over another</td>
<td>Experience and judgment slightly favor one activity over another</td>
</tr>
<tr>
<td>5</td>
<td>Essential or strong importance</td>
<td>Experience and judgment strongly favor one activity over another</td>
</tr>
<tr>
<td>7</td>
<td>Demonstrated importance</td>
<td>An activity is strongly favored and its dominance is demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Absolute importance</td>
<td>The evidence favoring one activity over another is of the highest possible order of affirmation</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate values between the two adjacent judgments</td>
<td>When compromise is needed</td>
</tr>
</tbody>
</table>

Reciprocals of above nonzero

If activity \( i \) has one of the above nonzero numbers assigned to it when compared with activity \( j \), then \( j \) has the reciprocal value when compared with \( i \)

Rationals

Ratios arising from the scale

If consistency were to be forced by obtaining a numerical values to span the matrix

\(^a\)On occasion in 2 by 2 problems, we have used \( 1 + \varepsilon, 0 < \varepsilon < 1/2 \) to indicate very slight dominance between two nearly equal activities.

\(^1\)For a discussion of the specific scale used and its justification, see Saaty (1979).
Returning to our problem of violating the stronger form of consistency, we compare the consistency index with what it would be if our numerical judgments were taken at random from the scale 1/9, 1/8, 1/7, ..., 1/2, 1, 2, ..., 9 (using a reciprocal matrix).

Using a sample of size 500 each, Saaty (1979) has established for different-order random-entry reciprocal matrices an average consistency index:

<table>
<thead>
<tr>
<th>Size of Matrix</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random Consistency</td>
<td>.00</td>
<td>.00</td>
<td>.58</td>
<td>.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

and regards consistency of about 10 percent or less as being very good. When the consistency is poor, one needs to get more information on the activities being compared with respect to the criterion of comparison.

The measure of consistency has also been extended to an entire hierarchy. The Analytic Hierarchy Process is concerned with decomposing a complex problem into a hierarchy, each of whose levels consists of a few manageable clusters and where each cluster is decomposed into subclusters represented in lower levels. The process is continued down to the simplest elements of the problem represented at the lowest level of the hierarchy.

A pairwise comparison reciprocal matrix is used to compare the relative contribution of the elements in each level of the hierarchy to an element in the adjacent upper level. The principal eigenvector of this matrix is then derived and weighted by the priority of the property with respect to which the comparison is made. (That weight is obtained by comparing the properties among themselves as to their contribution to the criteria of a still higher level. The weighted eigenvectors can now be added componentwise to obtain an overall weight or priority of contribution of each element to the entire hierarchy.)

This process of principal eigenvector extraction and hierarchical weighting and composition leads to a unidimensional scale for the priorities of the elements in any level of the hierarchy. The resulting priorities represent the intensity of the respondent's judgmental perception of the relative importance of the elements represented in the hierarchy considering the importance of and trade-offs among the criteria.

The specific steps involved in the development and analysis of an analytical hierarchy are specified in Table 2. The procedure is straightforward with a number of applications to resource allocation, forecasting, planning, conflict resolution, and other choice situations. One of the major attractions of the AHP is that it breaks down a complex process into a manageable series of data collection tasks.

The major applications of the AHP to date have been to five managerial problem areas:

a. **Allocation of resources/planning**

   Among the major applications in this area are:

   - The allocation of electricity to industry, in case of shortage, according to industries' contribution to health, welfare, employment, national defense, and other national objectives (Saaty and Mariano, 1979).
   - Allocating a country's resources -- the Sudan Project (Saaty, 1977a).
   - Allocating funds of the electric power research institute to research projects (Saaty and Bennett, 1977).
Table 2
An Outline of the Analytic Hierarchy Approach

1. Define the problem and specify the solution desired.

2. Structure the hierarchy from the overall managerial purposes (the highest levels) through relevant intermediate levels to the level where control would alleviate -- or solve -- the problem.

3. Construct a pairwise comparison matrix of the relative contribution or impact of each element on each governing objective or criterion in the adjacent upper level. In such a matrix of the elements by the elements, the elements are compared in a pairwise manner with respect to a criterion in the next level. In comparing the i,j elements, people prefer to give a judgment which indicates the dominance as an integer. Thus, if the dominance does not occur in the i,j position while comparing the ith element with the jth element, then it is given in the j,i position as a ji and its reciprocal is automatically assigned to aij.

4. Obtain all n(n-1)/2 judgments (step 2) specified by the set of matrices developed in step 3.

5. Having collected the pairwise comparison data and entered the reciprocals together with n unit entries down the main diagonal, the eigenvalue problem aw = λmax w is solved and consistency is tested.

6. Steps 3, 4, and 5 are repeated for all levels and clusters in the hierarchy.

7. Hierarchical composition is now used to weight the eigenvectors by the weights of the criteria and the sum is taken over all weighted eigenvectors entering corresponding to each element to obtain the composite priority of the elements in a level. These are then used to weight the eigenvectors corresponding to those in the next lower level and so on, resulting in a composite priority vector for the lowest level of the hierarchy.\textsuperscript{a}

8. Consistency is then evaluated for the entire hierarchy by simply multiplying each consistency index by the priority of the corresponding criterion and adding overall such products. The result is divided by the same type of expression using the random consistency index corresponding to the dimensions of each matrix weighted by the priorities as before. The ratio should be about 1.0 percent or less for acceptable overall consistency. Otherwise, the quality of the judgmental data should be improved.

\textsuperscript{a}The calculation of the priorities does not require judgment on all possible pairs. Various shortcuts can be undertaken. For a discussion of this, see Saaty (1979).

b. Conflict resolution
The approach has been applied in two key areas:
- The Northern Ireland Conflict (Alexander and Saaty, 1977). As a result of this study J. Alexander was invited to present the results to the Constitutional Convention held in 1976 and offer subsequent guidance to the parties involved.
- Terrorism (Saaty and Bennett, 1977), which suggests methods for negotiating with terrorists under alternative value systems.

c. Forecasting
The most dramatic AHP-based application in this area has been in the development of a methodology to predict the Fischer-Spatsky Championship Match of 1972 after the game, but most significantly to predict the outcome of the Karpov-Korthonoi match of 1978 before the outcome was clear (Saaty and Vargas, in press).

d. Input-Output Analysis
Estimates of input-output coefficients based on the AHP can be accomplished in a much simpler manner at a fraction of the monetary and time costs of conventional economic procedures. This was demonstrated in the Sudan Project, which included an explicit evaluation of the two approaches to input-output analysis and their results (see Saaty and Vargas, 1979a).
e. Choice Behavior

Conceptually the AHP is applicable to the modeling of any choice situation which lends itself to a hierarchical representation of at least two levels of objectives (criteria for evaluation) and activities (product, courses of action, etc.). To date, the AHP has been applied only to a limited number of cases. One of the early applications to the modeling of managerial choice behavior has been to the lease/buy-decision with respect to a fleet of cars conducted for NAFA -- the National Association of Fleet Administrators (Saaty and Vargas, 1979b).

An Analytic Hierarchy Formulation of the Portfolio Decisions

The objective of the AHP is to establish the weights for a set of activities reflecting their importance on multiple criteria ordered in a hierarchical structure. These weights -- the priorities of the hierarchy -- are used to allocate a resource such as money among the activities at the lower levels of the hierarchy. As such, the approach is ideally suited to the allocation of a firm's resources among the products in its target product portfolio. Most product portfolio approaches\(^2\) (such as the Boston Consulting Group share-growth matrix, the Shell International sector profitability and competitive position matrix, or the product performance four-factor matrix), although helpful in portraying the current position of the firm's products on the selected dimensions, do not provide explicit rules for the selection of the target portfolio (e.g., what combination of "dogs," "problem children," "cash cows," and "stars" should a company have?). Yet, the selection of a target portfolio, in particular, the determination of the desired allocation of resources among the components of the target portfolio, is a critical management function. In fact, one might argue that the allocation of resources is the primary responsibility of top management.

The current approaches to the resource allocation problem, such as industrial dynamics (Forester, 1961) and mathematical programming (see for example, Wagner, 1969), are rarely used by top management for the allocation of resources among the various products and businesses of the firm. The limitations of the current approaches to resource allocation at the corporate level are especially evident if one considers not a simple allocation among the current products but rather an allocation which would take a more realistic posture and incorporate the following alternatives:

- new products and business
- existing and potential new markets
- existing and new modes of distribution

and in the case of international operations:

- existing and new modes of entry (export, joint venture, etc.).

This focus on both the existing products/markets and distribution outlets as well as the new directions of growth, is a critical component of any strategic planning effort, since it incorporates not only the resource allocation under existing conditions but forces the managers/planners to incorporate their assessment of likely future scenarios and their anticipated impact on the firm and its ability to achieve its objectives.

\(^2\) For a discussion of the various product portfolio models, see Wind (1980).
From a conceptual point of view, therefore, the product portfolio of the firm should be extended to the portfolio of product/market/distribution options, which recognizes explicitly that most companies can grow by introducing (by internal development or acquisition) new products, entering new markets (domestic and foreign), utilizing new methods of distribution, or any combination of the above.

This reformulated view of a firm's portfolio options is illustrated in Figure 1. It is an extension of the more conventional product by market matrix proposed by many corporate strategists for the directions of growth (see for example, Johnson and Jones, 1957). It further suggests that the portfolio decision should be made within the context of these options and not limited only to products and, in particular, the existing products of the firm.

Accepting this basic departure from the more conventional product portfolio models also implies that most currently used approaches to resource allocation, such as mathematical programming, are inappropriate since they tend to:

- focus on the "optimization" of a single objective (in contrast to the realistic situation of multiple objectives such as profitability, market share, sales growth, etc.).

- focus on existing alternative courses of action (and not future courses of actions under alternative environmental scenarios).

- with the exception of game theory, focus on a single decision-maker (ignoring the fact that most corporate decisions involve a number of participants with diverse and often conflicting views and preferences).

- focus on the tangible and measurable dimensions (ignoring the intangible but critically relevant dimensions which encompass the "political" setting).
utilize researcher designed models (whether accurately reflecting the respondent's "true model" or not).

- use models, such as mathematical programming, which are assumed to be a priori fixed without allowance for model modification to reflect an intelligent respondent's experience or other idiosyncratic characteristics.

It is necessary, therefore, to formulate a different resource allocation model which is not subject to the limitations of the more conventional procedures and is not restricted to products. AHP is suitable for this situation. Its major attraction as the conceptual and measurement approach for the determination of the firm's target portfolio and allocation of resources within it are:

- Flexible formulation of the hierarchy to reflect management's value systems.
- A flexible hierarchy which can incorporate any objectives (of varying units of measurement) and any courses of action (both current and innovative) under any set of environmental scenarios.
- A measurement procedure based on the relevant managers' perceived relationship among the various forces, actors, actions, and personal and organizational objectives.
- A built-in extension to incorporate the judgments of any number of decision makers and resolve conflicting views among them (Frawley and Saaty, 1978).
- A flexible process allowing for iteration in both the structure of the problem (e.g., alternative hierarchy) and judgments.

An Illustrative Application

The Colonial Penn Insurance Company is a fast growing insurance company, specializing in developing and marketing auto and homeowner's policies to the over-50 market segment. Company management faces the key strategic question of determining the company's direction for future growth. Should the company continue to focus its efforts only on insurance products and, in particular, the over-50 market, or should they diversify into other products and markets? Furthermore, given the firm's historical strength in the direct-mail operation, should they focus their operation on products and markets that can be reached effectively by mail or should they consider developing new distribution vehicles, such as telephone, stores, agents, etc.?

We used the AHP to help guide the selection of the desired target portfolio of products/markets and distribution outlets, and help allocate resources among the portfolio's components.

Note that the application of the AHP is only to the selection of a target portfolio and the allocation of resources among its components. It is not concerned, at this stage, with the identification of the current portfolio of the firm (the portfolio analysis part which is at the core of the existing approaches to product portfolio, such as the Boston Consulting Group's model).

This application is based on a project conducted jointly with Thomas L. Saaty.
A hierarchy developed jointly with the company president is presented in a disguised form in Figure 2. This hierarchy was based on three major levels:

A Disguised Analytical Hierarchy for the Selection of a Target Product/Market/Distribution Portfolio for Colonial Penn Insurance Company

a. The environmental scenarios -- expressed as three summary scenarios reflecting:
   - an optimistic environment (low-risk and potentially high-return environmental conditions),
   - continuation of the status quo,
   - a pessimistic scenario (high-risk and potentially low-return environmental conditions).

b. Corporate objectives -- the criteria for the evaluation of the various courses of action. Five objectives were identified:
   - profit level
   - sales growth
   - market share
   - volatility
   - demand on resources.
c. The courses of action -- activities. These included the three sets of products, markets, and distribution outlets but went into considerably greater specificity of potential activities, including:

**Distribution**
- mail order (current mode)
- telephone
- store/office
- agents

**Customers**
- selected target market
  - current customers
  - potential customers
- other market segments
  - selected subsegments
  - other subsegments
    - specific subsegments 1
    - specific subsegments 2

**Products**
- current insurance products
  - auto
  - homeowners
  - others 1
  - others 2
- new insurance products
  - deluxe modifications
  - budget modifications
- new noninsurance products type A
  - a nonfinancial service
  - another nonfinancial service
- new noninsurance products type B
  - a specific financial service
  - other financial services
  - general mail-order merchandise

Given the sensitive nature of information on a firm's plans for allocation of its resources among alternative courses of action, the actual options are disguised in all subsequent sections and referred to by letters and numbers which do not correspond in any order to the items listed above.

Having selected the hierarchical structure outlined in Figure 2, the president of the company evaluated all pairwise comparisons using the 9-point scale described in Table 1. Some of his pairwise comparison matrices are illustrated in Table 3.

These data provided the input to the eigenvalue analysis discussed earlier. The calculation for one of the matrices is illustrated in Figure 3. The resulting eigenvalues for the entire hierarchy are presented in a disguised form in Figure 4.

The consistency calculations are illustrated in Figure 5 for the simple reciprocal matrix of the three classes of activities by profit level, illustrated in Figure 3.
Table 3
Illustrative Pairwise Comparison Matrices

<table>
<thead>
<tr>
<th>Internal Forces under</th>
<th>Profit Level</th>
<th>Sales Growth</th>
<th>Market Share</th>
<th>Volatility</th>
<th>Demand on Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Profit</td>
<td>Sales Growth</td>
<td>Market Share</td>
<td>Volatility</td>
<td>Demand on Resources</td>
</tr>
<tr>
<td>Profit Level</td>
<td>1,1,1</td>
<td>1/3,1,4</td>
<td>1/5,1/2,5</td>
<td>3,3,1/2</td>
<td>4,2,1</td>
</tr>
<tr>
<td>Sales Growth</td>
<td>3,1,1/4</td>
<td>1,1,1</td>
<td>1/3,3,1/2</td>
<td>2,2,1/4</td>
<td>4,2,1</td>
</tr>
<tr>
<td>Market Share</td>
<td>5,2,1/5</td>
<td>3,1/3,2</td>
<td>1,1,1</td>
<td>4,2,1/3</td>
<td>4,2,1</td>
</tr>
<tr>
<td>Volatility</td>
<td>1/3,1/3,2</td>
<td>1/2,1/2,4</td>
<td>1/4,1/3,2</td>
<td>1,1,1</td>
<td>2,3,2</td>
</tr>
<tr>
<td>Demand on Resources</td>
<td>1/4,1/2,1</td>
<td>1/4,1/2,1</td>
<td>1/4,1/2,1</td>
<td>1/2,1/3,1/2</td>
<td>1,1,1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Products</th>
<th>Customers</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit Level</td>
<td>1</td>
<td>1/3</td>
<td>1/5</td>
</tr>
<tr>
<td>Customers</td>
<td>3</td>
<td>1</td>
<td>1/4</td>
</tr>
<tr>
<td>Distribution</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Sales Growth</td>
<td>1</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>Products</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Customers</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Distribution</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Market Share</td>
<td>1</td>
<td>1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>Products</td>
<td>2</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>Customers</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Distribution</td>
<td>1/5</td>
<td>1/4</td>
<td>1</td>
</tr>
<tr>
<td>Volatility</td>
<td>1</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Products</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Customers</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Distribution</td>
<td>1/5</td>
<td>1/4</td>
<td>1</td>
</tr>
<tr>
<td>Demand on Resources</td>
<td>1</td>
<td>1/2</td>
<td>1/4</td>
</tr>
<tr>
<td>Products</td>
<td>2</td>
<td>1</td>
<td>1/2</td>
</tr>
<tr>
<td>Customers</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
The relative importance of (and allocation rule among) products, customers and distribution outlets with respect to the Profit level objective is the priority vector: Products .103 Customers .230 and Distribution = .667.
An examination of the resulting AHP (Figure 4) suggests explicit rules for allocating the firm's resources in development of products, markets, and distribution vehicles under three alternative scenarios. In the disguised example presented in Figure 4, the president has a significant preference for the development of distribution vehicles. In fact, the allocation of the current resources of the firm under this example should be as follows:

<table>
<thead>
<tr>
<th>Activities</th>
<th>Optimistic</th>
<th>Current</th>
<th>Pessimistic</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distribution</td>
<td>.1009</td>
<td>.1429</td>
<td>.2102</td>
<td>.4540</td>
</tr>
<tr>
<td>Customers</td>
<td>.0633</td>
<td>.0983</td>
<td>.1734</td>
<td>.3350</td>
</tr>
<tr>
<td>Products</td>
<td>.0327</td>
<td>.0588</td>
<td>.1344</td>
<td>.2259</td>
</tr>
<tr>
<td>Overall</td>
<td>.1969</td>
<td>.3000</td>
<td>.5180</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

These results suggest that the allocation of resources among the three components of the portfolio should be Distribution 45 percent, Customers 33 percent, and Products 22 percent. The actual results suggested an allocation of resources significantly different from the firm's current resource allocation pattern. It has led the president to reevaluate his firm's activities and assign task forces to those aspects of the portfolio (as suggested...
by the detailed priorities of Figure 4) that are not receiving the attention and resources they deserve.

\[
\begin{array}{ccc}
A & W & \lambda_{\text{max}} W \\
1 & 1/3 & 1/3 & .103 & 0.312 \\
3 & 1 & 1/4 & .230 & 0.706 \\
5 & 4 & 1 & .667 & 2.102 \\
\end{array}
\]

\[
\begin{array}{ccc}
\lambda_{\text{max}} W & W & \\
0.312 & 0.103 & 3.0291 \\
0.706 & 0.230 & 3.0696 \\
2.102 & 0.667 & 3.1514 \\
\end{array}
\]

\[
\lambda_{\text{max}} \approx \frac{3.0291 + 3.0696 + 3.1514}{3} = \frac{9.2501}{3} = 3.0834
\]

\[
\text{C.I. } \frac{\lambda_{\text{max}} - n}{n-1} = \frac{3.0834 - 3}{2} = \frac{.0834}{2} = .0417
\]

\[
\text{C.R. } = \frac{.0417}{.08} = .071
\]

where:

- C.I. = Consistency Index
- C.R. = Consistency Ratio

**Figure 5**

**Illustrative Consistency Calculations**

**Implementation and Extension**

Prior to implementing the product/market distribution portfolio and allocation rules suggested by the AHP, management must have confidence in the stability and sensibility of the results. This requires that one does not stop with the development of a single hierarchy and its evaluation by a single decision-maker at one given point in time but rather views the analytic hierarchy approach to portfolio determination as a process involving a number of iterations with all relevant decision-makers of the firm over time.

Given the sensitivity of disclosures of any substantive findings, we do not report on the results of the group evaluations and the resolution of conflicts but rather focus on three additional aspects of the AHP application to the portfolio decision based on the results for the president alone. These include:
a. an examination of the resulting portfolio and allocation rules under different hierarchical structures.

b. an examination of the sensitivity of results to different evaluations of the likelihood of occurrence of the various scenarios (the same procedure can obviously be applied to examine the sensitivity of the results to different evaluations of the relative importance of the various objectives).

c. an examination of other aspects related to the hierarchy but which are not an integral part of the hierarchical structure developed by management.

Some illustrative results for these additional analyses are presented next.

Figure 6 shows the results obtained under a somewhat different hierarchical structure, in which the president went directly from the three scenarios to the evaluation of the various courses of actions without going through the intermediate stage to evaluate the activities with respect to the five objectives and then the objectives with respect to the scenarios. The similarity of the results under the two hierarchical structures is clearly evident in Table 4. This suggests that although the rank order of allocation is stable across the two hierarchies, the specific allocation is sensitive to the hierarchical structure. In both of these disguised cases, most of the resources should be allocated to distribution and the least resources to products.

![Diagram showing an alternative hierarchical structure and its priorities]
The second extension is the examination of the sensitivity of the results to different assessments of the likely occurrence of the various environmental scenarios.

Figure 7 shows the results of the same hierarchy presented in Figure 6 but with a somewhat more pessimistic outlook reflecting the 15 percent likelihood of occurrence of an optimistic scenario (in contrast to 20 percent in Figures 4 and 6), the 25 percent likelihood of continuation of the status quo (in contrast to 30 percent in Figures 4 and 6) and the 60 percent likelihood of a pessimistic scenario (in contrast to 50 percent in Figures 4 and 6).

An examination of the resulting hierarchy (Figure 7) suggests that again the rank order of priorities does not change while the specific allocation rules are quite sensitive to the different assessments of likelihood.

Such sensitivity analysis should be conducted for all critical aspects of the hierarchy, establishing the range of priorities for the different activities. It is this range of priorities and not any specific priority that should ultimately govern management's allocation of resources among the components of its portfolio.

The third extension of the AHP is to other decisions and forces which if subjected to such an analysis would offer management valuable insights and diagnostic information but which are not directly critical for the hierarchical structure. An example of such an extended analysis is the examination of the external forces which affect and are affected by environmental scenarios. Table 5 illustrates the pairwise comparison matrix for this additional analysis and Figure 8 displays the resulting hierarchical output. An examination of these results suggests the president's perception that the future will be affected principally by economic developments (40 percent), followed by the legal/governmental forces (30 percent), competition (30 percent), and consumerism (10 percent).

In this specific case, one could, of course, develop alternative hierarchical structures based on these external forces instead of the three summary scenarios, or incorporate the external forces as another interim stage in the hierarchy. Operationally, however, it is desirable not to proliferate the number of levels of the hierarchy, since each additional level requires a considerable number of additional judgments for pairwise comparisons with all the elements in lower and higher levels in the hierarchy.
WILL-BEING OF COMPANY

OPTIMISTIC SCENARIO (.15)
CONTINUATION OF CURRENT STATUS (.25)
PESSIMISTIC SCENARIO (.60)

DISTRIBUTION
(.146,.129,.287)

CUSTOMERS
(.087,.101,.171)

PRODUCTS
(.022,.020,.043)

Figure 7
The Second Analytic Hierarchy under Different Likelihood of Occurrence of the Three Environmental Scenarios

Table 5
Pairwise Comparison Matrix of External Forces and Scenarios

<table>
<thead>
<tr>
<th>Optimistic, Current, Pessimistic Scenario</th>
<th>ECONOMIC</th>
<th>LEGAL/GOV'T</th>
<th>COMPETITION</th>
<th>CONSUMERISM</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECONOMIC</td>
<td>1,1,1</td>
<td>1,1/2,4</td>
<td>1/5,2,5</td>
<td>3,3,5</td>
</tr>
<tr>
<td>LEGAL/GOV'T</td>
<td>1,2,1/4</td>
<td>1,1,1</td>
<td>1/3,3,5</td>
<td>3,4,3</td>
</tr>
<tr>
<td>COMPETITION</td>
<td>5,1/2,1/5</td>
<td>3,1/3,1/5</td>
<td>1,1,1</td>
<td>5,3,1/3</td>
</tr>
<tr>
<td>CONSUMERISM</td>
<td>1/3,1/3,1/5</td>
<td>1/3,1/4,1/3</td>
<td>1/5,1/3,3</td>
<td>1,1,1</td>
</tr>
</tbody>
</table>
Furthermore, the AHP reveals that the president views economic developments as critical in a pessimistic scenario, whereas competition is predominant in an optimistic scenario, and the legal/government forces are principally important under current conditions and a pessimistic scenario.

Discussion and Direction for Future Research

The AHP for the product/market/distribution portfolio of the firm offers a model for and measurement approach to the determination of management's desired target portfolio and allocation of resources among the components of the portfolio (at any level of specificity).

As discussed earlier, none of the existing procedures for portfolio analysis nor the more general procedures for resource allocation can accomplish this task. Simple linear-in-log models (see for example, Carroll, Green, and DeSarbo, 1979) and conjoint analysis (Wind, 1978a) have been utilized in a few cases for the determination of the desired portfolio of products. Yet, neither of these procedures nor the other general allocation models have the flexibility and multistage advantages of the AHP.

The AHP when applied to the portfolio decision offers specific guidelines for allocation of resources among the firm's current and potential products, market, and distribution outlets under alternative environmental conditions reflecting management evaluation of the relative importance of its various objectives.

These specific guidelines can be subjected to any number of sensitivity analyses establishing the critical priority range for each of the portfolio components. This
approach can be taken at any level of specificity, including specific brands by specific market segments and distribution outlets.

The procedure described in this paper can and has been extended to cover other items:

1. The evaluation of various decision makers within the firm and the resolution of conflict among them (see, for example, Saaty, 1979).

2. The construction of nonlinear hierarchies; i.e., hierarchies with feedback loops where a level influences another level and at the same time is influenced by it (Saaty, 1979).

3. The construction of time-dependent judgments; i.e., the pairwise comparisons are generated for different time periods, or alternatives reflect the respondent's perceived time trend.

4. The weighting of the judgments by the respondents' perceived expertise, derived from a separate hierarchy for this purpose determined by the respondents themselves.

With respect to marketing applications, the AHP is not limited to the portfolio issue. Conceptually, it can be applied to any allocation of resources (e.g., selection of channels of distribution) and choice prediction (e.g., concept or product testing) situations. In addition, it can also be applied to cases in which the buying or marketing decision involves a number of participants (the buying center, Wind, 1979b) with conflicting perceptions or objectives.

The extent to which the analytical hierarchy would offer a better (more reliable and valid) procedure than some of the other existing approaches to these decisions is an empirical question. Managers have had no difficulty in completing pairwise matrices. In fact, many of them find the task challenging, interesting, and of intrinsic value in that it forces them to examine relationships which are often left unexamined. Will consumers be able to respond to such a demanding task? Time will tell. The conceptual advantage of the AHP approach, suggests, however, that further experimentation with this approach could lead to the establishment of an important addition to the arsenal of marketing models and measurement approaches.

References


Wind, Yoram, Product Policy (Reading, Massachusetts: Addison-Wesley, 1980).